

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} \left(\frac{0}{0} \text{ form} \right) = \lim_{x \rightarrow 0} \frac{\cos x}{1} = \lim_{x \rightarrow 0} \cos x = \cos 0 = 1$$

L'Hospital's Rule

$$\lim_{x \rightarrow a} \frac{f(x)}{g(x)} \left[\frac{f(a)}{g(a)} \text{ is either } \frac{0}{0} \text{ or } \frac{\infty}{\infty} \right]$$

$$\lim_{x \rightarrow a} \frac{f(x)}{g(x)}$$

$$\lim_{x \rightarrow 0^+} \frac{\cot x}{\ln x} \left(\frac{\infty}{\infty} \text{ form} \right)$$

$$\frac{\cot 0}{\ln 0} = \frac{\infty}{\infty}$$

$$= \lim_{x \rightarrow 0^+} \frac{-\csc^2 x}{\frac{1}{x}}$$

$$= \lim_{x \rightarrow 0^+} \frac{-x}{\sin^2 x}$$

$$= \lim_{x \rightarrow 0^+} \frac{-x}{1 - \cos^2 x}$$

$$= \lim_{x \rightarrow 0^+} \frac{-x}{(1 + \cos x)(1 - \cos x)}$$

$$= \lim_{x \rightarrow 0^+} \frac{1}{\frac{1 - \cos x}{x}}$$

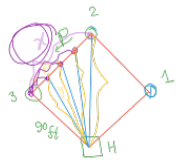
$$= \lim_{x \rightarrow 0^+} \frac{1}{\frac{1 - \cos x}{x}}$$

$$= \infty \left(\frac{1}{\frac{1}{2}} \right) = \infty$$

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$$

$$\lim_{x \rightarrow 0} \frac{1 - \cos x}{x} = 0$$

$$\frac{0}{0} \text{ or } \frac{\infty}{\infty}$$



$$x^2 + y^2 = 1$$

$$\frac{d}{dt}(x^2 + y^2) = \frac{d}{dt}(1)$$

$$\Rightarrow 2x \frac{dx}{dt} + 0 = 2y \frac{dy}{dt}$$

$$\Rightarrow \frac{dy}{dx} = \frac{x}{y} \frac{dx}{dt}$$

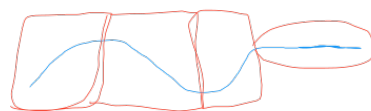
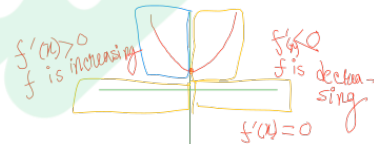
$$= \frac{-20}{\sqrt{8500}} \approx -0.0068$$

$$x = 20 \text{ ft}$$

$$\frac{dx}{dt} \Big|_{x=20} = -30 \text{ ft/s}$$

$$y = \sqrt{x^2 + 90^2} = \sqrt{20^2 + 90^2} = \sqrt{8500}$$

$$\frac{dy}{dt} \Big|_{x=20} = -6.51 \text{ ft/s}$$



Increasing $f(x) = x^2 + 2$

$x = 2, f(2) = 6$
 $x = 3, f(3) = 11$
 $x = 4, f(4) = 18$

Decreasing

$x = -4, f(-4) = 18$
 $x = -3, f(-3) = 11$
 $x = -2, f(-2) = 6$

$$f(x) = 5 + 12x - x^2$$

$$f'(x) = \frac{d}{dx} (5 + 12x - x^2)$$

$$= 12 - 2x$$

$$= 3(4 - x)$$

$$= 3(2+x)(2-x)$$

$$\text{We set, } f'(x) = 0$$

$$\Rightarrow 3(2+x)(2-x) = 0 \Rightarrow x = 2, -2$$

Intervals	Sign of $f'(x)$	Conclusion
$-\infty < x < -2$	$(+)(-)(+) = (-)$	f is decreasing
$-2 < x < 2$	$(+)(+)(+) = (+)$	f is increasing
$2 < x < \infty$	$(+)(+)(-) = (-)$	f is decreasing

$$f''(x) = -6x$$

$$\text{We set } f''(x) = 0$$

$$\Rightarrow 6x = 0 \Rightarrow x = 0$$

$$D_f = (-\infty, \infty)$$

$$\text{Interval}$$

$$\text{Sign of } f''(x) = -6x$$

$$\text{Conclusion}$$

$$-\infty < x < 0 \quad (-)(-) = (+) \quad f \text{ is concave up}$$

$$0 < x < \infty \quad (-)(+) = (-) \quad f \text{ is concave down}$$

$x = 0$ is a point of inflection.



Concavity:

$$f''(x) = 0$$

$$x \rightarrow \text{inflection point.}$$

