Homework #7
Algorithms I
600.463
Spring 2017

Due on: Tuesday, April 18th, 11:59pm
Late submissions: will NOT be accepted
Format: Please start each problem on a new page.
Where to submit: On Gradescope, under HW7
Please type your answers; handwritten assignments will not be accepted.
To get full credit, your answers must be explained clearly,
with enough details and rigorous proofs.

April 11, 2017

Problem 1 (20 points)

Let $G = (V, E)$ be a directed graph. Vertex $a \in V$ is a *central* vertex if for all $b \in V$ there exists a path from a to b. Design an algorithm to test whether graph G has a *central* vertex in $O(V + E)$ time. Prove the correctness of your algorithm and analyze the running time.

Problem 2 (20 points)

A "friendly" Airline has n flights [1]. In order to avoid "re-accommodation", a passenger must satisfy several requirements. Each requirement is of the form "you must take at least k_i flights from set F_i". The problem is to determine whether or not a given passenger will experience "re-accommodation". The hard part is that any given flight cannot be used towards satisfying multiple requirements. For example, if one requirement states that you must take at least two flights from $\{A, B, C\}$, and a second requirement states that you must take at least two flights

[1] Any relation to actual airlines of similar name is purely coincidental.

from $\{C, D, E\}$, then a passenger who had taken just $\{B, C, D\}$ would not yet be able to avoid "re-accommodation".

Your job is to give a polynomial-time algorithm for the following problem. Given a list of requirements $r_1, r_2, \ldots, r_m$ (where each requirement r_i is of the form: "you must take at least k_i flights from set F_i"), and given a list L of flights taken by some passenger, determine if that passenger will experience "re-accommodation".

Specifically, you just need to show how this can be reduced to a network flow problem and assume there is a given polynomial-time blackbox algorithm solving the flow problem. Prove that your reduction is correct.