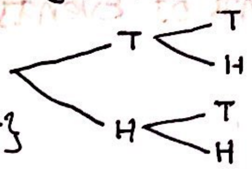
Lecture-1

1. **Sample space:** The sample space (S) of an experiment is a set consisting of all of the possible experimental outcomes.

*1 A usual six-sides dice has a sample space $S = \{1,2,3,4,5,6\}$

*2 toss a coin, the $S = \{H, T\}$

*3 a coin is tossed two times? → $S = \{HH, HT, TH, TT\}$



• If we throw 2 dice

	1	2	3	4	5	6
1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
2	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
3	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
4						(4,6)
5						(5,6)
6	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

event B

total → 36

$S = \{(1,1), (1,2), \ldots, (6,6)\}$

2. **Probablity Values:** Each outcome in the sample space has a probab-

*1 Probablity Value = $\dfrac{\text{the number of favourable outcomes}}{\text{The total number of possible outcomes}}$

*2 Possible out come of a coin toss are $\{H, T\}$. $\therefore P(H) = \frac{1}{2}$, $P(S) = \frac{1}{2}$.

* $S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$.

[?] If I toss a coin for 3 times, then calculate the number of probablity of exactly one head. → $\frac{3}{8}$

[?] atleast one head বললে → $\frac{7}{8}$

*3 If all the outcomes of a sample space have the same chance of occurance then probablity value = $\dfrac{1}{\text{the total number of possible outcomes}}$

* A fair dice have six (~~ছয়~~) (n) outcomes and each of the six outcomes must have a probablity of 1/6, i.e. $P(1) = P(2) = P(3) = P(4) = \cdots = \frac{1}{6}$

* 7 আসার probablity কত? → 0 [Impossible event-এর ক্ষেত্রে 0 হবে]

* 5 টিপ/ঢোল ফেলাইলে 6 আসার সম্ভাবনা কত? → 1.
[sure event-এর ক্ষেত্রে probablity value 1 আসবে]

*4 একটি red & blue dice throw করলে ।।. Atleast 1টি 6 পাওয়ার Possibility?
~~1টি 6~~ $\frac{11}{36}$. event B ≈ 11

*5. properties of Probablity Value : (1) $0 \leq P \leq 1$;
(2) $\sum P = 1$.

3. Events: An event is a subset of the sample space.

Ex: $S = \{1, 3, 4\}$ and $A = \{4, 1\}$. since A is a subset of the sample space S. so A in an event

$A = \{5, 1\}$ হলে event হইতো না।

4. Complements of Events: The complement of an event A, is the event consisting of everything in the sample space that is not contained within the event A.

* Dice $S = \{1, 2, 3, 4, 5, 6\}$, if event $A = \{1, 3, 6\}$ ~~then the compl~~ the complementary of event A $(A^c) = \{2, 4, 5\}$

5. Intersections of Events: contains common outcomes between two events

* $A = \{1, 3, 4\}$, $B = \{4\}$

$\therefore A \cap B = \{4\}$

6. Mutually Exclusive Events: Two events A & B are said to be mutu exclusive if they have no outcomes in com

(কোনো common outcome থাকবে না).

* Ex: $A = \{1, 3, 4\}$, $B = \{5\}$

7. Unions of Events: Consists of the outcomes that are contained within at least one of the events A&B

$A = \{1, 3, 4\}$ & $B = \{5\}$

$A \cup B = \{1, 3, 4, 5\}$

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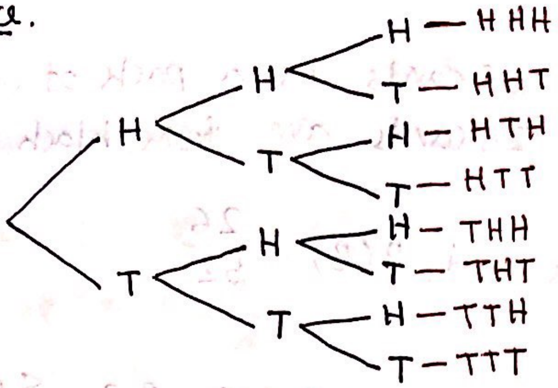
Assignment-1.

1. Sample Space.

Classwork: (1)



$S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$

(2) prime numbers less than 15 are: 2, 3, 5, 7, 11, 13

$S = \{2, 3, 5, 7, 11, 13\}$

(3) There might be no female,
only one female or
n female (All are female).

So, $S = \{0 \text{ female}, 1 \text{ female}, \dots, n \text{ female}\}$

(4) We know a pack of card contains 4 ace o
one might have no aces as well as might have 4 aces.

So, $S = \{0A, 1A, 2A, 3A, 4A\}$.

(5) One's birthday can be ~~from~~ 1st January to 31st December.

~~S = { 1st Jan, 2nd Jan, 29th Fe~~

$S = \{1^{st} \text{Jan}, 2^{nd} \text{Jan}, \dots, 29^{th} \text{Feb}, \dots, 31^{st} \text{Dec}\}$

(6) Let if a car repair on time noted by T, if late, noted by L, and if a car repaired is satisfactory denoted by S and if not denoted by U.

$\therefore S = \{TS, TU, LS, LU\}$.

2. Probablity Values:

<u>classwork: 1)</u> There are, 52 cards in a pack of cards.
26 cards are from black suits.

$\therefore$ Probablity Value of Black card, $P(B) = \frac{26}{52}$

2.2. Properties of Pr. Values: 1. $0 \leq P \leq 1$ & 2. $\Sigma P = 1$.

<u>classwork: 1)</u> $S = \{I, II, III, IV, V\}$

If $P(I) = 0.13$, $P(II) = 0.24$, $P(III) = \cancel{0.38}\ 0.07$.
$P(IV) = 0.38$.

$\therefore P(I) + P(II) + P(III) + P(IV) + P(V) = 1$. $[\because \Sigma P = 1]$

$\Rightarrow 0.13 + 0.24 + 0.07 + 0.38 + P(V) = 1$.

$\Rightarrow P(V) = 1 - 0.82$
$= 0.18$. Answer

2) $S = \{I, II, III, IV, V\}$

Given $P(I) = 0.08$, $P(II) = 0.20$ & $P(III) = 0.33$.

$\therefore P(I) + P(II) + P(III) + P(IV) + P(V) = 1$.

$\Rightarrow P(IV) + P(V) = 1 - 0.08 - 0.20 - 0.33$

$\Rightarrow P(IV) + P(V) = 0.39$.

$\therefore P(V) = 0.39 - P(IV)$.

So, the possible valeee for V are, $0 \leq P(V) \leq 0.39$. Answer

If outcomes IV and V are equally likely, then their probablity values are $\frac{39}{2} = 0.195$.

$\therefore P(V) = 0.195$ and $P(IV) = 0.195$.

6. Mutually Exclusive Events:

Classwork: (a)

We know, $A = \{13 ♡, 13 ◇\}$

$B = \{13 ♣, 13 ♠\}$

A and B are mutually exclusive events. Because they have no common events. Answer

(b) Here, $A = \{♡A, ◇A, ♣A, ♠A\}$

$B = \{13 ♡\}$

in the B set there is a ♡A.

So these two events A and B are not mutually exclusive events.

$$\boxed{P(A) = \frac{n}{N}}$$ Individual probablity.

Conditional Probablity: Conditional Probablity is the probablity of an event given that another event has already occurred. Conditional probablity is denoted by $\boxed{P(A|B) = \frac{P(A\cap B)}{P(B)}}$ [If one event is known what is the pro.. of happening another event? B is known what is the pr.. A? B জানা (known).]

Example: 3 Power Plants, plant X, plant Y, & plant Z. যদি electricity generate হয় (1) or not generating electricity (0). (0,1,0) → X & Z not generating & Y generating ele..

Let A be the event that plant X is not generating electricity and
B be the event that at least two out of the three plants are generating ele..

S

At most-1 count হবে

A:
(0,0,0) 0.07
(0,0,1) 0.04
(0,1,0) 0.03
(0,1,1) 0.18

(1,0,0) 0.16

B:
(1,0,1) 0.18
(1,1,0) 0.21
(1,1,1) 0.13

~~Atmost~~ At least -1 count হবে

What is the probablity of happening the event A?
Ans: $P(A) = \frac{n}{N} = \Sigma P(A) = 0.32$.

If event B is known what is the probablity of happening the event A?

Sol^n: We know, $P(A|B) = \frac{P(A\cap B)}{P(B)}$

$P(A\cap B) = P(0,1,1) = 0.18$

$P(B) = 0.18 + 0.21 + 0.13 + 0.18 = 0.70$

$\therefore P(A|B) = \cancel{\frac{0.70}{0.18}}\ \frac{0.18}{0.70} = 0.257$

$\therefore P(A) = 0.07 + 0.04 + 0.03 + 0.18 = 0.32$.

Example: A card is drawn at random from a pack of cards. Calculate.

(a) P(King | Card from red suit) = ? Clubs/diamonds/hearts/spades.

~~A = {K clubs, K diamonds, K hearts}~~ A = {K clubs, K diamonds, K hearts, K spades}

B = {13 diamonds, 13 hearts}

$P(A|B) = \frac{P(A\cap B)}{P(B)} = \frac{2/52}{26/52} = \frac{2}{26} = \frac{1}{13}$

(b) P(king | red picture card)?

A = {K clubs, K diamonds, K hearts, K spades}

B = {Jd, Jh, Qd, Qc, Kd, Kh}

$P(A|B) = \frac{P(A\cap B)}{P(B)} = \frac{2/52}{6/52} = \frac{1}{3}$

Classwork: P(heart | card from black suit)?

A = {13 heart}, B = {13 c, 13 s} $\therefore A\cap B = \{\}$

$\therefore P(A|B) = \frac{P(A\cap B)}{P(B)} = \frac{0/52}{26/52} = \cancel{0/26} = 0$

Question: A car repair is either on time or late and either satisfactory or unsatisfactory. If a repair is made on time, then there is a probablity of 0.85 that it is satisfa... There is a probablity of 0.77 that a repair will be made on time. ~~What i~~ what is the probablity that a repair is made on time and is satisfactory?

Soln: Let If a car repairment is satisfactory then it is denoted by S,
If a car " is unsatisfactory " " " " " S',
If ... is made on time " " " " " T,
If ... is made on late " " " " " T'.

For time-এ repair হলে Satisfactory হওয়ার Pr.. $0.85 = P(S|T)$
For time-এ repair হওয়ার Pro... $0.77 = P(T)$

$P(S|T) = 0.85$, $P(T) = 0.77$ & $P(S\cap T) = ?$

We know, $\boxed{P(S|T) = \dfrac{P(S\cap T)}{P(T)}}$

যার Probablity বের করতে হবে তাকে আগে বসাবো।

$\therefore P(S\cap T) = P(S|T) * P(T) = 0.85 * 0.77 = 0.6545$

Question: Three types of batteries are being tested, type I, type II, and type III. The outcome (I, II, III) denotes that the battery of type I fails first, battery of type II next, and the battery of type III lasts the longest.

(I, II, III) 0.11	(I, III, II) 0.07
(II, I, III) 0.24	(II, III, I) 0.39
(III, I, II) 0.16	(III, II, I) 0.03

S

Calculate the probablities

(a) A type I battery lasts longest conditional on a type II battery failing first. (I|II)

(b) A type I battery lasts longest conditional on a type II battery lasting the longest.

Soln: Let A denotes type I battery lasts longest and B denotes type II battery failing first.

$A = \{(II, III, I), (III, II, I)\}$

$B = \{(II, I, III), (II, III, I)\}$

we know, $P(A|B) = P(A\cap B)/P(B)$

Here, $A\cap B = $ II, III I.

$P(A\cap B) = 0.39$ and $P(B) = 0.24 + 0.39 = 0.63$

So, $P(A|B) = 0.39/0.63 = 0.62$

Classwork: A type I battery lasts longest conditional on a type II battery not failing first

(b) $A = \{(II, III, I), (III, II, I)\}$ $B = \{(I, III, II), (III, I, II)\}$

$\therefore A\cap B = \{\} = \phi$

$\therefore P(A|B) = \dfrac{0}{0.7+0.16} = 0$

$A = \{(II, III, I), (III, II, I)\}$

$B = \{(III, II, I), (I, II, III), (III, I, II), (I, III, II)\}$

A | conditional on ≡ given that | B

$\therefore A\cap B = \{(III, II, I)\}$ $\therefore P(A\cap B) = 0.03$

$P(B) = (0.03 + 0.11 + 0.16 + 0.07) = 0.37$

$\therefore P(A|B) = \dfrac{0.03}{0.37} = 0.08$

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Assignment-1

1) red denoted by r, blue by b and dull denoted by d and shiny by s.
$S = \{(r,s),(r,d),(b,s),(b,d)\}$ Answer

2) $A = \{\heartsuit A, \diamondsuit A, \clubsuit A, \spadesuit A\}$
$n = 4, \; N = 52.$
$\therefore P(A) = \frac{4}{52}$ Answer

3) $P(A) = 0.5$
$P(A \cap B) = 0.1$
$P(A \cup B) = 0.8$
$P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$\Rightarrow P(B) = P(A \cup B) + P(A \cap B) - P(A)$
$= 0.8 + 0.1 - 0.5$
$\therefore P(B) = 0.4.$ Answer

4) $P(I) = 2 \times P(II)$ ..... (i)
$P(II) = 3 \times P(III)$ ..... (ii)
$P(I) = 2 \times 3 \times P(III)$
$= 6 \times P(III)$
$\Sigma P = 1.$
$\Rightarrow P(I) + P(II) + P(III) = 1$
$\Rightarrow 6 \times P(III) + 3 \times P(III) + P(III) = 1$
$\Rightarrow P(III) = \frac{1}{10}$
$P(I) = 6 \times \frac{1}{10} = \frac{6}{10}$
$P(II) = 3 \times \left(\frac{1}{10}\right) = \frac{3}{10}$ Answer

5] $\Sigma P = 1$

$\therefore P = 1 - 0.28 - 0.55$

$= 0.17$ Answer

6] $A = \{c, d\}$ ~~0.48~~, $P(A) = 0.48 + 0.2 = 0.50$

$A' = \{a, b, e\}$, $P(A') = 0.13 + 0.22 + P(b)$.

(a) $P(A) + P(A') = 1$

$\Rightarrow 0.50 + 0.35 + P(b) = 1$

$\therefore P(b) = 1 - 0.85 = 0.15$ Answer

(b) $P(A) = 0.50$ Answer

(c) $P(A') = 1 - 0.50$

$= 0.50$ Answer

7] $P(A) = 0.27. = P(b) + P(c) + P(e)$

(a) $P(b) = 0.27 - 0.11 - 0.06$

$= 0.10$ Answer

(b) $P(A') = 1 - P(A) = 1 - 0.27$

$= 0.73.$ Answer

(c) $P(A') = P(a) + P(f) + P(d) = 0.73$

$\Rightarrow P(d) = 0.73 - 0.09 - 0.29$

$= 0.35$ Answer

8| ~~(a) A = {(III, I, II), (~~

(a) $A = \{(\text{III}, \text{II}, \text{I}), (\text{II}, \text{III}, \text{I})\}$

$P(A) = 0.03 + 0.39$

$= 0.42$ Answer

(b) ~~shorte~~.

Last shortest means type I battery fails first.

$A = \{(\text{I}, \text{II}, \text{III}), (\text{I}, \text{III}, \text{II})\}$

$P(A) = 0.11 + 0.07$

$= 0.18$ Answer

(c)

type I does not last long,

$\therefore A = \{(\text{III}, \text{I}, \text{II}), (\text{I}, \text{III}, \text{II}), (\text{II}, \text{I}, \text{III}), (\text{I}, \text{II}, \text{III})\}$

$P(A) = 0.16 + 0.07 + 0.24 + 0.11$

$= 0.58$. Answer

(d)

type I battery lasts longer than the type II battery. $\to$ II, I.

Let, $A = \{(\text{III}, \text{II}, \text{I}), (\text{II}, \text{I}, \text{III}), (\text{II}, \text{III}, \text{I})\}$

$P(A) = 0.03 + 0.24 + 0.39$

$= 0.66$ Answer

9) 1

(a) both assembly lines are shut down.

$\therefore A = \{(S,S)\}$

$P(A) = 0.02$ Answer

(b) neither assembly line is shut down.

$\therefore A = \{(P,P),(P,F),(F,P),(F,F)\}$

$P(A) = 0.14 + 0.20 + 0.21 + 0.19$

$= 0.74$ Answer

(c) at least one assembly line is at full capacity?.

$A = \{(F,S),(F,P),(F,F),(P,F),(S,F)\}$

$P(A) = 0.06 + 0.21 + 0.19 + 0.20 + 0.05$

$= 0.71$ Answer

(d) Exactly one assembly line is at full capacity?.

$A = \{(F,S),(F,P),(P,F),(S,F)\}$

$P(A) = 0.06 + 0.21 + 0.20 + 0.05$

$= 0.52$ Answer

If A is the ~~set~~ events of neither assembly line is shut down, so

$A = \{(P,P),(P,F),(F,P),(F,F)\}$

So the complement of A, $A^c = \{(S,S),(S,P),(S,F),(P,S),(F,S)\}$

Answer

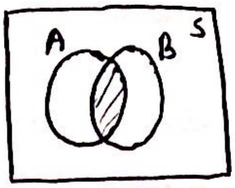
If B is the events set of at least one assembly line is at full capacity

$B = \{(F,S),(F,P),(F,F),(P,F),(S,F)\}$

So the complement of B, $B^c = \{(S,S),(S,P),(P,S),(P,P)\}$

Answer

10) $P(A) = 0.4$
$P(A \cap B) = 0.3$



We know, $P(A \cup B) \leq 1$

$\Rightarrow P(A) + P(B) - P(A \cap B) \leq 1$

$\Rightarrow 0.4 + P(B) - 0.3 \leq 1$

$\Rightarrow P(B) + 0.1 \leq 1$.

~~$\Rightarrow -0.9 \leq P(B) \leq 1$~~

~~$\Rightarrow -0.9 \leq P(B) \leq$~~

$\Rightarrow \cancel{-1.10} \leq P(B) \leq 0.9$ ..... (I)

But $P(B) \geq P(A \cap B)$

$\therefore P(B) \geq 0.3$ ..... (II)

From (I) and (II), we get $0.3 \leq P(B) \leq 0.09$

Answer

11) $A = \{13 \heartsuit\}$, $P(A) = \frac{13}{52} = \frac{1}{4}$

$B = \{13 \clubsuit\}$, $P(B) = 13/52$

$C = \{13 \diamondsuit\}$, $P(C) = 13/52$

Yes, these three events mutually exclusive. Because they have no common cards.

Answer

$P(A \cup B \cup C) = \frac{13}{52} + \frac{1}{4} + \frac{1}{4}$

$= \frac{3}{4}$ Answer

If $A = \{13 \heartsuit\}$, then $A' = \{13 \diamondsuit, 13 \clubsuit, 13 \spadesuit\}$.

and $B = \{13 \clubsuit\}$

So, B is a subset of A' or $B \subset A'$ Answer

12) ~~shiny ball, s = 55~~

shiny red ball, ~~(s, r)~~ (r, s) ⇒ (r ∩ s) = 55

shiny ball, s = 91

red ball, r = 79

$\therefore P(r \cap s) = \frac{55}{200}$, $P(s) = \frac{91}{200}$, $P(r) = \frac{79}{200}$

probablity that it is either a shiny ball or a red ball,

$\cancel{P(r \cup s) =}$ $P(s \cup r) = P(s) + P(r) - P(r \cap s)$

$= \frac{91}{200} + \frac{79}{200} - \frac{55}{200}$

$= \frac{115}{200}$ Answer

13) There are 150 balls.

$r \cap s = 36 \quad \therefore P(r \cap s) = \frac{36}{150}$

$b = 54. \quad \therefore P(b) = \frac{54}{150}$

$\therefore r = 150 - 54 = 96 \quad \therefore P(r) = \frac{96}{150}$

Probablity of the chosen ball being shiny conditional on it being red,

$P(A|B) = \frac{P(A \cap B)}{P(B}$

$P(s|r) = \frac{P(r \cap s)}{P(r)} = \frac{36/150}{96/150} = \frac{36}{96} = \frac{6}{16} = \frac{3}{8}$ Answer

=

Lecture-3

RANDOM VARIABLES

Random variable:

A random variable is obtained by assigning a numerical value to each outcome of a particlelar experiment.

Random variable	Possible Values	Random Events
$X = \begin{cases} 0 \\ 1 \end{cases}$	$0 \leftarrow$	Head
	$1 \leftarrow$	Tail

so:

- We have an experiment (such as tossing coin)
- We give values to each event.
- The set of values is a Random Variable.

Types of a Random Variables:

Random Variables can be either ~~de~~ Discrete or Continuous:

$\boxed{\text{Descrete}}$ random var can ~~tad~~ only take certain values (such as 0,1)

$\boxed{\text{Continuous}}$ random variable can ~~only~~ any value within a range (ex:

A company ~~manifac~~' manufactures metal cylinders that are used in the construction of a particlelar type of engine. The company discovers that the cylinders it manufactures can have a diameter anywhere between 49.5 and 50.5mm. Suppose that the random variable X is the diameter of a randomly chosen cylinder manufactured by the company. Since this random variable can take any value between 49.5 and 50.5, it is a continuous random variable.)

Discrete → $\boxed{X: 0, 1}$ | Continuous : $\boxed{49.5 \leq X \leq 50.5}$

ছক্কায় জেনে 1, 2, ..., 6 বাদে কিছু আসার chance নাই

কোনো Range-এর মধ্যে। money bag-1 20 - 50tk আছে।

Probablity mass function (PMF):

PMF is a function of a discrete random variable that gives the probablity that a discrete random variable is exactly equal to a value.

Note that by definition the PMF is a probablity measure, so it satisfies all properties of a prabablity. i.e The properties of PMF:

(i) $0 \leq P_i \leq 1$ (ii) $\sum P_i = 1$.

একটা কিছু possibility 0, 1 আসবে

PMF এ exact কিছু বলতে 0 আসবে

বোঝানো হচ্ছে & 1 আসবে না। must be descrete.

Example: I toss a coin twice. Let X be the number of heads. Find the PMF of X.

Soln: Here the random variable X be the number of heads.

Outcomes: HH, HT, TH, TT.

$S = \{HH, HT, TH, TT\}$

So, the value of the $X = \{0, 1, 2\}$.

Here, $P(HH) = P(HT) = P(TH) = P(TT) = \frac{1}{4}$

PMF is given by $P(0) = P(TT) = \frac{1}{4}$

$P(1) = P(HT) + P(TH) = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$

$P(2) = \frac{1}{4}$ Answer

Note: I toss a coin twice. Let X be the number of heads. Find the PMF of X

soln: $X : \{0,1\}$

$S : \{H, T\}$

$P(H) = 1/2$, $P(T) = 1/2$

$P(0) = P(T) = 1/2$

$P(1) = P(H) = 1/2$

$X : \{1,2,3,4,5,6\}$ (the score shown ~~up~~ on top face)

$S : \{1,2,3,4,5,6\}$

$P(1) = P(2) = P(3) = P(4) = P(5) = P(6) = 1/6$

PMF:

$P(1) = P(2) = P(3) = P(4) = P(5) = P(6) = 1/6$

*** If you throw a dice twice. Let X be the number of 6 comes on the t[wo] dice. Find the Probablity mass function of X.

	1	2	3	4	5	6
1	11	12	13	14	15	16
2	21	22	23	24	25	26
3	31	32	33	34	35	36
4	41	42	43	44	45	46
5	51	52	53	54	55	56
6	61	62	63	64	65	66

25

$X : \{0,1,2\}$

PMF:

$P(0) = \frac{25}{36}$ (the probablity that no 6 comes on the two dices)

$P(1) = 10/36$ (. . . . . . one . . . .)

$P(2) = 1/36$ (. . . . . . two . . . .)

10

1

$P(0) + P(1) + P(2) = 1.$

একটা case দিয়ে ask করলে তাকে random variable descrete/continuous

Cumulative distribution function (CDF):

CDF is a function that starts at a small value of x (numerical value of the random variable X) and increases to a large values of x.

Ex: I toss a coin twice. Let X be the number of $\boxed{\text{heads}}$. Find the CDF of X.

Soln: $X : \{0, 1, 2\}$ (from previous example)

PMF is given by $P(0) = P(TT) = 1/4$

$P(1) = P(HT) + P(TH)$

$= 1/4 + 1/4 = 1/2$

$P(2) = P(HH)$

$= 1/4.$

$F(0) = P(0) = 1/4$ (from previous solution) (probablity that no head occur)

$F(1) = P(0) + P(1) = 1/4 + 1/2 = \frac{3}{4}$ (probablity that not more 1 head ")

$F(2) = P(0) + P(1) + P(2) = 1$ (probablity that no more than two head ")

(0,0,0) 0.07	(1,0,0) 0.16
(0,0,1) 0.04	(1,0,1) 0.18
(0,1,0) 0.03	(1,1,0) 0.21
(0,1,1) 0.18	(1,1,1) 0.13

Question: A massenger supervises the operation of three power plants, plant X, plant Y & plant Z. At any given time, each of the three " can be classified as either generating electricity (1) or being idle (0). With the notation (0,1,0) used to represent the situation where plant Y is generating electricity but plants X and Z are not generating electricity. Here random var X be the number of plants generating electricity.

(1) Calculate the probablity mass func, (2) Draw PMF (3) Draw CDF.

Soln:

Here, random variable X be the number of plants generating electricity. $X : \{0, 1, 2, 3\}$

(1) $P(0) = 0.07$ (no....)

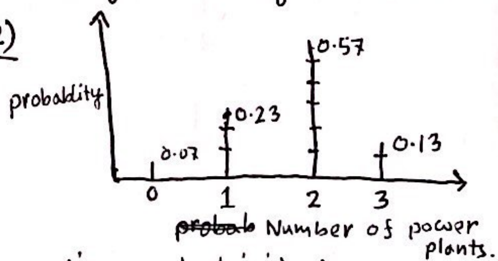
$P(1) = 0.23$ (one.....)

$P(2) = 0.57$ (two....)

$P(3) = 0.13$ (three....)

x_i	0	1	2	3
p_i	0.07	0.33	0.57	0.13

(2)

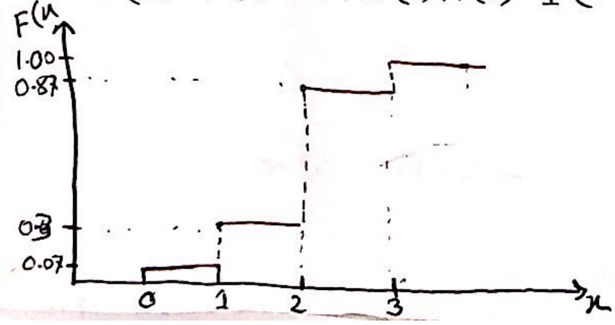


(3) $F(0) = P(0) = 0.07$ (Probablity that no plants are generating electricity)

$F(1) = P(0) + P(1) = 0.03$ (" " " more than one plant is " ")

$F(2) = P(0) + P(1) + P(2) = 0.87$ (" " " " " two plants are " ")

$F(3) = P(0) + P(1) + P(2) + P(3) = 1$ (" " " " " three " " " ")



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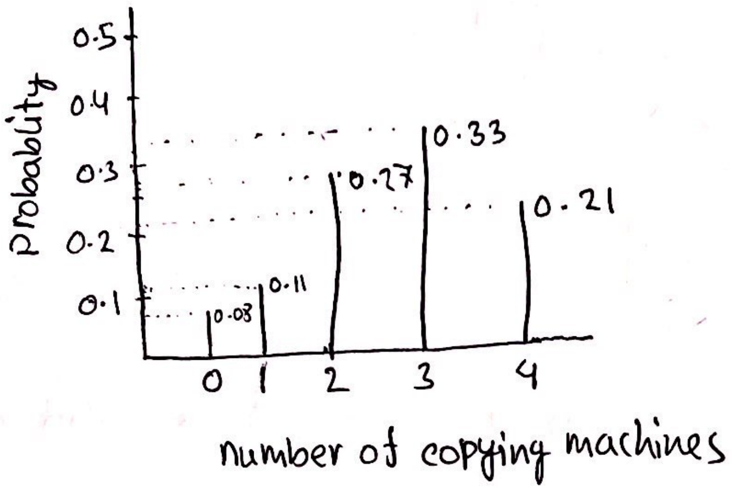
(a) $P(X=4) = 1 - P(x=0) + P(x=1) + P(X=2) + P(X=3)$

$= 1 - 0.08 - 0.11 - 0.27 - 0.33$

$= 1 - 0.79$

$= 0.21$ Answer

(b)



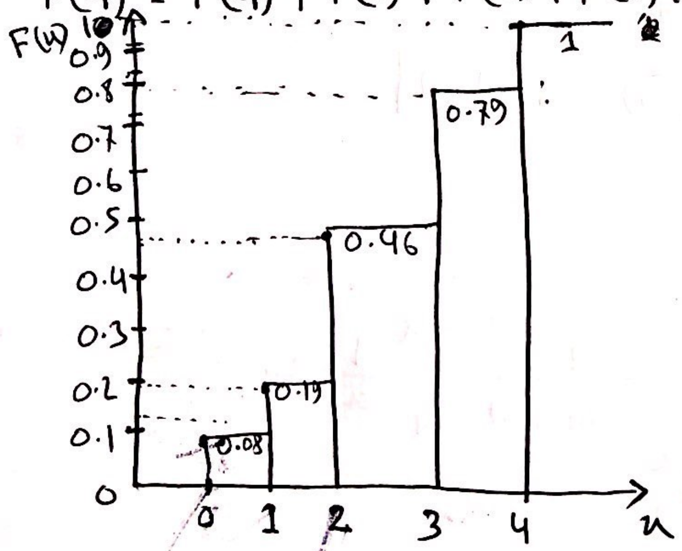
(c) $F(0) = P(0) = 0.08$

$F(1) = P(1) + P(0) = 0.08 + 0.11 = 0.19$

$F(2) = P(2) + P(1) + P(0) = 0.19 + 0.27 = 0.46$

$F(3) = P(3) + F(2) = 0.46 + 0.33 = 0.79$

$F(4) = P(4) + P(3) + P(2) + P(1) + P(0) = 0.21 + 0.79 = 1.$



Probablity Density Function: (within a range)

P.D.F. is a function of a continuous random variable. The probablity that the random variable lies between two values a & b is obtained by integrating the probablity density fun^c (PDF) $f(x)$ must integrate to one over the whole sample space, so that the total probablity is equal to 1.

(i) $f(x) \geq 0$
(ii) $a < x < b$, $\int_a^b f(x)dx = 1$

Question: Suppose that the diameter of a metal cylinder has a probablity density fun^c $\boxed{f(x) = 1.5 - 6(x-5.0)^2}$ for $\boxed{49.5 \leq x \leq 50.5}$.

(i) Show that total area under the probablity density function = 1 or
(ii) prove that this is a valid PDF.

Sol^n(i): $\int_{49.5}^{50.5} 1.5 - 6(x-50.0)^2 dx$

$= \left[1.5x - \frac{6(x-50.0)^3}{3}\right]_{49.5}^{50.5}$ $\left[\int c\,dx = cx, \int (x-a)^n dx = \frac{(x-a)^{n+1}}{n+1}\right.$

$= [1.5*50.5 - 2(50.5-50)^3 - [1.5*49.5 - 2(49.5-50)^3]$

$= 75.5 - 74.5 = 1$

That is 100% of the cylinders will have diameters within these limits.

Since over the whole sample space, the integration of $f(x) = 1$, so this is a valid PDF.

(ii) Calculate The Probablity that a metal cylinder has a diameter between 49.8 & 50.1 mm.

Sol^n: $\int_{49.8}^{50.1} 1.5 - 6(x-50.0)^2 dx$.

$= \left[1.5x - \frac{6(x-50)^3}{3}\right]_{49.8}^{50.1}$ $\left[\text{As}, \int c\,dx = cx, \int (x-a)^n dx = \frac{(x-a)^{n+1}}{n+1}\right]$

$= [1.5*50.1 - 2(50.1-50)^3] - [1.5*49.8 - 2(49.8-50)^3]$

$= 75.148 - 74.716$

$= 0.43$

That is, 43% of the cylinders will have diameters within these limits.

cumulative distribution fun^c starts from the lowest value of x (given) and increase to a ~~highest~~ certain value of x. (not given)

(iii) Find cumulative distribution function.

Sol^n: $F(x) = \int_{49.5}^{x} 1.5 - 6(x-50.0)^2 dx$

$= [1.5x - 2(x-50)^3]_{49.5}^{x}$

$= [1.5x - 2(x-50)^3] - [1.5*49.5 - 2(49.5-50)^3]$

Cumulative D. Fun^c; $F(x) = 1.5x - 2(x-50)^3 - 74.5$

$F(49.8) = 1.5 \times 49.8 - 2(49.8-50)^3 - 74.5 =$

(1.5)

Question: Let $f(x) = A(0.5 - (x-0.25)^2)$ for $0.125 \le x \le 0.5$

(?) fun^c-এর মধ্যে মান থাকলে আগে validity check কর।

(?) আর কিছু unknown থাকলে তা বের করতে বলা হবে।

(a) Find the value of A.

Sol^n: $\int_{0.125}^{0.5} A(0.5 - (x-0.25)^2)\,dx = 1.$

$$A\left[0.5x - \frac{(x-0.25)^3}{3}\right]_{0.125}^{0.5} = 1$$

$$\Rightarrow A\left[0.5x - \frac{(0.125-0.125)^3}{3}\right]\Big] = 1$$

$$\Rightarrow A\left[\left[0.5*0.5 - \frac{(0.5-0.25)^3}{3}\right] - \left[0.5*0.125 - \frac{(0.125-0.25)^3}{3}\right]\right]$$

$$\Rightarrow A\left[\left[\frac{1}{4} - \frac{1}{192}\right] - \left[\frac{1}{16} + \cancel{\frac{9}{512}}\ \frac{1}{1536}\right]\right] = 1$$

$$\Rightarrow A\left[\frac{47}{192} - \frac{97}{1536}\right] = 1 \Rightarrow A \cdot \frac{93}{512} = 1 \Rightarrow \boxed{A = \frac{512}{93}} \text{ Answer}$$

(b) Construct the cumulative distribution function. (cdf/CDF)

Sol^n: $F(x) = \int_{0.125}^{x} \frac{512}{93}(0.5 - (x-0.25)^2)\,dx$

$$= \frac{512}{93}\left[0.5x - \frac{(x-0.25)^3}{3}\right]_{0.125}^{x}$$

$$= \frac{512}{93}\left[\left[0.5x - \frac{(x-0.25)^3}{3}\right] - \left[0.5*0.125 - \frac{(0.125-0.25)^3}{3}\right]\right]$$

$$= \frac{512}{93}\left[\left[0.5x - \frac{(x-0.25)^3}{3}\right] - \frac{97}{1536}\right] \text{ Answer}$$

(c) What is the probablity that the paint thickness at a particular point is less than 0.2 mm?

$P(X<0.2)$

এটা বলা মানে $\int^{\boxed{0.2}}$

Sol^n: $\int_{0.125}^{0.2} \frac{512}{93}(0.5 - (x-0.25)^2)\,dx$

$$= \frac{512}{93}\left[\left[0.5x - \frac{(x-0.25)^3}{3}\right] - \cancel{\frac{97}{1536}}\right]_{0.125}^{0.2}$$

$$= \frac{512}{93}\left[\left[0.5*0.2 - \frac{(0.2-0.25)^3}{3}\right] - \frac{97}{1536}\right] = 0.203 \text{ Answer}$$

lower limit বলা না থাকলে সবসময় PDF-এর lower limit-ই হবে।

Continuous-এর ক্ষেত্রে,

Descrete হলে, P(0) & P(1) বের করতে হতো।

$$\cancel{\int_{0.125}^{0.5} x\,\frac{512}{93}\left[0.5 - (x-0.25)^2\right]dx}$$

$$= \frac{512}{93} \cdot \int_{0.125}^{0.5} x\left[0.5 - (x-0.25)^2\right]dx$$

PDF দিবে CDF বের করতে হবে।

আবার value-ও দিতে পারে।

Exception of a Random Variable: Expectation of a Random Variable is denoted by E(x) and represents an 'average' value of the random var.

For $\boxed{\text{discrete}}$ random variable, $\boxed{E(x) = \sum x\, p(x)}$ → PMF → values of the random variable

For $\boxed{\text{continuous}}$ random variable, $\boxed{E(x) = \int_{l}^{u} x f(x)\, dx}$

Example:

A manager supervises the operation of three power plants, plant X, Y & Z. At any given time, each of the three plants can be classified as either generating electricity (1) or being idle (0). With the notation (0,1,0) used to represent the situation where plant Y is generating electricity but plants X & Z are not generating electricity. Here, random variable X be the number of plants ge... electricity.

$(0,0,0)_{0.07}$	$(1,0,0)_{0.16}$
$(0,0,1)_{0.04}$	$(1,0,1)_{0.18}$
$(0,1,0)_{0.03}$	$(1,1,0)_{0.21}$
$(0,1,1)_{0.18}$	$(1,1,1)_{0.13}$

1. Calculate the probablity mass fun
2. Draw PMF
3. ~~Cal~~ Calculate CDF.
4. Draw CDF.
5. Calculate the expected number of pow plant generating electricity.

Solution: Sample O. : $X = \{0,1,2,3\}$

$P(0) = 0.07$ (Probablity that no plants are generating ele
$P(1) = 0.16+0.04+0.03 = 0.23$ (" " one " is " "
$P(2) = 0.21+0.18+0.18 = 0.57$ (" " two " are " "
$P(3) = 0.13$ (" " three " " "

$\boxed{5}$ $E(x) = \sum x P(x)$ ~~0*0.07~~

→ Exception/ average.

$= 0 * P(0) + 1 * P(1) + 2 * P(2) + 3 * P(3)$

$= (0 * 0.07) + (1 * 0.23) + (2 * 0.57) + (3 * 0.13)$

$= 1.76$ Answer, on avg two plants are generating electricity.

Example: Suppose that the diameter of a metal cylinder has a probablity density func $\boxed{f(x) = 1.5 - 6(x - 50.0)^2}$ for $\boxed{49.5 \le x \le 50.5}$

(i) Show that total area under the probablity density func = 1 or prove that this is a valid P.D.F.

(ii) calculate the probablity that a metal cylinder has a diameter between 49.8 and 50.1 mm.

(iii) Find ~~the~~ cumulative distribution func.

(iv) calculated the expected diameter of a metal cylinder.

Soln: We know,

$$\boxed{E(x) = \int_L^U x f(x)\,dx}$$

$$= \int_{49.5}^{50.5} x(1.5 - 6(x-50.0)^2)\,dx$$

$$= \int_{49.5}^{50.5} x(1.5 - 6x^2 + 600x - 15000x)\,dx$$

$$= \int_{49.5}^{50.5} (1.5x - 6x^3 + 600x^2 - 15000x)\,dx$$

$$= \left[1.5\frac{x^2}{2} - 6\cdot\frac{x^4}{4} + 600\frac{x^3}{3} - 15000\frac{x^2}{2}\right]_{49.5}^{50.5}$$

$$= -3123100 - (-3123150)$$

$$= 50 \text{ Answer}$$

□ Variance: Variance measures how far a set of numbers are spread out from their average value,

avg point থেকে অন্যান্য point-এর দূরত্ব variance measure করে।

For discrete random variable,

$$\boxed{\begin{aligned} V(x) &= E(x^2) - (E(x))^2 \\ &= \Sigma x^2 p(x) - (\Sigma x p(x))^2 \end{aligned}}$$

for continuous random variable,

$$\boxed{\begin{aligned} V(x) &= E(x^2) - (E(x))^2 \\ &= \int_L^U x^2 f(x)\,dx - \left(\int_L^U x f(x)\,dx\right)^2 \end{aligned}}$$

□ Standard deviation: Standard deviation is the square root of the varience.

$$\boxed{SD(x) = \sqrt{V(x)}}$$

Example: A manager supervises the operation of three power plants...

: : : -

(1) Calculate the probablity mass func (2) Draw PMF (3) Calculate CDF. (4) Draw CDF (5) Calculate the expected number of power plant generating electricity. (6) Calculate variance and standard deviation. **

Soln: $P(0) = 0.07$ (probablity that no plants are generating electricty)

$P(1) = 0.23$ (...

$P(2) = 0.57$ (...

$P(3) = 0.13$ (...

$\boxed{E(x) = \Sigma x p(x)} = 0 * 0.07 + 1 * 0.23 + 2 * 0.57 + 3 * 0.13 = 1.76$

$\boxed{E(x^2) = \Sigma x^2 p(x)} = 0^2 * 0.07 + 1^2 * 0.23 + 2^2 * 0.57 + 3^2 * 0.13 = 3.68$

* $\boxed{V(x) = E(x^2) - (E(x))^2} = 3.68 - (1.76)^2 = 0.5824$

** $\boxed{SD(x) = \sqrt{V(x)}}$

$= 0.7632$

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Assignment 2

Classwork 1
given,
$f(x) = \frac{1}{\ }$

Example: Suppose that the diameter of a metal cylinder has a probab... density fun^c $f(x) = 1.5 - 6(x-50.0)^2$ for $49.5 \le x \le 50.5$

(i) Show that total area under the probablity density fun^c = 1 or prove that this is a va... po...
(ii) Calculate the probablity that a metal cylinder has a diameter between 49.8 and 50...
(iii) Find cumulative distribution fun^c.
(iv) Calculated the expeted diameter of a metal cylinder.
(V) Calculate variance and standard deviation.

Soln: We know, $V(x) = E(x^2) - (E(x))^2$

$E(x) = 50$ [from previous soln]

$$E(x^2) = \int_L^U x^2 f(x)\,dx$$

$$= \int_{49.5}^{50.5} x^2(1.5 - 6(x-50)^2)dx$$

$$= \int_{49.5}^{50.5} x^2(1.5 - 6(x^2 - 100x + 2500))dx$$

$$= \int_{49.5}^{50.5} x^2(1.5 - 6x^2 + 600x - 15000x^2)dx$$

$$= \int_{49.5}^{50.5} (1.5x^2 - 6x^4 + 100x^3 - 15000x^2)dx$$

$$= \left[1.5\frac{x^3}{3} - 6\frac{x^5}{5} + 600\frac{x^4}{4} - 15000\frac{x^3}{3}\right]_{49.5}^{50.5} = -6243624\cdot1 + 6243874\cdot1$$

$$= 2500.$$

$$= -6243\,6240.6 - (-6243740.65)$$

$$= 2500.05$$

$$V(x) = 2500 - 50^2$$

$$= 0$$

$$Sd(x) = \sqrt{V(x)} = \sqrt{0} = 0 \text{ Answer}$$

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Assignment 2

Classwork 1.

given,

$$f(x) = \frac{1}{x \ln(1.5)}$$

(i) $\int_4^6 \frac{1}{x\ln(1.5)} dx = \frac{1}{\ln(1.5)} \int_4^6 \frac{1}{x} dx$

$= \frac{1}{\ln(1.5)} \times [\ln(x)]_4^6$

$= \frac{1}{\ln 1.5} \times [\ln(6) - \ln(4)]$

$= 1.$ [using calculator]

Since, the integration of $f(x) = 1$, so this is a valid probablity density function.

(ii) Here, $P(4.5 \leq X \leq 5.5)$

$= \int_{4.5}^{5.5} \frac{1}{x\ln(1.5)} dx$

$= \frac{1}{\ln(1.5)} \times [\ln(x)]_{4.5}^{5.5}$

$= \frac{1}{\ln(1.5)} \times [\ln(5.5) - \ln(4.5)]$

$= 0.495$

Answer

(iii) $F(x) = \int_4^x \frac{1}{x\ln(1.5)} dx = \frac{1}{\ln(1.5)} \int_4^x \frac{1}{x} dx$ [for $4 \leq x \leq 6$]

$= \frac{1}{\ln(1.5)} [\ln(x)]_4^x$

$= \frac{1}{\ln(1.5)} [\ln x - \ln 4]$

Answer

Classwork 2

Given, $f(x) = \frac{512}{93}\left[0.5 - (x-0.25)^2\right]$ for $0.125 \leq x \leq 0.5$

expected or average paint thickness,

$$E(x) = \int_{0.125}^{0.5} x \, f(x) \, dx$$

$$= \int_{0.125}^{0.5} x \frac{512}{93}\left[0.5 - (x-0.25)^2\right] dx$$

$$= \frac{512}{93} \int_{0.125}^{0.5} \left[0.5x - x^3 + 0.5x^2 - 0.0625x\right] dx$$

$$= \frac{512}{93} \int_{0.125}^{0.5} \left[-x^3 + 0.5x^2 + 0.4375x\right] dx$$

$$= \frac{512}{93}\left[\left(-\frac{x^4}{4}\right) + \left(0.5 \cdot \frac{x^3}{3}\right) + \left(0.4375 \cdot \frac{x^2}{2}\right)\right]_{0.125}^{0.5}$$

$$= \frac{512}{93}\left[\frac{23}{384} - \frac{181}{49152}\right]$$

$$= \frac{512}{93} \times \frac{921}{16384}$$

$$= \frac{307}{992} = 0.309$$

$$= 0.309$$

Answer