

$$f(x) = \frac{x^2 \ln x}{e^x \sin x}$$

$$\begin{aligned} f'(x) &= \frac{e^x \sin x \frac{d}{dx}(x^2 \ln x) - x^2 \ln x \frac{d}{dx}(e^x \sin x)}{(e^x \sin x)^2} \\ &= \frac{e^x \sin x \left(x^2 \frac{d}{dx}(\ln x) + \ln x \frac{d}{dx}(x^2) \right) - x^2 \ln x \left(e^x \frac{d}{dx}(\sin x) + \sin x \frac{d}{dx}(e^x) \right)}{e^{2x} \sin^2 x} \\ &= \frac{e^x \sin x \left(x^2 \frac{1}{x} + \ln x \cdot 2x \right) - x^2 \ln x (e^x \cos x + \sin x e^x)}{e^{2x} \sin^2 x} \\ &= \frac{e^x [x \sin x + 2x \ln x \sin x - x^2 \ln x \cos x - x^2 \ln x \sin x]}{e^{2x} \sin^2 x} \\ f'(x) &= \frac{x \sin x (1 + 2 \ln x) - x^2 \ln x (\cos x + \sin x)}{e^x \sin^2 x} \end{aligned}$$

$$f''(x) = \frac{e^x \sin^2 x \frac{d}{dx} [x \sin x (1 + 2 \ln x) - x^2 \ln x (\cos x + \sin x)]}{(e^x \sin^2 x)^2} - \frac{\frac{d}{dx}(e^x \sin^2 x)}{e^{2x} \sin^2 x}$$

Logarithmic differentiation:

$y = x^{2x}$ (circled 2x)
 $y = x^u \rightarrow \frac{dy}{du} = 2x$ (circled u)
 Power rule
 $y = x^{2x}$

Taking logarithm on both sides to find

$$\begin{aligned} \ln y &= \ln x^{2x} = 2x \ln x \\ \frac{d}{dx}(\ln y) &= \frac{d}{dx}(2x \ln x) \\ \Rightarrow \frac{1}{y} \frac{dy}{dx} &= 2x \frac{d}{dx}(\ln x) + \ln x \frac{d}{dx}(2x) \\ \Rightarrow \frac{dy}{dx} &= y \left[2x \frac{1}{x} + \ln x \cdot 2 \right] \\ \Rightarrow \frac{dy}{dx} &= x^{2x} [2 + 2 \ln x] \end{aligned}$$