Stabilizing Controller with Structured Norm-bounded Uncertainty for a Lower-Limb Exoskeleton

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***Abstract*— This paper describes accounting for time-varying uncertainty in a lower limb exoskeleton. Using a linear fractional transformation (LFT) of the isolated uncertainty blocks, the exoskeleton system can account for perturbations in the system parameters. Also an H_∞ optimal controller was synthesized to achieve system stability. The final result is a construction of the nominal system with a controller K that enforces this stability constraint. However, the resulting performance of the system did not show that implementing it into a tracking control framework to track a reference input θ_k would be successful.**

I. INTRODUCTION

Strokes are currently among the leading causes of prolonged disability in America [1] and these victims often suffer forms of motor impairment as a result. Independent living for these individuals is often not feasible since they lack the motor function to perform activities like walking. To compensate, patients often have to attend long sessions of gait rehabilitation. To improve rehabilitation outcomes for these patients, research into including robotic devices into gait training has been an active topic of discussion.

Specifically, lower-limb robotic exoskeletons are a subject of investigation due to their ability to provide gravity compensation, partial compensation of swing leg weight, feedforward movement assistance during swing and compensatory gait correction [2]. However, these systems are very dynamic and are subect to frequent disturbances that cannot be modeled. These disturbances can include patient effort exerted and design admittances.

The degree in which these exoskeletal systems operate varies from only a one degree of freedom actuator around the axis of rotation of the knee on a single leg to applying two to three degree of freedom actuators located at the hip and knee on both legs. This paper will be considering the former for simplicity and is similar in design to [3].

II. THE EXOSKELETON

The exoskeletal system considered is a one degree of freedom device with an actuator located at the same axis of rotation as the knee. The actuator is then connected to two links that are connected to the thigh and lower leg thereby allowing for the actuator to generate a supplemental torque around the knee. The net moment around the knee can be described by:

$$\tau_k = \tau_b + \tau_e + \tau_{Ie} + \tau_g$$

where τ_h is the net torque around the knee, τ_b is torque caused by the dampening of the knee, τ_e torque caused by exoskeleton, τ_{Ie} inertial torque caused by the rotation of the actuator and τ_g is torque caused by gravity. Due to rehabilitation sessions taking place at low speeds and the low inertial moment caused by the motor these forces are considered negligible. Therefore,

$$\tau_k = \tau_e + \tau_g$$

Expanding the definitions for each torque:

$$\tau_k = I\ddot{\theta}_k(t) = l_h gmsin(\theta_k) - l_s k(\theta_e - \theta_k)$$

Where the system parameters can be defined as the following:
θ_k: angle of knee displacement
θ_e: angle of exoskeleton displacement
I : Inertia of lower leg + exoskeleton
l_h : length from ankle to knee
l_s : length from applied $F_s hank$ to knee
m: mass of human+exoskeleton
k: rotary spring constant

and the uncertain parameter in the system are
$m = m_0(1 + \eta_m \delta_m)$
$k = k_0(1 + \eta_k \delta_k)$

Letting
$x_1(t) = \theta_k(t)$
$x_2(t) = \dot{x}_1(t)$
$\dot{x}_2(t) = \ddot{\theta}_k(t)$
$u(t) = \theta_e(t)$
We can derive a transfer function and block diagram for the system with isolated uncertainty blocks

$$x(s) = \frac{-l_s k}{m l_h^2 s^2 - (l_h g m + l_s k)} u(s)$$

III. THE LFT

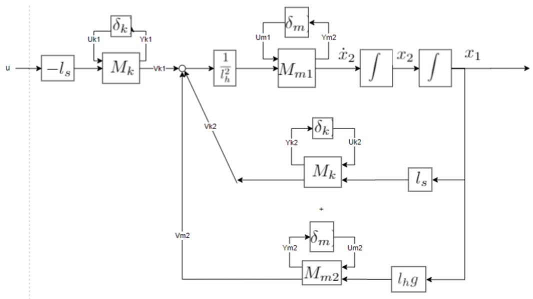

Fig. 1. Block Diagram with Isolated Uncertainty Blocks

The uncertain terms can be expressed by evaluating the upper star product $\bar{S}(M, \delta)$
for $\frac{1}{m_0(1+\eta_m\delta_m)}$ term

$$\frac{1}{m} = \frac{1}{m_0(1+\eta_m\delta_m)} = \bar{S}(M_{m1}, \delta_m)$$

$$M_{m1} = \begin{bmatrix} -\eta_m & \frac{1}{m_0} \\ -\eta_m & \frac{1}{m_0} \end{bmatrix}$$

for $m_0(1 + \eta_m\delta_m)$ term

$$m = m_0(1 + \eta_m\delta_m) = \bar{S}(M_{m1}, \delta_m)$$

$$M_{m2} = \begin{bmatrix} 0 & m_0 \\ \eta_m & m_0 \end{bmatrix}$$

for $k_0(1 + \eta_k\delta_k)$ term

$$k = k_0(1 + \eta_k\delta_k) = \bar{S}(M_k, \delta_k)$$

$$M_k = \begin{bmatrix} 0 & k_0 \\ \eta_k & k_0 \end{bmatrix}$$

Expressing the equations for all signals in the block diagram
$\dot{x}_1 = x_2$
$\dot{x}_2 = -\eta_m u_{m1} + \frac{1}{l_h^2 m_0}(v_{k1} + v_{k2} + v_{m2})$
$y_{m1} = -\eta_m u_{m1} + \frac{1}{l_h^2 m_0}(v_{k1} + v_{k2} + v_{m2})$
$y_{m2} = l_h g m_0 x_1$
$y_{k1} = -l_s k_0 w$
$y_{k2} = l_s k_0 x_1$
$z = x_1$

$v_{m2} = \eta_m u_{m2} + m_0 l_h g x_1$
$v_{k1} = \eta_k u_{k1} - k_0 l_s w$
$v_{k2} = \eta_k u_{k2} + k_0 l_s x_1$

$u_{m1} = \delta_m y_{m1}$
$u_{m1} = \delta_m y_{m1}$
$u_{k1} = \delta_k y_{k1}$
$u_{k2} = \delta_k y_{k2}$

The linear fractional transformation (LFT) can then be represented in a 9-matrix representation

$$\begin{bmatrix} \dot{x} \\ y \\ z \end{bmatrix} = \left[\begin{array}{c|c|c} A & B_1 & B_2 \\ \hline C_1 & D_{11} & D_{12} \\ \hline C_2 & D_{21} & D_{22} \end{array}\right] \begin{bmatrix} x \\ u \\ w \end{bmatrix}$$

$$\left[\begin{array}{cc|cccc|c} 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ \frac{k_0 l_s + m_0 l_h g}{l_h^2 m_0} & 0 & -\eta_m & \frac{\eta_m}{l_h^2 m_0} & \frac{\eta_k}{l_h^2 m_0} & \frac{\eta_k}{l_h^2 m_0} & -\frac{k_0 l_s}{l_h^2 m_0} \\ \hline \frac{k_0 l_s + m_0 l_h g}{l_h^2 m_0} & 0 & -\eta_m & \frac{\eta_m}{l_h^2 m_0} & \frac{\eta_k}{l_h^2 m_0} & \frac{\eta_k}{l_h^2 m_0} & -\frac{k_0 l_s}{l_h^2 m_0} \\ l_h g m_0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -l_s k_0 \\ l_s k_0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

Upon achieving this result, completing the Upper Feedback Interconnection

$$\bar{S}(P, \Delta) = (P_{22} + P_{21}(I - \Delta P_{11})^{-1}\Delta P_{12}) \begin{bmatrix} x \\ q \end{bmatrix}$$

and substituting values of $g = 9.8m/s^2$, $l_h = 1$ meters, $l_s = 0.5$ meters, $m_0 = 60$ kg, $\eta_m = 0.2$, $k_0 = 15$ Nm/rad, and $\eta_k = 0.1$ allows us to retrieve the following matrices for the controller synthesis LMI:

$$A = \begin{bmatrix} 0 & 1 \\ 598 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ -0.0017 \end{bmatrix} \; B2 = \begin{bmatrix} 0 \\ -0.0017 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \; D12 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \; D22 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$M = \begin{bmatrix} 0 & 0 & 0 & 0 \\ -0.2000 & 3.4708e-05 & 1.7354e-05 & 1.7354e-05 \end{bmatrix}$$

$$N = \begin{bmatrix} 0.1055 & 0 \\ 588 & 0 \\ 0 & 0 \\ 10 & 0 \end{bmatrix} \; Q = 0$$

IV. MATHEMATICAL FORMULATION

A. LMI 1: H_∞ Optimal State-Feedback Controller with Structured Norm-Bounded Uncertainty

There exists K such that the system with u(t) = Kx(t)

$$\dot{x}(t) = Ax(t) + Bu(t) + Mp(t) + B_2 w(t)$$
$$q(t) = Nx(t) + D_{12}u(t)$$
$$y(t) = Cx(t) + D_{22}u(t)$$

$$p(t) = \Delta(t)q(t)$$
$$\Delta \in \mathbf{\Delta}, ||\Delta|| \leq 1$$

satisfies $||y||_{L_2} \leq \gamma ||u||_{L_2}$ if there exists some $\Theta \in \mathbf{P\Theta}$, Z and $P > 0$ such that:

$$\begin{bmatrix} AP + BZ + PA^T + Z^T B^T + B_2 B_2^T + M\Theta M^T & (CP + D_{22}Z)^T & PN^T + Z^T D_{12}^T \\ CP + D_{22}Z & -\gamma^2 I & 0 \\ NP + D_{12}Z & 0 & -\Theta \end{bmatrix} < 0$$

where the recovered controller is $K = ZP^{-1}$

B. LMI 2: Structured Singular Value of the Closed Loop System

Suppose the system M has a transfer function $\hat{M}(s) = C(sI - A)^{-1}B + D$ with $\hat{M} \in H_\infty$. The following are equivalent

1. There exists $\Theta \in \mathbf{\Theta}$ such that $||\Theta M \Theta^{-1}||^2 < \gamma$
2. There exists $\Theta \in \mathbf{P\Theta}$ and $X > 0$ such that

$$\begin{bmatrix} A_{cl}^T X + X A_{cl} & X B_{cl} \\ B_{cl}^T X & -\Theta \end{bmatrix} + \frac{1}{\gamma^2} \begin{bmatrix} C_{cl}^T \\ D_{cl}^T \end{bmatrix} \Theta \begin{bmatrix} C_{cl} & D_{cl} \end{bmatrix} < 0$$

V. MATLAB RESULTS

A. Controller Sythesis

The controller sythesis performed using LMI 1 resulted in the following values for Z, Θ, X and $,\gamma$

$$Z = \begin{bmatrix} -0.6121 & 2.2434e+04 \end{bmatrix}$$

$$\Theta = \begin{bmatrix} 126.29 & 0 & 0 & 0 \\ 0 & 126.29 & 0 & 0 \\ 0 & 0 & 4.32e+06 & 0 \\ 0 & 0 & 0 & 4.32e+06 \end{bmatrix}$$

$$X = \begin{bmatrix} 0.0184 & -0.4633 \\ -0.4633 & 12.3419 \end{bmatrix}$$

$$K = \begin{bmatrix} 8.4048e+05 & 3.3369e+04 \end{bmatrix}$$

$$\gamma = 3.5941e+03$$

Successful synthesization of this controller on the nominal plant results in the system achieving stabilization. However, this value of gamma does not imply a very useful value for guaranteed performance since the upper-bounded L_2 gain of the output is approximately $3,594$ times the L_2 gain of the input. Having a gain that is close to this upper bound would likely result in low system performance or even failure.

B. Structured Singular Value of Closed Loop System

Closing the loop with controller K can be done by constructing the following closed loop matrices:
$A_{cl} = A + B_{cl}K$
$C_{1,cl} = C_1 + D_{cl}K$
$Bcl = B$
$D_{cl} = D_{12}$

Using bisection on γ in LMI 2
$\gamma = 0.3174$

Since the structured singular value μ is defined in terms of the scalings Θ:

$$\mu(M, \mathbf{\Delta}) = \inf_{\Theta \in \mathbf{\Theta}} ||\Theta M \Theta^{-1}||$$

and the condition from LMI 2 that states

$$||\Theta M \Theta^{-1}||^2 < \gamma$$

we can calculate the upper bound of μ to be approximately $\mu < 0.5634$. The structured singular value μ is a measure of robustness that describes the norm of the smallest allowable destabilization $||\delta||$
$\mu(M, \mathbf{\Delta}) = \frac{1}{\inf\limits_{\Delta \in \mathbf{\Delta}, I - M\Delta \text{ is singular}} ||\Delta||}$

It follows that the closed loop system can be increased increase by a factor $1/u$ =1.7 before losing stability.

VI. TRACKING FRAMEWORK

The resulting stabilized, closed loop system can now be considered our nominal system in a tracking control framework [4]. The results from implementing this is not included in this paper but the problem is formulated according to Figure 2 where $r = \theta_{kref}$ is the reference knee angle, $w = \theta_e$, $e = \theta_{kerror}$, and $K2$ is a compensator that that is designed to optimize the performance of e.

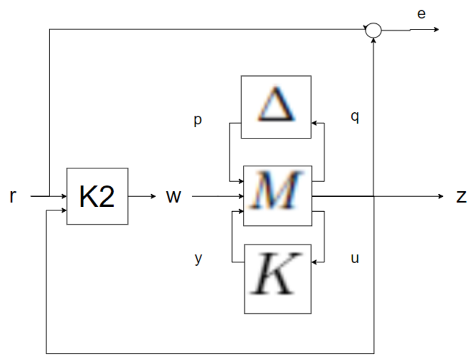

Fig. 2. Block Diagram for Nominal System in the Tracking Framework

VII. CONCLUSION

The proposed method yields fairly conservative results due to the restrictions put on $\mathbf{\Delta} \in \mathbb{R}^{4x4}$ instead of $\mathbb{R}^{2x2}$. If these uncertainty parameters were condensed, the results would likely be more accurate and yields tighter bounds. Also, the controller that was synthesized did not produce meaningful performance guarantees and adding this loosening the restrictions on Δ may improve this. Final performance metrics in the tracking framework would have to be tested to confirm this but the current state of the developed system does not provide enough evidence to support moving forward with this step.

References are important to the reader; therefore, each citation must be complete and correct. If at all possible, references should be commonly available publications.

REFERENCES

[1] M. Johnson, K. Wisneski, J. Anderson, D. Nathan and R. Smith, "Development of ADLER: The Activities of Daily Living Exercise Robot," in The First IEEE/RAS-EMBS International Conference on Biomedical Robotics and Biomechatronics, Milwaukee, 2006.

[2] Menga, Ghirardi. "Lower Limb Exoskeleton for Rehabilitation with Improved Postural Equilibrium." Robotics (Basel) 7.2 (2018): 28–.

[3] Karavas, Ajoudani. "Tele-Impedance Based Assistive Control for a Compliant Knee Exoskeleton." Robotics and autonomous systems 73 (2015): 78–90.

[4] Lucia, Franzè. "Stabilization and Reference Tracking for Constrained Switching Systems: A Predictive Control Approach." International journal of adaptive control and signal processing 31.12 (2017): 1871–1884.