Homework #6
Introduction to Algorithms/Algorithms 1
600.363/463
Spring 2016

Due on: Thursday, March 24th, 11.59pm
Late submissions: will NOT be accepted
Format: Please start each problem on a new page.
Where to submit: On blackboard, under student assessment
Please type your answers; handwritten assignments will not be accepted.
To get full credit, your answers must be explained clearly,
with enough details and rigorous proofs.

March 16, 2016

1   Problem 1 (25 points)

You are given a simple weighted graph $G = (V, E)$. Suppose that $e \in E$ is an edge that belongs to the cycle C inside G, and has the maximum weight among other edges in C. Prove that minimum spanning tree of G does not contain e.

2   Problem 2 (25 points)

You are given a simple weighted graph $G = (V, E)$ and its minimum spanning tree T_{mst}. Somebody added new edge e to the graph G. Let's call new graph $G' = (V, E \cup e)$. Devise an algorithm which checks if T_{mst} is also a minimum spanning tree for the graph G' or not. Prove correctness of your algorithm and provide running time analysis.

You can assume that all edge weights in G' are distinct. T_{mst} and G' are given to you as adjacency lists.

Your algorithm should work in $O(|V|)$ time for a full score. For the algorithm working in $O(|V|^{1+\varepsilon})$ time (for any positive ε) you will get 15 points.