Exponential Function

Defn :

The expression of the form $\boxed{a^r}$

where a is the base,

r is The exponent.

and a is any real number

r is any rational number.

But The meaning a^x is

The base a is positive real number

The exponent x is irrational number.

So The exponential is the form

$$\boxed{a^x \approx a^r}$$

The base is any real number

The exponent r select a rational number is form by truncating (removing) all but a finite number of digit from the irrational number x.

$$\boxed{f(x) = a^x}$$

Law of Exponents

If s, t, a and b are real numbers with $a > 0$ and $b > 0$ then

I) $\boxed{a^s \cdot a^t = a^{s+t}}$

II) $\boxed{(a^s)^t = a^{st}}$

(III) $(ab)^s = a^s \cdot b^s$.

(IV) $(1)^s = 1$

(V) $\boxed{a^{-s} = \frac{1}{a^s} = \left(\frac{1}{a}\right)^s}$

(VI) $a^0 = 1$.

Properties of the Exponential Function

$f(x) = a^x$ where $a > 1$

I) The Domain is the set of all real numbers or $(-\infty, \infty)$

II) The range is the set of positive real number $(0, \infty)$.

III) There are no x-intercept.

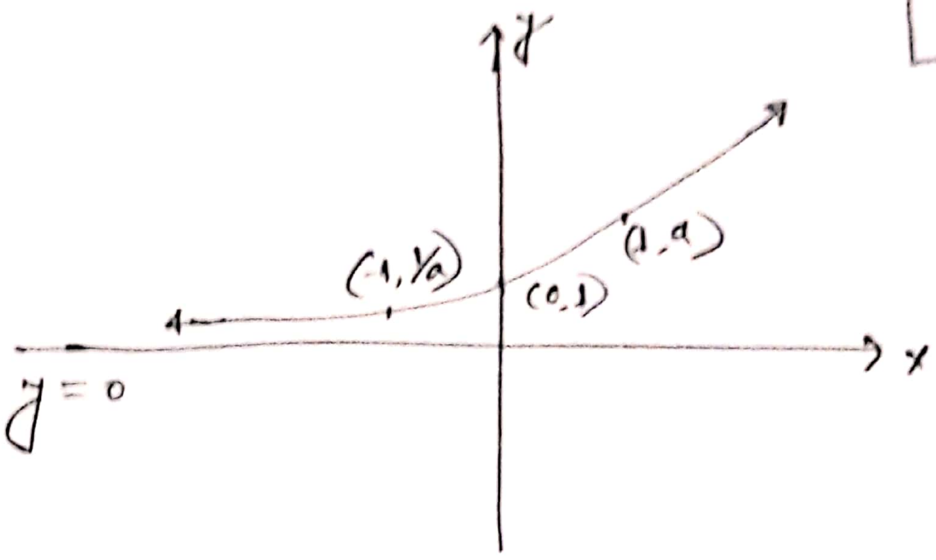
IV) The y intercept is 1.

V) The x-axis $(y=0)$ is a horizonal asymptote as $x \to -\infty$ $\left[\lim_{x \to -\infty} a^x = 0\right]$

VI) $f(x) = a^x$, where $a > 1$ is an increasing function and one-to-one.

VII) The graph of $f(x)$ contain the points $(0,1)$, $(1,a)$ and $(-1, 1/a)$

VIII) The graph of $f(x)$ is smooth and continuous with no corners or gaps.



Problem: Graph $f(x) = 2^{-x} - 3$ and determine the Domain, range and horizontal asymptote of $f(x)$.

Properties of the exponential function
$f(x) = a^x$ where $0 < a < 1$

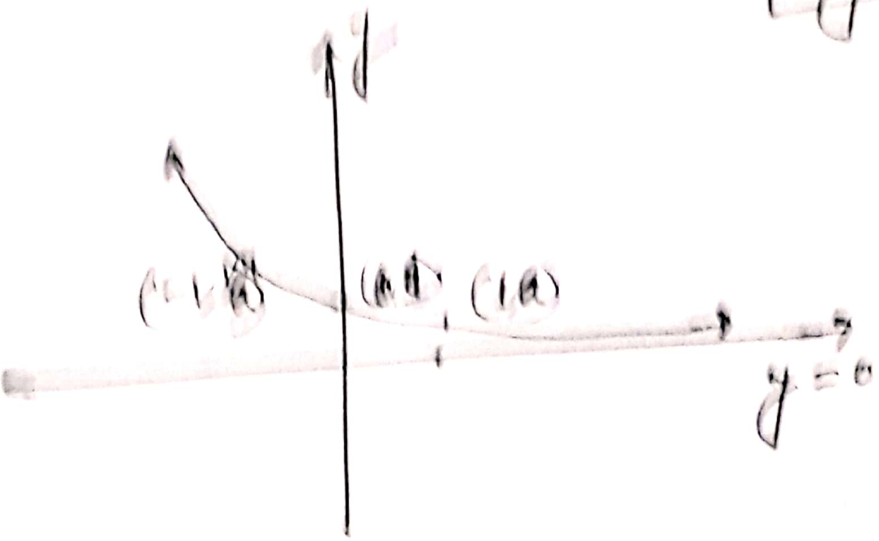
I) The Domain is the set of all real numbers $\mathbb{R}$
II) The range is the set of positive real numbers $(0, \infty)$

III) There is no x-intercept.
IV) The y-intercept is 1.
V) The x-axis $(y=0)$ is a horizontal asymptote as $x \to \infty$. $\left[\lim_{x\to\infty} a^x = 0\right]$

VI) $f(x) = a^x$, $0 < a < 1$ is a decreasing function and is one-to-one.

VII) The graph of $f(x)$ contain the points $(-1, 1/a)$, $(0, 1)$ and $(1, a)$

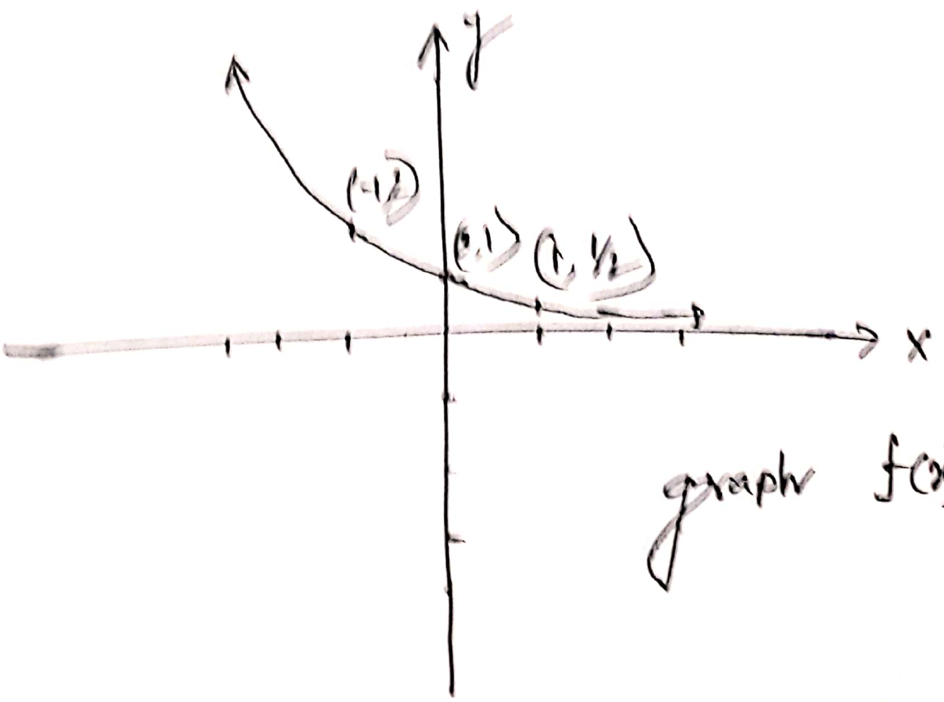
VII) The graph of f is smooth and continuous with no corners.



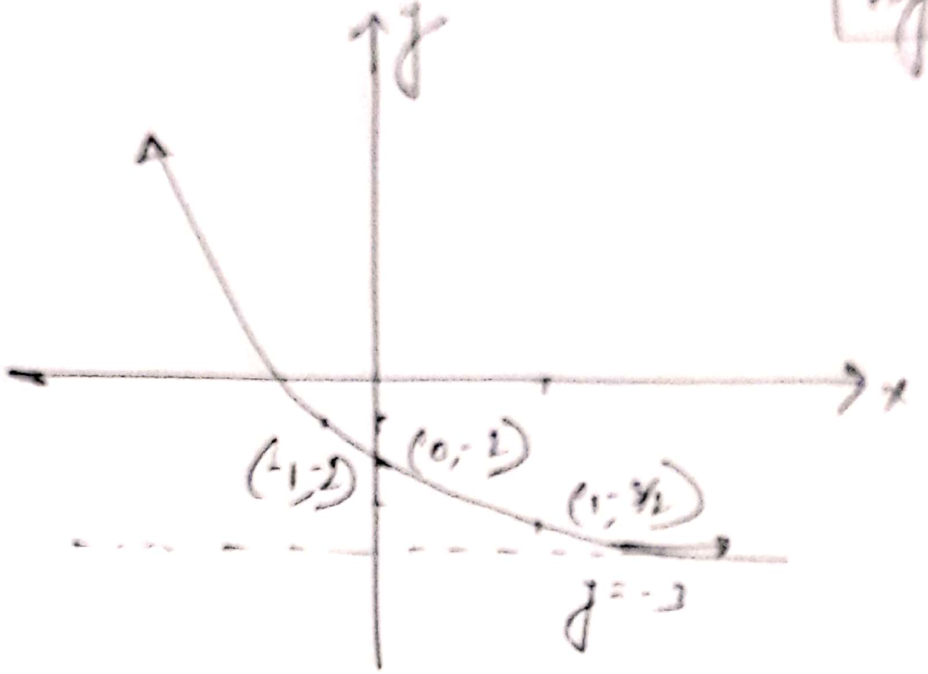
Problem: Graph $f(x) = 2^{-x} - 3$ and determine the domain, range and horizontal asymptote of $f(x)$.

Soln:

$$f(x) = 2^{-x} - 3$$
$$= \left(\tfrac{1}{2}\right)^{x} - 3.$$



graph $f(x) = \left(\tfrac{1}{2}\right)^{x}$.



graph $f(x) = \left(\frac{1}{2}\right)^x - 3$.

The Domain of $f(x) = \left(\frac{1}{2}\right)^x - 3$ is
$\mathbb{R}$ or $(-\infty, \infty)$

The Range of $f(x) = \left(\frac{1}{2}\right)^x - 3$ is
$(-3, \infty)$

The horizontal asymptote of $f(x)$ is the line $\boxed{y = -3}$.

Define The number e

The number e is defined as the number that the expression

$$\left(1+\frac{1}{n}\right)^n$$

$$\Rightarrow \boxed{e = \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n}$$

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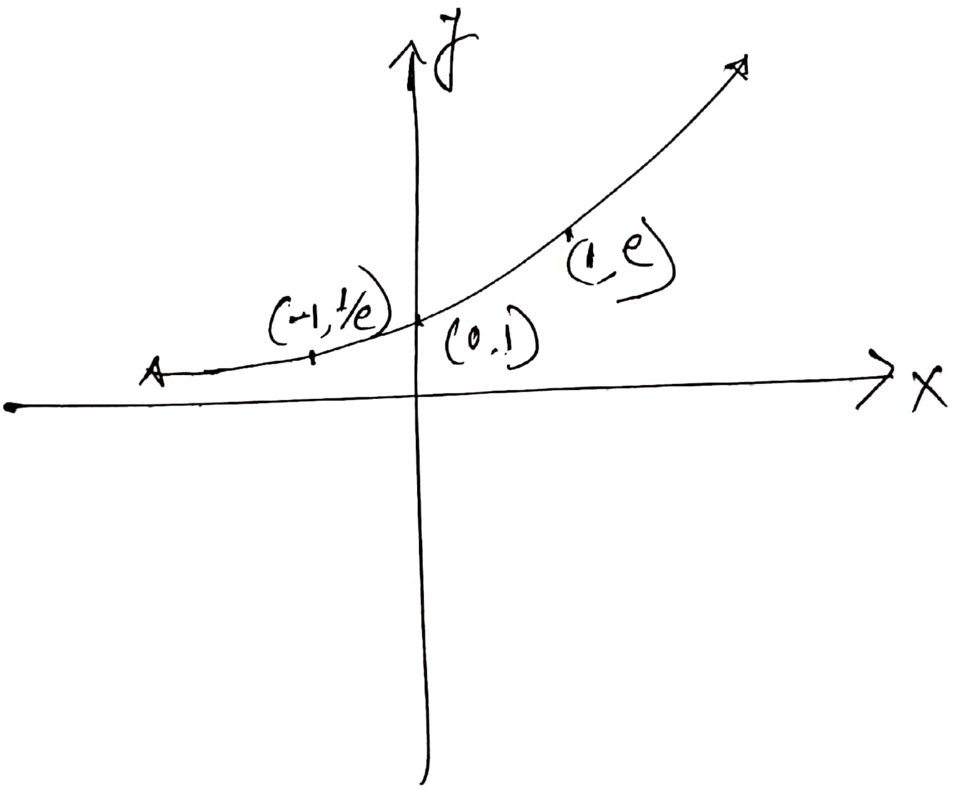


The exponential function $f(x) = e^x$ whose base is the number e, occur with such frequency in application that it is usually to as the exponential function & is denoted by

$$\boxed{f(x) = e^x}$$

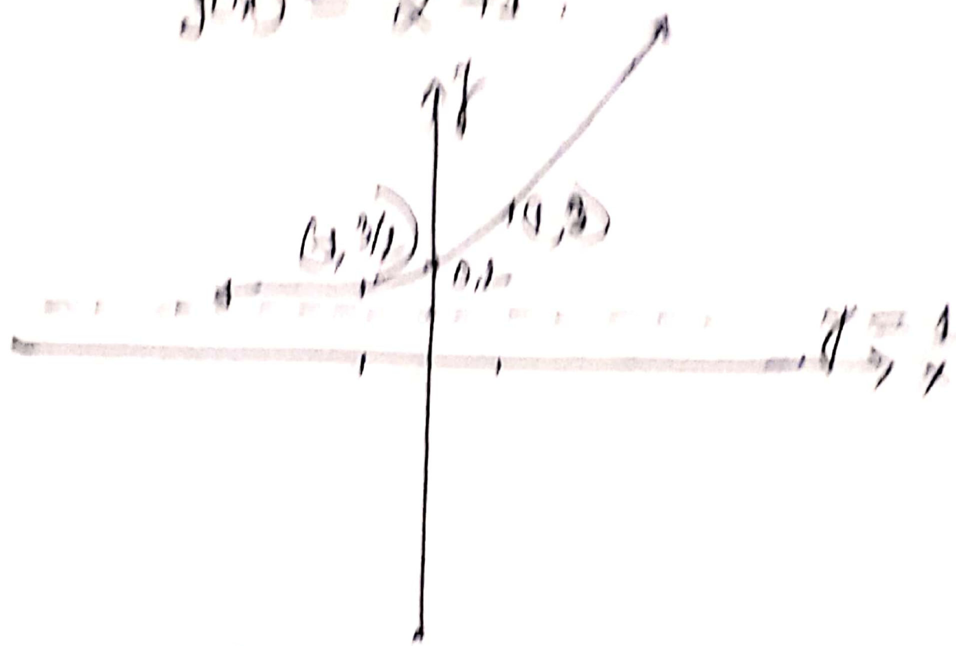
I) Domain $= \mathbb{R} = (-\infty, \infty)$

II) Range $= (0, \infty)$.

III) The x-axis $(y = 0)$ is a horizontal asymptote as $x \to -\infty$ $\left[\lim_{x \to -\infty} e^x = 0\right]$

(4) Problem: Use transform to graph each function. Determine the domain, range and horizontal asymptote of each function.

$f(x) = 2^x + 1$.



i) The Domain of the function $= (-\infty, \infty) = \mathbb{R}$

ii) The Range of the function $= (1, \infty)$

iii) The Horizontal asymptote $\boxed{y = 1}$ line.

Problem: Solve the exponential functional equation

a) $3^{x+1} = 81$, b) $4^{2x-1} = 2^{x+3}$

Soln $3^{x+1} = 3^4$

$\Rightarrow$ So the base is same, the exponents equal to each other to obtain

$\Rightarrow x+1 = 4$

$\Rightarrow x = 4-1$

$\therefore \boxed{x = 3}$

Problem Solve the equation

$$(3)^{x^2-7} = 27^{2x}$$

Soln $(3)^{x^2-7} = (3)^{3 \cdot 2x} = (3)^{6x}$

$\Rightarrow 3^{x^2-7} = 3^{6x}$

$\Rightarrow x^2 - 7 = 6x$

$\Rightarrow x^2 - 6x - 7 = 0$

$$x^2 - 7x + x - 7 = 0$$

$$\Rightarrow x(x-7) + 1(x-7) = 0$$

$$\Rightarrow (x-7)(x+1) = 0$$

$$\therefore \boxed{x = 7, -1}$$

<u>Problem</u> If $4^x = 7$ what does 4^{-2x} equal ?

<u>Solution</u> : Given that,

$$4^x = 7$$

$$\Rightarrow \frac{1}{4^x} = \frac{1}{7}$$

$$\Rightarrow 4^{-x} = \frac{1}{7}$$

$$\Rightarrow (4^{-x})^2 = \left(\frac{1}{7}\right)^2 \quad [\text{Squaring}]$$

$$\Rightarrow \boxed{4^{-2x} = \frac{1}{49}}$$

<u>Problem</u> : If $2^x = 3$ what does 4^{-x} equal?

Problem: Suppose that $f(x) = 2^x$

a) what is $f(4)$? what point is on the graph?

b) if $f(x) = 1/16$ what is x? what is the point on the graph of $f(x)$.

Solution: a) Given that,

$$f(x) = 2^x$$

$$\therefore f(4) = (2)^4 = 16$$

The point on the graph $(4, 16)$

b) $$f(x) = 1/16$$

$$\Rightarrow 2^x = \frac{1}{(2)^4} = 2^{-4}$$

$$\Rightarrow 2^x = 2^{-4}$$

$$\therefore \boxed{x = -4}$$

The point on the graph $(-4, 1/16)$

$\boxed{\text{Logarithmic Function}}$

$\boxed{\text{Defination}}$ The Logarithmic function to the base a, where $a > 0$ and $a \neq 1$, is denoted by

$$\boxed{y = \log_a x} \text{ if } \boxed{x = a^y}$$

The Domain of logarithmic $y = \log_a x$ is $(0, \infty)$

$\boxed{\text{Relating Logarithms to Exponents}}$

If $y = \log_a x$

$\Rightarrow \boxed{x = a^y}$

<u>Example</u> a) $y = \log_3 x$

$\therefore x = 3^y$.

b) $4 = \log_3 81$

$\Rightarrow \boxed{81 = 3^4}$

Changing Exponential statement to Logarithmic statement

[9]. $9 = 3^2$

Soln: Use the fact that

$y = \log_a x$ and $x = a^y$

where $a > 0$ and $a \neq 1$.

$\therefore 9 = 3^2$

$\Rightarrow \boxed{2 = \log_3 9}$

Problem: Solve the problem $\boxed{9-16}$

Problem 17. $\log_2 8 = 3$.

Solution: Use the fact $y = \log_a x$ and $x = a^y$ where $a > 0$, $a \neq 1$

$\Rightarrow \boxed{8 = 2^3}$

Problem 21. $\log_3 2 = x$ $\therefore \boxed{2 = 3^x}$

[Evaluate logarithmic expressions]

To find the exact value of a logarithmic, we can write the logarithmic in exponential function notation. Use the fact

$y = \log_a x$ and $x = a^y$

where $a > 0$ and $a \neq 1$.

Example: Find the exact value of $\log_2 16$.

Soln Let, $y = \log_2 16$

$\Rightarrow 2^y = 16$

$\Rightarrow 2^y = (2)^4$

$\Rightarrow y = 4$

$\therefore \boxed{\log_2 16 = 4}$

Domain of a logarithmic function

The logarithmic function $y = \log_a x$ has been defined as the inverse of the exponential function. ~~$y = a^x$~~

$$\boxed{y = \log_a x \quad \text{and} \quad x = a^y}$$

Domain $(0, \infty)$ — Range $(-\infty, \infty)$

$\boxed{0 < x < \infty}$ — $\boxed{-\infty < y < \infty}$

Domain of logarithmic function = Range of exponential function = $(0, \infty)$

Range of the logarithmic function = Domain of exponential function $(-\infty, \infty)$.

* Properties of the logarithmic function

$y = \log_a x$

I) The Domain is the set of positive real number $(0, \infty)$

II) The Range is the set of all real numbers $(-\infty, \infty) = \mathbb{R}$.

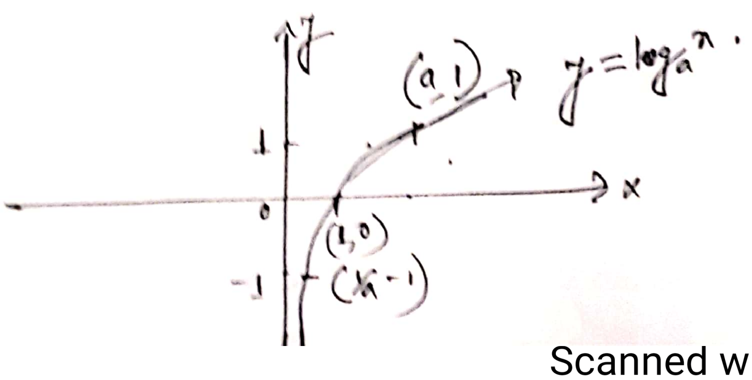
III) The x-intercept is $(1,0)$

IV) There is no y-intercept

V) The y-axis $(x=0)$ is a vertical asymptote of the graph

VI) The logarithmic function is decreasing if $0<a<1$ and increasing if $a>1$.

VII) The graph of $f(x)$ contain the points $(1,0)$, $(a,1)$ and $(1/a, -1)$



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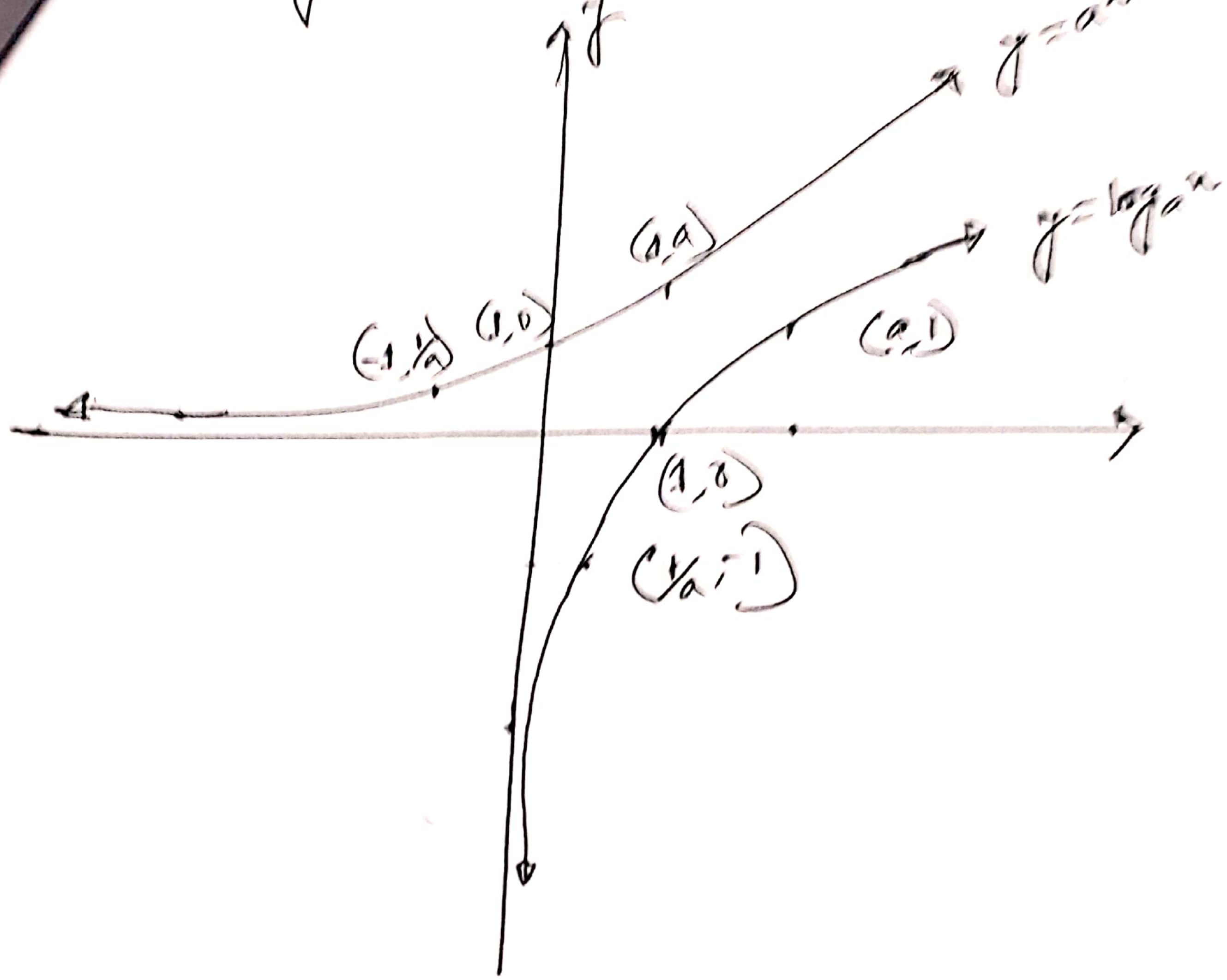


Common logarithmic function

<u>Defn</u>: If the base of a logarithmic function is the number 10 then we have the common logarithmic function. If the base a of the logarithmic function is not indicated, it is understood to be 10

$$\boxed{y = \log x \text{ if and only if } x = 10^y}$$

Problem: a) Find the domain of the logarithmic function
$f(x) = -\ln(x-2)$

b) Draw the graph

c) From the graph find range and vertical asymptote

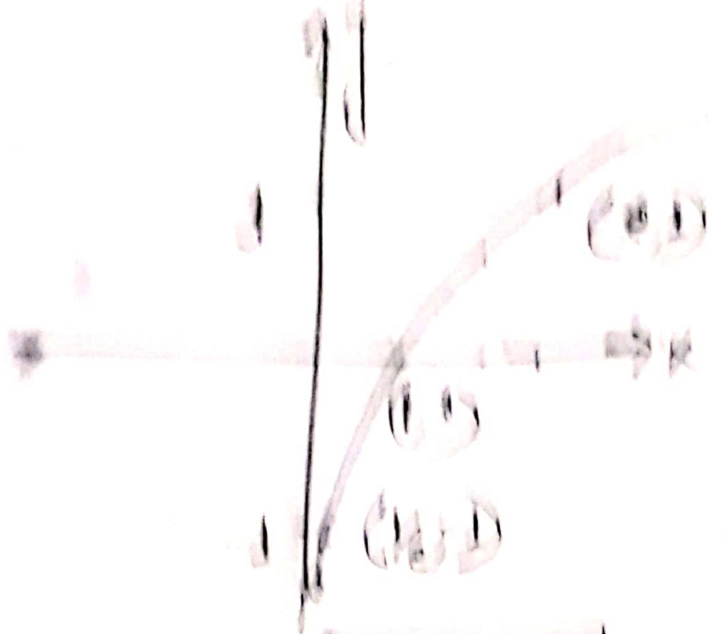
d) Find the inverse function.

e) Find the domain and range of the inverse function.

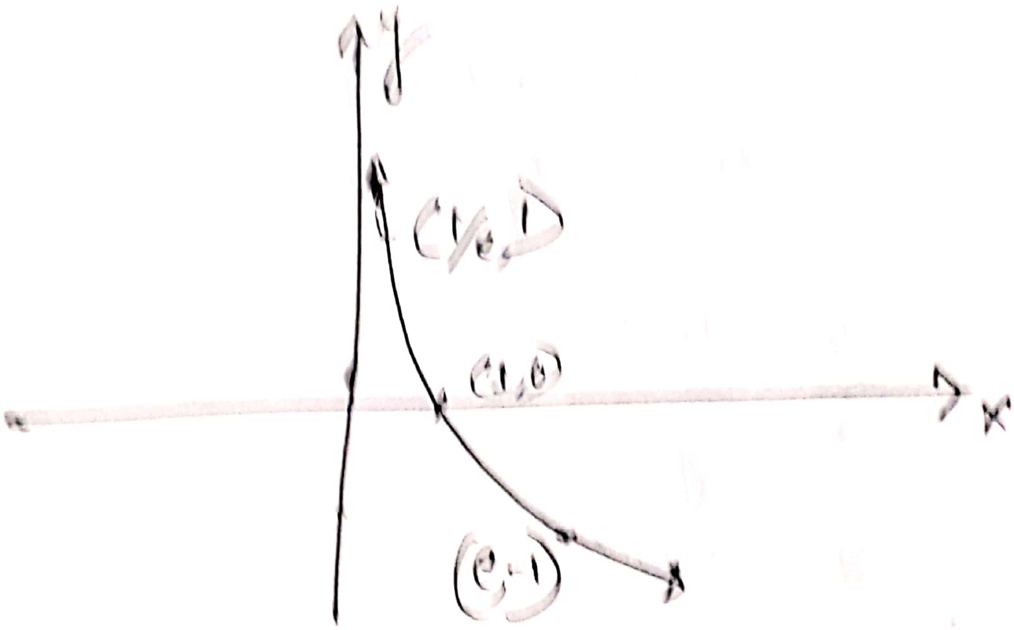
f) graph of the inverse function.

Solution: a) The domain of $f(x)$ consists of all x for $(x-2) > 0$
$\Rightarrow x > 2$.
The Domain $(2, \infty)$.

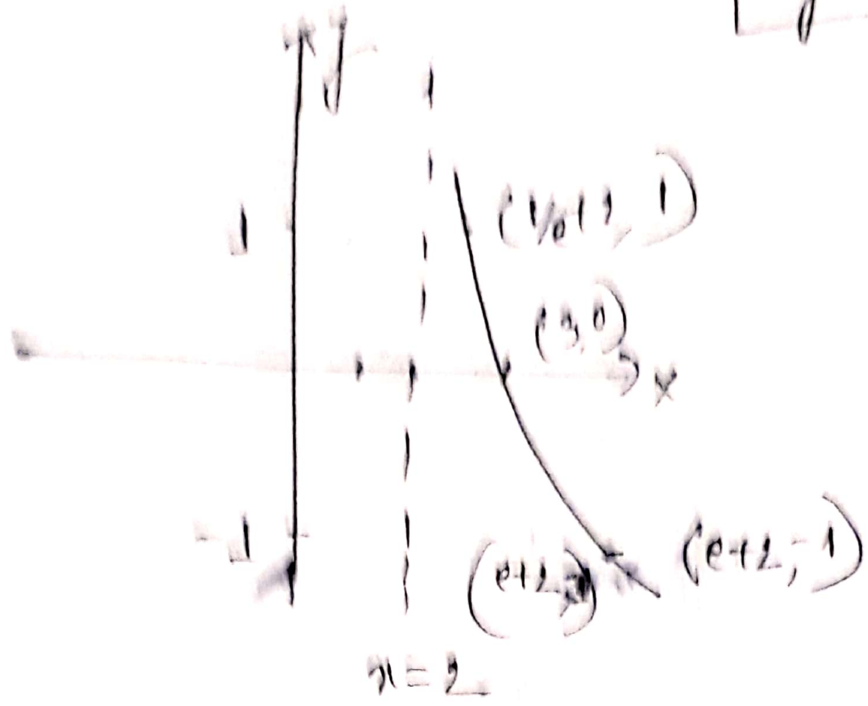
b) To obtain the graph $f(x) = -\ln(x-2)$. we begin The graph $f(x) = \ln x$ and the Transformation is given below.



$\boxed{y = \ln x}$



$\boxed{y = -\ln x}$



d) To find inverse function

$y = -\ln(x-2)$. The inverse function is defined implicitly by the equation

$\Rightarrow -x = \ln(y-2)$

$\Rightarrow e^{-x} = y-2$

$\Rightarrow y = 2+e^{-x}$

$\therefore \boxed{f^{-1}(x) = 2+e^{-x}}$

e) The Domain of $f^{-1}(x) = 2+e^{-x}$ is set of all real numbers $= \mathbb{R}$.