



Hypothesis Testing With Python

True Difference or Noise?

169.61

169.88

Which is better?

Noise?

That's a question.

Mosky



- Python Charmer at Pinkoi.
- Has spoken at: PyCons in TW, MY, KR, JP, SG, HK, COSCUPs, and TEDx, etc.
- Countless hours on teaching Python.
- Own the Python packages: ZIPCodeTW, MoSQL, Clime, etc.
- <http://mosky.tw/>

Outline

- Tests with simulated datasets
 - What methods work not well?
 - What methods work well?
 - Simple mean, etc.
- Tests with actual datasets
- How tests work
- Common tests
- Complete a test

The Packages

- \$ pip3 install jupyter numpy scipy sympy matplotlib ipython pandas seaborn statsmodels scikit-learn
- Or
- > conda install jupyter numpy scipy sympy matplotlib ipython pandas seaborn statsmodels scikit-learn
- Visit the Pipfile for the exact versions.

Tests with datasets

Go with the notebooks

- *01_tests_with_simulated_datasets.ipynb*
- The notebooks are available on <https://github.com/moskytw/hypothesis-testing-with-python>.

False positive rate, p-value, and α

false positive rate

Calculated with the actual answer.

p-value

Calculated false positive rate by
a reasonable hypothesis.

α

Predefined acceptable false positive rate.

Fair coin testing

- “The coin is fair.”
- Case 1: Toss the coin 100 times, and come up 53 heads.
 - “Hmmm ... somehow fair.”
- Case 2: Toss the coin 100 times, and come up 87 heads.
 - “Not fair! So extreme!”

Hypothesis testing

- “The means of two populations are equal.”
- Case 1: $p\text{-value} \geq 0.05$
 - “Hmmm ... somehow equal.”
- Case 2: $p\text{-value} < 0.05$
 - “Not equal! So extreme!”

Hypothesis testing in a “null” taste

- $\langle \text{null hypothesis} \rangle$
- Case 1: $p\text{-value} \geq \alpha$
 - Can't reject $\langle \text{null hypothesis} \rangle$.
- Case 2: $p\text{-value} < \alpha$
 - Reject $\langle \text{null hypothesis} \rangle$.

Hypothesis testing in an “alternative” taste

- *<alternative hypothesis>*, \equiv not *<null hypothesis>* usually.
- Case 1: $p\text{-value} \geq a$
 - Can't accept *<alternative hypothesis>*.
- Case 2: $p\text{-value} < a$
 - Accept *<alternative hypothesis>*.

Hypothesis testing in “-+” taste

- “The case is negative.”
- Case 1: $p\text{-value} \geq \alpha$
 - “Hmmm ... somehow negative.”
- Case 2: $p\text{-value} < \alpha$
 - “Positive! So extreme!”

P-value, α , wording, and summary

p-value & α	Wording	Summary
$p\text{-value} < 0.001$	Very significant	***
$p\text{-value} < 0.01$	Very significant	**
$p\text{-value} < 0.05$	Significant	*
$p\text{-value} \geq 0.05$	Not significant	ns

- *02_tests_with_actual_datasets.ipynb*

How tests work

Seeing is believing

- $p\text{-value} = 0.0027 (< 0.01)$
 - 
- $p\text{-value} = 0.0271 (0.01\text{--}0.05)$
 -  ?  ? ? ? ?
- $p\text{-value} = 0.2718 (\geq 0.05)$
 - ? ? ? ? ? ?
- *03_how_tests_work.ipynb*

Common tests

The cheat sheet

- If testing independence:
 - If total sample size < 1000, or more than 20% of cells have expected frequencies < 5, **Fisher's exact test**.
 - Else, **Chi-squared test**.
- If testing difference:
 - If median is better, don't want to trim outliers, variable is ordinal, or any group size < 20:
 - If groups are paired, **Wilcoxon signed-rank test**.
 - If groups are independent, **Mann–Whitney U test**.
 - Else:
 - If groups are paired, **Paired Student's t-test**.
 - If groups are independent, **Welch's t-test**, not Student's.

More cheat sheets & references

- More cheat sheets:
 - Selecting Commonly Used Statistical Tests – Bates College
 - Which statistical test should I use? – University of Sheffield
 - Choosing a statistical test – HBS
- References:
 - Fisher's exact test of independence – HBS
 - Statistical notes for clinical researchers – Restor Dent Endod
 - Nonparametric Test and Parametric Test – Minitab
 - Advantages and limitations – Welch's t-test – Wikipedia
 - Dependent t-test for paired samples – Student's t-test – Wikipedia

The tests in Python

- *04_common_tests.ipynb*

Complete a test

Are p-value & α enough?

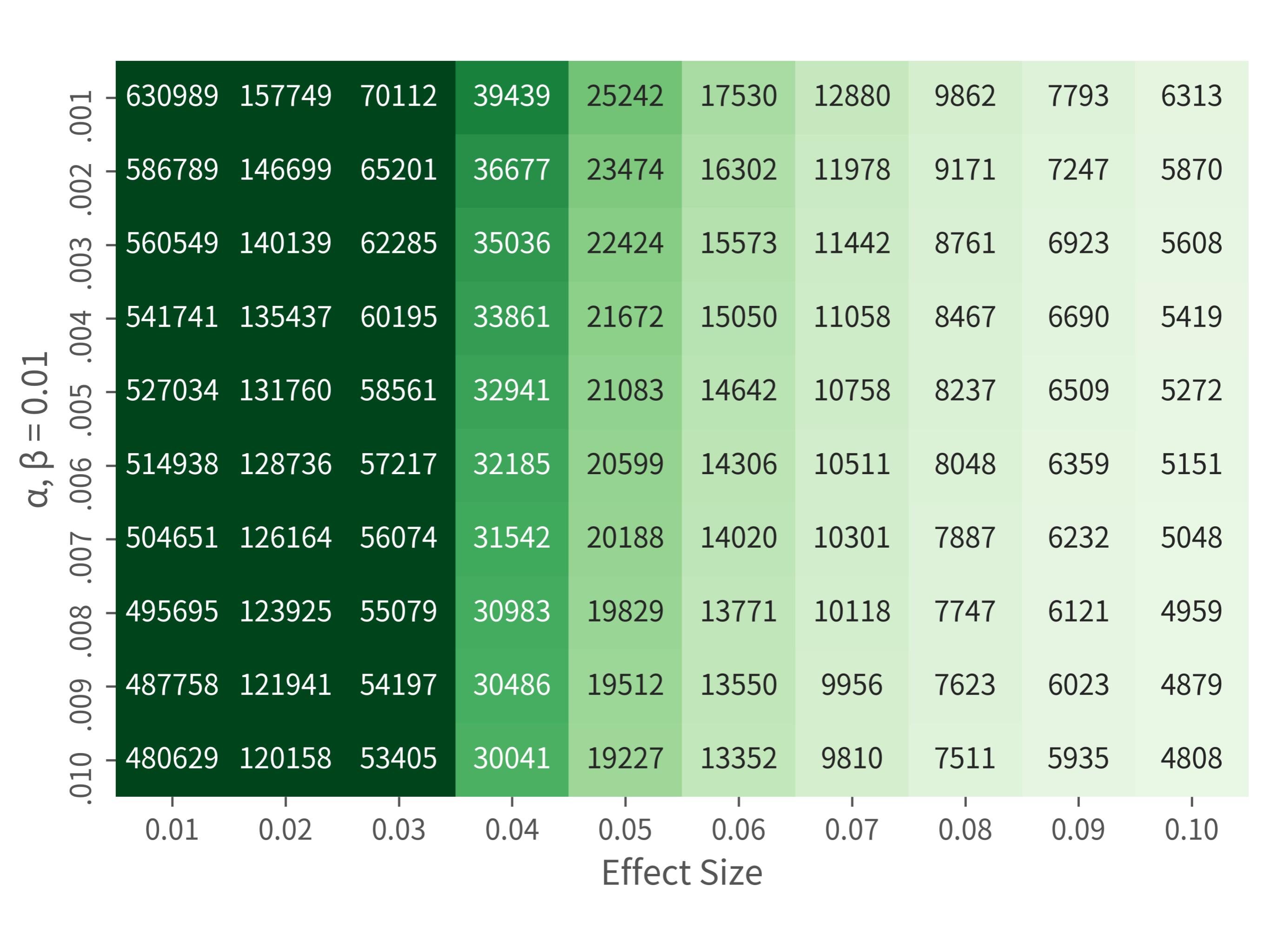
- power?
- sample size?
- β ?
- effect size?
- false discovery rate?
- ? ? ? ? ?

a, β , and effect size

- DSM5: The case for double standards – James Coplan, M.D.
 - The figures explain α , β , and *effect size* perfectly, but due to the copyright, only put the link here.
 - “FP”: α
 - “FN”: β
 - “The distance between the means”: *effect size*
 - $$\text{effect size} = \frac{\mu_1 - \mu_2}{\sigma}$$

- When α , β , *effect size* are defined, get the *sample size*.
- When α , β , *sample size* are defined, get the *effect size*.
- Increase *sample size* to decrease α , β , or *effect size*.





Confusion matrix, where $A = 00_2 = C[0, 0]$

		predicted negative AC	predicted positive BD
actual negative AB	true negative A	false positive B	
actual positive CD	false negative C	true positive D	

False positive rate = B/AB = 4/(96+4) = 4/100

.....

		predicted negative AC	predicted positive BD
actual negative AB	96 A	4 B	
	9 C	41 D	

Predefined acceptable confusion matrix

		predicted negative AC	predicted positive BD
actual negative AB	true negative A	false positive B	
	false negative C	true positive D	
actual positive CD			

$$a = B/AB = P(BD|AB)$$

.....

		predicted negative AC	predicted positive BD
actual negative AB	true negative A	false positive B	
	CD	false negative C	true positive D

$$\beta = C/CD = P(AC|CD)$$

.....

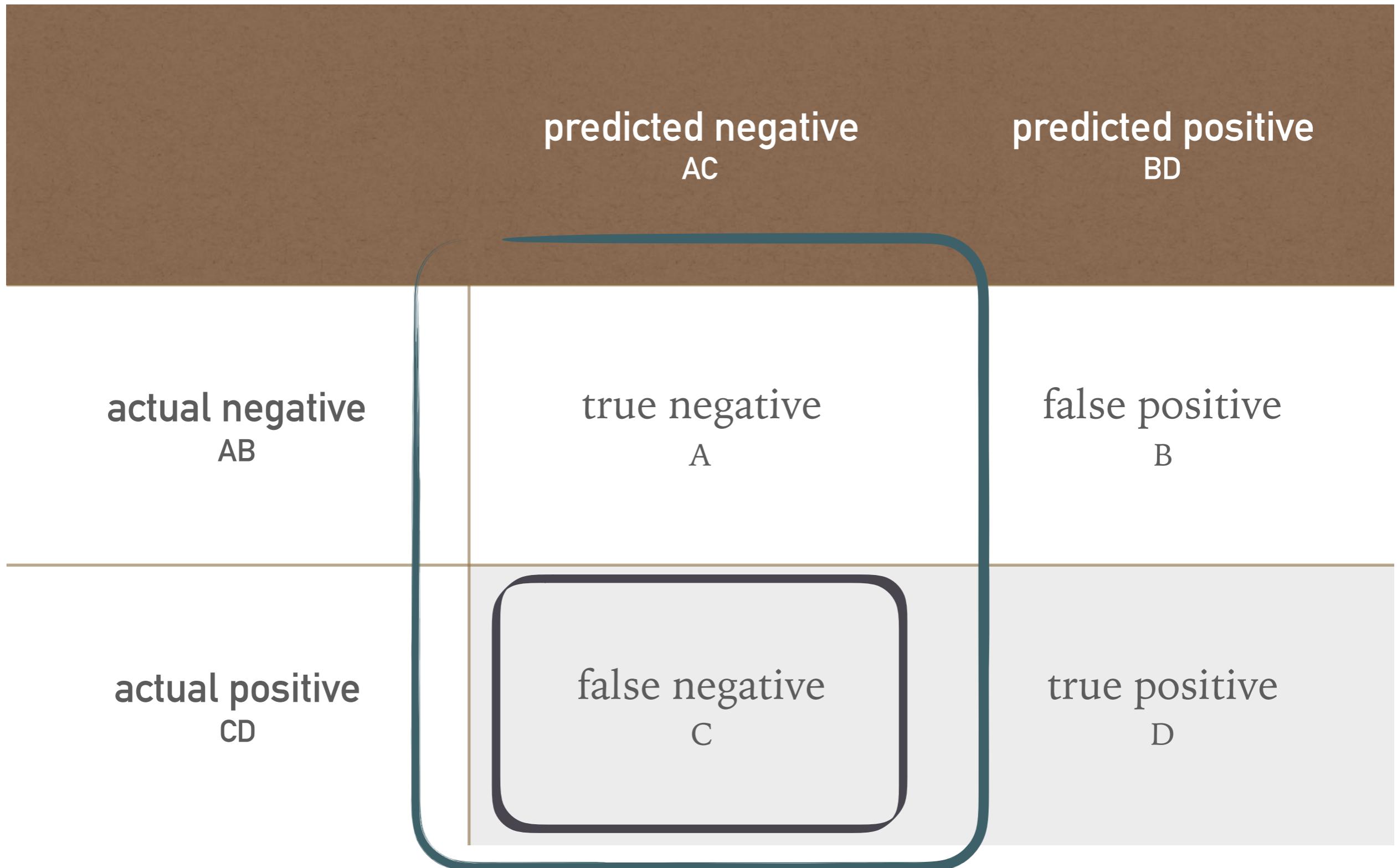
		predicted negative AC	predicted positive BD
actual negative AB	true negative A	false positive B	
actual positive CD	false negative C	true positive D	

$$\text{Inverse } a = B/BD = P(AB|BD)$$

	predicted negative AC	predicted positive BD
actual negative AB	true negative A	false positive B
actual positive CD	false negative C	true positive D

$$\text{Inverse } \beta = C/AC = P(CD|AC)$$

.....

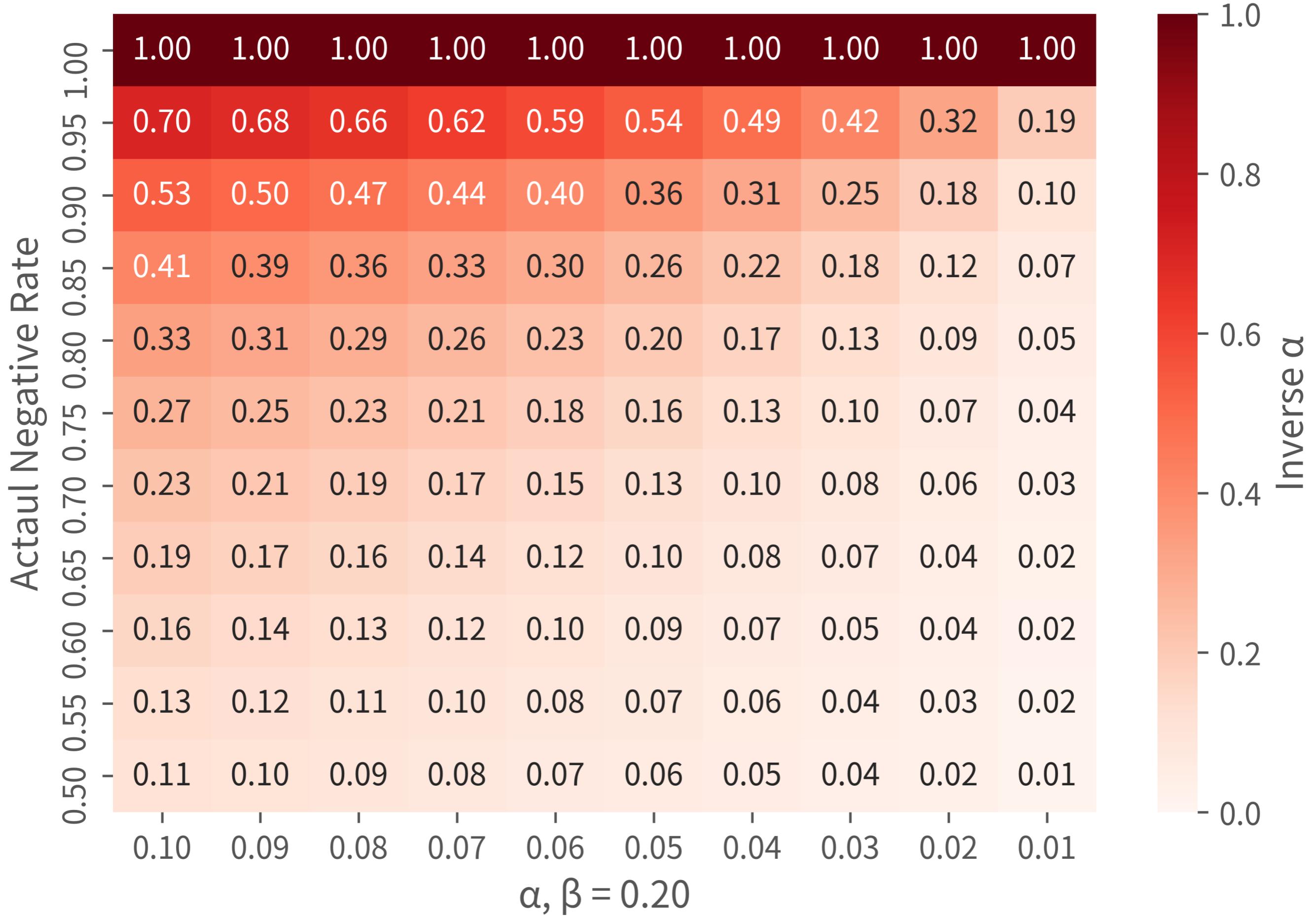


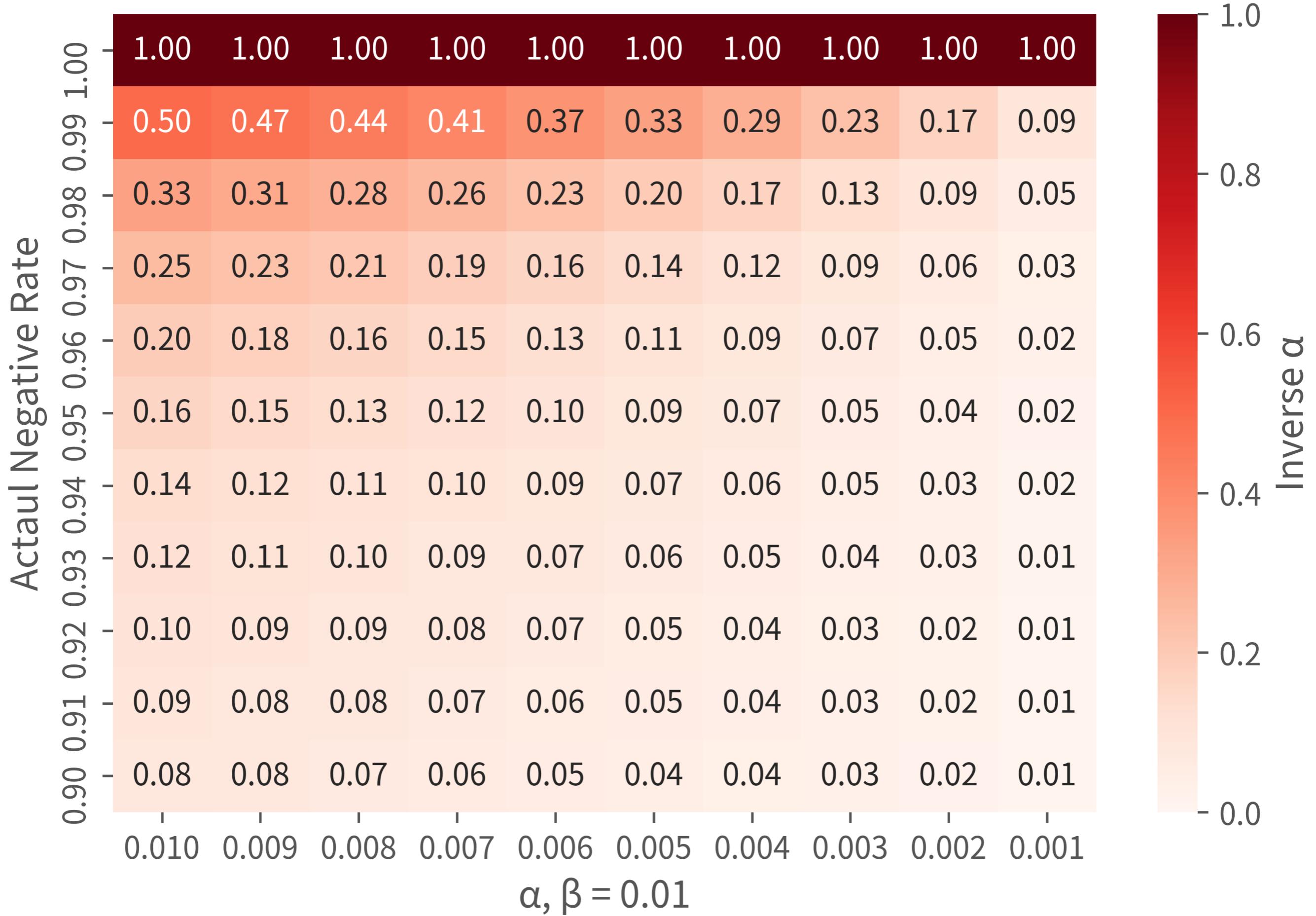
Derive inverse a

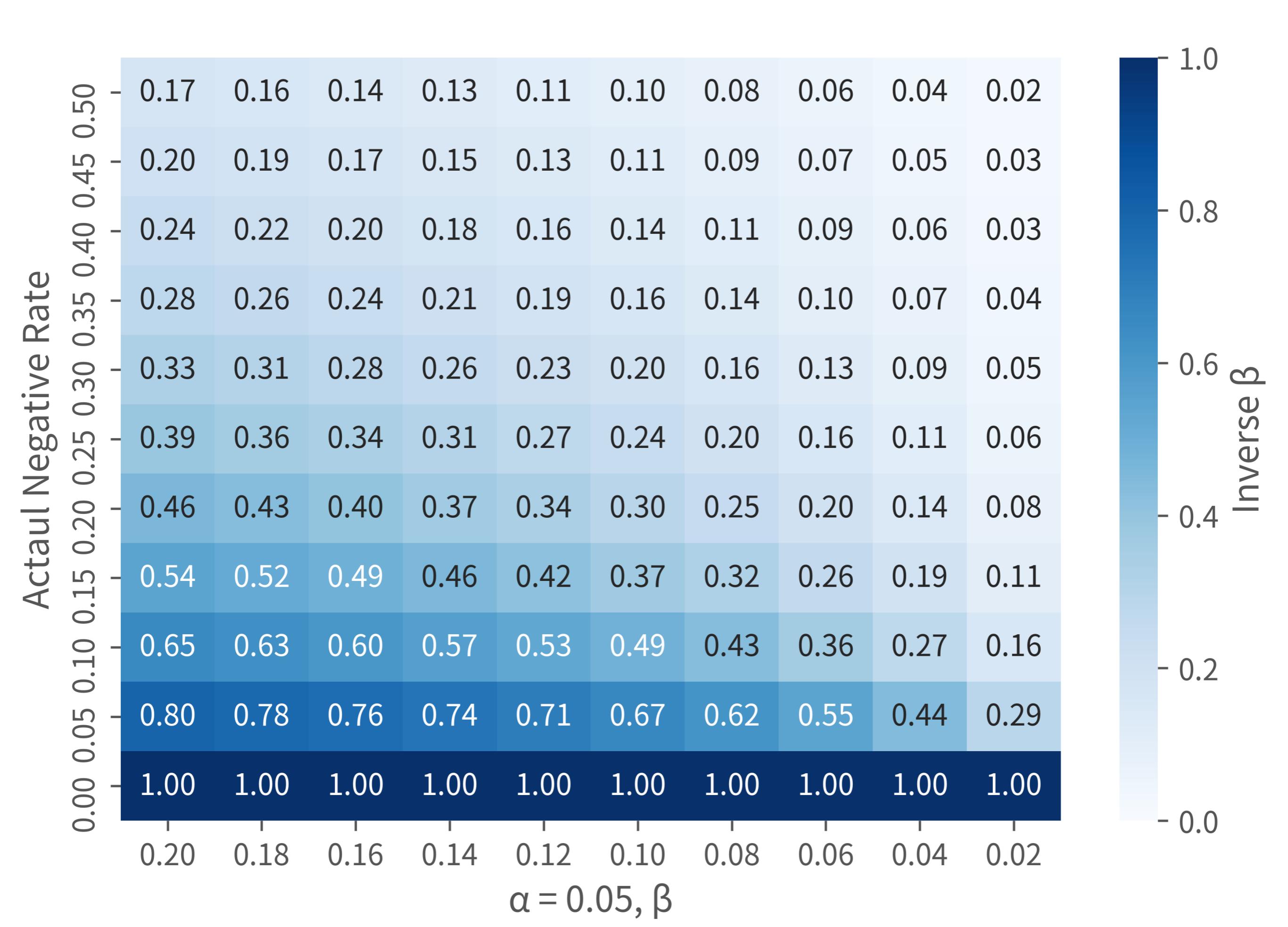
- A drug test:

- $a = 1\% = B/AB = P(BD | AB) = P(\text{predict} + | \text{not user})$
- $\beta = 1\% = C/CD = P(AC | CD) = P(\text{predict} - | \text{user})$
- $\text{actual negative rate} = 99.5\% = AB/ABCD = P(AB) = P(\text{not user})$
- $\text{inverse } a = B/BD = P(AB | BD) = P(\text{not user} | \text{predict} +)?$

$$\begin{aligned}P(AB | BD) &= \frac{P(BD | AB)P(AB)}{P(BD)} \\&= \frac{P(BD | AB)P(AB)}{P(BD | AB)P(AB) + P(BD | CD)P(CD)} \\&= \frac{0.01 \times 0.995}{0.01 \times 0.995 + 0.99 \times 0.005} \\&\approx 66.8 \%\end{aligned}$$







Rates in predefined acceptable confusion matrix

$=$ = predefined _____			
α	B/AB	significance level type I error rate	false positive rate
β	C/CD	type II error rate	false negative rate
inverse α	B/BD		false discovery rate
inverse β	C/AC		false omission rate
confidence level	A/AB	1- α	specificity
power	D/CD	1- β	sensitivity recall

Rates in confusion matrix

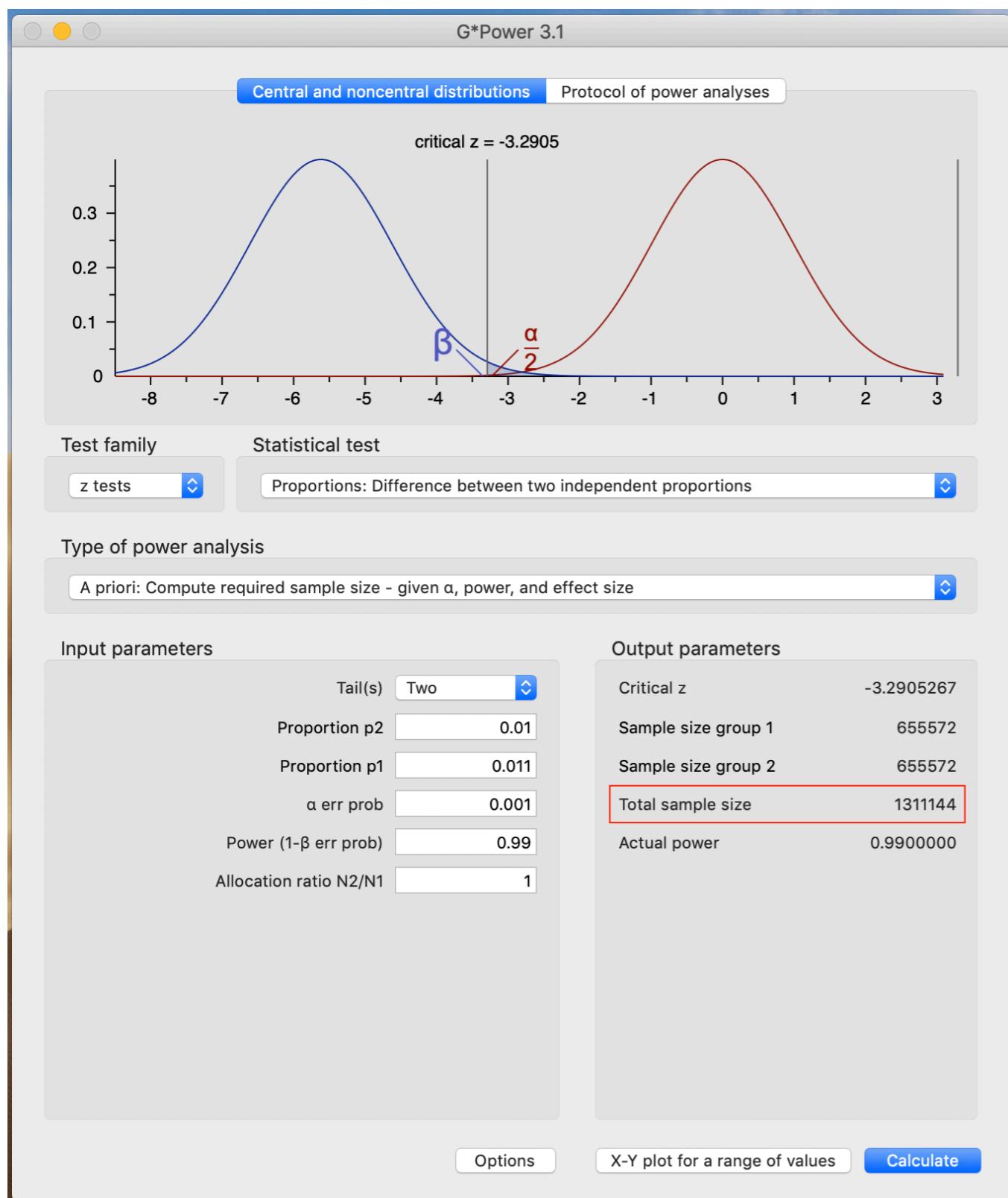
	=	=	= observed __
false positive rate	B/AB		α
false negative rate	C/CD		β
false discovery	B/BD		inverse α
false omission	C/AC		inverse β
actual negative	AB/ABCD		
sensitivity	D/CD	recall	power
specificity	A/AB		confidence level
precision	D/BD		inverse power
recall	D/CD	sensitivity	power

Most formal steps

- State the hypothesis → what *test*.
- Estimate the *actual negative rate*.
- The *actual negative rate* → how much α, β are required.
- The $\alpha, \beta, \text{effect size}$ → how large *sample size* is required.
- Still collect a sample as large as possible.
- Understand and preprocess the sample.
 - Missing data, outliers, Q–Q plot, transform, etc.
- Test and report fully.

Calculate by yourself

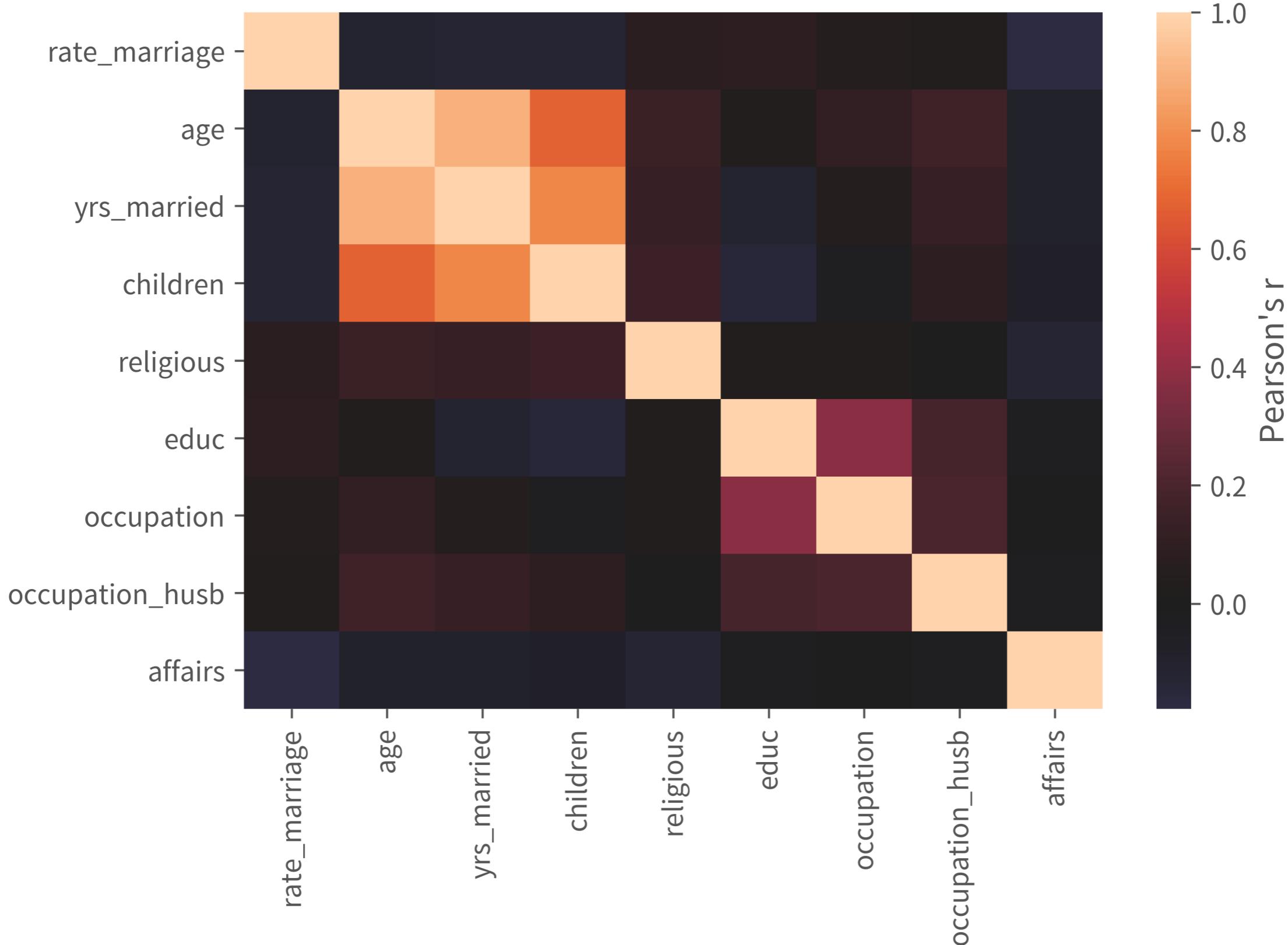
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- G*Power can calculate:
 - *sample size*
 - α
 - $power = 1 - \beta$
 - *effect size*
- <http://www.gpower.hhu.de/>
- *05_complete_a_test.ipynb*

Other statistical tools

Correlation analysis



Regression analysis

```
In [7]: fair_df = sm.datasets.fair.load_pandas().data  
ols_res = smf.ols('children ~ yrs_married', fair_df).fit()  
ols_res.summary()
```

Out[7]: OLS Regression Results

Dep. Variable:	children	R-squared:	0.597			
Model:	OLS	Adj. R-squared:	0.597			
Method:	Least Squares	F-statistic:	9437.			
Date:	Fri, 06 Jul 2018	Prob (F-statistic):	0.00			
Time:	00:40:17	Log-Likelihood:	-8430.3			
No. Observations:	6366	AIC:	1.686e+04			
Df Residuals:	6364	BIC:	1.688e+04			
Df Model:	1					
Covariance Type:	nonrobust					
	coef	std err	t	P> t	[0.025	0.975]
Intercept	0.0259	0.018	1.429	0.153	-0.010	0.062
yrs_married	0.1522	0.002	97.142	0.000	0.149	0.155
Omnibus:	449.258	Durbin-Watson:	1.972			
Prob(Omnibus):	0.000	Jarque-Bera (JB):	709.624			
Skew:	0.559	Prob(JB):	8.07e-155			
Kurtosis:	4.193	Cond. No.	18.5			

Keep learning

- Statistics
 - [Seeing Theory](#)
 - [Statistics – SciPy Tutorial](#)
 - [StatsModels](#)
 - [Biological Statistics](#)
 - [Research Methods](#)
- Machine Learning
 - [Scikit-learn Tutorials](#)
 - [Standford CS229](#)
 - [Hsuan-Tien Lin](#)

Recap

- *p-value*: “tail” probability given “actual negative” hypothesis.
- *actual negative rate* does matter.
- $\alpha, \beta, \text{effect size} \downarrow; \text{sample size} \uparrow.$
- Visualization and simulation do help.
- Bonus: `a1_figures.ipynb` .
- Let's identify noise efficiently!