Homework #5
Introduction to Algorithms/Algorithms 1
600.363/463
Spring 2016

Due on: Thursday, March 3rd, 11.59pm
Late submissions: will NOT be accepted
Format: Please start each problem on a new page.
Where to submit: On blackboard, under student assessment
Please type your answers; handwritten assignments will not be accepted.
To get full credit, your answers must be explained clearly,
with enough details and rigorous proofs.

February 26, 2016

1 Problem 1 (10 points)

In class we considered the algorithm SELECT, which determines the ith smallest element in the array of size n in $O(n)$ time for the worst case input. The first step of this algorithm is division into $n/5$ groups of 5 elements each. Consider other two versions of this algorithm: the first one uses division into $n/3$ groups of 3 elements each and the second one uses division into $n/7$ groups of 7 elements each. Otherwise both algorithms implement the same routine as SELECT. Which algorithm is asymptotically faster, the one with division into groups of 3, groups of 5 (SELECT) or groups of 7? Prove your statement.

2 Problem 2 (20 points)

Professor asked Bob to find the median of an integer array A, of size n, which is stored on the lab server. Bob has access to the server, however, his rights are very limited: he can only read data from the server, but cannot write to the server.
The array is so large that Bob can not just copy it to his machine. Bob's computer has only $O(logn)$ memory. Help Bob to develop an efficient algorithm which finds

the median of A. Provide a correctness proof and running time analysis. Full score will be given for $O(n \log n)$ expected time algorithm.

3 Problem 3 (20 points)

Bob is a presidential candidate. He is planning to visit every city in the state M to give a talk in every city. Bob's budget is very tight so he can buy only two flight tickets. Bob can land in any city A in the state M and he can depart from any city B in the state M, but inside the state he can only commute by car. Between any two cities in the state M there is exactly one highway (the network of roads can be represented by a complete graph). Because of Bob's tight budget, Bob cannot stop in any city more than once. To make things more complicated for Bob, each highway is a one-way highway. Prove that there exists cities A and B and a path from A to B, such that Bob will visit every city exactly once.

Note: you need to prove existence of such A, B and a path from A to B for any for any given configuration of the highway's one way directions.

A complete graph is a simple graph with an edge between any two vertices.