Methods of Analysis and Selected Topics (dc)

The circuits described in previous chapters had only one source or two or more sources in series or parallel. The step-by-step procedures outlined in those chapters can be applied only if the sources are in series or parallel. There will be an interaction of sources that will not permit the reduction techniques used to find quantities such as the total resistance and the source current.

The methods to be introduced in this chapter include branch-current analysis, mesh analysis, and nodal analysis. Each can be applied to the same network, although usually one is more appropriate than the other. The "best" method cannot be defined by a strict set of rules but can be determined only after developing an understanding of the relative advantages of each.

8.2 CURRENT SOURCES

In previous chapters, the voltage source was the only source appearing in the circuit analysis. This was primarily because voltage sources such as the battery and supply are the most common in our daily lives and in the laboratory environment.

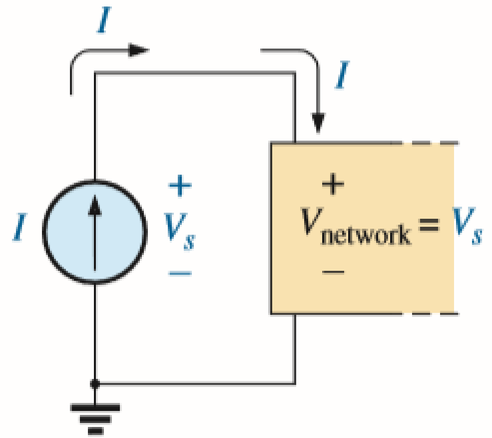
We now turn our attention to a second type of source called the **current source** which appears throughout the analyses in this chapter. Although current sources are available as labo-

The current source is often described as the *dual* of the voltage source. Just as a battery provides a fixed voltage to a network, a current source establishes a fixed current in the branch where it is located. Further, the

The symbol for a current source appears in Fig. 8.1(a). The arrow indicates the direction in which it is supplying current to the branch where it is located. The result is a current equal to the source current through the series resistor. In Fig. 8.1(b), we find that the voltage across a current source is determined by the polarity of the voltage drop caused by the current source. For single-source networks, it always has the polarity of Fig. 8.1(b), but for multisource networks it can have either polarity.

In general, therefore,

a current source determines the direction and magnitude of the current in the branch where it is located.



EXAMPLE 8.3 Determine the current I_1 and the voltage V_s for the network in Fig. 8.4.

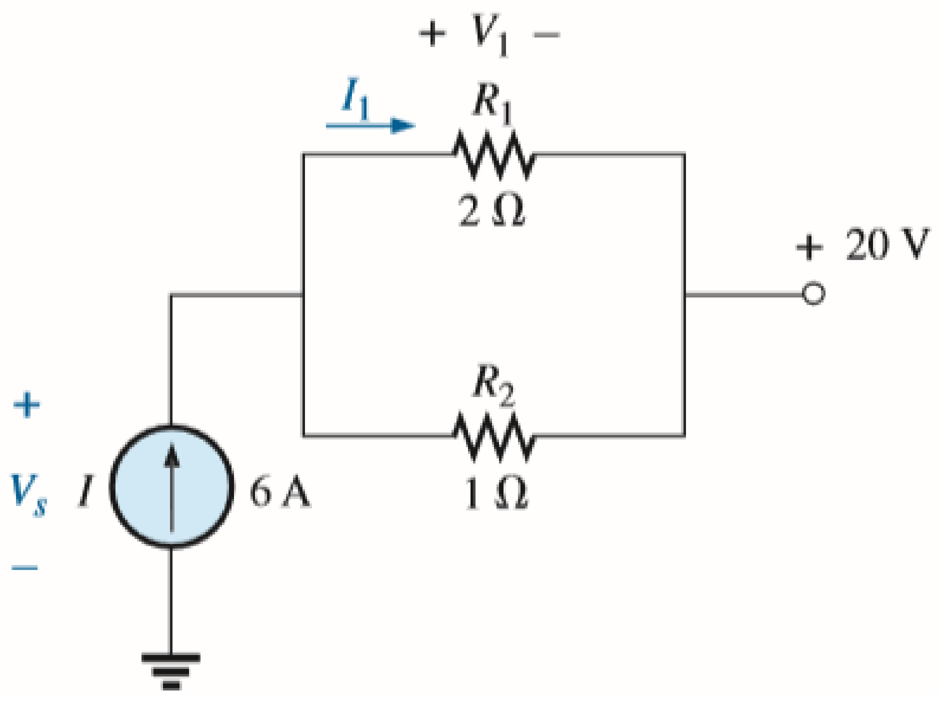


FIG. 8.4
Example 8.3.

Solution: First note that the current in the branch with the current source must be 6 A, no matter what the magnitude of the voltage source to the right. In other words, the currents of the network are defined by I, R_1, and R_2. However, the voltage across the current source is directly affected by the magnitude and polarity of the applied source.

Using the current divider rule:

$$I_1 = \frac{R_2 I}{R_2 + R_1} = \frac{(1\ \Omega)(6\ \text{A})}{1\ \Omega + 2\ \Omega} = \frac{1}{3}(6\ \text{A}) = \mathbf{2\ A}$$

The voltage V_1:

$$V_1 = I_1 R_1 = (2\ \text{A})(2\ \Omega) = 4\ \text{V}$$

Applying Kirchhoff's voltage rule to determine V_s:

$$+V_s - V_1 - 20\ \text{V} = 0$$

and

$$V_s = V_1 + 20\ \text{V} = 4\ \text{V} + 20\ \text{V} = \mathbf{24\ V}$$

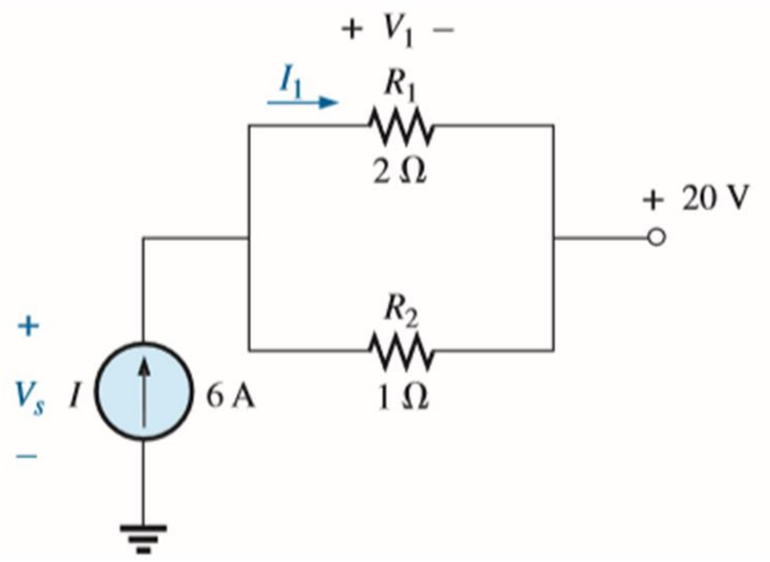


FIG. 8.4
Example 8.3.

8.3 SOURCE CONVERSIONS

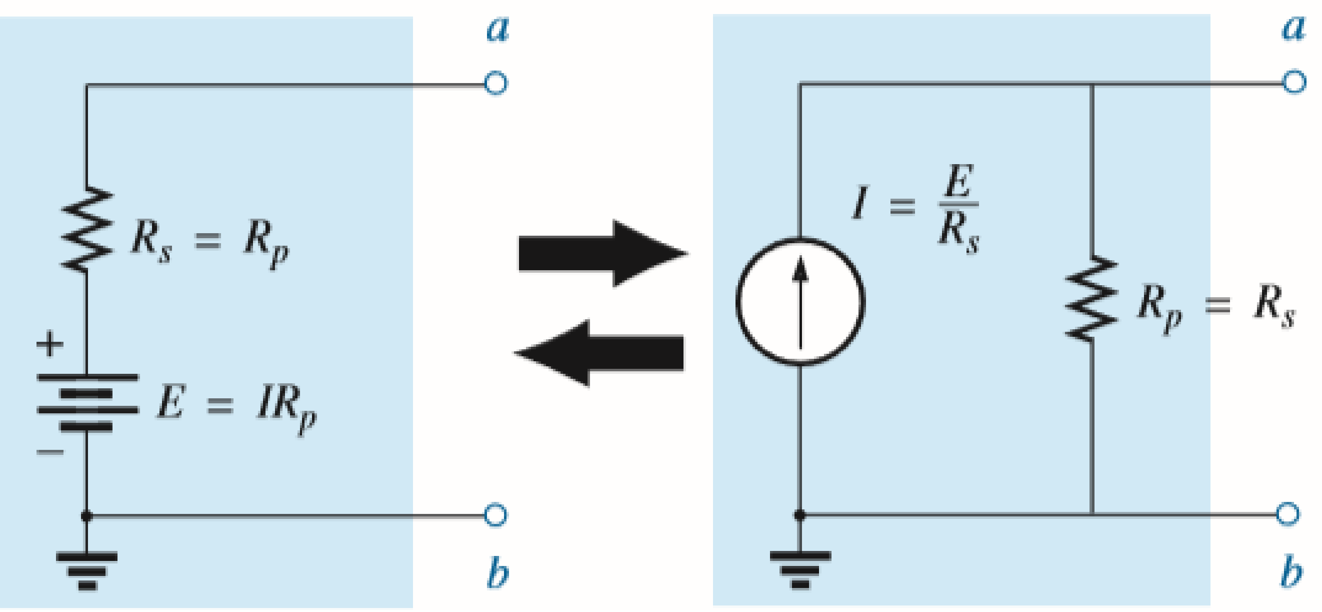


FIG. 8.6
Source conversion.

EXAMPLE 8.4 For the circuit in Fig. 8.7:

a. Determine the current I_L.
b. Convert the voltage source to a current source.
c. Using the resulting current source of part (b), calculate the current through the load resistor, and compare your answer to the result of part (a).

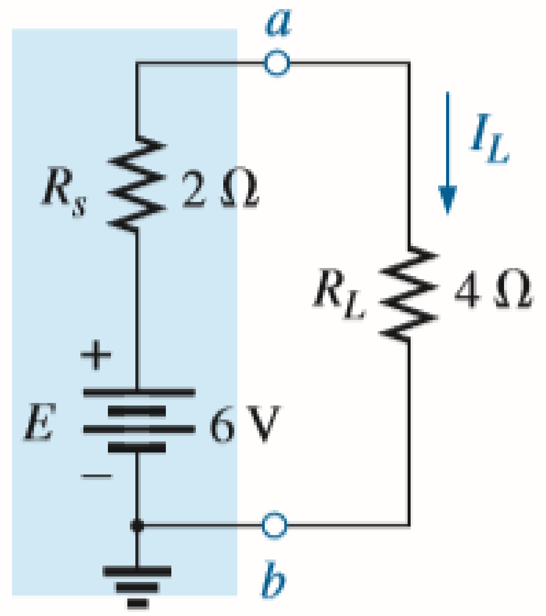


FIG. 8.7

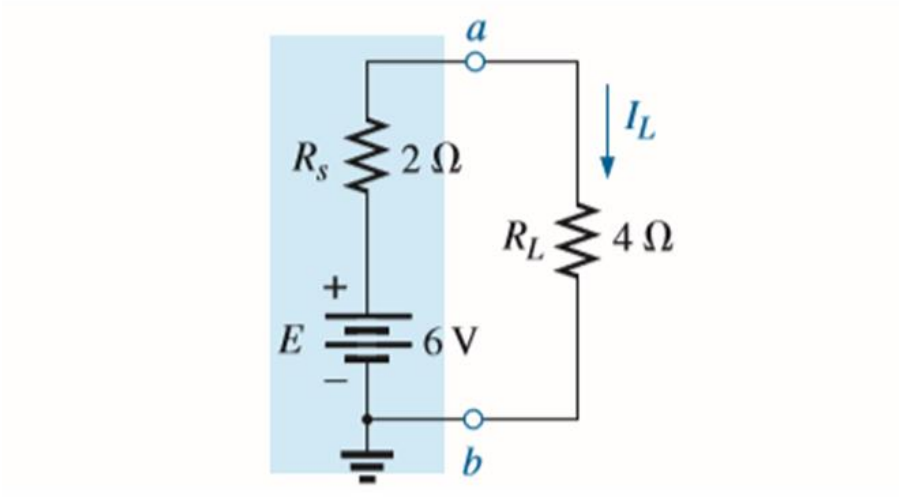


FIG. 8.7
Practical voltage source and load for Example 8.

EXAMPLE 8.4 For the circuit in Fig. 8.7:

a. Determine the current I_L.
b. Convert the voltage source to a current source.
c. Using the resulting current source of part (b), calculate the current through the load resistor, and compare your answer to the result of part (a).

Solutions:

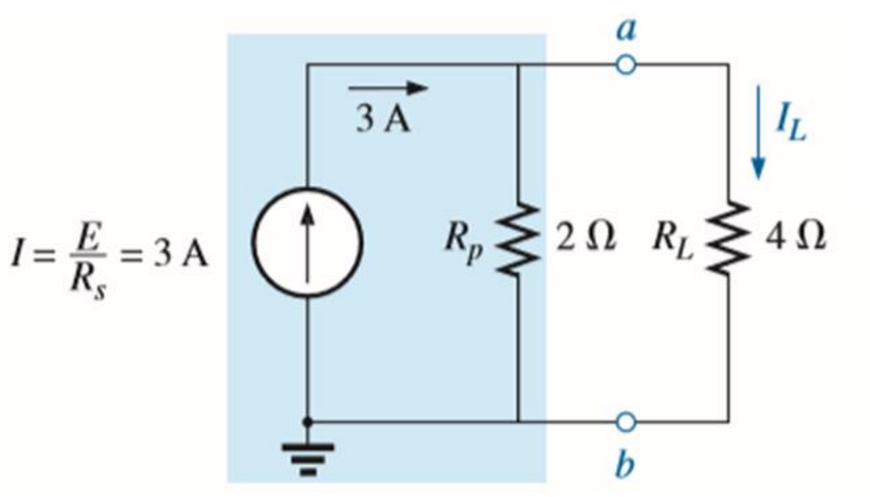
a. Applying Ohm's law:

$$I_L = \frac{E}{R_s + R_L} = \frac{6\text{ V}}{2\ \Omega + 4\ \Omega} = \frac{6\text{ V}}{6\ \Omega} = \mathbf{1\ A}$$

b. Using Ohm's law again:

$$I = \frac{E}{R_s} = \frac{6\text{ V}}{2\ \Omega} = \mathbf{3\ A}$$

and the equivalent source appears in Fig. 8.8 with the load reapplied.



c. Using the current divider rule:

$$I_L = \frac{R_p I}{R_p + R_L} = \frac{(2\ \Omega)(3\text{ A})}{2\ \Omega + 4\ \Omega} = \frac{1}{3}(3\text{ A}) = \mathbf{1\ A}$$

We find that the current I_L is the same for the voltage source as it was for the equivalent current source—the sources are therefore equivalent.

8.4 CURRENT SOURCES IN PARALLEL

We found that voltage sources of different terminal voltages cannot be placed in parallel because of a violation of Kirchhoff's voltage law. Similarly,

current sources of different values cannot be placed in series due to a violation of Kirchhoff's current law.

However, current sources can be placed in parallel just as voltage sources can be placed in series. In general,

two or more current sources in parallel can be replaced by a single current source having a magnitude determined by the difference of the sum of the currents in one direction and the sum in the opposite direction. The new parallel internal resistance is the total resistance of the resulting parallel resistive elements.

Consider the following examples.

EXAMPLE 8.6 Reduce the parallel current sources in Fig. 8.11 to a single current source.

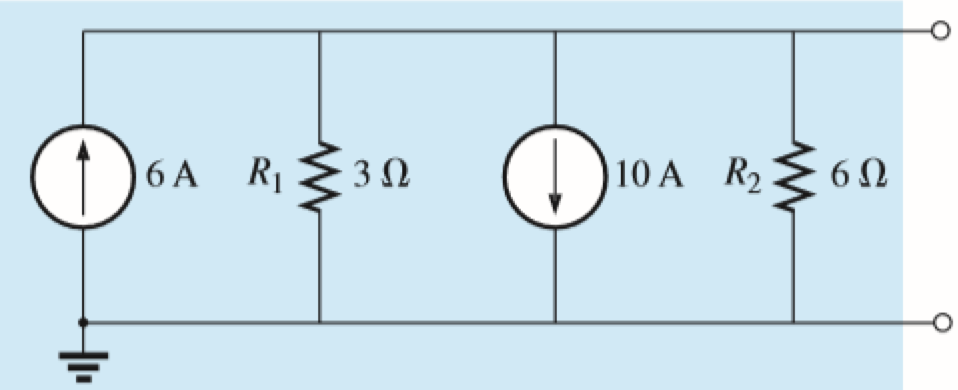


FIG. 8.11
Parallel current sources for Example 8.6.

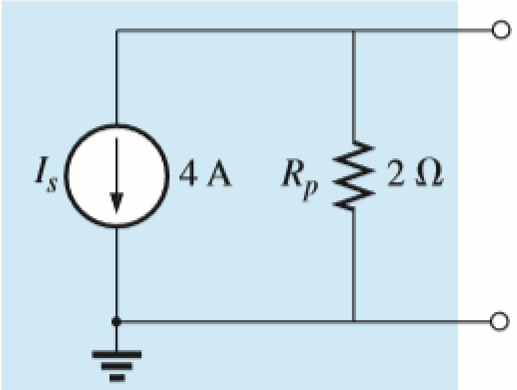


FIG. 8.12

Solution: The net source current is

$$I = 10\text{ A} - 6\text{ A} = \mathbf{4\ A}$$

with the direction of the larger.

The net internal resistance is the parallel combination of resistors, R_1 and R_2:

$$R_p = 3\ \Omega \| 6\ \Omega = \mathbf{2\ \Omega}$$

The reduced equivalent appears in Fig. 8.12.

EXAMPLE 8.7 Reduce the parallel current sources in Fig. 8.13 to a single current source.

Solution: The net current is

$$I = 7\ \text{A} + 4\ \text{A} - 3\ \text{A} = \mathbf{8\ A}$$

with the direction shown in Fig. 8.14. The net internal resistance remains the same.

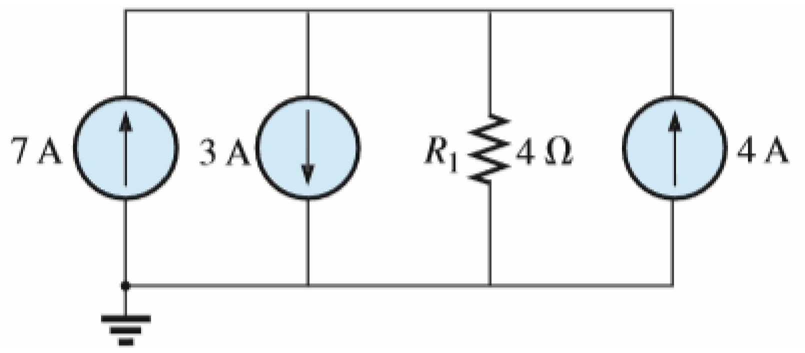


FIG. 8.13
Parallel current sources for Example 8.7.

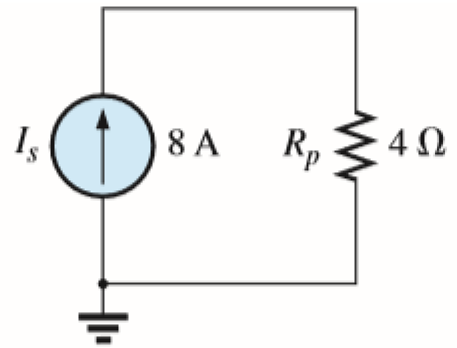


FIG. 8.14
Reduced equivalent for Fig. 8.13.

EXAMPLE 8.8 Reduce the network in Fig. 8.15 to a single current source, and calculate the current through R_L.

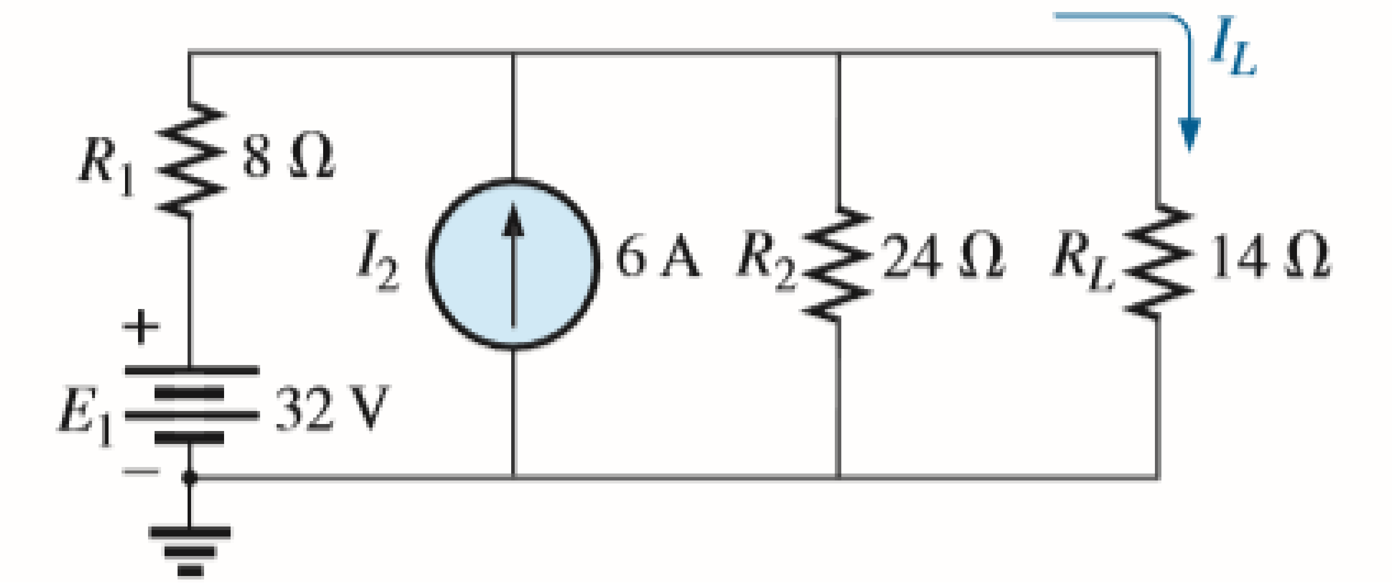


FIG. 8.15

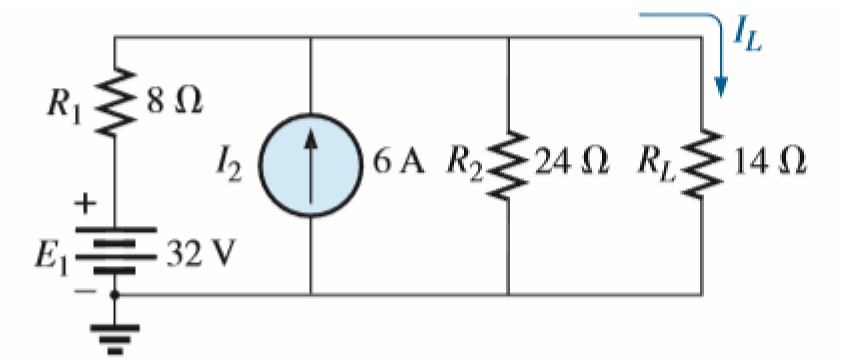


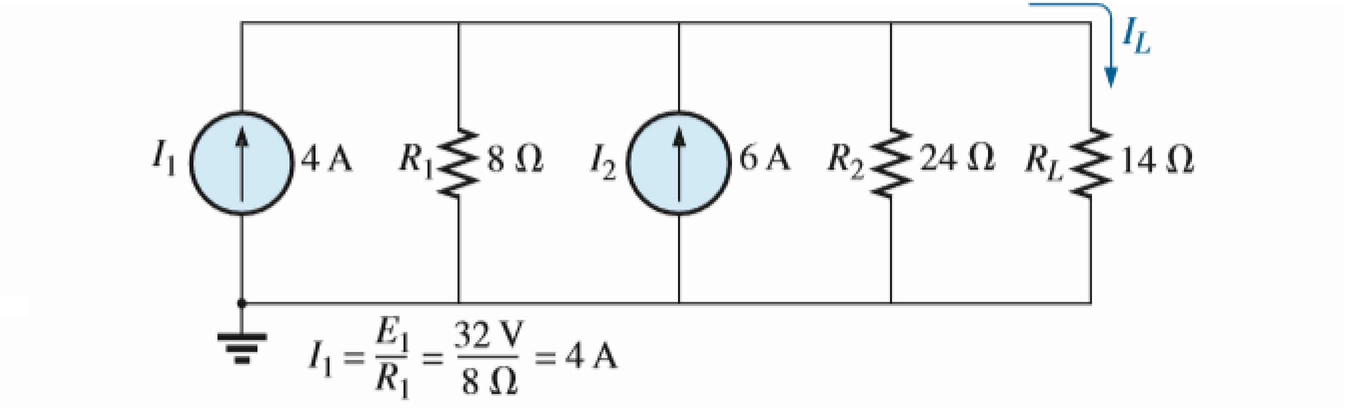
FIG. 8.15

Solution: In this example, the voltage source will first be converted to a current source as shown in Fig. 8.16. Combining current sources,

$$I_s = I_1 + I_2 = 4\ \text{A} + 6\ \text{A} = \mathbf{10\ A}$$

and

$$R_s = R_1 \| R_2 = 8\ \Omega \| 24\ \Omega = \mathbf{6\ \Omega}$$



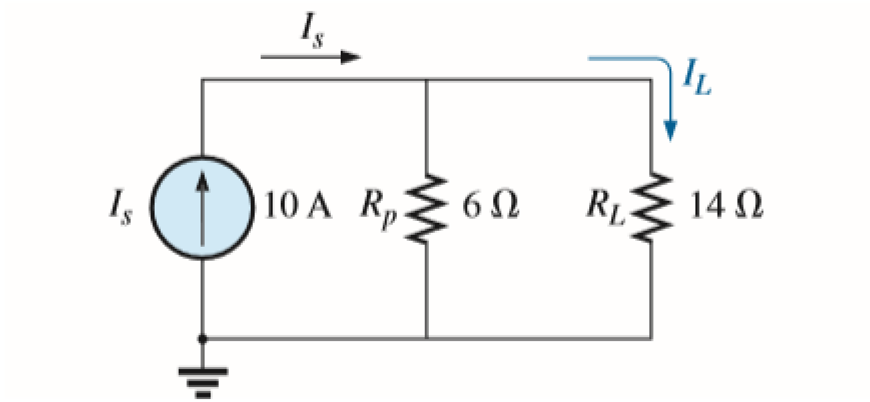


FIG. 8.17
Network in Fig. 8.16 reduced to its simplest form.

Applying the current divider rule to the resulting network in Fig. 8.17,

$$I_L = \frac{R_p I_s}{R_p + R_L} = \frac{(6\ \Omega)(10\ \text{A})}{6\ \Omega + 14\ \Omega} = \frac{60\ \text{A}}{20} = \mathbf{3\ A}$$

8.5 CURRENT SOURCES IN SERIES

The current through any branch of a network can be only single-valued. For the situation indicated at point *a* in Fig. 8.18, we find by application of Kirchhoff's current law that the current leaving that point is greater than that entering—an impossible situation. Therefore,

current sources of different current ratings are not connected in series,

just as voltage sources of different voltage ratings are not connected in parallel.

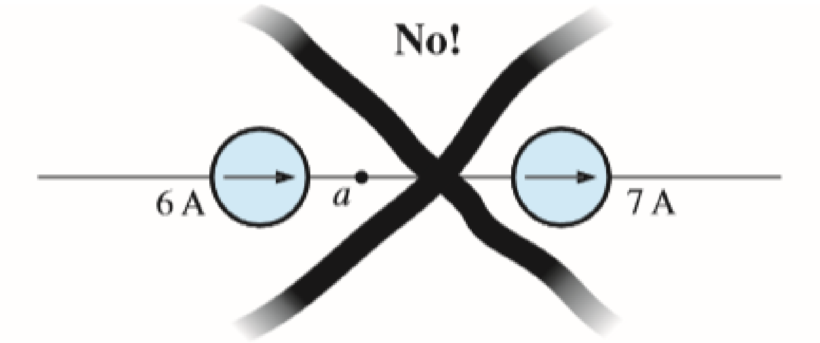


FIG. 8.18
Invalid situation.

It is assumed that the use of the **determinants method** to solve for the currents I_1, I_2, and I_3 is understood and is a part of the student's mathematical background. If not, a detailed explanation of the procedure is provided in Appendix D. Calculators and computer software packages such as Mathcad can find the solutions quickly and accurately.

DETERMINANTS

Determinants are used to find the mathematical solutions for the variables in two or more simultaneous equations. Once the procedure is properly understood, solutions can be obtained with a minimum of time and effort and usually with fewer errors than when using other methods.

Consider the following equations, where x and y are the unknown variables and a_1, a_2, b_1, b_2, c_1, and c_2 are constants:

Col. 1		Col. 2		Col. 3	
a_1x	$+$	b_1y	$=$	c_1	**(D.1a)**
a_2x	$+$	b_2y	$=$	c_2	**(D.1b)**

Col. 1		Col. 2		Col. 3
a_1x	$+$	b_1y	$=$	c_1
a_2x	$+$	b_2y	$=$	c_2

Using determinants to solve for x and y requires that the following formats be established for each variable:

$$
\begin{array}{cc}
 & \begin{array}{cc}\text{Col.} & \text{Col.} \\ 1 & 2\end{array} \\
x = & \dfrac{\begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}
\end{array}
\qquad
\begin{array}{cc}
 & \begin{array}{cc}\text{Col.} & \text{Col.} \\ 1 & 2\end{array} \\
y = & \dfrac{\begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}
\end{array}
\qquad \textbf{(D.2)}
$$

First note that only constants appear within the vertical brackets and that the denominator of each is the same. In fact, the denominator is simply the coefficients of x and y in the same arrangement as in Eqs. (D.1a) and (D.1b). When solving for x, replace the coefficients of x in the numerator by the constants to the right of the equal sign in Eqs. (D.1a) and (D.1b), and repeat the coefficients of the y variable. When solving for y, replace the y coefficients in the numerator by the constants to the right of the equal sign, and repeat the coefficients of x.

Each configuration in the numerator and denominator of Eq. (D.2) is referred to as a *determinant (D),* which can be evaluated numerically in the following manner:

$$\text{Determinant} = D = \begin{array}{c} \begin{array}{cc} \text{Col.} & \text{Col.} \\ \underline{1} & \underline{2} \end{array} \\ \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} \end{array} = a_1b_2 - a_2b_1 \qquad \textbf{(D.3)}$$

Expanding the entire expression for x and y, we have the following:

$$x = \frac{\begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}} = \frac{c_1b_2 - c_2b_1}{a_1b_2 - a_2b_1} \qquad \textbf{(D.4a)}$$

$$y = \frac{\begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}} = \frac{a_1c_2 - a_2c_1}{a_1b_2 - a_2b_1} \qquad \textbf{(D.4b)}$$

EXAMPLE D.1 Evaluate the following determinants:

a. $\begin{vmatrix} 2 & 2 \\ 3 & 4 \end{vmatrix} = (2)(4) - (3)(2) = 8 - 6 = \mathbf{2}$

b. $\begin{vmatrix} 4 & -1 \\ 6 & 2 \end{vmatrix} = (4)(2) - (6)(-1) = 8 + 6 = \mathbf{14}$

c. $\begin{vmatrix} 0 & -2 \\ -2 & 4 \end{vmatrix} = (0)(4) - (-2)(-2) = 0 - 4 = \mathbf{-4}$

d. $\begin{vmatrix} 0 & 0 \\ 3 & 10 \end{vmatrix} = (0)(10) - (3)(0) = \mathbf{0}$

EXAMPLE D.2 Solve for x and y:

$$\begin{aligned} 2x + y &= 3 \\ 3x + 4y &= 2 \end{aligned}$$

Solution:

$$x = \frac{\begin{vmatrix} 3 & 1 \\ 2 & 4 \end{vmatrix}}{\begin{vmatrix} 2 & 1 \\ 3 & 4 \end{vmatrix}} = \frac{(3)(4) - (2)(1)}{(2)(4) - (3)(1)} = \frac{12 - 2}{8 - 3} = \frac{10}{5} = \mathbf{2}$$

$$y = \frac{\begin{vmatrix} 2 & 3 \\ 3 & 2 \end{vmatrix}}{5} = \frac{(2)(2) - (3)(3)}{5} = \frac{4 - 9}{5} = \frac{-5}{5} = \mathbf{-1}$$

EXAMPLE D.4 Solve for x and y:

$$\begin{aligned} x &= 3 - 4y \\ 20y &= -1 + 3x \end{aligned}$$

$$\begin{aligned} x &= 3 - 4y \\ 20y &= -1 + 3x \end{aligned}$$

Solution: In this case, the equations must first be placed in the format of Eqs. (D.1a) and (D.1b):

$$\begin{aligned} x + 4y &= 3 \\ -3x + 20y &= -1 \end{aligned}$$

$$x = \frac{\begin{vmatrix} 3 & 4 \\ -1 & 20 \end{vmatrix}}{\begin{vmatrix} 1 & 4 \\ -3 & 20 \end{vmatrix}} = \frac{(3)(20) - (-1)(4)}{(1)(20) - (-3)(4)}$$

$$= \frac{60 + 4}{20 + 12} = \frac{64}{32} = \mathbf{2}$$

$$y = \frac{\begin{vmatrix} 1 & 3 \\ -3 & -1 \end{vmatrix}}{32} = \frac{(1)(-1) - (-3)(3)}{32}$$

$$= \frac{-1 + 9}{32} = \frac{8}{32} = \mathbf{\frac{1}{4}}$$

procedure for solving any number of simultaneous equations.
Consider the three following simultaneous equations:

Col. 1		Col. 2		Col. 3		Col. 4
a_1x	$+$	b_1y	$+$	c_1z	$=$	d_1
a_2x	$+$	b_2y	$+$	c_2z	$=$	d_2
a_3x	$+$	b_3y	$+$	c_3z	$=$	d_3

$$x = \frac{\begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix}}{D}, \quad y = \frac{\begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix}}{D}, \quad z = \frac{\begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}}{D}$$

where

$$D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

$$D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = \underset{\text{Multiplying factor}}{a_1} \underbrace{\left(+ \begin{vmatrix} b_2 & c_2 \\ \underbrace{b_3 \quad c_3}_{\text{Minor}} \end{vmatrix} \right)}_{\text{Cofactor}} + \underset{\text{Multiplying factor}}{b_1} \underbrace{\left(- \begin{vmatrix} a_2 & c_2 \\ \underbrace{a_3 \quad c_3}_{\text{Minor}} \end{vmatrix} \right)}_{\text{Cofactor}} + \underset{\text{Multiplying factor}}{c_1} \underbrace{\left(+ \begin{vmatrix} a_2 & b_2 \\ \underbrace{a_3 \quad b_3}_{\text{Minor}} \end{vmatrix} \right)}_{\text{Cofactor}}$$

$$a_{1(\text{minor})} = \begin{vmatrix} \cancel{a_1} & \cancel{b_1} & \cancel{c_1} \\ \cancel{a_2} & b_2 & c_2 \\ \cancel{a_3} & b_3 & c_3 \end{vmatrix} = \begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix}$$

$$b_{1(\text{minor})} = \begin{vmatrix} \cancel{a_1} & \cancel{b_1} & \cancel{c_1} \\ a_2 & \cancel{b_2} & c_2 \\ a_3 & \cancel{b_3} & c_3 \end{vmatrix} = \begin{vmatrix} a_2 & c_2 \\ a_3 & c_3 \end{vmatrix}$$

EXAMPLE D.7 Expand the following third-order determinants:

a. $$D = \begin{vmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{vmatrix} = 1\left(+\begin{vmatrix} 2 & 1 \\ 1 & 3 \end{vmatrix}\right) + 3\left(-\begin{vmatrix} 2 & 3 \\ 1 & 3 \end{vmatrix}\right) + 2\left(+\begin{vmatrix} 2 & 3 \\ 2 & 1 \end{vmatrix}\right)$$

$$= 1[6 - 1] + 3[-(6 - 3)] + 2[2 - 6]$$

$$= 5 + 3(-3) + 2(-4)$$

$$= 5 - 9 - 8$$

$$= \mathbf{-12}$$

b. $$D = \begin{vmatrix} 0 & 4 & 6 \\ 2 & 0 & 5 \\ 8 & 4 & 0 \end{vmatrix} = 0 + 2\left(-\begin{vmatrix} 4 & 6 \\ 4 & 0 \end{vmatrix}\right) + 8\left(+\begin{vmatrix} 4 & 6 \\ 0 & 5 \end{vmatrix}\right)$$

$$= 0 + 2[-(0 - 24)] + 8[(20 - 0)]$$

$$= 0 + 2(24) + 8(20)$$

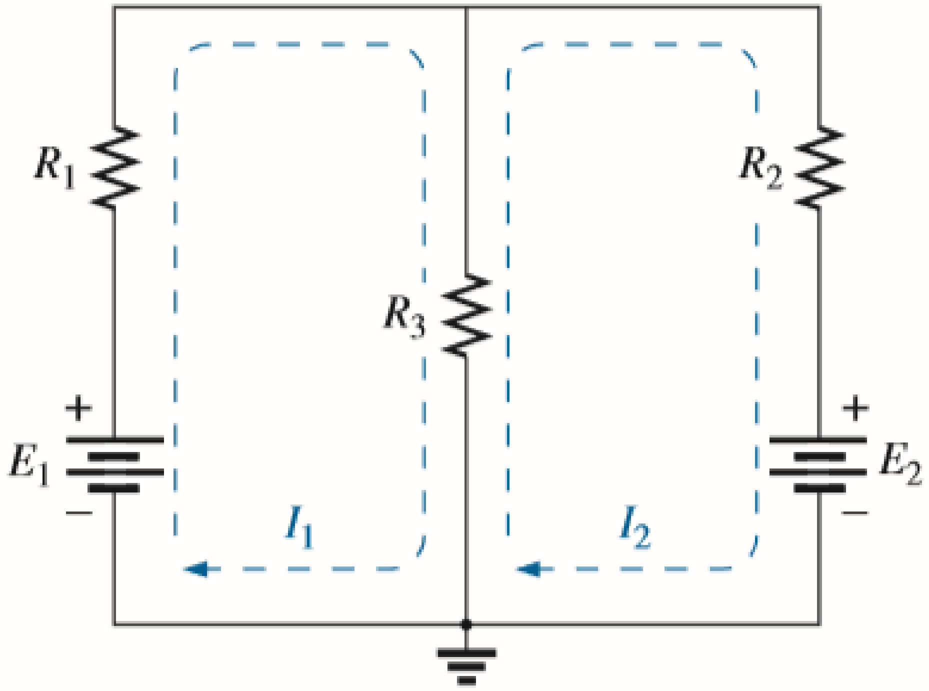
$$= 48 + 160$$

$$= \mathbf{208}$$

8.7 MESH ANALYSIS (GENERAL APPROACH)

The next method to be described—**mesh analysis**—is actually an extension of the branch-current analysis approach just introduced. By defining

The currents to be defined are called **mesh** or **loop currents.**

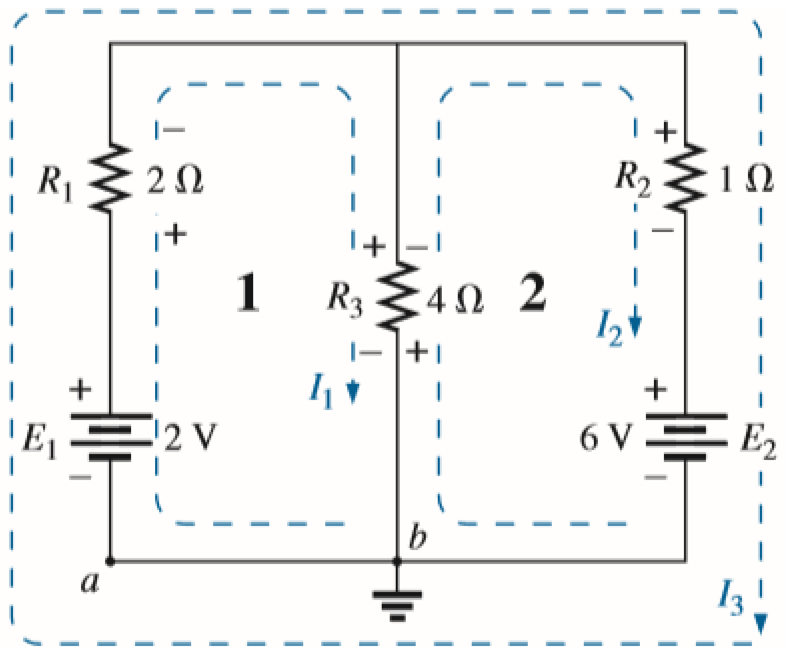


The defined mesh currents can initially be a little confusing because it appears that two currents have been defined for resistor R_3. There is no problem with E_1 and R_1, which have only current I_1, or with E_2 and R_2, which have only current I_2. However, defining the current through R_3 may seem a little troublesome. Actually, it is quite straightforward. The current through R_3 is simply the difference between I_1 and I_2, with the direction of the larger. This is demonstrated in the examples to follow.

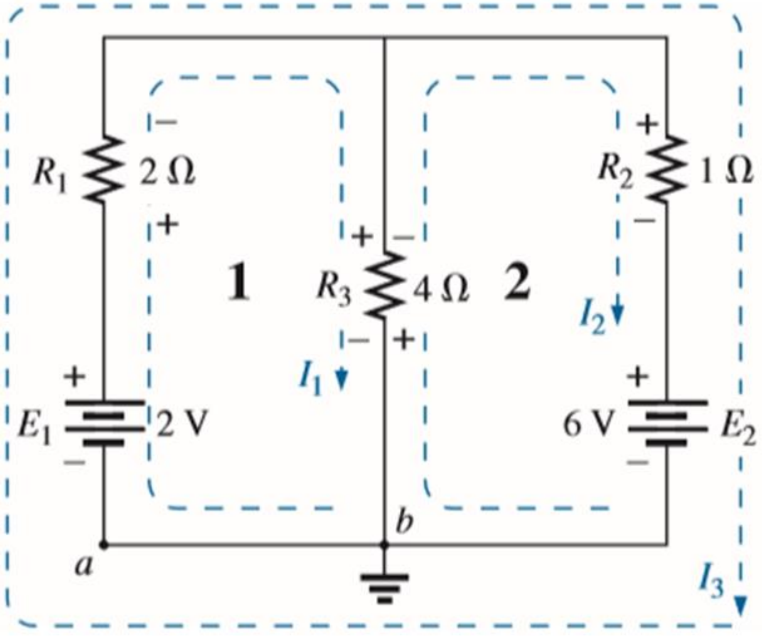
Because mesh currents can result in more than one current through an element, branch-current analysis was introduced first. Branch-current analysis is the straightforward application of the basic laws of electric circuits. Mesh analysis employs a maneuver ("trick," if you prefer) that removes the need to apply Kirchhoff's current law.

Mesh Analysis Procedure

1. ***Assign a distinct current in the clockwise direction to each independent, closed loop of the network. It is not absolutely necessary to choose the clockwise direction for each loop current. In fact, any direction can be chosen for each loop current with no loss in accuracy, as long as the remaining steps are followed properly. However, by choosing the clockwise direction as a standard, we can develop a shorthand method (Section 8.8) for writing the required equations that will save time and possibly prevent some common errors.***



2. *Indicate the polarities within each loop for each resistor as determined by the assumed direction of loop current for that loop. Note the requirement that the polarities be placed within each loop. This requires, as shown in Fig. 8.30, that the 4 Ω resistor have two sets of polarities across it.*
3. *Apply Kirchhoff's voltage law around each closed loop in the clockwise direction. Again, the clockwise direction was chosen to establish uniformity and prepare us for the method to be introduced in the next section.*
   a. *If a resistor has two or more assumed currents through it, the total current through the resistor is the assumed current of the loop in which Kirchhoff's voltage law is being applied, plus the assumed currents of the other loops passing through in the same direction, minus the assumed currents through in the opposite direction.*
   b. *The polarity of a voltage source is unaffected by the direction of the assigned loop currents.*
4. *Solve the resulting simultaneous linear equations for the assumed loop currents.*



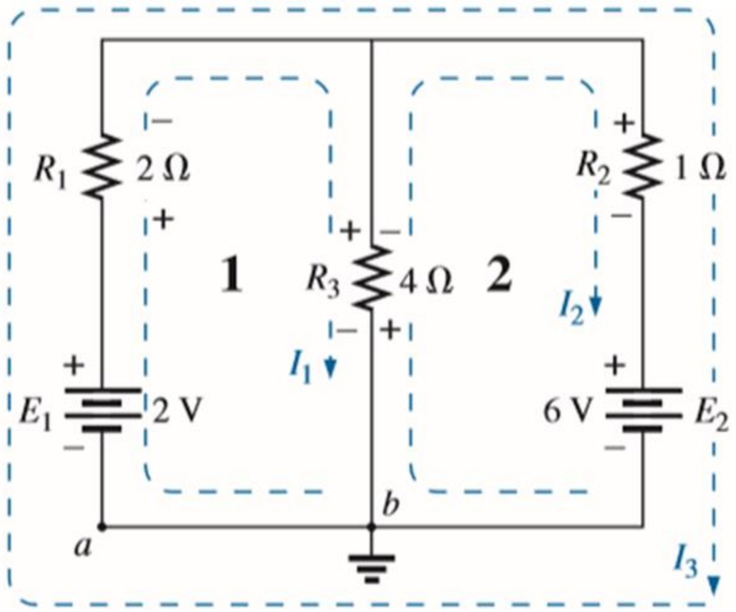
EXAMPLE 8.11 Consider the same basic network as in Example 8.9, now appearing as Fig. 8.30.

Solution:

Step 1: Two loop currents (I_1 and I_2) are assigned in the clockwise direction in the windows of the network. A third loop (I_3) could have been included around the entire network, but the information carried by this loop is already included in the other two.

Step 2: Polarities are drawn within each window to agree with assumed current directions. Note that for this case, the polarities across the 4 Ω resistor are the opposite for each loop current.

Step 3: Kirchhoff's voltage law is applied around each loop in the clockwise direction. Keep in mind as this step is performed that the law is con-



loop 1: $+E_1 - V_1 - V_3 = 0$ (clockwise starting at point a)

$$+2\text{ V} - (2\ \Omega)\,I_1 - \overbrace{(4\ \Omega)\underbrace{(I_1 - I_2)}_{\substack{\text{Total current}\\ \text{through}\\ 4\ \Omega\text{ resistor}}}}^{\substack{\text{Voltage drop across}\\ 4\ \Omega\text{ resistor}}} = 0$$

Subtracted since I_2 is opposite in direction to I_1.

loop 2: $-V_3 - V_2 - E_2 = 0$ (clockwise starting at point b)

$$-(4\ \Omega)(I_2 - I_1) - (1\ \Omega)I_2 - 6\text{ V} = 0$$

Step 4: The equations are then rewritten as follows (without units for clarity):

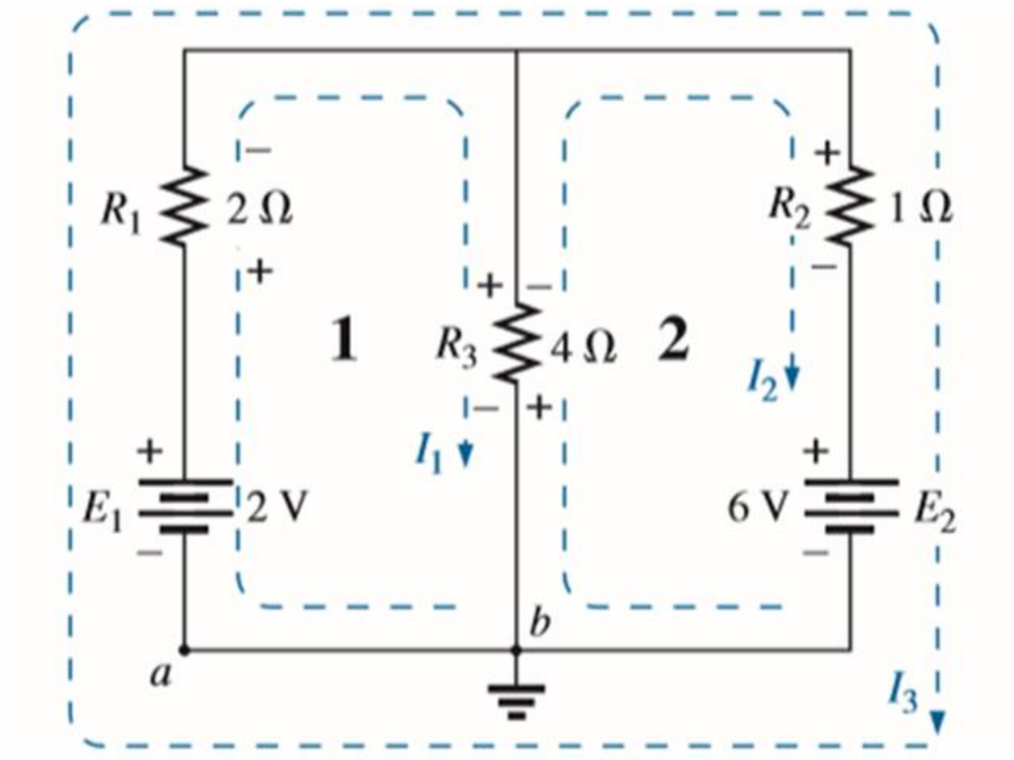
loop 1: $+2 - 2I_1 - 4I_1 + 4I_2 = 0$
loop 2: $-4I_2 + 4I_1 - 1I_2 - 6 = 0$

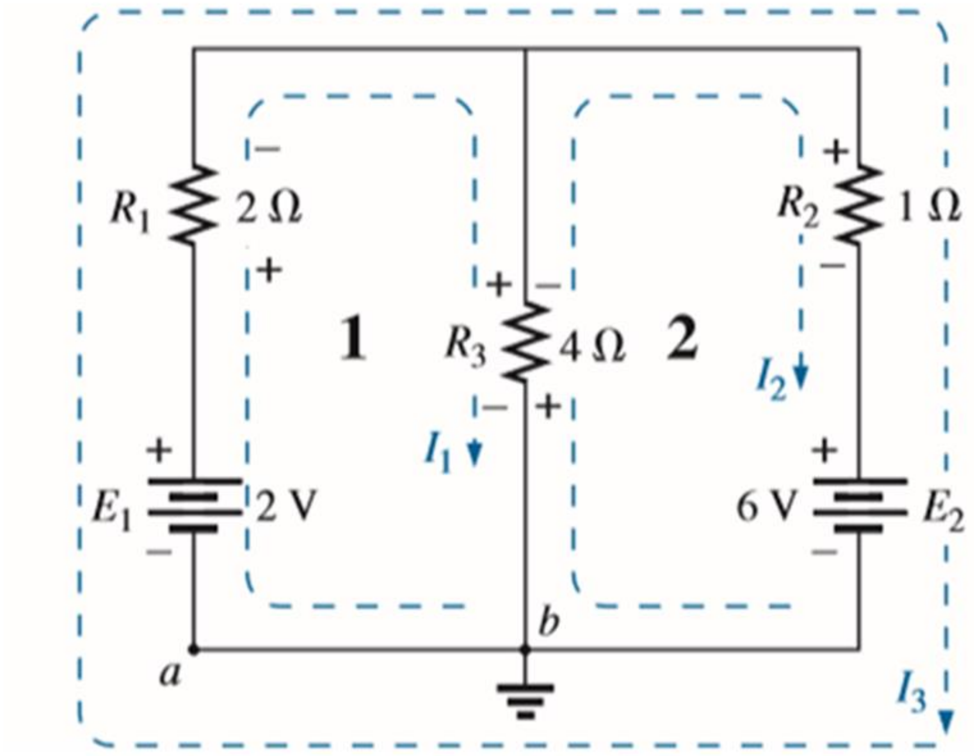
and

loop 1: $+2 - 6I_1 + 4I_2 = 0$
loop 2: $-5I_2 + 4I_1 - 6 = 0$

or

loop 1: $-6I_1 + 4I_2 = -2$
loop 2: $+4I_1 - 5I_2 = +6$





Applying determinants results in

$$I_1 = \mathbf{-1\,A} \quad \text{and} \quad I_2 = \mathbf{-2\,A}$$

The minus signs indicate that the currents have a direction opposite to that indicated by the assumed loop current.

$$\begin{aligned}\text{loop 1:}\quad I_{4\Omega} &= I_1 - I_2 = -1\text{ A} - (-2\text{ A}) = -1\text{ A} + 2\text{ A}\\ &= \mathbf{1\,A} \quad \text{(in the direction of } I_1\text{)}\end{aligned}$$

EXAMPLE 8.12 Find the current through each branch of the network in Fig. 8.31.

Solution:

Steps 1 and 2: These are as indicated in the circuit. Note that the polarities of the 6 Ω resistor are different for each loop current.

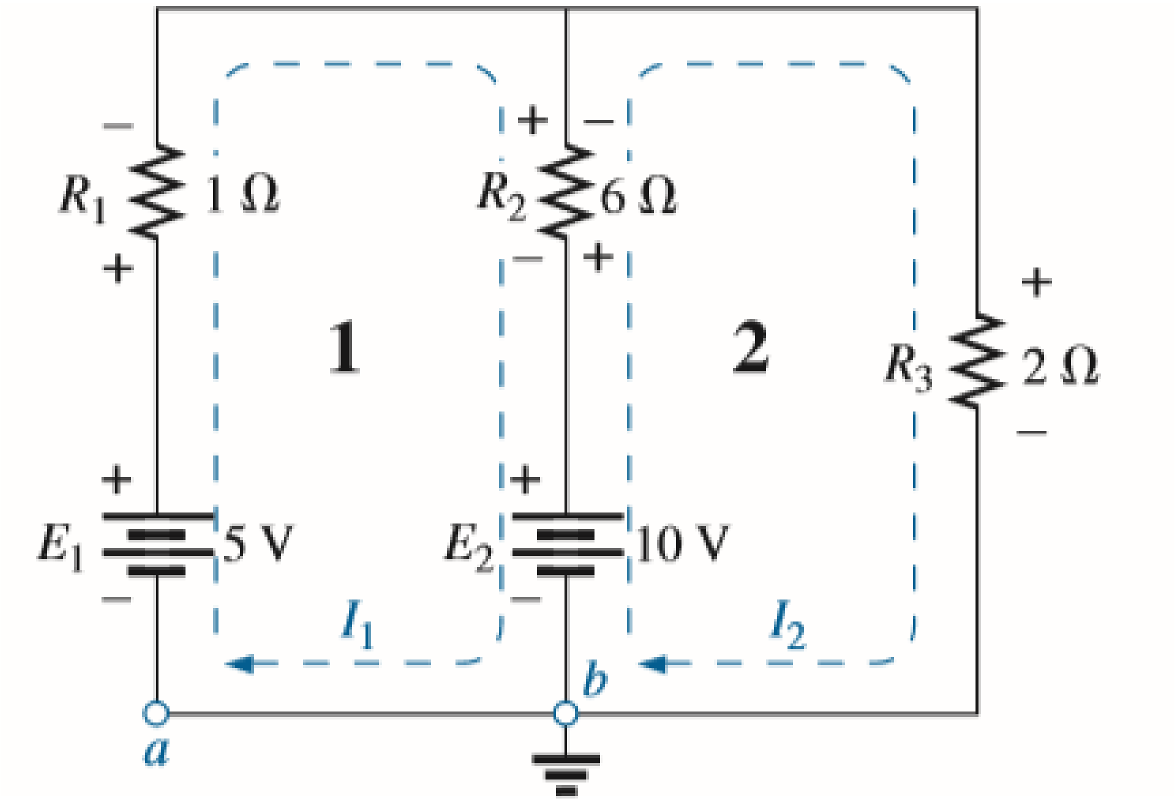


FIG. 8.31

Example 8.12.

Step 3: Kirchhoff's voltage law is applied around each closed loop in the clockwise direction:

loop 1: $+E_1 - V_1 - V_2 - E_2 = 0$ (clockwise starting at point a)

$$+5\text{ V} - (1\ \Omega)I_1 - (6\ \Omega)(I_1 \underset{\uparrow}{-} I_2) - 10\text{ V} = 0$$

I_2 flows through the 6 Ω resistor in the direction opposite to I_1.

loop 2: $E_2 - V_2 - V_3 = 0$ (clockwise starting at point b)

$$+10\text{ V} - (6\ \Omega)(I_2 - I_1) - (2\ \Omega)I_2 = 0$$

The equations are rewritten as

$$\left.\begin{aligned} 5 - I_1 - 6I_1 + 6I_2 - 10 &= 0 \\ 10 - 6I_2 + 6I_1 - 2I_2 &= 0 \end{aligned}\right\} \begin{aligned} -7I_1 + 6I_2 &= 5 \\ +6I_1 - 8I_2 &= -10 \end{aligned}$$

Step 4:

$$I_1 = \frac{\begin{vmatrix} 5 & 6 \\ -10 & -8 \end{vmatrix}}{\begin{vmatrix} -7 & 6 \\ 6 & -8 \end{vmatrix}} = \frac{-40 + 60}{56 - 36} = \frac{20}{20} = \mathbf{1\ A}$$

$$I_2 = \frac{\begin{vmatrix} -7 & 5 \\ 6 & -10 \end{vmatrix}}{20} = \frac{70 - 30}{20} = \frac{40}{20} = \mathbf{2\ A}$$

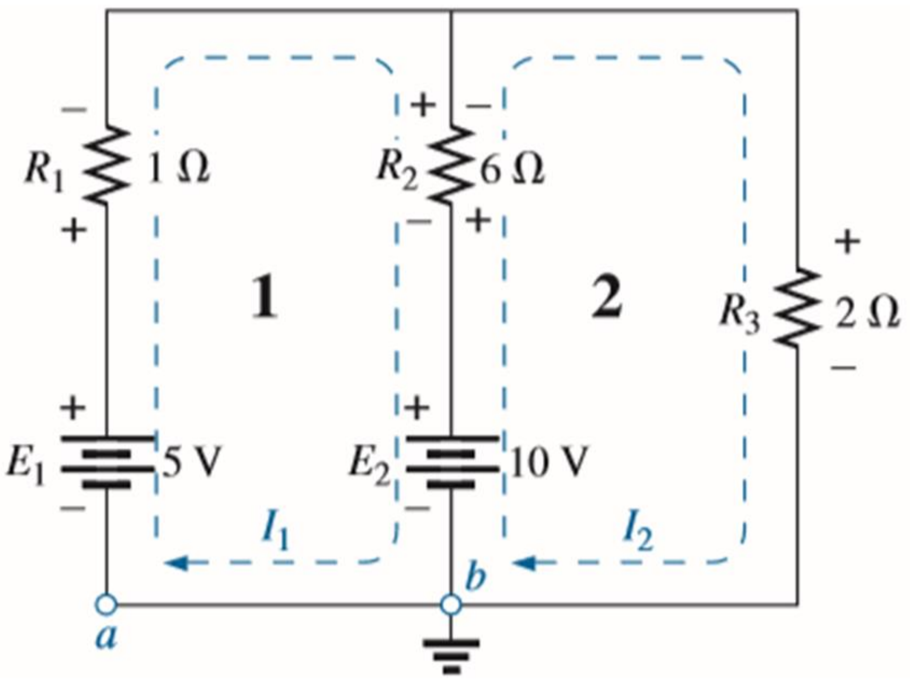


FIG. 8.31
Example 8.12.

Since I_1 and I_2 are positive and flow in opposite directions through the 6 Ω resistor and 10 V source, the total current in this branch is equal to the difference of the two currents in the direction of the larger:

$$I_2 > I_1 \quad (2\ \text{A} > 1\ \text{A})$$

Therefore,

$$I_{R_2} = I_2 - I_1 = 2\ \text{A} - 1\ \text{A} = \mathbf{1\ A} \quad \text{in the direction of } I_2$$

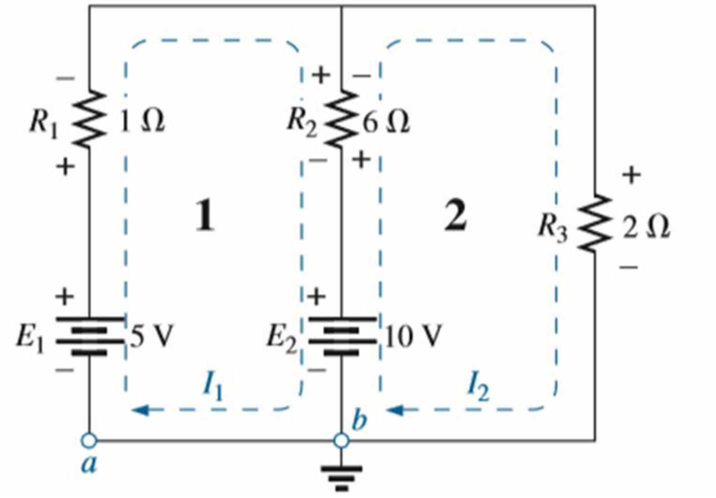


FIG. 8.31
Example 8.12.

It is sometimes impractical to draw all the branches of a circuit at right angles to one another. The next example demonstrates how a portion of a network may appear due to various constraints. The method of analysis is no different with this change in configuration.

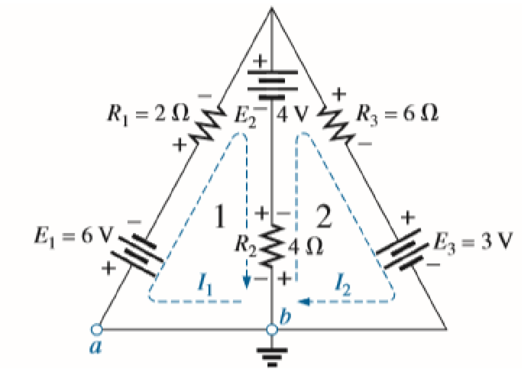
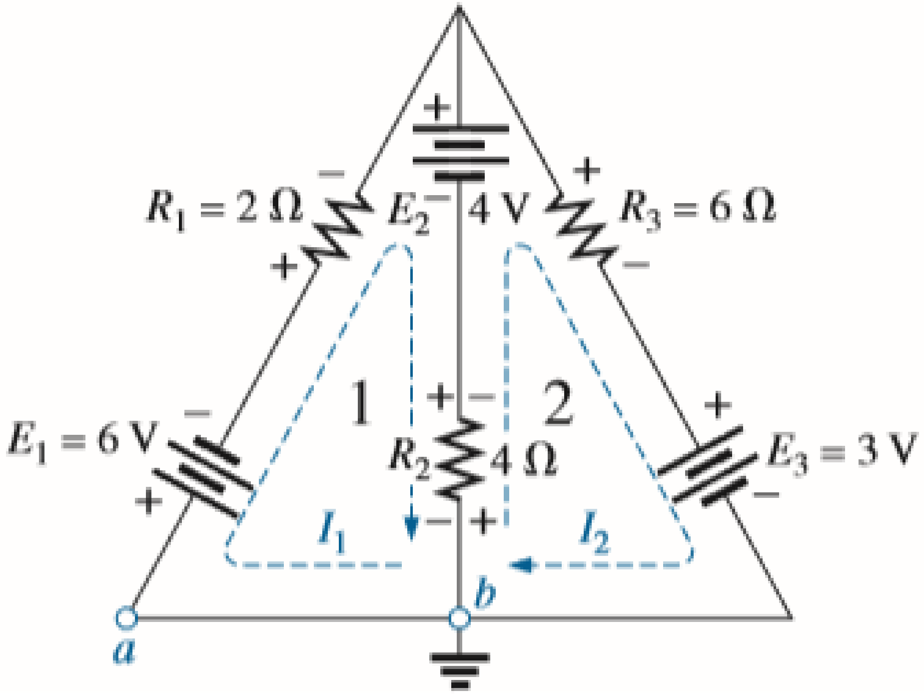


FIG. 8.32



$R_1 = 2\ \Omega$
E_2
$4\ \text{V}$
$R_3 = 6\ \Omega$
$E_1 = 6\ \text{V}$
R_2
$4\ \Omega$
$E_3 = 3\ \text{V}$
1
2
I_1
I_2
a
b

FIG. 8.32

Supermesh Currents

Occasionally, you will find current sources in a network without a parallel resistance. This removes the possibility of converting the source to a voltage source as required by the given procedure. In such cases, you

use the Supermesh approach

Start as before and assign a mesh current to each independent loop, including the current sources, as if they were resistors or voltage sources. Then mentally (redraw the network if necessary) remove the current sources (replace with open-circuit equivalents), and apply Kirchhoff's voltage law to all the remaining independent paths of the network using the mesh currents just defined. Any resulting path, including two or more mesh currents, is said to be the path of a **supermesh current.** Then relate the chosen mesh currents of the network to the independent current sources of the network, and solve for the mesh currents. The next example clarifies the definition of supermesh current and the procedure.

EXAMPLE 8.14 Using mesh analysis, determine the currents of the network in Fig. 8.33.

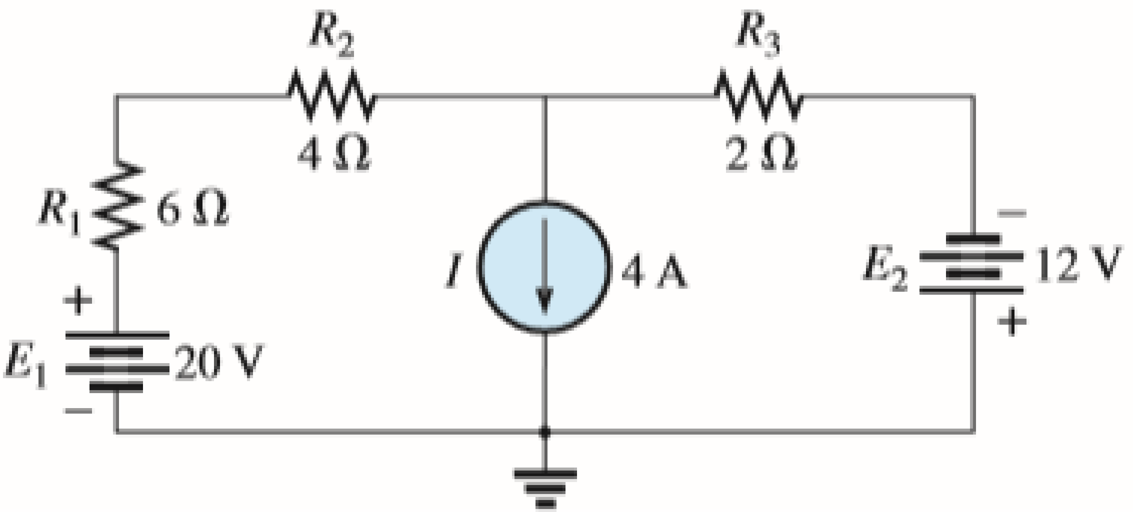


FIG. 8.33
Example 8.14.

Solution: First, the mesh currents for the network are defined, as shown in Fig. 8.34. Then the current source is mentally removed, as shown in Fig. 8.35, and Kirchhoff's voltage law is applied to the resulting network. The single path now including the effects of two mesh currents is referred to as the path of a *supermesh current.*

Applying Kirchhoff's law:

$$20\text{ V} - I_1(6\ \Omega) - I_1(4\ \Omega) - I_2(2\ \Omega) + 12\text{ V} = 0$$

or

$$10I_1 + 2I_2 = 32$$

Node a is then used to relate the mesh currents and the current source using Kirchhoff's current law:

$$I_1 = I + I_2$$

The result is two equations and two unknowns:

$$\begin{aligned} 10I_1 + 2I_2 &= 32 \\ I_1 - \ \ I_2 &= 4 \end{aligned}$$

Applying determinants:

$$I_1 = \frac{\begin{vmatrix} 32 & 2 \\ 4 & -1 \end{vmatrix}}{\begin{vmatrix} 10 & 2 \\ 1 & -1 \end{vmatrix}} = \frac{(32)(-1) - (2)(4)}{(10)(-1) - (2)(1)} = \frac{40}{12} = \mathbf{3.33\ A}$$

and

$$I_2 = I_1 - I = 3.33\text{ A} - 4\text{ A} = \mathbf{-0.67\ A}$$

In the above analysis, it may appear that when the current source was removed, $I_1 = I_2$. However, the supermesh approach requires that we stick with the original definition of each mesh current and not alter those definitions when current sources are removed.

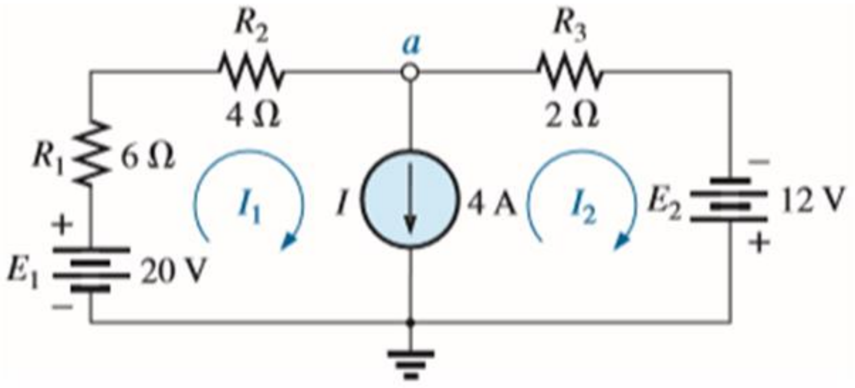


FIG. 8.34
Defining the mesh currents for the network in Fig. 8.33.

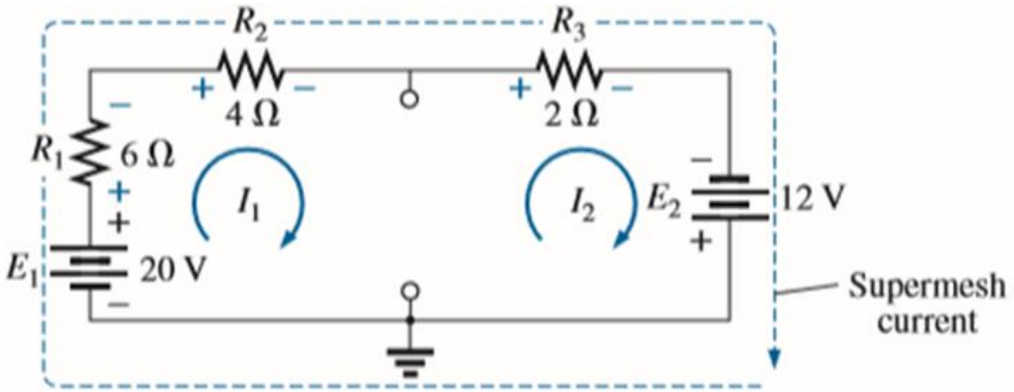


FIG. 8.35
Defining the supermesh current.

EXAMPLE 8.15 Using mesh analysis, determine the currents for the network in Fig. 8.36.

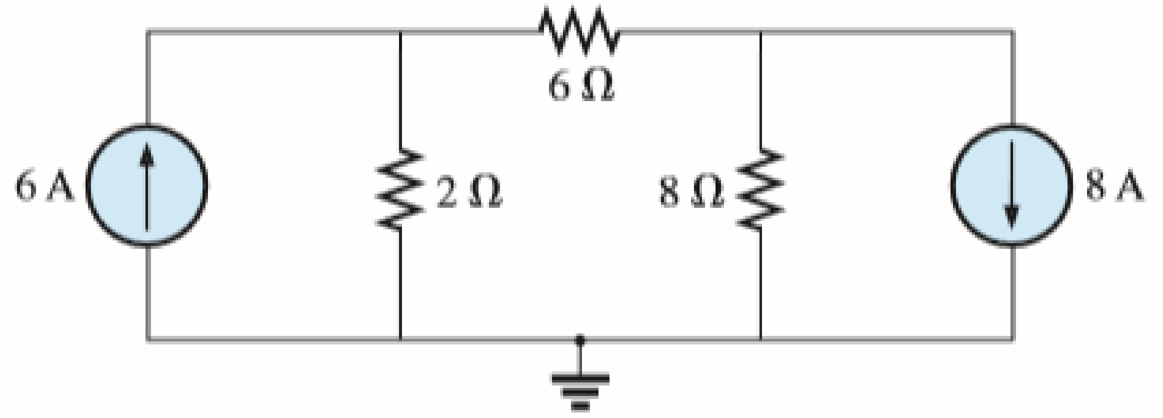


FIG. 8.36
Example 8.15.

Solution: The mesh currents are defined in Fig. 8.37. The current sources are removed, and the single supermesh path is defined in Fig. 8.38.

Applying Kirchhoff's voltage law around the supermesh path:

$$-V_{2\Omega} - V_{6\Omega} - V_{8\Omega} = 0$$
$$-(I_2 - I_1)2\ \Omega - I_2(6\ \Omega) - (I_2 - I_3)8\ \Omega = 0$$
$$-2I_2 + 2I_1 - 6I_2 - 8I_2 + 8I_3 = 0$$
$$2I_1 - 16I_2 + 8I_3 = 0$$

Introducing the relationship between the mesh currents and the current sources:

$$I_1 = 6\ \text{A}$$
$$I_3 = 8\ \text{A}$$

results in the following solutions:

$$2I_1 - 16I_2 + 8I_3 = 0$$
$$2(6\ \text{A}) - 16I_2 + 8(8\ \text{A}) = 0$$

and $$I_2 = \frac{76\ \text{A}}{16} = \mathbf{4.75\ A}$$

Then $$I_{2\Omega}\downarrow = I_1 - I_2 = 6\ \text{A} - 4.75\ \text{A} = \mathbf{1.25\ A}$$

and $$I_{8\Omega}\uparrow = I_3 - I_2 = 8\ \text{A} - 4.75\ \text{A} = \mathbf{3.25\ A}$$

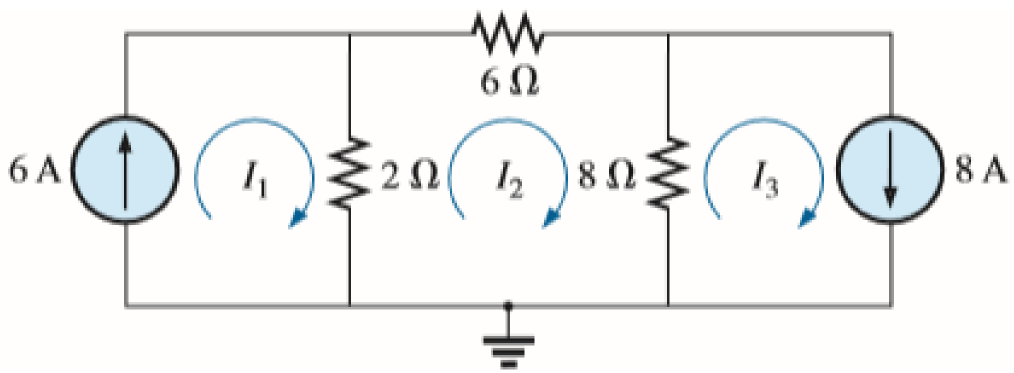


FIG. 8.37

Defining the mesh currents for the network in Fig. 8.36.

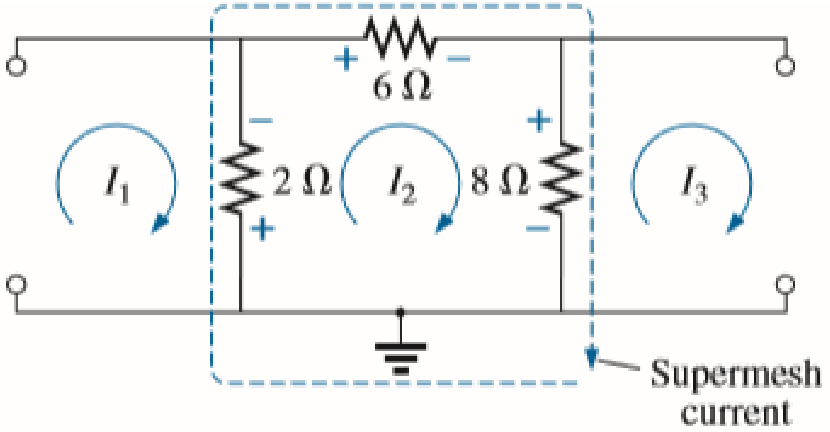


FIG. 8.38

Defining the supermesh current for the network in Fig. 8.36.

EXAMPLE 8.17 Write the mesh equations for the network in Fig. 8.41.

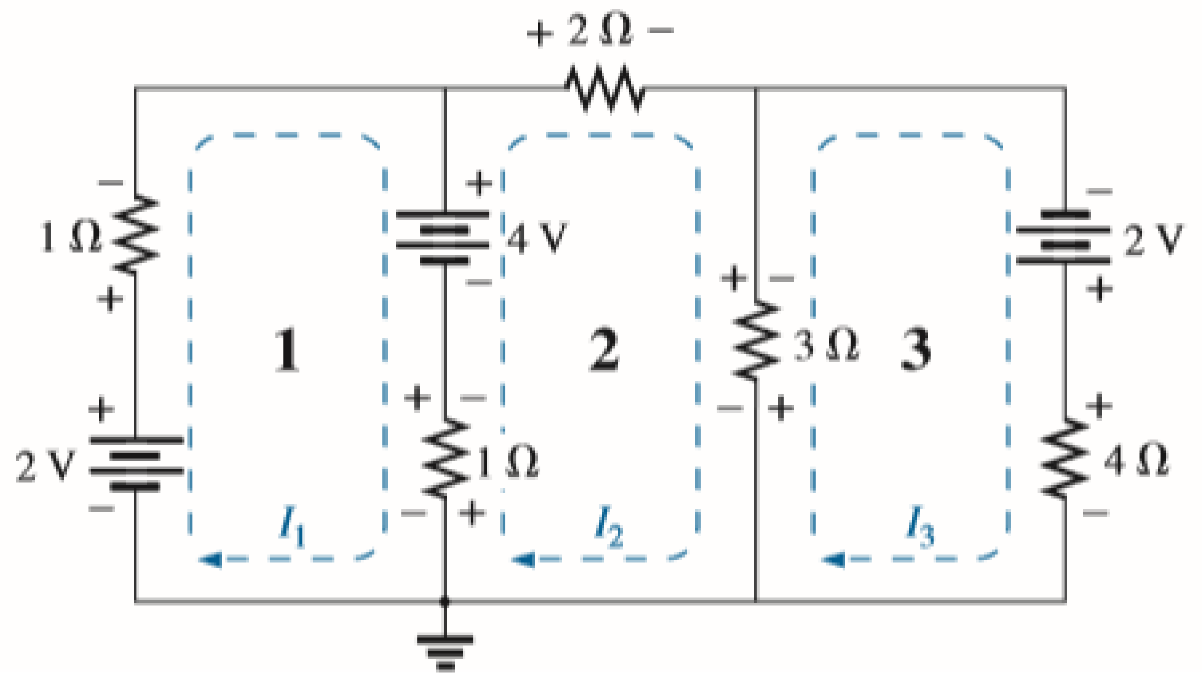


FIG. 8.41
Example 8.17.

EXAMPLE 8.18 Find the current through the 10 Ω resistor of the network in Fig. 8.42.

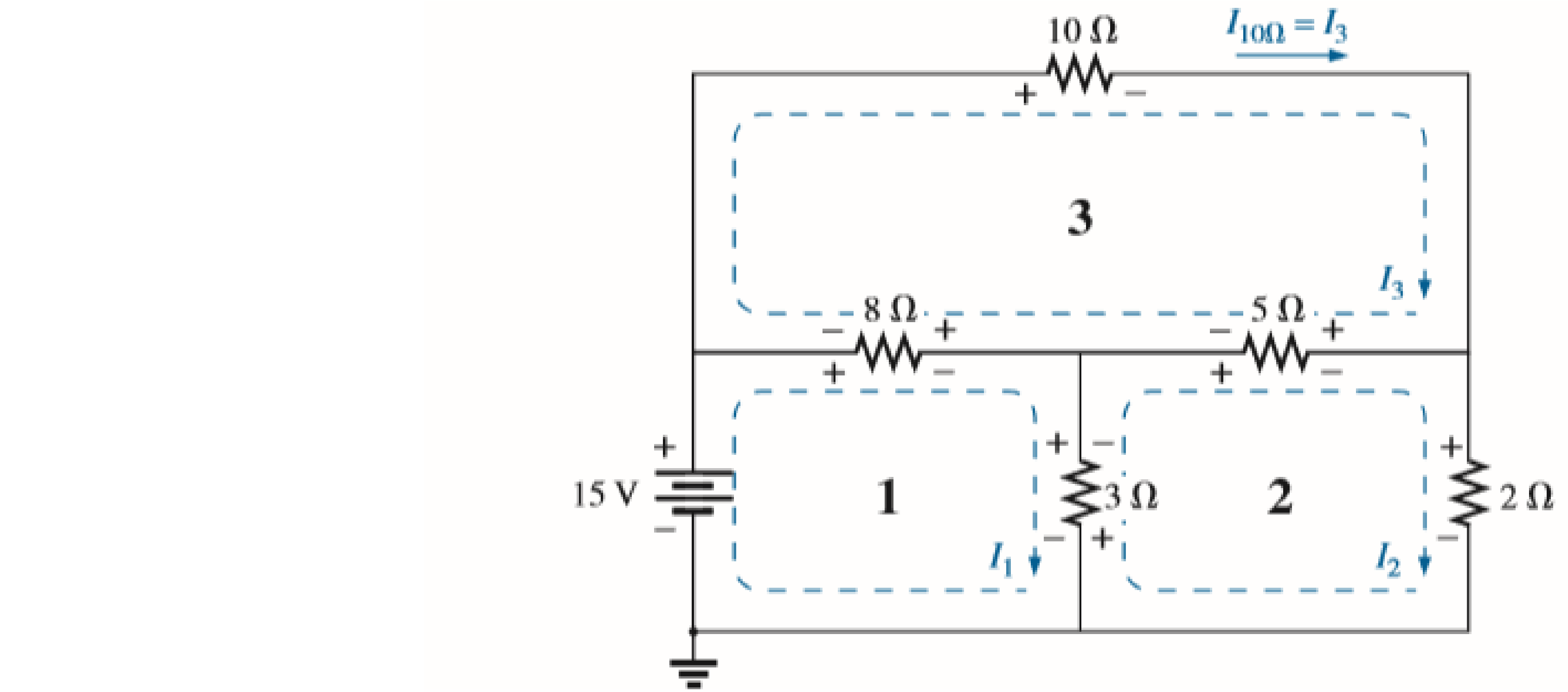


FIG. 8.42

8.9 NODAL ANALYSIS (GENERAL APPROACH)

The methods introduced thus far have all been to find the currents of the network. We now turn our attention to **nodal analysis**—a method that provides the nodal voltages of a network, that is, the voltage from the various **nodes** (junction points) of the network to ground. The method is developed through the use of Kirchhoff's current law in much the same manner as Kirchhoff's voltage law was used to develop the mesh analysis approach.

Although it is not a requirement, we make it a policy to make ground our reference node and assign it a potential level of zero volts. All the other voltage levels are then found with respect to this reference level. For

Nodal Analysis Procedure

1. Determine the number of nodes within the network.

2. Pick a reference node, and label each remaining node with a subscripted value of voltage: V_1, V_2, and so on.

3. Apply Kirchhoff's current law at each node except the reference. Assume that all unknown currents leave the node for each application of Kirchhoff's current law. In other words, for each node, don't be influenced by the direction that an unknown current for another node may have had. Each node is to be treated as a separate entity, independent of the application of Kirchhoff's current law to the other nodes.

4. Solve the resulting equations for the nodal voltages.

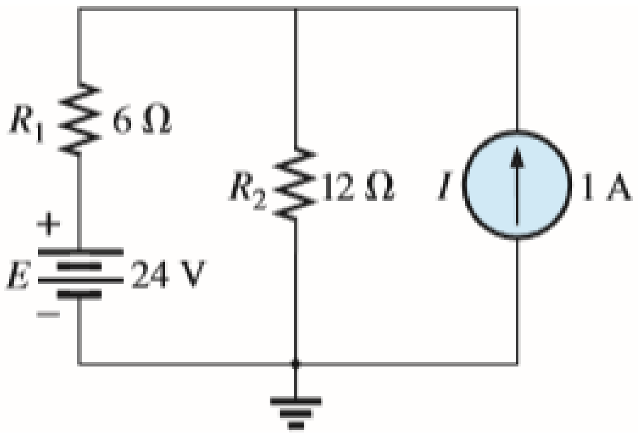


FIG. 8.46
Example 8.19.

A few examples clarify the procedure defined by step 3. It initially takes some practice writing the equations for Kirchhoff's current law correctly, but in time the advantage of assuming that all the currents leave a node rather than identifying a specific direction for each branch become obvious. (The same type of advantage is associated with assuming that all the mesh currents are clockwise when applying mesh analysis.)

EXAMPLE 8.19 Apply nodal analysis to the network in Fig. 8.46.

Solution:

Steps 1 and 2: The network has two nodes, as shown in Fig. 8.47. The lower node is defined as the reference node at ground potential (zero volts), and the other node as V_1, the voltage from node 1 to ground.

Step 3: I_1 and I_2 are defined as leaving the node in Fig. 8.48, and Kirchhoff's current law is applied as follows:

$$I = I_1 + I_2$$

The current I_2 is related to the nodal voltage V_1 by Ohm's law:

$$I_2 = \frac{V_{R_2}}{R_2} = \frac{V_1}{R_2}$$

The current I_1 is also determined by Ohm's law as follows:

$$I_1 = \frac{V_{R_1}}{R_1}$$

with $\qquad V_{R_1} = V_1 - E$

Substituting into the Kirchhoff's current law equation:

$$I = \frac{V_1 - E}{R_1} + \frac{V_1}{R_2}$$

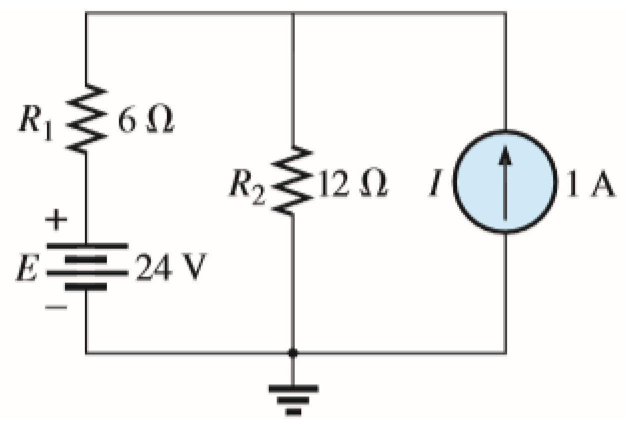
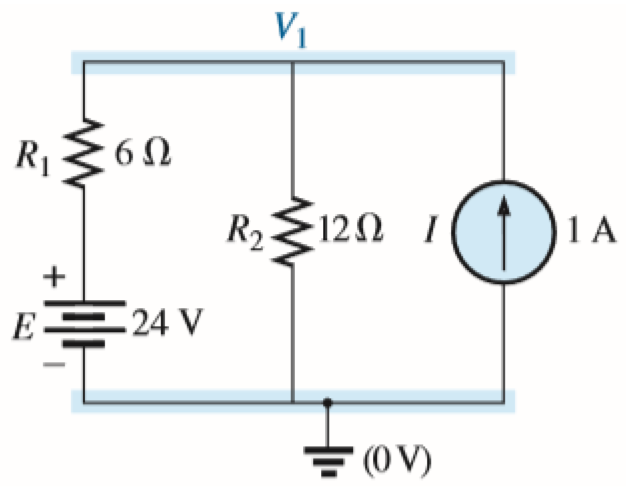


FIG. 8.46
Example 8.19.



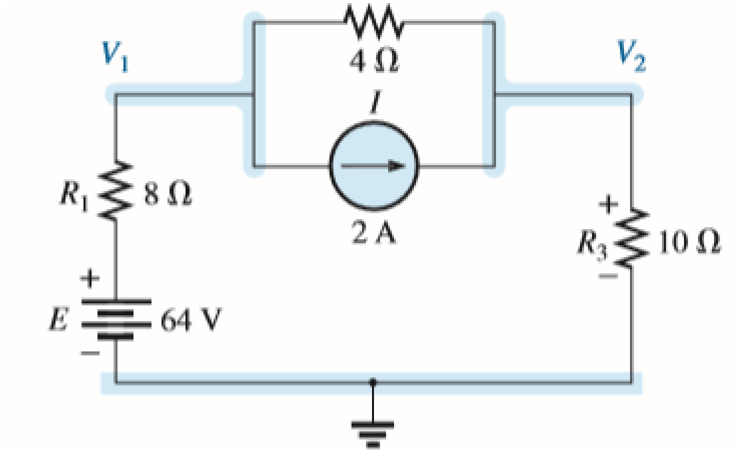


FIG. 8.50
Defining the nodes for the network in Fig. 8.49.

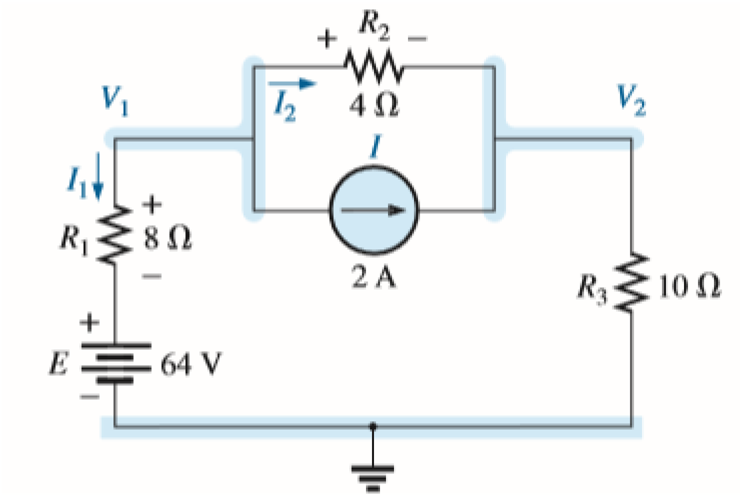


FIG. 8.51
Applying Kirchhoff's current law to node V_1.

EXAMPLE 8.20 Apply nodal analysis to the network in Fig. 8.49.

Solution:

Steps 1 and 2: The network has three nodes, as defined in Fig. 8.50, with the bottom node again defined as the reference node (at ground potential, or zero volts), and the other nodes as V_1 and V_2.

Step 3: For node V_1, the currents are defined as shown in Fig. 8.51 and Kirchhoff's current law is applied:

$$0 = I_1 + I_2 + I$$

with
$$I_1 = \frac{V_1 - E}{R_1}$$

and
$$I_2 = \frac{V_{R_2}}{R_2} = \frac{V_1 - V_2}{R_2}$$

so that
$$\frac{V_1 - E}{R_1} + \frac{V_1 - V_2}{R_2} + I = 0$$

or
$$\frac{V_1}{R_1} - \frac{E}{R_1} + \frac{V_1}{R_2} - \frac{V_2}{R_2} + I = 0$$

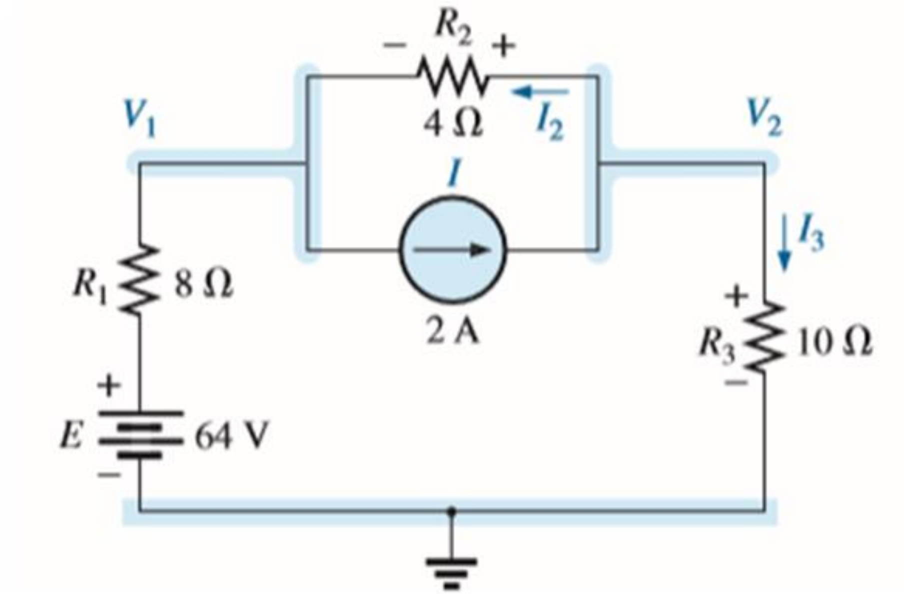


FIG. 8.52
Applying Kirchhoff's current law to node V_2.

For node V_2 the currents are defined as shown in Fig. 8.52, and Kirchhoff's current law is applied:

$$I = I_2 + I_3$$

with
$$I = \frac{V_2 - V_1}{R_2} + \frac{V_2}{R_3}$$

or
$$I = \frac{V_2}{R_2} - \frac{V_1}{R_2} + \frac{V_2}{R_3}$$

which become

$$\begin{array}{r} 0.375V_1 - 0.25V_2 = 6 \\ -0.25V_1 + 0.35V_2 = 2 \\ \hline \end{array}$$

Using determinants,

$$V_1 = \mathbf{37.82\ V}$$
$$V_2 = \mathbf{32.73\ V}$$

Since E is greater than V_1, the current I_1 flows from ground to V_1 and is equal to

$$I_{R_1} = \frac{E - V_1}{R_1} = \frac{64\text{ V} - 37.82\text{ V}}{8\ \Omega} = \mathbf{3.27\ A}$$

The positive value for V_2 results in a current I_{R_3} from node V_2 to ground equal to

$$I_{R_3} = \frac{V_{R_3}}{R_3} = \frac{V_2}{R_3} = \frac{32.73\text{ V}}{10\ \Omega} = \mathbf{3.27\ A}$$

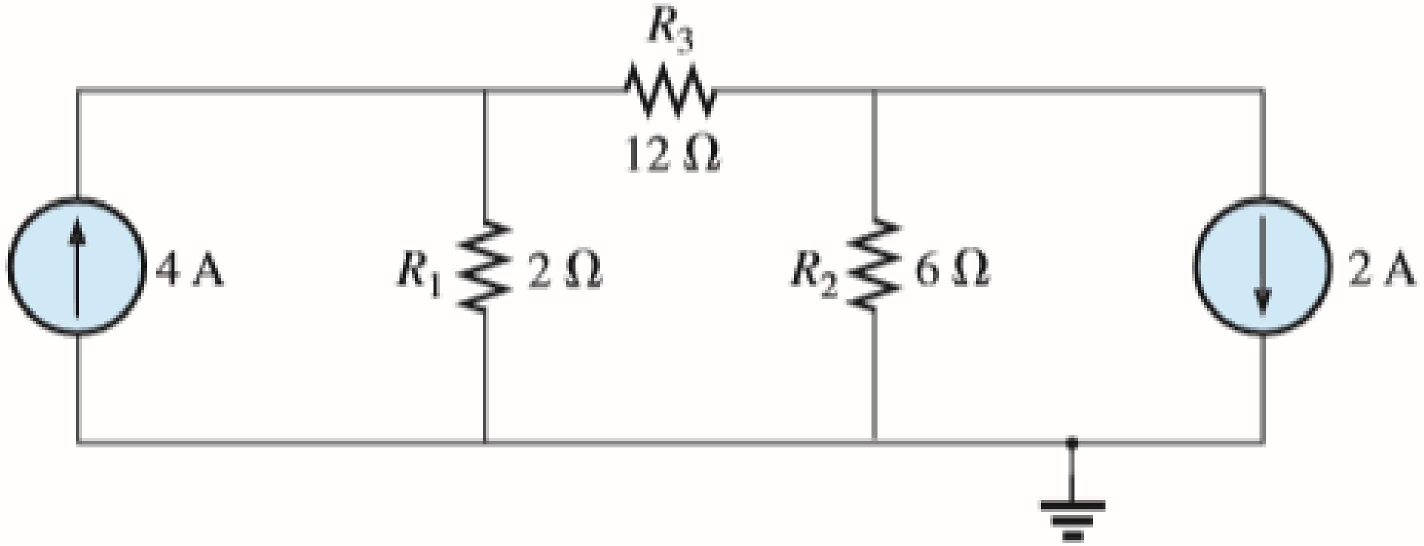
Since V_1 is greater than V_2, the current I_{R_2} flows from V_1 to V_2 and is equal to

$$I_{R_2} = \frac{V_1 - V_2}{R_2} = \frac{37.82\text{ V} - 32.73\text{ V}}{4\ \Omega} = \mathbf{1.27\ A}$$

Step 4: The result is two equations and two unknowns:

$$\begin{array}{r} V_1\left(\dfrac{1}{8\ \Omega} + \dfrac{1}{4\ \Omega}\right) - V_2\left(\dfrac{1}{4\ \Omega}\right) = 6\text{ A} \\ -V_1\left(\dfrac{1}{4\ \Omega}\right) + V_2\left(\dfrac{1}{4\ \Omega} + \dfrac{1}{10\ \Omega}\right) = 2\text{ A} \\ \hline \end{array}$$

EXAMPLE 8.21 Determine the nodal voltages for the network in Fig. 8.54.



Solution:

Steps 1 and 2: As indicated in Fig. 8.55:

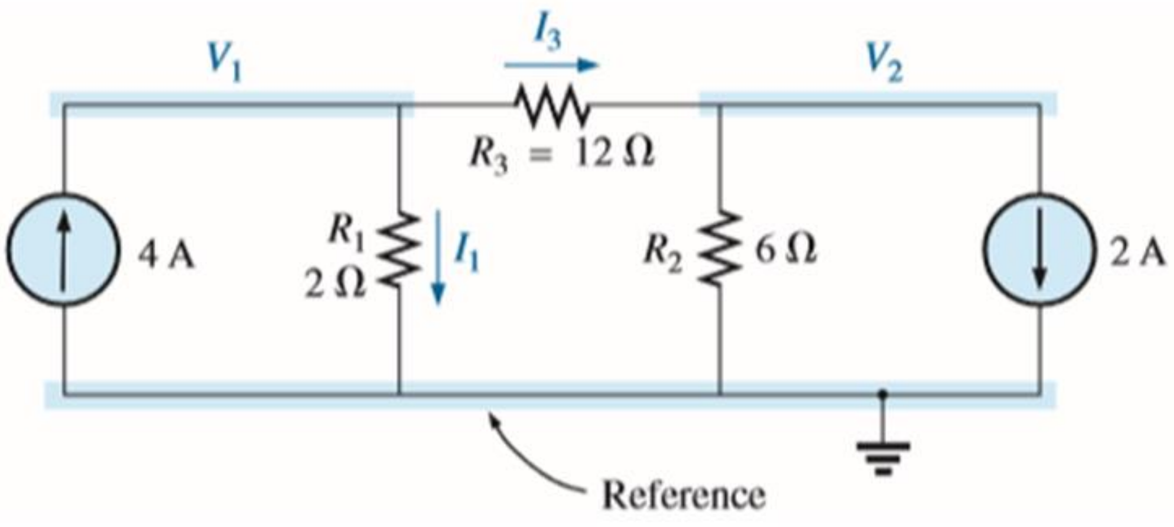


FIG. 8.55

Defining the nodes and applying Kirchhoff's current law to the node V_1.

Step 3: Included in Fig. 8.55 for the node V_1. Applying Kirchhoff's current law:

$$4\ \text{A} = I_1 + I_3$$

and

$$4\ \text{A} = \frac{V_1}{R_1} + \frac{V_1 - V_2}{R_3} = \frac{V_1}{2\ \Omega} + \frac{V_1 - V_2}{12\ \Omega}$$

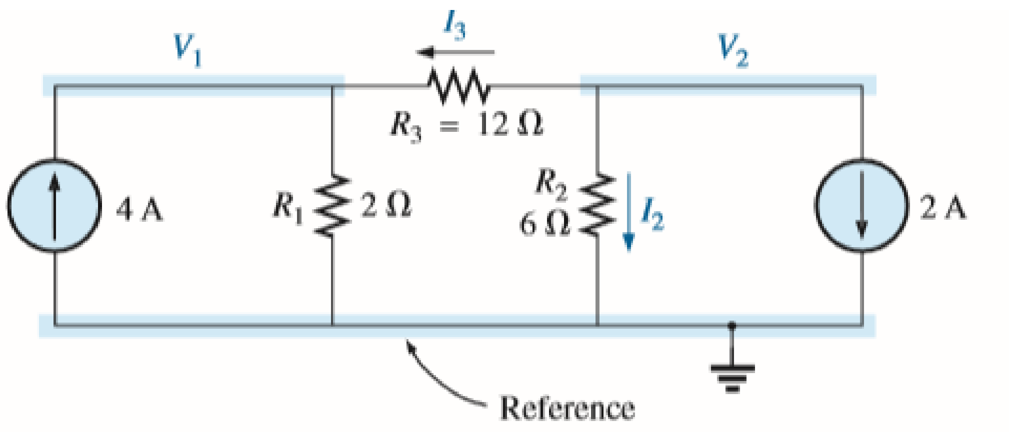


FIG. 8.56

Applying Kirchhoff's current law to the node V_2.

Applying Kirchhoff's current law:

$$0 = I_3 + I_2 + 2\text{ A}$$

and $$\frac{V_2 - V_1}{R_3} + \frac{V_2}{R_2} + 2\text{ A} = 0 \longrightarrow \frac{V_2 - V_1}{12\ \Omega} + \frac{V_2}{2\ \Omega} + 2\text{ A} = 0$$

Expanding and rearranging:

$$V_2\left(\frac{1}{12\ \Omega} + \frac{1}{6\ \Omega}\right) - V_1\left(\frac{1}{12\ \Omega}\right) = -2\text{ A}$$

resulting in Eq. (8.1), which consists of two equations and two unknowns:

$$\left.\begin{aligned} V_1\left(\frac{1}{2\ \Omega} + \frac{1}{12\ \Omega}\right) - V_2\left(\frac{1}{12\ \Omega}\right) &= +4\text{ A} \\ V_2\left(\frac{1}{12\ \Omega} + \frac{1}{6\ \Omega}\right) - V_1\left(\frac{1}{12\ \Omega}\right) &= -2\text{ A} \end{aligned}\right\} \qquad \textbf{(8.1)}$$

and $$V_1 = \frac{\begin{vmatrix} 48 & -1 \\ -24 & 3 \end{vmatrix}}{\begin{vmatrix} 7 & -1 \\ -1 & 3 \end{vmatrix}} = \frac{120}{20} = \mathbf{+6\ V}$$

$$V_2 = \frac{\begin{vmatrix} 7 & 48 \\ -1 & -24 \end{vmatrix}}{20} = \frac{-120}{20} = \mathbf{-6\ V}$$

Since V_1 is greater than V_2, the current through R_3 passes from V_1 to V_2. Its value is

$$I_{R_3} = \frac{V_1 - V_2}{R_3} = \frac{6\ \text{V} - (-6\ \text{V})}{12\ \Omega} = \frac{12\ \text{V}}{12\ \Omega} = \mathbf{1\ A}$$

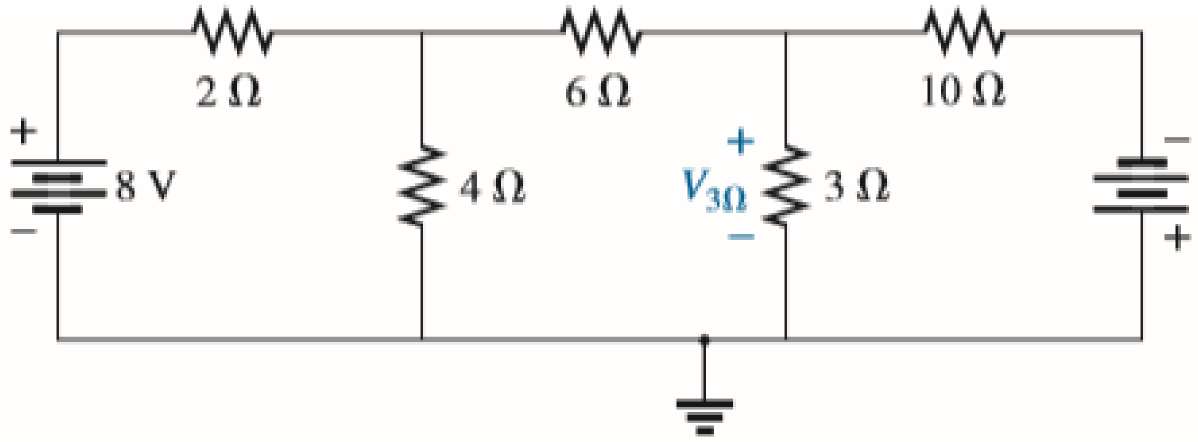
The fact that V_1 is positive results in a current I_{R_1} from V_1 to ground equal to

$$I_{R_1} = \frac{V_{R_1}}{R_1} = \frac{V_1}{R_1} = \frac{6\ \text{V}}{2\ \Omega} = \mathbf{3\ A}$$

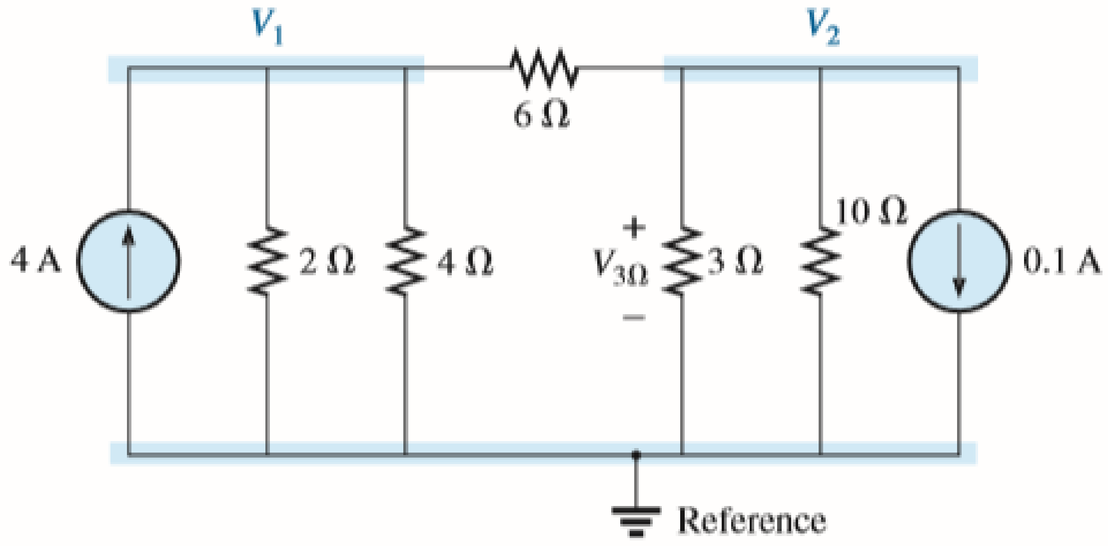
Finally, since V_2 is negative, the current I_{R_2} flows from ground to V_2 and is equal to

$$I_{R_2} = \frac{V_{R_2}}{R_2} = \frac{V_2}{R_2} = \frac{6\ \text{V}}{6\ \Omega} = \mathbf{1\ A}$$

EXAMPLE 8.24 Find the voltage across the 3 Ω resistor in Fig. 8.61 by nodal analysis.



Solution: Converting sources and choosing nodes (Fig. 8.62), we have



EXAMPLE 8.25 Using nodal analysis, determine the potential across the 4 Ω resistor in Fig. 8.63.

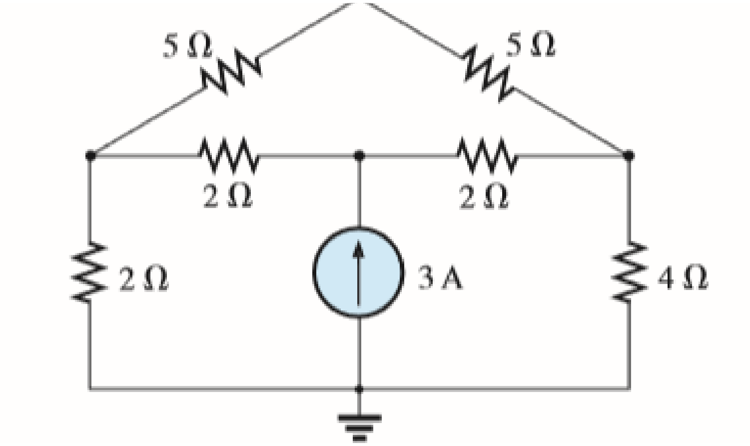
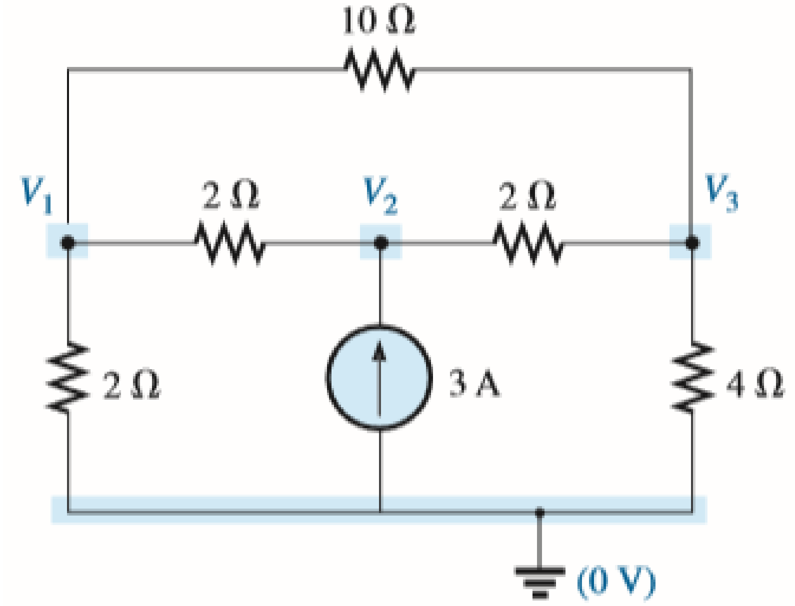
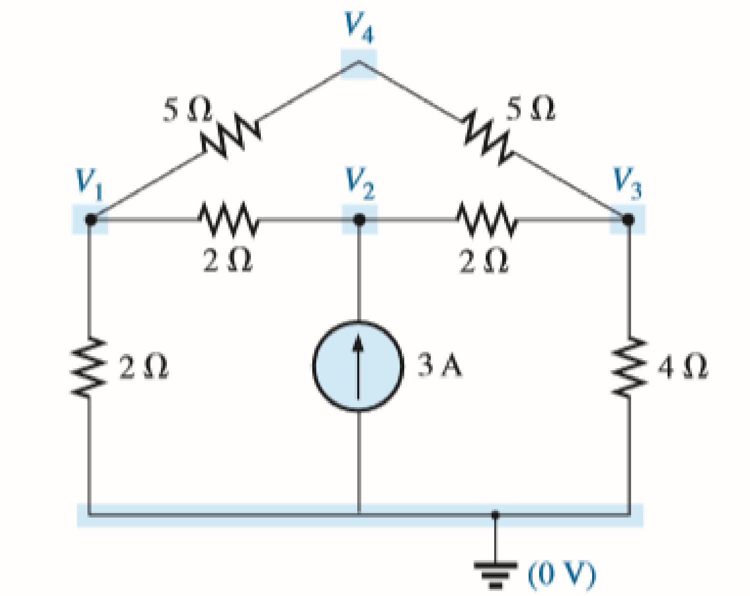


FIG. 8.63
Example 8.25.





$$
\begin{aligned}
1.1V_1 - 0.5V_2 - 0.1V_3 &= 0\\
V_2 - 0.5V_1 - 0.5V_3 &= 3\\
0.85V_3 - 0.5V_2 - 0.1V_1 &= 0
\end{aligned}
$$

$$
V_3 = V_{4\Omega} = \frac{\begin{vmatrix} 1.1 & -0.5 & 0 \\ -0.5 & +1 & 3 \\ -0.1 & -0.5 & 0 \end{vmatrix}}{\begin{vmatrix} 1.1 & -0.5 & -0.1 \\ -0.5 & +1 & -0.5 \\ -0.1 & -0.5 & +0.85 \end{vmatrix}} = \mathbf{4.65\ V}
$$

EXAMPLE 8.26 Write the nodal equations and find the voltage across the 2 Ω resistor for the network in Fig. 8.67.

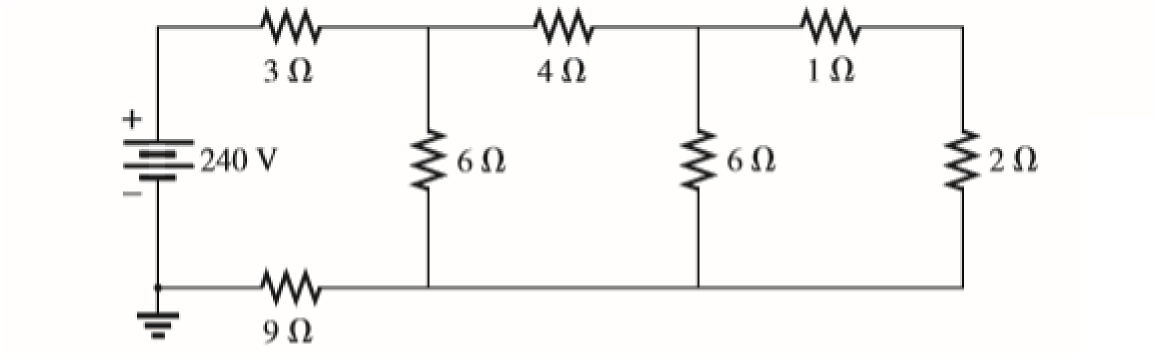
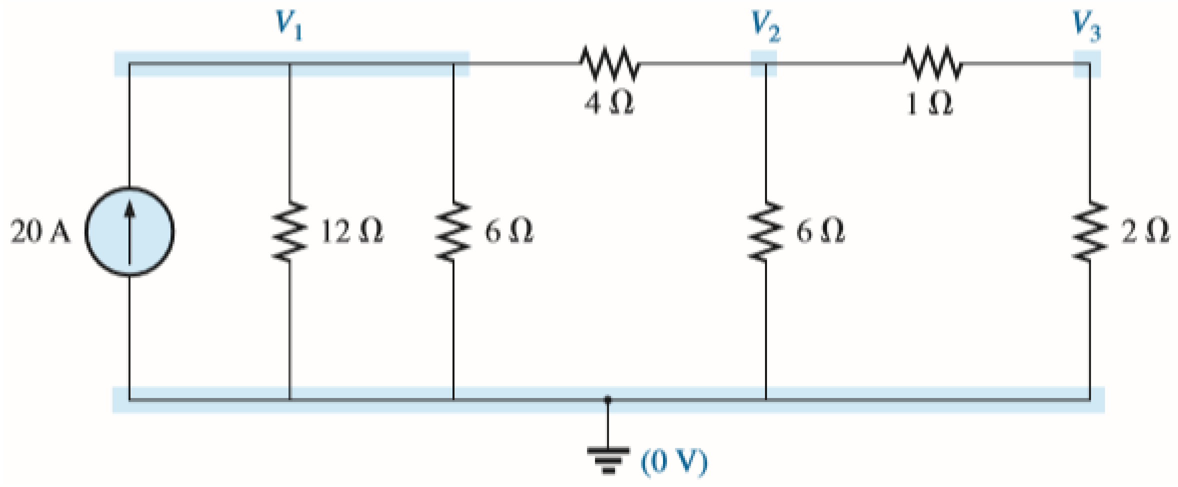


FIG. 8.67
Example 8.26.

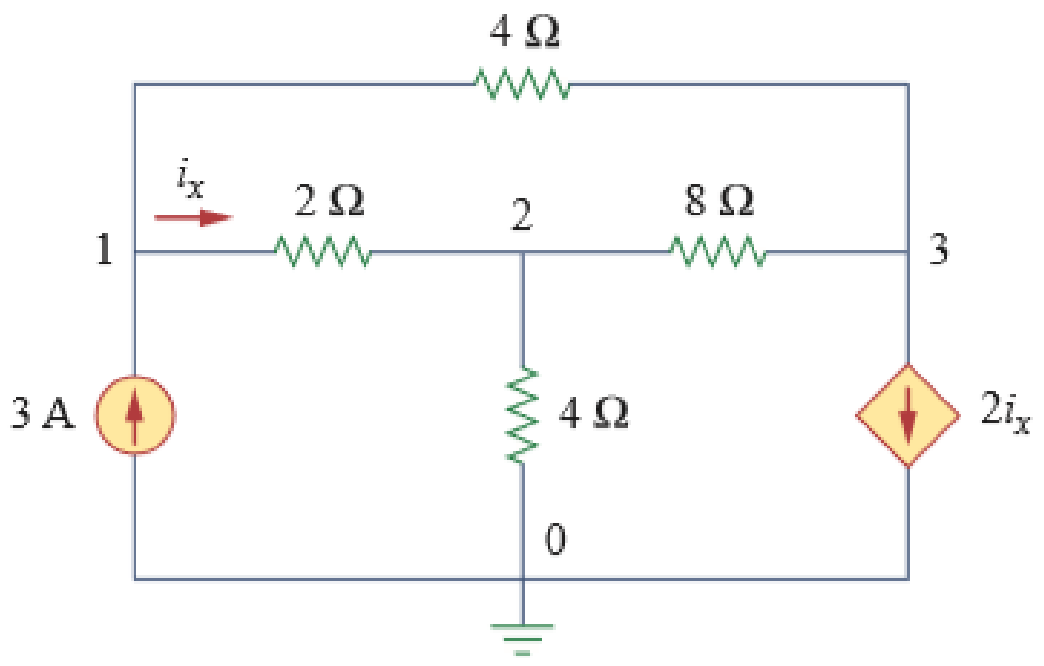
$$
\begin{aligned}
0.5V_1 - 0.25V_2 + \quad 0 &= 20 \\
-0.25V_1 + \frac{17}{12}V_2 - 1V_3 &= 0 \\
0 - 1V_2 + 1.5V_3 &= 0
\end{aligned}
$$

$$V_3 = V_{2\Omega} = \mathbf{10.67\ V}$$



Example 3.2

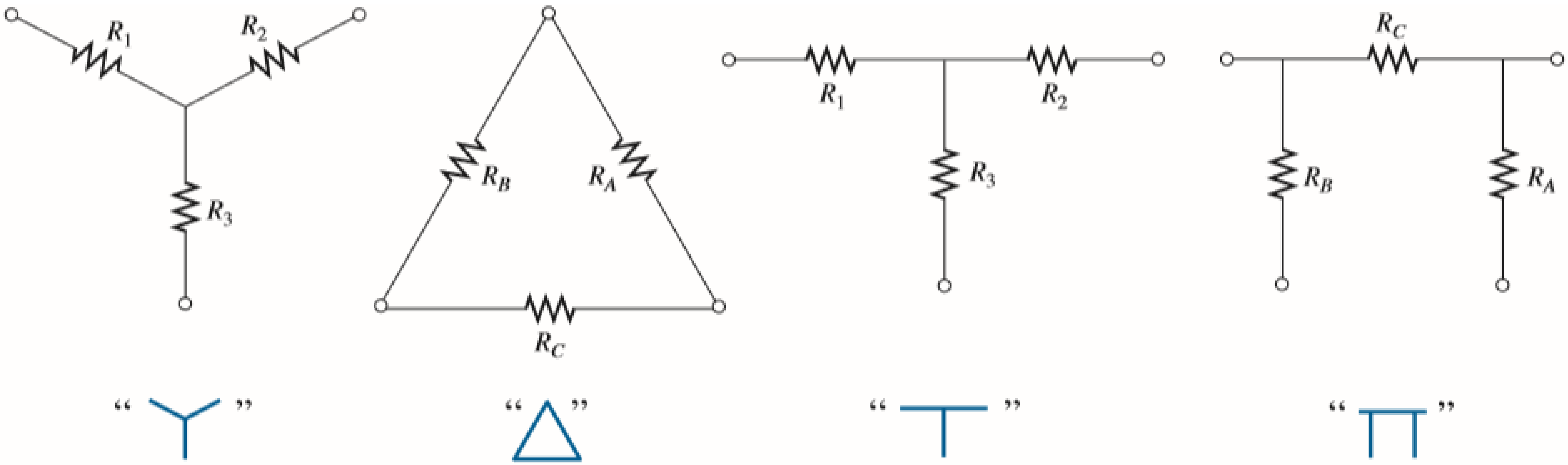
Determine the voltages at the nodes in Fig. 3.5(a).



(a)

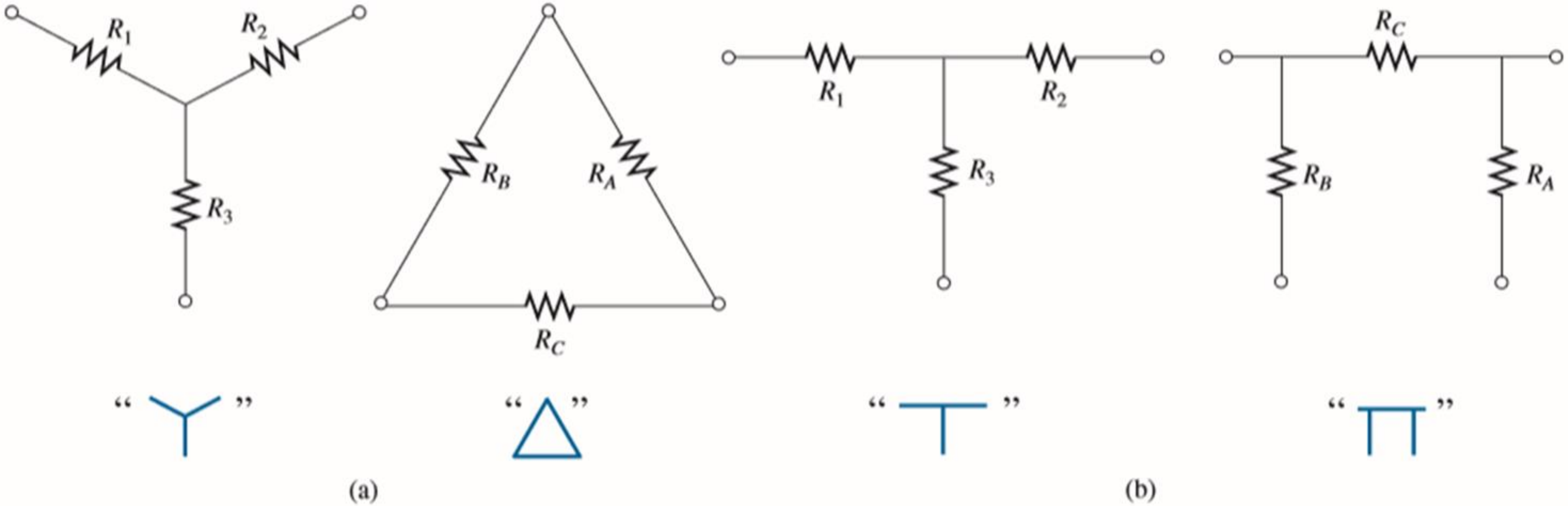
8.12 Y-Δ (T-π) AND Δ-Y (π-T) CONVERSIONS

Circuit configurations are often encountered in which the resistors do not appear to be in series or parallel. Under these conditions, it may be necessary to convert the circuit from one form to another to solve for any unknown quantities if mesh or nodal analysis is not applied. Two circuit configurations that often account for these difficulties are the **wye (Y)** and **delta (Δ) configurations** depicted in Fig. 8.81(a). They are also referred to as the **tee (T)** and **pi (π)**, respectively, as indicated in Fig. 8.81(b). Note that the pi is actually an inverted delta.



(a)

(b)



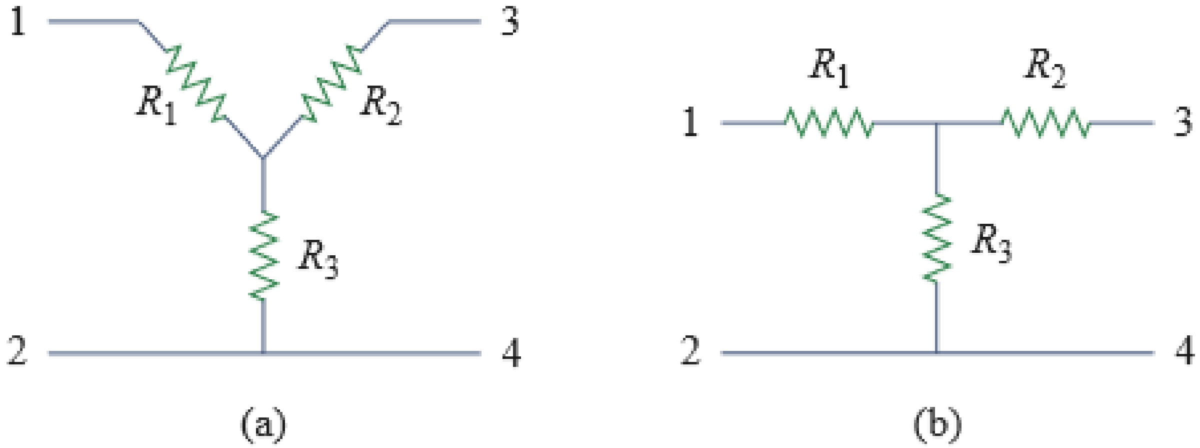


Figure 2.47
Two forms of the same network: (a) Y, (b) T.

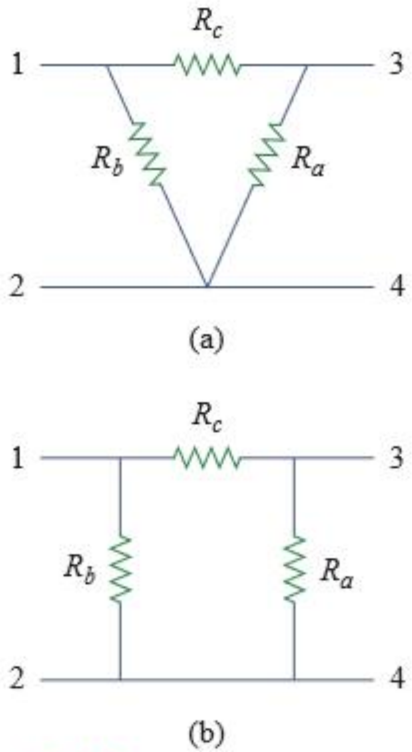


Figure 2.48
Two forms of the same network: (a) Δ, (b) Π.

Delta to Y

$$R_3 = \frac{R_A R_B}{R_A + R_B + R_C} \tag{8.4a}$$

Following the same procedure for R_1 and R_2, we have

$$R_1 = \frac{R_B R_C}{R_A + R_B + R_C} \tag{8.4b}$$

and

$$R_2 = \frac{R_A R_C}{R_A + R_B + R_C} \tag{8.4c}$$

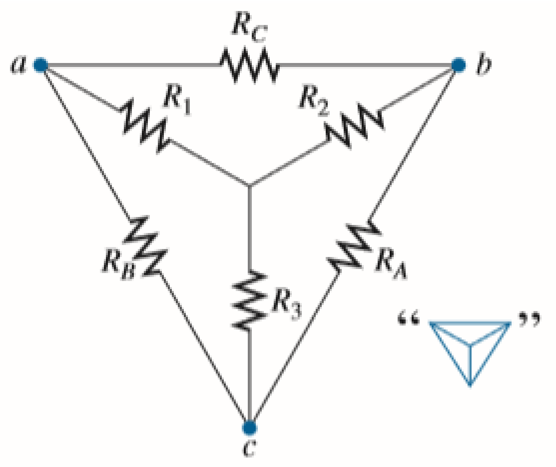


FIG. 8.82
Introducing the concept of Δ-Y or Y-Δ conversions.

Note that each resistor of the Y is equal to the product of the resistors in the two closest branches of the Δ divided by the sum of the resistors in the Δ.

Y to Delta

and $$R_C = \frac{R_1R_2 + R_1R_3 + R_2R_3}{R_3} \quad \text{(8.5a)}$$

We follow the same procedure for R_B and R_A:

$$R_A = \frac{R_1R_2 + R_1R_3 + R_2R_3}{R_1} \quad \text{(8.5b)}$$

and $$R_B = \frac{R_1R_2 + R_1R_3 + R_2R_3}{R_2} \quad \text{(8.5c)}$$

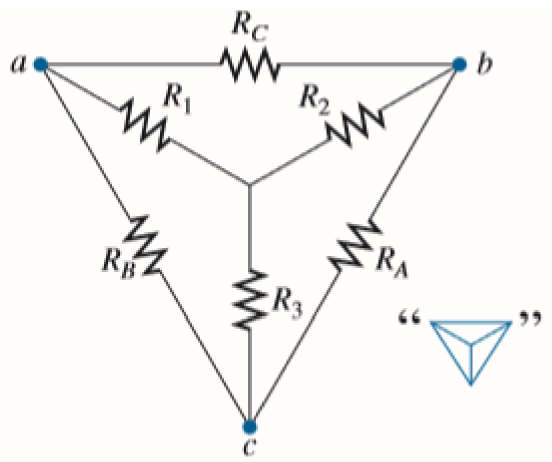


FIG. 8.82
Introducing the concept of Δ-Y or Y-Δ conversions.

Note that the value of each resistor of the Δ is equal to the sum of the possible product combinations of the resistances of the Y divided by the resistance of the Y farthest from the resistor to be determined.

The Y and Δ networks are said to be *balanced* when

$$R_1 = R_2 = R_3 = R_Y, \qquad R_a = R_b = R_c = R_\Delta \tag{2.56}$$

Under these conditions, conversion formulas become

$$\boxed{R_Y = \frac{R_\Delta}{3} \qquad \text{or} \qquad R_\Delta = 3R_Y} \tag{2.57}$$

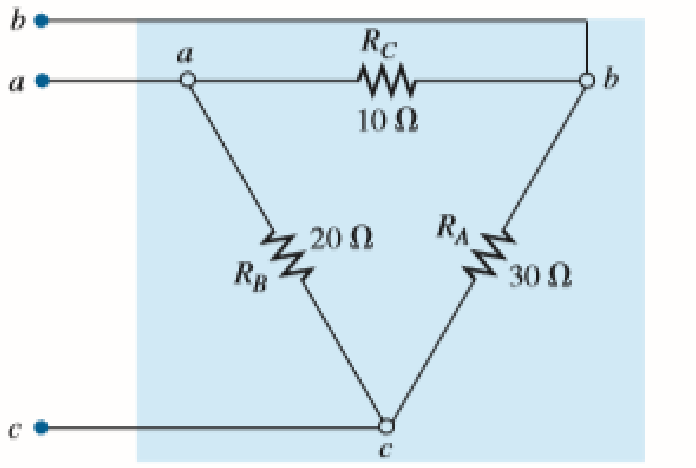


FIG. 8.85
Example 8.27.

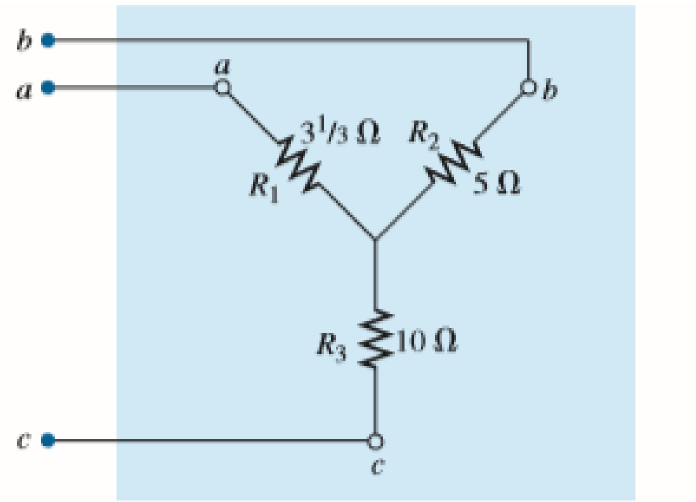


FIG. 8.86
The Y equivalent for the Δ in Fig. 8.85.

EXAMPLE 8.27 Convert the Δ in Fig. 8.85 to a Y.

Solution:

$$R_1 = \frac{R_B R_C}{R_A + R_B + R_C} = \frac{(20\ \Omega)(10\ \Omega)}{30\ \Omega + 20\ \Omega + 10\ \Omega} = \frac{200\ \Omega}{60} = \mathbf{3\tfrac{1}{3}\ \Omega}$$

$$R_2 = \frac{R_A R_C}{R_A + R_B + R_C} = \frac{(30\ \Omega)(10\ \Omega)}{60\ \Omega} = \frac{300\ \Omega}{60} = \mathbf{5\ \Omega}$$

$$R_3 = \frac{R_A R_B}{R_A + R_B + R_C} = \frac{(20\ \Omega)(30\ \Omega)}{60\ \Omega} = \frac{600\ \Omega}{60} = \mathbf{10\ \Omega}$$

The equivalent network is shown in Fig. 8.86.

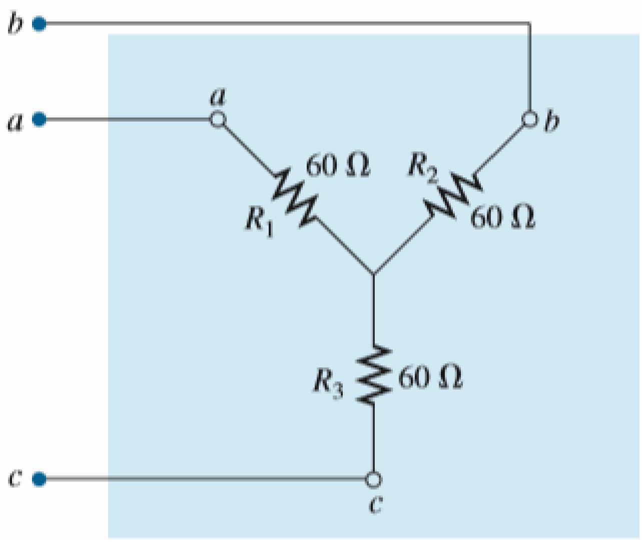


FIG. 8.87
Example 8.28.

EXAMPLE 8.28 Convert the Y in Fig. 8.87 to a Δ.

Solution:

$$R_A = \frac{R_1R_2 + R_1R_3 + R_2R_3}{R_1}$$

$$= \frac{(60\ \Omega)(60\ \Omega) + (60\ \Omega)(60\ \Omega) + (60\ \Omega)(60\ \Omega)}{60\ \Omega}$$

$$= \frac{3600\ \Omega + 3600\ \Omega + 3600\ \Omega}{60} = \frac{10{,}800\ \Omega}{60}$$

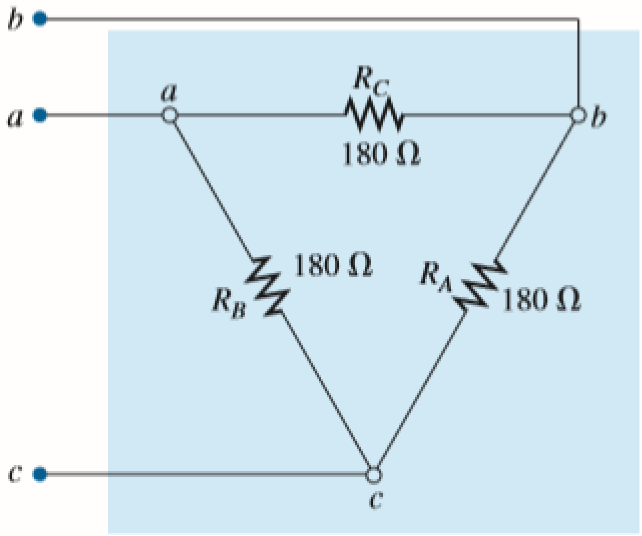
$$R_A = \mathbf{180\ \Omega}$$

However, the three resistors for the Y are equal, permitting the use of Eq. (8.6) and yielding

$$R_\Delta = 3R_Y = 3(60\ \Omega) = 180\ \Omega$$

and
$$R_B = R_C = \mathbf{180\ \Omega}$$

The equivalent network is shown in Fig. 8.88.



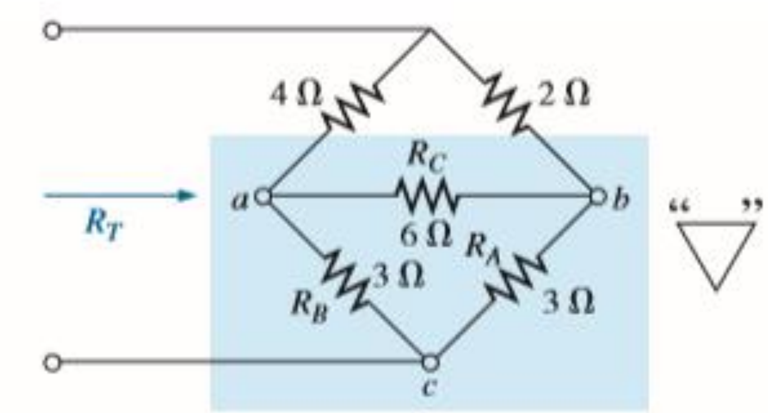


FIG. 8.89
Example 8.29.

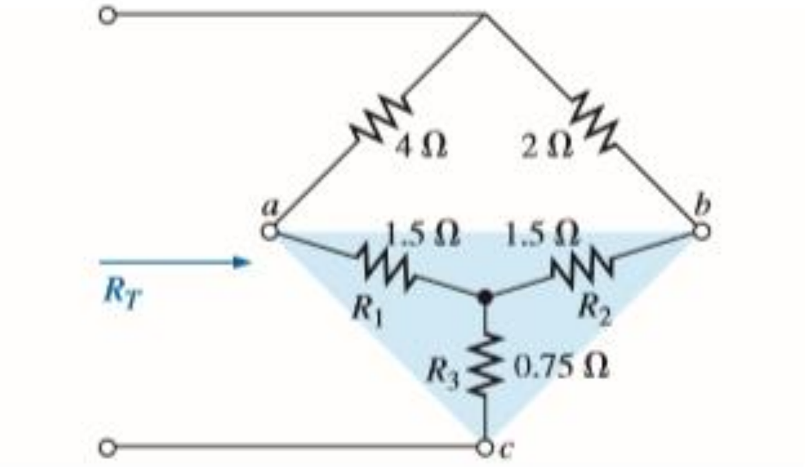


FIG. 8.90
Substituting the Y equivalent for the bottom Δ in Fig. 8.89.

EXAMPLE 8.29 Find the total resistance of the network in F
where $R_A = 3\ \Omega$, $R_B = 3\ \Omega$, and $R_C = 6\ \Omega$.

Solution:

Two resistors of the Δ w
therefore, two resistors c
be equal.

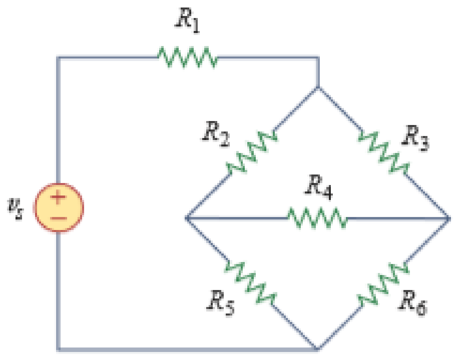
$$R_1 = \frac{R_B R_C}{R_A + R_B + R_C} = \frac{(3\ \Omega)(6\ \Omega)}{3\ \Omega + 3\ \Omega + 6\ \Omega} = \frac{18\ \Omega}{12} = \mathbf{1.}$$

$$R_2 = \frac{R_A R_C}{R_A + R_B + R_C} = \frac{(3\ \Omega)(6\ \Omega)}{12\ \Omega} = \frac{18\ \Omega}{12} = \mathbf{1.5\ \Omega} \leftarrow$$

$$R_3 = \frac{R_A R_B}{R_A + R_B + R_C} = \frac{(3\ \Omega)(3\ \Omega)}{12\ \Omega} = \frac{9\ \Omega}{12} = \mathbf{0.75\ \Omega}$$

Replacing the Δ by the Y, as shown in Fig. 8.90, yields

$$\begin{aligned} R_T &= 0.75\ \Omega + \frac{(4\ \Omega + 1.5\ \Omega)(2\ \Omega + 1.5\ \Omega)}{(4\ \Omega + 1.5\ \Omega) + (2\ \Omega + 1.5\ \Omega)} \\ &= 0.75\ \Omega + \frac{(5.5\ \Omega)(3.5\ \Omega)}{5.5\ \Omega + 3.5\ \Omega} \\ &= 0.75\ \Omega + 2.139\ \Omega \\ R_T &= \mathbf{2.89\ \Omega} \end{aligned}$$



Example 2.10

Calculate the equivalent resistance R_{ab} in the circuit in Fig. 2.37.

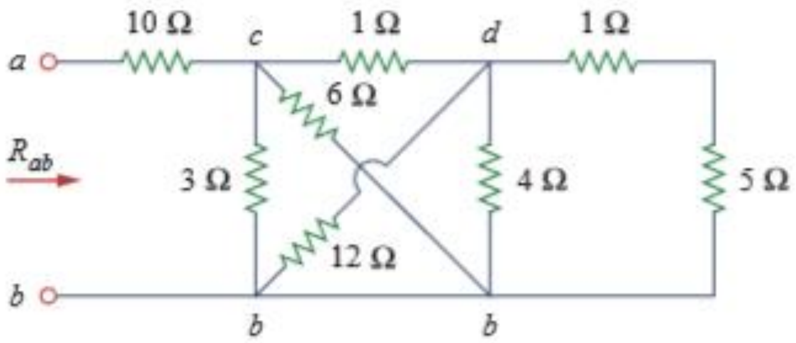


Figure 2.37
For Example 2.10.

Solution:

The 3-Ω and 6-Ω resistors are in parallel because they are connected to the same two nodes c and b. Their combined resistance is

$$3\ \Omega \parallel 6\ \Omega = \frac{3 \times 6}{3 + 6} = 2\ \Omega \qquad \textbf{(2.10.1)}$$

Similarly, the 12-Ω and 4-Ω resistors are in parallel since they are connected to the same two nodes d and b. Hence

$$12\ \Omega \,\|\, 4\ \Omega = \frac{12 \times 4}{12 + 4} = 3\ \Omega \tag{2.10.2}$$

Also the 1-Ω and 5-Ω resistors are in series; hence, their equivalent resistance is

$$1\ \Omega + 5\ \Omega = 6\ \Omega \tag{2.10.3}$$

With these three combinations, we can replace the circuit in Fig. 2.37 with that in Fig. 2.38(a). In Fig. 2.38(a), 3-Ω in parallel with 6-Ω gives 2-Ω, as calculated in Eq. (2.10.1). This 2-Ω equivalent resistance is now in series with the 1-Ω resistance to give a combined resistance of 1 Ω + 2 Ω = 3 Ω. Thus, we replace the circuit in Fig. 2.38(a) with that in Fig. 2.38(b). In Fig. 2.38(b), we combine the 2-Ω and 3-Ω resistors in parallel to get

$$2\ \Omega \,\|\, 3\ \Omega = \frac{2 \times 3}{2 + 3} = 1.2\ \Omega$$

This 1.2-Ω resistor is in series with the 10-Ω resistor, so that

$$R_{ab} = 10 + 1.2 = 11.2\ \Omega$$

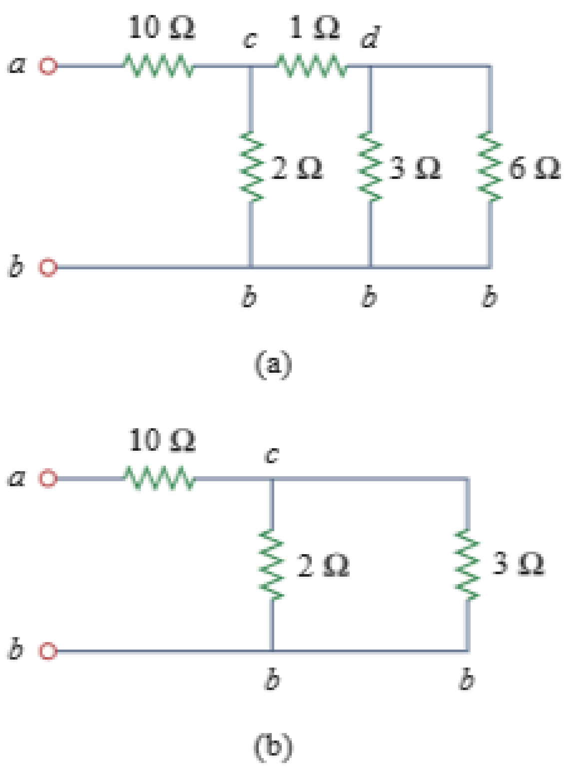


Figure 2.38
Equivalent circuits for Example 2.10.

Practice Problem 2.10

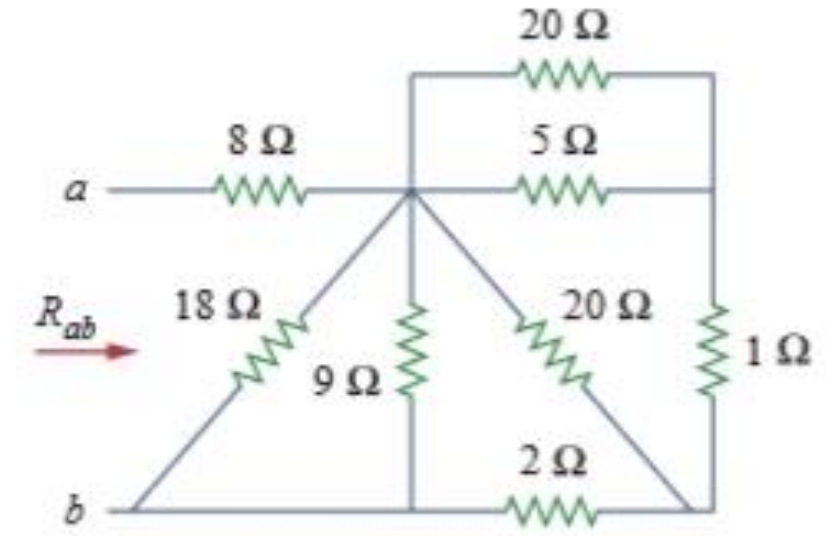


Figure 2.39
For Practice Prob. 2.10.

An **ideal independent source** is an active element that provides a specified voltage or current that is completely independent of other circuit elements.

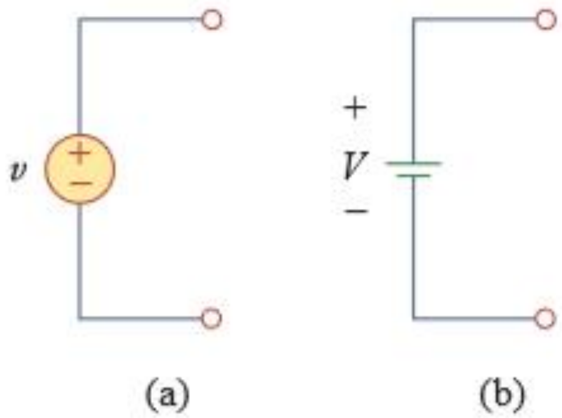


Figure 1.11
Symbols for independent voltage sources: (a) used for constant or time-varying voltage, (b) used for constant voltage (dc).

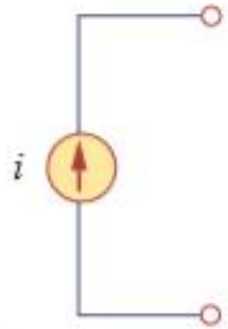


Figure 1.12
Symbol for independent current source.

An **ideal dependent** (or **controlled**) **source** is an active element in which the source quantity is controlled by another voltage or current.

Dependent sources are usually designated by diamond-shaped symbols, as shown in Fig. 1.13. Since the control of the dependent source is achieved by a voltage or current of some other element in the circuit, and the source can be voltage or current, it follows that there are four possible types of dependent sources, namely:

1. A voltage-controlled voltage source (VCVS).
2. A current-controlled voltage source (CCVS).
3. A voltage-controlled current source (VCCS).
4. A current-controlled current source (CCCS).

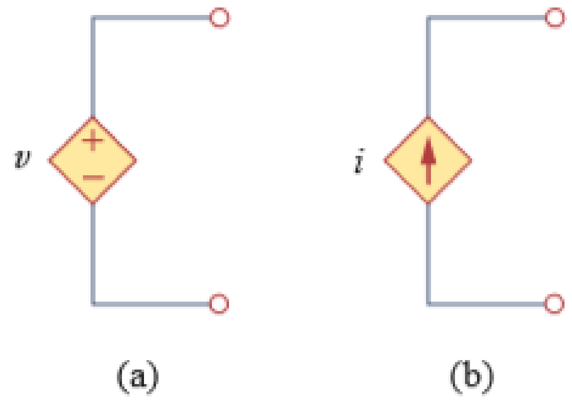
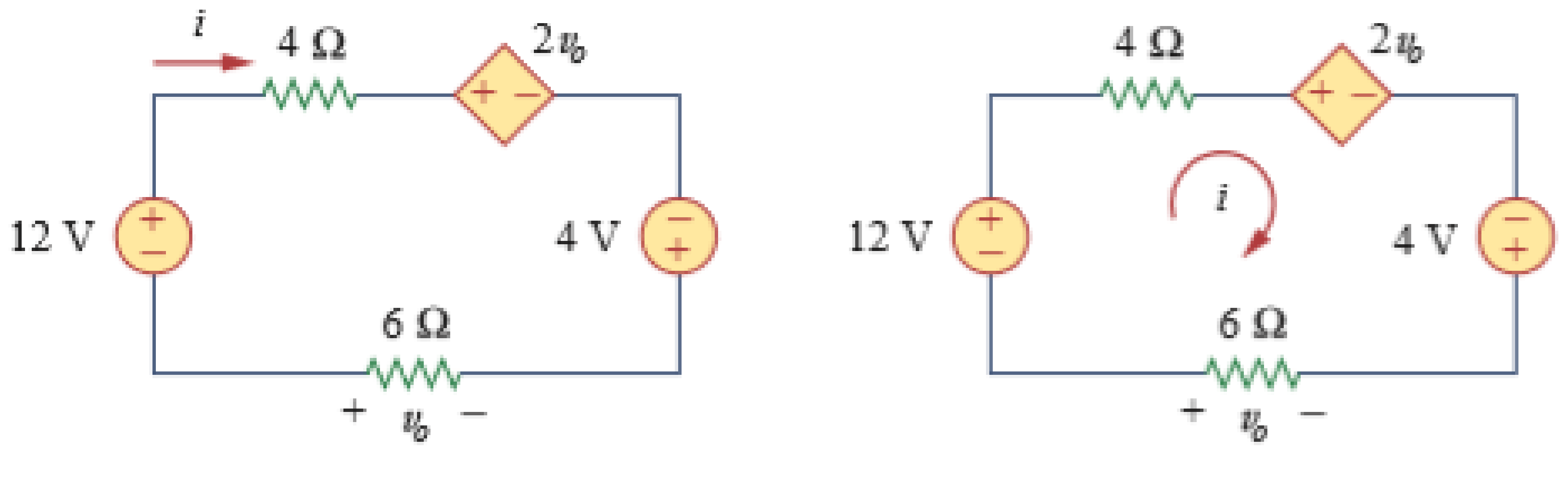


Figure 1.13
Symbols for: (a) dependent voltage source, (b) dependent current source.

Determine v_o and i in the circuit shown in Fig. 2.23(a).



(a) (b)

Solution:

We apply KVL around the loop as shown in Fig. 2.23(b). The result is

$$-12 + 4i + 2v_o - 4 + 6i = 0 \qquad \textbf{(2.6.1)}$$

Applying Ohm's law to the 6-Ω resistor gives

$$v_o = -6i \qquad \textbf{(2.6.2)}$$

Substituting Eq. (2.6.2) into Eq. (2.6.1) yields

$$-16 + 10i - 12i = 0 \quad \Rightarrow \quad i = -8 \text{ A}$$

and $v_o = 48$ V.

Practice Problem 2.6

Find v_x and v_o in the circuit of Fig. 2.24.

Answer: 10 V, -5 V.

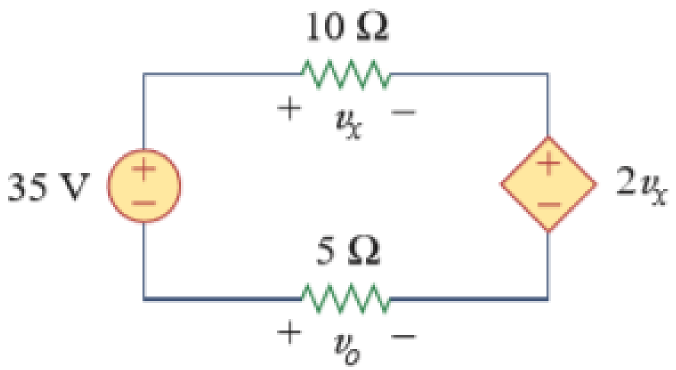


Figure 2.24
For Practice Prob. 2.6.

Example 2.7

Find current i_o and voltage v_o in the circuit shown in Fig. 2.25.

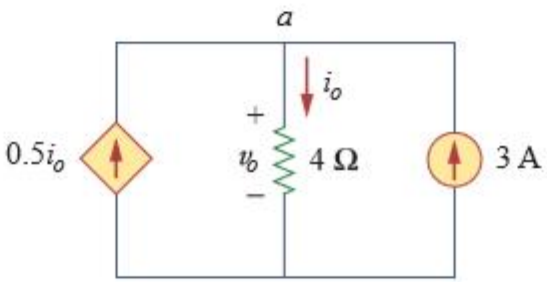


Figure 2.25
For Example 2.7.

Solution:

Applying KCL to node a, we obtain

$$3 + 0.5i_o = i_o \quad \Rightarrow \quad i_o = 6 \text{ A}$$

For the 4-Ω resistor, Ohm's law gives

$$v_o = 4i_o = 24 \text{ V}$$

Practice Problem 2.7

Find v_o and i_o in the circuit of Fig. 2.26.

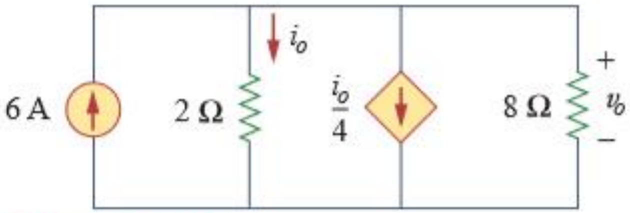


Figure 2.26
For Practice Prob. 2.7.

Answer: 8 V, 4 A.