



# Hypothesis Testing With Python

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*True Difference or Noise?*

**169.61**

**169.88**

**Which is better?**

# Noise?

**That's a question.**

# Mosky

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- Python Charmer at Pinkoi.
- Has spoken at: PyCons in TW, MY, KR, JP, SG, HK, COSCUPs, and TEDx, etc.
- Countless hours on teaching Python.
- Own the Python packages: ZIPCodeTW, MoSQL, Clime, etc.
- <http://mosky.tw/>

# Outline

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- Tests with simulated datasets
  - What methods work not well?
  - What methods work well?
  - Simple mean, etc.
- Tests with actual datasets
- How tests work
- Common tests
- Complete a test

# The Packages

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- \$ pip3 install jupyter numpy scipy sympy matplotlib ipython pandas seaborn statsmodels scikit-learn
- Or
- > conda install jupyter numpy scipy sympy matplotlib ipython pandas seaborn statsmodels scikit-learn
- Visit the Pipfile for the exact versions.

# Tests with datasets

# Go with the notebooks

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- *01\_tests\_with\_simulated\_datasets.ipynb*
- The notebooks are available on <https://github.com/moskytw/hypothesis-testing-with-python>.

# False positive rate, p-value, and $\alpha$

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false positive rate

Calculated with the actual answer.

p-value

Calculated false positive rate by  
a reasonable hypothesis.

$\alpha$

Predefined acceptable false positive rate.

# Fair coin testing

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- “The coin is fair.”
- Case 1: Toss the coin 100 times, and come up 53 heads.
  - “Hmmm ... somehow fair.”
- Case 2: Toss the coin 100 times, and come up 87 heads.
  - “Not fair! So extreme!”

# Hypothesis testing

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- “The means of two populations are equal.”
- Case 1:  $p\text{-value} \geq 0.05$ 
  - “Hmmm ... somehow equal.”
- Case 2:  $p\text{-value} < 0.05$ 
  - “Not equal! So extreme!”

# Hypothesis testing in a “null” taste

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- $\langle \text{null hypothesis} \rangle$
- Case 1:  $p\text{-value} \geq \alpha$ 
  - Can't reject  $\langle \text{null hypothesis} \rangle$ .
- Case 2:  $p\text{-value} < \alpha$ 
  - Reject  $\langle \text{null hypothesis} \rangle$ .

# Hypothesis testing in an “alternative” taste

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- *<alternative hypothesis>*,  $\equiv$  not *<null hypothesis>* usually.
- Case 1:  $p\text{-value} \geq a$ 
  - Can't accept *<alternative hypothesis>*.
- Case 2:  $p\text{-value} < a$ 
  - Accept *<alternative hypothesis>*.

# Hypothesis testing in “-+” taste

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- “The case is negative.”
- Case 1:  $p\text{-value} \geq \alpha$ 
  - “Hmmm ... somehow negative.”
- Case 2:  $p\text{-value} < \alpha$ 
  - “Positive! So extreme!”

# P-value, $\alpha$ , wording, and summary

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p-value & $\alpha$	Wording	Summary
$p\text{-value} < 0.001$	Very significant	***
$p\text{-value} < 0.01$	Very significant	**
$p\text{-value} < 0.05$	Significant	*
$p\text{-value} \geq 0.05$	Not significant	ns

- *02\_tests\_with\_actual\_datasets.ipynb*

# How tests work

# Seeing is believing

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- $p\text{-value} = 0.0027 (< 0.01)$ 
  - 
- $p\text{-value} = 0.0271 (0.01\text{--}0.05)$ 
  -  ?  ? ? ? ?
- $p\text{-value} = 0.2718 (\geq 0.05)$ 
  - ? ? ? ? ? ?
- *03\_how\_tests\_work.ipynb*

# Common tests

# The cheat sheet

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- If testing independence:
  - If total sample size < 1000, or more than 20% of cells have expected frequencies < 5, **Fisher's exact test**.
  - Else, **Chi-squared test**.
- If testing difference:
  - If median is better, don't want to trim outliers, variable is ordinal, or any group size < 20:
    - If groups are paired, **Wilcoxon signed-rank test**.
    - If groups are independent, **Mann–Whitney U test**.
  - Else:
    - If groups are paired, **Paired Student's t-test**.
    - If groups are independent, **Welch's t-test**, not Student's.

# More cheat sheets & references

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- More cheat sheets:
  - Selecting Commonly Used Statistical Tests – Bates College
  - Which statistical test should I use? – University of Sheffield
  - Choosing a statistical test – HBS
- References:
  - Fisher's exact test of independence – HBS
  - Statistical notes for clinical researchers – Restor Dent Endod
  - Nonparametric Test and Parametric Test – Minitab
  - Advantages and limitations – Welch's t-test – Wikipedia
  - Dependent t-test for paired samples – Student's t-test – Wikipedia

# The tests in Python

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- *04\_common\_tests.ipynb*

# Complete a test

# Are p-value & $\alpha$ enough?

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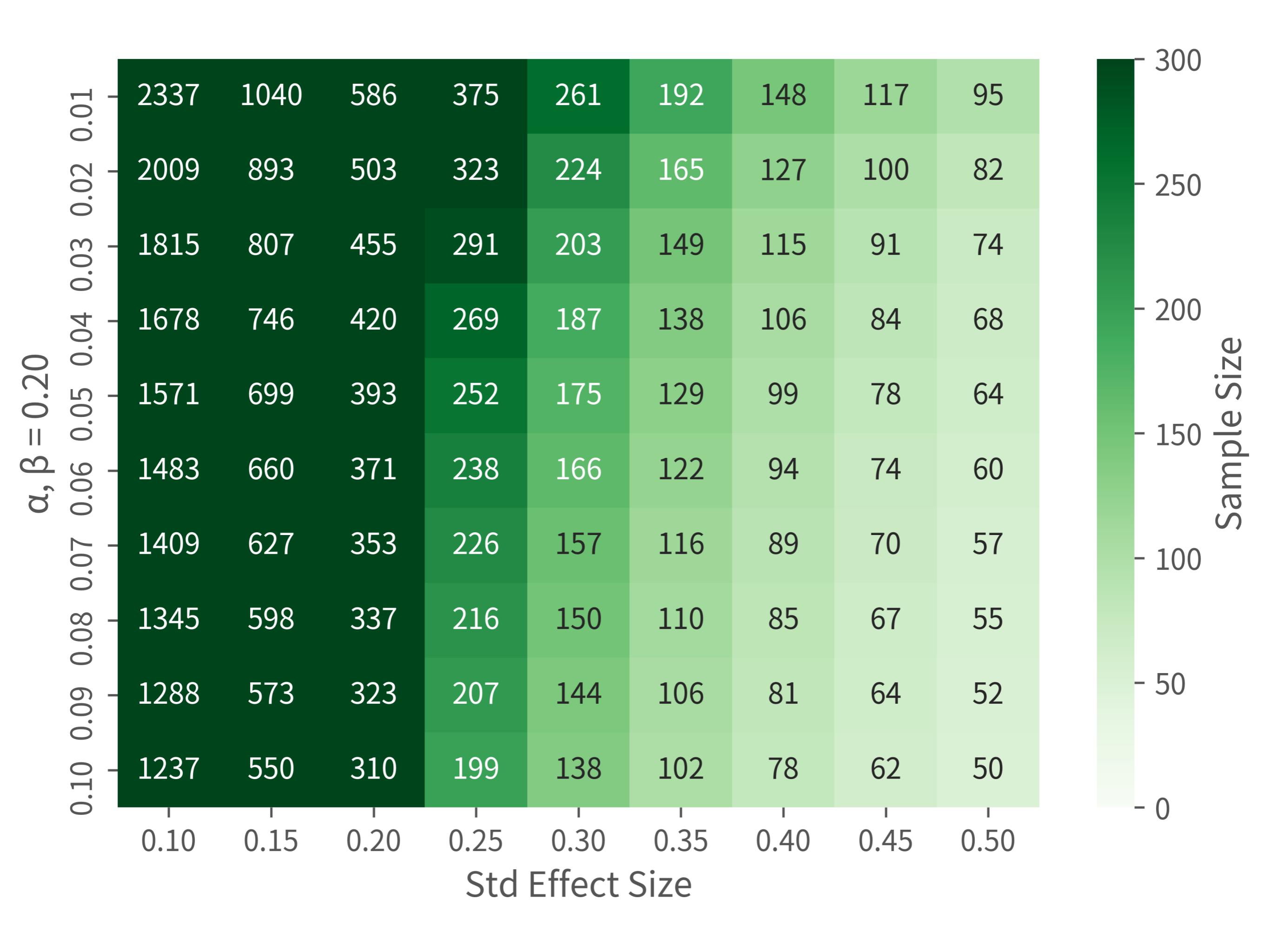
- power?
- sample size?
- $\beta$ ?
- effect size?
- false discovery rate?
- ? ? ? ? ?

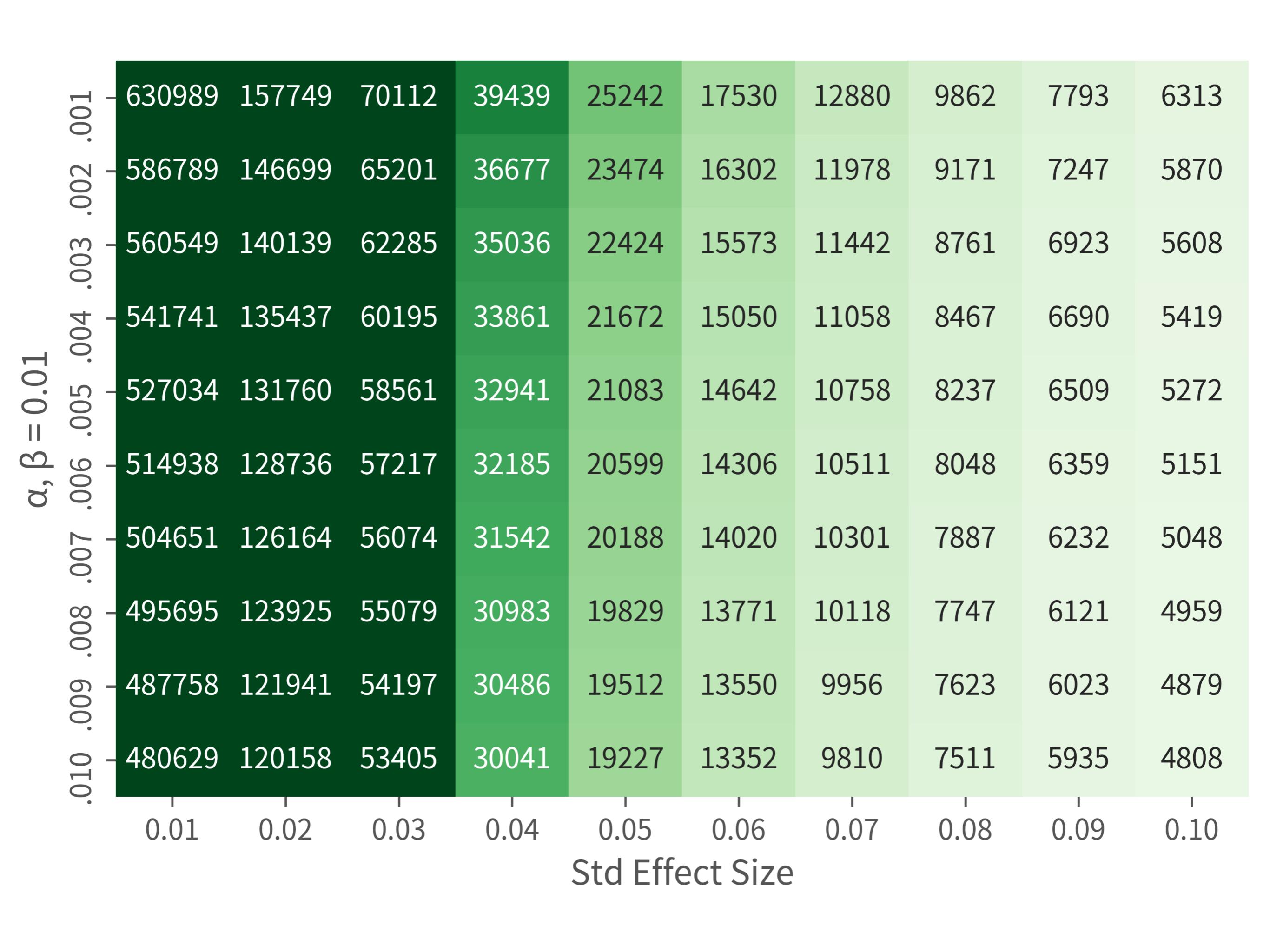
# $\alpha$ , $\beta$ , and effect size

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- DSM5: The case for double standards – James Coplan, M.D.
  - The figures explain  $\alpha$ ,  $\beta$ , and *effect size* perfectly, but due to the copyright, only put the link here.
  - “FP”:  $\alpha$
  - “FN”:  $\beta$
  - “The distance between the means”: *effect size*

- When  $\alpha$ ,  $\beta$ , *effect size* are defined, get the *sample size*.
- When  $\alpha$ ,  $\beta$ , *sample size* are defined, get the *effect size*.
- Increase *sample size* to decrease  $\alpha$ ,  $\beta$ , or *effect size*.





# Confusion matrix, where $A = 00_2 = C[0, 0]$

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		predicted negative AC	predicted positive BD
actual negative AB	true negative A	false positive B	
actual positive CD	false negative C	true positive D	

**False positive rate = B/AB = 4/(96+4) = 4/100**

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		predicted negative AC	predicted positive BD
actual negative AB	96 A	4 B	
	9 C	41 D	

# Predefined acceptable confusion matrix

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		predicted negative AC	predicted positive BD
actual negative AB	true negative A	false positive B	
	false negative C	true positive D	
actual positive CD			

$$a = B/AB = P(BD|AB)$$

.....

		predicted negative AC	predicted positive BD
actual negative AB	true negative A	false positive B	
	false negative C	true positive D	
actual positive CD			

$$\beta = C/CD = P(AC|CD)$$

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		predicted negative AC	predicted positive BD
actual negative AB	true negative A	false positive B	
actual positive CD	false negative C	true positive D	

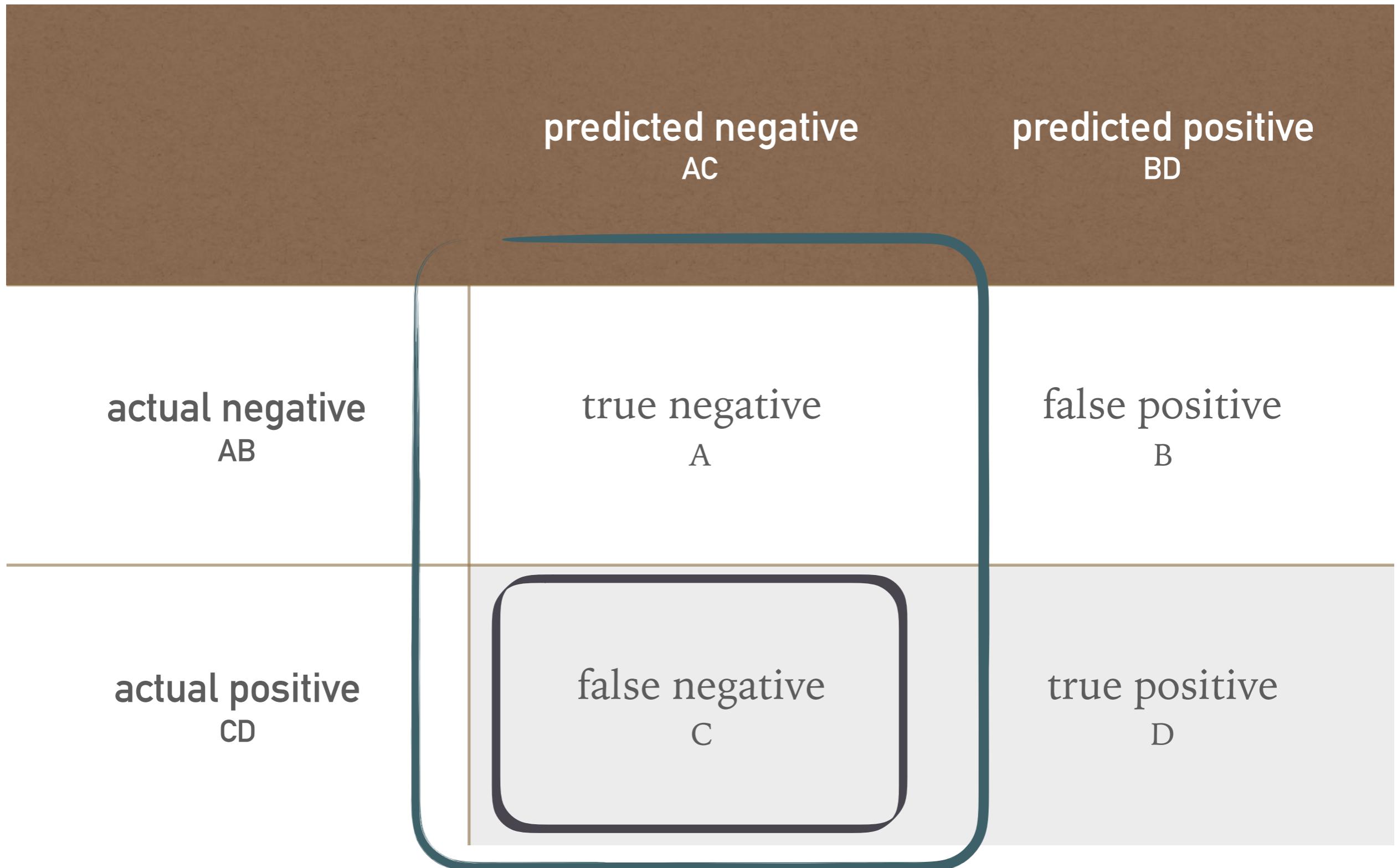
$$\text{Inverse } a = B/BD = P(AB|BD)$$

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	predicted negative AC	predicted positive BD
actual negative AB	true negative A	false positive B
actual positive CD	false negative C	true positive D

$$\text{Inverse } \beta = C/AC = P(CD|AC)$$

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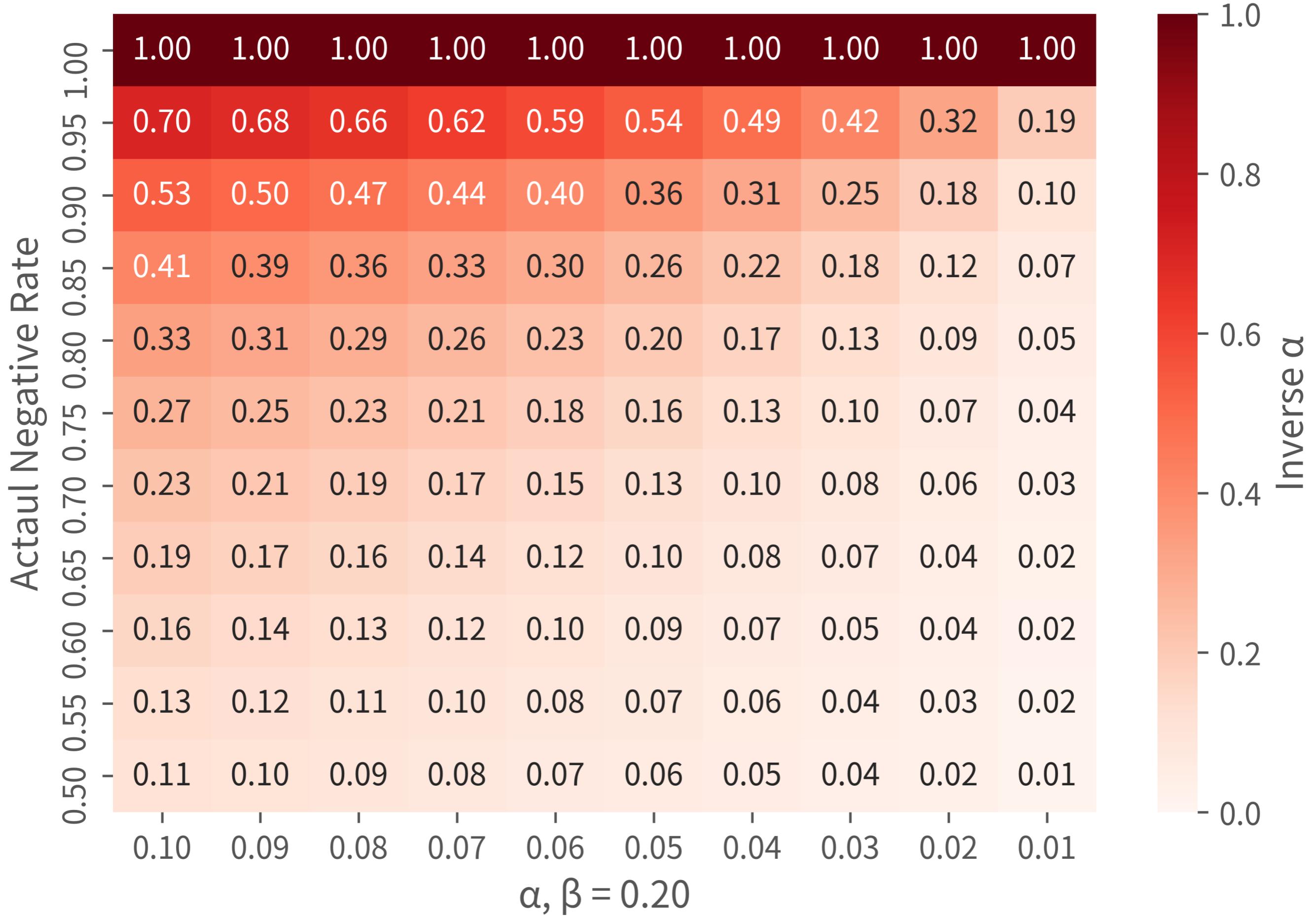
# Derive inverse a

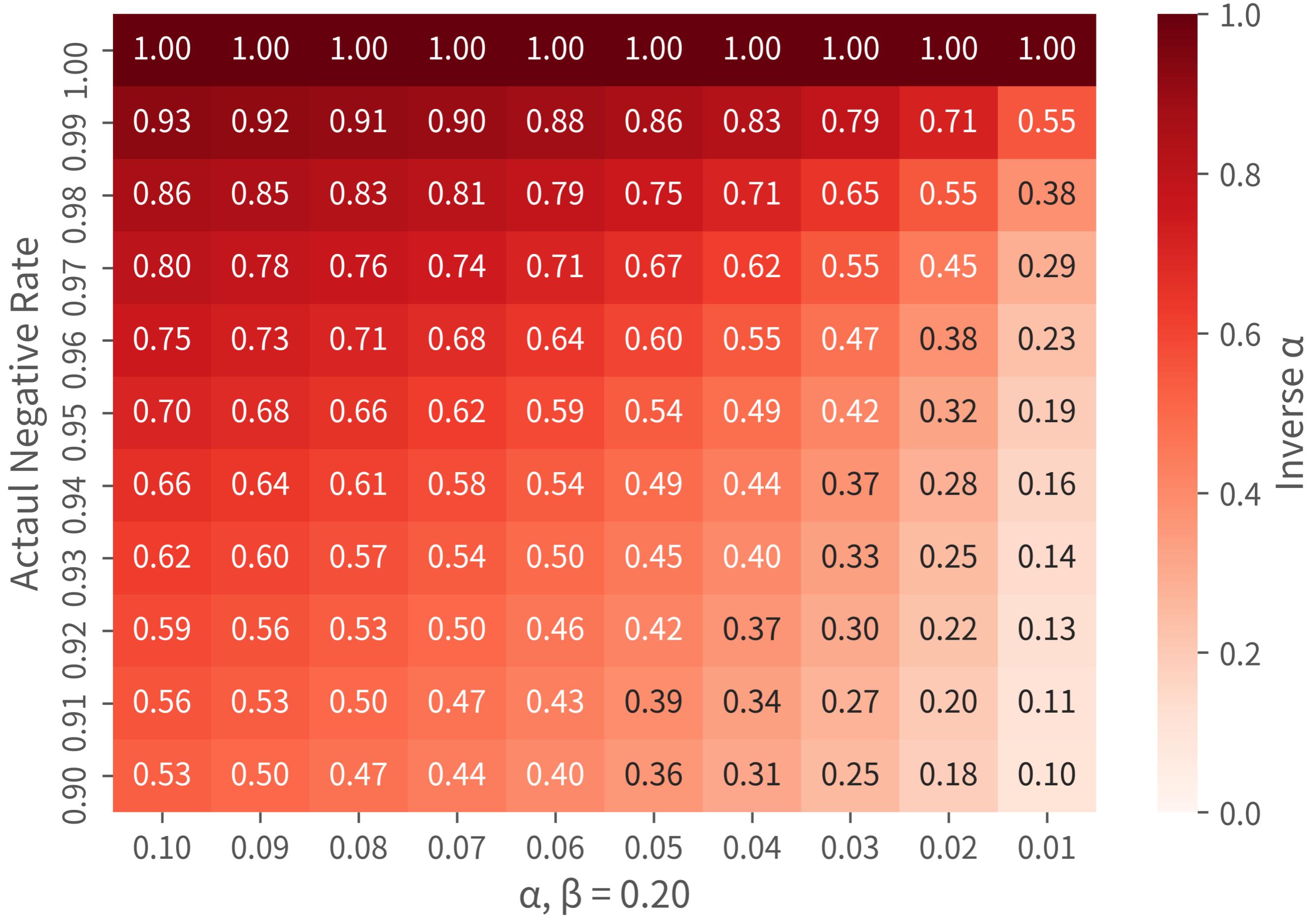
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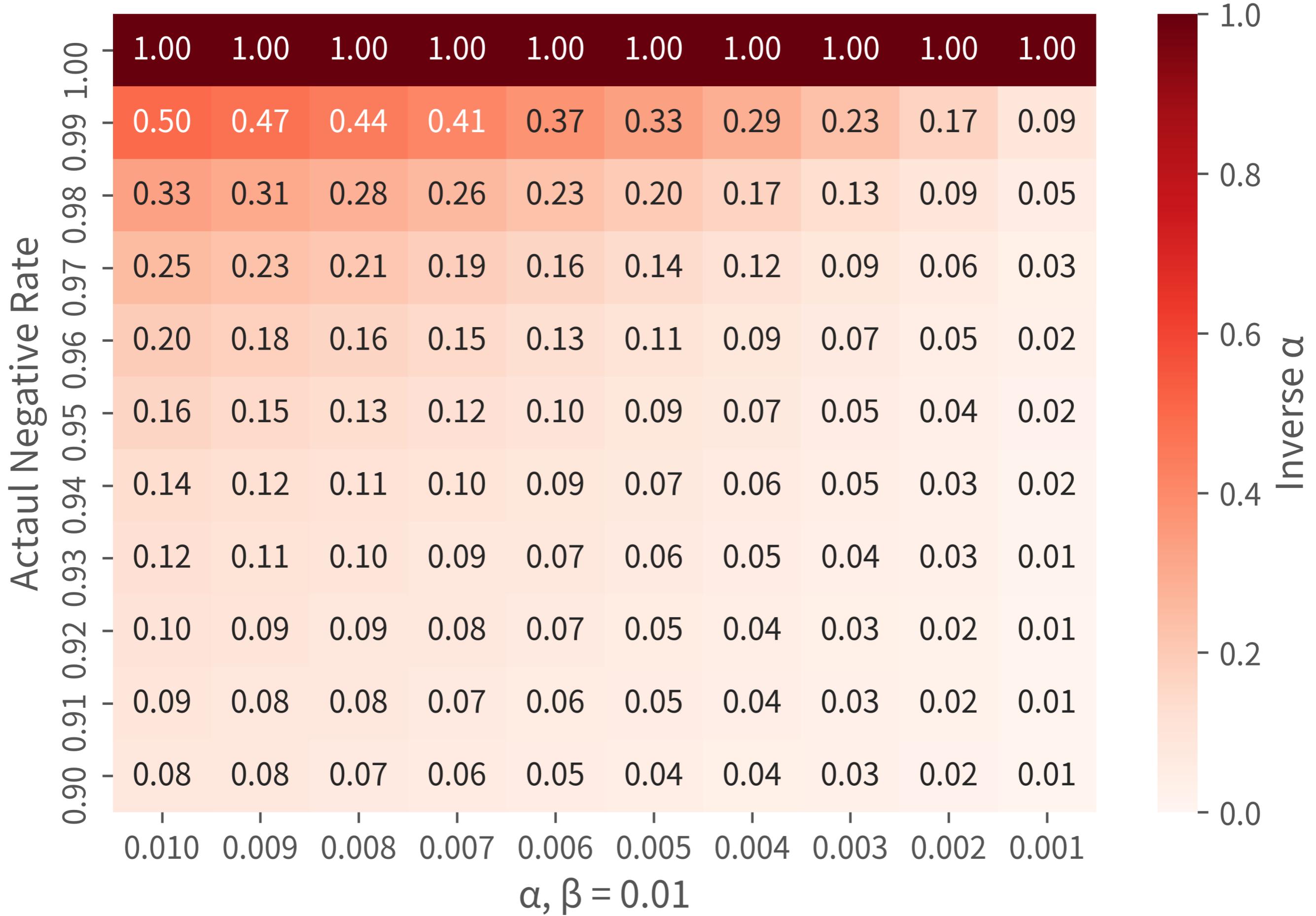
- A drug test:

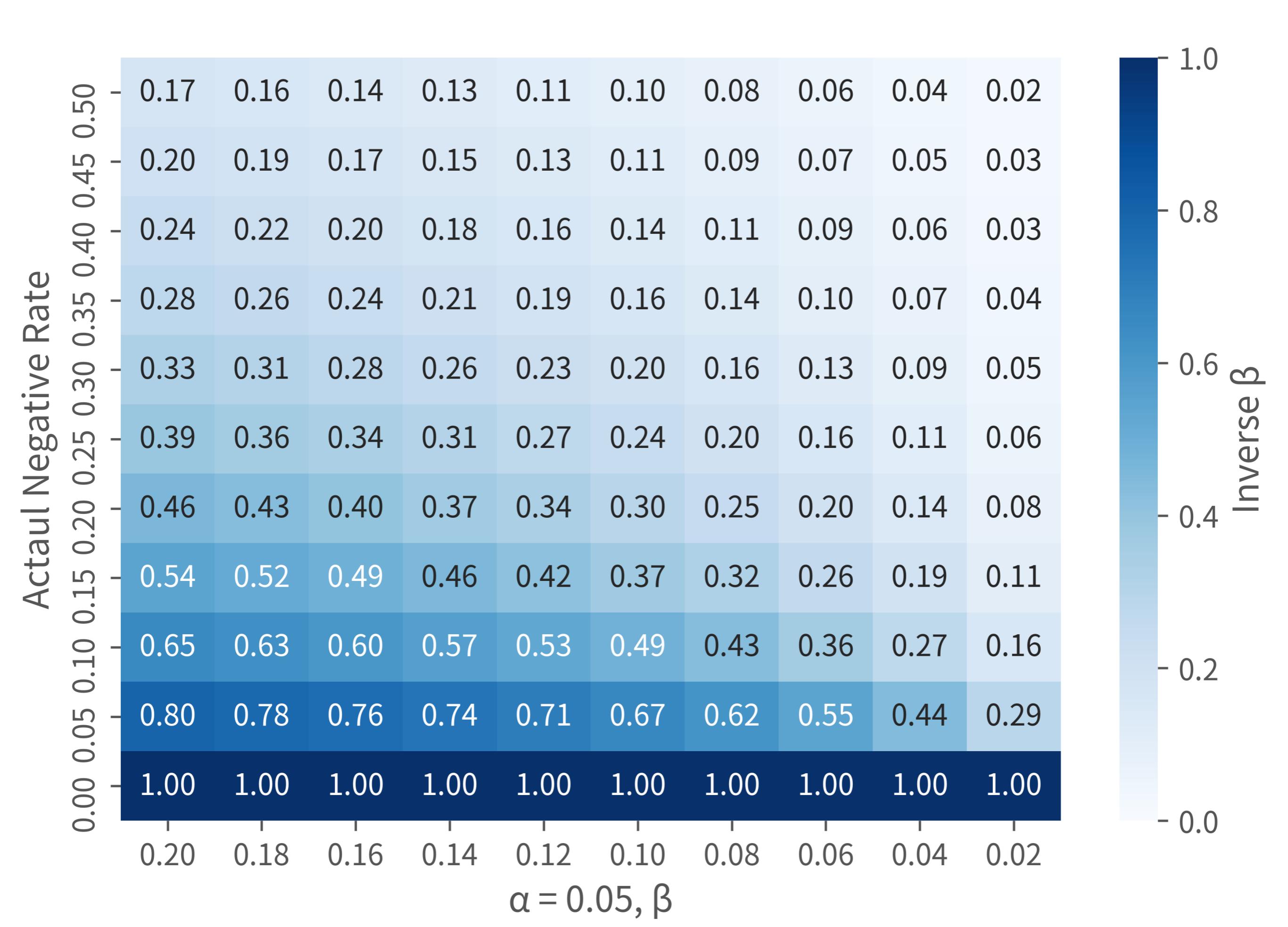
- $a = 1\% = B/AB = P(BD | AB) = P(\text{predict} + | \text{not user})$
- $\beta = 1\% = C/CD = P(AC | CD) = P(\text{predict} - | \text{user})$
- $\text{actual negative rate} = 99.5\% = AB/ABCD = P(AB) = P(\text{not user})$
- $\text{inverse } a = B/BD = P(AB | BD) = P(\text{not user} | \text{predict} +)?$

$$\begin{aligned}P(AB | BD) &= \frac{P(BD | AB)P(AB)}{P(BD)} \\&= \frac{P(BD | AB)P(AB)}{P(BD | AB)P(AB) + P(BD | CD)P(CD)} \\&= \frac{0.01 \times 0.995}{0.01 \times 0.995 + 0.99 \times 0.005} \\&\approx 66.8 \%\end{aligned}$$









# Rates in predefined acceptable confusion matrix

$=$ = predefined _____			
$\alpha$	B/AB	significance level type I error rate	false positive rate
$\beta$	C/CD	type II error rate	false negative rate
inverse $\alpha$	B/BD		false discovery rate
inverse $\beta$	C/AC		false omission rate
confidence level	A/AB	1- $\alpha$	specificity
power	D/CD	1- $\beta$	sensitivity recall

# Rates in confusion matrix

	=	=	= observed __
false positive rate	B/AB		$\alpha$
false negative rate	C/CD		$\beta$
false discovery	B/BD		inverse $\alpha$
false omission	C/AC		inverse $\beta$
actual negative	AB/ABCD		
sensitivity	D/CD	recall	power
specificity	A/AB		confidence level
precision	D/BD		inverse power
recall	D/CD	sensitivity	power

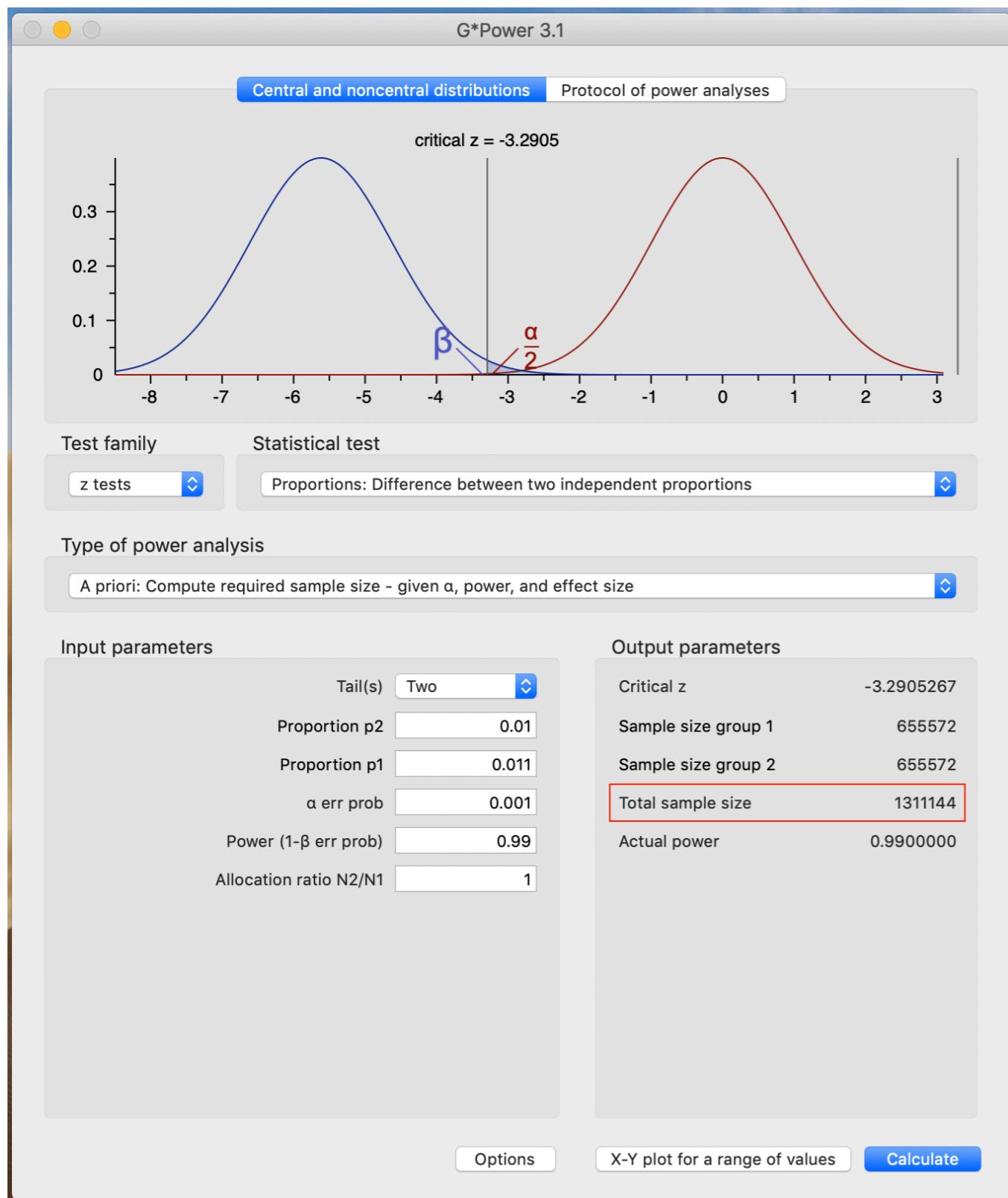
# Most formal steps

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- State the hypothesis → what *test*.
- Estimate the *actual negative rate*.
- The *actual negative rate* → how much  $\alpha, \beta$  are required.
- The  $\alpha, \beta, \text{effect size}$  → how large *sample size* is required.
- Still collect a sample as large as possible.
- Understand and preprocess the sample.
  - Missing data, outliers, Q–Q plot, transform, etc.
- Test and report fully.

# Calculate by yourself

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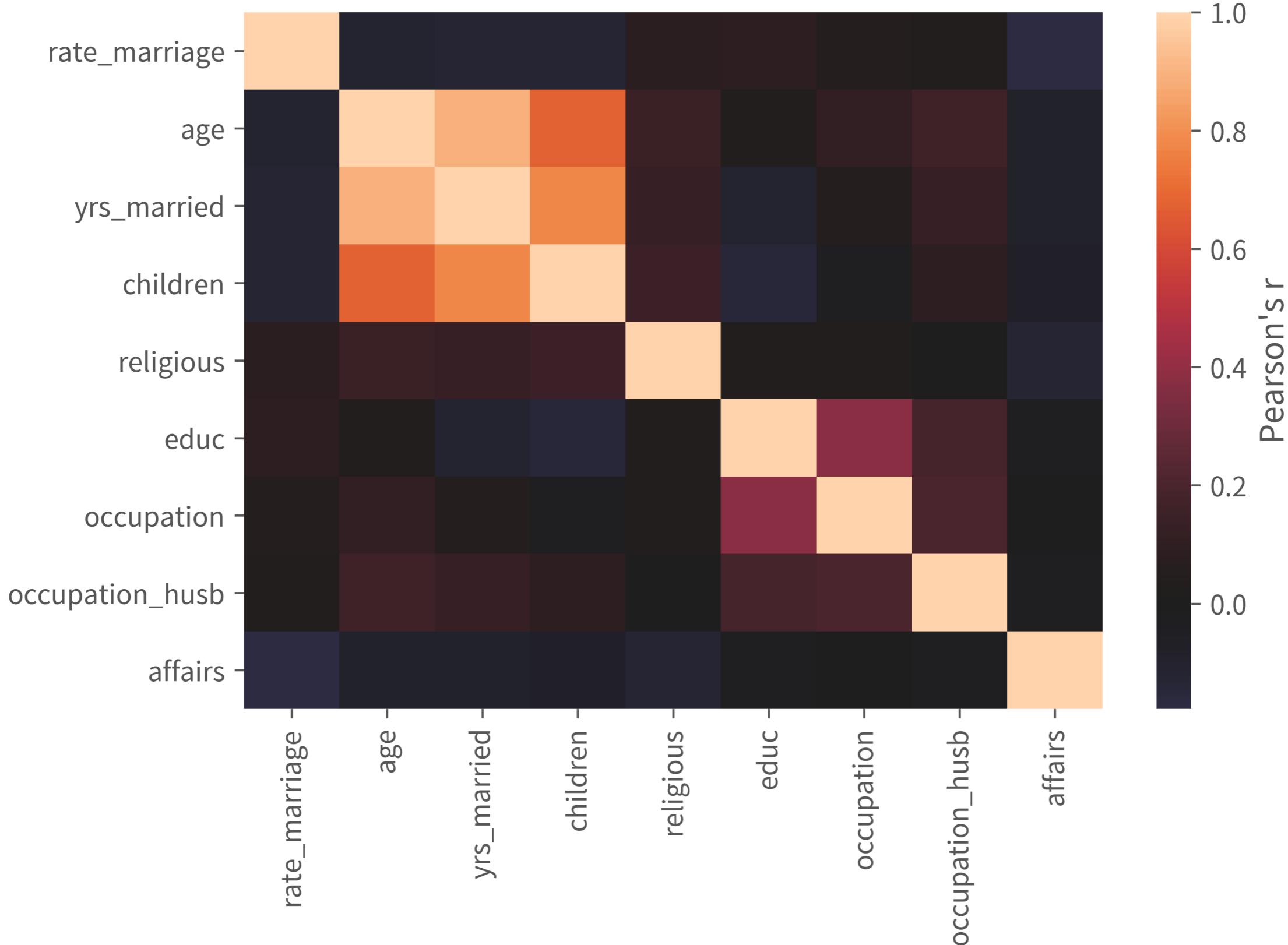


- G\*Power can calculate:
  - *sample size*
  - $\alpha$
  - $power = 1 - \beta$
  - *effect size*
- <http://www.gpower.hhu.de/>
- *05\_complete\_a\_test.ipynb*

# Other statistical tools

# Correlation analysis

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# Regression analysis

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```
In [7]: fair_df = sm.datasets.fair.load_pandas().data  
ols_res = smf.ols('children ~ yrs_married', fair_df).fit()  
ols_res.summary()
```

Out[7]: OLS Regression Results

Dep. Variable:	children	R-squared:	0.597			
Model:	OLS	Adj. R-squared:	0.597			
Method:	Least Squares	F-statistic:	9437.			
Date:	Fri, 06 Jul 2018	Prob (F-statistic):	0.00			
Time:	00:40:17	Log-Likelihood:	-8430.3			
No. Observations:	6366	AIC:	1.686e+04			
Df Residuals:	6364	BIC:	1.688e+04			
Df Model:	1					
Covariance Type:	nonrobust					
	coef	std err	t	P> t	[0.025	0.975]
Intercept	0.0259	0.018	1.429	0.153	-0.010	0.062
yrs_married	0.1522	0.002	97.142	0.000	0.149	0.155
Omnibus:	449.258	Durbin-Watson:	1.972			
Prob(Omnibus):	0.000	Jarque-Bera (JB):	709.624			
Skew:	0.559	Prob(JB):	8.07e-155			
Kurtosis:	4.193	Cond. No.	18.5			

# Keep learning

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- Statistics
  - [Seeing Theory](#)
  - [Statistics – SciPy Tutorial](#)
  - [StatsModels](#)
  - [Biological Statistics](#)
  - [Research Methods](#)
- Machine Learning
  - [Scikit-learn Tutorials](#)
  - [Standford CS229](#)
  - [Hsuan-Tien Lin](#)

# Recap

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- *p-value*: “tail” probability given “actual negative” hypothesis.
- *actual negative rate* does matter.
- $\alpha, \beta, \text{effect size} \downarrow; \text{sample size} \uparrow.$
- Visualization and simulation do help.
- Bonus: `a1_figures.ipynb` .
- Let's identify noise efficiently!