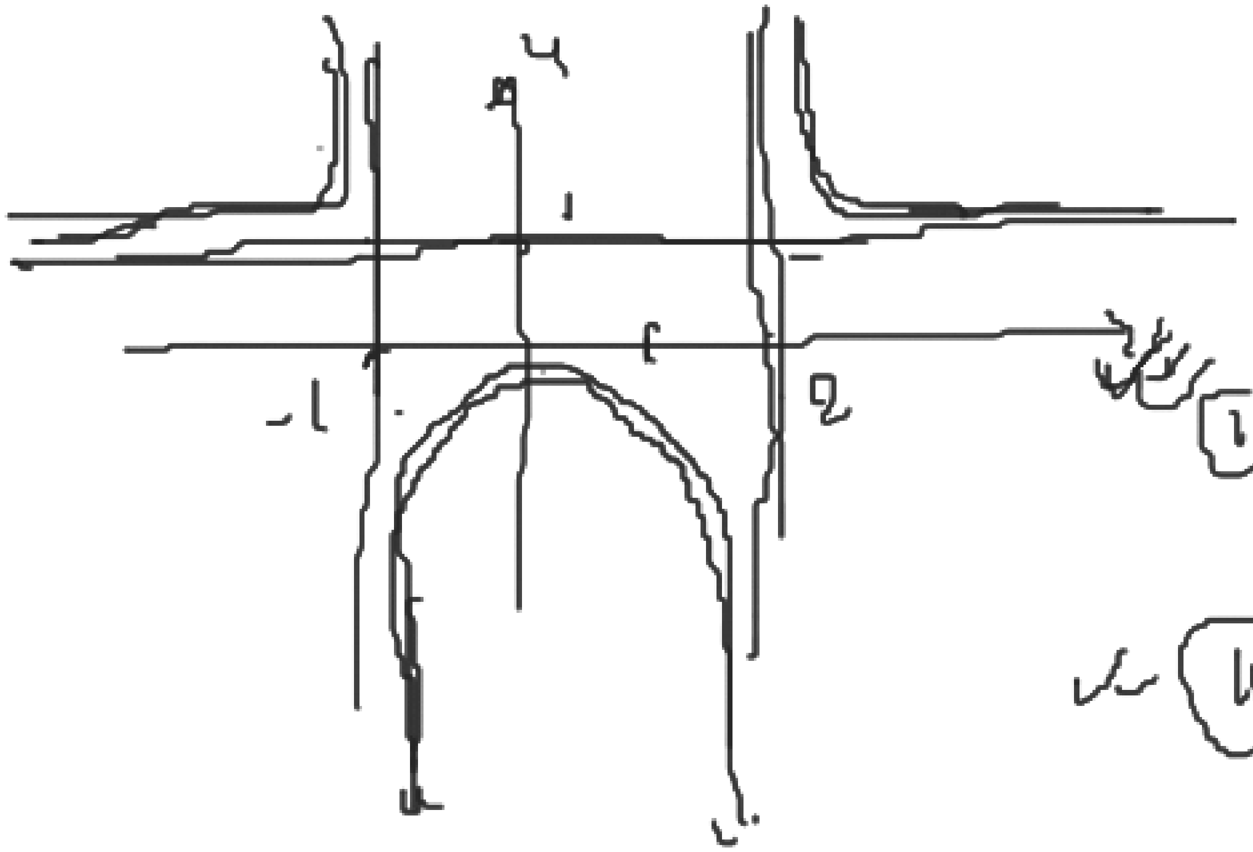
4.



(i) D: $(-\infty, -1) \cup (-1, 2) \cup (2, \infty)$

(ii) R: $(-\infty, 0] \cup (1, \infty)$

(iii) Horizontal asymptote: $y = 1$

(iv) Vertical " : $x = -1$ &

$x = 2$

5/4:

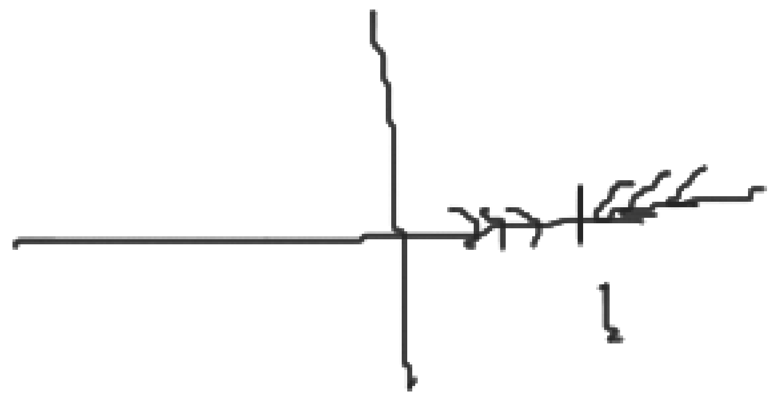
1.2 Computing Limit:

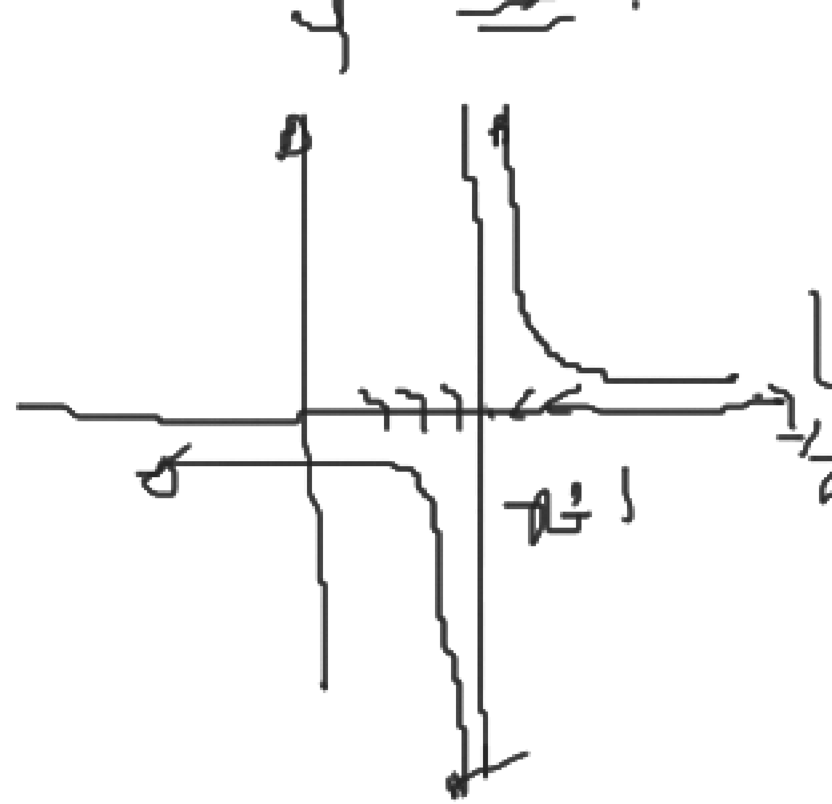
Theorem: a & k any real number

(i) $\lim_{x \to a} k = k$ (ii) $\lim_{x \to a} (x) = a$

(iii) $\lim_{x \to 0^-} \frac{1}{x} = -\infty$ (iv) $\lim_{x \to 0^+} \frac{1}{x} = +\infty$

Ex: $\lim_{x \to 5} (3) = 3$, $\lim_{x \to 2} x = 2$





$\lim_{x \to 0}$

$\lim_{x \to 1^-} \left(\frac{1}{x-1}\right) = -\infty$

$\lim_{x \to 1^+} \frac{1}{x-1} = +\infty$

$\therefore \lim_{x \to 1} \frac{1}{x-1}$ doesn't exist.

Theorem: $\lim_{x \to a} f(x) = L_1$, $\lim_{x \to a} g(x) = L_2$

(I) $\lim_{x \to a} \left[f(x) \pm g(x) \right] = \lim_{x \to a} f(x) \pm \lim_{x \to a} g(x) = L_1 \pm L_2$

(II) $\lim_{x \to a} \left[f(x) \cdot g(x) \right] = \lim_{x \to a} f(x) \cdot \lim_{x \to a} g(x) = L_1 L_2$

(III) $\lim_{x \to a} \left[\frac{f(x)}{g(x)} \right] = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)} = \frac{L_1}{L_2}$; $L_2 \neq 0$

(IV) $\lim_{x \to a} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x \to a} f(x)} = \sqrt[n]{L_1}$, $L_1 \geq 0$ if

n is even.

Ex. $\lim_{x\to 3} \{x^3 - 3x^2 + 9x\}$

Sol^n $\lim_{x\to 3} x^3 - 3.\lim_{x\to 3} x^2 + 9.\lim_{x\to 3} x$

$= (3)^3 - 3.(3)^2 + 9.3 = 27 - 27 + 27 = 27$

Theorem: $p(x) = c_0 + c_1 x + c_2 x^2 + \cdots\cdots + c_n x^n$

$\lim_{x\to a} p(x) = \lim_{x\to a} \left[c_0 + c_1 x + c_2 x^2 + \cdots + c_n x^n\right] = c_0 + c_1 a + c_2 a^2 + \cdots + c_n a^n$

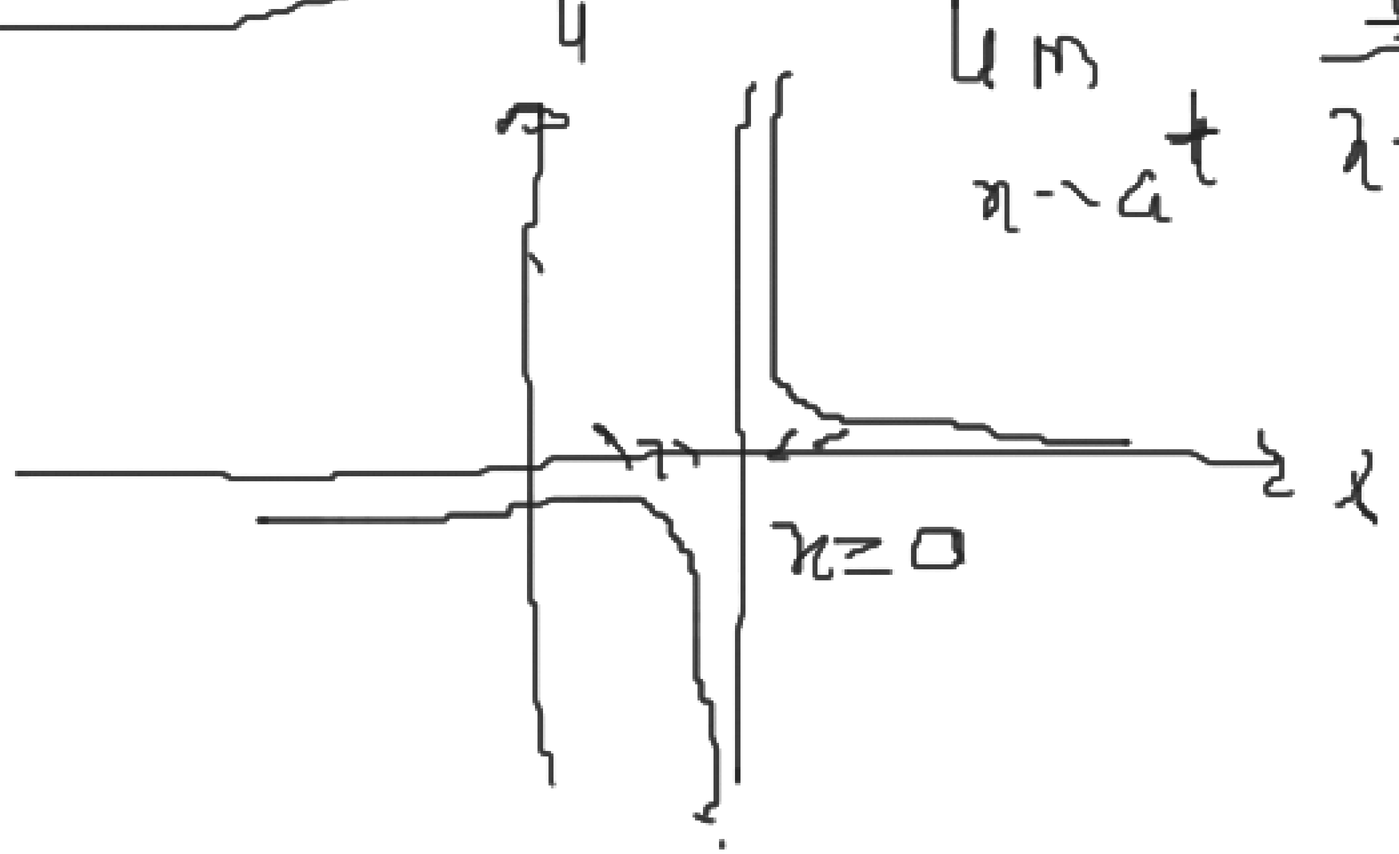
$= p(a)$

Ex. $\lim_{x\to 2} (x^3 + 4x^2 + 3) = 2^3 + 4(2)^2 + 3$

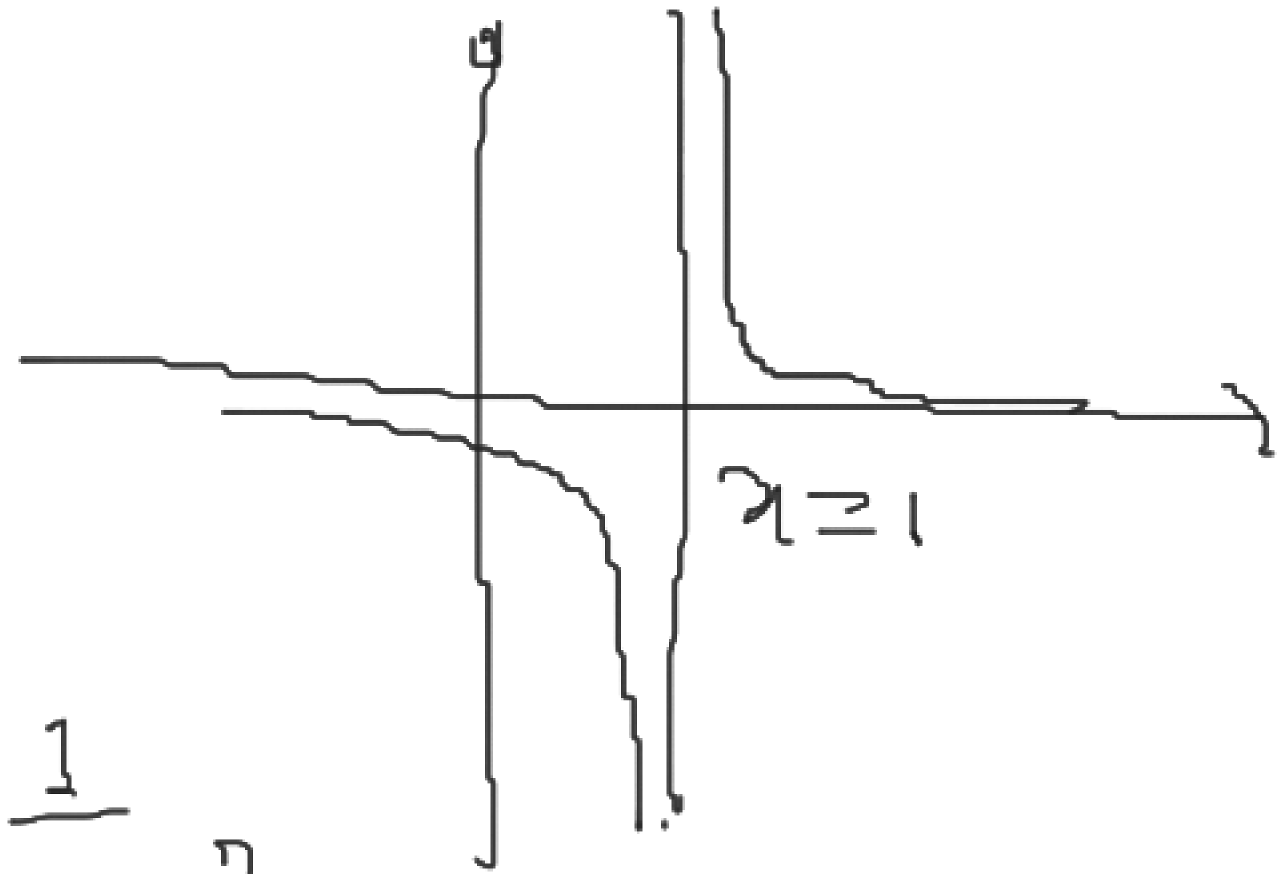
$= 8 + 16 + 3 = 35$

- $f(x) = \frac{1}{x-a}$, $\lim_{x \to a} f(x) = ?$
- $f(x) = \frac{1}{(x-a)^2}$, $\ldots = ?$
- $f(x) = -\frac{1}{(x-a)^2}$, $\ldots = ?$
- $\lim_{x \to a} \frac{1}{x-a} = ?$

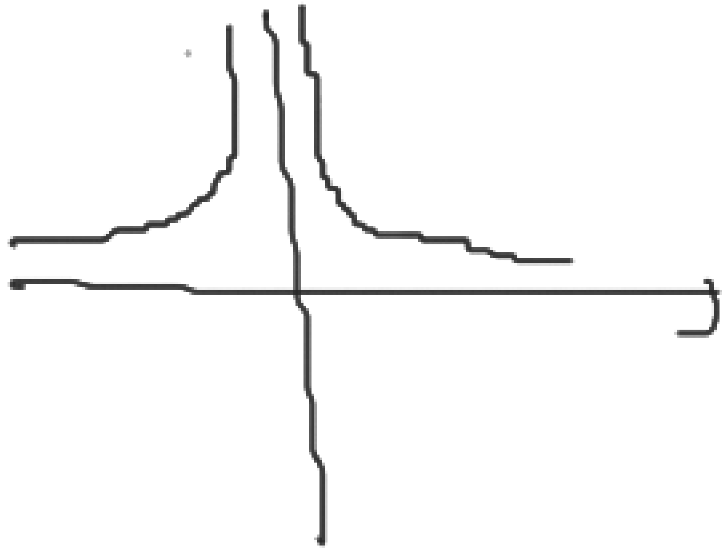
$$\left.\begin{aligned} \lim_{x \to a^-} \frac{1}{x-a} &= -\infty \\ \lim_{x \to a^+} \frac{1}{x-a} &= +\infty \end{aligned}\right\} \Rightarrow \lim_{x \to a} \frac{1}{x-a} \text{ does not exist}$$



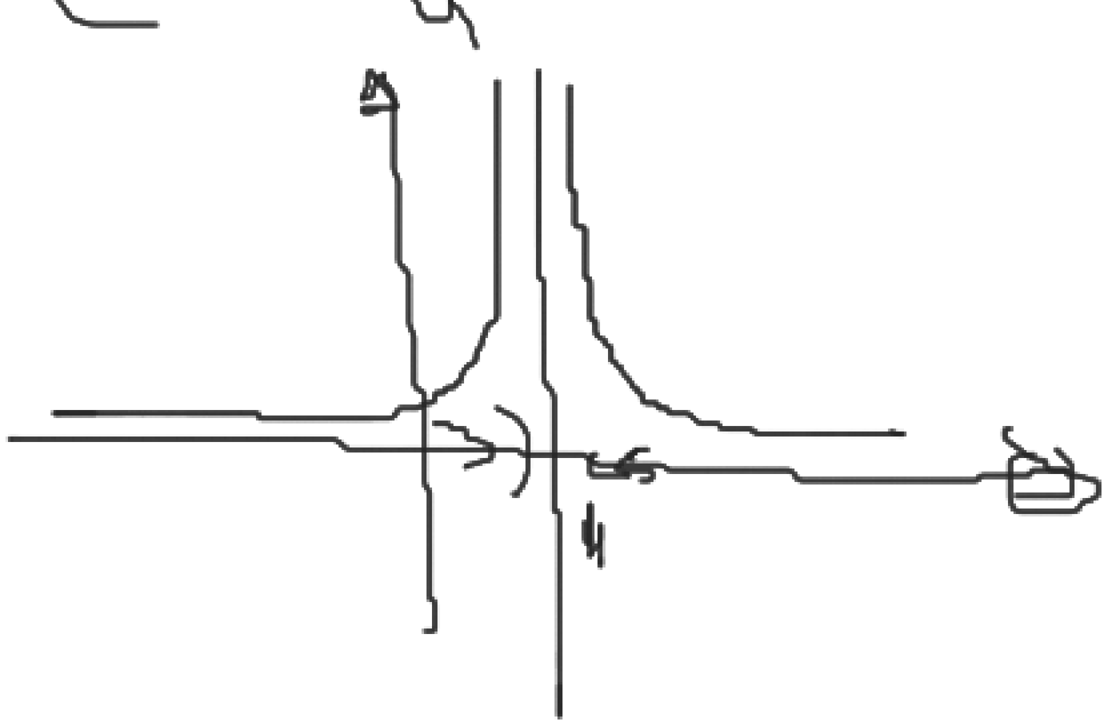
Ex: $f(x) = \frac{1}{x-1}$ $\qquad \lim_{x \to 1} f(x) = ?$



$f(x) = \frac{1}{x^2}$



Ex: $f(x) = \frac{1}{(x-1)^2}$



$$\lim_{x \to 1^-} \frac{1}{(x-1)^2} = +\infty$$

$$\lim_{x \to 1^+} \frac{1}{(x-1)^2} = +\infty$$

$$\therefore \lim_{x \to 1} \frac{1}{(x-1)^2} = +\infty$$

A

Ex 9 (a) $\lim_{x \to 3} \frac{x^2 - 6x + 9}{x - 3} = ?$

Sol $\lim_{x \to 3} \frac{(x-3)^{\cancel{2}}}{\cancel{(x-3)}} = \lim_{x \to 3} (x-3)$

$= 0$ A

(c) $\lim_{x \to 5} \frac{x^2 - 3x - 10}{x^2 - 10x + 25} = ?$

Sol. $\lim_{x \to 5} \frac{x^2 - 5x + 2x - 10}{(x-5)^2}$

$= \lim_{x \to 5} \frac{\cancel{(x-5)}(x+2)}{(x-5)^{\cancel{2}}}$

$= \lim_{x \to 5} \frac{x+2}{x-5}$ = doesn't exist A

$\lim_{x \to 5^-} \frac{x+2}{x-5} = -\infty$

$\lim_{x \to 5^+} \frac{x+2}{x-5} = +\infty$

$\therefore \lim_{x \to 5} \frac{x+2}{x-5}$ does not exist

Theorem : $f(x) = \frac{P(x)}{Q(x)}$; a is any real number

(i) If $Q(a) \neq 0$ then $\lim_{x \to a} f(x) = f(a)$

(ii) If $\boxed{Q(a) = 0}$ but $\boxed{P(a) \neq 0}$, $\lim_{x \to a} f(x)$ does not exist

Ex. $f(x) = \frac{x+2}{x-5}$ $\quad \lim_{x \to 5} f(x) = ?$

Soln

$P(x) = x+2$; $P(5) = 5+2 = 7 \neq 0$

$Q(x) = x-5$, $\therefore Q(5) = 5-5 = 0$

$\therefore \lim_{x \to 5} f(x)$ does not exist

Limit of Piecewise-Defined Function:

Ex 11:

$$f(x) = \begin{cases} \boxed{\dfrac{1}{x+2}} & ; \quad x < -2 \\ \boxed{x^2 - 5} & ; \quad -2 < x \le 3 \\ \sqrt{x+13} & ; \quad x > 3 \end{cases}$$

(a) $\boxed{\lim_{x \to -2} f(x) = }$?

(b) $\lim_{x \to 0} f(x) = ?$

(c) $\lim_{x \to 3} f(x) = ?$

Sol (a) $\lim_{x \to -2} f(x) = ?$

$$\begin{cases} \lim_{x \to -2^-} f(x) = \lim_{x \to -2^-} \dfrac{1}{x+2} = -\infty \\ \lim_{x \to -2^+} f(x) = \lim_{x \to -2^+} (x^2 - 5) = (-2)^2 - 5 = 4 - 5 = -1 \end{cases}$$

$\therefore \lim_{x \to -2} f(x)$ does not exist.

(b) $\lim_{x \to 0} f(x) = \lim_{x \to 0} (x^2 - 5) = 0^2 - 5 = -5$ Ans

(c) $\lim_{x \to 3} f(x) = ?$

Now. $\lim_{x \to 3^-} f(x) = \lim_{x \to 3^-} (x^2 - 5) = (3)^2 - 5 = 4$

$\lim_{x \to 3^+} f(x) = \lim_{x \to 3^+} \sqrt{x+13} = \sqrt{3+13} = 4$

$\therefore \lim_{x \to 3} f(x) = 4$ Ans

H.W Ex 1.2 : $\boxed{3-32}$ & $\boxed{37-40}$

31. $f(x) = \begin{cases} \boxed{x-1}, & x \le 3 \\ \boxed{3x-7}, & \boxed{x > 3} \end{cases}$

(a) $\lim_{x \to 3^-} f(x) = ?$

(b) $\lim_{x \to 3^+} f(x) = ?$

(c) $\lim_{x \to 3} f(x) = ?$

Sol: (a) $\lim_{x \to 3^-} f(x) = \lim_{x \to 3^-} (x-1) = 3-1 = 2$ Ans

(b) $\lim_{x \to 3^+} f(x) = \lim_{x \to 3^+} (3x-7) = 3(3)-7 = 2$ Ans

(c) $\lim_{x \to 3} f(x) = 2$ Ans