№ 1.3

a)

k += 1	1
i = n	1
while i > 0:	n
i -= 1	n+1

$T(n) = 1+1+n+n+1 = 3+2n$

b)

i = n	1
while i > 1	$(\log_2 n)+1$
k += 1	$\log_2 n$
i //= 2	$\log_2 n$

$\log_2 n = m$

$T(n) = 1 + \log_2 n + 1 + \log_2 n + \log_2 n =$

$= 2 + 3\log_2 n = 2 + 3m$

c)

i = 0	1
while i < n:	$\frac{n}{2}+1$
j = 0	$\frac{n}{2}$
while j < n:	$\frac{n}{2}\left(\frac{n}{2}+1\right)$
k += 1	$\frac{n^2}{4}$
j += 2	$\frac{n^2}{4}$
i += 2	$\frac{n}{2}$

$T(n) = 1 + \frac{n}{2} + 1 + \frac{n}{2} + \frac{n^2}{4} + \frac{n}{2} + \frac{n^2}{4} + \frac{n}{2} + \frac{n}{2} =$

$= 2 + \frac{4n}{2} + \frac{3n^2}{2}$

d)

i = 0	1
while i < n:	n+1
j = 0	n
while j < i*i:	$\Sigma i^2 + n$
k += 1	$\frac{(n-1)n(2n+1)}{6}$
j += 1	$\frac{(n-1)n(2n-1)}{6}$
i += 1	n

$\sum_{i=0}^{n-1} i^2 = \frac{(n-1)n(2n-1)}{6} = S$

$T(n) = 1+n+1+n+S+n+S+S+n = 2+4n+3S$

e) $i = 1$ — 1

while $i < n$: — $\log_2 n + 1$

$j = 1$ — $\log_2 n$

while $j < n$: — $\log_2 n (\log_2 n + 1)$

$k += 1$ — $\log_2 n \log_2 n$

$j *= 2$ — $\log_2 n \log_2 n$

$i *= 2$ — $\log_2 n$

$$T(n) = 1 + \log_2 n + 1 + \log_2 n + 3\log_2^2 n + \log_2 n + \log_2 n =$$

$$= 2 + 3\log_2 n + 4\log_2^2 n$$

f) $i = 1$ — 1

while $i < n$: — $\log_2 n + 1$

$j = i$ — $\log_2 n$

while $j < n$: — $\sum\limits_{i=1,2,4} \log_2 \frac{n}{i} + 1$

$k += 1$ — $\sum \log_2 \frac{n}{i}$

$j *= 2$ — S

$i *= 2$ — $\log_2 n$

$$\sum_{i=1,2,4} \log_2 \frac{n}{i} = \frac{\log_2 n(\log_2 n + 1)}{2} = S$$

$$\log_2 \frac{n}{i} = m$$

$$S = \frac{m(m+1)}{2}$$

$$T(n) = 1 + m + 1 + m + \frac{m(m+1)}{2} + m + \frac{m(m+1)}{2} + m$$

$$= 2 + \frac{2m(m+1)}{2} + 4m = 2 + m^2 + m + 4m =$$

$$= 2 + m^2 + 5m =$$

№ 1.4

а) $T(n) = \begin{cases} 1 & n \le a,\ a > 0 \\ T(n-a)+1 & n > a \end{cases}$

$n > a \quad T(n) = T(n-a)+1$

$T(n) = [T(n-2a)+1]+1 = T(n-2a)+2 =$

$= [T(n-3a)+1]+2 = T(n-3a)+3 = \dots$

$= T(n-ma)+m$

$n = ma+r \quad 0 \le r < a$

$T(n) = T(r)+m$

$T(n) = m+1, \quad r \le a \quad T(r) = 1$

$m = \frac{n}{a} \quad T(n) = \frac{n}{a}+1$

б) $T(n) = \begin{cases} 1 & n = 0 \\ T(n-1)+2^n & n \ge 1 \end{cases}$

$T(n) = [T(n-2)+2^{n-1}]+2^n = T(n-2)+2^{n-1}+2^n$

$T(n) = [T(n-3)+2^{n-2}]+2^{n-1}+2^n =$

$= T(n-3)+2^{n-2}+2^{n-1}+2^n$

$T(0) \quad T(n) = \underset{1}{\underset{\shortparallel}{T(0)}}+2^1+2^2+\dots+2^n$

$\sum_{i=1}^{n} 2^i = 2^{n+1}-2$

$T(n) = 1+(2^{n+1}-2) = 2^{n+1}-1$

d) $T(n) = \begin{cases} 1 & n = 1 \\ 2T(\lfloor \frac{n}{2} \rfloor) + 1 & n \geq 2 \end{cases}$

$n = 2^k$

$S(k) = T(2^k) \qquad S(0) = T(1) = 1$

$\cdot\, k \geq 1$

$S(k) = 2S(k-1) + 1$

$S(k) = 2[2S(k-2) + 1] + 1 = 2^2 S(k-2) + 2 + 1$

$S(k) = 2^2[2S(k-3) + 1] + 2 + 1 = 2^3 S(k-3) + 2^2 + 2 + 1$

$S(k) = 2^k S(0) + (2^{k-1} + 2^{k-2} + \dots + 2 + 1)$

$2^{k-1} + 2^{k-2} + \dots + 1 = 2^k - 1$

$S(0) = 1$

$S(k) = 2^k \cdot 1 + (2^k - 1) = 2 \cdot 2^k - 1 = 2^{k+1} - 1$

$n = 2^k \Rightarrow k = \log_2 n$

$T(n) = 2^{\log_2 n + 1} - 1 = 2n - 1.$

d) $T(n) = \begin{cases} 1 & n = 1 \\ aT(\lfloor n/a \rfloor) + n & n \geq 2,\ a \geq 2 \end{cases}$

$\cdot\, n = a^k \Rightarrow \lfloor \frac{n}{a} \rfloor = \frac{n}{a} = a^{k-1}$

$S(k) = T(a^k) \Rightarrow S(0) = T(1) = 1$

$\cdot\, k \geq 1$

$S(k) = aS(k-1) + a^k$

$S(k) = a[aS(k-2) + a^{k-1}] + a^k =$

$= a^2 S(k-2) + a^k + a^k = a^2 S(k-2) + 2a^k$

$S(k) = a^2[aS(k-3) + a^{k-2}] + 2a^k =$

$= a^3 S(k-3) + a^k + 2a^k = a^3 S(k-3) + 3a^k$

$S(k) = a^k \underbrace{S(0)}_{=1} + k a^k = a^k + k a^k = a^k(1+k)$

$n = a^k$

$k = \log_2 n$

$T(n) = n(\log_a n + 1) = n \log_a n + n$