HOMEWORK - 4

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Course : CSE495A

Section : 1

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Ans to the Q.NO - 1

Given,

$$G(i,j) = \sum_{u=0}^{k-1} \sum_{v=0}^{l-1} F(u,v) . \bar{I}(i+u, j+v)$$

Image, $I = \begin{bmatrix} 7 & 4 & 1 \\ 8 & 5 & 2 \\ 9 & 6 & 3 \end{bmatrix}$ and Image I after Padding,

$$\bar{I} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 7 & 4 & 1 & 0 \\ 0 & 8 & 5 & 2 & 0 \\ 0 & 9 & 6 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(a)

$$F = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$G = F \times I$ and $G \in \mathbb{R}^{3\times3}$

According to the formula, F start's from (0,0).

$G(1,1) = F(0,0).\bar{I}(1,1) + F(0,1).\bar{I}(1,2) + F(0,2).$
$\bar{I}(1,3) + F(1,0).\bar{I}(2,1) + F(1,1).\bar{I}(2,2) +$
$F(1,2).\bar{I}(2,3) + F(2,0).\bar{I}(3,1) + F(2,1).$
$\bar{I}(3,2) + F(2,2).\bar{I}(3,3).$

$= 0+0+0+0+7+0+0+0+0 = 7$

As other values of F are 0 without F(1,1) (according to the formula). Therefore, we can just calculate F(1,1).

$G(1,2) = F(1,1) . \bar{I}(2,3) = 1 . 4 = 4$

$G(1,3) = F(1,1) . \bar{I}(2,4) = 1 . 1 = 1$

$G(2,1) = F(1,1) . \bar{I}(3,2) = 1 . 8 = 8$

$G(2,2) = F(1,1) . \bar{I}(3,3) = 1 . 5 = 5$

$G(2,3) = F(1,1) . \bar{I}(3,4) = 1 . 2 = 2$

$G(3,1) = F(1,1) . \bar{I}(4,2) = 1 . 9 = 9$

$G(3,2) = F(1,1) . \bar{I}(4,3) = 1 . 6 = 6$

$G(3,3) = F(1,1) . \bar{I}(4,4) = 1 . 3 = 3$.

$\therefore$ Output Image, $G = \begin{bmatrix} 7 & 4 & 1 \\ 8 & 5 & 2 \\ 9 & 6 & 3 \end{bmatrix}$

(Ans).

(b)

$$F = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

As other values of F are 0 without F(0,0) (according to the formula). Therefore, we can just calculate F(0,0).

$G(1,1) = F(0,0).\,I(1,1) = 1.0 = 0$

$G(1,2) = F(0,0).\,I(1,2) = 1.0 = 0$

$G(1,3) = F(0,0).\,I(1,3) = 1.0 = 0$

$G(2,1) = F(0,0).\,I(2,1) = 1.0 = 0$

$G(2,2) = F(0,0).\,I(2,2) = 1.7 = 7$

$G(2,3) = F(0,0).\,I(2,3) = 1.4 = 4$

$G(3,1) = F(0,0).\,I(3,1) = 1.0 = 0$

$G(3,2) = F(0,0).\,I(3,2) = 1.8 = 8$

$G(3,3) = F(0,0).\,I(3,3) = 1.5 = 5$

$\therefore$ Output Image, $G = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 7 & 4 \\ 0 & 8 & 5 \end{bmatrix}$ (Ans)

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(c)

$$F = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ -1 & -1 & -1 \end{bmatrix}$$

$G(1,1) = F(0,0).\bar{I}(1,1) + F(0,1).\bar{I}(1,2) + F(0,2).$
$\bar{I}(1,3) + F(1,0).\bar{I}(2,1) + F(1,1).\bar{I}(2,2) +$
$F(1,2).\bar{I}(2,3) + F(2,0).\bar{I}(3,1) + F(2,1).\bar{I}(3,2)$
$+ F(2,2).\bar{I}(3,3)$

$= 0+0+0+0+0+0+0-8-5 = -13$

$G(1,2) = F(0,0).\bar{I}(1,2) + F(0,1).\bar{I}(1,3) + F(0,2).$
$\bar{I}(1,4) + F(1,0).\bar{I}(2,2) + F(1,1).\bar{I}(2,3) +$
$F(1,2).\bar{I}(2,4) + F(2,0).\bar{I}(3,2) + F(2,1).$
$\bar{I}(3,3) + F(2,2).\bar{I}(3,4)$

$= 0+0+0+0+0+0-8-5-2 = -15$

$G(1,3) = F(0,0).\bar{I}(1,3) + F(0,1).\bar{I}(1,4) + F(0,2).\bar{I}(1,5)$
$+ F(1,0).\bar{I}(2,3) + F(1,1).\bar{I}(2,4) + F(1,2).\bar{I}(2,5)$
$+ F(2,0).\bar{I}(3,3) + F(2,1).\bar{I}(3,4) + F(2,2).\bar{I}(3,5)$

$= 0+0+0+0+0+0-5-2+0 = -7$

$$G(2,1)=F(0,0).\bar{I}(2,1)+F(0,1).I(2,2)+F(0,2).\bar{I}(2,3)$$
$$+F(1,0).\bar{I}(3,1)+F(1,1).\bar{I}(3,2)+F(1,2).\bar{I}(3,3)$$
$$+F(2,0).\bar{I}(4,1)+F(2,1)\bar{I}(4,2)+F(2,2).\bar{I}(4,3)$$

$$=0+7+4+0+0+0+0-9-6=-4$$

$$G(2,2)=F(0,0).\bar{I}(2,2)+F(0,1).\bar{I}(2,3)+F(0,2)\bar{I}(2,4)$$
$$+F(1,0).\bar{I}(3,2)+F(1,1).\bar{I}(3,3)+F(1,2).\bar{I}(3,4)$$
$$+F(2,0).\bar{I}(4,2)+F(2,1)\bar{I}(4,3)+F(2,2).\bar{I}(4,4)$$

$$=7+4+1+0+0+0-9-6-3=-6$$

$$G(2,3)=F(0,0).\bar{I}(2,3)+F(0,1).\bar{I}(2,4)+F(0,2).\bar{I}(2,5)$$
$$+F(1,0).\bar{I}(3,3)+F(1,1).\bar{I}(3,4)+F(1,2).\bar{I}(3,5)$$
$$+F(2,0)\bar{I}(4,3)+F(2,1)\bar{I}(4,4)+F(2,2)\bar{I}(4,5)$$

$$=4+1+0+0+0+0-6-3+0=-4$$

$$G(3,1)=F(0,0).\bar{I}(3,1)+F(0,1).\bar{I}(3,2)+F(0,2)\bar{I}(3,3)$$
$$+F(1,0).\bar{I}(4,1)+F(1,1).\bar{I}(4,2)+F(1,2).\bar{I}(4,3)$$
$$+F(2,0)\bar{I}(5,1)+F(2,1)\bar{I}(5,2)+F(2,2)\bar{I}(5,3)$$

$$=0+8+5+0+0+0+0+0+0=13$$

$$G(3,2)=F(0,0).\bar{I}(3,2)+F(0,1).\bar{I}(3,3)+F(0,2).\bar{I}(3,4)$$
$$+F(1,0).\bar{I}(4,2)+F(1,1).\bar{I}(4,3)+F(1,2).\bar{I}(4,4)$$
$$+F(2,0)\bar{I}(5,2)+F(2,1)\bar{I}(5,3)+F(2,2)\bar{I}(5,4)$$
$$=8+5+2+0+0+0+0+0+0=15$$

$$G(3,3) = F(0,0).I(3,3) + F(0,1).I(3,4) + F(0,2).I(3,5) + F(1,0).I(4,3) + F(1,1).I(4,4) + F(1,2).I(4,5) + F(2,0).I(5,3) + F(2,1).I(5,4) + F(2,2).I(5,5)$$

$$= 5+2+0+0+0+0+0+0+0 = 7$$

$$\therefore \text{output Image, } G = \begin{bmatrix} -13 & -15 & -7 \\ -4 & -6 & -4 \\ 13 & 15 & 7 \end{bmatrix}$$

The filter is shown are mentioned in convolation. Image differentiation filtters taking the derivative of an image can be used to identify specific. The filter is called Sobel mask and it is used to calculate the derivative by convolution. That is the calculation of the derivative of the image along the y-axis. Certain features like the edges can be identified by using the derivative of the image. Here, F is intend to detect horizontal edges in image. It reacts strongly to horizontal edges when performing shifting operation from dark to light and light to dark. It emphasises features like horizontal boundaries and edges.

(d)

$$F' = \begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix}$$

As other values of F' are 0 without $F'(0,0)$, $F'(0,2)$, $F'(1,0)$, $F'(1,2)$, $F'(2,0)$, $F'(2,2)$ (according to the formula). Therefore, we can just calculate $F'(0,0)$, $F'(0,2)$, $F'(1,0)$, $F'(1,2)$, $F'(2,0)$, $F'(2,2)$.

$$G(1,1) = F'(0,0).\bar{I}(1,1) + F'(0,2).\bar{I}(1,3) + F'(1,0).\bar{I}(2,1) + F'(1,2).\bar{I}(2,3) + F'(2,0).\bar{I}(3,1) + F'(2,2)\bar{I}(3,3)$$

$$= 0+0+0+4+0+5 = 9.$$

$$G(1,2) = F'(0,0).\bar{I}(1,2) + F'(0,2).\bar{I}(1,4) + F'(1,0).\bar{I}(2,2) + F'(1,2)\bar{I}(2,4) + F'(2,0).\bar{I}(3,2) + F'(2,2).\bar{I}(3,4)$$

$$= 0+0-7+1-8+2 = -12.$$

$$G(1,3) = F'(0,0).\bar{I}(1,3) + F'(0,2).\bar{I}(1,5) + F'(1,0).\bar{I}(2,3) + F'(1,2).\bar{I}(2,5) + F'(2,0).\bar{I}(3,3) + F'(2,2).\bar{I}(3,5)$$

$$= 0+0-4+0-5+0 = -9$$

$$G(2,1) = F'(0,0).\bar{I}(2,1) + F'(0,2).\bar{I}(2,3) + F'(1,0).\bar{I}(3,1) + F'(1,2).\bar{I}(3,3) + F'(2,0).\bar{I}(4,1) + F'(2,2).\bar{I}(4,3)$$

$$= 0+4+0+5+0+6 = 15$$

$G(2,2) = F'(0,0).\bar{I}(2,2) + F'(0,2)\bar{I}(2,4) + F'(1,0).\bar{I}(3,2)$
$+ F'(1,2).\bar{I}(3,4) + F'(2,0).\bar{I}(4,2) + F'(2,2).\bar{I}(4,4)$
$= -7 + 1 - 8 + 2 - 9 + 3 = -18$

$G(2,3) = F'(0,0).\bar{I}(2,3) + F'(0,2).\bar{I}(2,5) + F'(1,0).\bar{I}(3,3)$
$+ F'(1,2).\bar{I}(3,5) + F'(2,0).\bar{I}(4,3) + F'(2,2).\bar{I}(4,5)$
$= -4 + 0 - 5 + 0 - 6 + 0 = -15$

$G(3,1) = F'(0,0).\bar{I}(3,1) + F'(0,2).\bar{I}(3,3) + F'(1,0).\bar{I}(4,1)$
$+ F'(1,2).\bar{I}(4,3) + F'(2,0).\bar{I}(5,1) + F'(2,2).\bar{I}(5,3)$

$= 0 + 5 + 0 + 6 + 0 + 0 = 11$

$G(3,2) = F'(0,0).\bar{I}(3,2) + F'(0,2)\bar{I}(3,4) + F'(1,0)\bar{I}(4,2)$
$+ F'(1,2)\bar{I}(4,4) + F'(2,0).\bar{I}(5,2) + F'(2,2)\bar{I}(5,4)$
$= -8 + 2 - 9 + 3 + 0 + 0 = -12$

$G(3,3) = F'(0,0)\bar{I}(3,3) + F'(0,2)\bar{I}(3,5) + F'(1,0).\bar{I}(4,3)$
$+ F'(1,2)\bar{I}(4,5) + F'(2,0)\bar{I}(5,3) + F'(2,2)\bar{I}(5,5)$
$= -5 + 0 - 6 + 0 + 0 + 0 = -11$

$\therefore$ output Image, $G = \begin{bmatrix} 9 & -12 & -9 \\ 15 & -18 & -15 \\ 11 & -12 & -11 \end{bmatrix}$

Here, the difference is in edge detection. It calculates the derivative of the Image along the x axis. F is intend to detect horizontal edges. It reacts to horizontal where a shift from dark to light. F' detects vertical changes where a transition from dark to light or vice versa. It looks the area in vertical direction. So, F and F' are designed for edge detection for defined operation. Those are detect edges in their direction and contributing to different aspect of image analysis.

(e)

$$F = \frac{1}{16}\begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$

$$\begin{aligned} G(1,1) &= F(0,0).I(1,1) + F(0,1).I(1,2) + F(0,2).I(1,3) \\ &\quad + F(1,0).I(2,1) + F(1,1).I(2,2) + F(1,2).I(2,3) \\ &\quad + F(2,0).I(3,1) + F(2,1)I(3,2) + F(2,2)I(3,3) \\ &= \frac{1}{16}(0+0+0+0+4.7+2.4+0+2.8+1.5) \\ &= \frac{57}{16} \end{aligned}$$

$G(1,2) = F(0,0).I(1,2) + F(0,1)I(1,3) + F(0,2).I(1,4) +$

$F(1,0)\bar{I}(2,2) + F(1,1)\bar{I}(2,3) + F(1,2)\bar{I}(2,4) +$
$F(2,0)\bar{I}(3,2) + F(2,1)\bar{I}(3,3) + F(2,2)\bar{I}(3,4)$

$= \frac{1}{16}(0+0+0+1.7+4.4+2.1+1.8+2.5+1.2)$

$= \frac{52}{16}$

$G(1,3) = F(0,0)\bar{I}(1,3) + F(0,1)\bar{I}(1,4) + F(0,2)\bar{I}(1,5) +$
$F(1,0)\bar{I}(2,3) + F(1,1).\bar{I}(2,4) + F(1,2).\bar{I}(2,5) +$
$F(2,0).\bar{I}(3,3) + F(2,1).\bar{I}(3,4) + F(2,2)\bar{I}(3,5)$

$= \frac{1}{16}(0+0+0+2.4+4.1+0+1.5+2.2+0)$

$= \frac{21}{16}$

$G(2,1) = F(0,0)\bar{I}(2,1) + F(0,1).\bar{I}(2,2) + F(0,2)\bar{I}(2,3)$
$+ F(1,0)\bar{I}(3,1) + F(1,1)\bar{I}(3,2) + F(1,2)\bar{I}(3,3) +$
$F(2,0)\bar{I}(4,1) + F(2,1)\bar{I}(4,2) + F(2,2)\bar{I}(4,3)$

$= \frac{1}{16}(0+2.7+1.4+0+4.8+2.5+0+1.9+1.6)$

$= \frac{84}{16}$

$G(2,2) = F(0,0)\bar{I}(2,2) + F(0,1)\bar{I}(2,3) + F(0,2)\bar{I}(2,4) +$
$F(1,0)\bar{I}(3,2) + F(1,1)\bar{I}(3,3) + F(1,2)\bar{I}(3,4) +$
$F(2,0)\bar{I}(4,2) + F(2,1)\bar{I}(4,3) + F(2,2)\bar{I}(4,4)$

$= \frac{1}{16}(1.7+2.4+1.1+2.8+4.5+2.2+1.9+2.6+1.3)$

$= 5$

$$G(2,3) = F(0,0)\bar{I}(2,3) + F(0,1)\bar{I}(2,4) + F(0,2)\bar{I}(2,5) + F(1,0)\bar{I}(3,3) + F(1,1)\bar{I}(3,4) + F(1,2)\bar{I}(3,5) + F(2,0)\bar{I}(4,3) + F(2,1)\bar{I}(4,4) + F(2,2)\bar{I}(4,5)$$

$$= \frac{1}{16}(1.4 + 2.1 + 0 + 2.5 + 4.2 + 0 + 1.6 + 2.3 + 0)$$

$$= \frac{36}{16}$$

$$G(3,1) = F(0,0)\bar{I}(3,1) + F(0,1)\bar{I}(3,2) + F(0,2)\bar{I}(3,3) + F(1,0)\bar{I}(4,1) + F(1,1)\bar{I}(4,2) + F(1,2)\bar{I}(4,3) + F(2,0)\bar{I}(5,1) + F(2,1)\bar{I}(5,2) + F(2,2)\bar{I}(5,3)$$

$$= \frac{1}{16}(0 + 2.8 + 1.5 + 0 + 4.9 + 2.6 + 0 + 0 + 0)$$

$$= \frac{69}{16}$$

$$G(3,2) = F(0,0)\bar{I}(3,2) + F(0,1)\bar{I}(3,3) + F(0,2)\bar{I}(3,4) + F(1,0)\bar{I}(4,2) + F(1,1)\bar{I}(4,3) + F(1,2)\bar{I}(4,4) + F(2,0)\bar{I}(5,2) + F(2,1)\bar{I}(5,3) + F(2,2)\bar{I}(5,4)$$

$$= \frac{1}{16}(1.8 + 2.5 + 1.2 + 2.9 + 4.6 + 2.3 + 0 + 0 + 0)$$

$$= \frac{68}{16}.$$

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$$G(3,3) = F(0,0)\bar{I}(3,3) + F(0,1)\bar{I}(3,4) + F(0,2)\bar{I}(3,5) + F(1,0)\bar{I}(4,3) + F(1,1)\bar{I}(4,4) + F(1,2)\bar{I}(4,5) + F(2,0)\bar{I}(5,3) + F(2,1)\bar{I}(5,4) + F(2,2)\bar{I}(5,5)$$

$$= \frac{1}{16}(1.5 + 2.2 + 0 + 2.6 + 4.3 + 0 + 0 + 0 + 0)$$

$$= \frac{33}{16}$$

$\therefore$ Output Image, $G = \begin{bmatrix} \frac{57}{16} & \frac{52}{16} & \frac{21}{16} \\ \frac{84}{16} & 5 & \frac{36}{16} \\ \frac{69}{16} & \frac{68}{16} & \frac{33}{16} \end{bmatrix}$

$$= \frac{1}{16}\begin{bmatrix} 57 & 52 & 21 \\ 84 & 80 & 36 \\ 69 & 68 & 33 \end{bmatrix}$$

Here, the filter is Gaussian smoothing filter. It is used to apply a smoothing effect to the image. It is working as smoothing and blurring tool. It is designed to smooth Image by averaging each pixel. It reduces high frequency noise and creates a less detailed appearance. The normalization factor $\frac{1}{16}$ ensure the smoothing is done in a way which preserves the overall brightness of input image. Moreover, It removes sharp filters and blurs the image.

(f)

$$F = \frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$\begin{aligned} G(1,1) &= F(0,0)\bar{I}(1,1) + F(0,1)\bar{I}(1,2) + F(0,2)\bar{I}(1,3) \\ &\quad + F(1,0).\bar{I}(2,1) + F(1,1)\bar{I}(2,2) + F(1,2)\bar{I}(2,3) + \\ &\quad F(2,0)\bar{I}(3,1) + F(2,1)\bar{I}(3,2) + F(2,2)\bar{I}(3,3) \\ &= \frac{1}{9}(0+0+0+0+7+4+0+8+5) \\ &= \frac{24}{9}. \end{aligned}$$

$$\begin{aligned} G(1,2) &= F(0,0)\bar{I}(1,2) + F(0,1)\bar{I}(1,3) + F(0,2)\bar{I}(1,4) + \\ &\quad F(1,0)\bar{I}(2,2) + F(1,1)\bar{I}(2,3) + F(1,2)\bar{I}(2,4) + \\ &\quad F(2,0)\bar{I}(3,2) + F(2,1)\bar{I}(3,3) + F(2,2)\bar{I}(3,4) \end{aligned}$$

$$= \frac{1}{9}(0+0+0+7+4+1+8+5+2)$$

$$= 3$$

$$G(1,3) = F(0,0)\bar{I}(1,3) + F(0,1)\bar{I}(1,4) + F(0,2)\bar{I}(1,5) +$$
$$F(1,0)\bar{I}(2,3) + F(1,1)\bar{I}(2,4) + F(1,2)\bar{I}(2,5) +$$
$$F(2,0)\bar{I}(3,3) + F(2,1)\bar{I}(3,4) + F(2,2)\bar{I}(3,5)$$

$$= \frac{1}{9}(0+0+0+4+1+0+5+2+0)$$

$$= \frac{12}{9}$$

$$G(2,1) = F(0,0)\bar{I}(2,1) + F(0,1)\bar{I}(2,2) + F(0,2)\bar{I}(2,3)$$
$$+ F(1,0)\bar{I}(3,1) + F(1,1)\bar{I}(3,2) + F(1,2)\bar{I}(3,3) +$$
$$F(2,0)\bar{I}(4,1) + F(2,1)\bar{I}(4,2) + F(2,2)\bar{I}(4,3)$$

$$= \frac{1}{9}(0+7+4+0+8+5+0+9+6)$$

$$= \frac{39}{9}$$

$$G(2,2) = F(0,0)\bar{I}(2,2) + F(0,1)\bar{I}(2,3) + F(0,2)\bar{I}(2,4) +$$
$$F(1,0)\bar{I}(3,2) + F(1,1)\bar{I}(3,3) + F(1,2)\bar{I}(3,4) +$$
$$F(2,0)\bar{I}(4,2) + F(2,1)\bar{I}(4,3) + F(2,2)\bar{I}(4,4)$$

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$$= \frac{1}{9}(7+4+1+8+5+2+9+6+3)$$

$$= 5$$

$$G(2,3) = F(0,0)\bar{I}(2,3) + F(0,1)\bar{I}(2,4) + F(0,2)\bar{I}(2,5) + F(1,0)\bar{I}(3,3) + F(1,1)\bar{I}(3,4) + F(1,2)\bar{I}(3,5) + F(2,0)\bar{I}(4,3) + F(2,1)\bar{I}(4,4) + F(2,2)\bar{I}(4,5)$$

$$= \frac{1}{9}(4+1+0+5+2+0+6+3+0)$$

$$= \frac{21}{9}$$

$$G(3,1) = F(0,0)\bar{I}(3,1) + F(0,1)\bar{I}(3,2) + F(0,2)\bar{I}(3,3) + F(1,0)\bar{I}(4,1) + F(1,1)\bar{I}(4,2) + F(1,2)\bar{I}(4,3) + F(2,0)\bar{I}(5,1) + F(2,1)\bar{I}(5,2) + F(2,2)\bar{I}(5,3)$$

$$= \frac{1}{9}(0+8+5+0+9+6+0+0+0)$$

$$= \frac{28}{9}$$

$$G(3,2) = F(0,0)\bar{I}(3,2) + F(0,1)\bar{I}(3,3) + F(0,2)\bar{I}(3,4) + F(1,0)\bar{I}(4,2) + F(1,1)\bar{I}(4,3) + I(1,2)\bar{I}(4,4)$$

$$+F(2,0)\bar{I}(5,2)+F(2,1)\bar{I}(5,3)+F(2,2)\bar{I}(5,4)$$

$$=\frac{1}{9}(8+5+2+9+6+3+0+0+0)$$

$$=\frac{33}{9}$$

$$G(3,3)=F(0,0)\bar{I}(3,3)+F(0,1)\bar{I}(3,4)+F(0,2)\bar{I}(3,5)$$
$$+F(1,0)\bar{I}(4,3)+F(1,1)\bar{I}(4,4)+F(1,2)\bar{I}(4,5)+$$
$$F(2,0)\bar{I}(5,3)+F(2,1)\bar{I}(5,4)+F(2,2)\bar{I}(5,5)$$

$$=\frac{1}{9}(5+2+0+6+3+0+0+0+0)$$

$$=\frac{16}{9}$$

$$\therefore \text{Output Image, } G=\begin{bmatrix} \frac{24}{9} & 3 & \frac{12}{9} \\ \frac{39}{9} & 5 & \frac{21}{9} \\ \frac{28}{9} & \frac{33}{9} & \frac{16}{9} \end{bmatrix}$$

$$=\frac{1}{9}\begin{bmatrix} 24 & 27 & 12 \\ 39 & 45 & 21 \\ 28 & 33 & 16 \end{bmatrix}$$

Here, the difference in functionality is that using the filter $F = \frac{1}{16}\begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}$ gives exactly the same weight to all pixels but Gaussian smoothing filter gives more weight to neighboring pixels. In part (f), F is average moving filter. Gaussian smoothing filter is used for smoothing filter used for smoothing or blurring Image. And Average moving filters are used for basic smoothing through pixels averaging. Gaussian smoothing emphasises the central pixel gradually diminishing weights towards the edges in a gaussian distribution, Moreover, it also provides better edges and a smoother image. But basic smoothing simplifies the smoothing and less focus on presenting details.

Ans to the QNO - 2

(a)

According to the formula the correlation,

$$G(i,j) = \sum_{u=0}^{k-1} \sum_{v=0}^{l-1} F(u,v) . \bar{I}(i+u, j+v)$$

where F is the filter, $\bar{I} \in \mathbb{R}^{(m+k-1) \times (n+l-1)}$ is the original image I, padded with zero along the edges.

vector representation of F, $f = \text{vector}(F)$

Let, $t(i,j)$ is the vector representation of $\bar{I}(i,j)$ of neighborhood patch of images.

$\therefore$ $t(i,j) = \text{vector}(\bar{I}(i-1 : i+2, j-1 : j+2))$

Applying vectorization which turns a matrix into a single column, we have to apply dot product. Dot product were need to use the neighborhood patch as the dot product of a row vector and a column vector can be expressed as a multiplication. The correlation operation involves corresponding in neighborhood and summing the result.

$G(i,j) = f^T t_{ij}$ reshaping f and neighborhood patch $\bar{I}(i,j)$. Therefore, correlation operation can perform as a vector dot products.

$$f = [F(0,0), \ldots, F(u,v)] \quad t_{ij} = \begin{bmatrix} \bar{I}(i,j) \\ \bar{I}(i,j+1) \\ \vdots \\ \bar{I}(i+u, j+v) \end{bmatrix}$$

Now,

$$f = \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix} \quad t_{ij} = \begin{bmatrix} t_{11} & t_{12} & t_{13} \\ t_{21} & t_{22} & t_{23} \\ t_{31} & t_{32} & t_{33} \end{bmatrix}$$

correlation calculation:

$$f_{11} t_{11} + f_{12} t_{12} + f_{13} t_{13} + f_{21} t_{21} + f_{22} t_{22} + f_{23} t_{23} + f_{31} t_{31} + f_{32} t_{32} + f_{33} t_{33} = G(i,j)$$

So,

R.H.S $\Rightarrow$ $f^T \begin{bmatrix} f_{11} & f_{21} & f_{31} \\ f_{12} & f_{22} & f_{32} \\ f_{13} & f_{23} & f_{33} \end{bmatrix}$

$\therefore$ dot product of $(f^T \cdot t_{ij}) = \begin{bmatrix} f_{11} & f_{21} & f_{31} \\ f_{12} & f_{22} & f_{32} \\ f_{13} & f_{23} & f_{33} \end{bmatrix} \begin{bmatrix} t_{11} & t_{12} & t_{13} \\ t_{21} & t_{22} & t_{23} \\ t_{31} & t_{32} & t_{33} \end{bmatrix}$

$= f_{11} t_{11} + f_{21} t_{21} + f_{31} t_{31} + f_{12} t_{12} + f_{22} t_{22} + f_{32} t_{32} + f_{13} t_{13} + f_{23} t_{23} + f_{33} t_{33}$.

$= f_{11} t_{11} + f_{12} t_{12} + f_{13} t_{13} + f_{21} t_{21} + f_{22} t_{22} + f_{23} t_{23} + f_{31} t_{31} + f_{32} t_{32} + f_{33} t_{33}$

$= G(i,j) =$ **L.H.S.**

$\therefore G(i,j) = f^T t_{ij}$

Therefore, the correlation calculation of $f^T t_{ij}$ = dot product of f^T and t_{ij}.

(showed).