Exam Introduction Robotics (4L160)

23/4/2009, Thursday, 9.00-12.00

General

- You are allowed to use the book, the slides of the lectures, your notes, and your laptop.
- There are some typographical errors in the book. It is completely your responsibility to make sure that what you quote from the book is accurate.
- Grades:

problem	points
1	20
2	25
3	25
4	30
5	20
total	120

1

The matrix $\mathbf{R}_n^0(t)$ defines orientation of the coordinate frame $o_n x_n y_n z_n$, attached to the tip of a robot relative to the coordinate frame of the base $o_0 x_0 y_0 z_0$. Variable t denotes time. Determine the vector of angular velocities $\boldsymbol{\omega}_n^0$ of the robot tip in the base frame if:

a) $\mathbf{R}_n^0(t) = \begin{bmatrix} \cos(3t) & 0 & \sin(3t) \\ 0 & 1 & 0 \\ -\sin(3t) & 0 & \cos(3t) \end{bmatrix}$,

b) $\mathbf{R}_n^0(t) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(2t) & \sin(2t) \\ 0 & -\sin(2t) & \cos(2t) \end{bmatrix}$,

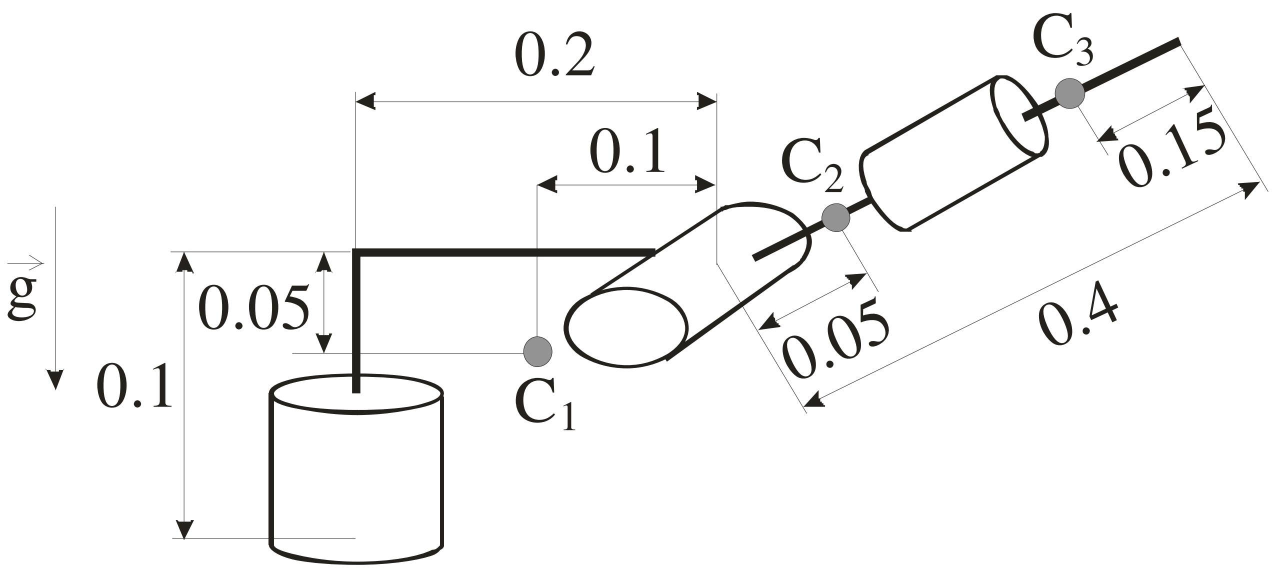
c) $\mathbf{R}_n^0(t) = \begin{bmatrix} \cos(t) & \sin(t) & 0 \\ -\sin(t) & \cos(t) & 0 \\ 0 & 0 & 1 \end{bmatrix}$,

d) $\mathbf{R}_n^0(t) =$

$$\begin{bmatrix} \frac{1+3\cos(t)}{4} & \frac{1-\cos(t)}{4} - \frac{\sqrt{2}}{2}\sin(t) & \frac{\sqrt{2}}{4}(1-\cos(t)) + \frac{1}{2}\sin(t) \\ \frac{1-\cos(t)}{4} + \frac{\sqrt{2}}{2}\sin(t) & \frac{1+3\cos(t)}{4} & \frac{\sqrt{2}}{4}(1-\cos(t)) - \frac{1}{2}\sin(t) \\ \frac{\sqrt{2}}{4}(1-\cos(t)) - \frac{1}{2}\sin(t) & \frac{\sqrt{2}}{4}(1-\cos(t)) + \frac{1}{2}\sin(t) & \frac{1+\cos(t)}{2} \end{bmatrix}.$$

Hint: the time-derivative of a rotation matrix relates this matrix with the corresponding vector of angular velocities.

Assign coordinate frames according to the Denavits-Hartenberg (DH) convention to the RRR robot manipulator shown in the figure below. Derive the forward kinematics equations (homogenous transformation matrix $\mathbf{H}_3^0$) for this manipulator.



3

For the manipulator considered in the problem 2, determine Jacobian matrix which relates the joint velocities with angular velocities of the coordinate frame $o_3x_3y_3z_3$ attached to the tip. For what manipulator configurations is this Jacobian matrix singular? What are the joint velocities in the following cases:

a) the joint positions are $\mathbf{q} = [\pi/2 \quad \pi/2 \quad \pi/2]^T$, while the vector of angular velocities $\boldsymbol{\omega}_3^0$ of the tip is equal to the solution of problem 1 a)?

b) the joint positions are $\mathbf{q} = [\pi \quad \pi/2 \quad \pi/2]^T$, while the vector of angular velocities of the tip $\boldsymbol{\omega}_3^0$ is equal to the solution of problem 1 b)?

c) the joint positions are $\mathbf{q} = [0 \quad 0 \quad \pi/2]^T$, while the vector of angular velocities of the tip $\boldsymbol{\omega}_3^0$ is equal to the solution of problem 1 c)?

d) the joint positions are $\mathbf{q} = [\pi/4 \quad 3\pi/4 \quad \pi/2]^T$, while the vector of angular velocities $\boldsymbol{\omega}_3^0$ of the tip is equal to the solution of problem 1 d)?

4

For the manipulator considered in the problem 2, masses of links 1, 2, and 3 are equal to 2 [kg], 1 [kg], and 2 [kg], respectively. The inertia tensor $\mathbf{I}_i$ of each link i (i=1,2,3), expressed relative to the coordinate frame attached to the link center of the mass, is equal to

$$\mathbf{I}_i = \begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 0.1 \end{bmatrix}.$$

The coordinates of the link centers of masses C_i, i=1,2,3, as well as direction of the gravity vector $\vec{g}$ ($g = 9.81$ [m/s^2]), are indicated in the figure shown in the problem 2.

1. Write down the total kinetic energy.
2. Write down the total potential energy.
3. Derive inertia matrix.
4. Derive elements of the vector of centripetal/Coriolis effects.
5. Derive elements of the gravity vector.

5

For the nonlinear system derived in the problem 4, determine an inverse dynamics control law so that the closed-loop system is linear and decoupled, with each subsystem having natural frequency of 2 [rad/s] and damping ratio of 0.1.