I

4(b)

$\int \sin^5 x \cos^6 x \, dx.$

$= \int \sin^4 x \cos^6 x \, \sin x \, dx.$

$= \int (\sin^2 x)^2 \cos^6 x \, \sin x \, dx.$

$= \int (1 - \cos^2 x)^2 \cos^6 x \, \sin x \, dx.$

Set, $u = \cos x$ then, $\dfrac{du}{dx} = - \sin x$

$\implies \sin x \, dx = - du.$

Now,

$\int \sin^5 x \cos^6 x \, dx = \int (1 - \cos^2 x)^2 \cos^6 x \, \sin x \, dx.$

$= \int (1 - u^2)^2 \, u^6 \cdot (-1) \, du.$

$$= -\int (1-u^2)^2 u^6 \, du.$$

$$= -\int [1 - 3u^2$$

$$= -\int [1 - 2u + u^2$$

$$= -\int [(1)^2 - 2 \cdot 1 \cdot u^2 + (u^2)^2] u^6 \, du$$

$$= -\int [1 - 2u^2 + u^4] u^6 \, du$$

$$= -\int [u^6 - 2u^8 + u^{10}] \, du.$$

$$= -\left[\frac{u^6}{6} - \frac{2}{8} \right]$$

$$= -\left[\frac{1}{7} u^7 - \frac{2}{9} u^9 + \frac{1}{11} u^{11} \right] + C$$

$$= -\left[\frac{1}{7}\cos 7x - \frac{2}{9}\cos 9x + \frac{1}{11}\cos 11x\right] + C.$$

(Ans).