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MATH 330 Homework #10
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Exercise 17.3. Use the division algorithm to find $q(x)$ and $r(x)$ such that $a(x) = q(x)b(x) + r(x)$ with $\deg r(x) < \deg b(x)$ for each of the following pairs of polynomials.

(c) $a(x) = 4x^5 - x^3 + x^2 + 4$ and $b(x) = x^3 - 2$ in $\mathbb{Z}_7[x]$

(d) $a(x) = x^5 + x^3 - x^2 - x$ and $b(x) = x^3 + x$ in $\mathbb{Z}_2[x]$

Proof. content... □

Exercise 17.4. Find the greatest common divisor of each of the following pairs $p(x)$ and $q(x)$ of polynomials. If $d(x) = \gcd(p(x), q(x))$, find two polynomials $a(x)$ and $b(x)$ such that $a(x)p(x) + b(x)q(x) = d(x)$.

(a) $p(x) = 7x^3 + 6x^2 - 8x + 4$ and $q(x) = x^3 + x - 2$ where $p(x), q(x) \in \mathbb{Q}[x]$

(b) $p(x) = x^3 + x^2 - x + 1$ and $q(x) = x^3 + x - 1$ where $p(x), q(x) \in \mathbb{Z}_2[x]$

Proof. content... □

Exercise 17.11. Prove or disprove: There exists a polynomial $p(x)$ in $\mathbb{Z}_6[x]$ of degree n with more than n distinct zeros.

Proof. content... □

Exercise 17.19. Let $\mathbb{Q}^*$ be the multiplicative group of positive rational numbers. Prove that $\mathbb{Q}^*$ is isomorphic to $(\mathbb{Z}[x], +)$.

Proof. content... □

Exercise 17.21. If F is a field, show that there are infinitely many irreducible polynomials in $F[x]$.

Proof. content... □