Homework #2
Introduction to Algorithms/Algorithms 1
600.363/463
Spring 2017

Due on: Tuesday, February 14th, 5pm
Late submissions: will NOT be accepted
Format: Please start each problem on a new page.
Where to submit: On Gradescope, a single PDF file.
Please type your answers; handwritten assignments will not be accepted.
To get full credit, your answers must be explained clearly,
with enough details and rigorous proofs.

February 16, 2017

1 Problem 1 (20 points)

1.1 (10 points)

Give tight asymptotic bounds (Θ) for $T(n)$ in each of the following recurrences. If you cannot provide tight bounds, provide upper and lower bounds, making them as tight as possible. Assume that $T(n)$ is a constant for $n \leq 8$ or other appropriately chosen small constant. Provide a short proof or justification of your answer. (Applying the master theorem is a proof.)

- $T(n) = T(3n/4) + 2n \log n - 4$
- $T(n) = 4T(n/2) + n^2 \log_{10} n + 10n \log n$
- $T(n) = 4T(n/3) + n \log^2 n$

1.2 (10 points)

Given a set A of n integers and an integer T, design an algorithm to test whether k of the integers in A add up to T. Prove the correctness of your algorithm and analyze the running time. (Note: full credit will be given to an $O(n^{k-1} \log n)$ algorithm).

2 Problem 2 (20 points)

Given an array (length $> k$) with positive and negative numbers, find the maximum average subarray whose length should be greater or equal to given length k. For example, given an array = $\{2, 11, -7, -6, 51, 3\}$, and $k = 3$, the maximum average subarray of length 3 begins at item -6, and the maximum average is $(-6 + 51 + 3)/3 = 16$. Please justify the correctness of your algorithm and analyze the running time.