Final

$f(D)y = X$

(IV) $X = \sin ax, \cos ax$

$$y_p = \frac{1}{f(D)} \sin ax$$

$$= \frac{1}{f(D^2)} \sin ax$$

$$= \frac{1}{f(-a^2)} \sin ax$$

Example 1:

$$y_p = \frac{1}{D^2-1} \sin x$$

$$= x \frac{1}{2D} \sin x$$

$$= \frac{x}{2} \cdot \left(\frac{1}{D} \sin x\right) \rightarrow \text{integration.}$$

$$= \frac{x}{2}(-\cos x) = \frac{-x\cos x}{2}.$$

(V) $X = e^{ax} V$

$$y_p = \frac{1}{f(D)} e^{ax} V \longrightarrow \text{constant / polynomial / exponential}$$

$$= e^{ax} \frac{1}{f(D+a)} V.$$

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Example 2:

$$y_p = \frac{1}{D^2-1} e^x \sin x$$

$$= e^x \frac{1}{(D+1)^2-1} \sin x$$

$$= e^x \frac{1}{D^2+2D} \sin x$$

$$= e^x \frac{1}{-1+2D} \sin x$$

$$= e^x \frac{1}{2D-1} \sin x$$

$$= e^x \frac{2D+1}{4D^2-1} \sin x$$

$$= e^x \frac{2D\sin x + \sin x}{4(-1)-1}$$

$$= e^x \frac{2\cos x + \cos x}{-5}$$ Ans.

3. Solve $\frac{d^2y}{dx^2} + 4y = \sin 2x$

Given that, $\frac{d^2y}{dx^2} + 4y = \sin 2x$

$\Rightarrow D^2y + 4y = \sin 2x$

$\Rightarrow (D^2+4)y = \sin 2x$

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Now A.E.,

$D^2+4=0$

$\Rightarrow D^2 = -4$

$\Rightarrow D = \pm 2i$

$\Rightarrow D = 0+2i,\ 0-2i$

$\therefore y_c = e^{ax}(c_1\cos bx + c_2 \sin bx)$

$= e^{0x}(c_1 \cos 2x + c_2 \sin 2x)$

$= c_1\cos 2x + c_2 \sin 2x$

$\therefore y_p = \frac{1}{D^2+4}(\sin 2x)$

$= x\frac{1}{2D}(\sin 2x)$

$= \frac{x}{2}\,\frac{-\cos 2x}{2}$

$= \frac{-x\cos 2x}{4}$

$\therefore$ Finally the solution is,

$$y = c_1\cos 2x + c_2 \sin 2x - \frac{x\cos 2x}{4}$$ Ans.

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4. Solve $\frac{d^2y}{dx^2} - y = e^x \sin x$

Given that,

$$\frac{d^2y}{dx^2} - y^2 = \cancel{\sin}\ e^x \sin x$$

$$\Rightarrow D^2y - y^2 = e^x \sin x$$

$$\Rightarrow (D^2-1)y = e^x \sin x$$

Now A.E, $D^2 - 1 = 0$

$$\Rightarrow D = \pm 1 \quad \therefore D = 1, -1$$

$$\therefore y_c = c_1 e^x + c_2 e^{-x}$$

$$y_p = \frac{1}{D^2-1}(e^x \sin x)$$

$$\cancel{= e^x \frac{1}{D^2-1} \sin}$$

$$= e^x \frac{1}{(D+1)^2-1} \sin x$$

$$= e^x \frac{1}{D^2+2D+1-1} \sin x$$

$$= e^x \frac{1}{D^2+2D} \sin x$$

$$= e^{x} \frac{1}{-1+2D} \sin x$$

$$= e^x \frac{1}{2D-1} \sin x$$

$$= e^x \frac{2D+1}{4D^2-1} \sin x$$

$$= e^x \frac{2D+1}{-4-1} \sin x$$

$$= e^x \frac{2D+1}{-5} \sin x$$

$$= \frac{-e^x}{5}(2D \sin x + \sin x)$$

$$= \frac{-e^x}{5}(2\cos x + \cancel{\sin x}\ \sin x)$$

Now the final solution is,

$$y = y_c + y_p$$

$$= c_1 e^x + c_2 e^{-x} - \frac{e^x}{5}(2\cos x + \cancel{\cos x}\ \sin x)$$

Ans.

5. Solve $(D^2+4)^2 y = \sin 2x$.

Given that $(D^2+4)^2 y = \sin 2x$.

Now A.E, $(D^2+4)^2 = 0$

$$\Rightarrow D^2 + 4 = 0$$

$$\Rightarrow D^2 = -4$$

$$\Rightarrow D = \pm 2i$$

$$\therefore y_c = e^{ax}(c_1 \cos bx + c_2 \sin bx)$$

$$= e^{0\cdot x}(c_1 \cos 2x + c_2 \sin 2x)$$

$$= c_1 \cos 2x + c_2 \sin 2x$$

$\therefore y_p = \frac{1}{D^2+4} \sin 2x$

$= x \frac{1}{2D} \sin 2x$

$= \frac{x}{2} (-\cos 2x/2)$

$= \frac{-x\cos 2x}{4}$ ~~Ans~~

~~8. Solve $D^4y - 81y = \sin 2x$.~~

$\therefore$ The final solution is,

$y = y_c + y_p = c_1 \cos 2x + c_2 \sin 2x - \frac{x\cos 2x}{4}$ Ans

8. Solve $D^4y - 81y = \sin 2x$.

Given that, $D^4y - 81y = \sin 2x$

$\Rightarrow (D^4 - 81)y = \sin 2x$

Now A.E., $D^4 - 81 = 0$

$\Rightarrow (D^2)^2 - (9)^2 = 0$

$\Rightarrow (D^2+9)(D^2-9) = 0$

$\Rightarrow D = \pm 3i, \pm 3.$

$\therefore y_c = c_1 e^{3x} + c_2 e^{-3x} + e^{0.x}(c_3 \cos 3x + c_4 \sin 3x)$

$= c_1 e^{3x} + c_2 e^{-3x} + c_3 \cos 3x + c_4 \sin 3x$

$y_p = \frac{1}{D^4-81} \sin 2x$

$= \frac{1}{(2)^4-81} \sin 2x$

$= \frac{1}{16-81} \sin 2x$

$= \frac{-\sin 2x}{65}$

$\therefore$ The final solution is,

$y = c_1 e^{3x} + c_2 e^{-3x} + c_3 \cos 3x + c_4 \sin 3x - \frac{\sin 2x}{65}$ Ans.

10. Solve $D^2y + 4Dy + 4y = e^{2x} + \cos 2x$

Given that, $D^2y + 4Dy + 4y = e^{2x} + \cos 2x$

$\Rightarrow (D^2 + 4D + 4)y = e^{2x} + \cos 2x$

Now A.E., $D^2 + 4D + 4 = 0$

$\Rightarrow D^2 + 2D + 2D + 4 = 0$

$\Rightarrow D(D+2) + 2(D+2) = 0$

$\Rightarrow (D+2)(D+2) = 0$

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$\therefore D = -2, -2$

$\therefore y_c = (c_1 + c_2 x) e^{-2x}$

$$y_p = \frac{1}{D^2+4D+4} (e^{2x} + \cos 2x)$$

$$= \frac{1}{D^2+4D+4} e^{2x} + \frac{1}{D^2+4D+4} \cos 2x.$$

$$= \frac{1}{(2)^2+4.2+4} e^{2x} + \frac{1}{4D} \cos 2x$$

$$= \frac{1}{16} e^{2x} + \frac{1}{4} \frac{\sin 2x}{2}$$

$$= \frac{1}{16} e^{2x} + \frac{1}{8} \sin 2x$$

$\therefore y = (c_1 + c_2 x) e^{-2x} + \frac{1}{16} e^{2x} + \frac{1}{8} \sin 2x$

Ans.

11. Solve $D^2y - 8Dy + 16y = \cos 3x$.

Given that, $D^2y - 8Dy + 16y = \cos 3x$.

$\Rightarrow (D^2 - 8D + 16) y = \cos 3x$.

Now A.E., $D^2 - 8D + 16 = 0$

$\Rightarrow D^2 - 4D - 4D + 16 = 0$

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$\Rightarrow D(D-4)-4(D-4)=0$

$\Rightarrow (D-4)(D-4)=0$

$\therefore D=4,4.$

$\therefore y_c=(c_1+c_2x)e^{4x}$

$$y_p=\frac{1}{D^2-8D+16}\cos 3x$$

$$=\frac{1}{-9-8D+16}\cos 3x$$

$$=\frac{1}{7-8D}\cos 3x$$

$$=\frac{(7+8D)}{49-64D^2}\cos 3x$$

$$=\frac{7+8D}{49-64\cdot(-9)}\cos 3x$$

$$=\frac{7+8D}{49+576}\cos 3x$$

$$=\frac{7+8D}{625}\cos 3x$$

$$=\frac{1}{625}(7\cos 3x+8D\cos 3x)$$

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$$=\frac{1}{625}\{7\cos 3x+8.(-3\sin 3x)\}$$

$$=\frac{1}{625}(7\cos 3x-24\sin 3x)$$

$$\therefore y=(c_1+c_2x)e^{4x}+\frac{1}{625}(7\cos 3x-24\sin 3x)$$

Ans.

12. Solve $D^3y-2Dy+4y=e^x\cos x.$

Given that, $D^3y-2Dy+4y=e^x\cos x$

$\Rightarrow (D^3-2D+4)y=e^x\cos x$

Now A.E., $D^3-2D+4=0$

$\Rightarrow D^2(D+2)-2D(D+2)+2(D+2)=0$

$\Rightarrow (D^2-2D+2)(D+2)=0$

$\therefore D=-2,$ $\frac{-(-2)\pm\sqrt{4-8}}{2.1}$

$=\frac{2\pm\sqrt{-4}}{2}$

$=\frac{2\pm 2i}{2}$

$=1\pm i$

$$\therefore y_c = c_1 e^{-2x} + e^{1\cdot x}(c_2\cos x + c_3 \sin x)$$

$$= c_1 e^{-2x} + e^{x}(c_2\cos x + c_3 \sin x)$$

$$y_p = \frac{1}{D^3 - 2D + 4} e^x \cos x$$

$$= \frac{e^x}{(D+1)^3 - 2(D+1) + 4} \cos x$$

$$= \frac{e^x}{D^3 + 3D^2\cdot 1 + 3D\cdot(1)^2 + (1)^3 - 2D - 2 + 4} \cos x$$

$$= \frac{e^x}{D^3 + 3D^2 + 3D + 1 - 2D + 2} \cos x$$

$$= \frac{e^x}{D^3 + 3D^2 + D + 3} \cos x$$

$$= \frac{e^x}{D^3 + 3(-1) + D + 3} \cos x$$

$$= \frac{e^x}{D^3 - 3 + D + 3} \cos x$$

$$= \frac{e^x}{D^3 + D} \cos x$$

$$\cancel{= \frac{e^x}{D\{-(1)\}\cdot D + D} \cos x}$$

$$\cancel{= x\frac{e^x}{3D^2+1}\cos x} \quad \cancel{= \frac{xe^x}{-3}\cos x}$$

$$\cancel{\frac{xe^x}{-3+1}\cos x}$$

$$= \frac{e^x}{D^3}\cos x + \frac{e^x}{D}\cos x$$

$$= x\frac{e^x}{3D^2}\cos x + e^x \sin x$$

$$= \frac{xe^x}{-3}\cos x + e^x \sin x$$

$$= e^x\left(\frac{x}{-3}\cos x + \sin x\right)$$

Ans.

$$\therefore y = c_1 e^{-2x} + e^x(c_2\cos x + c_3\sin x) + e^x\left(\frac{x}{-3}\cos x + \sin x\right)$$

Ans.

13. Solve $(D^2+D)y = e^x \sin x$

Given that, $(D^2+D)y = e^x \sin x$

Now A.E., $D^2 + D = 0$

$\Rightarrow D(D+1) = 0$

$\therefore D = 0, -1.$

∴ $y_c = c_1 e^{0.x} + c_2 e^{-1.x} = c_1 + c_2 e^{-x}$

$y_p = \frac{1}{D^2+D} e^x \sin x$

$= e^x \frac{1}{(D+1)^2+(D+1)} \sin x$

$= e^x \frac{1}{D^2+2D+1+D+1} \sin x$

$= e^x \frac{1}{D^2+3D+2} \sin x$

$= e^x \frac{1}{-1+3D+2} \sin x$

$= e^x \frac{1}{1+3D} \sin x$

$= e^x \frac{1-3D}{1-9D^2} \sin x$

$= e^x \frac{1-3D}{1-9.(-1)} \sin x$

$= e^x \frac{1-3D}{10} \sin x$

$= \frac{e^x}{10}(\sin x - 3D\sin x)$

$= \frac{e^x}{10}\{\sin x - 3\cos x)$

∴ $y = c_1 + c_2 e^{-x} + \frac{e^x}{10}(\sin x - 3\cos x)$.

14. Solve $(D^4 - 7D^2 - 18)y = \cos x$

Given that, $(D^4 - 7D^2 - 18)y = \cos x$

Now A.E., $D^4 - 7D^2 - 18 = 0$

$\Rightarrow D^3(D-3) + 3D^2(D-3) + 2D(D-3) + 6(D-3) = 0$

$\Rightarrow (D^3 + 3D^2 + 2D + 6)(D-3) = 0$

∴ $D = 3, -3, \sqrt{2}i, -\sqrt{2}i$ [Using calculator]

∴ $y_c = c_1 e^{3x} + c_2 e^{-3x} + e^{0.x}(c_3 \cos\sqrt{2}x + c_4 \sin\sqrt{2}x)$

$= c_1 e^{3x} + c_2 e^{-3x} + (c_3 \cos\sqrt{2}x + c_4 \sin\sqrt{2}x)$

$y_p = \frac{1}{D^4 - 7D^2 - 18} \cos x$

$= \frac{1}{D^4 - 7(-1) - 18} \cos x$

$= \frac{1}{D^4 - 10} \cos x$

$= \frac{1}{(-1).(-1) - 10} \cos x$

$= \frac{1}{-9} \cos x$

∴ $y = c_1 e^{3x} + c_2 e^{-3x} + (c_3 \cos\sqrt{2}x + c_4 \sin\sqrt{2}x) - \frac{1}{9}\cos x$ Ans.

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16. Solve $(D^3+D^2+D+1)y = \cos 3x \sin x$

Given that, $(D^3+D^2+D+1)y = \cos 3x \sin x$

Now A.E., $D^3+D^2+D+1=0$

$\cancel{\Rightarrow D^2(D+1)+2(D+1)-1(D+1)=0}$

$\cancel{\Rightarrow (D^2+} \Rightarrow D^2(D+1)+1(D+1)=0$

$\Rightarrow (D+1)(D^2+1)=0$

$\therefore D=-1$ or $D^2=-1$

$\Rightarrow D=\pm\sqrt{-1}=\pm 1i$

$\therefore D=-1, \pm 1i$

$\therefore y_c = c_1 e^{-1.x} + e^{0.x}(c_2\cos 1.x + c_3 \sin 1.x)$

$= c_1 e^{-x} + (c_2 \cos x + c_3 \sin x)$

$y_p = \frac{1}{D^3+D^2+D+1} \cos 3x \sin x$

$= \frac{1}{2}\left\{\frac{1}{D^3+D^2+D+1} 2\sin x \cos 3x\right\}$

$= \frac{1}{2}\left\{\frac{1}{D^3+D^2+D+1} \sin(x+3x)+\sin(x-3x)\right\}$

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$= \frac{1}{2}\left[\frac{1}{D^3+D^2+D+1}\{\sin 4x + \sin(-2x)\}\right]$

$= \frac{1}{2}\left[\frac{1}{D^3+D^2+D+1}\sin 4x - \frac{1}{D^3+D^2+D+1}\sin 2x\right]$

$= \frac{1}{2}\left[\frac{1}{-16D-16+D+1}\sin 4x - \frac{1}{-4D-4+D+1}\sin 2x\right]$

$= \frac{1}{2}\left[\frac{1}{-15D-15}\sin 4x - \frac{1}{-3D-3}\sin 2x\right]$

$= \frac{1}{2}\left[\frac{-1}{15}\cdot\frac{1}{1+D}\sin 4x + \frac{1}{3}\cdot\frac{1}{1+D}\sin 2x\right]$

$= \frac{-1}{30}\cdot\frac{1-D}{1-D^2}\sin 4x + \frac{1}{6}\cdot\frac{1-D}{1-D^2}\sin 2x$

$= \frac{-1}{30}\cdot\frac{1-D}{1+16}\sin 4x + \frac{1}{6}\cdot\frac{1-D}{1+4}\sin 2x$

$= \frac{-1}{510}(\sin 4x - D\sin 4x) + \frac{1}{30}(\sin 2x - D\sin 2x)$

$= \frac{-1}{510}(\sin 4x - 4\cos 4x) + \frac{1}{30}(\sin 2x - 2\sin 2x)$

$\therefore y = c_1 e^{-x} + (c_2\cos x + c_3 \sin x) - \frac{1}{510}(\sin 4x - 4\cos 4x) + \frac{1}{30}(\sin 2x - 2\sin 2x)$

Ay

<u>Laplace Transformation</u>

$\int uv\,dx = u\int v\,dx - \int\left(\frac{du}{dx}\int v\,dx\right)dx$

Shortcuts of this equation :-

(i) $\int \underset{u}{x^n}\, \underset{v}{\phi(x)}\, dx = u\int v - u_1 \iint v + u_2 \iiint v - \cdots$ 'u'

upto constant.

(ii) $\int e^{ax} \sin bx\, dx = \frac{e^{ax}(a\sin bx - b\cos bx)}{a^2+b^2}$

(iii) $\int e^{ax} \cos bx\, dx = \frac{e^{ax}(a\cos bx + b\sin bx)}{a^2+b^2}$

<u>Ex. of (i) :</u> $\int x^2 e^{2x}\, dx = x^2\left(\frac{e^{2x}}{2}\right) - 2x\left(\frac{e^{2x}}{4}\right) + 2\left(\frac{e^{2x}}{8}\right)$

$= \frac{x^2 e^{2x}}{2} - \frac{x e^{2x}}{2} + \frac{e^{2x}}{4}$ <u>Ans</u>.

*
$$
\begin{tikzcd}
F(t) \arrow[rr, bend left, "\mathcal{L}\{\}"] \arrow[rr] & & f(s) \arrow[ll, bend left, "\mathcal{L}^{-1}\{\}"]
\end{tikzcd}
$$
t কে s-এ রূপান্তর করা,

$\mathcal{L}\{F(t)\} = \int_0^\infty e^{-st} F(t)\, dx = [\phi(st)]_0^\infty = f(s)$

* Find the LT of $F(t)=1$.

Given that, $F(t)=1$

We have, $\mathcal{L}\{F(t)\} = \int_0^\infty e^{-st} F(t)\, dt$

$\therefore \mathcal{L}\{1\} = \int_0^\infty e^{-st} \cdot 1\, dt$

$= \int_0^\infty e^{-st}\, dt$

$= \left[\frac{e^{-st}}{-s}\right]_0^\infty$

$= -1/s\left[e^{-st}\right]_0^\infty$

$= -1/s\left[e^{-\infty} - e^0\right]$

$= -1/s[0-1]$ $\left[\because e^{-\infty} = \frac{1}{e^\infty} = \frac{1}{\infty} = 0\right]$

$= 1/s$ <u>Ans</u>

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* Find LT of $F(t) = t$

Given that, $F(t) = t$

We have, $\mathcal{L}\{F(t)\} = \int_0^\infty e^{-st} F(t)\, dt$

$$\therefore \mathcal{L}\{t\} = \int_0^\infty e^{-st} \cdot t\, dt$$

$$= \int_0^\infty t\, e^{-st}\, dt$$

$$= \left[t \cdot \left(\frac{e^{-st}}{-s}\right) - 1\left(-1/s\ \frac{e^{-st}}{-s}\right)\right]_0^\infty$$

$$= -\left[\frac{te^{-st}}{s} + \frac{e^{-st}}{s^2}\right]_0^\infty$$

$$= -\left\{(0+0) - (0 + 1/s^2)\right\}$$

$$= \frac{1}{s^2}$$

Ans.

Formulas:

1. $\mathcal{L}\{1\} = \frac{1}{s}$
2. $\mathcal{L}\{t\} = \frac{1}{s^2}$
3. $\mathcal{L}\{t^n\} = \frac{n!}{s^{n+1}}$ [পূর্ণসংখ্যা] $= \frac{\Gamma(n+1)}{s^{n+1}}$ [ভগ্নাংশ]

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4. $\mathcal{L}\{e^{at}\} = \frac{1}{s-a}$
5. $\mathcal{L}\{\sin at\} = \frac{a}{s^2+a^2}$
6. $\mathcal{L}\{\cos at\} = \frac{s}{s^2+a^2}$
7. $\mathcal{L}\{\sinh at\} = \frac{a}{s^2-a^2}$
8. $\mathcal{L}\{\cosh at\} = \frac{s}{s^2-a^2}$

Properties:

1. Linearity property:

* Evaluate $\mathcal{L}\{e^{2t} + t^2 + \sin t\}$

Given that, $\mathcal{L}\{e^{2t} + t^2 + \sin t\}$

$$= \mathcal{L}\{e^{2t}\} + \mathcal{L}\{t^2\} + \mathcal{L}\{\sin t\}$$

$$= \frac{1}{s-2} + \frac{2!}{s^{2+1}} + \frac{1}{s^2+1^2}$$

$$= \frac{1}{s-2} + \frac{2}{s^3} + \frac{1}{s^2+1}$$

Ans.

2. First shifting property:

$\mathcal{L}\{e^{at} \cdot F(t)\} = f(s-a)$

We know, $\mathcal{L}\{F(t)\} = f(s) = f(s-a)$

* Evaluate $\mathcal{L}\{e^{2t}\sin 3t\}$

Given that, $\mathcal{L}\{e^{2t}\sin 3t\}$

Here, $F(t) = \sin 3t$, and $a = 2$

$\therefore \mathcal{L}\{F(t)\} = \mathcal{L}\{\sin 3t\}$

$\Rightarrow f(s) = \frac{3}{s^2+3^2} = \frac{3}{s^2+9}$

Now replacing s by s−2.

$\therefore f(s-2) = \frac{3}{(s-2)^2+9} = \frac{3}{s^2-4s+13}$

We have, $\mathcal{L}\{e^{at}F(t)\} = f(s-a)$

$\therefore \mathcal{L}\{e^{2t}\sin 3t\} = f(s-2)$

$= \frac{3}{s^2-4s+13}$

Ans.

* Evaluate $\mathcal{L}\{e^{2t}\sin^2 t\}$

Given that, $\mathcal{L}\{e^{2t}\sin^2 t\}$

Here $F(t) = \sin^2 t$ and $a = 2$.

$\therefore \mathcal{L}\{F(t)\} = \mathcal{L}\{\sin^2 t\}$

$\Rightarrow f(s) = 1/2\, \mathcal{L}\{2\sin^2 t\}$

$= 1/2\, \mathcal{L}\{1-\cos 2t\}$

$= 1/2\, [\mathcal{L}\{1\} - \mathcal{L}\{\cos 2t\}]$

$= 1/2\, \left[1/s - \frac{s}{s^2+2^2}\right]$

$= 1/2\, \left[1/s - \frac{s}{s^2+4}\right]$

Replacing s by s−2,

$\therefore f(s-2) = 1/2\left[\frac{1}{s-2} - \frac{s-2}{(s-2)^2+4}\right]$

$= 1/2\left[\frac{1}{s-2} - \frac{s-2}{s^2-4s+8}\right]$

We have, $\mathcal{L}\{e^{at}F(t)\} = f(s-a)$

$\Rightarrow \mathcal{L}\{e^{2t}\sin^2 t\} = f(s-a)$

$= 1/2\left[\frac{1}{s-2} - \frac{s-2}{s^2-4s+8}\right]$

Ans.

H.W:

1. $\mathcal{L}\{e^{-2t}(\sin t + \cos t)\}$

Given that, $\mathcal{L}\{e^{-2t}(\sin t + \cos t)\}$

Here, $F(t) = \sin t + \cos t$ and $a = -2$.

$$\therefore \mathcal{L}\{F(t)\} = \mathcal{L}\{\sin t + \cos t\}$$

$$\Rightarrow f(s) = \mathcal{L}\{\sin t\} + \mathcal{L}\{\cos t\}$$

$$= \frac{1}{s^2+1} + \frac{s}{s^2+1}$$

$$= \frac{s+1}{s^2+1}$$

Replacing s by $s+2$,

$$\therefore f(s+2) = \frac{(s+2)+1}{(s+2)^2+1}$$

$$= \frac{s+3}{s^2+4s+5}$$

We have, $\mathcal{L}\{e^{at}F(t)\} = f(s-a)$

$$\Rightarrow \mathcal{L}\{e^{-2t}(\sin t + \cos t)\} = \frac{s+3}{s^2+4s+5}$$

Ans.

2. $\mathcal{L}\{e^{2t}(\sin^2 t + \cos^2 t)\}$

Given that, $\mathcal{L}\{e^{2t}(\sin^2 t + \cos^2 t)\}$

~~Here~~ $= \mathcal{L}\{e^{2t}.1\}$

$$= \frac{1}{s-2}$$

Ans.

3. $\mathcal{L}\{e^{2t}(\sin^2 t - \cos^2 t)\}$

Given that, $\mathcal{L}\{e^{2t}(\sin^2 t - \cos^2 t)\}$

Here, $F(t) = \sin^2 t - \cos^2 t$ and $a = 2$.

$$\therefore \mathcal{L}\{F(t)\} = \mathcal{L}\{\sin^2 t - \cos^2 t\}$$

$$\Rightarrow f(s) = -\mathcal{L}\{\cos^2 t - \sin^2 t\}$$

$$= -\mathcal{L}\{\cos 2A\}$$

$$= -\frac{s}{s^2+4}$$

Now, Replacing s by $s-2$,

$$f(s-2) = -\frac{(s-2)}{(s-2)^2+4}$$

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$$= \frac{-(s-2)}{s^2-4s+8}$$

We have, $\mathcal{L}\{e^{at}F(t)\} = f(s-a)$

$$\therefore \mathcal{L}\{e^{2t}(\sin^2 t - \cos^2 t)\} = f(s-a)$$

$$= \frac{-(s-2)}{s^2-4s+8}$$

Ans.

3. Multiplication by t^n:

* $\mathcal{L}\{t^n F(t)\} = (-1)^n \frac{d^n}{ds^n} f(s)$

* Evaluate LT of $\mathcal{L}\{t^2 \sin t\}$

Given that, $\mathcal{L}\{t^2 \sin t\}$

Here, $F(t) = \sin t$ and $n=2$.

$\therefore \mathcal{L}\{F(t)\} = \mathcal{L}\{\sin t\}$

$\Rightarrow F(s) = \frac{1}{s^2+1^2} = \frac{1}{s^2+1}$

We have, $\mathcal{L}\{t^n F(t)\} = (-1)^n \frac{d^n}{ds^n} f(s)$

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$$\therefore \mathcal{L}\{t^2 \sin t\} = (-1)^2 \frac{d^2}{ds^2} \frac{1}{s^2+1}$$

$$= \frac{d}{ds} \cdot \frac{d}{ds} \frac{1}{s^2+1}$$

$$= \frac{d}{ds} \cdot \frac{d}{ds} (s^2+1)^{-1}$$

$$= \frac{d}{ds} (-1)(s^2+1)^{-2} \cdot 2t$$

$$= -2 \frac{d}{ds} \frac{s}{(s^2+1)^2}$$

$$= -2 \frac{(s^2+1)\cdot 1 - s\, 2(s^2+1)\cdot 2s}{(s^2+1)^4}$$

$$= -2 \frac{(s^2+1)(s^2+1-4s^2)}{(s^2+1)^4}$$

$$= -2 \frac{1-3s^2}{(s^2+1)^3} = \frac{6s^2-2}{(s^2+1)^3}$$

Ans.

Inverse Laplace Transformation

~~$\mathcal{L}^{-1}$~~ # Formulas:

1. $\mathcal{L}^{-1}\{1/s\} = 1$

2. $\mathcal{L}^{-1}\{1/s^2\} = t$

3. $\mathcal{L}^{-1}\left\{\frac{n!}{s^{n+1}}\right\} = t^n$ or,

$\mathcal{L}^{-1}\left\{\frac{1}{s^{n+1}}\right\} = \frac{t^n}{n!}$

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4. $\mathcal{L}^{-1}\left\{\frac{1}{s-a}\right\} = e^{at}$

5. $\mathcal{L}^{-1}\left\{\frac{a}{s^2+a^2}\right\} = \sin at$

6. $\mathcal{L}^{-1}\left\{\frac{s}{s^2+a^2}\right\} = \cos at$

7. $\mathcal{L}^{-1}\left\{\frac{a}{s^2-a^2}\right\} = \sinh at$

8. $\mathcal{L}^{-1}\left\{\frac{s}{s^2-a^2}\right\} = \cosh at$

Properties:

1. Linearity property:

* Evaluate $\mathcal{L}^{-1}\left\{1/s + 1/s^2 + \frac{4}{s^2+9} + \frac{s-1}{s^2+2}\right\}$

Given that, $\mathcal{L}^{-1}\left\{1/s + 1/s^2 + \frac{4}{s^2+9} + \frac{s-1}{s^2+2}\right\}$

$$= \mathcal{L}^{-1}\{1/s\} + \mathcal{L}^{-1}\{1/s^2\} + \mathcal{L}^{-1}\left\{\frac{4}{s^2+9}\right\} + \mathcal{L}^{-1}\left\{\frac{s-1}{s^2+2}\right\}$$

$$= 1 + t + 4\mathcal{L}^{-1}\left\{\frac{1}{s^2+3^2}\right\} + \mathcal{L}^{-1}\left\{\frac{s}{s^2+2} - \frac{1}{s^2+2}\right\}$$

$$= 1 + t + \frac{4}{3}\mathcal{L}^{-1}\left\{\frac{3}{s^2+3^2}\right\} + \mathcal{L}^{-1}\left\{\frac{s}{s^2+(\sqrt{2})^2} - \frac{1}{\sqrt{2}}\frac{\sqrt{2}}{s^2+(\sqrt{2})^2}\right\}$$

$$= 1 + t + \frac{4}{3}\sin 3t + \cos\sqrt{2}t - \frac{1}{\sqrt{2}}\sin\sqrt{2}t$$ Ans.

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* Evaluate $\mathcal{L}^{-1}\left\{\frac{1}{s^2+4s+3}\right\}$.

Given that, $\mathcal{L}^{-1}\left\{\frac{1}{s^2+4s+3}\right\}$

$$= \mathcal{L}^{-1}\left\{\frac{1}{s^2+3s+s+3}\right\}$$

$$= \mathcal{L}^{-1}\left\{\frac{1}{s(s+3)+1(s+3)}\right\}$$

$$= \mathcal{L}^{-1}\left\{\frac{1}{(s+3)(s+1)}\right\}$$

$$= \mathcal{L}^{-1}\left\{\frac{-1/2}{s+3} + \frac{1/2}{s+1}\right\}$$

$$= 1/2\,\mathcal{L}^{-1}\left\{\frac{1}{s+1} - \frac{1}{s+3}\right\}$$

$$= 1/2\,\mathcal{L}^{-1}\left\{\frac{1}{s+1}\right\} - 1/2\,\mathcal{L}^{-1}\left\{\frac{1}{s+3}\right\}$$

$$= 1/2\,\mathcal{L}^{-1}\left\{\frac{1}{s-(-1)}\right\} - 1/2\,\mathcal{L}^{-1}\left\{\frac{1}{s-(-3)}\right\}$$

$$= 1/2\, e^{-t} - 1/2\, e^{-3t}$$

$$= 1/2\left(e^{-t} - e^{-3t}\right)$$

Ans.

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2. First shifting property:

$$\mathcal{L}^{-1}\{f(s-a)\} = e^{at}F(t)$$
$$= e^{at}\mathcal{L}^{-1}\{f(s)\}$$

*Evaluate $\mathcal{L}^{-1}\left\{\frac{1}{s^2+4s+3}\right\}$

Given that, $\mathcal{L}^{-1}\left\{\frac{1}{s^2+4s+3}\right\}$

$$= \mathcal{L}^{-1}\left\{\frac{1}{s^2+2.s.2+2^2-1}\right\}$$

$$= \mathcal{L}^{-1}\left\{\frac{1}{(s+2)^2-1}\right\}$$

$$= \mathcal{L}^{-1}\{f(s+2)\} \text{ with } f(s+2) = \frac{1}{(s+2)^2-1}$$

$$= \mathcal{L}^{-1}\{f(s-(-2))\} \text{ with } a=-2.$$

$$= e^{-2t}\mathcal{L}^{-1}\left\{\frac{1}{(s+2)^2-1}\right\}$$

$$= e^{-2t}\sinh t$$

Ans.

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3. Second shifting property:

$$\mathcal{L}^{-1}\{e^{-as}.f(s)\} = \begin{cases} F(t-a), & t>a \\ 0, & t<a \end{cases}$$

*Evaluate $\mathcal{L}^{-1}\left\{\frac{e^{-\pi s}}{s^2+4s+3}\right\}$

Given that, $\mathcal{L}^{-1}\left\{\frac{e^{-\pi s}}{s^2+4s+3}\right\}$

Here, $a=\pi$ and $f(s) = \frac{1}{s^2+4s+3}$

$\therefore \mathcal{L}^{-1}\{f(s)\} = \mathcal{L}^{-1}\left\{\frac{1}{s^2+4s+3}\right\}$

$\therefore F(t) = e^{-2t}\sinh t$ [Previous math]

Replacing t by $t-\pi$

$\therefore F(t-\pi) = e^{-2(t-\pi)}\sinh(t-\pi)$

We have, $\mathcal{L}^{-1}\{e^{-as}f(s)\} = \begin{cases} F(t-a); & t>a \\ 0; & t<a \end{cases}$

$$\therefore \mathcal{L}^{-1}\left\{e^{-\pi s}\frac{1}{s^2+4s+3}\right\} = \begin{cases} F(t-\pi); & t>\pi \\ 0; & t<\pi \end{cases}$$

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$$= \mathcal{L}^{-1}\left\{\frac{e^{-\pi s}}{s^2+4s+3}\right\} = \begin{cases} e^{-2(t-\pi)}\sinh(t-\pi); & t>\pi \\ 0 & ; t<\pi \end{cases}$$

Ans.

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Assignment :

1. Find the Laplace transformation of the following functions.

a. $f(t) = \sin(2t)\cos(2t)$

Given that, $f(t) = \sin(2t)\cos(2t)$

$$\therefore \mathcal{L}\{f(t)\} = \mathcal{L}\{\sin 2t \cos 2t\}$$

$$= \frac{1}{2}\mathcal{L}\{2\sin 2t \cos 2t\}$$

$$= \frac{1}{2}\mathcal{L}\{\sin 4t\}$$

$$= \frac{1}{2}\frac{4}{s^2+16}$$

$$= \frac{2}{s^2+16}$$

Ans.

b. $f(t) = \cos^2(3t)$

Given that, $\cos^2(3t) = f(t)$

$$\therefore \mathcal{L}\{f(t)\} = \mathcal{L}\{\cos^2(3t)\}$$

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$= \frac{1}{2} \mathcal{L}\left\{2\cos^2 3t\right\}$

$= \frac{1}{2} \mathcal{L}\left\{1 + \cos 6t\right\}$

$= \frac{1}{2}\left[\mathcal{L}\{1\} + \mathcal{L}\{\cos 6t\}\right]$

$= \frac{1}{2}\left[\frac{1}{s} + \frac{s}{s^2+36}\right]$

Ans.

2. Evaluate:

1. $\mathcal{L}^{-1}\left\{\frac{4}{s-2} - \frac{3}{s+5}\right\} = 4e^{2t} - 3e^{-5t}$

∴ Given that, $\mathcal{L}^{-1}\left\{\frac{4}{s-2} - \frac{3}{s+5}\right\} = 4e^{2t} - 3e^{-5t}$

L.H.S, $\mathcal{L}^{-1}\left\{\frac{4}{s-2} - \frac{3}{s+5}\right\}$

$= \mathcal{L}^{-1}\left\{\frac{4}{s-2}\right\} - \mathcal{L}^{-1}\left\{\frac{3}{s+5}\right\}$

$= 4\mathcal{L}^{-1}\left\{\frac{1}{s-2}\right\} - 3\mathcal{L}^{-1}\left\{\frac{1}{s-(-5)}\right\}$

$= 4e^{2t} - 3e^{-5t}$

$=$ R.H.S

∴ L.H.S = R.H.S

Ans.

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2. $\mathcal{L}^{-1}\left\{\frac{s}{s^2+9}+\frac{5}{s^2+9}\right\}=\cos(3t)+5/3\sin(3t)$

Given that, $\mathcal{L}^{-1}\left\{\frac{s}{s^2+9}+\frac{5}{s^2+9}\right\}=\cos(3t)+5/3\sin(3t)$

L.H.S, $\mathcal{L}^{-1}\left\{\frac{s}{s^2+9}+\frac{5}{s^2+9}\right\}$

$=\mathcal{L}^{-1}\left\{\frac{s}{s^2+9}\right\}+\mathcal{L}^{-1}\left\{\frac{5}{s^2+9}\right\}$

$=\mathcal{L}^{-1}\left\{\frac{s}{s^2+3^2}\right\}+5\mathcal{L}^{-1}\left\{\frac{1}{s^2+3^2}\right\}$

$=\cos(3t)+5/3\,\mathcal{L}^{-1}\left\{\frac{3}{s^2+3^2}\right\}$

$=\cos(3t)+5/3\sin(3t)$

$=$ R.H.S

$\therefore$ L.H.S = R.H.S

Ans.

3. $\mathcal{L}^{-1}\left\{\frac{8(s+2)}{(s+2)^2+25}-\frac{4}{(s+2)^2+25}\right\}=8e^{-2t}\cos(5t)-4/5\,e^{-2t}\sin(5t)$

Given that,

$\mathcal{L}^{-1}\left\{\frac{8(s+2)}{(s+2)^2+25}-\frac{4}{(s+2)^2+25}\right\}=8e^{-2t}\cos(5t)-4/5\,e^{-2t}\sin(5t)$

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L.H.S,

$$\mathcal{L}^{-1}\left\{\frac{8(s+2)}{(s+2)^2+25}-\frac{4}{(s+2)^2+25}\right\}$$

$$=\mathcal{L}^{-1}\left\{\frac{8(s+2)}{(s+2)^2+25}\right\}-\mathcal{L}^{-1}\left\{\frac{4}{(s+2)^2+25}\right\}$$

$$=\left[8\mathcal{L}^{-1}\{f(s+2)\} \text{ with } f(s+2)=\frac{s+2}{(s+2)^2+25}\right]-\left[4\mathcal{L}^{-1}\{f(s+2)\} \text{ with } f(s+2)=\frac{1}{(s+2)^2+25}\right]$$

$$=\left[8\mathcal{L}^{-1}\{f(s-(-2))\} \text{ with } a=-2\right]-\left[4\mathcal{L}^{-1}\{f(s-(-2))\} \text{ with } a=-2\right]$$

$$=8e^{-2t}\mathcal{L}^{-1}\left\{\frac{(s+2)}{(s+2)^2+(5)^2}\right\}-4e^{-2t}\mathcal{L}^{-1}\left\{\frac{1}{(s+2)^2+(5)^2}\right\}$$

$$=8e^{-2t}\cos(5t)-4e^{-2t}\sin(5t)$$

$=$R.H.S

$\therefore$ L.H.S $=$ R.H.S

Ans

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4. $\mathcal{L}^{-1}\left\{4/s-1/s^2+5/s^3+2/s^4\right\}=4-t+\frac{5}{2!}t^2+\frac{2}{3!}t^3$

Given that, $\mathcal{L}^{-1}\left\{4/s-1/s^2+5/s^3+2/s^4\right\}=4-t+\frac{5}{2!}t^2+\frac{2}{3!}t^3$

L.H.S, $\mathcal{L}^{-1}\left\{4/s-1/s^2+5/s^3+2/s^4\right\}$

$$=\mathcal{L}^{-1}\{4/s\}-\mathcal{L}^{-1}\{1/s^2\}+\mathcal{L}^{-1}\{5/s^3\}+\mathcal{L}^{-1}\{2/s^4\}$$

$$=4\mathcal{L}^{-1}\{s\}-\mathcal{L}^{-1}\{1/s^2\}+5/2!\,\mathcal{L}^{-1}\left\{\frac{2!}{s^{2+1}}\right\}+\frac{2}{3!}\mathcal{L}^{-1}\left\{\frac{3!}{s^{3+1}}\right\}$$

$$=4.-t+5/2!\,t^2+2/3!\,t^3$$

$=$R.H.S

$\therefore$ L.H.S $=$ R.H.S Ans

5. $\mathcal{L}^{-1}\left\{\frac{10}{(s-5)^2}+\frac{2}{(s-5)^3}\right\}=10te^{5t}+\frac{2}{2!}t^2e^{5t}=10te^{5t}+t^2e^{5t}$

Given that, $\mathcal{L}^{-1}\left\{\frac{10}{(s-5)^2}+\frac{2}{(s-5)^3}\right\}=10te^{5t}+\frac{2}{2!}t^2e^{5t}=10te^{5t}+te^{5t}$

L.H.S, $\mathcal{L}^{-1}\left\{\frac{10}{(s-5)^2}+\frac{2}{(s-5)^3}\right\}$

GOOD LUCK

GOOD LUCK

$= \left[10\mathcal{L}^{-1}\{f(s-5)\} \text{ with } f(s-5) = \frac{1}{(s-5)^2}\right] + \left[2\mathcal{L}^{-1}\{f(s-5)\} \text{ with } f(s-5) = \frac{1}{(s-5)^3}\right]$

$= 10e^{5t}\mathcal{L}^{-1}\left\{\frac{1}{(s-5)^2}\right\} + 2e^{5t}\mathcal{L}^{-1}\left\{\frac{1}{(s-5)^3}\right\}$

$= 10e^{5t}t + \frac{2}{2!}e^{5t}\mathcal{L}^{-1}\left\{\frac{2!}{(s-5)^{2+1}}\right\}$

$= 10te^{5t} + \frac{2}{2!}t^2e^{5t}$

$= 10te^{5t} + t^2e^{5t}$

= R.H.S

∴ L.H.S = R.H.S

Ans

6. $\mathcal{L}^{-1}\left\{\frac{1}{(s+3)^2+4}\right\} = \frac{1}{2}e^{-3t}\sin(2t)$

Given that, $\mathcal{L}^{-1}\left\{\frac{1}{(s+3)^2+4}\right\} = \frac{1}{2}e^{-3t}\sin(2t)$

L.H.S, $\mathcal{L}^{-1}\left\{\frac{1}{(s+3)^2+4}\right\}$

$= \mathcal{L}^{-1}\{f(s+3)\} \text{ with } f(s+3) = \frac{1}{(s+3)^2+4}$

$= \mathcal{L}^{-1}\{f(s-(-3))\} \text{ with } a = -3.$

$= e^{-3t}\mathcal{L}^{-1}\left\{\frac{1}{(s+3)^2+4}\right\}$

$= e^{-3t}\mathcal{L}^{-1}\left\{\frac{1}{2}\cdot\frac{2}{(s+3)^2+(2)^2}\right\}$

$= \frac{1}{2}e^{-3t}\sin(2t)$

= R.H.S

∴ L.H.S = R.H.S

Ans