Homework #9
Algorithms I
600.463
Spring 2017

Due on: Thursday, May 4th, 11:59pm
Late submissions: will NOT be accepted
Format: Please start each problem on a new page.
Where to submit: On Gradescope, under HW9
Please type your answers; handwritten assignments will not be accepted.
To get full credit, your answers must be explained clearly,
with enough details and rigorous proofs.

April 26, 2017

Problem 1 (20 points)

You are given a biased coin but you dont know what the bias is. Can you simulate a fair coin? Please prove the correctness of your solution.

Problem 2 (20 points)

The following approach is often called *reservoir sampling*. Suppose we have a sequence of items passing by one at a time. We want to maintain a sample of one item with the property that it is uniformly distributed over all the items that we have seen at each step. Moreover, we want to accomplish this without knowing the total number of items in advance or storing all of the items that we see. Consider the following algorithm, which stores just one item in memory at all times. When the first item appears, it is stored in the memory. When the k-th item appears, it replaces the item in memory with probability $1/k$. Explain why this algorithm solves the problem. (Exercise 2.18 from the Book "**Probability and Computing:** randomized algorithms and probabilistic analysis", Mitzenmacher and Upfal 2009. `https://catalyst.library.jhu.edu/catalog/bib_5738970`)