k
$f(x,y)$
k
$C = \{(x,y) \in k^2 \mid f(x,y) = 0\}$
$p =$
$(x,y) \in$
C
C
$\frac{\sigma f}{\sigma x}(p) =$
$\frac{\sigma f}{\sigma y}(p) =$
0
p
C
$f(x,y) =$
$y^2 -$
$x^3 +$
$3x$

$$\{ \ \sigma f \sigma x = -3x+3 = 0 \longrightarrow x = \pm 1 \frac{\sigma f}{\sigma y} = 2y = 0 \longrightarrow y = 0$$

$P(1,0)$
$Q(-1,0)$
P
Q
K
$y^2 + a_1 xy + a_3 = x^3 + a_2 x^2 + a_4 x + a_6$
$a_1, a_2, a_3, a_4, a_6 \in$
K
$y^2 =$
$x^3 +$
$Ax +$
B
$y^2 =$
$x^3 +$
1
R
$\Delta = -(4A^3 + 27B^2)$

$y^2 =$
$x^3 +$
$Ax +$
B
$\Delta \neq$
0
j
k
E
$E : y^2 = x^3 + Ax + B$

$\Delta = -(4A^3 + 27B^2)$

j
E

$$j(E) = -1728 \frac{4A^3}{\Delta}$$

$E_1 : y^2 = x^3 + x + 1$

$E_2 : y^2 = x^3 + 4x + 8$

E_1

$$A = 1, B = 1 \Longrightarrow \Delta(E_1) = -(4A^3 + 27B^2) = -31$$

$$j(E_1) = -1728 \frac{4A^3}{\Delta} = -1728 \frac{4}{-31} = \frac{6912}{31}$$

E_2

$$A = 4, B = 8 \Longrightarrow \Delta(E_2) = -(4A^3 + 27B^2) = -1984$$

$$j(E_2) = -1728 \frac{4A^3}{\Delta} = -1728 \frac{256}{-1984} = \frac{442368}{1984}$$

E_1
E_2
j
$E :$
$y^2 =$
$x^3 +$
$Ax +$
B
$E' :$
$y^2 =$
$x^3 +$

K

$E : y^2 = x^3+Ax+B, E' : y^2 = x^3+A'x+B'$

K

E
E'
$j(E) =$
$J'(E')$

E
K
$\varphi :$
$E(\bar{K}) \to$
$E(\bar{K})$

φ
$\varphi(x, y) =$
$(\varphi_1(x, y), \varphi_2(x, y))$
φ_1
φ_2
P
Q
$\varphi(P+$
$Q) =$
$\varphi(P)+$
$\varphi(Q)$
$\varphi(\infty) =$
∞

E
$y^2 =$
x^3+
$Ax+$
B
F_q
F_q

$E(F_q) = \{(x, y) \in F_q \times F_q | y^2 = x^3+Ax+B\}\{\infty\}$