Homework #2
Introduction to Algorithms/Algorithms 1
600.363/463
Spring 2014

Due on: Tuesday, February 11th, 5pm
Late submissions: will NOT be accepted
Format: Please start each problem on a new page.
Where to submit: On blackboard, under student assessment
Please type your answers; handwritten assignments will not be accepted.
To get full credit, your answers must be explained clearly,
with enough details and rigorous proofs.

February 4, 2014

1 Problem 1 (20 points)

1.1 15 points

Given a set of numbers $x_1, x_2, \ldots, x_n$, we define μ_∞ as the value of μ that minimizes the quantity $\max_i |x_i - \mu|$. That is,

$$\mu_\infty = \min_{\mu} \max_{1 \le i \le n} |x_i - \mu|.$$

Give an algorithm that computes μ_∞ for a set of n numbers in $O(n)$ time. Prove the correctness of your algorithm and prove that it runs in $O(n)$ time.

1.2 5 points

In class, we discussed the closest-pair problem for points in $\mathbb{R}^2$. What about when our points are in $\mathbb{R}$? That is, given a list of real numbers $x_1, x_2, \ldots, x_n$, find the value

$$\min_{i \ne j} |x_i - x_j|.$$

Give an algorithm that computes this value in $O(n \log n)$ time. Prove the correctness of your algorithm and prove that its runtime is indeed $O(n \log n)$.

2 Problem 2 (20 points)

2.1 15 points

Given a sequence of distinct integers $a_1, a_2, \ldots, a_n$ sorted in ascending order (so that $a_1 < a_2 < \cdots < a_n$), devise an algorithm that returns True if there exists an element in the sequence such that $a_i = i$, and returns false if no such element exists. Prove the correctness of your algorithm and give an analysis of its runtime. Your algorithm must run in $O(\log n)$ time for full credit.

2.2 5 points

An array of numbers A is *almost sorted* if for every $1 \leq i \leq \sqrt{n} \leq j$, we have $A[i] \leq A[j]$ and for every $\sqrt{n} \leq j \leq k \leq n$ we have $A[j] \leq A[k]$. Give an algorithm that takes as input an almost sorted array A and sorts A in $o(n)$ time.

3 Optional Problems

Solve the following problems from CLRS: $3.1, 3.5, 4.1, 7.2$