Moshe Mason Rubin
PHYS 230 Final Exam Formula Sheet
7 December 2016

Galilean Transformations:

Where $\mathcal{V}$ is the relative speed between frames.

$$\begin{aligned} x' &= x - \mathcal{V}t & v'_x &= \frac{\mathrm{d}x'}{\mathrm{d}t'} = \frac{\mathrm{d}(x-vt)}{dt} = v_x - \mathcal{V} \\ y' &= y & v'_y &= \frac{\mathrm{d}y'}{\mathrm{d}t'} = v_y \\ z' &= z & v'_z &= \frac{\mathrm{d}z'}{\mathrm{d}t'} = v_z \\ t' &= t \end{aligned}$$

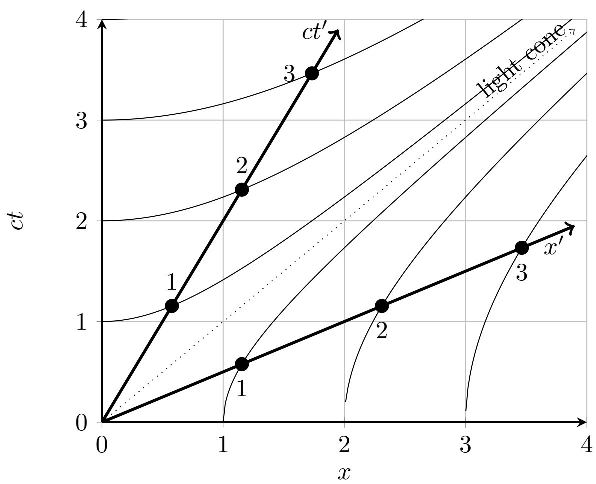
Lorentz Transformations:

Where $\gamma = \frac{1}{\sqrt{1-\frac{\mathcal{V}^2}{c^2}}}$ and $\mathcal{V}$ is the relative speed between frames.

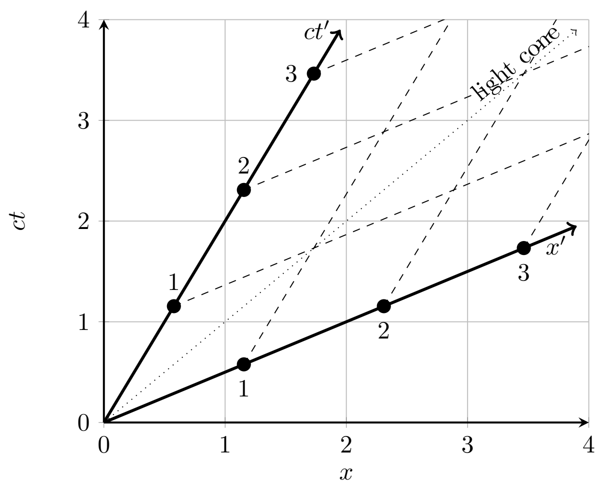
$$\begin{array}{cc|cc} x' = \gamma(x - \mathcal{V}t) & v'_x = \frac{\mathrm{d}x'}{\mathrm{d}t'} = \frac{\frac{\mathrm{d}x'}{\mathrm{d}t}}{\frac{\mathrm{d}t'}{\mathrm{d}t}} = \frac{v_x - \mathcal{V}}{1-\frac{v_x\mathcal{V}}{c^2}} & x = \gamma\left(x' + \mathcal{V}t'\right) & v_x = \frac{v'_x + \mathcal{V}}{1+\frac{v'_x\mathcal{V}}{c^2}} \\ y' = y & v'_y = \frac{\mathrm{d}y'}{\mathrm{d}t'} = \frac{\frac{\mathrm{d}y'}{\mathrm{d}t}}{\frac{\mathrm{d}t'}{\mathrm{d}t}} = \frac{v_y\sqrt{1-\frac{\mathcal{V}^2}{c^2}}}{1-\frac{v_x\mathcal{V}}{c^2}} & y = y' & v_y = \frac{v'_y\sqrt{1-\frac{\mathcal{V}^2}{c^2}}}{1+\frac{v'_x\mathcal{V}}{c^2}} \\ z' = z & v'_z = \frac{\mathrm{d}z'}{\mathrm{d}t'} = \frac{\frac{\mathrm{d}z'}{\mathrm{d}t}}{\frac{\mathrm{d}t'}{\mathrm{d}t}} = \frac{v_z\sqrt{1-\frac{\mathcal{V}^2}{c^2}}}{1-\frac{v_x\mathcal{V}}{c^2}} & z = z' & v_z = \frac{v'_z\sqrt{1-\frac{\mathcal{V}^2}{c^2}}}{1+\frac{v'_x\mathcal{V}}{c^2}} \\ t' = \gamma\left(t - \frac{\mathcal{V}D}{c^2}\right) & ct' = \gamma\left(ct - \frac{\mathcal{V}D}{c}\right) & t = \gamma\left(t' + \frac{\mathcal{V}D'}{c^2}\right) & ct = \gamma\left(ct' + \frac{\mathcal{V}D'}{c}\right) \\ \hline \multicolumn{2}{r|}{t_{\text{moving}} = t_{\text{stationary}}\sqrt{1-\frac{\mathcal{V}}{c^2}}} & \multicolumn{2}{l}{d_{\text{moving}} = D_{\text{rest frame}}\sqrt{1-\frac{\mathcal{V}}{c^2}}} \end{array}$$

Minkowski Spacetime:

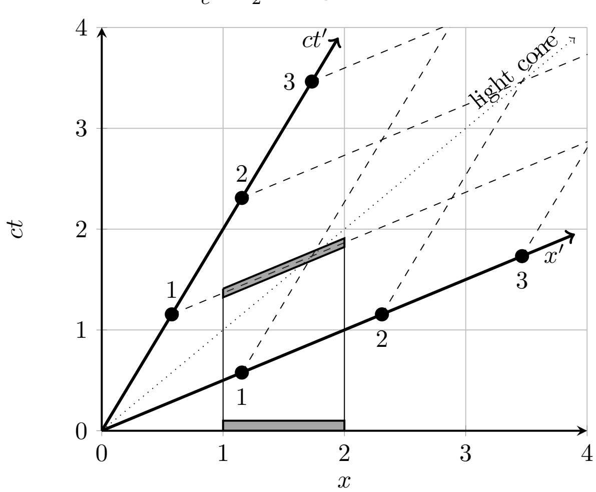
$\frac{\mathcal{V}}{c} = \frac{1}{2}$



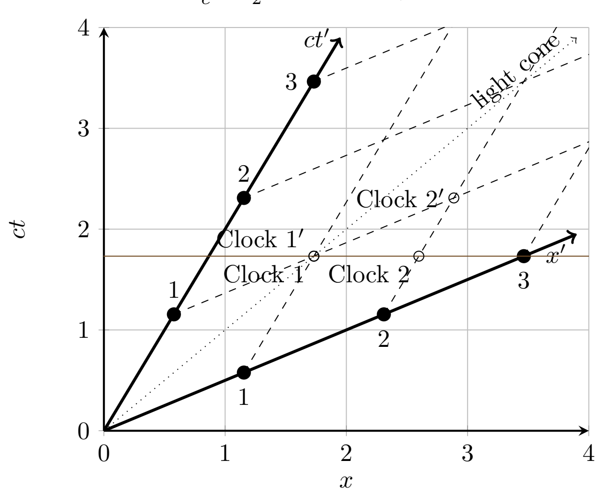
$\frac{\mathcal{V}}{c} = \frac{1}{2}$

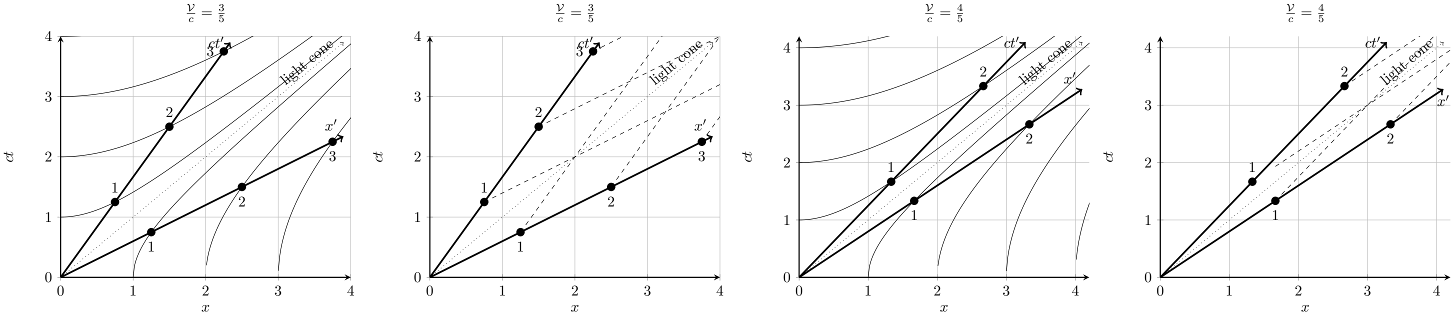


$\frac{\mathcal{V}}{c} = \frac{1}{2}$, Length Contraction



$\frac{\mathcal{V}}{c} = \frac{1}{2}$, Clocks in-sync in S'





Invariant:

$$(x')^2 - (ct')^2 = (x)^2 - (ct)^2 = \pm a^2$$
$$(E')^2 - (p'c)^2 = (E)^2 - (pc)^2 = m^2c^4$$

Einstein's Postulates:

1. Absolute uniform motion cannot be detected.
2. The velocity of light does not depend upon the velocity of its source.

Momentum and Energy:

Where v in $\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$ is given by $v = |\mathbf{v}| = \sqrt{v_x^2 + v_y^2 + v_z^2}$ the speed of the particle in that particular frame of reference and is different between two frames.

$$\begin{array}{c|c} P_\mu = (P_0, P_x, P_y, P_z) & P_0' = \frac{P_0 - \frac{\mathcal{V} P_x}{c}}{\sqrt{1-\frac{\mathcal{V}^2}{c^2}}} \\ P_\mu = \left(\frac{mc}{\sqrt{1-\frac{v^2}{c^2}}}, \frac{mv_x}{\sqrt{1-\frac{v^2}{c^2}}}, \frac{mv_y}{\sqrt{1-\frac{v^2}{c^2}}}, \frac{mv_z}{\sqrt{1-\frac{v^2}{c^2}}} \right) & P_x' = \frac{P_x - \frac{\mathcal{V} P_0}{c}}{\sqrt{1-\frac{\mathcal{V}^2}{c^2}}} \\ (P_\mu)_{\text{initial}} = (P_\mu)_{\text{final}} & P_y' = P_y \quad P_z' = P_z \end{array}$$

$$E_{\text{photon}} = hf = p_{\text{photon}} c$$

$$\begin{array}{c|c} E = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} mc^2 & E = mc^2 + KE \\ KE = \left(\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} - 1 \right) mc^2 & E^2 = m^2c^4 + p^2c^2 \end{array}$$

Relativistic Doppler Effect:

$$\begin{array}{c|c} \cos(\theta) = \frac{\cos(\theta') + \frac{\mathcal{V}}{c}}{1 + \frac{\mathcal{V}}{c}\cos(\theta')} & \cos(\theta') = \frac{\cos(\theta) - \frac{\mathcal{V}}{c}}{1 - \frac{\mathcal{V}}{c}\cos(\theta)} \\ f_{\text{observed}} = f_{\text{emit}} \frac{\sqrt{1-\frac{\mathcal{V}^2}{c^2}}}{1 - \frac{\mathcal{V}}{c}\cos(\theta)} = f_{\text{emit}} \sqrt{\frac{1+\frac{\mathcal{V}}{c}}{1-\frac{\mathcal{V}}{c}}} & \Delta\lambda = \frac{h}{m_e c}(1 - \cos(\theta)) \longrightarrow \text{“Compton Scattering”} \end{array}$$

Fixed Target Collisions:

If Particle 2 is a stationary target and Particle 1 strikes it in order to achieve energy E_T' in the center of momentum frame then

$$E_1 = \frac{{E_T'}^2 - (m_1^2 + m_2^2)c^4}{2m_2c^2}$$

$$(E_1)_{\text{threshold}} = \frac{\left(\sum_{\substack{\text{final} \\ \text{particles}}} mc^2 \right)^2 - (m_1^2 + m_2^2)c^4}{2m_2c^2}$$