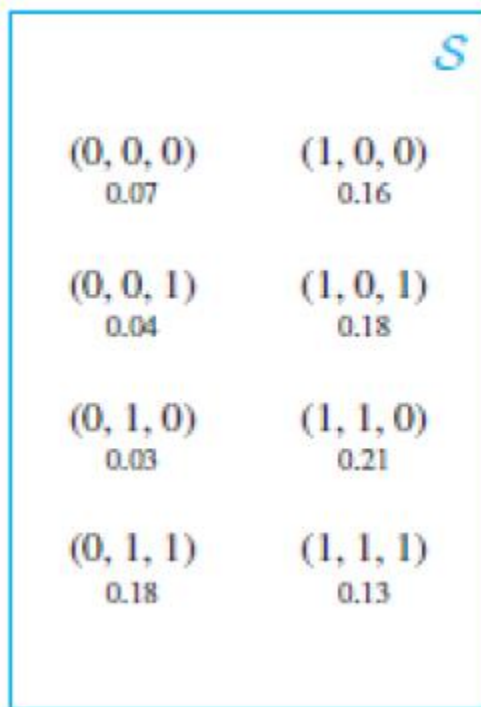


**Expectation of a Random Variable:** Expectation of a Random Variable is denoted by  $E(X)$  and represents an 'average' value of the random variable.

For discrete random variable,  $E(x) = \sum xp(x)$

For continuous random variable,  $E(x) = \int_L^U xf(x)dx$

**Example:** A manager supervises the operation of three power plants, plant X, plant Y, and plant Z. At any given time, each of the three plants can be classified as either generating electricity (1) or being idle (0). With the notation (0, 1, 0) used to represent the situation where plant Y is generating electricity but plants X and Z are not generating electricity. Here, random variable X be the number of plants generating electricity.



S

(0, 0, 0) 0.07	(1, 0, 0) 0.16
(0, 0, 1) 0.04	(1, 0, 1) 0.18
(0, 1, 0) 0.03	(1, 1, 0) 0.21
(0, 1, 1) 0.18	(1, 1, 1) 0.13

- 1) Calculate the probability mass function
- 2) Draw PMF
- 3) Calculate CDF
- 4) Draw CDF
- 5) Calculate the expected number of power plant generating electricity.

**Solution:  $X = \{0,1,2,3\}$**

$P(0) = 0.07$  (probability that no plants are generating electricity)

$P(1) = 0.23$  (probability that one plant is generating electricity)

$P(2) = 0.57$  (probability that two plants are generating electricity)

$P(3) = 0.13$  (probability that three plants are generating electricity)

$$E(x) = \sum xp(x) = 0*0.07 + 1*0.23 + 2*0.57 + 3*0.13 = 1.76$$

**Example:** Suppose that the diameter of a metal cylinder has a probability density function  $f(x) = 1.5 - 6(x - 50.0)^2$  for  $49.5 \leq x \leq 50.5$

i) Show that total area under the probability density function = 1 or Prove that this is a valid PDF.

ii) Calculate The probability that a metal cylinder has a diameter between 49.8 and 50.1 mm.

iii) Find cumulative distribution function.

**iv) Calculated the expected diameter of a metal cylinder**

**Solution:** We know,  $E(x) = \int_L^U xf(x)dx$

$$= \int_{49.5}^{50.5} x(1.5 - 6(x - 50.0)^2) dx$$

$$= \int_{49.5}^{50.5} x(1.5 - 6(x^2 - 100x + 2500)) dx$$

$$= \int_{49.5}^{50.5} (1.5x - 6x(x^2 - 100x + 2500)) dx$$

$$= \int_{49.5}^{50.5} (1.5x - 6x^3 + 600x^2 - 15000x) dx$$

$$= \left[ 1.5 \frac{x^2}{2} - 6 \frac{x^4}{4} + 600 \frac{x^3}{3} - 15000 \frac{x^2}{2} \right]_{49.5}^{50.5} \quad \left[ \int x^n dx = \frac{x^{n+1}}{n+1} \right]$$

$$= -3123100 - (-3123150)$$

$$= 50$$

**Variance:** Variance measures how far a set of numbers are spread out from their average value.

For discrete random variable,

$$V(x) = E(x^2) - (E(x))^2 = \sum x^2 p(x) - (\sum x p(x))^2$$

For continuous random variable,

$$V(x) = E(x^2) - (E(x))^2 = \int_L^U x^2 f(x) dx - \left( \int_L^U x f(x) dx \right)^2$$

**Standard deviation:** Standard deviation is the square root of the variance.

$$SD(x) = \sqrt{V(x)}$$

**Example:** A manager supervises the operation of three power plants, plant X, plant Y, and plant Z. At any given time, each of the three plants can be classified as either generating electricity (1) or being idle (0). With the notation (0, 1, 0) used to represent the situation where plant Y is generating electricity but plants X and Z are not generating electricity. Here, random variable X be the number of plants generating electricity.

<i>S</i>	
(0, 0, 0) 0.07	(1, 0, 0) 0.16
(0, 0, 1) 0.04	(1, 0, 1) 0.18
(0, 1, 0) 0.03	(1, 1, 0) 0.21
(0, 1, 1) 0.18	(1, 1, 1) 0.13

- 1) Calculate the probability mass function
- 2) Draw PMF
- 3) Calculate CDF
- 4) Draw CDF
- 5) Calculate the expected number of power plant generating electricity.

**6) Calculate variance and standard deviation**

**Solution:  $X = \{0,1,2,3\}$**

$P(0) = 0.07$  (probability that no plants are generating electricity)

$P(1) = 0.23$  (probability that one plant is generating electricity)

$P(2) = 0.57$  (probability that two plants are generating electricity)

$P(3) = 0.13$  (probability that three plants are generating electricity)

$$E(x) = \sum xp(x) = 0*0.07 + 1*0.23 + 2*0.57 + 3*0.13 = 1.76$$

$$E(x^2) = \sum x^2P(x) = 0^2*0.07 + 1^2*0.23 + 2^2*0.57 + 3^2*0.13 = 3.68$$

$$V(x) = E(x^2) - (E(x))^2 = 3.68 - (1.76)^2 = 0.5824$$

$$SD(x) = \sqrt{V(x)} = 0.7632$$

**Example:** Suppose that the diameter of a metal cylinder has a probability density function  $f(x) = 1.5 - 6(x - 50.0)^2$  for  $49.5 \leq x \leq 50.5$

i) Show that total area under the probability density function = 1 or Prove that this is a valid PDF.

ii) Calculate The probability that a metal cylinder has a diameter between 49.8 and 50.1 mm.

iii) Find cumulative distribution function.

iv) Calculated the expected diameter of a metal cylinder

**v) Calculate variance and standard deviation**

**Solution: v)** We know,  $V(x) = E(x^2) - (E(x))^2$

$E(x) = 50$  [from previous solution]

$$E(x^2) = \int_L^U x^2 f(x) dx$$

$$= \int_{49.5}^{50.5} x^2 (1.5 - 6(x - 50.0)^2) dx$$

$$= \int_{49.5}^{50.5} x^2 (1.5 - 6(x^2 - 100x + 2500)) dx$$

$$= \int_{49.5}^{50.5} (1.5x^2 - 6x^2(x^2 - 100x + 2500)) dx$$

$$= \int_{49.5}^{50.5} (1.5x^2 - 6x^4 + 600x^3 - 15000x^2) dx$$

$$= \left[ 1.5 \frac{x^3}{3} - 6 \frac{x^5}{5} + 600 \frac{x^4}{4} - 15000 \frac{x^3}{3} \right]_{49.5}^{50.5}$$

$$= -62436241 + 62438741$$

$$= 2500$$

$$V(x) = 2500 - 50^2 = 0$$

$$SD(x) = \sqrt{V(X)} = 0$$