03/09/23

TOPIC NAME: 10 th Class

DAY: Sunday

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= state of the system $\begin{pmatrix} x \\ y \\ \theta \end{pmatrix}$ (function of time)

= control of the system $\begin{pmatrix} v \\ \omega \end{pmatrix}$

ই course-এ শুধু 2D থাকবে।

~~State vector describes~~

State variables: Describes the state of system at any given ~~in~~ instance in time.

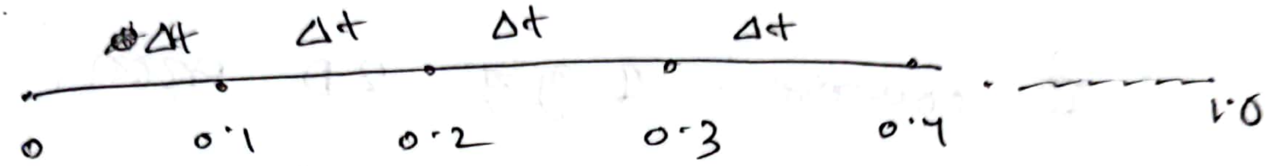
control variables: The input to the system

$$u \longrightarrow \boxed{\dot{x} = f(x,u)} \longrightarrow x$$

The state variables evolve as a function of time given an input.

~~এ~~ এ input দিলে state vector time এর সাথে change হবে।

$\Delta t = 0.1$



for loop

	for loop	
$\Delta t = 1 \rightarrow$	$10 \rightarrow$	$X[10], Y[10], \theta[10], V[10], \omega[10]$
$\Delta t = 0.1 \rightarrow$	$100 \rightarrow$	$X[100], Y[100], \theta[100], V[100], \omega[100]$
$\Delta t = 0.01 \rightarrow$	1000	

Mid Question:

Show the system is differentially flat.

If it is, then calculate differentially flat trajectory.

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Last class

the system is differentially flat

for $z = \begin{bmatrix} x \\ y \end{bmatrix}$.

Suppose, given $x(0), y(0), x(T), y(T)$

calculate a differentially flat trajectory.

T is final time

~~$T_{initial} = 0$~~ $t_{initial} = 0$

$t_{final} = T$

Solution: Approximate z as a linear combination of basis functions.

$z = \sum_{i=1}^{N} \alpha_i \; \varphi_i(t)$, here φ_i's are basis functions.

α_i are constants. we have to calculate next class solve

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=> Independent vector.

$$\begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

=> Dependent vector

$$\begin{bmatrix} 0 \\ 5 \end{bmatrix} \quad \begin{bmatrix} 0 \\ 20 \end{bmatrix}$$

Scalar multivector

$$5\begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad 20\begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

GOOD LUCK™

11th Class

TOPIC NAME :

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Term Paper Objective → Learning about a picked topic

Perception → Sensor information processing

Target → differentially flat trajectory.

$$\dot{x} = v\cos\theta$$

$$\dot{y} = v\sin\theta$$

$$\dot{\theta} = \omega$$

Initial position $\begin{cases} x(0) = a \\ y(0) = b \end{cases}$ Initial velocity $\begin{cases} \dot{x}(0) = e \\ \dot{y}(0) = f \end{cases}$

final position $\begin{cases} x(T) = c \\ y(T) = d \end{cases}$ final velocity $\begin{cases} \dot{x}(T) = g \\ \dot{x}(T) = h \end{cases}$

This system is flat for $z = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}$

$$\dot{x} = \dot{z}_1$$

$$\dot{y} = \dot{z}_2$$

Estimate the trajectory as,

$$z_1(t) = \sum_{i=1}^{N} a_{1i} \, \varphi_i(t)$$

$$z_2(t) = \sum_{i=1}^{N} a_{2i} \, \varphi_i(t)$$

Here φ_i's are basis functions. a_{1i} & a_{2i}'s are constants.

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$z_1(t)$ & $z_2(t)$ can be written as linear combination of the basis functions.

N = # of basis functions.
↳ number

Let's choose the following 4 basis functions.

$\psi_1(t) = 1,\ \psi_2(t) = t,\ \psi_3(t) = t^2,\ \psi_4(t) = t^3$

যত বেশি বেশি function, তত বেশি smooth হবে।

like: t^3 কে upto 0 পর্যন্ত differentiate করা যায়।

যত বেশি Differentiate করতে পারবো তত বেশি smoothness বাড়বে।

- Larger N means smoother function.

⟹ অবতো ২টি Page দেখো

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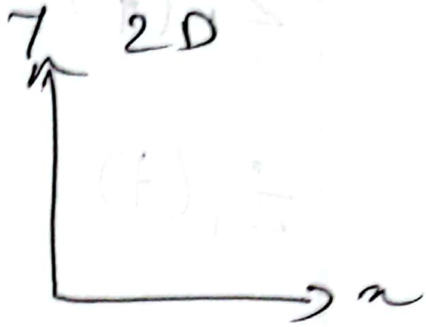
Basis vectors



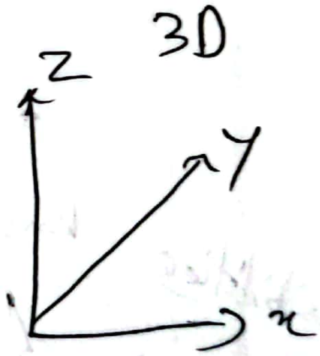
2D imension →

2 Basis vectors.

$$X = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad Y = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Basis vectors in 2D.

In 2-dimensions, all vectors can be written as linear combinations of x & y.



$$X = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \qquad Y = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \qquad Z = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Basis vector in 3D

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$$z_1(t) = a_{11} + a_{12}t + a_{13}t^2 + a_{14}t^3$$

$$\dot{z}_1(t) = a_{12} + 2a_{13}t + 3a_{14}t^2$$

$$z_2(t) = a_{21} + a_{22}t + a_{23}t^2 + a_{24}t^3$$

$$\dot{z}_2(t) = a_{22} + 2a_{23}t + 3a_{24}t^2$$

$$\underset{\text{known}}{A}\ \underset{\text{unknown}}{x} = \underset{\text{known}}{b}$$

2 possibilities of $\rightarrow$

$$x = \begin{cases} A^{-1}b & \text{if } A \text{ is full rank} \\ A^{+}b & \text{if } A \text{ is rank diffecient or not square.} \end{cases}$$

A^{+} is called the pseudo inverse.

$\rightarrow$ Test in code' inv (A), system auto decide [illegible] pseudo [illegible] normal inverse [illegible]

$$Ax = b$$

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unknowns

x a b

$$\begin{bmatrix} a_{11} \\ a_{12} \\ a_{13} \\ a_{14} \\ a_{21} \\ a_{22} \\ a_{23} \\ a_{324} \end{bmatrix} = \begin{bmatrix} z_1(0) \\ \dot{z}_1(0) \\ z_2(0) \\ \dot{z}_2(0) \\ z_1(T) \\ \dot{z}_1(T) \\ z_2(T) \\ \dot{z}_2(T) \end{bmatrix}$$

$z_1(0) = a_{11}$, $\dot{z}_1(0) = a_{12}$

$z_1(T) = a_{11} + a_{12}t +$
$a_{12}T^2 + a_{14}T^3$

X

A

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & T & T^2 & T^3 & 0 & 0 & 0 & 0 \\ 0 & 1 & 2T & 3T^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & T & T^2 & T^3 \\ 0 & 0 & 0 & 0 & 0 & 1 & 2T & 3T^2 \end{bmatrix}$$

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$$z_1(t) = \sum_{i=1}^{4} a_{1i}\, \psi_i(t)$$

$\cancel{x = A^{-1}b}$

$x = A^{-1}b$ if A is full rank.

HT $\rightarrow$ Problem 2 & Problem 3 বাসায় করা।

Straight Integration.

12th Class

DAY: 10/09/23 Sunday

TOPIC NAME: Assignment 1

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Problem 3: $r = 0.1$ and $L = 1$

$r \rightarrow$ wheel radius

$L \rightarrow$ car length –

1 : 10 ratio.

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{pmatrix} = \begin{pmatrix} \cos\theta \\ \sin\theta \\ 0 \end{pmatrix} v + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \omega$$

Program Writing

// Parameter declaration. $\longrightarrow$ $\left.\begin{matrix} x \\ y \\ \theta \\ v \\ \omega \end{matrix}\right\}$ arrays of size $\frac{t_{final} - t_{initial}}{\Delta t}$ +

// Parameter Initialization.

v [100]

ω [100]

v [10] → v (t = 1)

t = 0   0.1   0.2 … t = 1.0

i = 0   1   2 … i = 10

$v(1) = 1$, $v(2) = 1$, $v(3) = 1$ … $v(6) = 1$

v [30] → v (t = 3)

TOPIC NAME

// Parameter initialization

<u>One option</u>

```
for (i=0; i<=100; i++)
    v[i]=1
    w[i]=0
if (i>=5) && (i<=15)
    w[i]=3
}
```

<u>Another option</u>

```
for (i= ; i<=5; i++)
    w[i]=3
for(i=20;
```

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//main body

for(i=1;i<=100,i++)

~~x(i+~~ x(i+1) = x(i) + dt x cosθ(i) x v(i);

y(i+1) = y(i) + dt * sinθ(i) * v(i);

θ(i+1) = θ(i) + dt * w(i);

APPy Euler's method.

same

$$\dot{x} = \cos\theta v$$

$$\frac{x(t+\Delta t) - x(t)}{\Delta t} = \cos\theta(t)\, v$$

$$\Rightarrow x(t+\Delta t) = x(t) + \Delta t \cos\theta(t)\, v(t);$$

degree & radian conversion খেয়াল রাখতে হবে। আমরা radian দিয়ে কাজ করবো।

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coding - এ degree থেকে radian - এ convert করার সময় ~~যত্ন~~ করতে হবে।

Question 2 → 3 এর Ans help
[illegible] 8 page এ

$A x = b$ → known vector

A → known matrix

x → unknown vector

$A_{m\times n}$

① $m = n$; (square matrix) $x = A^{-1}b$ (if A is fullrank)

$x = A^{+}b$ (If A is rank

$A^{+} = A^{T}(AA^{T})^{-1}$ ② $m < n$; $x = A^{+}b$ (x is called ~~least~~ "least norm" solution.

③ $m > n$; fit of basis functions

(x is called "least squares" solution)

$A^{+} = (A^{T}A)^{-1} A^{T}$

Google করলে → Least-norm solution

in coding (Python etc),

inv (A) → প্রথমে নিজেই A এর ~~dim~~ dimension set করবে,

যত বেশি basis function use → তত বেশি Accuracy But আবার তত বেশি ~~calculation~~ ~~করতে হবে,~~ calculation - ও হবে।

Regression এর আরেক নাম least squares.

13th class 12/09/23

TOPIC NAME: Lecture 3 Slide DAY: Tuesday

Example: consider the system,

$\dot{x}_1 = u_1$

$\dot{x}_2 = u_2$

$\dot{x}_3 = x_2 u_1$

States	control
x_1	u_1
x_2	u_2
x_3	

① show this system is differentially flat for $z = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_3 \end{bmatrix}$

$x_1 = z_1$ $\qquad u_1 = \dot{x}_1 = \dot{z}_1$

$x_3 = z_2$ $\qquad u_2 = \dot{x}_2 = \frac{d}{dt}\left(\frac{\dot{z}_2}{\dot{z}_1}\right)$

$x_2 = \frac{\dot{x}_3}{u_1}$

$= \frac{\dot{z}_2}{\dot{z}_1}$

x & u are functions of $z, \dot{z}, \ddot{z}$

capital $T \to$ final time

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so, the system is DF for $z = \begin{bmatrix} x_1 \\ x_3 \end{bmatrix}$.

(Ans)

② Calculate the mission trajectory for the following conditions:

$x_1(0) = x_{10}$, $x_2(0) = x_{20}$, $x_3(0) = x_{30}$

$x_1(T) = x_{1f}$, $x_2(T) = x_{2f}$, $x(T) = x_{3f}$

use 4 basis functions $\varphi_1(t) = 1$,

$\varphi_2(t) = t$, $\varphi_3(t) = t^2$, $\varphi_4(t) = t^3$

$z_1 = \sum_{i=1}^{4} a_{1i}\varphi_i(t)$

$\Rightarrow z_1 = a_{11}\varphi_1(t) + a_{12}\varphi_2(t) + a_{13}\varphi_3(t)$
$+ a_{14}\varphi_4(t)$

$\Rightarrow \boxed{z_1 = a_{11} + a_{12}t + a_{13}t^2 + a_{14}t^3}$

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$$\dot{z}_1 = a_{12} + 2a_{13} + 3a_{14}t^2$$

$$\boxed{z_2 = a_{21} + a_{22} + a_{23}t^2 + a_{24}t^3}$$

$$\dot{z}_2 = a_{22} + 2a_{23}t + 3a_{24}t^2$$

$$\dot{z}_2(0) = a_{22}$$

$$\dot{z}_2(T) = a_{22} + 2a_{23}T + 3a_{24}T^2$$

$$x_1 = z_1$$

$$x_2 = \frac{\dot{z}_2}{\dot{z}_1}$$

$$x_3 = z_2$$

$$x_1(0) = z_1(0) = x_{10}$$

$$x_2(0) = \frac{\dot{z}_2(0)}{z_1(0)} = \frac{a_{22}}{a_{12}} = x_{20}$$

$$x_3(0) = z_2(0) = x_{30}$$

$$x_1(T) = z_1(T) = x_{1f}$$

TOPIC NAME: Slide Lecture 3, P-22
print it, keep it

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$Ax = b$

$$
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
1 & T & T^2 & T^3 & 0 & 0 & 0 & 0 \\
0 & 1 & 2T & 3T^2 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & T & T^2 & T^3 \\
0 & 0 & 0 & 0 & 0 & 1 & 2T & 3T^2
\end{bmatrix}
\begin{bmatrix} a_{11} \\ a_{12} \\ a_{13} \\ a_{14} \\ a_{21} \\ a_{22} \\ a_{23} \\ a_{24} \end{bmatrix}
=
\begin{bmatrix} x_{10} \\ 1 \\ x_{30} \\ x_{20} \\ x_{1f} \\ 1 \\ x_{3f} \\ x_{2f} \end{bmatrix}
\quad
\begin{array}{l} \text{for everyone} \\ z_1(0) \\ \dot{z}_1(0) \\ z_2(0) \\ \dot{z}_2(0) \\ z_1(T) \\ \dot{z}_1(T) \\ z_2(T) \\ \dot{z}_2(T) \end{array}
$$

$$x_2 = \frac{\dot{z}_2}{\dot{z}_1}$$

$$x_2(0) = \frac{\dot{z}_2(0)}{\dot{z}_1(0)} = \dot{z}_2(0)$$

$$x_2(T) = \frac{\dot{z}_2(T)}{\dot{z}_1(T)} = \dot{z}_2(T)$$

Set

$\dot{z}_1(0) = 1 \Rightarrow a_{12} = 1$

Set

$\dot{z}_1(T) = 1$

$\Rightarrow a_{12} + 2a_{13}T + 3a_{14}T^2 = 1$

Now,

$X = A^{+}b$

Inverse Spscific calculation

14th class

17/09/23
Sunday

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part b থেকে Matrix invarse করে,

Ass. Quiz 4.|| differential flatness use করতে হবে path calculate করতে।

(b) একবারে আসতে হবে $Ax = b$ create using IC

$$X_1 = z_1 = \sum_{i=1}^{4} a_{1i}\,\psi_i(t) = a_{11}\psi_1(t) + \dots + \dots + a_{14}\psi_4(t)$$

$$X_2 = z_2 = \sum_{i=1}^{4} a_{2i}\,\psi_i(t) =$$

$$X = \begin{bmatrix} a_{11} \\ a_{12} \\ a_{13} \\ a_{14} \\ a_{21} \\ a_{22} \\ a_{23} \\ a_{24} \end{bmatrix}$$

$\Rightarrow X = A^{+}b$

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$\dot{z}_1$ = differentiate of z_1

$\dot{z}_2$ = 〃 of z_2

$x_2 = \frac{\dot{z}_2}{\dot{z}_1}$

$x_2(0) = \frac{\dot{z}_2(0)}{\dot{z}_1(0)} = \dot{z}_2(0)$

$x_2 = \frac{\dot{z}_2}{\dot{z}_1} = \dot{z}_2$

$x_2(t) = \frac{\dot{z}_2(t)}{\dot{z}_1(t)}$

$\dot{x}_1(0) = 1$

$\dot{x}_1(T) = 1$

(*) plot(x_1, x_2, x_3)

$$b = \begin{bmatrix} z_1(0) \\ \dot{z}_1(0) \\ z_2(0) \\ \dot{z}_2(0) \\ z_1(T) \\ \dot{z}_1(T) \\ z_2(T) \\ \dot{z}_2(T) \end{bmatrix} = \begin{bmatrix} x_1(0) \\ 1 \\ x_3(0) \\ x_2(0) \\ x_1(T) \\ 1 \\ x_3(T) \\ x_2(T) \end{bmatrix}$$

GOOD LUCK™

C++ code

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ki libraries

numpy.linalg.pinv

numpy.random.normal (with noise)

pseudo inverse দ্বারা calculation করতে হয়,

$x_1 = a_{11} + a_{12}t + a_{13}t^2 + a_{14}t^3$

$\dot{x}_1 =$

$t = [0, 0.1, 0.2, \ldots\ldots, 10]$

start interval end

$t = 0 : 0.1 : 10$

$t = 0 : 0.01 : 10$

$$t = \begin{bmatrix} 0 \\ 0.1 \\ 0.2 \\ \vdots \\ 10 \end{bmatrix}$$

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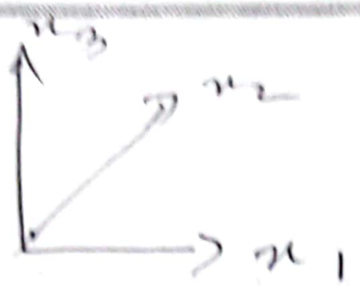
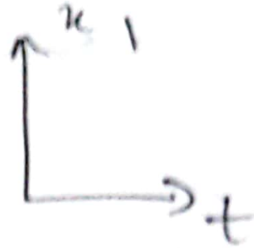
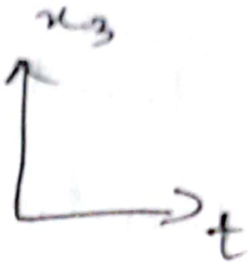
(1)


(2)


(3)


(4)


- code করার সময় 0 or infinity দিয়ে কিছু divide করা যাবে না, code crash করবে।
- exception দিয়ে handle করতে হবে।

v=velocity.

$$\dot{x}(t) = v(t)\cos\theta(t)$$
$$\dot{y}(t) = v(t)\sin\theta(t)$$
$$\dot{v}(t) = a(t)$$
$$\dot{\theta}(t) = \omega(t)$$

দুই হাতে control হচ্ছে।

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s = displacement.

$s(t)$

velocity, $v(t) = \frac{d}{dt}(s) = \dot{s}$
হচ্ছে [illegible] s এর derivative

acceleration, $a(t) = \dot{v}(t)$
হচ্ছে [illegible] v এর derivative

θ ~~[illegible]~~ angular displacement.
$\dot{\theta}$ 〃 〃 velocity

$Z = (x, y)$

$x = \sum_{i=1}^{4} a_{1i}\, \psi_i(t), \quad y = \sum_{i=1}^{4} a_{2i}\, \psi_i(t)$

$\dot{x} =$ $\dot{y} =$

$\ddot{x} =$ $\ddot{y} =$

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I want to know controls $a(t)$, $\omega(t)$

$$\begin{bmatrix} \ddot{x}(t) \\ \ddot{y}(t) \end{bmatrix} \begin{matrix} = \cancel{-v(t)\sin\theta(t)} \\ -v(t)\sin\theta(t)\,\dot{\theta}(t) + \dot{v}(t)\cos\theta(t) \\ = v(t)\cos\theta(t)\,\dot{\theta}(t) + \dot{v}(t)\sin\theta(t) \end{matrix}$$

$$= \begin{bmatrix} -v\sin\theta & \cos\theta \\ v\cos\theta & \sin\theta \end{bmatrix} \begin{bmatrix} \dot{\theta}(t) \\ \dot{v}(t) \end{bmatrix}$$

$Ax = b$

$b = Ax$

$$\Rightarrow \underbrace{\begin{bmatrix} \ddot{x}(t) \\ \ddot{y}(t) \end{bmatrix}}_{b} = \underbrace{\begin{bmatrix} -v\sin\theta & \cos\theta \\ v\cos\theta & \sin\theta \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} \omega(t) \\ a(t) \end{bmatrix}}_{x}$$

$A^{-1}b$

$$\Rightarrow \underbrace{\begin{bmatrix} \omega(t) \\ a(t) \end{bmatrix}}_{x} = \underbrace{\begin{bmatrix} -v(t)\sin\theta(t) & \cos\theta(t) \\ v(t)\cos\theta(t) & \sin\theta(t) \end{bmatrix}^{-1}}_{A^{-1}} \underbrace{\begin{bmatrix} \ddot{x}(t) \\ \ddot{y}(t) \end{bmatrix}}_{b}$$

15th class 19/09/23

Tuesday

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Homework 2 → Sunday, Oct 15
(1 ta python problem + সব theory)

Midterm → Tuesday, Oct 17

Syllabus : HW 1, HW 2 & class notes.

= noise কম হলে open loop system
ঠিকমত handle করতে পারবে না।

= noise বেশি হলে closed loop system
handle করতে পারে smoothly.
But extreme হলে আর পারবে না।

$$V = \sqrt{\dot{x}^2 + \dot{y}^2}$$

$$V = \frac{\dot{x}}{\cos\theta}$$ [But এইটা use করা যাবে না $\theta = \pi/2$ হলে $\cos \pi/2 = 0$]

$$V = \frac{\dot{y}}{\sin\theta}$$ [আবার $\theta = 0$ হবে $\sin 0 = 0$]

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same for $\tan\theta$ and $\cot\theta$.

(i) Gaussian Distribution & Normal Distribution is same.

* Term paper: Sensor related to drone / autonomous vehicle.

Sensors:
Lidar, Radar, GPS, IMU, wheel encoder etc.

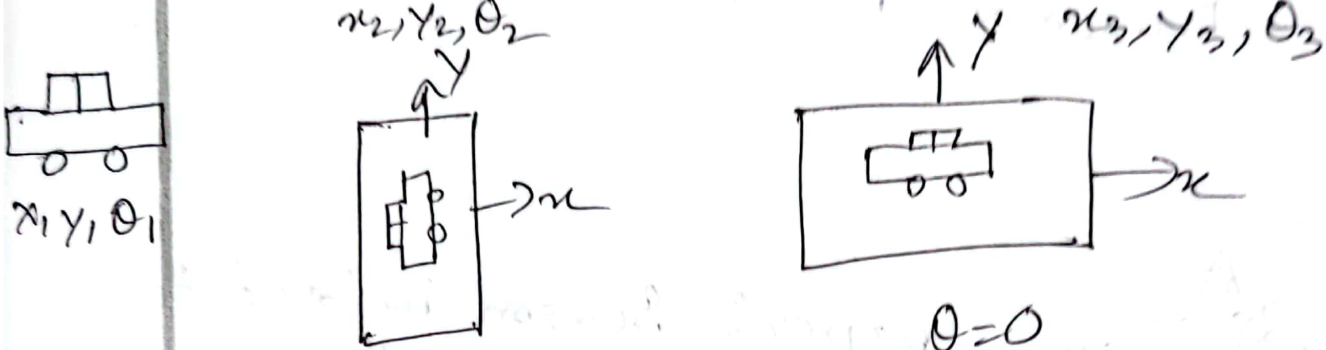
Regression in Basic Robotics.

Perception

Navigation

Lecture 5 left from next class.

Parking Spot



$\theta = 90^\circ$

$\theta = 0$

3rd situation

Lyapunov stability theory.

→ Self Study.

Given a system, $\dot{x} = f(x,u)$, if there exists a function $V(x,u)$, such that,

① $V(0,0) = 0$ [i.e. $x=0$ & $u=0$]

② $V(x,u) > 0$ for $x \neq 0$ & $u \neq 0$

③ $\dot{V}(x,u) < 0$ for $x \neq 0$ & $u \neq 0$.

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Then the system if stable.

The goal is to compute u which meets

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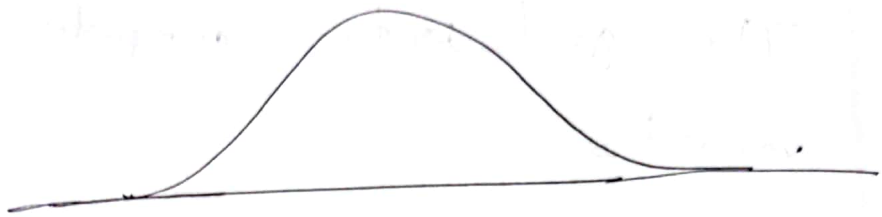
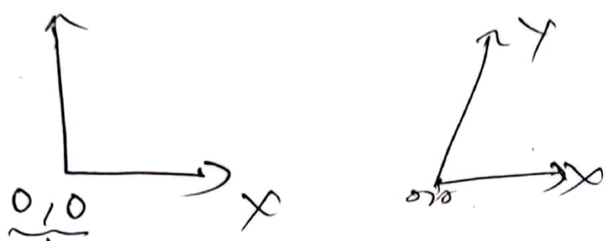
TIME : DATE : / /





Reference point.

Mean vector

$$\mu = \begin{bmatrix} \mu_x \\ \mu_x \end{bmatrix}$$

Covariance Matrix

$$\sigma = \begin{bmatrix} \sigma_x & \sigma_{xy} \\ \sigma_{yx} & \sigma_y \end{bmatrix}$$

TOPIC NAME : DAY : TIME : DATE : / /

x, y, z

$$\mu = \begin{bmatrix} \mu_x \\ \mu_y \\ \mu_z \end{bmatrix} \qquad \sigma = \begin{bmatrix} \sigma_x & \cancel{\sigma}\sigma_{xy} & \sigma_{xz} \\ x & \sigma_y & \\ & & \sigma_z \end{bmatrix}$$

Black & white Image / B&W = 2D Matrix

~~Colo~~ Colour Imag / RGB = 3D "

(*) for compression.

SVD = Singular Value Decomposition

CamScanner