# BJT AC Analysis

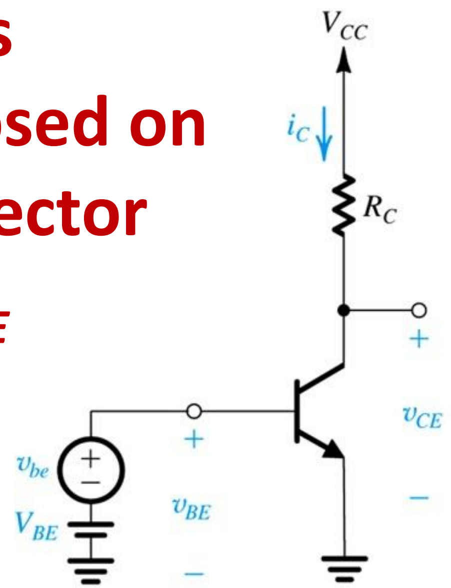
Topic 5 (Chapter 5)

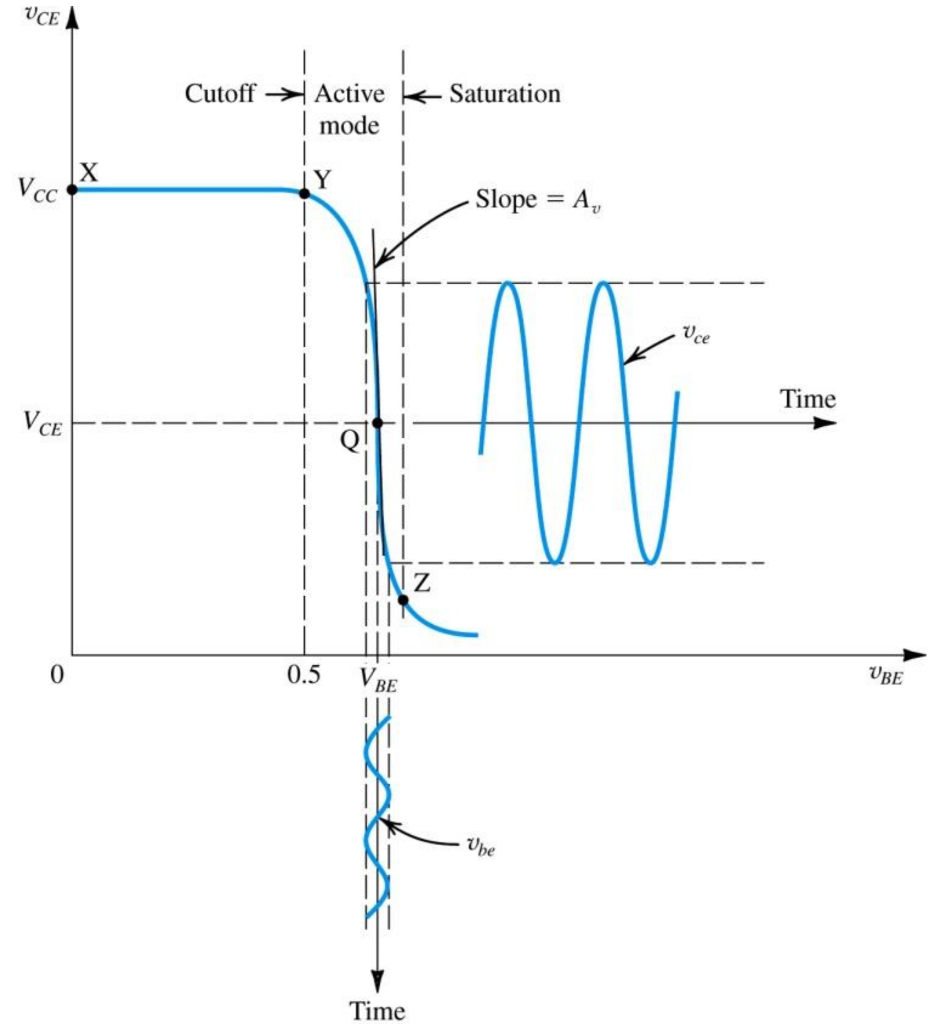
# AC Amplification

- After a transistor has been biased with the Q point near the middle of the load line, we can couple a small ac voltage into the base.
  - This will produce an ac collector voltage.
  - The ac collector voltage looks like the ac base voltage, except that it's a lot bigger.
    - In other words, the ac collector voltage is an *amplified* version of the ac base voltage.
- The invention of amplifying devices, first vacuum tubes and later transistors, was crucial to the evolution of electronics.
  - Without amplification, there would be no radio, no television, and no computers.

# BJT amplifier biased at a point Q

- A small signal voltage $v_{be}$ is applied
- The output signal $v_{ce}$ appears superimposed on the dc collector voltage $V_{CE}$



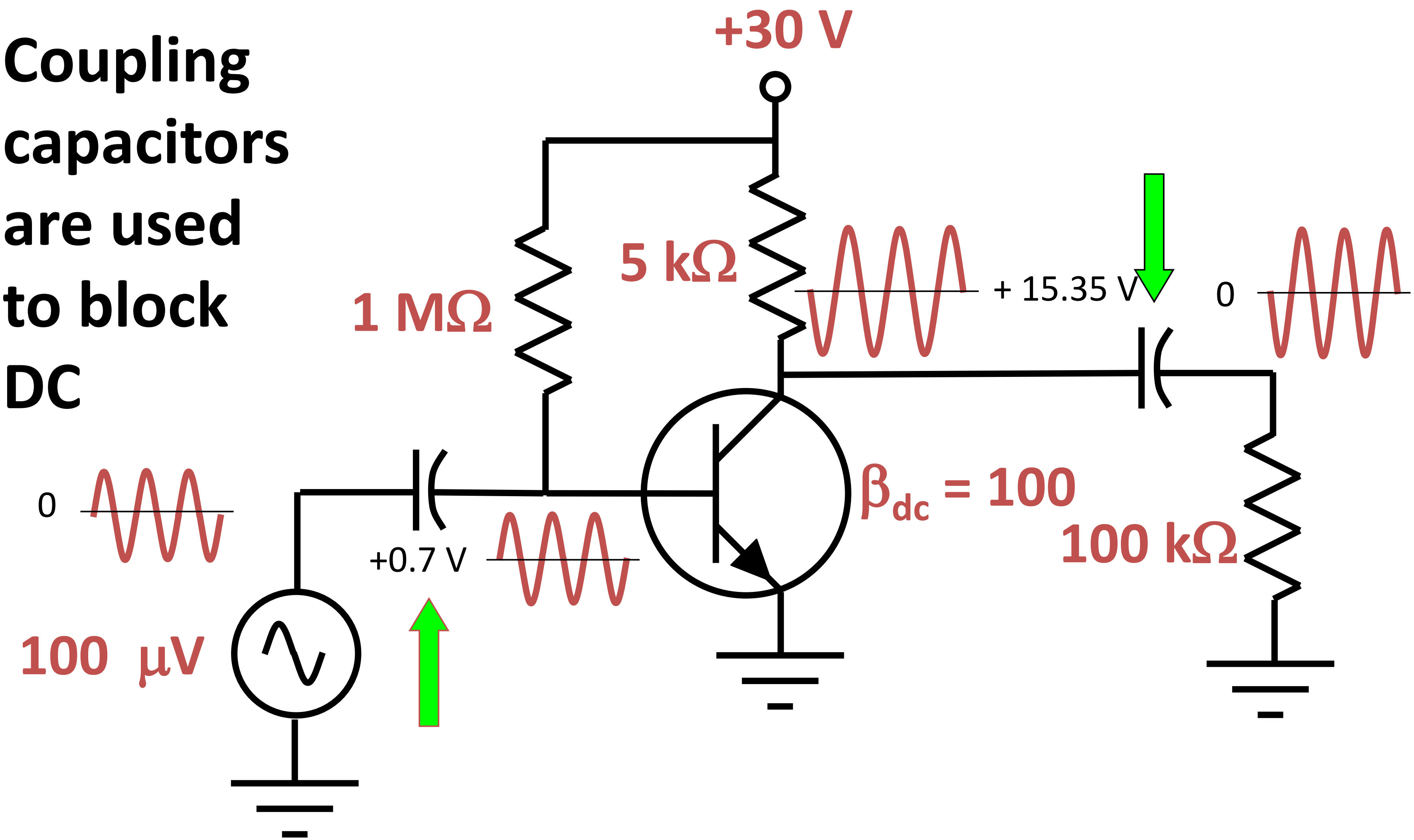

- The amplitude of $v_{ce}$ is larger than that of $v_{be}$ by the voltage gain $A_v$

# Base-biased amplifier

- **AC** input is applied into base
- Coupling capacitors are used to block DC
  - The **reactance** of a coupling capacitor is small for AC signal
- Amplified and inverted **output** at the collector
- **AC** output coupled to the load

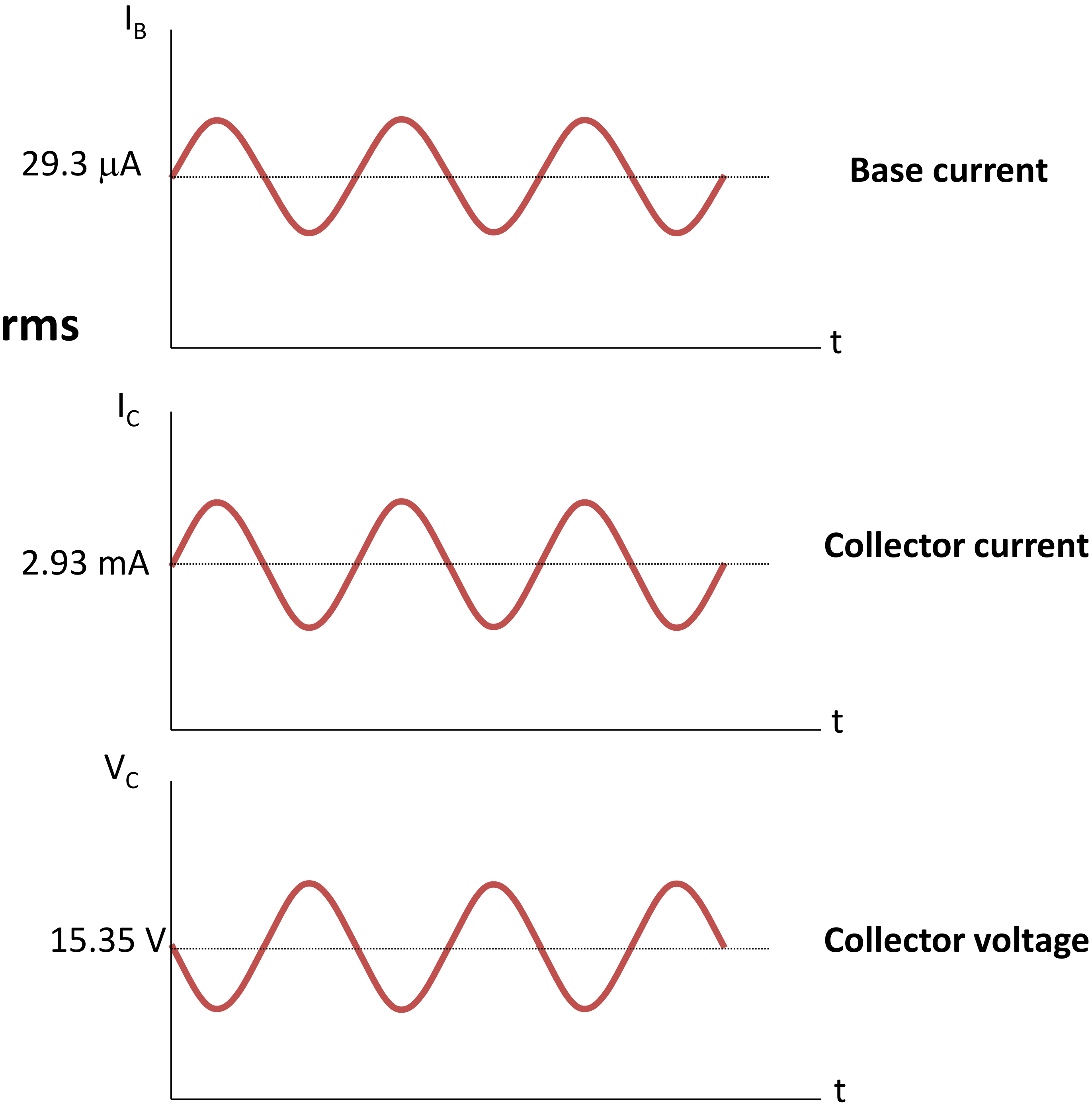
# A base-biased amplifier with capacitive coupling

**Coupling capacitors are used to block DC**

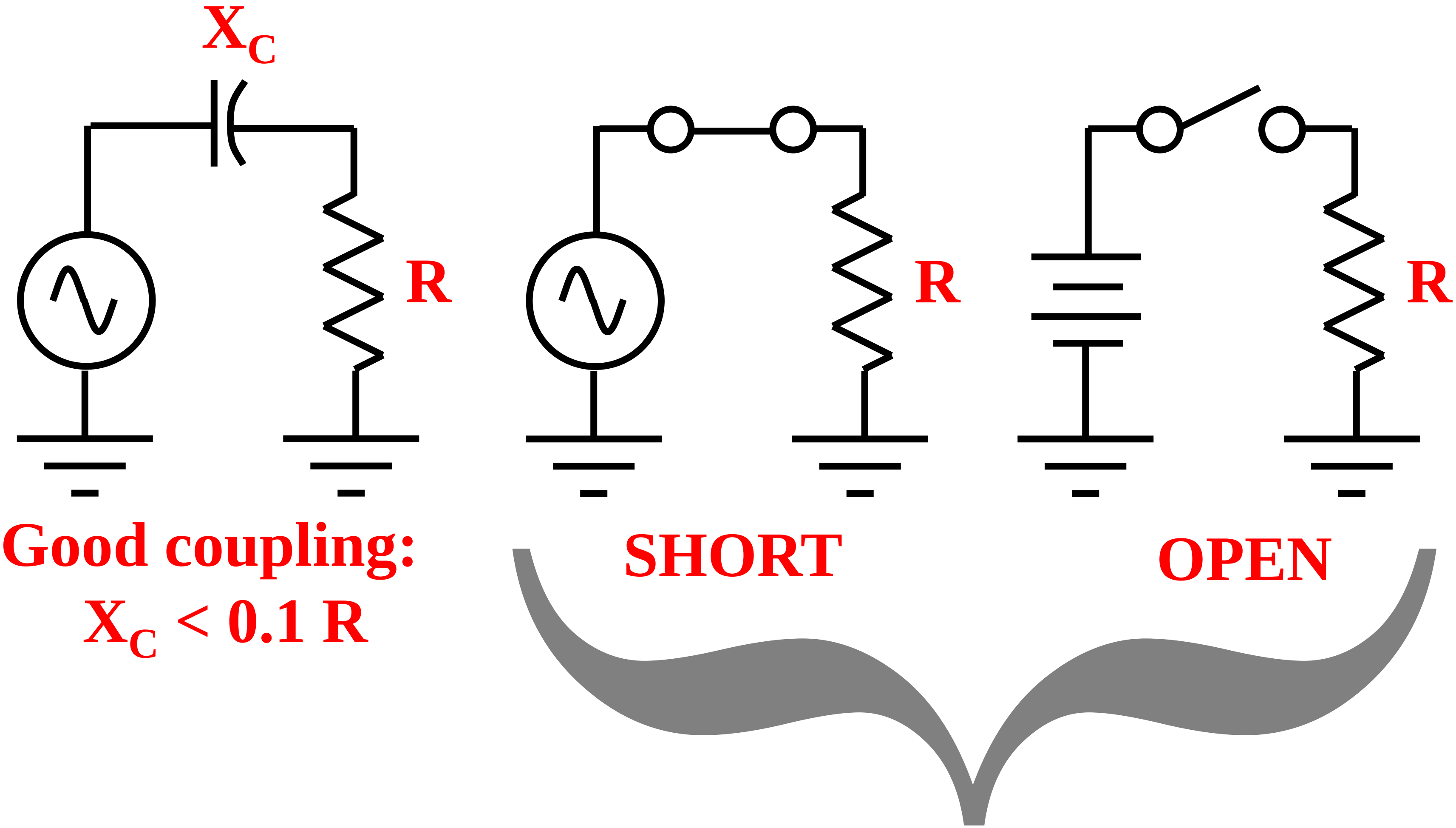



**DC analysis gives: $I_B = 29.3\ \mu A$, $I_C = 2.93\ mA$ and $V_C = 15.35\ V$**

# Current Waveforms



# The coupling capacitor



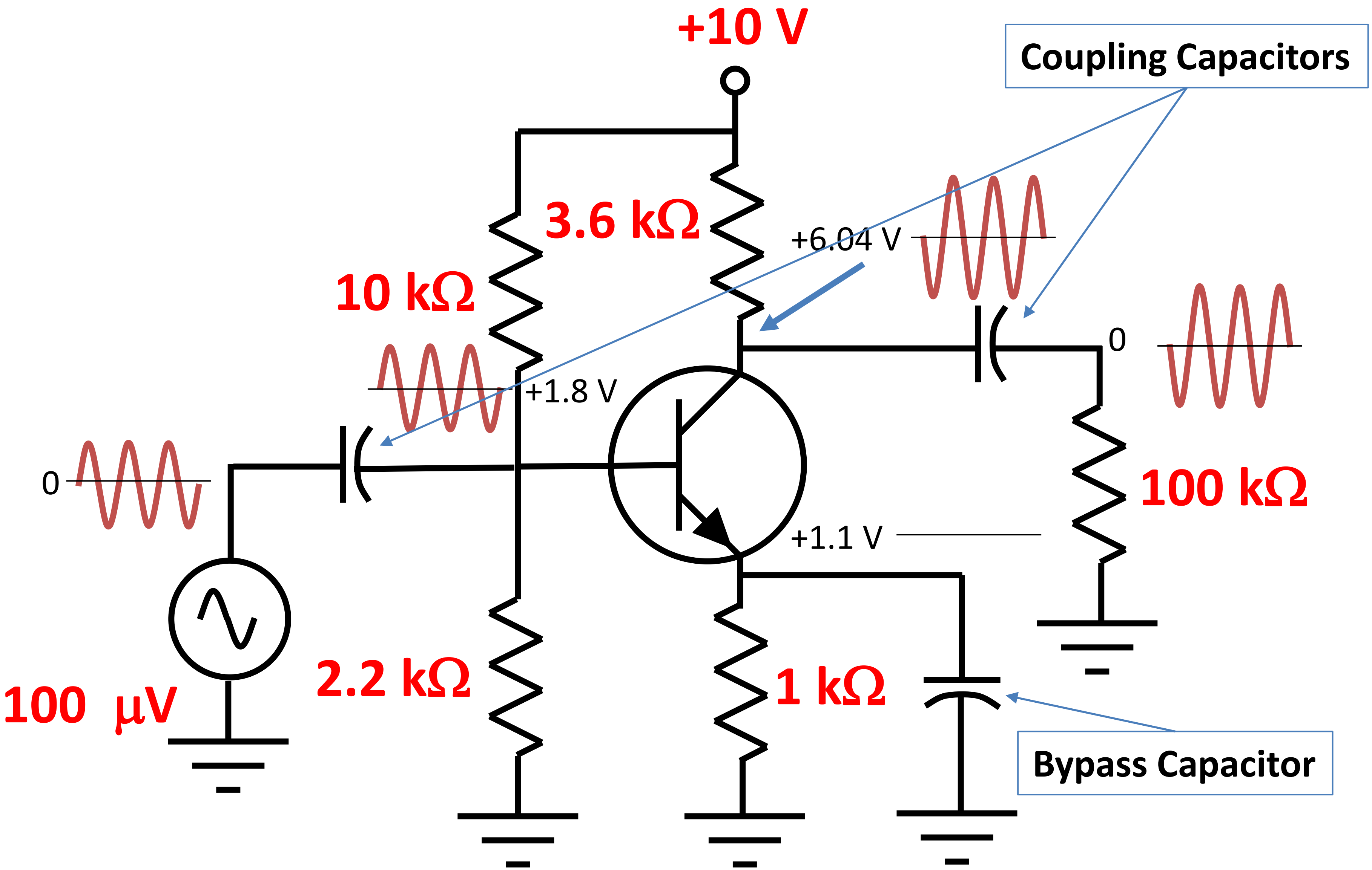

**Good coupling:**
$X_C < 0.1\ R$

1. For ac analysis, the capacitor is a short.
2. For dc analysis, the capacitor is open.

# VDB Amplifier

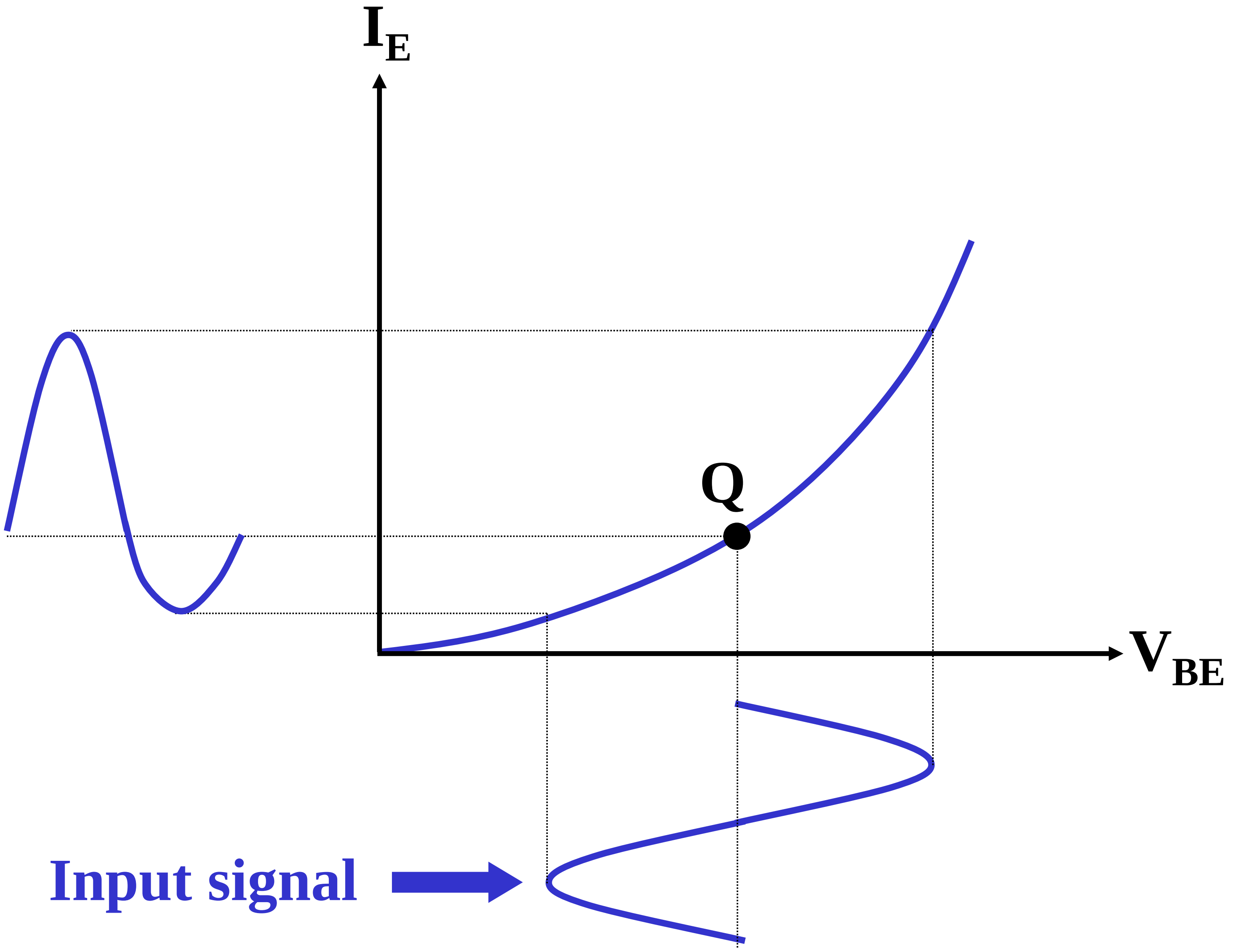
- Dc voltages and currents are calculated mentally by opening capacitors
- The ac signal is coupled via a coupling capacitor
- The bypass capacitor causes an ac signal to appear across the base-emitter junction and provides higher gain

# VDB Amplifier



# Distortion

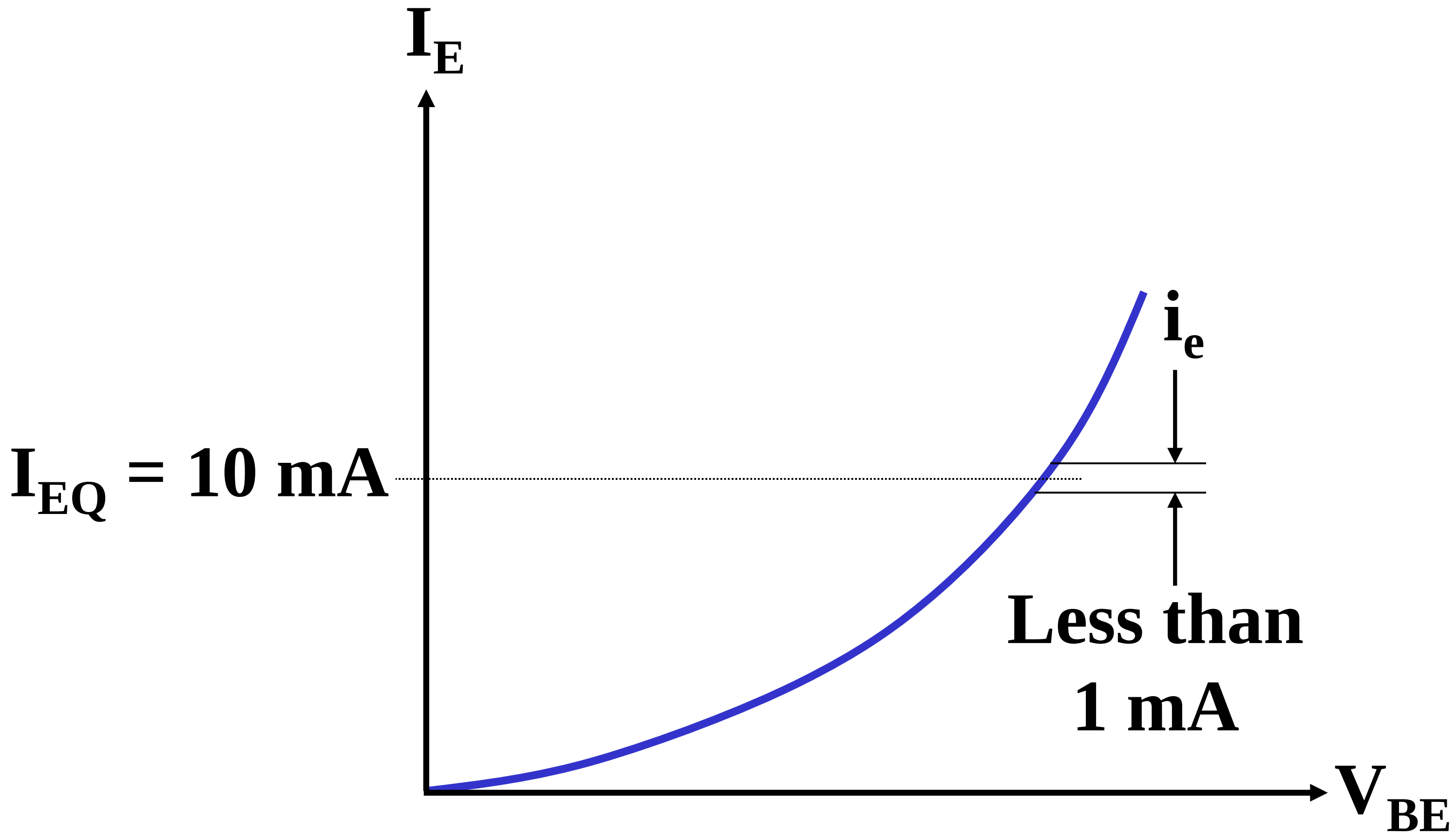
- The stretching and compressing of alternate half cycles
- Undesirable in high-fidelity amplifiers
- Can be minimized by keeping the ac input small




**<u>Large-signal</u> operation produces distortion**

# The 10 percent rule

- Total emitter current consists of dc and ac
- To minimize distortion, $i_e$ must be small compared to $I_{EQ}$
- The ac signal is small when the peak-to-peak ac emitter current is less than 10 percent of the dc emitter current




**Total emitter current:** $I_E = I_{EQ} + i_e$

**<u>Small-signal</u> operation:** $i_{e(PP)} < 0.1 I_{EQ}$

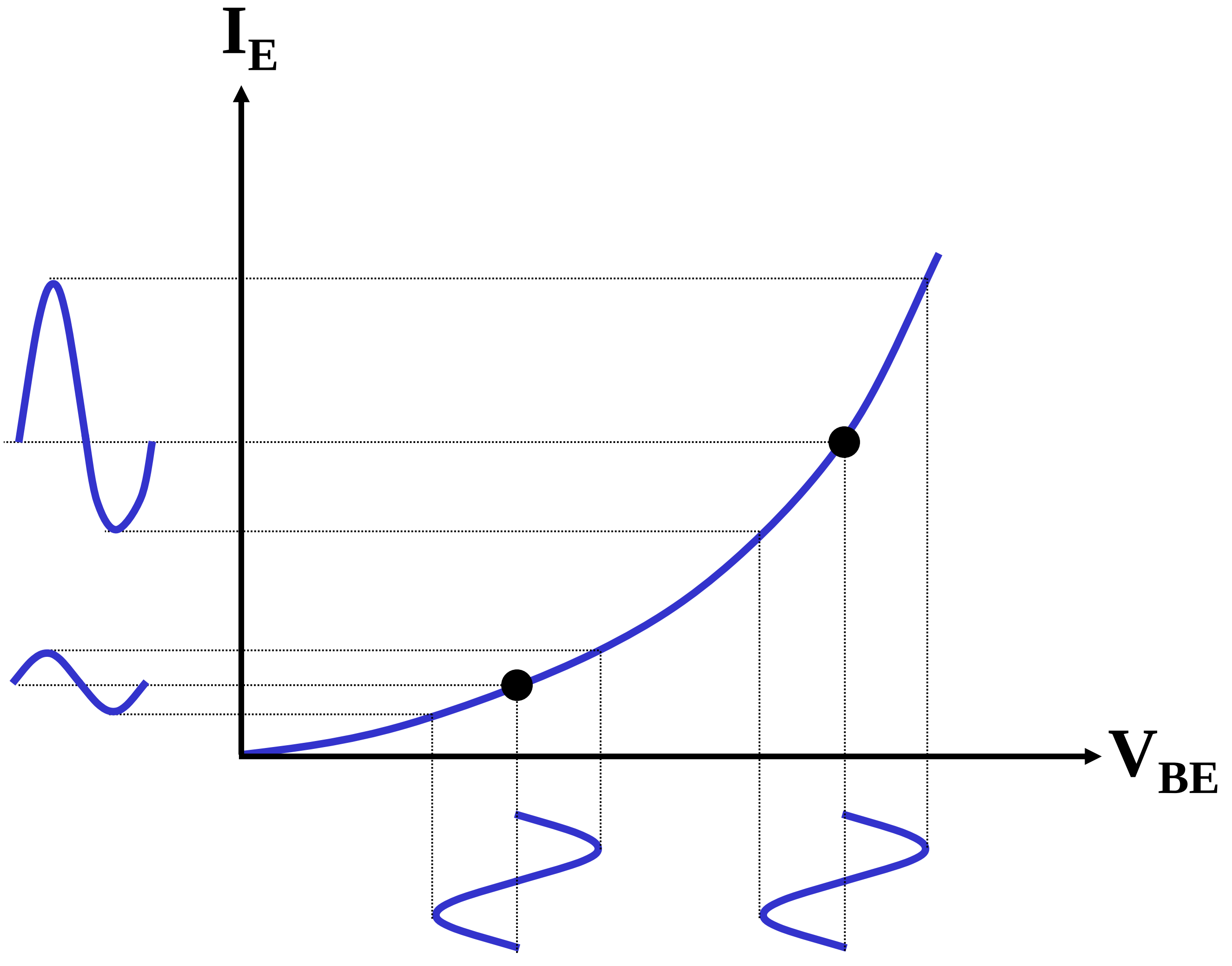
**The dc current gain is given as:**

$$\beta_{dc} = \frac{I_C}{I_B}$$

**The ac current gain is given as:**

$$\beta_{ac} = \frac{i_c}{i_b}$$

**Use CAPITAL letters for dc quantities and lowercase letters for ac.**




**The <u>size</u> of the ac emitter current depends on the Q point.**

**Total** emitter current: $I_E = I_{EQ} + i_e$

**Total** base-emitter voltage: $V_{BE} = V_{BEQ} + v_{be}$

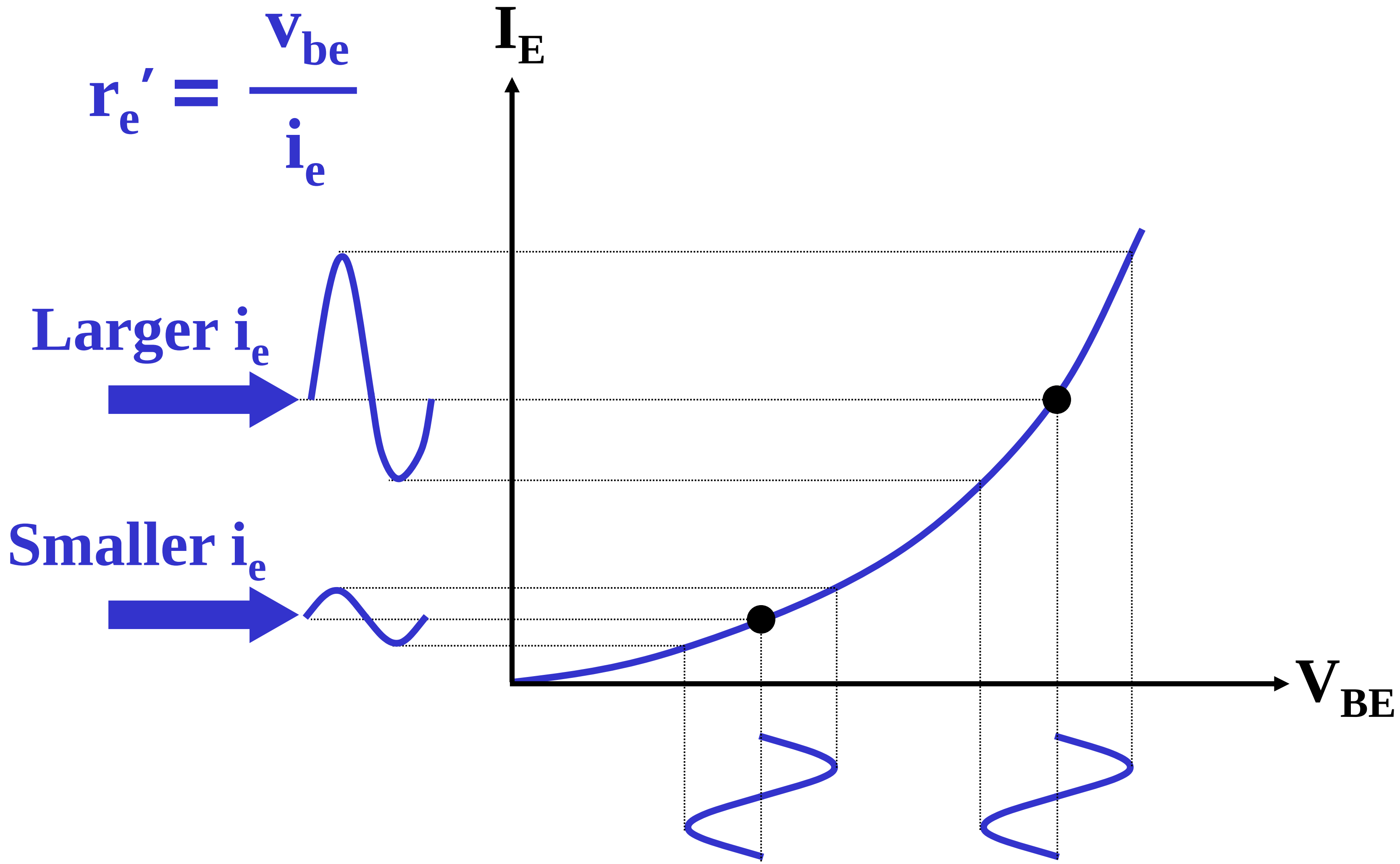
The ac resistance of the emitter diode is defined as:

$$r_e' = \frac{v_{be}}{i_e}$$

The ac resistance of the emitter diode decreases when the dc emitter current increases

# Ac resistance of the emitter diode

- Equals the ac base-emitter voltage divided by the ac emitter current
- The prime (') in $r_e$' indicates that the resistance is inside the transistor




**Note that $r_e'$ <u>varies</u> with the operating point.**

**This implies that $r_e'$ is a function of the <u>dc emitter</u> current.**

# Formula for ac emitter resistance

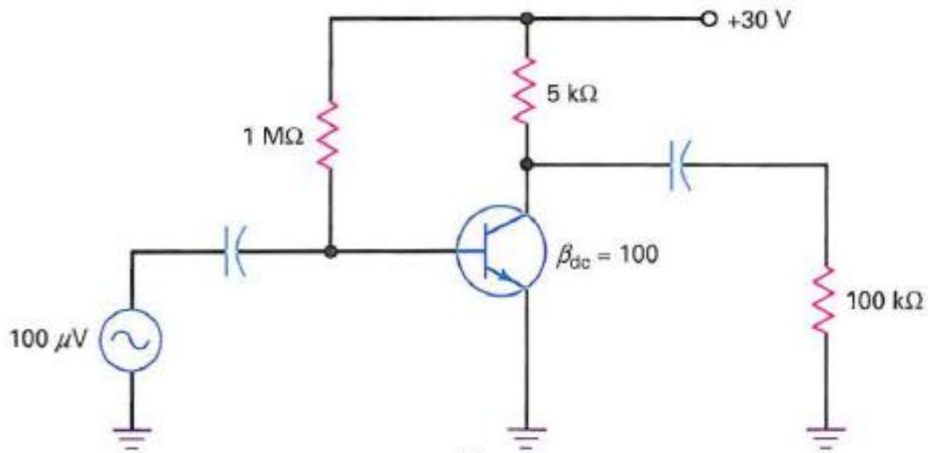
**Derived by using solid-state physics and calculus:**

$$r_e' = \frac{26\ \text{mV}}{I_E}$$

**Widely used in industry because of its simplicity and it applies to almost all commercial transistors**

# Example 9-4

What does $r'_e$ equal in the base-biased amplifier of Fig. 9-15*a*?

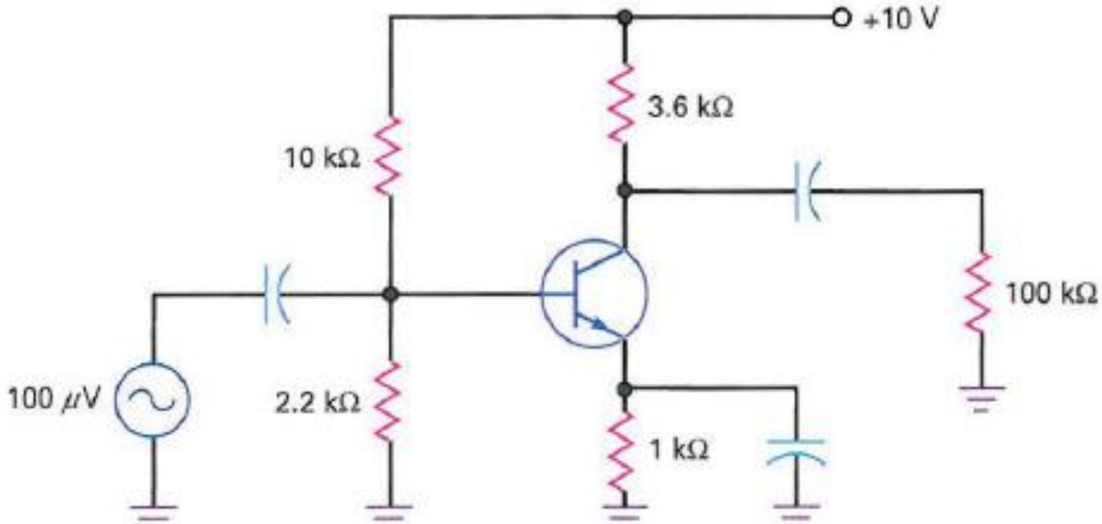



(a)

$$r'_e = \frac{25\ \text{mV}}{3\ \text{mA}} = 8.33\ \Omega$$

# Example 9-5

In Fig. 9-15*b*, what does $r'_e$ equal?

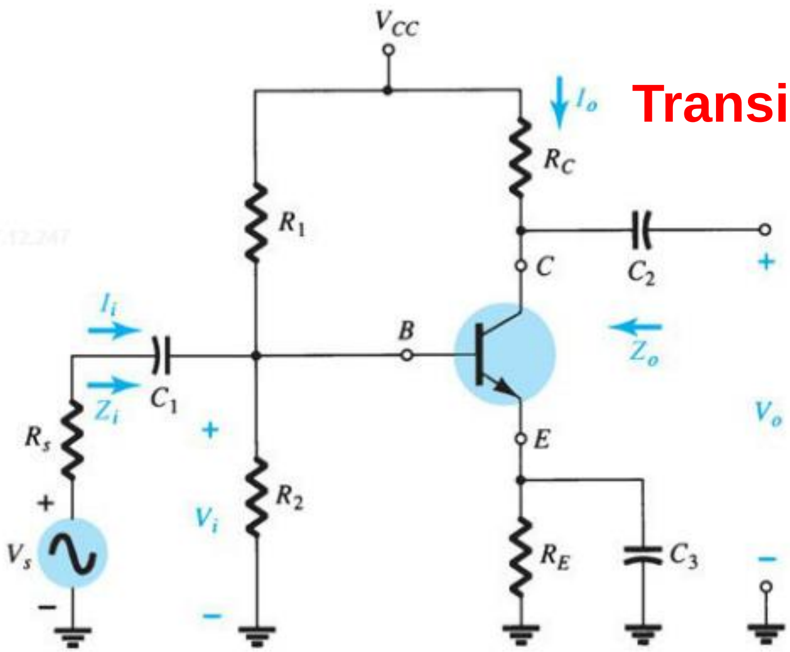



(*b*)

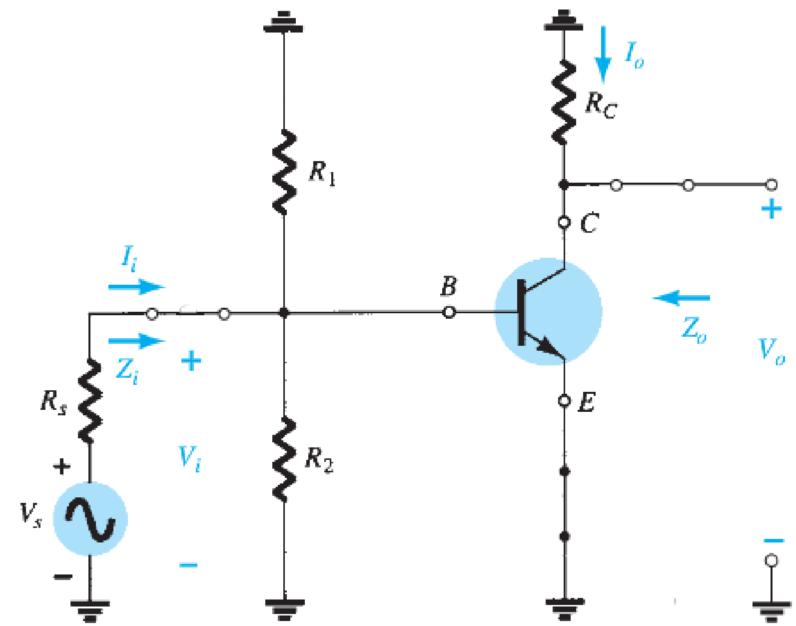
SOLUTION We analyzed this VDB amplifier earlier and calculated a dc emitter current of 1.1 mA. The ac resistance of the emitter diode is:

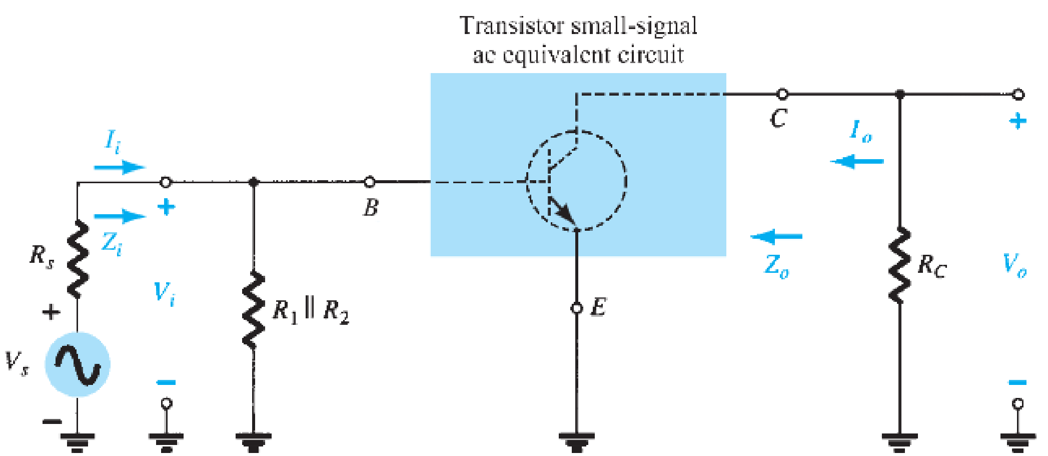
$$r'_e = \frac{25\ \text{mV}}{1.1\ \text{mA}} = 22.7\ \Omega$$

# BJT TRANSISTOR MODELING




Transistor Circuit




Redrawn for small-signal ac analysis

**AC Equivalent Circuit:**

After the removal of the dc supply and insertion of the short-circuit equivalent for the capacitors.

# Transistor models

- Ac equivalent circuit for a transistor
- Simulates how a transistor behaves when an ac signal is present
- There are two models commonly used in small signal AC analysis of a transistor:
  - $r_e$ model
    - Ebers-Moll (T model) and $\pi$ type models are widely used
  - Hybrid equivalent model (*h* parameter model)

# The $r_e$ Transistor Model

## (The Input Equivalent Circuit)

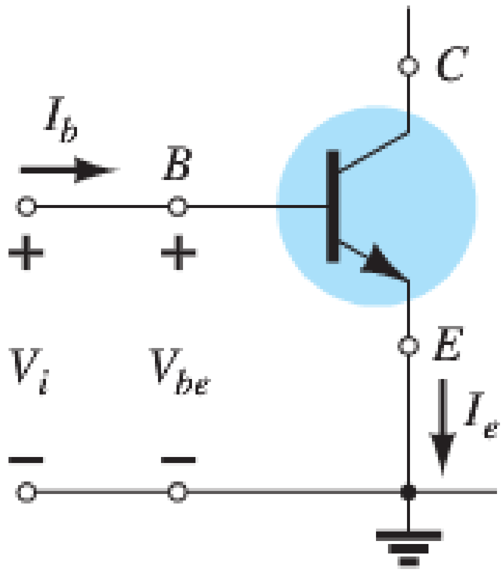



**FIG. 5.8**

*Finding the input equivalent circuit for a BJT transistor.*

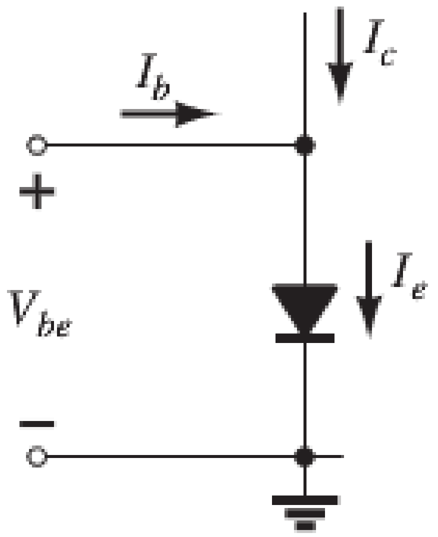



**FIG. 5.10**

*Equivalent circuit for the input side of a BJT transistor.*

# The $r_e$ Transistor Model

## (The BJT Equivalent Circuit – T Model)

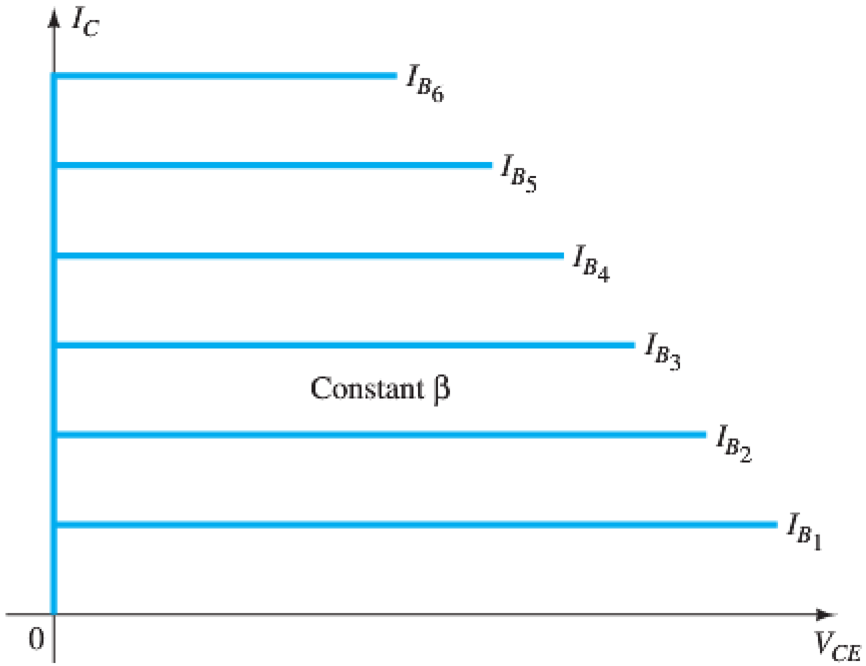



**FIG. 5.11**
*Constant β characteristics.*

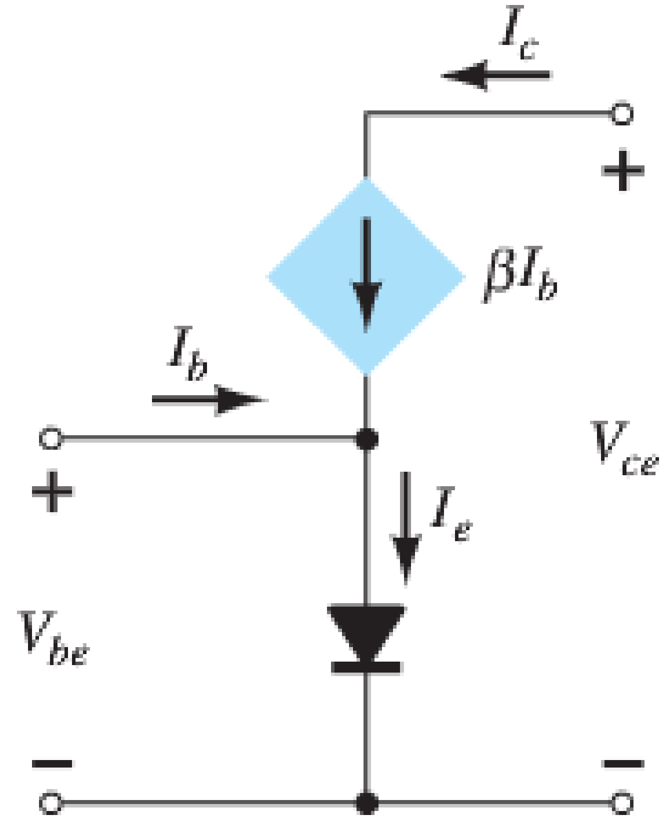



**FIG. 5.12**
*BJT equivalent circuit.*

# The T model of a transistor:

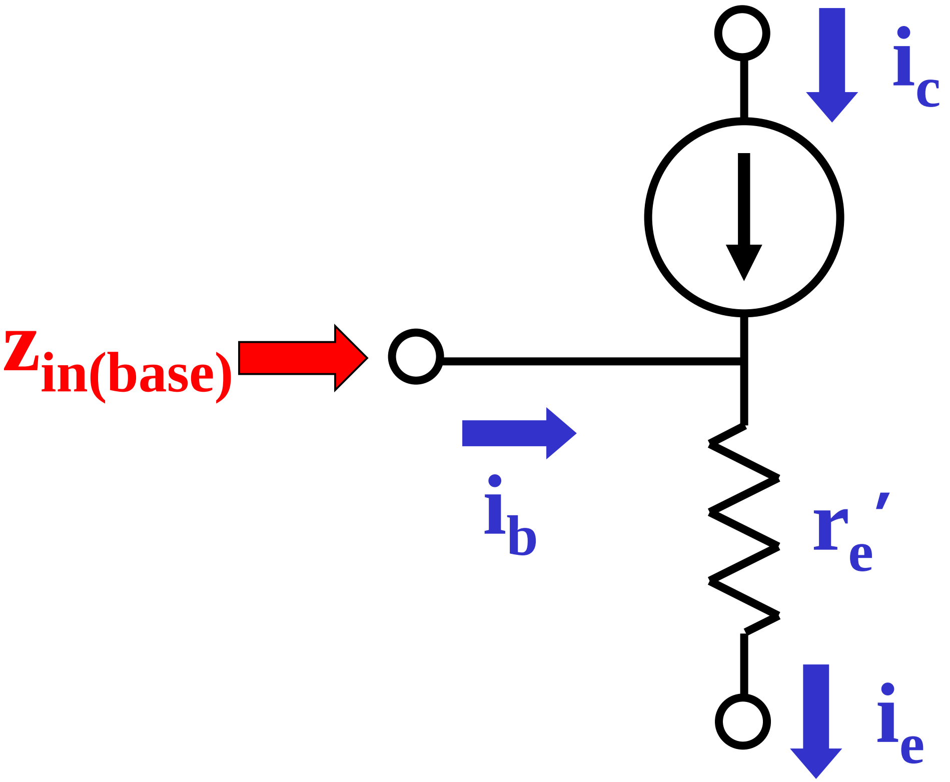
$$v_{be} = i_e r_e'$$

$$z_{in(base)} = \frac{v_{be}}{i_b}$$

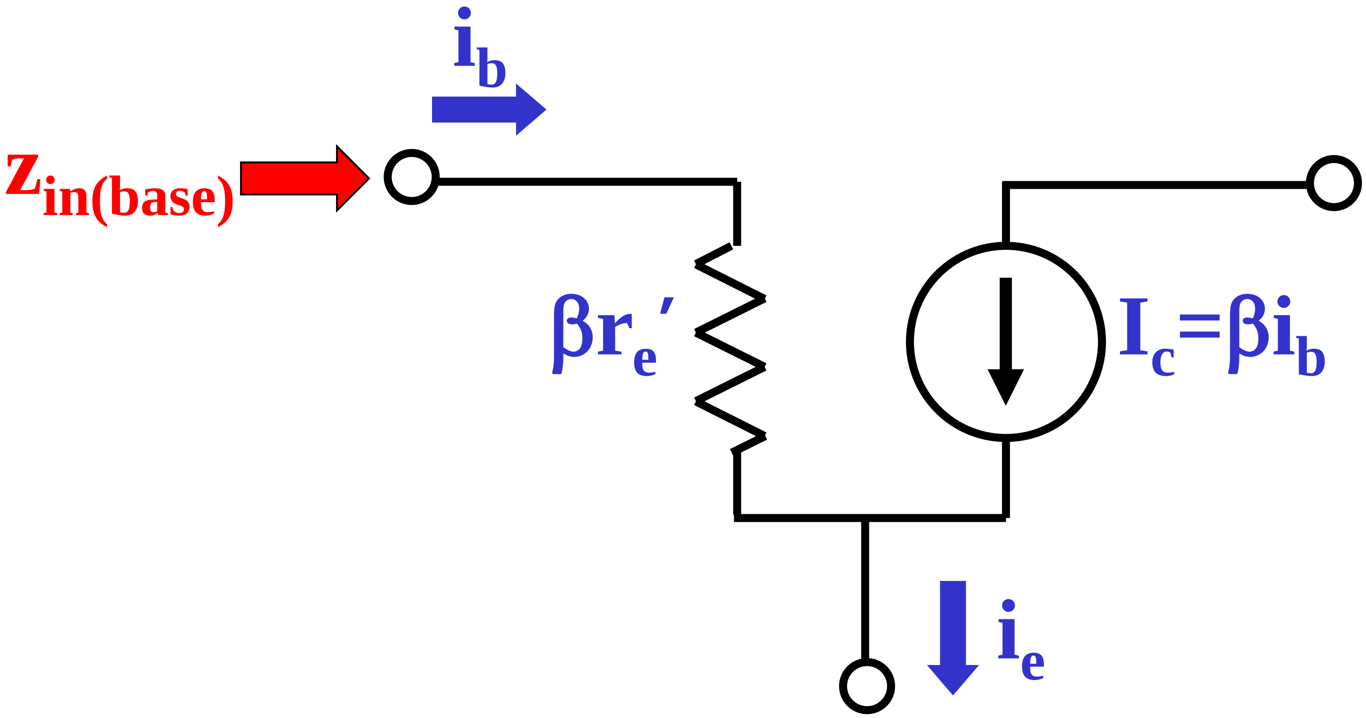
$$= \frac{i_e r_e'}{i_b}$$

$$= (\beta+1) r_e'$$

$$\approx \beta\, r_e'$$



# The π model of a transistor is based on $z_{in(base)} = \beta r_e'$ :




**Clearly shows the input impedance of $\beta r_e$' will load the ac voltage source driving the base**

# Overview of $r_e$ Transistor Models

## (T and π Model)

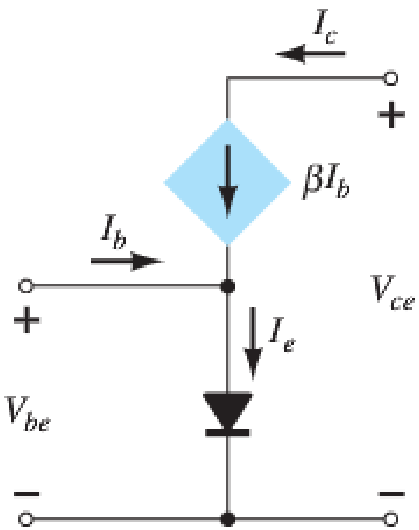
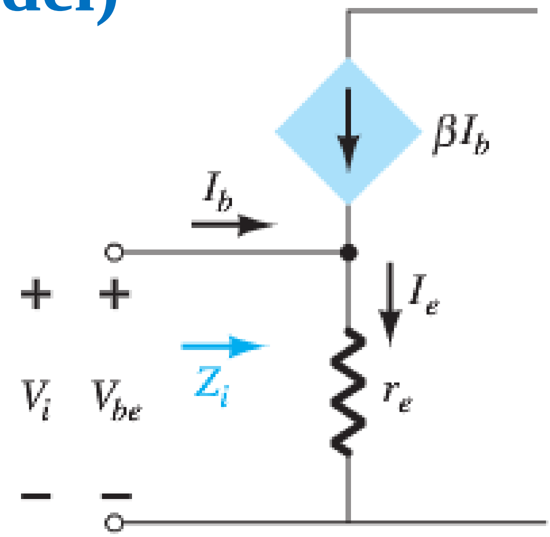
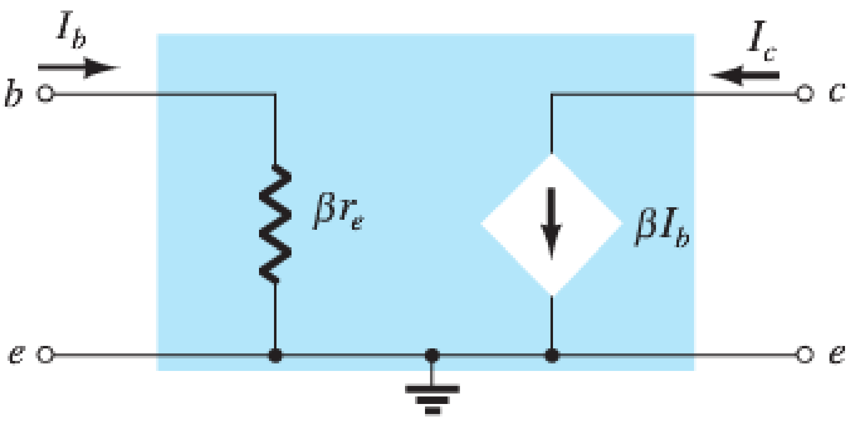



**FIG. 5.12**
*BJT equivalent circuit.*



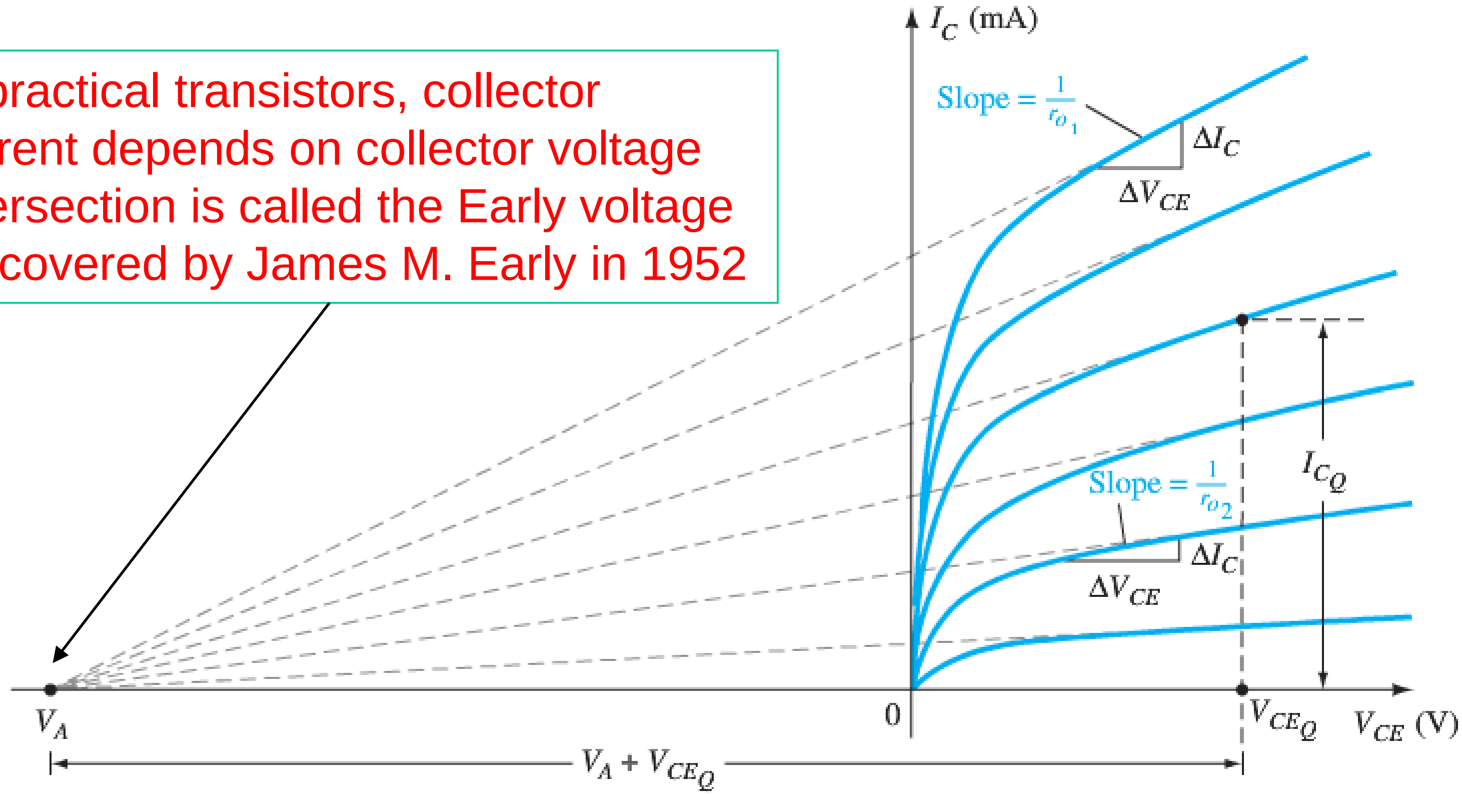

**T Model**




**π Model**

# Early voltage and modeling the output impedance

- In practical transistors, collector current depends on collector voltage
- Intersection is called the Early voltage
- Discovered by James M. Early in 1952




$$r_o = \frac{\Delta V}{\Delta I} = \frac{V_A + V_{CE_Q}}{I_{C_Q}}$$

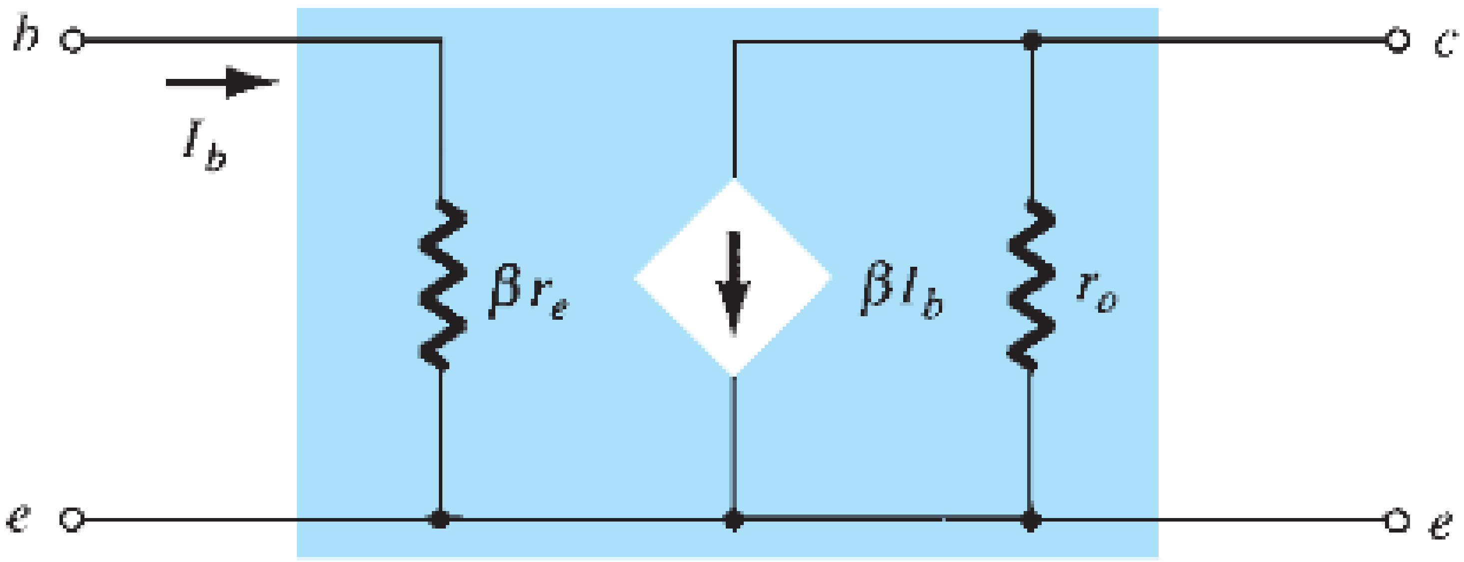
$$r_o \cong \frac{V_A}{I_{C_Q}}$$

If Early voltage is not available,

$$\text{Slope} = \frac{\Delta y}{\Delta x} = \frac{\Delta I_C}{\Delta V_{CE}} = \frac{1}{r_o}$$

$$r_o = \frac{\Delta V_{CE}}{\Delta I_C}$$

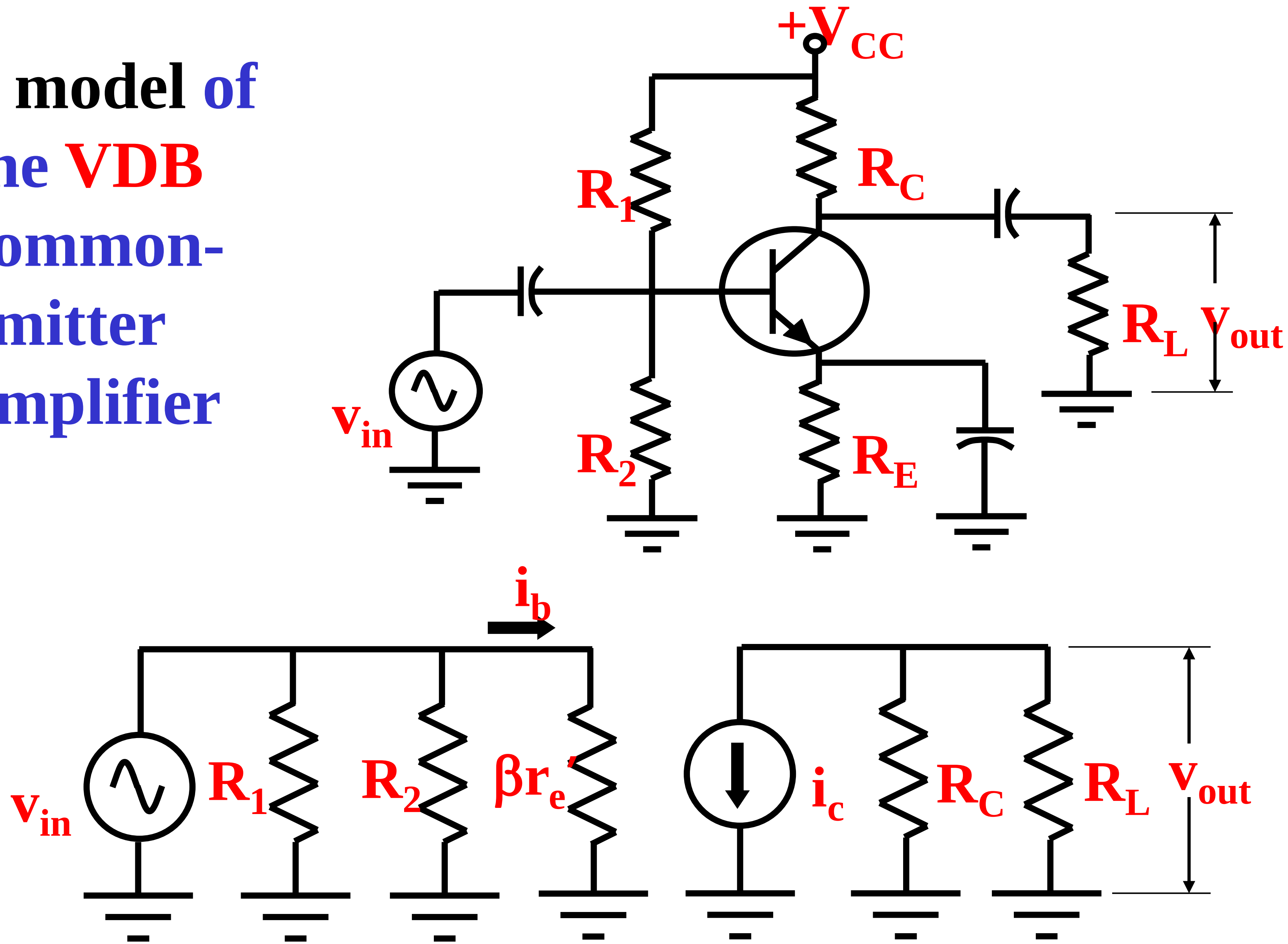
# $r_e$ model including effects of $r_o$



# Reasons for Slight Upward Slope of Collector Curves

- CB depletion layer widens with increasing $V_{CE}$
  - Narrows the base
  - Fewer holes available for recombination
- Results in smaller $I_B$ and higher $I_C$

# π model of the VDB common-emitter amplifier

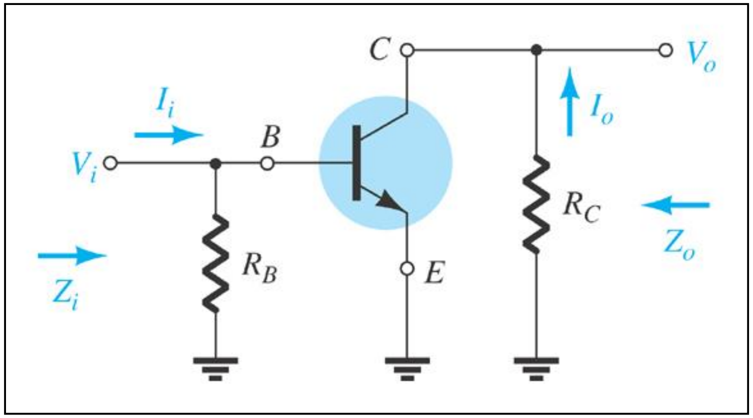



This model best illustrates that $z_{in(stage)} = R_1 \| R_2 \| \beta r_e'$

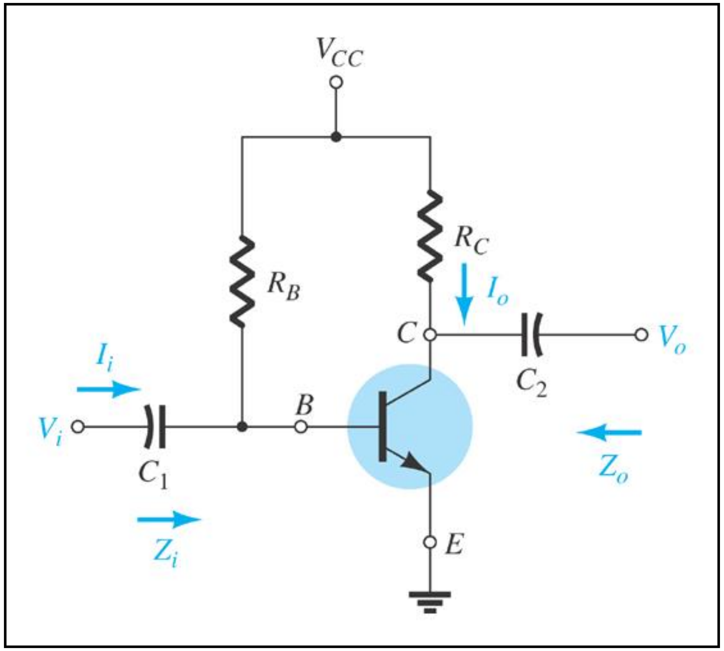
# Amplifier analysis

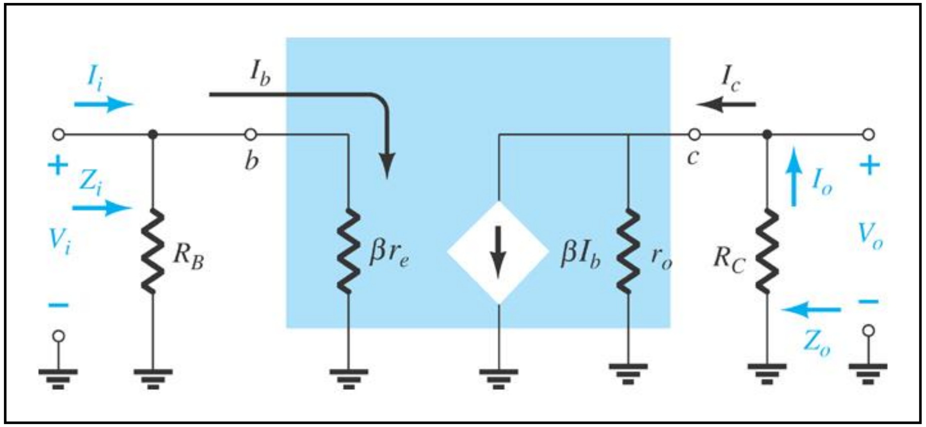
- **Perform** a complete dc analysis
- **Mentally** short all coupling and bypass capacitors for ac signals
- **Visualize** all dc supply voltages as ac grounds
- **Replace** the transistor by its $\pi$ or T model
- **Draw** the ac equivalent circuit

# Common-Emitter Fixed-Bias Configuration



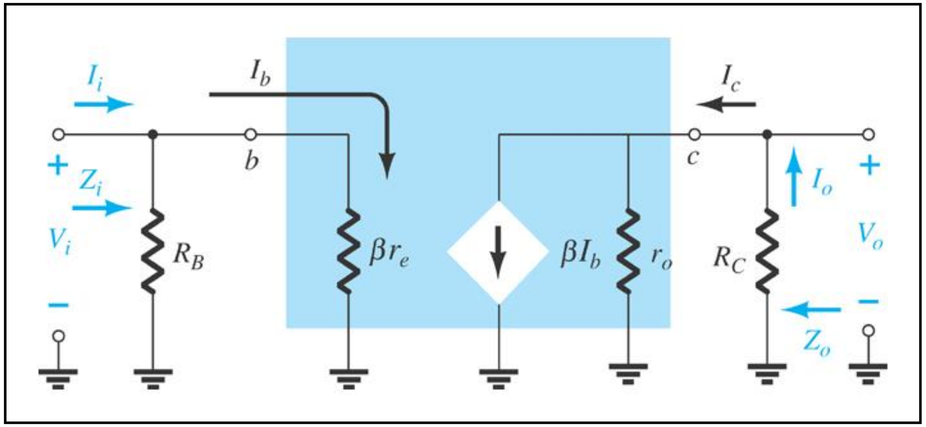

AC equivalent




$r_e$, model

# Common-Emitter Fixed-Bias Calculations




Input impedance:

$$Z_i = R_B || \beta |_e$$

$$Z_i \cong \beta r_e \Big|_{R_E \geq 10\beta r_e}$$

Output impedance:

$$Z_o = R_C || r_o$$

$$Z_o \cong R_C \Big|_{r_o \geq 10 R_C}$$

Voltage gain:

$$A_v = \frac{V_o}{V_i} = -\frac{(R_C || r_o)}{r_e}$$

$$A_v = -\frac{R_C}{r_e} \Big|_{r_o \geq 10 R_C}$$

Current gain:

$$A_i = \frac{I_o}{I_i} = \frac{\beta R_B r_o}{(r_o + R_C)(R_B + \beta r_e)}$$

$$A_i \cong \beta \Big|_{r_o \geq 10 R_C,\ R_B \geq 10 \beta r_e}$$

Current gain from voltage gain:

$$A_i = -A_v \frac{Z_i}{R_C}$$

**EXAMPLE 5.1** For the network of Fig. 5.25:

a. Determine $r_e$.
b. Find $Z_i$ (with $r_o = \infty\,\Omega$).
c. Calculate $Z_o$ (with $r_o = \infty\,\Omega$).
d. Determine $A_v$ (with $r_o = \infty\,\Omega$).
e. Repeat parts (c) and (d) including $r_o = 50\text{ k}\Omega$ in all calculations and compare results.

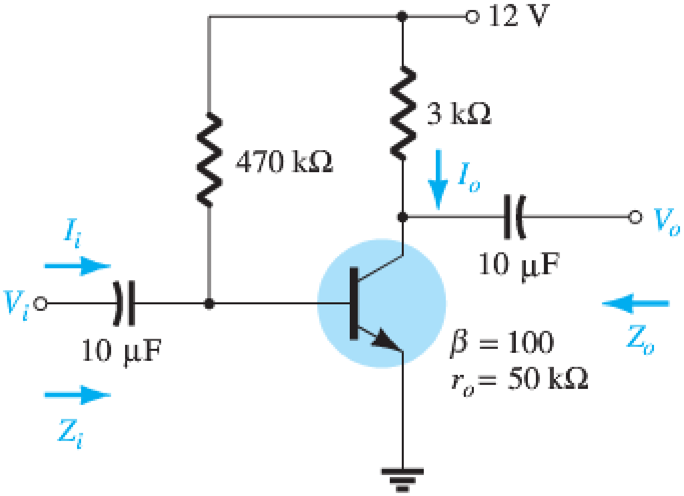



**FIG. 5.25**
*Example 5.1.*

**Solution:**

a. DC analysis:

$$I_B = \frac{V_{CC} - V_{BE}}{R_B} = \frac{12\text{ V} - 0.7\text{ V}}{470\text{ k}\Omega} = 24.04\ \mu\text{A}$$

$$I_E = (\beta + 1)I_B = (101)(24.04\ \mu\text{A}) = 2.428\text{ mA}$$

$$r_e = \frac{26\text{ mV}}{I_E} = \frac{26\text{ mV}}{2.428\text{ mA}} = \mathbf{10.71\ \Omega}$$

b. $\beta r_e = (100)(10.71\ \Omega) = 1.071\text{ k}\Omega$

$$Z_i = R_B \| \beta r_e = 470\text{ k}\Omega \| 1.071\text{ k}\Omega = \mathbf{1.07\text{ k}\Omega}$$

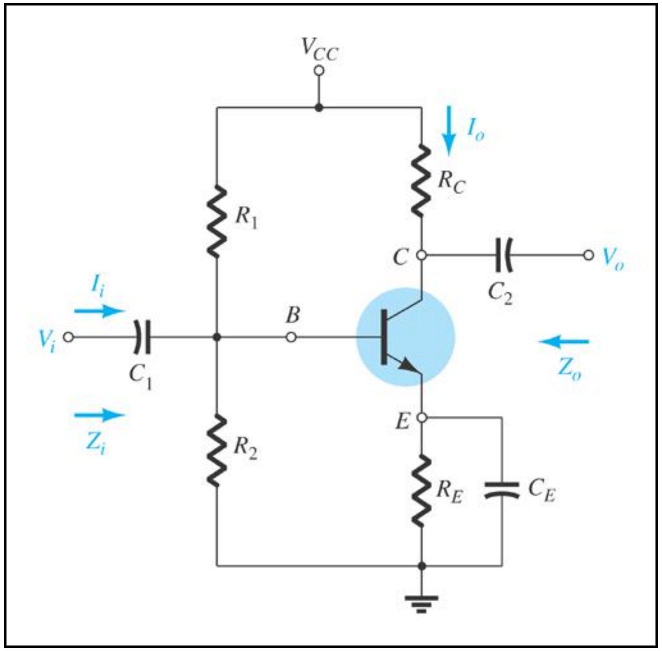
c. $Z_o = R_C = \mathbf{3\text{ k}\Omega}$

d. $A_v = -\dfrac{R_C}{r_e} = -\dfrac{3\text{ k}\Omega}{10.71\ \Omega} = \mathbf{-280.11}$

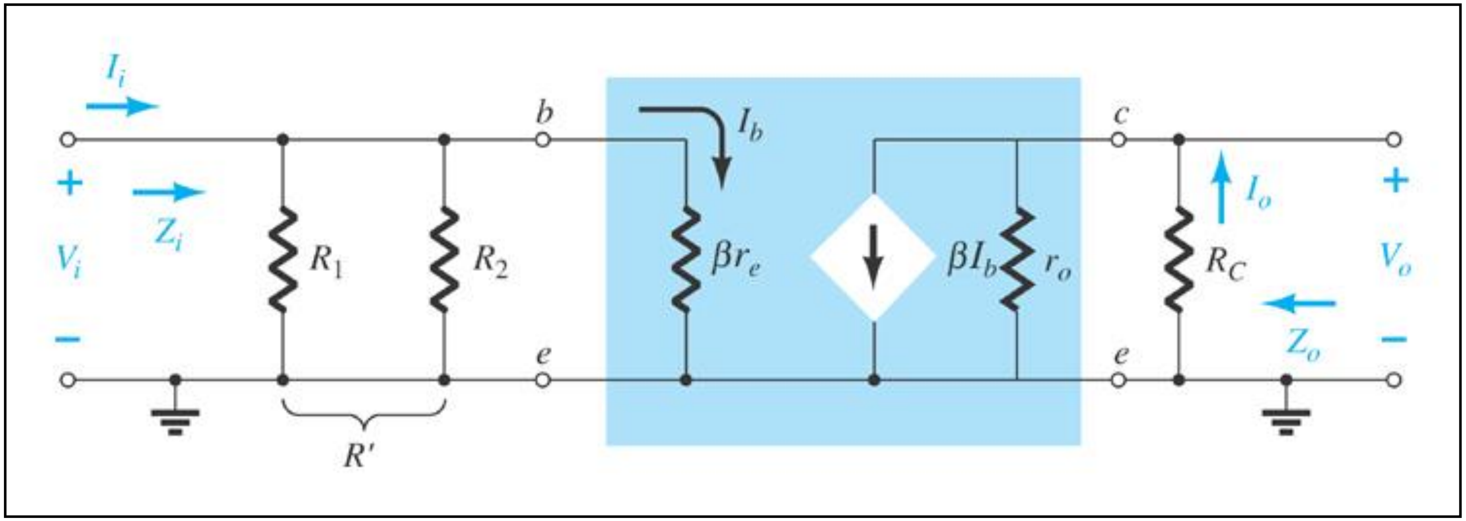
e. $Z_o = r_o \| R_C = 50\text{ k}\Omega \| 3\text{ k}\Omega = \mathbf{2.83\text{ k}\Omega}$ vs. $3\text{ k}\Omega$

$$A_v = -\frac{r_o \| R_C}{r_e} = \frac{2.83\text{ k}\Omega}{10.71\ \Omega} = \mathbf{-264.24} \text{ vs. } -280.11$$

# Common-Emitter Voltage-Divider Bias




$r_e$ model requires you to determine $\beta$, $r_e$, and $r_o$.



# Common-Emitter Voltage-Divider Bias

Input impedance

$$R' = R_1 \,||\, R_2$$
$$Z_i = R' \,||\, \beta r_e$$

Output impedance

$$Z_o = R_C \,||\, r_o$$
$$Z_o \cong R_C \big|_{r_o \geq 10R_C}$$

Voltage gain

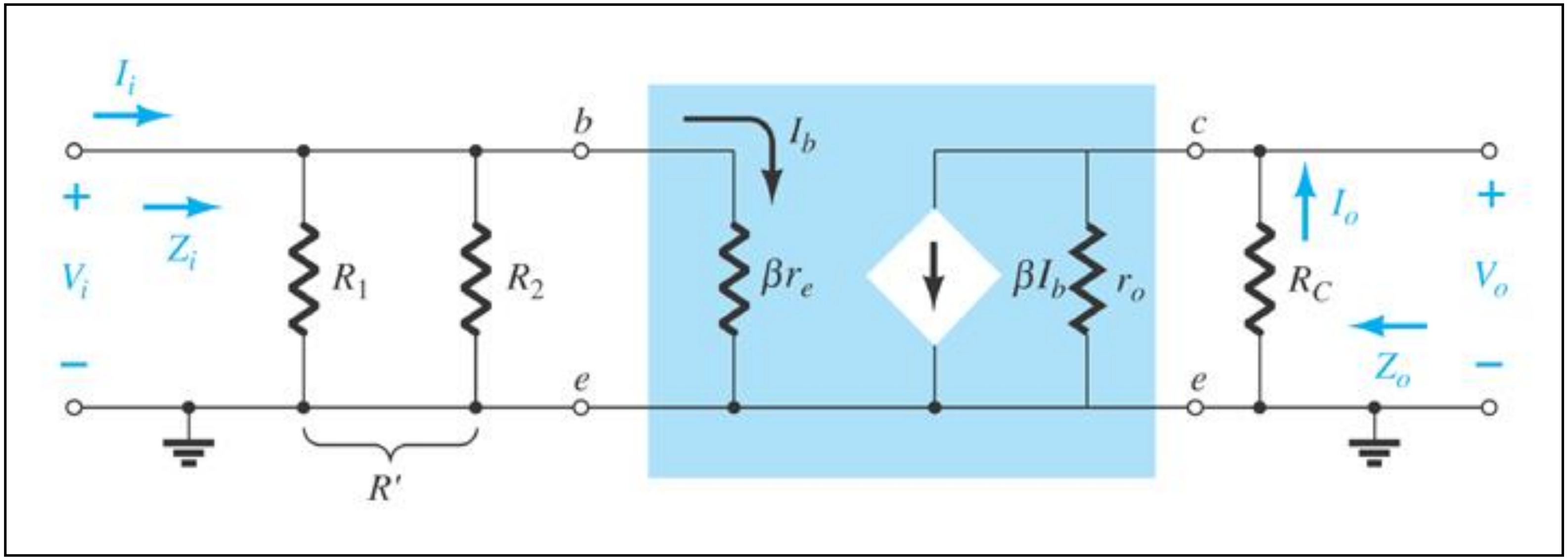
$$A_v = \frac{V_o}{V_i} = \frac{-R_C \,||\, r_o}{r_e}$$
$$A_v = \frac{V_o}{V_i} \cong -\frac{R_C}{r_e} \bigg|_{r_o \geq 10R_C}$$

Current gain

$$A_i = \frac{I_o}{I_i} = \frac{\beta R' r_o}{(r_o + R_C)(R' + \beta r_e)}$$
$$A_i = \frac{I_o}{I_i} \cong \frac{\beta R'}{R' + \beta r_e} \bigg|_{r_o \geq 10R_C}$$
$$A_i = \frac{I_o}{I_i} \cong \beta \big|_{r_o \geq 10R_C,\, R' \geq 10\beta r_e}$$

Current gain from $A_v$

$$A_i = -A_v \frac{Z_i}{R_C}$$



**EXAMPLE 5.2** For the network of Fig. 5.28, determine:

a. $r_e$.
b. $Z_i$.
c. $Z_o$ ($r_o = \infty\,\Omega$).
d. $A_v$ ($r_o = \infty\,\Omega$).
e. The parameters of parts (b) through (d) if $r_o = 50\,\text{k}\Omega$ and compare results.

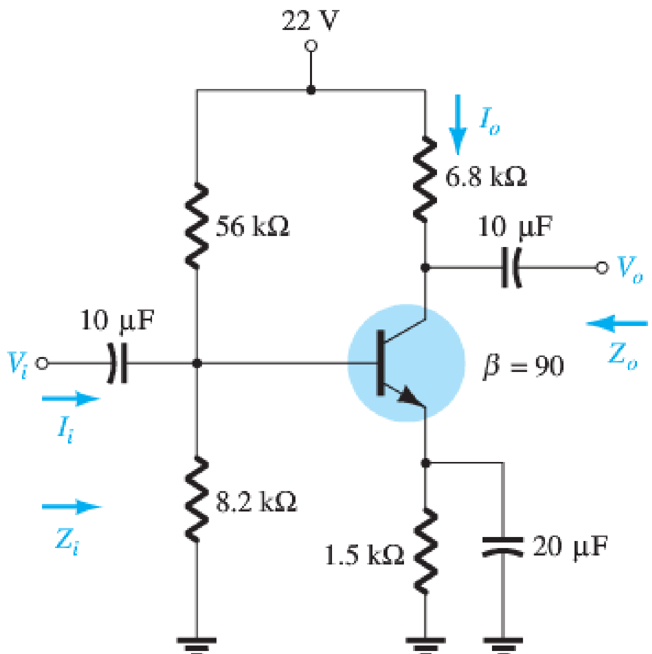



**FIG. 5.28**
*Example 5.2.*

Testing $\beta R_E > 10R_2$,

$$(90)(1.5\,\text{k}\Omega) > 10(8.2\,\text{k}\Omega)$$

$$135\,\text{k}\Omega > 82\,\text{k}\Omega \ (\textit{satisfied})$$

$$V_B = \frac{R_2}{R_1 + R_2}V_{CC} = \frac{(8.2\,\text{k}\Omega)(22\,\text{V})}{56\,\text{k}\Omega + 8.2\,\text{k}\Omega} = 2.81\,\text{V}$$

$$V_E = V_B - V_{BE} = 2.81\,\text{V} - 0.7\,\text{V} = 2.11\,\text{V}$$

$$I_E = \frac{V_E}{R_E} = \frac{2.11\,\text{V}}{1.5\,\text{k}\Omega} = 1.41\,\text{mA}$$

$$r_e = \frac{26\,\text{mV}}{I_E} = \frac{26\,\text{mV}}{1.41\,\text{mA}} = \mathbf{18.44\,\Omega}$$

b. $R' = R_1 \| R_2 = (56\,\text{k}\Omega)\|(8.2\,\text{k}\Omega) = 7.15\,\text{k}\Omega$

$$Z_i = R' \| \beta r_e = 7.15\,\text{k}\Omega \| (90)(18.44\,\Omega) = 7.15\,\text{k}\Omega \| 1.66\,\text{k}\Omega$$
$$= \mathbf{1.35\,k\Omega}$$

c. $Z_o = R_C = \mathbf{6.8\,k\Omega}$

d. $A_v = -\dfrac{R_C}{r_e} = -\dfrac{6.8\,\text{k}\Omega}{18.44\,\Omega} = \mathbf{-368.76}$

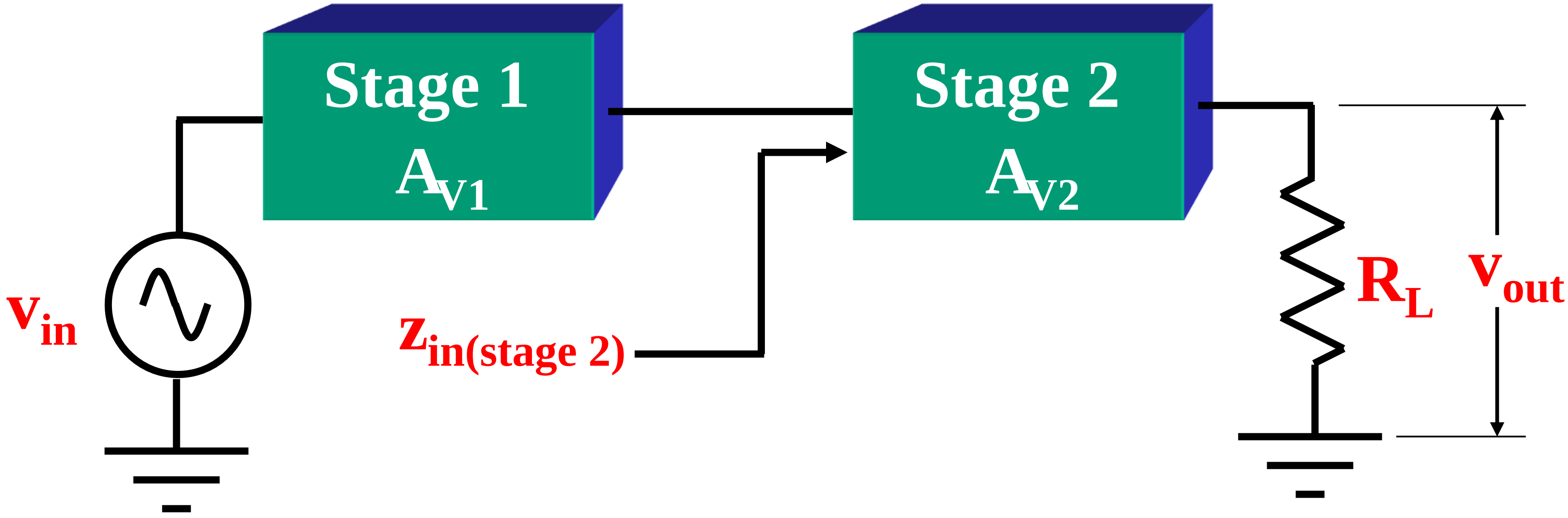
e. $Z_i = \mathbf{1.35\,k\Omega}$



$$Z_o = R_C \| r_o = 6.8\,\text{k}\Omega \| 50\,\text{k}\Omega = \mathbf{5.98\,k\Omega} \text{ vs. } 6.8\,\text{k}\Omega$$

$$A_v = -\frac{R_C \| r_o}{r_e} = -\frac{5.98\,\text{k}\Omega}{18.44\,\Omega} = \mathbf{-324.3} \text{ vs. } -368.76$$

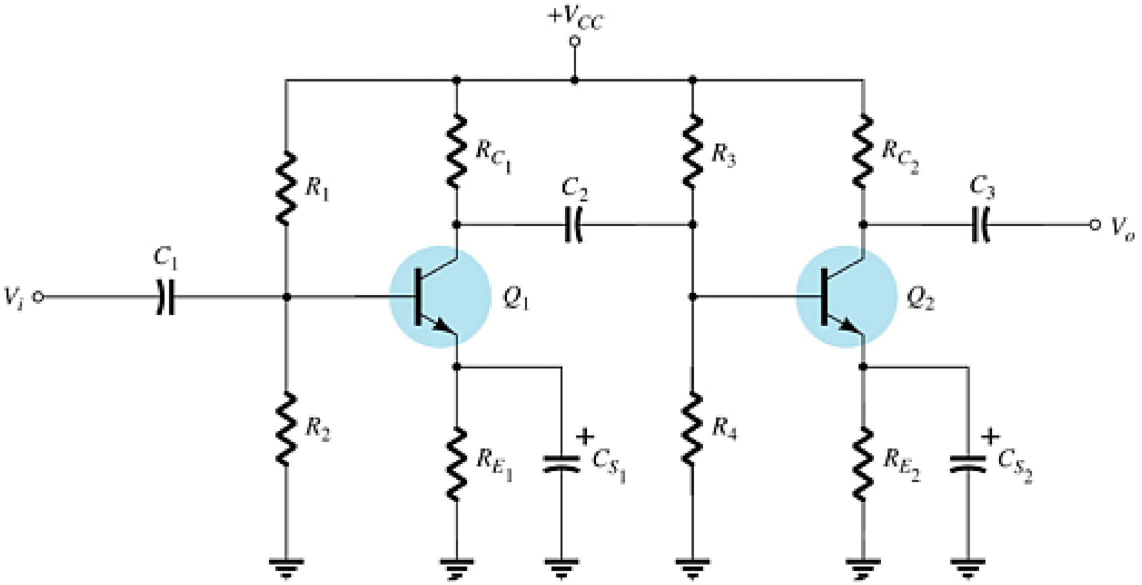
# To get more gain, a cascaded amplifier can be used.




**The overall voltage gain:** $A_V = A_{V1}A_{V2}$

- The output of one amplifier is the input to the next amplifier
- The DC bias circuits are isolated from each other by the coupling capacitors

# R-C Coupled BJT Amplifiers




**Voltage gain:**

$$A_{v1} = \frac{R_C \,||\, R_1 \,||\, R_2 \,||\, \beta R_e}{r_e}$$

$$A_{v2} = \frac{R_C}{r_e}$$

$$A_v = A_{v1} A_{v2}$$

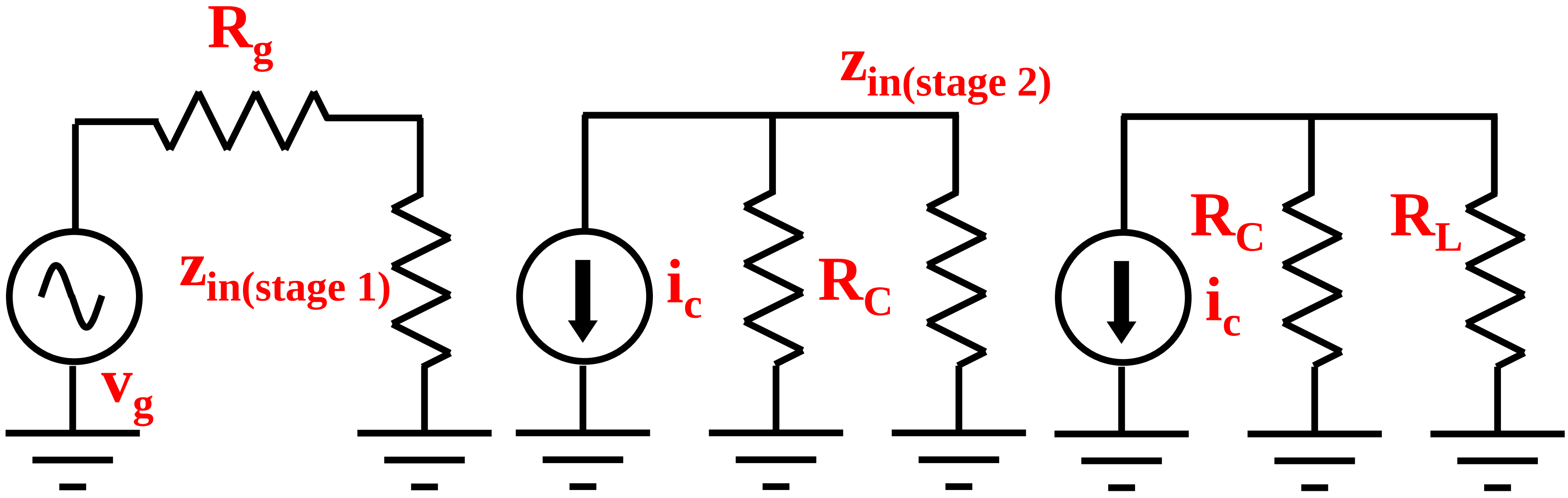
**Input impedance, first stage:**

$$Z_i = R_1 \,||\, R_2 \,||\, \beta R_e$$

**Output impedance, second stage:**

$$Z_o = R_C$$

# Ac equivalent circuit for the two-stage amplifier




The <u>2nd</u> stage loads the <u>1st</u> stage:

$$R_{c1} = R_C \parallel Z_{in}\ (\text{stage } 2)$$

# The Hybrid $\pi$ Model

The hybrid pi model is most useful for analysis of high-frequency transistor applications.

At lower frequencies the hybrid pi model closely approximate the $r_e$ parameters, and can be replaced by them.

# The Hybrid Equivalent Model

Hybrid parameters are developed and used for modeling the transistor. These parameters can be found on a transistor's specification sheet:

$h_i$ = input resistance
$h_r$ = reverse transfer voltage ratio ($V_i/V_o$) $\cong$ 0
$h_f$ = forward transfer current ratio ($I_o/I_i$)
$h_o$ = output conductance