Some Probablity Mass functions (Descrete random variable)

↳ random variable value are certain.

Bernoulli distribution: a single ~~trail~~ trial is conducted which takes a binary outcome. The probablity mass fun^c is,

only one trial and there are two possible outcome.

$$\boxed{p(X=x) = p^x (1-p)^{1-x}} \quad x = 0,1$$

At this case ran... var... x is called berno distribution.

p indicates probablity
x " values of the r. var..

single coin toss.
single dice roll and possibility of 6 come / even আসবে কি না।

Expectation: $\boxed{E(x) = p}$

Variance: $\boxed{V(x) = p(1-p)}$

Example: if you toss a coin one times. For example, Random variable X indicates the no of head.

Binomial Distribution: n trails $(n>1)$ are conducted which takes a binary outcome. The probability mass fun^c is

n trials $[n>1]$ &
2 possible outcome,
every trials are independent.

$$\boxed{p(X=x) = \binom{n}{x} p^x (1-p)^{n-x}} \quad x=0,1,2,\dots,n$$

toss a coin for 5 times

Expectation: $\boxed{E(x) = np}$

Variance: $\boxed{V(x) = np(1-p)}$

Example: If you toss a coin five times. For example, Random variable X indicates the no of tail. $n=5$ $x=0,1,2,3,4,5$

Example: Suppose a milk factory ~~contains~~ has $\boxed{\text{20 containers}}$ (trial) and $\boxed{\text{there}}$ is a propablity of $\boxed{0.261}$ that a milk container is underweight.

(a) What is the distribution of the number of underweight containers in abox?

(b) Calculate expected number of underweight cantons in a box and also calculate its variance.

(c) Calculate the probablity that a box cotains exactly seven underweight ~~contain exactly seven underweight containers~~ and also. 7

(d) Calculate the probablity that a box contains no more than three underweight containers. 3

(e) Calculate the probablity that a box contain at least two underweight containers. 2

possible out come হবে 2
তাহলে, হয় underweight or underweight না।

Solution: (a) Binomial distribution

$$P(X=x) = \binom{20}{x} p^x (1-p)^{20-x} \qquad x = 0,1,2,\dots,20$$

(b) $E(x) = np = 20 * 0.261 = 5.22,$

$V(x) = np(1-p)$

$= 20 * 0.261 * (1-0.261) = 3.857$

standard deviation $= \sqrt{3.857} = 1.96$

(c) $P(X=7) = \binom{20}{7} 0.261^7 (1-0.261)^{20-7}$ $\quad {}^nC_r = \binom{20}{7} = {}^{20}C_r$

$= 0.1254.$

(d) $P(X \le 3) = P(X=0) + P(X=1) + P(X=2) + P(X=3)$

$= \binom{20}{0} \times (0.261)^0 * 0.739^{20} + \binom{20}{1} \times 0.261^1 \times 0.739^{19}$

$+ \binom{20}{2} \times 0.261^2 \times 0.739^{18} + \binom{20}{3} \times 0.261^3 \times 0.739^{17}$

$= 0.0024 + 0.0167 + 0.0559 + 0.1185$

$= 0.1935$ Answer

(e) $P(X \ge 2) = P(2) + P(3) + \dots\dots + P(20) = ?$

We know that total probablity $= 1$.

$P(0) + P(1) + P(2) + P(3) + \dots\dots + P(20) = 1.$

$\Rightarrow P(2) + P(3) + \dots + P(20) = 1 - P(0) - P(1)$

$\Rightarrow P(2) + P(3) + \dots\dots + P(20) = 1 - 0.0024 - 0.0167$

$= 0.9809.$

Poisson distribution:

*counts the number of events that occur in an time interval
নির্দিষ্ট সময়ে কয়বার আসবে।

* trial সংখ্যা fixed না থাকলে তখন follow করবে।
যখন x কোনো number of events count করবে;

The poisson distribution is used when a random variable counts the number of events that occur in an time interval. For example,

1) the number of telephone calls received by an operator within a certain time limit.
2) The number of patients arriving in an emergency room between 10 and 11pm.

The probability mass function is,

$$\boxed{p(X=x) = \frac{e^{-\lambda}\lambda^{x}}{x!}} \qquad x = 0, 1, 2, 3, \ldots$$

λ হচ্ছে parameter/ unknown value. দেওয়া থাকবে।

Expectation: $\boxed{E(x) = \lambda}$ Average value কে indicate করে

Variance: $\boxed{V(x) = \lambda}$

Difference between ~~bion~~ binomial distribution and Poisson distribution:

1) In binomial distribution, number of trials are $\boxed{\text{fixed}}$. In poisson distribution, number of trials are $\boxed{\text{infinite}}$.
2) In binomial distribution, $\boxed{\text{Variance} \leq \text{Mean}}$. In Poisson distribution, $\boxed{\text{Mean} = \text{Variance}}$.
3) Example of binomial distribution: Coin tossing experiment. ~~Ex of Poisson~~

Example of Poisson distribution: The number of patients arriving in an emergency room between 10 and 11 pm.

Example: Suppose that the number of errors in a piece of software has a parameter $\lambda = 3$. This parameter immediately implies that the expected number of errors is three and that the variance in the number of errors is also equal to three.

(a) What is distribution of the number of errors in a piece of software.
(b) Calculate the probability that a piece of software has no errors.
(c) Calculate the " " ~~there the~~ are three or more errors in a piece of software.

Soln: (a) The number of errors in a piece of software follows pussion distribution

$$P(X=x) = \frac{e^{-3} 3^x}{x!} \qquad x = 0, 1, 2, 3, \ldots$$

(b) $P(X=0) = \frac{e^{-3} 3^0}{0!} = 0.05$

(c) $P(X=3) + P(X=4) + \ldots = ?$

We know,

$P(X=0) + P(X=1) + P(X=2) + P(X=3) + P(X=4) + \ldots = 1$

$\Rightarrow P(X=3) + P(X=4) + \ldots = 1 - P(X=0) - P(X=1) - P(X=2)$

$\Rightarrow P(X=3) + P(X=4) + \ldots = 1 - \frac{e^{-3} 3^0}{0!} - \frac{e^{-3} 3^1}{1!} - \frac{e^{-3} 3^2}{2!}$

$\Rightarrow P(X=3) + P(X=4) + \ldots = 0.577$ Answer

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on:

$\Rightarrow$ For bionomial distribution, Expectation: $E(x) = np$

Variance: $V(x) = np(1-p)$ $[0 \le p \le 1, n > 1]$

Mean = Variance (when $p = 0$)

$\Rightarrow n.0 = n.0(1-0)$

$\Rightarrow 0 = 0$

Mean $(n) >$ variance (0) when $p = 1$.

because when $p = 1$, mean $= n.1 = n$ $(n > 1)$

and variance $= n.1.(1-1) = 0$

$\therefore$ Mean $>$ variance $[0 < p < 1]$

$\Rightarrow np > np(1-p)$

Ex: $50 > 50(1-p)$ here, $(0 < (1-p) < 1)$

$\Rightarrow 50 > 50(1-0.2)$ if $p = 0.2 \Rightarrow 50 > 50 \times 0.8$

$\Rightarrow 50 > 40$

4.2 Geometric distribution:

* The number of trials ~~a~~ is not fixed.
  And experiment will continue until the first success occurs
* So, success fixed = 1.
  trial সংখ্যা unknown
  x indicates number of trial, $x \neq 0$

The bionomial distribution is the distribution of the number of success occuring in a fixed number of trials n, it is sometimes of interest to count instead the number of trials performed until the first success occurs. Such a random variable is said to have a geometric distribution.

The probablity mass function is,

$$\boxed{P(X=x) = (1-p)^{(x-1)} p} \quad x = 1, 2, 3, \ldots$$

Expectation $\boxed{E(X) = 1/p}$

Variance $\boxed{V(X) = \frac{1-p}{p^2}}$

 # Example: Suppose that a company wishes to hire one new workers and that each applicant interviewed has a probablity of 0.6 of being found acceptable.

Geo

P

1) What is the distribution of the total number of applicants that the company needs to interview?
2) Calculate the probablity that exactly six applicants need to be interviewed
3) Calculate the probablity that the company allows up to/at most six applicants to be interviewed.
4) Calculate the probablity that ~~the~~ at least six applicants need to be interviewed.
5) Calculate the expected numbers of interviews.

Soln: [1] The total number of applicants that the company needs to interview follows geometric distribution.

The probablity mass function is, ~~P(X) = x~~

$$\cancel{P(X)} \quad P(X=x) = (1-0.6)^{(x-1)} 0.6 \quad x = 1, 2, 3, \ldots$$

[2] $P(X=6) = (1-0.6)^{(6-1)} \cdot 0.6$

$= 6.144 \times 10^{-3}$

[3] $P(X \leq 6) = P(X=1) + P(X=2) + \ldots + P(X=6)$

$= 0.6 + 0.24 + 0.096 + 0.0384 + 0.01536 + 0.006144$

$= 0.996$. Answer

4. $P(X \geq 6) = P(X=6) + P(X=7) + \dots$

We know, $P(X=1) + \dots + P(X=5) + P(X=6) + P(X=7) + \dots = 1$

$\Rightarrow P(X=6) + P(X=7) + \dots = 1 - P(X=1) - \dots - P(X=5)$

$= 1 - 0.98976$

$= 0.01024$ Answer

5. $E(X) = \frac{1}{p} = \frac{1}{0.6} = 1.667$ Answer

Negative binomial distribution:

The binomial distribution is the distribution of the number of success occurring in a fixed number of trials n, it is sometimes of interest to count instead the number of trials performed until the rth $(r > 1)$ success occurs. Such a random variable is said to have a negative bionomial distribution.

Extention of geomatric dis...

* The experiment will continue untill the r^{th} success occur

The p.m.f. is $\boxed{P(X=x) = \binom{x-1}{r-1} (1-p)^{(x-r)} p^r}$ $x = r, r+1, r+2, \dots$

Expectation, $\boxed{E(x) = r/p}$

variance, $\boxed{V(x) = \frac{r(1-p)}{p^2}}$

Example: Suppose that a company wishes to hire $\boxed{\text{three}}$ new workers and that each applicant interviewed has a probablity of $\boxed{0.6}$ of being found acceptable.

1. what is the distribution of the total number of applicants that the company needs to interviewed?
2. Calculate the probablity that exactly six applicants need to be interviewed.
3. Calculate the probablity that the company allows upto/at most six applicants to be interviewed.
4. Calculate the probablity that at least six applicants need to be interviewed.
5. Calculate the expected number of interviews.

Soln: The total number of applicants that the company needs to interview follows negative

1. binomial distribution.

The p.m.f. is $P(X=x) = \binom{x-1}{3-1} \cdot (1-0.6)^{(x-3)} (0.6)^3$ $x = 3, 4, 5, 6, \dots$

2. $P(X=6) = \binom{6-1}{3-1} (1-0.6)^{(6-3)} (0.6)^3 = \binom{5}{2} (0.4)^3 (0.6)^3 = \binom{5}{2} \times 0.013824 = 0.138$ A

3. $P(X \leq 6) = P(X=3) + P(X=4) + P(X=5) + P(X=6) = \binom{2}{2}(1-0.6)^0 \cdot 0.6^3 + \binom{3}{2} 0.4^1 \cdot 0.6^3 + \binom{4}{2} 0.4^2 0.6^3 + \binom{5}{2} 0.4^3 0.6^3$

$= \boxed{0.216} + 0.2592 + 0.20736 + 0.13826 = 0.820$ Answer

4. $P(X \geq 6) = P(X=6) + P(X=7) + \dots = ?$

$P(X=3) + P(X=4) + P(X=5) + P(X=6) + P(X=7) + \dots = 1 \Rightarrow P(X=6) + P(X=7) + \dots = 1 - P(X=3) - \dots - P(X=5)$

$\Rightarrow P(X=6) + P(X=7) + \dots = 1 - P(X=3) - P(X=4) - P(X=5) = 1 - 0.68256 = 0.31744$ Answer

5. $E(X) = 3/0.6 = 5$

Normal Distribution: The probability density function of normal distribution is:

a funᶜ of a continuous Random variable. range of the random var is $(-\infty, \infty)$
range of the μ(mu) is $-\infty$ to $+\infty$
" " " variance (σ^2) is 0 to ∞

$$\boxed{f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}}$$

$-\infty < x < \infty$,
$-\infty < \mu < \infty$,
$0 \le \sigma^2 < \infty$

** N. Dis... এ বললে Data plot করলে। তবে যদি দেখি symmetry
হলে Normal
shaperior test-এ যদি P-এর value পাওয়া
P value > 0.5 then Normal
P " < 0.5 " not normal
exam এ বলা থাকলে।

Expectation: $\boxed{E(x) = \mu}$

Variance: $\boxed{V(x) = \sigma^2}$

Standard normal distribution: When $\boxed{\mu = 0}$ and $\boxed{\sigma^2 = 1}$ (variance) then the normal distribution is called standard normal distribution.

The probablity density function of standard normal distribution is:

$$\boxed{f(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2}} \qquad -\infty < z < \infty$$

Property of a normal distribution

Mean: Avegage value
Mode: Maximum "
Median: middle point

1) It is symmetric.
2) Mean = Mode = Median. (Mean: average value; Mode: maximum value; Median: middle point)
3) It is unimodal.
↳ only one peak point
4) The total area under the curve is equal to one. $\int_{-\infty}^{\infty} f(x)\,dx = 1$
5) The normal curve approaches, but never touches, the x-axis.

→ unimodal
→ bimodal
→ Multi

Transformation:

$$\int_{-\infty}^{\infty} f(x)\,dx = \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}\,dx$$

$$= \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{1}{2}z^2}\,\sigma\,dz$$

$$= \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2}\,dz$$

(Normal distribution থেকে standard normal distribution-এ convert করে)

Let, $\boxed{z = \frac{x-\mu}{\sigma}}$ → এই উপায় মাধ্যমে x এর integration কে z-এর " এ রূপান্তর করা হবে।

$\Rightarrow z = \frac{x}{\sigma} - \frac{\mu}{\sigma}$

$\Rightarrow \frac{dz}{dx} = \frac{1}{\sigma}$

$\Rightarrow dx = \sigma\,dz$

x	$-\infty$	∞
z	$-\infty$	∞

That is if $X \sim N(\mu, \sigma^2)$ and if you want to transform the normal distribution to standard distribution then the transform random variable is

$$\boxed{z = \frac{x-\mu}{\sigma} \text{ (z score)}}$$

F(x) মানে lower point (মানে $-\infty$) থেকে x point পর্যন্ত Area.
F(1) 2

Probablity Calculations for Normal Distributions:

$P(a<x<b)$
$= P\left(\frac{a-\mu}{\sigma} < \frac{x-\mu}{\sigma} < \frac{b-\mu}{\sigma}\right)$
$= P\left(\frac{a-\mu}{\sigma} < z < \frac{b-\mu}{\sigma}\right)$
$= F\left(\frac{b-\mu}{\sigma}\right) - F\left(\frac{a-\mu}{\sigma}\right)$

$P(x<b)$
$= P(-\infty < x < b)$
$= P\left(\frac{-\infty-\mu}{\sigma} < \frac{x-\mu}{\sigma} < \frac{b-\mu}{\sigma}\right)$
$= P\left(-\infty < z < \frac{b-\mu}{\sigma}\right)$
$= F\left(\frac{b-\mu}{\sigma}\right)$

$P(x>a)$
$= P(a<x<\infty)$
$= P\left(\frac{a-\mu}{\sigma} < \frac{x-\mu}{\sigma} < \frac{\infty-\mu}{\sigma}\right)$
$= P\left(\frac{a-\mu}{\sigma} < z < \infty\right)$
$= F(\infty) - F\left(\frac{a-\mu}{\sigma}\right)$
$= 1 - F\left(\frac{a-\mu}{\sigma}\right)$

Example: A company manufactures concrete blocks that are used for construction purposes. Suppose that the weights of the individual concrete blocks are normally distributed with a mean value of $\mu = 11.0$ kg and a standard deviation of $\sigma = 0.3$ kg.

$\boxed{\text{(i)}}$ Calculate the probablity that a concrete block weight is less than 10.5 kg

~~(ii) " " " "~~ " " is within 10 kg to 12 kg

$\boxed{\text{(ii)}}$ " " " " a " " " is greater than 10.5 kg

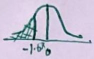
$\boxed{\text{(iii)}}$ " " " " a "

$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$

$= \frac{1}{\sqrt{2\pi} \cdot 0.3} e^{-\frac{(x-11)^2}{2 \cdot 0.3^2}}$

S.N.D.F. $f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2}$

Solⁿ: (i)



$P(x < 10.5)$

$= P(-\infty < x < 10.5)$

$= P\left(\frac{-\infty - 11}{0.3} < \frac{x-11}{0.3} < \frac{10.5-11}{0.3}\right)$

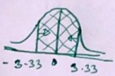
$= P(-\infty < z < -1.67)$

$= F(-1.67)$ [এই value page 787 থেকে পাবো]

$= 0.0475$

(ii) $P(10 < x < 12)$

$= P\left(\frac{10-11}{0.3} < \frac{x-11}{0.3} < \frac{12-11}{0.3}\right)$



$= P(-3.33 < z < 3.33)$

$= F(3.33) - F(-3.33)$

$= 0.99957 - 0.00043$

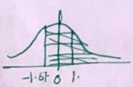
$= 0.99914$

* -3.4 -এর চেয়ে ছোট value আসলে 0 ধরে নিব।

* 3.4 এর চেয়ে বড় আসলে 1 ধরে নিব।

(iii) $P(x > 10.5) = P(10.5 < x) = P(10.5 < x < \infty)$

$= P\left(\frac{10.5-11}{0.3} < \frac{x-11}{0.3} < \frac{\infty - 11}{0.3}\right)$



$= P(-1.67 < z < \infty)$

$= F(\infty) - F(-1.67)$

$= 1 - 0.04746$

$= 0.95254$

Central Limit Theorem: When sample size is large (≥ 30), the average of a set of independent identically distributed random variables ($\bar{X}$) is always approximately normally distributed with mean population mean and variance population variance divided by sample size. ie

$x \sim$ (follows) any distribution, when the sample size is large হলে $\boxed{n \geq 30}$ then,
Average value of the random variable ($\bar{x}$) always follows ($\sim$) Normal distr^n.

$$\bar{X} \sim N\left(E(x), \frac{V(x)}{n}\right)$$

$E(x)$: population means (যাকে Expected value of x) $\to E(\bar{x})$

$\frac{V(x)}{n}$: Populalation Variance divided by n. $\to V(\bar{x})$

when n is large, $\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$

if $\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$ it indicates that

$$\boxed{E(\bar{X}) = \mu}$$

[$\because \bar{x} \to$ Average $\therefore \bar{X} = \frac{x_1 + x_2 + \dots + x_n}{n}$]

$$\Rightarrow E\left(\frac{x_1 + x_2 + \dots + x_n}{n}\right) = \mu$$

$$\Rightarrow \frac{1}{n} E(x_1 + x_2 + \dots + x_n) = \mu \quad [\text{Formula, } E(cx) = cE(x)]$$

$$\Rightarrow E(x_1 + x_2 + \dots + x_n) = n\mu$$

$$\Rightarrow \boxed{E\left(\sum_{i=1}^{n} x_i\right) = n\mu}$$

$$V(\bar{x}) = \frac{\sigma^2}{n}$$

$$\Rightarrow V\left(\frac{x_1 + x_2 + \dots x_n}{n}\right) = \frac{\sigma^2}{n}$$

$$\Rightarrow \frac{1}{n^2} V(x_1 + x_2 + \dots + x_n) = \frac{\sigma^2}{n} \quad [\text{Formula } V(cx) = c^2 V(x)]$$

$$\Rightarrow V(x_1 + x_2 + \dots + x_n) = \frac{n^2 \sigma^2}{n}$$

$$\therefore \boxed{V\left(\sum_{i=1}^{n} X_i\right) = n\sigma^2}$$

$$\therefore \boxed{\sum_{i=1}^{n} X_i \sim N(n\mu ; n\sigma^2)}$$

$$V(x) = E(x^2) - (E(x))^2$$
$$V(cx) = E((cx)^2) - (E(cx))^2$$
$$= E(c^2 x^2) - (cE(x))^2$$
$$= c^2 E(x^2) - c^2 (E(x))^2$$
$$= c^2 [E(x^2) - (E(x))^2]$$

~~$\therefore$ i.e. $V(\sum_{i=1}^{n} x_i$~~

for example, $x \sim$ Poisson(λ) with parameter

$E(x) = \lambda$, $V(x) = \lambda$

sample size, $n = 100$.

$$\therefore \bar{X} \sim N\left(\lambda, \frac{\lambda}{n}\right).$$

$$\Rightarrow \bar{X} \sim N\left(\lambda, \frac{\lambda}{100}\right)$$

* যে distribu^n-ই হোক না কেনো, যদি sample size $n \geq 30$ হয়, then the average value of x, always follows N dist. $\bar{X} \sim N\left(E(x), \frac{V(x)}{\lambda}\right)$

Example: The number of flaws in a glass sheet has a Poisson distribution with a parameter $\boxed{\lambda = 0.5}$. (a) What is the distribution of the average number of flaws per sheet in $\boxed{100 \text{ sheets}}$ (n) of glass?
(b) Calculate the probability that this average is between $\boxed{0.45}$ & $\boxed{0.55}$
(c) What is the distribu^n of the total number (Σ) of flaws in 100 sheets of glass?
(d) Calculate the probability that there are fewer than 40 flaws in 100 sheets of glass? ~~(d) $P(x < 40)$~~ (d) $P\left(\sum_{i=0}^{100} x_i \leq 40\right)$

(a) $\bar{X} \sim N\left(0.5, \frac{0.5}{100}\right)$

(b) $P(0.45 < \bar{x} < 0.55)$

(c) $\sum_{i=0}^{100} x_i \sim N(n\lambda, n\lambda)$

$\sum_{i=0}^{100} x_i \sim N(100 \times 0.5, 50)$

Soln: The probability mass func of Poisson distribun is

$$P(X=x) = \frac{e^{-0.5} \cdot 0.5^x}{x!} \qquad x = 0, 1, 2, 3, \ldots$$

Expectation, $E(X) = 0.5$

Variance, $V(x) = \cancel{0.5}$ 0.5

(a) The distribun of the avg number of flaws per sheet in 100 sheets of glass follows normal distribun with mean = 0.5.

Variance : $\cancel{V(\bar{x})} = \frac{0.5}{100}$

i.e. $\bar{X} \sim \bar{N}\left(0.5, \frac{0.5}{100}\right)$

(b) $P(0.45 < \bar{X} < 0.55) = P\left(\frac{0.45-0.5}{\sqrt{\frac{0.5}{100}}} < \frac{\bar{x}-0.5}{\sqrt{0.5/100}} < \frac{0.55-0.5}{\sqrt{0.5/100}}\right)$

~~P(-0.707<~~

$F\left(\frac{0.55-0.5}{\sqrt{\frac{0.5}{100}}}\right) - F\left(\frac{0.45-0.5}{\sqrt{\frac{0.5}{100}}}\right)$

$F(\text{sth}) = \left(\frac{\text{sth} - E(x)}{SD} = \frac{\text{sth} - E(x)}{\sqrt{V(x)}}\right)$

$= P(-0.707 < z < 0.707)$

$= F(.707) - F(-0.707)$

$= 0.7611 - 0.2389$

$= 0.5222$

(c) The distribun of the total number of flaws in 100 sheets of glass

$x + x_2 + \ldots + x_{100} \sim N(100 * 0.5, 100 * 0.5)$

$= \sum_{i=1}^{100} x_i \sim N(50, 50)$

(d) $P\left(\sum_{i=1}^{100} X_i < 40\right) = P\left(\frac{\sum_{i=1}^{100} x_i - 50}{\sqrt{50}} < \frac{40-50}{\sqrt{50}}\right)$

$F\left(\frac{40-50}{\sqrt{50}}\right)$

$= P(z < -1.41) = F(-1.41)$

$= 0.0793$

All distributions
2, 17

Bernoulli:
*1 1 trial
*2 2 Possible Outcome

P.M.F.: $P(X=x) = p^{x}(1-p)^{1-x}$
Expectation: $E(x) = p$
Variance: $V(x) = p(1-p)$

$x = 0, 1$

$^{20}C_7 = \binom{20}{7}$

single coin toss.
ছক্কায় 6 আসার সম্ভাবনা
dice trow করলে even আসার সম্ভাবনা। (counts number of success) (for both)

BiNomial:
*1 n trial
*2 two (2) possible outcome
*3 every trials are independent.
toss a coin for 5 times.
* highest value = trial সংখ্যার সমান।
*4 x indicates number of success

P.M.F.: $P(X=x) = \binom{n}{x} p^{x}(1-p)^{n-x}$
Expectation: $E(x) = np$
Variance: $V(x) = np(1-p)$

$x = 0, 1, 2, \dots n$

Number of trials are fixed.
Variance ≤ Mean

difference

① Number of trials are infinite
variance = Mean

Poisson:
*1 counts number of events that occur within in a time interval.
*2 নির্দিষ্ট সময়ে কয়বার আসলো/ উঠলো।
*3 যখন x কোনো number of events, অর্থাৎ Random variable-এর trial সংখ্যা fixed থাকবে না। তখন Poisson Dis- follow করবে।
*4 x indicates number of events/counts number of events.

P.M.F: $P(X=x) = \frac{e^{-\lambda}\lambda^{x}}{x!}$
Expectation: / Average Value $E(x) = \lambda$
Variance: $V(x) = \lambda$

$x = 0, 1, 2, 3, \dots \infty$
λ হচ্ছে parameter. unknown value/ quantity.

Geometric:
*1 number of trial is not fixed.
*2 The experiment will continue until the first success occurs.
*3 so success fixed = 1. trial সংখ্যা unknown.
*4 x indicates number of trial.
$x \neq 0$. P indecates the probablity.
1বার H না পাওয়া পর্যন্ত toss করে যাচ্ছি।

P.M.F: $P(X=x) = (1-p)^{(x-1)} p$
Expectation: $E(x) = 1/p$
Variance: $V(x) = \frac{1-p}{p^2}$

$x = 1, 2, 3, \dots \infty$

Negative binomial:
*1 Extention of geomatric distribution.
*2 The experiment will continue until the r^{th} success occur.
*3 $r > 1$. & r indicates number of success
*4 No. of trial is not fixed
*5 P indecates the probablity.

P.M.F: $P(X=x) = \binom{x-1}{r-1} \times (1-p)^{(x-r)} \times p^{r}$
Expectation: $E(x) = r/p$
Variance: $V(x) = \frac{r(1-p)}{p^2}$

$x = r, r+1, r+2, \dots \infty$
$r > 1$

Normal : $*_1$ a func of continuous random var.

Exam-এ জন্য থাকবে N. Distr...

$*_2$ range of random var $-\infty < x < \infty$

$*_3$ " " μ (mu) $-\infty < \mu < \infty$

$*_4$ " " variance (σ^2) $0 \le \sigma^2 < \infty$

P.D.F.: $f(x) = \dfrac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$

Expectation: $E(x) = \mu$

Variance: $V(x) = \sigma^2$

$-\infty < x < \infty$
$-\infty < \mu < \infty$
$0 \le \sigma^2 < \infty$

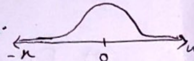
Standard Normal Distribution:

$*_1$ $\mu = 0$
$\sigma^2 = 0$

P.D.F: ~~f(z)~~
$$f(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2}$$

Property of Normal distribution

1) symmetric func. (দেখে বুঝা যাবে। আর নিচে check দিবে। যেখানে Pvalue > 0.5 হলে Normal
Pvalue < 0.5 " Not "
2) Mean (Average Value) = Mode (Manimam Value) = Median (middle point).
3) Unimodal (only one peak point.)
→ Bimodal
→ Multi modal
4) The total Area ($\int_{-\infty}^{\infty} f(x)\,dx$) under the curve is equal (=) to one (1) ($\int_{-\infty}^{\infty} f(x)\,dx = 1$)
5) The normal curve approaches, but not touches the x-axis.



Transformation:

$$f(x) = \boxed{\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx} \text{ Normal dist...}$$

$$= \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{1}{2}z^2} \sigma\,dz = \boxed{\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} dz} \text{ Standard N. dis}$$

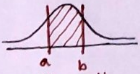
$z = \dfrac{x-\mu}{\sigma} = \dfrac{x}{\sigma} - \dfrac{\mu}{\sigma}$

$\dfrac{dz}{dx} = \dfrac{1}{\sigma}$

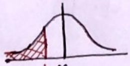
$\therefore dx = \sigma\,dz$.

Probablity Calculation for Normal Distribution.

(i) $P(a < x < b)$
$= P\left(\dfrac{a-\mu}{\sigma} < \dfrac{x-\mu}{\sigma} < \dfrac{b-\mu}{\sigma}\right)$
$= P\left(\dfrac{a-\mu}{\sigma} < z < \dfrac{b-\mu}{\sigma}\right)$
$= F\left(\dfrac{b-\mu}{\sigma}\right) - F\left(\dfrac{a-\mu}{\sigma}\right)$



(ii) $P(x < b)$
$= P(-\infty < x < b)$
$= P\left(\dfrac{-\infty-\mu}{\sigma} < \dfrac{x-\mu}{\sigma} < \dfrac{b-\mu}{\sigma}\right)$
$= P\left(-\infty < z < \dfrac{b-\mu}{\sigma}\right)$
$= F\left(\dfrac{b-\mu}{\sigma}\right)$



(iii) $P(x > a)$
$= P(a < x < \infty)$
$= P\left(\dfrac{a-\mu}{\sigma} < \dfrac{x-\mu}{\sigma} < \dfrac{\infty-\mu}{\sigma}\right)$
$= P\left(\dfrac{a-\mu}{\sigma} < z < \infty\right)$
$= F(\infty) - F\left(\dfrac{a-\mu}{\sigma}\right)$
$= 1 - F\left(\dfrac{a-\mu}{\sigma}\right)$

F(sth)-এর মান আমরা
page 787-795-এর মধ্যে পাবো
আর না পাইলে like $F(-4) = 0$
$F(4) = 1$

follows

$x \sim$ any distribution & the sample size is large $(n \geq 30)$, population mean or expected value of $x = E(x)$ & population variance $= V(x)$

$\therefore$ Average value of the random variable,

$$\bar{x} \sim N\left(E(x), \frac{V(x)}{n}\right) \equiv \bar{x} \sim N(E(\bar{x}), V(\bar{x}))$$

অর্থাৎ যে distribution-ই হোক x, কিন্তু $\bar{x}$ normal distribution follow করবে।

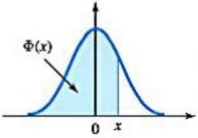
~~Exp~~

if, $\bar{x} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$ * $\boxed{E(cx) = cE(x)}$ * $\boxed{V(cx) = c^2 V(x)}$

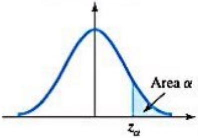
$\therefore$ Expectation, $\boxed{E(\bar{x}) = \mu \quad \therefore E\left(\sum_{i=1}^{n} x_i\right) = n\mu}$ Variance, $\boxed{V(x) = \frac{\sigma^2}{n} \quad \therefore V\left(\sum_{i=1}^{n} x_i\right) = \frac{n^2\sigma^2}{n} = n\sigma^2}$

$$\boxed{\begin{array}{c} \bar{x} \sim N\left(\mu, \frac{\sigma^2}{n}\right) \\ \hline \sum_{i=1}^{n} \bar{x}_i \sim N(n\mu, n\sigma^2) \end{array}}$$

Table I: Cumulative Distribution Function of the Standard Normal Distribution



Critical Points



α	z_α
0.10	1.282
0.05	1.645
0.025	1.960
0.01	2.326
0.005	2.576

x	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
−3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
−3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
−3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
−3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
−3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
−2.9	0.0019	0.0018	0.0017	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
−2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
−2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
−2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
−2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
−2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0061
−2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
−2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
−2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
−2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
−1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
−1.8	0.0359	0.0352	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
−1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
−1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
−1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
−1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0722	0.0708	0.0694	0.0681
−1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
−1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
−1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
−1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
−0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
−0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
−0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
−0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
−0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
−0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
−0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
−0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
−0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
−0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

(Continued on next page)

x	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5339
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9019	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9278	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9610	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998