Chapter - 3.1

3.1.4

here, $p = 0.09$

$n = 9$

ⓐ $$P(X=2) = {}^{n}C_{x} \cdot p^{x} \cdot (1-p)^{n-x}$$

$$= {}^{9}C_{2} \cdot (0.09)^{2} (1-0.09)^{9-0.09}$$

$$= 0.1507$$

ⓑ $$P(x \geq 2) = 1 - \{P(x<2)\}$$

$$= 1 - \{P(X=0) + P(X=1)\}$$

$$= 1 - 0.8080$$

$$= 0.1911$$

ⓒ Expected number of bull's eye scored, $E(X) = np$

$$= 9 \times 0.09$$

$$= 0.81$$

and,

$$Var(X) = np(1-p)$$

$$= 9 \times 0.09 (1-0.09)$$

$$= 0.7371$$

(Ans)

3.1.6

here,

$n = 10$ (n denotes the number of correct answer).

probability of ques is right, $p = \frac{1}{5}$

$= 0.2$

$\therefore \quad X \sim B(10, \frac{1}{5})$

ⓐ Probability of mass function is $(10, \frac{1}{5})$ or $(10, 0.02)$

probability that student guesses blindly and will pass the exam is

$$P(X \geq 7) = P(X=7) + P(X=8) + P(X=9) + P(X=10)$$

$$= {}^{10}C_7 (0.2)^7 (1-0.2)^{10-7} + {}^{10}C_8 (0.2)^8 \cdot (1-0.2)^{10-8}$$

$$+ {}^{10}C_9 (0.2)^9 (1-0.2)^{10-9} + {}^{10}C_{10} (0.2)^{10} (1-0.2)^{10-10}$$

$$= 0.000864 \quad \text{(Ans)}$$

(b) if students can eliminate three options. So, probability that a question is correct is $= \frac{1}{2}$

$= 0.5$

$\therefore X \sim B\left(10, \cancel{0.5}\,\frac{1}{2}\right)$

Probability mass function of $B(10, 0.5)$ student will pass the quiz.

So, here,

$n = 10$

$p = 0.5$

$$\therefore P(X \geq 7) = P(X=7) + P(X=8) + P(X=9) + P(X=10)$$

$$= {}^{10}C_{7}\,(0.5)^{7}(1-0.5)^{10-7} + {}^{10}C_{8}\,(0.5)^{8}(1-0.5)^{10-8}$$

$$+ {}^{10}C_{9}\,(0.5)^{9}(1-0.5)^{10-9} + {}^{10}C_{10}\,(0.5)^{10}(1-0.5)^{10-10}$$

$$= 0.171875$$

(Ans.)

3.1.9

let, X denotes the number of order received by the company.

where,

$n = 18$

$P = 80\% = 0.6$

ⓐ $P(8 \le X \le 10) = P(X=8) + P(X=9) + P(X=10)$

$= {}^{10}C_8 (0.6)^8 (1-0.6)^{10-8} + {}^{10}C_9 (0.6)^9 (1-0.6)^{10-9}$

$+ {}^{10}C_{10} (0.6)^{10} \cdot (1-0.6)^{10-10}$

$= 0.07707 + 0.12844 + 0.1734$

$= 0.37891$

ⓑ $P(X \le 4) = P(X=0) + P(X=1) + P(X=2) + P(X=3) + P(X=4)$

$= {}^{10}C_0 (0.6)^0 (1-0.6)^{10-0} + {}^{10}C_1 (0.6) (1-0.6)^{10-1} +$

${}^{10}C_2 (0.6)^2 (1-0.6)^{10-2} + {}^{10}C_3 (0.6)^3 (1-0.6)^{10-3} +$

${}^{10}C_4 (0.6)^4 (1-0.6)^{10-4}$

$= 0.0013$ (Ans)

3.1.11

here,

$n = 10$

$p = 0.65$

At least half of the investment delivers profit means at least 5 compony get the profit.

$\therefore$ Probability of mass function $X \sim B(10, 0.65)$ is

$$P(x \geq 5) = P(X=5) + P(X=6) + P(X=7) + P(X=8) + P(X=9) + P(X=10)$$

$$= {}^{10}C_5 (0.65)^5 (1-0.65)^{10-5} + {}^{10}C_6 (0.65)^6 (1-0.65)^{10-6} + \dots\dots + {}^{10}C_{10} (0.65)^{10} (1-0.65)^{10-10}$$

$$= 0.15357 + 0.2376 + 0.2522 + 0.1756 + 0.0724 + 0.0134$$

$$= 0.905$$ (Ans:)

Chapter - 3.2

3.2.3

$$E(X) = \sum_{x=1}^{\infty} x \; P(X=x) = \frac{1}{p}$$

$$\therefore E(X) = \sum_{x=1}^{\infty} x \; p \left(1-p\right)^{x-1}$$

let,

$$q = 1-p$$

$$\therefore E(X) = \sum_{x=1}^{\infty} x \cdot p \cdot q^{x-1}$$

$$= p + 2pq + 3pq^2 + 4pq^3 + \cdots\cdots \longrightarrow (i)$$

multiplying both side by q.

$$q \cdot E(X) = 0 + pq + 2pq^2 + 3pq^3 + 4pq^4 + \cdots \quad (ii)$$

Here, subtracting both side:

$$E(X) - q \cdot E(X) = (p-0) + (2pq - pq) + (3pq^2 - 2pq^2) + \cdots\cdots$$

$$\Rightarrow E(X)\left(1-q\right) = p + pq + pq^2 + pq^3 + \cdots\cdots$$

$$\Rightarrow E(X)\,(1-q) = \frac{1}{1-q}$$

$$\Rightarrow E(X) = \frac{p}{(1-q)^2}$$

as, $1-p = q$

so, $1-q = p$

$$\therefore E(X) = \frac{p}{p^2}$$

$$\therefore E(X) = \frac{1}{p}$$

3.2.5

given, $p = 0.09$

(a) let, x be the number of hits to get the first hit of the bull's eye.

$\therefore$ Probability $P(X=4) = p(1-p)^{x-1}$

$= 0.09(1-0.09)^{4-1}$

$= 0.0678$

(b) here,

$x = 10$

$p = 0.09$

$r = 3$

$\therefore P(X=10) = {}^{x-1}C_{r-1} \times p^r \times (1-p)^{x-r}$

$= {}^{10-1}C_{3-1} \times (0.09)^3 \times (1-0.09)^{10-3}$

$= {}^9C_2 \times (0.09)^3 \times (1-0.09)^7$

$= 0.0135$

(Ans)

(c) The expected number of arrows shot before the first bull's eye $= \frac{r}{p}$

here,
$r = 1$
$p = 0.09$

$= \frac{1}{0.09}$

$= 11.11$

(d) The expected number of arrow shot before the third bull's eye -

here,

$r = 3$

$p = 0.09$

$\therefore E(X) = \frac{r}{p}$

$= \frac{3}{0.09}$

$= 33.33$

(Ans)

3.2.4

Each of r components has a geometric distribution with parameters p.

$$P(X=x) = {}^{x-1}C_{r-1} \, (1-p)^{x-r} \cdot p^r$$

and for $y = x_1 + x_2 + \cdots + x_r$

Expected value is Mean, $E(X) = \frac{r}{p}$

So that,

$$Var(X) = E(X^2) - [E(X)]^2$$

$$= \frac{r}{p^2} - \frac{r}{p}$$

$$= \frac{r - rp}{p^2}$$

$$= \frac{r(1-p)}{p^2}$$

3.2.9

Probability of recieving orders over the internet is, $p = \frac{60}{100} = 0.6$

(a) The fifth order received is the first internet order.

Hence,

$x = 5$

and, $r = 1$

$$\therefore P(X=5) = {}^{5-1}C_{1-1} (0.6)^1 (1-0.6)^{5-1}$$

$$= 0.01536$$

(b) ~~Eight~~ Eighth order received is fourth internet order.

Hence,

$x = 8$

$r = 4$

$$\therefore P(X=8) = {}^{x-1}C_{r-1}\, p^r (1-p)^{x-r}$$

$$= {}^{7}C_{3} (0.6)^4 (1-0.6)^{8-4}$$

$$= 0.1161 \quad \text{(Ans!)}$$

Chapter - 3.3

3.3.2

Total member, $N = 15$

Choosen people / Sample taken, $n = 5$

Focus(Righ wing members), $r = 8$

The number of right wing members choosen from the committee members has a hypergeometric distribution.

we know,

$$P(X=x) = \frac{\binom{r}{x}\binom{N-r}{n-x}}{\binom{N}{n}}$$

$$\therefore P(X=0) = \frac{\binom{8}{0}\binom{15-8}{5-0}}{\binom{15}{5}} = \frac{21}{3003} = \frac{3}{429}$$

$$P(X=1) = \frac{\binom{8}{1}\binom{15-8}{5-1}}{\binom{15}{5}} = \frac{40}{429}$$

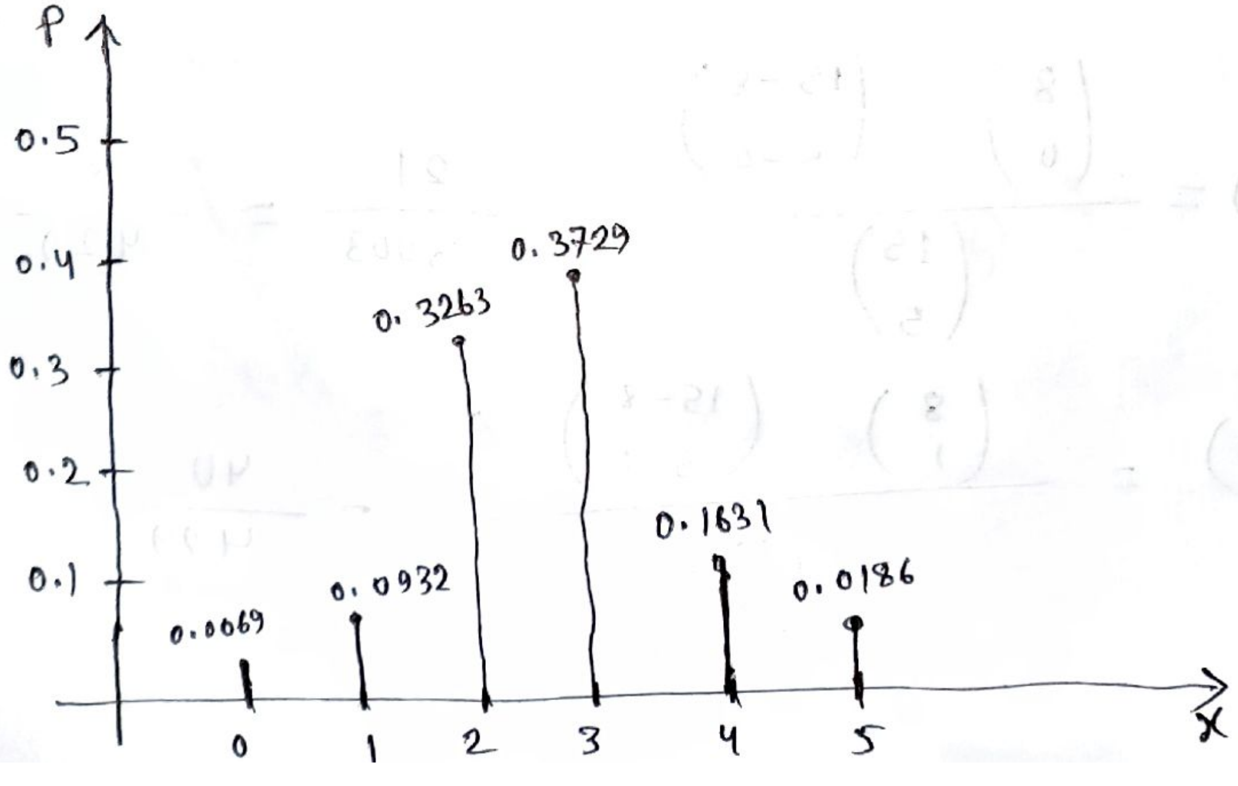
$$\therefore P(X=2) = \frac{\binom{8}{2}\binom{15-8}{5-2}}{\binom{15}{5}} = \frac{140}{429}$$

$$P(X=3) = \frac{\binom{8}{3}\binom{15-8}{5-3}}{\binom{15}{5}} = \frac{160}{429}$$

$$P(X=4) = \frac{\binom{8}{4}\binom{15-8}{5-4}}{\binom{15}{5}} = \frac{70}{429}$$

$$P(X=5) = \frac{\binom{8}{5}\binom{15-8}{5-5}}{\binom{15}{5}} = \frac{8}{429}$$

x	0	1	2	3	4	5
P	$\frac{3}{429}$	$\frac{40}{429}$	$\frac{140}{429}$	$\frac{160}{429}$	$\frac{70}{429}$	$\frac{8}{429}$



3.3.3

A box contains total balls, $N = 17$

A sample of balls chosen random, $n = 5$

Red balls are (focus), $r = 10$

(a) Jar contains exactly 3 red balls, $x = 3$

$$\therefore P(X=3) = \frac{\binom{10}{3}\binom{17-10}{5-3}}{\binom{17}{5}}$$

$$= 0.40723$$

(b) Exactly 1 red balls.

$$\therefore P(X=1) = \frac{\binom{10}{1}\binom{17-10}{5-1}}{\binom{17}{5}}$$

$$= 0.05656$$

(c) More blue balls then red balls.

Maximum 2 red balls in jar. Therefore probability is.

$$P(x \leq 2) = P(X=0) + P(X=1) + P(X=2)$$

$$= \frac{\binom{10}{0}\binom{17-10}{5-0}}{\binom{17}{5}} + \frac{\binom{10}{1}\binom{17-10}{5-1}}{\binom{17}{5}} + \frac{\binom{10}{2}\binom{17-10}{5-2}}{\binom{17}{5}}$$

$= 0.31447$

(Ans)

3.3.7

Total item of product, $N = 11$

Customer takes 3 items at random, $n = 3$

Outdated items in a product (focus), $r = 4$

(a) Probability of none of the outdated products are selected by the customer,

$$P(X=0) = \frac{\binom{4}{0}\binom{11-4}{3-0}}{\binom{11}{3}}$$

$= 0.2121$

(b) Exactly 2 of items taken by customer are outdated.

$$\therefore P(X=2) = \frac{\binom{4}{2}\binom{11-4}{3-2}}{\binom{11}{3}}$$

$$= 0.2545 \quad \text{(Ans.)}$$

3.3.8

Total Number of cupcakes, $N = 15$

Randomly select 5 cupcakes, $n = 5$

Chocolate in cupcakes, $r = 9$

$\therefore$ Probabily that the number of chocolate cupcakes remaing on the plate is between 5 and 7 is -

if remaining 5 then we select 4. Thus remaining 6 & 7. so, selected 3 & 2.

$$P(5 \leq X \leq 7) = P(X=5) + P(X=6) + P(X=7)$$

$$= \frac{\binom{9}{4}\binom{15-9}{5-5}}{\binom{15}{5}} + \frac{\binom{9}{3}\binom{15-9}{5-3}}{\binom{15}{5}} + \frac{\binom{9}{2}\binom{15-9}{5-2}}{\binom{15}{5}}$$

$$= 0.9111 \quad \text{(Ans.)}$$

(b) Exactly 2 of items taken by customer are outdated.

$$\therefore P(X=2) = \frac{\binom{4}{2}\binom{11-4}{3-2}}{\binom{11}{3}}$$

$= 0.2545$ (Ans.)

3.3.8

Total Number of cupcakes, $N = 15$

Randomly select 5 cupcakes, $n = 5$

Chocolate in cupcakes, $r = 9$

$\because$ Probability that the number of chocolate cupcakes remaing on the plate is between 5 and 7 is -

if remaining 5 then we select 4. Thus remaining 6 & 7. so, selected 3 & 2.

$$P(5 \leq x \leq 7) = P(x=5) + P(x=6) + P(x=7)$$

$$= \frac{\binom{9}{4}\binom{15-9}{5-5}}{\binom{15}{5}} + \frac{\binom{9}{3}\binom{15-9}{5-3}}{\binom{15}{5}} + \frac{\binom{9}{2}\binom{15-9}{5-2}}{\binom{15}{5}}$$

$= 0.9111$ (Ans.)

chapter – 3.4

3.4.3

$$E(X) = \sum_{0}^{\infty} x \cdot \frac{e^{-\lambda} \cdot \lambda^{x}}{x!}$$

$$= 0 + \sum_{1}^{\infty} x \frac{e^{-\lambda} \cdot \lambda^{x}}{x!}$$

$$= \sum_{x=1}^{\infty} \frac{x \cdot e^{-\lambda} \cdot \lambda^{x}}{x(x-1)!}$$

$$= e^{-\lambda} \cdot \lambda \sum_{x=1}^{\infty} \frac{\lambda^{x-1}}{(x-1)!}$$

$$= e^{-\lambda} \cdot \lambda \cdot e^{\lambda}$$

$$= \lambda$$

3.4.6

4 traffic accident occurs in during one hour in a city.

So, the probability in poission distribution can be distributed as, $\lambda = 4$.

ⓐ Probability that no accident

$$P(X=x) = \frac{e^{-\lambda} \cdot \lambda^{x}}{x!}$$

$$\therefore P(x=0) = \frac{e^{-4} \cdot (4)^{0}}{0!}$$

$$= 0.0183$$

ⓑ At least 6 accident occurs.

$$\therefore P(x \geq 6) = 1 - P(x<6)$$

$$= 1 - \{P(X=0) + P(X=1) + P(X=2) + P(X=3) + P(X=4) + P(X=5)\}$$

$$= 1 - \left\{ \frac{e^{-4} \cdot (4)^{0}}{0!} + \frac{e^{-4} \cdot (4)^{1}}{1!} + \frac{e^{-4} (4)^{2}}{2!} + \cdots + \frac{e^{-4} \cdot (4)^{5}}{5!} \right\}$$

$$= 1 - 0.78513$$

$$= 0.21487$$ (Ans)

3.4.8

here,

$\lambda = 9.2$

(a) Probability that there are between 6 and 10 misrecorded price of information is.

$$\therefore P(6 \leq x \leq 10) = P(X=6) + P(X=7) + P(X=8) + P(X=9) + P(X=10)$$

$$= \frac{e^{-9.2}(9.2)^6}{6!} + \frac{e^{-9.2}(9.2)^7}{7!} + \frac{e^{-9.2}(9.2)^8}{8!} + \frac{e^{-9.2}(9.2)^9}{9!} + \frac{e^{-9.2}(9.2)^{10}}{10!}$$

$= 0.57795$ (Ans)

No more than four misrecorded pieces of information.

(b) $$P(x \leq 4) = P(X=0) + P(X=1) + P(X=2) + P(X=3) + P(X=4)$$

$$= \frac{e^{-9.2}(9.2)^0}{0!} + \frac{e^{-9.2}(9.2)^1}{1!} + \frac{e^{-9.2}(9.2)^2}{2!} + \frac{e^{-9.2}(9.2)^3}{3!} + \frac{e^{-9.2}(9.2)^4}{4!}$$

$= 0.04858$ (Ans)

3.4.7

Box contains total electrical switches, $n = 500$

Probability of each defective switches, $P = 0.005$

$\therefore$ Probability of mass function is $B(n, p)$

$$= B(500, 0.005)$$

This function can be approximated by a poission distribution with $\lambda = np$

$$= 500 \times 0.005$$

$$= 2.5$$

Now,

Probability of no more than 3 defective switches is, $P(x \leq 3) = P(X=0) + P(X=1) + P(X=2) + P(X=3)$

$$= \frac{e^{-2.5}(2.5)^0}{0!} + \frac{e^{-2.5}(2.5)^1}{1!} + \frac{e^{-2.5}(2.5)^2}{2!} + \frac{e^{-2.5}(2.5)^3}{3!}$$

$$= 0.7576$$ (Ans:)

3.4.9

The number of error in a company's accounts has a poission distribution with a mean, $\lambda = 4.7$

$\therefore$ Probability that there will be exactly three errors in company accounts is given.

$$P(X=x) = \frac{e^{-\lambda} \cdot \lambda^{x}}{x!}$$

$$\therefore P(X=3) = \frac{e^{-4.7} (4.7)^{3}}{3!}$$

$$= 0.15738$$

So, option B is correct

(Ans)

chapter - 4.1

4.1.1

$X \sim U(-3,8)$

(a) $X \sim U(-3,8)$ is said to have a uniform distribution so that expected value is

$$E(X) = \frac{a+b}{2}$$

$$= \frac{-3+8}{2}$$

$$= 2.5$$

(b) Varience is $Var(X) = \frac{(b-a)^2}{12}$

$$= \frac{(8+3)^2}{12}$$

$$= \frac{121}{12}$$

$\therefore$ Standard deviation of $X = \sqrt{Var(X)}$

$$= \sqrt{\frac{121}{12}}$$

$$= 3.175$$

(c) The upper quantiles is, $p = \frac{3}{4}$

So, The upper quantiles of the distribution is,

$$A = (1-p)\,a + pb$$

$$= \left(1 - \frac{3}{4}\right)(-3) + \left(\frac{3}{4} \times 8\right)$$

$$= -\frac{3}{4} + 6$$

$$= \frac{21}{4}$$

$$= 5.25$$

(d) $p(0 \leq x \leq 4)$

The probability of uniform distribution function $X \sim U(a, b)$ is given by.

$$p(x_1 \leq X \leq x_2) = \int_{x_1}^{x_2} \frac{1}{b-a}\,dx$$

Hence,

$x_1 = 0$

$x_2 = 4$

$a = -3$

$b = 8$

$$\therefore \quad P(0 \leq x \leq 4) = \int_0^4 \frac{1}{8+3}\,dx$$

$$= \frac{1}{11}\int_0^4 dx$$

$$= \frac{1}{11}\left[x\right]_0^4$$

$$= \frac{1}{11}\left[4-0\right]$$

$$= \frac{4}{11}$$ (Ans)

4.1.2

here,

$a = 1.43$

$b = 1.60$

(a) Expected voltage, $E(X) = \frac{a+b}{2}$

$$= \frac{1.43 + 1.60}{2}$$

$$= 1.515$$

(b) Variance is $Var(X) = \frac{(b-a)^2}{12}$

$$= \frac{(1.60-1.43)^2}{12}$$

$$= \frac{(0.17)^2}{12}$$

$\therefore$ Standard deviation is $= \sqrt{Var(X)}$

$$= \sqrt{\frac{(0.17)^2}{12}}$$

$$= 0.0490$$

(c) Comulative distribution function is,

$$F(x) = \frac{x-a}{b-a} \quad \text{for } a \le x \le b$$

$$= \frac{x-1.43}{1.60-1.43}$$

$$= \frac{x-1.43}{0.17} \quad \text{for } 1.43 \le x \le 1.60$$

(d)

Probability that a battery has a voltage less than 1.48 volts is

$$P(x<1.48) = F(1.48)$$

$$= \frac{x-1.43}{0.17}$$

$$= \frac{1.48-1.43}{0.17}$$

$$= \frac{0.05}{0.17}$$

$$= 0.2941 \quad (\text{Ans})$$

(e) Box contains total battery, $n=50$

Now,

Probability of battery in the box with a voltage less than 1.5 volts is,

$$P(x<1.5) = F(1.5)$$

$$= \frac{x-1.43}{0.17}$$

$$= \frac{1.5-1.43}{0.17}$$

$$P = \frac{0.07}{0.17}$$

$$P = 0.4118$$

The numbers of batteries with a voltage less than 1.5 volts has a binomial distribution with

$n = 50$ ~~and~~

and, $p = 0.4118$.

So, the expected voltage is, $E(X) = np$

$$= 50 \times 0.4118$$

$$= 20.59$$

and, the variance in voltage is

$$Var(X) = np(1-p)$$

$$= 50 \times 0.4118\,(1-0.4118)$$

$$= 50 \times 0.4118 \times 0.5882$$

$$= 12.1110$$

(Ans)

4.1.5

Metal pin a has a diameter that has a uniform distribution between $X \sim U(4.182, 4.185)$

(a) The comulative distribution function is given

$$F(x) = \frac{x-a}{b-a} \quad \text{for } a \leq x \leq b$$

$$= \frac{x - 4.182}{4.185 - 4.182}$$

$$= \frac{x - 4.182}{0.003} \quad \text{for } 4.182 \leq x \leq 4.185$$

The probability that a pin will fit into hole that has a diameter of 4.184 mm.

$$P(X = 4.184) = \frac{x - 4.182}{0.003}$$

$$= \frac{4.184 - 4.182}{0.003}$$

$$= \frac{0.002}{0.003}$$

$$= \frac{2}{3} \quad \text{(Ans.)}$$

(b)

given that,

holes diameter is $= 4.184$ mm.

Difference between the diameter of the hole and the diameter of pin is less than 0.0005 m.m.

So,

the range of pin's diameter is

$$= 4.1835 \leq x \leq 4.1840$$

and,

the probability of in this diameter range of pin is.

$$p(4.1835 \leq x \leq 4.1840) = \int_{4.1835}^{4.1840} \frac{1}{4.185 - 4.182} dx$$

$$= \frac{1}{0.003} \int_{4.1835}^{4.1840} dx$$

$$= \frac{1}{0.003} \left[x \right]_{4.1835}^{4.1840}$$

$$= \frac{1}{0.003}\left[4.1840 - 4.1835\right]$$

$$= \frac{0.0005}{0.003}$$

$$= \frac{1}{6}$$

and,

the pin fits in hole when the diameter of holes is less than or equal to 4.1840. So, the probability of pin fits in hole

$$P(x \leq 4.1840) = F(4.1840)$$

$$= \frac{x - 4.1820}{4.185 - 4.182}$$

$$= \frac{4.1840 - 4.1820}{0.003}$$

$$= \frac{0.0020}{0.003}$$

$$= \frac{2}{3}$$

So, the probability as required is,

$$P(4.1835 \leq x \leq 4.1840 \mid x \leq 4.1840) = \frac{P(4.1835 \leq x \leq 4.1840)}{P(x \leq 4.1840}$$

$$= \frac{\frac{1}{6}}{\frac{2}{3}}$$

$$= \frac{1}{4} \quad \text{(Ans.)}$$

Chapter- 4.2

4.2.1

Expected value of an exponential distribution is.

$$E(X) = \int_0^\infty x\, f(x)\, dx$$

$$= \int_0^\infty x \lambda \exp(-\lambda x)\, dx$$

$$= \lambda x \int_0^\infty e^{-\lambda x} - \int_0^\infty \frac{dy}{dx} \int e^{-\lambda x}\, dx$$

$$= \frac{\lambda}{\lambda} \left| -x e^{-\lambda x} \right|_0^\infty + \frac{\lambda}{\lambda^2} \left| -e^{-\lambda x} \right|_0^\infty$$

$$= \frac{\lambda}{\lambda} \left[-\infty \times e^{-\infty} + 0 \times e^{0} \right] + \frac{\lambda}{\lambda^2} \left[-e^{-\infty} + e^{0} \right]$$

$$= [0+0] + \frac{1}{\lambda}[0+1]$$

$$= \frac{1}{\lambda}$$

And,

$$E(X^2) = \int_0^\infty x^2 \lambda e^{(-\lambda x)} \, dx$$

$$= \lambda \int_0^\infty x^2 e^{-\lambda x} \, dx$$

$$= \lambda \left[\frac{1}{\lambda}\left(-\infty e^{-\infty} + 0\right) + \frac{2}{\lambda} \left\{ x \int_0^\infty e^{-\lambda x} dx - \int_0^\infty \frac{dx}{dx} e^{-\lambda x} \right. \right.$$

$$= 0 + 2\left\{ \frac{1}{\lambda}\left(-\infty e^{-\infty} + 0\right) - \frac{1}{\lambda^2}\left(e^{-\infty} - e^{0}\right) \right\}$$

$$= 0 + 2\left(\frac{1}{\lambda} + \frac{1}{\lambda^2}\right)$$

$$= 2 \times \frac{1}{\lambda^2}$$

$$= \frac{2}{\lambda^2}$$

Ans

4.2.3

The time in days between breakdowns of a machine is exponentially distributed with $\lambda = 0.2$

(a) Expected time between machine breakdowns is,

$$E(X) = \frac{1}{\lambda}$$

$$= \frac{1}{0.2}$$

$$= 5$$

(b) Variance of machine breakdown is

$$Var(X) = \frac{1}{\lambda^2} = \frac{1}{(0.2)^2} = 25$$

$\therefore$ standard deviation $= \sqrt{Var(X)}$

$$= \sqrt{25}$$

$$= 5$$

(c) We know,

median of the exponencial distribution is,

$f(x) = 1 - e^{-\lambda x}$

$= 1 - e^{-0.2x}$

here,
$\lambda = 0.2$
$f(x) = 0.5$

To get the median value of x we have to set up

$F(x) = 0.5$

$\therefore 0.5 = 1 - e^{-0.2x}$

$\Rightarrow -0.5 = -e^{-0.2x}$

$\Rightarrow e^{-0.2x} = 0.5$

$\Rightarrow -0.2x = \ln(0.5)$

$\Rightarrow x = \frac{\ln(0.5)}{0.2}$

$\therefore x = 3.4657$

or

(c)

$\frac{-\ln(1-0.5)}{\lambda}$

$= \frac{0.693}{\lambda}$

$= \frac{0.693}{0.2}$

$= 3.465$ day

(d) the probability that after the machine is repaired it lasts at least a week before failing.

$$P(x \geq 7) = 1 - P(x < 7)$$

$$= 1 - F(7)$$

$$= 1 - \left\{ 1 - e^{-0.2 \times 7} \right\}$$

$$= 1 - 1 + e^{-1.4}$$

$$= e^{-1.4}$$

$$= 0.2465 \text{ (Ans)}$$

(e) Machine has performed satisfactory for six days.

Probability that it lasts at least two more days before breaking downs memoryless property of the exponencial distribution and given by

$$P(x \geq 2) = 1 - F(2)$$

$$= 1 - \left(1 - e^{-0.2 \times 2}\right)$$

$$= 1 - 1 + e^{-0.2 \times 2}$$

$$= e^{-0.4}$$

$$= 0.6703 \text{ (Ans)}$$

4.2.5

Probability density function is given as

$$f(x) = \frac{1}{2}\lambda e^{-\lambda|x-\theta|} \quad \text{for} \quad -\infty \leq x \leq \infty$$

So, the, commulative function of this distribution is

$$F(x) = \int_{-\infty}^{x} \frac{1}{2}\lambda e^{-\lambda|x-\theta|} dx \quad \text{for} \quad -\infty \leq x \leq \theta$$

$$= \frac{1}{2}\lambda \int_{-\infty}^{x} e^{-\lambda|x-\theta|} dx$$

$$= \frac{\lambda}{2}\int_{-\infty}^{\theta} e^{-\lambda|x-\theta|} dx + \frac{\lambda}{2}\int_{\theta}^{x} e^{-\lambda|x-\theta|} dx$$

$$= \frac{\lambda}{2} \times \frac{1}{\lambda}\left[-e^{-\lambda|x-\theta|}\right]_{-\infty}^{\theta} + \frac{\lambda}{2} \times \frac{1}{\lambda}\left[-e^{-\lambda|x-\theta|}\right]_{\theta}^{x}$$

$$= \frac{1}{2}\left[e^{-\infty} + e^{0}\right] + \left(-\frac{1}{2}\right)\left[-e^{0} + e^{-\lambda(x-\theta)}\right]$$

$$= \frac{1}{2} + \frac{\lambda}{2}\int_{0}^{x} e^{-\lambda(x-\theta)} dx$$

$$= \frac{1}{2} + \frac{1}{2} - \frac{1}{2}e^{-\lambda}(x-\theta)$$

$$F(x) = 1 - \frac{1}{2}e^{-\lambda(x-\theta)}$$

(a) $P(x \leq 0) = F(0)$

given,
$\lambda = 3$
$\theta = 2$

$= \frac{1}{2} e^{-\lambda(\theta - x)}$

$= \frac{1}{2} e^{-3(2-0)}$

$= \frac{1}{2} e^{-6}$

$= 0.00123$ (Ans)

(b) $P(x \geq 1) = 1 - P(x \leq 1)$

$= 1 - F(1)$

$= 1 - \left[\frac{1}{2} e^{-3(2-1)}\right]$

$= 1 - \frac{1}{2} e^{-3}$

$= 1 - 0.02489$

$= 0.97511$

(Ans)

4.2.7

(a) The value of the parameter λ of the poisson process is $\lambda = 1.8$

(b) Expectation of the time $= \frac{1}{\lambda}$

$$= \frac{1}{1.8}$$

$$= 0.5556$$

(c) $p(x \geq 1) = 1 - (1 - e^{-\lambda x})$

$$= 1 - (1 - e^{-1.8 \times 1})$$

$$= 0.1653$$

(d) A poission distribution with parameter $= 4\lambda$

$$= 4 \times 1.8$$

$$= 7.2$$

(e) $p(x \geq 4) = 1 - p(x=0) - p(x=1) - p(x=2) - p(x=3)$

$$= 1 - \frac{e^{-7.2} \times (7.2)^0}{0!} - \frac{e^{-7.2}(7.2)^1}{1!} - \frac{e^{-7.2}(7.2)^2}{2!} - \frac{e^{-7.2}(7.2)^3}{3!}$$

$$= 0.9281$$

(Ans:)

Chapter - 5.1

5.1.1

given that $Z \sim N(0,1)$ Here, $\sigma = 1$, $\sigma^2 = 1$

(a) $P(Z \leq 1.34) = P(-\infty \leq Z \leq 1.34)$

we know that,

$$P(a \leq x \leq b) = \phi\left(\frac{b-\mu}{\sigma}\right) - \phi\left(\frac{a-\mu}{\sigma}\right)$$

$$\Rightarrow P(-\infty \leq Z \leq 1.34) = \phi\left(\frac{1.34-0}{1}\right) - \phi\left(\frac{-\infty-0}{1}\right)$$

$$= \phi(1.34)$$

$$= 0.9099$$

(A

(b) $P(Z \geq -0.22) = 1 - P(Z \leq -0.22)$

$$= 1 - \left[\phi\left(\frac{-0.22-0}{1}\right) - \phi\left(\frac{-0-0}{1}\right)\right]$$

$= 1 - [\phi(-0.22)]$

$= 1 - 0.4129$

$= 0.5871$ (Ans)

(c)

$$P(-2.19 \leq z \leq 0.43) = \phi\left(\frac{0.43-0}{1}\right) - \phi\left(\frac{-2.19-0}{1}\right)$$

$= \phi(0.43) - \phi(-2.19)$

$= 0.6664 - \cancel{\phi(-2}\ 0.0143$

$= 0.6521$ (Ans)

(d)

$$P(0.09 \leq z \leq 1.76) = \phi\left(\frac{0.76-0}{1}\right) - \phi\left(\frac{0.09-0}{1}\right)$$

$= \phi(1.76) - \phi(0.09)$

$= 0.9608 - 0.5359$

$= 0.4249$

Ans

(e) $p(|z| \leq 0.38) = p(-0.38 \leq z \leq 0.38)$

$$= \phi\left(\frac{0.38 - 0}{1}\right) - \phi\left(\frac{-0.38 - 0}{1}\right)$$

$$= \phi(+0.38) - \phi(-0.38)$$

$$= 0.6480 - 0.3520$$

$$= 0.2960 \text{ (Ans)}$$

(f) The value of x for which $p(z \leq x) = 0.55$

we know that the cumulative function of a Normal distri.. is

$$\phi(x) = \int_{-\infty}^{x} \phi(x)\, dx$$

$$\Rightarrow 0.55 = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}\, dx$$

$$\Rightarrow 0.55 = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{x^2}{2}}\, dx$$

solving this,

$$x = 0.1257$$

(9)

The value of x which $P(X \geq x) = 0.04$

$$\Rightarrow P(X \geq x) = 1 - P(X \leq x)$$

$$\Rightarrow 0.04 = 1 - \phi(x)$$

$$\Rightarrow \phi(x) = 0.96$$

$$\Rightarrow \phi(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx$$

$$\frac{d}{dx}\left(\frac{x^2}{2}\right) = \frac{1}{2} \cancel{2}x$$

$$\Rightarrow 0.96 = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} \left[e^{-\frac{x^2}{2}} (-x) \right]_{-\infty}^{x}$$

$$= \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} \cdot x$$

Solving this,

$$x = 12.4758$$

5.1.3

$X \sim N(10.2)$

so, $\mu = 10$

and, $\sigma^2 = 2$

$\therefore \sigma = \sqrt{2}$

$= 1.4142$

(a) $P(x \leq 10.34) = P(-\infty \leq X \leq 5.0)$

$$= \phi\left(\frac{5.0-\mu}{\sigma}\right) - \phi\left(\frac{-\infty-\mu}{\sigma}\right)$$

$$= \phi\left(\frac{10.34-10}{1.4142}\right) - \phi\left(\frac{-\infty-10}{1.4142}\right)$$

$$= \phi\left(\frac{0.34}{1.4142}\right) - 0$$

$$= \phi\left(\frac{0.34}{1.4142}\right)$$

$$= \phi(0.2404)$$

$= 0.5948$

(b) $P(x \geq 11.98) = 1 - P(x \leq 11.98)$

$= 1 - P(-\infty \leq x \leq 11.98)$

$= 1 - \left[\phi\left(\frac{11.98-\mu}{\sigma}\right) - \phi\left(\frac{-\infty-\mu}{\sigma}\right)\right]$

$= 1 - \left[\phi\left(\frac{11.98-10}{1.4142}\right) - \phi\left(\frac{-\infty-10}{1.4142}\right)\right]$

$= 1 - \phi\left(\frac{1.98}{1.4142}\right)$

$= 1 - \phi(1.4001)$

$= 1 - 0.9192$

$= 0.0808$

(c) $P(7.67 \leq x \leq 9.90) = \phi\left(\frac{9.90-\mu}{1.4142}\right) - \phi\left(\frac{7.67-\mu}{\sigma}\right)$

$= \phi\left(\frac{9.90-10}{1.4142}\right) - \phi\left(\frac{7.67-10}{1.4142}\right)$

$= \phi(-0.0707) - \phi(-1.6476)$

$= 0.4721 - 0.0505$

$= 0.4216$ (Ans

(d) $$P(10.88 \leq x \leq 13.22) = \phi\left(\frac{13.22-10}{1.4142}\right) - \phi\left(\frac{10.88-10}{1.4142}\right)$$

$$= \phi\left(\frac{3.22}{1.4142}\right) - \phi\left(\frac{0.88}{1.4142}\right)$$

$$= \phi(2.2769) - \phi(0.6223)$$

$$= 0.9884 - 0.7324$$

$$= 0.256$$ (Ans

(e) $$P(|X-10| \leq 3) = P(7 \leq X \leq 13)$$

$$= \phi\left(\frac{13-10}{1.4142}\right) - \phi\left(\frac{7-10}{1.4142}\right)$$

$$= \phi(2.1213) - \phi(-2.1213)$$

$$= \phi(2.1213) - 1 + \phi(-2.1213)$$

$$= [2\phi(2.1213)] - 1$$

$$= (2 \times 0.983) - 1$$

$$= 1.9662 - 1$$

$$= 0.9662$$ (Ans

(f) The value of x which $P(X \leq x) = 0.81$

$$\therefore \phi(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx$$

$$\Rightarrow 0.81 = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{x^2}{2}} dx$$

$$\Rightarrow 0.81 = \frac{1}{\sqrt{2\pi}} \cdot e^{-\frac{x^2}{2}}$$

$$\Rightarrow \ln(0.81) = \frac{1}{\sqrt{2\pi}} \frac{-x^2}{2}$$

$$= 0.2107 = \frac{1}{\sqrt{2\pi}} \left(\frac{x^2}{2}\right)$$

$$=$$

(g) The value of x which $P(X \geq x) = 0.04$

$$\Rightarrow P(X \geq x) = 1 - P(X \leq x)$$

$$\Rightarrow 0.04 = 1 - \phi(x)$$

$$\Rightarrow \phi(x) = 0.96$$

$$\Rightarrow \phi(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} \, dx$$

(h) The value of x for which $P(|X-10| \geq x) = 0.63$

$$\Rightarrow P(|X-10| \geq x) = 1 - P(|X-10| \leq x)$$

$$= 1 - P(10 - x \leq X \leq 10 + x)$$

$$= 1 - [\phi(10+x) - \phi(10-x)]$$

$$= 1 - [\phi(10+x) - 1 + \phi(10+x)]$$

$$= 1 - [2\phi(10+x) - 1]$$

$$\Rightarrow 0.63 = 2 - 2\phi(10+x)$$

$$\Rightarrow -2\phi(10+x) = -1.37$$

$$\Rightarrow 2\phi(10+x) = 1.37$$

$$\Rightarrow \phi(10+x) = 0.685$$

5.1.9

let, x be the thickness of glass sheets produced by a process.

$\therefore X \sim (3, (0.12)^2)$

(a) $P(X \geqslant 3.2) = P\left(\frac{X-3.0}{0.12} \geqslant \frac{X-3.0}{0.12}\right)$

$= P\left(2 \geqslant \frac{X-3.0}{0.12}\right)$

$= 1 - P\left(2 \leqslant \frac{x-3.0}{0.12}\right)$

$= 1 - \phi\left(\frac{3.2-3.0}{0.12}\right)$

$= 1 - \phi(1.67)$

$= 1 - 0.9525$

$= 0.0475$

(b) $P(X \leqslant 2.7) = P(-\infty \leqslant X \leqslant 2.7)$

$= \phi\left(\frac{2.7-3.0}{0.12}\right) - \phi\left(\frac{-\infty-3.0}{0.12}\right)$

$$= \phi\left(\frac{-0.3}{0.12}\right) - \phi(-\infty)$$

$$= \phi(-2.5) - 0$$

$$= 0.00621 \quad \text{(Ans}$$

or,

$$P(X \leq 2.7) = P\left(\frac{X - 3.0}{0.12} \leq \frac{x - 3.0}{0.12}\right)$$

$$= P\left(Z \leq \frac{x - 3.0}{0.12}\right)$$

$$= \phi\left(\frac{2.7 - 3.0}{0.12}\right)$$

$$= \phi(-2.5)$$

$$= 0.00621 \quad \text{(Ans}$$

(d)

$$P(3.00 - c \leq x \leq 3.00 + c) = 0.99$$

$$\Rightarrow \phi\left(\frac{3.00 + c - 3.0}{0.12}\right) - \phi\left(\frac{3.00 - c - 3.0}{0.12}\right) = 0.99$$

$$\Rightarrow \phi\left(\frac{c}{0.12}\right) - 1 + \phi\left(\frac{c}{0.12}\right) = 0.99$$

$$\Rightarrow 2\phi\left(\frac{c}{0.12}\right) = 1.99$$

$$\Rightarrow \phi\left(\frac{c}{0.12}\right) = 0.995$$

$$\Rightarrow \frac{c}{0.12} = \phi^{-1}(0.995)$$

$$\Rightarrow c = \phi^{-1}(0.995) \times 0.12$$

$$\Rightarrow c = 2.5758 \times 0.12$$

$$\therefore c = 0.3091 \quad \text{(Ans.)}$$

5.1.11

here,

$$\mu = 4.3 \text{ mm}$$

$$\sigma = 0.12 \text{ mm}$$

(a) upper quantiles,

$$P\left(\frac{X-4.3}{0.12} \leq \frac{x-4.3}{0.12}\right) = 0.75$$

$$\Rightarrow P\left(Z \leq \frac{x-4.3}{0.12}\right) = 0.75$$

$$\Rightarrow \phi\left(\frac{x-4.3}{0.12}\right) = 0.75$$

$$\Rightarrow \frac{x-4.3}{0.12} = \phi^{-1}(0.75)$$

$$\Rightarrow x = \phi^{-1}(0.75) \times 0.12 + 4.3$$

$$\therefore x = 4.3816$$

lower quantiles, $P\left(\frac{X-4.3}{0.12} \leq \frac{x-4.3}{0.12}\right) = 0.25$

$$\Rightarrow P\left(z \leq \frac{x-4.3}{0.12}\right) = 0.25$$

$$\Rightarrow \phi\left(\frac{x-4.3}{0.12}\right) = 0.25$$

$$\Rightarrow \frac{x-4.3}{0.12} = \phi^{-1}(0.25)$$

$$\Rightarrow x = \phi^{-1}(0.25) \times 0.12 + 4.3$$

$$\therefore x = 4.2190 \text{ (Ans.)}$$

(b) $P(4.3 - c \leq X < 4.3 + c) = 0.80$

$$\Rightarrow \phi\left(\frac{4.3+c-4.3}{0.12}\right) - \phi\left(\frac{4.3-c-4.3}{0.12}\right) = 0.80$$

$$\Rightarrow \phi\left(\frac{c}{0.2}\right) - 1 + \left(\frac{c}{0.12}\right) = 0.80$$

$$\Rightarrow 2\phi\left(\frac{c}{0.12}\right) = 1.80$$

$$\Rightarrow \phi\left(\frac{c}{0.12}\right) = 0.90$$

$$\Rightarrow \frac{c}{0.12} = \phi^{-1}(0.90)$$

$$\Rightarrow c = \phi^{-1}(0.90) \times 0.12$$

$$\therefore c = 0.1536 \quad (\text{Ans})$$

5.1.13

here, $\mu = 23.8$

$\sigma^2 = 1.28$

$\therefore \sigma = 1.1313$

(a) $$P(x \leq 23.0) = P\left(\frac{x - 23.8}{1.1313} \leq \frac{x - 23.8}{1.1313}\right)$$

$$= P\left(z \leq \frac{23 - 23.8}{1.1313}\right)$$

$$= P\left(z \leq \frac{0.8}{1.1313}\right)$$

$$\Rightarrow \phi\left(\frac{-0.8}{1.1313}\right)$$

$$= \phi(-0.7071)$$

$$= 0.2389$$

(b) $P(X \geq 24.0) = 1 - P(X \leq 24.0)$

$$= 1 - P\left(\frac{X - 23.8}{1.1313} \leq \frac{x - 23.8}{1.1313}\right)$$

$$= 1 - \phi\left(\frac{24 - 23.8}{1.1313}\right)$$

$$= 1 - \phi(0.1767)$$

$$= 1 - 0.5714$$

$$= 0.4286$$

(c) $P(24.2 \leq X \leq 24.5)$

$$= \phi\left(\frac{24.2 - 23.8}{1.1313}\right) - \phi\left(\frac{24.5 - 23.8}{1.1313}\right)$$

$$= \phi(0.62) - \phi(0.35)$$

$$= 0.7324 - 0.6368$$

$$= 0.0956$$

(d) Upper quantiles,

$$P\left(\frac{X-23.8}{1.1313} \leq \frac{x-23.8}{1.1313}\right) = 0.75$$

$$\Rightarrow \phi\left(\frac{x-23.8}{1.1313}\right) = 0.75$$

$$\Rightarrow \frac{x-23.8}{1.1313} = \phi^{-1}(0.75)$$

$$\Rightarrow x = \phi^{-1}(0.75) \times 1.1313 + 23.8$$

$$\therefore x = 24.675$$

(e) 95th percentile:

$$P\left(\frac{X-23.8}{1.1313} < \frac{x-23.8}{1.1313}\right) = 0.95$$

$$\Rightarrow \phi\left(\frac{x-23.8}{1.1313}\right) = 0.95$$

$$\Rightarrow x = \phi^{-1}(0.95) \times 1.1313 + 23.8$$

$$\therefore x = 25.6667 \text{ (Ans)}$$

Chapter - 5.2

(1) 5.2.1

given,

$X \sim \mathcal{X}(3.2, 6.5)$, $\mathcal{Y}(-2.1, 3.5)$, $Z(12.0, 7.5)$

(a) For. $X+Y$ there is,

$\mu = \mu_1 + \mu_2$ and, $\sigma^2 = \sigma_1^2 + \sigma_2^2$

$= 3.2 - 2.1$

$= 1.1$

$= 6.5 + 3.5$

$\sigma^2 = 10$

$\therefore \sigma = \sqrt{10}$

$\therefore P(X+Y \geq 0) = P(0 \leq X+Y \leq 0)$

$= \phi\left(\frac{X-1.1}{\sqrt{10}}\right) - \phi\left(\frac{x-1.1}{\sqrt{10}}\right)$

$= \phi(z) - \phi\left(\frac{0-1.1}{\sqrt{10}}\right)$

$= -\phi\left(\frac{-1.1}{\sqrt{10}}\right)$

$$= 1 - \phi\left(\frac{-1.1}{\sqrt{10}}\right)$$

$$= 1 - \phi(-0.03478)$$

$$= 1 - 0.3639$$

$$= 0.6361 \quad \text{(Ans)}$$

(b) $X + Y - 2Z \leq -20$

here,

$$\mu = \mu_1 + \mu_2 - 2\mu_3$$

$$= -22.9$$

and, $\sigma^2 = \sigma_1^2 + \sigma_2^2 - 2\sigma_3^2$

$$= 6.5 + 3.5 + (-2)^2 \times 12$$

$$= 40$$

$$\therefore \sigma = \sqrt{40}$$

$$\therefore P(X + Y - 2Z \leq -20) = P(-\infty \leq X + Y - 2Z \leq -20)$$

$$= \phi\left(\frac{-20 + 22.9}{\sqrt{40}}\right)$$

$$= \phi(0.4585)$$

$= 0.6767$

(e) $P(3X + 5Y \geq 1) = P(1 \leq 3X + 5Y \leq \infty)$

$= 0 - \phi\left(\frac{1 - (-0.9)}{12.003}\right)$

$= 1 - \phi\left(\frac{1.9}{12.003}\right)$

$= 1 - 0.5625$

$= 0.4375$ (Ans)

(e) $P(|X + 6Y + Z| \geq 2) = 1 - P(|X+6Y+Z| \leq 2)$

$= 1 - P(-2 \leq X + 6Y + Z \leq 2)$

$= 1 - \phi(-2) - \phi(2)$

$= 2\phi(2) - 1$

$= 2 \times$

5.2.3

given,

$$x_i \sim (\mu_i, \delta_i^2)$$

independent variable, so, $\mu = 0$
$\delta^2 = 0$

so that,

$$P(|x_i| \leq 0.5) = P(-0.5 \leq x_i \leq 0.5)$$
$$= \phi(0.5) - \phi(0.5)$$
$$= \phi(0.5) - 1 + \phi(0.5)$$
$$= 2\phi(0.5) - 1$$
$$= 2 \times 0.6915 - 1$$
$$= 0.3830$$

(b) Normal distribution,

given,

$$\mu = 0$$

and, $\delta^2 = \frac{1}{8}$

$$\therefore \delta = \frac{1}{\sqrt{8}}$$

$\therefore P(|\bar{x}| \leq 0.5) = P(-0.5 \leq \bar{x} \leq 0.5)$

$= \phi\left(\frac{0.5-0}{0.3535}\right) - \phi\left(\frac{-0.5-0}{0.3535}\right)$

$= \phi\left(\frac{0.5}{0.3535}\right) - \phi\left(\frac{-0.5}{0.3535}\right)$

$= 2\phi\left(\frac{0.5}{0.3535}\right) - 1$

$= 2 \times 0.9214 - 1$

$= 0.8428$ (Ans)

ⓒ given, $\mu = 0$ and, $\sigma^2 = \frac{1}{n}$

$\therefore \sigma = \frac{1}{\sqrt{n}}$

$\therefore P(|\bar{x}| \leq 0.5) \geq 0.99$

$\Rightarrow P(-0.5 \leq \bar{x} \leq 0.5) = 0.99$

$$\Rightarrow \phi\left(\frac{0.5-0}{1/\sqrt{n}}\right) - \phi\left(\frac{-0.5-0}{1/\sqrt{n}}\right) = 0.99$$

$$\Rightarrow 2\phi\left(\frac{0.5}{1/\sqrt{n}}\right) - 1 = 0.99$$

$$\Rightarrow 2\phi\left(\frac{0.5}{1/\sqrt{n}}\right) = 1.99$$

$$\Rightarrow \phi\left(\frac{0.5}{1/\sqrt{n}}\right) = 0.995$$

$$\Rightarrow \phi\left(0.5\sqrt{n}\right) = \phi^{-1}\left(0.995\right)$$

$$\Rightarrow 0.5\sqrt{n} = 2.5758$$

$$\Rightarrow \sqrt{n} = \frac{2.5758}{0.5}$$

$$\Rightarrow \sqrt{n} = 5.1516$$

$$\therefore n = 26.54 \quad \text{(Ans)}$$

5.2.11

$$\bar{X} \sim N(4.5,\ 0.88)$$

$$N\left(4.5,\ \frac{0.88}{15}\right)$$

(a) $N(4.5,\ 0.0587)$

$\mu = 4.5$
$\sigma^2 = 0.0567$
$\sigma =$

(a)

$$P(4.2 \leq \bar{X} \leq 4.9)$$

$$= P\left(\frac{4.2-4.5}{\sqrt{0.0587}} \leq \frac{\bar{X}-4.5}{\sqrt{0.0587}} \leq \frac{4.9-4.5}{\sqrt{0.0587}}\right)$$

$$= P(-1.238 \leq Z \leq 1.651)$$

$$= \phi(1.651) - \phi(-1.238)$$

$$= 0.951 - 0.108$$

$$= 0.843$$

(b)

$$P(4.5 - c \leq \bar{X} \leq 4.5 + c) = 0.99$$

$$\Rightarrow \phi\left(\frac{4.5+c-4.5}{\sqrt{0.0587}}\right) - \phi\left(\frac{4.5-c-4.5}{\sqrt{0.0587}}\right) = 0.99$$

$$\Rightarrow \phi\left(\frac{c}{\sqrt{0.0587}}\right) - 1 + \phi\left(\frac{c}{\sqrt{0.0587}}\right) = 0.99$$

$$\Rightarrow \phi\left(\frac{c}{\sqrt{0.0587}}\right) = 0.995$$

$$\Rightarrow \frac{c}{\sqrt{0.0587}} = 2.5758$$

$$\Rightarrow c = 2.5758 \times \sqrt{0.0587}$$

$$\therefore c = 0.624$$

(Ans)

5.2.19

Company A has an expected return of

$$E(X) = \mu = \$30{,}000$$

and, $\sigma_1 = \$4000$

Company B has an expected return of

$$E(X) = \mu_2 = \$45000$$

$$\sigma_2 = \$3000$$

∴ Total return from both investment is,

$$X = X_1 + X_2$$

$$= 30000 + 45000$$

$$= 75000$$

and, $\sigma^2 = \sigma_1^2 + \sigma_2^2$

$$= (4000)^2 + (3000)^2$$

$$\sigma^2 = 25000000$$

$$\Rightarrow \sigma = \sqrt{25000000}$$

$$\Rightarrow \sigma = 5000$$

∴ $P(X \geq 85000) = P(85000 \leq X < \infty)$

$$= \phi\left(\frac{\infty - \cancel{85000}\ 75000}{5000}\right) - \phi\left(\frac{85000 - 75000}{5000}\right)$$

88.

$$= 0 - \phi\left(\frac{10000}{5000}\right)$$

$$= 0 - \phi(2)$$