Calculus & Analytic Geometry
MAT-120
Section:19,20

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Objective

The course is designed to introduce you

- the fundamentals of Calculus and
- Analytic Geometry

- It enables us to model real life situations

Objective...Continues

- This course will help the students to understand basic facts and terminology relating to numbers, absolute values, intervals etc.

- The students will be able to visualize the algebraic equations as geometric curves and **conversely**
- to present geometric curves by algebraic equations.

Objective...Continues

- This course will enable the students to define the fundamental concepts in mathematics, the notation of function, limit, continuity, differentiation, integration etc.

- Finally, the students will know how to apply this knowledge in many real life problems.

Origin of calculus

The word Calculus comes from the Greek name for pebbles

Pebbles were used for counting and doing simple algebra…

It was developed in seventeenth century.

Inventor

Inventors:
Calculus was developed by
Sir Isaac Newton (1642,1727) and
Sir Gottfried Wilhelm Leibnitz (1646, 1716)

Also:

Euler, D. Alemberts, Lagrange, Gauss, Cauchy, Rolle, Laplace, Taylor, Laurent are noteworthy.

What is Calculus?

- Calculus is the mathematical tool used to analyze changes in physical quantities.

- It is the mathematics of change
- It is the mathematics of tangent lines, slopes, areas, volumes

“A method of calculation in a special notation (like logic or symbolic logic)”

What is Calculus? Continued

- "The branch of mathematics that is concerned with limits and with the differentiation and integration of functions"

- "The branch of mathematics involving derivatives and integrals."

OUTLINE

Lecture 1:	Introduction, numbers, intervals and inequalities	Chapter 1
Lecture 2:	Absolute value, coordinate planes and graphs	**Chapter 1**
Lecture 3:	Operations and graphs of Functions	**Chapter 1**
Lecture 4:	Limit: Intuitive introduction and computational techniques	**Chapter 2**
Lecture 5:	Continuity,	**Chapter 2**
Lecture 6:	Limit and continuity of trigonometric functions & **Quiz I**	**Chapter 2**
Lecture 7:	**Review**	**Chapter 2**
Lecture 8:	**MIDTERM-1 (Chapter 1, 2)**	

Assignment 1

- **1.1**: 23, 27, 29, 33, 37,39, 41,
- **1.2**: 5, 7, 9, 19, 21,25, 29, 33, 35
- **1.3**: 13, 15, 17, 21, 23, 27, 31, 35,
- **1.4**: 25, 27, 29, 33, 37, 41, 47
- **1.5**: 23, 25, 29, 33, 37, 39, 43, 57, 61, 65, 67
- **2.1**: 3, 9, 17, 21, 25, 37, 39, 41,47,
- **2.2**: 7, 9, 29, 31, 35, 49,
- **2.3**: 9, 11, 19, 21, 27, 41, 43,
- **2.5**: 11, 17, 23, 29, 37, 41, 59, 67, 73,
- **2.6**: 3, 9, 15, 25,
- **2.7**: 15, 17, 23

Outline…Continues

Lecture 9:	**Differentiation, tangent lines and rates of change, derivatives**	**Chapter 3**
Lecture 10:	Techniques of differentiations, derivatives of trigonometric functions	**Chapter 3**
Lecture 11:	Chain rule Implicit differentiation	**Chapter 3**
Lecture 12:	Applications of differentiation, increasing, decreasing functions	**Chapter 4**
Lecture 13:	Relative extrema: First and second derivative tests	**Chapter 4**
Lecture 14:	Mean value theorem: Rolle's theorem, Lagrangr's MVT	**Chapter 4**
Lecture 15:	Review and **Quiz II**	**Chapter 4**
Lecture 16:	**MIDTERM-2 (Chapter 3, 4)**	

Assignment 2

- **3.1**: 9, 19,23,
- **3.2**: 5, 11, 43,
- **3.3**: 11, 27, 41,
- **3.4**: 5, 17, 27,
- **3.5**: 11, 21, 37, 39, 43, 47, 65,
- **3.6**: 9, 27, 29,
- **4.1**: 7, 9, 13, 17, 19, 25, 33, 37,
- **4.2**: 15, 19, 21, 25, 37,
- **4.3**: 3,13, 15, 25, 35, 43,
- **4.6**: 7, 21, 25, 31, 37,
- **4.9**: 3, 5,11,13

OUTLINE

Lecture 17:	**Introduction of Integration, Antiderivatives, Indefinite integrals,**	**Chapter 5**
Lecture 18:	Techniques of indefinite integrations	**Chapter 5**
Lecture 19:	Definite integrals, fundamental theorem of calculus	**Chapter 5**
Lecture 20:	Applications of integrations: areas, arc lengths	**Chapter 5**
Lecture 21:	Overview of logarithms and exponents	**Chapter 6**
Lecture 22:	Derivatives and integrals of logarithms and exponents	**Chapter 6**
Lecture 23:	First order differential equations and applications	**Chapter 6**
Lecture 24:	First order differential equations and applications continued and **Quiz III**	**Chapter 6**
	Final Exam- (Chapter 5, 6)	

Assignment 3

- **5.2**: 11, 23, 25, 27,29,47,49,
- **5.3**: 1c, 3d, 11,21,23,33,
- **5.6**: 19, 21, 23, 25, 33,35, 37,
- **5.7**: 9, 15, 21, 23,
- **5.8**: 9, 11, 15, 17, 21, 23, 25, 27,
- **5.9**: 17, 19

Assesment

Midterm 1	20%
Midterm 2	25%
Class Attendance	10%
Quiz (Average of 3)	10%
Assignment	10%
Final	25%
Total	100%

Office

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- Office Room: 1047
- Office Hour: 2:30 – 4:20.

Chapter 1: Introduction

Calculus is classified into two parts:

Differential Calculus:

The concept of infinite simal

The concept of function

The concept of limit

Integral Calculus:

A process of summation

A process which is the inverse of differentiation

Preliminaries

- **Number:**

Numbers are the foundation of all mathematics. So it is important to be familiar with the various kinds of numbers and the difference between them.

$$N \subset W \subset I \subset Q \subset \Re \subset C$$

- (i)The set of natural numbers N = $\{1, 2, 3........\}$
- (ii) The set of whole numbers W = $\{0, 1, 2, 3........\}$
- (iii)The set of integer numbers I or

Z = $\{.........-3, -2, -1, 0, 1, 2, 3........\}$

- (iv) The set of +ve integer numbers N $=\{1, 2, 3........\}$
- (v) The set of –ve integer numbers N $=\{..........,-3, -2, -1\}$
- (vi) The set of rational numbers Q = $\left\{\frac{p}{q},\quad q \neq 0\right\}$
- OR: The numbers which can be expressed in decimal form terminating and repeating form

(vii) The set of Irrational numbers: The numbers which can be expressed in decimal form in non-terminating and non-repeating form

(viii) The set of real numbers: The set of all the numbers we discussed above is called the set of real numbers.

$$\{.........-3, -2, -\sqrt{2}, -1, 0, 1, 1/2\sqrt{2},\ 2, 3........\}$$

(ix) The set of complex numbers C = the numbers of the form and x, y are real. $\{x+iy,\quad i=\sqrt{-1}\}$

Intervals:

- **Intervals**: An interval is a line segment on a coordinate line.
- **Types**:
- (i) **Closed interval**: If a, b be two real numbers such that a < b, then the closed interval is denoted by [a, b] or $a \le x \le b$
- (ii) **Open interval**: If a, b be two real numbers such that a < b, then the open interval is denoted by (a, b) or $a < x < b$

Interval continued

- (iii) **Open closed interval**: If a, b be two real numbers such that a < b, then the open interval is denoted by (a, b] or $a < x \le b$

- (iv) **Closed Open interval**: If a, b be two real numbers such that a < b, then the open interval is denoted by [a, b) or $a \le x < b$

- **Finite and infinite interval:** Interval of infinite extent are called infinite interval otherwise finite. $-\infty < x < \infty$

Solution of inequalities

- **Solution of inequalities** : A solution of an inequality in an unknown x is a value for x that makes the inequality a true statement.

Example:

$$4+5x \le 3x-7$$

$$4+5x \le 3x-7 \quad => \quad 5x \le 3x-11 \quad => \quad x \le \frac{-11}{2}$$

- Solve the following inequalities and sketch the solution on a coordinate line.

- 1. $\dfrac{3x+1}{x-2} < 1$ 2. $\dfrac{3}{x-5} \le 2$ 3. $x^2 - 9x + 20 \le 0$

- 4. $x^2 \le 5$

Absolute value

- **Absolute value**: The absolute value or magnitude of a real number x, is denoted by $|x|$ and is define by

$$|x| = \begin{cases} x & , x \geq 0 \\ -x & , x < 0 \end{cases}$$

- or

$$|x| = \begin{cases} x, & x > 0 \\ 0, & x = 0 \\ -x, & x < 0 \end{cases}$$

Square root

- **Square root:** A number whose square is x is called a square root of x. Every real number x has two square roots: one positive and one negative. The positive square root is denoted by $\sqrt{x}$.
- For any real number x, $\sqrt{x^2} = |x|$
- **Incorrect:** $\sqrt{x^2} = \pm x$
- Example: $\sqrt{4} = \pm 2$

Geometric interpretation of AV

- **Geometric interpretation of AV**: Let A(a, 0) and B(b, 0) be two points on the coordinate line. Because distance is con-negative, the distance d between A and B is

$$d = \begin{cases} b-a, & a<b \\ 0, & a=b \\ a-b, & a>b \end{cases}$$

Two important formulas

- (a) $|x-a|<k \quad => -k<x-a<k \quad => a-k<x<a+k$
- (b) $|x-a|>k \quad => x-a<-k \quad \& \quad x-a>k$

- Solve the following inequalities and sketch the solution on a coordinate line.

- 1. $\dfrac{1}{|2x-3|}>5$ 2. $\dfrac{3}{|2x-1|}\geq 4$

The End