North South University

Department of Electrical & Computer Engineering

Assignment

Assignment:	**MID**
Course Code:	**MAT361**
Course Section:	**04**
Course Name:	**Probability and Statistics**
Name:	**Rasheeq Ishmam**
ID	**1831350042**
Faculty:	**Israt Jahan Ma'am**
Date of Submission:	**19.04.2021**

① Red : R   Marbles : 2 ⇒ 1 Red, 1 Blue

Blue : B

Draw a marble from the box and then replace it in the box. Draw another marble.

Sample space, $S = \{RR, RB, BR, BB\}$

Drawing,
1st R, then B = RB
1st B, then R = BR
1st marble, then same marble = (RR, BB)

Draw a marble from the box without replacing.

Sample space, $S = \{RB; BR\}$

④

ⓐ $P(X=1) = 0.10 + 0.15 + 0 + 0.05 = 0.3$

$P(X=2) = 0.20 + 0.05 + 0.05 + 0.20 = 0.5$

$P(X=3) = 0.05 + 0 + 0.10 + 0.05 = 0.2$

$P(Y=0) = 0.10 + 0.20 + 0.05 = 0.35$

$P(Y=1) = 0.15 + 0.05 + 0 = 0.2$

$P(Y=2) = 0 + 0.05 + 0.10 = 0.15$

$P(Y=3) = 0.05 + 0.20 + 0.05 = 0.3$

//

ⓑ We know,

$$P(X \mid Y=J) = \frac{P(X, Y=J)}{P(Y=J)}$$

from ⓐ, ~~P(y Y~~

$$P(Y=1) = \cancel{0.35}\ 0.2$$

so,

$$P(X=1 \mid Y=1) = \frac{P(X=1, Y=1)}{P(Y=1)} = \frac{0.15}{\cancel{0.35}\ 0.2} = 0.75$$

$$P(X=2 \mid Y=1) = \frac{P(X=2, Y=1)}{P(Y=1)} = \frac{0.05}{0.2} = 0.25$$

$$P(X=3 \mid Y=1) = \frac{P(X=3, Y=1)}{P(Y=1)} = \frac{0}{0.2} = 0$$

ⓒ $E(X \mid Y=1) = \sum_{i=1}^{3} i P(X \mid Y=1)$

$$= (1 \times P(X=1 \mid Y=1)) + (2 \times P(X=2 \mid Y=1)) + (3 \times P(X=3 \mid Y=1))$$

$$= (1 \times 0.75) + (2 \times 0.25) + (3 \times 0)$$

$$= 0.75 + 0.5 + 0$$

$$= 1.25$$

(d)

$$E((X|Y=1)^2) = \sum_{i=1}^{3} i^2 P(X|Y=1)$$

$$= (1^2 \times 0.75) + (2^2 \times 0.25) + (3^2 \times 0)$$

$$= 0.75 + 1 + 0$$

$$= 1.75$$

from (c), $E(X|Y=1) = 1.25$

We know,

$$V(X) = E(X^2) - (E(X))^2$$

So, $V(X|Y=1) = E((X|Y=1)^2) - (E(X|Y=1))^2$

$$= 1.75 - (1.25)^2$$

$$= 0.1875$$

(Ans)

(e) $E(X,Y) = \sum_{i=1}^{3} \sum_{j=0}^{3} ij P_{ij}$

$E(X,Y) = (1\times0\times0.1) + (1\times1\times0.15) + (1\times2\times0) + (1\times3\times0.05) +$

$\cancel{(2\times0\times0.2) + (2\times1\times0.15) + (2\times}$

$(2\times0\times0.2) + (2\times1\times0.05) + (2\times2\times0.05) + (2\times3\times0.2) +$

$(3\times0\times0.05) + (3\times1\times0) + (3\times2\times0.10) + (3\times3\times0.05)$

$E(X,Y) = 2.85$ (Ans)

3

(a) $f(x,y) = x + y \qquad 0 < x < c, \; 0 < y < 1$

$\Rightarrow \int_0^1 \int_0^c f(x,y)\, dx\, dy = 1$

$\Rightarrow \int_0^1 \int_0^c (x+y)\, dx\, dy = 1$

$\Rightarrow \int_0^1 \left[\frac{x^2}{2} + xy\right]_0^c dy = 1$

$\Rightarrow \int_0^1 \left(\left(\frac{c^2}{2} + cy\right) - 0\right) dy = 1$

$\Rightarrow \int_0^1 \left(\frac{c^2}{2}\cancel{y} + cy\right) dy = 1$

$\Rightarrow \left[\frac{c^2}{2} y + \frac{cy^2}{2}\right]_0^1 = 1$

$\Rightarrow \left(\frac{c^2}{2}\cancel{y}.1 + \frac{c.1^2}{2}\right) = 1$

$\Rightarrow \frac{c^2}{2} + \frac{c}{2} = 1$

$\Rightarrow \frac{c^2+c}{2} = 1$

$\Rightarrow c^2 + c = 2$

$\Rightarrow c^2 + c - 2 = 0$

$\Rightarrow c^2 + 2c - c - 2 = 0$

$\Rightarrow c(c+2) - 1(c+2) = 0$

$\Rightarrow (c+2)(c-1) = 0$

$c = -2, \; c = 1$

But, $0 < x < c$

So, $c = 1$ (Ans)

(b) We have,

$$f(x,y) = x+y \qquad 0<x<1,\ 0<y<1$$

$$g(x) = \int_0^1 x+y \; dy$$

$$= \left[xy + \frac{y^2}{2}\right]_0^1$$

$$= \left(x+\frac{1}{2}\right) - 0$$

$$= x+\frac{1}{2}$$

$$h(y) = \int_0^1 x+y \; dx$$

$$= \left[\frac{x^2}{2} + xy\right]_0^1$$

$$= \frac{1}{2}+y$$

$$= y+\frac{1}{2}$$

(c) Two random variable are said to be independent if $g(u).h(y) = f(u,y)$

$$g(u).h(y) = \left(x+\frac{1}{2}\right)\left(y+\frac{1}{2}\right) = xy + \frac{x}{2} + \frac{y}{2} + \frac{1}{4}$$

$$f(u,y) = u+y$$

So, $g(u).h(y) \neq f(u,y)$

Random variable X and Y are not independent.

(d) $$f(x \mid y=0.5) = \frac{f(u,y=0.5)}{h(y=0.5)}$$

$$= \frac{x+0.5}{0.5+\frac{1}{2}}$$

$$= \frac{u+0.5}{1}$$

$$= u+0.5$$

$$= u+\frac{1}{2}$$

(Ans)

2

ⓐ $f(x) = cxe^{-\frac{x}{2}} \quad x > 0$

$\Rightarrow \int_0^\infty cxe^{-\frac{x}{2}} dx = 1$

$\Rightarrow c \int_0^\infty xe^{-\frac{x}{2}} dx = 1$

let, $u = -\frac{x}{2} \quad , x = -2u$

$\frac{du}{dx} = -\frac{1}{2}$

$dx = -2 du$

$I = \int xe^{-\frac{x}{2}} dx$

$= \int (-2u) e^u (-2 du)$

$= 4 \int ue^u du$

$= 4 (ue^u - \int e^u du)$

$= 4 (ue^u - e^u)$

$= 4e^u (u-1)$

$= 4e^{-\frac{x}{2}} (-\frac{x}{2} - 1) \quad [u = -\frac{x}{2}]$

$= e^{-\frac{x}{2}} (-2x - 4)$

$\Rightarrow c \left[e^{-\frac{x}{2}} (-2x-4) \right]_0^\infty = 1$

$$\Rightarrow c\left[\frac{-2x-4}{e^{x/2}}\right]_0^{\infty} = 1$$

$$\Rightarrow c\left[-0-\left(\frac{-0-4}{1}\right)\right] = 1$$

$$\Rightarrow c\,(-(-4)) = 1$$

$$\Rightarrow c\,.\,4 = 1$$

$$\Rightarrow c = \frac{1}{4}$$

(Ans)

2

(b)

$$CDF = F(x) = \int_0^x f(x)\, du$$

$$= \int_0^x \frac{1}{4} x e^{-\frac{x}{2}}\, du$$

$$\cancel{\frac{1}{4}\left[e^{-2x}\right]}$$

$$= \frac{1}{4}\int_0^x x e^{-\frac{x}{2}}\, du$$

$$= \frac{1}{4}\left[e^{-\frac{x}{2}}(-2u-4)\right]_0^x \quad \left[\begin{array}{l}\text{we get} \\ \int x e^{-\frac{u}{2}} du \text{ from (a)}\end{array}\right]$$

$$= \frac{1}{4}\left[\frac{(-2u-4)}{e^{x/2}}\right]_0^u$$

$$= \frac{1}{4}\left(\frac{-2u-4}{e^{u/2}} - (-4)\right)$$

$$= \frac{1}{4}\left(\frac{-2(u+2)}{e^{u/2}} + 4\right)$$

$$= -\frac{x+2}{2e^{x/2}} + 1$$

$$= -\frac{x}{2e^{x/2}} - \frac{2}{2e^{x/2}} + 1$$

$$= 1 - \frac{xe^{-\frac{x}{2}}}{2} - e^{-\frac{x}{2}}$$

(Ans)