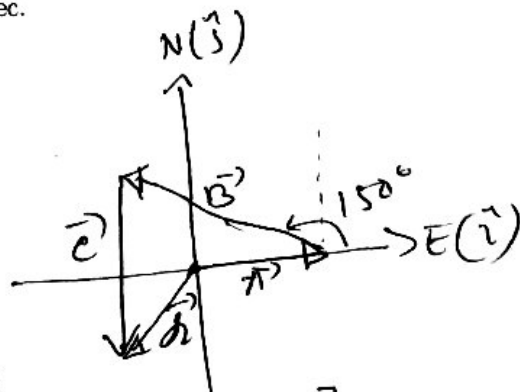


Problems: Marks are as indicated

- (5 marks) You started walking from your home and proceeded towards North 10m, then a distance of 20m at an angle of  $150^\circ$  counterclockwise from the East, and finally 30 m towards South. Find the final position from your home both in Cartesian and Polar forms. Note that East is the x-axis.
- (5 marks) The velocity of a toy car is given by  $v = 2t - 3$ , where  $v$  is in meters per second and  $t$  is in second. Find the displacement and average velocity from  $t = 0$  to  $t = 2.0$  sec.

#1 Let  $\vec{r}_0 = 0$  (origin)  
 $\therefore \Delta \vec{r} = \vec{A} + \vec{B} + \vec{C} \equiv \vec{r} - \vec{r}_0 = \vec{r}$

Now,  $\vec{A} = 10\text{m}\hat{j}$   
 $\vec{B} = (20\text{m}, 150^\circ)$   
 $\vec{C} = -30\text{m}\hat{j}$



$$\therefore \vec{r} = (A_x + B_x + C_x)\hat{i} + (A_y + B_y + C_y)\hat{j}$$

$$= [(0 + 20\cos 150^\circ + 0)\hat{i} + (10 + 20\sin 150^\circ - 30)\hat{j}]\text{m}$$

$$\Rightarrow \boxed{\vec{r} = -17.32\text{m}\hat{i} - 10\text{m}\hat{j}} = \boxed{(20\text{m}, 210^\circ)}$$

Cartesian form Polar form

#2  $\Delta x = \int_0^2 v dt = \int_0^2 (2t - 3) dt = \left[ \frac{2t^2}{2} - 3t \right]_{t=0}^{t=2}$   
 $= (2^2 - 3 \cdot 2) \text{m} = -2\text{m}$

$$\therefore \boxed{\Delta x = -2\text{m} \Rightarrow \text{Displacement}}$$

and  $V_{av} = \frac{\Delta x}{\Delta t} = \frac{-2\text{m}}{2\text{sec}} \Rightarrow \boxed{V_{av} = -1\text{m/s}}$

Problems: Marks are as indicated

1. (5 marks) Consider three vectors:  $\vec{A} = (-1, 2)$ ,  $\vec{B} = (3, 45^\circ)$  and  $\vec{C} = (2, -1)$ . Compute:  $\vec{A} - 2\vec{B} + \vec{C}$  both in Cartesian and Polar forms.
2. (5 marks) The position of a particle moving along the  $x$ -axis is given by  $x = 12t^2 - 2t^3$ , where  $x$  is in meters and  $t$  is in seconds. Determine: (a) the velocities at  $t = 2.0$  s and  $t = 4.0$  s, and (b) the average velocity from  $t = 2.0$  s to  $t = 4.0$  s.

#1  $\vec{A} - 2\vec{B} + \vec{C} = (A_x - 2B_x + C_x)\hat{i} + (A_y - 2B_y + C_y)\hat{j}$   
 $= [-1 - 2(3\cos 45^\circ) + 2]\hat{i} + [2 - 2(3\sin 45^\circ) - 1]\hat{j}$   
 $\Rightarrow \vec{A} - 2\vec{B} + \vec{C} = -3.2\hat{i} - 3.2\hat{j} = (4.5, 225^\circ)$  ✓

#2 (a)  $v_x = \frac{dx}{dt} \Big|_{t=2} = (24t - 6t^2) \Big|_{t=2} \text{ m/s} = \boxed{24 \text{ m/s}}$  ✓  
 $v_x = \frac{dx}{dt} \Big|_{t=4} = (24 \times 4 - 6 \times 4^2) \text{ m/s} = \boxed{0}$  ✓

(b) Since  $x(t)$  is cubic polynomial at constant  $s$ .  
 $\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x_4 - x_2}{t_4 - t_2} = \frac{[(12 \cdot 4^2 - 2 \cdot 4^3) - (12 \cdot 2^2 - 2 \cdot 2^3)] \text{ m}}{4 - 2}$   
 $\Rightarrow \boxed{\bar{v} = 16 \text{ m/s}}$  ✓

**Problems: Marks are as indicated**

1. (5 marks) Consider three vectors:  $\vec{A} = (-1, 2)$ ,  $\vec{B} = (3, 45^\circ)$  and  $\vec{C} = (2, -1)$ . Compute  $\vec{A} + 2\vec{B} - \vec{C}$  both in Cartesian and Polar forms.
2. (5 marks) The position of a particle is given by  $x = 20t - 5t^3$ , where  $x$  is in meters and  $t$  is in seconds. When and where the particle changes the direction of it's velocity?

$$\#1) \vec{A} + 2\vec{B} - \vec{C} = (A_x + 2B_x - C_x)\hat{i} + (A_y + 2B_y - C_y)\hat{j}$$

$$= (-1 + 2 \cdot 3 \cos 45^\circ - 2)\hat{i} + (2 + 2 \cdot 3 \sin 45^\circ + 1)\hat{j}$$

$$\Rightarrow \vec{A} + 2\vec{B} - \vec{C} = 1.24\hat{i} + 7.24\hat{j} = (7.35, 80.5^\circ) \quad \checkmark$$

#2 The direction of velocity changes at the turning point.

Now,  $v = \frac{dx}{dt} = 0 \Rightarrow 20 - 15t^2 = 0 \Rightarrow t = \pm \left(\frac{20}{15}\right)^{1/2} \quad (t \neq 0)$

$$\Rightarrow \boxed{t = 1.158 \text{ s}} \quad \checkmark$$

and  $x(t = 1.158) = [20(1.158) - 5(1.158)^3] \text{ m}$

$$\Rightarrow \boxed{x(1.158) = 15.4 \text{ m}} \quad \checkmark$$

MCQ: Choose Only One Answer.

1. The Inertia of an object completely depends on the

- A. size of the object.    B. amount of Push applied.    C. Both of these.    D. None of these.

1. D

2. An object moving in a uniform circular motion in clockwise direction. At time  $t$ , it's location is in the 2<sup>nd</sup> quadrant. At that time, it's velocity is directed in the

- A. 1<sup>st</sup> quadrant.    B. 2<sup>nd</sup> quadrant.    C. 3<sup>rd</sup> quadrant.    D. 4<sup>th</sup> quadrant.

2. A

Problems: Marks are as indicated

3. (4 marks) A particle leaves the origin at  $t = 0$  with an initial velocity  $\vec{v} = 3.00\hat{i}$  m/s and a constant acceleration of  $\vec{a} = (-1.00\hat{i} - 0.500\hat{j})$  m/s<sup>2</sup>. Find: (a) when it reaches its maximum x coordinate, (b) what are it's velocity and position vector at that time?

4. (4 marks) A 0.340 kg particle moves in an  $xy$ -plane according to  $x(t) = -15.00 + 2.00t - 4.00t^3$  and  $y(t) = 25.00 + 7.00t - 9.00t^2$ , with  $x$  and  $y$  in meters and  $t$  in seconds. At  $t = 0.700$  sec, find the magnitude and direction of the force on the particle.

#3  $v_{0x} = 3 \text{ m/s}$ ;  $v_{0y} = 0$  &  $a_x = -1 \text{ m/s}^2$ ;  $a_y = -0.5 \text{ m/s}^2$ .

(a) Max. x-coord:  $v_x = 0 = v_{0x} + a_x t = 3 - t \Rightarrow t = 3 \text{ sec}$

(b)  $v_x = v_{0x} + a_x t = 3 - 1(3) = 0$   
 $v_y = v_{0y} + a_y t = [0 + (-0.5)(3)] \text{ m/s} = -1.5 \text{ m/s}$  }  $\Rightarrow \vec{v} = -1.5 \text{ m/s } \hat{j}$

Now  $x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2 = [0 + 3(3) + \frac{1}{2}(-1)(3)^2] \text{ m} = 4.5 \text{ m}$   
 $y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2 = [25 + 7(3) + \frac{1}{2}(-9)(3)^2] = -2.25 \text{ m}$  }  $\Rightarrow \vec{r} = (4.5\hat{i} - 2.25\hat{j}) \text{ m}$

#4  $a_x = \frac{d^2x}{dt^2} = -24t \Rightarrow \text{At } t = 7 \text{ sec: } a_x = -16.8 \text{ m/s}^2$

&  $a_y = \frac{d^2y}{dt^2} = -18 \text{ m/s}^2$ .

So,  $F_x = m a_x = -50.7 \text{ N}$   
 &  $F_y = m a_y = -61 \text{ N}$  }  $\Rightarrow F = \sqrt{F_x^2 + F_y^2} = 8.3 \text{ N}$   
 and  $\theta_p = 180 + \tan^{-1}(\frac{61}{50.7}) = 227^\circ$

Finally at  $t = 7 \text{ sec: } \vec{P} = (8.3 \text{ N}, 227^\circ)$  ←

### MCQ: Choose Only One Answer.

1. Two different object has the same inertia, but one is bigger in size. Which one requires stronger push to change the velocity?
- A. The bigger object.    B. The smaller object.    C. Both need the same Push.  
D. It can not be determined.

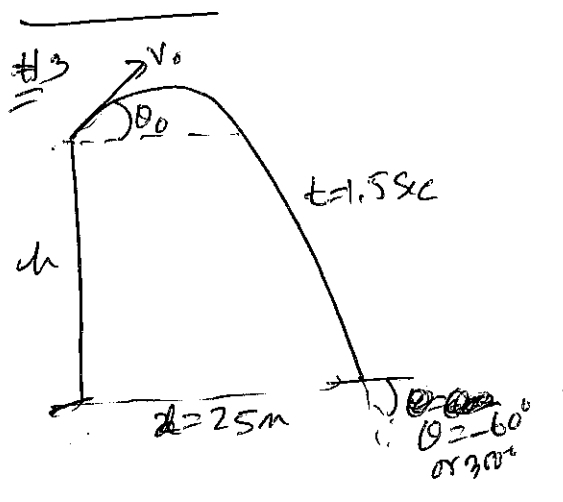
 1. C

2. An object moving in a uniform circular motion obeys the
- A. 1<sup>st</sup> law.    B. 2<sup>nd</sup> law.    C. 3<sup>rd</sup> law.    D. It can not be determined.

 2. B

### Problems: Marks are as indicated

3. (4 marks) A ball is thrown rightward from the top of the tower, at height  $h$  above the ground. The ball hits the ground 1.50 sec later, at distance  $d = 25.0$  m from the building and at angle  $\theta = 60.0^\circ$  with the horizontal. Find:  
(a) the height  $h$  (b) the initial velocity of the ball.
4. (4 marks) A 2.00 kg object is subjected to three forces that give it an acceleration  $\vec{a} = -(8.00 \text{ m/s}^2)\hat{i} + (6.00 \text{ m/s}^2)\hat{j}$ . If two of the three forces are  $\vec{F}_1 = (30.0 \text{ N})\hat{i} + (16.0 \text{ N})\hat{j}$  and  $\vec{F}_2 = -(12.0 \text{ N})\hat{i} + (8.00 \text{ N})\hat{j}$ , find the third force.



$$x = x_0 + v_{0x}t = x_0 + v_x t$$

$$d = 25 \text{ m} = x - x_0 = v_x (1.5) \Rightarrow v_{0x} = 16.7 \text{ m/s} = v_x$$

$$\text{Now } \tan \theta = \frac{v_y}{v_x} \Rightarrow v_y = v_x \tan 30^\circ = -28.9 \text{ m/s}$$

$$\therefore v_{0y} = v_y - a_y t = -14.2 \text{ m/s}$$

hence,

$$\vec{v}_0 = (16.7 \text{ m/s}, -14.2 \text{ m/s}) = (21.9 \text{ m/s}, 320^\circ)$$

Cartesian form                      Polar form

The height  $h$

$$h = |y - y_0| = v_{0y}t + \frac{1}{2} a_y t^2$$

$$\Rightarrow \boxed{h = 32.3 \text{ m}}$$

#4  $\Sigma \vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 = m\vec{a} \Rightarrow \vec{F}_3 = m\vec{a} - \vec{F}_1 - \vec{F}_2$

substituting the values:

$$\vec{F}_3 = [2(-8\hat{i} + 6\hat{j}) - (30\hat{i} + 16\hat{j}) - (-12\hat{i} + 8\hat{j})] \text{ N}$$

$$\Rightarrow \boxed{\vec{F}_3 = (-34\hat{i} - 12\hat{j}) \text{ N}}$$

MCQ: Choose Only One Answer.

1. You are carrying a crate of weight 147 N on your shoulder and walking with constant velocity towards home. Your weight is 580 N. The net force on you is

A. zero. B. 147 N. C. 433 N. D. 727 N.

1. A

2. An object is moving in a uniform circular motion of radius 5 m. It completes a cycle in 2 seconds. What is the magnitude of the velocity of the object?

A. 5 m/s. B.  $\pi$  m/s. C.  $5\pi$  m/s D.  $10\pi$  m/s

2. C

Problems: Marks are as indicated

3. (4 marks) In Figure-1, a stone is projected at a cliff of height  $h$  with an initial speed of 42.0 m/s directed at angle  $\theta_0 = 60^\circ$  above the horizontal. The stone strikes 5.50 sec after launching. Find (a) the speed of the stone just

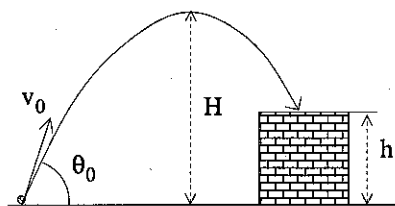


Figure 1: Diagram for Question-3

before the impact on the cliff, (b) the maximum height  $H$  reached above the ground.

4. (4 marks) A 2.00 kg object is subjected to three forces that give it an acceleration  $\vec{a} = -(8.00 \text{ m/s}^2)\hat{i} + (6.00 \text{ m/s}^2)\hat{j}$ . If two of the three forces are  $\vec{F}_1 = (30.0 \text{ N})\hat{i} + (16.0 \text{ N})\hat{j}$  and  $\vec{F}_2 = -(12.0 \text{ N})\hat{i} + (8.00 \text{ N})\hat{j}$ , find the third force.

#3 (a)  $V_x = v_{0x} = v_0 \cos \theta_0 = 42 \cos 60^\circ \text{ m/s} = 21 \text{ m/s}$   
 $V_y = v_{0y} + a_y t = [42 \sin 60^\circ - (9.8)(5.5)] \text{ m/s} = -17.5 \text{ m/s}$   
 $V = \sqrt{V_x^2 + V_y^2} \Rightarrow \boxed{V = 27.3 \text{ m/s}}$

(b)  $H = v_{0y}^2 / 2g = (42 \sin 60^\circ)^2 / (2 \times 9.8) \Rightarrow \boxed{H = 67.5 \text{ m}}$

#4  $\sum \vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 = m\vec{a} \Rightarrow \vec{F}_3 = m\vec{a} - \vec{F}_1 - \vec{F}_2$   
 $\therefore \vec{F}_3 = [2(-8\hat{i} + 6\hat{j}) - (30\hat{i} + 16\hat{j}) - (-12\hat{i} + 8\hat{j})] \text{ N}$   
 $\Rightarrow \boxed{\vec{F}_3 = (-34\hat{i} - 12\hat{j}) \text{ N}}$

## MCQ: Choose Only One Answer.

1. Which of the following statements is/are true?

(a) The number of the potential energy terms in the total energy can not be more than the number of the force acting on a body.

(b) The potential energy of an elastic system can never have a negative value.

A. Only (1a). B. Only (1b). C. Both (1a) and (1b). D. None of these.

1. C

2. Consider free-fall phenomena of three identical balls (except colors): red ball is dropped, white ball is thrown downward and the green ball is thrown upward from the same position above the ground. Ignore air resistance. Which of the following statements are true?

(a) They have the same final potential energy.

(b) The work done on the green ball is the maximum.

(c) They have the same change in kinetic energy.

A. Only (2a) and (2b). B. Only (2a) and (2c). C. Only (2b) and (2c). D. All are true.

2. B

## Problems: Marks are as indicated

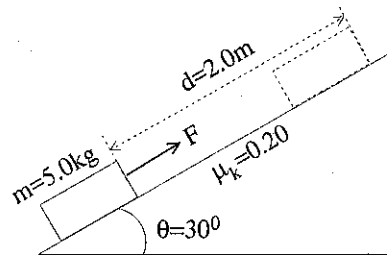
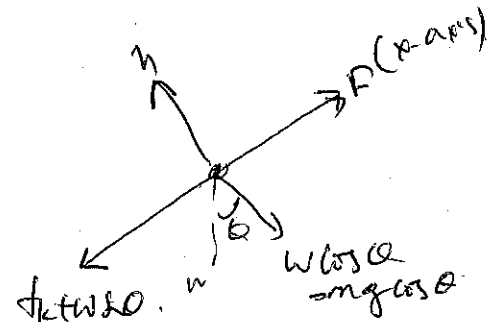
3. (8 marks) Figure-1 shows a crate of mass 5.0kg is pulled by a Force  $F$  on an inclined rough surface at an angle  $\theta = 30^\circ$ . Starting from rest, the crate moved a distance  $d = 2.0\text{m}$  with final velocity 2.0m/s up the inclined

Figure 1: Diagram for Question-3

surface. The coefficient of kinetic friction is  $\mu_k = 0.20$ . Find the magnitude of the force  $F$ .

$$W_{\text{net}} = \Delta K = W_{\text{net}} + W_g + W_f + W_p = mgd \cos(90^\circ + \theta) - \mu_k mgd + Fd$$

$$\Delta K = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = \frac{1}{2}mv_f^2$$

$$\therefore \frac{1}{2}mv_f^2 = -mgd \sin \theta - \mu_k (mg \cos \theta) d + Fd$$

$$\Rightarrow F = \frac{mv_f^2 + 2mgd(\sin \theta + \mu_k \cos \theta)}{2d}$$

$$= \frac{5(2)^2 + 2(5)(9.8)(2)[\sin 30 + 0.2(\cos 30)]}{2 \times 2} \text{ N}$$

$$\Rightarrow \boxed{F = 38.0 \text{ N}} \quad \leftarrow$$

MCQ: Choose Only One Answer.

1. The Work Energy Theorem requires that

- A. the body must obey Newton's 1<sup>st</sup> law. B. the system must be a conservative system.  
C. no frictional force be present. D. None of these.

1. D

2. Figure-1 shows three paths by which an object can move from point a to point b. The work done along paths I and II are as shown. If the work done is path dependent, the total work done to go from point a to point b along path I and back to point a by path III will be

- A. zero. B. positive. C. negative. D. It can not be determined.

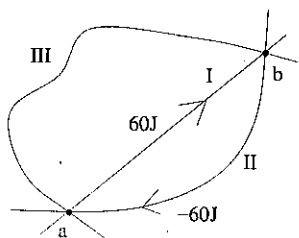
2. C

Figure 1: Diagram for Question-2

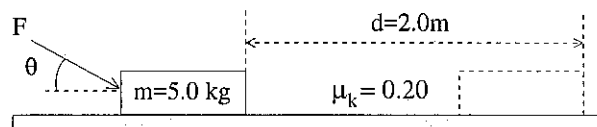


Figure 2: Diagram for Question-3

Problems: Marks are as indicated

3. (8 marks) Figure-2 shows a crate of mass 5.0 kg is pushed by a Force  $F$  that is directed at an angle  $\theta = 30^\circ$ . Starting from rest, the crate moved a distance  $d$  with final velocity 2.0 m/s on a frictional horizontal surface with  $\mu_k = 0.20$ . Find the magnitude of the force  $F$ .

$$W_{\text{net}} = \Delta K = K_f - K_i = K_f = \frac{1}{2}mv_f^2 = \frac{1}{2}(5)(2)^2 = 10 \text{ J}$$

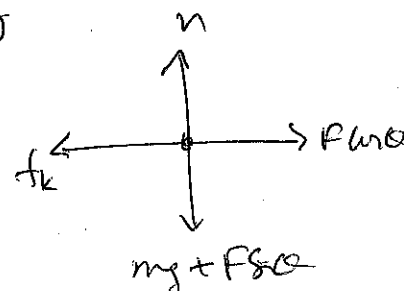
$$\begin{aligned} \text{Now, } W_{\text{net}} &= W_n + W_f + W_g + W_p \\ &= -\mu_k n d + F d \cos \theta \\ &= -\mu_k (mg + F \sin \theta) d + F d \cos \theta \\ &= -\mu_k mg d + F d (\cos \theta - \mu_k \sin \theta) \end{aligned}$$

$$\text{Therefore, } \frac{1}{2}mv_f^2 = -\mu_k mg d + F d (\cos \theta - \mu_k \sin \theta) = 10 \text{ J}$$

$$\Rightarrow F = \frac{10 + \mu_k mg d}{d (\cos \theta - \mu_k \sin \theta)}$$

$$= \frac{10 + (0.2)(5)(9.8)(2.0)}{2.0 (\cos 30 - 0.2 \sin 30)} \text{ N}$$

$$\Rightarrow \boxed{F = 19.3 \text{ N}}$$





MCQ: Choose Only One Answer.

1. The Work Energy Theorem requires that

- A. the body must obey Newton's 2<sup>nd</sup> law. B. all forces must be conservative.  
C. no frictional force be present. D. None of these.

1. D

2. Figure-1 shows three paths by which an object can move from point a to point b. The work done along paths I and II are as shown. If the work done is path independent, the work done to go from point b to point a along path III will be

- A. exactly -60 J. B. exactly 60 J. C. more than 60 J. D. less than -60 J.

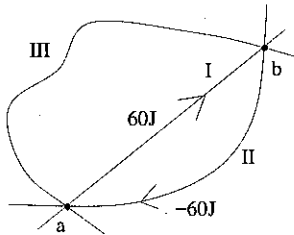
2. A

Figure 1: Diagram for Question-2

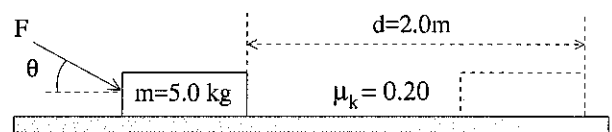


Figure 2: Diagram for Question-3

Problems: Marks are as indicated

3. (8 marks) Figure-2 shows a crate of mass 5.0 kg is pushed by a Force  $F$  that is directed at an angle  $\theta = 30^\circ$ . The crate moved a distance  $d$  at constant velocity on the frictional horizontal surface with  $\mu_k = 0.20$ . Find the magnitude of the force  $F$ .

$$W_{\text{net}} = \Delta K = \cancel{W_n} + \cancel{W_f} + \cancel{W_g} + W_P = 0 \quad (v = \text{const.} \Rightarrow \Delta K = 0)$$

$$\Rightarrow -\mu_k n d + F d \cos \theta = 0$$

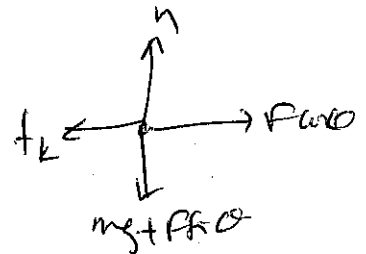
$$\Rightarrow -\mu_k (mg + F \sin \theta) + F \cos \theta = 0$$

$$\Rightarrow F (\cos \theta - \mu_k \sin \theta) = \mu_k mg$$

$$\Rightarrow F = \frac{\mu_k mg}{\cos \theta - \mu_k \sin \theta}$$

$$= \frac{(0.2)(5)(9.8)}{\cos 30 - 0.2(\sin 30)} \text{ N}$$

$$\Rightarrow \boxed{F = 12.79 \text{ N}}$$



Solution

Name: \_\_\_\_\_

ID # \_\_\_\_\_

Quiz # 4 (November 26, 2018)

PHY 107 (5)

Marks: \_\_\_\_\_

/10

MCQ: Choose Only One Answer.

1. You drop a ball from 1.5 m above a hard floor. The ball bounced up from the floor and rises 1.2 m above the floor. Ignore air resistance. The collision between the ball and the hard floor must be  
 A. completely inelastic. B. completely elastic. C. partially elastic.  
 D. It can not be determined.

1. c

2. Figure-1 shows linear momentum of an object as a function of time. Which point in the graph is a turning point?  
 A. Points a and d. B. Points d and f. C. Points b and c. D. Points e and f.

2. B

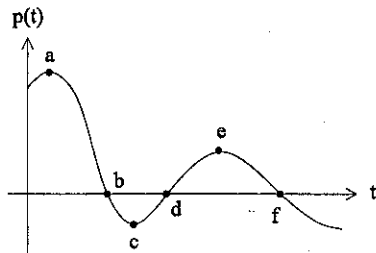


Figure 1: Diagram for Question-2

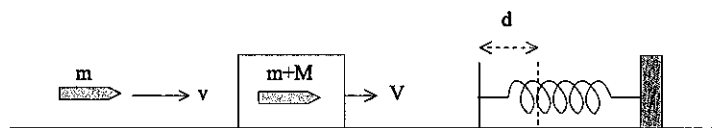


Figure 2: Diagram for Question-3

3. A torque on an object is given by  $\vec{\tau} = \vec{r} \times \vec{F}$ , and it can be zero when  
 A. the axis of rotation passes through the object. B. the applied force is zero.  
 C. the force and the position vector are parallel. D. All of the above.

3. D

Problems: Marks are as indicated

4. A bullet of mass  $m = 300$  grams is fired with speed  $v = 500$  m/s by an air rifle towards a wooden block on mass  $M = 30$  kg which is at rest on a frictionless horizontal floor. The bullet remains inside block after collision, and the bullet-block runs into an ideal spring of force constant  $200$  kN/m, and compress it by a distance  $d$  as shown in Figure-2. Find the following:

- (a) (2 marks) The speed of the wooden block just after the collision.  
 (b) (2 marks) The distance  $d$ .

$$\begin{aligned} \text{(a)} \quad P_f &= P_i \Rightarrow (m+M)V = mv \\ \Rightarrow V &= \frac{mv}{m+M} = \frac{(0.3)(500)}{30+0.3} \text{ m/s} \\ \Rightarrow V &= 4.95 \text{ m/s} \quad \checkmark \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad \frac{1}{2}kd^2 &= \frac{1}{2}(m+M)V^2 \\ \Rightarrow d &= \sqrt{\frac{(30.3)(4.95)^2}{200 \times 10^3}} \text{ m} \end{aligned}$$

$$\Rightarrow d = 0.061 \text{ m} = 6.1 \text{ cm} \quad \checkmark$$

5. (3 marks) The angular position of a point on a wheel is given by

$$\theta = 2t^3 - 2t^2 + 4,$$

where  $\theta$  is in radians and  $t$  is in seconds. Find the turning point (if any).

Need  $\omega = 0$  &  $\neq 0$ .

$$\therefore \omega = \frac{d\theta}{dt} = 6t^2 - 4t = 0 \Rightarrow t = 0, \frac{2}{3} \text{ Sec}$$

can't be turning point

$$\therefore t = \frac{2}{3} \text{ Sec}$$

$$\& \theta(t = \frac{2}{3}) = \left[ 2\left(\frac{2}{3}\right)^3 - 2\left(\frac{2}{3}\right)^2 + 4 \right] \text{ rad} = \underline{3.7 \text{ rad.}}$$

MCQ: Choose Only One Answer.

- You drop a ball of mass 200 grams from a height 1.5 m above a hard floor. The ball bounced up from the floor and rises 1.2 m above the floor. Ignore air resistance. The collision between the ball and the hard floor is inelastic because  
 A. 0.588 J of kinetic energy is lost.    B. 1.96 J of kinetic energy is lost.  
 C. 1.08 J of kinetic energy is lost.    D. None of these. 1. A
- A skater holds her arms outstretched as she spins at 180 rpm. What is the speed of her hands if they are 22.3 cm apart? Note that 'rpm' means 'revolution per minute'.  
 A. 0.220 m/s.    B. 13.2 m/s.    C. 132 cm/s.    D. ~~120~~<sup>210</sup> m/s. 2. D/any.

Problems: Marks are as indicated

- A bullet of mass  $m = 300$  grams is fired with speed  $v = 500$  m/s by an air rifle towards a wooden block on mass  $M = 30$  kg which is hanging at rest from the ceiling. The bullet remains inside block after collision, and the

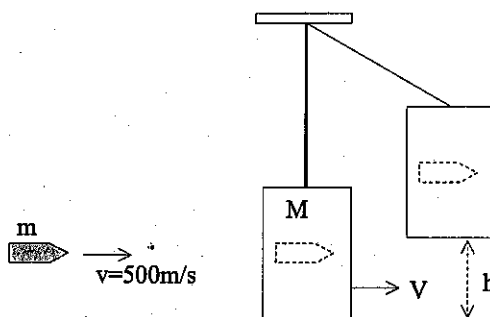


Figure 1: Diagram for Question-3

bullet-block raises a height of  $h$  as shown in Figure-1. Find the following:

- (2 marks) The energy loss during collision.
- (2 marks) The height  $h$ .

(a)  $P_f = P_i \Rightarrow (m+M)V = mv \Rightarrow V = \frac{mv}{m+M} = \frac{(0.3)(500)}{0.3+30} \text{ m/s} = 4.95 \text{ m/s}$

$\therefore K_{\text{lost}} = | \Delta K | = \left| \frac{1}{2}(m+M)V^2 - \frac{1}{2}mv^2 \right|$

$= \left[ 0.5(30.3)(4.95)^2 - 0.5(0.3)(500)^2 \right] \text{ J}$

$= 37129 \text{ J} \approx 37 \text{ kJ.}$

(b)  $\frac{1}{2}(m+M)V^2 = (m+M)gh \Rightarrow h = \frac{V^2}{2g} = 1.25 \text{ m.}$

4. A force  $\vec{F} = (2\hat{i} - 3\hat{j})\text{N}$  is applied to an object of mass 2.0 kg which is located at  $\vec{r} = (\hat{j} + 2\hat{k})\text{m}$ .

(a) (2 marks) What is the moment of inertia.

(b) (2 marks) Find the torque in Cartesian form.

$$(a) \quad I = mr^2 = 2(0+1+4) \Rightarrow I = 10 \text{ kg}\cdot\text{m}^2 \quad \checkmark$$

$$(b) \quad \vec{\tau} = \vec{r} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x & y & z \\ F_x & F_y & F_z \end{vmatrix} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 1 & 2 \\ 2 & -3 & 0 \end{vmatrix} \text{ N}\cdot\text{m}$$

$$= [\hat{i}(0+6) - \hat{j}(0-4) + \hat{k}(0-2)] \text{ N}\cdot\text{m}$$

$$\Rightarrow \underline{\vec{\tau} = (6\hat{i} + 4\hat{j} - 2\hat{k}) \text{ N}\cdot\text{m}} \quad \checkmark$$

# Solution

Name: \_\_\_\_\_

ID # \_\_\_\_\_

Quiz # 4 (November 20, 2018)

PHY 107 (7)

Marks: \_\_\_\_\_

/10

MCQ: Choose Only One Answer.

1. Which of the following is equivalent to Newton's 1<sup>st</sup> law?

A.  $\vec{p} = \text{constant}$ . B.  $\vec{J} = 0$ . C. Both of these.

D. ALAS!!! I don't know the answer. I usually do not pay attention in the class. Even though I am telling the truth (meaning the answer is correct), I will accept zero credit for this answer gleefully.

1. C

2. Figure-1 shows linear momentum of an object as a function of time. Where in the graph the object is in Equilibrium?

A. At points a and d. B. At points b and f. C. At points c and e. D. At points b and e.

2. C

3. Figure-3 shows the magnitude of the angular momentum  $L$  of a wheel versus time  $t$ . In which region, the magnitude of the torques is the greatest?

A.  $\tau_A$ . B.  $\tau_B$ . C.  $\tau_C$ . D.  $\tau_D$ .

3. D

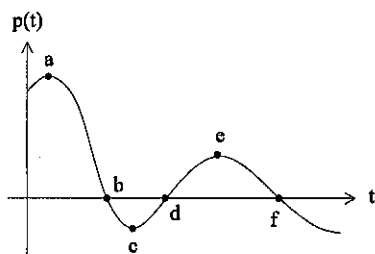


Figure 1: Diagram for Question-2

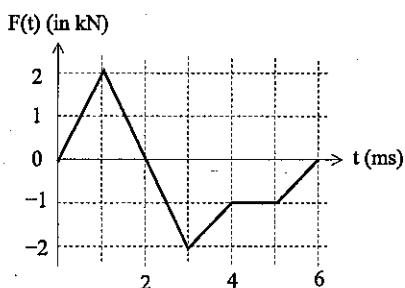


Figure 2: Diagram for Question-4

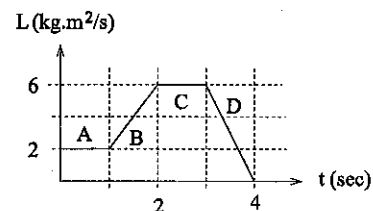


Figure 3: Diagram for Question-(4)

Problems: Marks are as indicated

4. (8 marks) Figure-2 shows a force applied on an object of mass 200 grams for 6.0 ms. Find the following:
- (a) (2 marks) The impulse on the object.
- (b) (2 marks) The average force applied on the object.

(a)  $J = \int F dt = \text{Area under the curve}$   
 Each square =  $(1 \text{ kN})(1 \text{ ms}) = 1.0 \text{ Ns}$ .  
 # of squares under the graph =  $(2-4) = -2$   
 $\therefore J = (-2)(1 \text{ Ns}) = \underline{\underline{-2 \text{ Ns}}}$

(b)  $F_{av} = \frac{J}{\Delta t} = \frac{-2}{6 \text{ ms}} \Rightarrow \underline{\underline{F_{av} = 333 \text{ N}}}$

5. (3 marks) An object of mass 200 grams is rotating whose angular speed is given by

$$\omega(t) = 6t^2 - 12t,$$

where  $\omega$  is in radian per seconds and  $t$  is in seconds. Compute the angular displacement and average angular velocity from  $t = 0$  to  $t = 2.0$  sec.

$$\begin{aligned}\Delta\theta &= \int_0^2 \omega dt = \int_0^2 (6t^2 - 12t) dt \\ &= \left[ 2t^3 - 6t^2 \right]_0^2 \text{ rad} = \left[ 2(2)^3 - 6(2)^2 \right] \text{ rad}\end{aligned}$$

$$\Rightarrow \boxed{\Delta\theta = -8 \text{ rad}} \quad \checkmark$$

$$\omega_{av} = \frac{\Delta\theta}{\Delta t} = \frac{-8}{2} \text{ rad/s}$$

$$\Rightarrow \boxed{\omega_{av} = -4 \text{ rad/s}} \quad \checkmark$$

MCQ: Choose Only One Answer.

1. Two spaceship, one is double in mass than the other, are launched from earth surface. For which spaceship the escape velocity is higher? Ignore air resistance.

A. Both have the same escape velocity. B. The heavier one. C. The lighter one. D. None of these.

1. A

2. The free-fall acceleration at the surface of planet 1 is  $20 \text{ m/s}^2$ . The radius and the mass of planet 2 are twice those of planet 1. What is  $g$  on planet 2?

A.  $80 \text{ m/s}^2$ . B.  $10 \text{ m/s}^2$ . C.  $20 \text{ m/s}^2$  D.  $40 \text{ m/s}^2$ .

2. B

Problems: Marks are as indicated

3. (4 marks) After a long space travel you reached an unknown planet in a remote part in the galaxy. BY performing free-fall experiment, you found the acceleration due to gravity to be  $8.00 \text{ m/s}^2$ . The average circumference of the planet is  $2.47 \times 10^7 \text{ m}$ . What is the mass of the planet? You may use  $G = 6.673 \times 10^{-11} \text{ N.m}^2/\text{kg}^2$ .

$$g = 8 \text{ m/s}^2 \quad \text{and Planet radius, } R = \frac{\text{Circumference}}{2\pi} = \frac{2.47 \times 10^7 \text{ m}}{2\pi} = 3.93 \times 10^6 \text{ m.}$$

$$\text{Now } g = \frac{GM}{R^2}$$

$$\Rightarrow M = \frac{gR^2}{G} = \frac{8 \times (3.93 \times 10^6)^2}{6.67 \times 10^{-11}} \text{ kg}$$

$$\Rightarrow \boxed{M = 1.85 \times 10^{24} \text{ kg}} \quad \text{✓}$$



4. (4 marks) A 5.00 kg object on a horizontal frictionless surface is attached to a spring with  $k = 1000 \text{ N/m}$ . The object is displaced from equilibrium 40.0 cm horizontally and given an initial velocity of 10.0 m/s back toward the equilibrium position.

- (a) (2 marks) Find the time period of the system.  
(b) (2 marks) What is the total energy of the system?

$$(a) \omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{1000}{5}} \text{ rad/s.}$$

$$\therefore T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{1000/5}} \Rightarrow \boxed{T = 0.44 \text{ sec}} \quad \checkmark$$

$$(b) E = \text{constant} = K_{\text{initial}} + U_{\text{initial}}$$

$$= \frac{1}{2}mv_i^2 + \frac{1}{2}kx_i^2$$

$$= \left[ \frac{1}{2}(5)(10)^2 + \frac{1}{2}(1000)(0.40)^2 \right] \text{ J}$$

$$\Rightarrow \boxed{E = 330 \text{ J}} \quad \checkmark$$

MCQ: Choose Only One Answer.

1. Two spaceship, one is double in mass than the other, are launched from earth surface. For which spaceship the escape velocity is higher? Ignore air resistance.

A. Both have the same escape velocity. B. The heavier one. C. The lighter one. D. None of these.

1. A

2. If the amplitude of a SHM is doubled, then

A. it's frequency is also doubled. B. it's kinetic energy is also doubled.  
C. it's total energy is also doubled. D. it's velocity amplitude is also doubled.

2. D

Problems: Marks are as indicated

3. (4 marks) Figure-1 shows two masses are separated by a distance of 20 m. At what distance from the smaller

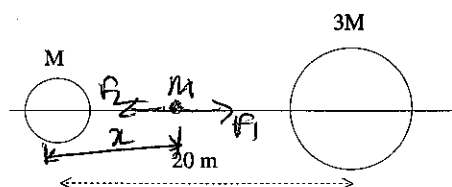


Figure 1: Diagram for Question-3

mass a third mass  $m$  can be put so that the net force on  $m$  is zero?

$$\sum F = 0 \Rightarrow F_1 = F_2 \quad (\text{See the diagram above})$$

$$\Rightarrow \frac{GmM}{x^2} = \frac{Gm(3M)}{(20-x)^2}$$

$$\Rightarrow \frac{1}{x^2} = \frac{3}{(20-x)^2}$$

$$\Rightarrow \frac{1}{x} = \frac{\sqrt{3}}{20-x}$$

$$\Rightarrow \sqrt{3}x = 20-x$$

$$\Rightarrow x(1+\sqrt{3}) = 20$$

$$\Rightarrow x = \frac{20}{1+\sqrt{3}} \text{ m}$$

$$\Rightarrow \boxed{x = 7.32 \text{ m}} \quad \leftarrow$$

4. (4 marks) Figure-2 shows a graph of a simple harmonic motion of a mass  $m$  that is attached with an ideal spring of constant  $25 \text{ N/m}$ . Answer the following questions:

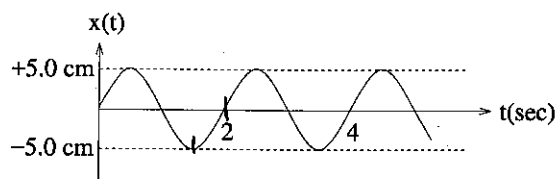


Figure 2: Diagram for Question-(4)

- (a) (3 marks) What is the kinetic energy at  $t = 2 \text{ sec}$ ?  
 (b) (3 marks) Find the acceleration at  $t = 1.5 \text{ sec}$ .

(a) At  $t = 2 \text{ sec}$ :  $x = 0$ . So  $v = v_m = \omega A$ .  
 $\therefore K_{t=2} = \frac{1}{2} m v_m^2 = \frac{1}{2} m \omega^2 A^2 = \frac{1}{2} k A^2$   
 $= \frac{1}{2} (25) (0.05)^2 \text{ J}$   
 $\Rightarrow \boxed{K_{at t=2} = 0.03125 \text{ J}}$  ✓

(b) At  $t = 1.5 \text{ sec}$ :  $x = -x_m = -0.05 \text{ m}$   
 $\therefore a = -\omega^2 x = -\left(\frac{2\pi}{T}\right)^2 x$   
 $= -\left(\frac{2\pi}{2}\right)^2 (-0.05) \text{ m/s}^2$   
 $\Rightarrow \boxed{a = 0.49 \text{ m/s}^2}$  ✓

MCQ: Choose Only One Answer.

1. A small mass  $M$  applies a force of  $F$  on a larger mass  $5M$ . How much force the larger mass applies on the smaller mass?

A.  $F$ . B.  $5F$ . C.  $25F$ . D.  $F/5$ .

1. A

2. You move a ball of mass  $m$  away from a sphere of mass  $M$ . The gravitational potential energy of the system of ball and sphere

A. increase. B. decrease. C. remain same. D. The potential energy is irrelevant.

2. A

Problems: Marks are as indicated

3. (4 marks) Compute the radius of the orbit of a 7.5 ton geosynchronized communication satellite? You may use  $G = 6.67 \times 10^{-11} \text{ N.m}^2/\text{Kg}^2$ , earth mass is  $M = 5.998 \times 10^{24} \text{ kg}$  and earth radius is  $R = 6730 \text{ km}$ .

$$T = 24 \text{ hr} = 24 \times 60 \times 60 \text{ sec.}$$

$$\text{Now } T^2 = \left( \frac{4\pi^2}{GM} \right) r^3$$

$$\Rightarrow r = \left( \frac{GMT^2}{4\pi^2} \right)^{1/3}$$

$$= \left[ \frac{(6.67 \times 10^{-11}) (5.998 \times 10^{24}) (24 \times 60 \times 60)^2}{4\pi^2} \right]^{1/3} \text{ m}$$

$$\Rightarrow r = 4.2 \times 10^7 \text{ m}$$

4. (4 marks) Figure-1 shows a graph of a simple harmonic motion of a mass  $m$  that is attached with an ideal spring of constant  $25 \text{ N/m}$ . Answer the following questions:

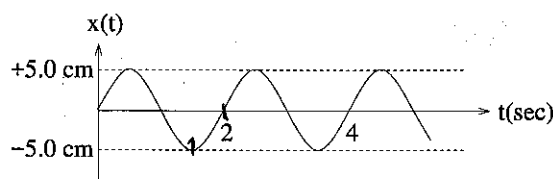


Figure 1: Diagram for Question-(3)

- (a) (2 marks) What is the kinetic energy at  $t = 2 \text{ sec}$ ?  
 (b) (2 marks) Find the acceleration at  $t = 1.5 \text{ sec}$ .

(a) At  $t = 2 \text{ sec}$ ,  $v_m = \omega x_m$  (here  $\omega = 2\pi/T$ )  
 $\therefore K = \frac{1}{2} m v_m^2 = \frac{1}{2} m \omega^2 x_m^2 = \frac{1}{2} k x_m^2$   
 $= \frac{1}{2} (25) (5 \times 10^{-2})^2 \text{ J}$   
 $\Rightarrow \boxed{K = 0.03125 \text{ J}}$  ✓

(b) At  $t = 1.5$ ,  $x = -x_m = -0.05 \text{ m}$   
 $\therefore a = -\omega^2 x = -\left(\frac{2\pi}{T}\right)^2 x$   
 $= -\left(\frac{2\pi}{4}\right)^2 (-0.05) \text{ m/s}^2$   
 $\Rightarrow \boxed{a = 0.49 \text{ m/s}^2}$  ✓