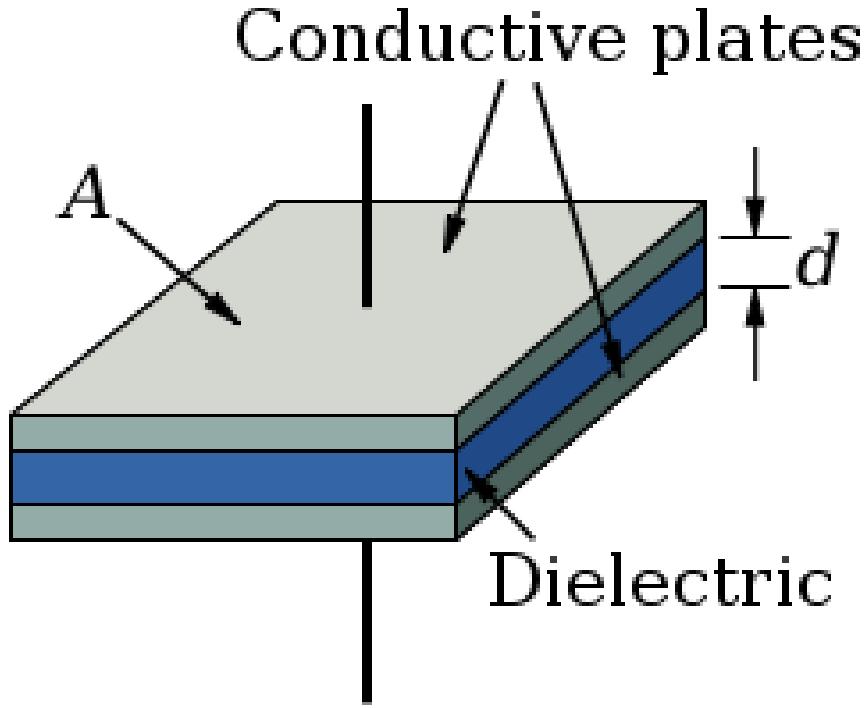


# CAPACITORS



The capacitor is a component which has the ability or “capacity” to store energy in the form of an electrical charge producing a potential difference across its plates, much like a small rechargeable battery.

In a way, a capacitor is a little like a battery. Although they work in completely different ways, capacitors and batteries both store electrical energy. If you have read How Batteries Work, then you know that a battery has two terminals. Inside the battery, chemical reactions produce electrons on one terminal and absorb electrons on the other terminal. A capacitor is much simpler than a battery, as it can't produce new electrons -- it only stores them.

In its basic form, a capacitor consists of two or more parallel conductive (metal) plates which are not connected or touching each other, but are electrically separated either by air or by some form of a good insulating material such as waxed paper, mica, ceramic, plastic or some form of a liquid gel as used in electrolytic capacitors. The insulating layer between a capacitors plates is commonly called the Dielectric.

Due to this insulating layer, DC current can not flow through the capacitor as it blocks it allowing instead a voltage to be present across the plates in the form of an electrical charge.

The conductive metal plates of a capacitor can be either square, circular or rectangular, or they can be of a cylindrical or spherical shape with the general shape, size and construction of a parallel plate capacitor depending on its application and voltage rating.

When used in a direct current or DC circuit, a capacitor charges up to its supply voltage but blocks the flow of current through it because the dielectric of a capacitor is non-conductive and basically an insulator.

By applying a voltage to a capacitor and measuring the charge on the plates, the ratio of the charge  $Q$  to the voltage  $V$  will give the capacitance value of the capacitor and is therefore given as:  $C = Q/V$  this equation can also be re-arranged to give the familiar formula for the quantity of charge on the plates as:  $Q = C \times V$

$$C = \frac{Q}{V}$$

$C$  = farads (F)

$Q$  = coulombs (C)

$V$  = volts (V)

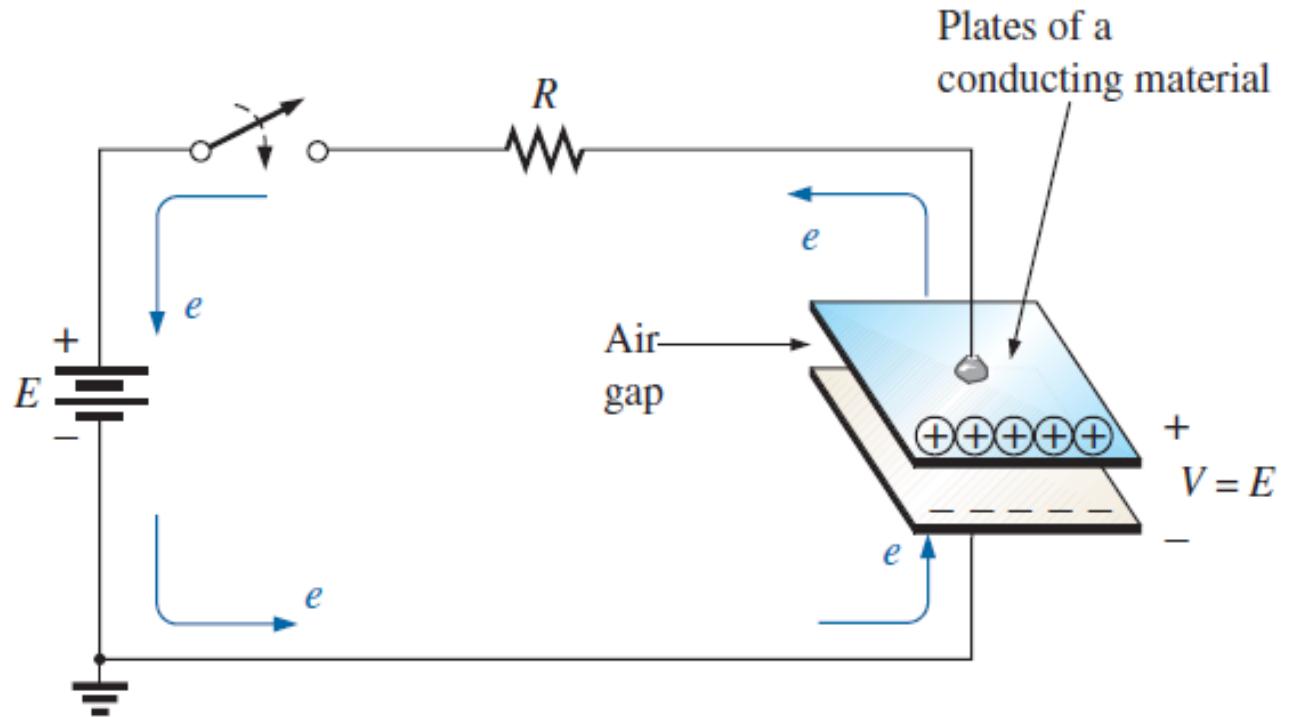
Because capacitors store the energy of the electrons in the form of an electrical charge on the plates the larger the plates and/or smaller their separation the greater will be the charge that the capacitor holds for any given voltage across its plates. In other words, larger plates, smaller distance, more capacitance.

## 10.1 INTRODUCTION

Thus far, the resistor has been the only network component appearing in our analyses. In this chapter, we introduce the **capacitor**, which has a significant impact on the types of networks that you will be able to design and analyze. Like the resistor, it is a two-terminal device, but its characteristics are totally different from those of a resistor. In fact, *the capacitor displays its true characteristics only when a change in the voltage or current is made in the network.* All the power delivered to a resistor is dissipated in the form of heat. An ideal capacitor, however, stores the energy delivered to it in a form that can be returned to the system.

## 10.3 CAPACITANCE

Thus far, we have examined only isolated positive and negative spherical charges, but the description can be extended to charged surfaces of any shape and size. In Fig. 10.4, for example, two parallel plates of a material such as aluminum (the most commonly used metal in the construction of capacitors) have been connected through a switch and a resistor to a battery. If the parallel plates are initially uncharged and the switch is left open, no net positive or negative charge exists on either plate. The instant the switch is closed, however, electrons are drawn from the upper plate through the resistor to the positive terminal of the battery. There will be a surge of current at first, limited in magnitude by the resistance present. The level of flow then declines, as will be demonstrated in the sections to follow. This action creates a net positive charge on the top plate. Electrons are being repelled by the negative terminal through the lower conductor to the bottom plate at the same rate they are being drawn to the positive terminal. This transfer of electrons continues until the potential difference across the parallel plates is exactly equal to the battery voltage. The final result is a net positive charge on the top plate and a negative charge on the bottom plate, very similar in many respects to the two isolated charges in Fig. 10.3(a).



**FIG. 10.4**  
*Fundamental charging circuit.*

Before continuing, it is important to note that the entire flow of charge is through the battery and resistor—not through the region between the plates. In every sense of the definition, *there is an open circuit between the plates of the capacitor.*

This element, constructed simply of two conducting surfaces separated by the air gap, is called a **capacitor**.

*Capacitance is a measure of a capacitor's ability to store charge on its plates—in other words, its storage capacity.*

In addition,

*the higher the capacitance of a capacitor, the greater the amount of charge stored on the plates for the same applied voltage.*

The unit of measure applied to capacitors is the farad (F), named after an English scientist, Michael Faraday, who did extensive research in the field (Fig. 10.5). In particular,

*a capacitor has a capacitance of 1 F if 1 C of charge ( $6.242 \times 10^{18}$  electrons) is deposited on the plates by a potential difference of 1 V across its plates.*

The farad, however, is generally too large a measure of capacitance for most practical applications, so the microfarad ( $10^{-6}$ ) or picofarad( $10^{-12}$ ) are more commonly encountered.

The relationship between the applied voltage, the charge on the plates, and the capacitance level is defined by the following equation:

$$C = \frac{Q}{V} \quad \begin{aligned} C &= \text{farads (F)} \\ Q &= \text{coulombs (C)} \\ V &= \text{volts (V)} \end{aligned} \quad (10.5)$$

Eq. (10.5) reveals that for the same voltage ( $V$ ), the greater the charge ( $Q$ ) on the plates (in the numerator of the equation), the higher the capacitance level ( $C$ ).

If we write the equation in the following form:

$$Q = CV \quad (\text{coulombs, C}) \quad (10.6)$$

it becomes obvious through the product relationship that the higher the capacitance ( $C$ ) or applied voltage ( $V$ ), the greater the charge on the plates.

## EXAMPLE 10.1

- a. If  $82.4 \times 10^{14}$  electrons are deposited on the negative plate of a capacitor by an applied voltage of 60 V, find the capacitance of the capacitor.
- b. If 40 V are applied across a  $470 \mu\text{F}$  capacitor, find the charge on the plates.

### Solutions:

- a. First find the number of coulombs of charge as follows:

$$82.4 \times 10^{14} \text{ electrons} \left( \frac{1 \text{ C}}{6.242 \times 10^{18} \text{ electrons}} \right) = 1.32 \text{ mC}$$

and then

$$C = \frac{Q}{V} = \frac{1.32 \text{ mC}}{60 \text{ V}} = 22 \mu\text{F} \quad (\text{a standard value})$$

- b. Applying Eq. (10.6):

$$Q = CV = (470 \mu\text{F})(40 \text{ V}) = 18.8 \text{ mC}$$

The **electric field strength** between the plates is determined by the voltage across the plates and the distance between the plates as follows:

$$\mathcal{E} = \frac{V}{d}$$

(10.7)

$\mathcal{E}$  = volts/m (V/m)  
 $V$  = volts (V)  
 $d$  = meters (m)

Note that the distance between the plates is measured in meters, not centimeters or inches.

The equation for the electric field strength is determined by two factors only: *the applied voltage and the distance between the plates*. The

Different materials placed between the plates establish different amounts of additional charge on the plates. All, however, must be insulators and must have the ability to set up an electric field within the structure. A list of common materials appears in Table 10.1 using air as the reference level of 1.\* All of these materials are referred to as **dielectrics**, the “di” for *opposing*, and the “electric” from *electric field*. The symbol  $\epsilon_r$  in Table 10.1 is called the **relative permittivity** (or **dielectric constant**). The term **permittivity** is applied as a measure of how easily a material “permits” the establishment of an electric field in the material. The relative permittivity compares the permittivity of a material to that of air. For instance, Table 10.1 reveals that mica, with a relative permittivity of 5, “permits” the establishment of an opposing electric field in the material five times better than in air. Note the ceramic material at the bottom

**TABLE 10.1**  
*Relative permittivity (dielectric constant)  $\epsilon_r$  of various dielectrics.*

Dielectric	$\epsilon_r$ (Average Values)
Vacuum	1.0
Air	1.0006
Teflon®	2.0
Paper, paraffined	2.5
Rubber	3.0
Polystyrene	3.0
Oil	4.0
Mica	5.0
Porcelain	6.0
Bakelite®	7.0
Aluminum oxide	7
Glass	7.5
Tantalum oxide	30
Ceramics	20–7500
Barium-strontium titanite (ceramic)	7500.0

Defining  $\epsilon_o$  as the permittivity of air, the relative permittivity of a material with a permittivity  $\epsilon$  is defined by Eq. (10.8):

$$\epsilon_r = \frac{\epsilon}{\epsilon_o} \quad (\text{dimensionless}) \quad (10.8)$$

Note that  $\epsilon_r$ , which (as mentioned previously) is often called the **dielectric constant**, is a dimensionless quantity because it is a ratio of similar quantities. However, permittivity does have the units of farads/meter (F/m) and is  $8.85 \times 10^{-12}$  F/m for air. Although the relative

## 10.4 CAPACITORS

### Capacitor Construction

We are now aware of the basic components of a capacitor: conductive plates, separation, and dielectric. However, the question remains, How do all these factors interact to determine the capacitance of a capacitor?

*Larger plates* permit an increased area for the storage of charge, so the area of the plates should be in the numerator of the defining equation. *The smaller the distance between the plates*, the larger the capacitance so this factor should appear in the numerator of the equation. Finally, since *higher levels of permittivity* result in higher levels of capacitance, the factor  $\epsilon$  should appear in the numerator of the defining equation.

The result is the following general equation for capacitance:

$$C = \epsilon \frac{A}{d}$$

(10.9)

$C$ = farads (F)
$\epsilon$ = permittivity (F/m)
$A$ = m <sup>2</sup>
$d$ = m

If we substitute Eq. (10.8) for the permittivity of the material, we obtain the following equation for the capacitance:

$$C = \epsilon_0 \epsilon_r \frac{A}{d} \quad (\text{farads, F}) \quad (10.10)$$

or if we substitute the known value for the permittivity of air, we obtain the following useful equation:

$$C = 8.85 \times 10^{-12} \epsilon_r \frac{A}{d} \quad (\text{farads, F}) \quad (10.11)$$

If we form the ratio of the equation for the capacitance of a capacitor with a specific dielectric to that of the same capacitor with air as the dielectric, the following results:

$$\frac{C = \epsilon \frac{A}{d}}{C_o = \epsilon_o \frac{A}{d}} \Rightarrow \frac{C}{C_o} = \frac{\epsilon}{\epsilon_o} = \epsilon_r$$

and

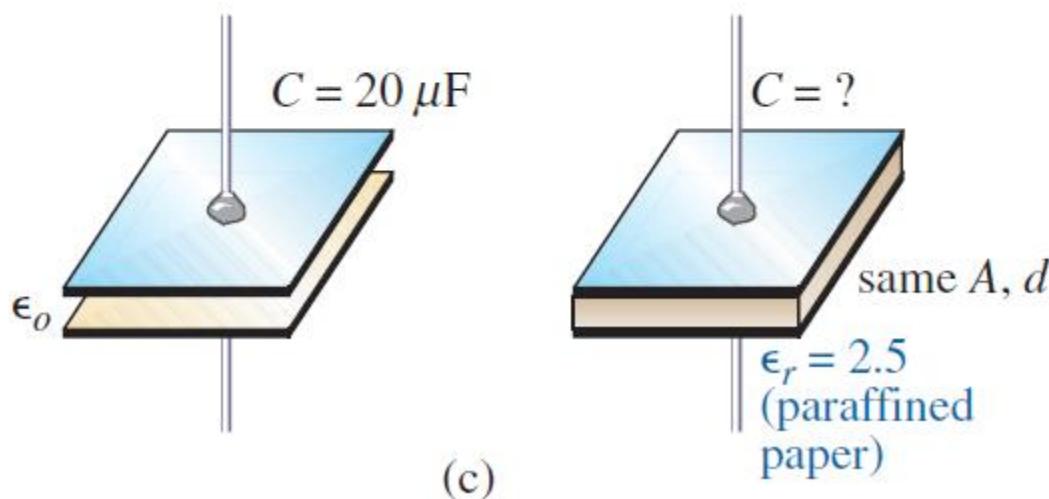
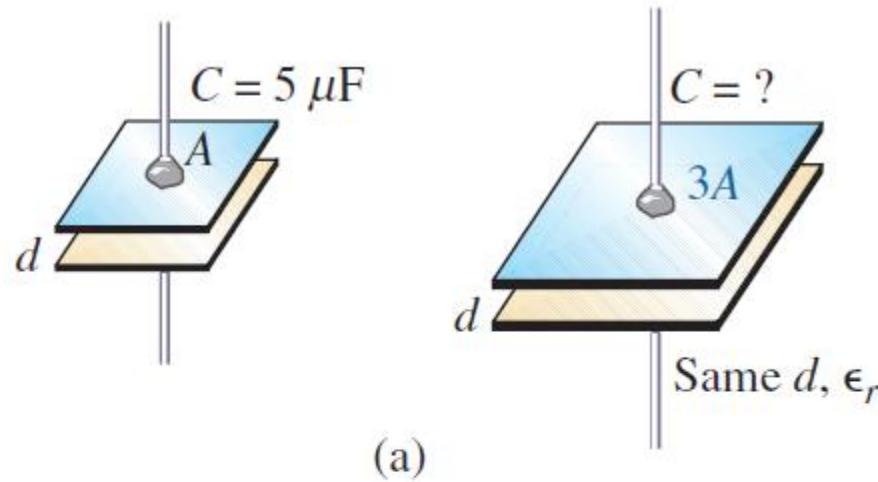
$$C = \epsilon_r C_o \quad (10.12)$$

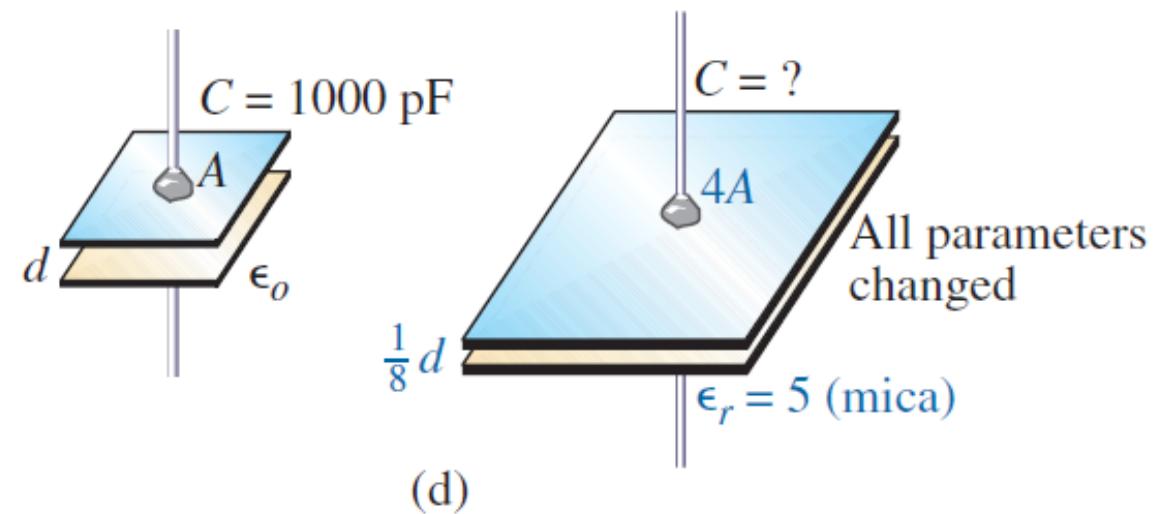
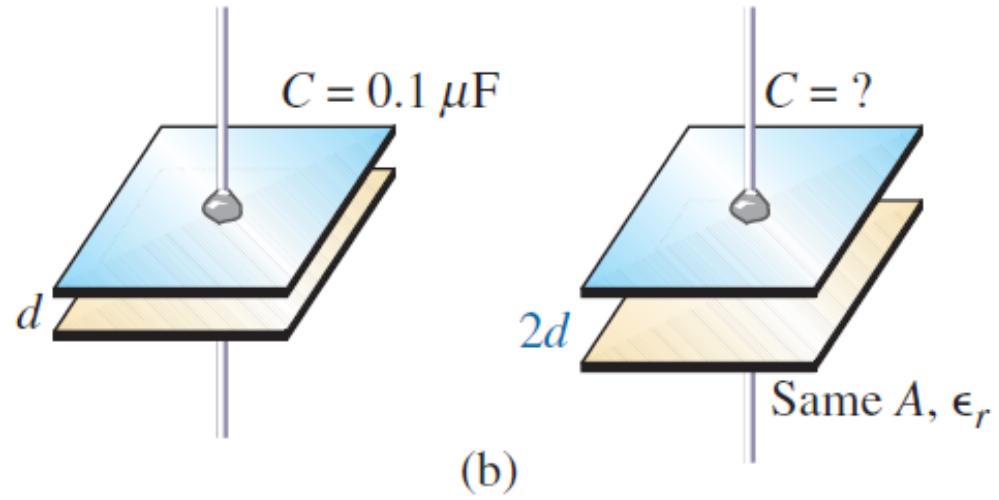
The result is that

*the capacitance of a capacitor with a dielectric having a relative permittivity of  $\epsilon_r$  is  $\epsilon_r$  times the capacitance using air as the dielectric.*

The next few examples review the concepts and equations just presented.

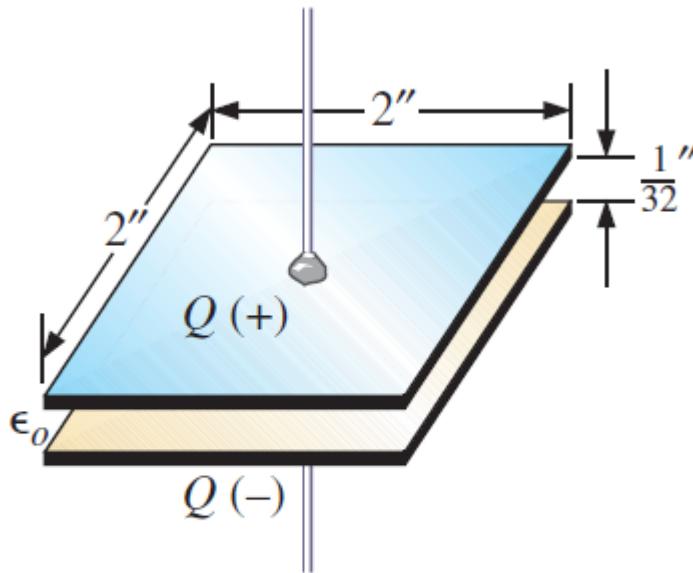
**EXAMPLE 10.2** In Fig. 10.9, if each air capacitor in the left column is changed to the type appearing in the right column, find the new capacitance level. For each change, the other factors remain the same.





**EXAMPLE 10.3** For the capacitor in Fig. 10.10:

- a. Find the capacitance.
- b. Find the strength of the electric field between the plates if 48 V are applied across the plates.
- c. Find the charge on each plate.



**FIG. 10.10**  
Air capacitor for Example 10.3.

**Solutions:**

- a. First, the area and the distance between the plates must be converted to the SI system as required by Eq. (10.11):

$$d = \frac{1}{32} \text{ in.} \left( \frac{1 \text{ m}}{39.37 \text{ in.}} \right) = 0.794 \text{ mm}$$

and  $A = (2 \text{ in.})(2 \text{ in.}) \left( \frac{1 \text{ m}}{39.37 \text{ in.}} \right)^2 = 2.581 \times 10^{-3} \text{ m}^2$

Eq. (10.11):

$$C = 8.85 \times 10^{-12} \epsilon_r \frac{A}{d} = 8.85 \times 10^{-12} (1) \frac{(2.581 \times 10^{-3} \text{ m}^2)}{0.794 \text{ mm}} = \mathbf{28.8 \text{ pF}}$$

- b. The electric field between the plates is determined by Eq. (10.7):

$$\mathcal{E} = \frac{V}{d} = \frac{48 \text{ V}}{0.794 \text{ mm}} = \mathbf{60.5 \text{ kV/m}}$$

- c. The charge on the plates is determined by Eq. (10.6):

$$Q = CV = (28.8 \text{ pF})(48 \text{ V}) = \mathbf{1.38 \text{ nC}}$$

In the next example, we will insert a ceramic dielectric between the plates of the air capacitor in Fig. 10.10 and see how it affects the capacitance level, electric field, and charge on the plates.

---

### EXAMPLE 10.4

- a. Insert a ceramic dielectric with an  $\epsilon_r$  of 250 between the plates of the capacitor in Fig. 10.10. Then determine the new level of capacitance. Compare your results to the solution in Example 10.3.
- b. Find the resulting electric field strength between the plates, and compare your answer to the result in Example 10.3.
- c. Determine the charge on each of the plates, and compare your answer to the result in Example 10.3.

a. Using Eq. (10.12), the new capacitance level is

$$C = \epsilon_r C_o = (250)(28.8 \text{ pF}) = \mathbf{7200 \text{ pF}} = \mathbf{7.2 \text{ nF}} = \mathbf{0.0072 \mu\text{F}}$$

which is *significantly higher* than the level in Example 10.3.

b.  $\mathcal{E} = \frac{V}{d} = \frac{48 \text{ V}}{0.794 \text{ mm}} = \mathbf{60.5 \text{ kV/m}}$

Since the applied voltage and the distance between the plates did not change, *the electric field between the plates remains the same*.

c.  $Q = CV = (7200 \text{ pF})(48 \text{ V}) = \mathbf{345.6 \text{ nC}} = \mathbf{0.35 \mu\text{C}}$

We now know that the insertion of a dielectric between the plates increases the amount of charge stored on the plates. In Example 10.4, since the relative permittivity increased by a factor of 250, the charge on the plates *increased by the same amount*.

## Types of Capacitors

Capacitors, like resistors, can be listed under two general headings: **fixed** and **variable**. The symbol for the fixed capacitor appears in Fig. 10.11(a). Note that the curved side is normally connected to ground or to the point of lower dc potential. The symbol for variable capacitors appears in Fig. 10.11(b).

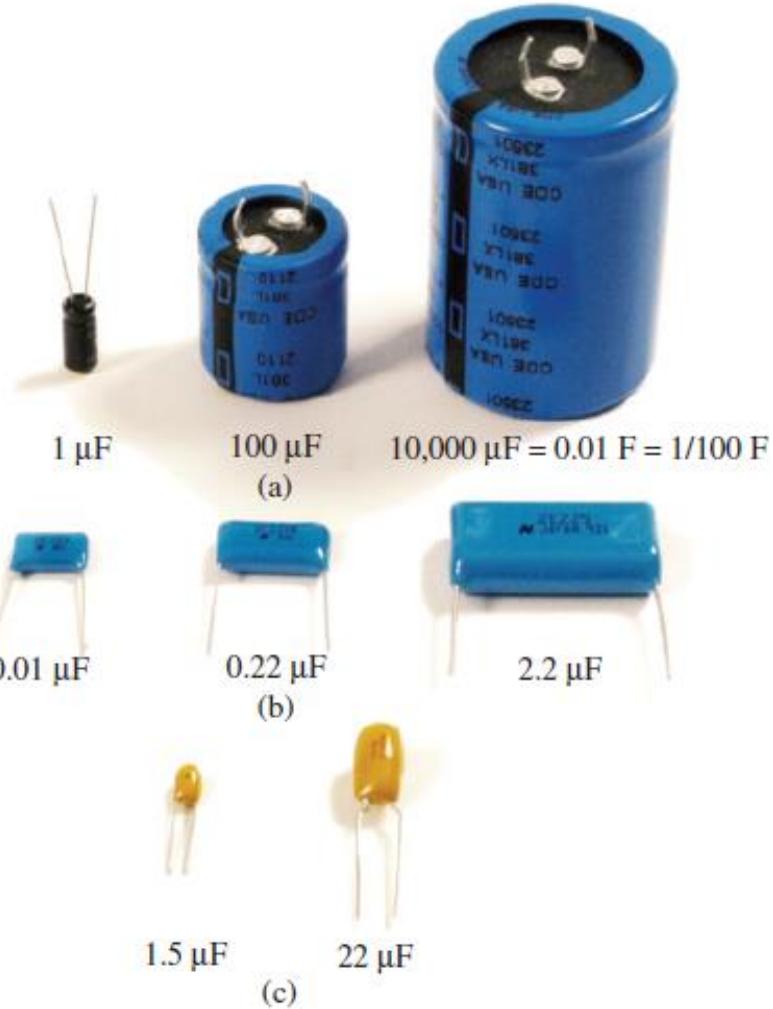
**Fixed Capacitors** Fixed-type capacitors come in all shapes and sizes. However,

*in general, for the same type of construction and dielectric, the larger the required capacitance, the larger the physical size of the capacitor.*



**FIG. 10.11**

*Symbols for the capacitor: (a) fixed; (b) variable.*

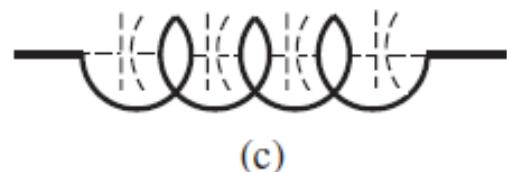
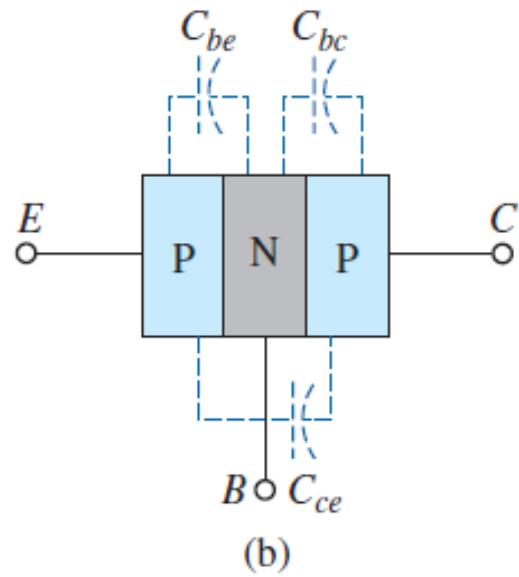
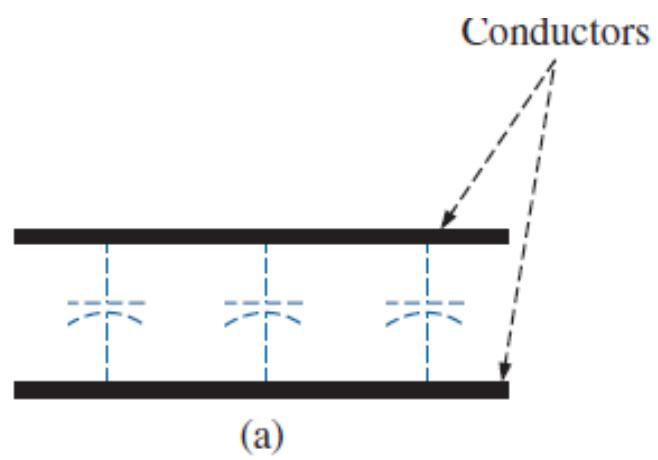


**FIG. 10.12**

Demonstrating that, in general, for each type of construction, the size of a capacitor increases with the capacitance value: (a) electrolytic; (b) polyester-film; (c) tantalum.

## 10.13 STRAY CAPACITANCES

In addition to the capacitors discussed so far in this chapter, there are **stray capacitances** that exist not through design but simply because two conducting surfaces are relatively close to each other. Two conducting wires in the same network have a capacitive effect between them, as shown in Fig. 10.77(a). In electronic circuits, capacitance levels exist between conducting surfaces of the transistor, as shown in Fig. 10.77(b). In Chapter 11, we will discuss another element called the *inductor*, which has capacitive effects between the windings [Fig. 10.77(c)]. Stray capacitances can often lead to serious errors in system design if they are not considered carefully.



**FIG. 10.77**  
*Examples of stray capacitance.*

## 10.12 ENERGY STORED BY A CAPACITOR

$$W_C = \frac{1}{2}CE^2$$

In general,

$$W_C = \frac{1}{2}CV^2 \quad (\text{J}) \quad (10.34)$$

where  $V$  is the steady-state voltage across the capacitor. In terms of  $Q$  and  $C$ ,

$$W_C = \frac{1}{2}C\left(\frac{Q}{C}\right)^2$$

or

$$W_C = \frac{Q^2}{2C} \quad (\text{J}) \quad (10.35)$$

**EXAMPLE 10.20** For the network in Fig. 10.75(a), determine the energy stored by each capacitor.

**Solution:** For  $C_1$ :

$$\begin{aligned}W_C &= \frac{1}{2} CV^2 \\&= \frac{1}{2} (2 \times 10^{-6} \text{ F})(16 \text{ V})^2 = (1 \times 10^{-6})(256) = \mathbf{256 \mu J}\end{aligned}$$

For  $C_2$ :

$$\begin{aligned}W_C &= \frac{1}{2} CV^2 \\&= \frac{1}{2} (3 \times 10^{-6} \text{ F})(56 \text{ V})^2 = (1.5 \times 10^{-6})(3136) = \mathbf{4704 \mu J}\end{aligned}$$

Due to the squared term, the energy stored increases rapidly with increasing voltages.

## 10.11 CAPACITORS IN SERIES AND IN PARALLEL

Capacitors, like resistors, can be placed in series and in parallel. Increasing levels of capacitance can be obtained by placing capacitors in parallel, while decreasing levels can be obtained by placing capacitors in series.

For capacitors in series, the charge is the same on each capacitor (Fig. 10.67):

$$Q_T = Q_1 = Q_2 = Q_3 \quad (10.28)$$

Applying Kirchhoff's voltage law around the closed loop gives

$$E = V_1 + V_2 + V_3$$

However,

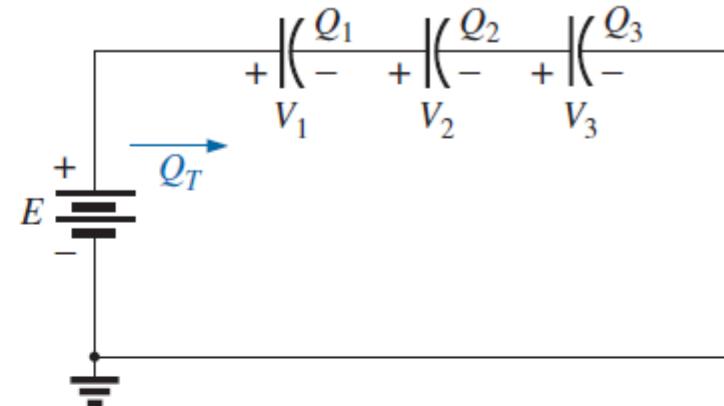
$$V = \frac{Q}{C}$$

so that

$$\frac{Q_T}{C_T} = \frac{Q_1}{C_1} + \frac{Q_2}{C_2} + \frac{Q_3}{C_3}$$

Using Eq. (10.28) and dividing both sides by  $Q$  yields

$$\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \quad (10.29)$$



**FIG. 10.67**  
*Series capacitors.*

which is similar to the manner in which we found the total resistance of a parallel resistive circuit. The total capacitance of two capacitors in series is

$$C_T = \frac{C_1 C_2}{C_1 + C_2} \quad (10.30)$$

The voltage across each capacitor in Fig. 10.67 can be found by first recognizing that

$$Q_T = Q_1$$

or

$$C_T E = C_1 V_1$$

Solving for  $V_1$ :

$$V_1 = \frac{C_T E}{C_1}$$

and substituting for  $C_T$ :

$$V_1 = \left( \frac{1/C_1}{1/C_1 + 1/C_2 + 1/C_3} \right) E \quad (10.31)$$

A similar equation results for each capacitor of the network.

For capacitors in parallel, as shown in Fig. 10.68, the voltage is the same across each capacitor, and the total charge is the sum of that on each capacitor:

$$Q_T = Q_1 + Q_2 + Q_3 \quad (10.32)$$

However,  
Therefore,  
but

$$Q = CV$$

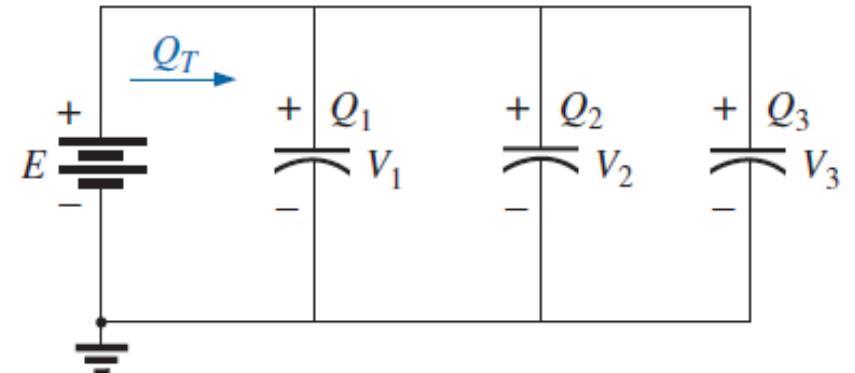
$$C_T E = C_1 V_1 = C_2 V_2 = C_3 V_3$$

$$E = V_1 = V_2 = V_3$$

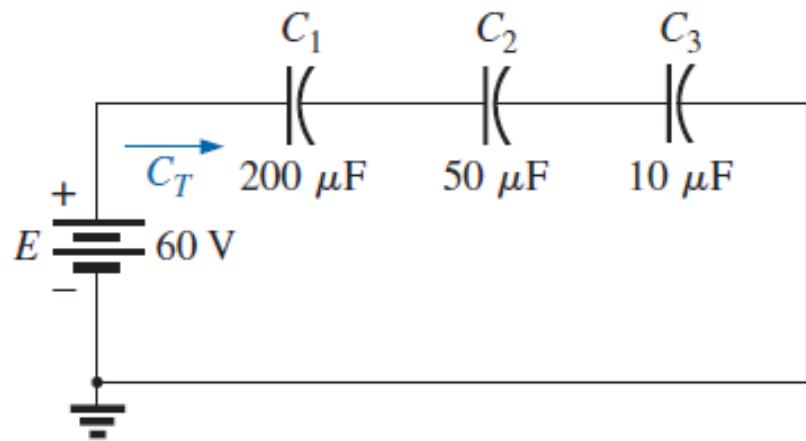
Thus,

$$C_T = C_1 + C_2 + C_3 \quad (10.33)$$

which is similar to the manner in which the total resistance of a series circuit is found.



**FIG. 10.68**  
*Parallel capacitors.*



**FIG. 10.69**  
*Example 10.15.*

---

**EXAMPLE 10.15** For the circuit in Fig. 10.69:

- Find the total capacitance.
- Determine the charge on each plate.
- Find the voltage across each capacitor.

**Solutions:**

$$\begin{aligned}
 \text{a. } \frac{1}{C_T} &= \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \\
 &= \frac{1}{200 \times 10^{-6} \text{ F}} + \frac{1}{50 \times 10^{-6} \text{ F}} + \frac{1}{10 \times 10^{-6} \text{ F}} \\
 &= 0.005 \times 10^6 + 0.02 \times 10^6 + 0.1 \times 10^6 \\
 &= 0.125 \times 10^6
 \end{aligned}$$

$$\text{and } C_T = \frac{1}{0.125 \times 10^6} = 8 \mu\text{F}$$

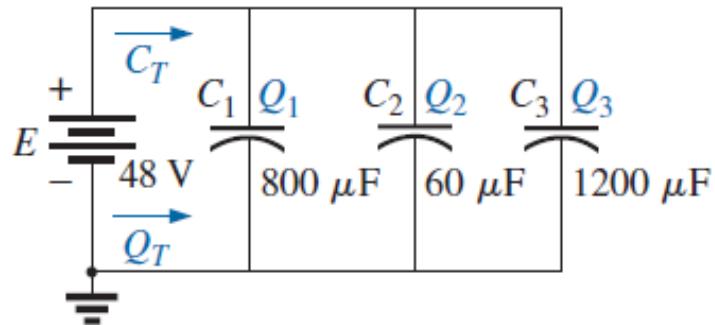
b.  $Q_T = Q_1 = Q_2 = Q_3$   
 $= C_T E = (8 \times 10^{-6} \text{ F}) (60 \text{ V}) = 480 \mu\text{C}$

c.  $V_1 = \frac{Q_1}{C_1} = \frac{480 \times 10^{-6} \text{ C}}{200 \times 10^{-6} \text{ F}} = 2.4 \text{ V}$

$$V_2 = \frac{Q_2}{C_2} = \frac{480 \times 10^{-6} \text{ C}}{50 \times 10^{-6} \text{ F}} = 9.6 \text{ V}$$

$$V_3 = \frac{Q_3}{C_3} = \frac{480 \times 10^{-6} \text{ C}}{10 \times 10^{-6} \text{ F}} = 48.0 \text{ V}$$

and  $E = V_1 + V_2 + V_3 = 2.4 \text{ V} + 9.6 \text{ V} + 48 \text{ V} = 60 \text{ V}$



**FIG. 10.70**

*Example 10.16.*

---

**EXAMPLE 10.16** For the network in Fig. 10.70:

- Find the total capacitance.
- Determine the charge on each plate.
- Find the total charge.

**Solutions:**

- $C_T = C_1 + C_2 + C_3 = 800 \mu\text{F} + 60 \mu\text{F} + 1200 \mu\text{F} = \mathbf{2060 \mu\text{F}}$
- $Q_1 = C_1 E = (800 \times 10^{-6} \text{ F})(48 \text{ V}) = \mathbf{38.4 \text{ mC}}$   
 $Q_2 = C_2 E = (60 \times 10^{-6} \text{ F})(48 \text{ V}) = \mathbf{2.88 \text{ mC}}$   
 $Q_3 = C_3 E = (1200 \times 10^{-6} \text{ F})(48 \text{ V}) = \mathbf{57.6 \text{ mC}}$
- $Q_T = Q_1 + Q_2 + Q_3 = 38.4 \text{ mC} + 2.88 \text{ mC} + 57.6 \text{ mC} = \mathbf{98.88 \text{ mC}}$

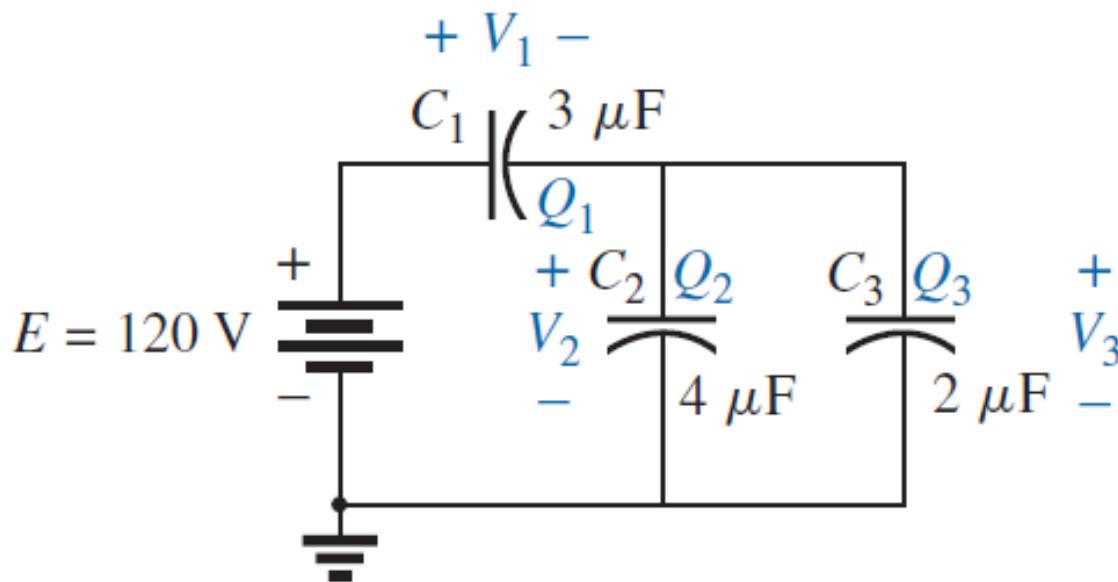
**EXAMPLE 10.17** Find the voltage across and the charge on each capacitor for the network in Fig. 10.71.

**Solution:**

$$C'_T = C_2 + C_3 = 4 \mu\text{F} + 2 \mu\text{F} = 6 \mu\text{F}$$

$$C_T = \frac{C_1 C'_T}{C_1 + C'_T} = \frac{(3 \mu\text{F})(6 \mu\text{F})}{3 \mu\text{F} + 6 \mu\text{F}} = 2 \mu\text{F}$$

$$Q_T = C_T E = (2 \times 10^{-6} \text{F})(120 \text{ V}) = 240 \mu\text{C}$$



An equivalent circuit (Fig. 10.72) has

$$Q_T = Q_1 = Q'_T$$

and, therefore,

$$Q_1 = 240 \mu\text{C}$$

and

$$V_1 = \frac{Q_1}{C_1} = \frac{240 \times 10^{-6} \text{ C}}{3 \times 10^{-6} \text{ F}} = 80 \text{ V}$$

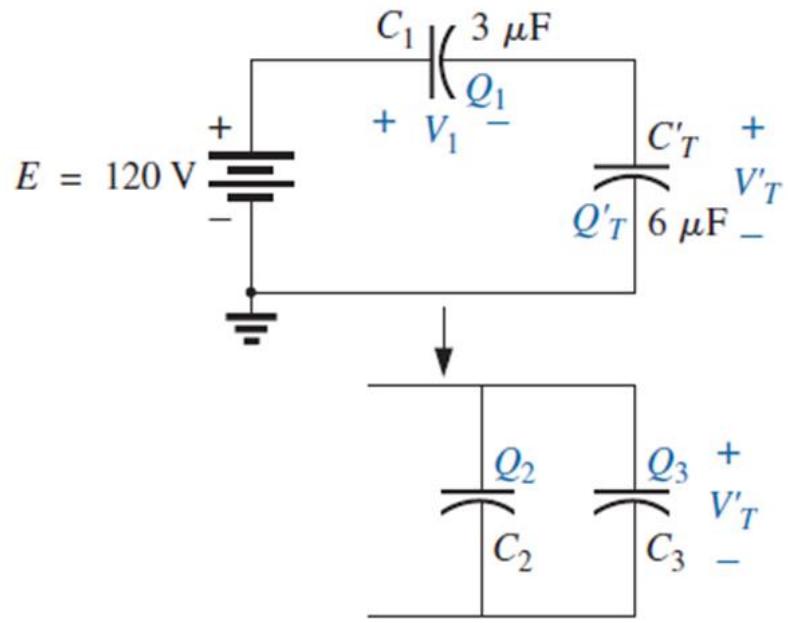
$$Q'_T = 240 \mu\text{C}$$

$$\text{Therefore, } V'_T = \frac{Q'_T}{C'_T} = \frac{240 \times 10^{-6} \text{ C}}{6 \times 10^{-6} \text{ F}} = 40 \text{ V}$$

and

$$Q_2 = C_2 V'_T = (4 \times 10^{-6} \text{ F})(40 \text{ V}) = 160 \mu\text{C}$$

$$Q_3 = C_3 V'_T = (2 \times 10^{-6} \text{ F})(40 \text{ V}) = 80 \mu\text{C}$$



**FIG. 10.72**

Reduced equivalent for the network in Fig. 10.71.

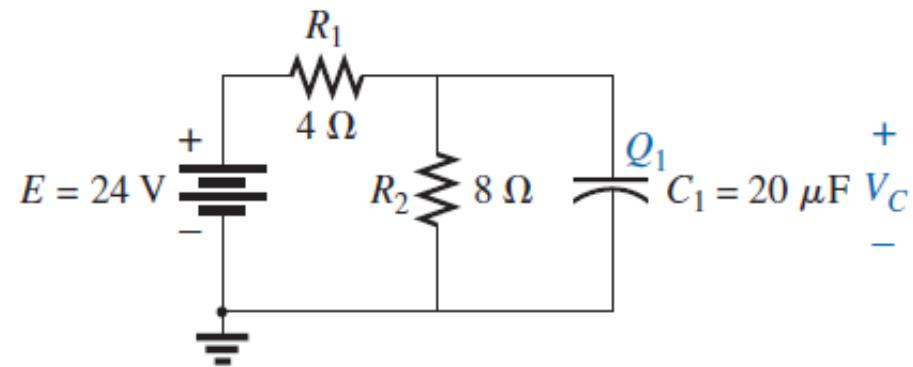
**EXAMPLE 10.18** Find the voltage across and the charge on capacitor  $C_1$  in Fig. 10.73 after it has charged up to its final value.

**Solution:** As previously discussed, the capacitor is effectively an open circuit for dc after charging up to its final value (Fig. 10.74).

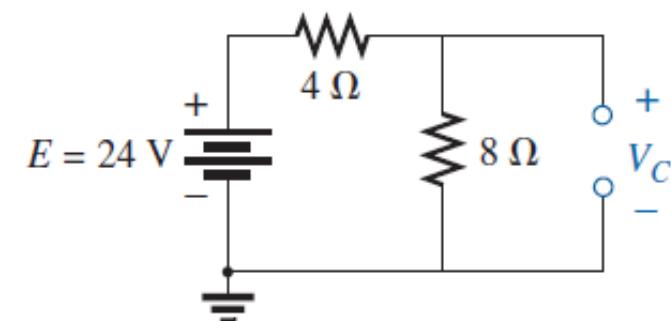
Therefore,

$$V_C = \frac{(8\ \Omega)(24\text{ V})}{4\ \Omega + 8\ \Omega} = 16\text{ V}$$

$$Q_1 = C_1 V_C = (20 \times 10^{-6}\text{ F})(16\text{ V}) = 320\ \mu\text{C}$$



**FIG. 10.73**  
*Example 10.18.*



**FIG. 10.74**  
*Determining the final (steady-state) value for  $v_C$ .*

**EXAMPLE 10.19** Find the voltage across and the charge on each capacitor of the network in Fig. 10.75(a) after each has charged up to its final value.

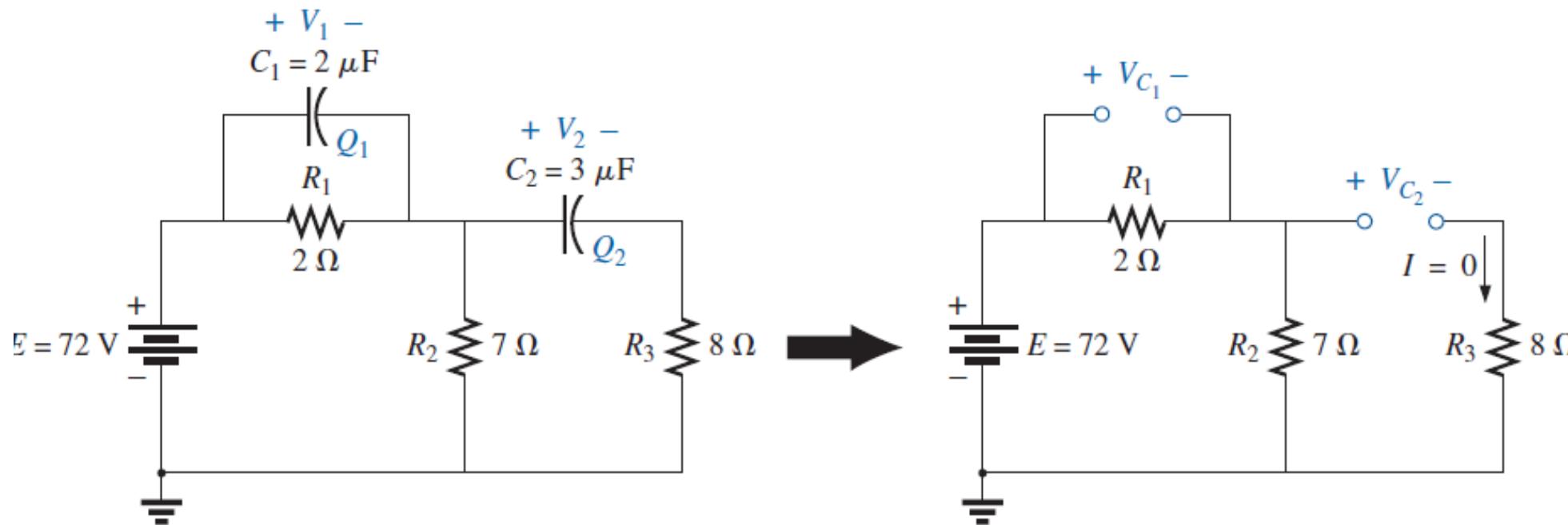
**Solution:** See Fig. 10.75(b).

$$V_{C_2} = \frac{(7\ \Omega)(72\text{ V})}{7\ \Omega + 2\ \Omega} = 56\text{ V}$$

$$V_{C_1} = \frac{(2\ \Omega)(72\text{ V})}{2\ \Omega + 7\ \Omega} = 16\text{ V}$$

$$Q_1 = C_1 V_{C_1} = (2 \times 10^{-6}\text{ F})(16\text{ V}) = 32\ \mu\text{C}$$

$$Q_2 = C_2 V_{C_2} = (3 \times 10^{-6}\text{ F})(56\text{ V}) = 168\ \mu\text{C}$$



## 10.5 TRANSIENTS IN CAPACITIVE NETWORKS: THE CHARGING PHASE

The placement of charge on the plates of a capacitor does not occur instantaneously. Instead, it occurs over a period of time determined by the components of the network. The charging phase, the phase during which charge is deposited on the plates, can be described by reviewing the response of the simple series circuit in Fig. 10.4. The circuit has been re-

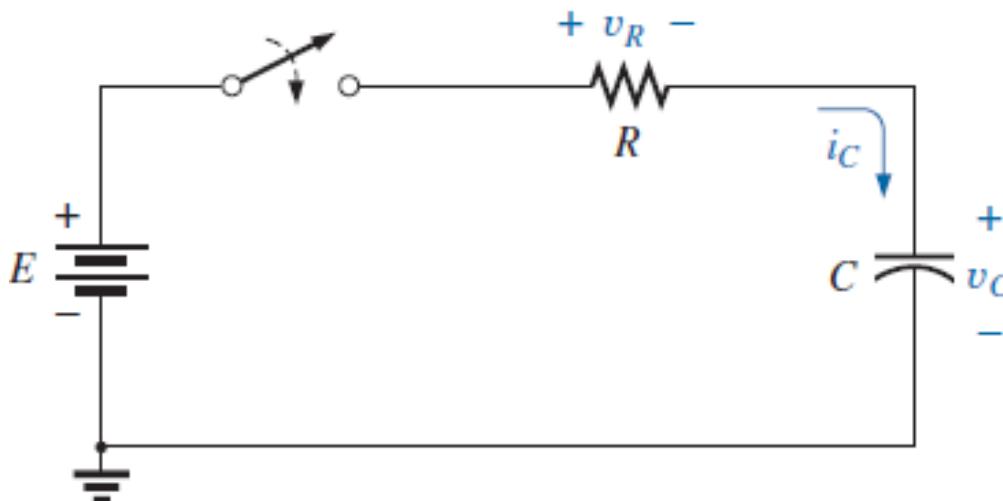
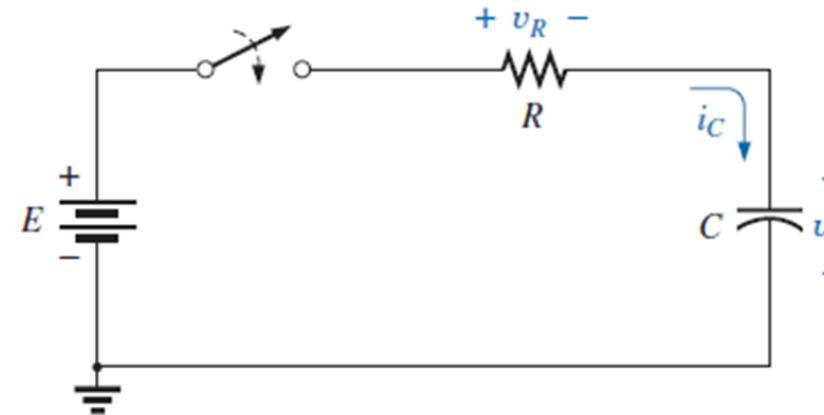


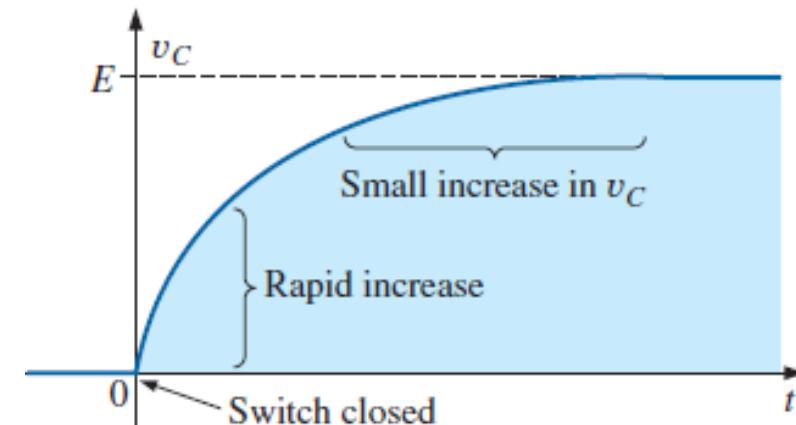
FIG. 10.26  
*Basic R-C charging network.*

Recall that the instant the switch is closed, electrons are drawn from the top plate and deposited on the bottom plate by the battery, resulting in a net positive charge on the top plate and a negative charge on the bottom plate. The transfer of electrons is very rapid at first, slowing down as the potential across the plates approaches the applied voltage of the battery. Eventually, when the voltage across the capacitor equals the applied voltage, the transfer of electrons ceases, and the plates have a net charge determined by  $Q = CV_C = CE$ . This period of time during which charge is being deposited on the plates is called the **transient period**—a period of time where the voltage or current changes from one steady-state level to another.

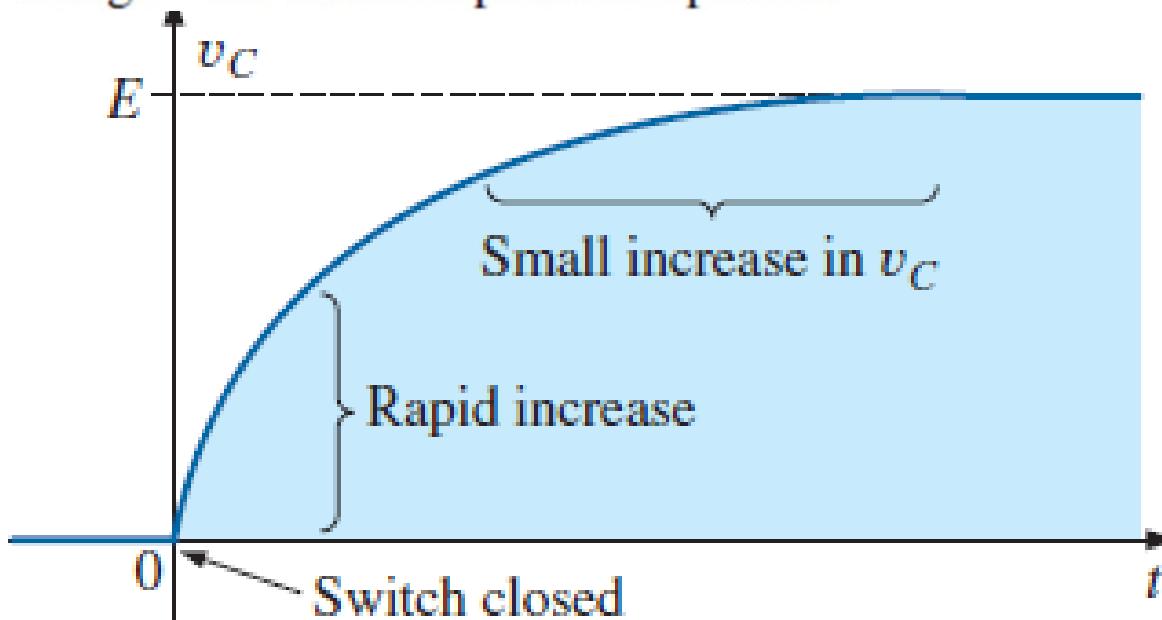


**FIG. 10.26**  
Basic R-C charging network.

Since the voltage across the plates is directly related to the charge on the plates by  $V = Q/C$ , a plot of the voltage across the capacitor will have the same shape as a plot of the charge on the plates over time. As shown in Fig. 10.27, the voltage across the capacitor is zero volts when the switch is closed ( $t = 0$  s). It then builds up very quickly at first since charge is being deposited at a very high rate of speed. As time passes, the charge is deposited at a slower rate, and the change in voltage drops off. The voltage continues to grow, but at a much slower rate. Eventually, as the voltage across the plates approaches the applied voltage, the charging rate is very slow, until finally the voltage across the plates is equal to the applied voltage—the transient phase has passed.



Since the voltage across the plates is directly related to the charge on the plates by  $V = Q/C$ , a plot of the voltage across the capacitor will have the same shape as a plot of the charge on the plates over time. As shown in Fig. 10.27, the voltage across the capacitor is zero volts when the switch is closed ( $t = 0 \text{ s}$ ). It then builds up very quickly at first since charge is being deposited at a very high rate of speed. As time passes, the charge is deposited at a slower rate, and the change in voltage drops off. The voltage continues to grow, but at a much slower rate. Eventually, as the voltage across the plates approaches the applied voltage, the charging rate is very slow, until finally the voltage across the plates is equal to the applied voltage—the transient phase has passed.



$$v_C = E(1 - e^{-t/\tau}) \text{ charging} \quad (\text{volts, V}) \quad (10.13)$$

First note in Eq. (10.13) that the voltage  $v_C$  is written in *lowercase (not capital) italic* to point out that it is a function that will change with time—

The quantity  $\tau$  is defined by

$$\tau = RC \quad (\text{time, s}) \quad (10.14)$$

The factor  $\tau$ , called the **time constant** of the network, has the units of time as shown below using some of the basic equations introduced earlier in this text:

$$\tau = RC = \left(\frac{V}{I}\right)\left(\frac{Q}{V}\right) = \left(\frac{V}{\mathcal{Q}/t}\right)\left(\frac{\mathcal{Q}}{V}\right) = t \text{ (seconds)}$$

In Eq. (10.13), if we substitute  $t = 0$  s, we find that

$$e^{-t/\tau} = e^{-0/\tau} = e^{-0} = \frac{1}{e^0} = \frac{1}{1} = 1$$

and

$$v_C = E(1 - e^{-t/\tau}) = E(1 - 1) = 0 \text{ V}$$

want to know the voltage across the plates after one time constant, we simply plug  $t = 1\tau$  into Eq. (10.13). The result is

$$e^{-t/\tau} = e^{-\tau/\tau} = e^{-1} \cong 0.368$$

and

$$v_C = E(1 - e^{-t/\tau}) = E(1 - 0.368) = 0.632E$$

as shown in Fig. 10.29.

At  $t = 2\tau$ :

$$e^{-t/\tau} = e^{-2\tau/\tau} = e^{-2} \cong 0.135$$

and

$$v_C = E(1 - e^{-t/\tau}) = E(1 - 0.135) = 0.865E$$

as shown in Fig. 10.29.

As the number of time constants increases, the voltage across the capacitor does indeed approach the applied voltage.

At  $t = 5\tau$ :

$$e^{-t/\tau} = e^{-5\tau/\tau} = e^{-5} \cong 0.007$$

and

$$v_C = E(1 - e^{-t/\tau}) = E(1 - 0.007) = 0.993E \cong E$$

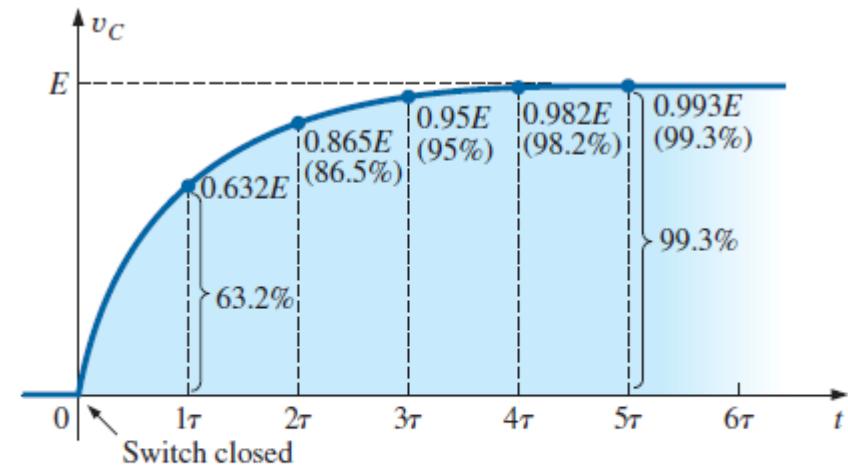
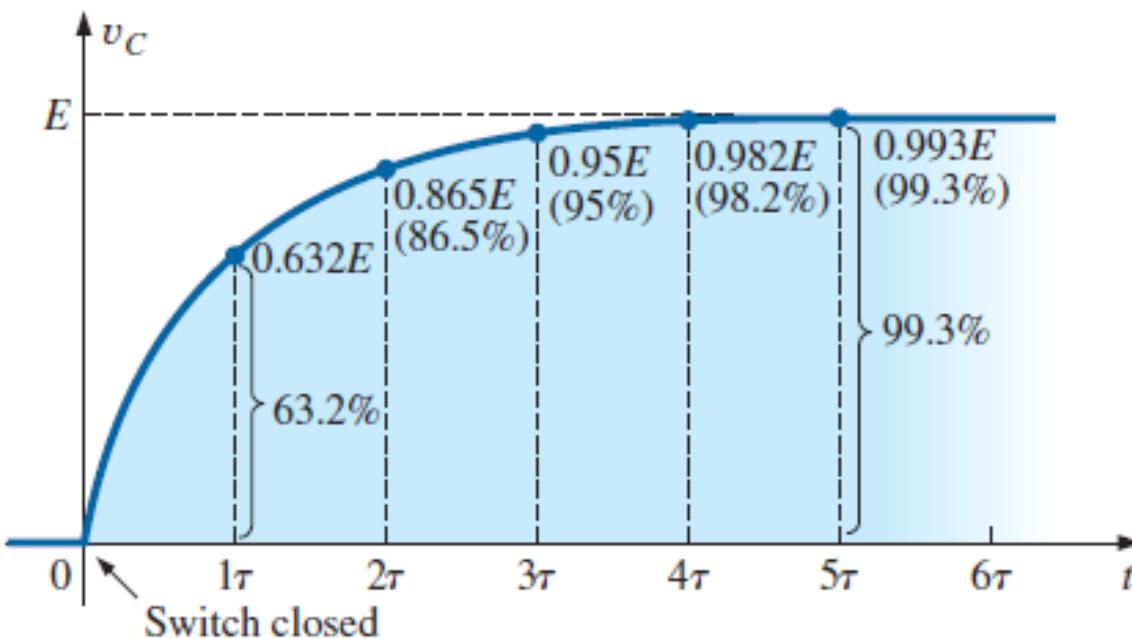


FIG. 10.29

Plotting the equation  $v_C = E(1 - e^{-t/\tau})$  versus time ( $t$ ).



**FIG. 10.29**

*Plotting the equation  $v_C = E(1 - e^{-t/\tau})$  versus time ( $t$ ).*

In fact, we can conclude from the results just obtained that

*the voltage across a capacitor in a dc network is essentially equal to the applied voltage after five time constants of the charging phase have passed.*

Or, in more general terms,

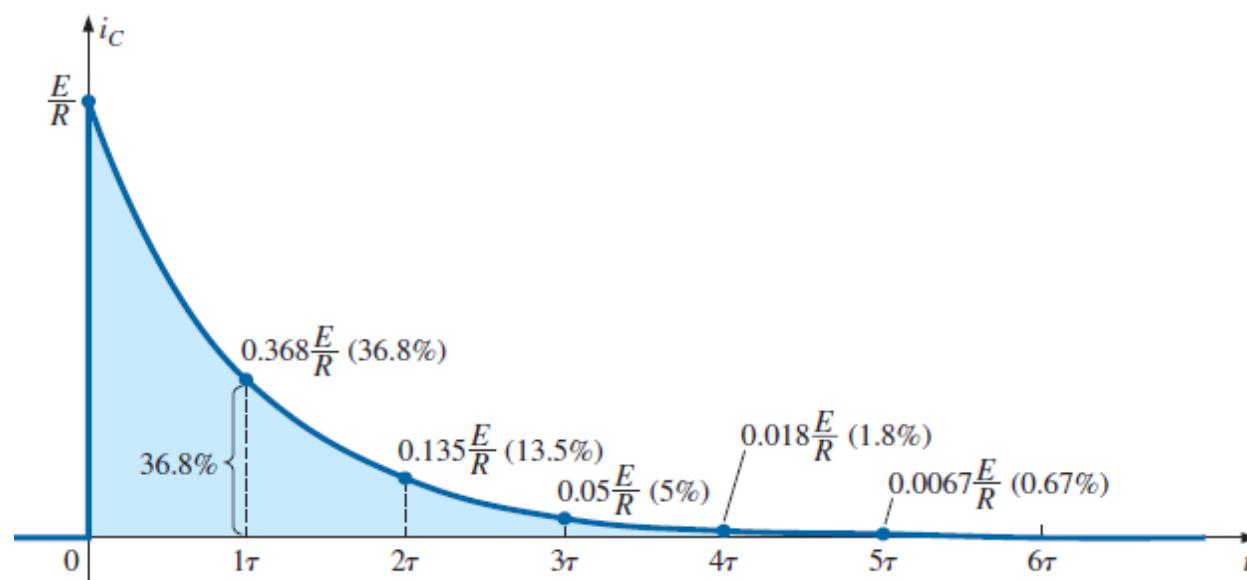
*the transient or charging phase of a capacitor has essentially ended after five time constants.*

It is indeed fortunate that the same exponential function can be used to plot the current of the capacitor versus time. When the switch is first closed, the flow of charge or current jumps very quickly to a value limited by the applied voltage and the circuit resistance, as shown in Fig. 10.30. The rate of deposit, and hence the current, then decreases quite rapidly, until eventually charge is not being deposited on the plates and the current drops to zero amperes.

The equation for the current is:

$$i_C = \frac{E}{R} e^{-t/\tau} \quad (\text{amperes, A}) \quad (10.15)$$

charging



At  $t = 0$  s:

$$e^{-t/\tau} = e^{-0} = 1$$

and

$$i_C = \frac{E}{R} e^{-t/\tau} = \frac{E}{R}(1) = \frac{E}{R}$$

At  $t = 1\tau$ :

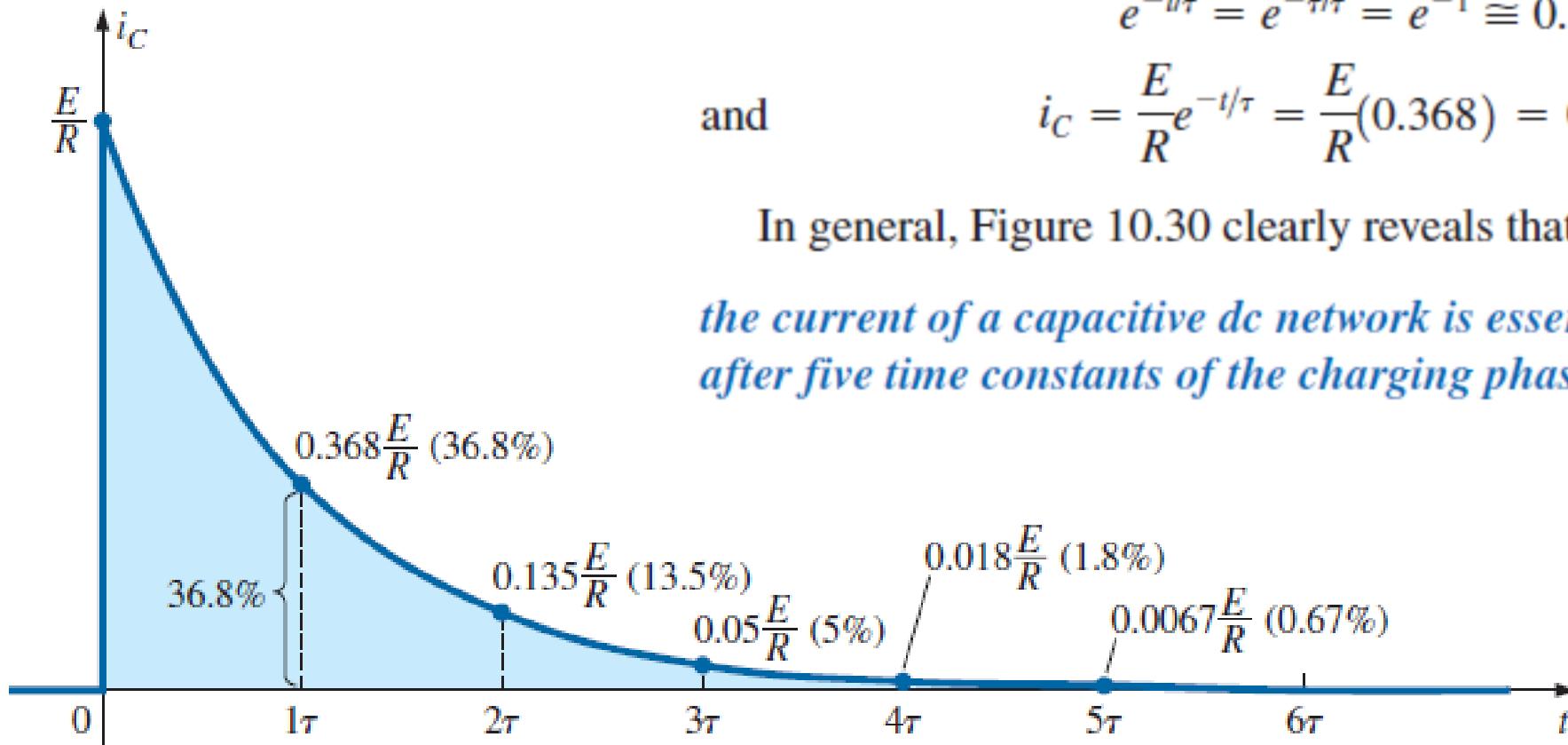
$$e^{-t/\tau} = e^{-\tau/\tau} = e^{-1} \cong 0.368$$

and

$$i_C = \frac{E}{R} e^{-t/\tau} = \frac{E}{R}(0.368) = 0.368 \frac{E}{R}$$

In general, Figure 10.30 clearly reveals that

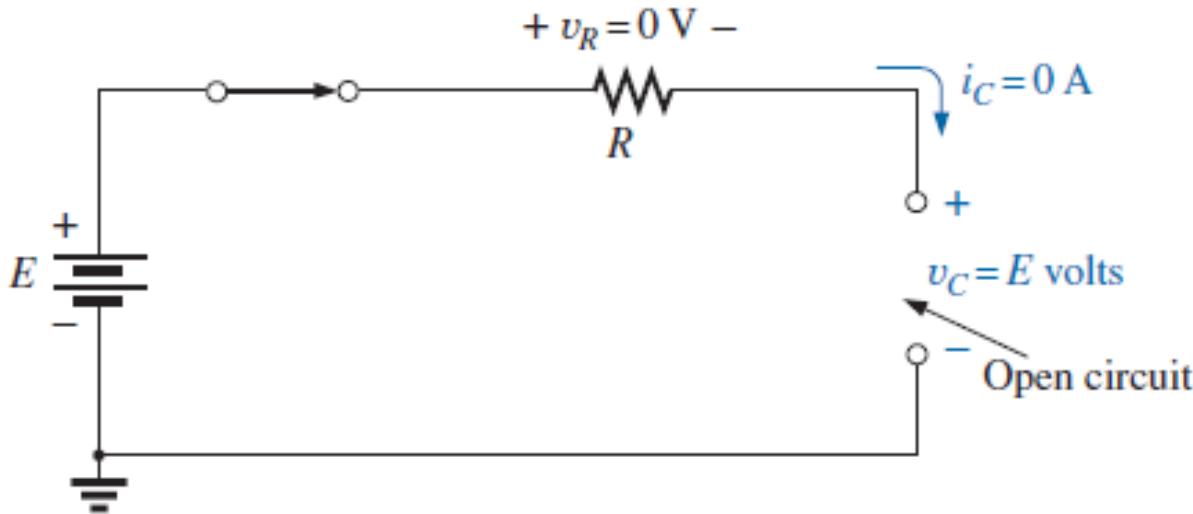
*the current of a capacitive dc network is essentially zero amperes after five time constants of the charging phase have passed.*



*during the charging phase, the major change in voltage and current occurs during the first time constant.*

The voltage across the capacitor reaches about 63.2% (about 2/3) of its final value, whereas the current drops to 36.8% (about 1/3) of its peak value. During the next time constant, the voltage increases only about 23.3%, whereas the current drops to 13.5%. The first time constant is therefore a very dramatic time for the changing parameters. Between the fourth and fifth time constants, the voltage increases only about 1.2%, whereas the current drops to less than 1% of its peak value.

*A capacitor can be replaced by an open-circuit equivalent once the charging phase in a dc network has passed.*



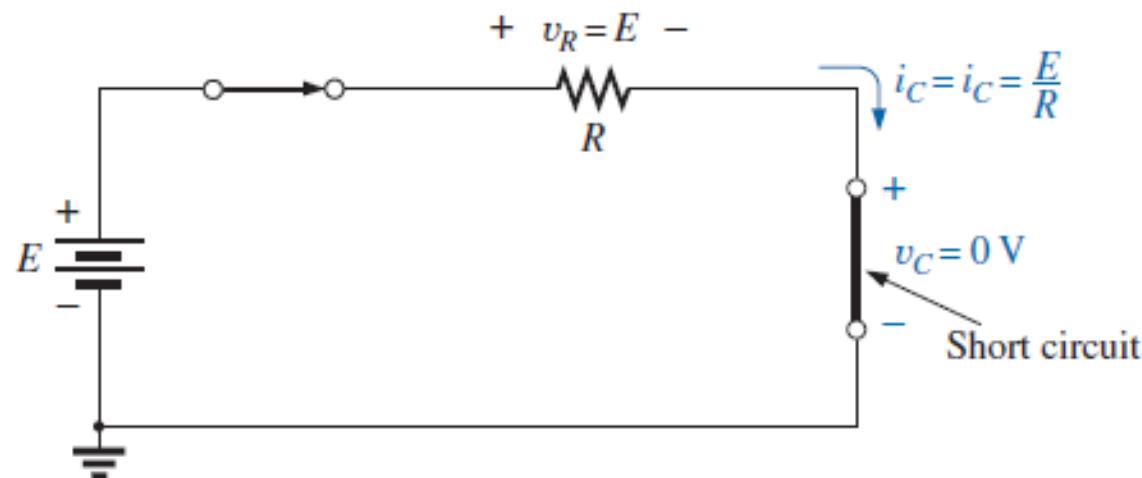
**FIG. 10.31**

*Demonstrating that a capacitor has the characteristics of an open circuit after the charging phase has passed.*

This conclusion will be particularly useful when analyzing dc networks that have been on for a long period of time or have passed the transient phase that normally occurs when a system is first turned on.

A similar conclusion can be reached if we consider the instant the switch is closed in the circuit we find that the current is a peak value at  $t = 0$  s, whereas the voltage across the capacitor is 0 V,

*a capacitor has the characteristics of a short-circuit equivalent at the instant the switch is closed in an uncharged series R-C circuit.*



**FIG. 10.32**

*Revealing the short-circuit equivalent for the capacitor that occurs when the switch is first closed.*

In Eq. (10.13), the time constant  $\tau$  will always have some value because some resistance is always present in a capacitive network. In some cases, the value of  $\tau$  may be very small, but five times that value of  $\tau$ , no matter how small, must therefore always exist; it cannot be zero. The result is the following very important conclusion:

*The voltage across a capacitor cannot change instantaneously.*

In fact, we can take this statement a step further by saying that the capacitance of a network is a measure of how much it will oppose a change in voltage in a network. The larger the capacitance, the larger the time constant, and the longer it will take the voltage across the capacitor to reach the applied value. This can prove very helpful when lightning arresters and surge suppressors are designed to protect equipment from unexpected high surges in voltage.

Since the resistor and the capacitor in Fig. 10.26 are in series, the current through the resistor is the same as that associated with the capacitor. The voltage across the resistor can be determined by using Ohm's law in the following manner:

$$v_R = i_R R = i_C R$$

so that

$$v_R = \left( \frac{E}{R} e^{-t/\tau} \right) R$$

and

$$v_R = E e^{-t/\tau} \quad \text{charging} \quad (\text{volts, V}) \quad (10.16)$$

and

$$v_R = E e^{-t/\tau}$$

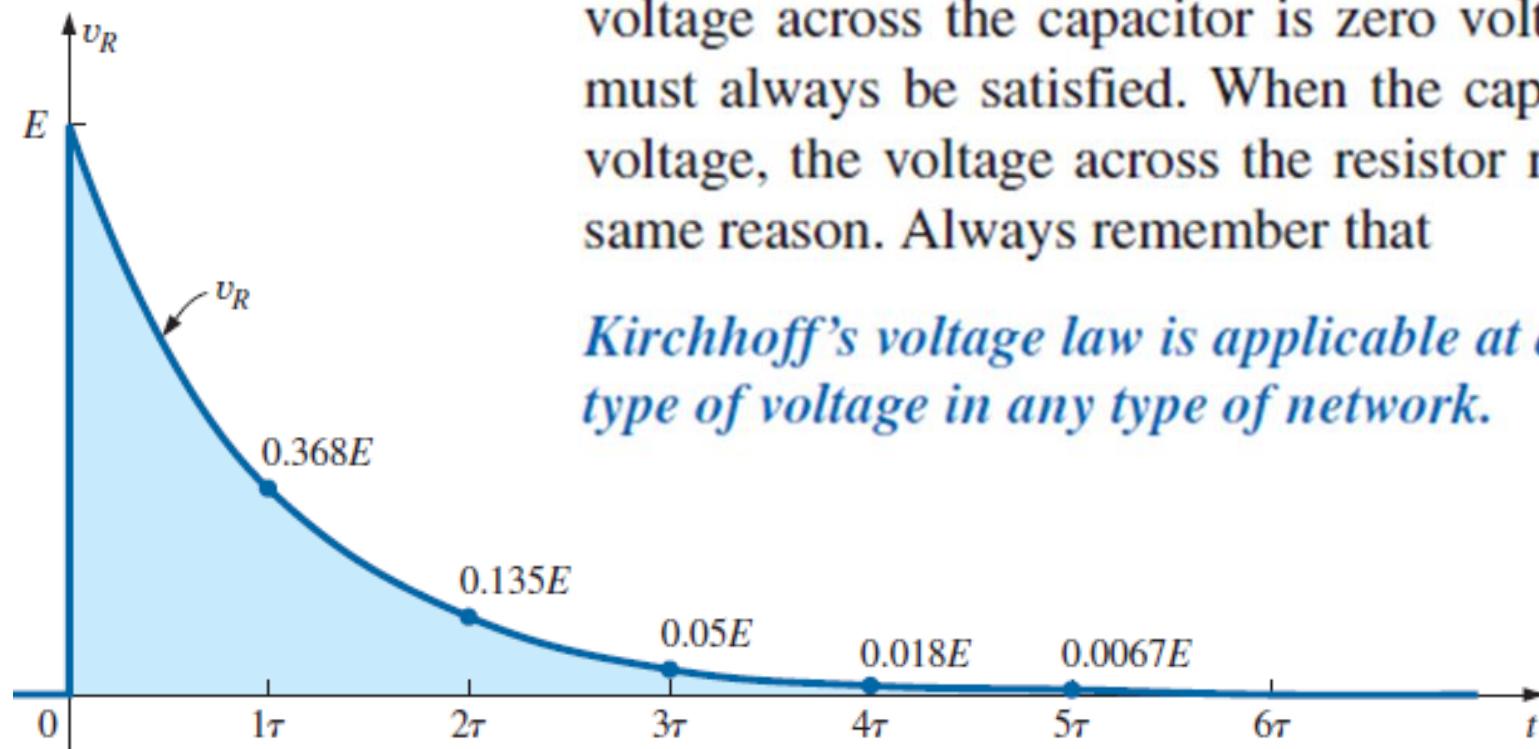
charging

(volts, V)

(10.16)

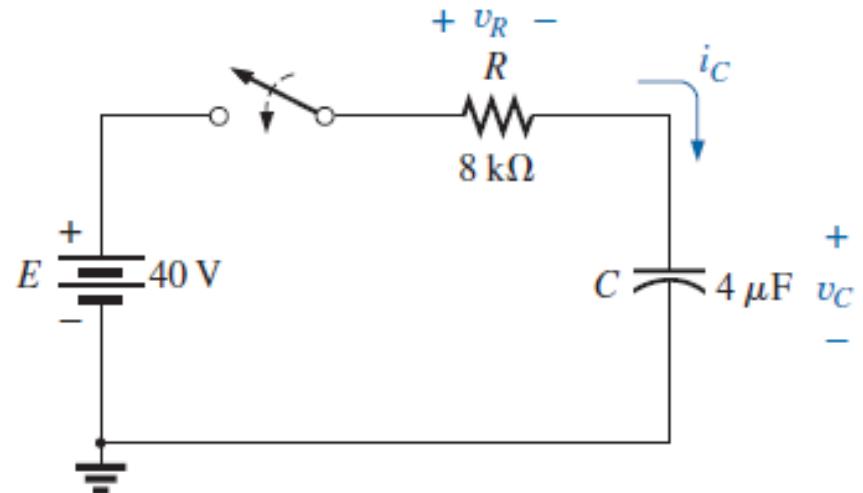
A plot of the voltage as shown in Fig. 10.33 has the same shape as that for the current because they are related by the constant  $R$ . Note, however, that the voltage across the resistor starts at a level of  $E$  volts because the voltage across the capacitor is zero volts and Kirchhoff's voltage law must always be satisfied. When the capacitor has reached the applied voltage, the voltage across the resistor must drop to zero volts for the same reason. Always remember that

*Kirchhoff's voltage law is applicable at any instant of time for any type of voltage in any type of network.*



**EXAMPLE 10.6** For the circuit in Fig. 10.35:

- a. Find the mathematical expression for the transient behavior of  $v_C$ ,  $i_C$ , and  $v_R$  if the switch is closed at  $t = 0$  s.
- b. Plot the waveform of  $v_C$  versus the time constant of the network.
- c. Plot the waveform of  $v_C$  versus time.
- d. Plot the waveforms of  $i_C$  and  $v_R$  versus the time constant of the network.
- e. What is the value of  $v_C$  at  $t = 20$  ms?
- f. On a practical basis, how much time must pass before we can assume that the charging phase has passed?
- g. When the charging phase has passed, how much charge is sitting on the plates?
- h. If the capacitor has a leakage resistance of  $10,000\text{ M}\Omega$ , what is the initial leakage current? Once the capacitor is separated from the circuit, how long will it take to totally discharge, assuming a linear (unchanging) discharge rate?



**FIG. 10.35**  
Transient network for Example 10.6.

a. The time constant of the network is

$$\tau = RC = (8 \text{ k}\Omega)(4 \mu\text{F}) = 32 \text{ ms}$$

resulting in the following mathematical equations:

$$v_C = E(1 - e^{-t/\tau}) = 40 \text{ V}(1 - e^{-t/32\text{ms}})$$

$$i_C = \frac{E}{R}e^{-t/\tau} = \frac{40 \text{ V}}{8 \text{ k}\Omega}e^{-t/32\text{ms}} = 5 \text{ mA}e^{-t/32\text{ms}}$$

$$v_R = Ee^{-t/\tau} = 40 \text{ V}e^{-t/32\text{ms}}$$

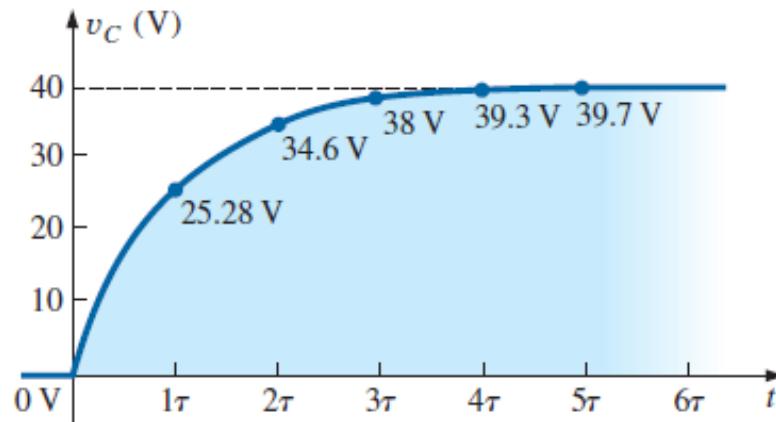


FIG. 10.36

$v_C$  versus time for the charging network in Fig. 10.35.

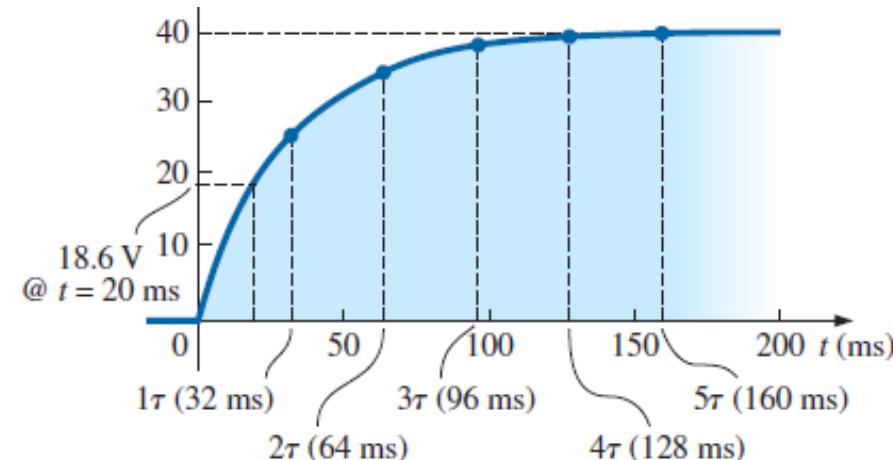
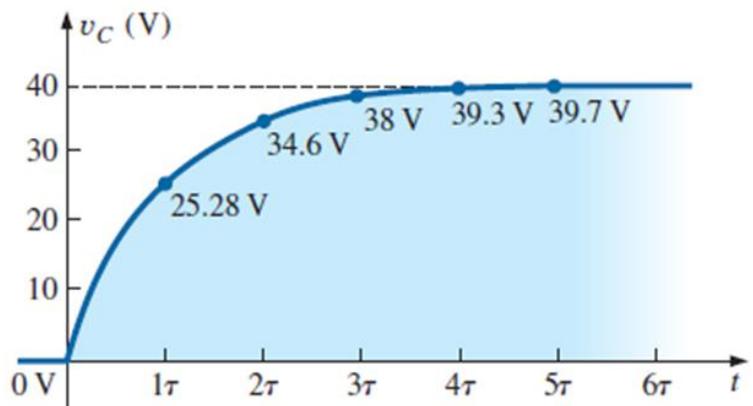


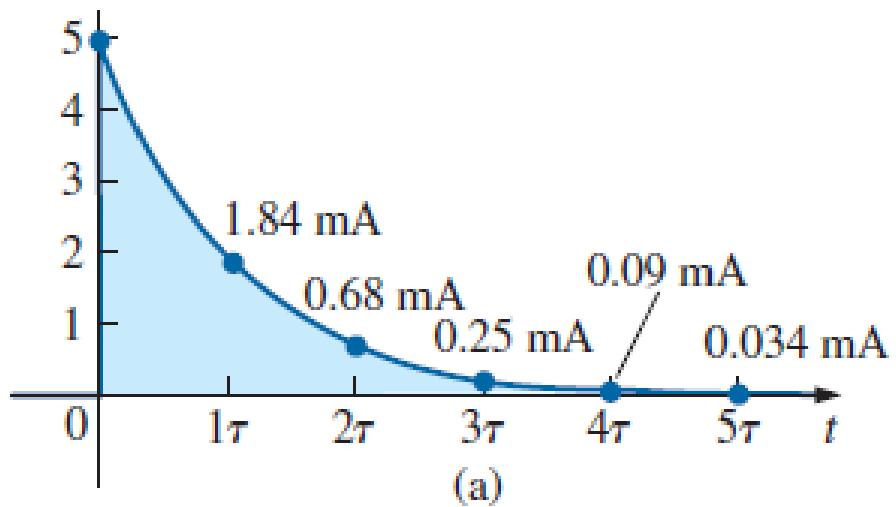
FIG. 10.37

Plotting the waveform in Fig. 10.36 versus time ( $t$ ).

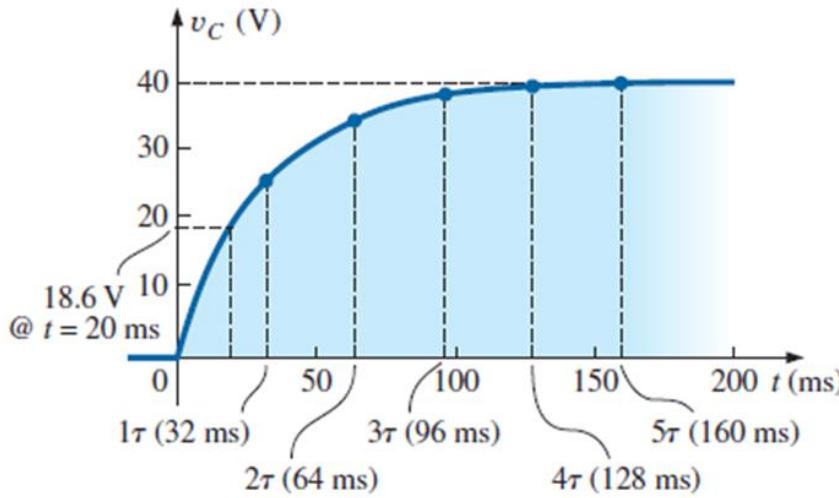


**FIG. 10.36**

$v_C$  versus time for the charging network in Fig. 10.35.

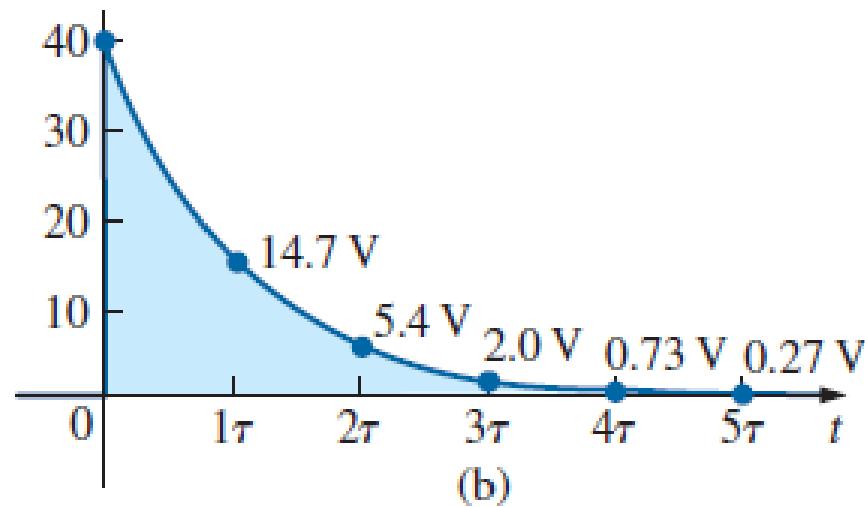


(a)



**FIG. 10.37**

Plotting the waveform in Fig. 10.36 versus time ( $t$ ).



(b)

**FIG. 10.38**

$i_C$  and  $v_R$  for the charging network in Fig. 10.36.

- e. Substituting the time  $t = 20$  ms results in the following for the exponential part of the equation:

$$e^{-t/\tau} = e^{-20\text{ms}/32\text{ms}} = e^{-0.625} = 0.535 \text{ (using a calculator)}$$

$$\begin{aligned}\text{so that } v_C &= 40 \text{ V}(1 - e^{-t/32\text{ms}}) = 40 \text{ V} (1 - 0.535) \\ &= (40 \text{ V})(0.465) = \mathbf{18.6 \text{ V}} \text{ (as verified by Fig. 10.37)}\end{aligned}$$

- f. Assuming a full charge in five time constants results in

$$5\tau = 5(32 \text{ ms}) = \mathbf{160 \text{ ms}} = \mathbf{0.16 \text{ s}}$$

- g. Using Eq. (10.6):

$$Q = CV = (4 \mu\text{F})(40 \text{ V}) = \mathbf{160 \mu\text{C}}$$

- h. Using Ohm's law:

$$I_{\text{leakage}} = \frac{40 \text{ V}}{10,000 \text{ M}\Omega} = 4 \text{ nA}$$

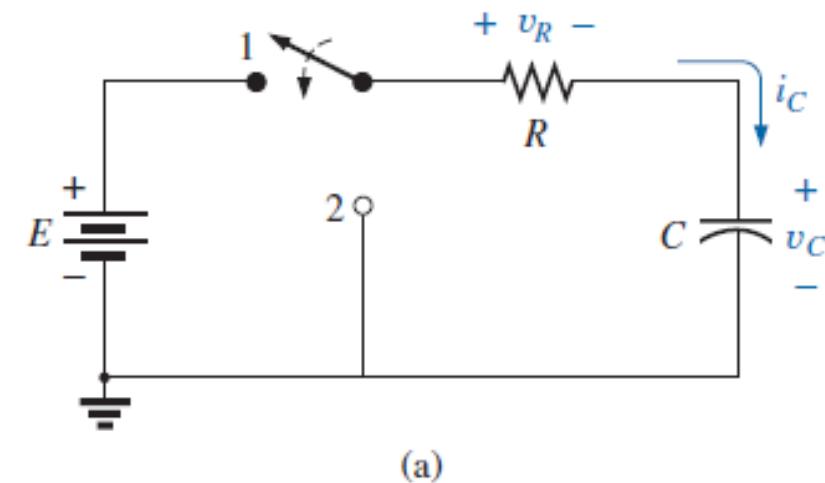
Finally, the basic equation  $I = Q/t$  results in

$$t = \frac{Q}{I} = \frac{160 \mu\text{C}}{4 \text{ nA}} = (40,000 \text{ s}) \left( \frac{1 \text{ min}}{60 \text{ s}} \right) \left( \frac{1 \text{ h}}{60 \text{ min}} \right) = \mathbf{11.11 \text{ h}}$$

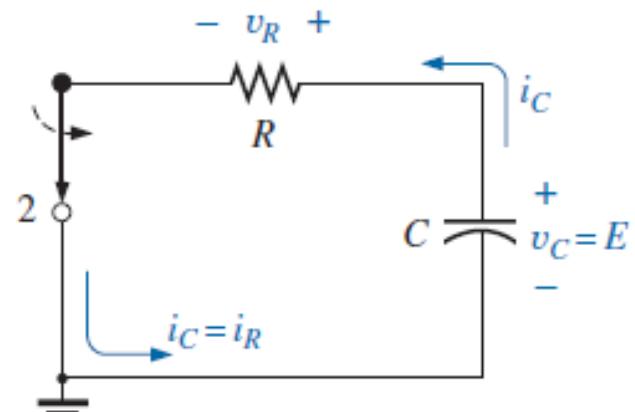
- e. What is the value of  $v_C$  at  $t = 20$  ms?
- f. On a practical basis, how much time must pass before we can assume that the charging phase has passed?
- g. When the charging phase has passed, how much charge is sitting on the plates?
- h. If the capacitor has a leakage resistance of  $10,000 \text{ M}\Omega$ , what is the initial leakage current? Once the capacitor is separated from the circuit, how long will it take to totally discharge, assuming a linear (unchanging) discharge rate?

## 10.6 TRANSIENTS IN CAPACITIVE NETWORKS: THE DISCHARGING PHASE

In Fig. 10.39(a), a second contact for the switch was added to the circuit in Fig. 10.26 to permit a controlled discharge of the capacitor. With the switch in position 1, we have the charging network described in the last section. Following the full charging phase, if we move the switch to position 2, the capacitor can be discharged through the resulting circuit in Fig. 10.39(b). In Fig. 10.39(b), the voltage across the capacitor appears directly across the resistor to establish a discharge current. Initially, the current jumps to a relatively high value; then it begins to drop. It drops with time because charge is leaving the plates of the capacitor, which in turn reduces the voltage across the capacitor and thereby the voltage across the resistor and the resulting current.



(a)



(b)

**FIG. 10.39**  
i) Charging network; (b) discharging configuration.

For the voltage across the capacitor that is decreasing with time, the mathematical expression is:

$$v_C = Ee^{-t/\tau} \quad \text{discharging} \quad (10.17)$$

For this circuit, the time constant  $\tau$  is defined by the same equation as used for the charging phase. That is,

$$\tau = RC \quad \text{discharging} \quad (10.18)$$

Since the current decreases with time, it will have a similar format:

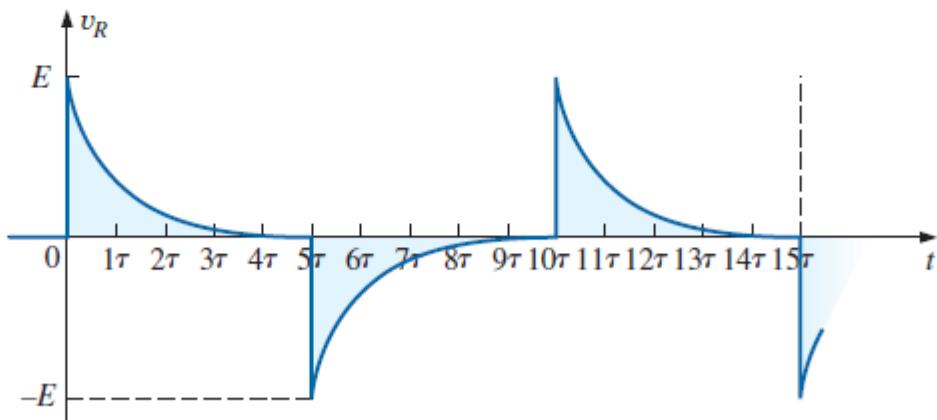
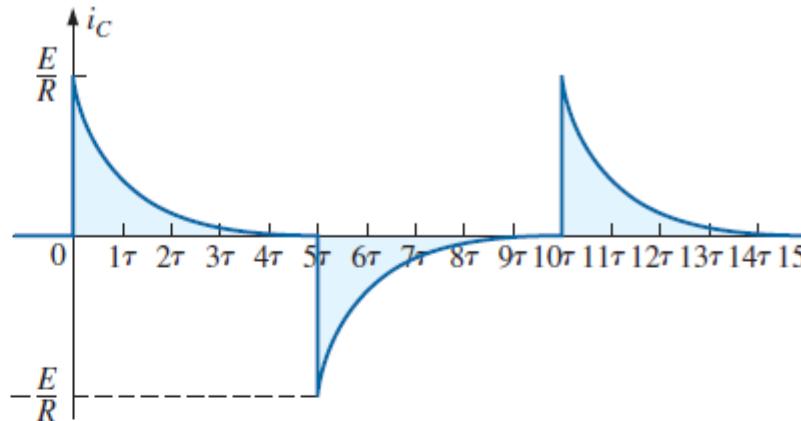
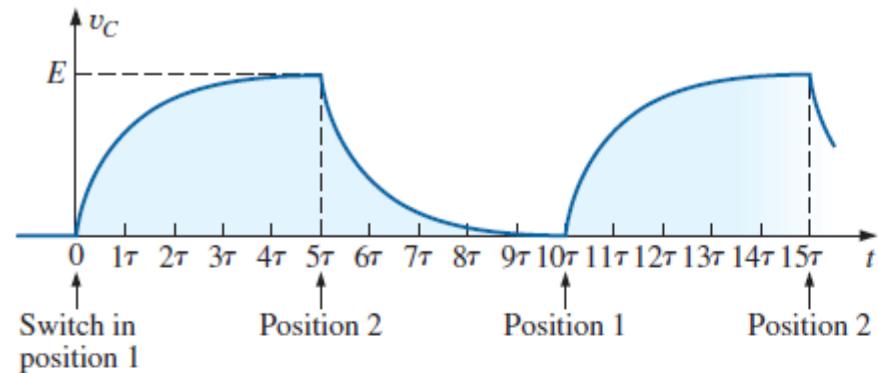
$$i_C = \frac{E}{R} e^{-t/\tau} \quad \text{discharging} \quad (10.19)$$

For the configuration in Fig. 10.39(b), since  $v_R = v_C$  (in parallel), the equation for the voltage  $v_R$  has the same format:

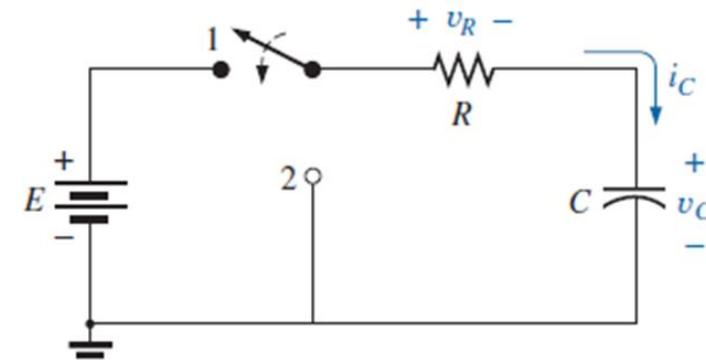
$$v_R = Ee^{-t/\tau} \quad \text{discharging} \quad (10.20)$$

The complete discharge will occur, for all practical purposes, in five time constants. If the switch is moved between terminals 1 and 2 every five time constants, the wave shapes in Fig. 10.40 will result for  $v_C$ ,  $i_C$ ,

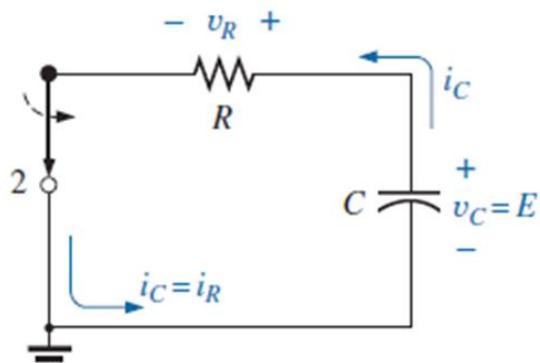
Before looking at the wave shapes for each quantity of interest, note that current  $i_C$  has now reversed direction as shown in Fig. 10.39(b). As shown in parts (a) and (b) in Fig. 10.39, the voltage across the capacitor does not reverse polarity, but the current reverses direction. We will show the rever-



**FIG. 10.40**  
 $v_C$ ,  $i_C$ , and  $v_R$  for  $5\tau$  switching between contacts in Fig. 10.39(a).



(a)



(b)

**FIG. 10.39**  
 i) Charging network; (b) discharging configuration.

**EXAMPLE 10.7** Using the values in Example 10.6, plot the waveforms for  $v_C$  and  $i_C$  resulting from switching between contacts 1 and 2 in Fig. 10.39 every five time constants.

**Solution:** The time constant is the same for the charging and discharging phases. That is,

$$\tau = RC = (8 \text{ k}\Omega)(4 \mu\text{F}) = 32 \text{ ms}$$

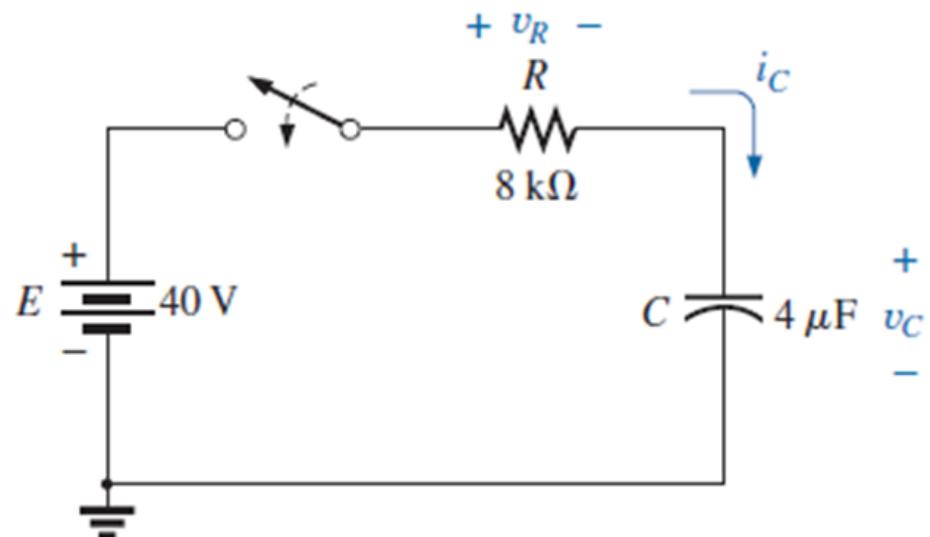
For the discharge phase, the equations are

$$v_C = Ee^{-t/\tau} = 40 \text{ V}e^{-t/32\text{ms}}$$

$$i_C = -\frac{E}{R}e^{-t/\tau} = \frac{40 \text{ V}}{8 \text{ k}\Omega}e^{-t/32\text{ms}} = -5 \text{ mA}e^{-t/32\text{ms}}$$

$$v_R = v_C = 40 \text{ V}e^{-t/32\text{ms}}$$

A continuous plot for the charging and discharging phases  
Fig. 10.41.



**FIG. 10.35**  
*Transient network for Example 10.6.*

A continuous plot for the charging and discharging phases appears in Fig. 10.41.

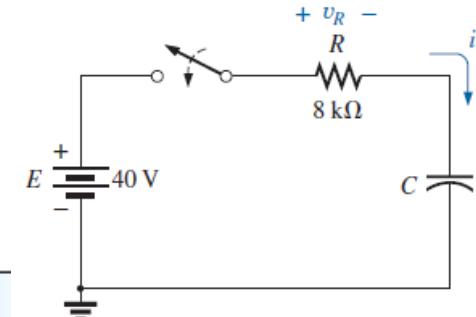
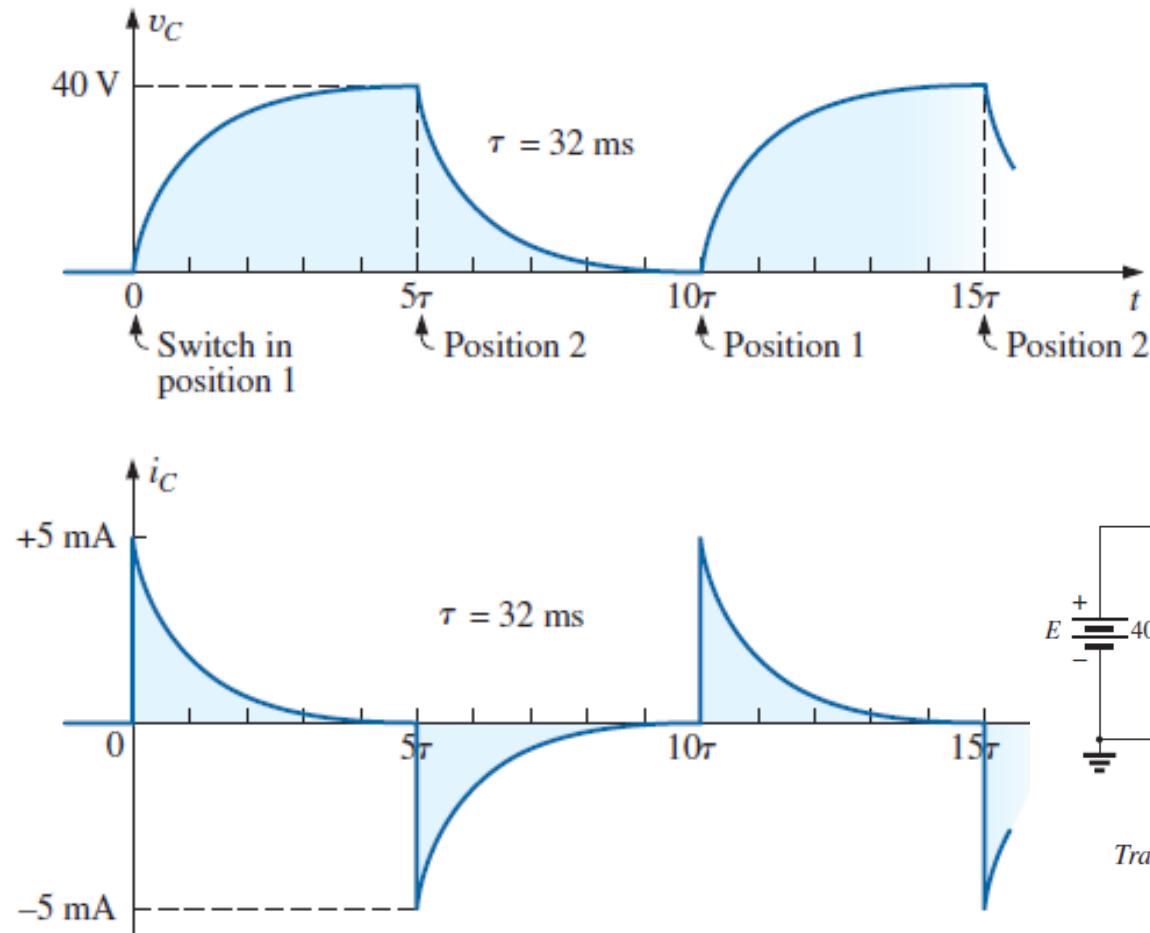


FIG. 10.35  
Transient network for Example 10.6.

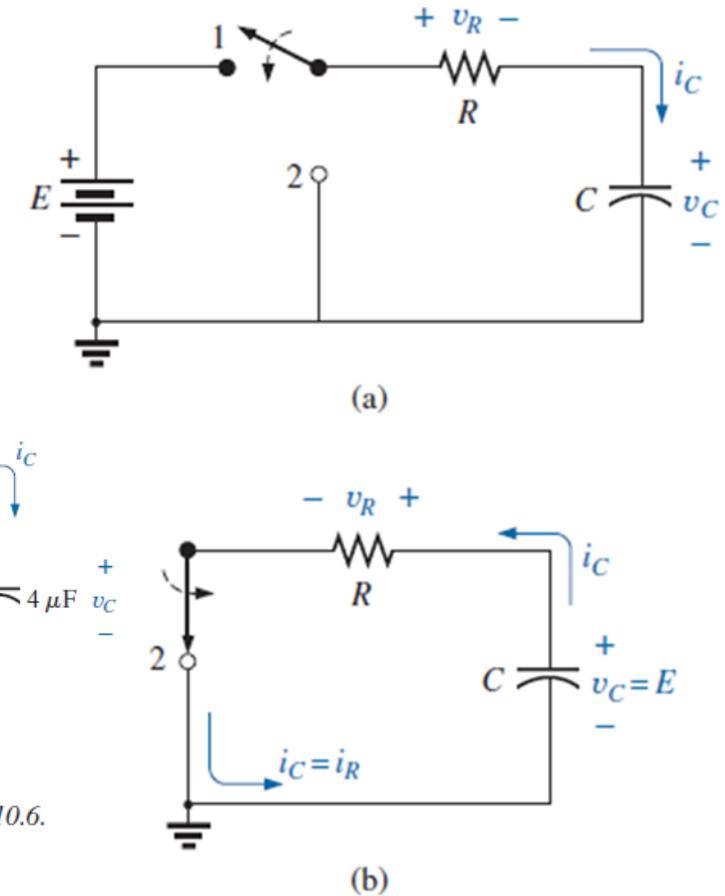
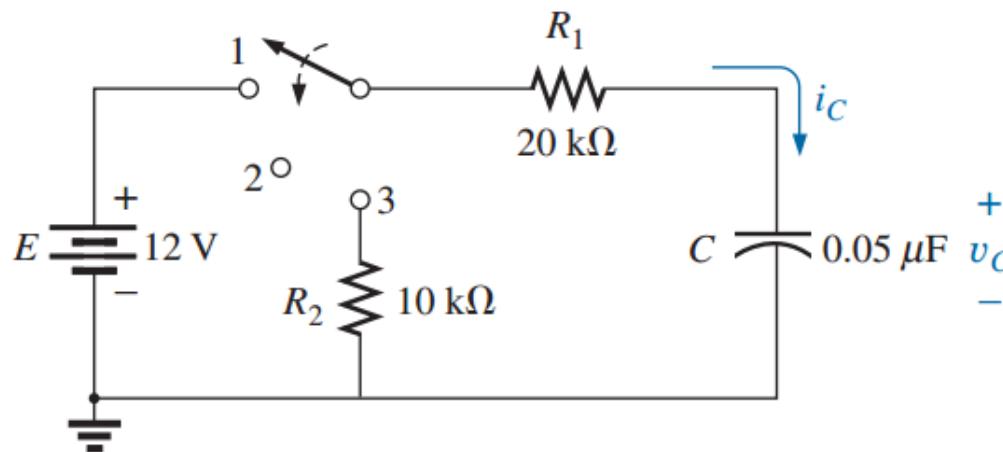


FIG. 10.39

i) Charging network; (b) discharging configuration.

**EXAMPLE 10.8** For the circuit in Fig. 10.44:

- a. Find the mathematical expressions for the transient behavior of the voltage  $v_C$  and the current  $i_C$  if the capacitor was initially uncharged and the switch is thrown into position 1 at  $t = 0$  s.



- b. Find the mathematical expressions for the voltage  $v_C$  and the current  $i_C$  if the switch is moved to position 2 at  $t = 10$  ms. (Assume that the leakage resistance of the capacitor is infinite ohms; that is, there is no leakage current.)
- c. Find the mathematical expressions for the voltage  $v_C$  and the current  $i_C$  if the switch is thrown into position 3 at  $t = 20$  ms.
- d. Plot the waveforms obtained in parts (a)–(c) on the same time axis using the defined polarities in Fig. 10.44.

**Solutions:**

a. *Charging phase:*

$$\tau = R_1 C = (20 \text{ k}\Omega)(0.05 \mu\text{F}) = 1 \text{ ms}$$

$$v_C = E(1 - e^{-t/\tau}) = 12 \text{ V}(1 - e^{-t/1\text{ms}})$$

$$i_C = \frac{E}{R_1} e^{-t/\tau} = \frac{12 \text{ V}}{20 \text{ k}\Omega} e^{-t/1\text{ms}} = 0.6 \text{ mA} e^{-t/1\text{ms}}$$

b. *Storage phase:* At 10 ms, a period of time equal to  $10\tau$  has passed, permitting the assumption that the capacitor is fully charged. Since  $R_{\text{leakage}} = \infty \Omega$ , the capacitor will hold its charge indefinitely. The result is that both  $v_C$  and  $i_C$  will remain at a fixed value of

$$v_C = 12 \text{ V}$$

$$i_C = 0 \text{ A}$$

c. *Discharge phase* (using 20 ms as the new  $t = 0$  s for the equations):

The new time constant is

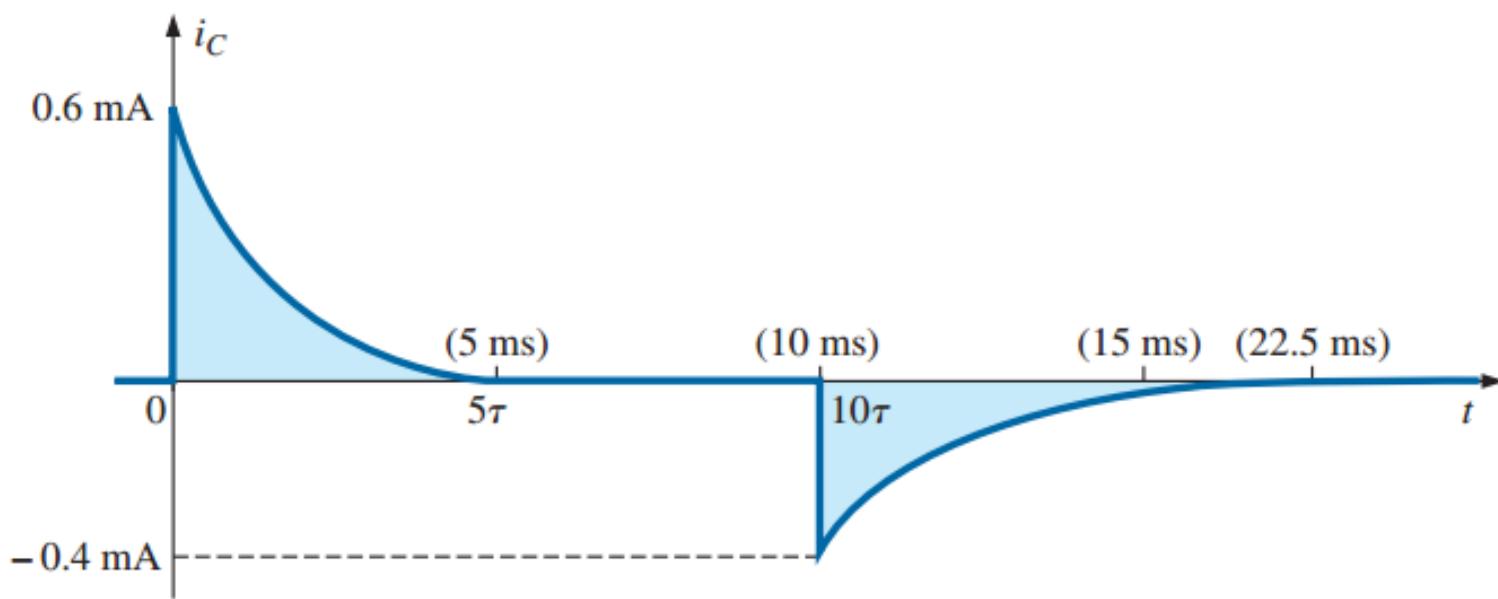
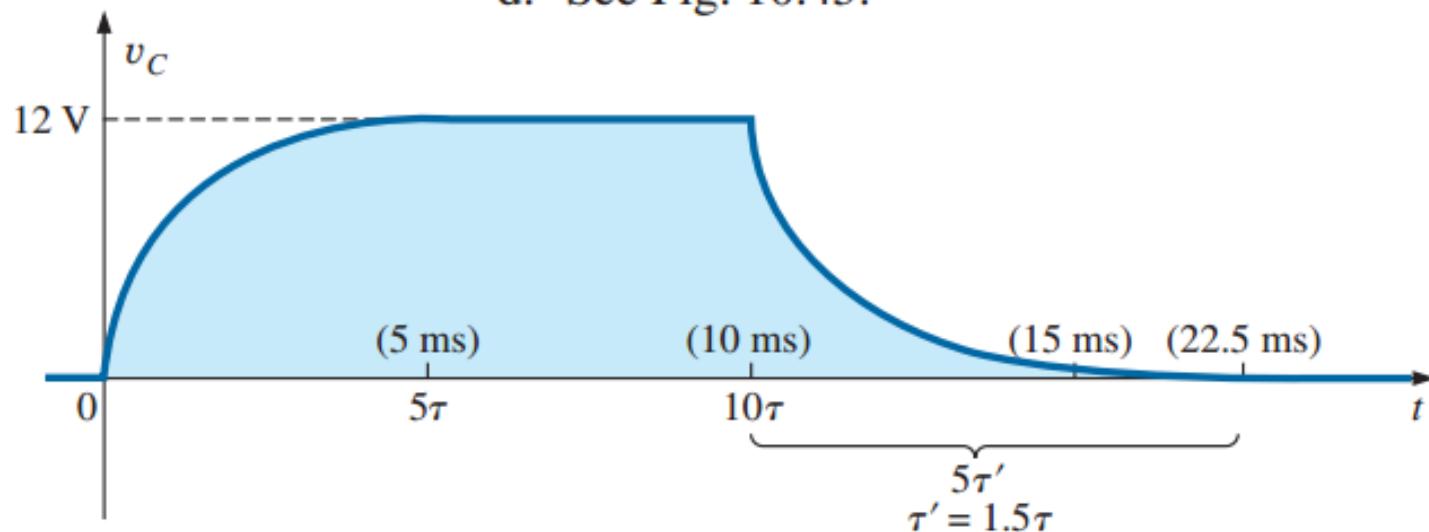
$$\tau' = RC = (R_1 + R_2)C = (20 \text{ k}\Omega + 10 \text{ k}\Omega)(0.05 \mu\text{F}) = 1.5 \text{ ms}$$

$$v_C = Ee^{-t/\tau'} = 12 \text{ V}e^{-t/1.5\text{ms}}$$

$$i_C = -\frac{E}{R}e^{-t/\tau'} = -\frac{E}{R_1 + R_2}e^{-t/\tau'}$$

$$= -\frac{12 \text{ V}}{20 \text{ k}\Omega + 10 \text{ k}\Omega} e^{-t/1.5\text{ms}} = -0.4 \text{ mA} e^{-t/1.5\text{ms}}$$

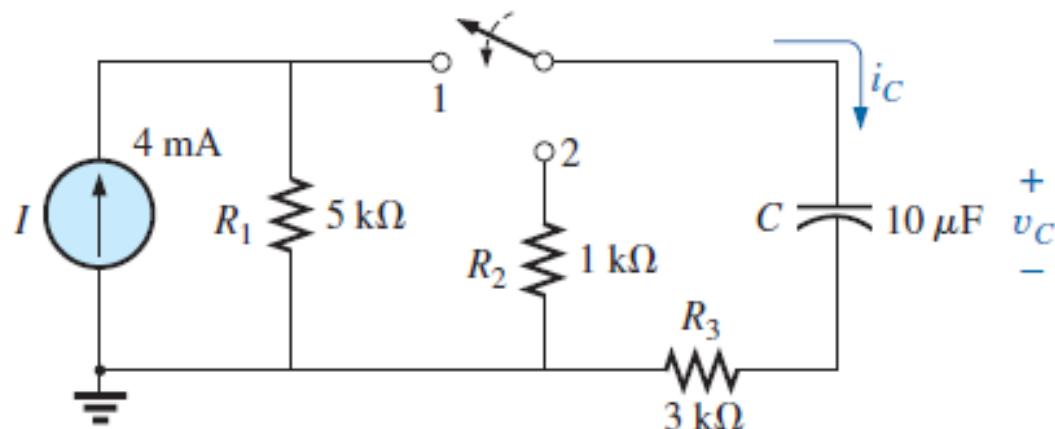
d. See Fig. 10.45.



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**EXAMPLE 10.9** For the network in Fig. 10.46:

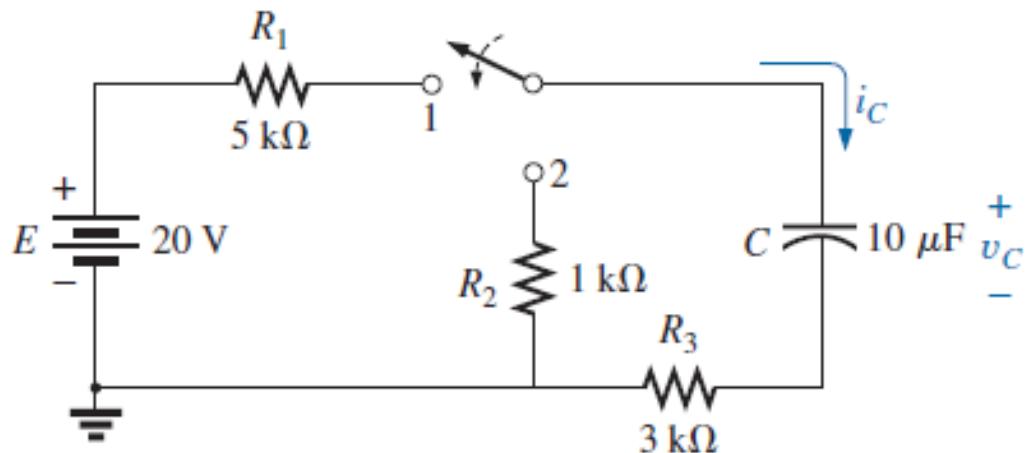
- a. Find the mathematical expression for the transient behavior of the voltage across the capacitor if the switch is thrown into position 1 at  $t = 0$  s.
- b. Find the mathematical expression for the transient behavior of the voltage across the capacitor if the switch is moved to position 2 at  $t = 1\tau$ .
- c. Plot the resulting waveform for the voltage  $v_C$  as determined by parts (a) and (b).
- d. Repeat parts (a)–(c) for the current  $i_C$ .



**FIG. 10.46**  
Network to be analyzed in Example 10.9.

**Solutions:**

- a. Converting the current source to a voltage source results in the configuration in Fig. 10.47 for the charging phase.



**FIG. 10.47**

*The charging phase for the network in Fig. 10.46.*

For the source conversion:  $E = IR = (4 \text{ mA})(5 \text{ k}\Omega) = 20 \text{ V}$

and

$$R_s = R_p = 5 \text{ k}\Omega$$

$$\tau = RC = (R_1 + R_3)C = (5 \text{ k}\Omega + 3 \text{ k}\Omega)(10 \mu\text{F}) = 80 \text{ ms}$$

$$v_C = E(1 - e^{-t/\tau}) = 20 \text{ V} (1 - e^{-t/80\text{ms}})$$

b. With the switch in position 2, the network appears as shown in Fig. 10.48. The voltage at  $1\tau$  can be found by using the fact that the voltage is 63.2% of its final value of 20 V, so that  $0.632(20 \text{ V}) = 12.64 \text{ V}$ . Or you can substitute into the derived equation as follows:

$$e^{-t/\tau} = e^{-t/\tau} = e^{-1} = 0.368$$

and  $v_C = 20 \text{ V}(1 - e^{-t/80\text{ms}}) = 20 \text{ V}(1 - 0.368)$   
 $= (20 \text{ V})(0.632) = 12.64 \text{ V}$

Using this voltage as the starting point and substituting into the discharge equation results in

$$\tau' = RC = (R_2 + R_3)C = (1 \text{ k}\Omega + 3 \text{ k}\Omega)(10 \mu\text{F}) = 40 \text{ ms}$$

$$v_C = Ee^{-t/\tau'} = 12.64 \text{ V}e^{-t/40\text{ms}}$$

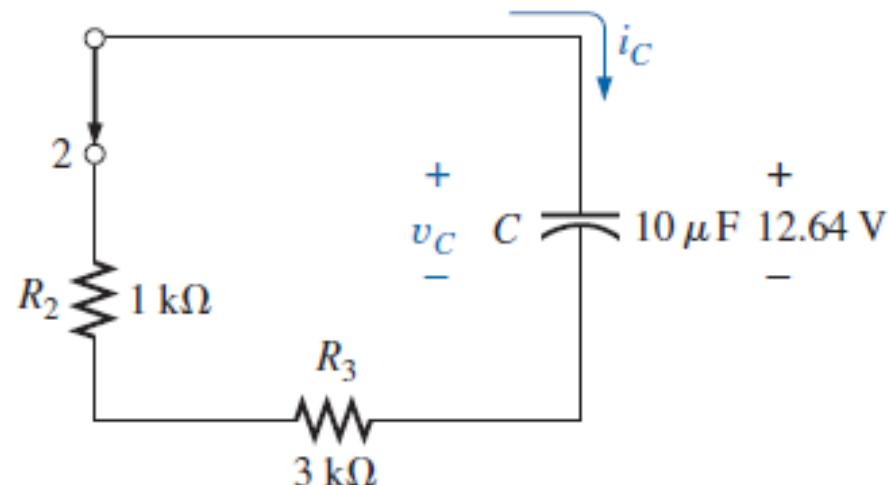
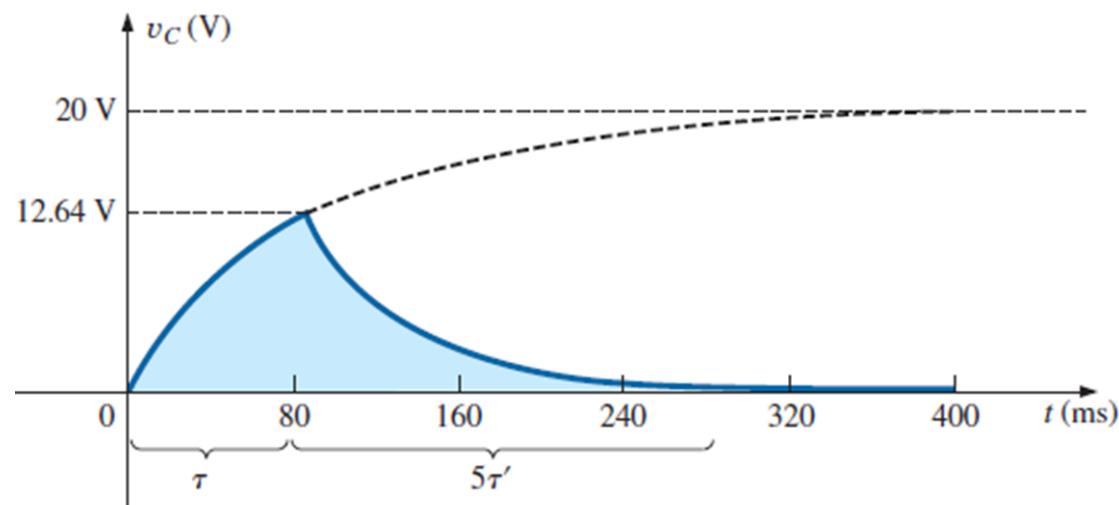


FIG. 10.48

Network in Fig. 10.47 when the switch is moved to position 2 at  $t = 1\tau_1$ .



d. The charging equation for the current is

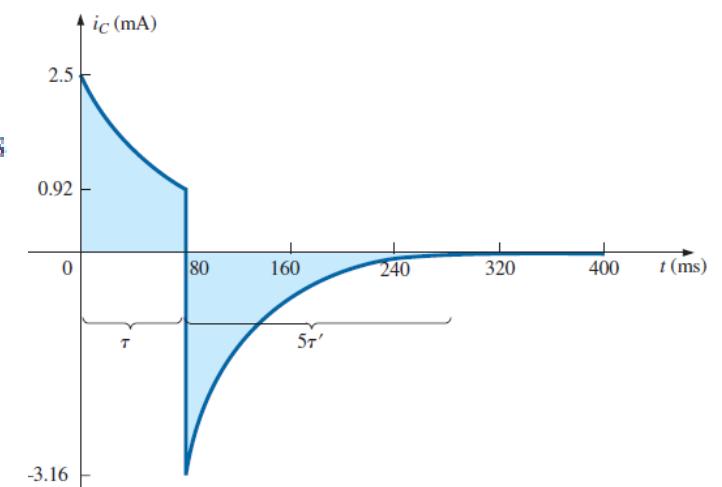
$$i_C = \frac{E}{R} e^{-t/\tau} = \frac{E}{R_1 + R_3} e^{-t/\tau} = \frac{20 \text{ V}}{8 \text{ k}\Omega} e^{-t/80\text{ms}} = 2.5 \text{ mA} e^{-t/80\text{ms}}$$

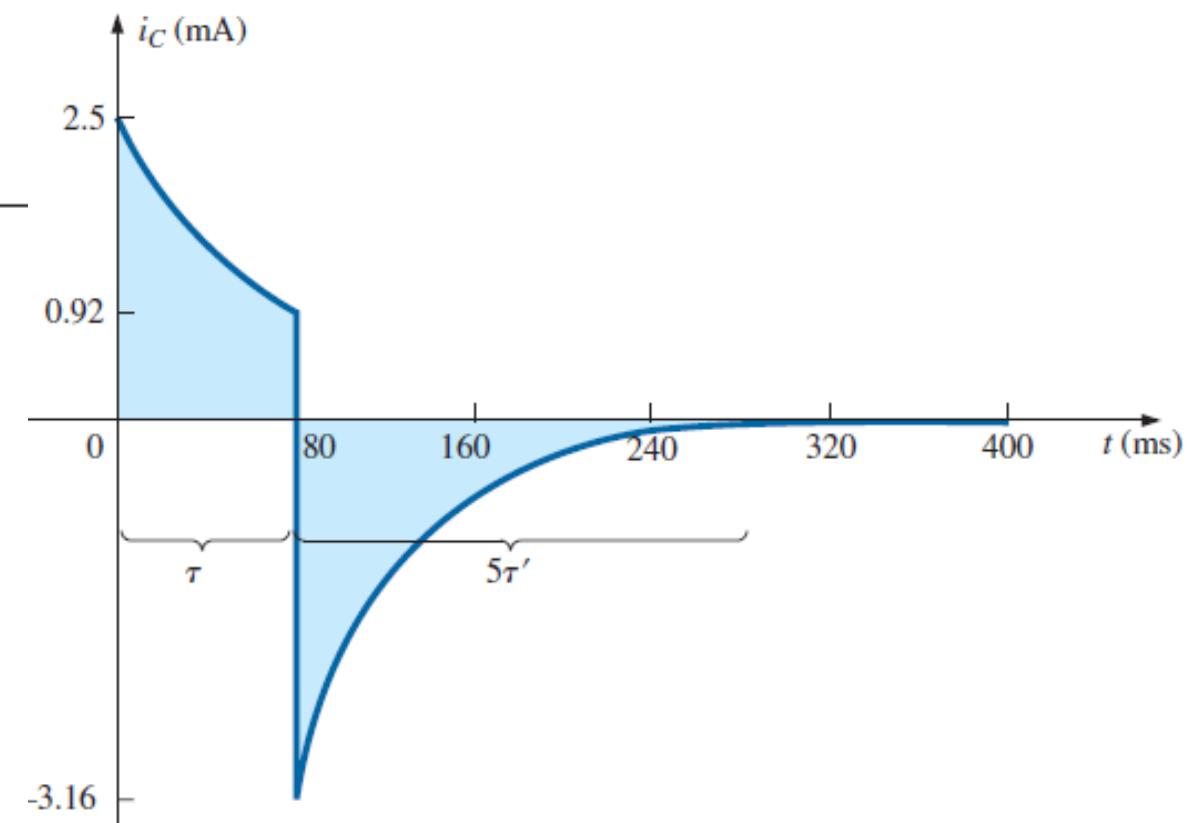
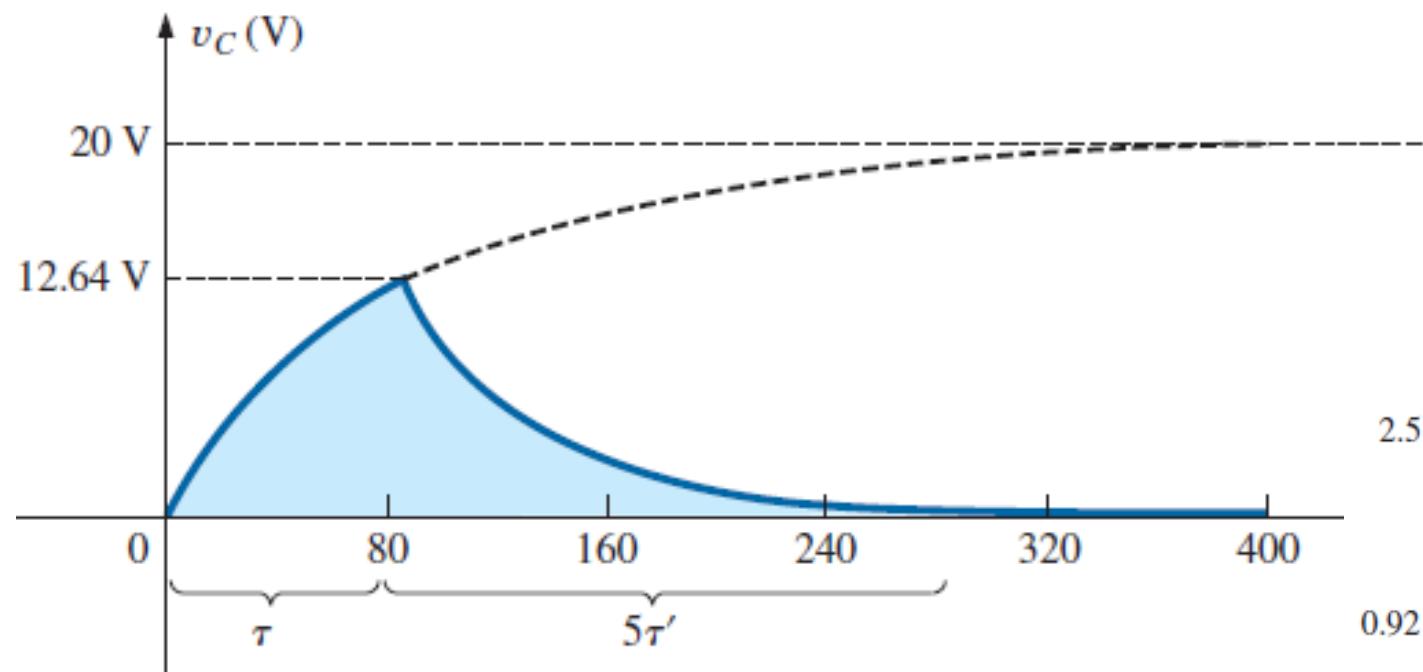
which, at  $t = 80 \text{ ms}$ , results in

$$i_C = 2.5 \text{ mA} e^{-80\text{ms}/80\text{ms}} = 2.5 \text{ mA} e^{-1} = (2.5 \text{ mA})(0.368) = 0.92 \text{ mA}$$

When the switch is moved to position 2, the  $12.64 \text{ V}$  across the capacitor appears across the resistor to establish a current of  $12.64 \text{ V}/4 \text{ k}\Omega = 3.16 \text{ mA}$ . Substituting into the discharge equation with  $V_i = 12.64 \text{ V}$  and  $\tau' = 40 \text{ ms}$  yields

$$\begin{aligned} i_C &= -\frac{V_i}{R_2 + R_3} e^{-t/\tau'} = -\frac{12.64 \text{ V}}{1 \text{ k}\Omega + 3 \text{ k}\Omega} e^{-t/40\text{ms}} \\ &= -\frac{12.64 \text{ V}}{4 \text{ k}\Omega} e^{-t/40\text{ms}} = -3.16 \text{ mA} e^{-t/40\text{ms}} \end{aligned}$$





## 10.10 THE CURRENT $i_C$

There is a very special relationship between the current of a capacitor and the voltage across it. For the resistor, it is defined by Ohm's law:  $i_R = v_R/R$ . The current through and the voltage across the resistor are related by a constant  $R$ —a very simple direct linear relationship. For the capacitor, it is the more complex relationship defined by

$$i_C = C \frac{dv_C}{dt} \quad (10.26)$$

The factor  $C$  reveals that the higher the capacitance, the greater the resulting current. Intuitively, this relationship makes sense, because higher capacitance levels result in increased levels of stored charge, providing a source for increased current levels. The second term,  $dv_C/dt$ , is sensitive to the *rate of change* of  $v_C$  with time. The function  $dv_C/dt$  is called the **derivative** (calculus) of the voltage  $v_C$  with respect to time  $t$ . The faster the voltage  $v_C$  changes with time, the larger the factor  $dv_C/dt$  will be and the larger the resulting current  $i_C$  will be. That is why the current jumps to its maximum of  $E/R$  in a charging circuit the instant the switch is closed. At that instant, if you look at the charging curve for  $v_C$ , the voltage is *changing* at its greatest rate. As it approaches its final value, the rate of change decreases, and, as confirmed by Eq. (10.26), the level of current decreases.

*The capacitive current is directly related to the rate of change of the voltage across the capacitor, not the levels of voltage involved.*

For example, the current of a capacitor will be *greater* when the voltage changes from 1 V to 10 V in 1 ms than when it changes from 10 V to 100 V in 1 s; in fact, it will be 100 times more.

calculate the **average current** associated with a capacitor for various voltages impressed across the capacitor. The average current is defined by the equation

$$i_{C_{av}} = C \frac{\Delta v_C}{\Delta t} \quad (10.27)$$

**Thank You**