Date : ___/___/___

- Differential equation (any order) that describes natural phenomena.

$\downarrow$

Convert to state space representation.

Reason: Highly efficient numerical method to compute matrix-vector formulations (e.g numpy, others)

- State Space representation,

$$\dot{X} = AX + Bu$$
$$Y = Cx + Du$$

X = State vector
U = input
Y = output vector
A = Dynamics
B = input matrix
C, D = weight matrix

Example:

$$\dddot{x} + 6\ddot{x} + 11\dot{x} + 6x = \underset{\text{input}}{u}$$

Lets take the states,

$$x_1 = x, \quad x_2 = \dot{x}, \quad x_3 = \ddot{x}$$
$$\dot{x}_1 = \dot{x}, \quad \dot{x}_2 = \ddot{x}, \quad \dot{x}_3 = \dddot{x}.$$

Date : ....../....../......

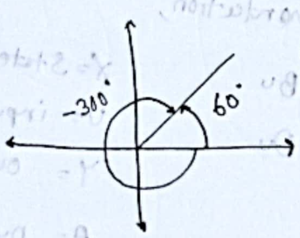
State vector

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}_{3\times1}$$

input vector $[U]_{1\times1}$

$$\dddot{x} = -6\ddot{x} - 11\dot{x} - 6x + u$$

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{pmatrix} = \underbrace{\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{pmatrix}}_{A} \underbrace{\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}}_{x} + \underbrace{\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}}_{B} \underbrace{U}_{u}$$



$\sin 60^\circ = \sin(-300^\circ)$

Unicycle and differential drive models

Translational

Rotational

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{pmatrix} = \begin{pmatrix} \sin\theta \\ \cos\theta \\ 0 \end{pmatrix} v + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \omega$$

$\dot{x} = \cos\theta \; v$

$\dot{y} = \sin\theta \; v$

$\dot{\theta} = \omega$

OR

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{pmatrix}_{3\times1} = \begin{pmatrix} \sin\theta & 0 \\ \cos\theta & 0 \\ 0 & 1 \end{pmatrix}_{3\times2} \begin{pmatrix} v \\ \omega \end{pmatrix}_{2\times1}$$

Date : ....../....../......

- Translational velocity is movements along a line mean back and forth or up and down

- Rotational velocity is moving along a curve but doesn't have to be a circle, any curve

- Differential drive mode,

L, R = constant
↓ ↓
Length Radius.

- Equation to matrix vector format

$\omega_r = \omega_l$	constant
$\omega_r > \omega_l$	Turn left
$\omega_r < \omega_l$	Turn Right

Date : ..../......../........

- For unicycle and differential drive model

↓

For

- state variable
- control variable
- Constant variable

↳ Doesn't changes with time

Matrix vector, of differential drive models;

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{pmatrix}_{3\times1} = \begin{pmatrix} \frac{r}{2}(\omega_L + \omega_R)\cos\theta \\ \frac{r}{2}(\omega_L + \omega_R)\sin\theta \\ \frac{r}{L}(\omega_R - \omega_L) \end{pmatrix} =$$

$$= \begin{bmatrix} \frac{r}{2}\cos\theta & \frac{1}{2}\cos\theta \\ \frac{r}{2}\sin\theta & \frac{1}{2}\sin\theta \\ -\frac{r}{L} & \frac{r}{L} \end{bmatrix}_{3\times2} \begin{bmatrix} \omega_L \\ \omega_R \end{bmatrix}_{2\times1}$$

Date : / /

- Square matrix has eigenvalues
- Rectangular matrix has ~~eig~~ Singular values.
  - Square matrix,

    number of eigenvalues = rank of the matrix

  - Rank of matrix is = linearly independent rows and colums of the matrix
  - if none of them can be written as linear combination of others is called linearly independent

Simplified car model

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{pmatrix} = \begin{pmatrix} v \cos\theta \\ v \sin\theta \\ \frac{v}{L} \tan\phi \end{pmatrix}$$

Date : ...../...../.....

ϕ = Stearing of the car model

$+\phi$ = turn left , $-\phi$ = turn Right

$|v| \leq V_{max}$, $|\phi| \leq \phi_{max} < \frac{\pi}{2}$] constraints

☒ Kinematic/ Dynamic models

$\dot{x}(t) = f(x, u, t)$ f = non linear function x, u, t

$\dot{x} = \frac{dx}{dt} \approx \frac{x(i+1) - x(i)}{\Delta t}$ Array index

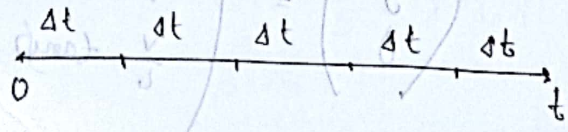
time rate of change of the function

x = State vector

u = control vector

t = time

Discretize the time using time step Δt (Sampling frequency)



Date : ____/____/____

$$\dot{x} = \frac{dx}{dt} = \frac{x(i+1) - x(i)}{\Delta t} = f(x, u, t)$$

$$\boxed{x(i+1) = x(i) + \Delta t \cdot f(x, u, t)}$$ euler equation

Example,

$\dot{x} = 3$, $x(0) = 0.2$

integrate this equation with euler methods with $\Delta t = 0.1$ till $t = 2$ sec

$\dot{x} = 3$

$$\frac{x(i+1) - x(i)}{\Delta t} = 3$$

or, $\underline{x(i+1) = x(i) + \Delta t \cdot 3}$

t	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9 ...	1.9	2.0
i	0	1	2	3	4	5	6	7	8	9 ...	19	20

Date : ...../...../.....

$i=0,$

$x(1) = x(0) + \Delta t \cdot 3$

$= 0.2 + \Delta t \cdot 3 \quad (0.2 + 0 \times 3)$

$i=1 \quad x(i=1), = x(\Delta t = 0.1) \rightarrow$

$x(2) = x(1) + \Delta t \cdot 3 \quad (0.2 + 0.1 \times 3)$

$x(i=2) = x(\Delta t = 0.2)$

$i=2$

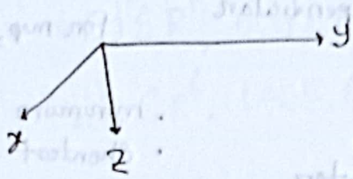
$x(3) = x(2) + \Delta t \cdot 3 \quad (0.5 + 0.2 \times 3)$

$x(i=3) = x(\Delta t = 0.3) \quad = 1.1$

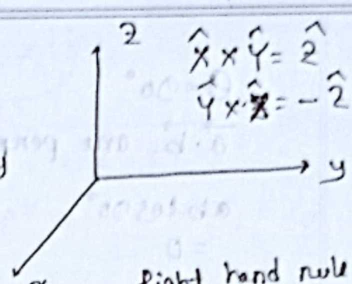
```
for (i=0, i<=20; i++)
    x(i+1) = x(i) + Δt.3
```

condition.

Date : / /



violates Right hand rule



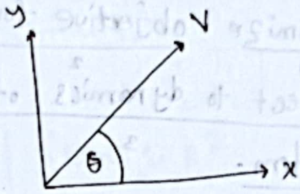
$\hat{x} \times \hat{y} = \hat{z}$

$\hat{y} \times \hat{x} = -\hat{z}$

Right hand rule

$V = V_x\hat{x} + V_y\hat{y} + V_z\hat{z}$

$\|V\| = \sqrt{V_x^2+V_y^2+V_z^2}$



atan → call always for Quadrant 1

$\boxed{\text{a-tan2}}$ → Right answer → for all Quadrant

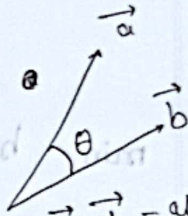
$V_x = V\cos\theta$

$V_y = V_y \sin\theta$

$V = V_x\hat{x} + V_y\hat{y}$

$\frac{V_y}{V_x} = \tan\theta$

$\theta = \tan^{-1}\left(\frac{V_y}{V_x}\right)$



$\vec{a}\cdot\vec{b} = ab\cos\theta$

$\vec{a}\cdot\vec{b} = a_xb_x + a_yb_y$

$\cos\theta = \frac{a_xb_x+a_yb_y}{ab}$

Date : / /

$\theta = 90^\circ$

$\vec{a} \cdot \vec{b}$ are perpendicular.

$ab \cos 90^\circ$
$= 0$

For map,

- minimum time
- Shortest distance
- average between minimum time and distance

Planar quadrator

optimal control → The problem

minimize objective function
Subject to dynamics of the System.

$$\min \; h\left(x(t_f), t_f\right) + \int_{t_0}^{t_f} g\left(x(t), u(t), t\right) dt$$

Subject to

$$\dot{x}(t) = a\left(\underset{\text{state}}{x(t)}, \underset{\text{control}}{u(t)}, \underset{\text{time}}{t}\right)$$

$$x(t) \in \mathcal{X}, \; u(t) \in \mathcal{U}$$

where,

Real number family

dimension (6x1)

Where,

$x(t) \in \mathbb{R}^n,\ u(t) \in \mathbb{R}^m$ and $x(t_0) = x_0$

$x(t) \in \mathbb{R}^6,\ u(t) \in \mathbb{R}^m$ and $x(t_0) = x_0$

$$\min \int_{t_0}^{t_f} \left(T_1(t)^2 + T_2(t)^2\right) dt$$

minimizing energy consumption means maximizing battery life.

$$\begin{array}{ll} \text{horizontal} & \\ \text{velocity of } x & \\ \text{vertical} & \\ \text{velocity of } y & \\ T_1 \neq T_2 & \\ & \end{array} \begin{bmatrix} x \\ v_x \\ y \\ v_y \\ \varphi \\ \omega \end{bmatrix}$$

$x(t) \in \mathbb{R}^6$

controls,

$$U(t) = \begin{bmatrix} T_1 \\ T_2 \end{bmatrix}$$

$U(t) \in \mathbb{R}^2$

constant terms,

l, m, g, I_{zz}

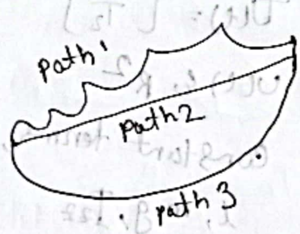
Date : ..../........./.........

2.

Open loop motion planning and control.

Objective:

(i) compute the trajectories between point A and point B. In general many points exists between point A and B.



(ii) Compute the controls (inputs) for the vehicle to move on the path.

Date : ____/____/____

. There in a trade off between the compartional cost (processor, memory, time requirements) of computing path version the optimality and ~~feab~~ feasibility of the path.

The optimal control problem

$$\min_{U} \text{ Objective function}$$

Subject to robot dynamics and physical constrains

The objective function in specified by user/ engine

For example:

(I) Minimize time $\left(\min_{U} t\right)$

(ii) Minimize distance $\left(\min_{U} d\right) = \sum_{i=1}^{n} \sqrt{x_i^2 + y_i^2}$ line segments

(iii) minimize energy

(V) Some function (possibly hybrid)

chapter; 2
→ one exam offline
→ two online, October 4,5
Date : ..../......./.......

Thrust created $\propto$ propeller speed $\propto$ Battery consumption

$$Ax = b$$

A is a known matrix $n \times n$

x is a unknown vector $n \times 1$

b is a known vector $n \times 1$

$$x = A^{-1} b$$

if A is full rank

$$\det(A) \neq 0 \qquad \left[\text{rank}(A) = n\right]$$

full rank of A means n linearly independent $\sum_{i=1}^{n}$ colums or rows.

$$\text{rank}(A) = n$$

Date : ....../....../......

What if,

A is not a full rank,

$x = A^+ b$ [+ dagger

A^+ is called the pseudo inverse]

For example,

A is 5x5

Trajectory in a linear combination of basis function

if, rank of A = 5 — Linearly independent

can calculate $= A^{-1}$

if, rank of A < 5 — Not linearly independent

then, calculate $= A^+$

Another case,

$Ax = b$

A is a known m×n matrix

m>n m<n

x is a unknown vector n×1

b is a known vector m×1

then, $x = A^+ b$

Date : ____/____/____

Example, with boundry condition

$$\text{States}\begin{bmatrix} \dot{x}_1 = u_1 \\ \dot{x}_2 = u_2 \\ \dot{x}_3 = x_2 u_1 \end{bmatrix} \text{inputs / controls} \quad \begin{matrix} x_1(0) & x_1(T) \\ x_2(0) & x_2(T) \\ x_3(0) & x_3(T) \end{matrix}$$

initial time $t=0$

final time $t=T$

This system is differentially flat for

$$z = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_3 \end{bmatrix}$$

but it not differentially flat for

$$z = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \text{ or. } z_1 = \begin{bmatrix} x_2 \\ x_3 \end{bmatrix}$$

$\boxed{x_1 = z_1}$ $\boxed{x_3 = z_2}$

$\dot{x}_1 = \dot{z}_1$ $\dot{x}_3 = \dot{z}_2$

$$x_2 = \frac{\dot{x}_3}{u_1} = \frac{\dot{x}_3}{\dot{x}_1} = \frac{\dot{z}_2}{\dot{z}_1}$$

Date : ______ / ______ / ______

$\dot{x}_1 = u_1$ so, $u_1 = \dot{z}_1$

$$\boxed{u_2 = \dot{x}_2 = \left(\frac{\dot{z}_2}{z_1}\right)}$$

So, we can write,

$$x_1 = z_1 = \sum_{i=1}^{n} a_{1i}\varphi_i(t)$$

$$x_3 = z_3 = \sum_{i=1}^{n} a_{2i}\varphi_i(t)$$

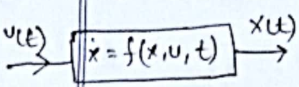
Date : ____ / ____ / ____

$\Delta t \downarrow$:
- Accuracy $\uparrow$
- Memory use $\uparrow$
- Computional time $\uparrow$
- errore $\downarrow$

4.

Principles of Robot Anatomy I

Open-loop control



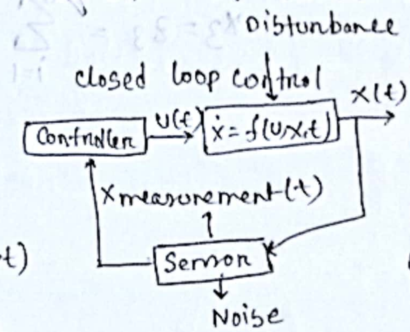
where,

$u(t) = u^{*}(t) = g(x(0), t)$; $0 < t < T$

The open loop control is calculated using initial condition $x(0)$

The open loop controll is calculated "A" priori at $t = 0$

closed loop control



Robot equation

$\dot{x} = f(x(t), u(t))$

x = State

u = control

In closed loop control u(t) is calculated at each time step Δt based on measured States.

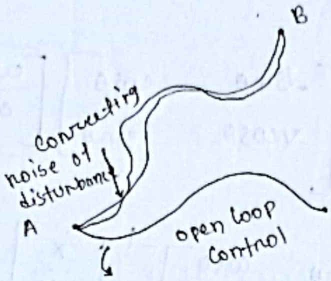
The closed loop control is calculated in real time

Eigenvalues = square matrix

Singularvalues = Non square matrix

Step:1

Calculate the desired trajectory



For example, calculate differential flatness trajectory

$$Z_j = \sum_{i=1}^{N} \alpha_i \varphi_i(t)$$

Open loop controllers due to noise and disturbance.

Step:2

Calculate the control $u(t)$, to make on this trajectory

Calculating open loop controll

Method:1 from the differential flatness equations, we have

$$U = \beta(z, \dot{z}, \ddot{z} \dots)$$

Method:2

Directly from the robot dynamics equations

Date : ....../....../......

$\dot{x} = V\cos\theta$, $\ddot{x} = -V\sin\theta \cdot \dot{\theta} + \dot{V}\cos\theta = -V\sin\theta\,\omega + a\cos\theta$

$\dot{y} = V\sin\theta$. $\ddot{y} = V\cos\theta + \dot{V}\sin\theta = V\cos\theta \cdot \omega + a\sin\theta$

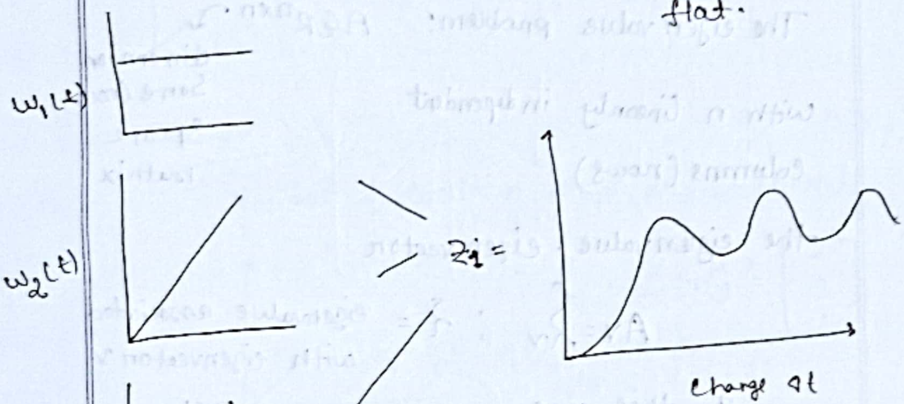
$$\begin{bmatrix} \ddot{x} \\ \ddot{y} \end{bmatrix} = \begin{bmatrix} -V\sin\theta & \cos\theta \\ V\cos\theta & \sin\theta \end{bmatrix} \begin{bmatrix} \omega \\ a \end{bmatrix}$$

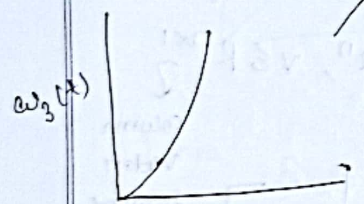
$$\begin{bmatrix} \omega \\ a \end{bmatrix} = \begin{bmatrix} -V\sin\theta & \cos\theta \\ V\cos\theta & \sin\theta \end{bmatrix}^{-1} \begin{bmatrix} \ddot{x} \\ \ddot{y} \end{bmatrix}$$

Basis function $\boxed{\text{MID term; Homework 2}}$

$$z_j = \sum_{i=1}^{n} a_{ij}^{[j]} \, \omega_i(t)$$

Linear combination of basis function when, differentially flat.





$n=3$

$$z_1 = \sum_{i=1}^{n} a_{1i}\,\omega_i(t)$$

$$= a_{11}\omega_1 + a_{12}\omega_2 + a_{13}\omega_3$$

$z_2, z_3 =$ same.

Eigenvalue decomposition

Date : ..../............/............

Eigenvalue = For square matrix

Singular value = For all type of matrix

The eigen value problem: $A \in R^{n \times n}$ dimension Same $(n \times n)$ Square matrix

with n linearly independent columns (rows)

the eigenvalue . eigen vector

$$Av = \lambda v \; ; \; \lambda = \text{eigenvalue associated with eigenvector } v.$$

In this case, $\lambda \in R^n$, $v \in R^{n \times 1}$ column Vector because 1 column

$Av_1 = \lambda_1 v_1$

$Av_2 = \lambda_2 v_2$

$Av_3 = \lambda_3 v_3$

$\vdots$

$Av_n = \lambda_n v_n$

$$A \underbrace{\begin{bmatrix} v_1 & v_2 \cdots v_n \end{bmatrix}}_{V} = \begin{bmatrix} v_1 & v_2 \cdots v_n \end{bmatrix} \underbrace{\begin{bmatrix} \lambda_1 & 0 & 0 & 0 \\ 0 & \lambda_2 & & 0 \\ 0 & 0 & \lambda_3 & 0 \\ \vdots & & \ddots & 0 \\ 0 & 0 & 0 & \lambda_n \end{bmatrix}}_{\Lambda}$$

Diaganoal matrix of eigenvalues

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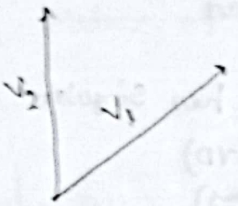
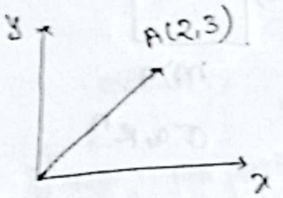


indication of amplication along v_1 direction (eigenvector)

Example

Vector (euclidean norm or 2-norm)



$$A = 2\hat{x} + 3\hat{y} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

$$\|A\| = \sqrt{2^2+3^2}$$

$$= \sqrt{13}$$

Form. $Av = \lambda v$ matrix,

$$AV = V\Lambda$$

$$AVV^{-1} = AVV^{-1}\Lambda \quad VV^{-1} =$$

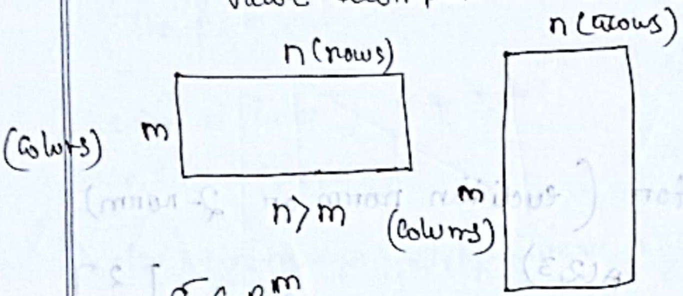
$$\boxed{A = A \; V \Lambda V^{-1}}$$

The eigen decomposition of a matrix A

Date : ..../........../..........

Singular value decomposition

Any matrix $A \in R^{m \times n}$ has singular value decomposition (SVD)



$\sigma \in R^m$

$\sigma \in R^n$

$AU = \sigma V$; σ singular value

$$\left.\begin{array}{l} U \in R^{n \times 1} \\ V \in R^{m \times 1} \end{array}\right\} \text{Singular vector}$$

For $n > m$

$A = V \Sigma U^T$

$A U_1 = \sigma_1 v_1$

$A U_2 = \sigma_2 v_2$

$\vdots$

$A U_m = \sigma_m v_m$

Date : / /

Matrix of Siregulan values,

$$\Sigma = \begin{bmatrix} \sigma_1 & 0 & 0 & \\ 0 & \sigma_2 & 0 & \\ 0 & 0 & \sigma_3 & \\ \vdots & \vdots & \vdots & \ddots \\ 0 & 0 & 0 & \sigma_m \end{bmatrix}$$

When $\sigma = 0$ null space

$AV = \lambda V$

If there is system A, along direction v, the amplification on direction v.

$A \in R^{n \times n}$ linearly independent eigen vectors.

$V^{-1}AV = V^{-1}V\Lambda = \Lambda$ [multiply V^{-1} from left]

$$\boxed{\Lambda = V^{-1}AV}$$

$AVV^{-1} = V\Lambda V^{-1}$ [multiply V^{-1} from right]

$$\boxed{A = V\Lambda V^{-1}}$$

Date : / /

$$B = \begin{bmatrix} \ddots & & \\ & \ddots & \\ 0 & & \ddots \end{bmatrix}$$ upper triangular matrix

$$C = \begin{bmatrix} \ddots & & 0 \\ & c_{ij} & \\ & & \ddots \end{bmatrix}$$ lower triangular matrix

~~diagonal~~ Diagonal values are the eigenvalues.

$$A = V \Lambda V^{-1}$$

$$AA^{-1} = V \Lambda V^{-1} \left(V \Lambda^{-1} V^{-1} \right)$$

$$= I$$

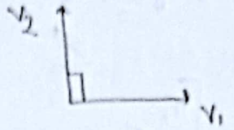
$$\Lambda^{-1} = \begin{bmatrix} \frac{1}{\lambda_1} & & & \\ & \frac{1}{\lambda_2} & & 0 \\ 0 & & \ddots & \\ & & & \frac{1}{\lambda_n} \end{bmatrix}$$

Singular value decomposition (SVD)

$$A = U \Sigma V^{T}$$ orthogonal matrix

$A \in R^{m \times n}$, $U \in R^{m \times m}$ orthogonal matrix

Orthogonal matrix has angle 90°



v_1, v_2 angle 90°. This is orthogonal

For Singular value decomposition vectors are orthogonal:

$$A = U \Sigma V^T$$

$A \in R^{m \times n}$, $U \in R^{m \times m}$ orthogonal matrix

$\Sigma \in R^{m \times n}$; diagonal matrix of singular matrix

$V \in R^{n \times n}$; Orthogonal matrix.

$m > n$; n singular values.

$$A = \underbrace{\begin{bmatrix} & & \\ & U & \\ & & \end{bmatrix}}_{m \times m} \underbrace{\begin{bmatrix} \sigma_1 & & 0 \\ & \sigma_2 & \\ 0 & & \sigma_n \\ \hline & 0 & \end{bmatrix}}_{m \times n} \underbrace{\begin{bmatrix} & & \\ & V^T & \\ & & \end{bmatrix}}_{n \times n}$$

graph $x_1, x_3 \vee T$ $\quad x_1 \vee x_3$

$u_1 \vee T$

$u_2 \vee T$

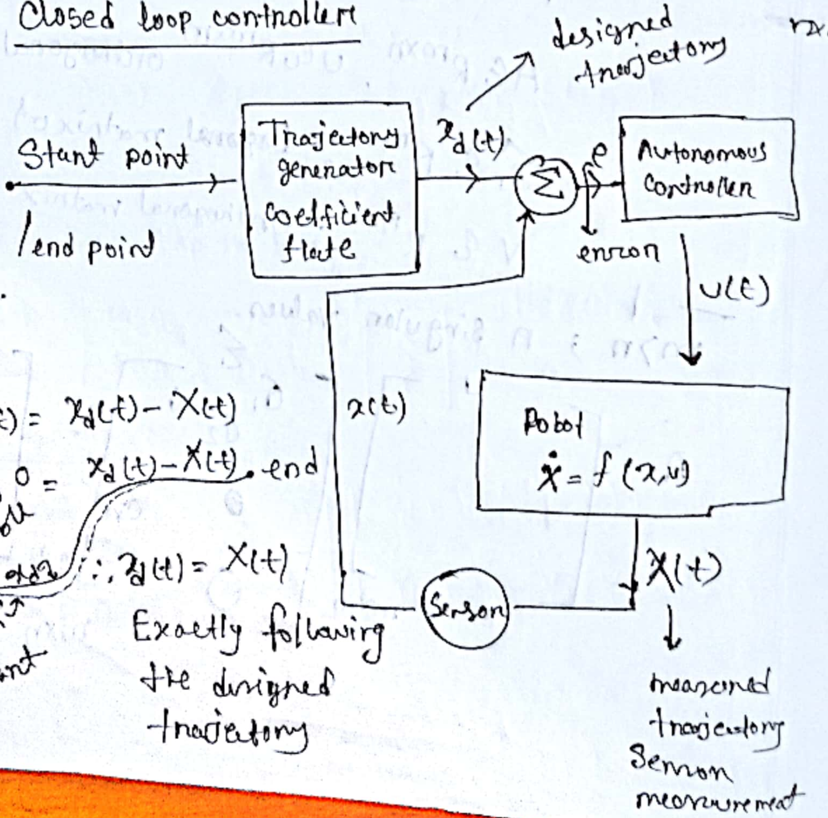
Date : / /

$n > m$; m singulan values

$$A = \underbrace{\begin{bmatrix} & \\ & \end{bmatrix}_{m \times m}}_{U} \underbrace{\left[\begin{array}{cccc|c} \sigma_1 & & & & \\ & \sigma_2 & & & \\ & & \sigma_3 & & 0 \\ & & & \ddots \sigma_n & \end{array}\right]}_{\Sigma} \underbrace{\begin{bmatrix} & \\ & \end{bmatrix}_{n \times n}}_{V^T}$$

$m \qquad n-m \quad m \times n$

~~Closed loop controller~~



$e(t) = x_d(t) - X(t)$

Small as possible $0 = x_d(t) - X(t)$. end

x_d, X

$\therefore x_d(t) = X(t)$

Start

Exactly following the designed trajectory

Date : / /

Example

$\ddot{y} + K_d \dot{y} + K_p y = 0$

$y(0), \dot{y}(0)$ are given ; K_d, K_p are constant.

Solution

$y(t) = A e^{r_1 t} + B e^{r_2 t}$

where A, B depends on $y(0), \dot{y}(0)$

r_1, r_2 depends on K_d, K_p.

$\ddot{y} - 6\dot{y} + 8y = 0$ $\qquad$ $y = e^{rx}$

$\qquad$ $\dot{y} = re^{rx}$

$r^2 - 6r + 8 = 0$ $\qquad$ $\ddot{y} = r^2 e^{rx}$

$(r-2)(r-4) = 0$

$r_1 = 2, r_2 = 4$ (depends on K_d, K_p)

There,

$\ddot{e}(t) + K_d \dot{e}(t) + K_p e(t) = 0$

The control problem is to choose K_p and K_d.

Such that as

$e(t) \longrightarrow 0$ and $t \longrightarrow \infty$

K_p: Propotional gain

K_d: derivative gain

Date : / /

$$\dot{e}(t) = \dot{x}_d(t) - \dot{x}_p(t)$$

$$\ddot{e}(t) = \ddot{x}_d(t) - \ddot{x}(t)$$

Then,

$$\ddot{x}_d(t) - \ddot{x}(t) + K_d\left(\dot{x}_d(t) - \dot{x}(t)\right) + K_p\left(x_d(t) - x(t)\right) = 0$$

$$\therefore \ddot{x}(t) = \ddot{x}_d(t) - K_d\left(\dot{x}_d(t) - \dot{x}(t)\right) + K_p\left(x_d(t) - x(t)\right)$$

For open loop controll

$$\begin{bmatrix} \dot{a} \\ \omega \end{bmatrix} = \begin{bmatrix} \cos\theta & -v\sin\theta \\ \sin\theta & v\cos\theta \end{bmatrix}^{-1} \begin{bmatrix} \ddot{x} \\ \ddot{y} \end{bmatrix}$$

"No error"

Closed loop controll

$$\begin{bmatrix} a \\ \omega \end{bmatrix} = \begin{bmatrix} \cos\theta & -v\sin\theta \\ \sin\theta & +v\cos\theta \end{bmatrix} \begin{bmatrix} \ddot{x}_d + K_d(\dot{x}_d - \dot{x}) + K_p(x_d - x) \\ \ddot{y}_d + K_d(\dot{y}_d - \dot{y}) + K_p(y_d - y) \end{bmatrix}$$

"There is noise"

Sensor has error / disturbance. That's why there is noise

Date : ____/____/____

Identifying States, controls and State space equations of different models

· State variables represent the systems current ~~pos~~ status (position, orientation, velocity ~~etc~~)

· Controls refer to inputs we apply to a system to influence its behaviour

Given,

~~[illegible]~~

$$\dot{x} = \frac{dx}{dt} = \frac{x(i+1) - x(i)}{\Delta t} = f(x,u,t) \rightarrow \dot{x}$$

or, $$\boxed{x(i+1) = \Delta t \cdot \dot{x} + x(i)}$$

~~or~~, euler equation

There,

Date : ...../...../.....

Identifying States, controls and State space equations of different models

· State variables represent the systems current ~~pos~~ status (position, orientation, velocity etc)

· Controls refer to inputs we apply to a system to influence its behaviour

Given,

~~$t_{init}=0, t_f=t_{fin}, \Delta t=0.1$~~

$$\dot{x} = \frac{dx}{dt} = \frac{x(i+1)-x(i)}{\Delta t} = \underbrace{f(x,u,t)}_{\dot{x}}$$

$$\text{or, } \boxed{x(i+1) = \Delta t \cdot \dot{x} + x(i)}$$

~~or~~,

↳ euler equation

There,

Date : / /

$\dot{x} = 3, \ \Delta t = 0.1 \ ; \ t_i = 0, \ t_f = 2$

$x(0) = 0.2$

$i = 0$ $\quad x(1) = x(0) + \Delta t \cdot 3$

$= 0.2 + (0 \times 3)$

$= 0.2$

$i = 1$ $\quad x(2) = x(1) + \Delta t \cdot 3$

$= 0.2 + 1 \times 3$

$= 3.2$

$i = 2$ $\quad x(3) = x(2) + \Delta t \cdot 3$

$= 3.2 + 2 \times 3$

$= 9.2$

Date : / /

Assignment : 1

Problem 2

Given,

for unicycle model

$$\dot{x} = v\cos\theta, \quad \dot{y} = v\sin\theta$$

$$\dot{\theta} = \omega$$

where,

$$x(0) = 0, \quad y(0) = 0, \quad \theta(0) = 1, \quad v = 1$$

$$\Delta t = 0.1$$

so, Euler equations are,

$$x(i+1) = x(i) + \Delta t \cdot \dot{x}(\theta(i)) \quad (v\cos\theta(i))$$

$$y(i+1) = y(i) + \Delta t \cdot \dot{y}(\theta(i)) \; (v\sin\theta(i))$$

$$\dot{\theta}(i+1) = \theta(i) + \Delta t \cdot \omega$$

$i = 0$

at $t = 0.1$, $\omega = 0$

$$x(1) = x(0) + 0.1 \cdot \cos(1) \cdot 1$$

$$= 0 + 0.1 \cdot 1$$