Some Probablity Mass functions (Descrete random variable)

↳ random variable value are certain.

Bernoulli distribution:

a single ~~trail~~ trial is conducted which takes a binary outcome. The probablity mass fun^c is,

only one trial and there are two possible outcome.

$$\boxed{p(X=x) = p^{x}(1-p)^{1-x}}$$

$x = 0,1$

p indecates probablity
x " values of the r. var..

At this case ran... var... x is called berno distrilution.

\# single coin toss.
\# single dice roll and possibility of 6 come / even আসবে "

Expectation: $\boxed{E(x) = p}$

Variance: $\boxed{V(x) = p(1-p)}$

Example: if you toss a coin one times. For example, Random variable X indicates the no of head.

Binomial Distribution:

n trails (n>1) are conducted which takes a binary outcome. The probability mass fun^c is

n trials [n>1] &
2 possible outcome,
every trials are independent.

$$\boxed{p(X=x) = \binom{n}{x} p^{x}(1-p)^{n-x}}$$

$x = 0,1,2,\dots,n$

\# toss a coin for 5 times

Expectation: $\boxed{E(x) = np}$

Variance: $\boxed{V(x) = np(1-p)}$

Example: If you toss a coin five times. For example, Random variable X indicates the no of tail. $n=5$ $x=0,1,2,3,4,5$

\# Example: Suppose a milk factory ~~contains~~ has $\boxed{\text{20 containers}}$ (trial) and $\boxed{\text{there}}$ is a propablity of $\boxed{0.261}$ that a milk container is underweight.

(a) What is the distribution of the number of underweight containers in a box?

(b) Calculate expected? number of underweight cantons in a box and also calculate its variance.

(c) Calculate the probablity that a box cotains exactly seven underweight ~~contain exactly seven underweight containers~~ and also.

(d) Calculate the probablity that a box contains no more than three underweight containers.

(e) Calculate the probablity that a box contain at least two underweight containers.

possible out come হবে 2
কারণ, হয় underweight or underweight না।

Solution: (a) Binomial distribution

$$P(X=x) = \binom{20}{x} p^x (1-p)^{20-x} \qquad x = 0, 1, 2, \ldots, 20$$

(b) $E(x) = np = 20 * 0.261 = 5.22,$

$V(x) = np(1-p)$

$= 20 * 0.261 * (1-0.261) = 3.857$

standard deviation $= \sqrt{3.857} = 1.96$

(c) $P(X=7) = \binom{20}{7} 0.261^7 (1-0.261)^{20-7}$ $\qquad {}^nC_r = \binom{20}{7} = {}^{20}C_r$

$= 0.1254.$

(d) $P(X \leq 3) = P(X=0) + P(X=1) + P(X=2) + P(X=3)$

$= \binom{20}{0} \times (0.261)^0 * 0.739^{20} + \binom{20}{1} \times 0.261^1 \times 0.739^{19}$

$+ \binom{20}{2} \times 0.261^2 \times 0.739^{18} + \binom{20}{3} \times 0.261^3 \times 0.739^{17}$

$= 0.0024 + 0.0167 + 0.0559 + 0.1185$

$= 0.1935$ Answer

(e) $P(X \geq 2) = P(2) + P(3) + \ldots\ldots + P(20) = ?$

We know that total probablity $= 1$.

$P(0) + P(1) + P(2) + P(3) + \ldots\ldots + P(20) = 1.$

$\Rightarrow P(2) + P(3) + \ldots + P(20) = 1 - P(0) - P(1)$

$\Rightarrow P(2) + P(3) + \ldots\ldots + P(20) = 1 - 0.0024 - 0.0167$

$= 0.9809.$

Poisson distribution:

The poisson distribution is used when a random variable counts the number of events that occur in an time interval. For example,

1) the number of telephone calls received by an operator within a certain time limit.

2) The number of patients arriving in an emergency room between 10 and 11pm.

*counts the number of events that occur in an time interval
নির্দিষ্ট সময়ে কয়বার আসলো।

* trial সংখ্যা fixed না থাকলে তখন follow করবে।
যখন x কোনো number of events count করবে

The probability mass function is,

$$\boxed{p(X=x) = \frac{e^{-\lambda}\lambda^{x}}{x!}} \qquad x = 0, 1, 2, 3, \ldots$$

λ হচ্ছে parameter / unknown value. দেয়া থাকবে।

Expectation: $\boxed{E(x) = \lambda}$ Average value কে indicate করে

Variance: $\boxed{V(x) = \lambda}$

Difference between binomial distribution and Poisson distribution:

1) In binomial distribution, number of trials are fixed. In poisson distribution, number of trials are infinite.
2) In binomial distribution, Variance $\leq$ Mean. In Poisson distribution, Mean = Variance.
3) Example of binomial distribution: Coin tossing experiment.

Example of Poisson distribution: The number of patients arriving in an emergency room between 10 and 11 pm.

Example: Suppose that the number of errors in a piece of software has a parameter $\lambda = 3$. This parameter immediately implies that the expected number of errors is three and that the variance in the number of errors is also equal to three.

(a) What is distribution of the number of errors in a piece of software.
(b) Calculate the probability that a piece of software has no errors.
(c) Calculate the " " are three or more errors in a piece of software.

Soln: (a) The number of errors in a piece of software follows poission distribution

$$p(X=x) = \frac{e^{-3}3^x}{x!} \quad x = 0, 1, 2, 3, \ldots$$

(b) $p(X=0) = \dfrac{e^{-3}3^0}{0!} = 0.05$

(c) $p(X=3) + p(X=4) + \ldots = ?$

We know,

$P(X=0) + P(X=1) + P(X=2) + P(X=3) + P(X=4) + \ldots = 1$

$\Rightarrow P(X=3) + P(X=4) + \ldots = 1 - P(X=0) - P(X=1) - P(X=2)$

$\Rightarrow P(X=3) + P(X=4) + \ldots = 1 - \dfrac{e^{-3}3^0}{0!} - \dfrac{e^{-3}3^1}{1!} - \dfrac{e^{-3}3^2}{2!}$

$\Rightarrow P(X=3) + P(X=4) + \ldots = 0.577$ Answer

ón:

$\Rightarrow$ For bionomial distribution, Expectation: $E(x) = np$

Variance: $V(x) = np(1-p)$ $[0 \le p \le 1, n > 1]$

Mean = Variance (when $p = 0$)

$\Rightarrow n.0 = n.0(1-0)$

$\Rightarrow 0 = 0$

Mean $(n) >$ variance (0) when $p = 1$.

because when $p = 1$, mean $= n.1 = n$ $(n > 1)$

and variance $= n.1.(1-1) = 0$

$\therefore$ Mean $>$ variance $[0 < p < 1]$

$\Rightarrow np > np(1-p)$

Ex: $50 > 50(1-p)$ here, $(0 < (1-p) < 1)$

$\Rightarrow 50 > 50(1-0.2)$ if $p = 0.2 \Rightarrow 50 > 50 \times 0.8$

$\Rightarrow 50 > 40$

Geometric distribution:

The bionomial distribution is the distribution of the number of success occuring in a fixed number of trials n, it is sometimes of interest to count instead the number of trials performed until the first success occurs. Such a random variable is said to have a geometric distribution.

* The number of trials a is not fixed. And experiment will continue untill the first success occurs
* So, success fixed = 1. trial [illegible] unknown. x indecates number of trial, $x \neq 0$

The probablity mass function is,

$$\boxed{P(X=x) = (1-p)^{(x-1)}\, p} \quad x = 1, 2, 3, \ldots$$

Expectation $\boxed{E(X) = 1/p}$

Variance $\boxed{V(X) = \dfrac{1-p}{p^2}}$

 # Example: Suppose that a company wishes to hire one new workers and that each applicant interviewed has a probablity of 0.6 of being found acceptable.

Geo

P

1) What is the distribution of the total number of applicants that the company needs to interview?
2) Calculate the probablity that exactly six applicants need to be interviewed
3) Calculate the problablity that the company allows up to/at most six applicants to be interviewed.
4) Calculate the probablity that at least six applicants need to be interviewed.
5) Calculate the expected numbers of interviews.

Soln: [1] The total number of applicants that the company needs to interview follows geometric distribution.

The problablity mass function is, ~~P(X) x~~

$$\cancel{P(X)} \quad P(X=x) = (1-0.6)^{(x-1)}\, 0.6 \quad x = 1, 2, 3, \ldots$$

[2] $P(X=6) = (1-0.6)^{(6-1)} \cdot 0.6$

$= 6.144 \times 10^{-3}$

[3] $P(X \le 6) = P(X=1) + P(X=2) + \ldots + P(X=6)$

$= 0.6 + 0.24 + 0.096 + 0.0384 + 0.01536 + 0.006144$

$= 0.996$. Answer

4. $P(X \geq 6) = P(X=6) + P(X=7) + \dots$

We know, $P(X=1) + \dots + P(X=5) + P(X=6) + P(X=7) + \dots = 1$

$\Rightarrow P(X=6) + P(X=7) + \dots = 1 - P(X=1) - \dots - P(X=5)$

$= 1 - 0.98976$

$= 0.01024$ Answer

5. $E(X) = \frac{1}{p} = \frac{1}{0.6} = 1.667$ Answer

Negative binomial distribution: The binomial distribution is the distribution of the number of success occurring in a fixed number of trials n, it is sometimes of interest to count instead the number of trials performed until the rth $(r>1)$ success occurs. Such a random variable is said to have a negative bionomial distribution.

Extention of geomatric dis...

* The experiment will continue untill the r^{th} success occur

The p.m.f. is $\boxed{P(X=x) = \binom{x-1}{r-1}(1-p)^{(x-r)} p^r}$ $x = r, r+1, r+2, \dots$

Expectation, $\boxed{E(x) = r/p}$

variance, $\boxed{V(x) = \frac{r(1-p)}{p^2}}$

Example: Suppose that a company wishes to hire $\boxed{\text{three}}$ r new workers and that each applicant interviewed has a probablity of $\boxed{0.6}$ p of being found acceptable.

1) What is the distribution of the total number of applicants that the company needs to interviewed?
2) Calculate the probablity that exactly six applicants need to be interviewed.
3) Calculate the probablity that the company allows upto/at most six applicants to be interviewed.
4) Calculate the probablity that at least six applicants need to be interviewed.
5) Calculate the expected number of interviews.

Soln: The total number of applicants that the company needs to interview follows negative

1. binomial distribution.

The p.m.f. is $P(X=x) = \binom{x-1}{3-1} \cdot (1-0.6)^{(x-3)} (0.6)^3$ $x = 3, 4, 5, 6, \dots$

2. $P(X=6) = \binom{6-1}{3-1}(1-0.6)^{(6-3)}(0.6)^3 = \binom{5}{2}(0.4)^3(0.6)^3 = \binom{5}{2} \times 0.013824 = 0.138$ A

3. $P(X \leq 6) = P(X=3) + P(X=4) + P(X=5) + P(X=6) = \binom{2}{2}(1-0.6)^0 \cdot 0.6^3 + \binom{3}{2} 0.4^1 \cdot 0.6^3 + \binom{4}{2} 0.4^2 0.6^3 + \binom{5}{2} 0.4^3 0.6^3$

$= \boxed{0.216} + 0.2592 + 0.20736 + 0.13826 = 0.820$ Answer

4. $P(X \geq 6) = P(X=6) + P(X=7) + \dots = ?$

$P(X=3) + P(X=4) + P(X=5) + P(X=6) + P(X=7) + \dots = 1 \Rightarrow P(X=6) + P(X=7) + \dots = 1 - P(X=3) - \dots - P(X=5)$

$\Rightarrow P(X=6) + P(X=7) + \dots = 1 - P(X=3) - P(X=4) - P(X=5) = 1 - 0.68256 = 0.31744$ Answer

5. $E(X) = 3/0.6 = 5$

Normal Distribution: The probability density function of normal distribution is:

a fun^c of a continuous Random variable. range of the random var is $(-\infty, \infty)$
range of the μ(mu) is $-\infty$ to $+\infty$
" " " variance (σ^2) is 0 to ∞

$$\boxed{f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}}$$

$-\infty < x < \infty$,
$-\infty < \mu < \infty$,
$0 \le \sigma^2 < \infty$

** N.Dis... এ যখন Data plot করলে। করে যদি দেখি symmetry হলে Normal
shaperior test-এ যদি p-এর value আসে
P value > 0.5 then Normal
P " < 0.5 " not normal
exam এ বলা থাকবে।

Expectation: $\boxed{E(x) = \mu}$

Variance: $\boxed{V(x) = \sigma^2}$

Standard normal distribution: When $\boxed{\mu = 0}$ and $\boxed{\sigma^2 = 1}$ (variance) then the normal distribution is called standard normal distribution.

The probablity density function of standard normal distribution is:

$$\boxed{f(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2}}$$

$-\infty < z < \infty$

Property of a normal distribution

Mean: Avegage value
Mode: Maximum "
Median: middle point

1) It is symmetric.
2) Mean = Mode = Median. (Mean: average value; Mode: maximum value; Median: middle point)
3) It is unimodal. ↳ only one peak point
4) The total area under the curve is equal to one. → $\int_{-\infty}^{\infty} f(x)\,dx = 1$
5) The normal curve approaches, but never touches, the x-axis.

→ unimodal
→ bimodal
→ Multi

Transformation:

$$\int_{-\infty}^{\infty} f(x)\,dx = \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx$$
$$= \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{1}{2}z^2} dz$$
$$= \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2} dz$$

(Normal distribution যেটা standard normal distribution-এ convert করে)

Let, $\boxed{z = \frac{x-\mu}{\sigma}}$ → এই টেকনিক মাধ্যমে x এর integration কে z-এর " এ রূপান্তর করা হবে।

$\Rightarrow z = \frac{x}{\sigma} - \frac{\mu}{\sigma}$

$\Rightarrow \frac{dz}{dx} = \frac{1}{\sigma}$

$\Rightarrow dx = \sigma\, dz$

x	$-\infty$	∞
z	$-\infty$	∞

That is if $X \sim N(\mu, \sigma^2)$ and if you want to transform the normal distribution to standard distribution then the transform random variable is

$$\boxed{z = \frac{x-\mu}{\sigma} \text{ (z score)}}$$

$F(x)$ মানে lower point (মানে $-\infty$) থেকে x point পর্যন্ত Area.
$F(1)$ 2

Probablity Calculations for Normal Distributions:

$P(a < x < b)$ → $P(a<x<b)$ দিয়ে এটাকে বের করা হয়
$= P\left(\frac{a-\mu}{\sigma} < \frac{x-\mu}{\sigma} < \frac{b-\mu}{\sigma}\right)$
$= P\left(\frac{a-\mu}{\sigma} < z < \frac{b-\mu}{\sigma}\right)$
$= F\left(\frac{b-\mu}{\sigma}\right) - F\left(\frac{a-\mu}{\sigma}\right)$

$P(x < b)$
$= P(-\infty < x < b)$
$= P\left(\frac{-\infty-\mu}{\sigma} < \frac{x-\mu}{\sigma} < \frac{b-\mu}{\sigma}\right)$
$= P\left(-\infty < z < \frac{b-\mu}{\sigma}\right)$
$= F\left(\frac{b-\mu}{\sigma}\right)$

$P(x > a)$
$= P(a < x < \infty)$
$= P\left(\frac{a-\mu}{\sigma} < \frac{x-\mu}{\sigma} < \frac{\infty-\mu}{\sigma}\right)$
$= P\left(\frac{a-\mu}{\sigma} < z < \infty\right)$
$= F(\infty) - F\left(\frac{a-\mu}{\sigma}\right)$
$= 1 - F\left(\frac{a-\mu}{\sigma}\right)$

Example: A company manufactures concrete blocks that are used for construction purposes. Suppose that the weights of the individual concrete blocks are normally distributed with a mean value of $\mu = 11.0\,kg$ and a standard deviation of $\sigma = 0.3\,kg$.

$\boxed{(i)}$ Calculate the probablity that a concrete block weight is less than 10.5kg.

~~(ii) " " " "~~ " " a " " " is within 10 kg to 12kg

$\boxed{(ii)}$ " " " " a " " " is greater than 10.5kg

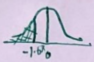
$\boxed{(iii)}$ " " " " a "

Soln: (i)

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$$= \frac{1}{\sqrt{2\pi}\cdot 0.3} e^{-\frac{(x-11)^2}{2\cdot 0.3^2}}$$

S.N.D.F. $f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2}$



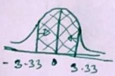
$$\begin{aligned}
& P(x < 10.5) \\
&= P(-\infty < x < 10.5) \\
&= P\left(\frac{-\infty - 11}{0.3} < \frac{x-11}{0.3} < \frac{10.5-11}{0.3}\right) \\
&= P(-\infty < z < -1.67) \\
&= F(-1.67)
\end{aligned}$$

[এই value page 787 থেকে দেখো]

$$= 0.0475$$

(ii) $P(10 < x < 12)$

$$= P\left(\frac{10-11}{0.3} < \frac{x-11}{0.3} < \frac{12-11}{0.3}\right)$$



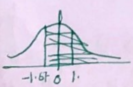
$$\begin{aligned}
&= P(-3.33 < z < 3.33) \\
&= F(3.33) - F(-3.33) \\
&= 0.99957 - 0.00043 \\
&= 0.99914
\end{aligned}$$

* -3.4 এর চেয়ে ছোট value আসলে 0 ধরে নিবো।

* 3.4 এর চেয়ে বড় আসলে 1 ধরে নিবো।

(iii) $P(x > 10.5) = P(10.5 < x) = P(10.5 < x < \infty)$

$$= P\left(\frac{10.5-11}{0.3} < \frac{x-11}{0.3} < \frac{\infty - 11}{0.3}\right)$$



$$\begin{aligned}
&= P(-1.67 < z < \infty) \\
&= F(\infty) - F(-1.67) \\
&= 1 - 0.04746 \\
&= 0.95254
\end{aligned}$$

All distributions
2, 17

Bernoulli:
*1 1 trial
*2 2 Possible Outcome

$\boxed{\begin{array}{ll} \text{P.m.F.:} & P(X=x) = p^x(1-p)^{1-x} \\ \text{Expectation:} & E(x) = p \\ \text{Variance:} & V(x) = p(1-p) \end{array}}$

$x = 0, 1$

$^{20}C_7 = \binom{20}{7}$

single coin toss.
ছক্কার 6 আসার সম্ভাবনা
dice trow করলে even আসার সম্ভাবনা।

(counts number of success) (for both)

BiNomial:
*1 n trial
*2 two (2) possible outcome
*3 every trials are independent.

$\boxed{\begin{array}{ll} \text{P.M.F.:} & P(X=x) = \binom{n}{x} p^x (1-p)^{n-x} \\ \text{Expectation:} & E(x) = np \\ \text{Variance:} & V(x) = np(1-p) \end{array}}$

$x = 0, 1, 2, \dots n$

toss a coin for 5 times.
* highest value = trial সংখ্যার সমান।
*4 x indicates number of success

Number of trials are fixed.
Variance ≤ Mean

difference

① Number of trials are infinite
variance = Mean

Poisson:
*1 counts number of events that occur within in a time interval.
*2 নির্দিষ্ট সময়ে কয়বার আসলো/ উঠলো।
*3 যখন x কোনো number of events, অর্থাৎ Random variable-এর trial সংখ্যা fixed থাকবে না। তখন Poisson Dis-follow করবে।
*4 x indicates number of events/counts number of events.

$\boxed{\begin{array}{ll} \text{P.M.F:} & P(X=x) = \dfrac{e^{-\lambda}\lambda^x}{x!} \\ \text{Expectation: /Average Value} & E(x) = \lambda \\ \text{Variance:} & V(x) = \lambda \end{array}}$

$x = 0, 1, 2, 3, \dots \infty$

λ হচ্ছে parameter. unknown value/ quantity.

Geometric:
*1 number of trial is not fixed.
*2 The experiment will continue until the first success occurs.
*3 so success fixed = 1. trial সংখ্যা unknown.
*4 x indicates number of trial.
$x \neq 0$. P indecates the probablity.

1বার H না পাওয়া পর্যন্ত toss করে যাচ্ছি।

$\boxed{\begin{array}{ll} \text{P.M.F:} & P(X=x) = (1-p)^{(x-1)} p \\ \text{Expectation:} & E(x) = 1/p \\ \text{Variance:} & V(x) = \dfrac{1-p}{p^2} \end{array}}$

$x = 1, 2, 3, \dots \infty$

Negative binomial:
*1 Extention of geomatric distribution.
*2 The experiment will continue until the r^{th} success occur.
*3 $r > 1$. & r indicates number of success
*4 No. of trial is not fixed
*5 P indecales the probablity.

$\boxed{\begin{array}{ll} \text{P.M.F:} & P(X=x) = \binom{x-1}{r-1} \times (1-p)^{(x-r)} \times p^r \\ \text{Expectation:} & E(x) = r/p \\ \text{Variance:} & V(x) = \dfrac{r(1-p)}{p^2} \end{array}}$

$x = r, r+1, r+2, \dots \infty$
$r > 1$

Normal : $*_1$ a func of continuous random var.
Exam-এ জন্য থাকবে এ N. Distr... $*_2$ range of random var $-\infty < x < \infty$
$*_3$ " " μ (mu) $-\infty < \mu < \infty$
$*_4$ " " variance(σ^2) $0 \le \sigma^2 < \infty$

P.D.F.: $\boxed{f(x) = \dfrac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}}$ $\quad -\infty < x < \infty$, $-\infty < \mu < \infty$, $0 \le \sigma^2 < \infty$

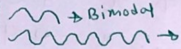
Expectation: $E(x) = \mu$

Variance: $V(x) = \sigma^2$

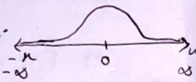
Standard Normal Distribution:
$*_1$ $\mu = 0$
$\sigma^2 = 0$

P.D.F: ~~f(z)~~
$$f(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2}$$

Property of Normal distribution
1) symmetric func. (দেখে বুঝা যায় আর কিছু check দিয়ে। যেকোনো P value > 0.5 হলে Normal
P value < 0.5 " Not ")
2) Mean (Average Value) = Mode (Maximum Value) = Median (middle point).
3) Unimodal (only one peak point.)



4) The total Area ($\int_{-\infty}^{\infty} f(x)\,dx$) under the curve is equal (=) to one (1) ($\int_{-\infty}^{\infty} f(x)\,dx = 1$)
5) The normal curve approaches, but not touches the x-axis.



Transformation:
$$f(x) = \boxed{\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx}$$ Normal dist...

$z = \frac{x-\mu}{\sigma} = \frac{x}{\sigma} - \frac{\mu}{\sigma}$

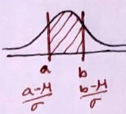
$\frac{dz}{dx} = \frac{1}{\sigma}$

$\therefore dx = \sigma dz$.

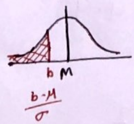
$$= \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{1}{2}z^2} \sigma dz = \boxed{\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} dz}$$ Standard N. dis

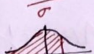
Probability Calculation for Normal Distribution.

(i) $P(a < x < b)$
$= P\left(\frac{a-\mu}{\sigma} < \frac{x-\mu}{\sigma} < \frac{b-\mu}{\sigma}\right)$
$= P\left(\frac{a-\mu}{\sigma} < z < \frac{b-\mu}{\sigma}\right)$
$= F\left(\frac{b-\mu}{\sigma}\right) - F\left(\frac{a-\mu}{\sigma}\right)$

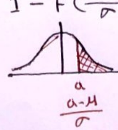


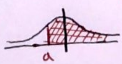
(ii) $P(x < b)$
$= P(-\infty < x < b)$
$= P\left(\frac{-\infty-\mu}{\sigma} < \frac{x-\mu}{\sigma} < \frac{b-\mu}{\sigma}\right)$
$= P\left(-\infty < z < \frac{b-\mu}{\sigma}\right)$
$= F\left(\frac{b-\mu}{\sigma}\right)$





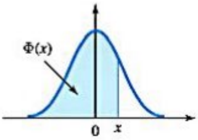
(iii) $P(x > a)$
$= P(a < x < \infty)$
$= P\left(\frac{a-\mu}{\sigma} < \frac{x-\mu}{\sigma} < \frac{\infty-\mu}{\sigma}\right)$
$= P\left(\frac{a-\mu}{\sigma} < z < \infty\right)$
$= F(\infty) - F\left(\frac{a-\mu}{\sigma}\right)$
$= 1 - F\left(\frac{a-\mu}{\sigma}\right)$



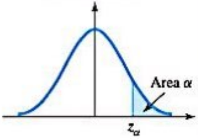


F(sth)-এর মান আমরা
page 787-795-এর মধ্যে পাবো
আর না পাইলে like $F(-4) = 0$
$F(4) = 1$

Table I: Cumulative Distribution Function of the Standard Normal Distribution



Critical Points



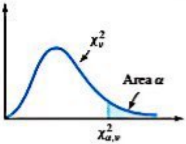
α	z_α
0.10	1.282
0.05	1.645
0.025	1.960
0.01	2.326
0.005	2.576

x	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
−3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
−3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
−3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
−3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
−3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
−2.9	0.0019	0.0018	0.0017	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
−2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
−2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
−2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
−2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
−2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0061
−2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
−2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
−2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
−2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
−1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
−1.8	0.0359	0.0352	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
−1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
−1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
−1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
−1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0722	0.0708	0.0694	0.0681
−1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
−1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
−1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
−1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
−0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
−0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
−0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
−0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
−0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
−0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
−0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
−0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
−0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
−0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

(Continued on next page)

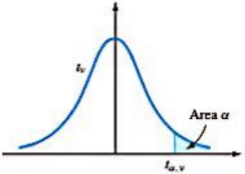
x	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5339
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9019	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9278	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9610	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

Table II: Critical Points of the Chi-Square Distribution



Degrees of freedom ν	α 0.995	0.99	0.975	0.95	0.90	0.10	0.05	0.025	0.01	0.005
1	0.000	0.000	0.001	0.004	0.016	2.706	3.841	5.024	6.635	7.879
2	0.010	0.020	0.051	0.103	0.211	4.605	5.991	7.378	9.210	10.597
3	0.072	0.115	0.216	0.352	0.584	6.251	7.815	9.348	11.345	12.838
4	0.207	0.297	0.484	0.711	1.064	7.779	9.488	11.143	13.277	14.860
5	0.412	0.554	0.831	1.145	1.610	9.236	11.071	12.833	15.086	16.750
6	0.676	0.872	1.237	1.635	2.204	10.645	12.592	14.449	16.812	18.548
7	0.989	1.239	1.690	2.167	2.833	12.017	14.067	16.013	18.475	20.278
8	1.344	1.646	2.180	2.733	3.490	13.362	15.507	17.535	20.090	21.955
9	1.735	2.088	2.700	3.325	4.168	14.684	16.919	19.023	21.666	23.589
10	2.156	2.558	3.247	3.940	4.865	15.987	18.307	20.483	23.209	25.188
11	2.603	3.053	3.816	4.575	5.578	17.275	19.675	21.920	24.725	26.757
12	3.074	3.571	4.404	5.226	6.304	18.549	21.026	23.337	26.217	28.299
13	3.565	4.107	5.009	5.892	7.042	19.812	22.362	24.736	27.688	29.819
14	4.075	4.660	5.629	6.571	7.790	21.064	23.685	26.119	29.141	31.319
15	4.601	5.229	6.262	7.261	8.547	22.307	24.996	27.488	30.578	32.801
16	5.142	5.812	6.908	7.962	9.312	23.542	26.296	28.845	32.000	34.267
17	5.697	6.408	7.564	8.672	10.085	24.769	27.587	30.191	33.409	35.718
18	6.265	7.015	8.231	9.390	10.865	25.989	28.869	31.526	34.805	37.156
19	6.844	7.633	8.907	10.117	11.651	27.204	30.144	32.852	36.191	38.582
20	7.434	8.260	9.591	10.851	12.443	28.412	31.410	34.170	37.566	39.997
21	8.034	8.897	10.283	11.591	13.240	29.615	32.671	35.479	38.932	41.401
22	8.643	9.542	10.982	12.338	14.042	30.813	33.924	36.781	40.289	42.796
23	9.260	10.196	11.689	13.091	14.848	32.007	35.172	38.076	41.638	44.181
24	9.886	10.856	12.401	13.848	15.659	33.196	36.415	39.364	42.980	45.559
25	10.520	11.524	13.120	14.611	16.473	34.382	37.652	40.646	44.314	46.928
26	11.160	12.198	13.844	15.379	17.292	35.563	38.885	41.923	45.642	48.290
27	11.808	12.879	14.573	16.151	18.114	36.741	40.113	43.194	46.963	49.645
28	12.461	13.565	15.308	16.928	18.939	37.916	41.337	44.461	48.278	50.993
29	13.121	14.257	16.017	17.708	19.768	39.087	42.557	45.722	49.588	52.336
30	13.787	14.954	16.791	18.493	20.599	40.256	43.773	46.979	50.892	53.672
40	20.707	22.164	24.433	26.509	29.051	51.805	55.758	59.342	63.691	66.766
50	27.991	29.707	32.357	34.764	37.689	63.167	67.505	71.420	76.154	79.490
60	35.534	37.485	40.482	43.188	46.459	74.397	79.082	83.298	88.379	91.952
70	43.275	45.442	48.758	51.739	55.329	85.527	90.531	95.023	100.425	104.215
80	51.172	53.540	57.153	60.391	64.278	96.578	101.879	106.629	112.329	116.321
90	59.196	61.754	65.647	69.126	73.291	107.565	113.145	118.136	124.116	128.299
100	67.328	70.065	74.222	77.929	82.358	118.498	124.342	129.561	135.807	140.169

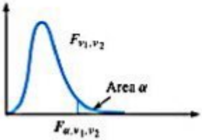
Table III: Critical Points of the t-Distribution



Degrees of freedom ν	α = 0.10	0.05	0.025	0.01	0.005	0.001	0.0005
1	3.078	6.314	12.706	31.821	63.657	318.31	636.62
2	1.886	2.920	4.303	6.965	9.925	22.326	31.598
3	1.638	2.353	3.182	4.541	5.841	10.213	12.924
4	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	1.319	1.714	2.069	2.500	2.807	3.485	3.767
24	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	1.311	1.699	2.045	2.462	2.756	3.396	3.659
30	1.310	1.697	2.042	2.457	2.750	3.385	3.646
40	1.303	1.684	2.021	2.423	2.704	3.307	3.551
60	1.296	1.671	2.000	2.390	2.660	3.232	3.460
120	1.289	1.658	1.980	2.358	2.617	3.160	3.373
∞	1.282	1.645	1.960	2.326	2.576	3.090	3.291

Table IV: Critical Points of the F-Distribution

$$F_{\nu_1,\nu_2} \sim \frac{\chi^2_{\nu_1}/\nu_1}{\chi^2_{\nu_2}/\nu_2}$$



$\alpha = 0.10$

ν_2 \ ν_1	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	∞
1	39.86	49.50	53.59	55.84	57.24	58.20	58.90	59.44	59.85	60.20	60.70	61.22	61.74	62.00	62.27	62.53	62.79	63.05	63.33
2	3.53	9.00	9.16	9.24	9.29	9.33	9.35	9.37	9.38	9.39	9.41	9.42	9.44	9.45	9.46	9.47	9.47	9.48	9.49
3	5.54	5.46	5.39	5.34	5.31	5.28	5.27	5.25	5.24	5.23	5.22	5.20	5.18	5.18	5.17	5.16	5.15	5.14	5.13
4	4.54	4.32	4.19	4.11	4.05	4.01	3.98	3.95	3.94	3.92	3.90	3.87	3.84	3.83	3.82	3.80	3.79	3.78	3.76
5	4.06	3.78	3.62	3.52	3.45	3.40	3.37	3.34	3.32	3.30	3.27	3.24	3.21	3.19	3.17	3.16	3.14	3.12	3.10
6	3.78	3.46	3.29	3.18	3.11	3.05	3.01	2.98	2.96	2.94	2.90	2.87	2.84	2.82	2.80	2.78	2.76	2.74	2.72
7	3.59	3.26	3.07	2.96	2.88	2.83	2.78	2.75	2.72	2.70	2.67	2.63	2.59	2.58	2.56	2.54	2.51	2.49	2.47
8	3.46	3.11	2.92	2.81	2.73	2.67	2.62	2.59	2.56	2.54	2.50	2.46	2.42	2.40	2.38	2.36	2.34	2.32	2.29
9	3.36	3.01	2.81	2.69	2.61	2.55	2.51	2.47	2.44	2.42	2.38	2.34	2.30	2.28	2.25	2.23	2.21	2.18	2.16
10	3.28	2.92	2.73	2.61	2.52	2.46	2.41	2.38	2.35	2.32	2.28	2.24	2.20	2.18	2.16	2.13	2.11	2.08	2.06
11	3.23	2.86	2.66	2.54	2.45	2.39	2.34	2.30	2.27	2.25	2.21	2.17	2.12	2.10	2.08	2.05	2.03	2.00	1.97
12	3.18	2.81	2.61	2.48	2.39	2.33	2.28	2.24	2.21	2.19	2.15	2.10	2.06	2.04	2.01	1.99	1.96	1.93	1.90
13	3.14	2.76	2.56	2.43	2.35	2.28	2.23	2.20	2.16	2.14	2.10	2.05	2.01	1.98	1.96	1.93	1.90	1.88	1.85
14	3.10	2.73	2.52	2.39	2.31	2.24	2.19	2.15	2.12	2.10	2.05	2.01	1.96	1.94	1.91	1.89	1.86	1.83	1.80
15	3.07	2.70	2.49	2.36	2.27	2.21	2.16	2.12	2.09	2.06	2.02	1.97	1.92	1.90	1.87	1.85	1.82	1.79	1.76
16	3.05	2.67	2.46	2.33	2.24	2.18	2.13	2.09	2.06	2.03	1.99	1.94	1.89	1.87	1.84	1.81	1.78	1.75	1.72
17	3.03	2.64	2.44	2.31	2.22	2.15	2.10	2.06	2.03	2.00	1.96	1.91	1.86	1.84	1.81	1.78	1.75	1.72	1.69
18	3.01	2.62	2.42	2.29	2.20	2.13	2.08	2.04	2.00	1.98	1.93	1.89	1.84	1.81	1.78	1.75	1.72	1.69	1.66
19	2.99	2.61	2.40	2.27	2.18	2.11	2.06	2.02	1.98	1.96	1.91	1.86	1.81	1.79	1.76	1.73	1.70	1.67	1.63
20	2.97	2.59	2.38	2.25	2.16	2.09	2.04	2.00	1.96	1.94	1.89	1.84	1.79	1.77	1.74	1.71	1.68	1.64	1.61
21	2.96	2.57	2.36	2.23	2.14	2.08	2.02	1.98	1.95	1.92	1.87	1.83	1.78	1.75	1.72	1.69	1.66	1.62	1.59
22	2.95	2.56	2.35	2.22	2.13	2.06	2.01	1.97	1.93	1.90	1.86	1.81	1.76	1.73	1.70	1.67	1.64	1.60	1.57
23	2.94	2.55	2.34	2.21	2.11	2.05	1.99	1.95	1.92	1.89	1.84	1.80	1.74	1.72	1.69	1.66	1.62	1.59	1.55
24	2.93	2.54	2.33	2.19	2.10	2.04	1.98	1.94	1.91	1.88	1.83	1.78	1.73	1.70	1.67	1.64	1.61	1.57	1.51
25	2.92	2.53	2.32	2.18	2.09	2.02	1.97	1.93	1.89	1.87	1.82	1.77	1.72	1.69	1.66	1.63	1.59	1.56	1.52
26	2.91	2.52	2.31	2.17	2.08	2.01	1.96	1.92	1.88	1.86	1.81	1.76	1.71	1.68	1.65	1.61	1.58	1.54	1.50
27	2.90	2.51	2.30	2.17	2.07	2.00	1.95	1.91	1.87	1.85	1.80	1.75	1.70	1.67	1.64	1.60	1.57	1.53	1.49
28	2.89	2.50	2.29	2.16	2.06	2.00	1.94	1.90	1.87	1.84	1.79	1.74	1.69	1.66	1.63	1.59	1.56	1.52	1.48
29	2.89	2.50	2.28	2.15	2.06	1.99	1.93	1.89	1.86	1.83	1.78	1.73	1.68	1.65	1.62	1.58	1.55	1.51	1.47
30	2.88	2.49	2.28	2.14	2.05	1.98	1.93	1.88	1.85	1.82	1.77	1.72	1.67	1.64	1.61	1.57	1.54	1.50	1.46
40	2.84	2.44	2.23	2.09	2.00	1.93	1.87	1.83	1.79	1.76	1.71	1.66	1.61	1.57	1.54	1.51	1.47	1.42	1.35
60	2.79	2.39	2.18	2.04	1.95	1.87	1.82	1.77	1.74	1.71	1.66	1.60	1.54	1.51	1.48	1.44	1.40	1.35	1.29
120	2.75	2.35	2.13	1.99	1.90	1.82	1.77	1.72	1.68	1.65	1.60	1.55	1.48	1.45	1.41	1.37	1.32	1.26	1.18
∞	2.71	2.30	2.08	1.94	1.85	1.77	1.72	1.67	1.63	1.60	1.55	1.49	1.42	1.38	1.34	1.30	1.24	1.17	1.10

(Continued on next page)

$\alpha = 0.05$

ν_2 \ ν_1	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	∞
1	161.44	199.50	215.69	224.57	230.16	233.98	236.78	238.89	240.55	241.89	243.91	245.97	248.02	249.04	250.07	251.13	252.18	253.27	254.31
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.39	19.40	19.41	19.43	19.45	19.45	19.46	19.47	19.48	19.49	19.50
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.74	8.70	8.66	8.64	8.62	8.59	8.57	8.55	8.53
4	7.71	6.94	6.59	6.39	6.20	6.16	6.09	6.04	6.00	5.96	5.91	5.86	5.80	5.77	5.75	5.72	5.69	5.66	5.63
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68	4.62	4.56	4.53	4.50	4.46	4.43	4.40	4.36
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.00	3.94	3.87	3.84	3.81	3.77	3.74	3.70	3.67
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57	3.51	3.44	3.41	3.38	3.34	3.30	3.27	3.23
8	5.52	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.28	3.22	3.15	3.12	3.08	3.04	3.01	2.97	2.93
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07	3.01	2.94	2.90	2.86	2.83	2.79	2.75	2.71
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.91	2.85	2.77	2.74	2.70	2.66	2.62	2.58	2.54
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.79	2.72	2.65	2.61	2.57	2.53	2.49	2.45	2.40
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.69	2.62	2.54	2.51	2.47	2.43	2.38	2.34	2.30
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.60	2.53	2.46	2.42	2.38	2.34	2.30	2.25	2.21
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.53	2.46	2.39	2.35	2.31	2.27	2.22	2.18	2.13
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.48	2.40	2.33	2.29	2.25	2.20	2.16	2.11	2.07
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.42	2.35	2.28	2.24	2.19	2.15	2.11	2.06	2.01
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.38	2.31	2.23	2.19	2.15	2.10	2.06	2.01	1.96
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.34	2.27	2.19	2.15	2.11	2.06	2.02	1.97	1.92
19	4.38	3.52	3.13	2.90	3.74	2.63	2.54	2.48	2.42	2.38	2.31	2.23	2.16	2.11	2.07	2.03	1.98	1.93	1.88
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.30	2.35	2.28	2.20	2.12	2.08	2.04	1.99	1.95	1.90	1.84
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.25	2.18	2.10	2.05	2.01	1.96	1.92	1.87	1.81
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.23	2.15	2.07	2.03	1.98	1.94	1.89	1.84	1.78
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	2.20	2.13	2.05	2.01	1.96	1.91	1.86	1.81	1.76
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.18	2.11	2.03	1.98	1.94	1.89	1.84	1.79	1.73
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24	2.16	2.09	2.01	1.96	1.92	1.87	1.82	1.77	1.71
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22	2.15	2.07	1.99	1.95	1.90	1.85	1.80	1.75	1.69
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.20	2.13	2.06	1.97	1.93	1.88	1.84	1.79	1.73	1.67
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19	2.12	2.04	1.96	1.91	1.87	1.82	1.77	1.71	1.65
29	4.18	3.33	2.93	3.70	2.55	2.43	2.35	2.28	2.22	2.18	2.10	2.03	1.94	1.90	1.85	1.81	1.75	1.70	1.64
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.09	2.01	1.93	1.89	1.84	1.79	1.74	1.68	1.62
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.00	1.92	1.84	1.79	1.74	1.69	1.64	1.58	1.51
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.92	1.84	1.75	1.70	1.65	1.59	1.53	1.47	1.39
120	3.92	3.09	2.68	2.45	2.29	2.18	2.09	2.02	1.96	1.91	1.83	1.75	1.66	1.61	1.55	1.50	1.43	1.35	1.25
∞	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83	1.75	1.67	1.57	1.52	1.46	1.39	1.32	1.22	1.00

Table IV: (*Continued*)

$\alpha = 0.01$

ν_1

ν_2	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	∞
1	4052	4999	5403	5625	5764	5859	5929	5981	6023	6055	6107	6157	6209	6235	6260	6287	6312	6339	6366
2	98.51	99.00	99.17	99.25	99.30	99.33	99.35	99.38	99.39	99.40	99.41	99.43	99.44	99.45	99.47	99.47	99.48	99.49	99.50
3	34.12	30.82	29.46	28.71	28.24	27.91	27.67	27.49	27.35	27.23	27.05	26.87	26.69	26.60	26.51	26.41	26.32	26.22	26.13
4	21.20	18.00	16.69	15.98	15.52	15.21	14.98	14.80	14.66	14.55	14.37	14.20	14.02	13.93	13.84	13.75	13.65	13.56	13.46
5	16.26	13.27	12.06	11.39	10.97	10.67	10.46	10.29	10.16	10.05	9.89	9.72	9.55	9.47	9.38	9.29	9.20	9.11	9.02
6	13.75	10.92	9.78	9.15	8.75	8.47	8.26	8.10	7.98	7.87	7.72	7.56	7.40	7.31	7.23	7.14	7.06	6.97	6.88
7	12.25	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.72	6.62	6.47	6.31	6.16	6.07	5.99	5.91	5.82	5.74	5.65
8	11.26	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.91	5.81	5.67	5.52	5.36	5.28	5.20	5.12	5.03	4.95	4.86
9	10.56	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.35	5.26	5.11	4.96	4.81	4.73	4.65	4.57	4.48	4.40	4.31
10	10.04	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.94	4.85	4.71	4.56	4.41	4.33	4.25	4.17	4.08	4.00	3.91
11	9.65	7.21	6.22	5.67	5.32	5.07	4.89	4.74	4.63	4.54	4.40	4.25	4.10	4.02	3.94	3.86	3.78	3.69	3.60
12	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.39	4.30	4.16	4.01	3.86	3.78	3.70	3.62	3.54	3.45	3.36
13	9.07	6.70	5.74	5.21	4.86	4.62	4.44	4.30	4.19	4.10	3.96	3.82	3.66	3.59	3.51	3.43	3.34	3.25	3.17
14	8.86	6.51	5.56	5.04	4.69	4.46	4.28	4.14	4.03	3.94	3.80	3.66	3.51	3.43	3.35	3.27	3.18	3.09	3.00
15	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.89	3.80	3.67	3.52	3.37	3.29	3.21	3.13	3.05	2.96	2.87
16	8.53	6.23	5.29	4.77	4.44	4.20	4.03	3.89	3.78	3.69	3.55	3.41	3.26	3.18	3.10	3.02	2.93	2.84	2.75
17	8.40	6.11	5.18	4.67	4.34	4.10	3.93	3.79	3.68	3.59	3.46	3.31	3.16	3.08	3.00	2.92	2.83	2.75	2.65
18	8.29	6.01	5.09	4.58	4.25	4.01	3.84	3.71	3.60	3.51	3.37	3.23	3.08	3.00	2.92	2.84	2.75	2.66	2.57
19	8.19	5.93	5.01	4.50	4.17	3.94	3.77	3.63	3.52	3.43	3.30	3.15	3.00	2.92	2.84	2.76	2.67	2.58	2.49
20	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.46	3.37	3.23	3.09	2.94	2.86	2.78	2.69	2.61	2.52	2.42
21	8.02	5.78	4.87	4.37	4.04	3.81	3.64	3.51	3.40	3.31	3.17	3.03	2.88	2.80	2.72	2.64	2.55	2.46	2.36
22	7.95	5.72	4.82	4.31	3.99	3.76	3.59	3.45	3.35	3.26	3.12	2.98	2.83	2.75	2.67	2.58	2.50	2.40	2.31
23	7.88	5.66	4.76	4.26	3.94	3.71	3.54	3.41	3.30	3.21	3.07	2.93	2.78	2.70	2.62	2.54	2.45	2.35	2.26
24	7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.26	3.17	3.03	2.89	2.74	2.66	2.58	2.49	2.40	2.31	2.21
25	7.77	5.57	4.68	4.18	3.85	3.63	3.46	3.32	3.22	3.13	2.99	2.85	2.70	2.62	2.54	2.45	2.36	2.27	2.17
26	7.72	5.53	4.64	4.14	3.82	3.59	3.42	3.29	3.18	3.09	2.96	2.81	2.66	2.58	2.50	2.42	2.33	2.23	2.13
27	7.68	5.49	4.60	4.11	3.78	3.56	3.39	3.26	3.15	3.06	2.93	2.78	2.63	2.55	2.47	2.38	2.29	2.20	2.10
28	7.64	5.45	4.57	4.07	3.75	3.53	3.36	3.23	3.12	3.03	2.90	2.75	2.60	2.52	2.44	2.35	2.26	2.17	2.06
29	7.60	5.42	4.54	4.04	3.73	3.50	3.33	3.20	3.09	3.00	2.87	2.73	2.57	2.49	2.41	2.33	2.23	2.14	2.03
30	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	3.07	2.98	2.84	2.70	2.55	2.47	2.39	3.30	2.31	2.11	2.01
40	7.31	5.18	4.31	3.83	3.51	3.29	3.12	2.99	2.89	2.80	2.66	2.52	2.37	2.29	2.20	2.11	2.02	1.92	1.80
60	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.72	2.63	2.50	2.35	2.20	2.12	2.03	1.94	1.84	1.73	1.60
120	6.85	4.79	3.95	3.48	3.17	2.96	2.79	2.66	2.56	2.47	2.34	2.19	2.03	1.95	1.86	1.76	1.66	1.53	1.38
∞	6.63	4.61	3.78	3.32	3.02	2.80	2.64	2.51	2.41	2.32	2.18	2.04	1.88	1.79	1.70	1.59	1.47	1.32	1.00

$\alpha = 0.05$

Degrees of freedom v	k = 2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
5	3.64	4.60	5.22	5.67	6.03	6.33	6.58	6.80	6.99	7.17	7.32	7.47	7.60	7.72	7.83	7.93	8.03	8.12	8.21
6	3.46	4.34	4.90	5.30	5.63	5.90	6.12	6.32	6.49	6.65	6.79	6.92	7.03	7.14	7.24	7.34	7.43	7.51	7.59
7	3.34	4.16	4.68	5.06	5.36	5.61	5.82	6.00	6.16	6.30	6.43	6.55	6.66	6.76	6.85	6.94	7.02	7.10	7.17
8	3.26	4.04	4.53	4.89	5.17	5.40	5.60	5.77	5.92	6.05	6.18	6.29	6.39	6.48	6.57	6.65	6.73	6.80	6.87
9	3.20	3.95	4.41	4.76	5.02	5.24	5.43	5.59	5.74	5.87	5.98	6.09	6.19	6.28	6.36	6.44	6.51	6.58	6.64
10	3.15	3.88	4.33	4.65	4.91	5.12	5.30	5.46	5.60	5.72	5.83	5.93	6.03	6.11	6.19	6.27	6.34	6.40	6.47
11	3.11	3.82	4.26	4.57	4.82	5.03	5.20	5.35	5.49	5.61	5.71	5.81	5.90	5.98	6.06	6.13	6.20	6.27	6.33
12	3.08	3.77	4.20	4.51	4.75	4.95	5.12	5.27	5.39	5.51	5.61	5.71	5.80	5.88	5.95	6.02	6.09	6.15	6.21
13	3.06	3.73	4.15	4.45	4.69	4.88	5.05	5.19	5.32	5.43	5.53	5.63	5.71	5.79	5.86	5.93	5.99	6.05	6.11
14	3.03	3.70	4.11	4.41	4.64	4.83	4.99	5.13	5.25	5.36	5.46	5.55	5.64	5.71	5.79	5.85	5.91	5.97	6.03
15	3.01	3.67	4.08	4.37	4.59	4.78	4.94	5.08	5.20	5.31	5.40	5.49	5.57	5.65	5.72	5.78	5.85	5.90	5.96
16	3.00	3.65	4.05	4.33	4.56	4.74	4.90	5.03	5.15	5.26	5.35	5.44	5.52	5.59	5.66	5.73	5.79	5.84	5.90
17	2.98	3.63	4.02	4.30	4.52	4.70	4.86	4.99	5.11	5.21	5.31	5.39	5.47	5.54	5.61	5.67	5.73	5.79	5.84
18	2.97	3.61	4.00	4.28	4.49	4.67	4.82	4.96	5.07	5.17	5.27	5.35	5.43	5.50	5.57	5.63	5.69	5.74	5.79
19	2.96	3.59	3.98	4.25	4.47	4.65	4.79	4.92	5.04	5.14	5.23	5.31	5.39	5.46	5.53	5.59	5.65	5.70	5.75
20	2.95	3.58	3.96	4.23	4.45	4.62	4.77	4.90	5.01	5.11	5.20	5.26	5.36	5.43	4.49	5.55	5.61	5.66	5.71
24	2.92	3.53	3.90	4.17	4.37	4.54	4.68	4.81	4.92	5.01	5.10	5.16	5.25	5.32	5.38	5.44	5.49	5.55	5.59
30	2.89	3.49	3.85	4.10	4.30	4.46	4.60	4.72	4.82	4.92	5.00	5.08	5.15	5.21	5.27	5.33	5.38	5.43	5.47
40	2.86	3.44	3.79	4.04	4.23	4.39	4.52	4.63	4.73	4.82	4.90	4.98	5.04	5.11	5.16	5.22	5.27	5.31	5.36
60	2.83	3.40	3.74	3.98	4.16	4.31	4.44	4.55	4.65	4.73	4.81	4.88	4.94	5.00	5.06	5.11	5.15	5.20	5.24
120	2.80	3.36	3.68	3.92	4.10	4.24	4.36	4.47	4.56	4.64	4.71	4.78	4.84	4.90	4.95	5.00	5.04	5.09	5.13
∞	2.77	3.31	3.63	3.86	4.03	4.17	4.29	4.39	4.47	4.55	4.62	4.68	4.74	4.80	4.85	4.89	4.93	4.97	5.01

$\alpha = 0.01$

Degrees of freedom v	k = 2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
5	5.70	6.97	7.80	8.42	8.91	9.32	9.67	9.97	10.2	10.5	10.7	10.9	11.1	11.2	11.4	11.6	11.7	11.8	11.9
6	5.24	6.33	7.03	7.56	7.97	8.32	8.61	8.87	9.10	9.30	9.49	9.65	9.81	9.95	10.1	10.2	10.3	10.4	10.5
7	4.95	5.92	6.54	7.01	7.37	7.68	7.94	8.17	8.37	8.55	8.71	8.86	9.00	9.12	9.24	9.35	9.46	9.55	9.65
8	4.74	5.63	6.20	6.63	6.96	7.24	7.47	7.68	7.87	8.03	8.18	8.31	8.44	8.55	8.66	8.76	8.85	8.94	9.03
9	4.60	5.43	5.96	6.35	6.66	6.91	7.13	7.32	7.49	7.65	7.78	7.91	8.03	8.13	8.23	8.32	8.41	8.49	8.57
10	4.48	5.27	5.77	6.14	6.43	6.67	6.87	7.05	7.21	7.36	7.48	7.60	7.71	7.81	7.91	7.99	8.07	8.15	8.22
11	4.39	5.14	5.62	5.97	6.25	6.48	6.67	6.84	6.99	7.13	7.25	7.36	7.46	7.56	7.65	7.73	7.81	7.88	7.95
12	4.32	5.04	5.50	5.84	6.10	6.32	6.51	6.67	6.81	6.94	7.06	7.17	7.26	7.36	7.44	7.52	7.59	7.66	7.73
13	4.26	4.96	5.40	5.73	5.98	6.19	6.37	6.53	6.67	6.79	6.90	7.01	7.10	7.19	7.27	7.34	7.42	7.48	7.55
14	4.21	4.89	5.32	5.63	5.88	6.08	6.26	6.41	6.54	6.66	6.77	6.87	6.96	7.05	7.12	7.20	7.27	7.33	7.39
15	4.17	4.83	5.25	5.56	5.80	5.99	6.16	6.31	6.44	6.55	6.66	6.76	6.84	6.93	7.00	7.07	7.14	7.20	7.26
16	4.13	4.78	5.19	5.49	5.72	5.92	6.08	6.22	6.35	6.46	6.56	6.66	6.74	6.82	6.90	6.97	7.03	7.09	7.15
17	4.10	4.74	5.14	5.43	5.66	5.85	6.01	6.15	6.27	6.38	6.48	6.57	6.66	6.73	6.80	6.87	6.94	7.00	7.05
18	4.07	4.70	5.09	5.38	5.60	5.79	5.94	6.08	6.20	6.31	6.41	6.50	6.58	6.65	6.72	6.79	6.85	6.91	6.96
19	4.05	4.67	5.05	5.33	5.55	5.73	5.89	6.02	6.14	6.25	6.34	6.43	6.51	6.58	6.65	6.72	6.78	6.84	6.89
20	4.02	4.64	5.02	5.29	5.51	5.69	5.84	5.97	6.09	6.19	6.29	6.37	6.45	6.52	6.59	6.65	6.71	6.76	6.82
24	3.96	4.54	4.91	5.17	5.37	5.54	5.69	5.81	5.92	6.02	6.11	6.19	6.26	6.33	6.39	6.45	6.51	6.56	6.61
30	3.89	4.45	4.80	5.05	5.24	5.40	5.54	5.65	5.76	5.85	5.93	6.01	6.08	6.14	6.20	6.26	6.31	6.36	6.41
40	3.82	4.37	4.70	4.93	5.11	5.27	5.39	5.50	5.60	5.69	5.77	5.84	5.90	5.96	6.02	6.07	6.12	6.17	6.21
60	3.76	4.28	4.60	4.82	4.99	5.13	5.25	5.36	5.45	5.53	5.60	5.67	5.73	5.79	5.84	5.89	5.93	5.98	6.02
120	3.70	4.20	4.50	4.71	4.87	5.01	5.12	5.21	5.30	5.38	5.44	5.51	5.56	5.61	5.66	5.71	5.75	5.79	5.83
∞	3.64	4.12	4.40	4.60	4.76	4.88	4.99	5.08	5.16	5.23	5.29	5.35	5.40	5.45	5.49	5.54	5.57	5.61	5.65

n	$\alpha = 0.20$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.02$	$\alpha = 0.01$
10	0.323	0.369	0.409	0.457	0.489
11	0.308	0.352	0.391	0.437	0.468
12	0.296	0.338	0.375	0.419	0.449
13	0.285	0.325	0.361	0.404	0.432
14	0.275	0.314	0.349	0.390	0.418
15	0.268	0.304	0.338	0.377	0.404
16	0.258	0.295	0.327	0.366	0.392
17	0.250	0.286	0.318	0.355	0.381
18	0.244	0.279	0.309	0.346	0.371
19	0.237	0.271	0.301	0.337	0.361
20	0.232	0.265	0.294	0.329	0.362
21	0.226	0.259	0.287	0.321	0.344
22	0.221	0.253	0.281	0.314	0.337
23	0.216	0.247	0.275	0.307	0.330
24	0.212	0.242	0.269	0.301	0.323
25	0.208	0.238	0.264	0.295	0.317
26	0.204	0.233	0.259	0.290	0.311
27	0.200	0.229	0.254	0.284	0.305
28	0.197	0.226	0.250	0.279	0.300
29	0.193	0.221	0.246	0.275	0.295
30	0.190	0.218	0.242	0.270	0.290
31	0.187	0.214	0.238	0.266	0.285
32	0.184	0.211	0.234	0.262	0.281
33	0.182	0.208	0.231	0.258	0.277
34	0.179	0.205	0.227	0.254	0.273
35	0.177	0.202	0.224	0.251	0.269
36	0.174	0.199	0.221	0.247	0.265
37	0.172	0.196	0.218	0.244	0.262
38	0.170	0.194	0.215	0.241	0.258
39	0.168	0.191	0.213	0.238	0.255
40	0.165	0.189	0.210	0.235	0.252
Approximation for $n > 40$:	$\frac{1.07}{\sqrt{n}}$	$\frac{1.22}{\sqrt{n}}$	$\frac{1.36}{\sqrt{n}}$	$\frac{1.52}{\sqrt{n}}$	$\frac{1.63}{\sqrt{n}}$