



Hypothesis Testing With Python

True Difference or Noise?

0.7196

0.7552

Which is better?

Noise?

That's a question.

Mosky



- Python Charmer at Pinkoi.
- Has spoken at: PyCons in TW, MY, KR, JP, SG, HK, COSCUPs, and TEDx, etc.
- Countless hours on teaching Python.
- Own the Python packages: ZIPCodeTW, MoSQL, Clime, etc.
- <http://mosky.tw/>

Outline

- Introduction
- Welch's t-test
- Chi-squared test
- Power analysis
- More tests
- Complete steps
- Theory
 - P-value & α
 - Raw effect size,
 β , sample Size
 - Actual negative rate,
inverse α , inverse β

The PDF, Notebooks, and Packages

- The PDF and notebooks are available on <https://github.com/moskytw/hypothesis-testing-with-python> .
- The packages:
 - \$ pip3 install jupyter numpy scipy sympy matplotlib ipython pandas seaborn statsmodels scikit-learn
- Or:
 - > conda install jupyter numpy scipy sympy matplotlib ipython pandas seaborn statsmodels scikit-learn

Define assumptions

- The hypothesis testing:
- Suitable to answer a **yes–no question**:
 - “**Means** or medians of two populations are equal?”
 - E.g., “The order counts of A and B are equal?”
 - “**Proportions** of two populations are equal?”
 - E.g., “The conversion rates of A and B are equal?”

To buy, or not to buy

- Going to buy a **bulb** on an online store.
- If see 10/100 bad reviews? Hmm ...
- If see 5/100 bad reviews? **Good to buy.**
- If see 1/100 bad reviews? **Good to buy.**

- Going to buy a notebook computer on an online store.
- If see 10/100 bad reviews? Hmm ...
- If see 5/100 bad reviews? Hmm ...
- If see 1/100 bad reviews? Maybe good enough.
- Context matters.

Build our “bad reviews” in statistics

- Build a statistical model by a hypothesis.
 - “The means of two populations are equal.”
- Put the data into the model, get a probability, *p-value*.
 - “How compatible the data and the model are.”
- If see *p-value* = 0.10?
- If see *p-value* = 0.05?
- If see *p-value* = 0.01?
- Depends on your research assumption.

Null, alternative, and p-value

- Null hypothesis: can build a model directly:
 - “The means of two populations are equal.”
 - \equiv “The expected value of difference is zero.”
 - E.g., GMV didn't change.
- Alternative hypothesis: can build a model by negating it.
 - “The means of two populations are different.”
 - $\equiv \neg$ “The expected value of difference is zero.”
 - E.g., GMV changed.
- P-value: the probability to observe the data, given null.

Misunderstandings of p-values

- “Buy or not” is a decision based on the research context.
 - In other words, “accept the alternative hypothesis or not”.
 - Not “prove a hypothesis is true or false.”
- Misunderstandings of p-values – Wikipedia

Suggested formatting

| p-value & α | Wording | Summary |
|----------------------------|------------------|---------|
| $p\text{-value} < 0.001$ | Very significant | *** |
| $p\text{-value} < 0.01$ | Very significant | ** |
| $p\text{-value} < 0.05$ | Significant | * |
| $p\text{-value} \geq 0.05$ | Not significant | ns |

- Many researchers also suggest to report **without** formatting.
 - Since the largely misunderstandings, e.g.,
 - $p\text{-value} < 0.05 \equiv$ the hypothesis is false (**wrong**)
- Scientists rise up against statistical significance – Natural
 - “We are not calling for a ban on P values. Nor are we saying they cannot be used as a decision criterion in certain specialized applications.”
 - “We are calling for a stop to the use of P values in the conventional, dichotomous way — to decide whether a result refutes or supports a scientific hypothesis.”

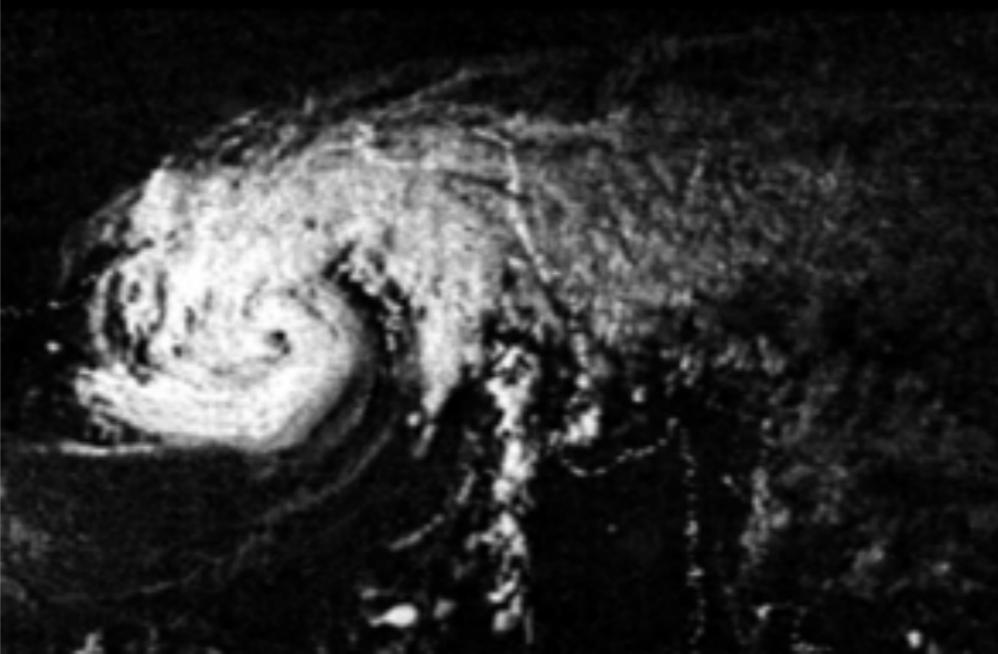
Today's assumptions

- “Poor or non-poor marriage has different affair times?”
- “Occupations have different affair times?”
- “Poor or non-poor marriage has different affair proportion?”
- “Occupations have different affair proportion?”

Validate Assumptions

- Collect data ...
- The “Fair” dataset:
 - Fair, Ray. 1978. “A Theory of Extramarital Affairs,” Journal of Political Economy, February, 45-61.
 - A dataset from 1970s.
 - Rows: 6,366
 - Columns: (next slide)
- The full version of the analysis steps:
<http://bit.ly/analysis-steps> .

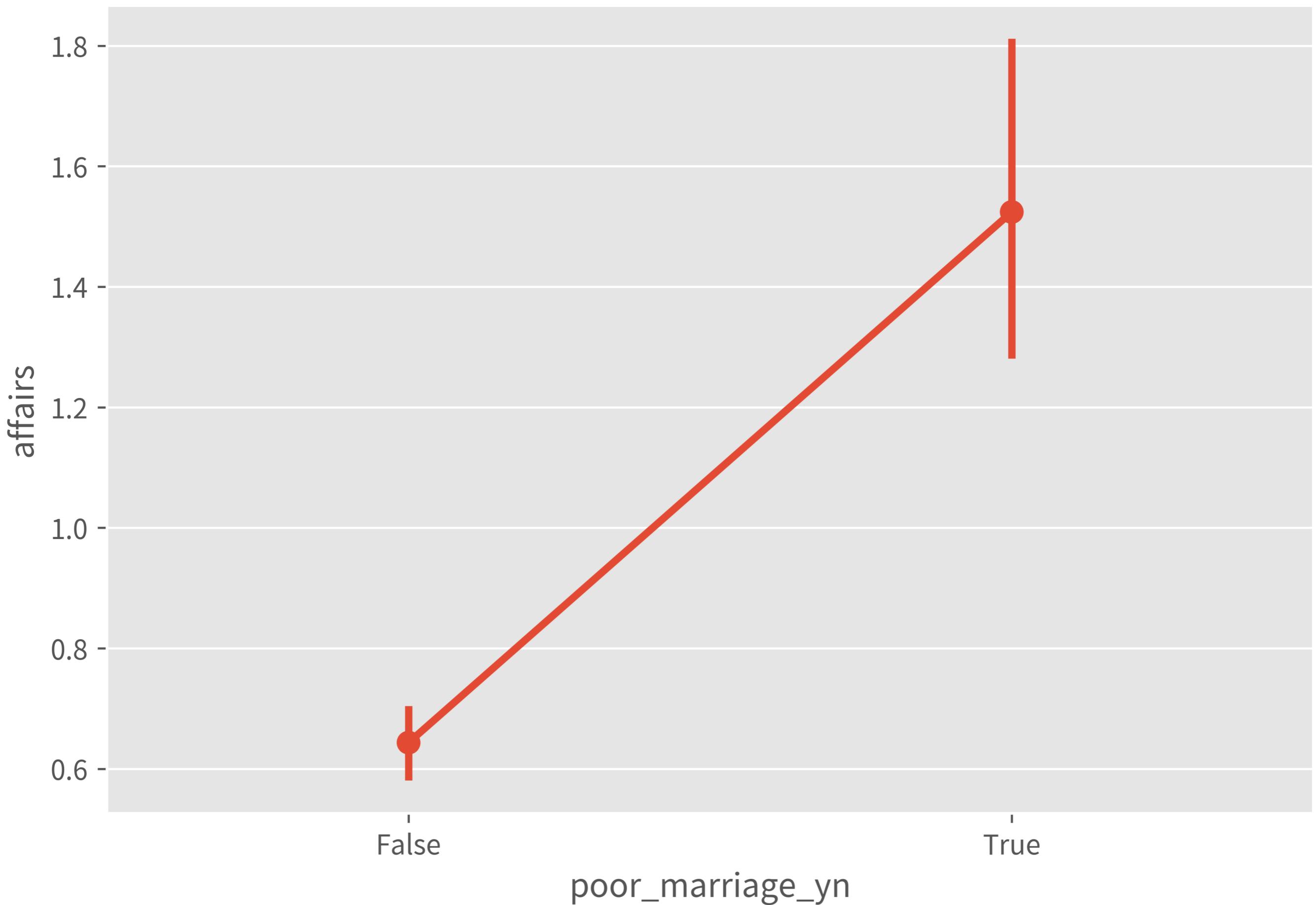
1. *rate_marriage*: 1~5; very poor, poor, fair, good, very good.
2. *age*
3. *yrs_married*
4. *children*: number of children.
5. *religious*: 1~4; not, mildly, fairly, strongly.
6. *educ*: 9, 12, 14, 16, 17, 20; grade school, some college, college graduate, some graduate school, advanced degree.
7. *occupation*: 1, 2, 3, 4, 5, 6; student, farming-like, white-collar, teacher-like, business-like, professional with advanced degree.
8. *occupation_husb*
9. *affairs*: n times of extramarital affairs per year since marriage.



Welch's t-test #1

| | count | mean | std |
|--|--------|----------|----------|
| poor_marriage_yn | | | |
| False | 5919.0 | 0.643549 | 2.116982 |
| True | 447.0 | 1.524038 | 3.015937 |
| p-value: 2.7446844166802127e-09 | | | |

- Preprocess:
 - Group into poor or not.
- Describe.
- Test:
 - The *p-value* is super low.
 - We accept the means are different at 99.9% confidence level ($= 1-\alpha$):
 - Non-poor: 0.6435
 - Poor: 1.5240



```
import statsmodels.api as sm
import scipy as sp

print(sm.datasets.fair.SOURCE,
      sm.datasets.fair.NOTE)

# -> Pandas's Dataframe
df_fair = sm.datasets.fair.load_pandas().data

df = df_fair
# 2: poor
# 3: fair
df = df.assign(poor_marriage_yn
                  =(df.rate_marriage <= 2))
df_fair_2 = df
```

```
df = df_fair_2

display(df
        .groupby('poor_marriage_yn')
        .affairs
        .describe())

a = df[df.poor_marriage_yn].affairs
b = df[~df.poor_marriage_yn].affairs

# ttest_ind(...) === Student's t-test
# ttest_ind(..., equal_var=False) === Welch's t-test
print('p-value:',
      sp.stats.ttest_ind(a, b, equal_var=False)[1])
```

```
df = df_fair_2
```

```
sns.pointplot(x=df.poor_marriage_yn,  
               y=df.affairs)
```

Why Welch's t-test, not Student's t-test?

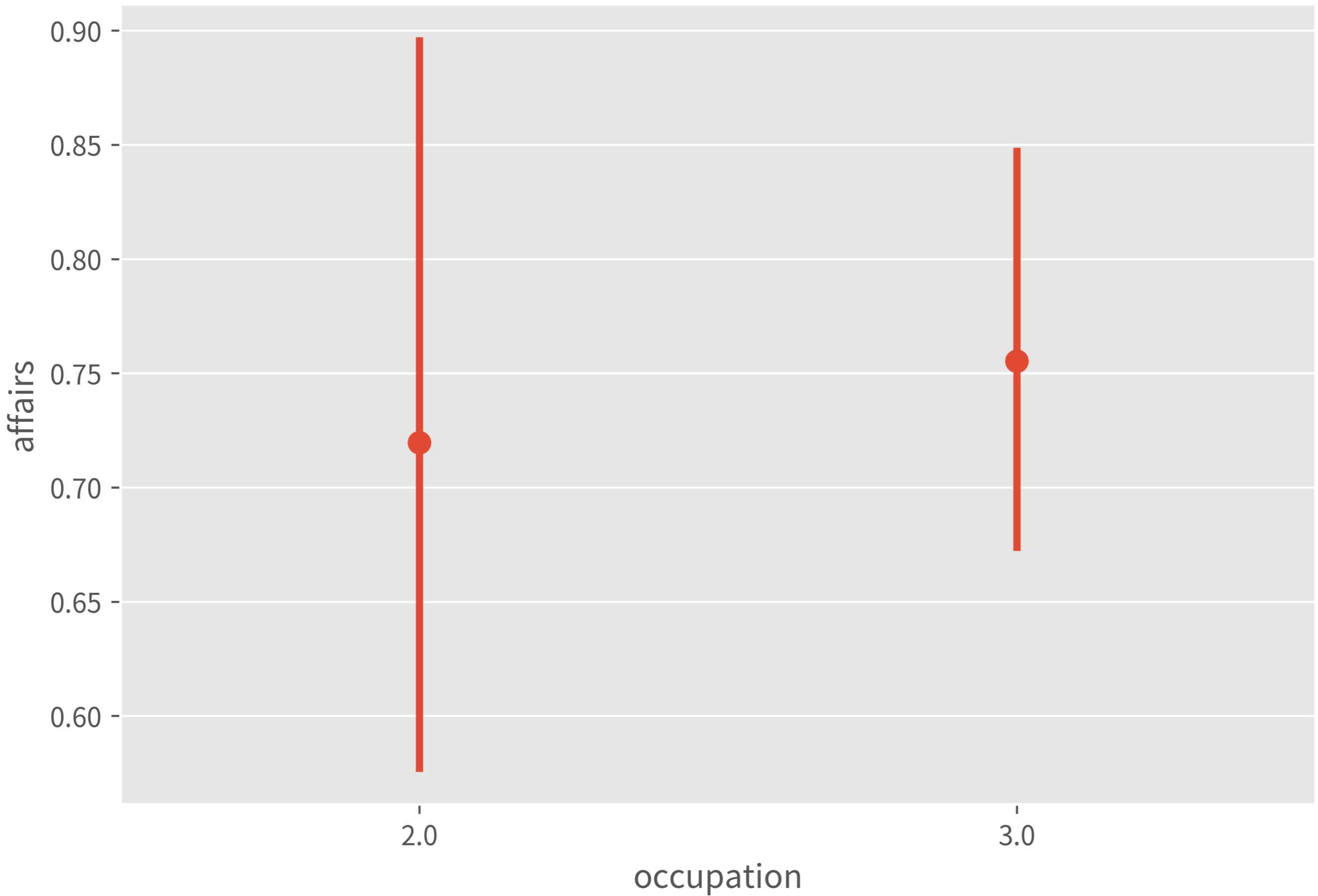
- Student's t-test assumed the two populations have the same variance, which may not be true in most cases.
- Welch's t-test relaxed this assumption without side effects.
- So, just use Welch's t-test directly. [ref]

Welch's t-test #2

| occupation | count | mean | std |
|------------|--------|----------|----------|
| 2.0 | 859.0 | 0.719556 | 2.375644 |
| 3.0 | 2783.0 | 0.755248 | 2.305594 |

p-value: 0.698381462473247

- Preprocess:
 - Select the two occupations.
 - Group by occupations.
- Describe.
- Test:
 - The *p-value* is high.
 - We can't accept the means are different at 99% confidence level.

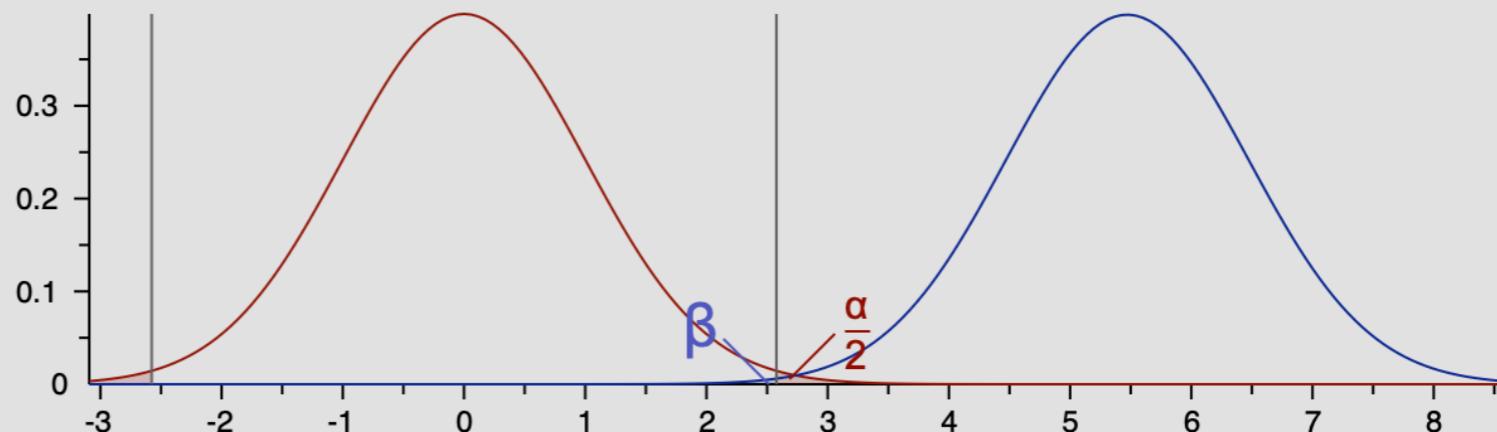


Power analysis for Welch's t-test #2

- The α is given null, accept alter, = *1-confidence level*.
- The β is given alter, don't accept alter, = *1-power*.
- Hope to detect ≥ 0.5 times difference at 99% confidence level:
 - *raw effect size* = 0.5
 - α = 0.01
- Use G*Power or StatsModels:
 - *power* = 0.9981
 - β = 0.0019
- The β is completely acceptable, so the test is valid.

Central and noncentral distributions

Protocol of power analyses

critical $t = 2.5772$ 

Test family

t tests

Statistical test

Means: Difference between two independent means (two groups)

 $n_1 \neq n_2$

Mean group 1

0

Mean group 2

1

SD σ within each group

0.5

 $n_1 = n_2$

Mean group 1

0.719556

Mean group 2

1.219557

SD σ group 1

2.375644

SD σ group 2

2.305594

Calculate

Effect

0.2135952

Calculate and transfer to main window

Close effect size drawer

Input parameters

Determine

Tail(s) Two

Effect size d 0.2135952

 α err prob 0.01

Sample size group 1 859

Sample size group 2 2783

Output parameters

Noncentrality parameter δ 5.4723605

Critical t 2.5771807

Df 3640

Power (1- β err prob) 0.9980984

X-Y plot for a range of values

Calculate

```
df = df_fair
# 2: farming-like
# 3: white-colloar
df = df[df.occupation.isin([2, 3])]
df_fair_3 = df
```

```
df = df_fair_3
```

```
display(df
    .groupby('occupation')
    .affairs
    .describe())
```

```
a = df[df.occupation == 2].affairs
b = df[df.occupation == 3].affairs
```

```
print('p-value: ',
      sp.stats.ttest_ind(a, b, equal_var=False)[1])
```

```
df = df_fair_3

sns.pointplot(x=df.occupation,
               y=df.affairs,
               join=False)

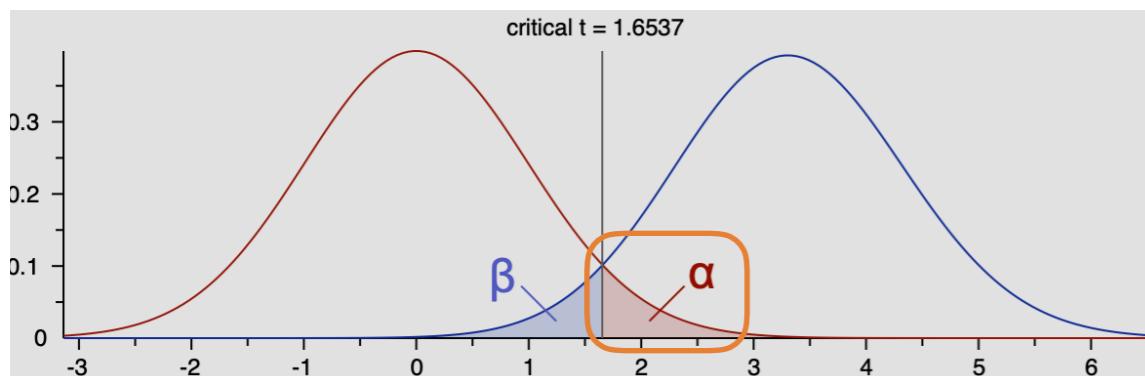
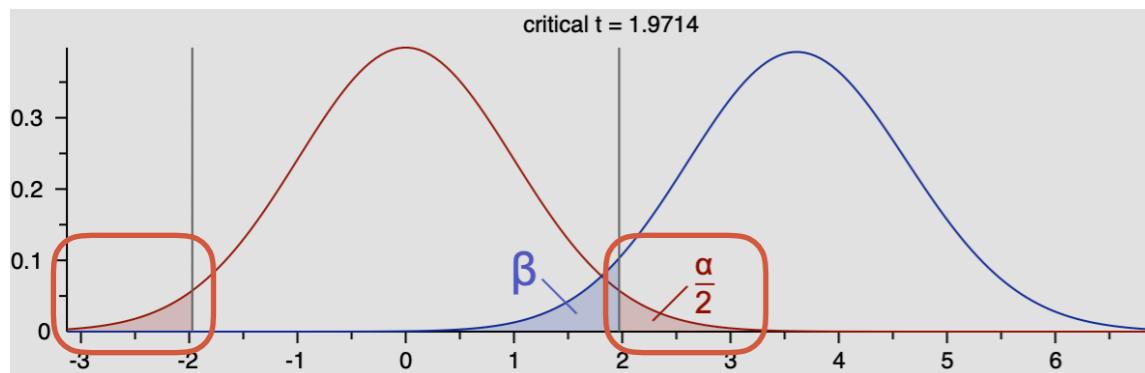
print('p-value: ',
      sp.stats.ttest_ind([1, 2, 3, 4, 5, 6],
                          [1, 2, 3, 4, 5, 60],
                          equal_var=False)[1])
```

Two tails or one tail

.....

- “B is **different** from A.”
- Regardless of directions.
- Use **two** tails.
- “B is **greater** than A.”
“B is **lesser** than A.”
- One of directions.
- Use **one** tail.
- If p-value function doesn't support and the distribution is symmetrical, just /2 to obtain one-tailed p-value.

Two tails



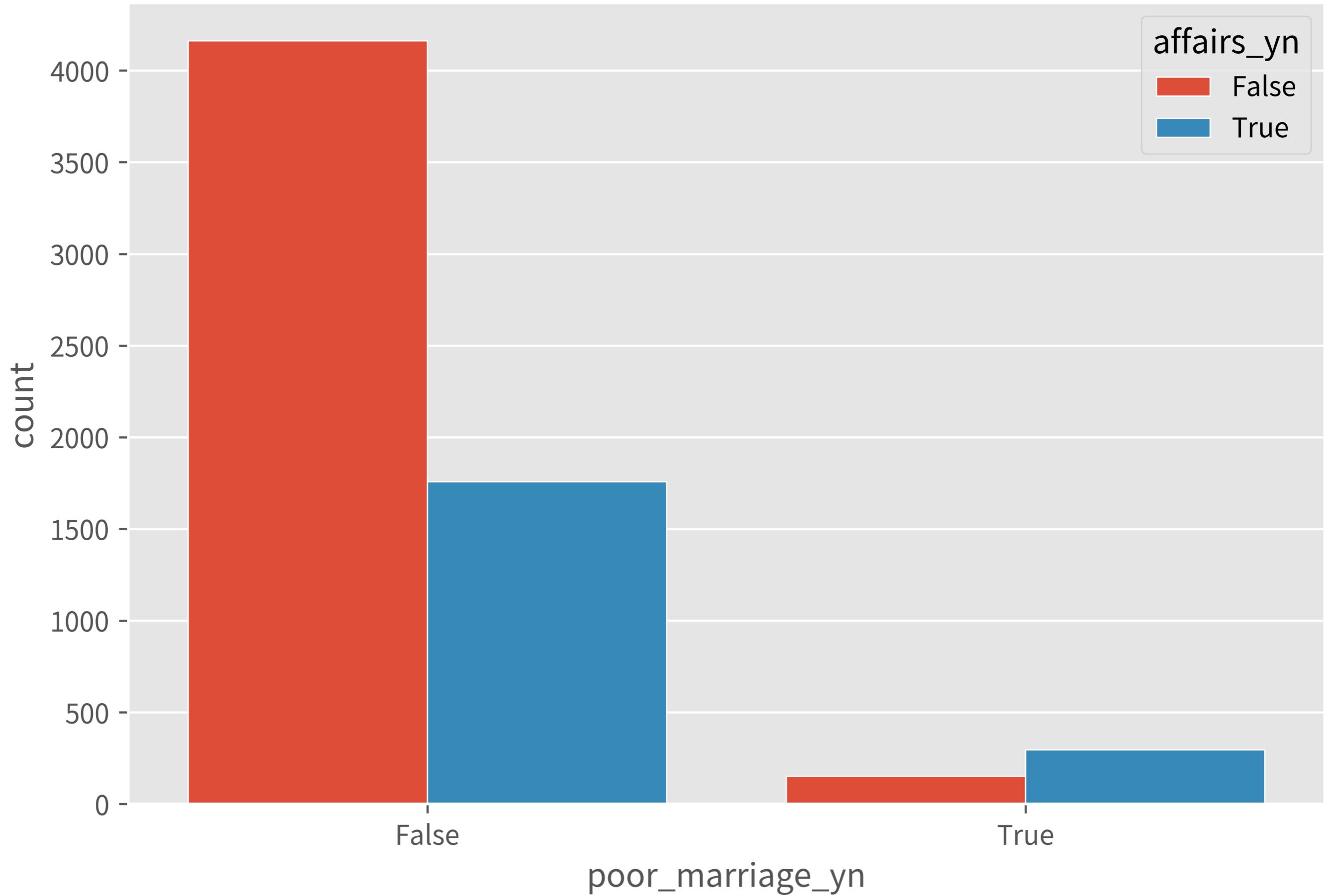
One tail

Chi-squared test #1

| | affairs_yn | False | True |
|-------------------------|-------------------|--------------|-------------|
| poor_marriage_yn | | | |
| False | 4161 | 1758 | |
| True | 152 | 295 | |
| | affairs_yn | False | True |
| poor_marriage_yn | | | |
| False | 0.702990 | 0.297010 | |
| True | 0.340045 | 0.659955 | |

p-value: 1.9460298519537103e-56

- Preprocess:
 - Transform affairs > 0 as true.
 - Select the two occupations.
 - Group by occupations.
- Describe.
- Test:
 - The *p-value* is super low.
 - We accept the proportions are different at 99.9% confidence level ($= 1-a$):
 - Non-poor: 30%
 - Poor: 66%



```
df = df_fair
# 2: poor
# 3: fair
df = df.assign(poor_marriage_yn
                =(df.rate_marriage <= 2),
                affairs_yn=(df.affairs > 0))
df_fair_2 = df
```

```
df = df_fair_2

df = (df
      .groupby(['poor_marriage_yn', 'affairs_yn'])
      [['affairs']]
      .count()
      .unstack()
      .droplevel(axis=1, level=0))

df_pct = df.apply(axis=1, func=lambda r: r/r.sum())

display(df, df_pct)

print('p-value:',
      sp.stats.chi2_contingency(
          df,
          correction=False
      )[1])
```

```
df = df_fair_2
sns.countplot(data=df,
               x='poor_marriage_yn', hue='affairs_yn',
               saturation=0.95, edgecolor='white')
```

Chi-squared test #2

.....

| affairs_yn | False | True |
|-------------------|--------------|-------------|
|-------------------|--------------|-------------|

| occupation | | |
|-------------------|--|--|
|-------------------|--|--|

| | | |
|------------|------|-----|
| 2.0 | 607 | 252 |
| 3.0 | 1818 | 965 |

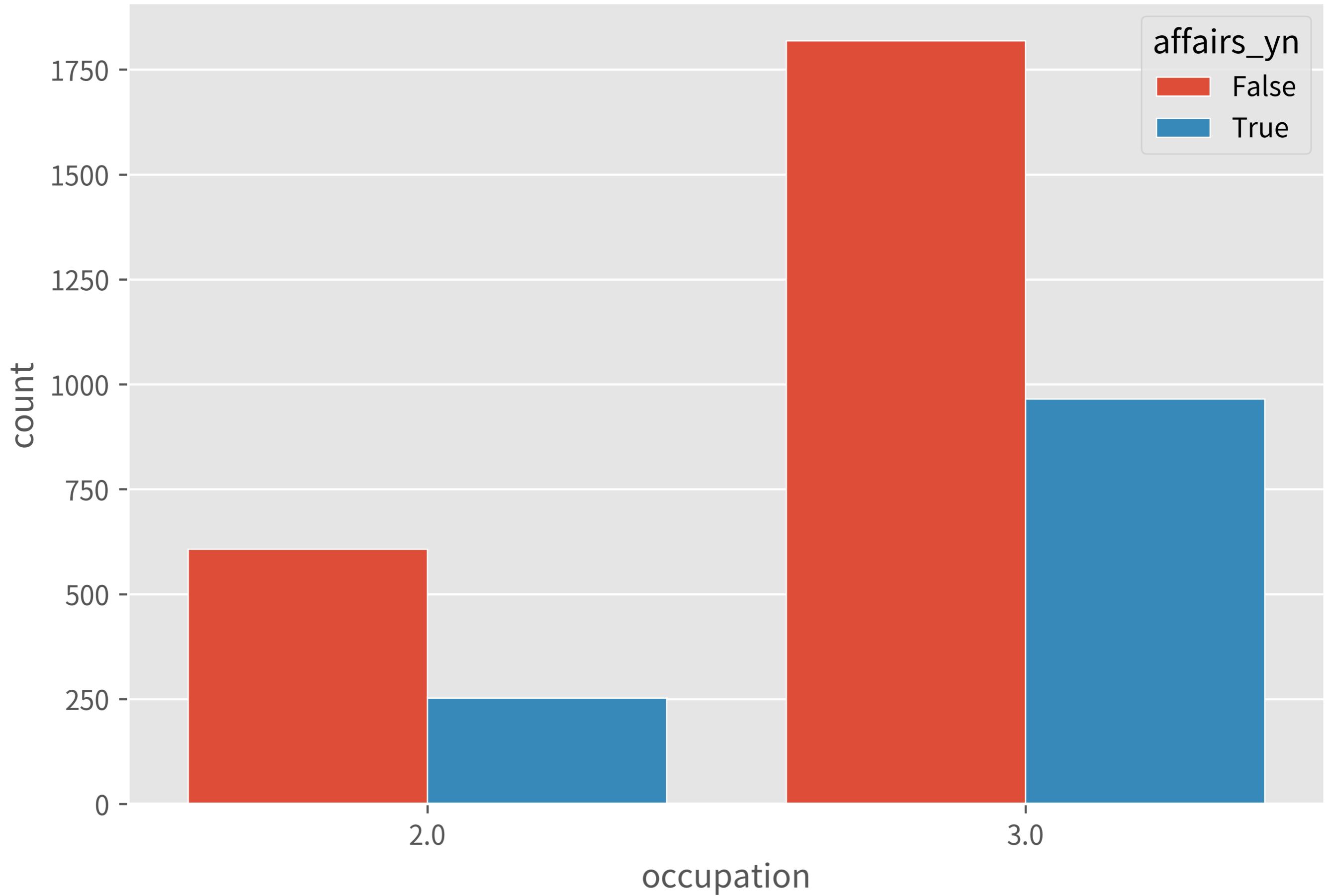
| affairs_yn | False | True |
|-------------------|--------------|-------------|
|-------------------|--------------|-------------|

| occupation | | |
|-------------------|--|--|
|-------------------|--|--|

| | | |
|------------|----------|----------|
| 2.0 | 0.706636 | 0.293364 |
| 3.0 | 0.653252 | 0.346748 |

p-value: 0.0037369587127306517

- Preprocess:
 - Transform affairs > 0 as true.
 - Group by occupation.
- Describe.
- Test:
 - The *p-value* is low.
 - We accept the proportions are different at 99.9% confidence level (= $1-\alpha$):
 - Farming-like: 29%
 - White-colloar: 35%



```
df = df_fair_2
# 2: farming-like
# 3: white-colloar
df = df[df.occupation.isin([2, 3])]
df_fair_3 = df
```

```
df = df_fair_3

df = (df
      .groupby(['occupation', 'affairs_yn'])
      [['affairs']]
      .count()
      .unstack()
      .droplevel(axis=1, level=0))

df_pct = df.apply(axis=1, func=lambda r: r/r.sum())

display(df, df_pct)

print('p-value: ',
      sp.stats.chi2_contingency(
          df,
          correction=False
      )[1])
```

```
df = df_fair_3
sns.countplot(data=df,
               x='occupation', hue='affairs_yn',
               saturation=0.95, edgecolor='white')
```

Power analysis

- $f(\alpha, \text{effect size}, \beta) = \text{sample size}$
- Before collecting data,
define α , effect size, β to calculate required sample size.
- After test,
 - If $p\text{-value} < \alpha$, good!
 - If $p\text{-value} > \alpha$ or closes to α , may investigate the β .
- The α , effect size, β here are “to-achieve”, not “observed”.
- Two-proportion z-test $\equiv 2 \times 2$ chi-squared test. [ref]
 - The power analysis of two-proportion z-test is much easier.

Confidence level, α , β , and power

| | don't accept alter | accept alter |
|---|--|--|
| null | <p>confidence level $= P(\text{don't accept alter} \text{null})$</p> | <p>$\alpha$ $= P(\text{accept alter} \text{null})$</p> |
| alter $= \text{null} + \text{effect size}$ | <p>β $= P(\text{don't accept alter} \text{alter})$</p> | <p>power $= P(\text{accept alter} \text{alter})$</p> |

The mini cheat sheet

- If testing **means**, Welch's t-test.
- If testing **medians**, Mann–Whitney U test.
- If testing **proportions**, chi-squared test.

The cheat sheet

- If testing homogeneity:
 - If total sample size < 1000, or more than 20% of cells have expected frequencies < 5, **Fisher's exact test**.
 - Else, **chi-squared test**, or **two-proportion z-test** ($\equiv 2 \times 2$ chi-squared test).
- If testing equality:
 - If median is better, don't want to trim outliers, variable is ordinal, or any group size ≤ 20 :
 - If groups are paired, **Wilcoxon signed-rank test**.
 - If groups are independent, **Mann–Whitney U test**.
 - Else:
 - If groups are paired, **Paired Student's t-test**.
 - If groups are independent, **Welch's t-test**, not Student's.

- More cheat sheets:
 - Common statistical tests are linear models – Lindeloev
 - Selecting Commonly Used Statistical Tests – Bates College
 - Which statistical test should I use? – University of Sheffield
 - Choosing a statistical test – HBS
- References:
 - Fisher's exact test of independence – HBS
 - Statistical notes for clinical researchers – Restor Dent Endod
 - Chi-squared test of proportions is identical to z-test squared – Rinterested
 - Nonparametric Test and Parametric Test – Minitab
 - Advantages and limitations – Welch's t-test – Wikipedia
 - Dependent t-test for paired samples – Student's t-test – Wikipedia

Complete steps

1. Decide what test.
2. Decide α , effect size, β to achieve.
3. Calculate sample size.
4. Still collect a sample as large as possible.
5. Test.
6. Investigate β if need.
7. Report fully, not only significant or not.

Keep learning

1. Seeing Theory
2. Statistics – SciPy Tutorial
3. StatsModels
4. Biological Statistics
5. Research design

Recap

- The null hypothesis is for building a model.
- The p-value is:
 - Given null hypothesis, the probability to observe the data.
 - Not the probability of null hypothesis.
 - “How compatible the data and the model are.”
- Power analysis to understand sample size or β .
- Welch's t-test, Mann–Whitney U test, and chi-squared test.
- Report fully, not only significant or not.
- Let's identify true difference or noise efficiently! 

P-value & α

Theory

Seeing is believing

- $p\text{-value} = 0.0027 (< 0.01)$
 - 
- $p\text{-value} = 0.0271 (0.01\text{--}0.05)$
 -  ?  ? ? ?
- $p\text{-value} = 0.2718 (\geq 0.05)$
 - ? ? ? ? ? ?
- *appendices/theory_01_how_tests_work.ipynb*

Confusion matrix, where $A = 00_2 = C[0, 0]$

| | | predicted negative AC | predicted positive BD |
|-----------------------|---------------------|--------------------------|--------------------------|
| actual negative AB | true negative A | false positive B | |
| actual positive CD | false negative C | true positive D | |

False positive rate = $P(BD|AB) = B/AB = 4/(96+4) = 4/100$

.....

| | | predicted negative AC | predicted positive BD |
|-----------------------|---------|--------------------------|--------------------------|
| actual negative AB | 96 A | 4 B | |
| | 9 C | 41 D | |

$$\alpha = P(\text{accept alter} | \text{null}) = P(\text{predicted positive} | \text{actual negative})$$

.....

| | | predicted negative | predicted positive |
|-----------------|----------------|--------------------|--------------------|
| actual negative | AB | AC | BD |
| | true negative | A | false positive |
| actual positive | CD | C | true positive |
| | false negative | D | |

Predefined acceptable confusion matrix

| | | predicted negative AC | predicted positive BD |
|-----------------------|---------------------|--------------------------|--------------------------|
| actual negative AB | true negative A | false positive B | |
| | false negative C | true positive D | |
| actual positive CD | | | |

False positive, p-value, and α

false positive rate

Calculated
with the actual answer.

p-value

Calculated false positive rate
by a null hypothesis.

α

Predefined acceptable
false positive rate.

Raw effect size, β , sample size

Theory

The elements of a complete test

1. The null hypothesis, data, p-value, α .
 2. The raw effect size, β , sample size.
 3. The false negative rate, inverse α , inverse β .
- Will introduce them by the confusion matrix.

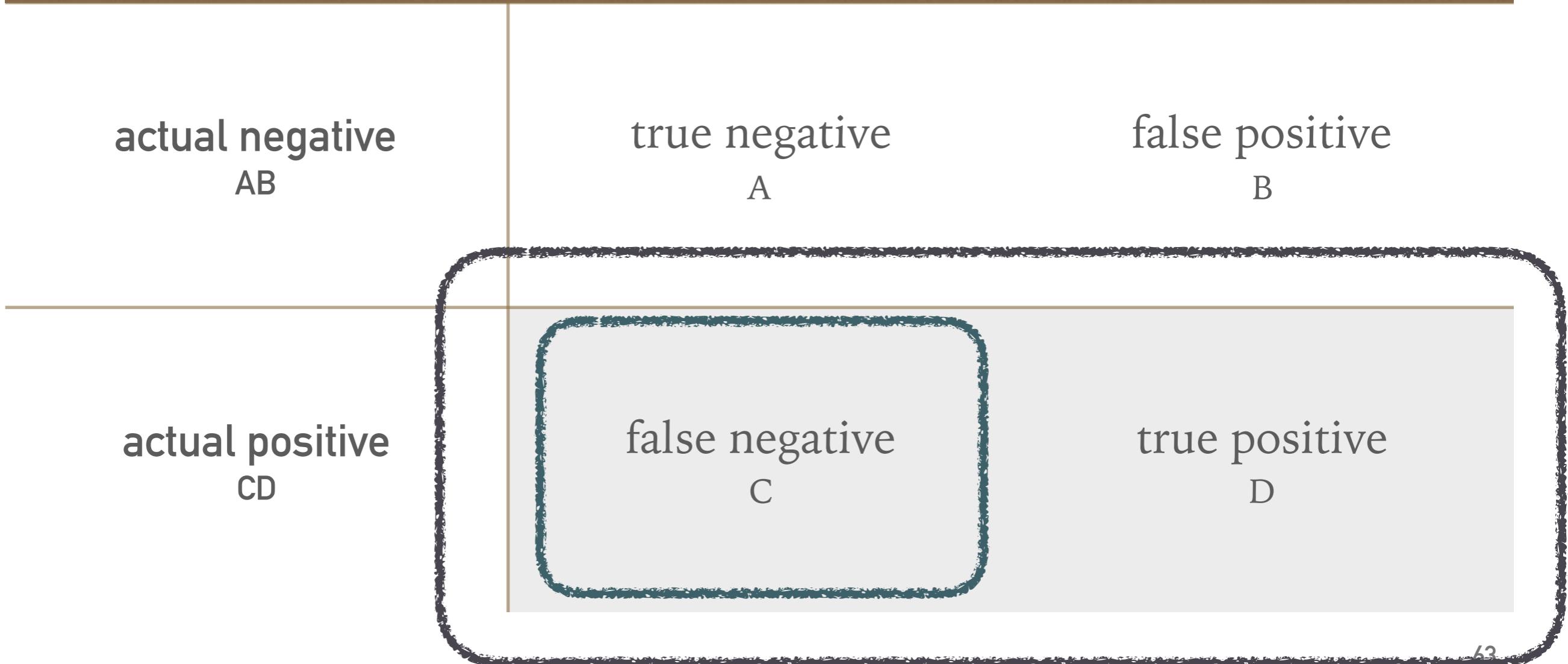
Raw effect size, and β

- DSM5: The case for double standards – James Coplan, M.D.
 - The figures explain *raw effect size*, a , and β and perfectly, but due to the copyright, only put the link here.
 - “FP”: a
 - “The distance between the means”: *raw effect size*
 - “FN”: β

$$\beta = P(AC|CD) = C/CD$$

.....

| | predicted negative AC | predicted positive BD |
|-----------------------|--------------------------|--------------------------|
| actual negative AB | true negative A | false positive B |
| actual positive CD | false negative C | true positive D |



- Given α , β , raw effect size, get the sample size.
- Given α , raw effect size, sample size, get the β .
- Increase sample size to decrease α , β , or raw effect size.

Actual negative rate,
inverse α , inverse β

Theory

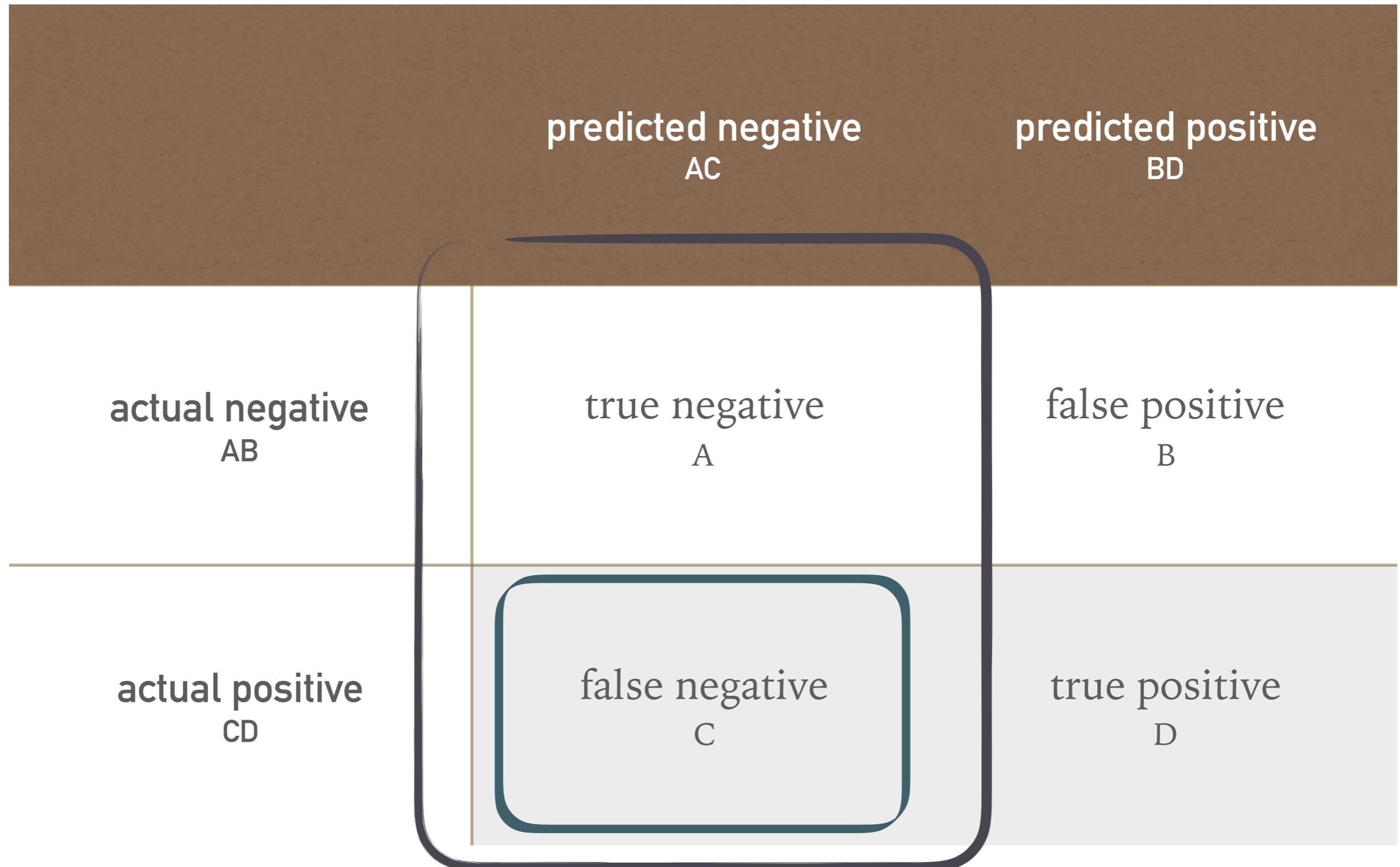
$$\text{Inverse } a = P(AB|BD) = B/BD$$

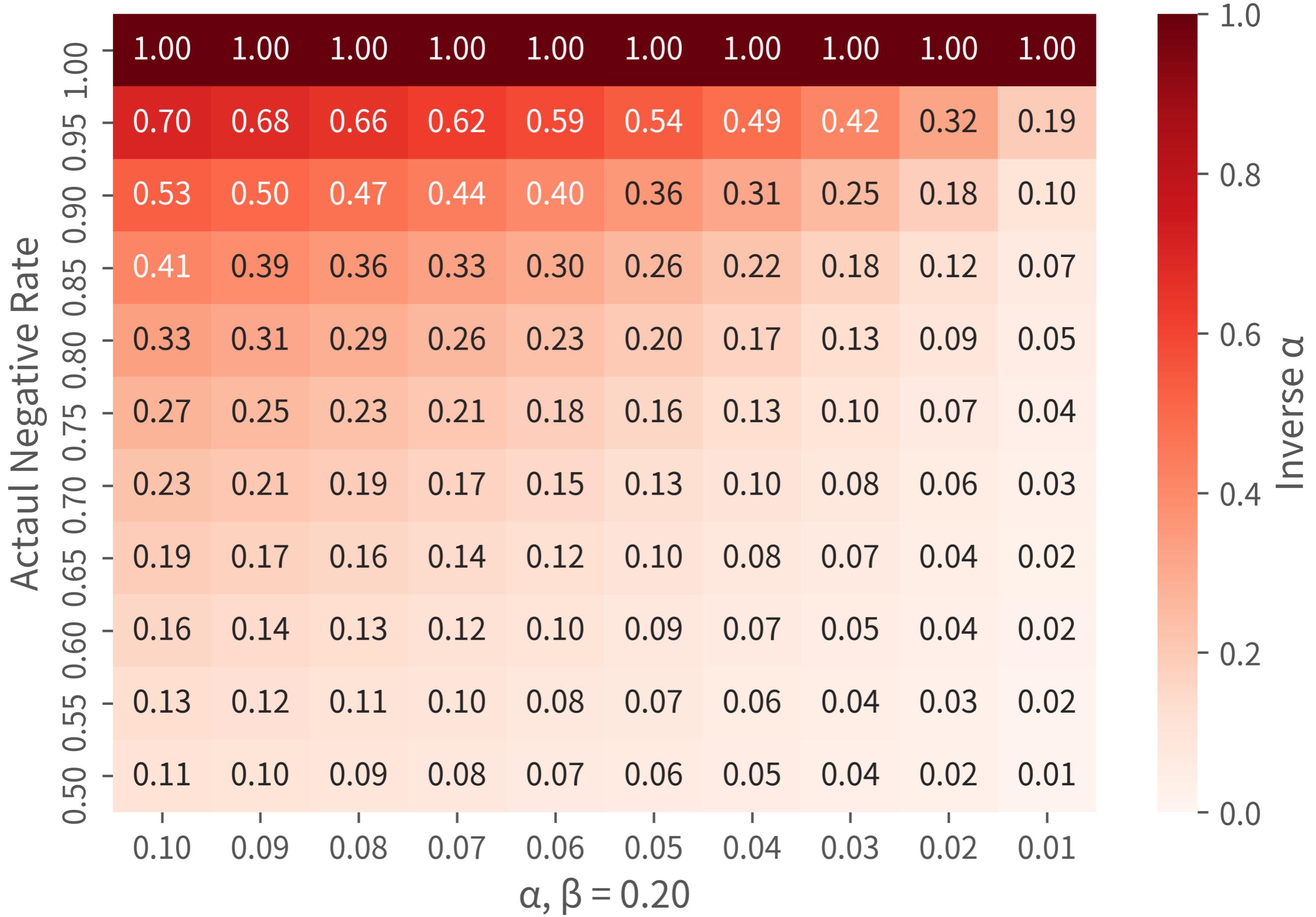
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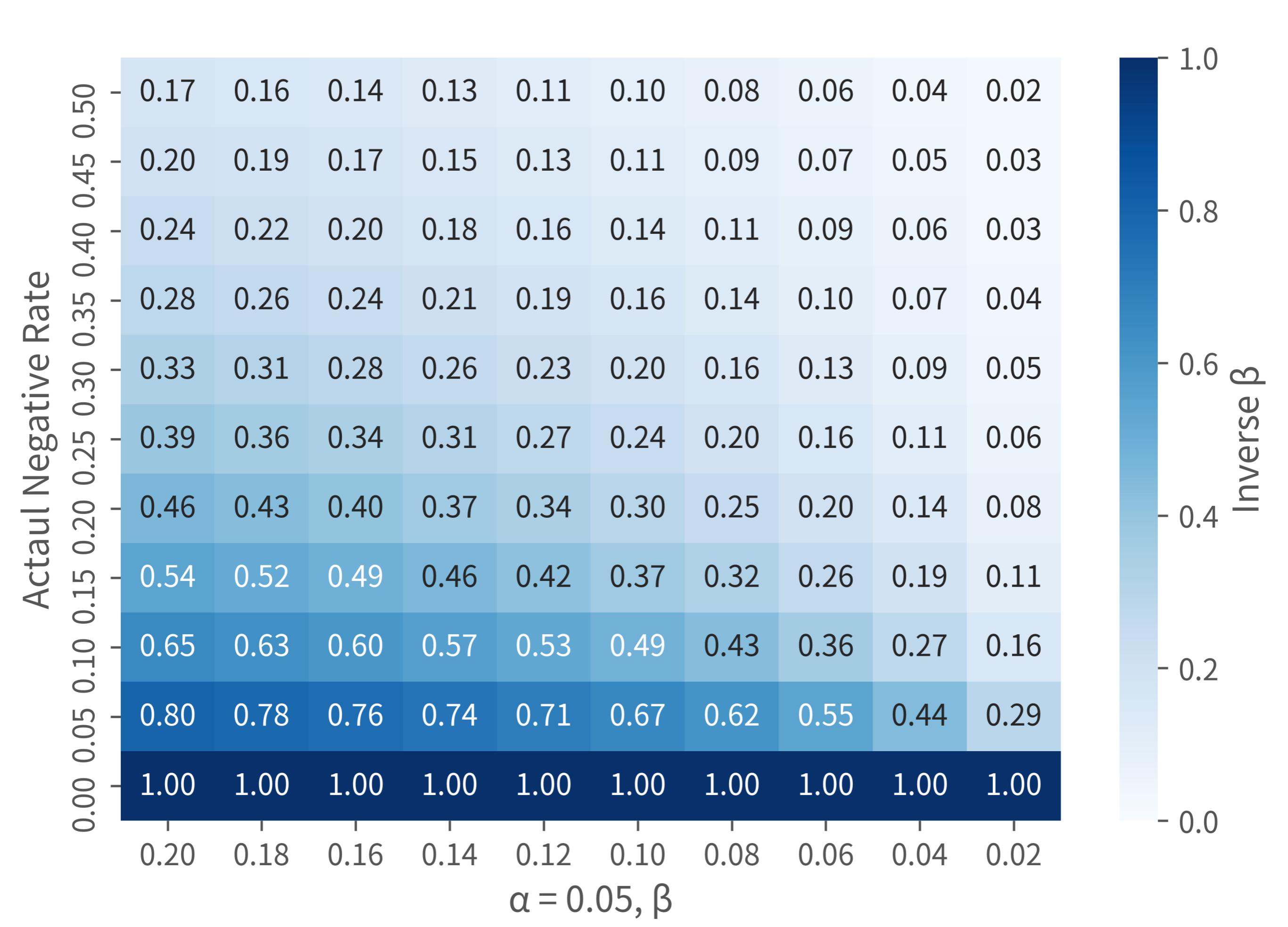
| | | predicted negative AC | predicted positive BD |
|-----------------------|---------------------|--------------------------|--------------------------|
| actual negative AB | true negative A | false positive B | |
| | false negative C | true positive D | |
| actual positive CD | | | |

$$\text{Inverse } \beta = P(CD|AC) = C/AC$$

.....







Rates in predefined acceptable confusion matrix

= = = predefined

| | | | |
|------------------|------|---|-----------------------|
| α | B/AB | significance level type I error rate | false positive rate |
| β | C/CD | type II error rate | false negative rate |
| inverse α | B/BD | | false discovery rate |
| inverse β | C/AC | | false omission rate |
| confidence level | A/AB | 1- α | specificity |
| power | D/CD | 1- β | sensitivity recall |

Rates in confusion matrix

| | = | = | = observed |
|----------------------|---------|-------------|------------------|
| false positive rate | B/AB | | α |
| false negative rate | C/CD | | β |
| false discovery rate | B/BD | | inverse α |
| false omission rate | C/AC | | inverse β |
| actual negative rate | AB/ABCD | | |
| sensitivity | D/CD | recall | power |
| specificity | A/AB | | confidence level |
| precision | D/BD | | inverse power |
| recall | D/CD | sensitivity | power |

- *appendices/theory_02_complete_a_test.ipynb*
- *appendices/theory_03_figures.ipynb*
- That's all. 