

ANSWER TO THE QUESTION NO. 4 C(i)

```
In [38]: import numpy as np
import matplotlib.pyplot as plt
```

```
In [39]: t = np.linspace(0, 10, 100)
```

```
In [40]: t
```

```
Out[40]: array([ 0.          ,  0.1010101 ,  0.2020202 ,  0.3030303 ,  0.4040404 ,
  0.50505051,  0.60606061,  0.70707071,  0.80808081,  0.90909091,
  1.01010101,  1.11111111,  1.21212121,  1.31313131,  1.41414141,
  1.51515152,  1.61616162,  1.71717172,  1.81818182,  1.91919192,
  2.02020202,  2.12121212,  2.22222222,  2.32323232,  2.42424242,
  2.52525253,  2.62626263,  2.72727273,  2.82828283,  2.92929293,
  3.03030303,  3.13131313,  3.23232323,  3.33333333,  3.43434343,
  3.53535354,  3.63636364,  3.73737374,  3.83838384,  3.93939394,
  4.04040404,  4.14141414,  4.24242424,  4.34343434,  4.44444444,
  4.54545455,  4.64646465,  4.74747475,  4.84848485,  4.94949495,
  5.05050505,  5.15151515,  5.25252525,  5.35353535,  5.45454545,
  5.55555556,  5.65656566,  5.75757576,  5.85858586,  5.95959596,
  6.06060606,  6.16161616,  6.26262626,  6.36363636,  6.46464646,
  6.56565657,  6.66666667,  6.76767677,  6.86868687,  6.96969697,
  7.07070707,  7.17171717,  7.27272727,  7.37373737,  7.47474747,
  7.57575758,  7.67676768,  7.77777778,  7.87878788,  7.97979798,
  8.08080808,  8.18181818,  8.28282828,  8.38383838,  8.48484848,
  8.58585859,  8.68686869,  8.78787879,  8.88888889,  8.98989899,
  9.09090909,  9.19191919,  9.29292929,  9.39393939,  9.49494949,
  9.5959596 ,  9.6969697 ,  9.7979798 ,  9.8989899 ,  10.        ])
```

```
In [41]: #check the Length of t
len(t)
```

```
Out[41]: 100
```

```
In [42]: # def calculate_coefficients():
T = 10
Tsq = T ** 2
Tcb = T ** 3
```

```
In [43]: A = np.array([[1, 0, 0, 0, 0, 0, 0, 0],
                  [0, 1, 0, 0, 0, 0, 0, 0],
                  [0, 0, 0, 0, 1, 0, 0, 0],
                  [0, 0, 0, 0, 0, 1, 0, 0],
                  [1, T, Tsq, Tcb, 0, 0, 0, 0],
                  [0, 1, 2*T, 3*Tsq, 0, 0, 0, 0],
                  [0, 0, 0, 0, 1, T, Tsq, Tcb,],
                  [0, 0, 0, 0, 0, 1, 2*T, 3*Tsq]
                 ])
A
```

```
Out[43]: array([[ 1,  0,  0,  0,  0,  0,  0,  0],
   [ 0,  1,  0,  0,  0,  0,  0,  0],
   [ 0,  0,  0,  0,  1,  0,  0,  0],
   [ 0,  0,  0,  0,  0,  1,  0,  0],
   [ 1,  10,  100, 1000, 0,  0,  0,  0],
   [ 0,  1,  20,  300, 0,  0,  0,  0],
   [ 0,  0,  0,  0,  1,  10,  100, 1000],
   [ 0,  0,  0,  0,  0,  1,  20,  300]]))
```

```
In [44]: Ainv = np.linalg.pinv(A)
```

```
In [45]: Ainv
```

```
Out[45]: array([[ 1.00000000e+00, -5.27355937e-16, -1.02995148e-17,
   0.00000000e+00, -9.09645623e-17,  1.11455983e-16,
   0.00000000e+00,  0.00000000e+00],
  [-2.77555756e-16,  1.00000000e+00, -6.86054248e-17,
   0.00000000e+00,  2.25514052e-16,  1.49186219e-16,
   0.00000000e+00,  0.00000000e+00],
  [-3.00000000e-02, -2.00000000e-01,  1.41792078e-17,
   0.00000000e+00,  3.00000000e-02, -1.00000000e-01,
   0.00000000e+00,  0.00000000e+00],
  [ 2.00000000e-03,  1.00000000e-02, -7.21157187e-19,
   0.00000000e+00, -2.00000000e-03,  1.00000000e-02,
   0.00000000e+00,  0.00000000e+00],
  [-9.05861232e-17,  7.83765075e-17,  1.00000000e+00,
   -9.86988269e-14,  9.05861232e-17, -3.02976149e-16,
   -1.23382207e-16, -5.29958022e-16],
  [ 5.72673486e-18, -7.25563308e-18,  1.94289029e-16,
   1.00000000e+00, -5.72673486e-18,  1.91538275e-17,
   3.72965547e-17, -1.80411242e-16],
  [ 1.57396595e-18, -6.90020030e-19, -3.00000000e-02,
   -2.00000000e-01, -1.57396595e-18,  5.26429558e-18,
   3.00000000e-02, -1.00000000e-01],
  [-1.24073118e-19,  6.37581422e-20,  2.00000000e-03,
   1.00000000e-02,  1.24073118e-19, -4.14975976e-19,
   -2.00000000e-03,  1.00000000e-02]])
```

```
In [46]: # X1_θ = 1
# X2_θ = 1
# X3_θ = 0
# X1_T = 5
# X2_T = 5
# X3_T = 1
```

```
In [47]: #initialized b
b = np.array([[1],#x1(θ)
              [1],#1
              [0],#x3(θ)
              [1],#x2(θ)
              [5],#x1(T)
              [1],#1
              [5],#x3(T)
              [5] #x2(T)
            ]
          )
b
```

```
Out[47]: array([[1],  
                 [1],  
                 [0],  
                 [1],  
                 [5],  
                 [1],  
                 [5],  
                 [5]])
```

```
In [48]: #matrix multiplication x = Ainv * b  
x = np.matmul(Ainv, b)  
x
```

```
Out[48]: array([[ 1.00000000e+00],  
                 [ 1.00000000e+00],  
                 [-1.80000000e-01],  
                 [ 1.20000000e-02],  
                 [-1.01827783e-13],  
                 [ 1.00000000e+00],  
                 [-5.50000000e-01],  
                 [ 5.00000000e-02]])
```

```
In [49]: a11 = x[0]  
a11
```

```
Out[49]: array([1.])
```

```
In [50]: a12 = x[1]  
a12
```

```
Out[50]: array([1.])
```

```
In [51]: a13 = x[2]  
a13
```

```
Out[51]: array([-0.18])
```

```
In [52]: a14 = x[3]  
a14
```

```
Out[52]: array([0.012])
```

```
In [53]: a21 = x[4]  
a21
```

```
Out[53]: array([-1.01827783e-13])
```

```
In [54]: a22 = x[5]  
a22
```

```
Out[54]: array([1.])
```

```
In [55]: a23 = x[6]  
a23
```

```
Out[55]: array([-0.55])
```

```
In [56]: a24 = x[7]  
a24
```

```
Out[56]: array([0.05])
```

```
In [57]: t
```

```
Out[57]: array([ 0.          ,  0.1010101 ,  0.2020202 ,  0.3030303 ,  0.4040404 ,
  0.50505051,  0.60606061,  0.70707071,  0.80808081,  0.90909091,
  1.01010101,  1.11111111,  1.21212121,  1.31313131,  1.41414141,
  1.51515152,  1.61616162,  1.71717172,  1.81818182,  1.91919192,
  2.02020202,  2.12121212,  2.22222222,  2.32323232,  2.42424242,
  2.52525253,  2.62626263,  2.72727273,  2.82828283,  2.92929293,
  3.03030303,  3.13131313,  3.23232323,  3.33333333,  3.43434343,
  3.53535354,  3.63636364,  3.73737374,  3.83838384,  3.93939394,
  4.04040404,  4.14141414,  4.24242424,  4.34343434,  4.44444444,
  4.54545455,  4.64646465,  4.74747475,  4.84848485,  4.94949495,
  5.05050505,  5.15151515,  5.25252525,  5.35353535,  5.45454545,
  5.55555556,  5.65656566,  5.75757576,  5.85858586,  5.95959596,
  6.06060606,  6.16161616,  6.26262626,  6.36363636,  6.46464646,
  6.56565657,  6.66666667,  6.76767677,  6.86868687,  6.96969697,
  7.07070707,  7.17171717,  7.27272727,  7.37373737,  7.47474747,
  7.57575758,  7.67676768,  7.77777778,  7.87878788,  7.97979798,
  8.08080808,  8.18181818,  8.28282828,  8.38383838,  8.48484848,
  8.58585859,  8.68686869,  8.78787879,  8.88888889,  8.98989899,
  9.09090909,  9.19191919,  9.29292929,  9.39393939,  9.49494949,
  9.5959596 ,  9.6969697 ,  9.7979798 ,  9.8989899 ,  10.        ])
```

```
In [58]: for i in t:
```

```
X1 = a11 + (a12*t) + (a13 * np.power(t,2)) + (a14 * np.power(t,3))

# X2 = a22 + (2*a23*t) + (3*a24* np.power(t,2))

X2 = (a22 + (2*a23*t) + (3*a24* np.power(t,2)))/(a12 + (2*a13*t) + (3*a14* np.

X3 = a21 + (a22*t) + (a23 * np.power(t,2)) + (a24 * np.power(t,3) )

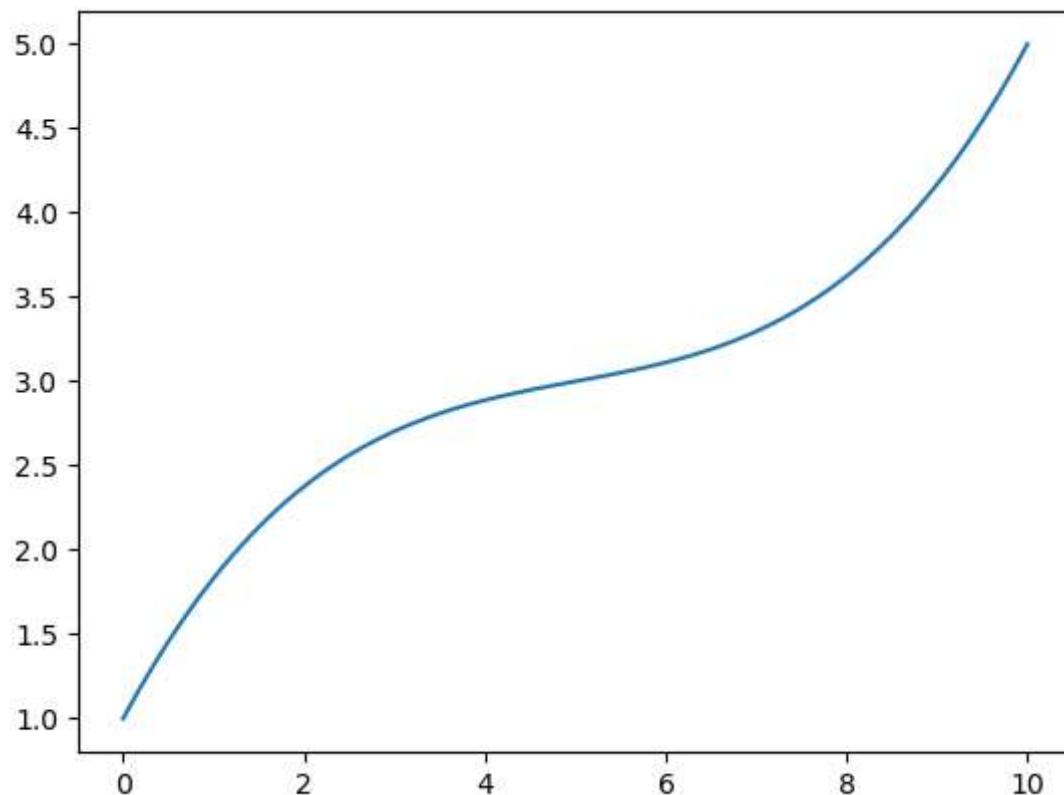
X1, X2, X3
```

```
Out[58]: (array([1.          , 1.09918592, 1.19477295, 1.2868353 , 1.37544716,
       1.46068274, 1.54261625, 1.62132188, 1.69687385, 1.76934636,
       1.8388136 , 1.90534979, 1.96902913, 2.02992583, 2.08811408,
       2.14366809, 2.19666206, 2.2471702 , 2.29526672, 2.34102581,
       2.38452168, 2.42582853, 2.46502058, 2.50217201, 2.53735704,
       2.57064987, 2.60212471, 2.63185575, 2.6599172 , 2.68638327,
       2.71132816, 2.73482607, 2.75695121, 2.77777778, 2.79737998,
       2.81583203, 2.83320811, 2.84958245, 2.86502923, 2.87962267,
       2.89343697, 2.90654633, 2.91902496, 2.93094706, 2.94238683,
       2.95341848, 2.96411622, 2.97455424, 2.98480675, 2.99494795,
       3.00505205, 3.01519325, 3.02544576, 3.03588378, 3.04658152,
       3.05761317, 3.06905294, 3.08097504, 3.09345367, 3.10656303,
       3.12037733, 3.13497077, 3.15041755, 3.16679189, 3.18416797,
       3.20262002, 3.22222222, 3.24304879, 3.26517393, 3.28867184,
       3.31361673, 3.3400828 , 3.36814425, 3.39787529, 3.42935013,
       3.46264296, 3.49782799, 3.53497942, 3.57417147, 3.61547832,
       3.65897419, 3.70473328, 3.7528298 , 3.80333794, 3.85633191,
       3.91188592, 3.97007417, 4.03097087, 4.09465021, 4.1611864 ,
       4.23065364, 4.30312615, 4.37867812, 4.45738375, 4.53931726,
       4.62455284, 4.7131647 , 4.80522705, 4.90081408, 5.        ]),
array([ 1.          , 0.923668 , 0.84404456, 0.76093654, 0.67413732,
       0.58342582, 0.48856549, 0.38930318, 0.2853681 , 0.17647059,
       0.06230105, -0.05747126, -0.18319929, -0.31525917, -0.45405111,
      -0.6          , -0.75355564, -0.91519257, -1.08540925, -1.26472626,
      -1.45368344, -1.65283541, -1.8627451 , -2.08397469, -2.31707317,
      -2.56255969, -2.82090156, -3.09248555, -3.37758112, -3.67629362,
      -3.98850575, -4.31380526, -4.65139729, -5.          , -5.35772358,
      -5.72193437, -6.08910891, -6.45468689, -6.81293778, -7.15686275,
      -7.47816092, -7.7672956 , -8.01369863, -8.20614732, -8.33333333,
      -8.38461538, -8.35090803, -8.22561341, -8.00546448, -7.6911315 ,
      -7.28746177, -6.80327869, -6.25074806, -5.64440539, -5.          ,
      -4.33333333, -3.65924748, -2.99086758, -2.33913885, -1.71264368,
      -1.11764706, -0.55830243, -0.0369506 , 0.44554455, 0.88946459,
      1.29593496, 1.66666667, 2.00374532, 2.30946569, 2.5862069 ,
      2.83634176, 3.06217381, 3.26589595, 3.44956539, 3.61509074,
      3.76422764, 3.89858047, 4.01960784, 4.12863071, 4.22684172,
      4.31531532, 4.39501779, 4.46681721, 4.53149277, 4.58974359,
      4.64219685, 4.68941519, 4.73190349, 4.77011494, 4.80445657,
      4.83529412, 4.86295646, 4.88773956, 4.90990991, 4.92970773,
      4.9473497 , 4.96303142, 4.97692967, 4.98920429, 5.        ]),
array([-1.01827783e-13, 9.54499592e-02, 1.79985757e-01, 2.53916576e-01,
       3.17551600e-01, 3.71200012e-01, 4.15170994e-01, 4.49773730e-01,
       4.75317402e-01, 4.92111195e-01, 5.00464290e-01, 5.00685871e-01,
       4.93085121e-01, 4.77971223e-01, 4.556533360e-01, 4.26440716e-01,
       3.90642472e-01, 3.48567813e-01, 3.00525920e-01, 2.46825978e-01,
       1.87777170e-01, 1.23688677e-01, 5.48696845e-02, -1.83706260e-02,
      -9.57230709e-02, -1.76878467e-01, -2.61527632e-01, -3.49361382e-01,
      -4.40070535e-01, -5.33345907e-01, -6.28878315e-01, -7.26358576e-01,
      -8.25477507e-01, -9.25925926e-01, -1.02739465e+00, -1.12957449e+00,
      -1.23215627e+00, -1.33483081e+00, -1.43728892e+00, -1.53922142e+00,
      -1.64031912e+00, -1.74027284e+00, -1.83877341e+00, -1.93551163e+00,
      -2.03017833e+00, -2.12246431e+00, -2.21206041e+00, -2.29865742e+00,
      -2.38194618e+00, -2.46161750e+00, -2.53736219e+00, -2.60887108e+00,
      -2.67583497e+00, -2.73794470e+00, -2.79489106e+00, -2.84636488e+00,
      -2.89205698e+00, -2.93165818e+00, -2.96485929e+00, -2.99135112e+00,
      -3.01082450e+00, -3.02297024e+00, -3.02747916e+00, -3.02404207e+00,
      -3.01234980e+00, -2.99209316e+00, -2.96296296e+00, -2.92465003e+00,
      -2.87684518e+00, -2.81923922e+00, -2.75152298e+00, -2.67338728e+00,
      -2.58452292e+00, -2.48462072e+00, -2.37337151e+00, -2.25046609e+00,
      -2.11559530e+00, -1.96844993e+00, -1.80872082e+00, -1.63609877e+00,
      -1.45027461e+00, -1.25093914e+00, -1.03778320e+00, -8.10497589e-01,
      -5.68773131e-01, -3.12300641e-01, -4.07709376e-02, 2.46125163e-01,
      5.48696845e-01, 8.67253290e-01, 1.20210368e+00, 1.55355720e+00,
```

```
1.92192304e+00, 2.30751037e+00, 2.71062837e+00, 3.13158624e+00,
3.57069316e+00, 4.02825830e+00, 4.50459085e+00, 5.00000000e+00]))
```

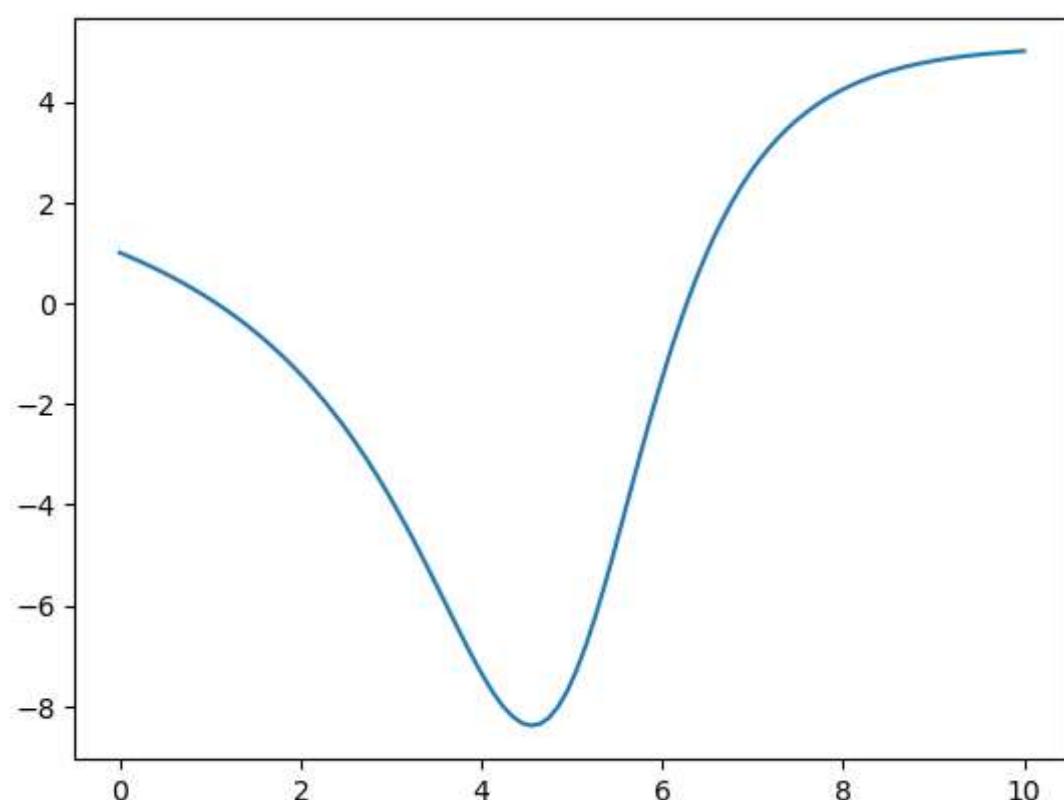
In [59]: `plt.plot(t,X1)`

Out[59]: [`<matplotlib.lines.Line2D at 0x205aa66e6d0>`]



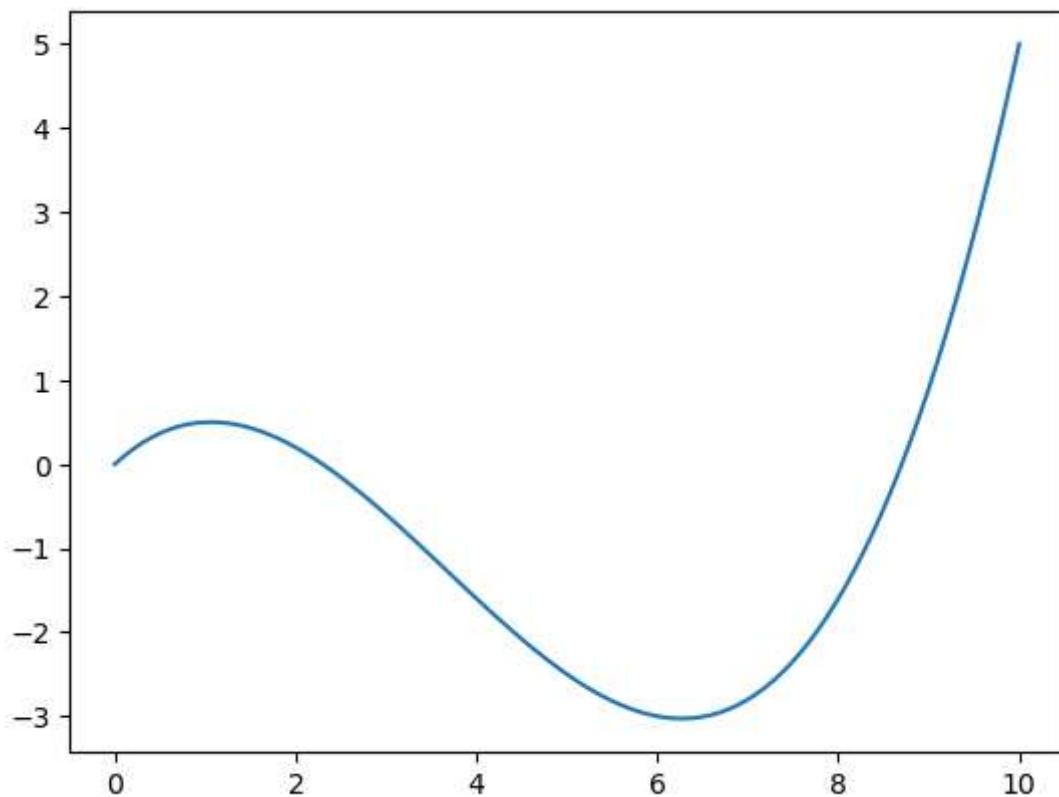
In [60]: `plt.plot(t,X2)`

Out[60]: [`<matplotlib.lines.Line2D at 0x205aa6caa30>`]



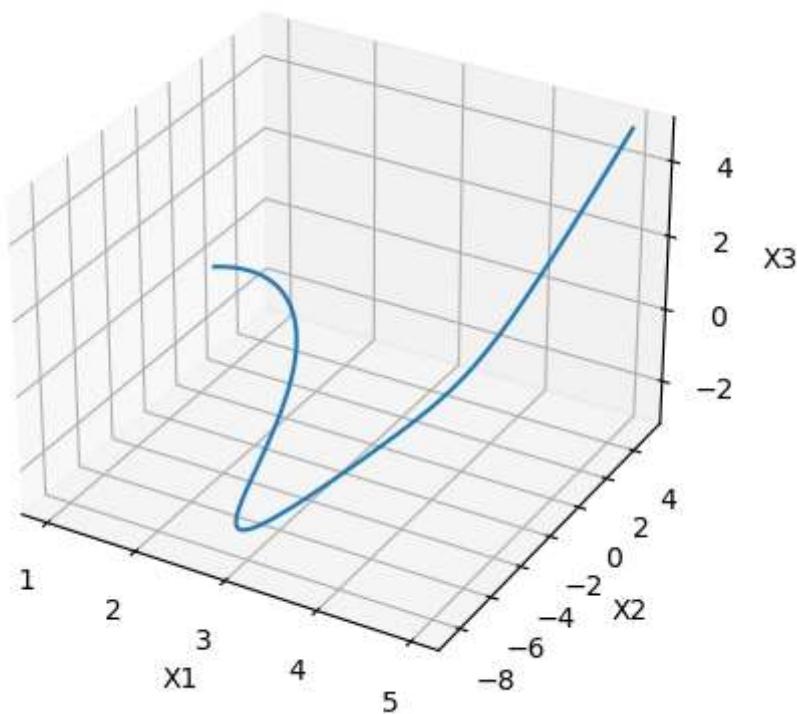
```
In [61]: plt.plot(t, X3)
```

```
Out[61]: [<matplotlib.lines.Line2D at 0x205aa73b190>]
```



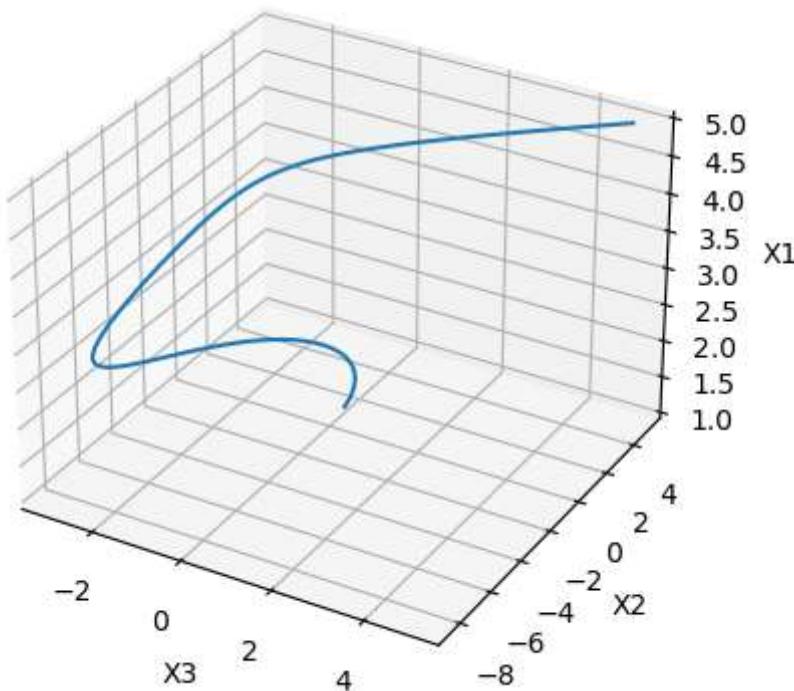
```
In [62]: ax = plt.figure().add_subplot(projection='3d')
ax.plot(X1, X2, X3)
ax.set_xlabel('X1')
ax.set_ylabel('X2')
ax.set_zlabel('X3')
```

```
Out[62]: Text(0.5, 0, 'X3')
```



```
In [63]: ax = plt.figure().add_subplot(projection='3d')
ax.plot(X3, X2, X1)
ax.set_xlabel('X3')
ax.set_ylabel('X2')
ax.set_zlabel('X1')
```

```
Out[63]: Text(0.5, 0, 'X1')
```



```
In [ ]:
```