Choose only one answer Marks = /12

1. The position as a function of time is given by

$$x(t) = -c + \frac{a}{t} + bt^2 ,$$

where a, b and c are constants. What is the dimension of a?

A. L. **B.** LT. **C.** LT^{-1}. **D.** LT^{-2}.

1. B

2. Figure-1 shows three vectors. $\vec{A} - \vec{B} - \vec{C}$ equals to

A. 0. **B.** $4\hat{i}$. **C.** $2\hat{i} + 4\hat{j}$. **D.** $2\hat{i} - 4\hat{j}$.

2. A

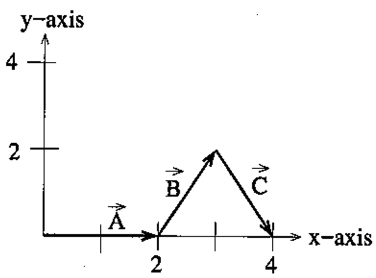


Figure 1: Diagram for Question-(2)

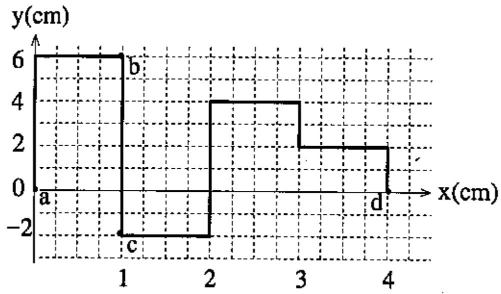


Figure 2: Diagram for Question-(3)

3. An insect is stalling on the floor slowly. It's positions at different times are shown in Figure-2. What is the total displacement from point a to point c?

A. 0. **B.** 15 cm. **C.** $(\hat{i} - 2\hat{j})$ cm. **D.** $-2\hat{j}$ cm.

3. C

4. Figure-3 gives the velocity of a cat chasing a mouse along the x-axis. Where is the cat at rest?

A. $a \to d$. **B.** $d \to h$.
C. $d \to e$. **D.** $f \to g$.

4. C

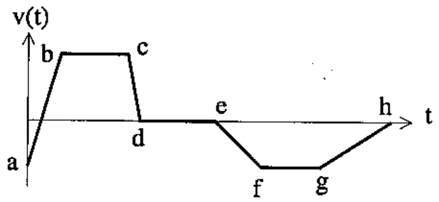


Figure 3: Diagram for Question-(4)

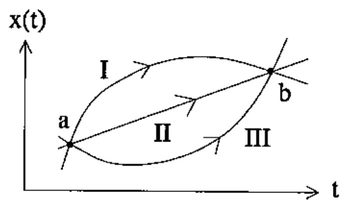


Figure 4: Diagram for Question-(5)

5. Figure-4 show three paths along which a body can move from the point a to the point b. Along which path the magnitude of the average velocity is the greatest?

A. Path-I. **B.** Path-II. **C.** Path-III. **D.** All paths are same.

5. D

6. You drop a ball downward from the top of a 100 m tall building and it lands on to to ground with speed v. 10 m below the dropping point, it's velocity is

A. 14.0 m/s upward. **B.** 14.0 m/s downward.
C. 28.0 m/s upward. **D.** 28.0 m/s downward.

6. B

7. Which of the following is/are always true for a projectile motion?

A. v_x is constant. **B.** $v_y = 0$ at the point of maximum height. **C.** $|\vec{a}| = 9.80\,\text{m/s}^2$.
D. All of these.

7. D

8. Water is poured into a container that has a small leak. The mass m of the water is given as a function of time t by $m = 5.00\,t^{0.8} - 3.00\,t + 20.00$, with $t \geq 0$, m in grams, and t in seconds. At what time is the water mass greatest?

A. 0.24 sec. **B.** 1.2 sec. **C.** 4.2 sec. **D.** 3.6 sec.

8. C

9. An 3.0 kg object is undergoing uniform circular motion of radius 25 m in the clockwise direction. At time t, it's velocity is found to be $(3\hat{\imath} - 4\hat{\jmath})$m/s. Compute the speed of the object.

A. 3 m/s. **B.** 4 m/s. **C.** 5 m/s. **D.** 7 m/s.

9. C

10. In Question-~~10~~ 9, in which quadrant the object is at that instant?

A. First quadrant. **B.** Second quadrant. **C.** Third quadrant. **D.** Fourth quadrant.

10. A

11. What is the angular speed of the object in Question-~~10~~ 9?

A. 0.2 rad/s. **B.** 2.0 rad/s. **C.** 0.5 rad/s. **D.** 5.0 rad/s.

11. A

12. Find the angle the centripetal acceleration makes with the positive x-axis (counterclockwise) in Question-10.

A. 53° **B.** 143° **C.** 217° **D.** 323°

12. C

You may do rough work below

Part-II: Show detail calculation to get full credit.

13. The x-components of two vectors, $\vec{A}$ and $\vec{B}$, are equal. Their magnitudes are related by $2A = B$, and also $\theta_A = 30°$.

(a) (3 marks) Find θ_B.

$A_x = B_x \Rightarrow \cancel{A}\cos\theta_A = B\cos\theta_B = 2\cancel{A}\cos\theta_B$

$\Rightarrow \cos 30° = 2\cos\theta_B \Rightarrow \boxed{\theta_B = 64.3°}$

(b) (3 marks) If $A = \sqrt{2}$, compute $\vec{A}\cdot\vec{B}$.

$\vec{A}\cdot\vec{B} = AB\cos(\theta_B - \theta_A)$

$= 2A^2\cos(64.3 - 30)$

$= 2(\sqrt{2})^2\cos 34.3°$

$\Rightarrow \boxed{\vec{A}\cdot\vec{B} = 3.36}$

B	D
A	C
C	C
C	A
D	A
B	C

14. If the position of a particle is given by $x = 20t - 5t^3$, where x is in meters and t is in seconds.

(a) (3 marks) When does the turning point occur?

$$v = \frac{dx}{dt} = 20 - 15t^2 = 0$$

$$\Rightarrow t = +\sqrt{\frac{20}{15}} \Rightarrow \boxed{t = 1.15 \text{ sec}}$$

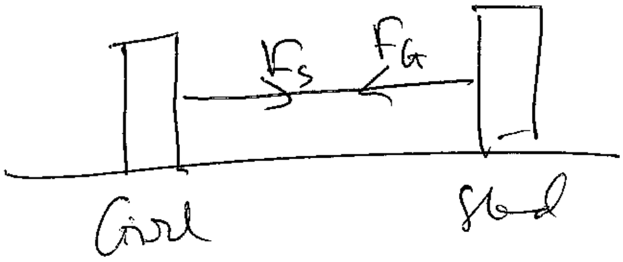
(b) (3 marks) Compute the average velocity between the time interval 0 sec and 2.0 sec.

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_0}{t_2 - t_0}$$

$$= \frac{[20(2) - 5(2)^3] - 0}{2}$$

$$\Rightarrow \boxed{\bar{v} = 0}$$

15. (6 marks) A 40 kg girl and an 8.4 kg sled are on the frictionless ice of a frozen lake, 15 m apart but connected by a rope of negligible mass. The girl exerts a horizontal 5.2 N force on the rope. Compute the magnitudes of the accelerations of the girl and the sled.



3rd law: $F_s = F_G = 5.2\ \text{N}$.

$\therefore \quad a_{girl} = \dfrac{F_G}{M} = 0.13\ \text{m/s}^2$

$a_{sled} = \dfrac{F_s}{m} = 0.62\ \text{m/s}^2$

3/3

Choose only one answer **Marks = /12**

1. The position as a function of time is given by

$$x(t) = -c + \frac{a}{t} + bt^2 ,$$

where a, b and c are constants. What is the dimension of b?

A. T. **B.** L. **C.** LT^{-1}. **D.** LT^{-2}.

1. D

2. Figure-1 shows three vectors. $\vec{A} + \vec{B} + \vec{C}$ equals to

A. 0. **B.** $4\hat{i}$. **C.** $2\hat{i} + 4\hat{j}$. **D.** $2\hat{i} - 4\hat{j}$.

2. B

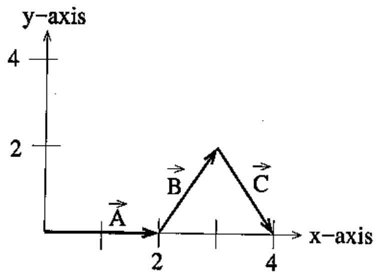


Figure 1: Diagram for Question-(2)

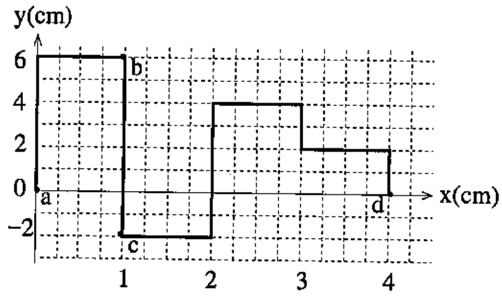


Figure 2: Diagram for Question-(3)

3. An insect is stalling on the floor slowly. It's positions at different times are shown in Figure-2. What is the total displacement from point a to point d?

A. 0. **B.** $4\,\text{cm}\,\hat{i}$. **C.** 28 cm. **D.** $28\,\text{cm}\,\hat{i}$.

3. B

4. Figure-3 gives the velocity of a cat chasing a mouse along the x-axis. In which time intervals, the cat slowing down?

A. $a \to b$ and $c \to d$. **B.** $c \to d$ and $g \to h$.
C. $e \to f$ and $e \to f$. **D.** $a \to b$ and $f \to g$.

4. B

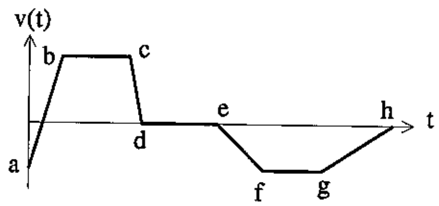


Figure 3: Diagram for Question-(4)

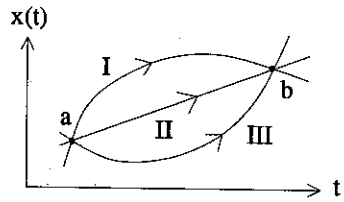


Figure 4: Diagram for Question-(5)

5. Figure-4 show three along which an body can move from point a to point b. Along which path the average velocity is the greatest?

A. Path-I. **B.** Path-II. **C.** Path-III. **D.** All paths are same.

5. D

6. You dropped a ball from the top of a 100 m tall building and it lands on to to ground with speed v. 10 m below the dropping point, its speed is 14.0 m/s. If you now throw the ball upward with speed v, what will be the velocity 90 m above the ground?

A. 14.0 m/s upward. **B.** 14.0 m/s downward.
C. 28.0 m/s upward. **D.** 28.0 m/s downward.

6. A

7. Which of the following is/are always true for a projectile motion?

   A. v_x is constant. **B.** $v_y = 0$ at the point of maximum height. **C.** $|\vec{a}| = 9.80\,\text{m/s}^2$.
   D. All of these.

7. D

8. Water is poured into a container that has a small leak. The mass m of the water is given as a function of time t by $m = 5.00\,t^{0.8} - 3.00\,t + 20.00$, with $t \geq 0$, m in grams, and t in seconds. At what time is the water mass greatest?

   A. 0.24 sec. **B.** 4.2 sec. **C.** 3.6 sec. **D.** 1.2 sec.

8. B

9. An 3.0 kg object is undergoing uniform circular motion of radius 10 m. At time t, it's velocity is found to be $(3\hat{i} + 4\hat{j})$m/s. Compute the speed of the object.

   A. 3 m/s. **B.** 4 m/s. **C.** 5 m/s. **D.** 7 m/s.

9. C

10. In Question-~~10~~9, in which quadrant the object is at that instant?

    A. First quadrant. **B.** Second quadrant. **C.** Third quadrant. **D.** Fourth quadrant.

10. D

11. What is the angular speed of the object in Question-~~10~~9?

    A. 2.0 rad/s **B.** 0.5 rad/s **C.** 5.0 rad/s **D.** 0.2 rad/s

11. B

12. Find the angle the centripetal acceleration makes with the positive x-axis (counterclockwise) in Question-10.

    A. 53° **B.** 143° **C.** 233° **D.** 323°

12. B

You may do rough work below

Part-II: Show detail calculation to get full credit.

13. In a meeting of mimes, mime 1 goes through a displacement $\vec{d_1} = (4.0m)\hat{\imath} + (5.0m)\hat{\jmath}$ and mime 2 goes through a displacement $\vec{d_2} = (-3.0m)\hat{\imath} + (4.0m)\hat{\jmath}$. What are:

(a) (3 marks) $\vec{d_1} \times \vec{d_2}$?

$$\vec{d_1} \times \vec{d_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 5 & 0 \\ -3 & 4 & 0 \end{vmatrix} m^2 = \underline{31\, m^2\, \hat{k}}$$

(b) (3 marks) $\vec{d_1} \cdot \vec{d_2}$?

$$\vec{d_1} \cdot \vec{d_2} = [4(-3)+5(4)]\,m^2 = \underline{8\, m^2}$$

D	D
B	B
B	C
B	D
D	B
A	B

14. The position of an insect flying along the x-axis is given by

$$x(t) = 2.0\,\mathrm{m} + (3.0\,\mathrm{m/s})t + (4.0\,\mathrm{m/s^2})t^2 - (5.0\,\mathrm{m/s^3})t^3 ,$$

where x is in meters and t is in seconds.

(a) (3 marks) Compute the turning point of the insect.

$v = \frac{dx}{dt} = 0+3+8t-15t^2 = 0$

$\Rightarrow 15t^2 - 8t - 3 = 0$

$\Rightarrow t = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(15)(-3)}}{2 \times 15}$ sec

$\Rightarrow t = 0.7873$ sec ($t>0$ only)

(b) (3 marks) Compute the average velocity between the time interval 1.0 sec and 3.0 sec.

$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x_3 - x_1}{t_3 - t_1}$

$= \frac{[2+3(3)+4(3)^2-5(3)^3] - [2+3+4-5]}{2}$ m/s

$\Rightarrow \bar{v} = -46$ m/s

15. In the overhead view of Figure-5, a 2.0 kg cookie tin is accelerated at $3.0\,\text{m/s}^2$ in the direction shown by $\vec{a}$, over a frictionless horizontal surface. The acceleration is caused by three horizontal forces, only two of which are shown: $\vec{F}_1$ of magnitude 10 N and $\vec{F}_2$ of magnitude 20 N.

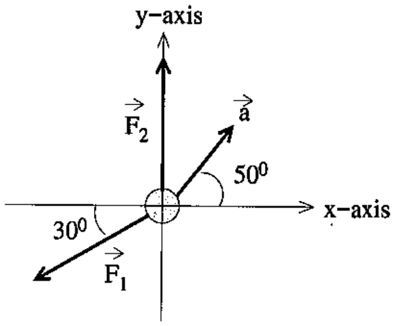


Figure 5: Diagram for Questions-(15)

(a) (2 marks) Using your understanding of Newton's laws, compute the net force on the cookie tin.

$$\sum \vec{F} = m\vec{a} = (2.0\,kg)(3.0\,m/s^2, 50^\circ)$$

$$\Rightarrow \boxed{\sum \vec{F} = (6\,N, 50^\circ)}$$

(b) (2 marks) In unit vector notation, find the third force $\vec{F}_3$.

$$\vec{F}_3 = \sum\vec{F} - \vec{F}_1 - \vec{F}_2 = \left[(6\cos 50 - 10\cos 210 - 20\cos 90)\hat{i} + (6\sin 50 - 10\sin 210 - 20\sin 90)\hat{j}\right] N$$

$$\Rightarrow \vec{F}_3 = (12.5\hat{i} - 10.4\hat{j})\,N$$

(c) (2 marks) What angle does $\vec{F}_3$ make with the x-axis?

$$\theta_3 = 360 - \tan^{-1}\left(\frac{10.4}{12.5}\right) \Rightarrow \theta_3 = 320^\circ.$$

Choose only one answer Marks = /14

1. Figure-1 shows three vectors. $\vec{A} - \vec{B} + \vec{C}$ equals to

   A. 0. **B.** $4\hat{\imath}$. **C.** $2\hat{\imath} + 4\hat{\jmath}$. **D.** $2\hat{\imath} - 4\hat{\jmath}$.

   1. D

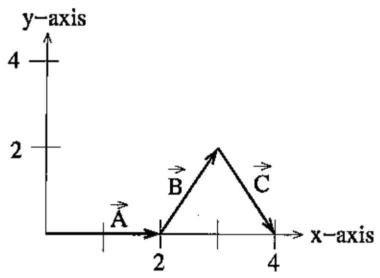


Figure 1: Diagram for Question-(1)

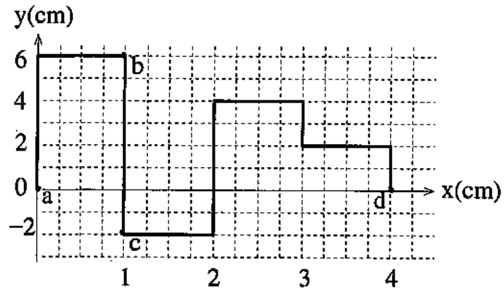


Figure 2: Diagram for Question-(2)

2. An insect is stalling on the floor slowly. It's positions at different times are shown in Figure-2. It takes 3 seconds to go from point b to point d. What is the average velocity for this time interval?

   A. $(\hat{\imath} - 2\hat{\jmath})$cm/s. **B.** $(-2\hat{\imath} + 4\hat{\jmath})$cm/s. **C.** $(2\hat{\imath} + 1.3\hat{\jmath})$cm/s. **D.** $(\hat{\imath} + 2\hat{\jmath})$cm/s.

   2. A

3. Which of the following statements is/are true?

   (a) The displacement is path independent.

   (b) The frictional force is path dependent.

   A. Only (a). **B.** Only (b). **C.** Both (a) and (b). **D.** None of these.

   3. C

4. The fastest growing plant on record is a Hesperoyucca whipplei that grew 3.70 m in 14 days. What was its growth rate in micrometers per second?

   A. $3\,\mu$m/s. **B.** $3.1\,\mu$m/s. **C.** $3.06\,\mu$m/s. **D.** $3.059\,\mu$m/s.

   4. C

5. The following equations give the position $x(t)$ of a particle in four situations (in each equation, x is in meters, t is in seconds, and $t > 0$); (i) $x = 3t - 2$; (ii) $x = -4t^2 - 2$; (iii) $x = 2/t^2$; and (iv) $x = -2$. In which case, the particle is slowing down?

   A. In (i). **B.** In (ii). **C.** In (iii). **D.** In (iv).

   5. C

6. Two objects, a tennis ball and a heavy metal ball, are released simultaneously. They have the same size. Which one will reach the ground first? Ignore air resistance.

   A. The tennis ball. **B.** The metal ball. **C.** They have the same acceleration.
   D. It can not be determined.

   6. C/Any

7. Which of the following conditions represents 'momentarily at rest'?

   A. $v = 0$ and $a = 0$. **B.** $v \neq 0$ and $a \neq 0$.
   C. $v = 0$ and $a \neq 0$. **D.** $v \neq 0$ and $a = 0$.

   7. C

8. The velocity of a projectile 10 m above the ground is found to be $(3\hat{\imath} + 4\hat{\jmath})$m/s. What is the maximum height of the projectile from the ground?

   A. 0.8 m. **B.** 5.6 m. **C.** 10.8 m. **D.** 18.5 m.

   8. C

9. At a certain instant, a fly ball has velocity $\vec{v} = (-25\hat{\imath} + 4.9\hat{\jmath})$m/s. Has the ball passed its highest point?

   A. No. **B.** Yes. **C.** It is at the highest point. **D.** It can not be determined.

   9. A

10. A projectile is fired with initial velocity $\vec{v}_0 = (5.0\hat{\imath} + 9.0\hat{\jmath})$m/s from the ground, and after some time it returns to the ground. Ignore air resistance. How long does it take to reach the maximum height?

A. One second. **B.** Two seconds. **C.** 1.84 sec. **D.** 0.92 sec.

10. D

11. What is the displacement of the projectile when it reaches the maximum height?

A. $4.6\,\mathrm{m}\,\hat{\imath}$. **B.** $9.2\,\mathrm{m}\,\hat{\imath}$. **C.** $4.6\mathrm{m}\,\hat{\imath} + 4.1\,\mathrm{m}\,\hat{\jmath}$. **D.** $9.2\mathrm{m}\,\hat{\imath} + 8.30\,\mathrm{m}\,\hat{\jmath}$.

11. C

12. An object is pushed down an inclined rough surface with constant velocity. How many forces are acting on the object?

A. 1. **B.** 2. **C.** 3. **D.** 4.

12. D

You may do rough below

Part-II: Show detail calculation to get full credit.

13. (6 marks) A velocity of toy car is given by: $v(t) = 3 + 2t - 3t^2$, where v is in m/s and t is in second. Compute the displacement and average velocity from $t = 0$ to $t = 2$ sec.

4+2

$$\Delta x = \int_0^2 (3+2t-3t^2)\,dt$$
$$= (3t+t^2-t^3)\Big|_0^2$$
$$= (3\times 2+4-8)\,m$$
$$\Rightarrow \boxed{\Delta x = 2m}$$

$$V_{av} = \frac{\Delta x}{\Delta t} \Rightarrow \boxed{V_{av} = 1\,m/s}$$

D D
A C
C D
C
C
C/Any
C
C
A

14. A car moves along an x-axis through a distance of 900m, starting at rest (at $x = 0$) and ending at rest (at $x = 900$m). Through the first one-fourth ($\frac{1}{4}$) of that distance, its acceleration is $+2.25\text{m/s}^2$. Through the rest of that distance, its acceleration is -0.750m/s^2.

(a) (4 marks) What is its travel time through the 900m distance?

$v_1 = 0$ $\quad a_1 = 2.25\,m/s^2$ $\quad v$ $\quad a_2 = -0.75\,m/s^2$ $\quad v_2 = 0$

$x_1 = 0$ $\quad d_1 = 225\,m$ $\quad x$ $\quad d_2 = 675\,m$ $\quad x_2 = 900\,m$

$t_1 = ?$ $\qquad t_2 = ?$

$d_1 = (x - x_1) = \cancel{v_1 t_1} + \frac{1}{2} a_1 t_1^2$

$\Rightarrow t_1 = \left(\frac{2d_1}{a_1}\right)^{1/2} = \left(\frac{2 \times 225}{2.25}\right)^{1/2}$ sec

$\Rightarrow t_1 = 14.14$ sec

$d_2 = \frac{1}{2} a_2 t_2^2$

$\Rightarrow t_2 = \left(\frac{2d_2}{a_2}\right)^{1/2} = 42.43$ sec

$\therefore \boxed{t = t_1 + t_2 = 56.57\,\text{sec}}$

(b) (2 marks) Find the average speed of the car.

$$\bar{v} = \frac{d}{t_1 + t_2} = \frac{900\,m}{56.57\,\text{sec}}$$

$$\Rightarrow \boxed{\bar{v} = 15.9\,m/s}$$

15. Figure-3 shows an overhead view of a 0.0250 kg lemon half and two of the three horizontal forces that act on it as it is on a frictionless table. Force $\vec{F}_1$ has a magnitude of 6.00 N and is at $\theta_1 = 30.0°$. Force $\vec{F}_2$ has a magnitude of

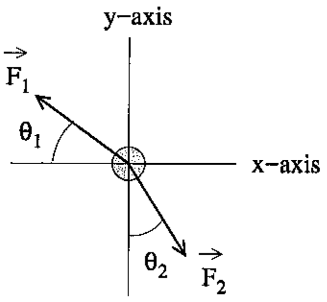


Figure 3: Diagram for Questions-(15)

7.00 N and is at $\theta_2 = 30.0°$.

(a) (3 marks) In unit-vector notation, what is the third force if the lemon half is at rest.

$\sum \vec{F} = 0 \Rightarrow \vec{F}_3 = -(\vec{F}_1 + \vec{F}_2)$

$= [(6 \cos 150 + 7 \cos 300)\hat{i} - (6 \sin 150 + 7 \sin 300)\hat{j}]$ N

$\Rightarrow \boxed{\vec{F}_3 = (1.7\hat{i} + 3.1\hat{j}) \text{ N}}$

(b) (3 marks) What would be the third force if the lemon half has velocity $\vec{v} = (13.0\hat{i} - 14.0\hat{j})$m/s? Explain or show calculation.

Since $\vec{v}$ = const $\Rightarrow \sum \vec{F} = 0 \Rightarrow$ $\boxed{\vec{F}_3 = (1.7\hat{i} + 3.1\hat{j}) \text{ N}}$

Solution

Choose only one answer **Marks = /9**

1. Which of the following statements is/are true?
   (a) The number of the potential energy terms in the total energy must be the same as the number of the conservative forces acting on a body.
   (b) To compute the total work done on a body, it is not necessary to know the number of forces acting.

   A. Only (1a). **B.** Only (1b). **C.** Both (1a) and (1b). **D.** None of these.

   1. C

2. Consider free-fall phenomena of three identical balls (except colors): red ball is dropped, white ball is thrown downward and the green ball is thrown upward from the same position above the ground . Ignore air resistance. Which of the following statements are true?
   (a) The final kinetic energy of the red ball is the lowest.
   (b) The change in potential energy is same for all.
   (c) The work done by all three are same.

   A. Only (2a) and (2b). **B.** Only (2b) and (2c). **C.** Only (2a) and (2c). **D.** All are true.

   2. D

3. During a collision the total linear momentum is always conserved because

   A. the net external force on the system is zero. **B.** the sum of internal forces is zero.
   C. the total kinetic energy is conserved. **D.** None of these

   3. A

4. Which of the following is equivalent to Newton's 1st law?

   A. $\frac{d\vec{p}}{dt} \neq 0$. **B.** $\vec{J} = 0$. **C.** $\sum \vec{F}_{\text{internal}} \neq 0$.
   D. ALAS!!! I don't know the answer. I usually do not pay attention in the class. Even though I am telling the truth (meaning the answer is correct), I will accept zero credit for this answer gleefully.

   4. B

5. A ceiling fan with 80-cm diameter blades is turning at 30 rpm. Suppose the fan coasts to a stop 25 sec after being turned off. What is the angular acceleration of the blade while stopping?

   A. $-0.126\,\text{rad/s}^2$. **B.** $1.5\,\text{rad/s}^2$. **C.** $3.8\,\text{rad/s}$. **D.** $-1.20\,\text{rad/s}^2$.

   5. A

6. Through how many revolutions does the fan turn while stopping?

   A. 25.0 rev. **B.** 6.25 rev. **C.** 12.5 rev. **D.** 37.5 rev.

   6. B

7. Figure-1 shows a tennis ball bounces off the hard floor. If the collision is elastic, the impulse on the tennis ball will be along

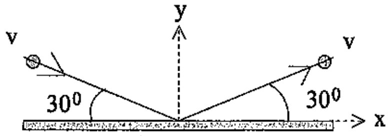


Figure 1: Diagram for Questions-(7)

   A. $\hat{\imath}$. **B.** $-\hat{\imath}$. **C.** $\hat{\jmath}$. **D.** $-\hat{\jmath}$.

   7. C

8. Figure-2 shows three small spheres that rotate about a vertical axis at points a, b and c. The perpendicular distance between the axis and the center of each sphere is given in the diagram. Rank the three moment of inertia, greatest first.

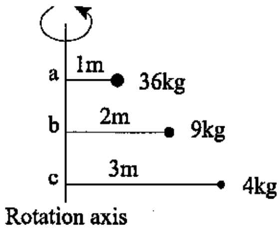


Figure 2: Diagram for Question-(8)

A. $I_a = I_b < I_c$. **B.** $I_a < I_b = I_c$. **C.** $I_a < I_b < I_c$. **D.** $I_a = I_b = I_c$.

8. D

9. Two masses $m_1 = 200$ grams and $m_2 = 100$ grams are joint by a thin massless rod. They are 6.0 m apart. Find the center of the mass of the system.

A. 1.0 m from m_1. **B.** 1.0 m from m_2. **C.** 2.0 m from m_2. **D.** 2.0 m from m_1.

9. D

Part-II: Show detail calculation to get full credit.

10. (7 marks) Figure-3 shows a crate of mass 5.0 kg is pulled by a Force F on an inclined rough surface at

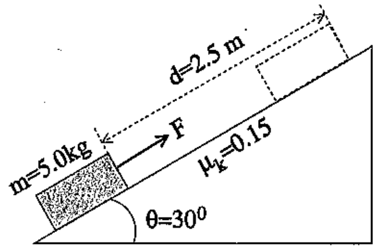


Figure 3: Diagram for Question-10

an angle $\theta = 30°$. Starting from rest the crate moved a distance $d = 2.5$m with final speed 3.0 m/s. The coefficient of kinetic friction is $\mu_k = 0.15$. Find the magnitude of the force F.

$$W_{net} = \Delta K = \frac{1}{2} m v_f^2 = 22.5 \text{ J}.$$

$$\text{But } W_{net} = \frac{1}{2} m v_f^2 = \overset{0}{\cancel{W_n}} + W_f + W_s + W_F$$

$$= -\mu_k mg\cos\theta\, d - mgd\sin\theta + Fd$$

$$\Rightarrow F = \frac{mv_f^2 + 2mgd(\sin\theta + \mu_k\cos\theta)}{2d}$$

$$\Rightarrow \boxed{F = 39.87 \text{ N}}$$

C D
D D
A
B
A
B
C

11. Figure-4 shows a force applied on an object of mass 200 grams for 6.0 ms. Find the following:

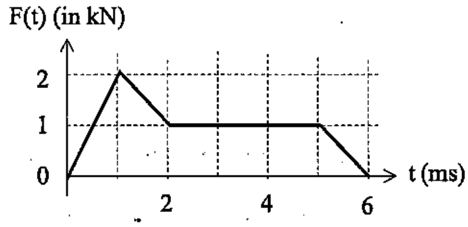


Figure 4: Diagram for Question-11

(a) (4 marks) The impulse on the object.

(b) (3 marks) The average force applied on the object.

(a) $J = \int_0^t F\,dt$ = Area under the curve = 6 squares.

Area of a square = 1 N.S.

$\therefore J = 6 \times 1\,NS \Rightarrow \boxed{J = 6\,NS}$

(b) $f_{av} = \frac{J}{\Delta t} = \frac{6\,NS}{6\times10^{-3}\,sec} \Rightarrow \boxed{F_{av} = 1000\,N = 1.0\,kN}$

12. An object of mass 200 grams is rotating whose angular speed is given by

$$\omega(t) = 6t^2 - 12t\ ,$$

where ω is in radian per seconds and t is in seconds.

(a) (4 marks) Find the turning point.

(b) (3 marks) What is the average angular velocity from $t = 0$ to $t = 2.0$ sec.

(a) $\omega = 0 \Rightarrow 6t(t-2) = 0 \Rightarrow \boxed{t = 2\,sec}$ ($t=0$ is not turning pt)

$\therefore \alpha(t=2) \neq 0$.

So $\boxed{t = 2.0\,sec}$.

(b) $\bar{\omega} = \frac{\Delta\theta}{\Delta t} = \frac{1}{\Delta t}\int_0^t \omega\, dt$

Now $\int_0^2 [6t^2 - 12t]\,dt = [2t^3 - 6t^2]_0^2 = (16-24)\,rad = -8\,rad$

$\therefore \bar{\omega} = -\frac{8}{2}\,r/s \Rightarrow \boxed{\bar{\omega} = -4\,r/s}$

Solution

Choose only one answer **Marks = /9**

1. $W_{tot} = \Delta K$ is valid only when

   A. F is constant. **B.** for closed path. **C.** no frictional force present. **D.** None of these.

   1. D

2. Figure-1 shows work done along three paths from the initial point to the final point. If an object goes along path-II and return along path-III, the work done will be

   A. exactly 60 J. **B.** more than 60 J. **C.** less than −60 J. **D.** exactly zero.

   2. D

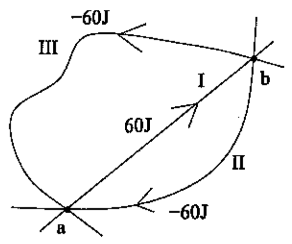


Figure 1: Diagram for Question-2

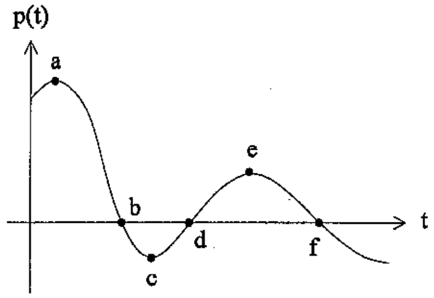


Figure 2: Diagram for Question-3

3. Figure-2 shows linear momentum of an object as a function of time. At which point the velocity changes direction from negative to positive?

   A. Point d. **B.** Point c. **C.** Point b. **D.** Point a.

   3. A

4. You drop a tennis ball of mass 300 grams from 1.5 m above a hard floor. The ball bounced up from the floor and rises 1.2 m above the floor. Ignore air resistance. The energy loss as heat and sound due to collision is

   A. 0.882 J. **B.** 3.528 J. **C.** 4.410 J. **D.** 3.969 J.

   4. A

5. A torque on an object is given by $\vec{\tau} = \vec{r} \times \vec{F}$, and it can be zero when

   A. the axis of rotation passes through the object. **B.** the applied force is zero.
   C. the force and the position vector are parallel. **D.** All of the above.

   5. D

6. In an experiment it is found that the rate of change of total angular momentum is zero. This implies that

   A. the total torque is zero. **B.** the angular acceleration is zero.
   C. the angular velocity is constant. **D.** All of the above.

   6. D

7. A ceiling fan with 80-cm diameter blades is turning at 30 rpm. Suppose the fan coasts to a stop 25 sec after being turned off. What is the angular acceleration of the blade while stopping?

   A. $-0.126\,\text{rad/s}^2$. **B.** $1.5\,\text{rad/s}^2$. **C.** $3.8\,\text{rad/s}$. **D.** $-1.20\,\text{rad/s}^2$.

   7. A

8. A tennis ball is dropped onto the floor, and it bounced up again. The direction of the impulse on the ball is

   A. upward. **B.** downward. **C.** There is no impulse. **D.** It is parallel to the floor.

   8. A

9. A small metal ball of mass $M = 500$ grams is rotating in a circular path of radius $R = 5.0$ m on the yz-plane under a torques τ. What is the direction of the torque?

   A. $\hat{\imath}$. **B.** $\hat{\jmath}$. **C.** $\hat{k}$. **D.** $\hat{\imath} + \hat{\jmath} + \hat{k}$.

   9. A

Part-II: Show detail calculation to get full credit.

10. (7 marks) Figure-3 shows a crate of mass 5.0 kg is pushed by a Force F that is directed at an angle $\theta = 30°$. The

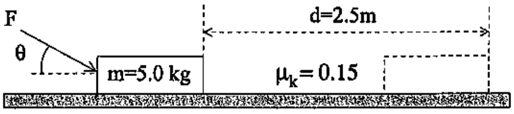


Figure 3: Diagram for Question-10

crate moved a distance $d = 2.5\,\mathrm{m}$ at a constant velocity on the frictional horizontal surface with $\mu_k = 0.15$. Find the magnitude of the force F.

$$W_{net} = \Delta K = 0 = \cancel{W_n}^{0} + W_f + W_F + \cancel{W_g}^{0} = 0$$

$$\Rightarrow -\mu_k \cancel{d}(mg + F\sin\theta) + F\cancel{d}\cos\theta = 0$$

$$\Rightarrow F(\cos\theta - \mu_k \sin\theta) = \mu_k mg$$

$$\Rightarrow F = \frac{\mu_k mg}{\cos\theta - \mu_k \sin\theta}$$

$$\Rightarrow \boxed{F = 9.29\,N}$$

11. A bullet of mass $m = 300$ grams is fired with speed $v = 500\,\text{m/s}$ by an air rifle towards a wooden block on mass $M = 30\,\text{kg}$ which is at rest on a frictionless horizontal floor. The bullet remains inside block after collision, and

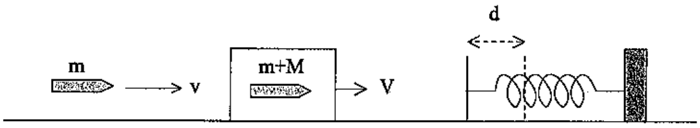


Figure 4: Diagram for Question-11

the bullet-block runs into an ideal spring of force constant $200\,\text{kN/m}$, and compress it by a distance d as shown in Figure-4. Find the following:

(a) (4 marks) The energy loss due to collision.

(b) (3 marks) The distance d.

(a) $mv = (m+M)V \Rightarrow V = \left(\frac{m}{m+M}\right)v = 4.95\,\text{m/s}$

$= \frac{(.3)(500)}{30.3}$

$\therefore \Delta K = \frac{1}{2}mv^2 - \frac{1}{2}(m+M)V^2$

$= \frac{1}{2}\left[(.3)(500)^2 - (30.3)(4.95)^2\right]\,\text{J}$

$\Rightarrow \boxed{\Delta K = 37129\,\text{J} = 3.7\times10^4\,\text{J}}$

(b) $\cancel{\frac{1}{2}}(m+M)V^2 = \cancel{\frac{1}{2}}kd^2$

$\Rightarrow d = \sqrt{\frac{(m+M)V^2}{k}} = \left[\frac{(30.3)(4.95)^2}{200\times10^3}\right]^{1/2}\,\text{m}$

$\Rightarrow \boxed{d = 0.061\,\text{m} = 6.1\,\text{cm}}$

DDAAADDAAAA

12. (7 marks) The angular position of a point on a wheel is given by

$$\theta = 2t^3 - 2t^2 + 4 ,$$

where θ is in radians and t is in seconds.

(a) (4 marks) Find the turning point (if any).

(b) (3 marks) Find the angular velocity when the angular acceleration is zero.

(a) $\omega = 6t^2 - 4t = 0 \Rightarrow 2t(3t - 2) = 0$

$\Rightarrow t = 0, \frac{2}{3}$ sec

$\alpha = 12t - 4 \neq 0$ at $t = 2/3$

$\therefore t = 2/3$ sec. is a turning point

(b) $\alpha = 0 \Rightarrow t = \frac{4}{12} = \frac{1}{3}$ sec.

$\therefore \omega_{1/3} = 6(\frac{1}{3})^2 - 4(\frac{1}{3}) = \left(\frac{6}{9} - \frac{4}{3}\right)$ sec

$= \frac{6 - 12}{9}$ sec $= -\frac{6}{9}$ sec

$\Rightarrow \boxed{\omega_{1/3} = -\frac{2}{3} \text{ r/s}}$

Solution

Choose only one answer Marks = /9

1. Which of the following statements is/are true?
   (a) The number of the potential energy terms in the total energy must be the same as the number of the conservative forces acting on a body.
   (b) To compute the total work done on a body, it is not necessary to know the number of forces acting.

   A. Only (1a). **B.** Only (1b). **C.** Both (1a) and (1b). **D.** None of these.

   1. C

2. Consider free-fall phenomena of three identical balls (except colors): red ball is dropped, white ball is thrown downward and the green ball is thrown upward from the same position above the ground . Ignore air resistance. Which of the following statements are true?
   (a) They have the same initial potential energy.
   (b) They fall on to the ground with same speed.
   (c) The work done by all three are same.

   A. Only (2a) and (2b). **B.** Only (2b) and (2c). **C.** Only (2a) and (2c). **D.** All are true.

   2. C

3. During a collision the total linear momentum is always conserved because

   A. the sum of internal forces is zero. **B.** the total kinetic energy is conserved.
   C. the net external force on the system is zero. **D.** None of these

   3. C

4. Figure-1 shows linear momentum of an object as a function of time. Which point is a turning point?

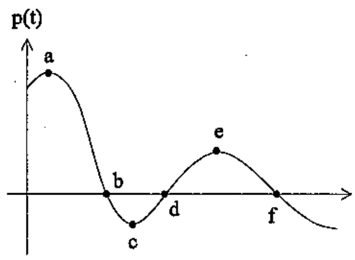


Figure 1: Diagram for Question-4

   A. a. **B.** b. **C.** c. **D.** None of these..

   4. B

5. A ceiling fan with 80-cm diameter blades is turning at 30 rpm. Suppose the fan coasts to a stop 25 sec after being turned off. What is the angular acceleration of the blade while stopping?

   A. $1.5\,\text{rad/s}^2$. **B.** $-0.126\,\text{rad/s}^2$. **C.** $3.8\,\text{rad/s}$. **D.** $-1.20\,\text{rad/s}^2$.

   5. B

6. Through how many revolutions does the fan turn while stopping?

   A. 25.0 rev. **B.** 6.25 rev. **C.** 12.5 rev. **D.** 37.5 rev.

   6. B

7. Figure-2 shows a tennis ball bounces off the hard floor. If the collision is elastic, the impulse on the tennis ball will be along

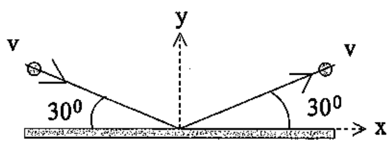


Figure 2: Diagram for Questions-(7)

   A. $\hat{j}$. **B.** $-\hat{j}$. **C.** $\hat{i}$. **D.** $-\hat{i}$.

   7. A

8. Figure-3 shows three small spheres that rotate about a vertical axis at points a, b and c. The perpendicular distance between the axis and the center of each sphere is given in the diagram. Rank the three moment of inertia, greatest first.

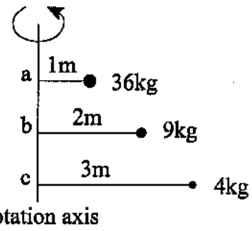


Figure 3: Diagram for Question-(8)

A. $I_a = I_b = I_c$. **B.** $I_a = I_b < I_c$. **C.** $I_a < I_b = I_c$. **D.** $I_a < I_b < I_c$.

8. A

9. Two masses $m_1 = 200$ grams and $m_2 = 100$ grams are joint by a thin massless rod. They are 6.0 m apart. Find the center of the mass of the system.

A. 1.0 m from m_1. **B.** 1.0 m from m_2. **C.** 2.0 m from m_2. **D.** 2.0 m from m_1.

9. D

Part-II: Show detail calculation to get full credit.

10. (7 marks) Figure-4 shows a crate of mass 5.0 kg is pulled by a Force F on an inclined rough surface at an angle

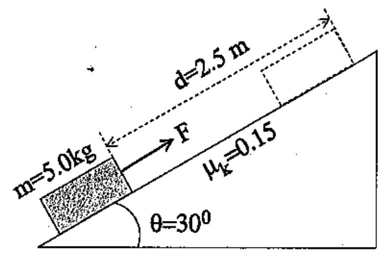


Figure 4: Diagram for Question-10

$\theta = 30°$. The crate moved a distance $d = 2.5$m at constant velocity. The coefficient of kinetic friction is $\mu_k = 0.15$. Find the magnitude of the force F.

$W_{net} = \Delta K = 0$ ($\vec{v}$ = constant)

$\therefore W_{net} = \overset{0}{\cancel{W_n}} + W_f + W_g + W_F = 0$

$\Rightarrow -\mu_k n \cancel{d} + F\cancel{d} + mg\cancel{d}\cos(90+\theta) = 0$

$\Rightarrow F = \mu_k n + mg\sin\theta = \mu_k mg\cos\theta + mg\sin\theta$

$= mg(\sin\theta + \mu_k\cos\theta)$

$= (5)(9.8)[\sin 30 + 0.15\cos 30]$ N

$\Rightarrow \boxed{F = 30.86\,N}$

C	A
C	D
C	
B	
B	
B	
A	

11. Two masses $m_a = 200$ grams and $m_b = 400$ grams are travelling with velocities $v_a = (2\hat{i} - 4\hat{j})$m/s and $v_b = (\hat{i} + 2\hat{j})$m/s as shown in Figure-5. The lighter mass is scattered with velocity $v_c = (1.5\hat{i} + 3\hat{j})$m/s after collision. Find the following:

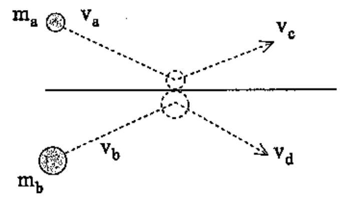


Figure 5: Diagram for Question-11

(a) (4 marks) The velocity of the heavier mass after collision.

(b) (3 marks) The linear momentum loss by the lighter particle.

(a) $\Rightarrow \sum \vec{P}_f = \sum \vec{P}_i \Rightarrow m_b \vec{V}_d + m_a \vec{V}_c = m_b \vec{V}_b + m_a \vec{V}_a$

$\Rightarrow \vec{V}_d = \vec{V}_b + \left(\frac{m_a}{m_b}\right)(\vec{V}_a - \vec{V}_c)$

$\Rightarrow \boxed{\vec{V}_d = (1.25\hat{i} - 1.5\hat{j})\,m/s}$

(b) $\Delta \vec{P} = m_a(\vec{V}_c - \vec{V}_a)$

$= 0.2(1.5\hat{i} + 3\hat{j} - 2\hat{i} + 4\hat{j})$ kgm/s

$\Rightarrow \boxed{\Delta \vec{p} = (-0.1\hat{i} + 1.4\hat{j})\,kg \cdot m/s}$

12. (7 marks) A force $\vec{F} = (-2\hat{\imath} + 3\hat{\jmath})$N is applied to an object of mass 2.0 kg which is located at $\vec{r} = (-\hat{\jmath} + 2\hat{k})$m.
   (a) (4 marks) Find the magnitude of the torque.
   (b) (3 marks) Find the magnitude of the angular acceleration.

(a) $\vec{\tau} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & -1 & 2 \\ -2 & 3 & 0 \end{vmatrix} = (-6\hat{i} - 4\hat{j} - 2\hat{k})$ N.m

$\therefore \tau = \sqrt{6^2+4^2+2^2} \Rightarrow \boxed{\tau = 7.48 \text{ N.m}}$

(b) $\alpha = \frac{\tau}{I} = \frac{\tau}{mr^2}$

$= \frac{7.48}{2(1+4)}$ r/s²

$\Rightarrow \boxed{\alpha = 0.748 \text{ r/s}^2}$