Homework #8
Algorithms I
600.463
Spring 2017

Due on: Thursday, April 27th, 11:59pm
Late submissions: will NOT be accepted
Format: Please start each problem on a new page.
Where to submit: On Gradescope, under HW8.
Please type your answers; handwritten assignments will not be accepted.
To get full credit, your answers must be explained clearly,
with enough details and rigorous proofs.

April 19, 2017

1 Problem 1 (20 points)

A bipartite graph $G = (V, E)$, where $V = L \cup R$, is *d-regular* if every vertex $v \in V$ has degree exactly d.

(a) Show that for every *d-regular* bipartite graph, $|L| = |R|$.

(b) Model the maximum *d-regular* bipartite matching as a max-flow problem. Show that the max-flow value from s to t in the formulation is $|L|$.

(c) Prove that every *d-regular* bipartite graph has a matching of cardinality $|L|$.

2 Problem 2 (20 points)

In the *maximum k-cut* problem, we are given an undirected graph $G = (V, E)$, and non-negative weights $w_{ij} \geq 0$ for all $(i, j) \in E$. The goal is to partition the vertex set V into k parts $V_1, \ldots, V_k$ so as to maximize the weight of all edges whose end-points are in different parts (i.e., $\max_{(i,j)\in E: i\in V_a, j\in V_b, a\neq b} w_{ij}$).
Give a randomized $\frac{k-1}{k}$-approximation algorithm for the MAX k-CUT problem.

Hint: please review Chapter 5.1 of the book—"The Design of Approximation Algorithms" (Williamson and Shmoys 2010), and try to solve this problem using similar ideas.