



ANSWER ANY FOUR

1. (a) Find the rank and nullity of the following system of linear equations:

$$x_1 + 2x_2 - x_3 + 4x_4 = 0$$

$$2x_1 - x_2 + 3x_3 + 3x_4 = 0$$

$$4x_1 + x_2 + 3x_3 + 9x_4 = 0$$

$$x_2 - x_3 + x_4 = 0$$

$$2x_1 + 3x_2 - x_3 + 7x_4 = 0$$

(5)

2. Show that the vectors $(1,1,0), (1,0,1)$ and $(0,1,1)$ form a basis or not for \mathbb{R}^3

(5)

3. (a) Define subspace and Verify whether is subspace or not:

$$W = \{(x, y, z) : x, y, z \in \mathbb{R}^3 \text{ and } x + y = 3z\}$$

- (b) Let u, v and w are independent vectors, show that $u+v, u-v$ and $u-2v+w$ are independent

4. Define eigen values and eigen vectors. Compute eigen values and corresponding eigen vectors of

$$A = \begin{bmatrix} 3 & 3 \\ 1 & 5 \end{bmatrix}. \text{ Verify } A^{-5} \text{ is diagonalizable or not.}$$

5. (a) Find the equation of the circle passing through the points $(2, -2), (3, 5)$ and $(-4, 6)$. Also find its center and radius.

- (b) Solve the linear program by graphical method:

$$\text{Maximize, } Z = x_1 + 2x_2$$

Subject to :

$$2x_1 + x_2 \leq 4$$

$$x_1 + x_2 \geq 1$$

$$x_2 \leq 5$$

$$x_1 \leq 4$$

$$x_1, x_2 \geq 0.$$