ASSIGNMENT-05

Name : Monowar Mofiz Arman

ID : 1921079642

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Submitted to : Dr. Mohammad Moniruzzaman Khan

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Digital Logic Design

* Rules of Boolean Algebra:

There are 12 basic rules useful in manipulating and simplifying Boolean expressions;

(1) $A+0=A$

(2) $A+1=1$

(3) $A.0=0$

(4) $A.1=A$

(5) $A+A=A$

(6) $A+\bar{A}=1$

(7) $A.A=A$

(8) $A.\bar{A}=0$

(9) $\bar{\bar{A}}=A$

(10) $A+AB=A$; $A+AB=A(1+B)=A.1=A$.

(11) $A+\bar{A}B=A+B$; $A+\bar{A}B=A+AB+\bar{A}B$ [$\because A=A+AB$]

$=AA+AB+\bar{A}B$ [$\because A=A.A$]

$=AA+AB+A.\bar{A}+\bar{A}.B$ [$\because A.\bar{A}=0$] [$A+0=A$]

$=A+B$

$$(11.1)\ A+\bar{A}B = (A+\bar{A}).(A+B) \quad [\text{Factorizing}]$$
$$= 1.(A+B) \quad [\bar{A}+A=1]$$
$$= A+B. \quad [A.1=A]$$

$$(12)\ (A+B)(A+C) = A+BC$$

Proof: $(A+B)(A+C) = AA+AC+AB+BC$ [Distributive law]

$$= A+AC+AB+BC \quad [\because A.A=A]$$

$$= A(1+C)+AB+BC \quad [\text{Distributive law}]$$

$$= A.1+AB+BC \quad [\because 1+A=1]$$

$$= A+AB+BC \quad [A.1=A].$$

$$= A(1+B)+BC.$$

$$= A.1+BC \quad [\because 1+A=1]$$

$$= A+BC \quad [\because A.1=A]$$

Demorgan's Theorems:

- first theorem: The complement of a product of variables is equal to the sum of the complements of the variables.

$\overline{AB} = \bar{A} + \bar{B}.$

* <u>Second theorem</u>: The complement of a sum of variables is equal to the product of the complements of the variables.

$\overline{A+B} = \bar{A}.\bar{B}.$

Apply De Morgan's theorems to the following expressions:

$\overline{A+B+C+D} = \bar{A}.\bar{B}.\bar{C}.\bar{D}$

$\overline{ABCD} = \bar{A}+\bar{B}+\bar{C}+\bar{D}$

$\overline{\bar{A}\bar{B}\bar{C}\bar{D}} = \bar{\bar{A}}+\bar{\bar{B}}+\bar{\bar{C}}+\bar{\bar{D}} = A+B+C+D.$

$\overline{(AB+C)(A+BC)} = \overline{(AB+C)} + \overline{(A+BC)} = \overline{(AB)}.\bar{C}+\bar{A}.\overline{(BC)}$

$= (\bar{A}+\bar{B}).\bar{C}+\bar{A}(\bar{B}+\bar{C})$

$\overline{\overline{(A+B)}+\bar{C}} = \overline{\overline{(A+B)}}.\bar{\bar{C}} = (A+B).C.$

$\overline{(A+B+C)D} = \overline{A+B+C}+\bar{D} = \bar{A}.\bar{B}.\bar{C}+\bar{D}$

$\overline{A\bar{B}+\bar{C}D+EF} = \overline{(A\bar{B})}.\overline{(\bar{C}D)}.\overline{(EF)} = (\bar{A}+\bar{\bar{B}}).(\bar{\bar{C}}+\bar{D})(\bar{E}+\bar{F})$

$= (\bar{A}+B)(C+\bar{D})(\bar{E}+\bar{F})$

$$\overline{(A+B)\overline{C}\,\overline{D}+E+\overline{F}} = \left(\overline{(A+B)\,\overline{C}\,\overline{D}}\right)\overline{E}\,\overline{\overline{F}}$$

$$= \left(\overline{(A+B)}+\overline{\overline{C}}+\overline{\overline{D}}\right)\overline{E}\,.\,F$$

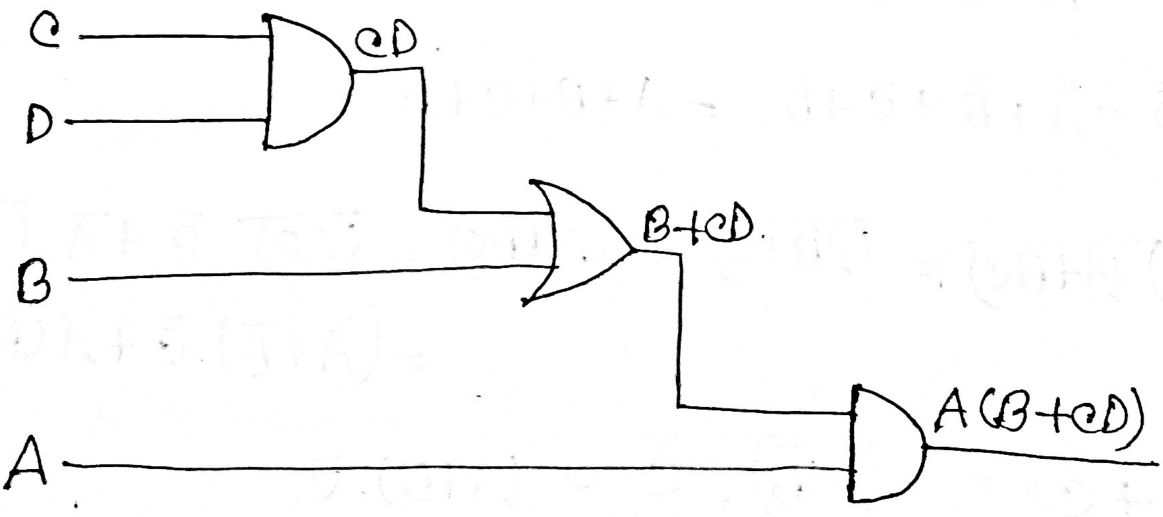
$$= (\overline{A}.\overline{B}+C+D)\,\overline{E}\,.\,F.$$

$$\overline{\overline{A+B\overline{C}}+D\,\overline{(E+\overline{F})}} = \overline{\overline{(A+B\overline{C})}}\;\overline{\left(D\,\overline{(E+\overline{F})}\right)}$$

$$= (A+B\overline{C})\left(\overline{D}+\overline{\overline{(E+\overline{F})}}\right)$$

$$= (A+B\overline{C})(\overline{D}+E+\overline{F}).$$

→ Boolean Expression for a logic circuit:



Simplification using Boolean Algebra:

(a) $AB + A(B+C) + B(B+C) = AB+AB+AC+BB+BC$

$= AB+AC+B+BC \quad [\because BB=B]$

$$= AB + AC + B(1+C)$$

$$= AB + AC + B \underset{[1+C=1]}{} = B(A+1) + AC = B + AC \quad [1+A=1]$$

(b) $\overline{AB} + \overline{AC} + \bar{A}\bar{B}\bar{C} = \bar{A} + \bar{B} + \bar{A} + \bar{C} + \bar{A}\bar{B}\bar{C}$

$$= \bar{A} + \bar{B} + \bar{C} + \bar{A}\bar{B}\bar{C} \quad [\bar{A} + \bar{A} = \bar{A}]$$

$$= \bar{A} + \bar{B} + \bar{C}(1 + \bar{A}\bar{B})$$

$$= \bar{A} + \bar{B} + \bar{C} \quad [\because 1 + \bar{A}\bar{B} = 1]$$

$$= \overline{ABC}$$

(c) $[A\bar{B}(C+BD) + \bar{A}\bar{B}]C = (A\bar{B}C + A\bar{B}BD + \bar{A}\bar{B})C$

$$= (A\bar{B}C + A.0.D + \bar{A}\bar{B})C \quad [\because \bar{B}.B = 0]$$

$$= (A\bar{B}C + 0 + \bar{A}\bar{B})C \quad [\because A.0 = 0]$$

$$= (A\bar{B}C + \bar{A}\bar{B})C \quad [\because A + 0 = A]$$

$$= A\bar{B}C.C + \bar{A}\bar{B}C.$$

$$= A\bar{B}C + \bar{A}\bar{B}C \quad [\because C.C = C]$$

$$= \bar{B}C(A + \bar{A})$$

$$= \bar{B}C.1 \quad [\because A + \bar{A} = 1]$$

$$= \bar{B}C \quad [\because A.1 = A]$$

Standard form of Boolean Expressions:

All Boolean expressions can be expressed in two standard forms:

(1) The Sum of Product (SOP) form,

(2) The Product of Sum (POS) form.

(1) The SOP form:

Examples: $AB + B\bar{C}$

$ABC + CDE + \bar{B}C\bar{D}$

Convert the following expressions into SOP form:

(a) $AB + B(CD + EF) = AB + BCD + BEF$

(b) $\overline{\overline{A+B} + C} = \overline{\overline{(A+B)}} . \bar{C} = (A+B)\bar{C} = A\bar{C} + B\bar{C}$

(2) The POS Form:

Examples: $(\bar{A}+B)(A+\bar{B}+C)$

$(A+B)(A+\bar{B}+C)(\bar{A}+C)$

The standard SOP form: A standard SOP expression is one in which all the variables in the domain appear in each product term in the expression.

<u>Example</u>: $A\bar{B}CD + \bar{A}\bar{B}C\bar{D} + AB\bar{C}\bar{D}$

Convert the following Boolean expression into standard SOP form:

$A\bar{B}\bar{C} + \bar{A}\bar{B} + AB\bar{C}D$

$A\bar{B}C = A\bar{B}C(D+\bar{D}) = A\bar{B}CD + A\bar{B}C\bar{D}$

$\bar{A}\bar{B} = \bar{A}\bar{B}(C+\bar{C}) = \bar{A}\bar{B}C + \bar{A}\bar{B}\bar{C}$

$= \bar{A}\bar{B}C(D+\bar{D}) + \bar{A}\bar{B}\bar{C}(D+\bar{D})$

$= \bar{A}\bar{B}CD + \bar{A}\bar{B}C\bar{D} + \bar{A}\bar{B}\bar{C}D + \bar{A}\bar{B}\bar{C}\bar{D}$

$\therefore A\bar{B}C + \bar{A}\bar{B} + AB\bar{C}D = A\bar{B}CD + A\bar{B}C\bar{D} + \bar{A}\bar{B}CD +$

$\bar{A}\bar{B}C\bar{D} + \bar{A}\bar{B}\bar{C}D + \bar{A}\bar{B}\bar{C}\bar{D} + AB\bar{C}D$

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The standard POS form: A standard POS expression is one in which all the variables in the domain appear in each sum term in the expression.

example: $(A+\bar{B}+C+\bar{D})(\bar{A}+B+C+\bar{D})(\bar{A}+B+\bar{C}+D)$

Convert the following Boolean expression into standard POS form:

$(A+\bar{B})(B+\bar{C}+D)$

now,

$A+\bar{B} = A+\bar{B}+C.\bar{C} = (A+\bar{B}+C)(A+\bar{B}+\bar{C})$ [Rule 12: $A+BC = (A+B)(A+C)$]

$= (A+\bar{B}+C+D.\bar{D})(A+\bar{B}+\bar{C}+D.\bar{D})$

$= (A+\bar{B}+C+D)(A+\bar{B}+C+\bar{D})(A+\bar{B}+\bar{C}+D)(A+\bar{B}+\bar{C}+\bar{D})$

$B+\bar{C}+D = B+\bar{C}+D+A.\bar{A} = (A+B+\bar{C}+D)(\bar{A}+B+\bar{C}+D)$

$(A+\bar{B})(B+\bar{C}+D) = (A+\bar{B}+C+D)(A+\bar{B}+C+\bar{D})(A+\bar{B}+\bar{C}+D)$
$(A+\bar{B}+\bar{C}+\bar{D})(A+B+\bar{C}+D)(\bar{A}+B+\bar{C}+D)$

Developing truth table for standard SOP expression:

let $X = \bar{A}\bar{B}C + A\bar{B}\bar{C} + ABC$

Inputs			Output	Output	Output
A	B	C	X	Y	Z
0	0	0	0	0	1
0	0	1	1	1	0
0	1	0	0	0	1
0	1	1	0	0	1
1	0	0	1	1	0
1	0	1	0	0	0
1	1	0	0	0	1
1	1	1	1	1	1

Converting POS Expression to truth table format:

let $Y = (A+B+C)(A+\bar{B}+C)(A+\bar{B}+\bar{C})(\bar{A}+B+\bar{C})(\bar{A}+\bar{B}+C)$

Determining Standard Expression from a truth table

standard SOP expression:

$Z = \bar{A}\bar{B}\bar{C} + \bar{A}B\bar{C} + \bar{A}BC + AB\bar{C} + ABC$

Standard POS expression:

$$Z = (A+B+\bar{C})\ (\bar{A}+B+C)(\bar{A}+B+\bar{C}).$$

3 variable Karnaugh map:

AB \ C	0	1
00	0	1
01	0	0
11	0	1
10	1	0

Map following SOP expression in the karnaugh map:

$$\bar{A}\bar{B}C + A\bar{B}\bar{C} + ABC.$$

Map the following SOP expression on a K map:

$$\bar{A}\bar{B}C + \bar{A}B\bar{C} + AB\bar{C} + ABC.$$

001 010 110 111

AB \ C		
00	0	1
01	1	0
11	1	1
10	0	0

Map the following SOP expression on a K map:

$\bar{A} + A\bar{B} + AB\bar{C}$

$\bar{A}$	$A\bar{B}$	$AB\bar{C}$
000	100	110
001	101	
010		
011		

AB \ C	0	1
00	1	1
01	1	1
11	1	0
10	1	1

K map simplification of SOP expression:

Use a K map to minimize the following expression:

$A\bar{B}C + \bar{A}BC + \bar{A}\bar{B}C + \bar{A}\bar{B}\bar{C} + A\bar{B}\bar{C}$

101   011   001   000   100

AB \ C	0	1
00	1	1
01	0	1
11	0	0
10	1	1

Simplified SOP expression:

$\bar{B} + \bar{A}C$.

1's in adjacent cells can be grouped. Each group can have 2, 4, 8, 16 .... ones.

Use K map to minimize the following expression:

$\bar{A}\bar{B}\bar{C}\bar{D} + \bar{A}\bar{B}\bar{C}D + \bar{A}B\bar{C}\bar{D} + \bar{A}B\bar{C}D + \bar{A}BCD + \bar{A}BC\bar{D}$

0000 0001 0100 0101 0111 0110

AB \ CD	00	01	11	10
00	1	1	0	0
01	1	1	1	1
11	0	0	0	0
10	0	1	1	0

Simplified SOP expression:

$\bar{A}\bar{C} + \bar{A}B + A\bar{B}D$

Simplify the following SOP expression using K map:

$\bar{D}\bar{C}\bar{B}\bar{A} + \bar{D}\bar{C}B\bar{A} + \bar{D}\bar{C}BA + \bar{D}C\bar{B}A + \bar{D}CB\bar{A} + \bar{D}CBA + D\bar{C}\bar{B}\bar{A} +$

0000 0010 0011 0101 0110 0111 1000

$D\bar{C}B\bar{A} + DCBA + DCB\bar{A} + D\bar{C}BA + D\bar{C}\bar{B}A$

1001

DC \ BA	00	01	11	10
00	1		1	1
01		1	1	1
11			1	1
10	1	1	1	1

Simplified expression:

$B + D\bar{C} + \bar{C}\bar{A} + \bar{D}CA$

Simplify the function expressed in sum of minterms form $F(W, X, Y, Z) = \Sigma(0, 1, 2, 3, 4, 5, 6, 8, 9, 12, 13, 14)$

no. of interms = " (minterms means standard product, ref: morris mano page 48)

WX \ YZ	00	01	11	10
00	0: 1	1: 1	3:	2: 1
01	4: 1	5: 1	7:	6: 1
11	12: 1	13: 1	15:	14: 1
10	8: 1	9: 1	11:	10:

$\therefore F = \bar{Y} + \bar{W}\bar{Z} + X\bar{Z}$

Use a k map to minimize the following:

POS expression:

$(A+B+C)\quad (A+B+\overline{C})\quad (A+\overline{B}+C)\quad (A+\overline{B}+\overline{C})\quad (\overline{A}+\overline{B}+C)$

$000\qquad 001\qquad 010\qquad 011\qquad 110$

AB \ C	0	1
00	0	0
01	0	0
11	0	1
10	1	1

Simplified POS expression:

$\overline{F} = \overline{A} + B\overline{C}$

$F = \overline{\overline{A} + B\overline{C}}$

$= \overline{\overline{A}}\,(\overline{B\overline{C}})$

$= A(\overline{B}+C)$

$F = A\overline{B} + AC = A(\overline{B}+C)$

Simplify the function expressed in product of maxterm form.

$F(x,y,z) = \Pi(0,2,5,7)$

no. of maxterm or sum = 4

$xy \backslash z$	0	1
00	$_0$ 0	$_1$
01	$_2$ 0	$_3$
11	$_6$	$_7$ 0
10	$_4$	$_5$ 0

$$\bar{F} = \bar{X}\bar{Z} + XZ.$$

$$F = \overline{\bar{X}\bar{Z} + XZ}$$

$$= \overline{\bar{X}\bar{Z}} + \overline{XZ}$$

$$= (\bar{\bar{X}} + \bar{\bar{Z}})(\bar{X}+\bar{Z}).$$

$$= (X+Z)(\bar{X}+\bar{Z}).$$

The '1' functions represents the minterms and

The '0' functions represents the maxterms.