1) i) Write down the state space equations (differ...

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Question

(0)

1) i) Write down the state space equations (differential equations) for the following robots. For each robot identify the state variables and the control variables. Describe the differences in dynamics between these robot models.

a) Unicycle model
b) Differential drive robot
c) Simplified car model

ii) For a given robot, what do the state variables and control variables represent? Please explain.

2) Simulate the differential equations for the unicycle robot model using Euler's method for $t \in [0, 10]$ and time step $\Delta t = 0.1$. Use the following control input:

$$v = 1 \quad 0 \le t \le 10$$

$$\omega = \begin{cases} 3 & 0.5 \le t \le 1.5 \\ -3 & 2 \le t \le 3 \\ -3 & 4 \le t \le 5 \\ 3 & 6 \le t \le 7 \\ -3 & 8 \le t \le 9 \\ 0 & all\ other\ t \end{cases}$$

With initial conditions $x(0) = 0$, $y(0) = 0$, $\theta(0) = 1$.

Plot x vs y, x vs t, y vs t and θ vs t. Submit your code and plots.

Feel free to simulate different initial conditions and control input sequences to gain a greater understanding.

3) Simulate the differential equations for the differential drive robot model using Euler's method for $t \in [0, 10]$ and time step $\Delta t = 0.1$. Use the following control input:

$$\omega_l = \begin{cases} 12 & 4 \le t \le 6 \\ 12 & 6 \le t \le 8 \\ 1 & all\ other\ t \end{cases} \quad \omega_r = \begin{cases} 12 & 0.5 \le t \le 1.5 \\ 12 & 2 \le t \le 4 \\ 1 & all\ other\ t \end{cases}$$

With initial conditions $x(0) = 0$, $y(0) = 0$, $\theta(0) = 1$.

Plot x vs y, x vs t, y vs t and θ vs t. Submit your code and plots.

Feel free to simulate different initial conditions and control input sequences to gain a greater understanding.

4) The equations for a **nonholonomic integrator** are given by,

$$\dot{x_1} = u_1$$
$$\dot{x_2} = u_2$$
$$\dot{x_3} = x_2 u_1$$

a) Show that this system is differentially flat for the flat output $z = (x_1, x_3)$.
b) Assume the system moves from initial time $t_i = 0$ to final time $t_f = T$. Assume the following initial conditions:

At $t_i = 0$, $x_1(0)$, $x_2(0)$, $x_3(0)$, $\dot{x_1}(0) = 1$
At $t_f = T$, $x_1(T)$, $x_2(T)$, $x_3(T)$, $\dot{x_1}(T) = 1$

Using four basis functions, $\psi_1 = 1$, $\psi_2 = t$, $\psi_3 = t^2$, $\psi_4 = t^3$, write down the matrix-vector equations for this system. [**Hint:** Slide 22 in the differential flatness presentation might be helpful]

c) Plot the trajectory for this system for the following initial conditions. Submit your code and screenshots.
   i) At $t_i = 0$, $x_1(0) = 1$, $x_2(0) = 1$, $x_3(0) = 0$, $\dot{x_1}(0) = 1$
   At $t_f = 10$, $x_1(10) = 5$, $x_2(10) = 5$, $x_3(10) = 5$, $\dot{x_1}(10) = 1$

   ii) At $t_i = 0$, $x_1(0) = 1$, $x_2(0) = 1$, $x_3(0) = 0$, $\dot{x_1}(0) = 1$
   At $t_f = 15$, $x_1(15) = 10$, $x_2(15) = 10$, $x_3(15) = 5$, $\dot{x_1}(15) = 1$

5) Trajectory tracking using an open loop controller

In this problem, first we will generate a trajectory using the differential flatness technique. Next, we will generate the control inputs for the robot to navigate this trajectory using open loop controller techniques.

Consider the equations for the dynamically extended unicycle robot,

$$\dot{x}(t) = V(t)\cos\theta(t)$$
$$\dot{y}(t) = V(t)\sin\theta(t)$$
$$\dot{V}(t) = a(t)$$
$$\dot{\theta}(t) = \omega(t)$$

The initial conditions are: $x(0) = 0$, $y(0) = 0$, $V(0) = 0.5$, $\theta(0) = -\pi/2$
$x(t_f) = 5$, $y(t_f) = 5$, $V(t_f) = 0.5$, $\theta(t_f) = -\pi/2$ where $t_f = 15$

Here the controls are $a(t)$ and $\omega(t)$.

a) Show this system is differentially flat with flat output $z = (x, y)$.

b) Generate a differentially flat trajectory using the four basis functions of the previous problem. Plot your generated trajectory.

c) We will now navigate our robot along this generated trajectory. To do that we need to compute the controls $a(t)$ and $\omega(t)$. Notice that differentiating the velocities $\dot{x}, \dot{y}$ we get the following equations, Introduction to Robotics

$$\begin{bmatrix} \ddot{x}(t) \\ \ddot{y}(t) \end{bmatrix} = \begin{bmatrix} \cos\theta\,(t) & -V(t)\sin\theta\,(t) \\ \sin\theta\,(t) & V(t)\cos\theta(t) \end{bmatrix} \begin{bmatrix} a(t) \\ \omega(t) \end{bmatrix}$$

You can estimate $\ddot{x}$, $\ddot{y}$ from your generated differentially flat trajectory. Then a(t) and ω(t) can be calculated by taking the matrix inverse in the equation above.

Once the controls a(t) and ω(t) are calculated, you can navigate the robot by numerically integrating the unicycle equations using Euler's method. See slide 5 of the "Open loop motion control and differential flatness" presentation.

Integrate the unicycle equations using Euler's method and generating the control inputs as described above. Plot the result for time steps $\Delta t = 0.1$ and 0.01. You can also try experimenting with other values for the time step. The smaller the time step, the more accurate the result but greater the computation.

6) Open loop controller performance in the presence of noise or disturbance

In problem 5(c), your robot should track the trajectory perfectly. This is because there is no noise or disturbance in the system. If noise and disturbance is present, which is the case for real systems, the robot won't be able to track the trajectory with an open loop controller. We will see this in this exercise.

We can randomly inject noise or disturbances into the system using the Gaussian (Normal) distribution. Inject noise in the V and θ equations in the following way,

$$V(t+\Delta t) = V(t) + \Delta t\, a(t) + 0.01 * randn$$
$$\theta(t+\Delta t) = \theta(t) + \Delta t\, \omega(t) + 0.001 * randn$$

Here "0.01 * randn" is a random number generated from a Gaussian distribution with mean 0 and standard deviation 0.01. Similarly, "0.001 * randn" is a random number generated from a Gaussian distribution with mean 0 and standard deviation 0.001.

With this noise injection, integrate the unicycle equations using Euler's method and the open loop controller of problem 5(c). Plot the result for time step $\Delta t = 0.01$. You should be able to see the robot deviate from the desired trajectory. Submit your code and screenshots.

Expert Answer

This solution was written by a subject matter expert. It's designed to help students like you learn core concepts.

Step-by-step

1st step

All steps

Answer only

Step 1/5

State space equations describe the dynamics of a system in terms of state variables and control variables. Each of the mentioned robot models has distinct dynamics and corresponding state variables and control variables

a) Unicycle Model:

State Variables:

(x) and (y) (position in 2D space)

(θ)(orientation angle)

Control Variables:

(v) (linear velocity)

(ω) (angular velocity)

Explanation:

The unicycle model represents a robot as a point moving in a 2D plane with an orientation angle. It can move forward and rotate in place.

State Equations are

$\dot{x}=v\cos(\theta)$

$\dot{y}=v\sin(\theta)$

$\dot{\theta}=\omega$

Step 2/5

b) Differential Drive Robot:

State Variables

(x) and (y) (position in 2D space)

(θ) (orientation angle)

Control Variables:

(v) (linear velocity)

(ω) (angular velocity)

Explanation:

The differential drive robot is similar to the unicycle model but with the ability to independently control its wheel speeds, allowing it to move more flexibly.

State Equations (assuming (r) is the wheel radius and (L) is the distance between the wheels)

$\dot{x}=v\cos(\theta)$

$\dot{y}=v\sin(\theta)$

$\dot{\theta}=vL\tan(\omega)$

Step 3/5

c) Simplified Car Model

State Variables:

(x) and (y) (position in 2D space)

(θ)(orientation angle)

(v) (velocity)

Control Variables:

(a) (acceleration)

(steering angle)

The simplified car model represents a carlike robot with additional state variables for velocity and control variables for acceleration and steering angle.

State Equations (assuming (L) is the distance between the front and rear axles)

$\dot{x}=v\cos(\theta)$

$\dot{y}=v\sin(\theta)$
$\dot{\theta}=vL\tan(\alpha)$

$\dot{v}=a$

Differences in Dynamics

Explanation:

The unicycle model and the differential drive robot are simpler and have fewer state variables (position and orientation), whereas the simplified car model includes velocity as an additional state variable.

The unicycle model assumes the robot moves without slippage, while the differential drive model considers wheel slippage and differential control.

The simplified car model introduces more complexity with a velocity state and control variables for acceleration and steering, making it more suitable for modeling carlike robots.

These models capture different aspects of robot dynamics, allowing for varying levels of complexity and accuracy in describing their behavior.

Step 4/5

ii) In robotics, the state variables and control variables represent essential aspects of a robot's behavior and control. An explanation of what these variables typically represent for a robot:

I) State Variables:

State variables describe the current state or configuration of the robot. They capture information about the robot's position, orientation, and other relevant characteristics at a specific point in time. The interpretation of state variables can vary depending on the robot model, but in general

1) Position Variables (e.g., (x) and (y)):

Explanation:

These represent the robot's location in space, typically in a 2D or 3D Cartesian coordinate system. For a mobile robot, (x) and (y) might denote its coordinates in the horizontal plane, while for a flying drone, they might represent both horizontal and vertical positions.

2) Orientation Variables (e.g., (θ)or(ϕ))

Explanation:

These variables specify the robot's orientation or heading in space. They often represent angles measured relative to a reference axis or direction. For example, (θ) might represent the heading angle (yaw) of a ground robot, while (ϕ) could represent the roll angle of an aerial vehicle.

3) Velocity Variables (e.g., (v))

Explanation:

In some robot models, state variables include velocity components. Velocity represents the rate of change of position and may include linear and angular velocities. Linear velocity (v) can denote how fast the robot is moving along its current path, while angular velocity (ω)can represent its rotational speed.

4) Other Relevant State Information

Explanation:

Depending on the robot's complexity and the specific modeling requirements, state variables can include additional information. For example, for a robot arm, joint angles and velocities would be crucial state variables.

Step 5/5

2) Control Variables

Control variables are parameters that the robot's controller can adjust to influence the robot's behavior or trajectory. They represent the commands or inputs that the robot can receive to change its state or perform tasks. Control variables typically represent as

1) Linear and Angular Velocity Commands (e.g., (v) and ω

Explanation:

These control variables dictate how fast the robot should move in translation and rotation. The controller adjusts these values to navigate or follow a desired trajectory.

2) Acceleration and Deceleration Commands (e.g., (a))

Explanation:

n some cases, control variables include acceleration commands, allowing the robot to change its velocity over time.

3) Steering or Path-Tracking Commands (e.g., (α))

Explanation:

For robots with steering mechanisms, such as cars, bicycles, or mobile robots, control variables like steering angle (α) control the direction the robot should follow.

4) Manipulator Joint Commands

Explanation:

For robotic arms and manipulators, control variables might include commands for each joint's position or velocity. These variables enable the robot to perform tasks that require precise manipulation or positioning.

5) Other Task-Specific Commands

Explanation:

Depending on the robot's application, there may be additional control variables designed to achieve specific goals. For example, in drones, control variables may include commands for altitude, camera tilt, or waypoint navigation.

State variables represent the current state or configuration of the robot, such as its position, orientation, and velocity, while control variables are the inputs that the robot's controller can adjust to control its behavior and achieve desired tasks.

Final answer

State variables represent the current characteristics of a robot, including its position, orientation, and velocity, while control variables are the inputs used by the robot's controller to adjust its behavior, such as linear and angular velocity commands, steering, and other task-specific commands.

Was this answer helpful?

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Q:

can someone help me fill in my data table. for run 4& 5 we used the same substances in run #1. concentrations for run 4 and 5 are the same a run #1 0.050M KI-5.0 ml 0.050M Na2S2O3-1.0ml 1% starch-1.0 ml pH 4.7 buffer-5.0ml 0.080M H2O2 1.0 ml DI H2O-7.0ML

A:

See answer

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