Introduction to Divide and Conquer Approach Searching Binary Search Sorting, Merge Sort

Week-01, Lecture-02

Course Code: CSE221

Course Title: Algorithms

Program: B.Sc. in CSE

Course Teacher: Tanzina Afroz Rimi

Designation: Lecturer

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Binary Search

- Binary search. Given `value` and sorted array `a[]`, find index `i` such that `a[i]` = `value`, or report that no such index exists.

- Invariant. Algorithm maintains `a[lo]` ≤ `value` ≤ `a[hi]`.

6	**13**	**14**	**25**	**33**	**43**	**51**	**53**	**64**	**72**	**84**	**93**	**95**	**96**	**97**
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
lo														**hi**

- Ex. Binary search for 33.

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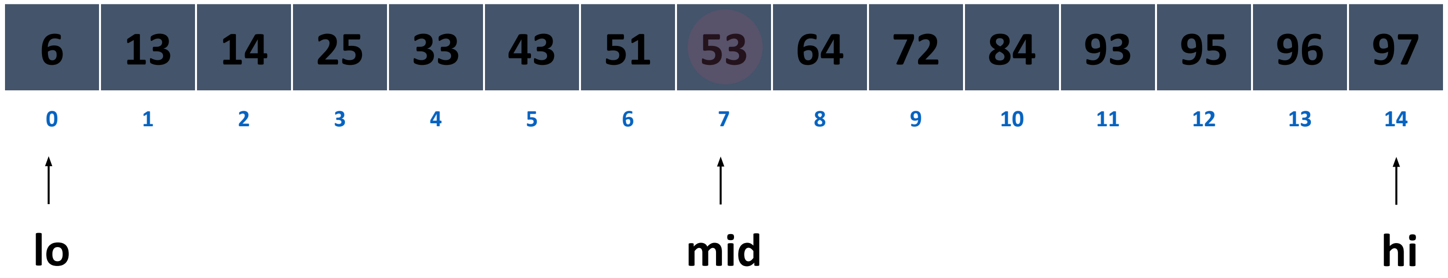

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6	13	14	25	33	43	51	53	64	72	84	93	95	96	97
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lo **hi**

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lo
hi

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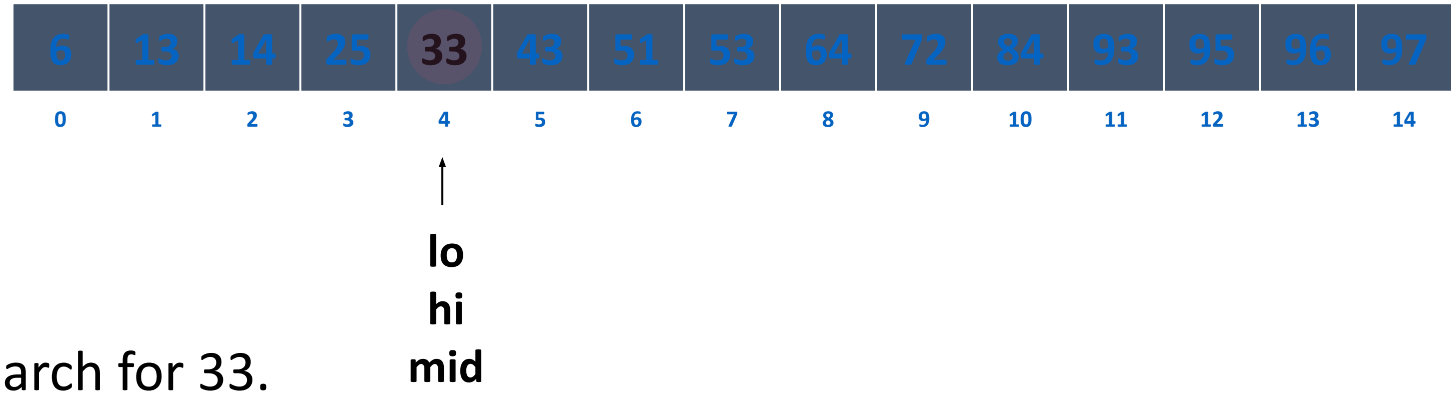

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Binary Search

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0	1	2	3	4	5	6	7	8	9	10	11	12	13	14

lo
hi
mid

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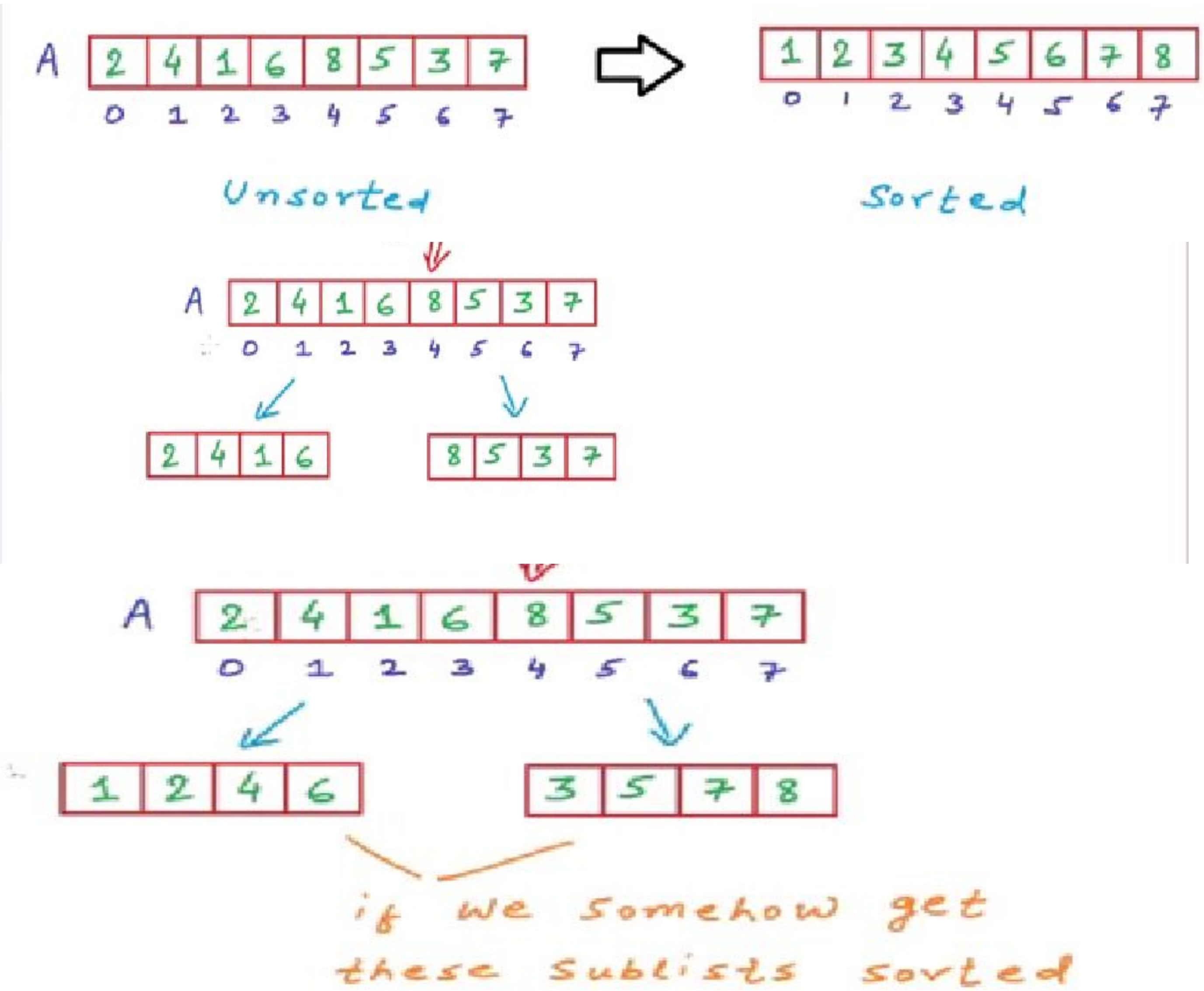
Merge Sort

Divide & Conqure

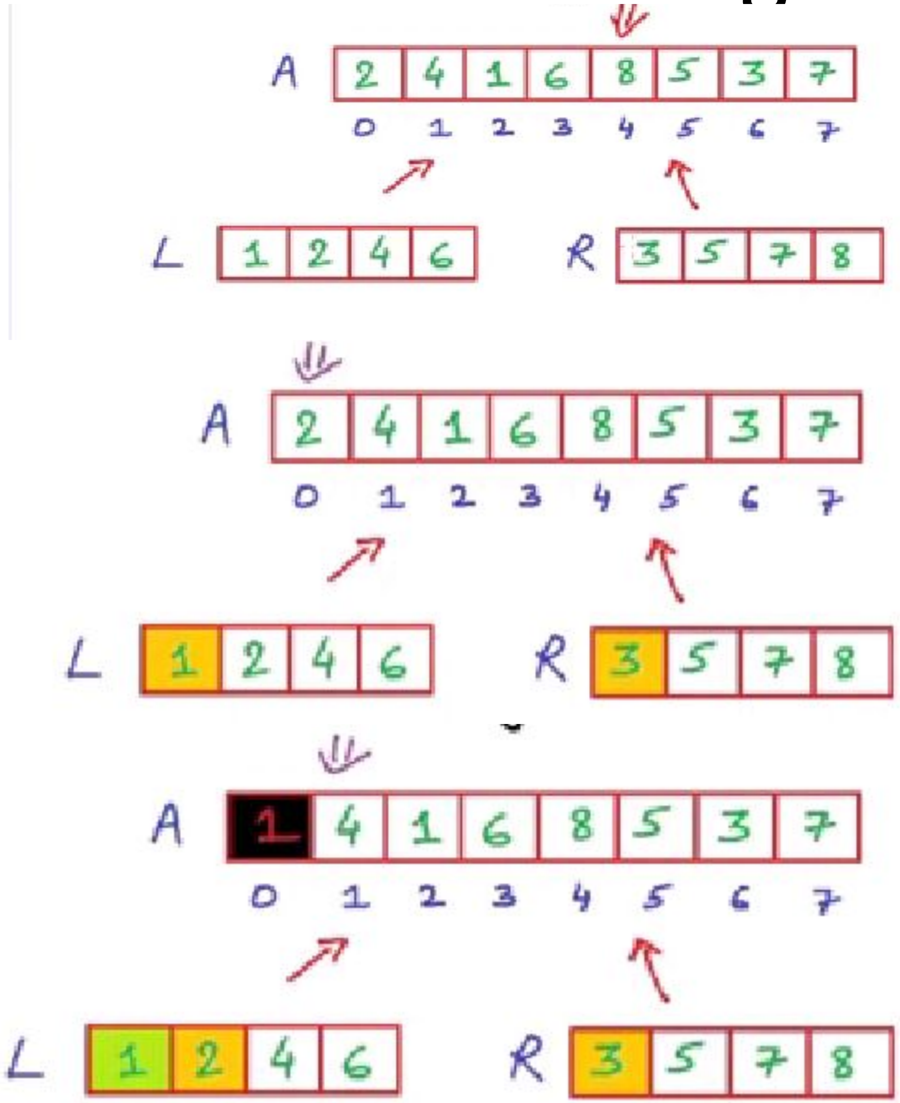
- The divide-and-conquer paradigm involves three steps at each level of the recursion:
  - **Divide** the problem into a number of subproblems.
  - **Conquer** the subproblems by solving them recursively. If the subproblem sizes are small enough, however, just solve the subproblems in a straightforward manner.
  - **Combine** the solutions to the subproblems into the solution for the original problem.
- The ***merge sort*** algorithm closely follows the divide-and-conquer paradigm. Intuitively, it operates as follows.
  - **Divide:** Divide the n-element sequence to be sorted into two subsequences of $n/2$ elements each.
  - **Conquer:** Sort the two subsequences recursively using merge sort.
  - **Combine:** Merge the two sorted subsequences to produce the sorted answer.

Merge Sort



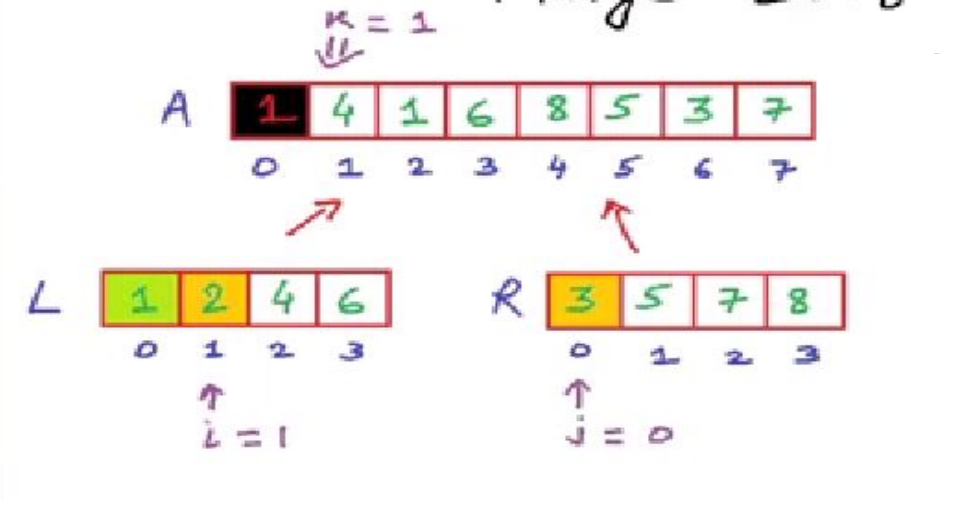
A
2 4 1 6 8 5 3 7
0 1 2 3 4 5 6 7
Unsorted
1 2 3 4 5 6 7 8
0 1 2 3 4 5 6 7
Sorted
A
2 4 1 6 8 5 3 7
0 1 2 3 4 5 6 7
2 4 1 6
8 5 3 7
A
2 4 1 6 8 5 3 7
0 1 2 3 4 5 6 7
1 2 4 6
3 5 7 8
if we somehow get
these sublists sorted

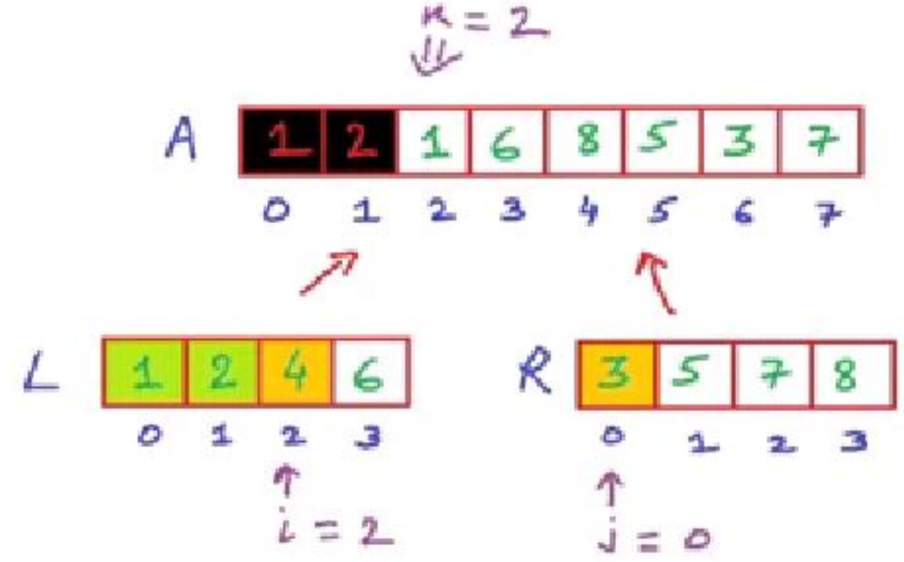
Merge..



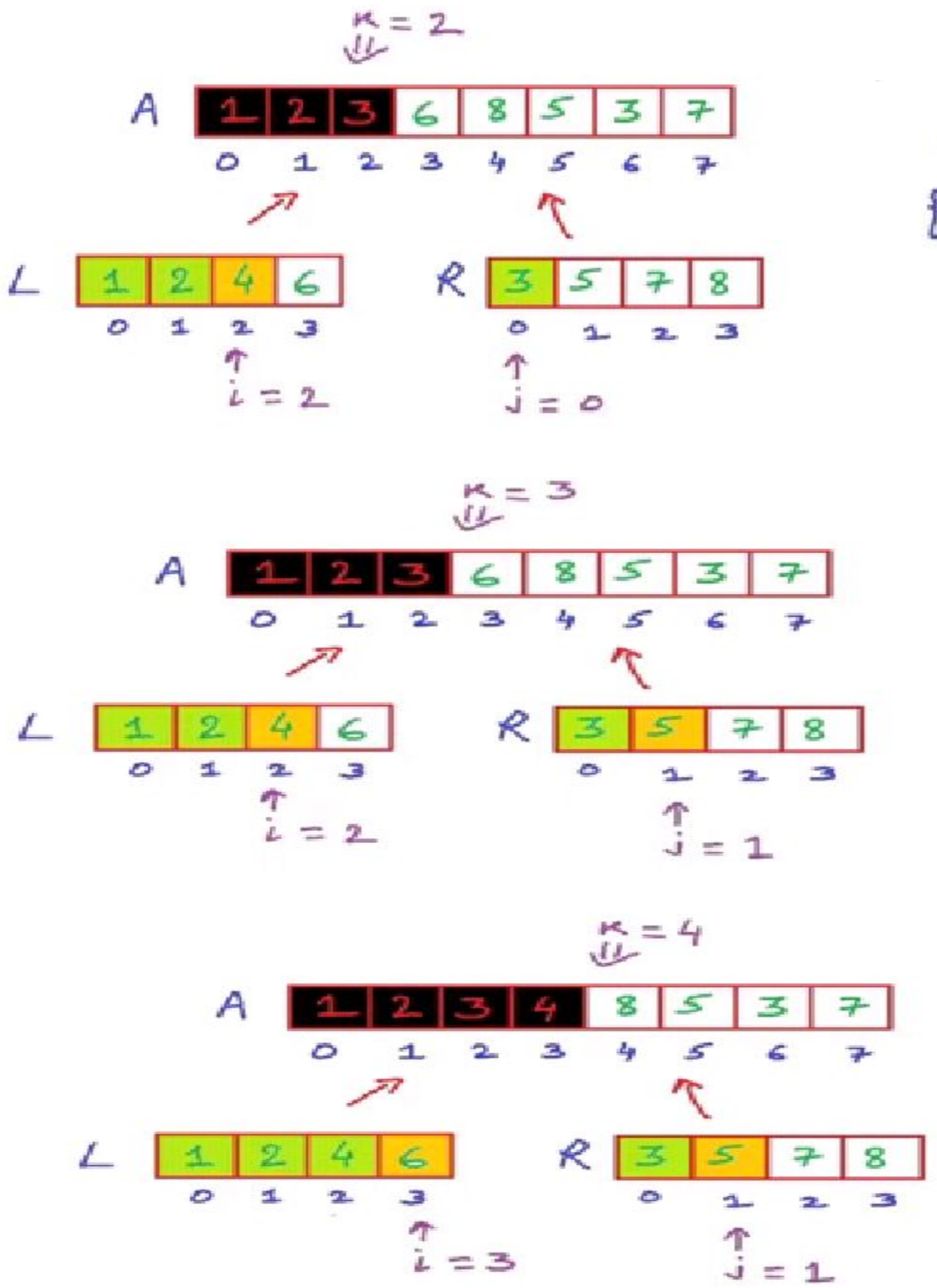
A
2 4 1 6 8 5 3 7
0 1 2 3 4 5 6 7
L
1 2 4 6
R
3 5 7 8
A
2 4 1 6 8 5 3 7
0 1 2 3 4 5 6 7
L
1 2 4 6
R
3 5 7 8
A
1 4 1 6 8 5 3 7
0 1 2 3 4 5 6 7
L
1 2 4 6
R
3 5 7 8

Merge Sort






```
Merge(L, R, A)
{
   nL ← length(L)
   nR ← length(R)
   i ← j ← k ← 0
  while(i < nL && j < nR)
  {
    if( L[i] <= R[j])
    {
       A[k] ← L[i]
       i ← i+1
    }
    else
    {
       A[k] ← R[j]
       j ← j+1
    }
    k ← k+1
  }
```

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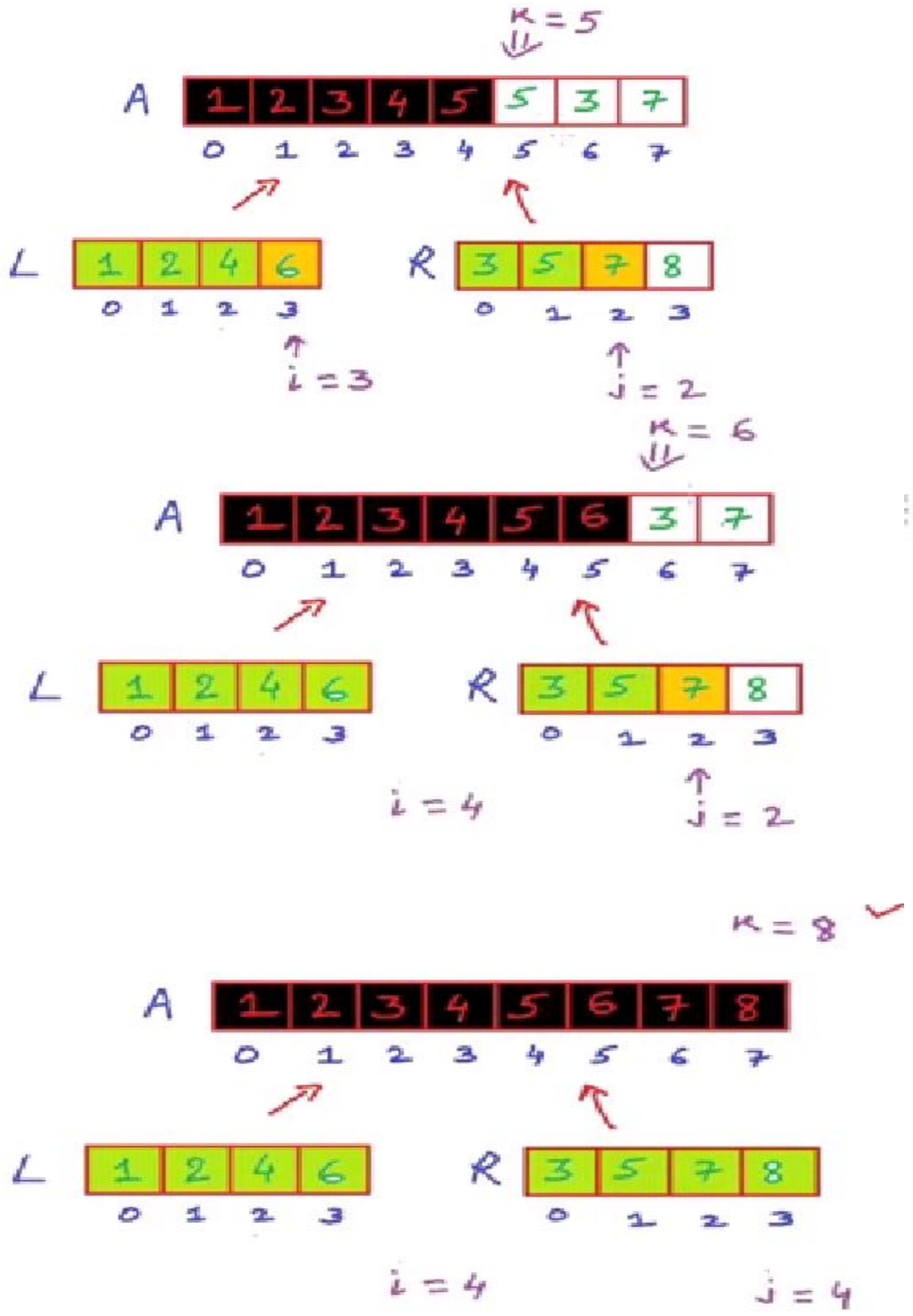
    }
    else
    {
      A[k] ← R[j]
      j ← j+1
    }
    k ← k+1
}
```

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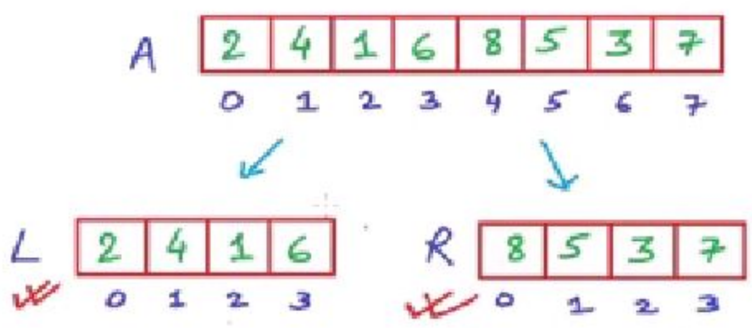
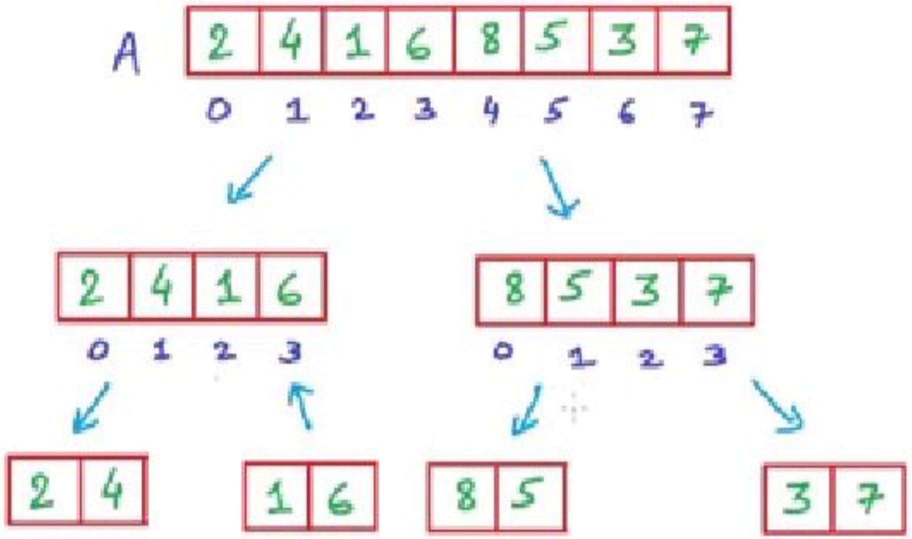
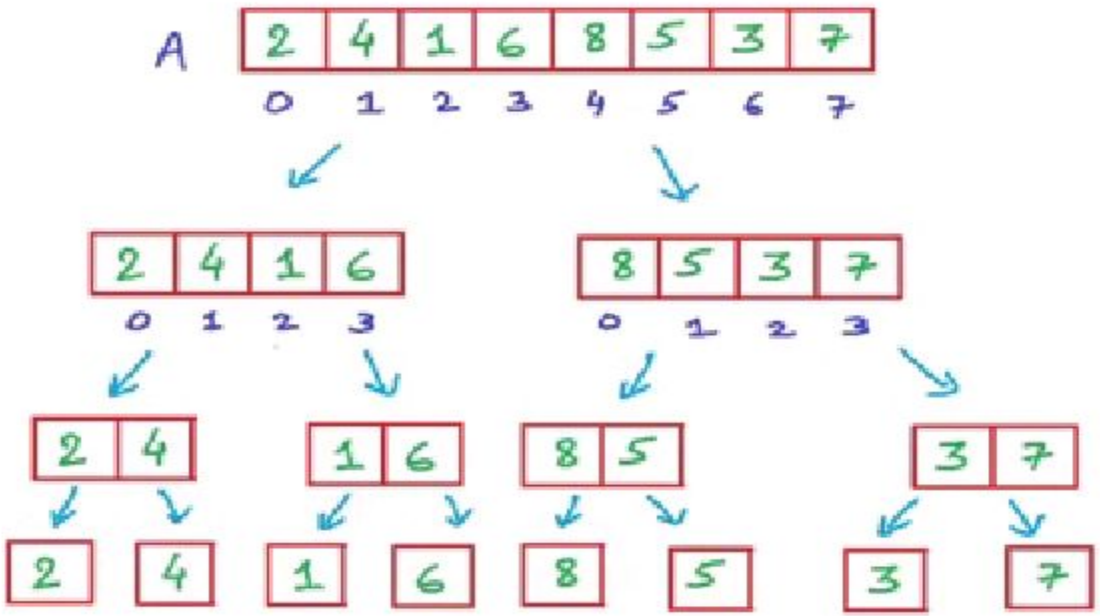
    {
      A[k] ← L[i]; i ← i+1
    }
    else
    {
      A[k] ← R[j]; j ← j+1
    }
    k ← k+1
  }
  while (i < nL)
  { A[k] ← L[i]; i←i+1; k←k+1;
  }
  while(j < nR)
  { A[k] ← R[j]; j←j+1; k←k+1;
  }
```

Merge Sort..


A
2 4 1 6 8 5 3 7
0 1 2 3 4 5 6 7
L
2 4 1 6
0 1 2 3
R
8 5 3 7
0 1 2 3


A
2 4 1 6 8 5 3 7
0 1 2 3 4 5 6 7
2 4 1 6
0 1 2 3
8 5 3 7
0 1 2 3
2 4
1 6
8 5
3 7


A
2 4 1 6 8 5 3 7
0 1 2 3 4 5 6 7
2 4 1 6
0 1 2 3
8 5 3 7
0 1 2 3
2 4
1 6
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2
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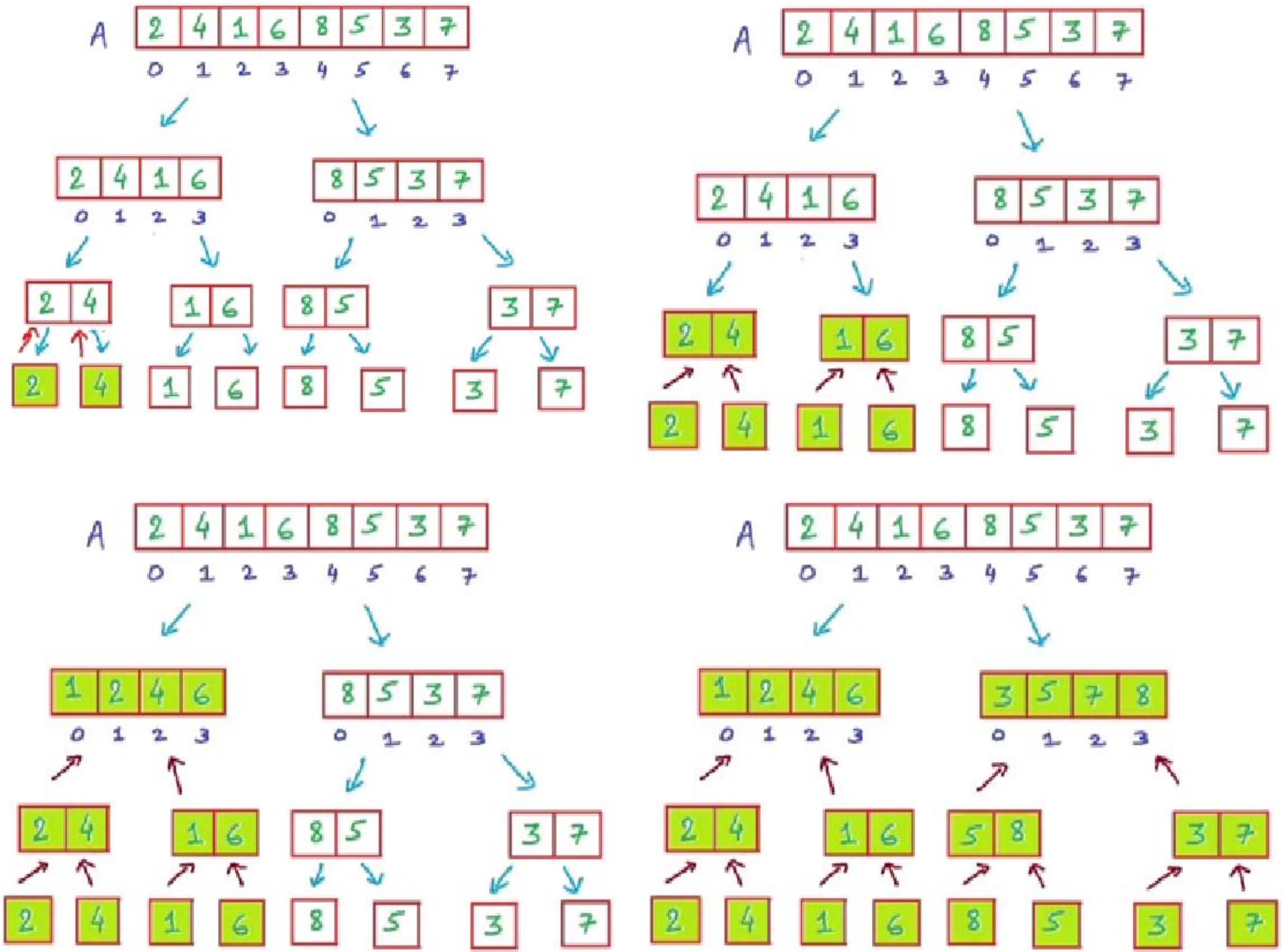
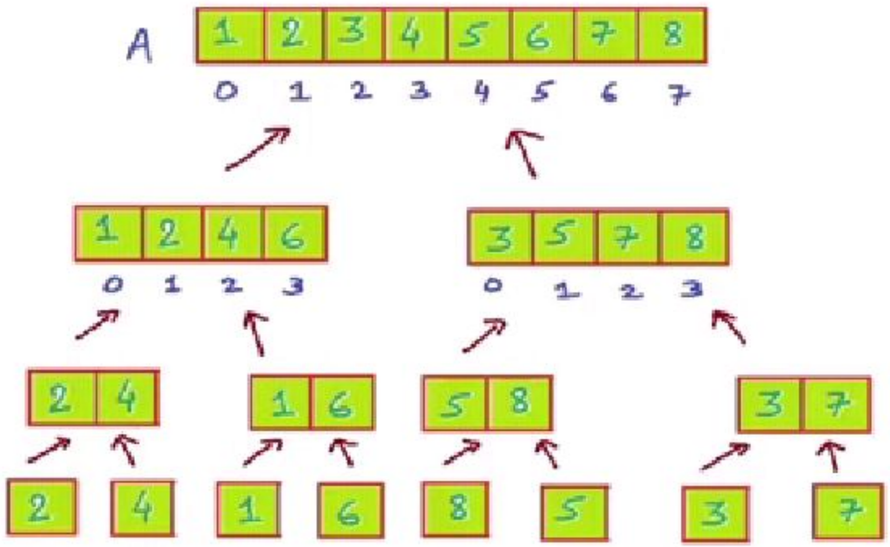
6
8
5
3
7


A
2 4 1 6 8 5 3 7
0 1 2 3 4 5 6 7
2 4 1 6
8 5 3 7
1 2 4 6
3 5 7 8
2 4
1 6
8 5
5 8
3 7


A
1 2 3 4 5 6 7 8
0 1 2 3 4 5 6 7
1 2 4 6
0 1 2 3
3 5 7 8
0 1 2 3
2 4
1 6
5 8
3 7
2
4
1
6
8
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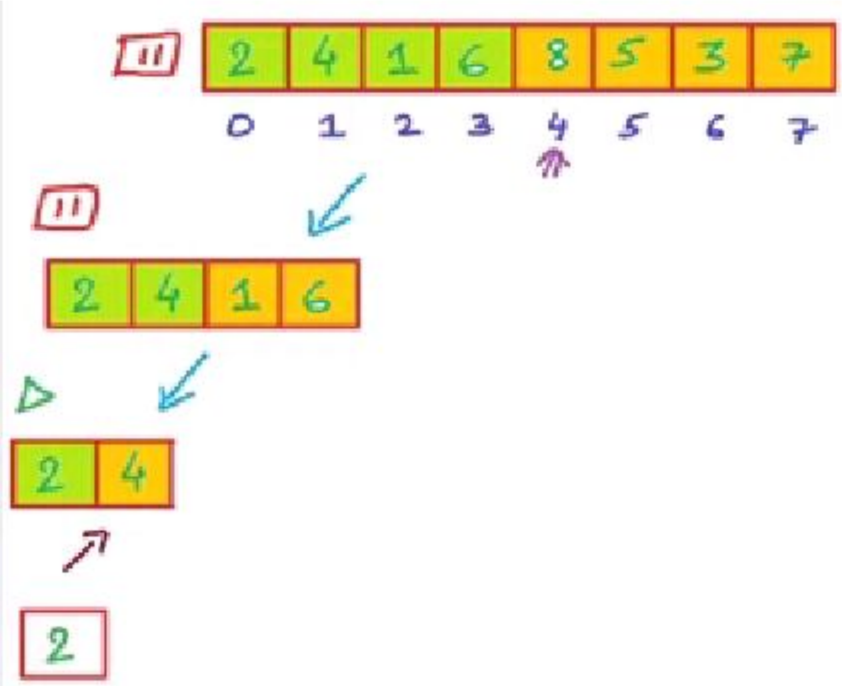
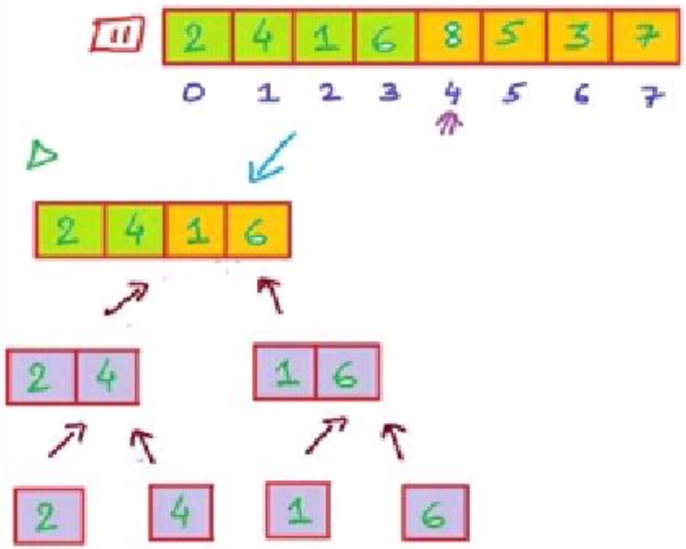
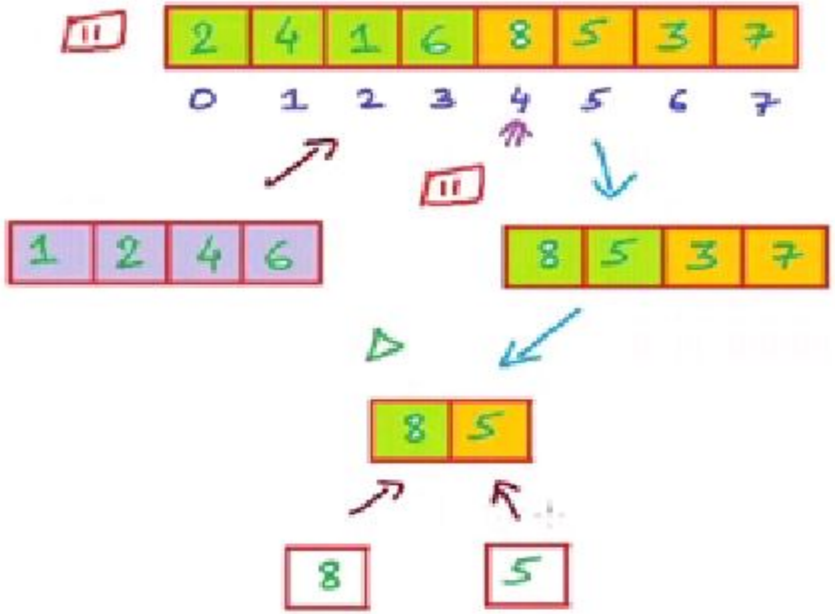
3
7



Base Condition

```
Mergesort (A)
{ n ← length(A)
  if (n < 2) return
  mid ← n/2
  left ← array of size(mid)
  right ← array of size(n-mid)
  for i ← 0 to mid-1
     left[i] ← A[i]
  for i ← mid to n-1
     right[i-mid] ← A[i]
  MergeSort(left)
  Mergesort(right)
} Merge(left, right, A)
```


2 4 1 6 8 5 3 7
0 1 2 3 4 5 6 7
2 4 1 6
2 4
1 6
2
4
1
6


2 4 1 6 8 5 3 7
0 1 2 3 4 5 6 7
8 5 3 7
1 2 4 6
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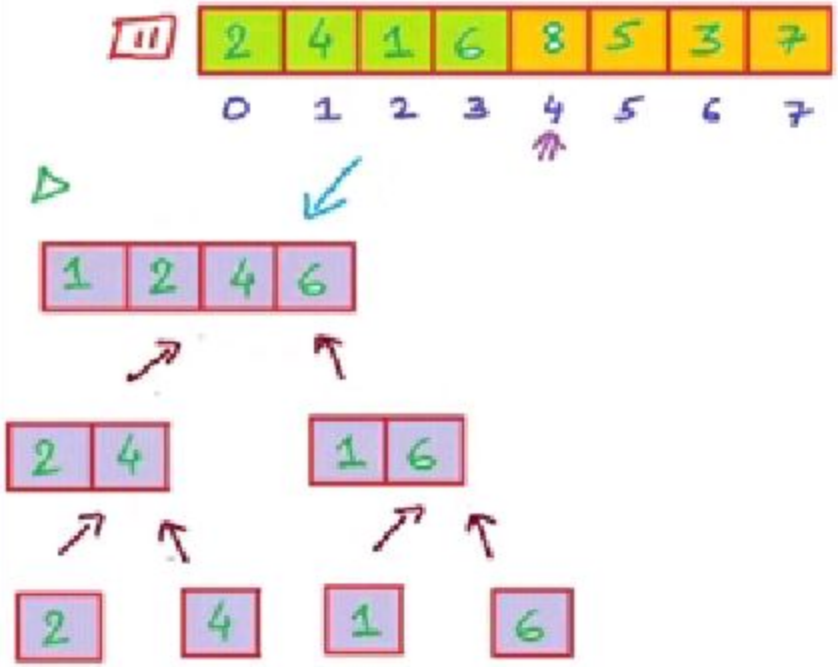
5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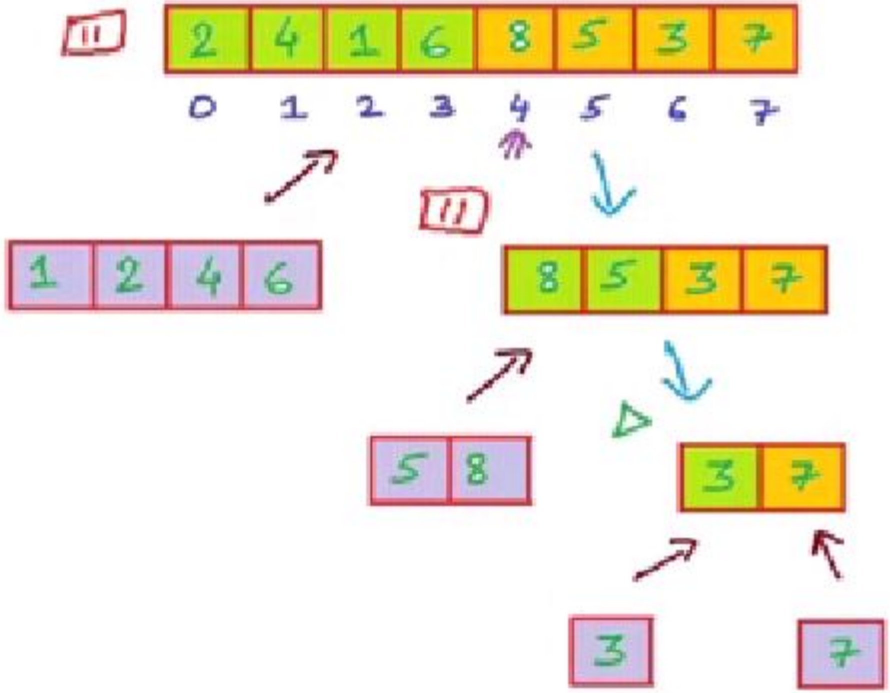
2 4
1 6
2
4
1
6


2 4 1 6 8 5 3 7
0 1 2 3 4 5 6 7
1 2 4 6
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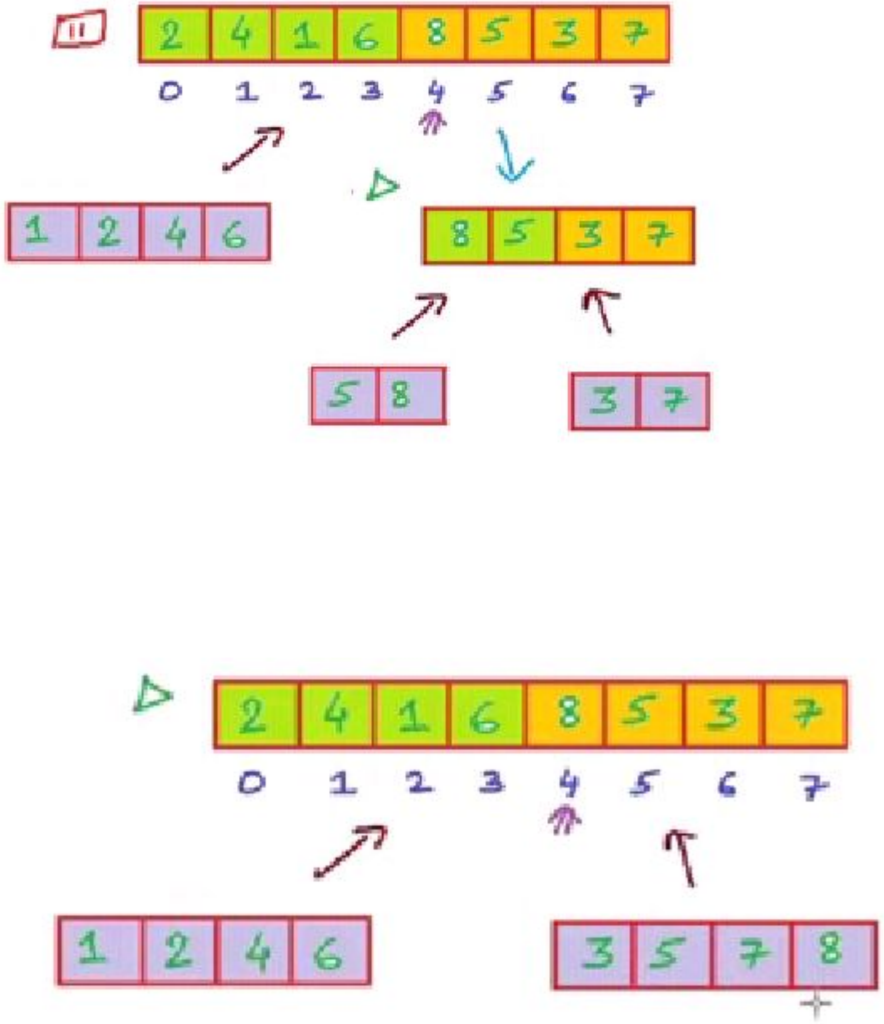
5 8
3 7
3
7

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   for i ← 0 to mid-1
      left[i] ← A[i]
   for i ← mid to n-1
      right[i-mid] ← A[i]
   Mergesort(left)
   Mergesort(right)
}  Merge(left, right, A)
```

1	2	3	4	5	6	7	8
0	1	2	3	4	5	6	7

```
Mergesort (A)
{ n ← length(A)
  if (n < 2) return                     ← base Condition
  mid ← n/2
  left ← array of size(mid)
  right ← array of size(n-mid)
  for i ← 0 to mid-1
      left[i] ← A[i]
  for i ← mid to n-1
      right[i-mid] ← A[i]
  Mergesort (left)                      ← Recursive calls
  Mergesort (right)
  Merge(left, right, A)                 ← Merge sorted halves
}
```

98	23	45	14	6	67	33	42

98	23	45	14	6	67	33	42

98	23	45	14

6	67	33	42

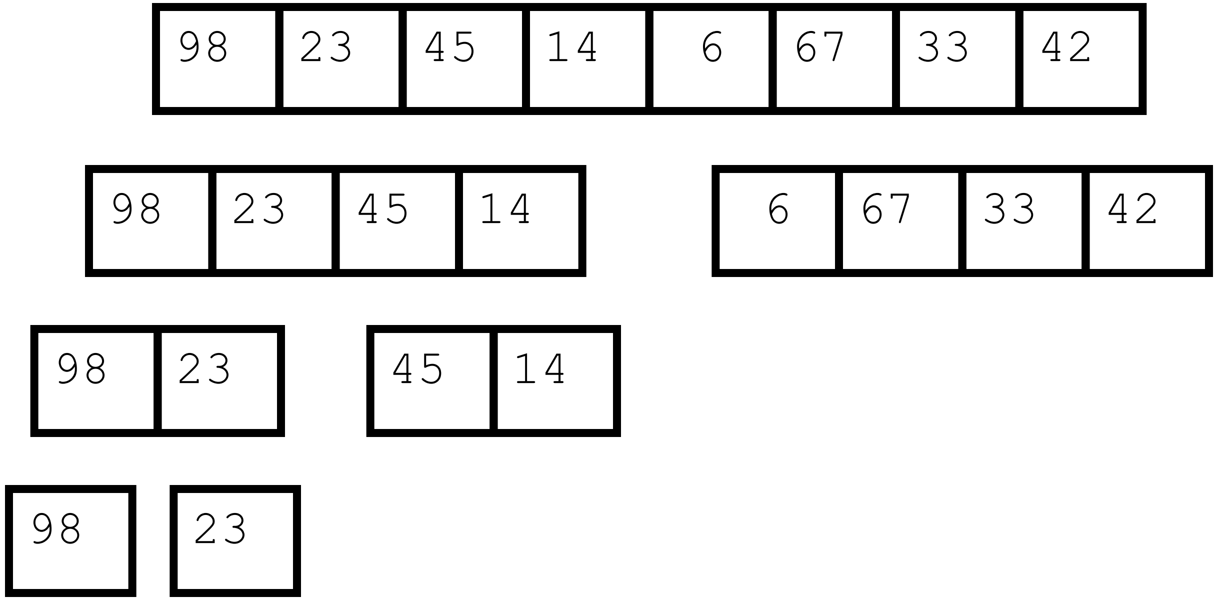
98	23	45	14	6	67	33	42

98	23	45	14

6	67	33	42

98	23

45	14


98 23 45 14 6 67 33 42
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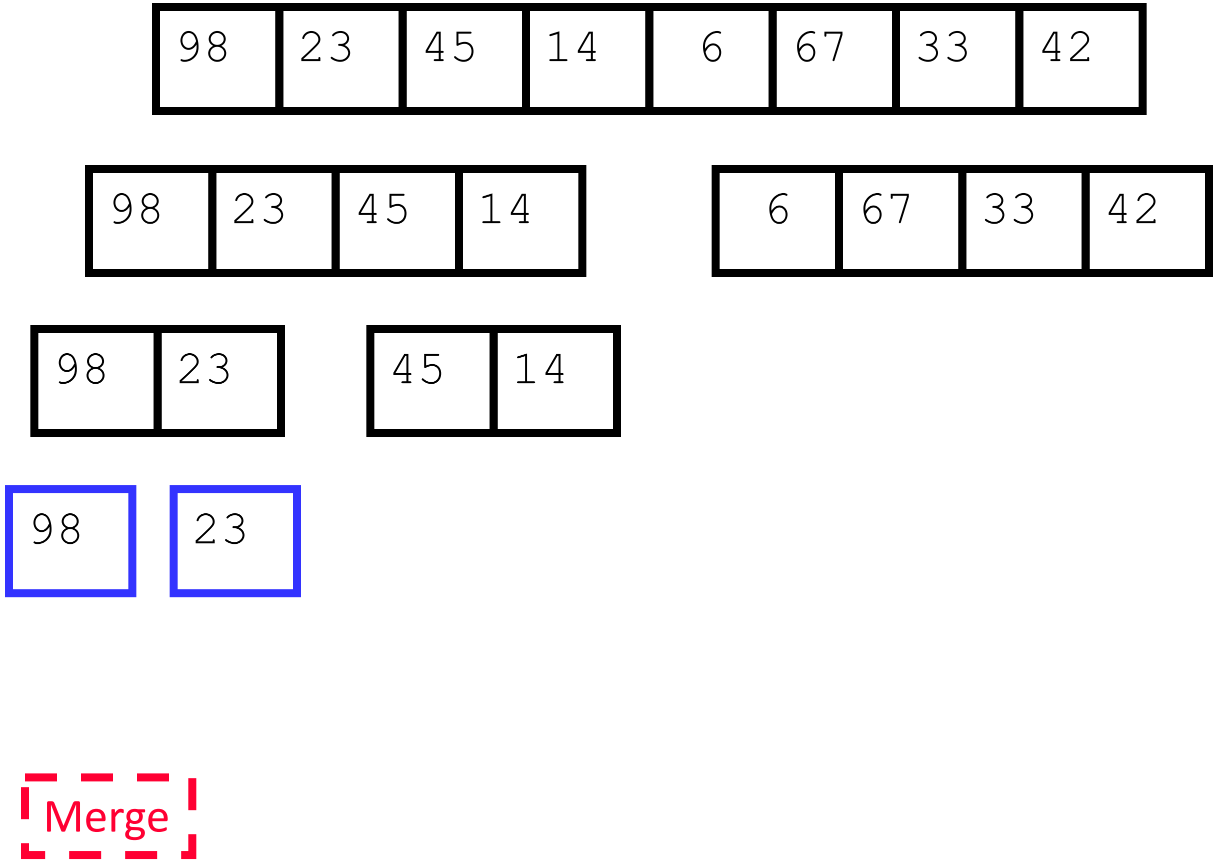
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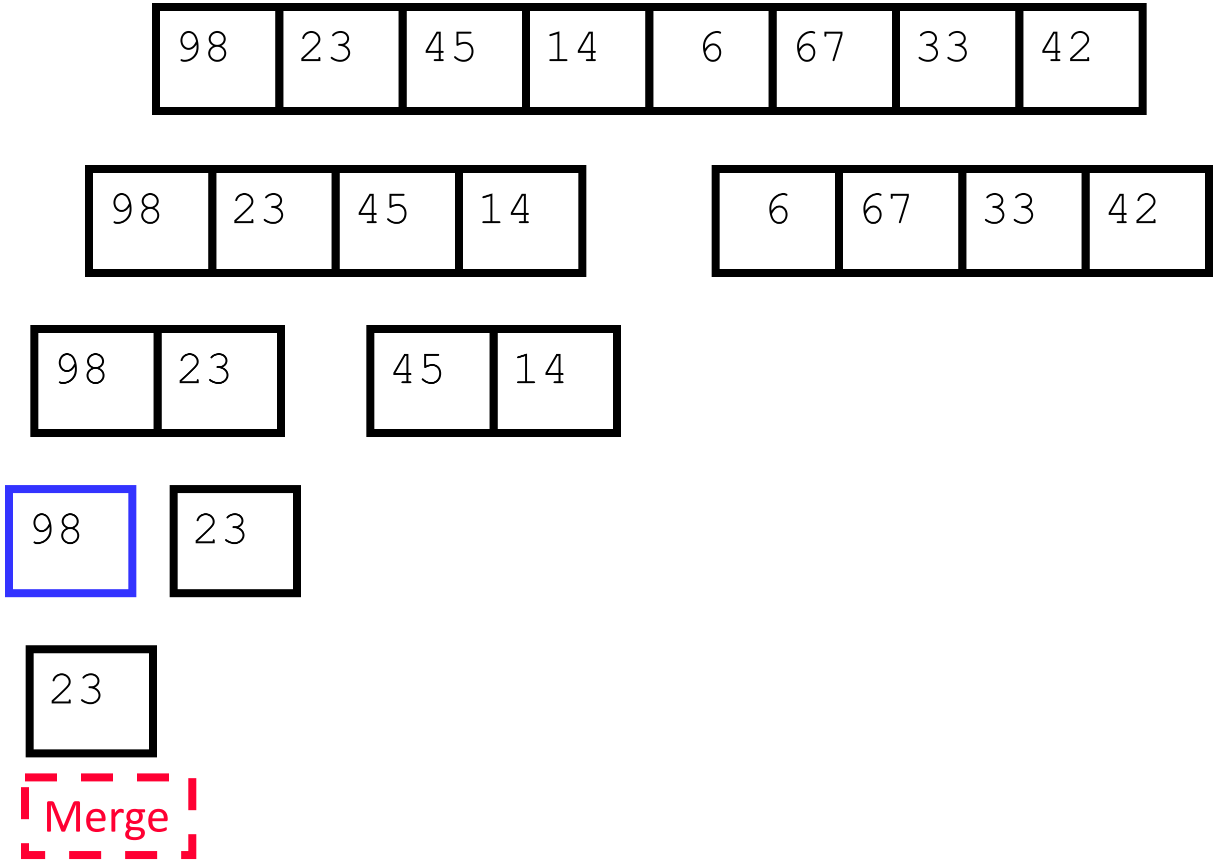
Merge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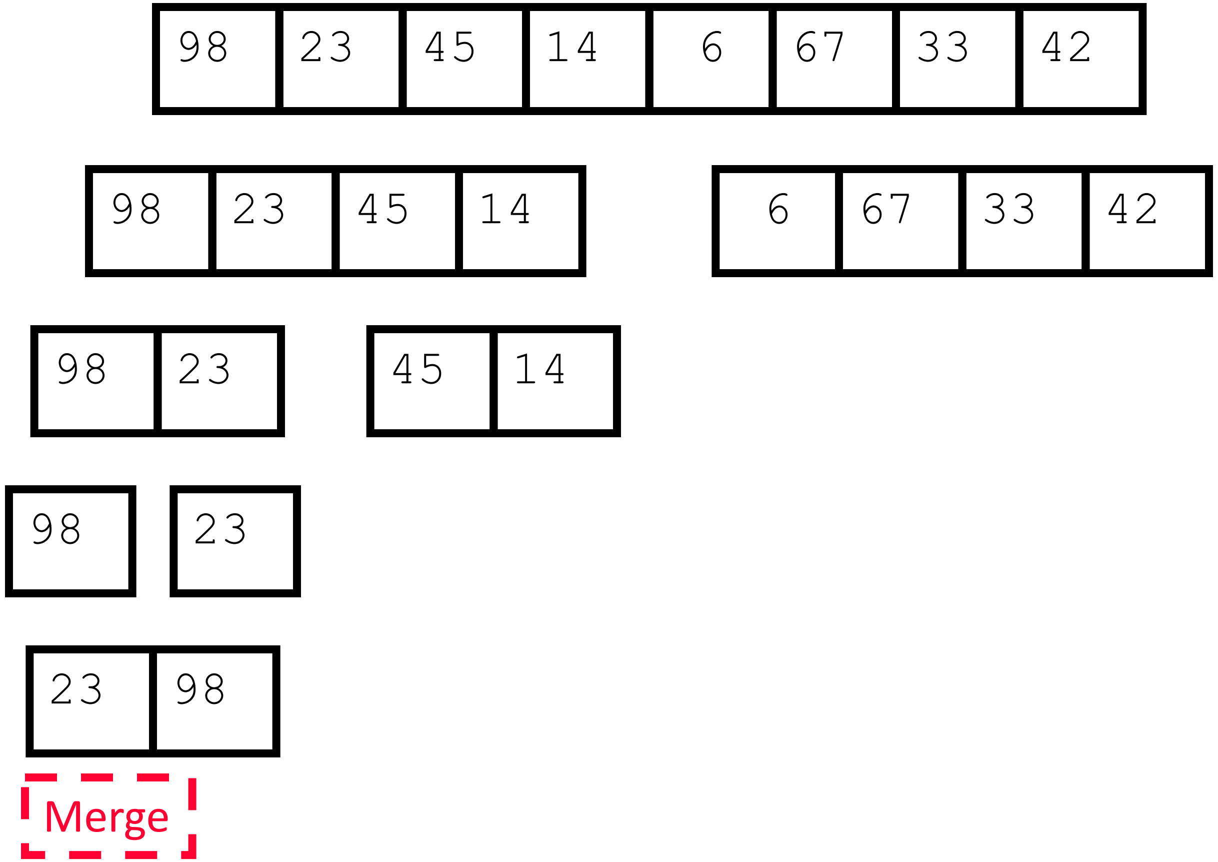
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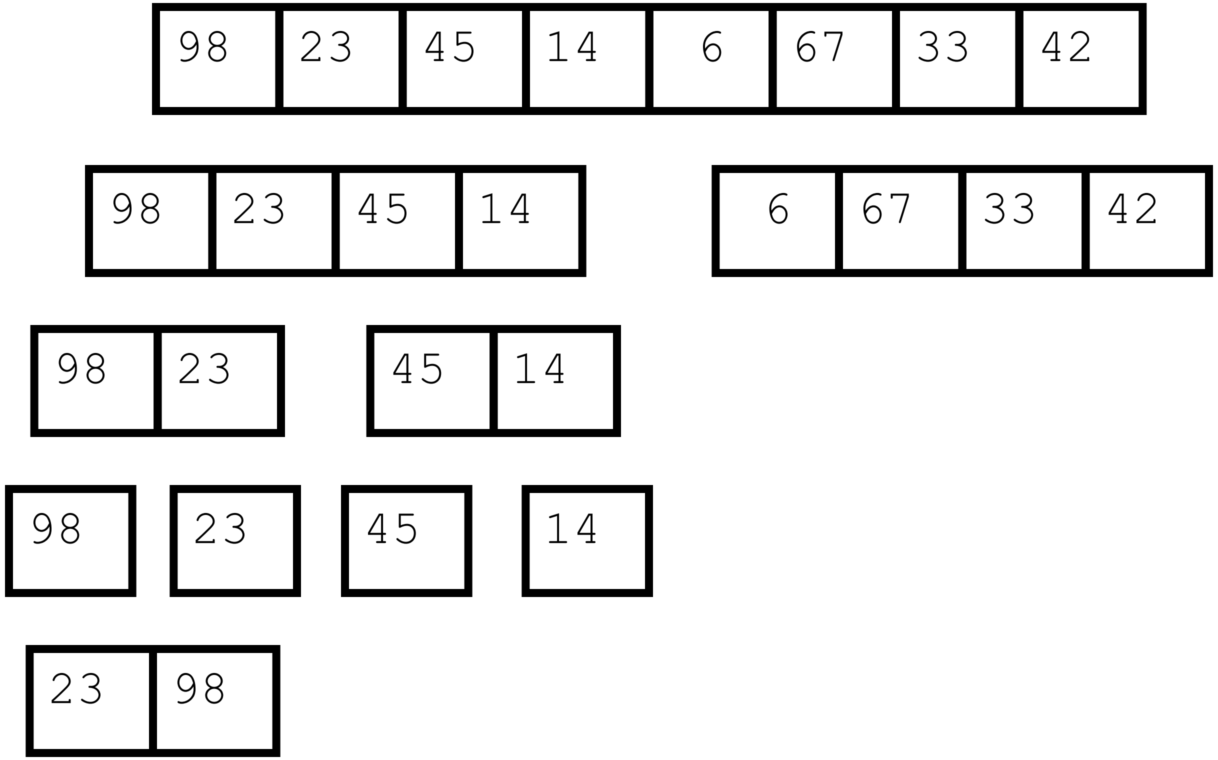
Merge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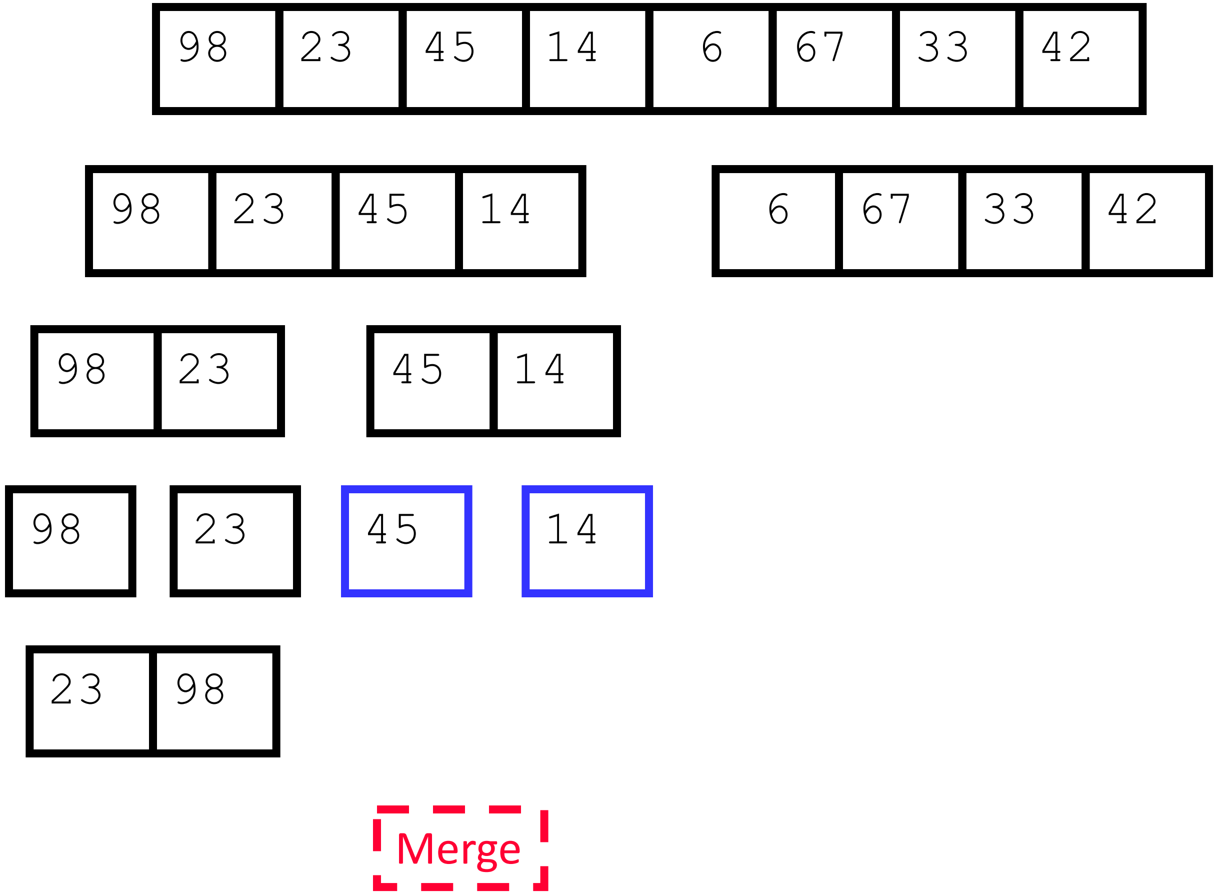
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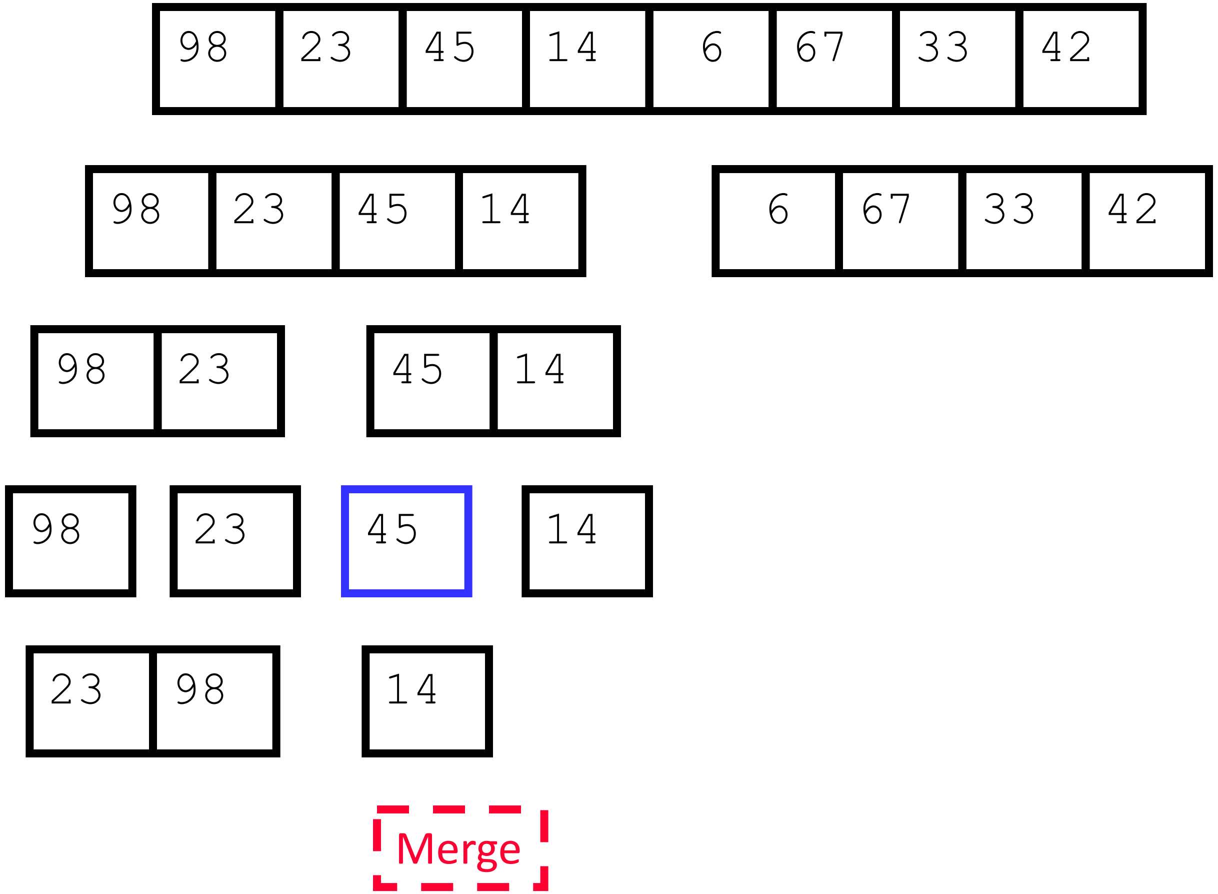
Merge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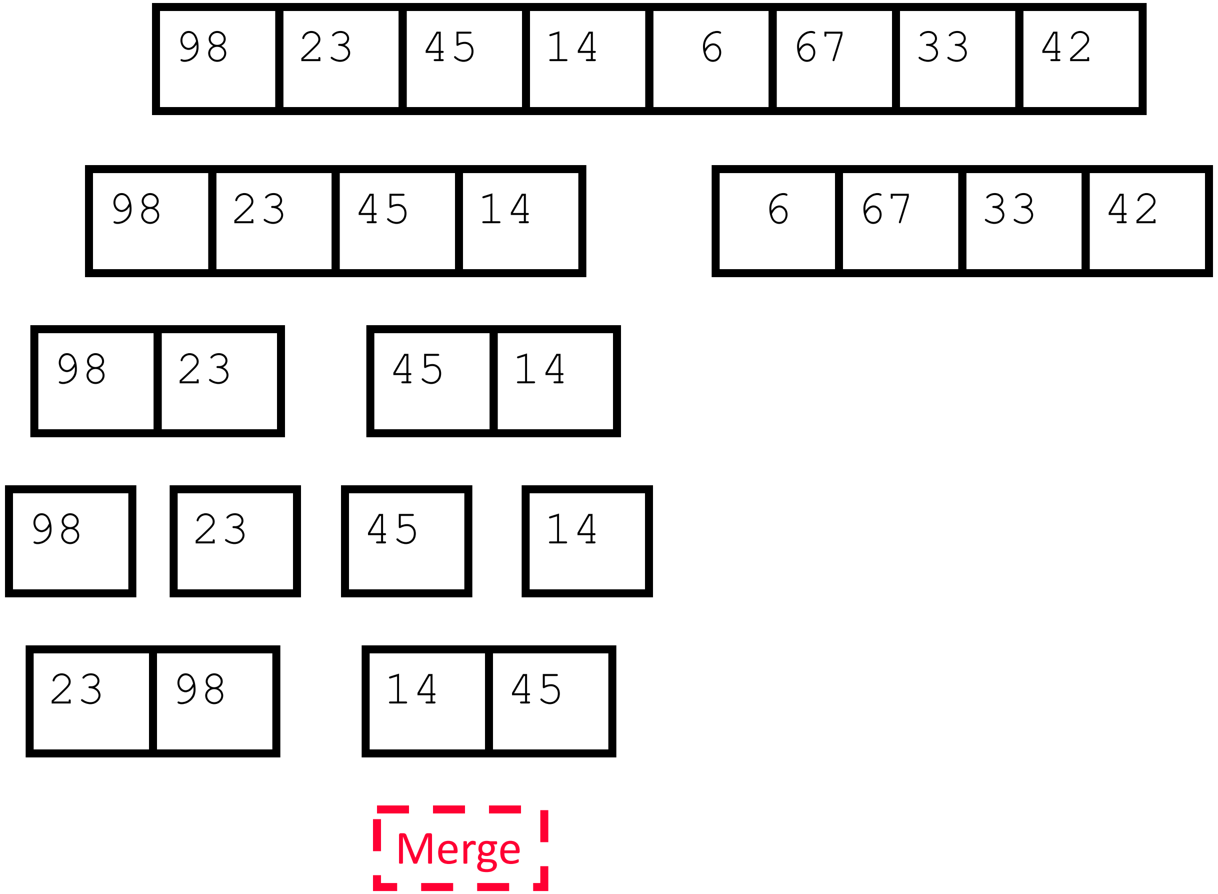
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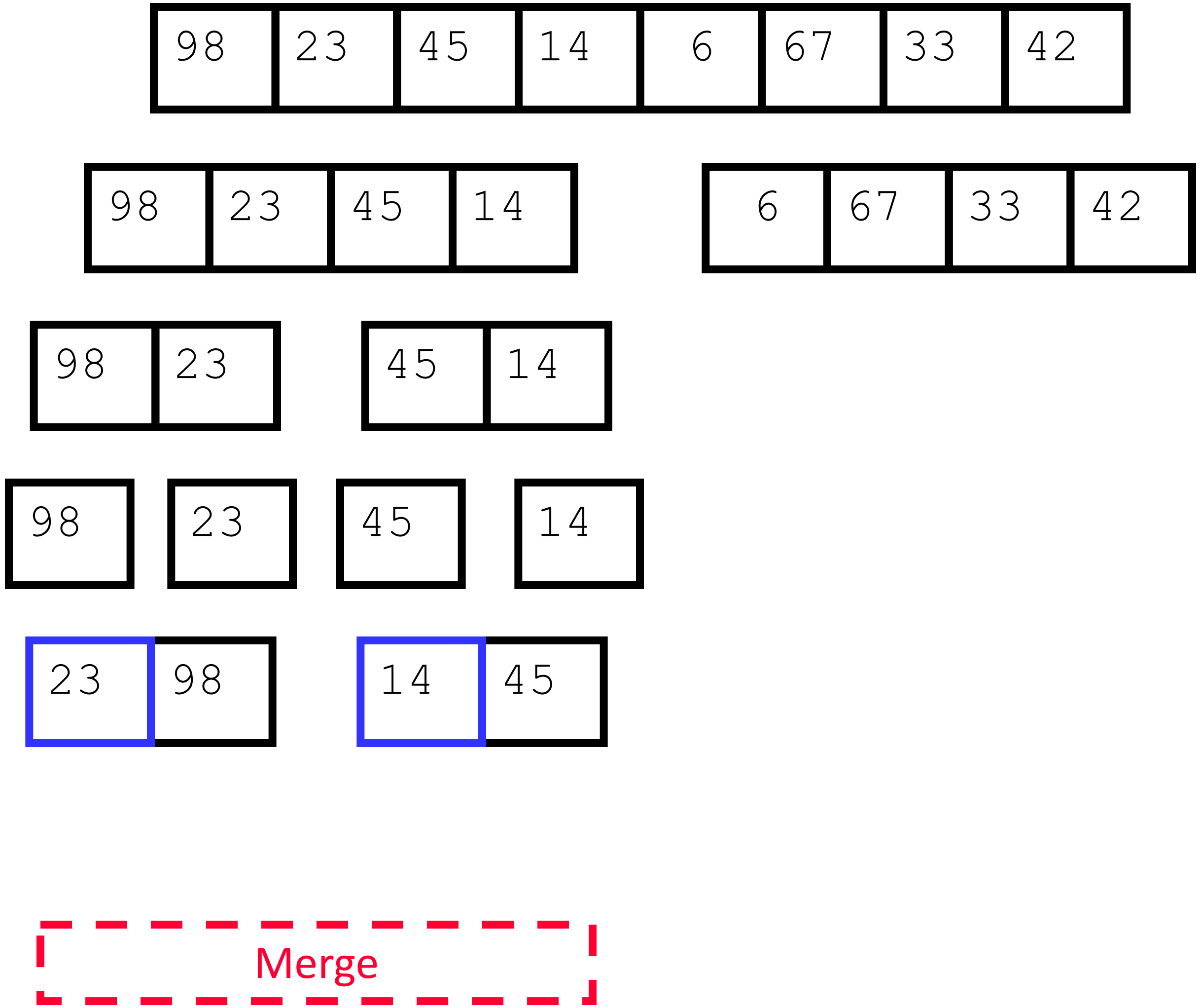
Merge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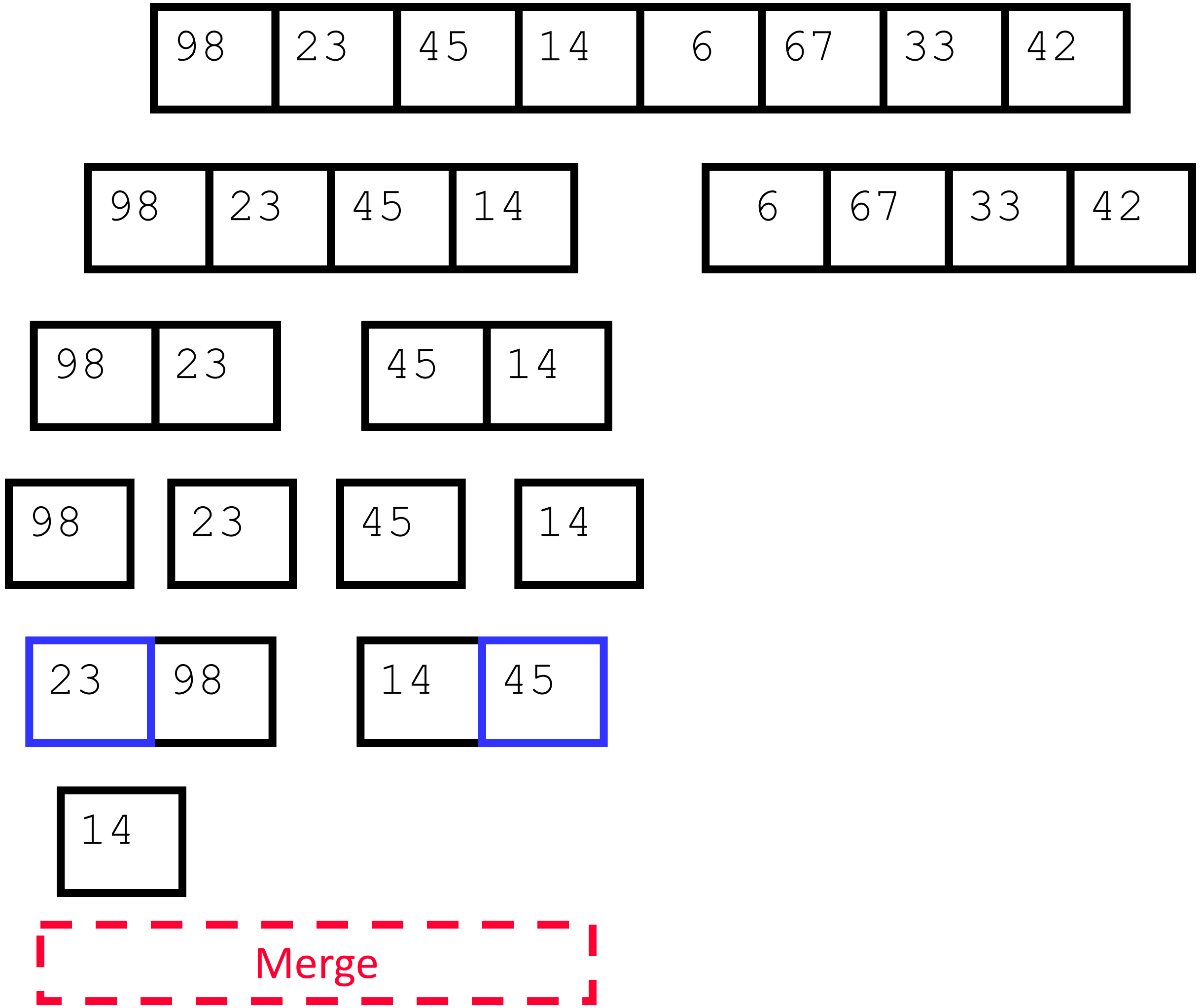
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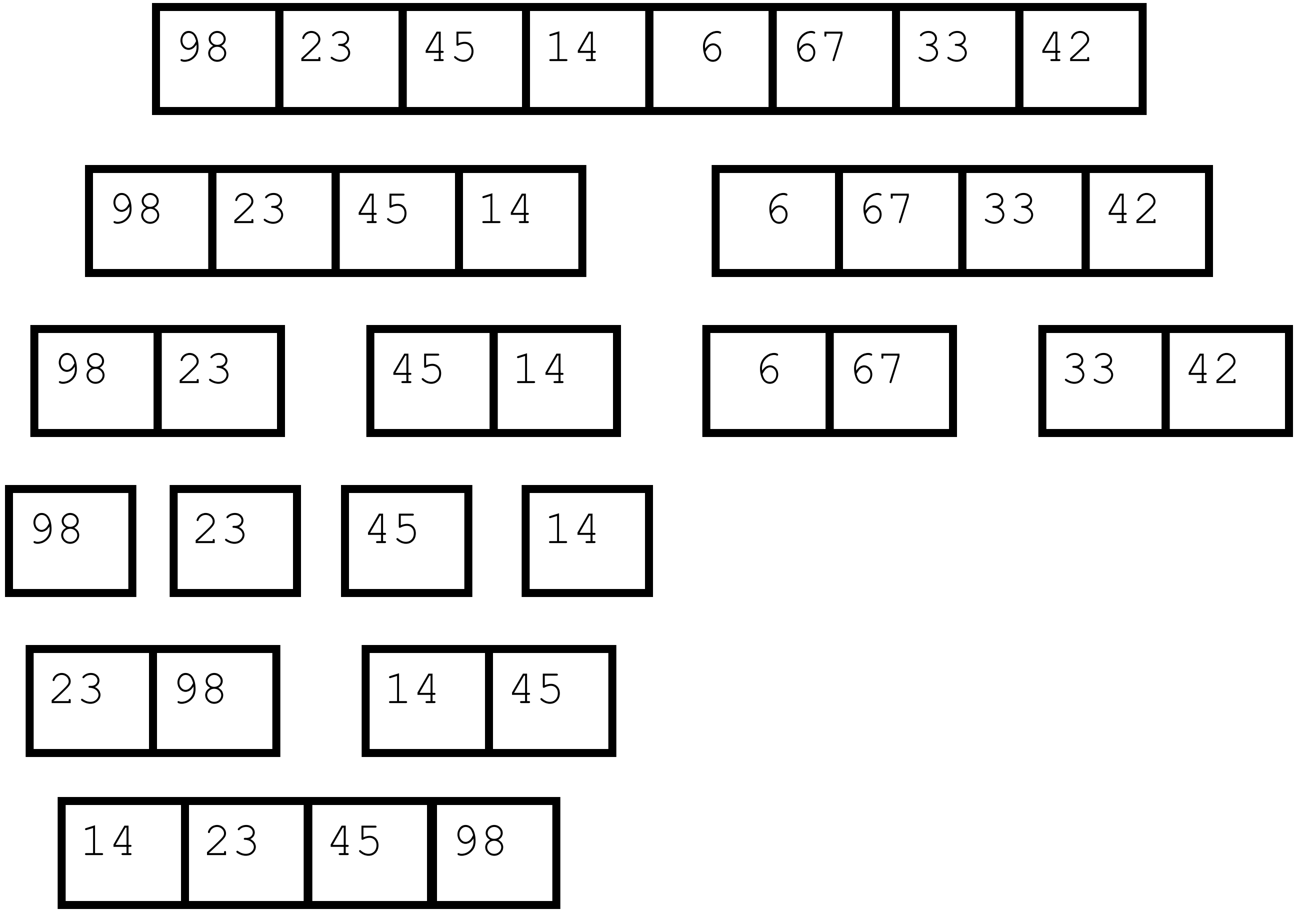
Merge



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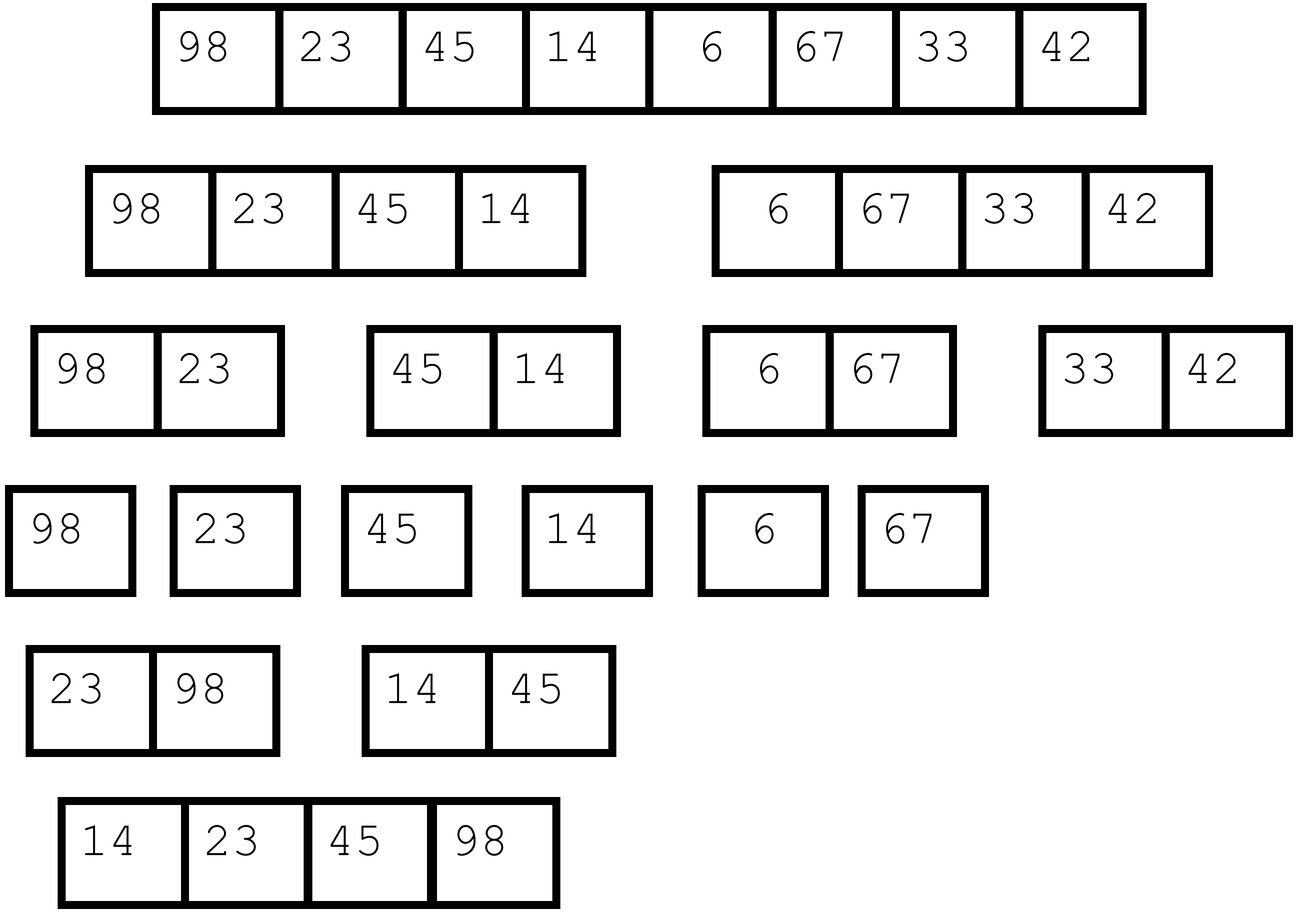
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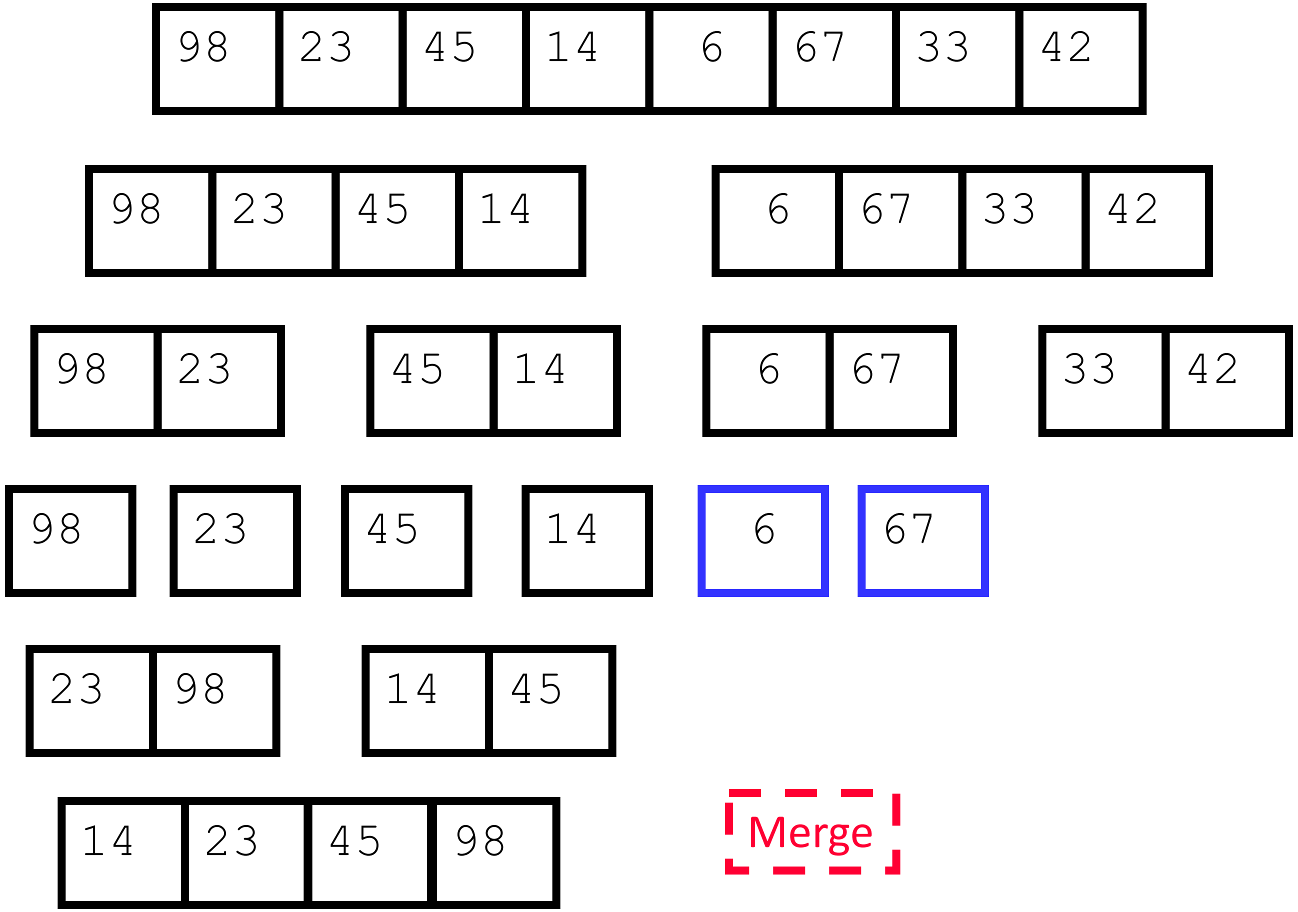
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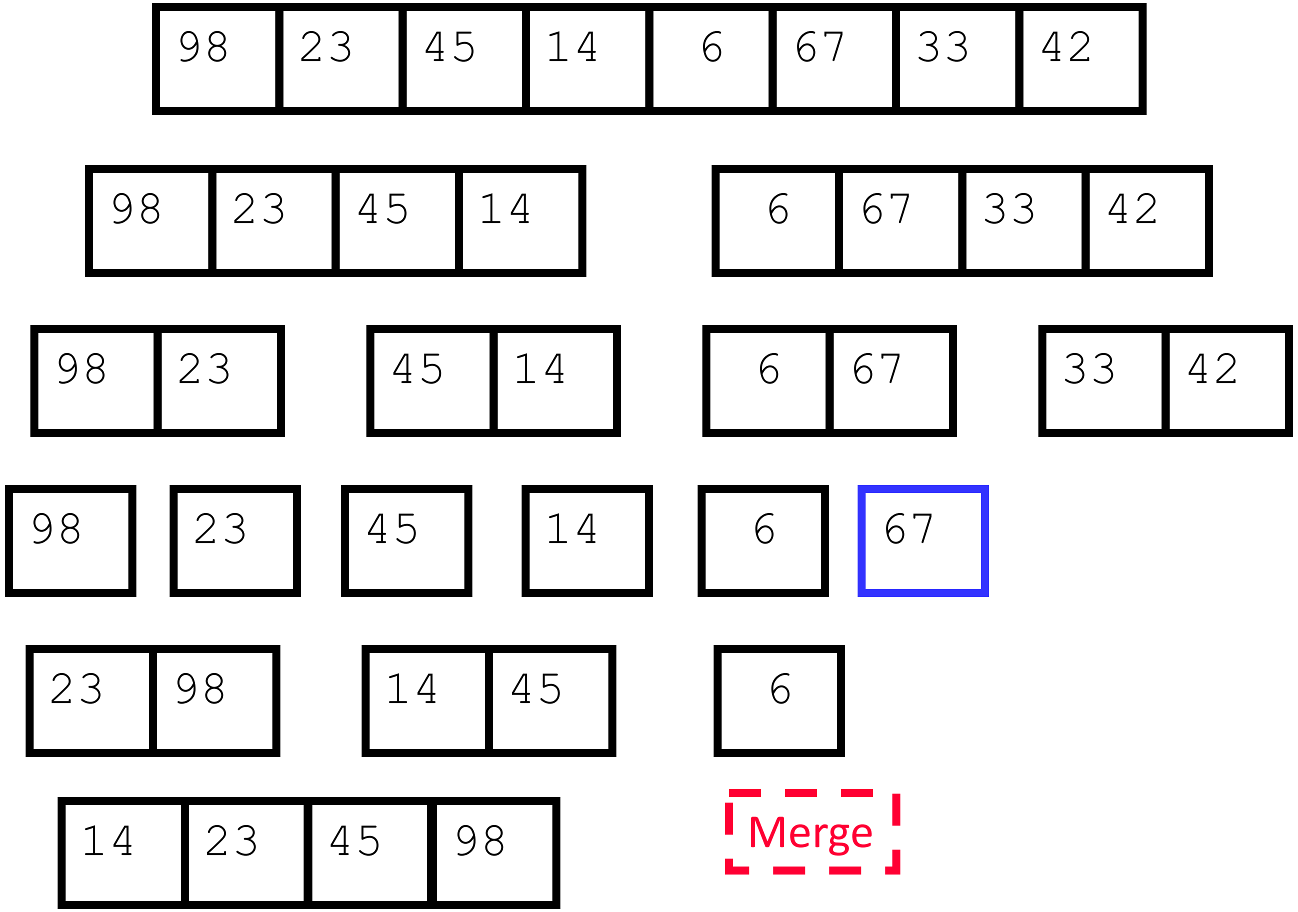
14 23 45 98
Merge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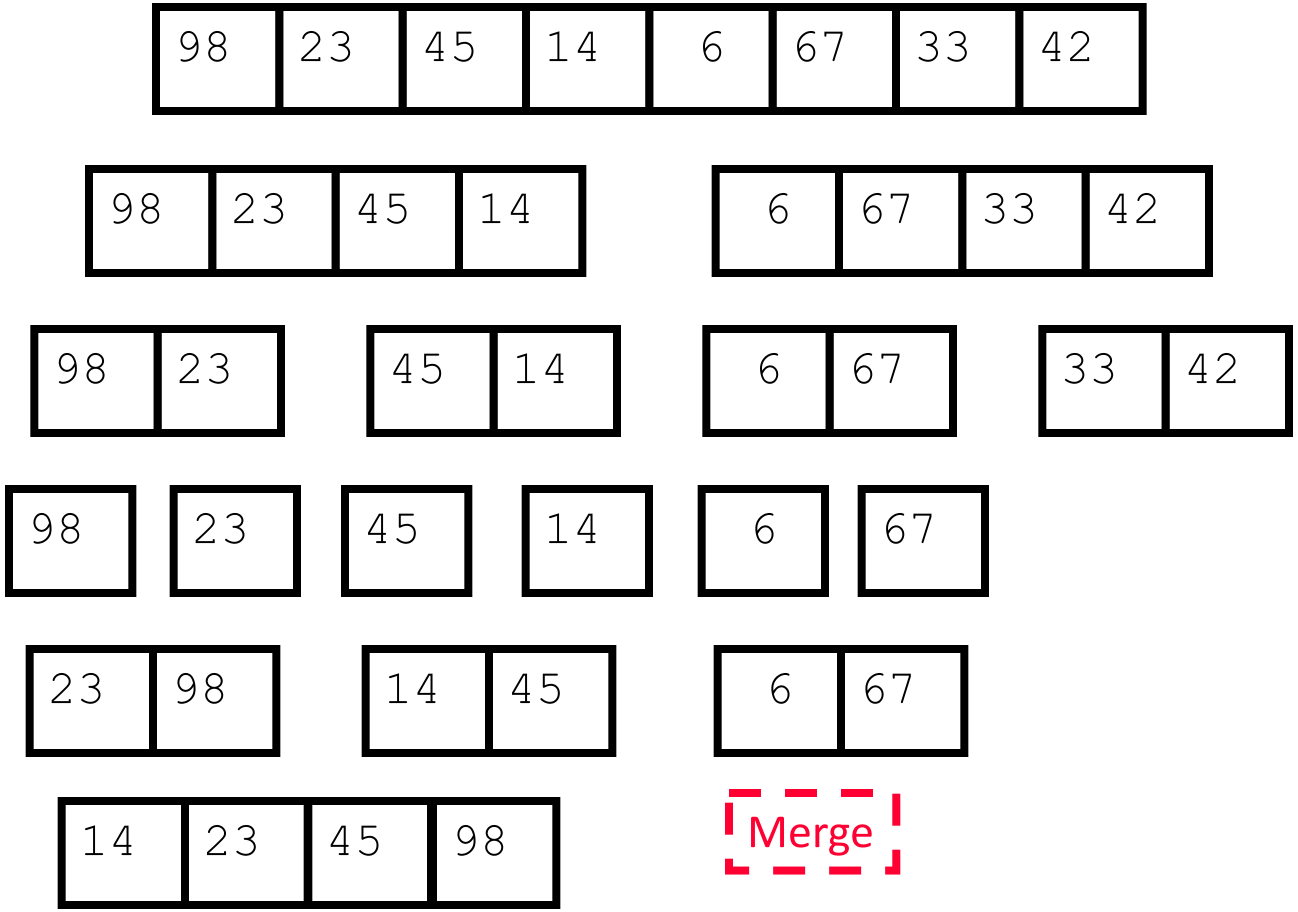
Merge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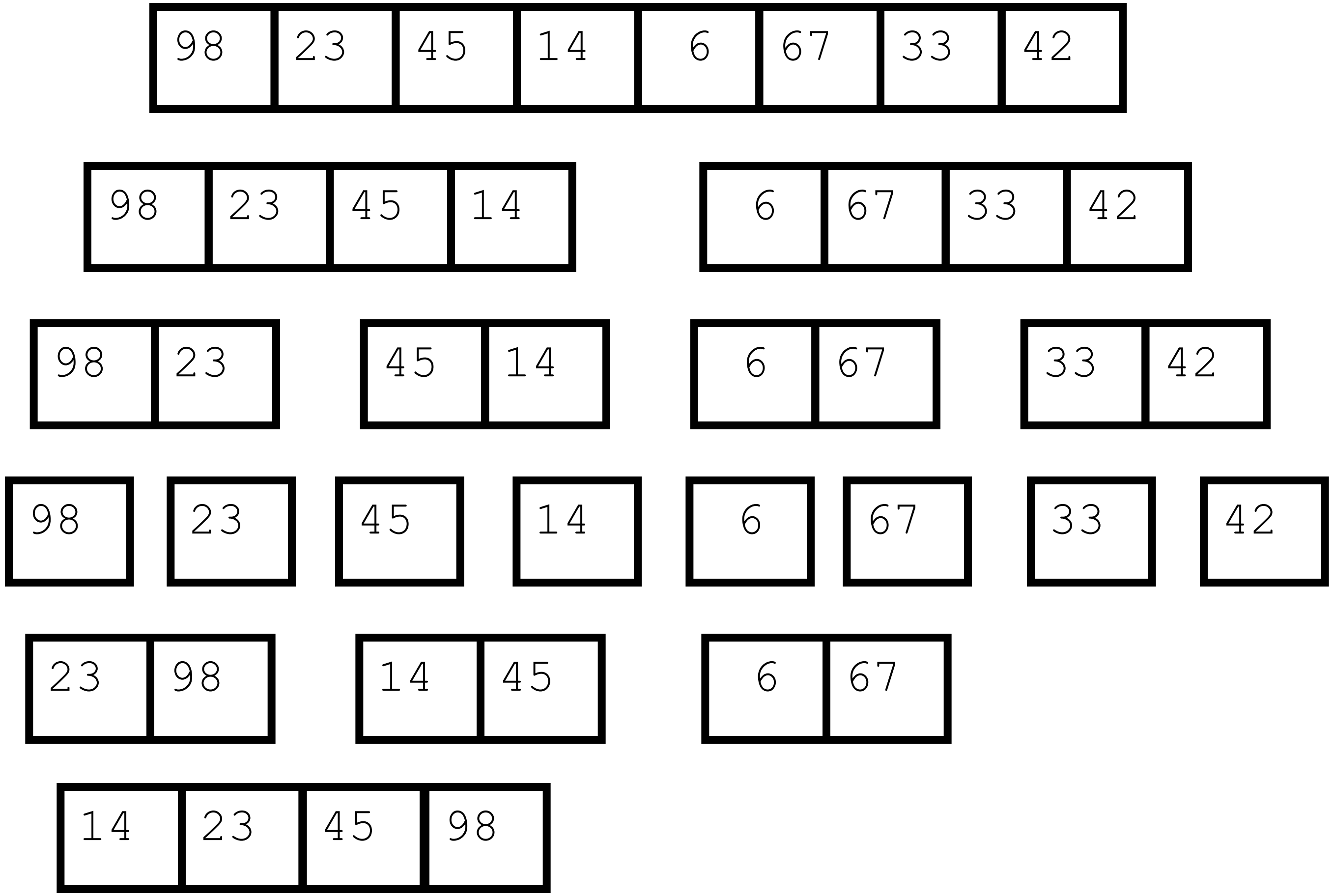
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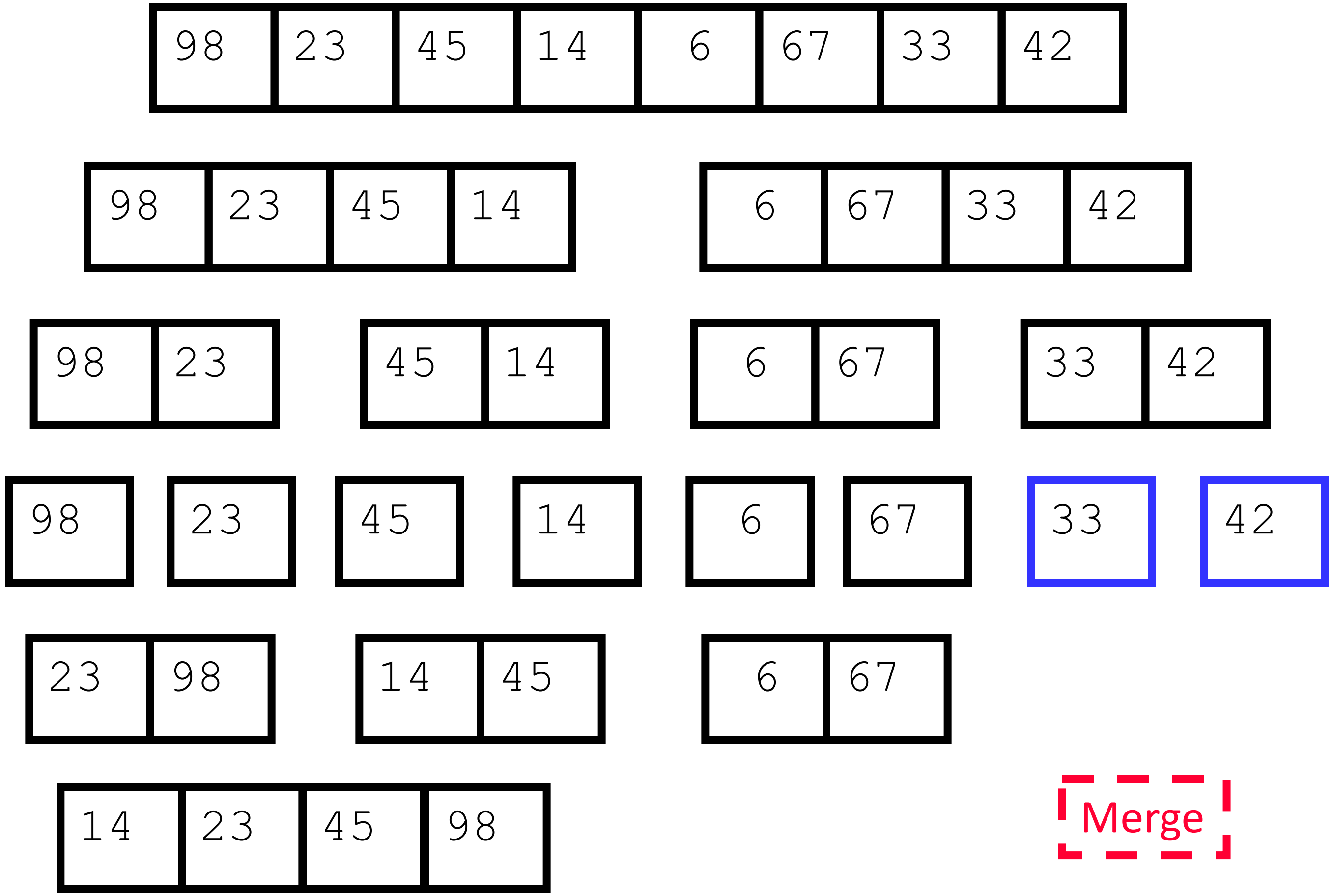
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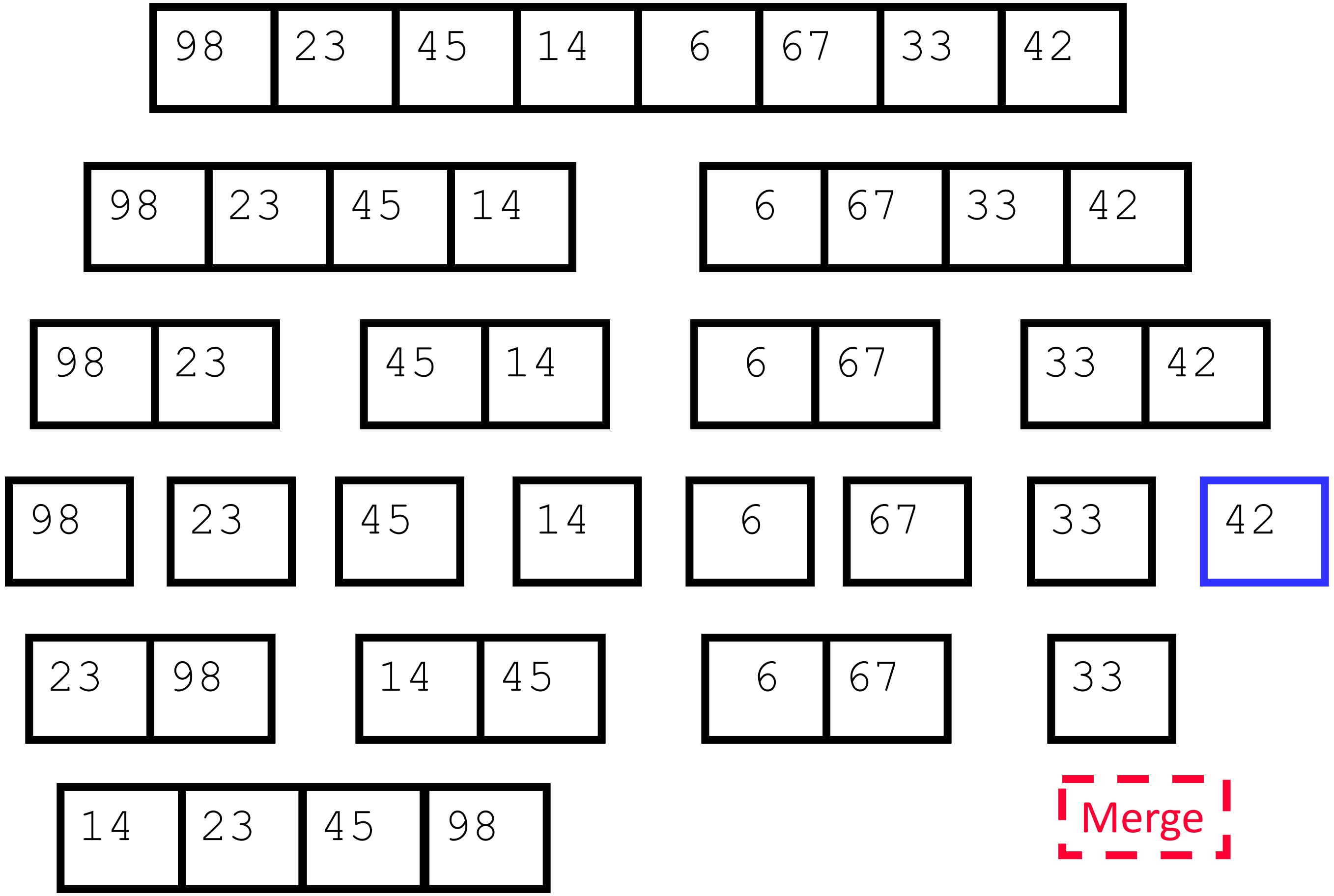
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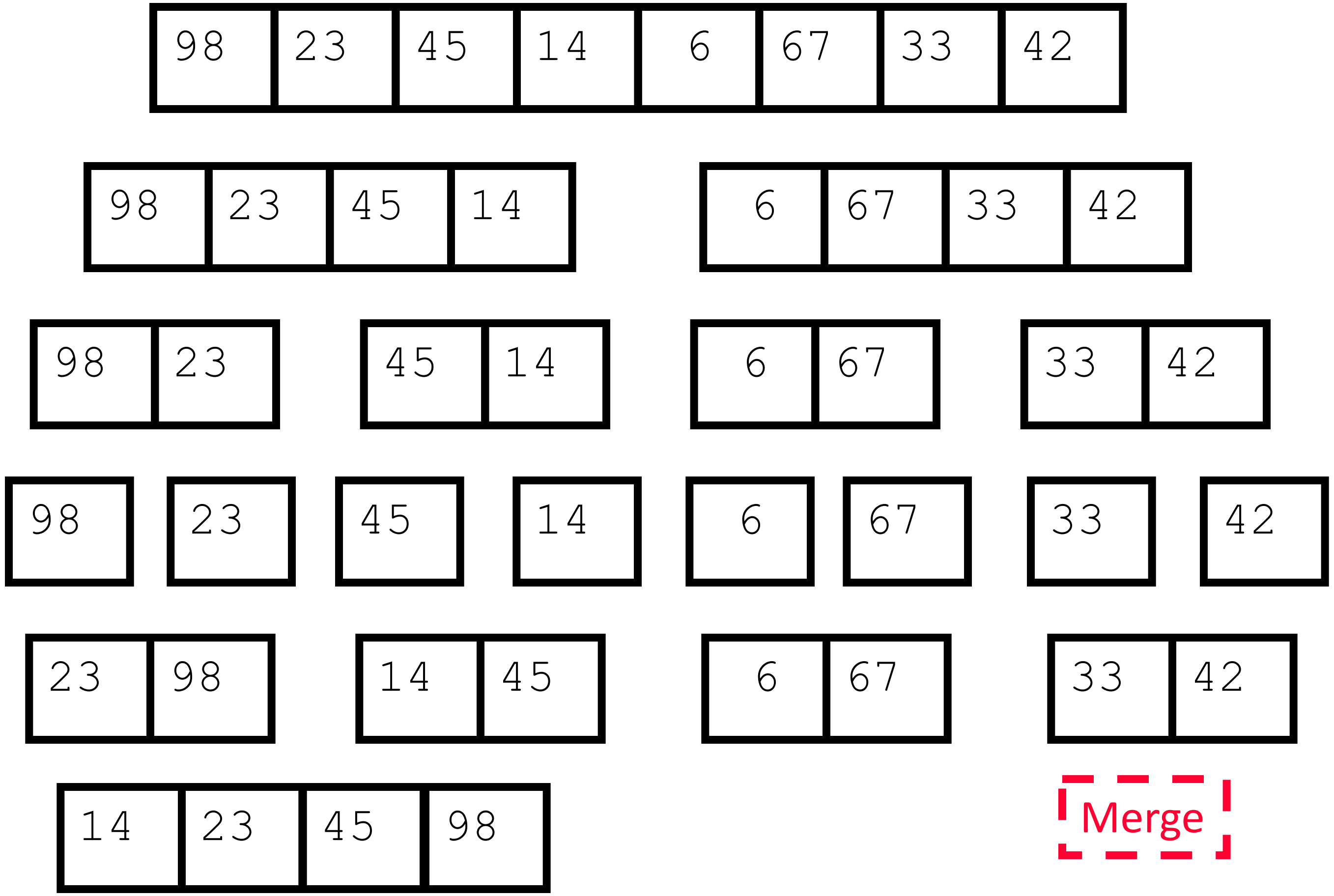
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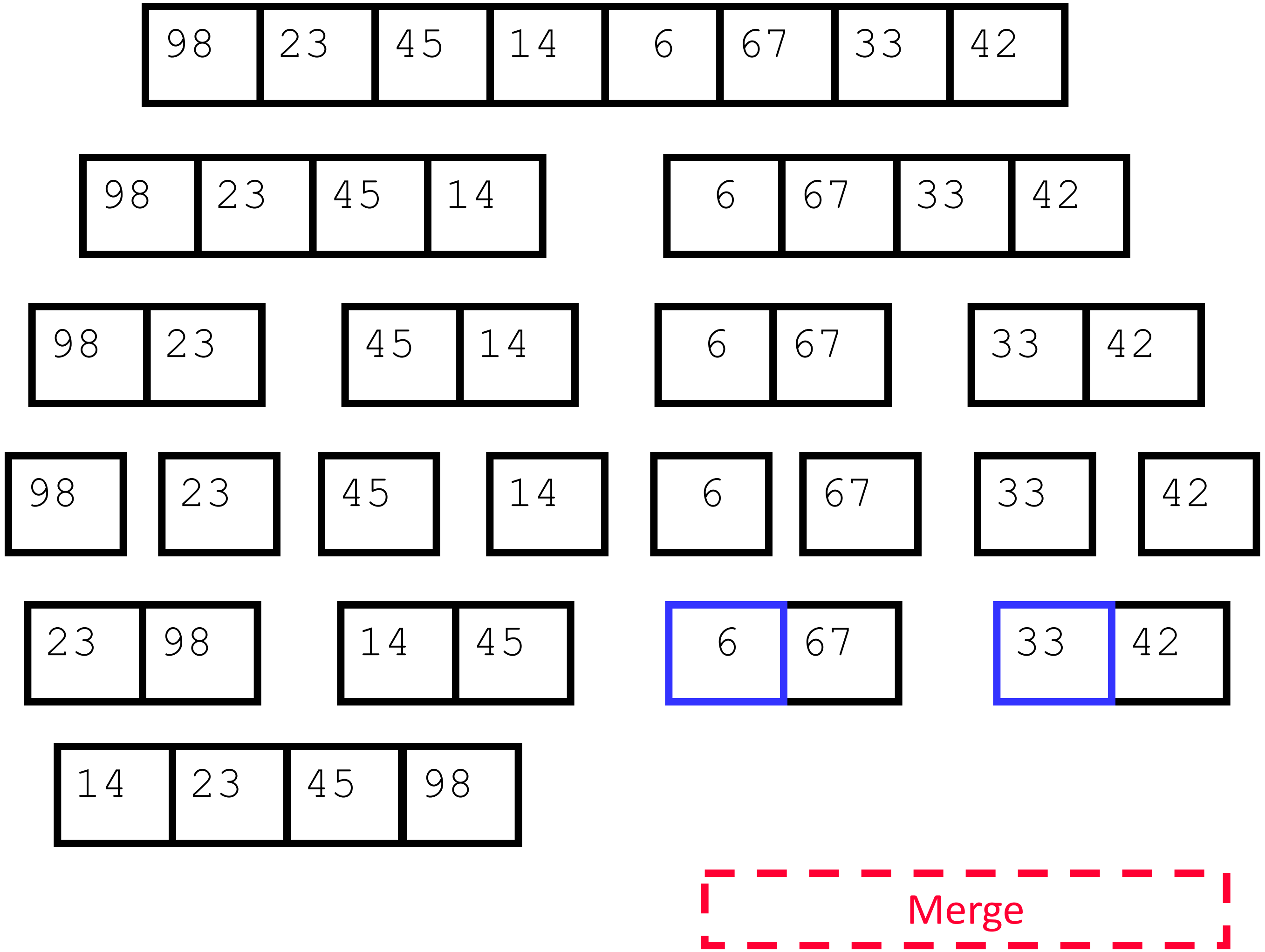
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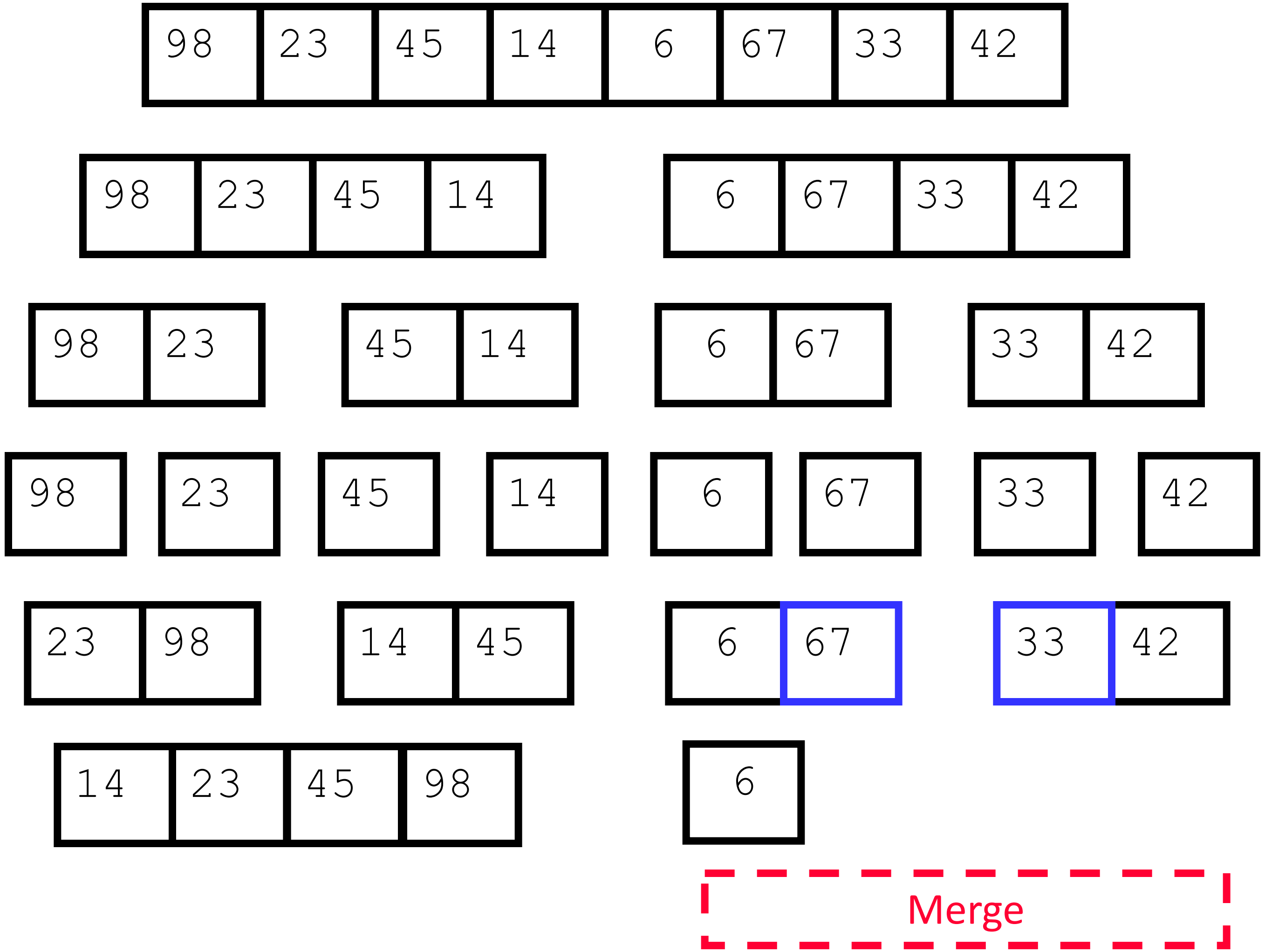

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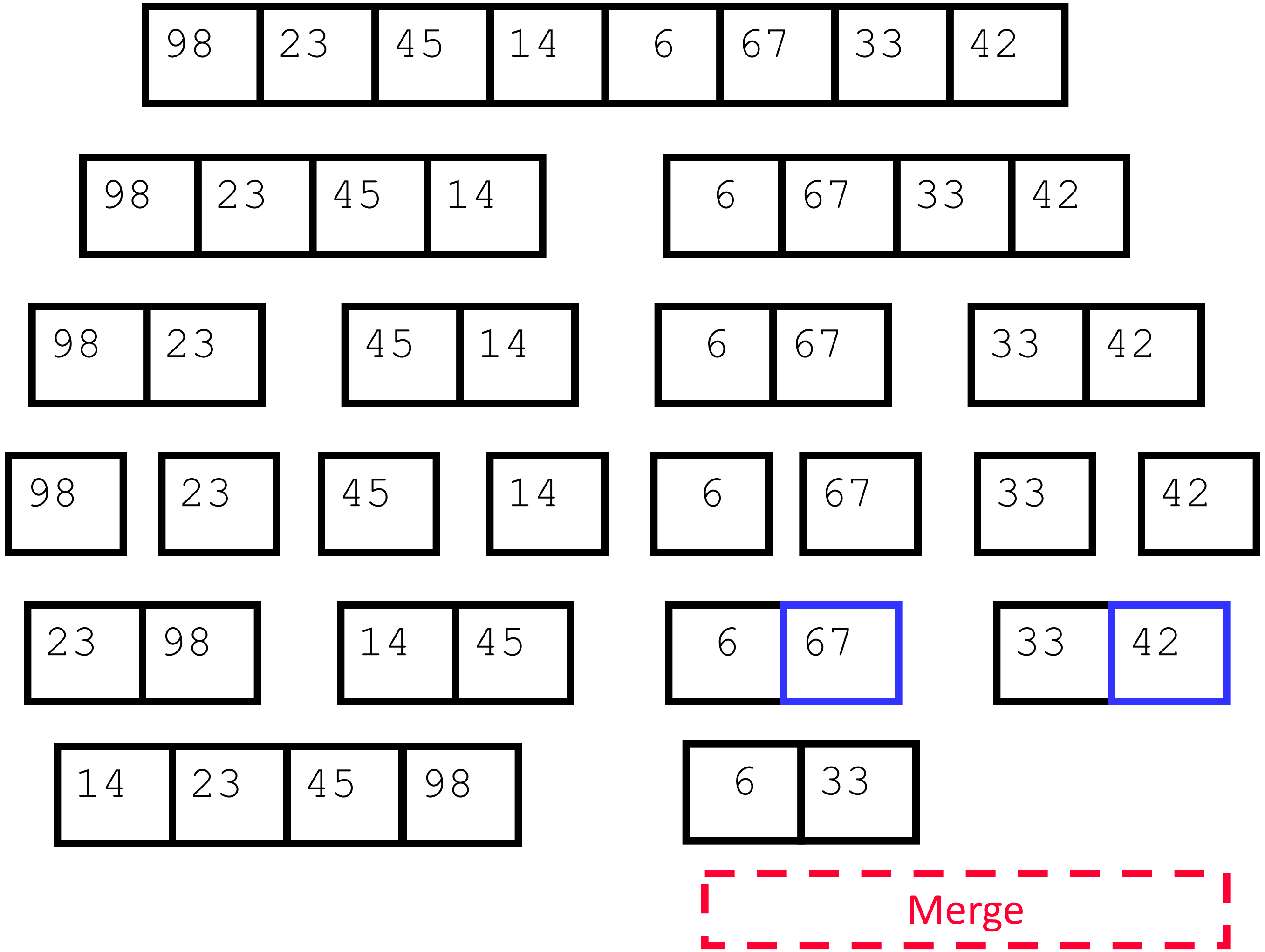
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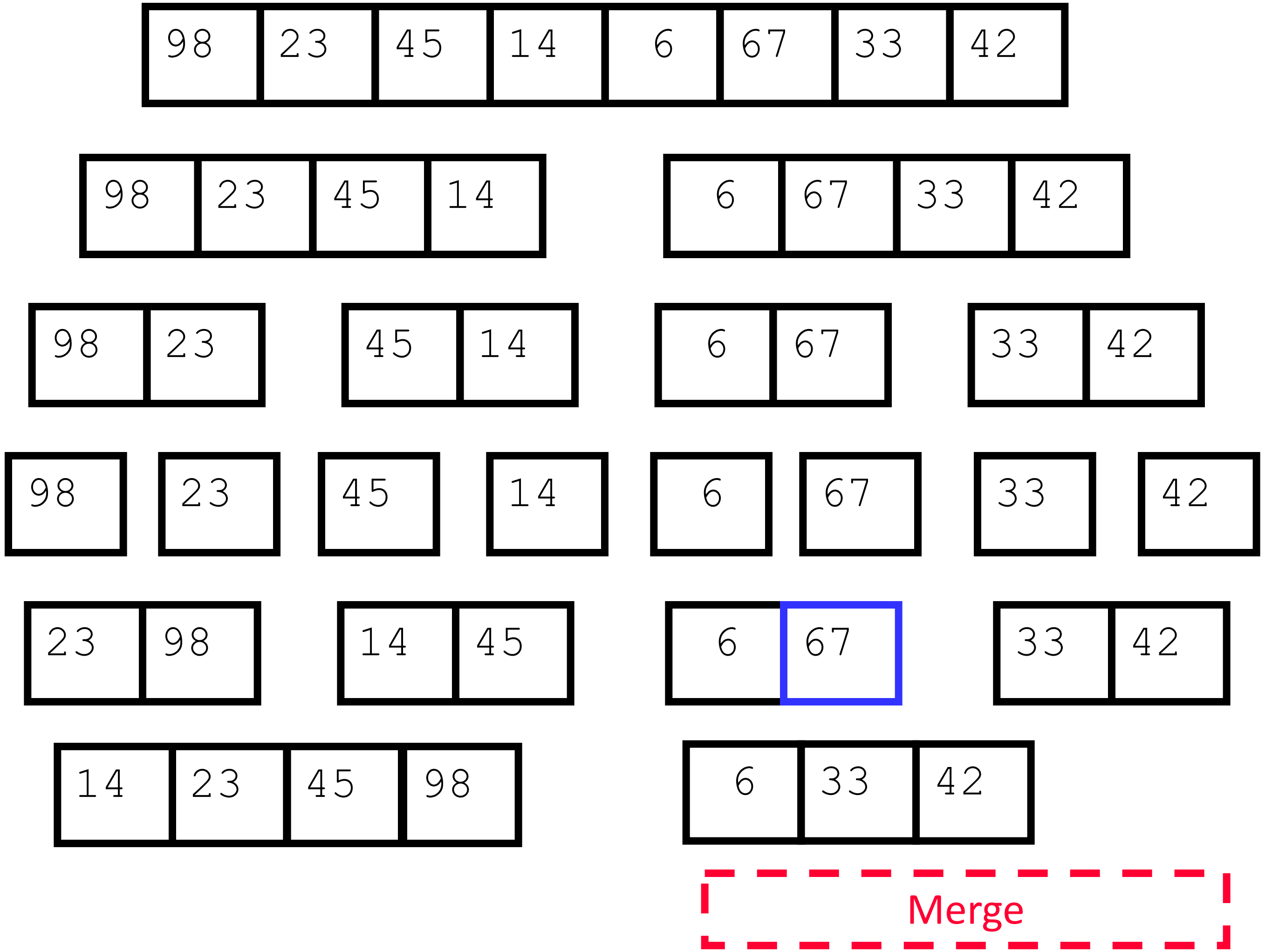
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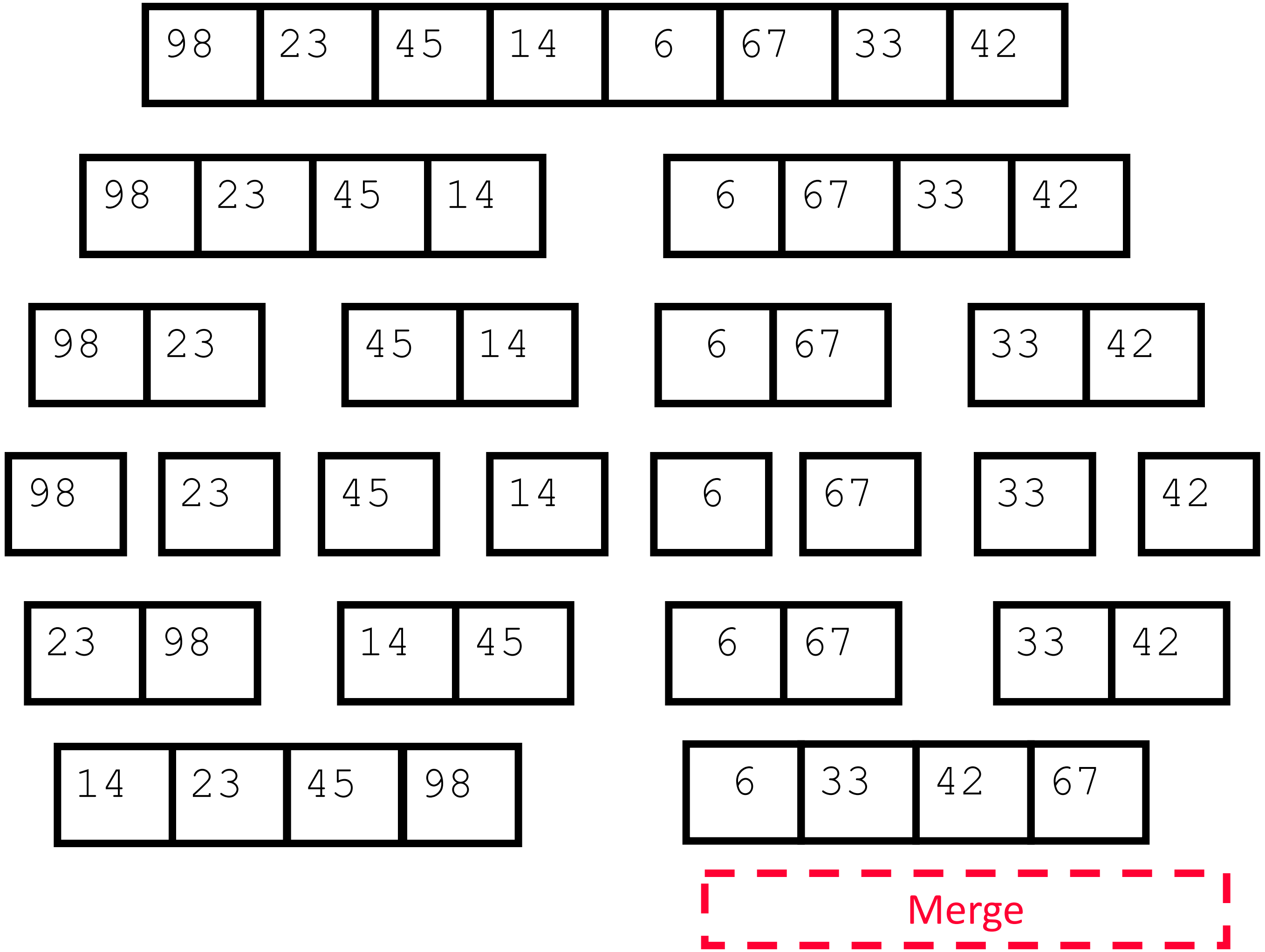
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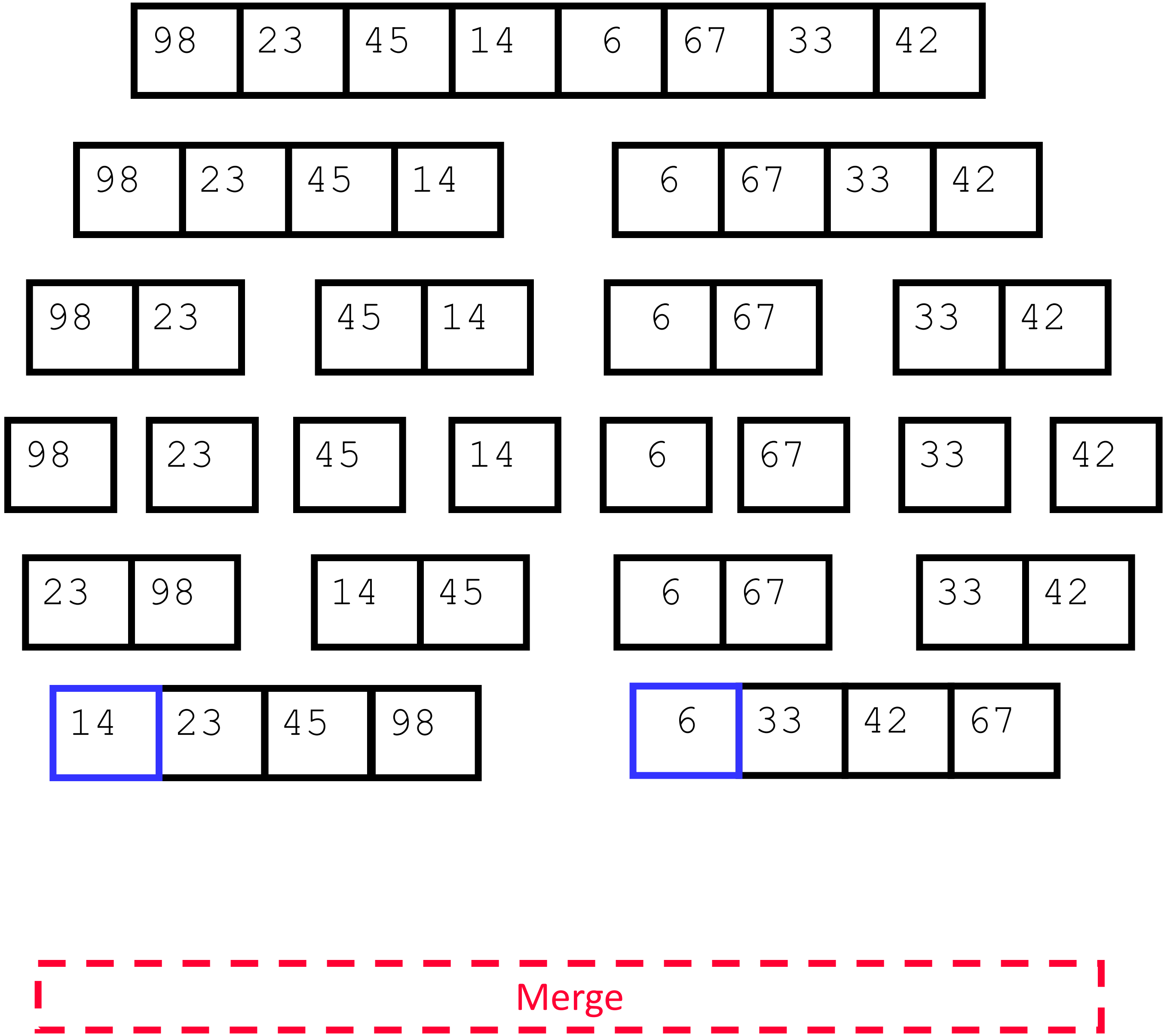
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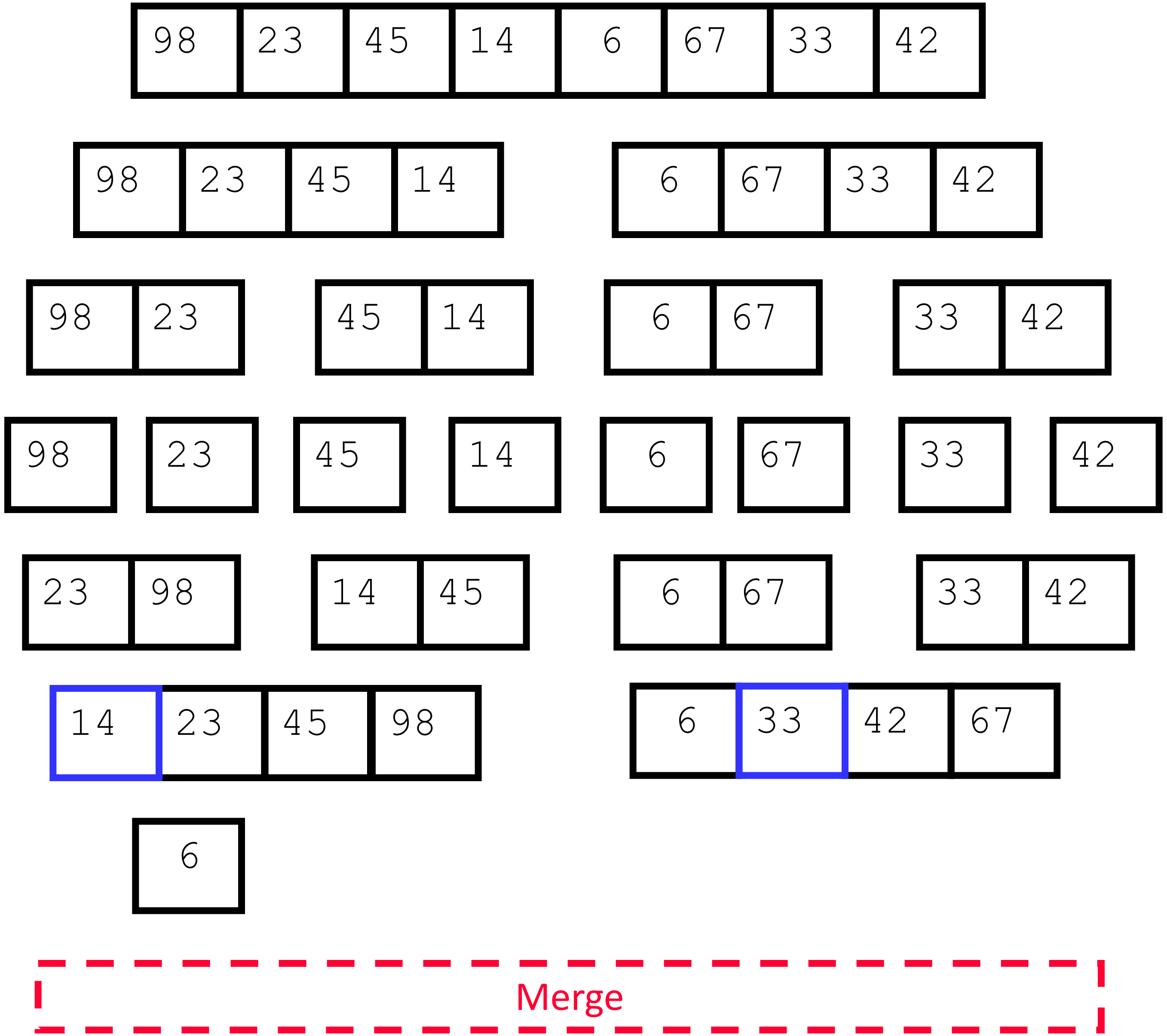
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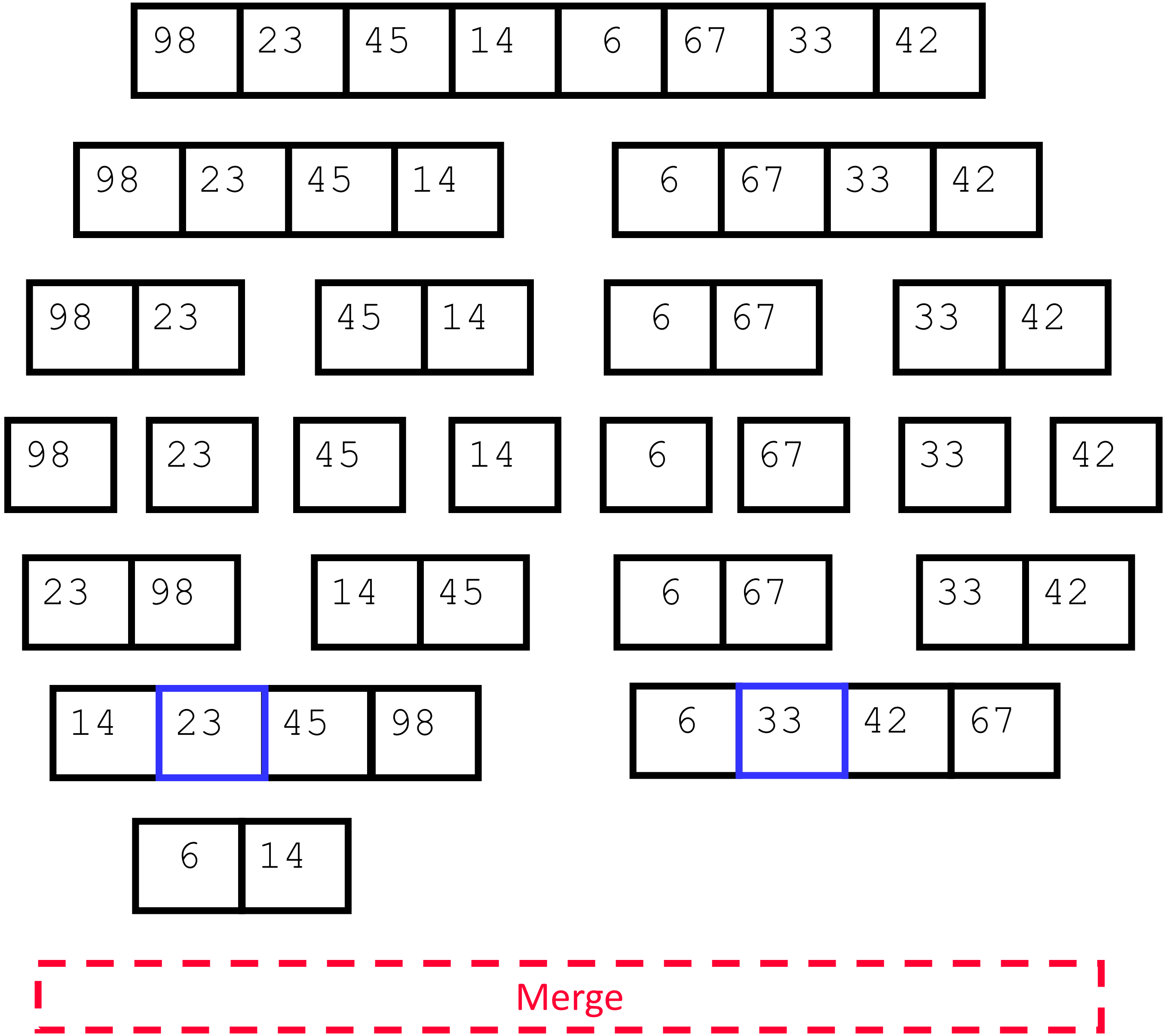
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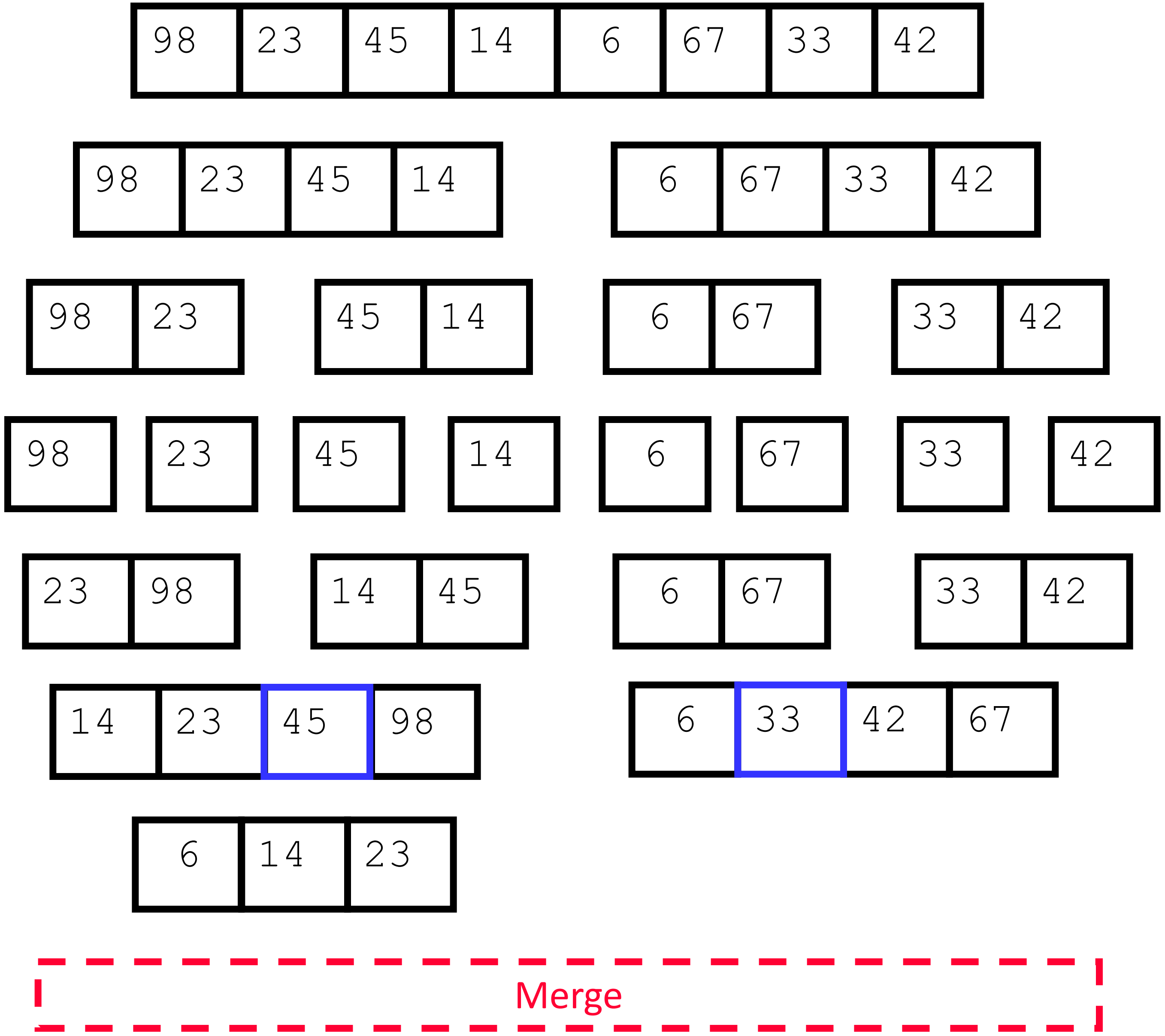
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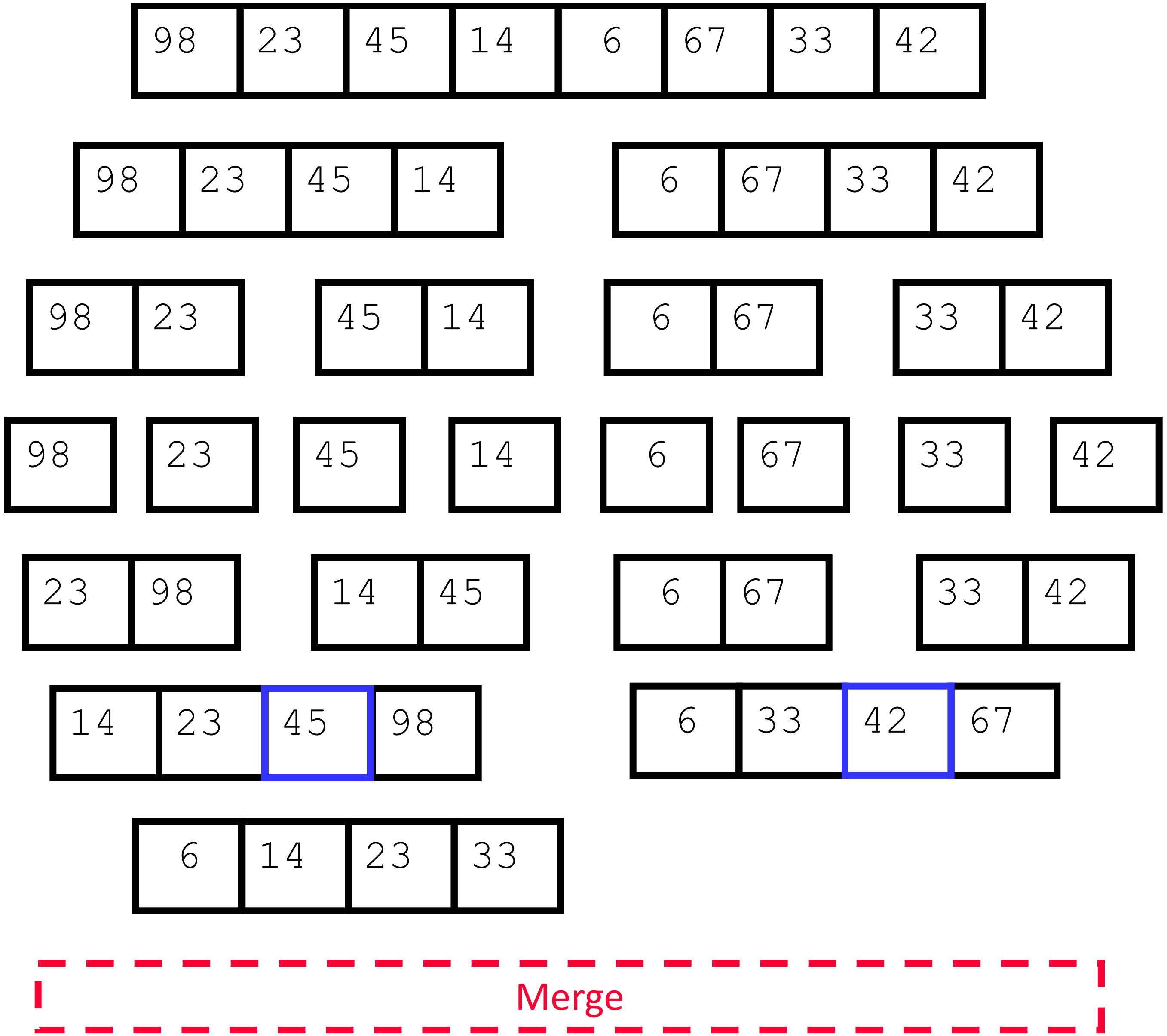
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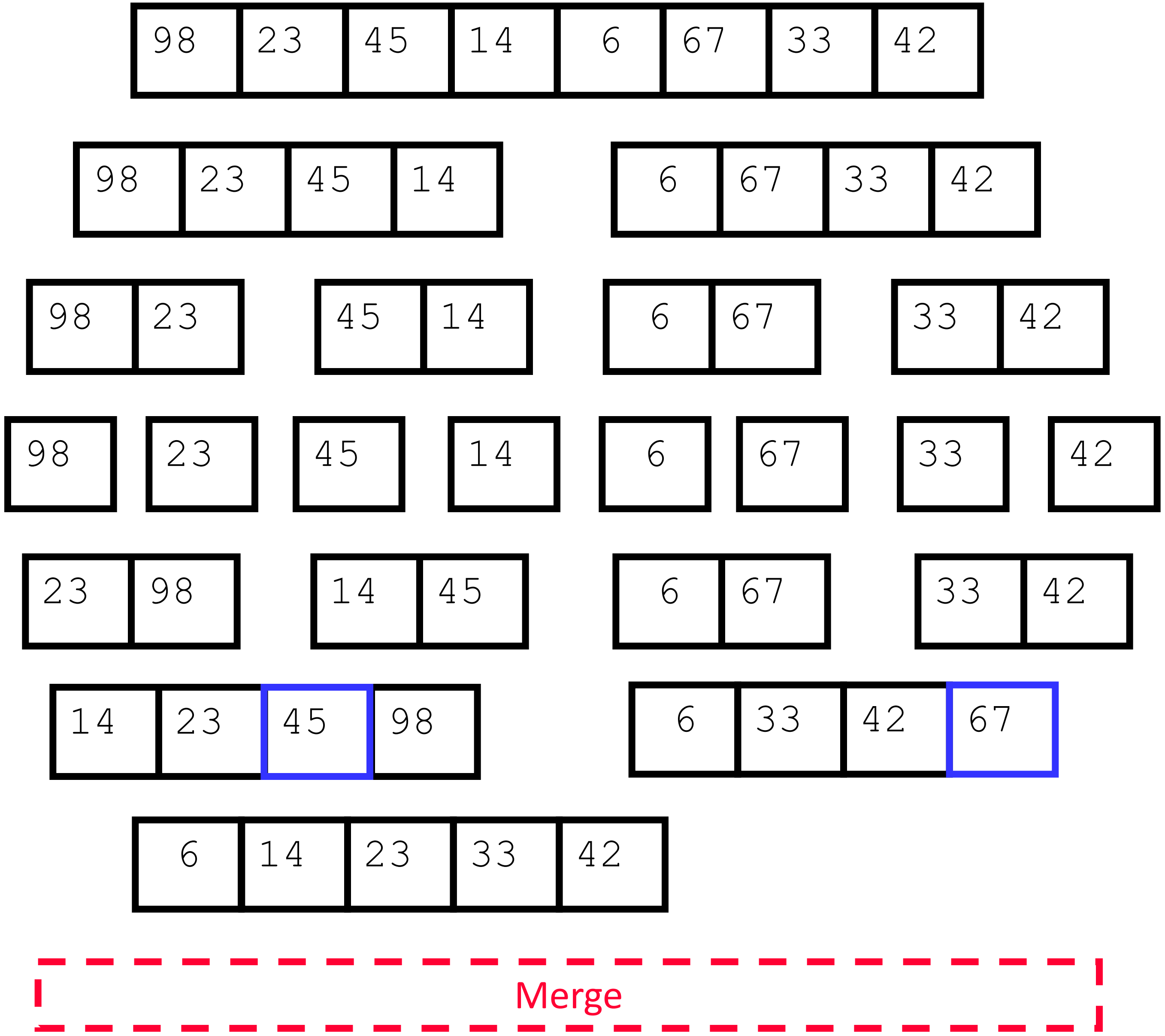
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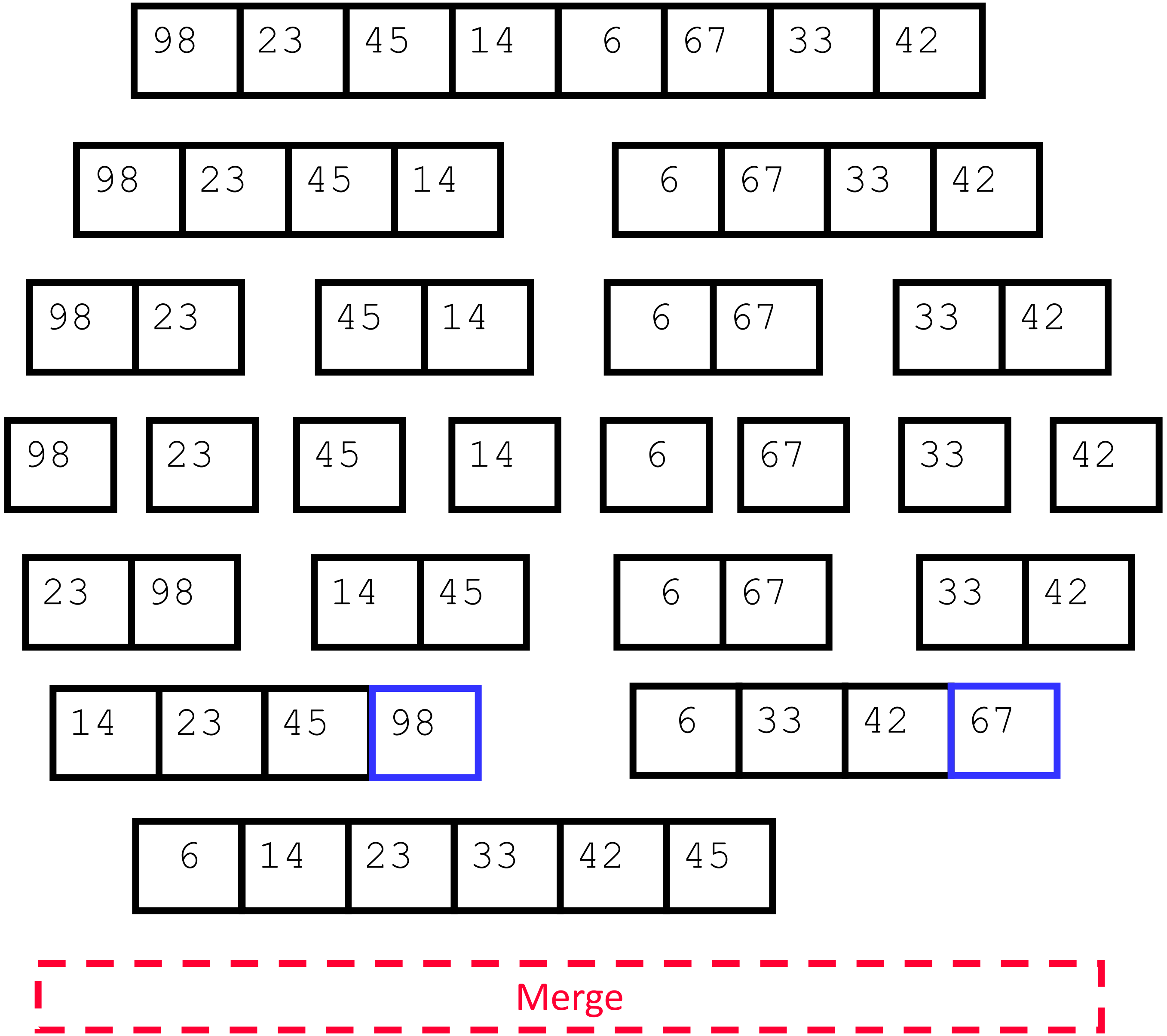
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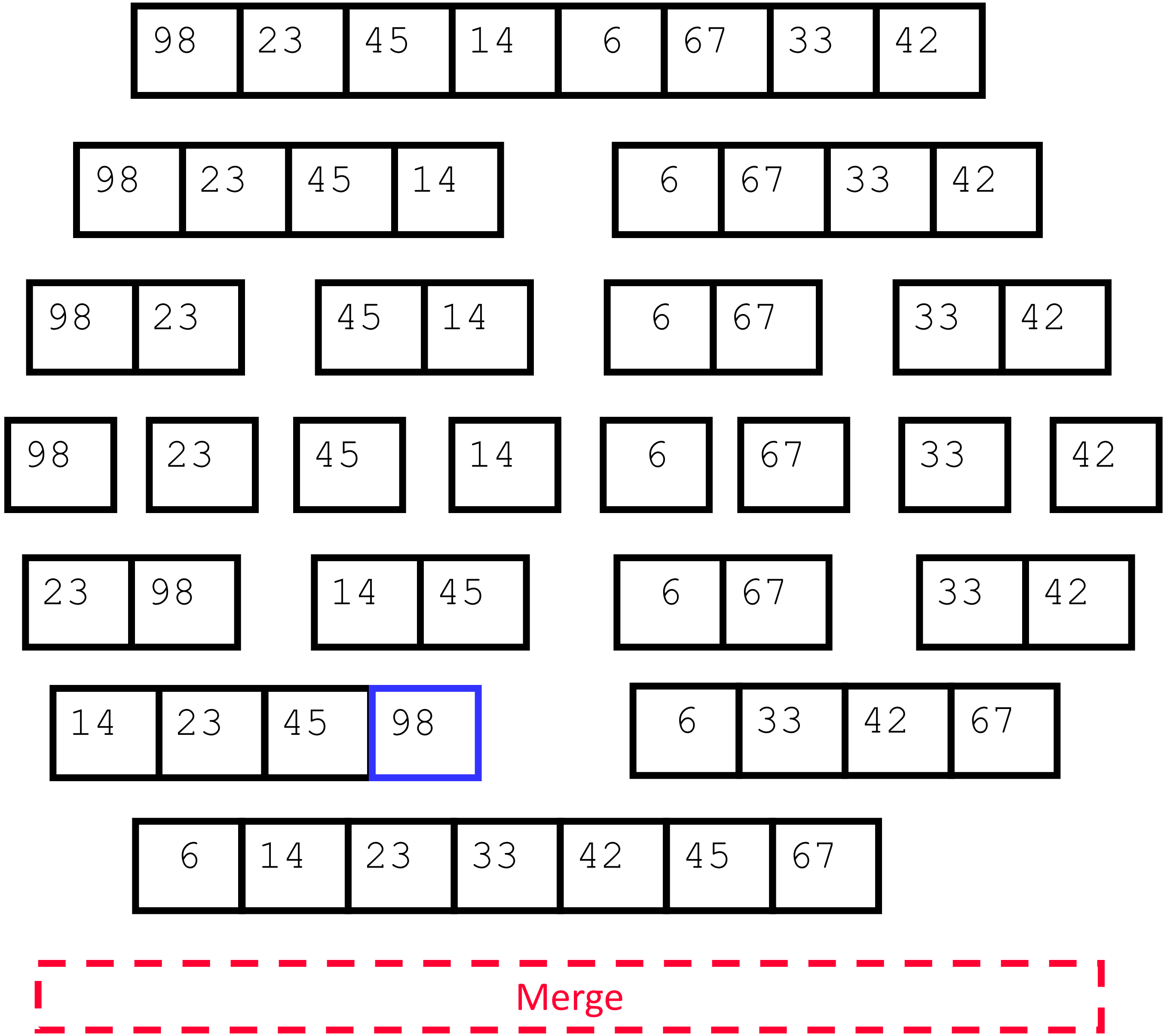
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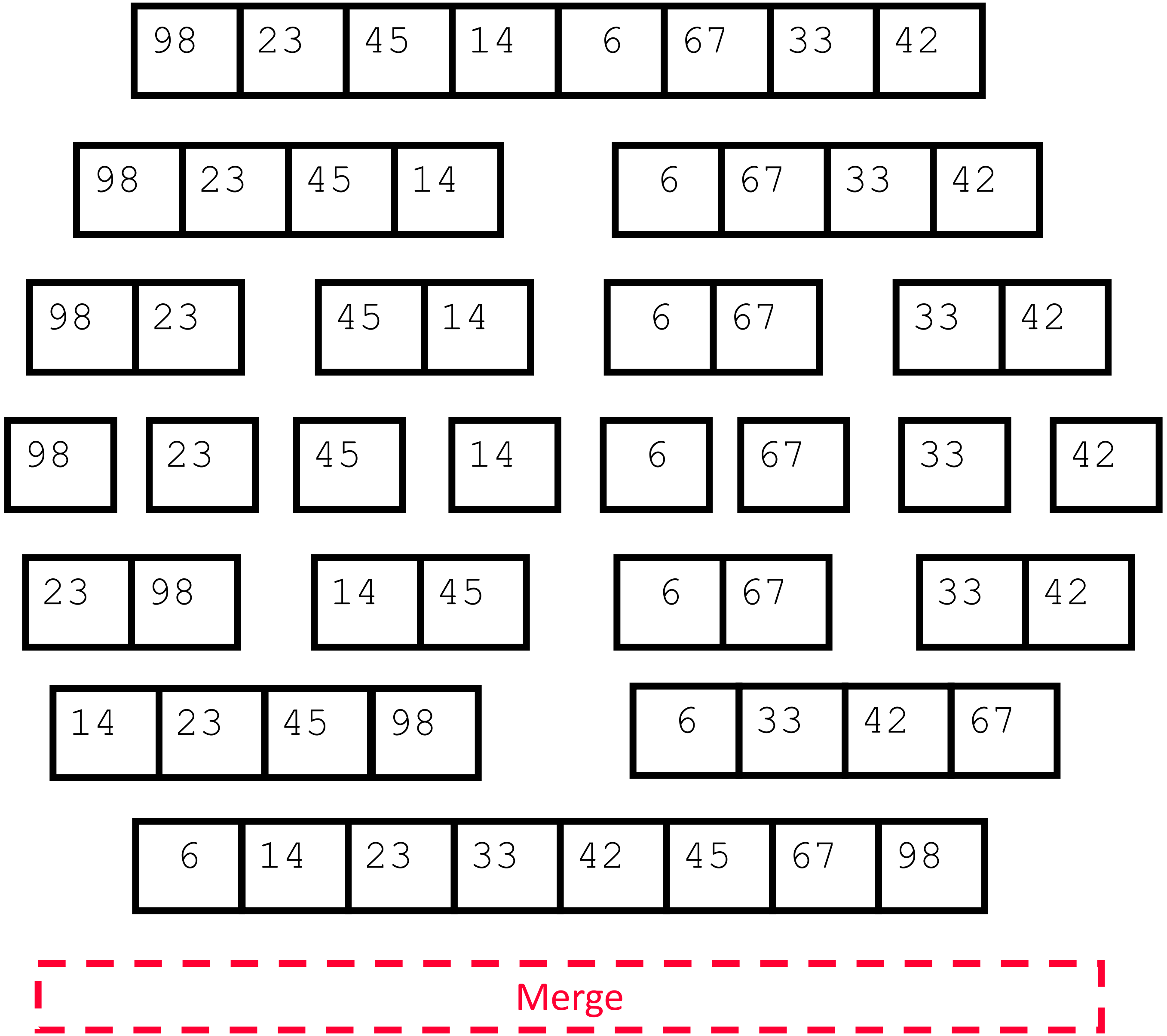
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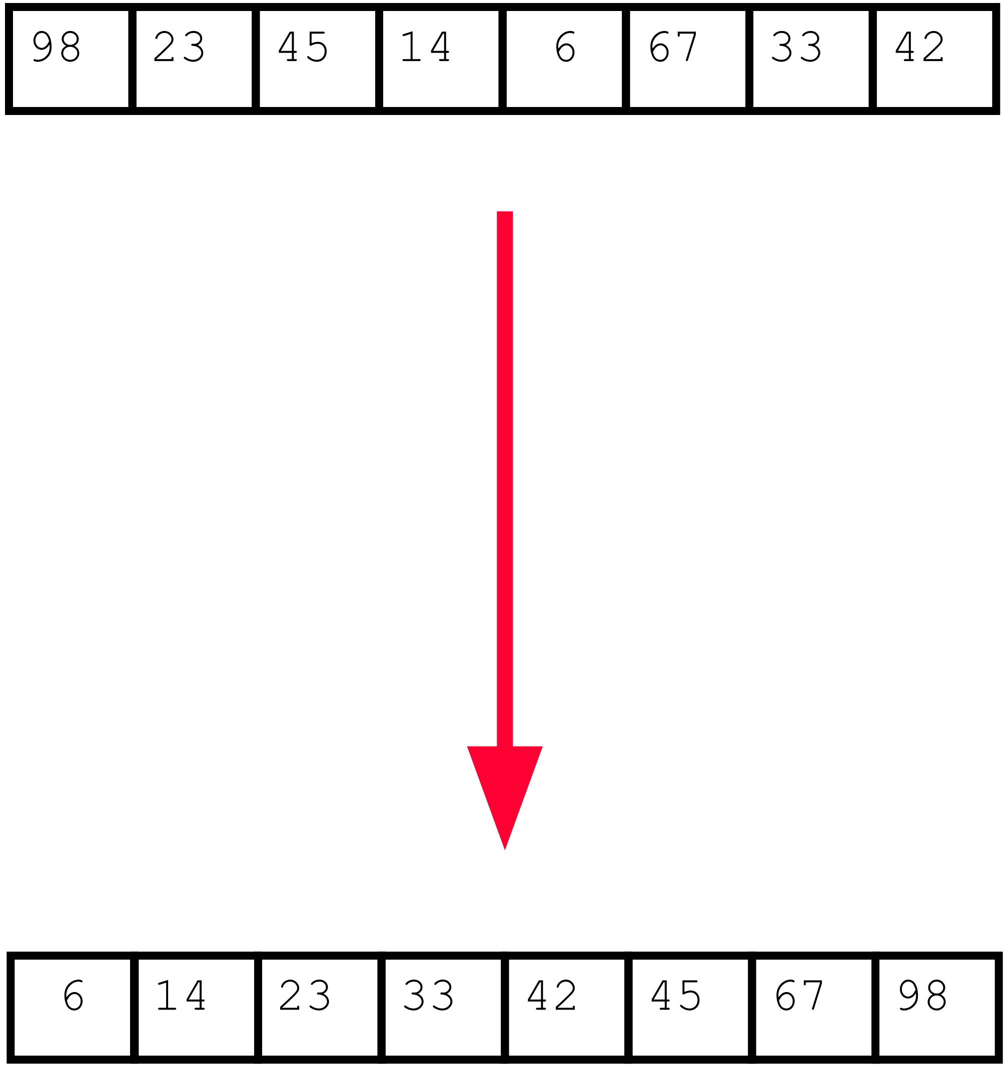
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Textbooks & Web References

- Text Book (Chapter 2)
- Reference book i (Chapter 2)
- Reference book ii (Chapter 5)
- www.visualgo.net

Thank you

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Any question?