Lecture 6: Introduction to Planning
MAE 345/549

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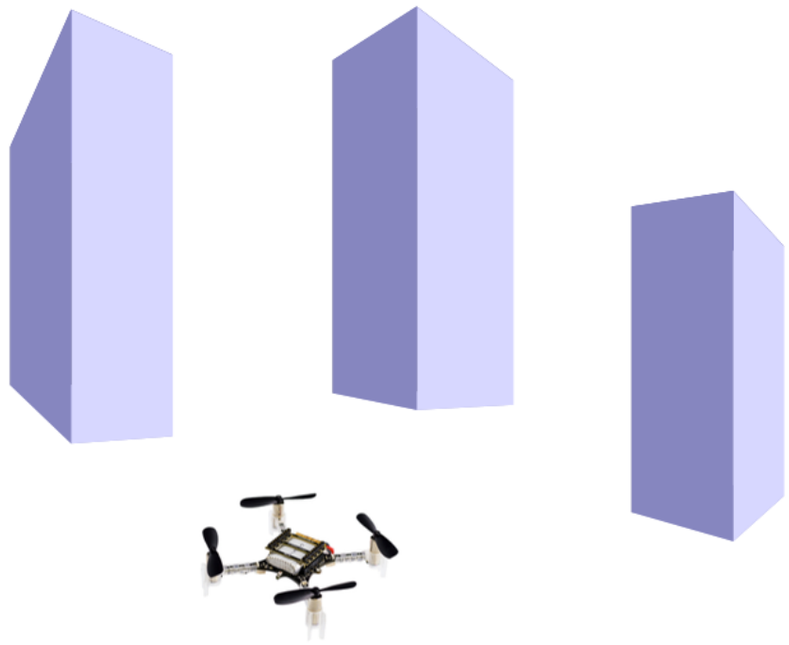
PRINCETON
UNIVERSITY

Mechanical and
Aerospace
Engineering

Motion Planning

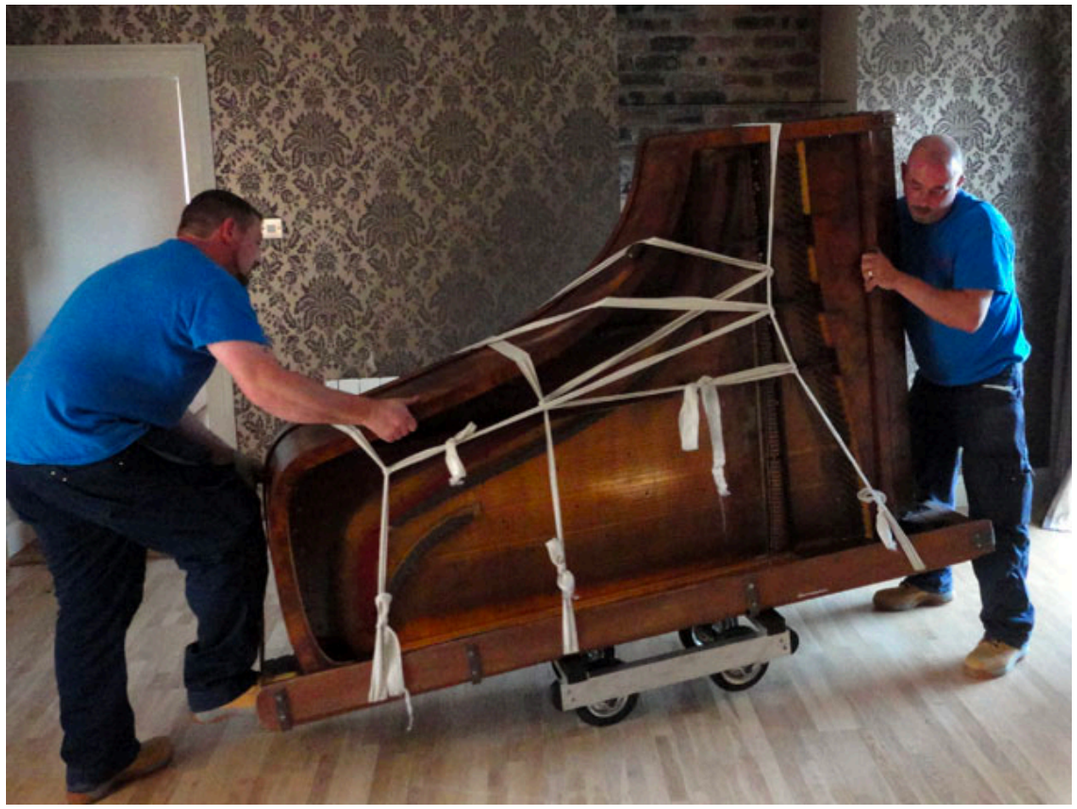
- Recall our overall goal for the course: Make quadrotor navigate through obstacle environments



- Next 5 lectures: focus on particular aspect of this
  - **Motion Planning**

Piano Mover's Problem

- Suppose we have a geometric model (e.g., CAD model) of piano and apartment

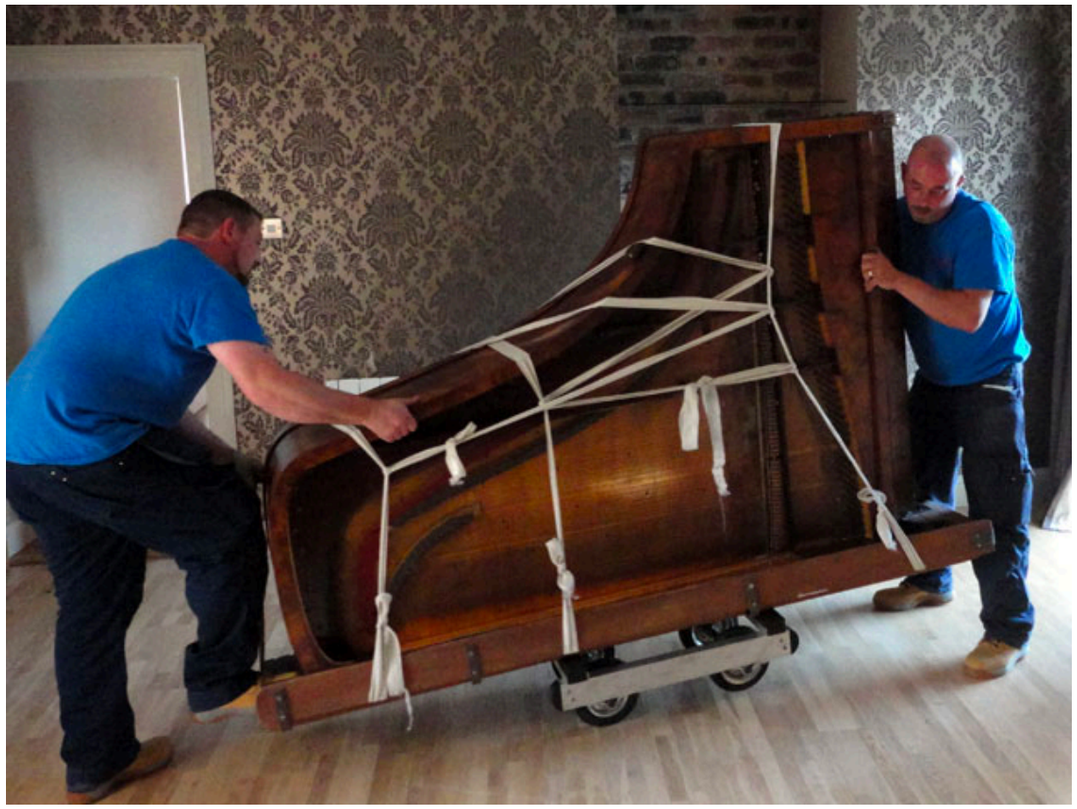


- Question: how can we get piano from point A to point B without hitting anything?

- Complex geometric problem!

Piano Mover's Problem

- Suppose we have a geometric model (e.g., CAD model) of piano and apartment



- Question: how can we get piano from point A to point B without hitting anything?

- Complex geometric problem!

Piano Mover's Problem

https://www.youtube.com/watch?v=cXm3WW-geD8

Piano Mover's Problem

- Piano mover's problem: long history in robotics (since ~1980s)

- Lots of early work on understanding computational complexity of problem (PSPACE-complete)

- Today, the term "piano mover's problem" is synonymous with "motion planning"

Piano Mover's Problem

Motion Planning: problem of finding path from configuration A to configuration B (without collisions)

(Assumes that geometric model of environment is provided. We will think about how to obtain such a model later in course.)

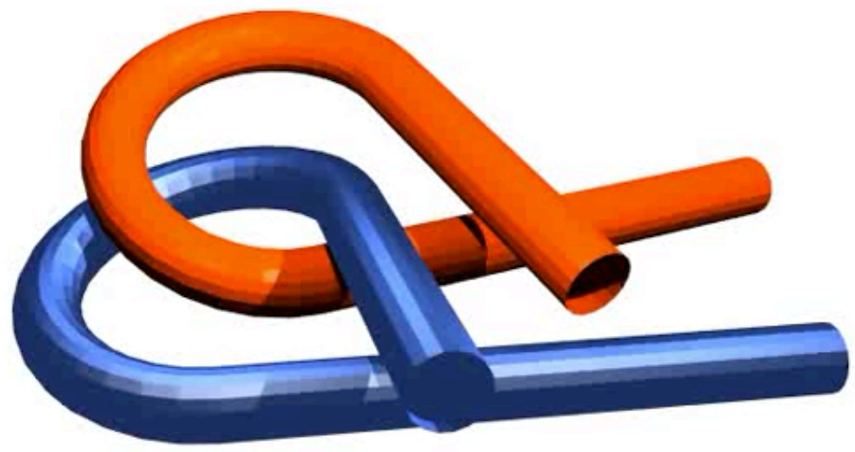
Examples of Planning

- Video from Real-Time robotics: http://rtr.ai/autonomous-vehicles/

Examples of Planning

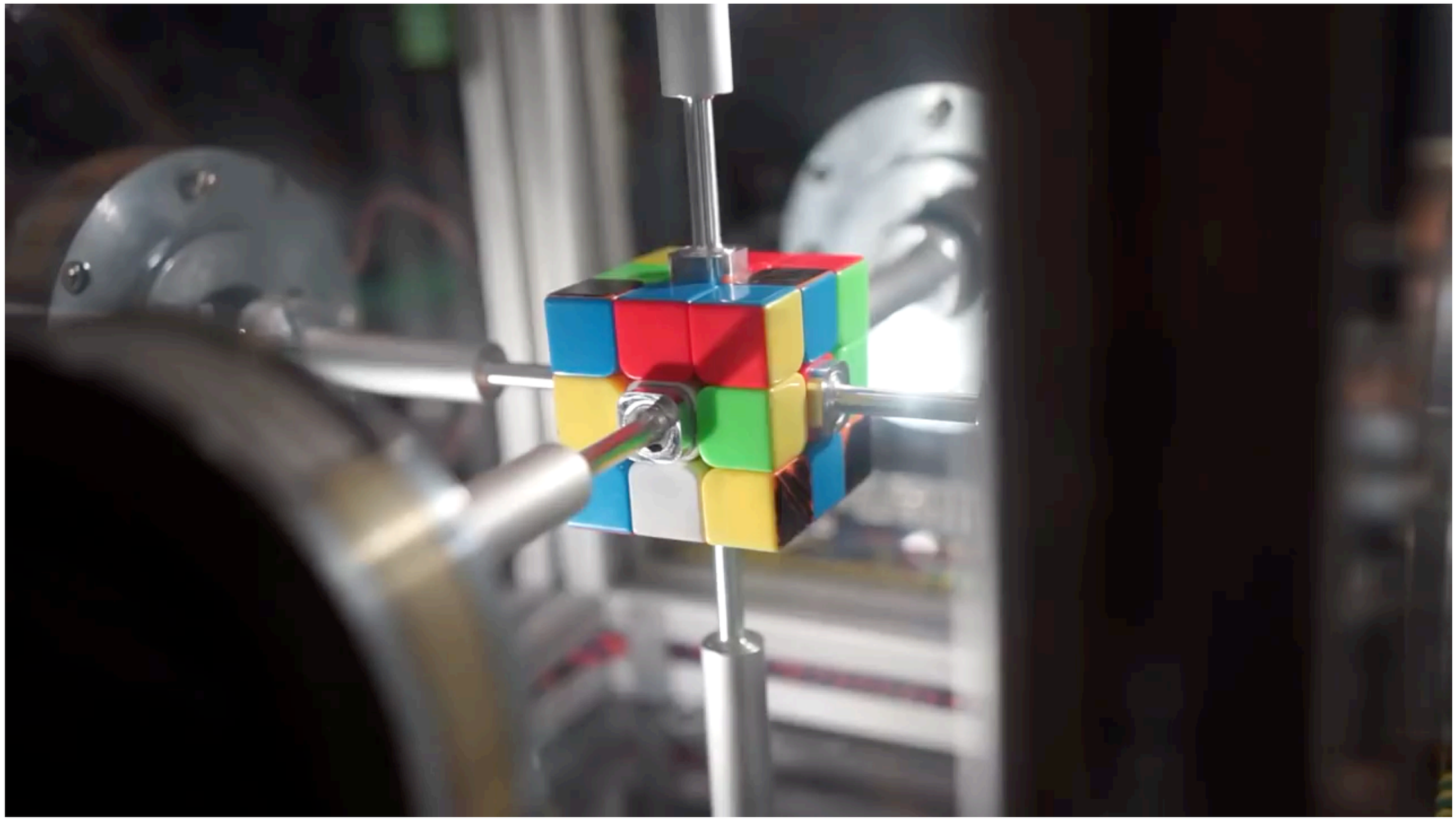
Intelligent and Mobile Robotics Group
http://imr.felk.cvut.cz

Alpha Puzzle 1.0



https://www.youtube.com/watch?v=UTbiAu8IXas

Examples of Planning



https://www.youtube.com/watch?v=OZu9gjQJUQs

Feasibility and Optimality

- As with control, there are two considerations with motion planning:

  - Feasibility: find me *any* path that gets from A to B without collisions

    Challenging by itself!

  - Optimality: find me *best* path (according to some performance criterion, e.g., distance)

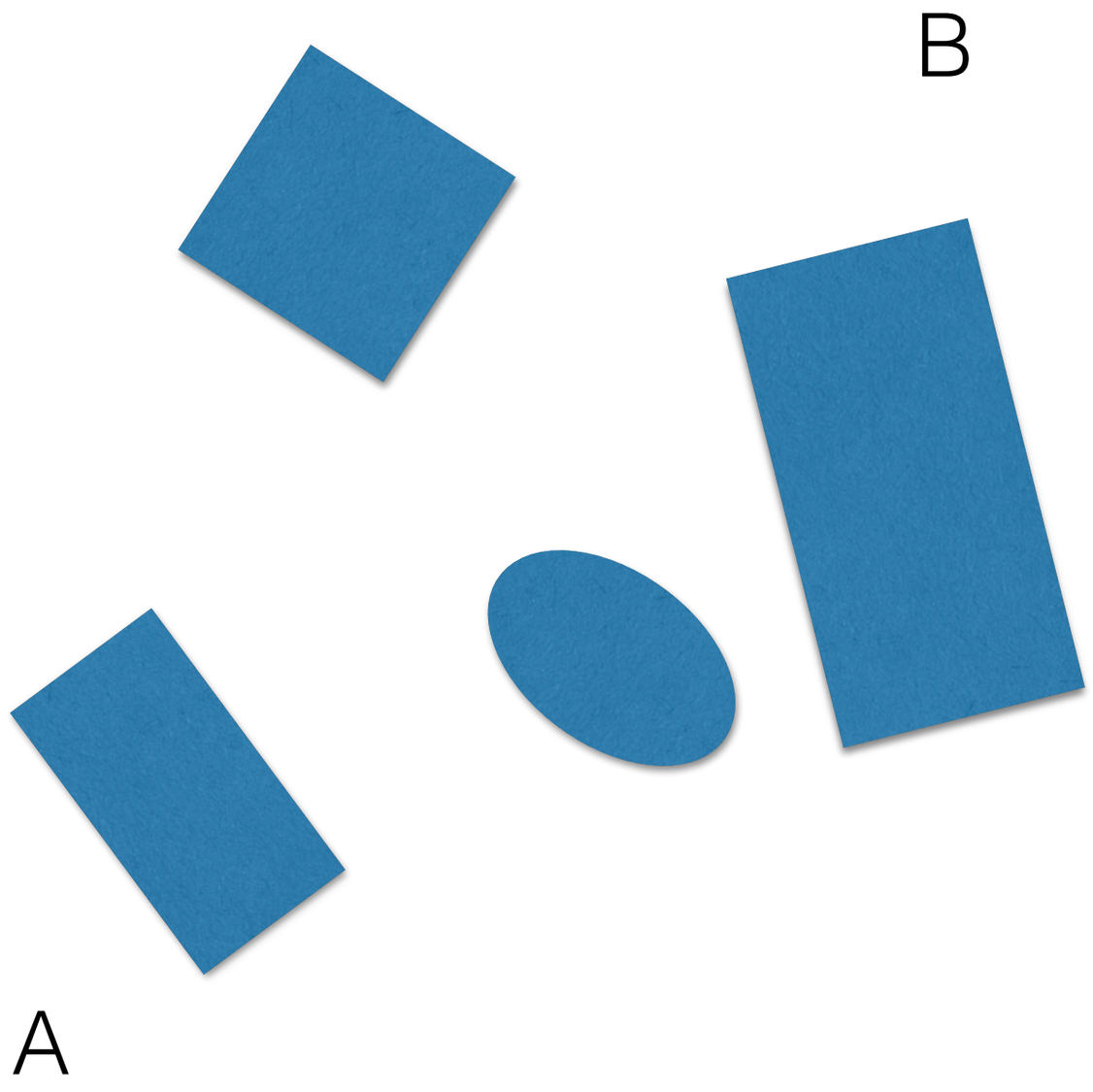
    Nice to have

Assumptions

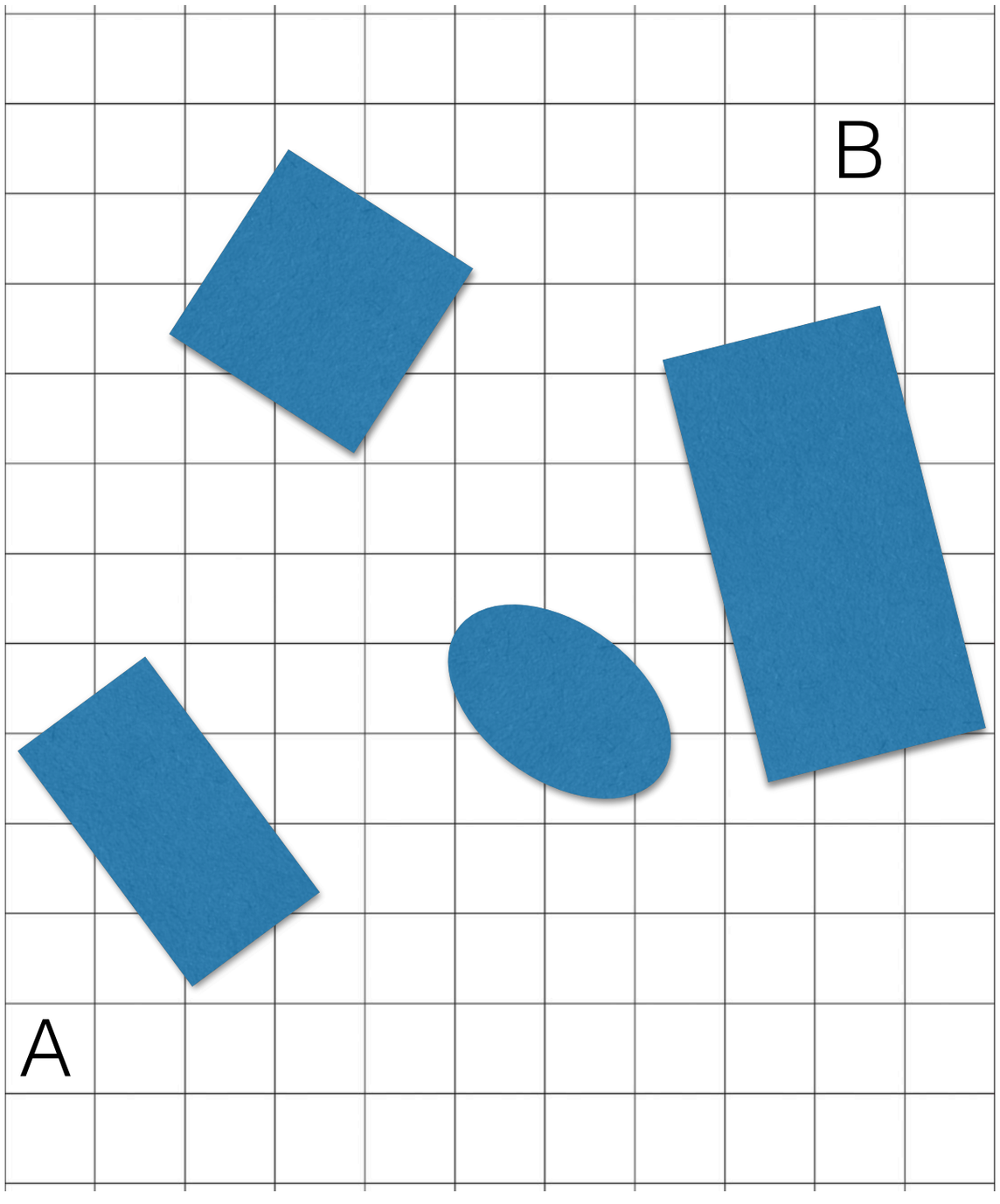
- Assumptions we will make for now:
    - Geometry of environment and robot is given to us
    - Any path can actually be executed by robot (big assumption; not true in general)
    - Any path can be perfectly followed by robot (not true, but feedback control helps with this)

Discrete Planning

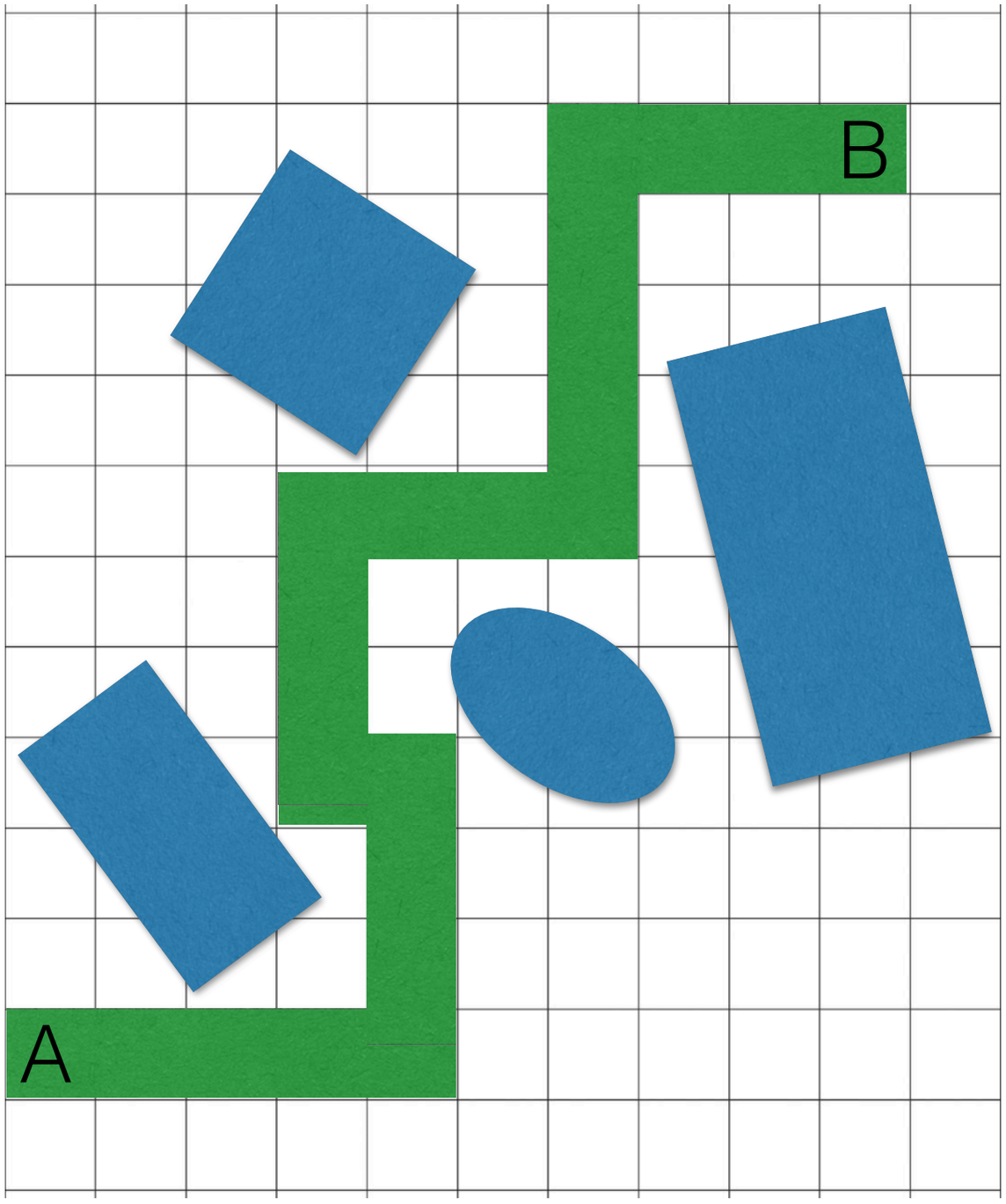
Discretization



Discretization

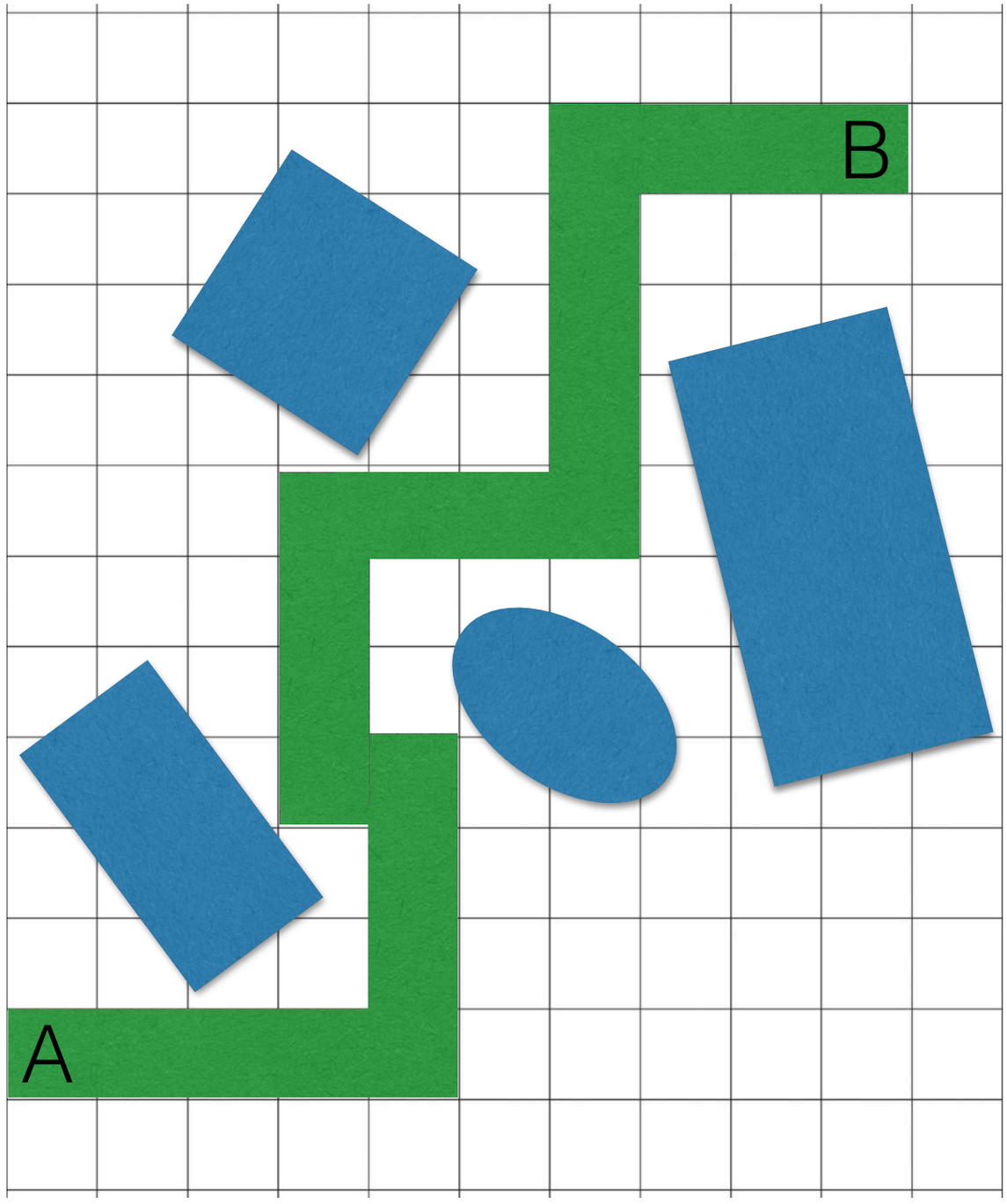


Discretization



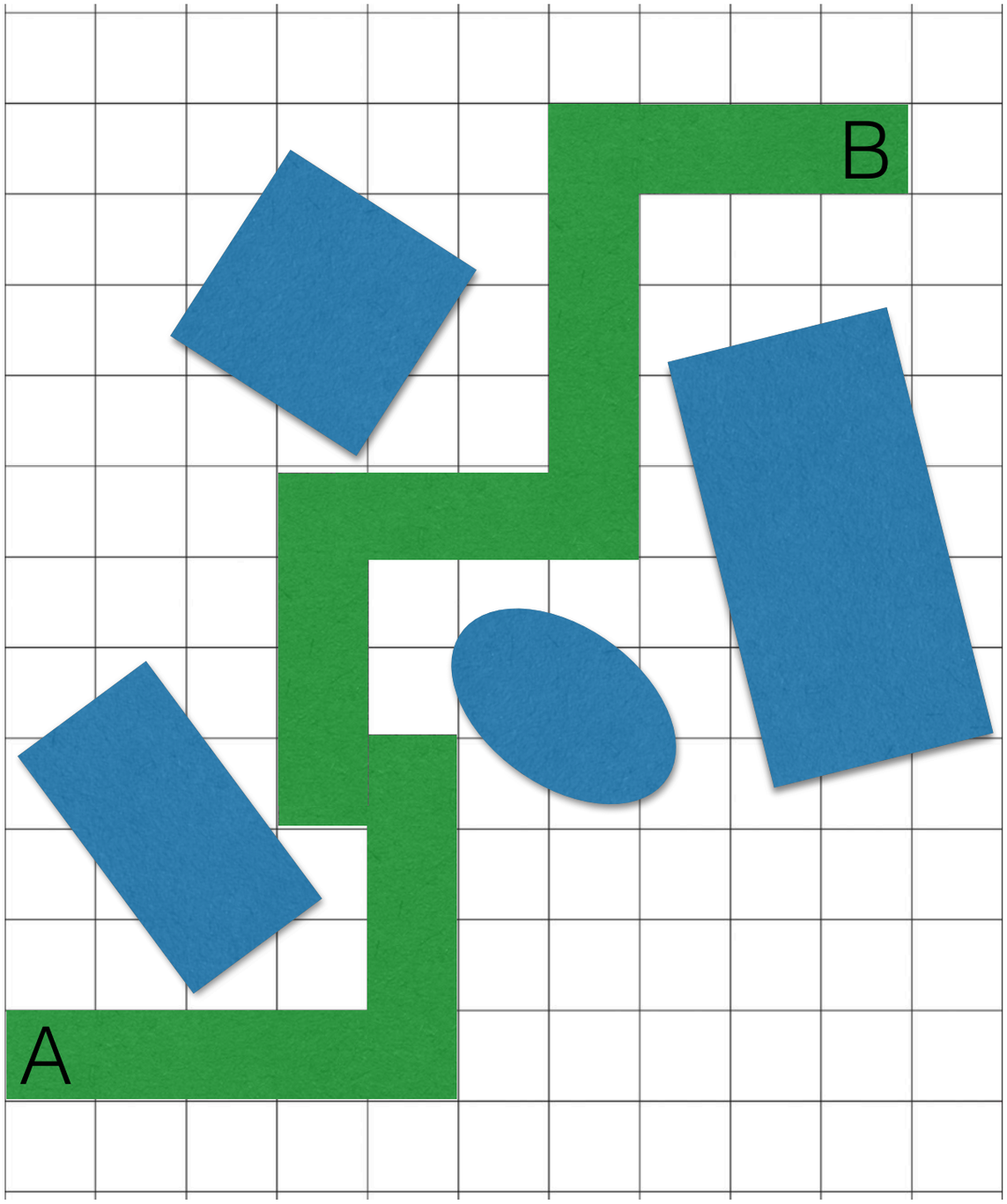
Discretization

- Discretization allows us to apply powerful algorithms for graph search to solve planning problem



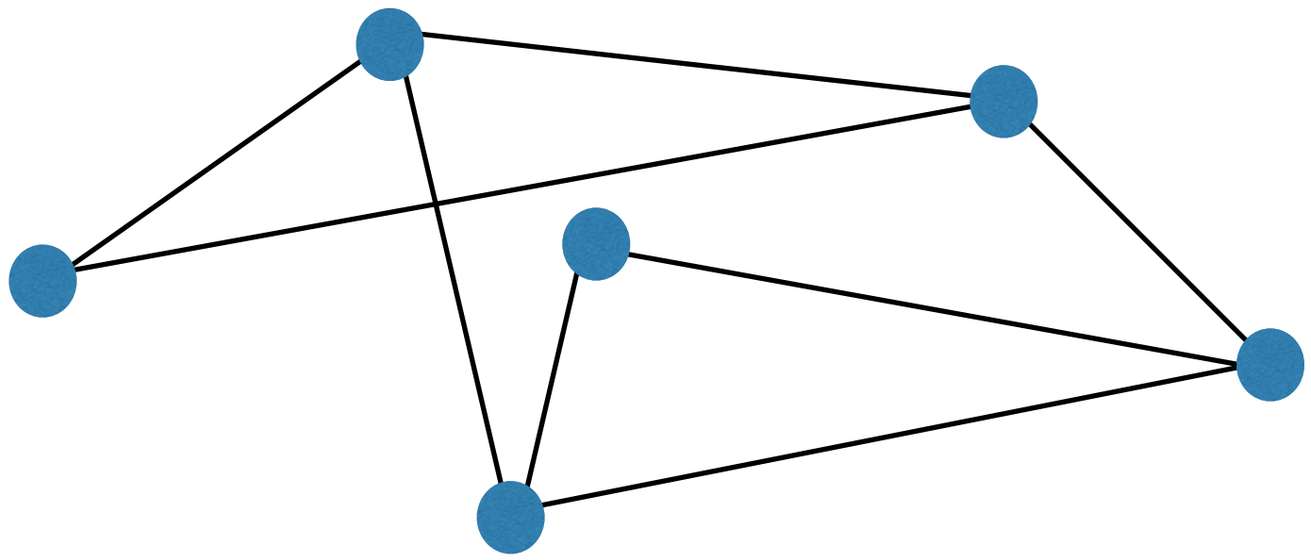
Discretization

- Choices we made:
- Uniform grid
- 4-connected (e.g., instead of allowing diagonal motion, i.e., 8-connected)
- Resolution of grid (finer grid is better approximation, but makes planning harder)



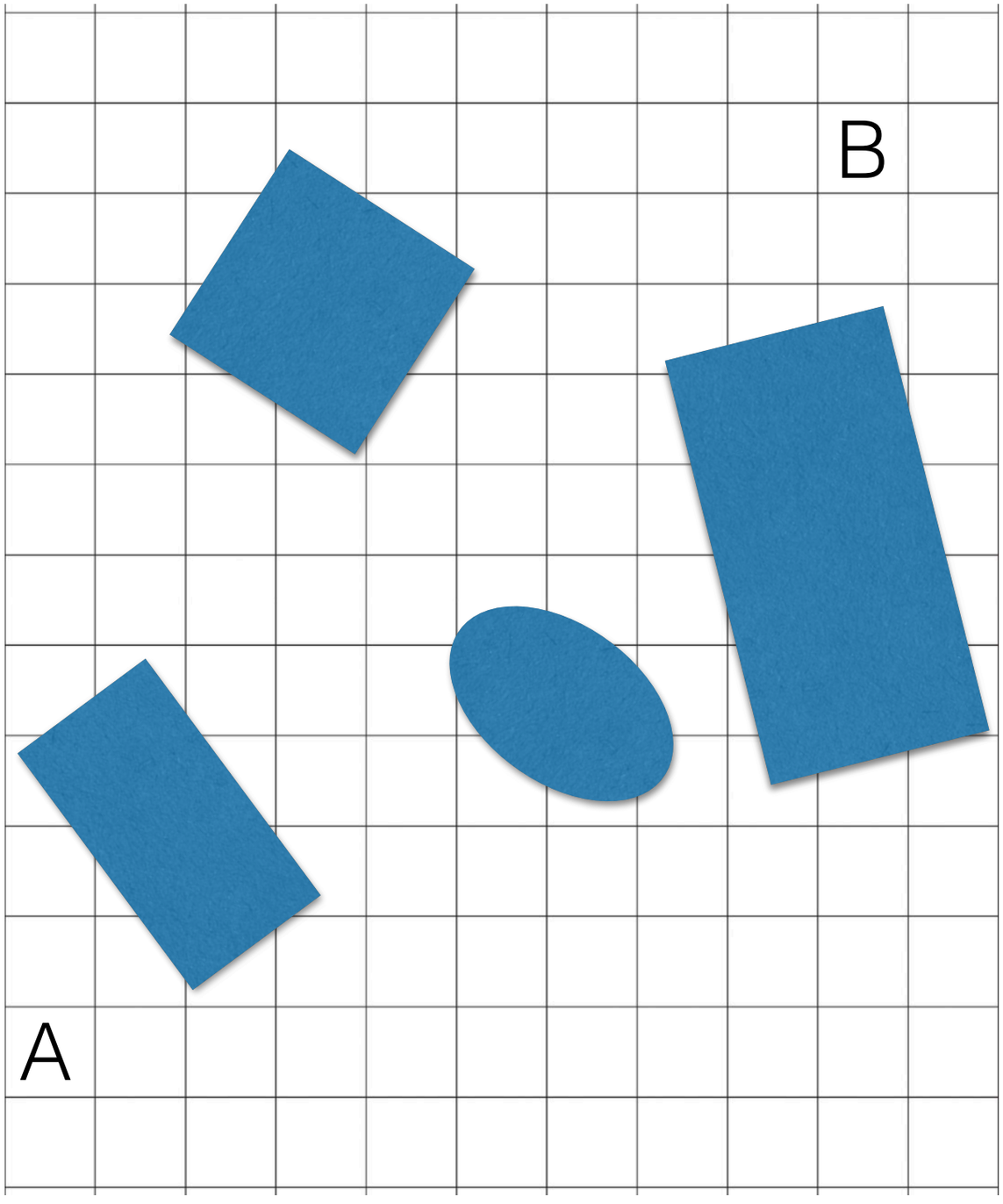
Graph Search

- Graph: collection of vertices (nodes) and edges
  - Edges define connectivity



Planning as Graph Search

- Associate a vertex with each cell in grid
- Connect vertices with edges based on 4-connectivity (or 8-connectivity)
- Delete any vertices (and corresponding edges) that have obstacle



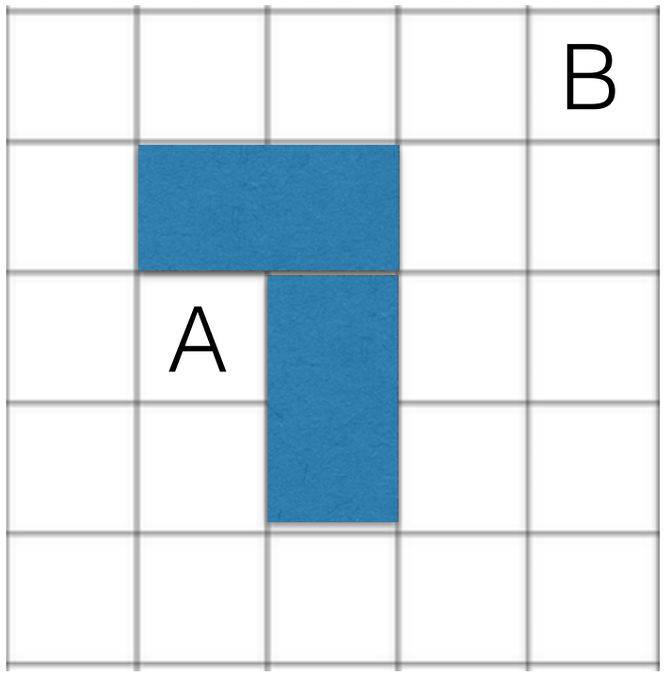
Planning as graph search: Example

Planning as graph search: Example

References: "Planning Algorithms" by S. LaValle

- Ch. 1.1, 1.2, 1.3
- Ch 2.1, 2.2

Example

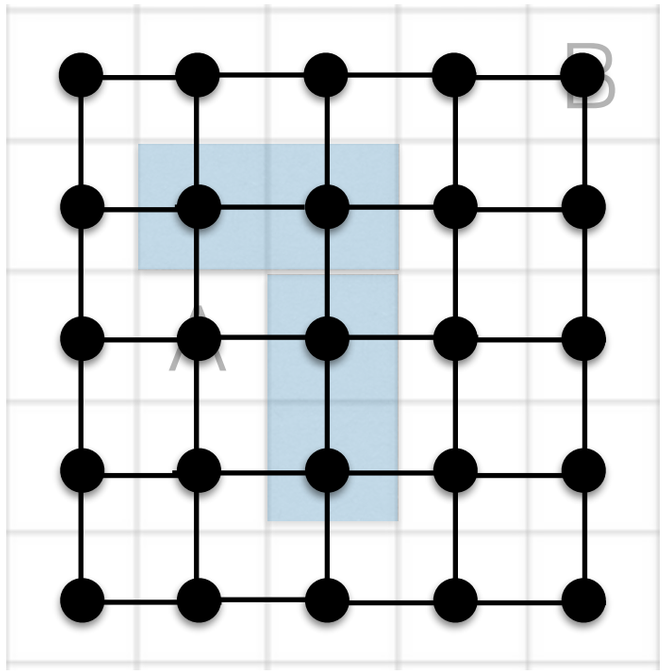


1. Associate a vertex with each cell

2. Edges connect vertices that we can travel between in one step

3. Remove vertices (and associated edges) that are occupied by obstacles

Example



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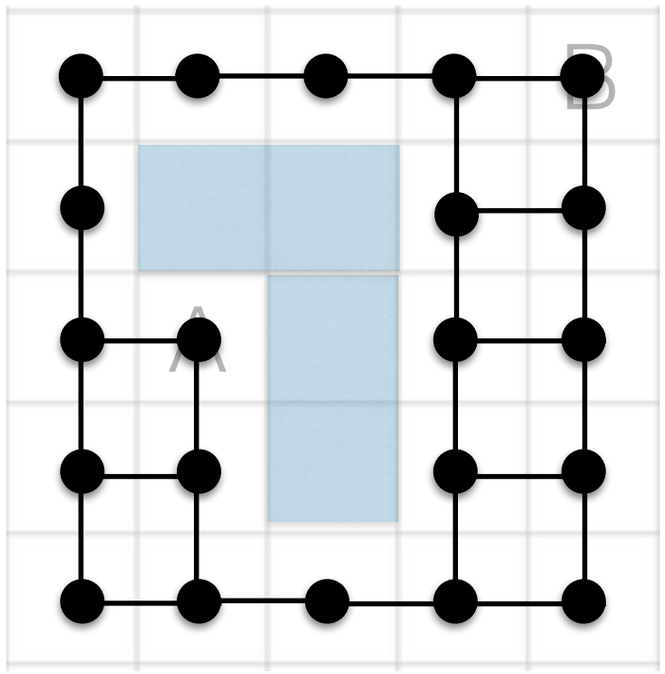

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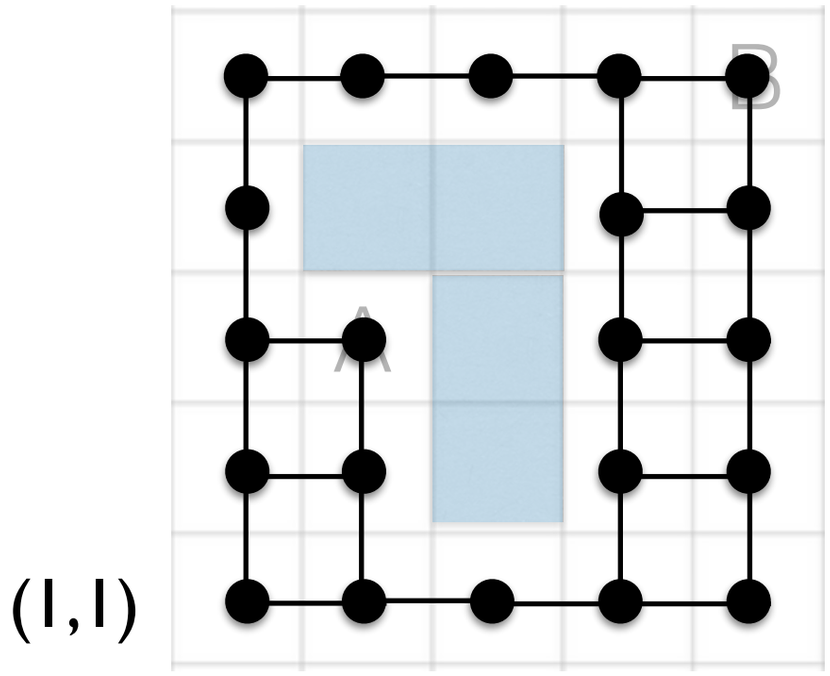
Example



We will label vertices: (i,j) = (x, y) = (column, row)

Examples: A : (2,3)

B : (5,5)

Graph Search

- We will start at A, and incrementally explore the graph until we have arrived at B

- Today: focus only on finding feasible plans

  - We will discuss optimal plans in the next lecture

Algorithm: Forward Search

Algorithm 1: Forward Search

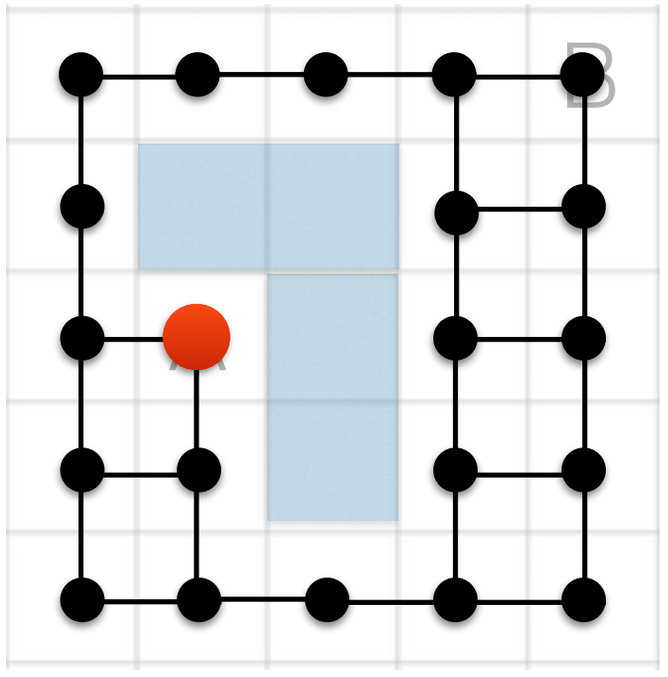
```
Q.Insert(A) and mark A as visited;
while Q not empty do
    x ← Q.GetVertex() ;
    if x ∈ x_G then
        // x_G is goal set, e.g., {B} ;
        Return SUCCESS;
    end
    for all neighbors x' of x do
        if x' not visited then
            Parent(x') ← x ;
            Mark x' as visited ;
            Q.Insert(x') ;
        end
    end
end
Return FAILURE ;
```

Example

Q = {(2,3)} = {A}

Mark A as visited

(visited vertices will be marked in red)

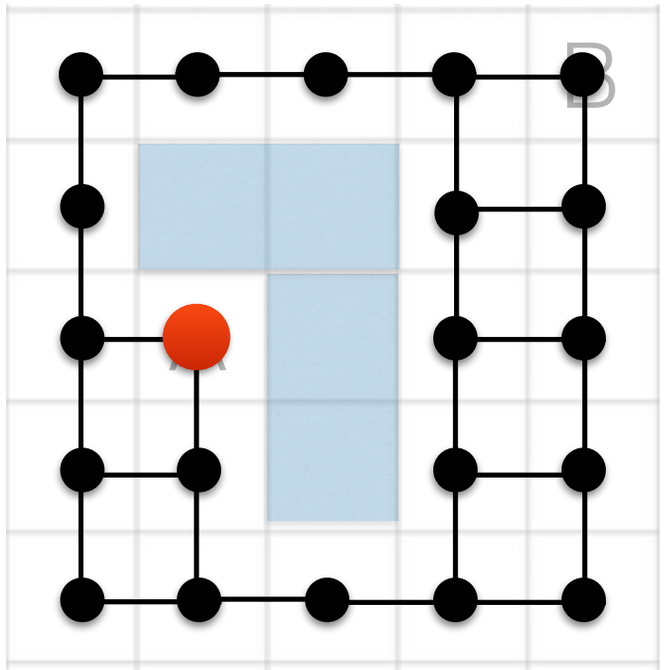


Example

Iteration 1

Q = {(2,3)} = {A}

$x \leftarrow Q.\mathrm{GetVertex}()$



Example

Iteration 1

Q = {(2,3)} = {A}

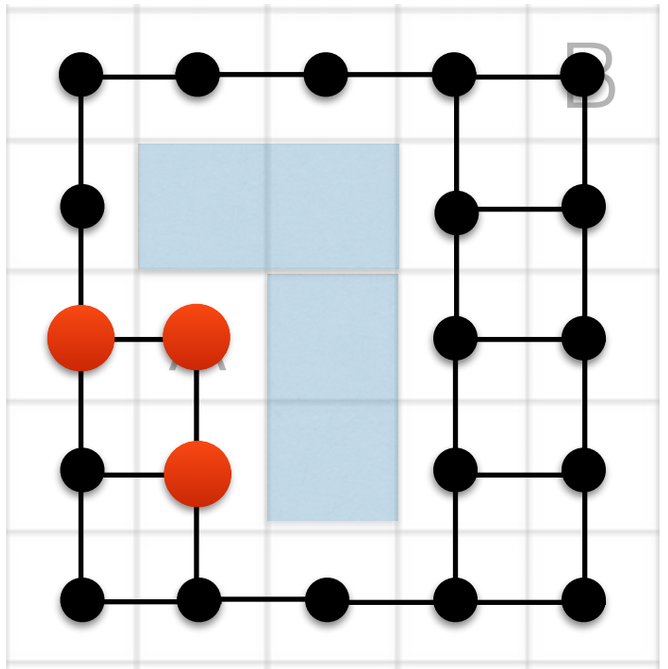
$x \leftarrow Q.\mathrm{GetVertex}()$

x = (2,3)

Neighbors of x: {(1,3), (2,2)}

Mark these as visited and add to Q

Q = {(1,3), (2,2)}

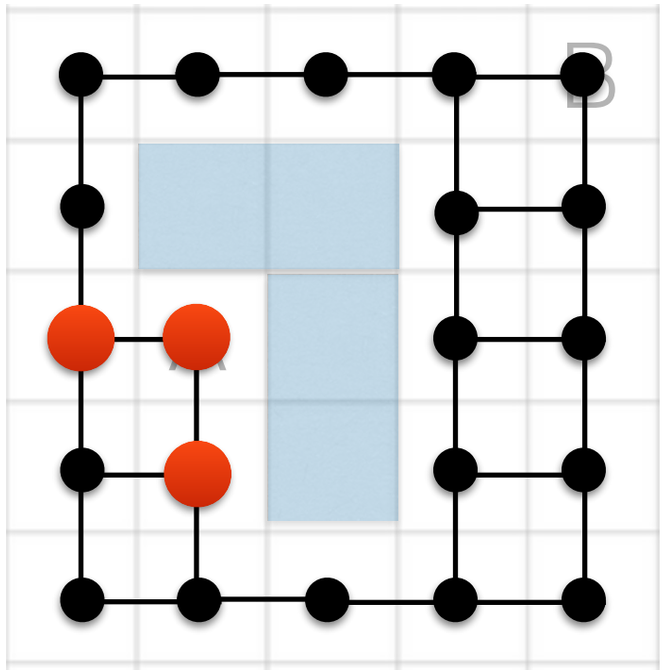


Example

Iteration 2

Q = {(1,3), (2,2)}

$x \leftarrow Q.\mathrm{GetVertex}()$



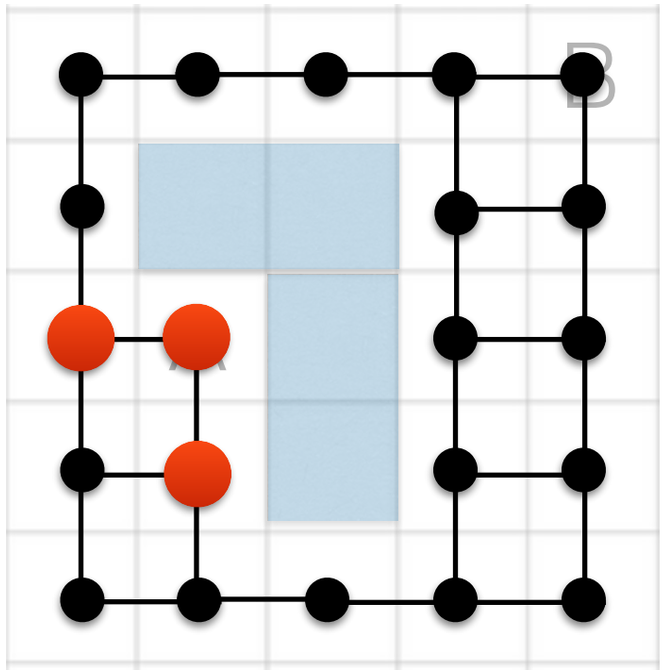
Example

Iteration 2

Q = {(1,3), (2,2)}

$x \leftarrow Q.\mathrm{GetVertex}()$

x = (1,3) [Here, we implement "First-in-first-out"]



Example

Iteration 2

Q = {(1,3), (2,2)}

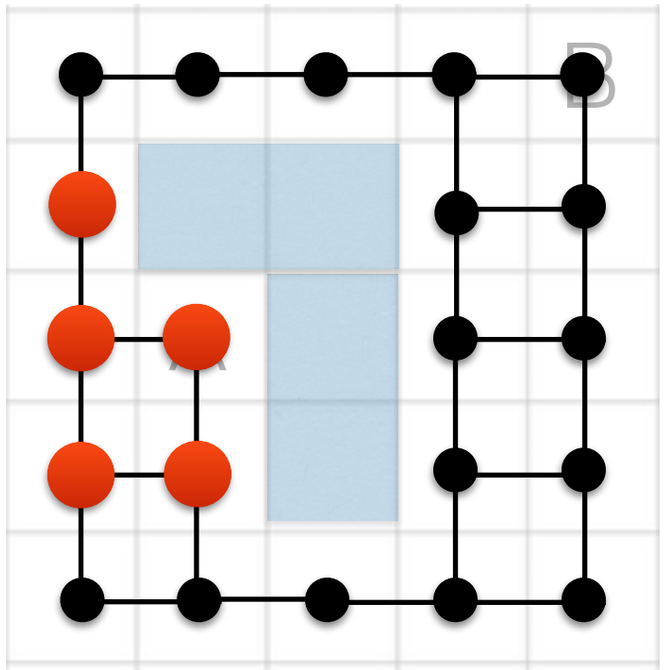
$x \leftarrow Q.\mathrm{GetVertex}()$

x = (1,3) [Here, we implement "First-in-first-out"]

Neighbors of x: {(1,4), (1,2), ~~(2,3)~~}

Mark unvisited as visited and add to Q

Q = {(2,2), (1,4), (1,2)}



Example

Iteration 3

Q = {(2,2), (1,4), (1,2)}

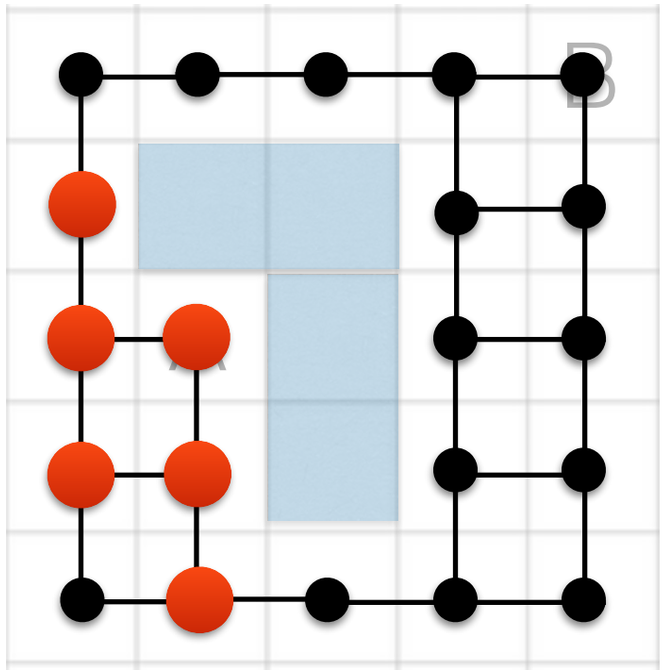
$x \leftarrow Q.\mathrm{GetVertex}()$

x = (2,2) ["First-in-first-out"]

Neighbors of x: {~~(1,2)~~, (2,1), ~~(2,3)~~}

Mark unvisited as visited and add to Q

Q = {(1,4), (1,2), (2,1)}



(5,5)

(1,1)

Example

Iteration 4

Q = {(1,4), (1,2), (2,1)}

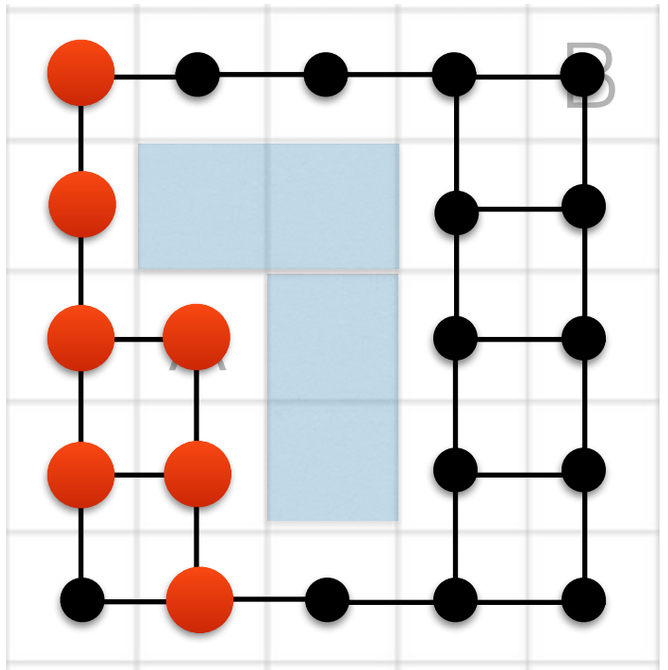
$x \leftarrow Q.\mathrm{GetVertex}()$

x = (1,4) ["First-in-first-out"]

Neighbors of x: {(1,5), ~~(1,3)~~}

Mark unvisited as visited and add to Q

Q = {(1,2), (2,1), (1,5)}



(5,5)

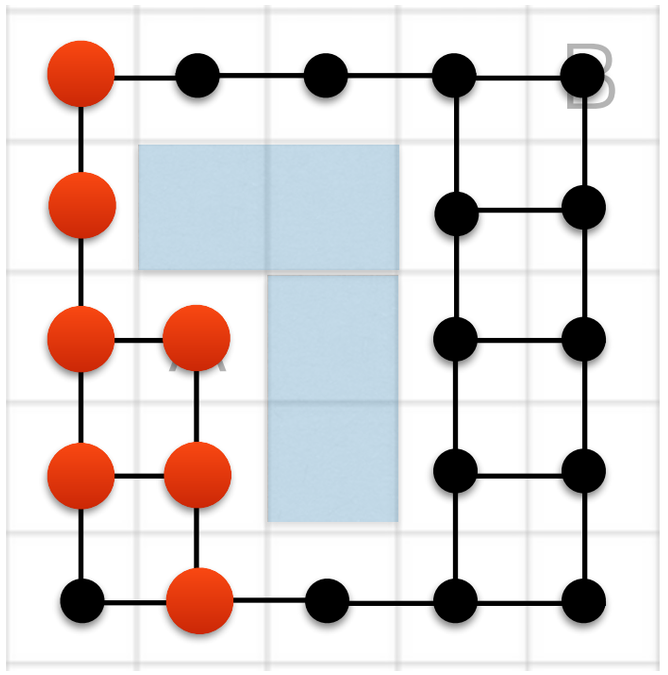
(1,1)

Example

Iteration 5

… and so on …

(until x hits goal vertex B)

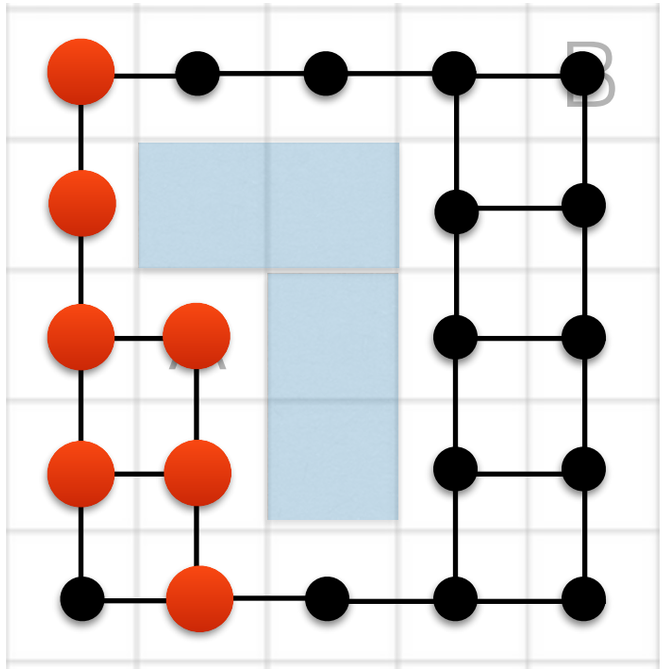


(5,5)

(1,1)

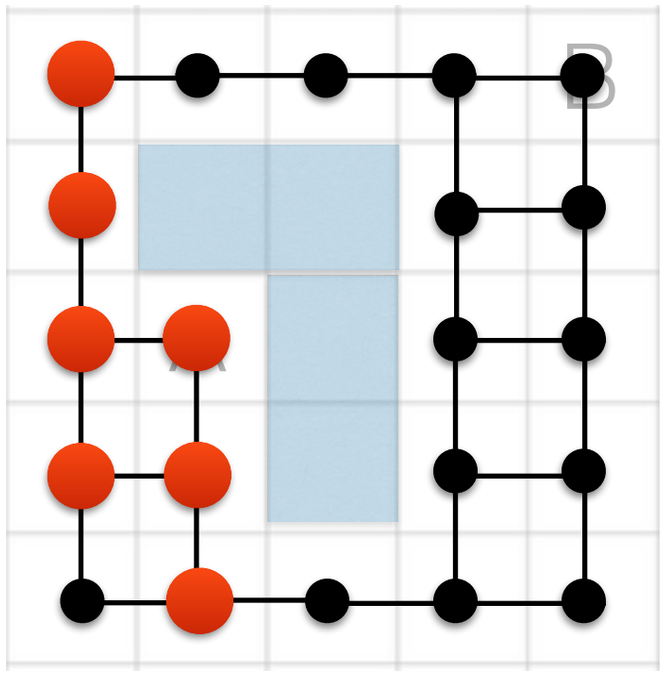
Example

- Note:
  - We can back out path from A to B by storing the "parent" of x' (i.e., the vertex of which x' was a neighbor). We can start by looking at the parent of the goal, then looking at the parent of the parent, and so on… until we hit A.



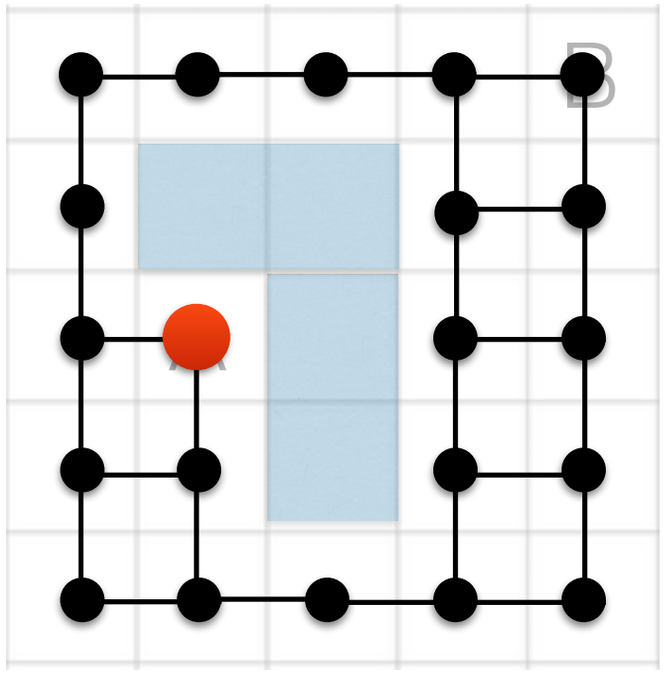
Example

- Note:
  - We implemented Q.GetVertex() using "first-in-first-out (FIFO)"
  - This corresponds to Breadth First Search (BFS)
  - BFS searches all paths of length k before searching paths of length k+1



Depth First Search (DFS)

Let's explore another way to implement Q.GetVertex():"Last-in-first-out" (LIFO)



Depth First Search (DFS)

Iteration 1

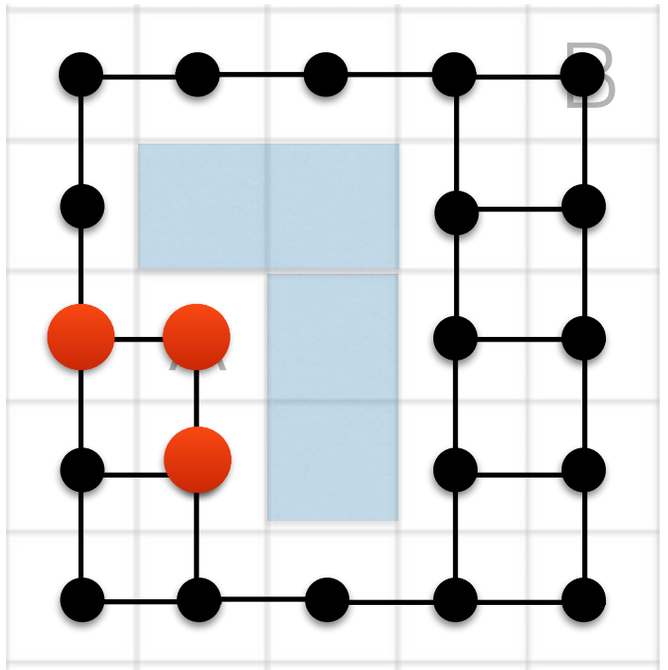
Q = {(2,3)} = {A}

$x \leftarrow Q.GetVertex()$

x = (2,3)

Neighbors: {(2,2), (1,3)}

Q = {(2,2), (1,3)}



Depth First Search (DFS)

Iteration 2

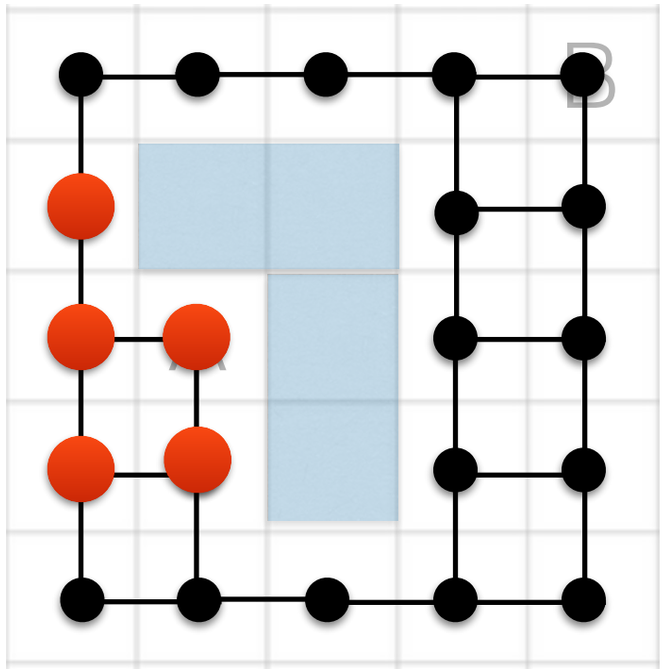
Q = {(2,2), (1,3)}

$x \leftarrow Q.\mathrm{GetVertex}()$

x = (1,3)

Neighbors: {~~(2,3)~~, (1,2), (1,4)}

Q = {(2,2), (1,2), (1,4)}



Depth First Search (DFS)

Iteration 3

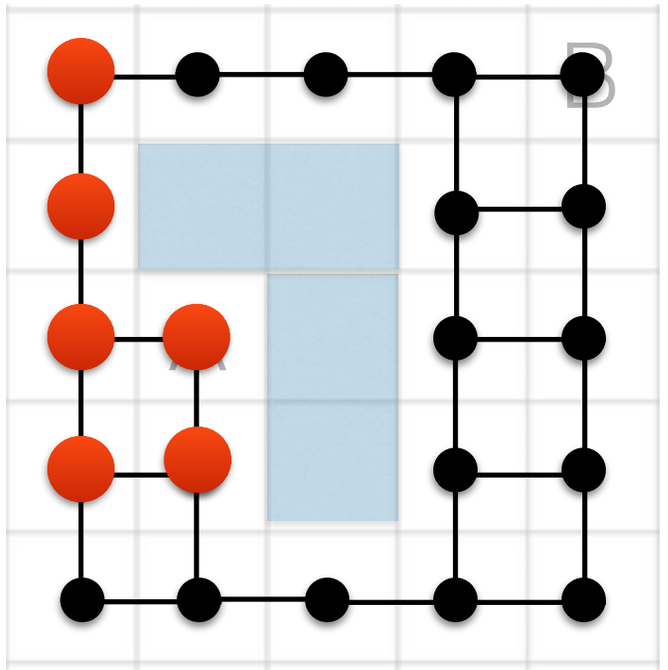
Q = {(2,2), (1,2), (1,4)}

$x \leftarrow Q.\mathrm{GetVertex}()$

x = (1,4)

Neighbors: {~~(1,3)~~, (1,5)}

Q = {(2,2), (1,2), (1,5)}



Depth First Search (DFS)

Iteration 4

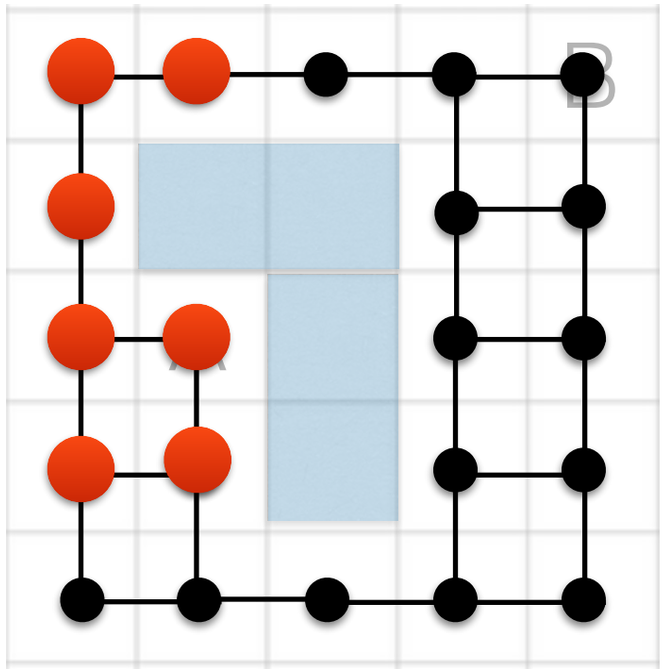
Q = {(2,2), (1,2), (1,5)}

$x \leftarrow Q.\mathrm{GetVertex}()$

x = (1,5)

Neighbors: {~~(1,4)~~, (2,5)}

Q = {(2,2), (1,2), (2,5)}



Depth First Search (DFS)

Iteration 5

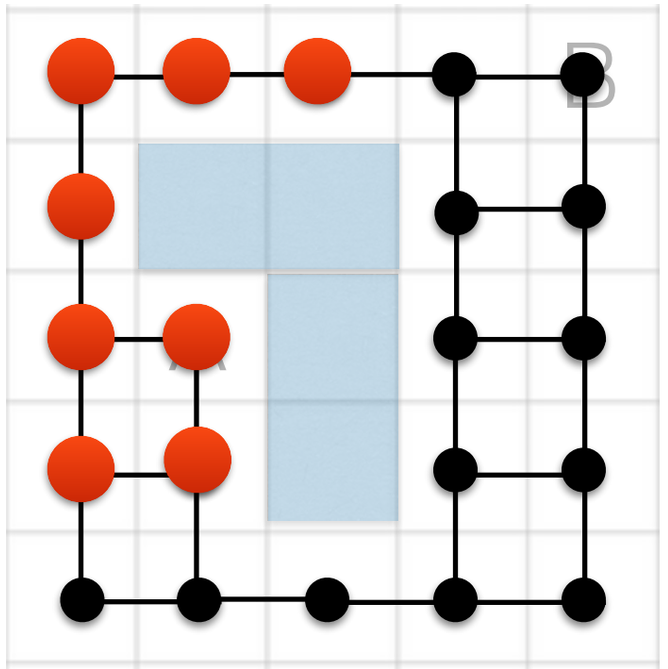
Q = {(2,2), (1,2), (2,5)}

$x \leftarrow Q.\mathrm{GetVertex}()$

x = (2,5)

Neighbors: {~~(1,5)~~, (3,5)}

Q = {(2,2), (1,2), (3,5)}



Depth First Search (DFS)

Iteration 6

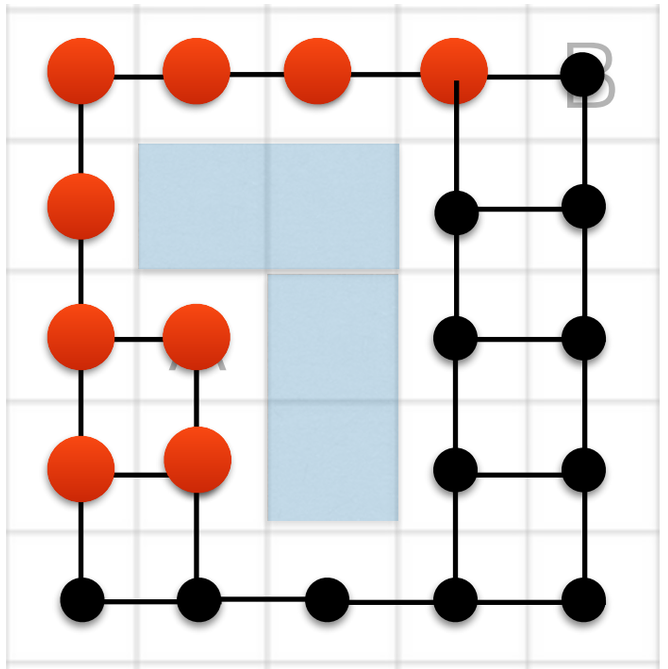
Q = {(2,2), (1,2), (3,5)}

$x \leftarrow Q.\mathrm{GetVertex}()$

x = (3,5)

Neighbors: {~~(2,5)~~, (4,5)}

Q = {(2,2), (1,2), (4,5)}



Depth First Search (DFS)

Iteration 7

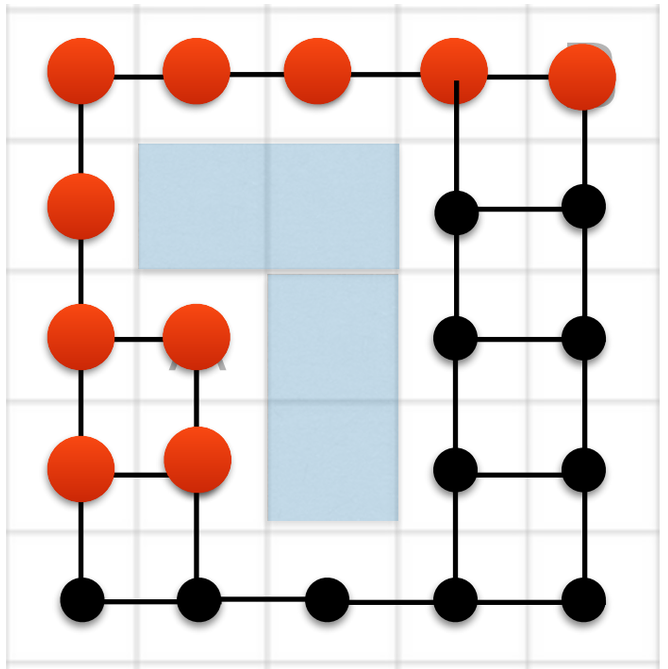
Q = {(2,2), (1,2), (4,5)}

$x \leftarrow Q.\mathrm{GetVertex}()$

x = (4,5)

Neighbors: {~~(3,5)~~, (5,5)}

Q = {(2,2), (1,2), (5,5)}



Depth First Search (DFS)

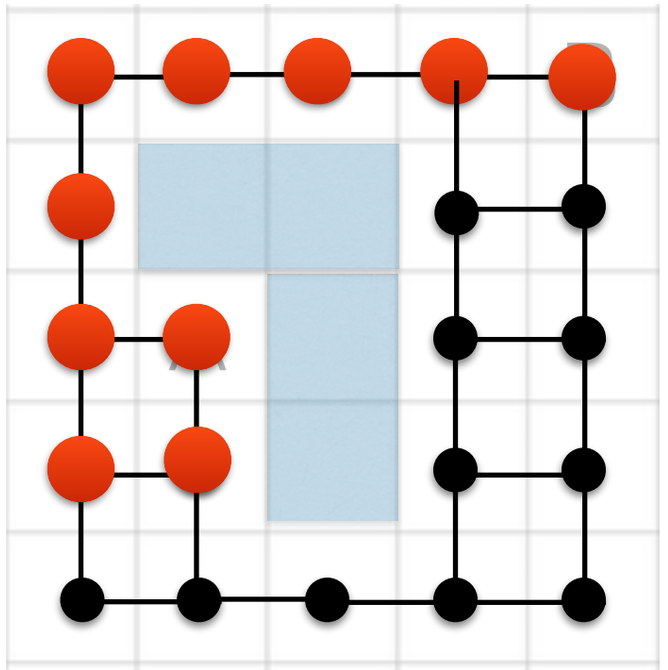
Iteration 8

Q = {(2,2), (1,2), (5,5)}

$x \leftarrow Q.\mathrm{GetVertex}()$

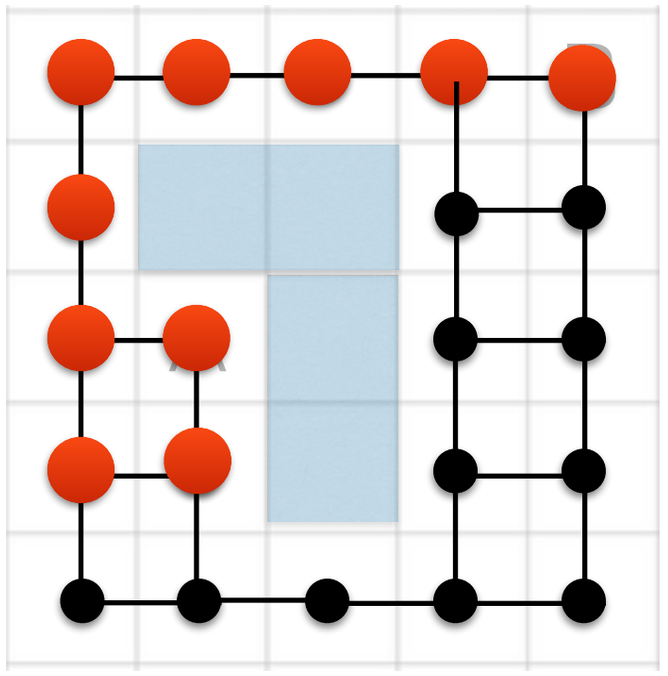
x = (5,5)

Done!



Depth First Search (DFS)

- In general, DFS will keep exploring some path deeply until it hits a dead-end or the goal
- If it hits a dead-end, it will pick some other avenue and explore until the end, and so on…
- DFS may work well in certain cases (as compared to BFS)
- e.g., when the path is long and there are only few ways to get to the goal



Concluding thoughts

- Can you think of other ways to implement Q.GetVertex()?
- Maybe we should bias our search towards the goal? (i.e., choose the element of Q that has the closest Euclidean distance to the goal)
  - We will see this in the next lecture
- We also haven't said much about optimality (we will discuss this in the next lecture too)