

Periodic function A function  $f(x)$  is said to be a periodic function if

$$f(x+T) = f(x)$$

where,  $T =$  be the period of the given function.

Example:  $f(x) = \sin x$

$$\therefore f(x+2\pi) = \sin(x+2\pi) = \sin x = f(x)$$

So  $2\pi$  be the period of  $\sin x$

The general form of  $\sin x / \cos x$

and 
$$\begin{aligned} f(x) &= a \sin(bx-c) + d \\ f(x) &= a \cos(bx-c) + d \end{aligned}$$

where  $a =$  amplitude

$bx-c =$  angular momentum

$d =$  shifting phase

\* The general form of Sinusoidal function

$$f(x) = a \sin(wx) \quad \text{where } w = bx - c \text{ is angular momentum.}$$

or  $f(x) = a \cos(wx)$

where  $a$  = amplitude

$$(bx - c) = w = \text{angular momentum}$$

$$T = \text{period} = \frac{2\pi}{w} = \frac{2\pi}{b}$$

$$\text{Starting point} \Rightarrow bx - c = w = 0$$

$$\therefore \boxed{bx - c = 0}$$

$$x\text{-scale point} = \frac{\text{Period}}{4}$$

$$\text{Ending point} \Rightarrow bx - c = 2\pi \Rightarrow w = 2\pi$$

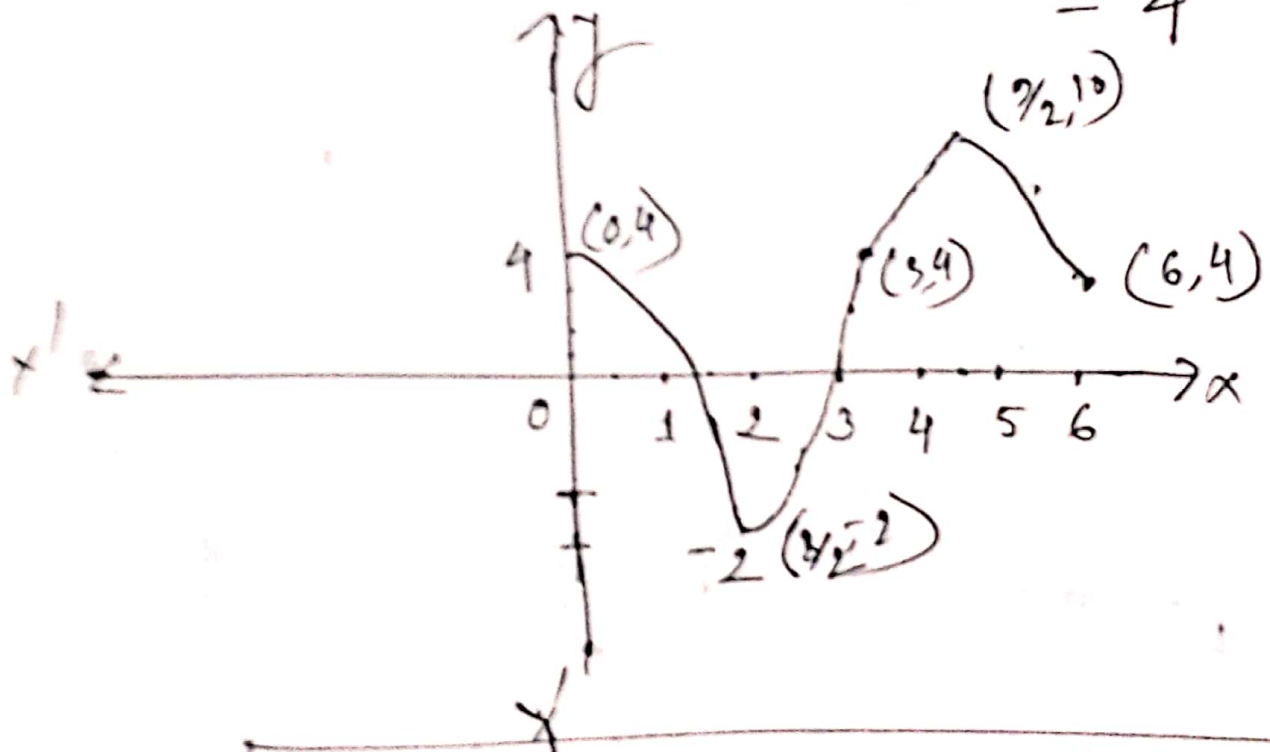
$$\therefore \boxed{bx - c = 2\pi}$$

The above properties is used for graphing Sinusoidal Function of the form

$$y = a \sin(wx) = a \sin(bx - c)$$

$$y = a \cos(wx) = a \cos(bx - c)$$

Starting point  $x=0$ ,  $f(0)=4$   
 next point  $0+\frac{3}{2}=\frac{3}{2}$   $\therefore f(\frac{3}{2})=-6+4=-2$   
 next point  $\frac{3}{2}+\frac{3}{2}=3$   $\therefore f(3)=-6\cdot 0+4=4$   
 next point  $3+\frac{3}{2}=\frac{9}{2}$   $\therefore f(\frac{9}{2})=6+4=10$   
 next point  $\frac{9}{2}+\frac{3}{2}=6$   $f(6)=-6\cdot 0+4=4$



graph of  $y = -6 \ln\left(\frac{\pi}{3}x\right) + 4$

Domain  $= (-\infty, \infty)$   
 Range  $= [-2, 10]$

$$\text{starting point} = -\pi/2 ; f(-\pi/2) = -3 \cos(-\pi/2 + \pi) = -3$$

2nd point : starting point +  $\pi$ -scale point

$$= \boxed{-\pi/2 + \pi/4 = -\pi/4}$$

$$\therefore f(-\pi/4) = -3 \cos(2(-\pi/4) + \pi) = 0$$

Next point :  $\boxed{-\pi/4 + \pi/4 = 0}$

$$\therefore f(0) = -3 \cos(0 + \pi) = 3$$

Next point :  $\boxed{0 + \pi/4 = \pi/4}$

$$\therefore f(\pi/4) = -3 \cos(2 \cdot \pi/4 + \pi) = -3 \cos(3\pi/2) = 0$$

Next point :  $\boxed{\pi/4 + \pi/4 = 2\pi/4 = \pi/2}$

$$\therefore f(\pi/2) = -3 \cos(2 \cdot \pi/2 + \pi) = -3$$