2. Assume that the following hexadecimal values are 16-bit twos-complement. Convert each to the corresponding decimal value.

i. `0x0013`
ii. `0x0444`
iii. `0x1234`
iv. `0xffff`
v. `0x8000`

Xxxx xxxx xxxx xxxx

1 → negative → 2's complement

0 → positive + normal

0x0013

0000 0000 0001 0011

$2^4\ 2^3\ 2^2\ 2^1\ 2^0$

$2^1 + 2^1 + 2^0$

$= 16 + 2 + 1 = 19$

0xffff

15 decimal

$2^3\ 2^2\ 2^1\ 2^0$

1111 1111 1111 1111

invert bits + add 1

0000 0000 0001 0000 + 1

0000 0000 0000 0001

$= -1$

0x8000

$= 1000\ 0000\ 0000\ 0000$

2^3

negative

2's complement:

invert

$= 0111\ 1111\ 1111\ 1111$

$+\ 1$

$= 1000\ 0000\ 0000\ 0000$

$2_{10} = 10_2$

$= 2^{15} = -32768$

3. Give a representation for each of the following decimal values in 16-bit twos-complement bit-strings. Show the value in binary, octal and hexadecimal.

i. 1
ii. 100
iii. 1000
iv. 10000
v. 100000
vi. -5
vii. -100

positive

convert as usual

negative

- convert as usual
- then apply 2's complement
- set MSB to 1

100

100/2 = 50 r 0

0000 0000 0110 0100

50/2 = 25 r 0

25/2 = 12 r 1

12/2 = 6 r 0

6/2 = 3 r 0

3/2 = 1 r 1

1/2 = 0 r 1

-5

convert 5 to binary:

5/2 = 2 r 1 0000 0000 0000 0101

2/2 = 1 r 0 invert:

1/2 = 0 r 1 1111 1111 1111 1010 +1

1111 1111 1111 1011

-100

0000 0000 0110 0100

two's complement

invert:

1111 1111 1001 1011

add 1

+ 1

1111 1111 1001 1100

4. What decimal numbers do the following single-precision IEEE 754-encoded bit-strings represent?

a. 0 00000000 00000000000000000000000
b. 1 00000000 00000000000000000000000
c. 0 01111111 10000000000000000000000
d. 0 01111110 00000000000000000000000
e. 0 01111110 11111111111111111111111
f. 0 10000000 01100000000000000000000
g. 0 10010100 10000000000000000000000
h. 0 01101110 10100000101000001010000

sign exponent fraction

$$\text{Value} = (-1)^{\text{Sign}} \times 2^{\text{Exp} - 127} \times (1 + \text{Frac})$$

float x = 3.4

a) sign = 0
exponent = 0
frac = 0

$$(-1)^0 \times 2^{0-127} \times (1 + 0)$$
$$= 1 \times 2^{-127}$$

f) sign 0

exp = 1000 0000 = 2^7 = 128

frac = .0110 0000 0000 0000 0000 000 $_2$

$2^{-1}\ 2^{-2}\ 2^{-3} \ldots$

$= 2^{-2} + 2^{-3}$

$= \frac{1}{4} + \frac{1}{8} = 0.375$

0000 0000 . 00000

$$(-1)^0 \times (1 + 0.375) \times 2^{128-127}$$
$$= 2 \times 1.375 = 2.75$$

5. Convert the following decimal numbers into IEEE 754-encoded bit-strings:

a. 2.5

b. 0.375 = k

c. 27.0

d. 100.0

$2^n \times (1 + frac)$

$\frac{k}{2^{\lfloor \log_2(k) \rfloor}}$

$\lfloor 2.05 \rfloor = 2$

$\lfloor 2.9 \rfloor = 2$

$\lfloor -2.3 \rfloor = -3$

$\frac{0.375}{2^{(-2)}} = 1.5 \times 2^{-2}$

$\times 2^{-2}$

$0.375 = 1.5 \times 2^{-2}$

$= (1 + 0.5) \times 2^{-2}$

1 0 0

0.5 × 2 = 1.0

0.2 × 2 = 0.4

.4 × 2 = 0.8

.8 × 2 = 1.6

6 × 2 = 12

→ 0011

exp − 127 = n

exp − 127 = −2   + 127

exp = −2 + 127

= 125

0 0111 1101 100 . . . - - - 0

22 0

sign

125/2 = 62 r1

62/2 = 31 r0

31/2 = 15 r1

15/2 = 7 r1

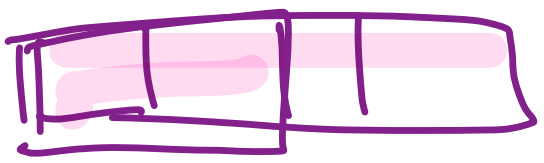
7/2 = 3 r1

3/2 = 1 r1

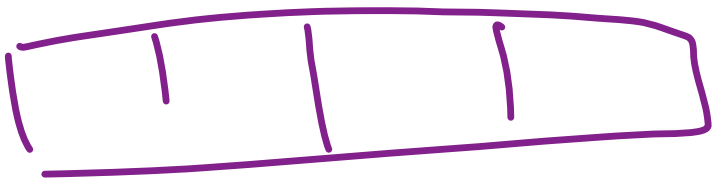
= 1

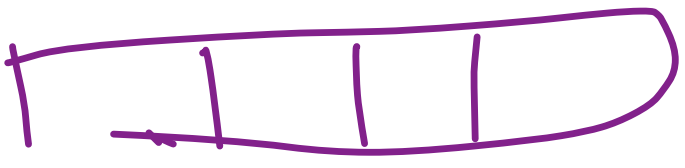
0111 1101

unions

a,b 

struct

a 

b 

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union

short, char 

100.0

$\Rightarrow \quad 2^n (1 + \boxed{frac})$

$$\frac{100}{2^{\lfloor \log_2(100) \rfloor}}$$

$\log_2(100) \approx 6.6$

$\lfloor 6.6 \rfloor = 6$

$$\frac{100}{2^6} = n = 1.5625$$

$100 = \boxed{1.5625} * 2^6$

exponent $-127 = 6$

exponent $= 6 + 127$

$= \boxed{133}$

133 in binary:

0b10 0001

133/2 = 66 r1

66/2 = 33 r0

33/2 =

frac

0.5625 x2 = 1.125 — 1001...

0.125 x2 = 0.25

0.25 x2 = 0.5

0.5 x2 = 1.0

sign

0 because it positive

result:

0 0010 0001 1001 0000000...