My notes on STAT 330 – Mathematical Statistics

Dillon Chan

Preliminaries

Sample space The set of all possible outcomes of an experiment, S.

σ-field Let S be a sample space with power set $\mathbb{P}(S)$. A collection of sets $\mathcal{B} \subseteq \mathbb{P}(S)$ is a σ-field / σ-algebra on S if

1. $\emptyset, S \in \mathcal{B}$
2. $\mathcal{B}$ closed under complementation
3. $\mathcal{B}$ closed under countable unions

Measurable space Let S be a sample space and $\mathcal{B}$ be a σ-algebra. Then the pair $(S, \mathcal{B})$ is a measurable space.

Measure Let X be a set and Σ be a σ-algebra over X. A function

$$\mu : X \to \mathbb{R} \cup \{+\infty\} \cup \{-\infty\}$$

is called a measure if it satisfies:

- Non-negativity: For all $E \in \Sigma$, $\mu(E) \geq 0$
- Null empty set: $\mu(\emptyset) = 0$
- Countable additivity: For all countable collections $\{E_i\}_{i=1}^{\infty}$ of pairwise disjoint sets[1] in Σ,

$$\mu\left(\cup_{k=1}^{\infty} E_k\right) = \sum_{k=1}^{\infty} \mu(E_k)$$

Definition from wikipedia

Probability Measure Let S be a sample space for a random experiment. Let $\mathcal{B} = \{A_1, A_2, \cdots\}$ be a σ-field on S. A probability measure is a function $\Pr : \mathcal{B} \to [0, 1]$ that satisfies:

- $\Pr(A) \geq 0$ for all $A \in \mathcal{B}$.
- $\Pr(S) = 1$
- If $A_1, A_2, \cdots \in \mathcal{B}$ are disjoint events then $\Pr(\cup_{i=1}^{\infty} A_i) = \sum_{i=1}^{\infty} \Pr(A_i)$.

Measure space A measure space is a measurable space equipped with a measure.

Probability space A probability space is a measureable space equipped with a probability measure.

[1] For any two elements in the set, them being disjoint is pairwise disjoint.