Home work - 1

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Ans to the QNO-1(i)

(a)

Unicycle Model:

State space equation (Differential equation):

$$\dot{x} = v\cos\theta$$

$$\dot{y} = v\sin\theta$$

$$\dot{\theta} = \omega$$

Here,

State variables:

(i) x and y : position in 2D space

(ii) θ : orientation of the robot.

Control variables:

(i) v : linear velocity

(ii) ω : angular velocity.

(b)

Differential drive robot:

State space equation (Differential equation):

The linear velocity v and angular velocity ω of the robot are related to the wheel velocities as,

$$v = \frac{V_r + V_l}{2}$$

$$\omega = \frac{V_r - V_l}{d}$$; where d is the distance between the two wheels.

Therefore, the state-space equations are:

$$\dot{x} = v\cos\theta = \frac{V_r + V_l}{2}\cos\theta$$

$$\dot{y} = v\sin\theta = \frac{V_r + V_l}{2}\sin\theta$$

$$\dot{\theta} = \omega = \frac{V_r - V_l}{d}$$

Here,

State variables:

(i) x and y: position in 2D space

(ii) θ : orientation of the robot.

Control variables:

(i) V_r : velocity of the right wheel.

(ii) V_l : velocity of the left wheel.

(c)

Simplified car model:

State-space equation (Differential equation):

$\dot{x} = v\cos\theta$

$\dot{y} = v\sin\theta$

$\dot{\theta} = \frac{v}{L}\tan\phi$; where L is the distance between the front and rear axles of the car.

Here,

State variables:

(i) x and y: position in 2D space.

(ii) θ : orientation of the car.

Control variables:

(i) v : linear velocity

(ii) ϕ : steering angle of the car.

(d)

Planar Quadrotor:

State-space equation (Differential equation):

The planar quadrotor is a simplified 2D model of a quad copter where the quadrotor moves in a plane. The dynamics are governed by Newton's second law of motion and rotational dynamics.

Translation motion:

$$m\ddot{x} = -(T_1 + T_2) \cdot \sin\theta$$

$$m\ddot{y} = (T_1 + T_2)\cos\theta - mg$$

Rational motion:

$I\ddot{\theta} = l(T_1 - T_2)$; where m is the mass of the quadrotor, I is the moment of inertia, l is the distance from the center to each rotor and g is the gravitational acceleration.

Therefore, the state-space equations are:

$$\ddot{x} = -\frac{T_1 + T_2}{m}\sin\theta$$

a

$$\ddot{y} = \frac{T_1 + T_2}{m} \cos\theta - g$$

$$\ddot{\theta} = \frac{l(T_1 - T_2)}{I}$$

Here,

State variables:

(i) x and y : position in 2D space.

(ii) θ : orientation or pitch angle of the quadrotor.

(iii) $\dot{x}$: horizontal velocity.

(iv) $\dot{y}$: vertical velocity.

(v) $\dot{\theta}$: The angular velocity or the rate of change of pitch angle.

Control variables :

(i) T_1 : the thrust generated by the left motor.

(ii) T_2 : the thrust generated by the right motor.

Ans to the QNO-1 (ii)

State variables:

(i) State variables represent the quantities that define the current "state" or condition of the robot at any given time.

(ii) position of the state variables describe the robot's location in space. Example: x, y and sometimes z for 3D systems.

(iii) orientation of the state variables decribe the direction in which the robot is facing. Example; angle θ.

(iv) velocities of the state variables decribe the rates of charge of position and orientation. Example: linear velocity $\dot{x}, \dot{y}$ and angular velocity $\dot{\theta}$.

Example: For unicycle model: State variable: x and y where x, y position of the robot in 2D space. θ (robot's orientation).

Control variables:

(i) Control variables represent the inputs to the robot that can be directly manipulated by an operator or control system to influence the robot's behavior. These inputs control how the state variables evolve over time.

(ii) velocities of the control variables decribe linear velocity v and angular velocity ω which directly affect the robot's motion and orientation.

(iii) Thrusts of the control variables decribe the control inputs can be the force or thrust applied by the rotors. Examples: T_1 and T_2 thrusts.

(iv) Steering angles of the control variables decribe the wheeled robots like cars, the steering angle ϕ is a control input.

<u>Example</u>: For unicycle model: Control variables:

<u>v</u> $\rightarrow$ linear velocity.

<u>ω</u> $\rightarrow$ angular velocity.

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Ans to the QNO-1(iii)

The differences in dynamics between unicycle model, differential drive robot and simplified car model are given below:

Aspect	Unicycle Model	Differential Drive Model	Simplified Car Model
Control inputs	Linear velocity (v) and angular velocity (ω)	Right and left wheel velocities (V_R, V_L)	Linear velocity (v) and steering angle (ϕ).
Turning Mechanism	Rotation is controlled by angular velocity(ω)	Turning is controlled by difference in wheel velocities.	Steering affects turning radius through steering angle.
Lateral Motion	No lateral motion, must rotate to change direction.	No lateral motion must rotate to change direction.	No lateral motion, steering must be used to change direction.
Realism for car like motion	Simplistic, not suited for car like motion	Not flexible than unicycle, but not suited for car like systems.	Realistic for car like systems with front wheel steering.

Turning Flexibility	Turns are controlled via ω which is less flexible	Flexible turning via independent wheel velocities	Turning constrained by steering angle and wheelbase.