Hypothesis Testing With Python

True Difference or Noise?

169.61

169.88

Which is better?

Noise?

That's a question.

Mosky

- Python Charmer at Pinkoi.
- Has spoken at: PyCons in TW, MY, KR, JP, SG, HK, COSCUPs, and TEDx, etc.
- Countless hours on teaching Python.
- Own the Python packages: ZIPCodeTW, MoSQL, Clime, etc.
- http://mosky.tw/

Outline

1. Tests with datasets
2. How tests work
3. Common tests
4. Complete a test

The Packages

- `$ pip3 install jupyter numpy scipy sympy matplotlib ipython pandas seaborn statsmodels scikit-learn`
- Or:
- `> conda install jupyter numpy scipy sympy matplotlib ipython pandas seaborn statsmodels scikit-learn`
- Or:
- `$ curl -fLo Pipfile.lock` https://raw.githubusercontent.com/moskytw/data-science-with-python/master/Pipfile.lock
- `$ pipenv sync`

Tests with datasets

To buy, or not to buy

- Going to buy a bulb on an online store.
- If see *10/100* bad reviews? Hmm …
- If see *5/100* bad reviews? Good to buy.
- If see *1/100* bad reviews? Good to buy.

To buy, or not to buy (cont.)

- Going to buy a notebook computer on an online store.
- If see *10/100* bad reviews? Hmm ...
- If see *5/100* bad reviews? Hmm ...
- If see *1/100* bad reviews? Maybe good enough.

Build our "bad reviews" in statistics

- Build a statistical model.
  - "The means of two populations are equal."
- Put the data into the model, get a probability, *p-value*.
  - "How compatible the data and the model are."
  - If see *p-value = 0.10*?
  - If see *p-value = 0.05*?
  - If see *p-value = 0.01*?
  - Depends on your research question.
- It is just the "Hypothesis Testing".

Null, alternative, and p-value

- **Null hypothesis**: can build a model directly:
  - "The means of two populations are equal."
  - ≡ "The expected value of difference is zero."
  - E.g., GMV didn't change.
- **Alternative hypothesis**: can build a model by negating it.
  - "The means of two populations are different."
  - ≡ Not "The expected value of difference is zero."
  - E.g., GMV changed.
- **P-value**: how compatible data and **a null hypothesis** are.

Misunderstandings of p-values

- "Buy or not" is a decision based on the study context.
    - In other words, "accept" or "reject" a hypothesis.
    - Not "prove a hypothesis is true or false."
- Misunderstandings of p-values – Wikipedia

Suggested formatting

p-value & α	Wording	Summary
p-value < 0.001	Very significant	***
p-value < 0.01	Very significant	**
p-value < 0.05	Significant	*
p-value ≥ 0.05	Not significant	ns

- Many researchers also suggest to report **without** formatting.
  - Since the largely misunderstandings, e.g.,
    - *p-value* $< 0.05 \equiv$ the hypothesis is false **(wrong)**
- Scientists rise up against statistical significance – Natural
  - "We are not calling for a ban on P values. Nor are we saying they cannot be used as a decision criterion in certain specialized applications."
  - "We are calling for a stop to the use of P values in the conventional, dichotomous way — to decide whether a result refutes or supports a scientific hypothesis."

Go with the notebooks

- The notebooks are available on https://github.com/moskytw/hypothesis-testing-with-python.
- *01_tests_with_simulated_datasets.ipynb* [skip]
- *02_tests_with_actual_datasets.ipynb*

How tests work

Seeing is believing

- *p-value = 0.0027* (< 0.01)
  - 👏🏻👏🏻👏🏻
- *p-value = 0.0271* (0.01–0.05)
  - 👏🏻❓👏🏻❓❓❓
- *p-value = 0.2718* (≥ 0.05)
  - ❓❓❓❓❓❓
- *03_how_tests_work.ipynb*

Common tests

The cheat sheet

- If testing homogeneity:
  - If total sample size < 1000, or
    more than 20% of cells have expected frequencies < 5, Fisher's exact test.
  - Else, **chi-squared test**, or **two-proportion z-test** (≡ 2×2 chi-squared test).
- If testing equality:
  - If median is better, don't want to trim outliers,
    variable is ordinal, or any group size ≤ 20:
    - If groups are paired, Wilcoxon signed-rank test.
    - If groups are independent, **Mann–Whitney U test**.
  - Else:
    - If groups are paired, Paired Student's t-test.
    - If groups are independent, **Welch's t-test**, not Student's.

- More cheat sheets:
  - Common statistical tests are linear models – Lindeloev
  - Selecting Commonly Used Statistical Tests – Bates College
  - Which statistical test should I use? – University of Sheffield
  - Choosing a statistical test – HBS
- References:
  - Fisher's exact test of independence – HBS
  - Statistical notes for clinical researchers – Restor Dent Endod
  - Chi-squared test of proportions is identical to z-test squared – Rinterested
  - Nonparametric Test and Parametric Test – Minitab
  - Advantages and limitations – Welch's t-test – Wikipedia
  - Dependent t-test for paired samples – Student's t-test – Wikipedia

Complete a test

The elements of a complete test

1. The null hypothesis, data, p-value, α.
2. The raw effect size, β, sample size.
3. The false negative rate, inverse α, inverse β.

- Will introduce them by the confusion matrix.

Confusion matrix, where $A = 00_2 = C[0, 0]$

	predicted negative AC	predicted positive BD
actual negative AB	true negative A	false positive B
actual positive CD	false negative C	true positive D

False positive rate = P(BD|AB) = B/AB = 4/(96+4) = 4/100

	predicted negative AC	predicted positive BD
actual negative AB	96 A	4 B
actual positive CD	9 C	41 D

How similar! Let's try to rename:

null hypothesis is ture	actual negative
alternative hypothesis is true	actual positive
accept null hypothesis	predict negative
accept alternative hypothesis	predict positive

α = P(accept alt|null) = P(predicted positive|actual negative)

	predicted negative AC	predicted positive BD
actual negative AB	true negative A	false positive B
actual positive CD	false negative C	true positive D

Predefined acceptable confusion matrix

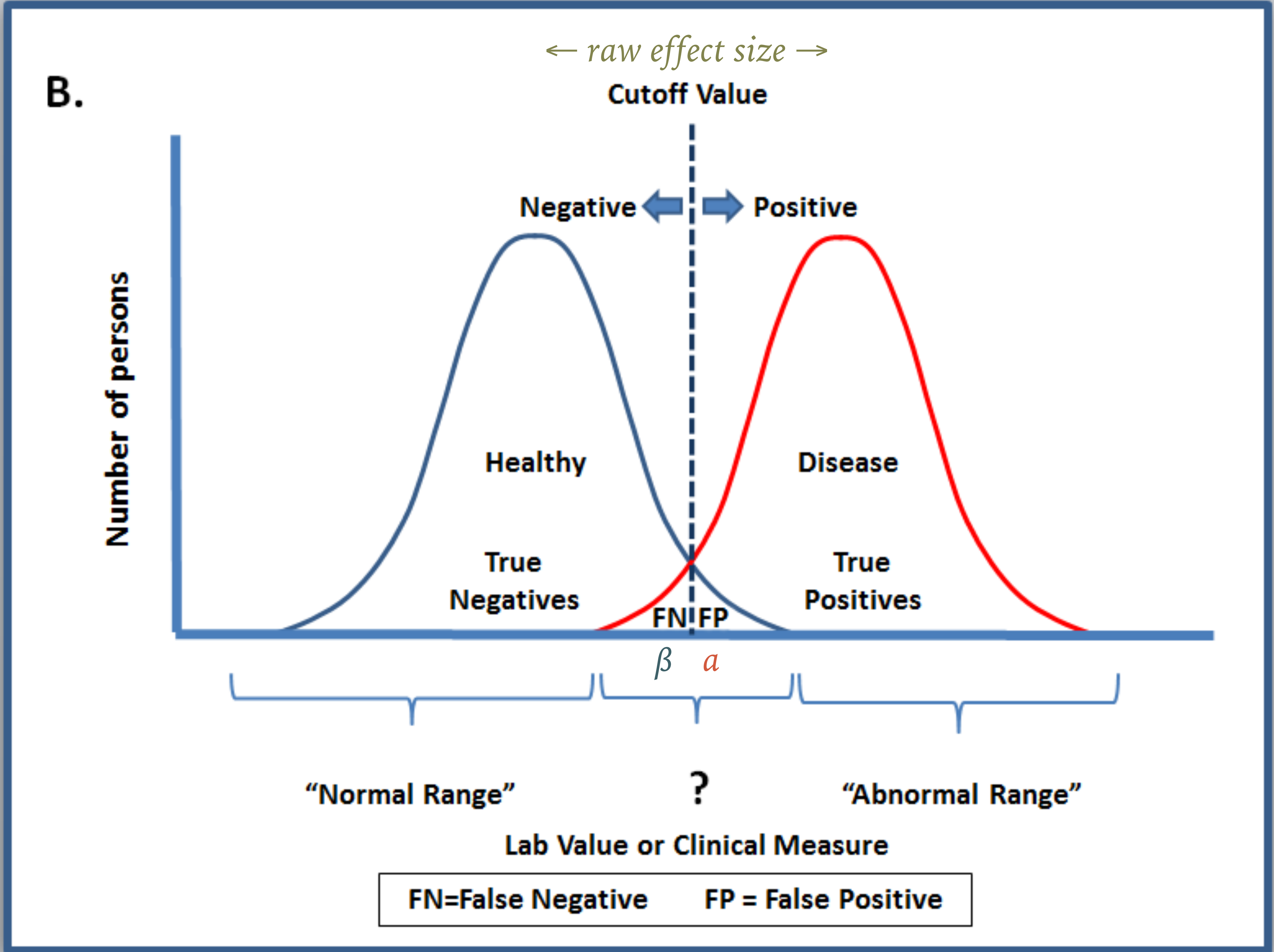
	predicted negative AC	predicted positive BD
actual negative AB	true negative A	false positive B
actual positive CD	false negative C	true positive D

False positive, p-value, and α

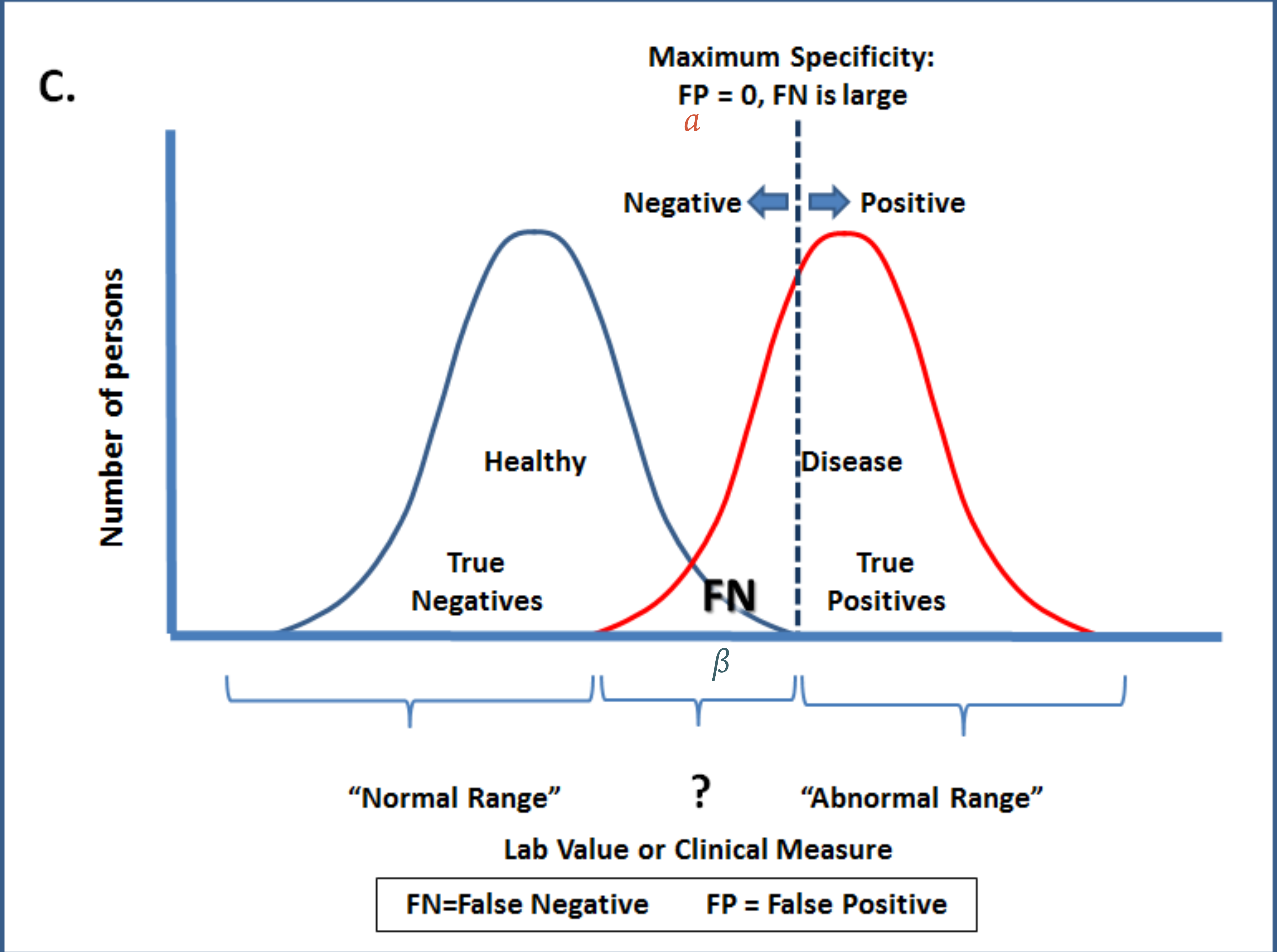
false positive rate	Calculated with the actual answer.
p-value	Calculated false positive rate by a null hypothesis.
α	Predefined acceptable false positive rate.

Raw effect size, and β

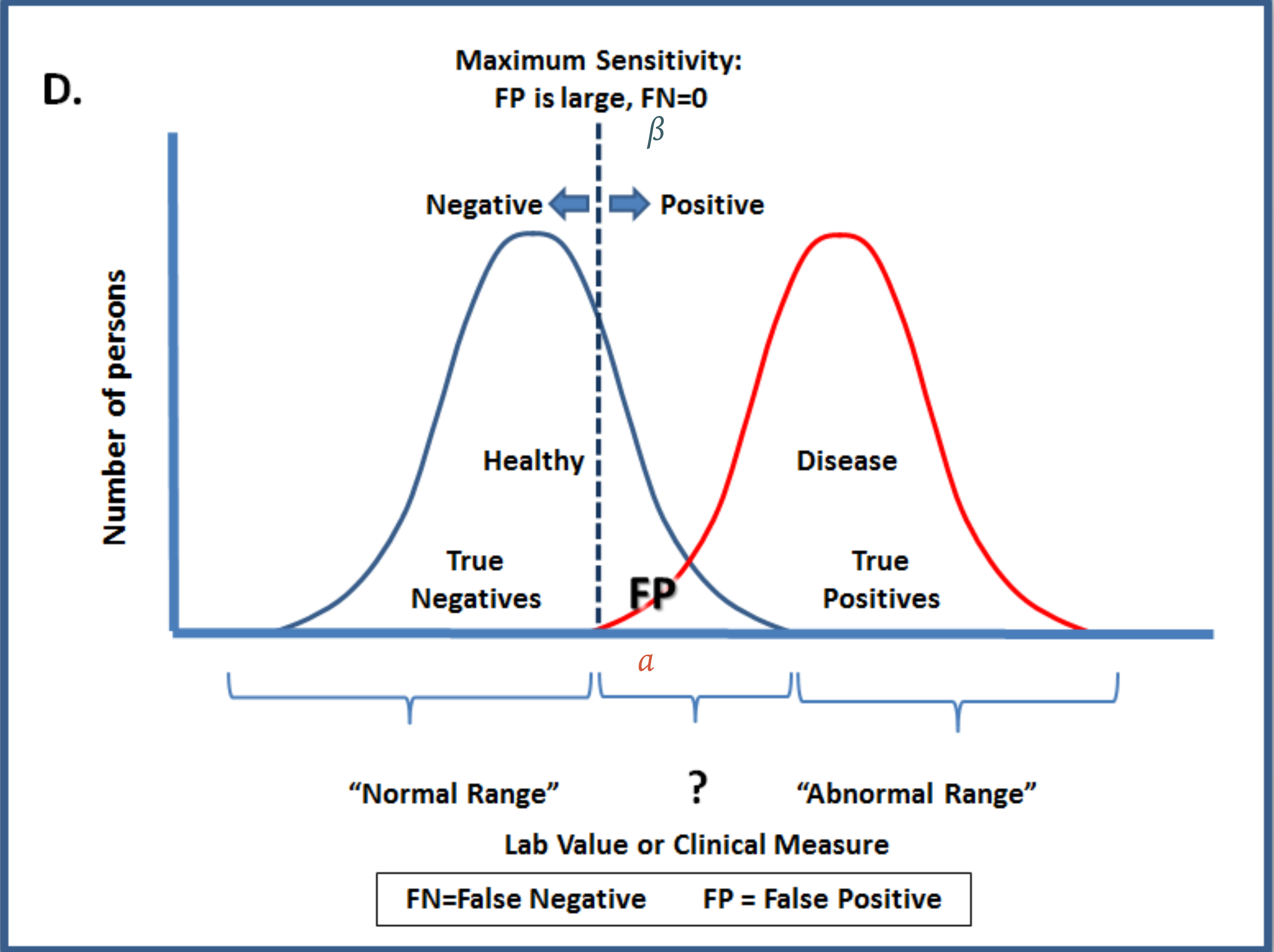
- DSM5: The case for double standards – James Coplan, M.D.
  - The figures explain *raw effect size*, *a*, and *β* and perfectly, but due to the copyright, only put the link here.
  - “FP”: *a*
  - “The distance between the means”: *raw effect size*
  - “FN”: *β*



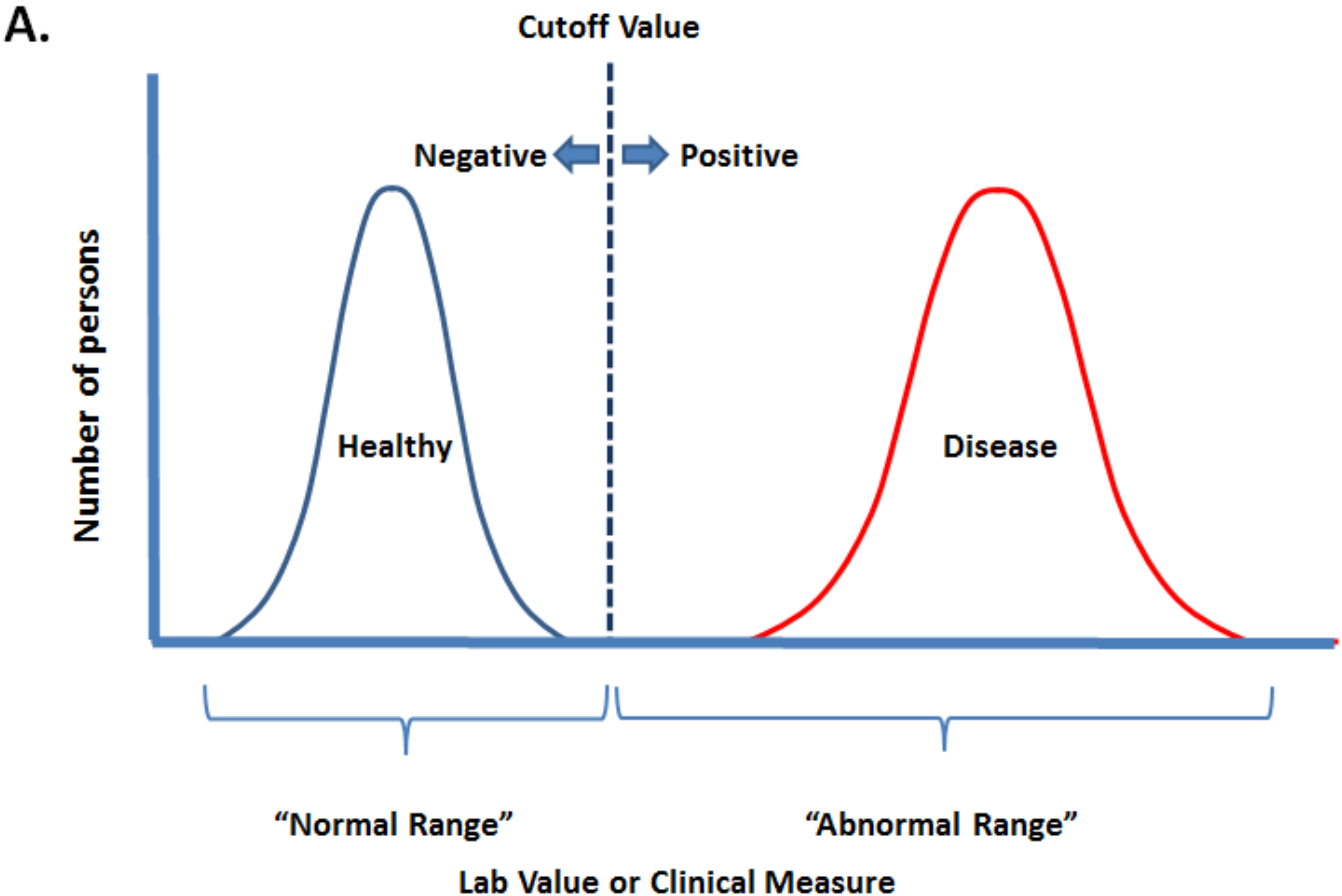
← raw effect size →
B.
Cutoff Value
Negative
Positive
Number of persons
Healthy
Disease
True
Negatives
True
Positives
FN
FP
β
a
"Normal Range"
?
"Abnormal Range"
Lab Value or Clinical Measure
FN=False Negative
FP = False Positive



C.
Maximum Specificity:
FP = 0, FN is large
a
Negative
Positive
Number of persons
Healthy
Disease
True
Negatives
FN
True
Positives
β
"Normal Range"
?
"Abnormal Range"
Lab Value or Clinical Measure
FN=False Negative
FP = False Positive


D.
Maximum Sensitivity:
FP is large, FN=0
β
Negative
Positive
Number of persons
Healthy
Disease
True
Negatives
FP
True
Positives
a
"Normal Range"
?
"Abnormal Range"
Lab Value or Clinical Measure
FN=False Negative
FP = False Positive

sample size ↑


A.
Cutoff Value
Negative
Positive
Number of persons
Healthy
Disease
“Normal Range”
“Abnormal Range”
Lab Value or Clinical Measure

β = P(AC|CD) = C/CD

	predicted negative AC	predicted positive BD
actual negative AB	true negative A	false positive B
actual positive CD	false negative C	true positive D

- Given α, β, raw effect size, get the sample size.
- Given α, raw effect size, sample size, get the β.
- Increase sample size to decrease α, β, or raw effect size.

Given α, raw effect size, sample size, get the β

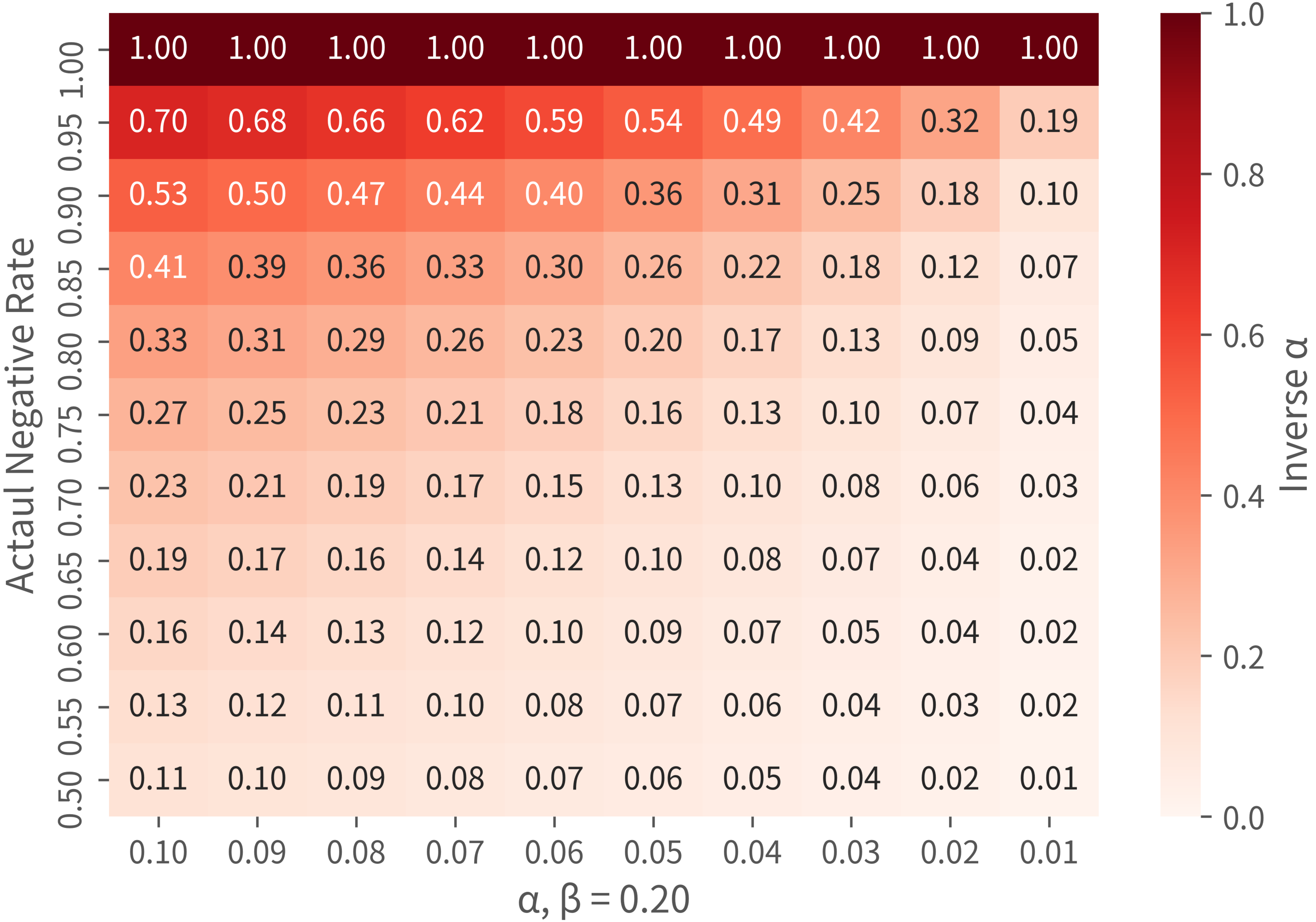
- "The conversion rates are the same."
- $a = 0.05$
- *raw effect size* = 4% - 3% = 1%
- With two-proportion z-test, given:
  - *sample size* = 1,000, get $\beta = 0.86$
  - *sample size* = 10,000, get $\beta = 0.22$
  - *sample size* = 17,550, get $\beta = 0.05$

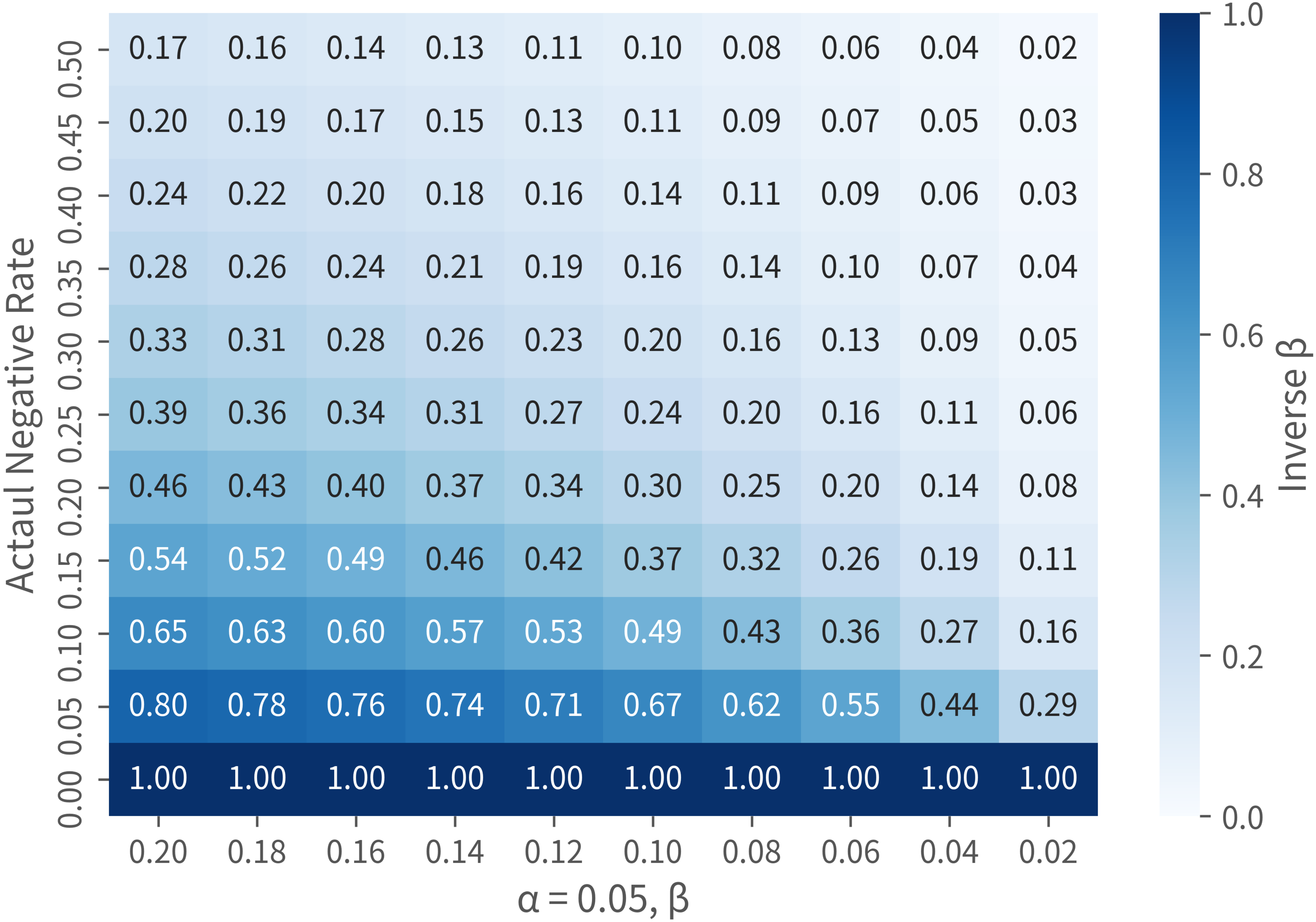
Inverse α = P(AB|BD) = B/BD

	predicted negative AC	predicted positive BD
actual negative AB	true negative A	false positive B
actual positive CD	false negative C	true positive D

Inverse β = P(CD|AC) = C/AC

	predicted negative AC	predicted positive BD
actual negative AB	true negative A	false positive B
actual positive CD	false negative C	true positive D


Actaul Negative Rate
1.00
0.95
0.90
0.85
0.80
0.75
0.70
0.65
0.60
0.55
0.50
1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00
0.70 0.68 0.66 0.62 0.59 0.54 0.49 0.42 0.32 0.19
0.53 0.50 0.47 0.44 0.40 0.36 0.31 0.25 0.18 0.10
0.41 0.39 0.36 0.33 0.30 0.26 0.22 0.18 0.12 0.07
0.33 0.31 0.29 0.26 0.23 0.20 0.17 0.13 0.09 0.05
0.27 0.25 0.23 0.21 0.18 0.16 0.13 0.10 0.07 0.04
0.23 0.21 0.19 0.17 0.15 0.13 0.10 0.08 0.06 0.03
0.19 0.17 0.16 0.14 0.12 0.10 0.08 0.07 0.04 0.02
0.16 0.14 0.13 0.12 0.10 0.09 0.07 0.05 0.04 0.02
0.13 0.12 0.11 0.10 0.08 0.07 0.06 0.04 0.03 0.02
0.11 0.10 0.09 0.08 0.07 0.06 0.05 0.04 0.02 0.01
0.10
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
α, β = 0.20
Inverse α
1.0
0.8
0.6
0.4
0.2
0.0


0.17 0.16 0.14 0.13 0.11 0.10 0.08 0.06 0.04 0.02
0.20 0.19 0.17 0.15 0.13 0.11 0.09 0.07 0.05 0.03
0.24 0.22 0.20 0.18 0.16 0.14 0.11 0.09 0.06 0.03
0.28 0.26 0.24 0.21 0.19 0.16 0.14 0.10 0.07 0.04
0.33 0.31 0.28 0.26 0.23 0.20 0.16 0.13 0.09 0.05
0.39 0.36 0.34 0.31 0.27 0.24 0.20 0.16 0.11 0.06
0.46 0.43 0.40 0.37 0.34 0.30 0.25 0.20 0.14 0.08
0.54 0.52 0.49 0.46 0.42 0.37 0.32 0.26 0.19 0.11
0.65 0.63 0.60 0.57 0.53 0.49 0.43 0.36 0.27 0.16
0.80 0.78 0.76 0.74 0.71 0.67 0.62 0.55 0.44 0.29
1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00
Actaul Negative Rate
0.50 0.45 0.40 0.35 0.30 0.25 0.20 0.15 0.10 0.05 0.00
0.20 0.18 0.16 0.14 0.12 0.10 0.08 0.06 0.04 0.02
α = 0.05, β
Inverse β
1.0 0.8 0.6 0.4 0.2 0.0

Rates in predefined acceptable confusion matrix

	=	=	= predefined
α	B/AB	significance level type I error rate	false positive rate
β	C/CD	type II error rate	false negative rate
inverse α	B/BD		false discovery rate
inverse β	C/AC		false omission rate
confidence level	A/AB	1-α	specificity
power	D/CD	1-β	sensitivity recall

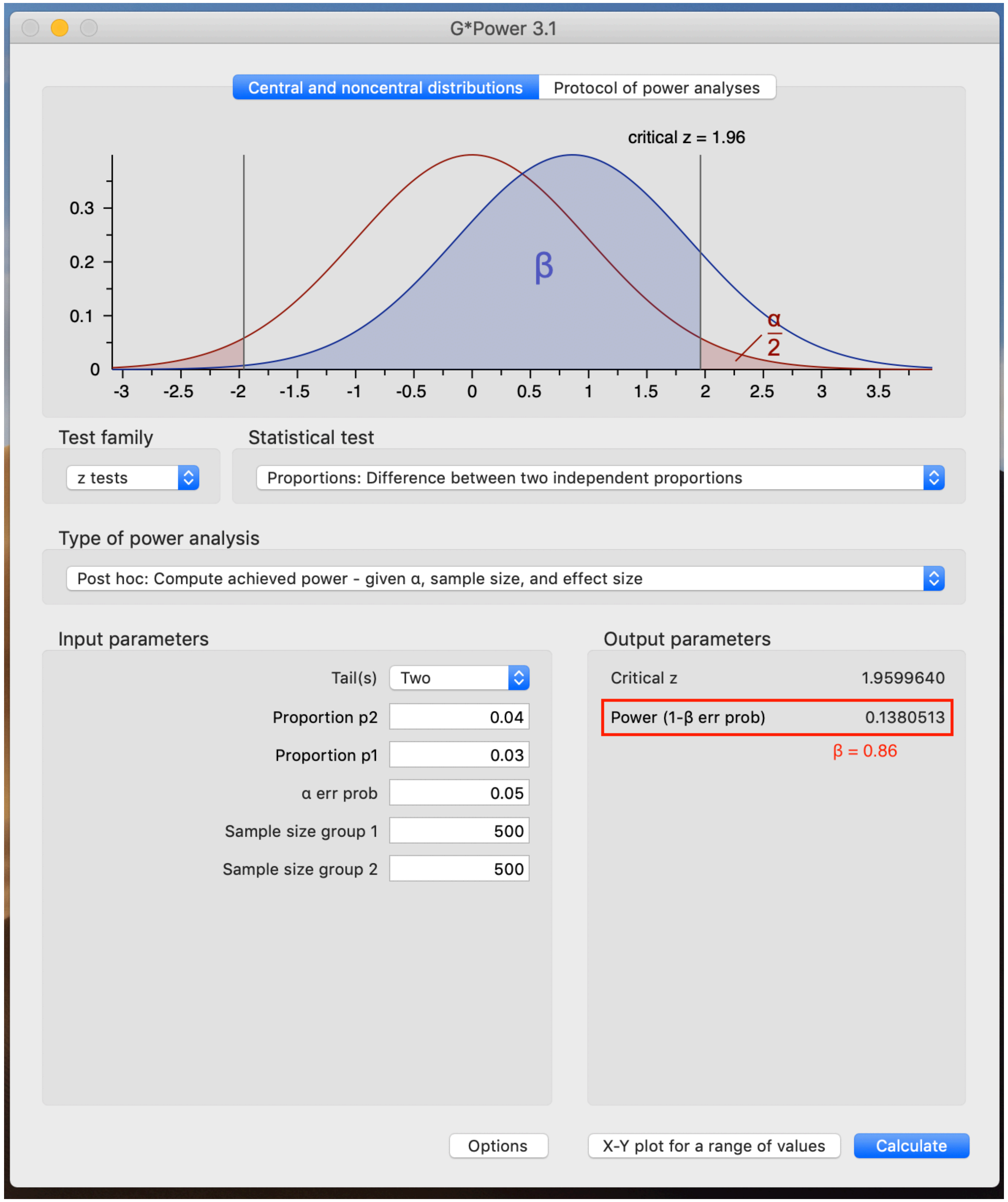
Rates in confusion matrix

	=	=	= observed
false positive rate	B/AB		α
false negative rate	C/CD		β
false discovery rate	B/BD		inverse α
false omission rate	C/AC		inverse β
actual negative rate	AB/ABCD		
sensitivity	D/CD	recall	power
specificity	A/AB		confidence level
precision	D/BD		inverse power
recall	D/CD	sensitivity	power

Most formal steps

1. The hypothesis → what test.
2. The *actual negative rate* → how much α, β are required.
3. The α, β, *raw effect size* → how large *sample size* is required.
4. Still collect a sample as large as possible.
5. Understand and preprocess the sample.
   - Plotting, missing data, outliers, transform, etc.
6. Test.
7. Report fully.
   - The *mean*, *confidence interval*, *p-value*, study context, etc.

Calculate by yourself



- G*Power is awesome.
  - http://www.gpower.hhu.de/

Keep learning

1. Seeing Theory
2. Statistics – SciPy Tutorial
3. StatsModels
4. Biological Statistics
5. Research method
    - Study design, experimental design, survey design etc.

Recap

1. *p-value*:
   - How compatible data and a null hypothesis are.
   - The false positive rate by a null hypothesis.
2. β does matter.
3. *actual negative rate* does matter.
4. The study context does matter.
5. Visualization and simulation do help.
   - More: 04, 05, and a1 notebooks in handouts.
6. Let's accept or reject a hypothesis efficiently!