PHY 107
Work and Energy

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OUTLINE

- Energy and Kinetic Energy
- Work and Kinetic Energy
- Work-Kinetic Energy Theorem
- Work done by a spring force
- Work done by a general variable force

Energy and Kinetic Energy

Energy is a scalar quantity associated with the state (or condition) of one or more objects

Focus is on only one type of energy (kinetic energy) and on only one way in which energy can be transferred (work).

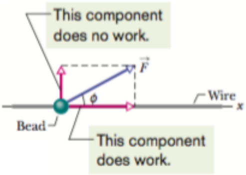
Kinetic energy K is energy associated with the state of motion of an object.

$$K.E = \frac{1}{2}mv^2$$

Work W is energy transferred to or from an object by means of a force acting on the object. Energy transferred to the object is positive work, and energy transferred from the object is negative work

Work and Kinetic Energy

$F_x = ma_x$
$v^2 = v_0^2 + 2a_x d$
$0.5mv^2 - 0.5mv_0^2 = F_x d$
To calculate the work a force does on an object as the object moves through some displacement, we use only the force component along the object's displacement. The force component perpendicular to the displacement does zero work.



Work done by a constant force: $W = Fd\cos(\phi) = \vec{F}.\vec{d}$

Work-Kinetic Energy Theorem

Let ΔK be the change in the kinetic energy of the object, and let W be the net work done on it.

$$\Delta K = K_f - K_i = W$$

$$K_f = K_i + W$$

KE after the net work= KE before the net work + Net work done
If the kinetic energy of a particle is initially 5 J and there is a net transfer of 2 J to the particle (positive net work), the final kinetic energy is 7 J. If, instead, there is a net transfer of 2 J from the particle (negative net work), the final kinetic energy is 3 J

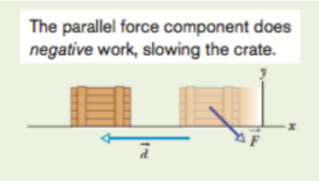
Work-Kinetic Energy Theorem

EXAMPLE Work done by a constant force in unit-vector notation

During a storm, a crate of crepe is sliding across a slick, oily parking lot through a displacement $\vec{d} = (3.0m)\hat{i}$ while a steady wind pushes against the crate with a force $F = (2.0N)\hat{i} + (-6.0N)\hat{j}$.

(a) How much work does this force do on the crate during the displacement?

(b) If the crate has a kinetic energy of 10 J at the beginning of displacement $\vec{d}$, what is its kinetic energy at the end of $\vec{d}$?



Work Energy Theorem

Solution:

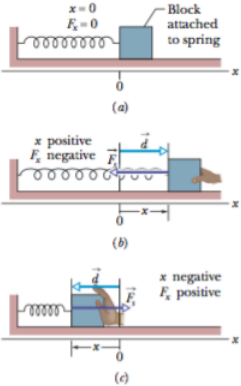
a) $W = \overrightarrow{F}.\overrightarrow{d} = [2\hat{i} - 6\hat{j}].[-3\hat{i}] = -6J$

b) Because the force does negative work on the crate, it reduces the crate's kinetic energy.

$K_f = K_i + W = 10 + -6 = 4J$

Work done by a spring force

Force in a spring (a variable force)
The force $\overrightarrow{F_s}$ from a spring is proportional to the displacement $\overrightarrow{d}$ of the free end from its position when the spring is in the relaxed state: $F_s = -k\overrightarrow{d}$ (Hooke's Law)

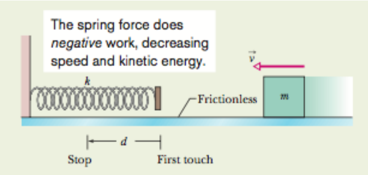


The constant k is called the spring constant (a measure of the stiffness of the spring).

Work done by a spring force

$W_s = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2$
This work W_s done by the spring force can have a positive or negative value, depending on whether the net transfer of energy is to or from the block as the block moves from x_i to x_f

EXAMPLE Work done by spring to change kinetic energy A cumin canister of mass $m = 0.40kg$ slides across a horizontal frictionless counter with speed $v = 0.50m/s$. It then runs into and compresses a spring of spring constant $k = 750N/m$. When the canister is momentarily stopped by the spring, by what distance d is the spring compressed?

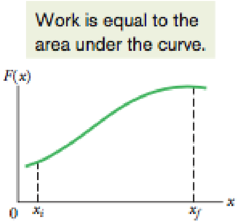


Work-Energy Theorem :

$K_f - K_i = -0.5kd^2$
$0 - \frac{1}{2}mv^2 = -\frac{1}{2}kd^2$

Work done by a general variable force

Check the bead in one of the previous slides
Now consider the force to be in the +ve x direction and the force magnitude to vary with position x
Magnitude of force changes with position, BUT, magnitude at a certain position does not change with time



$W = \int_{x_i}^{x_f} F(x)dx$ (Work;variable force)

Work done by a general variable force

EXAMPLE Work, 2D integration

Force $\overrightarrow{F} = (3x^2N)\hat{i} + 4N\hat{j}$, with x in meters, acts on a particle, changing only the kinetic energy of the particle. How much work is done on the particle as it moves from coordinates (2 m, 3 m) to (3 m, 0 m)? Does the speed of the particle increase, decrease, or remain the same?

Solution: $W = \int_2^3 3x^2dx + \int_3^0 4dy = 3[\frac{1}{3}x^3]_2^3 + 4[y]_3^0 = 7J$

Energy is transferred to the particle and KE of the particle increases.

Reference

Fundamentals of Physics by Halliday and Resnik