



Hypothesis Testing With Python

True Difference or Noise?

169.61

169.88

Which is better?

Noise?

That's a question.

Mosky



- Python Charmer at Pinkoi.
- Has spoken at: PyCons in TW, MY, KR, JP, SG, HK, COSCUPs, and TEDx, etc.
- Countless hours on teaching Python.
- Own the Python packages: ZIPCodeTW, MoSQL, Clime, etc.
- <http://mosky.tw/>

Outline

- Tests with simulated datasets
- Tests with actual datasets
- How tests work
- Common tests
- Complete a test

The Packages

- \$ pip3 install jupyter numpy scipy sympy matplotlib ipython pandas seaborn statsmodels scikit-learn
- Or
- > conda install jupyter numpy scipy sympy matplotlib ipython pandas seaborn statsmodels scikit-learn
- Visit the Pipfile for the exact versions.

Tests with datasets

Go with the notebooks

- *01_tests_with_simulated_datasets.ipynb*
- *02_tests_with_actual_datasets.ipynb*
- The notebooks are available on <https://github.com/moskytw/hypothesis-testing-with-python>.

P-value & α

.....

P-value & α

Wording

p-value < 0.01

Very significant

p-value < 0.05

Significant

p-value ≥ 0.05

Not significant

False positive rate, p-value, and α

false positive rate

Calculate with the ground truth.

p-value

Calculate with a hypothesis.

α

Predefine an acceptable false positive rate.

How tests work

Seeing is believing

- $p\text{-value} = 0.0027 (< 0.01)$
 - 
- $p\text{-value} = 0.0271 (0.01\text{--}0.05)$
 -  ?  ? ? ? ?
- $p\text{-value} = 0.2718 (\geq 0.05)$
 - ? ? ? ? ? ?
- *03_how_tests_work.ipynb*

Fair coin testing

- “The coin is fair.”
- Case 1: Toss the coin 100 times, and come up 53 heads.
 - “Hmmm ... somehow fair.”
- Case 2: Toss the coin 100 times, and come up 87 heads.
 - “Not fair! So extreme!”

Hypothesis testing

- “The means of two populations are equal.”
- Case 1: $p\text{-value} \geq 0.05$
 - “Hmmm ... somehow equal.”
- Case 2: $p\text{-value} < 0.05$
 - “Not equal! So extreme!”

Hypothesis testing in a “null” taste

- $\langle \text{null hypothesis} \rangle$
- Case 1: $p\text{-value} \geq \alpha$
 - Can't reject $\langle \text{null hypothesis} \rangle$.
- Case 2: $p\text{-value} < \alpha$
 - Reject $\langle \text{null hypothesis} \rangle$.

Hypothesis testing in an “alternative” taste

- *<alternative hypothesis>*, \equiv not *<null hypothesis>* usually.
- Case 1: $p\text{-value} \geq \alpha$
 - Can't accept *<alternative hypothesis>*.
- Case 2: $p\text{-value} < \alpha$
 - Accept *<alternative hypothesis>*.

Hypothesis testing in “-+” taste

- “The case is negative.”
- Case 1: $p\text{-value} \geq \alpha$
 - “Hmmm ... somehow negative.”
- Case 2: $p\text{-value} < \alpha$
 - “Positive! So extreme!”

Common tests

The cheat sheet

- If testing independence:
 - If total size < 1000, or more than 20% of cells have expected frequencies < 5, **Fisher's exact test**.
 - Else, **Chi-squared test**.
- If testing difference:
 - If median is better, don't want to trim outliers, variable is ordinal, or any group size < 20:
 - If groups are paired, **Wilcoxon signed-rank test**.
 - If groups are independent, **Mann–Whitney U test**.
 - Else:
 - If groups are paired, **Paired Student's t-test**.
 - If groups are independent, **Welch's t-test**, not Student's.

More cheat sheets & references

- More cheat sheets:
 - Selecting Commonly Used Statistical Tests – Bates College
 - Which statistical test should I use? – University of Sheffield
 - Choosing a statistical test – HBS
- References:
 - Fisher's exact test of independence – HBS
 - Statistical notes for clinical researchers – Restor Dent Endod
 - Nonparametric Test and Parametric Test – Minitab
 - Advantages and limitations – Welch's t-test – Wikipedia
 - Dependent t-test for paired samples – Student's t-test – Wikipedia

The tests in Python

- *04_common_tests.ipynb*

Complete a test

Is p-value enough?

- sample size?
- alpha?
- beta?
- effect size?
- ? ? ? ? ?

Confusion matrix, where $A = 00_2 = C[0, 0]$

		predicted - AC	predicted + BD
actual - AB	true -	A	false + B
	false -	C	true + D
actual + CD			

False positive rate = B / AB = observed α

		predicted - AC	predicted + BD
actual - AB	true -	A	false + B
	false -	C	true + D
actual + CD			

False negative rate = C / CD = observed β

		predicted - AC	predicted + BD
actual - AB	true -	false + B	
	A		
actual + CD	false - C		true + D

The diagram illustrates a 2x2 matrix for a diagnostic test, showing the relationship between predicted and actual outcomes. The columns represent the predicted outcome (predicted - or predicted +) and the rows represent the actual outcome (actual - or actual +). The cells are labeled A, B, C, and D, representing true negatives, false positives, false negatives, and true positives respectively. A large rounded rectangle highlights cell C (false negatives), and a smaller rounded rectangle highlights the entire row for 'actual +'.

Sample size ↑; α , β , effect size ↓

- Increase *sample size* to decrease α , β , *effect size*.
 - The *effect size* is the distance between groups.
$$\text{Effect Size} = \frac{\mu_1 - \mu_2}{\sigma}$$
 - DSM5: The case for double standards – James Coplan, M.D.
 - The figures explain α , β perfectly,
but due to the copyright, only put the link here.
 - When α , β , *effect size* are defined, get the *sample size*.
 - When α , β , *sample size* are defined, get the *effect size*.





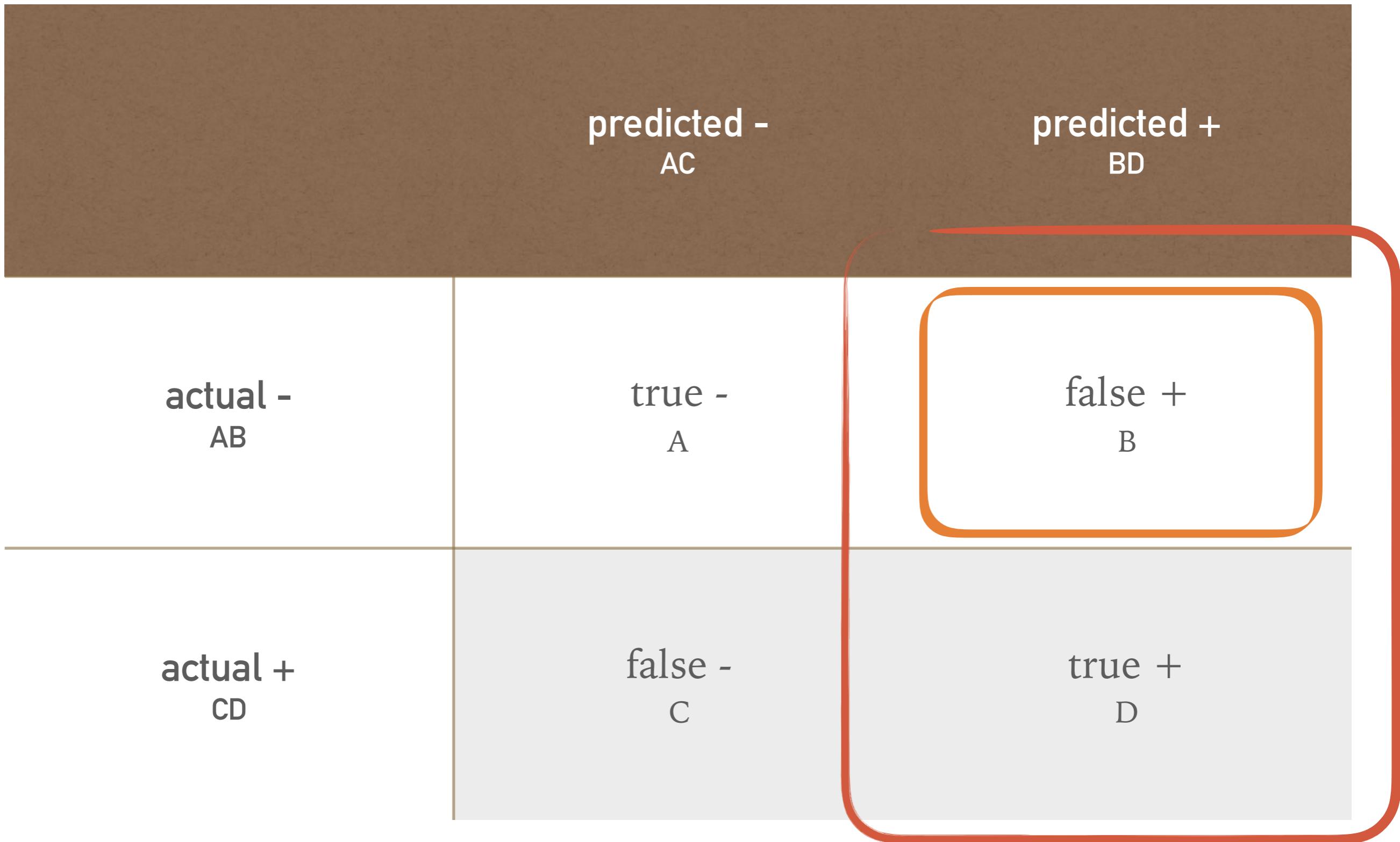
- If the n is large enough, a reasonable approximation to a binomial distribution is given by the normal distribution:
 - $X \sim B(n, p)$
 - When $n \rightarrow \infty$:
 - $\mu = np$
 - $\sigma = \sqrt{np(1-p)}$
 - $X \sim N(\mu, \sigma)$
- See also: <https://www.ncbi.nlm.nih.gov/pmc/articles/PMC3775042/>.

False discovery rate

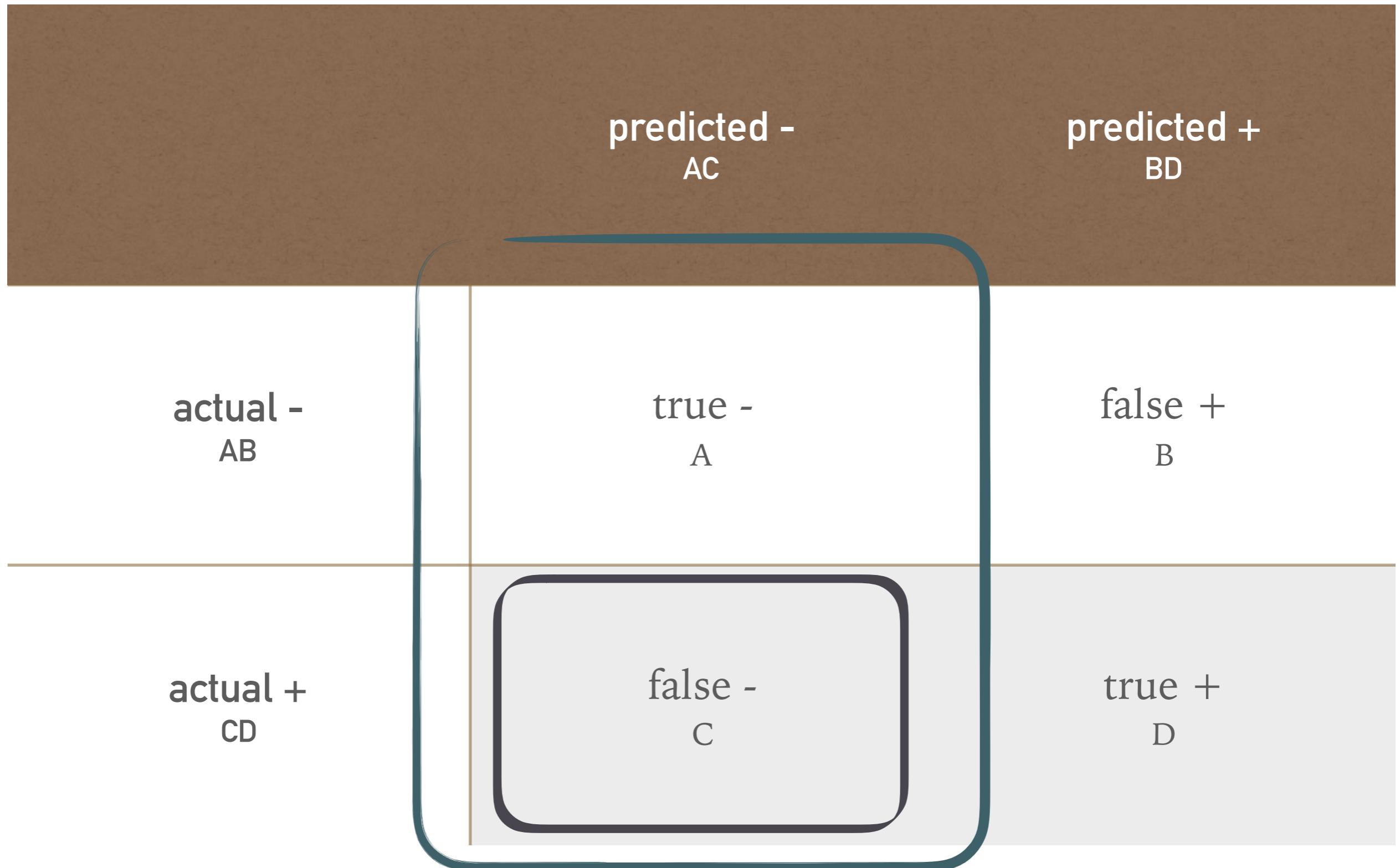
- Suppose:
 - A drug test has observed $\alpha = 1\%$ and observed $\beta = 1\%$
 - 99.5% of people are *not* drug users.
- What is the probability that a person with a positive test is not a drug user?

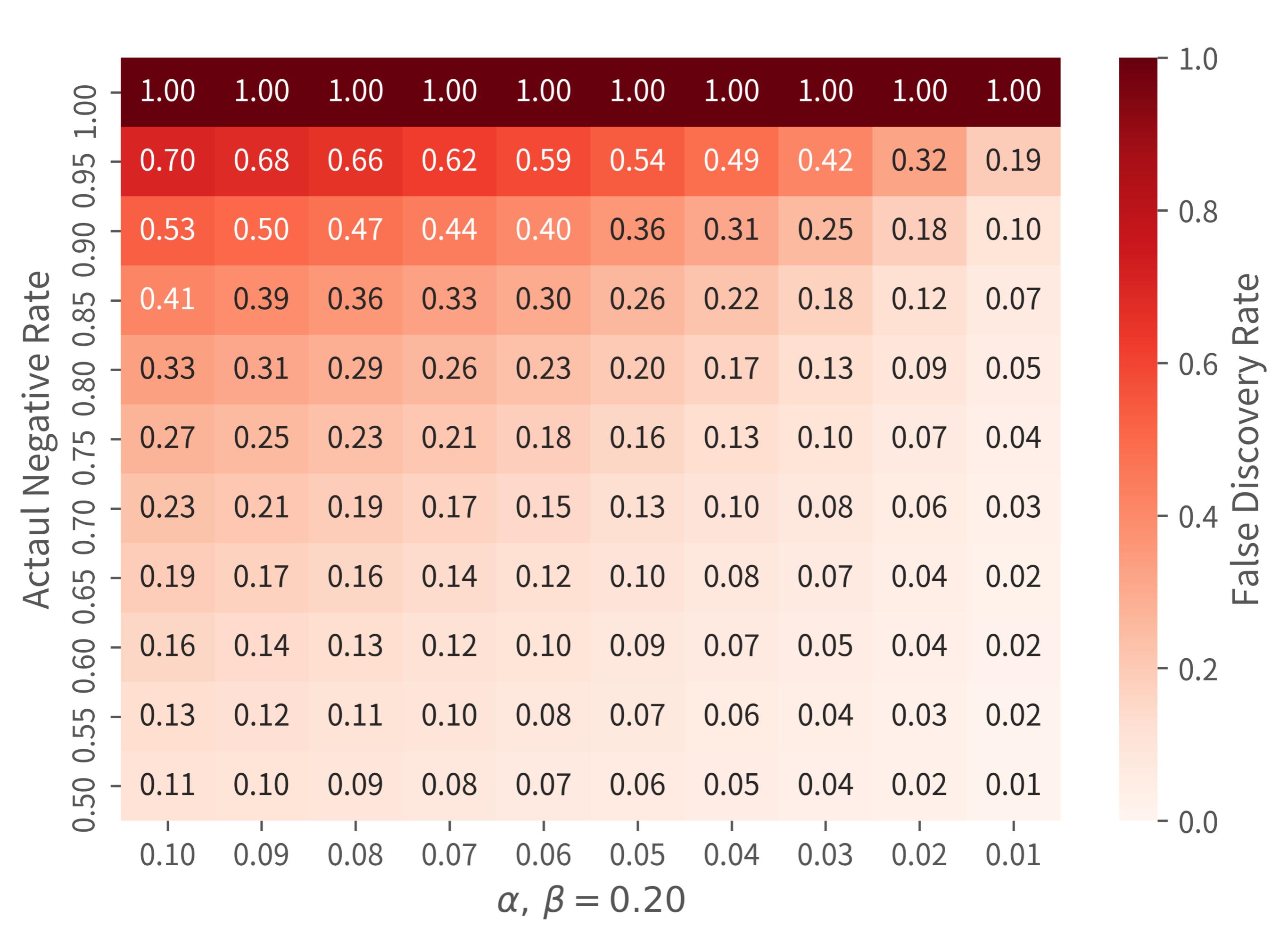
$$\begin{aligned}P(\text{non-user} \mid +) &= \frac{P(+ \mid \text{non-user})P(\text{non-user})}{P(+)} \\&= \frac{P(+ \mid \text{non-user})P(\text{non-user})}{P(+ \mid \text{non-user})P(\text{non-user}) + P(+ \mid \text{user})P(\text{user})} \\&= \frac{0.01 \times 0.995}{0.01 \times 0.995 + 0.99 \times 0.005} \\&\approx 66.8\%\end{aligned}$$

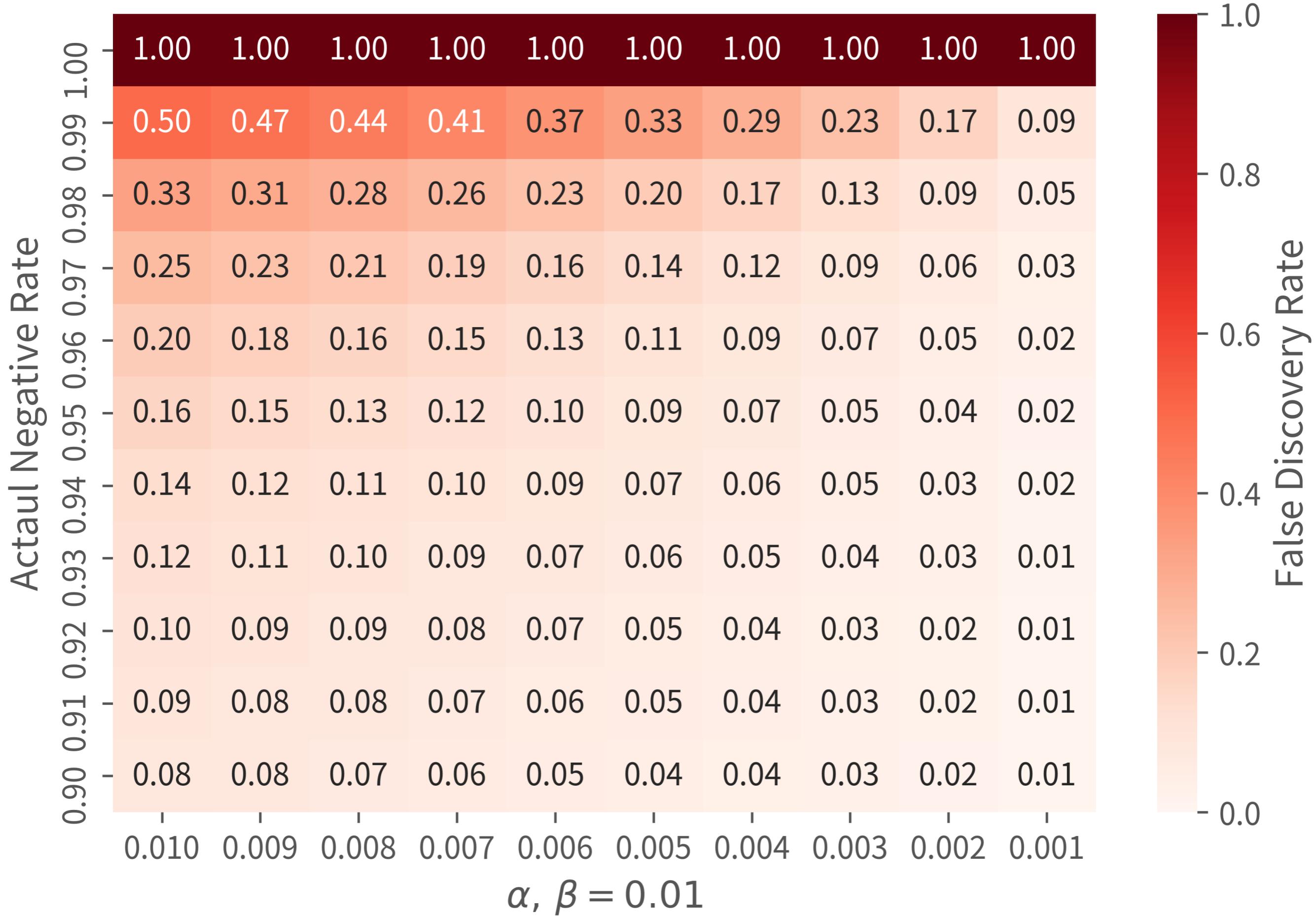
False discovery rate = B / BD

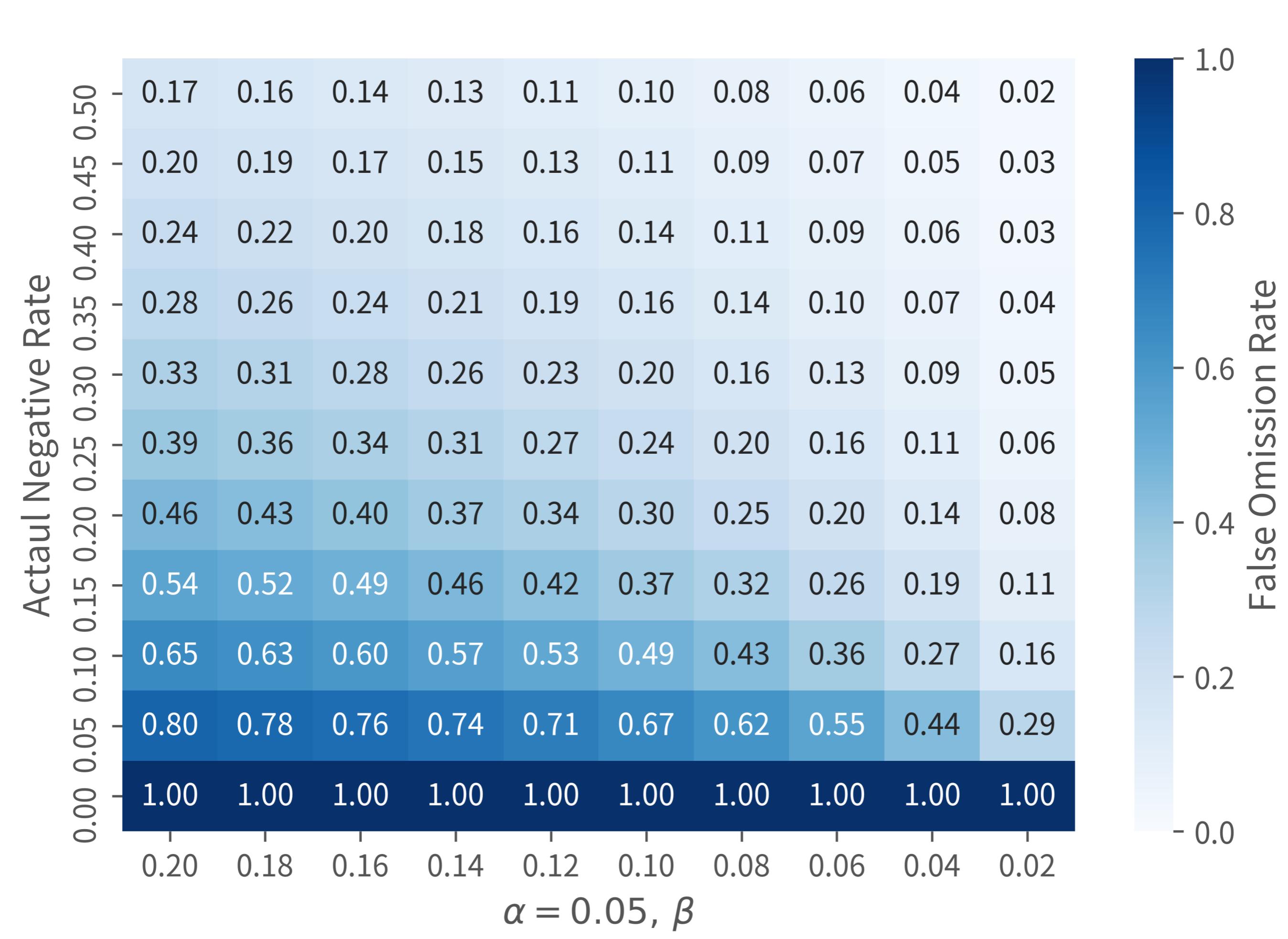


False omission rate = C / AC









Common “rates” in confusion matrix

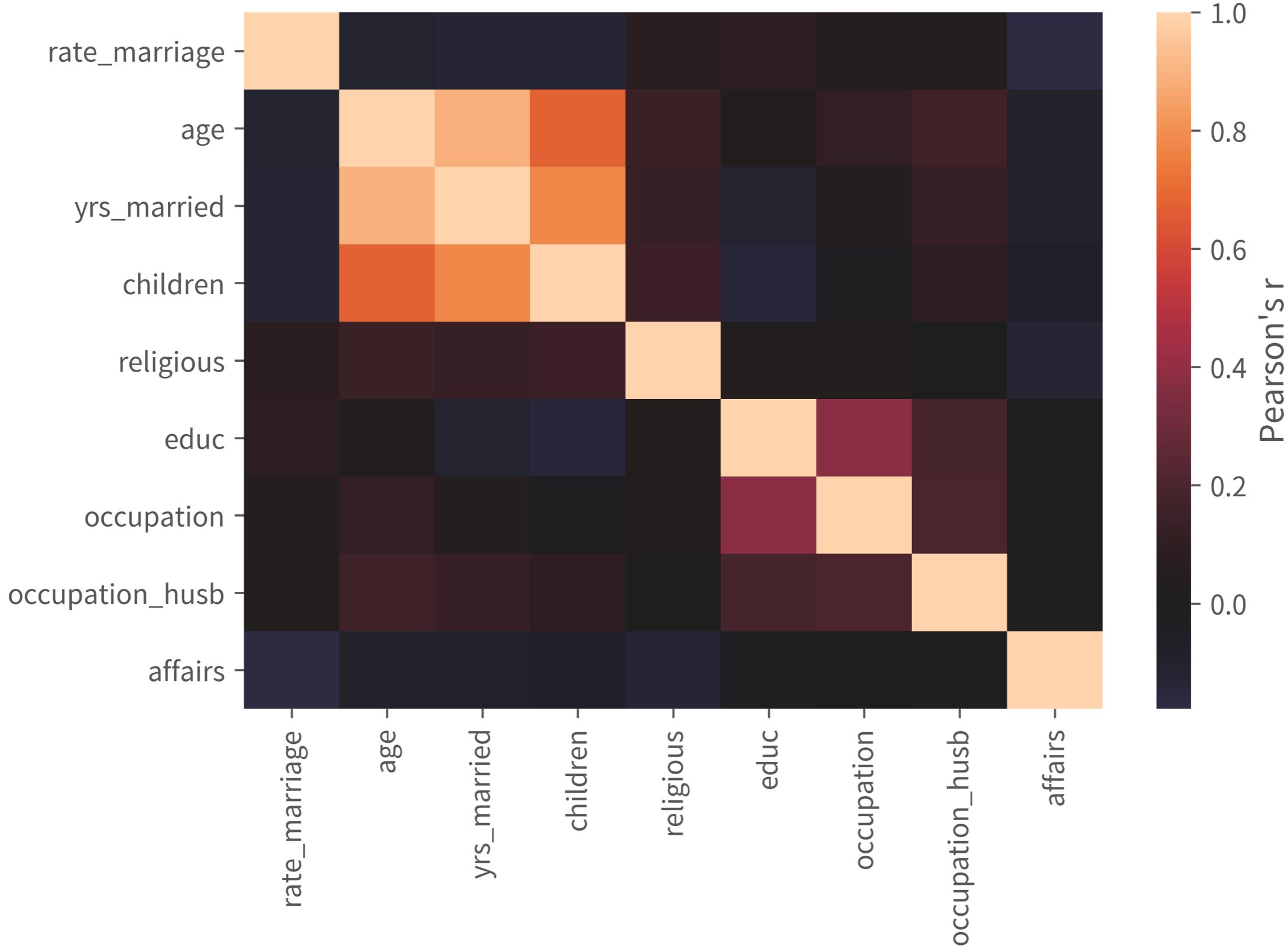
- false positive rate = B / AB = observed α
- false negative rate = C / CD = observed β
- actual negative rate = $AB / ABCD$
- false discovery rate = B / BD = observed inverse α
- false omission rate = C / AC = observed inverse β
- sensitivity = D / CD = observed power = recall
- specificity = A / AB = observed confidence level
- precision = D / BD = observed inverse power
- recall = D / CD = observed power = sensitivity

Most formal steps

- State the hypothesis → what *test*.
- Estimate the *actual negative rate*.
- The *actual negative rate* → how much α, β are required.
- The $\alpha, \beta, \text{effect size}$ → how large *sample size* is required.
- Still collect a sample as large as possible.
- Understand the sample.
 - Missing data, outliers, Q–Q plot, transform, etc.
- Test and report fully.
- *05_complete_a_test.ipynb*

Other statistical tools

Correlation analysis



Regression analysis

```
In [7]: fair_df = sm.datasets.fair.load_pandas().data  
ols_res = smf.ols('children ~ yrs_married', fair_df).fit()  
ols_res.summary()
```

Out[7]: OLS Regression Results

Dep. Variable:	children	R-squared:	0.597			
Model:	OLS	Adj. R-squared:	0.597			
Method:	Least Squares	F-statistic:	9437.			
Date:	Fri, 06 Jul 2018	Prob (F-statistic):	0.00			
Time:	00:40:17	Log-Likelihood:	-8430.3			
No. Observations:	6366	AIC:	1.686e+04			
Df Residuals:	6364	BIC:	1.688e+04			
Df Model:	1					
Covariance Type:	nonrobust					
	coef	std err	t	P> t	[0.025	0.975]
Intercept	0.0259	0.018	1.429	0.153	-0.010	0.062
yrs_married	0.1522	0.002	97.142	0.000	0.149	0.155
Omnibus:	449.258	Durbin-Watson:	1.972			
Prob(Omnibus):	0.000	Jarque-Bera (JB):	709.624			
Skew:	0.559	Prob(JB):	8.07e-155			
Kurtosis:	4.193	Cond. No.	18.5			

Keep learning

- Statistics
 - [Seeing Theory](#)
 - [Statistics – SciPy Tutorial](#)
 - [StatsModels](#)
 - [Biological Statistics](#)
 - [Research Methods](#)
- Machine Learning
 - [Scikit-learn Tutorials](#)
 - [Standford CS229](#)
 - [Hsuan-Tien Lin](#)

Recap

- *p-value*: the “tail” probability given “actual negative”.
- *confidence interval*: the values the middle probability maps to.
- *actual negative rate* does matter.
- $\alpha, \beta, \text{effect size} \downarrow; \text{sample size} \uparrow$.
- Visualization and simulation do help.
- Bonus: *a1_figures.ipynb* .
- Let's identify noise efficiently!