$y = f(x)$

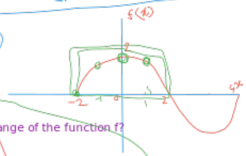
$$\frac{dy}{dx} = f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Fundamental rule of differentiation.

1. $\frac{d}{dx}(a) = 0$ 2. $\frac{d}{dx}(x^n) = nx^{n-1}$

3. $\frac{d}{dx}(e^x) = e^x$ 4. $\frac{d}{dx}(\ln x) = \frac{1}{x}$

$\frac{d}{dx}(5x)$



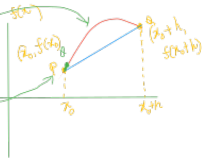
Derivative: Rate of change

What is the rate of change of the function f?

Average rate of change

Instantaneous rate of change

$$\boxed{\frac{f(x_0+h) - f(x_0)}{x_0+h-x_0}}$$



$$\lim_{h \to 0} \frac{f(x_0+h) - f(x_0)}{x_0+h-x_0}$$

$y = x^2+1$ (a) Avg. r. c. $[3, 5]$

$y = x^2+1$ (b) Inst. r. c. $x = -4$

$$r_{avg} = \frac{f(x_0+h) - f(x_0)}{x_0+h-x_0} = \frac{f(5) - f(3)}{5-3}$$

$$= \frac{26-10}{2} = \frac{16}{2} = 8$$

$$r_{inst} = \lim_{h \to 0} \frac{f(x_0+h) - f(x_0)}{x_0+h-x_0}$$

$$= \lim_{h \to 0} \frac{f(-4+h) - f(-4)}{-4+h+4}$$

$y = x^2+1$

$f(-4+h)$

$f(-4)$

$$= \lim_{h \to 0} \frac{(-4+h)^2+1-(-4)^2-1}{h}$$

$$= \lim_{h \to 0} \frac{16-8h+h^2-16}{h} = \lim_{h \to 0} (-8+h)$$

$$= -8+0 = -8$$

$y = x^2+1$

Find the instantaneous rate of change of "y" w.r.t. "x" at x = -4.

$$\boxed{\left.\frac{dy}{dx}\right|_{x=-4}}$$

$$\frac{dy}{dx} = \frac{d}{dx}(x^2+1)$$

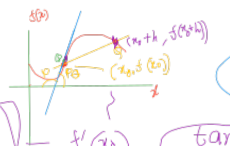
$$= \frac{d}{dx}(x^2) + \frac{d}{dx}(1)$$

$$= 2x^{2-1} + 0 = 2x$$

$$\left.\frac{dy}{dx}\right|_{x=-4} = 2(-4) = -8$$

slope

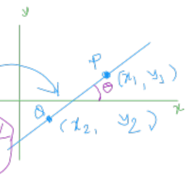
$\boxed{\tan \theta} = ??$



$$\boxed{\lim_{h \to 0} \frac{f(x_0+h) - f(x_0)}{x_0+h-x_0}} = f'(x_0)$$

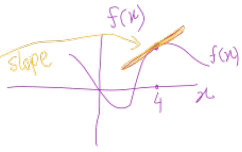
$$m = \frac{y_2-y_1}{x_2-x_1}$$

$$\tan\theta = m = \frac{y_2-y_1}{x_2-x_1}$$



$f'(x)$

$\boxed{f'(4)}$



$y = f(x) = 2\sqrt{x} + 5x^2$

① Find $f'(x)$

② Find the slope of a tangent line to the curve at $x = 4$.

$\frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}}$

$f(x) = 2\sqrt{x} + 5x^2$

$$\frac{d}{dx}[f(x)] = \frac{d}{dx}[2\sqrt{x} + 5x^2]$$

$$\Rightarrow f'(x) = \frac{d}{dx}(2\sqrt{x}) + \frac{d}{dx}(5x^2)$$

$$\boxed{\frac{d}{dx}(x^n) = nx^{n-1}}$$

$$\Rightarrow f'(x) = 2\frac{d}{dx}(x^{1/2}) + 5\frac{d}{dx}(x^2)$$

$$= 2 \cdot \frac{1}{2} x^{1/2-1} + 5 \cdot 2x^{2-1}$$

$$= x^{-1/2} + 10x$$

①

$$f'(x) = \frac{1}{\sqrt{x}} + 10x$$

②

The slope is given by

$$f'(4) = \frac{1}{\sqrt{4}} + 10 \cdot 4 = \frac{1}{2} + 40$$

$$= \boxed{40.5}$$