

$$\begin{array}{l} f_k(x,y) \\ C=\{(x,y)\in k^2\mid f(x,y)=0\} \\ p= \\ (x,y)\in \\ C \\ \frac{\sigma f}{\sigma x}(p)= \\ \frac{\sigma f}{\sigma y}(p)= \\ 0 \\ p \\ C \\ f(x,y)= \\ y^2- \\ x^3+ \\ 3x \\ \{\sigma f\sigma x=-3x+3=0\longrightarrow x=\pm 1\frac{\sigma f}{\sigma y}=2y=0\longrightarrow y=0 \end{array}$$

$$\begin{array}{l} P(1,0) \\ Q(-1,0) \\ \tilde{P} \\ \tilde{Q} \\ K \\ y^2+a_1xy+a_3=x^3+a_2x^2+a_4x+a_6 \\ a_1,a_2,a_3,a_4,a_6\in \\ K \\ y^2= \\ x^3+ \\ Ax+ \\ B \\ y^2= \\ x^3+ \\ 1 \\ R \\ \Delta=-(4A^3+27B^2) \end{array}$$

$$\begin{array}{l} y^2= \\ x^3+ \\ Ax+ \\ B \\ \Delta \neq \\ 0 \\ j \\ k \\ E \\ E: y^2 = x^3 + Ax + B \\ \Delta = -(4A^3 + 27B^2) \end{array}$$

$$\begin{array}{l} j \\ E \\ j(E)=-1728\frac{4A^3}{\Delta} \end{array}$$

$$\begin{array}{l} E_1: y^2 = x^3 + x + 1 \\ E_2: y^2 = x^3 + 4x + 8 \\ E_1 \end{array}$$

$$A=1,B=1\Longrightarrow \Delta(E_1)=-(4A^3+27B^2)=-31$$

$$\begin{array}{l} j(E_1)=-1728\frac{4A^3}{\Delta}=-1728\frac{4}{-31}=\frac{6912}{31} \\ E_2 \end{array}$$

$$A=4,B=8\Longrightarrow \Delta(E_2)=-(4A^3+27B^2)=-1984$$

$$j(E_2)=-1728\frac{4A^3}{\Delta}=-1728\frac{256}{-1984}=\frac{442368}{1984}$$

$$\begin{array}{l} E_1 \\ E_2 \\ j \\ E: \\ y^2= \\ x^3+ \\ Ax+ \\ B' \\ E': \\ y^2= \\ x^3+ \end{array}$$

$$K$$

$$E: y^2 = x^3 + Ax + B, E': y^2 = x^3 + A'x + B'$$

$$K$$

$$\frac{E'}{E'}\frac{j(E)}{J'(E')}=$$

$$\frac{E}{K}:\frac{E(\bar{K})}{E(\bar{K})}\rightarrow$$

$$\begin{array}{l} \varphi \\ \varphi(x,y)= \\ (\varphi_1(x,y),\varphi_2(x,y)) \\ \varphi_1 \\ \varphi_2 \\ P \\ Q \\ \varphi(P+ \\ Q)= \\ \varphi(P)+ \\ \varphi(Q) \\ \varphi_\infty(\infty)= \end{array}$$

$$\begin{array}{l} E_2 \\ y^2= \\ x^3+ \\ Ax+ \\ B \\ F_q \\ F_q \end{array}$$

$$E(F_q)=\{(x,y)\in F_q\times F_q|y^2=x^3+Ax+B\}\{\infty\}$$