0.5 Exponential and Logarithmic Function :

$2^{17} = ?$

$(3,4)^{2\pi} \rightarrow$

Exponential Function : $\boxed{f(x) = b^{x}}$; $b > 0$, $b \neq 1$

Ex. $f(x) = 2^{x}$. $f(x) = \left(\frac{1}{2}\right)^{x}$, $f(x) = \pi^{x}$, $f(x) = e^{x}$

$f(x) = -2^{x} + 4$

$y = b^{0} = 1$

Graph of $y = b^{x}$:

(i) $(0,1)$ — passes. $\boxed{x\text{-axis}}$

(ii) $b > 1$, $y = b^{x}$ increases

(iii) $0 < b < 1$. decreases

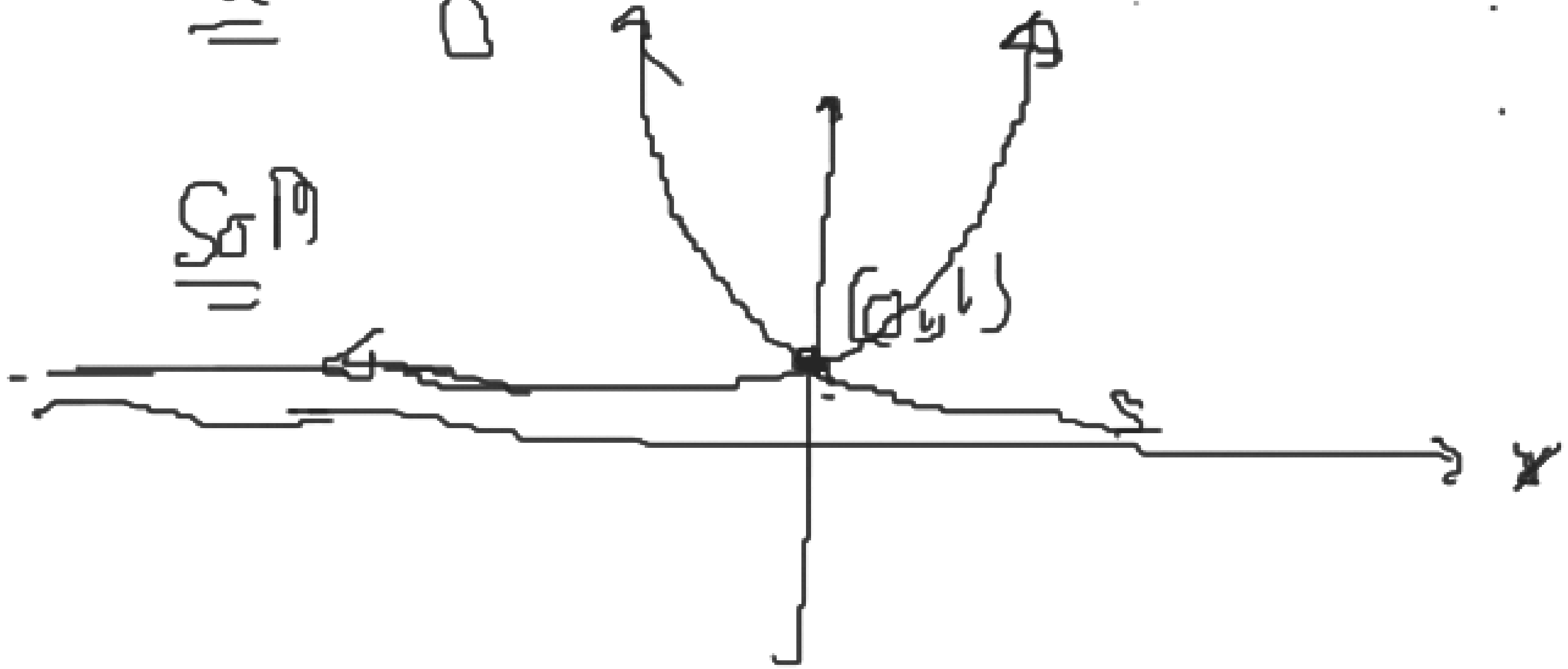
(iv) $b = 1$, $y = 1$, constant

(v) $D : (-\infty, +\infty)$, $R : (0, \infty)$

Ex. $y = 2^x$ graph = ? D = ? R = ?

$b = 2 > 1$

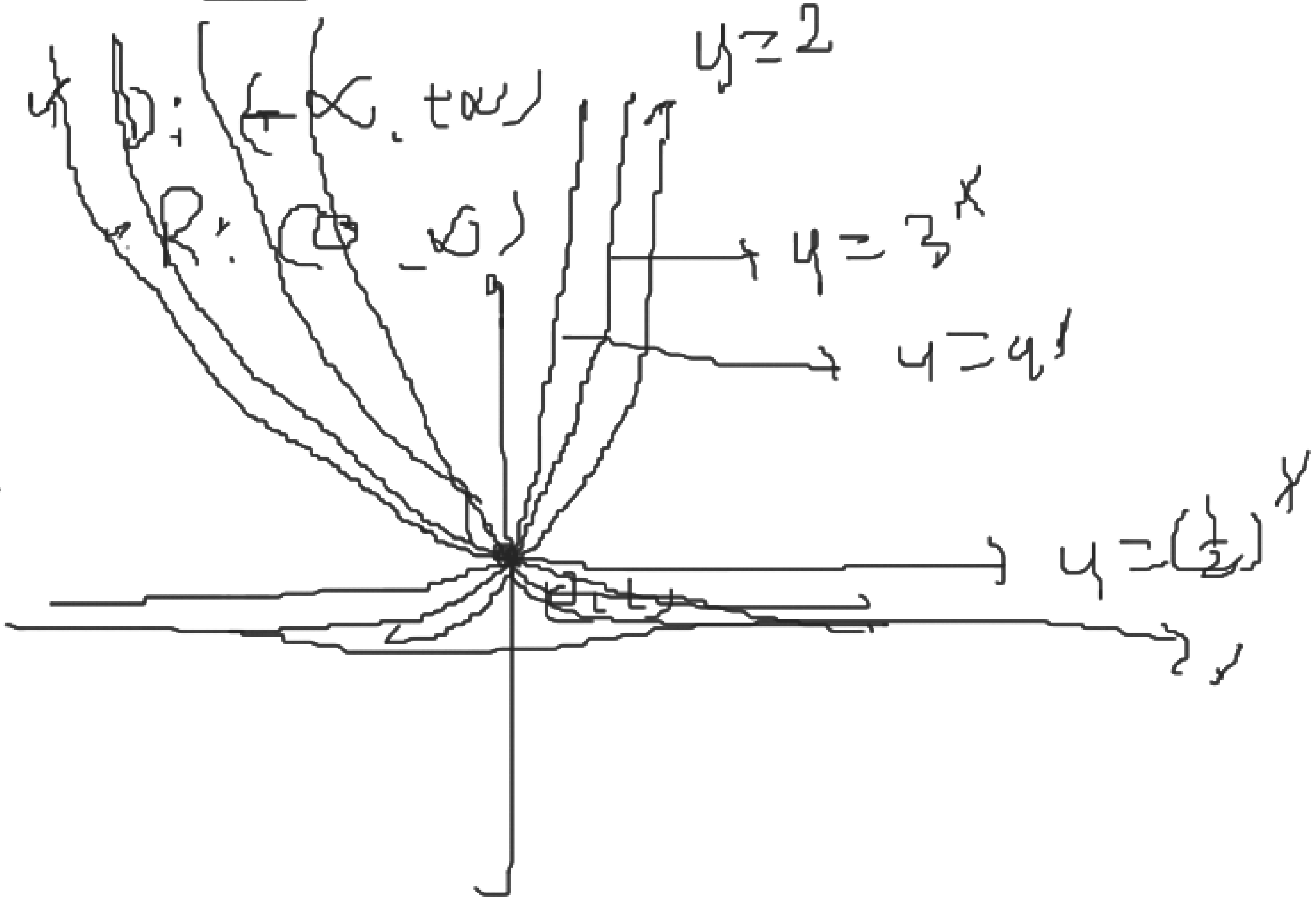
Sol



vs $y = \left(\frac{1}{2}\right)^x$

D: $(-\infty, +\infty)$

R: $(0, \infty)$

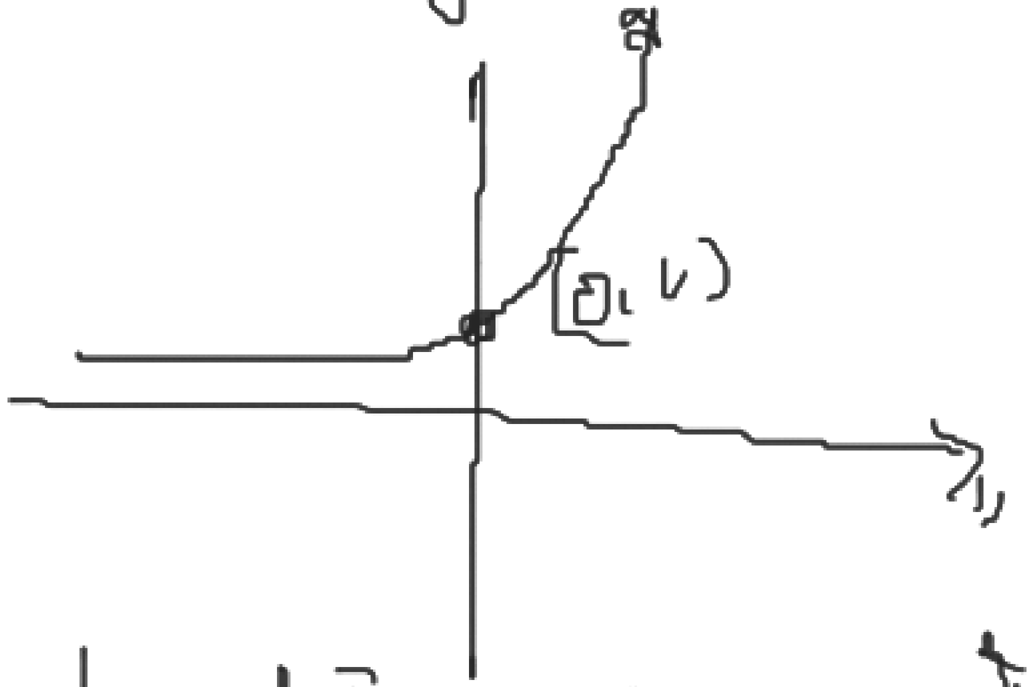


Ex

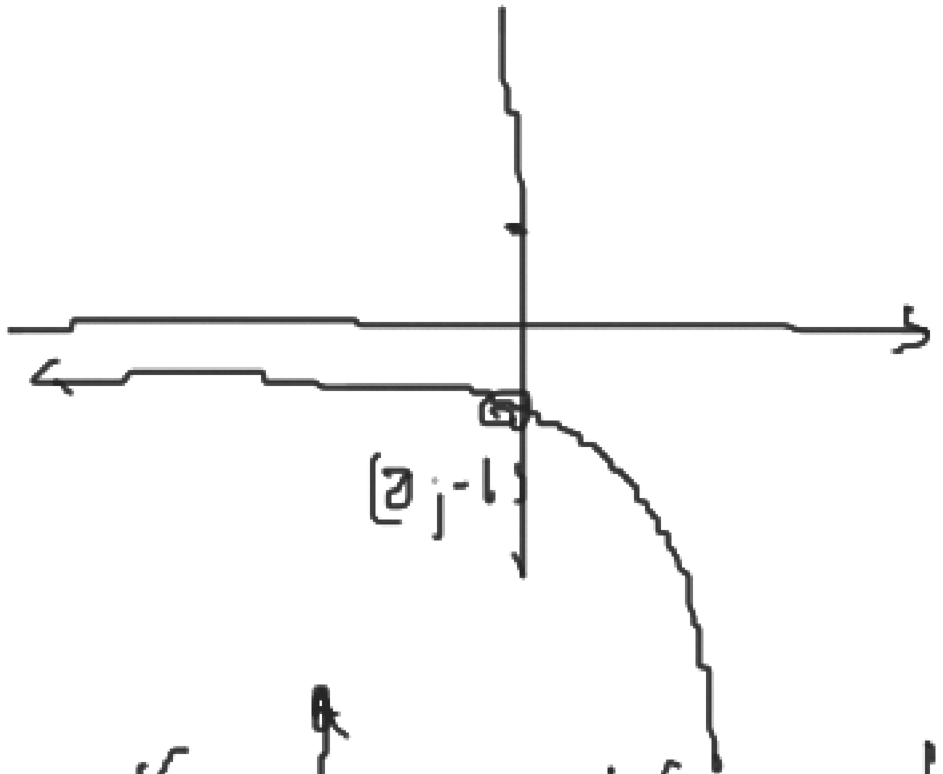
Ex1: $f(x) = 1 - 2^x$, Graph = ?, D = ? R = ?

Sol^n

$y = 2^x$

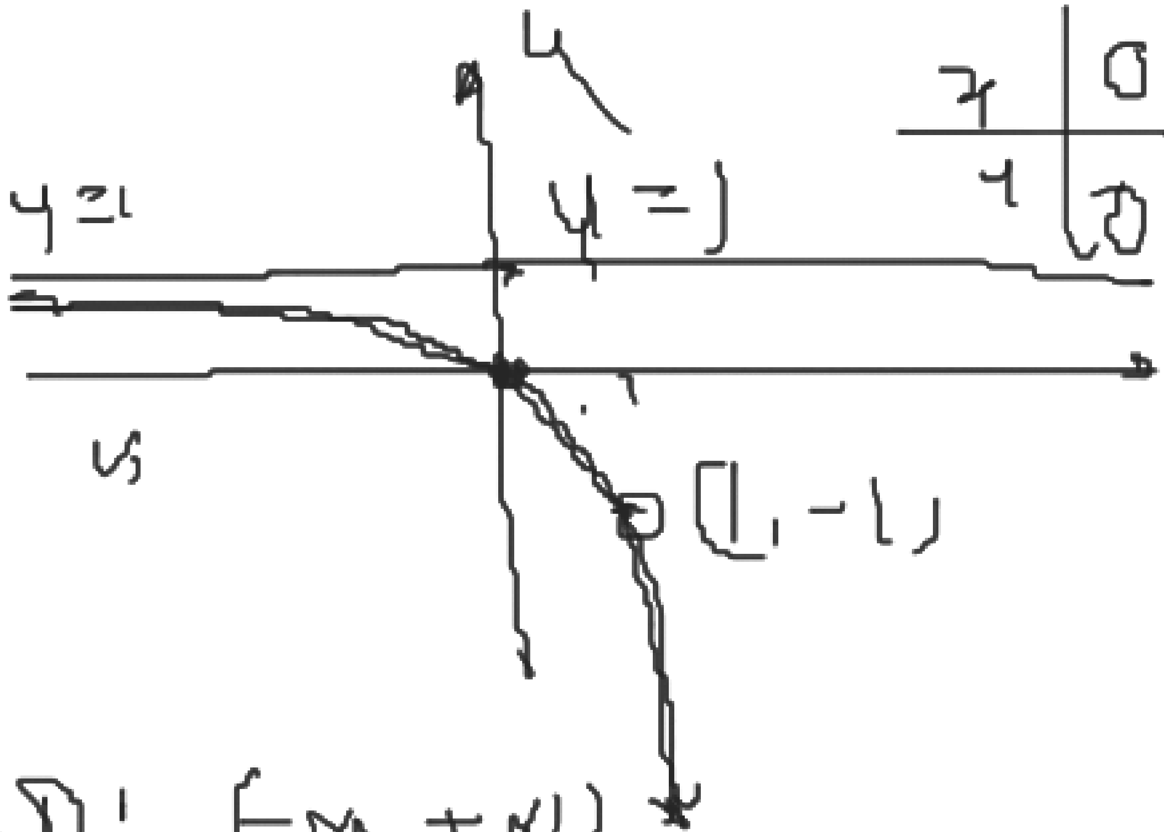


$y = -2^x$



$y = 1 - 2^x$

x	0	1
y	0	-1



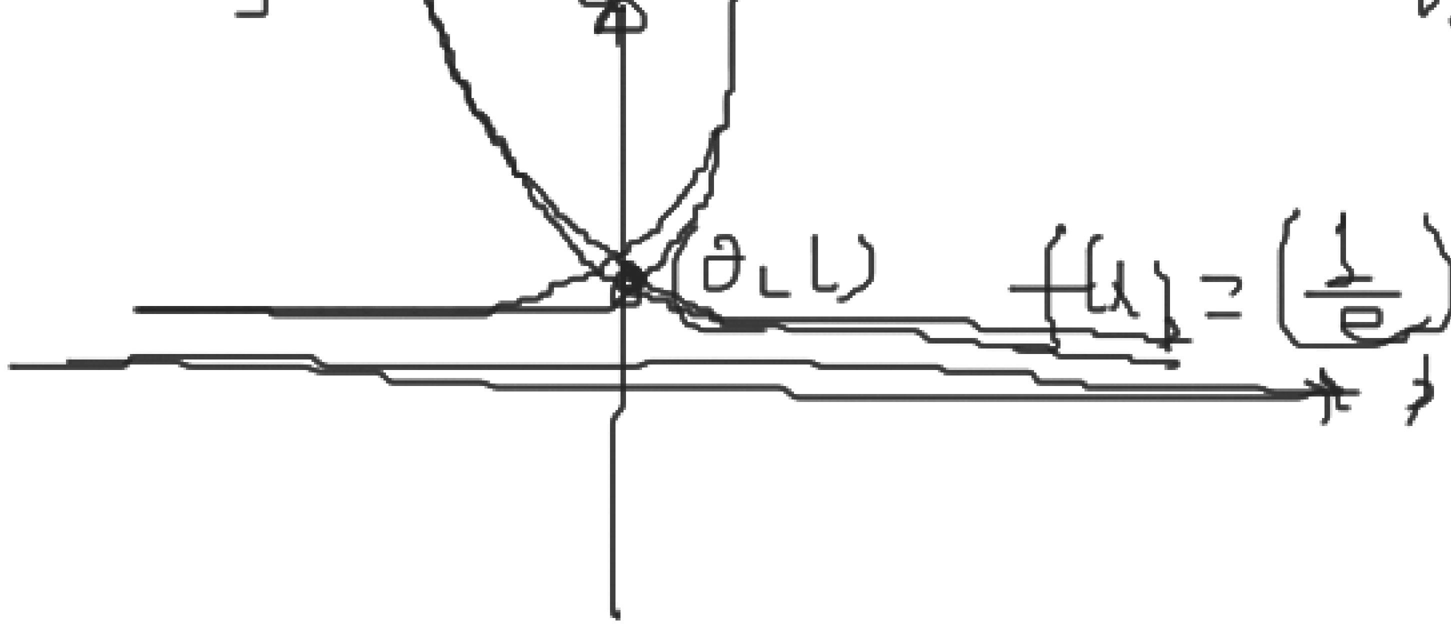
D: $(-\infty, +\infty)$

R: $(-\infty, 1)$

Natural Exp. Fn:

$f(x) = e^x$

$f(x) = \left(\frac{1}{e}\right)^x$



$e^x = \exp(x)$

$\exp(x_1+x_2) = e^{(x_1+x_2)} = e^{x_1} \cdot e^{x_2} = \exp(x_1) \cdot \exp(x_2)$ ✓

$e^{x_1+x_2+x_3} = e^{x_1} \times e^{x_2} \times e^{x_3}$

Logarithmic Function: If $b > 0$, $b \neq 1$, $x > 0$.

✓ $f(x) = \log_b x$

$$\boxed{y = \log_b x}$$

$\Rightarrow x = b^y$

$f^{-1}(y) = b^y$ $\quad f^{-1}(x) = b^x$ ✓

$2^4 = 16 \Rightarrow \log_2 16 = 4$

$$\boxed{y = \log_b x \Rightarrow \boxed{b^y = x}}$$

$$\boxed{b^y = x \Rightarrow y = \log_b x}$$

$3^2 = 9$

$\Rightarrow 2 = \log_3 9$

$4^2 = 16$

$\Rightarrow 2 = \log_4 16$

$\log_{10} 100 = 2$

$\Rightarrow 10^2 = 100$

$\log_5 5 = 1$

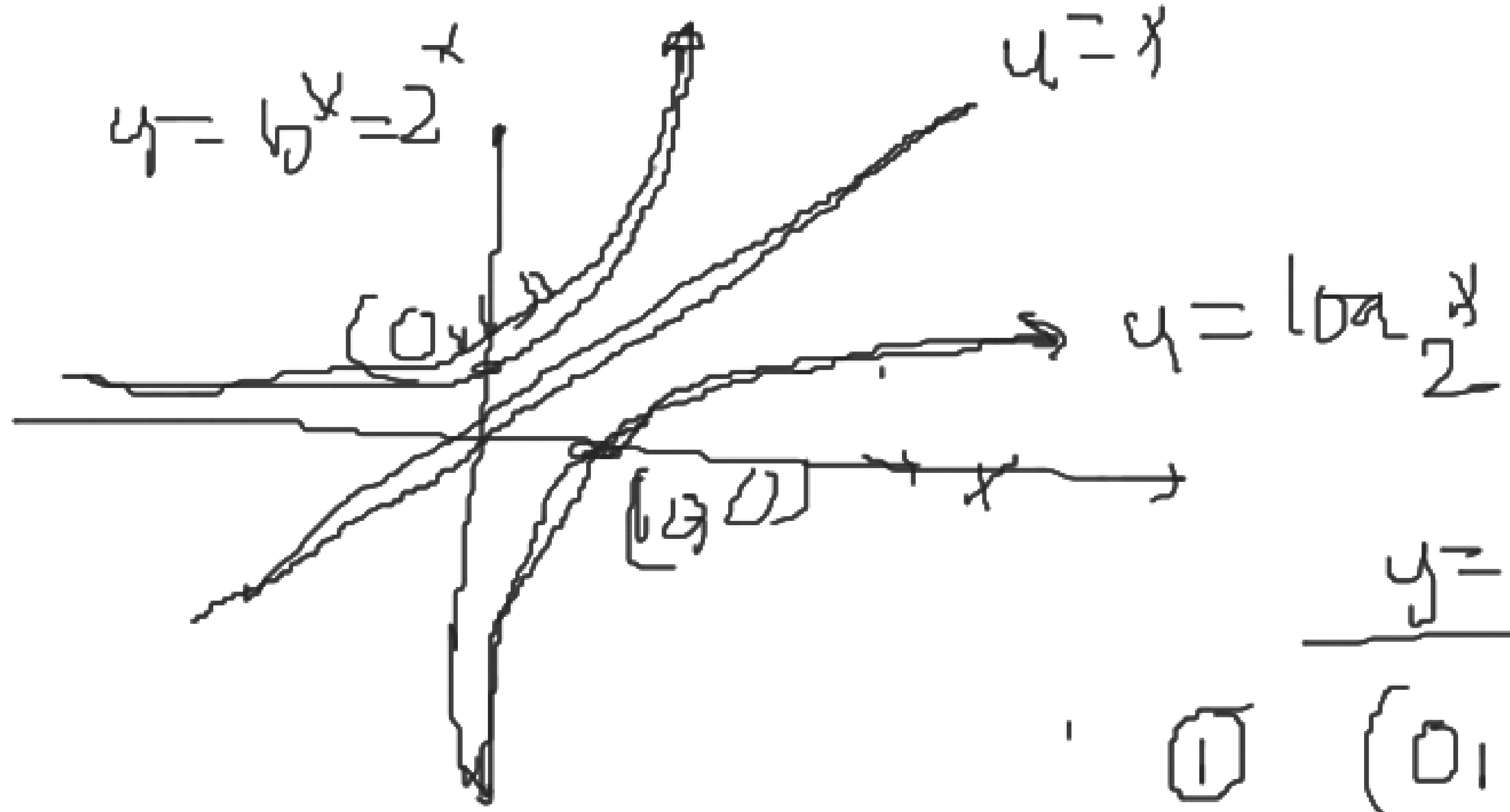
$\Rightarrow 5^1 = 5$

$\log_a 9 = 1$

$\Rightarrow \boxed{a^1 = 9}$

$10^{-2} = \frac{1}{100}$

$\Rightarrow -2 = \log_{10}\left(\frac{1}{100}\right)$



$y = 2^x \Rightarrow x = \log_2 y$

$\boxed{y = \log_2 x}$

	$y = b^x$	$y = \log_b x$
(i)	$(0,1)$	$(1,0)$
(ii)	x-axis horizontal	y-axis vertical asymptote
(iii)	$D: (-\infty, +\infty)$	$R: (-\infty, +\infty)$
(iv)	$R: (0, \infty)$	$D: (0, \infty)$

Properties:

(i) $\log_a (M \times N) = \log_a M + \log_a N$

(ii) $\log_a \left(\frac{M}{N}\right) = \log_a M - \log_a N$

(iii) $\log_a M^n = n \log_a M$

(iv) $\log_a \left(\frac{1}{c}\right) = -\log_a c$

Common Logarithm:

$\log_{10} x$

Natural

$\ln_e x$

Change of Base Rule:

$$\log_b x = \frac{\ln x}{\ln b}$$

$\boxed{\log_{3.1} 5.6} = ?$ $\log_{3.1} 5.6 = \frac{\ln 5.6}{\ln 3.1} = ?$ Ans

Ex 2: Find x: (a) $\log x = \sqrt{2}$ (b) $\ln(x+1) = 5$

Sol: $\log x = \sqrt{2}$

$\Rightarrow x = 10^{\sqrt{2}}$

Ans

Sol: $x+1 = e^5$

$\therefore x = e^5 - 1$ Ans

(c) $5^x = 7 \Rightarrow x = \log_5 7 = \dfrac{\ln 7}{\ln 5} = (\ )$ Ans

H.W. Ex 0.5: 5, 6, 16-29, 57 & 58

Chapter # 01 (Limit & Continuity)

IRC: Instantaneous Rate of Change

1st - 1"

1: Limit: $f(x) \longrightarrow L$ as $x \to a$

$x \to 0$

0.0000 .... 1 →

← 0.0000 .... 1 →

$$\boxed{\lim_{x \to a} f(x) = L}$$

$\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x)$ | Left Hand Limit (at $x = a$) : $\lim_{x \to a^-} f(x) = L^-$

Right Hand Limit (at $x = a$) : $\lim_{x \to a^+} f(x) = L^+$

$\lim_{x \to a} f(x) = \text{exist}$

$= L$

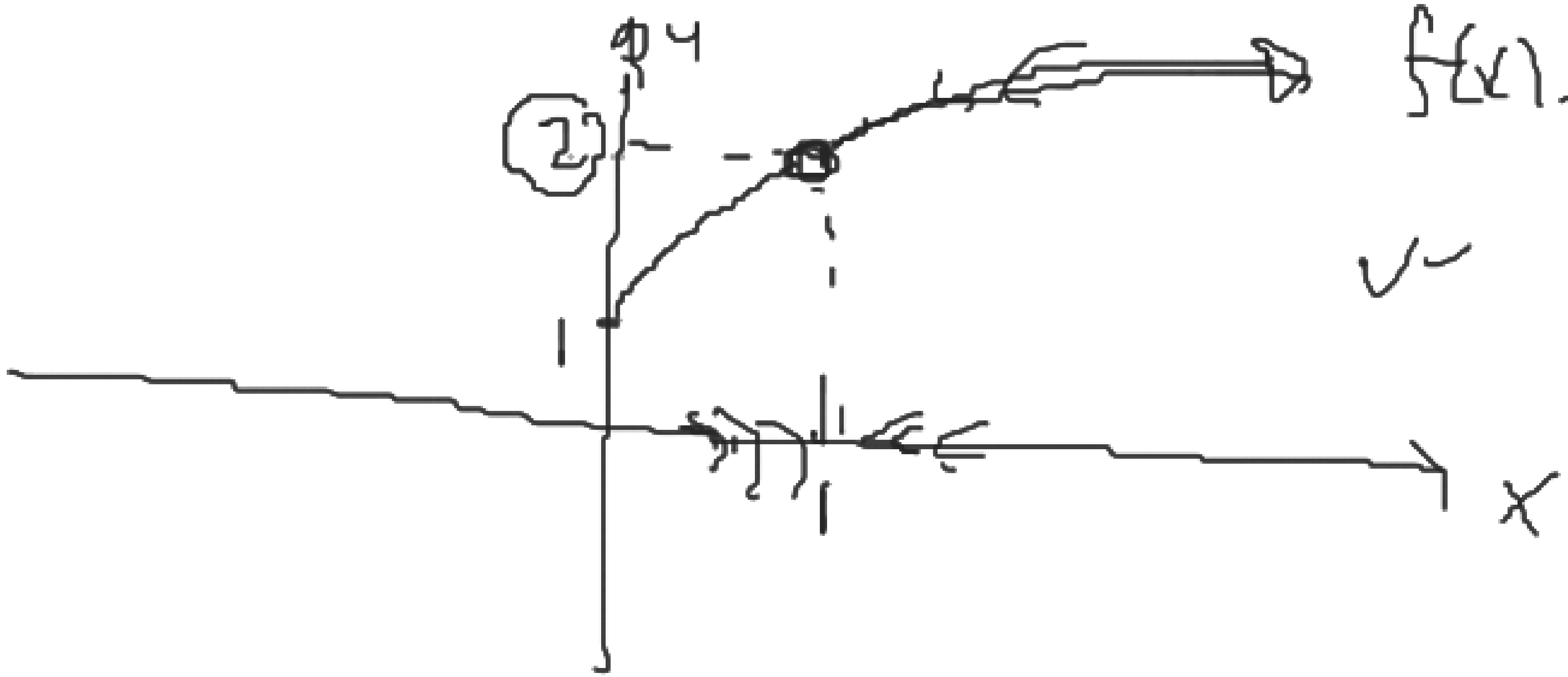
Ex 3 : $\lim_{x \to 1} \dfrac{x-1}{\sqrt{x}-1}$ $\boxed{\frac{0}{0}}$ X

Sol^n :

$$\lim_{x \to 1} \frac{x-1}{\sqrt{x}-1} = \lim_{x \to 1} \frac{(\sqrt{x})^2 - (1)^2}{(\sqrt{x}-1)}$$

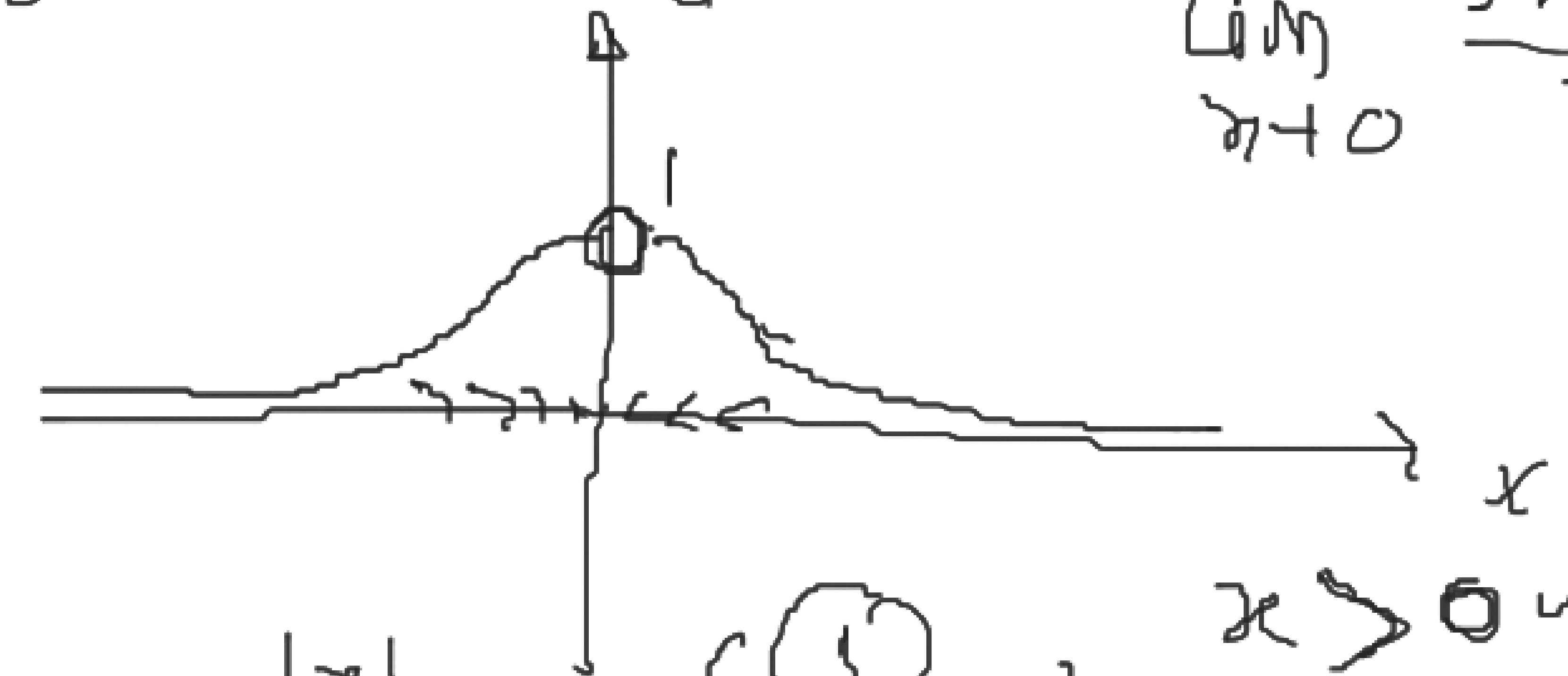
$$= \lim_{x \to 1} \frac{(\sqrt{x}+1)\cancel{(\sqrt{x}-1)}}{\cancel{(\sqrt{x}-1)}} = \lim_{x \to 1} (\sqrt{x}+1) = 1+1 = 2$$

A



$f(x) = \dfrac{x-1}{\sqrt{x}-1}$

Ex 3 $\lim_{x\to 0} \frac{\sin x}{x} = ?$



$\lim_{x\to 0} \frac{\sin x}{x} = 1$

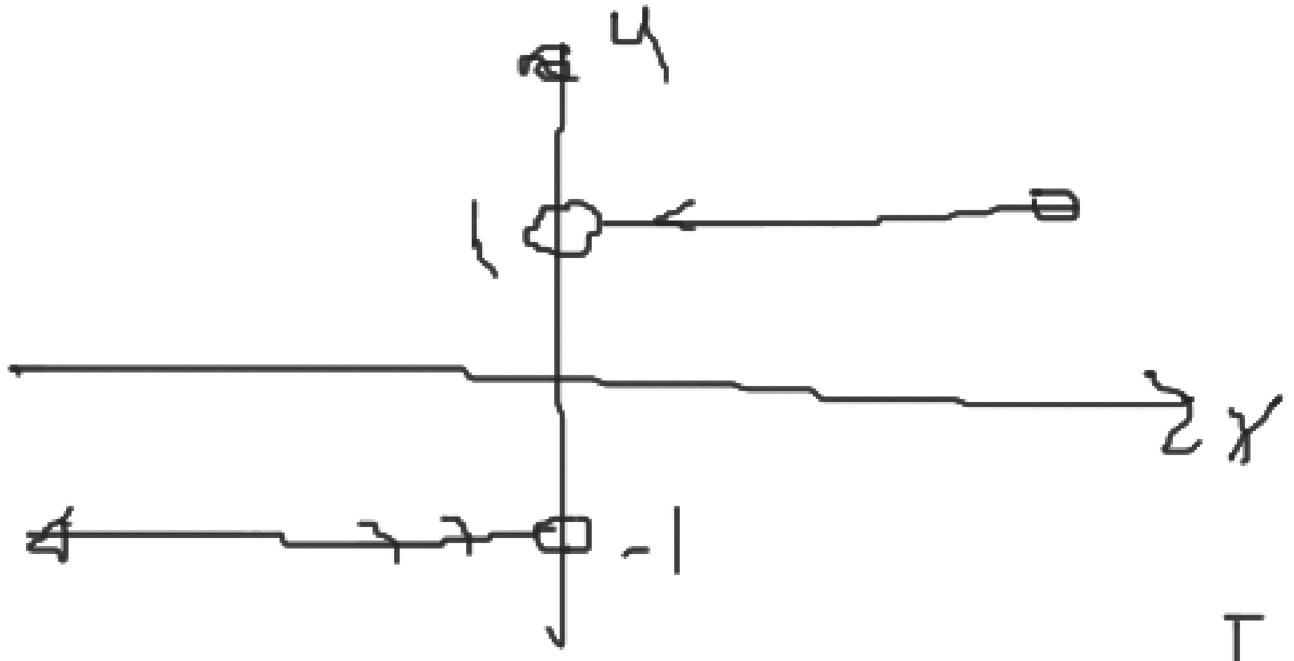
$|x| = \frac{x}{-x} = -1$

Ex: $f(x) = \frac{|x|}{x} = \begin{cases} 1, & x > 0 \\ -1, & x < 0 \end{cases}$ $\boxed{\lim_{x\to 0} f(x) = ?}$

Sol L.H. Limit (at $x=0$): $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} (-1) = -1$

R.H. Limit (at $x=0$): $\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} (1) = 1$

Since $\lim_{x\to 0^-} f(x) \neq \lim_{x\to 0^+} f(x)$. So $\lim_{x\to 0} f(x)$ doesn't exists



$$f(x) = \frac{|x|}{x}$$

If $\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x) = L$

$\therefore \lim_{x \to a} f(x) = L$

$\underline{\underline{Ex}}$ $f(x) = \begin{cases} \boxed{2x+1} & ; \ x > 1 \\ \boxed{x^2+2} & \ x < 1 \end{cases}$ $\boxed{\lim_{x \to 1} f(x)} = ?$ $\boxed{0.9999\cdots 9}$

$1.0000\cdots 1$

$\underline{\underline{Soln}}$: L.H. Limit (at $x = 1$): $\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} (x^2+2)$

$= (1)^2 + 2 = \boxed{3}$

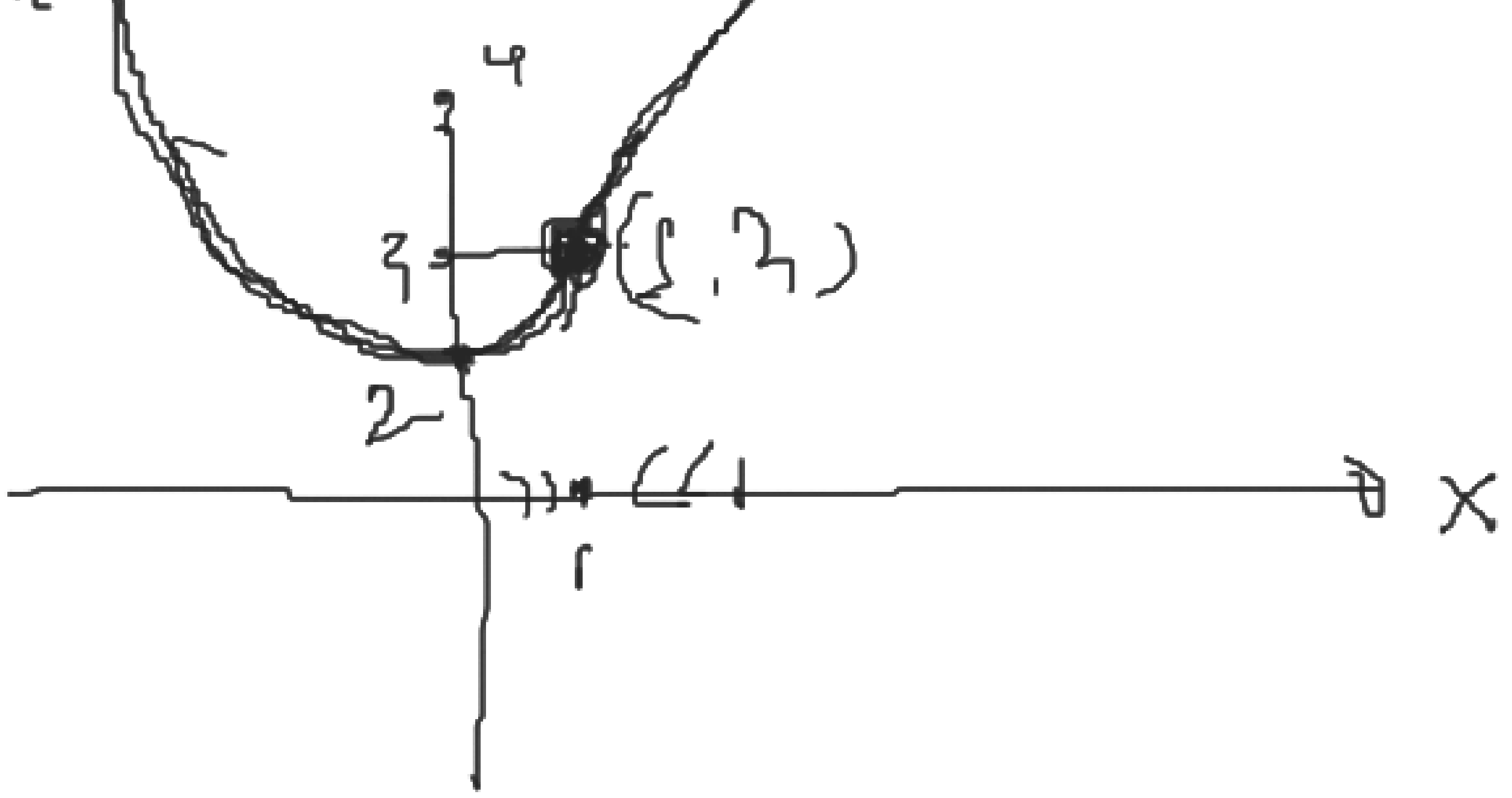
R.H. Limit (at $x = 1$): $\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} (2x+1) = 2 \cdot 1 + 1 = \boxed{3}$

$\because \lim_{x \to 1^-} f(x) = 3 = \lim_{x \to 1^+} f(x)$ $\therefore \lim_{x \to 1} f(x) = 3$ Ans

$$f(x) = \begin{cases} 2x+1 & ; \boxed{x > 1} \\ x^2+2 & ; x < 1 \end{cases}$$

$f(x) = 2x+1$

x	1	2
y	3	5



Infinite Limit : $\boxed{f(x) \to \infty}$ or $\boxed{f(x) \to -\infty}$ as $\boxed{x \to a}$

(I) If $\lim\limits_{x \to a^-} f(x) = +\infty$ and $\lim\limits_{x \to a^+} f(x) = +\infty$

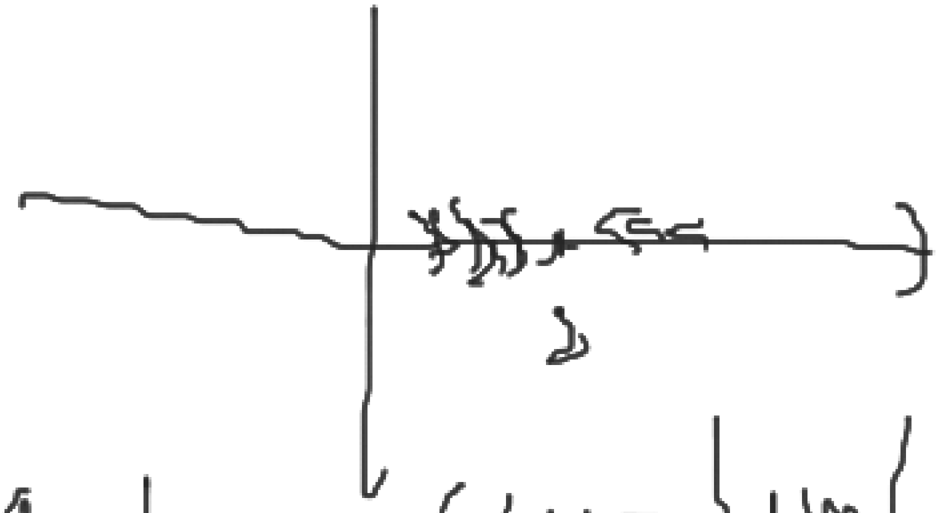
$\therefore \lim\limits_{x \to a} f(x) = +\infty$

(II) If $\lim\limits_{x \to a^-} f(x) = -\infty$ & $\lim\limits_{x \to a^+} f(x) = -\infty$

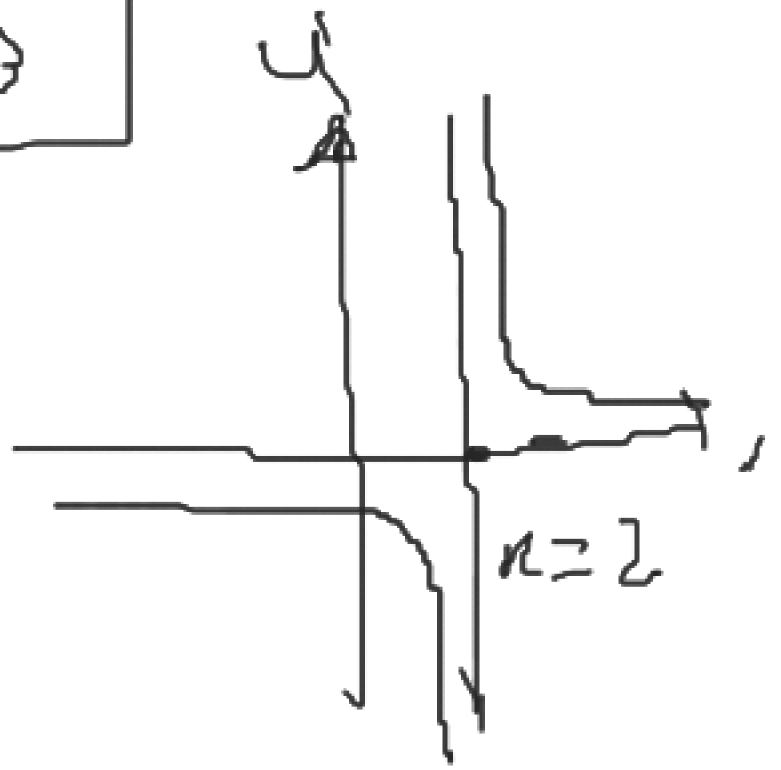
$\therefore \boxed{\lim\limits_{x \to a} f(x) = -\infty}$

Ex 1 $f(x) = \frac{1}{x-2}$ $\boxed{\lim_{x \to 2} f(x) = ?}$ $2^- = \boxed{1.9999\ldots 9}$
$2^+ = \boxed{2.000\ldots 1}$

Sol^n L.H. Limit (at $x = 2$) : $\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} \left(\frac{1}{x-2}\right) = \boxed{-\infty}$



R.H. Limit (at $x=2$) : $\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} \left(\frac{1}{x-2}\right) = \boxed{+\infty}$



$\therefore \lim_{x \to 2} \frac{1}{x-2}$ does not exist.

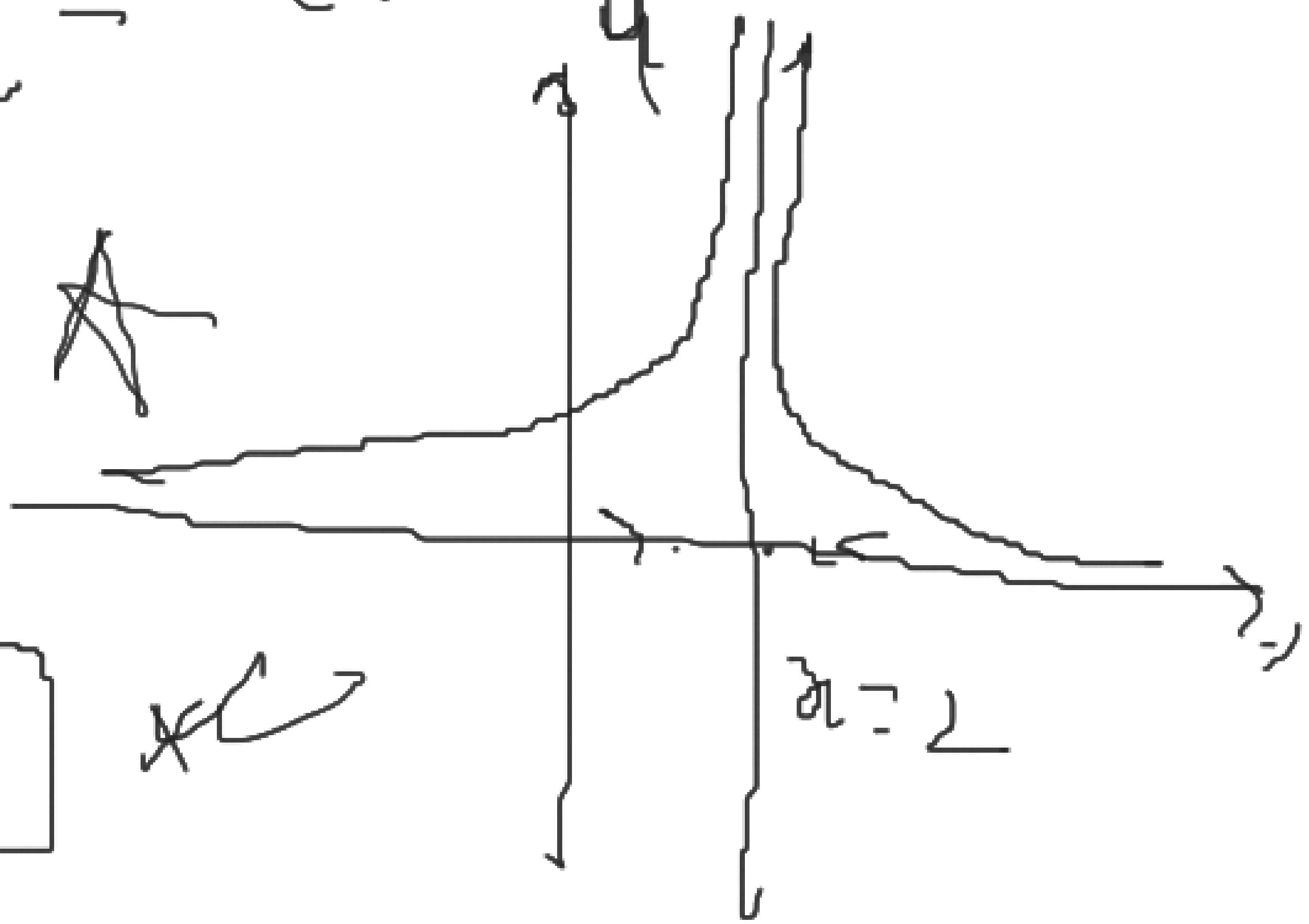
(4) $f(x) = \frac{1}{(x-2)^2}$ $\lim_{x \to 2} f(x) = ?$

$\frac{1.999 \ldots 9}{2.000 \ldots 1}$

Sol: L.H.L $(x=2)$: $\lim_{x \to 2^-} \frac{1}{(x-2)^2} = +\infty$

R.H.L $(x=2)$: $\lim_{x \to 2^+} \frac{1}{(x-2)^2} = +\infty$

$\therefore \lim_{x \to 2} \frac{1}{(x-2)^2} = +\infty$



H.W. Ex 2.1: $\boxed{3-7, 8 \quad 13-16}$