Quiz 01

Name : Mosnoor Mofiz Arnon

ID : 1921079642

Course : MAT130

Section : 09

Date : 10-03-2021

Ans to the Q.NO: 03

$$\int e^x \cos(2x)\,dx$$

Set,

$$I = \int e^x \cos(2x)\,dx.$$

Now set, $u = \cos 2x$. and $dv = e^x dx$.

$\Rightarrow \frac{du}{dx} = -2\sin(2x)$ $\Rightarrow \int 1.dv = \int e^x dx$

$\therefore du = -2\sin(2x)dx$ $\therefore v = e^x$.

$\therefore I = \int u\,dv.$

$= uv - \int v\,du$

$= \cos 2x.\, e^x - \int e^x.(-2)\sin(2x)\,dx$

$= e^x.\cos 2x + 2\int e^x \sin(2x)\,dx$

Set, $u = \sin 2x$ and $dv = e^x dx$.

$\Rightarrow \frac{du}{dx} = 2\cos(2x) \quad \Rightarrow \int 1.\,du = \int e^x dx.$

$\therefore du = 2\cos(2x)dx \quad \therefore v = e^x$

$= e^x \cos(2x) + 2\int \left[\sin(2x)e^x - \int e^x . 2\cos(2x)dx\right]$

$= e^x \cos(2x) + 2\sin(2x).e^x - 2\int e^x 2\cos(2x)dx.$

$= e^x \cos(2x) + 2\sin(2x).e^x - 4\int e^x \cos(2x)dx.$

$\therefore I = e^x\cos(2x) + 2e^x\sin(2x) - 4I.$

$\Rightarrow I + 4I = e^x(\cos(2x) + 2\sin(2x)).$

$\Rightarrow 5I = e^x(\cos(2x) + 2\sin(2x)).$

$\therefore I = \frac{e^x}{5}[\cos(2x) + 2\sin(2x)] + c$

(Ans).

Ans to the QNO: 02

$$\int \sin^3 x \cos^9 x \, dx.$$

$$= \int \cos^9 x \sin^3 x \, dx.$$

$$\Rightarrow \int -\cos^9(x)\,(\cos^2(x)-1)\cdot \sin(x)\cdot dx$$

Let,

$$u = \cos x.$$

$$\Rightarrow \frac{du}{dx} = -\sin x$$

$$\therefore du = -\sin x \, dx.$$

Then,

$$\int \sin^3 x \cos^9 x \, dx.$$

$$= -\int u^9 (u^2-1)\, du.$$

$$= \int u^{11} du - \int u^9 du.$$

$$= \frac{u^{12}}{12} - \frac{u^{10}}{10} + C$$

$$= \frac{\cos^{12}(x)}{12} - \frac{\cos^{10}(x)}{10} + C.$$

$$= \frac{1}{2}\left[\frac{1}{6}(\cos^{12}x) - \frac{1}{5}(\cos^{10}x)\right] + C.$$

(Ans).

Ans to the ONO: 01.

$\int_0^1 \cot^{-1} x \, dx$. Let, $\int \cot^{-1} x \, dx$.

$= \cot^{-1} x \int 1 \cdot dx - \int \left[\frac{d}{dx} \cot^{-1} x \int 1 \, dx\right] dx$.

$= x \cot^{-1} x - \int \left[-\frac{1}{1+x^2} x\right] dx$.

$= x \cot^{-1} x + \int \frac{x}{1+x^2} dx$.

$= x \cot^{-1} x + \frac{1}{2} \int \frac{2x}{1+x^2} dx$.

$= x \cot^{-1} x + \frac{1}{2} \ln(1+x^2) + C$.

~~$= x \cot^{-1} x + \frac{1}{2} \ln(1+x$~~

$\int_0^1 \cot^{-1} x \, dx = x \cot^{-1} x + \frac{1}{2} \ln(1+x^2) + C$.

$$= \left[1 \cot^{-1} 1 + \frac{1}{2} \ln(1+1)\right] - \left[0 + \frac{1}{2} \ln 1\right]$$

$$= \frac{\pi}{4} + \frac{1}{2} \ln 2 - 0$$

$$= \frac{\pi}{4} + \ln \sqrt{2}$$

(Ans).