Homework #6
Algorithms I
600.463
Spring 2017

Due on: Thursday, April 6th, 11:59pm
Late submissions: will NOT be accepted
Format: Please start each problem on a new page.
Where to submit: On Gradescope under HW6
Please type your answers; handwritten assignments will not be accepted.
To get full credit, your answers must be explained clearly,
with enough details and rigorous proofs.

March 30, 2017

Problem 1 (20 points)

Suppose you are given an undirected graph G with weighted edges and a minimum spanning tree T of G.

- Design an algorithm to update the minimum spanning tree when the weight of a single edge e is *increased*.

- Design an algorithm to update the minimum spanning tree when the weight of a single edge e is *decreased*.

In both cases, the input to your algorithm is the edge e and its new weight; your algorithms should modify T so that it is still a minimum spanning tree. Analyze the running time of your algorithms and prove the correctness.

Problem 2 (20 points)

Problem 2.1 (10 points)

Let $G = (V, E)$ be a directed, weighted graph with weight function $w : E \to \mathbb{R}$. Give an $O(VE)$-time algorithm to find, for each vertex $v \in V$, the value $\delta^*(v) = min_{u \in V}\{\delta(u, v)\}$, where $\delta(u, v)$ is the *shortest-path weight* from u to v defined in the textbook.

Problem 2.2 (10 points)

Let $G = (V, E)$ be a directed, weighted graph with nonnegative weight function $w : E \to \{1, 2, \ldots, W\}$ for some nonnegative integer W. Devise an algorithm to compute the shortest path distances from a given source vertex s in $O(WV + E)$ time.