Homework #7
Introduction to Algorithms/Algorithms 1
600.363/463
Spring 2016

Due on: Apr 4th, 11:59pm Late submissions: will NOT be accepted
Format: Please start each problem on a new page.
Where to submit: On blackboard, under student assessment
Please type your answers; handwritten assignments will not be accepted.
To get full credit, your answers must be explained clearly,
with enough details and rigorous proofs.

March 27, 2016

1 Problem 1 (20 points)

You are given a graph $G = (V, E)$ which represents a computer network. For each edge $e_i \in E$ its weight $w(e_i)$ represents the probability of failure of this edge $w(e_i) = p_i$, where $0 < p_i < 1$. All failures happen independently, thus the probability that there will be a failure on the path $e_{i_1} \to e_{i_2} \to e_{i_3}$ is $1 - (1 - p_{i_1})(1 - p_{i_2})(1 - p_{i_3})$. You want to develop an algorithm which finds NOT the shortest path but the path with the lowest probability of failure. Prove correctness and provide running time analysis. Full score will be given for algorithm working in $O(|V|^2)$ time.

2 Problem 2 (20 points)

You are given a graph $G = (V, E)$. The graph is connected and has at most $|V| + 10$ edges. Provide an algorithm with running time $O(|V|)$, which finds the minimum spanning tree of G. You may assume all edges have distinct weights. Prove correctness and provide running time analysis.

3 Problem 3 (10 points)

You are given an unweighted directed graph $G = (V, E)$ and nodes s and t. All nodes in the graph are colored either green, yellow or white. Provide an algorithm which determines if there is a path (might be not simple path) from s to t which goes through both green and yellow vertices at least once, and if so outputs such a path. Full score will be given for running time $O(|V| + |E|)$. Prove correctness and provide running time analysis.