Homework #7
Introduction to Algorithms/Algorithms 1
600.363/463
Spring 2013

Due on: Tuesday, March 25th, 5pm
Late submissions: will NOT be accepted
Format: Please start each problem on a new page.
Where to submit: On blackboard, under student assessment
Please type your answers; handwritten assignments will not be accepted.
To get full credit, your answers must be explained clearly,
with enough details and rigorous proofs.

March 11, 2014

Problem 1 (20 points)

Suppose that we have an undirected graph $G = (V, E)$ with all edge weights distinct. Prove that G has a unique minimum spanning tree.

Problem 2 (20 points)

Let $G = (V, E)$ be a connected, unweighted, undirected graph and let $u, v \in V$ be two vertices in graph G. Since G is connected, there exists a shortest path between nodes u and v, with some length $\delta(u, v)$. Of course, it is possible that there are many paths from u to v that all have this length. Call the number of $u - v$ paths of this length the *connection strength* between vertices u and v. That is, the connection strength between vertices u and v is the number of paths from u to v of length $\delta(u, v)$. Since G is connected, the connection strength between two vertices must be at least 1. Give an algorithm that takes a connected, unweighted, undirected graph G along with two vertices u and v in G, and returns the connection strength between vertices u and v. Your algorithm should run in $O(|V| + |E|)$ time. Prove the correctness of your algorithm and prove its runtime.