Engineer maths MAT211

Formula:

1. $\frac{d}{dx}(c) = 0$

2. $\frac{d}{dx}(x) = 1$

3. $\frac{d}{dx}(x^n) = nx^{n-1}$

4. $\frac{d}{dx}(e^x) = e^x$

5. $\frac{d}{dx}(a^x) = a^x \ln a$

6. $\frac{d}{dx}(\ln x) = 1/x$

7. $\frac{d}{dx}(\sin x) = \cos x$

8. $\frac{d}{dx}(\cos x) = -\sin x$

9. $\frac{d}{dx}(\tan x) = \sec^2 x$

10. $\frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$

11. $\frac{d}{dx}(\sec x) = \sec x . \tan x$

12. $\frac{d}{dx}(\operatorname{cosec} x) = -\operatorname{cosec} x . \cot x$

$\sin^2 x + \cos^2 x = 1$

$\sec^2 x = 1 + \tan^2 x$

$\operatorname{cosec}^2 x = 1 + \cot^2 x$

13. $\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$

14. $\frac{d}{dx}(\cos^{-1} x) = \frac{-1}{\sqrt{1-x^2}}$

Engineering math:

mid 1Chp. ODE → ordinary differential equation 01, 2, 3

Final 5Chp.
- LT → Laplace transformation - 1,2,3
- FT → Fourier " - 1
- FS → Fourier Series. - 1

$\cos C + \cos D = 2\cos\frac{C+D}{2}\cos\frac{C-D}{2}$

$\sin C + \sin D = 2\cos\frac{C+D}{2}\sin\frac{C-D}{2}$

15. $\frac{d}{dx}(\tan^{-1} x)$

16. $\frac{d}{dx}(\cot^{-1} x) =$

Eng. Math

ODE :

CH #01 : Formation of ODE

$\frac{dy}{dx}$ → ordinary differential operator

$\frac{\partial u}{\partial x}$ → partial differential operator

লবটির ← $\frac{dy}{dx} + y = x$ → ode

হরটির ← dx

$\frac{\partial u}{\partial x} + u = x$ → P.D.: pde

DE

Fraction থাকলে,

(1) $$\left(\frac{d^3y}{dx^3}\right)^2 + y\left(\frac{d^2y}{dx^2}\right)^6 + y = x$$

order - 3 [derivative এর সর্বোচ্চ সংখ্যা]

degree - 2 [order এর power]

(2) $$\sqrt{\frac{d^3y}{dx^3} + 9y} = \sqrt[3]{\left(\frac{d^5y}{dx^5}\right)^2 + y^2}$$

[Rational করতে হবে।]

$$\Rightarrow \left(\frac{d^3y}{dx^3} + 9y\right)^{1/2} = \left\{\left(\frac{d^5y}{dx^5}\right)^2 + y^2\right\}^{1/3}$$

$$\Rightarrow \left\{\left(\frac{d^3y}{dx^3} + 9y\right)^{1/2}\right\}^6 = \left[\left\{\left(\frac{d^5y}{dx^5}\right)^2 + y^2\right\}^{1/3}\right]^6$$

$$\Rightarrow \left(\frac{d^3y}{dx^3} + 9y\right)^3 = \left\{\left(\frac{d^5y}{dx^5}\right)^2 + y^2\right\}^2$$

order $\to 5$ degree $\to 4$

Linear ODE:

1. Dependent variable power must be 1 if it is seperate from $\frac{dy}{dx}$

2. $y\frac{dy}{dx}$ থাকবে না।

3. $\left(\frac{dy}{dx}\right)^1, \left(\frac{d^2y}{dx^2}\right)^1$ etc.

$$y = \underbrace{mx + c}_{\text{arbitary constant}}$$

$$y = \underbrace{ax^2 + bx + c}_{\text{arbitary constant}}$$

eq $\to$ soln : $x^2 - 5x + 6 = 0$; $x = 3$, $x = 2$

no. of arbitary constant = no. of derivative

$y = mx + c$

$\Rightarrow \frac{dy}{dx} = m\frac{d}{dx}(x) + 0 \Rightarrow \frac{dy}{dx} = m$

$\Rightarrow \frac{d^2y}{dx^2} = 0$ [ODE]

$\downarrow$

$y = mx + c \to$ solution

Eng. Ma

Find the ODE of the family of curves $y = Ae^{2x} + Be^{-2x}$

$$\frac{dy}{dx} = A(2e^{2x}) + B(-2e^{-2x})$$

$$= 2Ae^{2x} - 2Be^{-2x}$$

Again differentiation,

$$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}\left(2Ae^{2x} - 2Be^{-2x}\right)$$

$$\Rightarrow \frac{d^2y}{dx^2} = 2A\frac{d}{dx}(e^{2x}) - 2B\frac{d}{dx}(e^{-2x})$$

$$= 2A(2e^{2x}) - 2B(-2e^{-2x})$$

$$= 4Ae^{2x} + 4Be^{-2x}$$

$$= 4(Ae^{2x} + Be^{-2x})$$

$$= 4y$$

$$\frac{d^2y}{dx^2} - 4y = 0$$ (As desired)

5. Derive the differential equation for $c(y+c)^2 = x^3$

$$c(y+c)^2 = x^3 \quad \ldots (i)$$

$\Rightarrow$ <u>Solⁿ:</u> $\frac{d}{dx}\{c(y+c)^2\} = \frac{d}{dx}x^3$

$$\Rightarrow c\,2(y+c).\frac{dy}{dx} = 3x^2 \quad \ldots (ii)$$

$\Rightarrow$ (i) $\div$ (ii)

$$\frac{y+c}{2.\frac{dy}{dx}} = \frac{x}{3} \qquad \Rightarrow y+c = \frac{2x}{3}.\frac{dy}{dx}$$

$$\Rightarrow \frac{2x}{3}.\frac{dy}{dx} - y = c$$

From eq (i)

$$\frac{2x}{3}.\frac{dy}{dx} - y\left(y + \frac{2x}{3}\frac{dy}{dx} - y\right)^2 = x^3$$

$$\Rightarrow \left(\frac{2xy}{3}.\frac{dy}{dx} - y\right)\left(\frac{2x}{3}.\frac{dy}{dx}\right)^2 = x^3$$

$$\Rightarrow \left(\frac{2xy}{3}.\frac{dy}{dx} - y\right)\left\{\frac{4x^2}{9}\left(\frac{d^2y}{dx^2}\right)^2\right\} = x^3$$

$$\Rightarrow \frac{8x^3y}{27}.\left(\frac{dy}{dx}\right)^3 - \frac{4x^2y}{9}.\left(\frac{dy}{dx^2}\right)^2 = x^3$$

$$\Rightarrow 8xy\left(\frac{dy}{dx}\right)^3 - 12y\left(\frac{dy}{dx}\right)^2 = x$$

(As desired)

6. Derive an ODE for $y^2 = A(B+x)(B-x)$

Soln: Given that, $y^2 = A(B+x)(B-x)$

$$= A(B^2 - x^2)$$

$$= AB^2 - Ax^2$$

$$\frac{d}{dx}(y^2) = \frac{d}{dx}(AB^2 - Ax^2)$$

$$\Rightarrow 2y\frac{dy}{dx} = 0 - A2x$$

$$\Rightarrow 2y\frac{dy}{dx} = -A2x \quad \text{——(i)} \Rightarrow y\left(\frac{dy}{dx}\right) = -Ax$$

$$\frac{d}{dx}\left(y\frac{dy}{dx}\right) = \frac{d}{dx}(-Ax)$$

$$\Rightarrow y\frac{d^2y}{dx^2} + \frac{dy}{dx}\cdot\frac{dy}{dx} = -A$$

$$\Rightarrow y\cdot\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = -A$$

$\Rightarrow$ multiplying above eqn with x.

$$xy\frac{d^2y}{dx^2} + x\left(\frac{dy}{dx}\right)^2 = -Ax$$

using eq(i),

$$xy\frac{d^2y}{dx^2} + x\left(\frac{dy}{dx}\right)^2 = y\frac{dy}{dx}$$

(As desired)