Lyapunov

Given.

$$\dot{x}_1 = -x_1 + x_2^3 \quad \text{—— (I)}$$

$$\dot{x}_2 = -x_2 + u \quad \text{—— (II)}$$

$$V = \frac{1}{2}x_1^2 + \frac{1}{4}x_2^4 \quad \text{—— (III)}$$

$$\dot{V} = \frac{1}{\cancel{2}} \times \cancel{2} x_1 \dot{x}_1 + \frac{1}{\cancel{4}} \cancel{4} x_2^3 \cdot \dot{x}_2$$

$$\Rightarrow \dot{V} = x_1 \dot{x}_1 + x_2^3 \dot{x}_2$$

$$\Rightarrow \dot{V} = x_1(-x_1 + x_2^3) + x_2^3(-x_2 + u)$$

$$\Rightarrow \dot{V} = -x_1^2 + x_1 x_2^3 - x_2^4 + x_2^3 u \quad \text{—— (IV)}$$

We want to show that the system is stable —

$\boxed{\text{Condition - 1}}$

$$V(x_1 = 0, x_2 = 0) = 0 \quad \text{for } x_1 = 0,\ x_2 = 0$$

$$V = \frac{1}{2} \times 0^2 + \frac{1}{4} \times 0^4 = 0$$

$\boxed{\text{Condition 2}}$

$V(x_1 \neq 0; x_2 \neq 0) > 0$

$V = \frac{1}{2}x_1^2 + \frac{1}{4}x_2^4$

Here,

$V > 0$ for all cases of x_1 and x_2.

So condition 2 is satisfied.

$\boxed{\text{Condition 3}}$

$\dot{V}(x_1 \neq 0; x_2 \neq 0) < 0$ for $x_1 \neq 0; x_2 \neq 0$

$\dot{V} = -x^2 + x_1 x_2^3 - x_2^4 + x_2^3 u$

putting $u = -x_1$,

$\dot{V} = -x^2 + \cancel{x_1 x_2^3} - x_2^4 - \cancel{x_2^3 x_1}$

$\Rightarrow V = -x^2 - x_2^4 < 0$

Condition 3. is also met. The system is stable when $u = -x_1$.

Camera Frame coordinates $\longrightarrow$ Image frame coordinates

P_c $\qquad\qquad$ P

Given,

$P_c = (X_c, Y_c, Z_c)$

f = focal length

Image frame coordinates —

$$x = f \cdot \frac{X_c}{Z_c} \qquad y = f \cdot \frac{Y_c}{Z_c}$$

New reference frame given — $(\tilde{x}_0, \tilde{y}_0)$

New image frame coordinates —

$$\tilde{x} = f \cdot \frac{X_c}{Z_c} + \tilde{x}_0 \quad , \quad \tilde{y} = f \cdot \frac{Y_c}{Z_c} + \tilde{y}_0$$

Camera Frame coordinates $\longrightarrow$ pixel coordinates

P_c $\qquad\qquad$ (u, v)

$$u = \alpha \frac{X_c}{Z_c} + u_0$$

$$v = \beta \frac{Y_c}{Z_c} + v_0$$

[Everything will be given]

Least Square Regression

Given,

$x \longrightarrow \boxed{A} \xrightarrow{y}$

$A_{m \times n}$, $m > n$

$Y_{m \times 1} \Rightarrow$ observations

$X_{n \times 1} \rightarrow$ inputs

$e \rightarrow$ error $(m \times 1)$

↳ unknown

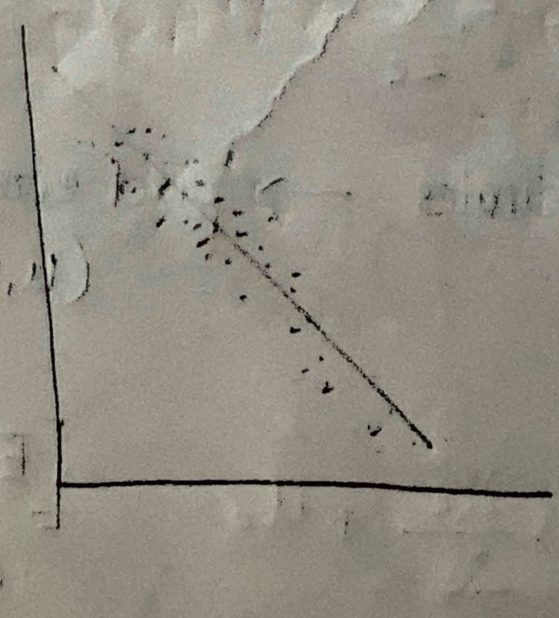
$y = Ax$

$\Rightarrow x = A^{+} y$

$A^{+} \longrightarrow (A^T A)^{-1} A^T$

The least squares problem is to minimize

$\|e\|^2$



$e = y - Ax$

First prove that $||e||^2 = e^T e$

$$e = \begin{bmatrix} e_1 \\ e_2 \\ \vdots \\ \vdots \\ e_m \end{bmatrix}_{m\times 1} \qquad e^T = \begin{bmatrix} e_1 & e_2 & e_3 & \cdots & e_m \end{bmatrix}_{1\times m}$$

$$e^T e = \begin{bmatrix} e_1 & e_2 & e_3 & \cdots & e_m \end{bmatrix}_{1\times m} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ \vdots \\ e_m \end{bmatrix}_{m\times 1}$$

$$= e_1^2 + e_2^2 + e_3^2 + \cdots + e_m^2$$

$$= ||e||^2$$

—x—

Minimizing

$$||e||^2 = e^T e = (y - Ax)^T (y - Ax)$$

$$= (y^T - x^T A^T)(y - Ax)$$

$$= y^T - y^T Ax - x^T A^T y + x^T A^T Ax$$

[Maintain order when multiplying

$y^T A x$

$\underset{1\times m}{y^T} \; \underset{m\times n}{A} \; \underset{n\times 1}{x}$

$= 1\times 1$

equal

$= y^T y - \boxed{2x^T A^T y} + x^T A^T A x$

$x^T A^T y$

$\underset{1\times n}{x^T} \; \underset{n\times m}{A^T} \; \underset{m\times 1}{y}$

$= 1$

Taking gradient —

$$\nabla_x \|e\|^2 = 0 - 2A^T y + 2A^T A x$$

$$2A^T y + 2A^T A x = 0$$

$$\Rightarrow 2A^T A x = 2A^T y$$

$$\Rightarrow x_{LS} = (A^T A)^{-1} A^T y.$$

We have sensor data, we want to fit a model to it.

$$y = Ax$$

y is sensor data

$y \in R^{m \times 1}$ [y is column matrix with m elements]

x is unknown parameters

$x \in R^{n \times 1}$

A is the model

$A \in R^{m \times n}$

Assumption: A is full rank

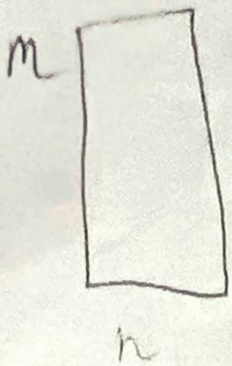
Solution: $x = A^{+} y$

A^{+} is called the pseudo inverse.

Three cases:

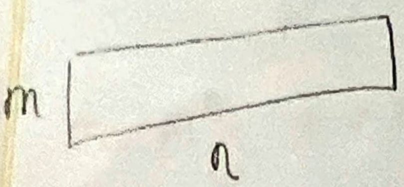
① $m = n \Rightarrow A^{+} = A^{-1}$

② $m > n \Rightarrow A^+ = (A^TA)^{-1}_{n\times n} A^T$



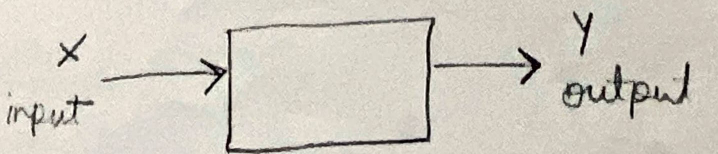
This is called the least squares solution.

③ $m < n \Rightarrow A^+ = A^+ (AA^T)^{-1}_{m\times m}$



This is called the minimum norm solution.

Example



Suppose someone gives you the input-output data below. Your job is to fit a model to it.

index i	Input x	Output y
1	2	3
2	1	2
3	3	4
4	7	8
5	5	3

Scatter plot → You do a scatter plot & decide to fit a line. Unknowns - (m, b)

$$y = mx + b$$

$$y = \begin{bmatrix} 3 \\ 2 \\ 4 \\ 8 \\ 3 \end{bmatrix} \quad x = \begin{bmatrix} 2 \\ 1 \\ 3 \\ 7 \\ 5 \end{bmatrix}$$

$$\begin{bmatrix} 3 \\ 2 \\ 4 \\ 8 \\ 3 \end{bmatrix} = m \begin{bmatrix} 2 \\ 1 \\ 3 \\ 7 \\ 5 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} b$$

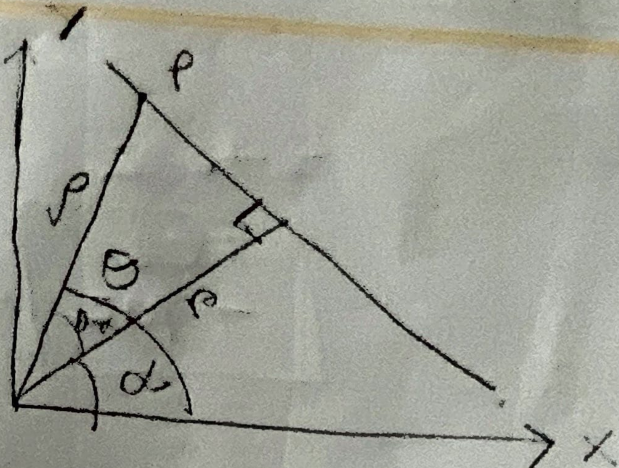
$$\Rightarrow \underbrace{\begin{bmatrix} 3 \\ 2 \\ 4 \\ 8 \\ 3 \end{bmatrix}}_{y} = \underbrace{\begin{bmatrix} 2 & 1 \\ 1 & 1 \\ 3 & 1 \\ 7 & 1 \\ 5 & 1 \end{bmatrix}}_{A} \begin{bmatrix} m \\ b \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} m \\ b \end{bmatrix} = (A^T A)^{-1} A^T y$$

$$y = a_1 + a_2 x + a_3 x^2$$

Unknowns : a_1, a_2, a_3

Data $\rightarrow$ Segmentation $\rightarrow$ Find best fit



$\cos(\theta - \alpha) = \frac{r}{\rho}$

$\Rightarrow \boxed{\rho \cos(\theta - \alpha) = r}$

→ eqⁿ of line in polar coordinates.

→ (r, α) are the parameter in polar coordinates

Split and merge segmentation.