

ANSWER TO THE QUESTION NO. 4 C(ii)

```
In [1]: import numpy as np
import matplotlib.pyplot as plt

In [2]: t = np.linspace(0, 15, 100)

In [3]: t

Out[3]: array([ 0.          ,  0.15151515,  0.3030303 ,  0.45454545,  0.60606061,
   0.75757576,  0.90909091,  1.06060606,  1.21212121,  1.36363636,
   1.51515152,  1.66666667,  1.81818182,  1.96969697,  2.12121212,
   2.27272727,  2.42424242,  2.57575758,  2.72727273,  2.87878788,
   3.03030303,  3.18181818,  3.33333333,  3.48484848,  3.63636364,
   3.78787879,  3.93939394,  4.09090909,  4.24242424,  4.39393939,
   4.54545455,  4.6969697 ,  4.84848485,  5.          ,  5.15151515,
   5.3030303 ,  5.45454545,  5.60606061,  5.75757576,  5.90909091,
   6.06060606,  6.21212121,  6.36363636,  6.51515152,  6.66666667,
   6.81818182,  6.96969697,  7.12121212,  7.27272727,  7.42424242,
   7.57575758,  7.72727273,  7.87878788,  8.03030303,  8.18181818,
   8.33333333,  8.48484848,  8.63636364,  8.78787879,  8.93939394,
   9.09090909,  9.24242424,  9.39393939,  9.54545455,  9.6969697 ,
   9.84848485,  10.          , 10.15151515, 10.3030303 , 10.45454545,
 10.60606061, 10.75757576, 10.90909091, 11.06060606, 11.21212121,
 11.36363636, 11.51515152, 11.66666667, 11.81818182, 11.96969697,
 12.12121212, 12.27272727, 12.42424242, 12.57575758, 12.72727273,
 12.87878788, 13.03030303, 13.18181818, 13.33333333, 13.48484848,
 13.63636364, 13.78787879, 13.93939394, 14.09090909, 14.24242424,
 14.39393939, 14.54545455, 14.6969697 , 14.84848485, 15.        ])
```

```
In [4]: #check the Length of t
len(t)

Out[4]: 100
```

```
In [5]: # def calculate_coefficients():
T = 15
Tsq = T ** 2
Tcb = T ** 3
```

```
In [6]: A = np.array([[1, 0, 0, 0, 0, 0, 0, 0],
   [0, 1, 0, 0, 0, 0, 0, 0],
   [0, 0, 0, 0, 1, 0, 0, 0],
   [0, 0, 0, 0, 0, 1, 0, 0],
   [1, T, Tsq, Tcb, 0, 0, 0, 0],
   [0, 1, 2*T, 3*Tsq, 0, 0, 0, 0],
   [0, 0, 0, 0, 1, T, Tsq, Tcb,],
   [0, 0, 0, 0, 0, 1, 2*T, 3*Tsq]
])
A
```

```
Out[6]: array([[ 1,  0,  0,  0,  0,  0,  0,  0],
   [ 0,  1,  0,  0,  0,  0,  0,  0],
   [ 0,  0,  0,  0,  1,  0,  0,  0],
   [ 0,  0,  0,  0,  0,  1,  0,  0],
   [ 1,  15,  225, 3375,  0,  0,  0,  0],
   [ 0,  1,  30,  675,  0,  0,  0,  0],
   [ 0,  0,  0,  0,  1,  15,  225, 3375],
   [ 0,  0,  0,  0,  0,  1,  30,  675]]))
```

```
In [7]: Ainv = np.linalg.pinv(A)
```

```
In [8]: Ainv
```

```
Out[8]: array([[ 1.00000000e+00,  6.58070820e-13,  0.00000000e+00,
   0.00000000e+00, -8.64325972e-16,  3.91841507e-15,
   0.00000000e+00,  0.00000000e+00],
   [ 4.16333634e-17,  1.00000000e+00,  0.00000000e+00,
   0.00000000e+00, -1.00180281e-16,  1.21430643e-17,
   0.00000000e+00,  0.00000000e+00],
   [-1.33333333e-02, -1.33333333e-01,  0.00000000e+00,
   0.00000000e+00,  1.33333333e-02, -6.66666667e-02,
   0.00000000e+00,  0.00000000e+00],
   [ 5.92592593e-04,  4.44444444e-03,  0.00000000e+00,
   0.00000000e+00, -5.92592593e-04,  4.44444444e-03,
   0.00000000e+00,  0.00000000e+00],
   [ 4.38264820e-17, -1.56199568e-17,  1.00000000e+00,
   4.13738488e-13, -4.38264820e-17,  2.19459069e-16,
   3.73236598e-16, -9.14524532e-16],
   [-3.59628264e-18, -8.65405541e-17, -1.66533454e-16,
   1.00000000e+00,  3.59628264e-18, -1.80076414e-17,
   -3.85975973e-17,  7.75421394e-16],
   [-1.03997980e-19,  1.18523646e-17, -1.33333333e-02,
   -1.33333333e-01,  1.03997980e-19, -5.20842611e-19,
   1.33333333e-02, -6.66666667e-02],
   [ 9.93178042e-21, -4.00814858e-19,  5.92592593e-04,
   4.44444444e-03, -9.93178042e-21,  4.97355363e-20,
   -5.92592593e-04,  4.44444444e-03]])
```

```
In [9]: #initialized b
b = np.array([[1],#x1(0)
              [1],#1
              [0],#x3(0)
              [1],#x2(0)
              [10],#x1(T)
              [1],#1
              [5],#x3(T)
              [10] #x2(T)
            ]
          )
b
```

```
Out[9]: array([[ 1],
   [ 1],
   [ 0],
   [ 1],
   [10],
   [ 1],
   [ 5],
   [10]]))
```

```
In [10]: #matrix multiplication x = Ainv * b
x = np.matmul(Ainv, b)
x
```

```
Out[10]: array([[ 1.00000000e+00],  
 [ 1.00000000e+00],  
 [-8.00000000e-02],  
 [ 3.55555556e-03],  
 [ 4.06268826e-13],  
 [ 1.00000000e+00],  
 [-7.33333333e-01],  
 [ 4.59259259e-02]])
```

```
In [11]: a11 = x[0]  
a11
```

```
Out[11]: array([1.])
```

```
In [12]: a12 = x[1]  
a12
```

```
Out[12]: array([1.])
```

```
In [13]: a13 = x[2]  
a13
```

```
Out[13]: array([-0.08])
```

```
In [14]: a14 = x[3]  
a14
```

```
Out[14]: array([0.00355556])
```

```
In [15]: a21 = x[4]  
a21
```

```
Out[15]: array([4.06268826e-13])
```

```
In [16]: a22 = x[5]  
a22
```

```
Out[16]: array([1.])
```

```
In [17]: a23 = x[6]  
a23
```

```
Out[17]: array([-0.73333333])
```

```
In [18]: a24 = x[7]  
a24
```

```
Out[18]: array([0.04592593])
```

```
In [19]: t
```

```
Out[19]: array([ 0.          ,  0.15151515,  0.3030303 ,  0.45454545,  0.60606061,
   0.75757576,  0.90909091,  1.06060606,  1.21212121,  1.36363636,
   1.51515152,  1.66666667,  1.81818182,  1.96969697,  2.12121212,
   2.27272727,  2.42424242,  2.57575758,  2.72727273,  2.87878788,
   3.03030303,  3.18181818,  3.33333333,  3.48484848,  3.63636364,
   3.78787879,  3.93939394,  4.09090909,  4.24242424,  4.39393939,
   4.54545455,  4.6969697 ,  4.84848485,  5.          ,  5.15151515,
   5.3030303 ,  5.45454545,  5.60606061,  5.75757576,  5.90909091,
   6.06060606,  6.21212121,  6.36363636,  6.51515152,  6.66666667,
   6.81818182,  6.96969697,  7.12121212,  7.27272727,  7.42424242,
   7.57575758,  7.72727273,  7.87878788,  8.03030303,  8.18181818,
   8.33333333,  8.48484848,  8.63636364,  8.78787879,  8.93939394,
   9.09090909,  9.24242424,  9.39393939,  9.54545455,  9.6969697 ,
   9.84848485,  10.         ,  10.15151515,  10.3030303 ,  10.45454545,
  10.60606061,  10.75757576,  10.90909091,  11.06060606,  11.21212121,
  11.36363636,  11.51515152,  11.66666667,  11.81818182,  11.96969697,
  12.12121212,  12.27272727,  12.42424242,  12.57575758,  12.72727273,
  12.87878788,  13.03030303,  13.18181818,  13.33333333,  13.48484848,
  13.63636364,  13.78787879,  13.93939394,  14.09090909,  14.24242424,
  14.39393939,  14.54545455,  14.6969697 ,  14.84848485,  15.        ])
```

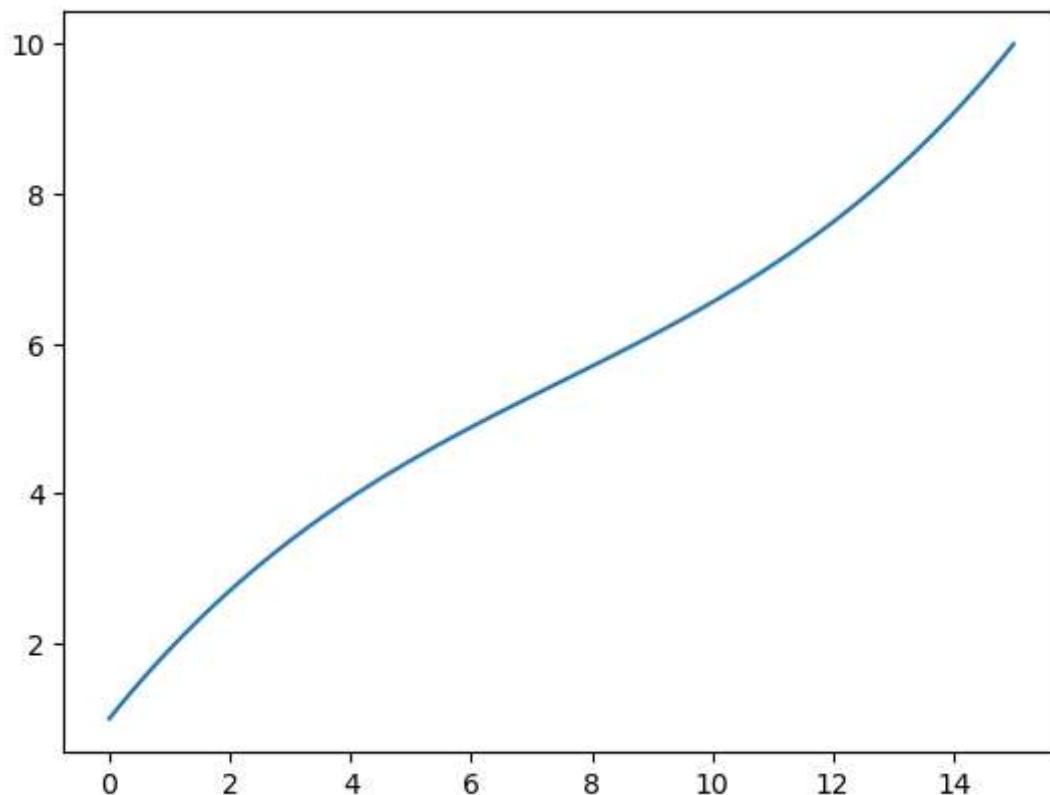
```
In [20]: for i in t:
    X1 = a11 + (a12*t) + (a13 * np.power(t,2)) + (a14 * np.power(t,3))
    # X2 = a22 + (2*a23*t) + (3*a24* np.power(t,2))
    X2 = (a22 + (2*a23*t) + (3*a24* np.power(t,2)))/(a12 + (2*a13*t) + (3*a14* np.
    X3 = a21 + (a22*t) + (a23 * np.power(t,2)) + (a24 * np.power(t,3) )
X1, X2, X3
```

```
Out[20]: (array([
  1.          , 1.14969097, 1.29578305, 1.43835045, 1.57746736,
  1.71320799, 1.84564655, 1.97485723, 2.10091425, 2.22389181,
  2.34386411, 2.46090535, 2.57508974, 2.68649148, 2.79518478,
  2.90124384, 3.00474287, 3.10575606, 3.20435763, 3.30062177,
  3.39462269, 3.48643459, 3.57613169, 3.66378817, 3.74947825,
  3.83327613, 3.91525602, 3.99549211, 4.07405861, 4.15102973,
  4.22647967, 4.30048263, 4.37311282, 4.44444444, 4.5145517 ,
  4.58350879, 4.65138993, 4.71826932, 4.78422115, 4.84931964,
  4.91363899, 4.9772534 , 5.04023708, 5.10266423, 5.16460905,
  5.22614576, 5.28734854, 5.34829161, 5.40904917, 5.46969542,
  5.53030458, 5.59095083, 5.65170839, 5.71265146, 5.77385424,
  5.83539095, 5.89733577, 5.95976292, 6.0227466 , 6.08636101,
  6.15068036, 6.21577885, 6.28173068, 6.34861007, 6.41649121,
  6.4854483 , 6.55555556, 6.62688718, 6.69951737, 6.77352033,
  6.84897027, 6.92594139, 7.00450789, 7.08474398, 7.16672387,
  7.25052175, 7.33621183, 7.42386831, 7.51356541, 7.60537731,
  7.69937823, 7.79564237, 7.89424394, 7.99525713, 8.09875616,
  8.20481522, 8.31350852, 8.42491026, 8.53909465, 8.65613589,
  8.77610819, 8.89908575, 9.02514277, 9.15435345, 9.28679201,
  9.42253264, 9.56164955, 9.70421695, 9.85030903, 10.        ]),
array([
  1.          , 0.80014217, 0.59654648, 0.38925114, 0.17831027,
  -0.03620431, -0.25420123, -0.47556751, -0.70016638, -0.92783505,
  -1.15838226, -1.39158576, -1.62718974, -1.86490208, -2.10439165,
  -2.34528552, -2.58716623, -2.82956904, -3.07197943, -3.31383065,
  -3.55450158, -3.79331493, -4.02953586, -4.26237112, -4.4909688 ,
  -4.71441895, -4.93175493, -5.14195584, -5.34395006, -5.53662 ,
  -5.71880819, -5.88932473, -6.04695622, -6.19047619, -6.31865688,
  -6.43028249, -6.52416357, -6.59915258, -6.65416018, -6.68817204,
  -6.70026589, -6.68962822, -6.65557035, -6.59754339, -6.51515152,
  -6.40816327, -6.27652033, -6.12034355, -5.93993581, -5.73578169,
  -5.50854374, -5.25905548, -4.98831123, -4.69745304, -4.3877551 ,
  -4.06060606, -3.71748968, -3.35996449, -2.98964287, -2.60817017,
  -2.2172043 , -1.81839634, -1.41337244, -1.00371747, -0.59096045,
  -0.17656215, 0.23809524, 0.65171629, 1.0630984 , 1.47113594,
  1.87482248, 2.27325142, 2.66561514, 3.05120287, 3.4293975 ,
  3.79967159, 4.16158265, 4.51476793, 4.85893898, 5.19387585,
  5.51942142, 5.83547558, 6.14198965, 6.43896096, 6.72642762,
  7.00446364, 7.2731743 , 7.53269183, 7.78317152, 8.024788 ,
  8.25773196, 8.48220716, 8.6984277 , 8.90661561, 9.10699873,
  9.29980875, 9.48527959, 9.66364591, 9.83514186, 10.        ]),
array([
  4.06268826e-13, 1.34839879e-01, 2.36968192e-01, 3.07343407e-01,
  3.46923989e-01, 3.56668408e-01, 3.37535131e-01, 2.90482624e-01,
  2.16469356e-01, 1.16453794e-01, -8.60559477e-03, -1.57750343e-01,
  -3.30021983e-01, -5.24462047e-01, -7.40112069e-01, -9.76013579e-01,
  -1.23120811e+00, -1.50473720e+00, -1.79564237e+00, -2.10296517e+00,
  -2.42574712e+00, -2.76302975e+00, -3.11385460e+00, -3.47726319e+00,
  -3.85229708e+00, -4.23799777e+00, -4.63340682e+00, -5.03756574e+00,
  -5.44951608e+00, -5.86829936e+00, -6.29295712e+00, -6.72253089e+00,
  -7.15606220e+00, -7.59259259e+00, -8.03116359e+00, -8.47081673e+00,
  -8.91059354e+00, -9.34953556e+00, -9.78668431e+00, -1.02210813e+01,
  -1.06517682e+01, -1.10777863e+01, -1.14981774e+01, -1.19119828e+01,
  -1.23182442e+01, -1.27160030e+01, -1.31043008e+01, -1.34821792e+01,
  -1.38486796e+01, -1.42028437e+01, -1.45437128e+01, -1.48703286e+01,
  -1.51817326e+01, -1.54769664e+01, -1.57550714e+01, -1.60150892e+01,
  -1.62560613e+01, -1.64770292e+01, -1.66770346e+01, -1.68551189e+01,
  -1.70103236e+01, -1.71416903e+01, -1.72482606e+01, -1.73290759e+01,
  -1.73831778e+01, -1.74096078e+01, -1.74074074e+01, -1.73756182e+01,
  -1.73132818e+01, -1.72194396e+01, -1.70931331e+01, -1.69334040e+01,
  -1.67392938e+01, -1.65098439e+01, -1.62440959e+01, -1.59410914e+01,
  -1.55998718e+01, -1.52194787e+01, -1.47989537e+01, -1.43373383e+01,
  -1.38336739e+01, -1.32870023e+01, -1.26963647e+01, -1.20608029e+01,
  -1.13793583e+01, -1.06510725e+01, -9.87498699e+00, -9.05014331e+00,
  -8.17558299e+00, -7.25034757e+00, -6.27347859e+00, -5.24401757e+00,
]),
```

```
-4.16100604e+00, -3.02348554e+00, -1.83049761e+00, -5.81083769e-01,  
7.25714445e-01, 2.09085550e+00, 3.51529786e+00, 5.00000000e+00]))
```

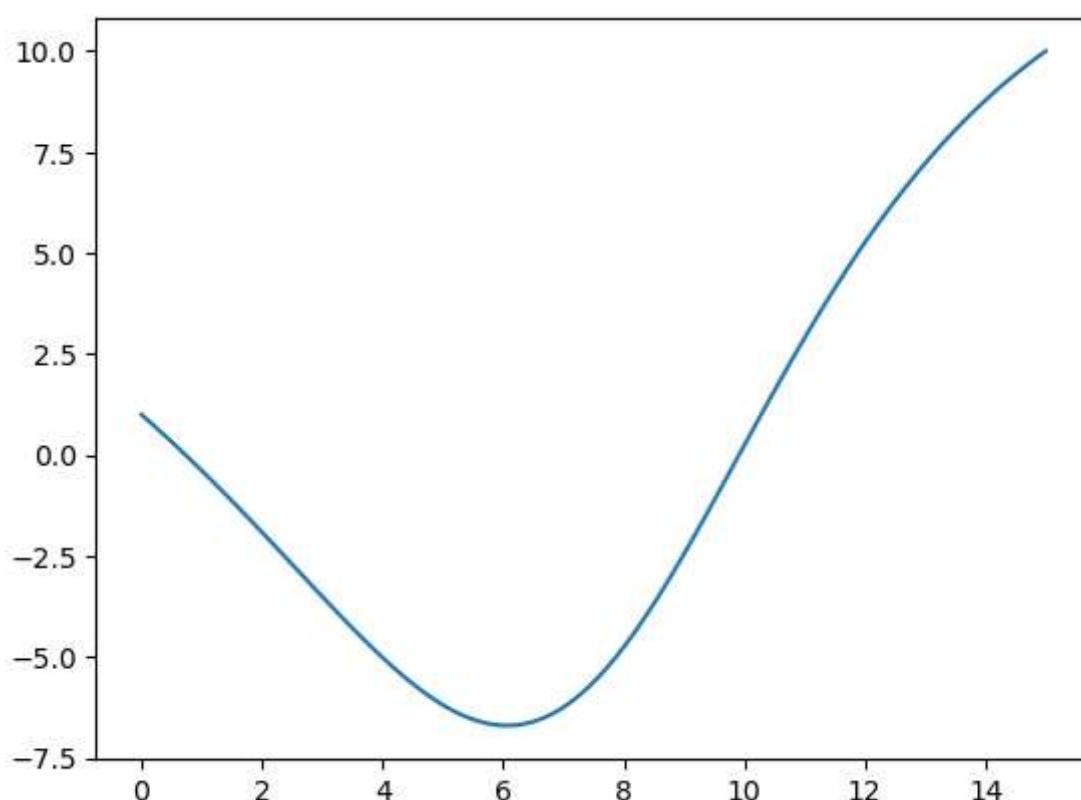
In [21]: `plt.plot(t,X1)`

Out[21]: [`<matplotlib.lines.Line2D at 0x1fa25f39f70>`]



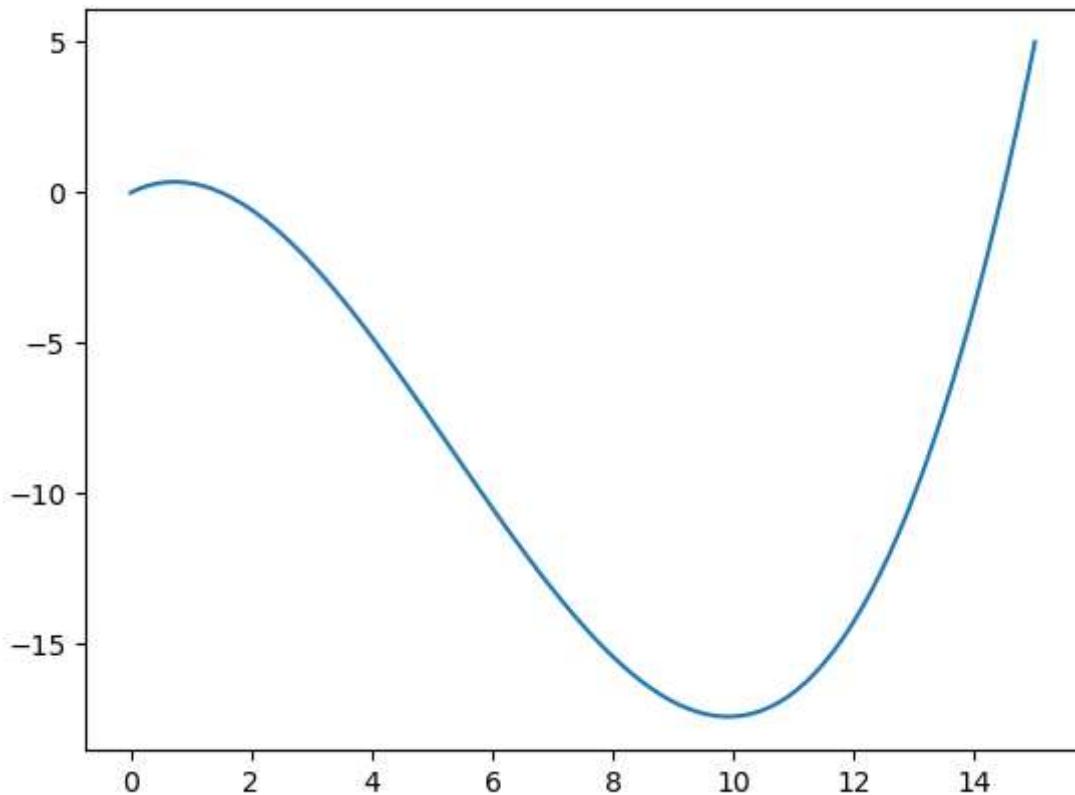
In [22]: `plt.plot(t,X2)`

Out[22]: [`<matplotlib.lines.Line2D at 0x1fa2616c580>`]



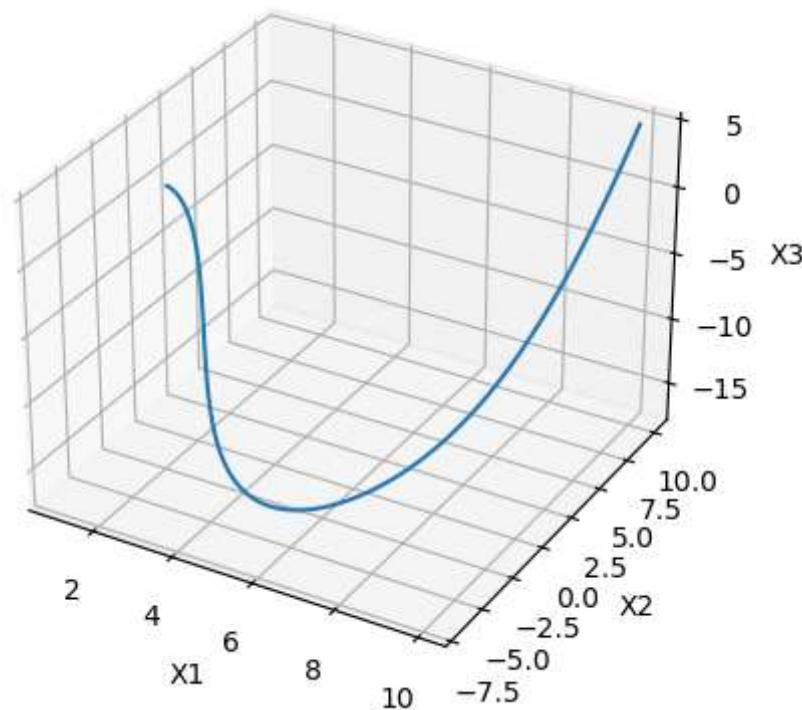
```
In [26]: plt.plot(t,X3)
```

```
Out[26]: [<matplotlib.lines.Line2D at 0x1fa261f70d0>]
```



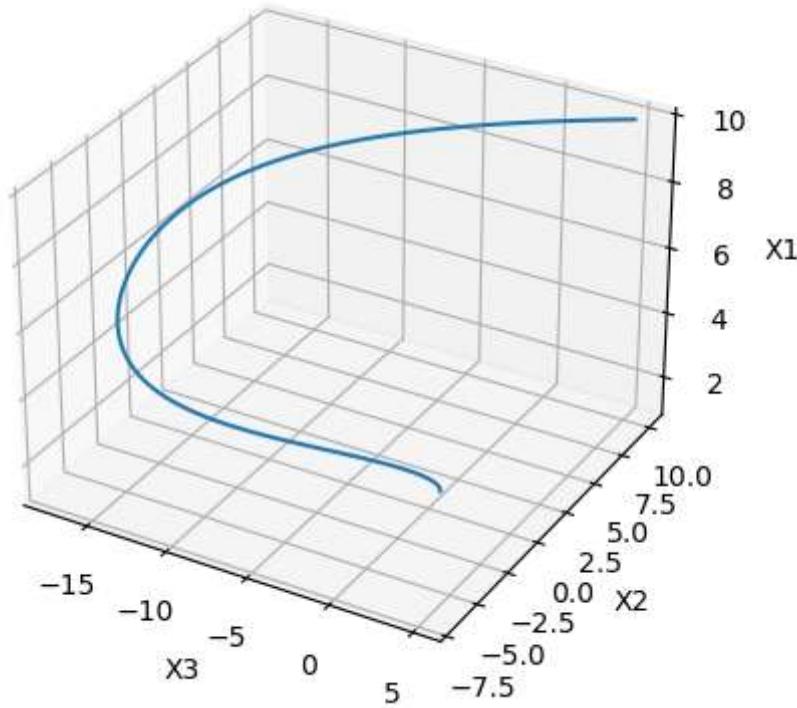
```
In [30]: ax = plt.figure().add_subplot(projection='3d')
ax.plot(X1, X2, X3)
ax.set_xlabel('X1')
ax.set_ylabel('X2')
ax.set_zlabel('X3')
```

```
Out[30]: Text(0.5, 0, 'X3')
```



```
In [28]: ax = plt.figure().add_subplot(projection='3d')
ax.plot(X3, X2, X1)
ax.set_xlabel('X3')
ax.set_ylabel('X2')
ax.set_zlabel('X1')
```

```
Out[28]: Text(0.5, 0, 'X1')
```



```
In [ ]:
```