Searching: Linear Search and brute force techniques, Sorting: Insertion Sort

Week-03, Lecture-01

Course Code: CSE221

Course Title: Algorithms

Program: B.Sc. in CSE

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Linear Search

- The linear search is a sequential search, which uses a loop to step through an array, starting with the first element.
- It compares each element with the value being searched for, and stops when either the value is found or the end of the array is encountered.
- If the value being searched is not in the array, the algorithm will unsuccessfully search to the end of the array.

Linear Search

- Since the array elements are stored in linear order searching the element in the linear order make it easy and efficient.
- The search may be successful or unsuccessfully. That is, if the required element is found them the search is successful other wise it is unsuccessfully.

Advantages

- The linear search is simple - It is very easy to understand and implement
- It does not require the data in the array to be stored in any particular order

Disadvantages

- It has very poor efficiency because it takes lots of comparisons to find a particular record in big files
- The performance of the algorithm scales linearly with the size of the input
- Linear search is slower then other searching algorithms

Linear Search Example

-23	97	18	21	5	-86	64	0	-37

element

Searching for -86.

Linear Search Example

-23	97	18	21	5	-86	64	0	-37

element

Searching for -86.

Linear Search Example

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Searching for -86.

Linear Search Example

-23	97	18	21	5	-86	64	0	-37

element

Searching for -86: found!

Analysis of Linear Search

How long will our search take?

In the best case, the target value is in the first element of the array.

So the search takes some tiny, and constant, amount of time.

In the worst case, the target value is in the last element of the array.

So the search takes an amount of time proportional to the length of the array.

Analysis of Linear Search

In the average case, the target value is somewhere in the array.

In fact, since the target value can be anywhere in the array, any element of the array is equally likely.

So on average, the target value will be in the middle of the array.

So the search takes an amount of time proportional to half the length of the array

Pseudocode

For all elements

Check if it is equal to element being searched for.

If it is ,return its position.

else continue.

Insertion Sort

- Iteration i. Repeatedly swap element i with the one to its left if smaller.

- Property. After ith iteration, `a[0]` through `a[i]` contain first i+1 elements in ascending order.

Array index	0	1	2	3	4	5	6	7	8	9
Value	2.78	7.42	0.56	1.12	1.17	0.32	6.21	4.42	3.14	7.71

Iteration 0: step 0.

Insertion Sort

- Iteration i. Repeatedly swap element i with the one to its left if smaller.

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Iteration 1: step 0.

Insertion Sort

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Iteration 2: step 0.

Insertion Sort

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Value	0.56	2.78	7.42	1.12	1.17	0.32	6.21	4.42	3.14	7.71

Iteration 2: step 1.

Insertion Sort

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- Property. After ith iteration, `a[0]` through `a[i]` contain first i+1 elements in ascending order.

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Value	0.56	2.78	7.42	1.12	1.17	0.32	6.21	4.42	3.14	7.71

Iteration 2: step 2.

Insertion Sort

- Iteration i. Repeatedly swap element i with the one to its left if smaller.

- Property. After ith iteration, `a[0]` through `a[i]` contain first i+1 elements in ascending order.

Array index	0	1	2	3	4	5	6	7	8	9
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Iteration 3: step 0.

Insertion Sort

- Iteration i. Repeatedly swap element i with the one to its left if smaller.

- Property. After ith iteration, `a[0]` through `a[i]` contain first i+1 elements in ascending order.

Array index	0	1	2	3	4	5	6	7	8	9
Value	0.56	1.12	2.78	7.42	1.17	0.32	6.21	4.42	3.14	7.71

Iteration 3: step 1.

Insertion Sort

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Iteration 3: step 2.

Insertion Sort

- Iteration i. Repeatedly swap element i with the one to its left if smaller.

- Property. After ith iteration, `a[0]` through `a[i]` contain first i+1 elements in ascending order.

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Value	0.56	1.12	2.78	1.17	7.42	0.32	6.21	4.42	3.14	7.71

Iteration 4: step 0.

Insertion Sort

- Iteration i. Repeatedly swap element i with the one to its left if smaller.

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Iteration 4: step 1.

Insertion Sort

- Iteration i. Repeatedly swap element i with the one to its left if smaller.

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Value	0.56	1.12	1.17	2.78	7.42	0.32	6.21	4.42	3.14	7.71

Iteration 4: step 2.

Insertion Sort

- Iteration i. Repeatedly swap element i with the one to its left if smaller.

- Property. After ith iteration, `a[0]` through `a[i]` contain first i+1 elements in ascending order.

Array index	0	1	2	3	4	5	6	7	8	9
Value	0.56	1.12	1.17	2.78	0.32	7.42	6.21	4.42	3.14	7.71

Iteration 5: step 0.

Insertion Sort

- Iteration i. Repeatedly swap element i with the one to its left if smaller.

- Property. After ith iteration, `a[0]` through `a[i]` contain first i+1 elements in ascending order.

Array index	0	1	2	3	4	5	6	7	8	9
Value	0.56	1.12	1.17	0.32	2.78	7.42	6.21	4.42	3.14	7.71

Iteration 5: step 1.

Insertion Sort

- Iteration i. Repeatedly swap element i with the one to its left if smaller.

- Property. After ith iteration, `a[0]` through `a[i]` contain first i+1 elements in ascending order.

Array index	0	1	2	3	4	5	6	7	8	9
Value	0.56	1.12	0.32	1.17	2.78	7.42	6.21	4.42	3.14	7.71

Iteration 5: step 2.

Insertion Sort

- Iteration i. Repeatedly swap element i with the one to its left if smaller.

- Property. After ith iteration, `a[0]` through `a[i]` contain first i+1 elements in ascending order.

Array index	0	1	2	3	4	5	6	7	8	9
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Iteration 5: step 3.

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Iteration 5: step 4.

Insertion Sort

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- Property. After ith iteration, `a[0]` through `a[i]` contain first i+1 elements in ascending order.

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Iteration 5: step 5.

Insertion Sort

- Iteration i. Repeatedly swap element i with the one to its left if smaller.

- Property. After ith iteration, `a[0]` through `a[i]` contain first i+1 elements in ascending order.

Array index	0	1	2	3	4	5	6	7	8	9
Value	0.32	0.56	1.12	1.17	2.78	6.21	7.42	4.42	3.14	7.71

Iteration 6: step 0.

Insertion Sort

- Iteration i. Repeatedly swap element i with the one to its left if smaller.

- Property. After ith iteration, `a[0]` through `a[i]` contain first i+1 elements in ascending order.

Array index	0	1	2	3	4	5	6	7	8	9
Value	0.32	0.56	1.12	1.17	2.78	6.21	7.42	4.42	3.14	7.71

Iteration 6: step 1.

Insertion Sort

- Iteration i. Repeatedly swap element i with the one to its left if smaller.

- Property. After ith iteration, `a[0]` through `a[i]` contain first i+1 elements in ascending order.

Array index	0	1	2	3	4	5	6	7	8	9
Value	0.32	0.56	1.12	1.17	2.78	6.21	4.42	7.42	3.14	7.71

Iteration 7: step 0.

Insertion Sort

- Iteration i. Repeatedly swap element i with the one to its left if smaller.

- Property. After ith iteration, `a[0]` through `a[i]` contain first i+1 elements in ascending order.

Array index	0	1	2	3	4	5	6	7	8	9
Value	0.32	0.56	1.12	1.17	2.78	4.42	6.21	7.42	3.14	7.71

Iteration 7: step 1.

Insertion Sort

- Iteration i. Repeatedly swap element i with the one to its left if smaller.

- Property. After ith iteration, `a[0]` through `a[i]` contain first i+1 elements in ascending order.

Array index	0	1	2	3	4	5	6	7	8	9
Value	0.32	0.56	1.12	1.17	2.78	4.42	6.21	7.42	3.14	7.71

Iteration 7: step 2.

Insertion Sort

- Iteration i. Repeatedly swap element i with the one to its left if smaller.

- Property. After ith iteration, `a[0]` through `a[i]` contain first i+1 elements in ascending order.

Array index	0	1	2	3	4	5	6	7	8	9
Value	0.32	0.56	1.12	1.17	2.78	4.42	6.21	3.14	7.42	7.71

Iteration 8: step 0.

Insertion Sort

- Iteration i. Repeatedly swap element i with the one to its left if smaller.

- Property. After ith iteration, `a[0]` through `a[i]` contain first i+1 elements in ascending order.

Array index	0	1	2	3	4	5	6	7	8	9
Value	0.32	0.56	1.12	1.17	2.78	4.42	3.14	6.21	7.42	7.71

Iteration 8: step 1.

Insertion Sort

- Iteration i. Repeatedly swap element i with the one to its left if smaller.

- Property. After ith iteration, `a[0]` through `a[i]` contain first i+1 elements in ascending order.

Array index	0	1	2	3	4	5	6	7	8	9
Value	0.32	0.56	1.12	1.17	2.78	3.14	4.42	6.21	7.42	7.71

Iteration 8: step 2.

Insertion Sort

- Iteration i. Repeatedly swap element i with the one to its left if smaller.

- Property. After ith iteration, `a[0]` through `a[i]` contain first i+1 elements in ascending order.

Array index	0	1	2	3	4	5	6	7	8	9
Value	0.32	0.56	1.12	1.17	2.78	3.14	4.42	6.21	7.42	7.71

Iteration 8: step 3.

Insertion Sort

- Iteration i. Repeatedly swap element i with the one to its left if smaller.

- Property. After ith iteration, `a[0]` through `a[i]` contain first i+1 elements in ascending order.

Array index	0	1	2	3	4	5	6	7	8	9
Value	0.32	0.56	1.12	1.17	2.78	3.14	4.42	6.21	7.42	7.71

Iteration 9: step 0.

Insertion Sort

- Iteration i. Repeatedly swap element i with the one to its left if smaller.

- Property. After ith iteration, `a[0]` through `a[i]` contain first i+1 elements in ascending order.

Array index	0	1	2	3	4	5	6	7	8	9
Value	0.32	0.56	1.12	1.17	2.78	3.14	4.42	6.21	7.42	7.71

Iteration 10: DONE.

Insertion Sort Complexity

INSERTION-SORT(A)	*cost*	*times*
for $j = 2$ **to** $A.length$	c_1	n
$key = A[j]$	c_2	$n - 1$
// Insert $A[j]$ into the sorted sequence $A[1..j-1]$.	0	$n - 1$
$i = j - 1$	c_4	$n - 1$
while $i > 0$ and $A[i] > key$	c_5	$\sum_{j=2}^{n} t_j$
$A[i+1] = A[i]$	c_6	$\sum_{j=2}^{n} (t_j - 1)$
$i = i - 1$	c_7	$\sum_{j=2}^{n} (t_j - 1)$
$A[i+1] = key$	c_8	$n - 1$

Insertion Sort Complexity

$$
\begin{aligned}
T(n) &= c_1 n + c_2(n-1) + c_4(n-1) + c_5 \sum_{j=2}^{n} t_j + c_6 \sum_{j=2}^{n} (t_j - 1) \\
&\quad + c_7 \sum_{j=2}^{n} (t_j - 1) + c_8(n-1) .
\end{aligned}
$$

Even for inputs of a given size, an algorithm's running time may depend on *which* input of that size is given. For example, in INSERTION-SORT, the best case occurs if the array is already sorted. For each $j = 2, 3, \ldots, n$, we then find that $A[i] \leq key$ in line 5 when i has its initial value of $j - 1$. Thus $t_j = 1$ for $j = 2, 3, \ldots, n$, and the best-case running time is

$$
\begin{aligned}
T(n) &= c_1 n + c_2(n-1) + c_4(n-1) + c_5(n-1) + c_8(n-1) \\
&= (c_1 + c_2 + c_4 + c_5 + c_8)n - (c_2 + c_4 + c_5 + c_8) .
\end{aligned}
$$

Insertion Sort Complexity

$$\sum_{j=2}^{n} j = \frac{n(n+1)}{2} - 1$$

and

$$\sum_{j=2}^{n} (j-1) = \frac{n(n-1)}{2}$$

(see Appendix A for a review of how to solve these summations), we find that in the worst case, the running time of INSERTION-SORT is

$$\begin{aligned} T(n) &= c_1 n + c_2(n-1) + c_4(n-1) + c_5\left(\frac{n(n+1)}{2} - 1\right) \\ &\quad + c_6\left(\frac{n(n-1)}{2}\right) + c_7\left(\frac{n(n-1)}{2}\right) + c_8(n-1) \\ &= \left(\frac{c_5}{2} + \frac{c_6}{2} + \frac{c_7}{2}\right)n^2 + \left(c_1 + c_2 + c_4 + \frac{c_5}{2} - \frac{c_6}{2} - \frac{c_7}{2} + c_8\right)n \\ &\quad - (c_2 + c_4 + c_5 + c_8) . \end{aligned}$$

We can express this worst-case running time as $an^2 + bn + c$ for constants a, b, and c that again depend on the statement costs c_i; it is thus a ***quadratic function*** of n.

Textbooks & Web References

- Text Book (Chapter 2)
- Reference book iii (Chapter 11)
- www.visualgo.net

Thank you

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Any question?