Exercise 6.9

15. $y = \sqrt{4x + \cosh^2 5x}$

$$\frac{dy}{dx} = \frac{4 + 2\cosh 5x \sinh 5x \times 5}{2\sqrt{4x + \cosh^2 5x}} = \frac{2 + 5\cosh(5x)\sinh(5x)}{\sqrt{4x + \cosh^2(5x)}}$$

16. $y = \sinh^3(2x)$

$$\frac{dy}{dx} = 3\sinh^2(2x)\cosh(2x) \times 2 = 6\sinh^2(2x)\cosh(2x)$$

17. $y = x^3 \tanh^2(\sqrt{x})$

$$\frac{dy}{dx} = 3x^2 \tanh^2(\sqrt{x}) + x^3 \times 2\tanh(\sqrt{x}) \times \frac{1}{2}x^{-1/2} \times \operatorname{sech}^2(\sqrt{x})$$

$$= 3x^2 \tanh^2(\sqrt{x}) + x^{5/2} \tanh(\sqrt{x}) \operatorname{sech}^2(\sqrt{x})$$

18. $y = \sinh(\cos 3x)$

$$\frac{dy}{dx} = \cosh(\cos 3x) \times -\sin 3x \times 3 = -3\cosh(\cos 3x)\sin 3x$$

19. $y = \sinh^{-1}\left(\frac{1}{3}x\right)$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1 + \left(\frac{x}{3}\right)^2}} \times \frac{1}{3} = \frac{1}{\sqrt{9 + x^2}}$$

20. $y = \sinh^{-1}\left(\frac{1}{x}\right)$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1 + \left(\frac{1}{x}\right)^2}} \times \left(\frac{-1}{x^2}\right) = -\frac{1}{x^2\sqrt{x^2 + 1}}$$

21. $y = \ln(\cosh^{-1}x)$

$$\frac{dy}{dx} = \frac{1}{\cosh^{-1}x} \times \frac{1}{\sqrt{x^2-1}} = \frac{1}{\cosh^{-1}x\sqrt{x^2-1}}$$

22. $y = \cosh^{-1}(\sinh^{-1}x)$

$$\frac{dy}{dx} = \frac{1}{\sqrt{(\sinh^{-1}x)^2-1}} \times \frac{1}{\sqrt{1+x^2}} = \frac{1}{\sqrt{[(\sinh^{-1}x)^2-1][1+x^2]}}$$

23. $y = \frac{1}{\tanh^{-1}x}$

$$\frac{dy}{dx} = -(\tanh^{-1}x)^{-2} \times \frac{1}{1-x^2} = \frac{-(\tanh^{-1}x)^2}{1-x^2}$$

24. $y = (\coth^{-1}x)^2$

$$\frac{dy}{dx} = 2\coth^{-1}x \times \frac{1}{1-x^2} = \frac{2\coth^{-1}x}{1-x^2}$$

25. $y = \cosh^{-1}(\cosh x)$

$$\frac{dy}{dx} = \frac{1}{\sqrt{(\cosh x)^2-1}} \times \sinh x = \frac{\sinh x}{|\sinh x|}$$

29. $\int \sinh^6 x \cosh x\, dx$

Put, $u = \sinh x \Rightarrow du = \cosh x\, dx$

So, $\int u^6\, du = \frac{u^7}{7} + C = \frac{\sinh^7 x}{7} + C$

30. $\int \cosh(2x-3)\,dx$

Put, $u = 2x-3 \Rightarrow du = 2dx \Rightarrow dx = \frac{du}{2}$

So, $\frac{1}{2}\int \cosh u\,du = \frac{1}{2}\sinh u + C = \frac{1}{2}\sinh(2x-3) + C$

31. $\int \sqrt{\tanh x}\,\operatorname{sech}^2 x\,dx$

Put, $\tanh x = u \Rightarrow \operatorname{sech}^2 x\,dx = du$

So, $\int u^{1/2}\,du = \frac{2}{3}u^{3/2} + C = \frac{2}{3}(\tanh x)^{3/2} + C$

32. $\int \operatorname{cosech}^2(3x)\,dx$

Put, $3x = u \Rightarrow dx = \frac{du}{3}$

So, $\frac{1}{3}\int \operatorname{cosech}^2 u\,du = -\frac{1}{3}\coth(3x) + C$

33. $\int \tanh x\,dx$

$= \int \frac{\sinh x}{\cosh x}\,dx =$

Put, $u = \cosh x \Rightarrow du = \sinh x\,dx$

So, $\int \frac{du}{u} = \ln u + C = \ln|\cosh x| + C$

39. $\int \coth^2 x \operatorname{cosech}^2 x \, dx$

Put, $u = \coth x \quad \Rightarrow du = -\operatorname{cosech}^2 x \, dx$

So, $-\int u^2 du = -\frac{1}{3}\coth^3 x + C$

35. $\int_{\ln 2}^{\ln 3} \tanh x \operatorname{sech}^3 x \, dx$

Put, $u = \operatorname{sech} x \quad \Rightarrow du = -\operatorname{sech} x \tanh x \, dx$

$-\int u^2 du = -\frac{1}{3}\operatorname{sech}^3 x + C$

So, $\int_{\ln 2}^{\ln 3} \tanh x \operatorname{sech}^3 x \, dx = -\frac{1}{3}\left[\operatorname{sech}^3 x\right]_{\ln 2}^{\ln 3} = \frac{37}{375}$

36. $\int_0^{\ln 3} \frac{e^x - e^{-x}}{e^x + e^{-x}} dx$

$= \int_0^{\ln 3} \tanh x \, dx$

$= \int_0^{\ln 3} \frac{\sinh x \, dx}{\cosh x}$

$= \left[\ln(\cosh x)\right]_0^{\ln 3} = \ln 5 - \ln 3$

37. $\int \frac{dx}{\sqrt{1+9x^2}}$

Put, $u = 3x \Rightarrow \frac{du}{3} = dx$

So, $\frac{1}{3}\int \frac{du}{\sqrt{1+u^2}} = \frac{1}{3}\sinh^{-1} u + C = \frac{1}{3}\sinh^{-1} 3x + C$

38. $\int \frac{dx}{\sqrt{x^2-2}}$

Put, $x = \sqrt{2}u \Rightarrow dx = \sqrt{2}\,du$

So, $\int \frac{\sqrt{2}\,du}{\sqrt{2u^2-2}} = \int \frac{du}{\sqrt{u^2-1}} = \cosh^{-1} u + C = \cosh^{-1}\left(\frac{x}{\sqrt{2}}\right) + C$

39. $\int \frac{dx}{\sqrt{1-e^{2x}}}$

Put, $u = e^x \Rightarrow du = e^x dx$

$\int \frac{e^x dx}{e^x\sqrt{1-e^{2x}}}$

$= \int \frac{du}{u\sqrt{1-u^2}} = -\operatorname{sech}^{-1} u + C = -\operatorname{sech}^{-1}(e^x) + C$

40. $\int \frac{\sin\theta \, d\theta}{\sqrt{1+\cos^2\theta}}$

Put, $u = \cos\theta \Rightarrow du = -\sin\theta \, d\theta$

So, $-\int \frac{du}{\sqrt{1+u^2}} = -\sinh^{-1}(\cos\theta) + C$

41. $\int \frac{dx}{2\sqrt{1+9x^2}}$

Put, $u = 2x \Rightarrow du = 2dx$

So, $\int \frac{2dx}{2x\sqrt{1+9x^2}} = \int \frac{du}{u\sqrt{1+u^2}} = -\operatorname{cosech}^{-1}(2x) + C$

42. $\int \frac{dx}{\sqrt{9x^2-25}}$

$= \frac{1}{3}\int \frac{dx}{\sqrt{x^2-\frac{25}{9}}} = \frac{1}{3}\cosh^{-1}\left(\frac{3x}{5}\right) + C$

43. $\int_0^{1/2} \frac{dx}{1-x^2}$

$= \left[\tanh^{-1}x\right]_0^{1/2} = \tanh^{-1}\frac{1}{2} - \tanh^{-1}0$

44. $\int_0^{\sqrt{3}} \frac{dt}{\sqrt{t^2+1}}$

$= \left[\sinh^{-1}t\right]_0^{\sqrt{3}} = \sinh^{-1}\sqrt{3} - \sinh^{-1}0$