MAT361.4

Assignment (mid)

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(1)

Let define,

Red marbles with R

and, Blue marbels with B

when we take 1 marbels from the box and replace it, possible outcomes are:

drawing 1 red marble, then blue marble : RB

" 1 blue " , " red " : BR

" a marble , " same " : RR, BB

∴ Sample space, S = {RR, RB, BR, BB}

Repeat when the second marble is drawn without replacing the first marble, possible outcomes:

drawing 1 marble , then another one : RB, BR

∴ sample space, S = {RB, BR}

(2)

a) $f(x) = c x e^{-\frac{x}{2}}$ $\quad x > 0$

let, $I = \int c x e^{-\frac{x}{2}} \, dx$

$= \int c(-2z) e^{z} (-2dz)$

| let, $z = -\frac{x}{2}$
| $\therefore x = -2z$
| $\Rightarrow dx = -2dz$

$= 4c \int z e^{z} dz$

$= 4c [z e^{z} - \int e^{z} dz]$

$= 4c (z e^{z} - e^{z})$

$= 4c e^{z} (z-1)$

| let, $u = z$ | $dv = e^{z}$
| $\therefore du = dz$ | $\therefore v = e^{z}$

$= 4c e^{-\frac{x}{2}} (-\frac{x}{2} - 1)$ $\quad [\because z = -\frac{x}{2}]$

$= c e^{-\frac{x}{2}} (-2x - 4)$

here, $\int_0^{\infty} f(x) \, dx = 1$

$\Rightarrow \int_0^{\infty} c x e^{-\frac{x}{2}} \, dx = 1$

$\Rightarrow \left[c e^{-\frac{x}{2}} (-2x - 4) \right]_0^{\infty} = 1$ $\quad$ [from the value of I]

$\Rightarrow c [0 - (-4)] = 1$

$\Rightarrow 4c = 1$ $\quad \therefore c = \frac{1}{4}$

b)

$$CDF = \int_0^x \frac{1}{4} x e^{-\frac{x}{2}} \, dx$$

:

$$= \frac{1}{4}\left[(-2x - 4)\, e^{-\frac{x}{2}} \right]_0^x$$

$$= \frac{1}{4}\left[(-2x-4)e^{-\frac{x}{2}} - (-4) \right]$$

$$= \frac{(-2x-4)e^{-\frac{x}{2}}}{4} + \frac{4}{4}$$

$$= \frac{-2(x+2)e^{-\frac{x}{2}}}{4} + 1$$

$$\therefore CDF = -\frac{(x+2)e^{-\frac{x}{2}}}{2} + 1$$

(3)

a) $f(x,y) = x+y \qquad 0<x<c \;, 0<y<1$

$$\int_0^1 \int_0^c (x+y)\, dx\, dy = 1$$

$$\Rightarrow \int_0^1 \left[\frac{x^2}{2} + yx\right]_0^c dy = 1$$

$$\Rightarrow \int_0^1 \left[\frac{c^2}{2} + yc\right] dy = 1$$

$$\Rightarrow \left[\frac{c^2 y}{2} + \frac{c y^2}{2}\right]_0^1 = 1$$

$$\Rightarrow \frac{c^2}{2} + \frac{c}{2} = 1$$

$$\Rightarrow c^2 + c = 2$$

$$\Rightarrow c^2 + c - 2 = 0$$

$$\Rightarrow c^2 + 2c - c - 2 = 0$$

$$\Rightarrow c(c+2) - 1(c+2) = 0$$

$$\Rightarrow (c+2)(c-1) = 0$$

$\therefore c = 1, \; c \neq -2$ [$c \neq -2$; because here c's value must be bigger then 0]

$\therefore$ value of c is 1.

b) Marginal Probability Density functions:

$$g(x) = \int_0^1 (x+y)\, dy$$

$$= \left[xy + \frac{y^2}{2}\right]_{y=0}^{1}$$

$$= x + \frac{1}{2}$$

$$h(y) = \int_0^1 (x+y)\, dx$$

$$= \left[\frac{x^2}{2} + xy\right]_{x=0}^{1}$$

$$= \frac{1}{2} + y$$

c) Random variables X and Y will be independent,

if, $g(x).h(y) = f(x,y)$

here, $g(x).h(y) = (x+\frac{1}{2})(\frac{1}{2}+y)$

$$= \frac{x}{2} + \frac{1}{4} + xy + \frac{y}{2}$$

$$= \frac{2x+1+4xy+2y}{4} \neq f(x,y)$$

$\therefore$ not independent.

d)

$$f(x|y=0.5) = \frac{f(x, y=0.5)}{h(y=0.5)}$$

$$= \frac{x+0.5}{\frac{1}{2}+0.5}$$

$$= \frac{x+0.5}{1}$$

$$= x+0.5$$

(4)

Marginal Probability mass function:

a) $P(X=1) = 0.10 + 0.15 + 0 + 0.05 = 0.3$

$P(X=2) = 0.20 + 0.05 + 0.05 + 0.20 = 0.5$

$P(X=3) = 0.05 + 0 + 0.10 + 0.05 = 0.2$

$P(Y=0) = 0.10 + 0.20 + 0.05 = 0.35$

$P(Y=1) = 0.15 + 0.05 + 0 = 0.2$

$P(Y=2) = 0 + 0.05 + 0.10 = 0.15$

$P(Y=3) = 0.05 + 0.20 + 0.05 = 0.3$

b) $P(x \mid Y=1) = \dfrac{P(x, Y=1)}{P(Y=1)}$

$P(Y=1) = 0.15 + 0.05 + 0$
$= 0.2$

$\therefore P(X=1 \mid Y=1) = \dfrac{P(1,1)}{P(Y=1)} = \dfrac{0.15}{0.2} = 0.75$

$\therefore P(X=2 \mid Y=1) = \dfrac{P(2,1)}{P(Y=1)} = \dfrac{0.05}{0.2} = 0.25$

$\therefore P(X=3 \mid Y=1) = \dfrac{P(3,1)}{P(Y=1)} = \dfrac{0}{0.2} = 0$

c) we know

$$E(X \mid Y=1) = \sum_{i=1}^{3} i\, p(X \mid Y=1)$$

$$= 1 \cdot P(X=1 \mid Y=1) + 2\, P(X=2 \mid Y=1) + 3\, P(X=3 \mid Y=1)$$

$$= (1 \times 0.75) + (2 \times 0.25) + (3 \times 0)$$

[Values from 'b']

$$= 1.25$$

d) $V(X \mid Y=1) = E((x \mid Y=1)^2) - (E(X \mid Y=1))^2$

$$= 1.75 - (1.25)^2$$

$$= 0.19$$

here,

$E(X \mid Y=1) = 1.25$ [from 'c']

$$E((X \mid Y=1)^2) = \sum_{i=1}^{3} i^2\, P(X \mid Y=1)$$

$$= (1 \times 0.75) + (2^2 \times 0.25) + (3^2 \times 0)$$

$$= 1.75$$

e) $E(XY) = \sum_{i=1}^{3} \sum_{j=0}^{3} ij\, P_{ij}$

$= (1 \times 0 \times 0.10) + (1 \times 1 \times 0.15) + (1 \times 2 \times 0) + (1 \times 3 \times 0.05)$

$+ (2 \times 0 \times 0.20) + (2 \times 1 \times 0.05) + (2 \times 2 \times 0.05) + (2 \times 3 \times 0.20)$

$+ (3 \times 0 \times 0.05) + (3 \times 1 \times 0) + (3 \times 2 \times 0.10) + (3 \times 3 \times 0.05)$

$= 2.85$