Homework #5
Introduction to Algorithms/Algorithms 1
600.363/463
Spring 2013

Due on: Tuesday, March 4th, 5pm
Late submissions: will NOT be accepted
Format: Please start each problem on a new page.
Where to submit: On blackboard, under student assessment
Please type your answers; handwritten assignments will not be accepted.
To get full credit, your answers must be explained clearly,
with enough details and rigorous proofs.

February 26, 2014

1 Problem 1 (20 points)

Suppose we have a complete directed binary tree $T = (V, E)$ with n nodes, in which all leaves are at the same depth (a complete binary tree is one in which every node has either two children or none). Then we must have $n = 2^d - 1$ for some positive integer d. Suppose further that we have a function $f : V \mapsto \mathbb{R}$, and $f(u) \neq f(v)$ for all $u, v \in V$ for which $u \neq v$. A node $u \in V$ is *golden* if $f(u)$ is such that $f(u) > f(v)$ for all $v \in V$ for which $(u, v) \in E$ or $(v, u) \in E$. That is, node u is golden if the value assigned to u by function f is strictly greater than the value assigned to the parent of u (if it exists) and to the children of u (if they exist). Given any node $u \in V$, we can find out the value of $f(u)$ by *querying* u. Devise an algorithm that finds a golden node in T using $O(\log n)$ queries. Prove the correctness of your algorithm, and prove that it does indeed require $O(\log n)$ queries.

2 Problem 2 (20 points)

Devise an algorithm that takes an undirected graph $G = (V, E)$ encoded as an adjacency list and returns `True` if graph G is a tree and `False` if not. Prove the correctness of your algorithm and prove its runtime. Recall that a graph G is a tree if it is connected and contains no cycles. Your algorithm should run in $O(|V|+|E|)$ time for full credit. You may assume that the input graph G does not contain any double edges and does not contain any self-loops– that is, there is at most one edge between any pair of nodes u and v, and there are no edges of the form $\{u, u\}$.

Optional exercises

Solve the following problems and exercises from CLRS: 22.2-5, 22.2-8, 22.4-5, 22.5-4, 22-4, 22-1.