Exercise 6.4

3. $y = 3x^{3/2}$ $x=0$, $x=1$

$\Rightarrow y' = 3 \times \frac{3}{2} x^{\frac{1}{2}} = \frac{9}{2} x^{1/2}$

$\Rightarrow 1 + (y')^2 = 1 + \frac{81}{4} x$

$L = \int_0^1 \sqrt{1 + [f'(x)]^2}\, dx$

$= \int_0^1 \sqrt{1 + \frac{81x}{4}}\, dx$

Put, $u = 1 + \frac{81}{4} x$ $\Rightarrow du = \frac{81}{4} dx$ $\Rightarrow \frac{4du}{81} = dx$

$x = 1,\ u = \frac{85}{4}$

$x = 0,\ u = 1$

So, $L = \int_1^{\frac{85}{4}} \frac{4}{81} \times u^{1/2}\, du = \frac{4}{81} \times \frac{2}{3} \times \left[u^{3/2}\right]_1^{\frac{85}{4}}$

$= 3.19$

4. $x = \frac{1}{3}(y^2+2)^{3/2}$ $\quad y=0, \; y=1$

$$g(y) = \frac{1}{3}(y^2+2)^{3/2}$$

$$\Rightarrow g'(y) = \frac{1}{3} \times \frac{3}{2}(y^2+2)^{\frac{1}{2}} \times 2y$$

$$= y(y^2+2)^{1/2}$$

$$\Rightarrow 1 + [g'(y)]^2 = y^2(y^2+2)+1 = 1+y^4+2y^2 = (y^2+1)^2$$

$$\text{So, } L = \int_0^1 \sqrt{1+[g'(y)]^2}\, dy$$

$$= \int_0^1 \sqrt{(y^2+1)^2}\, dy$$

$$= \int_0^1 (y^2+1)\, dy$$

$$= \left[\frac{y^3}{3}+y\right]_0^1$$

$$= \frac{4}{3}$$

5. $y = x^{2/3}$ $x = 1$, $x = 8$

$f(x) = x^{2/3}$

$\Rightarrow f'(x) = \frac{2}{3} x^{-\frac{1}{3}}$

$\Rightarrow 1 + [f'(x)]^2 = 1 + \frac{4}{9} x^{-\frac{2}{3}} \quad = \frac{9x^{\frac{2}{3}} + 4}{9x^{2/3}}$

So, $L = \int_1^8 \sqrt{1 + [f'(x)]^2}\, dx$

$= \int_1^8 \sqrt{\frac{9x^{2/3}+4}{9x^{2/3}}}\, dx$

$= \int_1^8 \frac{\sqrt{9x^{2/3}+4}}{3x^{1/3}}\, dx$

Put, $9x^{2/3} + 4 = u \Rightarrow 9 \times \frac{2}{3} x^{-\frac{1}{3}}\, dx = du \Rightarrow \frac{dx}{3x^{1/3}} = \frac{du}{18}$

$x = 8,\ u = 40 \qquad x = 1,\ u = 13$

So, $L = \int_{13}^{40} \frac{1}{18} u^{1/2}\, du$

$= \frac{1}{18} \times \frac{2}{3} \times \left[u^{3/2} \right]_{13}^{40} \quad = 206.11$

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6. $y = \frac{x^6+8}{16x^2}$. $x=2$, $x=3$

$f(x) = \frac{x^6}{16x^2} + \frac{8}{16x^2} = \frac{1}{16}x^4 + \frac{1}{2}x^{-2}$

$\Rightarrow f'(x) = \frac{1}{16} \times 4x^3 + \frac{1}{2} \times (-2) \times x^{-3} = \frac{1}{4}x^3 - x^{-3}$

$\Rightarrow 1 + [f'(x)]^2 = 1 + \left(\frac{1}{4}x^3 - x^{-3}\right)^2$

$= 1 + \frac{1}{16}x^6 - \frac{1}{2} + x^{-6}$

$= \frac{1}{16}x^6 + \frac{1}{2} + x^{-6}$

$= \left(\frac{1}{4}x^3 + x^{-3}\right)^2$

So, $L = \int_2^3 \sqrt{1 + [f'(x)^2}\, dx = \int_2^3 \sqrt{\left(\frac{1}{4}x^3 + x^{-3}\right)^2}\, dx$

$= \left[\frac{1}{4} \times \frac{1}{4} \times x^4 - \frac{1}{2}x^{-2}\right]_2^3$

$= 4.13$

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7. $24xy = y^4 + 48 \qquad y=2,\ y=4$

$$x = \frac{y^4+48}{24y} = \frac{y^4}{24y} + \frac{48}{24y} = \frac{1}{24}y^3 + 2y^{-1}$$

$$\Rightarrow g'(y) = \frac{1}{24} \times 3 \times y^2 + 2 \times (-1) \times y^{-2}$$

$$\Rightarrow 1 + [g'(y)]^2 = 1 + \left(\frac{1}{8}y^2 - 2y^{-2}\right)^2$$

$$= 1 + \frac{1}{64}y^4 - \frac{1}{2} + 4y^{-4}$$

$$= \frac{1}{64}y^4 + \frac{1}{2} + 4y^{-4}$$

$$= \left(\frac{1}{8}y^2 - 2y^{-2}\right)^2$$

So, $L = \int_2^4 \sqrt{1 + [g'(y)]^2}\, dy$

$$= \int_2^4 \sqrt{\left(\frac{1}{8}y^2 - 2y^{-2}\right)^2}\, dy$$

$$= \int_2^4 \left(\frac{1}{8}y^2 - 2y^{-2}\right) dy$$

$$= \left[\frac{1}{8} \times \frac{1}{3}y^3 - 2 \times (-1) \times y^{-1}\right]_2^4$$

8. $x = \frac{1}{8}y^4 + \frac{1}{4}y^{-2}$ $\quad y = 1,\ y = 4$

$$g(y) = \frac{1}{8}y^4 + \frac{1}{4}y^{-2}$$

$$\Rightarrow g'(y) = \frac{1}{8} \times 4y^3 + \frac{1}{4} \times (-2)y^{-3}$$

$$\Rightarrow 1 + [g'(y)]^2 = 1 + \left(\frac{1}{2}y^3 - \frac{1}{2}y^{-3}\right)^2$$

$$= 1 + \frac{1}{4}y^6 - \frac{1}{2} + \frac{1}{4}y^{-6}$$

$$= \frac{1}{4}y^6 + \frac{1}{2} + \frac{1}{4}y^{-6}$$

$$= \left(\frac{1}{2}y^3 - \frac{1}{2}y^{-3}\right)^2$$

So, $L = \int_1^4 \sqrt{1 + [g'(y)]^2}\, dy$

$$= \int_1^4 \sqrt{\left(\frac{1}{2}y^3 - \frac{1}{2}y^{-3}\right)^2}\, dy$$

$$= \int_1^4 \left(\frac{1}{2}y^3 - \frac{1}{2}y^{-3}\right) dy$$

$$= \left[\frac{1}{2} \times \frac{1}{4}y^4 - \frac{1}{2} \times (-2)y^{-2}\right]_1^4$$

$= 32.11$.