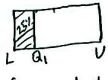
cumulative distribution function, CDF $= F(x) = \int_L^x f(x)\,dx$

Quartiles:

Quartiles are ~~three~~ three (3) cut points that will divide a dataset into four (4) equal-sized groups.

1) 1st quartile / lower quartile (Q_1) is known as lower quartile and 25% of the data lies below this point.



$F(x) = \int_L^{Q_1} f(x)\,dx = 25\%$

$F(Q_1) = 25\%$

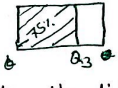
For 1st quartile, $\boxed{F(x) = 0.25}$ = cdf

2) 2nd quartile / $\boxed{\text{Median } (Q_2)}$ is the median of a data set and 50% of the data lies below this point.

$F(Q_2) = \int_L^{Q_2} f(x)\,dx = 0.50$

For median, $\boxed{F(x) = 0.50}$ = cdf

3) 3rd quartile / upper quartile (Q_3) is known as upper quartile and 75% of the data lies below this point.



For 3rd quartile, $\boxed{F(x) = 0.75}$ = cdf

Interquartile range (IQR):

Interquartile range is the difference between upper quartile and lower quartile.

$$\boxed{IQR = Q_3 - Q_1}$$

#Example: Suppose that the diameter of a metal cylinder has a probablity density funⁿ $f(x) = 1.5 - 6(x - 50.0)^2$ for $49.5 \le x \le 50.5$

(i) Show that total area under the probablity density funⁿ = 1 or Prove that this is a valid PDF.

(ii) Calculate the probablity that a metal cylinder has a diameter between 49.8 and 50.1 mm

(iii) Find cumulative distribution function.

(iv) Calculated the expected diameter of a metal cylinder.

(v) Find 1st, 2nd and 3rd quartile of a metal cylinder diameter.

Solⁿ: 1st quartile : $F(x) = 0.25$ [from solⁿ iii]

$\Rightarrow 1.5x - 2(x-50)^3 - 74.5 = 0.25 \Rightarrow 1.5x - 2(x^3 - 150x^2 + 7500x - 125000) - 74.5 = 0.25 = 0$

$\Rightarrow 1.5x - 2x^3 + 300x^2 - 15000x + 25000 - 74.75 = 0$

$\Rightarrow -2x^3 + 300x^2 - 14998.5x + 24992.25 = 0$

calculator

$\Rightarrow x = 50.9$ or 49.8 or 49.2, Since $49.5 \le x \le 50.5$. & So, 1st quartile $= 49.8$.

(Note: Model set up $\Rightarrow$ 5 : EQN $\Rightarrow$ 4 $\Rightarrow (-2) \Rightarrow = \Rightarrow (300) \Rightarrow = \Rightarrow (-14998.5) \Rightarrow = \Rightarrow 24992.25 \Rightarrow =$)

2nd quartile : $F(x) = 0.50$

$\Rightarrow 1.5x - 2(x-50)^3 - 74.5 - 0.50 = 0$

$\Rightarrow 1.5x - 2(x^3 - 150x^2 + 7500x - 125000) - 74.5 - 0.50 = 0$

$\cancel{\Rightarrow -2x^3 + 1.5x - 2x^3 + 300x^2 - 14998.5x + 249925.25 = 0}$

$\cancel{\Rightarrow 1.5x - 2x^3 + 300x^2 - 15000x + 250000 - 74.75 = 0}$

$\Rightarrow -2x^3 + 300x^2 - 14998.5x + 249925.25 = 0$

$\cancel{\therefore x = 50.9 \text{ or } 49.8 \text{ or } 49.2}$ $\Rightarrow -2x^3 + 300x^2 - 14998.5x + 249925 = 0$

since, $49.5 \le x \le 50.5$ $\therefore x = 49.1$ or 50.8 or 50

So 2nd quartile $= 50$ / Median $= 50$

range = 2.0
$Q_1 = 49.8$

3rd quartile : $F(x) = 0.75$

$\Rightarrow 1.5x - 2(x-50)^3 - 74.5 - 0.75 = 0$

$\Rightarrow 1.5x - 2(x^3 - 150x^2 + 7500x - 125000) - 74.5 - 0.75 = 0$

$\Rightarrow -2x^3 + 300x^2 - 14998.5x + 249924.25 = 0$

$\therefore x = 49$ or 50.7 or 50.2

since $49.5 \le x \le 50.5$

So 3rd quartile $= 50.2$

(vi) Find interquartile range.

Soln : IQR = 3rd quartile − 1st quartile $= 50.2 - 49.8 = 0.4$

$0.25 = Q_1$

$0.50 = Q_2$ / median

$0.75 = Q_3$

$IQR = Q_3 - Q_1$

Jointly distributed random Variable

Joint Probablity Distribution

The funⁿ $f(x,y)$ is a joint probablity density function of the continuous & random variables X and Y if

1. $f(x,y) \geq 0$, for all (x,y),
2. $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\,dx\,dy = 1$.

JPDF

two or more variable हो तो JPDF

i) $f(x,y) \geq 0$

ii) $\int_c^d\int_a^b f(x,y)\,dx\,dy$ ~~[a<x<b, c<y<d]~~ $[a<x<b, c<y<d]$

Marginal probablity distribution

The marginal distribution of a random variable X is obtained from the joint probablity distribution of two random variables X and Y by integrating over the value of the random variable Y. The marginal distribution is the individual probablity distribution of the random variable X considered alone. Similarly, The marginal distribution of a random variable Y is obtained from the joint probablity distribution of two random variable X and Y by integrating over the values of the random variable X. The marginal distribution is the individual probablity distribution of the random variable Y considered alone.

$f(x,y)$

$g(x) = \int_{-\infty}^{\infty} f(x,y)\,dy$ and $h(y) = \int_{-\infty}^{\infty} f(x,y)\,dx$

$g(x)$ M.P.D.F of x $\quad$ $h(y)$ M.P.D.F of y $= \int_a^b f(x,y)\,dx$

$g(x) = \int_c^d f(x,y)\,dy$

$f(x,y) = 4xy$

$h(y) = \int 4xy\,dx = 4y$

$g(x) = \int 4xy\,dy = 4x$

Conditional probablity

Let X and Y be two random variables, discrete or continuous. The conditional distribution of the random variable Y given that $X = x$ is,

$P(A|B) = \frac{P(A\cap B)}{P(B)}$

$$f(y|x) = \frac{f(x,y)}{g(x)}, \text{ provided } g(x) > 0$$

$f(y|x) = \frac{f(x,y)}{g(x)}$

Similarly, the condition distribution of X given that $Y = y$ is

$$f(x|y) = \frac{f(x,y)}{h(y)}, \text{ provided } h(y) > 0$$

Statical Independence

Let X and Y be two random variables, discrete or continuous, with joint probability distribution $f(x,y)$ and marginal distributions $g(x)$ and $h(y)$, respectively. The random variables X and Y are said to be ~~स्ता~~ statistically independent if and only if

$$f(x,y) = g(x)h(y)$$

$=$ हो तो statistically independent।

$\neq$ " " " नहीं।

Covariance

The covarience of two random variables ~~or~~ X and Y is defined to be

$$\boxed{\begin{aligned} Cov(X,Y) &= E((X-E(X))(Y-E(Y))) \\ &= E(XY) - E(X)E(Y) \end{aligned}}$$

If range of the covarience is $-\infty$ to ∞ then range " " correlation is -1 to 1

The covariance can be any positive or negative number, and independent random variables have a covariance of 0. (zero).

Correlation

The correlation between two random variables X and Y is defined to be

$$\boxed{Corr(X,Y) = \frac{Cov(X,Y)}{\sqrt{Var(X)Var(Y)}}}$$

The correlation takes value between -1 and 1, and independent random variables have a correlation of 0. (zero).

Class Work

Suppose that two continuous random variables X and Y have a joint probablity density func

$$f(x,y) = A(x-3)y$$

for $-2 \le x \le 3$ and $4 \le y \le 6$

(*1) if covariance range : $-\infty < cov < +\infty$ mean value
then correlation " " : $-1 < corr < +1$

(*) If covariance ; correlation = 0. then it means that there is no relation between the two variable.

(*2) - cov one var , corr = +ve, " two variable are positively related
so ~~cov~~ increase↑ then other variable also increase↑

(*4) If one var decreese " " also decrease

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1921826642
MAT 361.4

Assignment 03

$$f(x,y) = A(x-3)y \qquad -2 \le x \le 3 \; ; \; 4 \le y \le 6$$

*, A এর value কে বের করতে হবে/validity check.

(a) $\int_4^6 \int_{-2}^3 f(x,y)\,dx\,dy = 1$ $\quad$ [$\because$ pdf = 1, for total area]

$\Rightarrow A \int_4^6 \int_{-2}^3 (x-3)y \, dx \, dy = 1$

$\Rightarrow A \int_4^6 \left[\frac{x^2}{2} - 3x\right]_{-2}^3 y \, dy = 1$

$\Rightarrow A \int_4^6 \left[\frac{9}{2} - 9 - \frac{4}{2} - 6\right] y \, dy = 1.$

$\Rightarrow \left(-\frac{25A}{2}\right) \left[\frac{y^2}{2}\right]_4^6 = 1$

$\Rightarrow -\frac{25}{2} \cdot A \cdot \frac{20}{2} = 1$

$\therefore A = -1/125$ Answer ✓

(b) $P(0 \le x \le 1, \; 4 \le y \le 5)$ And

$= \int_4^5 \int_0^1 -\frac{1}{125} (x-3)y \, dx \, dy$

$= -\frac{1}{125} \int_4^5 \left[\frac{x^2}{2} - 3x\right]_0^1 y \, dy$

$= -1/125 \cdot \left[\frac{1}{2} - 3\right] \left[\frac{y^2}{2}\right]_4^5$

$= \left(-\frac{1}{125}\right) \cdot \left(-\frac{1}{5}\right) \cdot \left(\frac{9}{2}\right)$

$= 0.09.$ Answer

P (x এর range, y-এর range)
= value বসাতে।

(c) Marginal probablity density functions,

$$g(x) = \int_4^6 -\frac{1}{125}(x-3)y\,dy$$

$$= -\frac{1}{125}(x-3)\left[\frac{y^2}{2}\right]_4^6$$

$$= -\frac{1}{125}\cdot 10\,(x-3)$$ ~~Answer~~

$$\therefore g(x) = -\frac{2}{25}(x-3).$$ Answer

$$h(y) = \int_{-2}^{3} -\frac{1}{125}(x-3)y\,dx$$

$$= -\frac{1}{125}y\left[\frac{x^2}{2} - 3x\right]_{-2}^{3}$$

$$= -\frac{y}{125}\left[\frac{9}{2} - 9 - 2 - 6\right]$$

$$\therefore h(y) = y/10$$ Answer

marginal density fun^ বের করতে বলা আছে

$$g(x) = \int_{y=L/c}^{H/d} f(x,y)\,dy$$

= x এর কিছু expression

usually;

$$h(y) = \int_{x=L/a}^{H/b} f(x,y)\,dx$$

= y এর কিছু ...

[equation আসবে]

(d) Two random variable are said to be independent if ~~g(x) h~~

$$g(x).h(y) = f(x,y)$$

$$\therefore g(x).h(y) = -\frac{2}{25}(x-3)\cdot\frac{y}{10}$$

$$= -\frac{1}{125}(x-3)y$$

$$= f(x,y).$$

So, x and y are independent. Answer

independent/statical independent বললে, $g(x).h(y) = f(x,y)$ হবে।

$\neq$ হলে not independent

(e) $$f(y=5) = \frac{f(x, y=5)}{h(y=5)} = \frac{-\frac{1}{125}(x-3)\cdot 5}{5/10}$$

$$= -\frac{2}{25}(x-3)$$ Answer

y=□ conditional p... of x

$$f(y=\square) = \frac{f(x, y=\square)}{h(y=\square)}$$

$$f(x|y=\square) = \frac{f(x, y=\square)}{h(y=\square)}$$

[* equation আসবে]

(f) $E(x) = \int_{-2}^{3} x\, g(x)\, dx$

$= \int_{-2}^{3} -\frac{2x}{25}(x-3)\, dx$

$= -\frac{2}{25}\int_{-2}^{3}(x^2-3x)\, dx$

$= -\frac{2}{25}\left[\frac{x^3}{3} - \frac{3x^2}{2}\right]_{-2}^{3}$

$= -\frac{2}{25}\left[9 - \frac{27}{2} + \frac{8}{3} + 6\right]$

$= -\frac{2}{25} \cdot \frac{25}{6}$

$\therefore E(x) = -1/3$. Answer

$E(y) = \int_4^6 y\, h(y)\, dy = \int_4^6 y \cdot y/10\, dy$

$= \frac{1}{10}\left[\frac{y^3}{3}\right]_4^6$

$= \frac{1}{10}\left[\frac{6^3}{3} - \frac{4^3}{3}\right] = \frac{1}{10} \cdot \frac{152}{3}$

$E(y) = 76/15$ Answer

$E(x^2) = \int_{-2}^{3} x^2 g(x)\, dx = -\frac{2}{25}\int_{-2}^{3} x^2 (x-3)\, dx$

$= -\frac{2}{25}\left[\frac{x^4}{4} - \frac{3x^3}{3}\right]_{-2}^{3}$

$= -\frac{2}{25}\left[\frac{3^4}{4} - 3^3 - \frac{(-2)^4}{4} + (-2)^3\right]$

$= \left(-\frac{2}{25}\right) \cdot \left(-\frac{75}{4}\right)$

$\therefore E(x^2) = 3/2$

Expectation / average বলাকে

~~$E(x) = \int_L^H x\, g(x,y)\, dx$~~

$E(x) = \int_L^H x\, g(x)\, dx$

and

$E(y) = \int_L^H y\, h(y)\, dy$

দুইটাই বের করতে হবে।

আর variance বলাকে $E(x)$, $E(y)$ এর লাগে

$E(x^2) = \int_L^H x^2 g(x)\, dx$

& $E(y^2) = \int_L^H y^2 h(y)\, dy$

বের করতে হবে।

কারণ,

$V(x) = E(x^2) - (E(x))^2$

& $V(y) = E(y^2) - (E(y))^2$

[$E(x)$, $E(y)$ & $V(x)$, $V(y)$ are values]

$$E(y^2) = \int_4^6 y^2 h(y)\,dy$$
$$= \int_4^6 y^2 \cdot \frac{y}{10}\,dy = \int_4^6 \frac{1}{10} y^3\,dy$$
$$= \frac{1}{10}\left[\frac{y^4}{4}\right]_4^6$$
$$= \frac{1}{10}\cdot\left[6^4/4 - 4^3\right] = \frac{1}{10}\cdot 260$$

$\therefore E(y^2) = 26.$

$\therefore$ varience, $V(x) = E(x^2) - (E(x))^2$
$$= \frac{3}{2} - \left(-\tfrac{1}{3}\right)^2 = \frac{3}{2} - \frac{1}{9}$$
$$= \frac{25}{18} \quad \text{Answer}$$

$$V(y) = E(y^2) - (E(y))^2$$
$$= 26 - \left(\frac{76}{15}\right)^2$$
$$= \frac{5774}{225} \quad \text{Answer}$$

(g) Covariance of X and Y, $\text{cov}(x,y) = E(xy) - E(x)\cdot E(y)$

Here, $E(xy) = \int_4^6\int_{-2}^3 (xy)\left(-\frac{1}{125}(x-3)y\right)dx\,dy.$

$$= -\frac{1}{125}\int_4^6\int_{-2}^3 (x^2-3x)y^2\,dx\,dy$$
$$= -\frac{1}{125}\int_4^6 y^2\left[\frac{x^3}{3} - \frac{3x^2}{2}\right]_{-2}^3 dy$$
$$= -\frac{1}{125}\int_4^6 y^2\,dy\cdot\left[9 - \frac{27}{2} + \frac{8}{3} + 6\right]$$
$$= -\frac{1}{125}\cdot\frac{25}{6}\cdot\left[y^3/3\right]_4^6$$
$$= -\frac{1}{125}\cdot\frac{25}{6}\cdot\frac{152}{3}.$$
$$= -\frac{76}{45} \quad \text{Answer}$$

যদি $\boxed{\text{covariance}}$ বলতে, $E(x), E(y)$ & $E(xy) = \int_c^d\int_a^b xy f(x,y)\,dx\,dy$ বের করতে হবে। [প্রতিটি value]

covariance,
$\text{cov}(x,y) = E(xy) - E(x)E(y)$

$$\text{cov}(x,y) = E(xy) - E(x)E(y) = -\frac{76}{45} - \left[\left(-\tfrac{1}{3}\right)\cdot\frac{76}{15}\right] = -\frac{76}{45} + \frac{76}{45} = 0 \quad \text{Ans}$$

(h) correlation between X and Y,

$$corr(x,y) = \frac{cov(x,y)}{\sqrt{v(x).v(y)}}$$

$$= \frac{0}{\sqrt{v(x).v(y)}}$$

$$= 0$$ Answer

correlation বের করতে বলালে covariance [$cov(x,y) = E(xy) - E(x)E(y)$ রূপ], $v(x) = E(x^2) - (E(x))^2$ & $v(y) = E(y^2) - (E(y))^2$ বের করতে হবে।

correlation, $corr(x,y) = \frac{cov(x,y)}{\sqrt{v(x).v(y)}}$

0 হলে কোনো relation নেই। +ve হলে +vely related, -ve হলে negatively

Classwork:

Given, $f(x) = \frac{1}{x \ln(1.5)}$; $4 \le x \le 6$ and $f(x) = 0$ elsewhere

(a) ~~median~~

$$CDF = F(x) = \int_4^x \frac{1}{x \cdot \ln(1.5)} dx$$

$$= \frac{1}{\ln(1.5)} [\ln x]_4^x$$

$$= \frac{1}{\ln(1.5)} [\ln x - \ln 4]$$

$$= \frac{1}{\ln(1.5)} \times [\ln \frac{x}{4}]$$

q_2 মানে median.

q_2 বের করতে CDF মানে ফাংশন $f(x)$ কে integrate করতে হয় L থেকে x interval -1.

2nd quartile = median = $F(x) = 0.50$

$$\frac{1}{\ln(1.5)} \times \ln \frac{x}{4} = 0.5$$

$$\Rightarrow \ln \frac{x}{4} = 0.5 \times \ln 1.5$$

$$\Rightarrow \ln \frac{x}{4} = 0.203 \Rightarrow \frac{x}{4} = e^{0.203}$$

$$\Rightarrow x = 4e^{0.203}$$

$\therefore$ median = 4.899 Answer

(b) ~~For up~~ lower ~~upper~~ quartiles,

$$F(x) = 0.25$$

$$\Rightarrow \frac{1}{\ln(1.5)} \cdot \ln\left(\frac{x}{4}\right) = 0.25$$

lower quartile $= Q_1$

Upper " $= Q_3$

$$\Rightarrow \ln(x/4) = 0.25 \times \ln(1.5)$$

$$\Rightarrow \ln\left(\frac{x}{4}\right) = 0.1014$$

$$\Rightarrow x = 4e^{0.1014}$$

$$\therefore x = 4.43$$

$\therefore$ lower quartile $= 4.43$ Answer

For upper quartiles,

$$F(x) = 0.75$$

$$\Rightarrow \ln(x/4) = 0.75 \times \ln(1.5)$$

$$\Rightarrow x = 4e^{(0.75 \times \ln(1.5))}$$

$$= 5.42$$

$\therefore$ upper quartiles $= 5.422$ Answer

(c) Interquartile range,

$$IQR = \text{upper quartile} - \text{lower quartile}$$

$$= Q_3 - Q_1$$

$$= 5.422 - 4.43$$

$$= \frac{124}{125}$$

$$= 0.992$$ Answer

$IQR = Q_3 - Q_1$

Joint probablity mass function JPMF | JPDF → continuous

If the random variables are $\boxed{\text{discrete}}$ (certain), then the joint probablity mass function consists of probablity values $P(X=i, Y=j) = P_{ij}$ satisfying (↳ Probablity)

$$\boxed{\checkmark (1)\ 0 \le P_{ij} \le 1 \qquad \checkmark (2)\ \sum\sum P_{ij} = 1}$$

Marginal Probablity Distribution:

Even though two random variables X and Y may be jointly distributed, if interest is focused on only one of the random variables, then it is appropriate to consider the probablity distribution of that random variable alone. This is known as the marginal distribution of the random variable and can be obtained quite simply by summing the joint probablity distribution over the values of the other random variable.

For example, for two discrete random variables X and Y, the probablity values of the marginal distribution of X are $\boxed{P(X=i) = \sum_j P_{ij} \quad [i=1,2,3,4 \ \& \ j=1,2,3]}$

$$\checkmark P(X=i) = \sum_{j=1}^{3} P_{ij} = P_{i1} + P_{i2} + P_{i3}$$

$$\checkmark P(X=1) = \sum_{j=1}^{3} P_{1j} = P_{11} + P_{12} + P_{13}$$

$P(x, y)$ ← JPMF (suppose)

$P(x)$ → marginal probablity Distribution of x.

$P(y)$ → marginal probablity Distribution of y

Conditional Probablity Distributions:

If two random variables X and Y are jointly distributed, then it is sometimes useful to consider the distribution of one random variable conditional on the other random variable having taken a particular value.

The conditional probablity distribution of random variable X conditional on the event $Y=j$ consists of the probablity values $\boxed{P(X \mid Y=j) = \frac{P(X, Y=j)}{P(Y=j)}}$

$$P(x \mid y=2) = \frac{P(x, y=2)}{P(y=2)} \ ; \ P(x=1 \mid y=2) = \frac{P(x=1, y=2)}{P(y=2)} = \frac{0.08}{0.57}$$

নির্দিষ্ট value দেয়া থাকলে

Covariance: $\boxed{Cov(X,Y) = E(XY) - E(X)E(Y)}$

Correlation: $\boxed{Corr(X,Y) = \frac{Cov(X,Y)}{\sqrt{V(X)V(Y)}}}$ −1 to 1

Question: A company that services air conditioner units in residences. If the random variable X, taking the values 1, 2, 3 and 4, is the service time in hours taken at a particular location, and the random variable Y, taking the values 1, 2 and 3, is the number of air conditioner units at the location, then these two random variables can be thought of as jointly related.

Suppose that their joint probablity mass fun^c P_{ij} is given in the following Figure. The figure indicates, for example, that there is a probablity of 0.12 that ~~a p~~ X = 1 & Y = 1, so that there is a probablity of 0.12 that a particular location chosen at random has one air conditioner unit that takes a technician one hour to service. Similarly, there is a probablity of 0.07 that a location has three air conditioner units that take four hours to service.

X = service time (hrs)

Y = number of air conditioner units	1 hour	2h	3h	4h	marginal distribution of Y
1	0.12	0.08	0.07	0.05	→ 0.32 P(Y=1)
2	0.08	0.15	0.21	0.13	→ 0.57 P(Y=2)
3	0.01	0.01	0.02	0.07	→ 0.11 P(Y=3)
	0.21 P(X=1)	0.24 P(X=2)	0.30 P(X=3)	0.25 P(X=4)	← Marginal distribution of X.

∴ certain so desired ফলাফল function হবে JPMF

11 means, indicates a technician take 1 hour to service 1 air conditioner → probablity value $P_{ij} = P_{11} = 0.12$

Q 1) Show that this is a valid joint probablity mass function

Soln: $\sum_{i=1}^{4}\sum_{j=1}^{3} P_{ij} = 0.12 + 0.08 + \cdots + 0.07 = 1.00$.

validity check, সব যোগ করে 1 আসলে valid.

Q2) Calculate the probablity that a location has no more than two air conditioner units that take no more than two hours to service.

২টা বেশি না এবং location ২ঘণ্টার বেশি সময় নিবে না

Soln: $P_{11} + P_{12} + P_{21} + P_{22} = 0.12 + 0.08 + 0.08 + 0.15 = 0.43$

simply যোগ করতে হবে

Q3) Find marginal probablity mass func.

Soln:

$P(X=1) = 0.12 + 0.08 + 0.01 = 0.21$

$P(X=2) = 0.08 + 0.15 + 0.01 = 0.24$

$P(X=3) = 0.07 + 0.21 + 0.02 = 0.30$

$P(X=4) = 0.05 + 0.13 + 0.07 = 0.25$

X এর জন্য column যোগ
Y এর জন্য ROW যোগ সেরিয়াল

Similarly, you also need to calculate ($P(Y=1)$, $P(Y=2)$ & $P(Y=3)$)

$P(Y=1) = 0.12 + 0.08 + 0.07 + 0.05 = 0.32$

$P(Y=2) = 0.08 + 0.15 + 0.21 + 0.13 = 0.57$

$P(Y=3) = 0.01 + 0.01 + 0.02 + 0.07 = 0.11$

Q4) Calculate the expected $\boxed{\text{service time}}$ and the variance in the service times.

Sol^n: $E(x) = \sum x P(x)$, $E(x^2) = \sum x^2 P(x)$

The expected service time is $E(x) = \sum_{i=1}^{4} i P(x=i)$ $\quad [x = 1,2,3,4]$ $[P(x=1) = P_{11} + P_{21} + P_{31} + P_{41}$, $P(x=2) = P_{12} + P_{22} + P_{32}$.

$= 1P(x=1) + 2P(x=2) + 3P(x=3) + 4P(x=4)$

$= (1 \times 0.21) + (2 \times 0.24) + (3 \times 0.30) + (4 \times 0.25)$

$= 2.59.$

We know, $V(x) = E(x^2) - (E(x))^2$

Since $E(x^2) = \sum_{i=1}^{4} i^2 P(x=i)$

$= 1^2 P(x=1) + 2^2 P(x=2) + 3^2 P(x=3) + 4^2 P(x=4)$

$= (1 \times 0.21) + (4 \times 0.24) + (9 \times 0.30) + (16 \times 0.25)$

$= 7.87.$

The variance in the service times is:-

$V(x) = E(x^2) - (E(x))^2 = 7.87 - (2.59)^2 = 1.162$ Answer

Q5) Calculate the expected $\boxed{\text{number of units}}$ serviced and the variance in the ~~number~~ number of units serviced.

Sol^n: The expected number of units serviced,

$E(Y) = \sum_{j=1}^{3} j P(Y=j)$

$= 1P(Y=1) + 2P(Y=2) + 3P(Y=3)$

$= 1 \times 0.32 + 2 \times 0.57 + 3 \times 0.11$

$= 1.79$

We know, $V(Y) = E(Y^2) - (E(Y))^2$

Here, $E(Y^2) = \sum_{j=1}^{3} j^2 P(Y=j)$

$= 1 \times 0.32 + 4 \times 0.57 + 9 \times 0.11$

$= 3.59$

The variance in the number of units serviced $V(Y) = 3.59 - (1.79)^2$

$= 0.3859$

Q6) Suppose that a technician is visiting a location that is known to have three air conditioner units, what is the conditional distribution of the service time X.

$y = 3$

Sol^n: We know, $P(X|Y=j) = \dfrac{P(X, Y=j)}{P(Y=j)} \Rightarrow P(X|Y=3) = P(X, Y=3)/P(Y=3)$

Here, $P(Y=3) = 0.01 + 0.01 + 0.02 + 0.07 = 0.11$

$P(X=1|Y=3) = P(X=1, Y=3)/P(Y=3) = \dfrac{0.01}{0.11} = 0.091$

$P(X=2|Y=3) = P(X=2, Y=3)/P(Y=3) = 0.01/0.11 = 0.091$

$P(X=3|Y=3) = P(X=3, Y=3)/P(Y=3) = 0.02/0.11 = 0.182$

$P(X=4|Y=3) = P(X=4, Y=3)/P(Y=3) = 0.07/0.11 = 0.636$ Answer

$P(X|Y=3)$

*1 $P(Y=3)$ আমাদের কাছে থাকতে হবে।

*2 এখন এখন কাজ

$P(X=1|Y=3)$

$P(X=2|Y=3)$

$\vdots$

7) Calculate the conditional expectation of the service time.

Soln:

$$E(X|Y=3) = \sum_{i=1}^{4} \overset{=i}{i} P(X|Y=3)$$

$$= (1 \times 0.091) + (2 \times 0.091) + (3 \times 0.182) + (4 \times 0.636)$$

$$= 3.36$$

$$\cancel{E(Y|X=3) = \sum_{j=1}^{3} \overset{=j}{j} P(Y|X=3)}$$

$$\cancel{= (1 \times}$$

8) Calculate the covariance an correlation.

Soln:

$$Cov(X,Y) = E(X,Y) - E(X)E(Y)$$

The expected service time is $E(X) = 2.59$ hours, and the expected number of units serviced is $E(Y) = 1.79$.

$$E(X,Y) = \sum_{i=1}^{4} \sum_{j=1}^{3} ij P_{ij} = (1 \times 1 \times 0.12) + (1 \times 2 \times 0.08) + \dots + (4 \times 3 \times 0.07)$$

$$= 4.86.$$

$$Cov(X,Y) = 4.86 - 2.56 \times 1.79 = 0.224$$

We know,

$$Corr(X,Y) = \frac{Cov(X,Y)}{\sqrt{V(X)V(Y)}} = \frac{0.224}{\sqrt{1.162 \times 0.384}} = 0.34$$

1 টি $_1$ এর expectation বের করে,

$$E(x) = \sum_{i=1}^{4} i P(x=i)$$

1 টি $_2$ conditional expectation

$$E(x|Y=k) = \sum_{i=1}^{4} i P(x=i|y=k)$$