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1. Given,

$$x_1 + 2x_2 - 3x_3 + 4x_4 = 2$$

$$2x_1 + 5x_2 - 2x_3 + x_4 = 1$$

$$5x_1 + 12x_2 - 7x_3 + 6x_4 = 3$$

The augmented matrix for this system of equations

is

$$\begin{bmatrix} 1 & 2 & -3 & 4 & 2 \\ 2 & 5 & -2 & 1 & 1 \\ 5 & 12 & -7 & 6 & 3 \end{bmatrix}$$

Adding -2 times the first row to second and adding  
-5 times the first row to third, we get

$$\begin{bmatrix} 1 & 2 & -3 & 4 & 2 \\ 0 & 1 & 4 & -7 & -3 \\ 0 & 2 & 8 & -14 & -7 \end{bmatrix}$$

second

Adding -2 times the ~~first~~ row to third, we get

$$\begin{bmatrix} 1 & 2 & -3 & 4 & 2 \\ 0 & 1 & 4 & -7 & -3 \\ 0 & 0 & 0 & 0 & -1 \end{bmatrix}$$

~~Multi~~ Multiplying third row by -1, we get

$$\begin{bmatrix} 1 & 2 & -3 & 4 & 2 \\ 0 & 1 & 4 & -7 & -3 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

~~At~~ Adding -2 times the first third row to first and adding 3 times the third row to second, we get

$$\begin{bmatrix} 1 & 2 & -3 & 4 & 0 \\ 0 & 1 & 4 & -7 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Adding -2 times the second row to first, we get

$$\begin{bmatrix} 1 & 0 & -11 & 18 & 0 \\ 0 & 1 & 4 & -7 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

The corresponding system of equations is

$$x_1 - 11x_3 + 18x_4 = 0$$

$$x_2 + 4x_3 - 7x_4 = 0$$

$$0 = 1$$

Since the third equation is contradictory, the system of equations has no solution.



2. Given,

$$A = \begin{bmatrix} 5 & -7 & 1 \\ -7 & 8 & 2 \\ 1 & 2 & -4 \end{bmatrix}$$

here Now,

$$A^T = \begin{bmatrix} 5 & -7 & 1 \\ -7 & 8 & 2 \\ 1 & 2 & -4 \end{bmatrix}$$

$$\therefore \text{tr}(A^T) = 5 + 8 - 4 = 9$$

$$\begin{aligned} A^2 + 2A + \text{tr}(A^T)I &= \begin{bmatrix} 5 & -7 & 1 \\ -7 & 8 & 2 \\ 1 & 2 & -4 \end{bmatrix} \begin{bmatrix} 5 & -7 & 1 \\ -7 & 8 & 2 \\ 1 & 2 & -4 \end{bmatrix} \\ &= A^2 + 2A + 9I = \begin{bmatrix} 5 & -7 & 1 \\ -7 & 8 & 2 \\ 1 & 2 & -4 \end{bmatrix} + 2 \begin{bmatrix} 5 & -7 & 1 \\ -7 & 8 & 2 \\ 1 & 2 & -4 \end{bmatrix} + 9 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{aligned}$$

$$= \begin{bmatrix} 25+49+1 & -35-56+2 & 5-14-4 \\ -35-56+2 & 49+64+4 & -7+16-8 \\ 5-14-4 & -7+16-8 & 1+4+16 \end{bmatrix}$$

$$+ \begin{bmatrix} 10 & -14 & 2 \\ -14 & 16 & 4 \\ 2 & 4 & -8 \end{bmatrix} + \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

$$= \begin{bmatrix} 75 & -89 & -13 \\ -89 & 117 & 1 \\ -13 & 1 & 21 \end{bmatrix} + \begin{bmatrix} 10 & -14 & 2 \\ -14 & 16 & 4 \\ 2 & 4 & -8 \end{bmatrix} = \begin{bmatrix} 85 & -103 & -11 \\ -103 & 131 & 5 \\ -11 & 5 & 13 \end{bmatrix}$$

$$+ \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

$$= \begin{bmatrix} 94 & -103 & -11 \\ -103 & 142 & 5 \\ -11 & 5 & 22 \end{bmatrix}$$