PHY 107
Rotation

Mohammad Murshed
Department of Math and Physics

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OUTLINE

- The rotational variables
- Rotation with constant angular acceleration
- Relation between the linear and angular variables
- Calculating the Rotational Inertia
- Torque
- Newton's Second Law in Angular Form

Angular Position and Displacement

Angular Position The angular position of this line is the angle of the line relative to a fixed direction, which we take as the zero angular position.

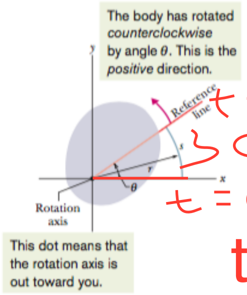
$\theta = \frac{s}{r}$ (radian measure)

1 rev$= 360^\circ = \frac{2\pi r}{r} = 2\pi rad$

What is a radian?

Angular displacement: $\Delta\theta = \theta_2 - \theta_1$

An angular displacement in the counterclockwise direction is positive, and one in the clockwise direction is negative.



t = 1

30

t = 0

theta_(t=0)=0

theta_(t=1)=30 deg

Angular Velocity and Acceleration

$$\omega_{avg} = \frac{\theta_2 - \theta_1}{t_2 - t_1}$$

$$\omega = \frac{d\theta}{dt}$$

$$\alpha_{avg} = \frac{\omega_2 - \omega_1}{t_2 - t_1}$$

$$\alpha = \frac{d\omega}{dt}$$

theta=t^3

omega=3t^2

alpha=3(2t)=6t

EXAMPLE Angular velocity derived from angular acceleration
A child's top is spun with angular acceleration $\alpha = 5t^3 - 4t$, with t in seconds and α in radians per second squared. At t = 0, the top has angular velocity 5 rad/s, and a reference line on it is at angular position $\theta = 2rad$.

(a) Obtain an expression for the angular velocity $\omega(t)$ of the top.

(b) Obtain an expression for the angular position $\theta(t)$ of the top.

Solution

a)$\int d\omega = \int \alpha dt$

b)$\int d\theta = \int \omega dt$

Use the initial conditions

Rotation with constant angular acceleration

Pure translation $\rightarrow$ Motion with a **constant linear acceleration**
Pure rotation $\rightarrow$ Motion with a **constant angular acceleration**

Linear Equation	Missing Variable		Angular Equation
$v = v_0 + at$	$x - x_0$	$\theta - \theta_0$	$\omega = \omega_0 + \alpha t$
$x - x_0 = v_0 t + \frac{1}{2}at^2$	v	ω	$\theta - \theta_0 = \omega_0 t + \frac{1}{2}\alpha t^2$
$v^2 = v_0^2 + 2a(x - x_0)$	t	t	$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$
$x - x_0 = \frac{1}{2}(v_0 + v)t$	a	α	$\theta - \theta_0 = \frac{1}{2}(\omega_0 + \omega)t$
$x - x_0 = vt - \frac{1}{2}at^2$	v_0	ω_0	$\theta - \theta_0 = \omega t - \frac{1}{2}\alpha t^2$

Rotation with constant angular acceleration

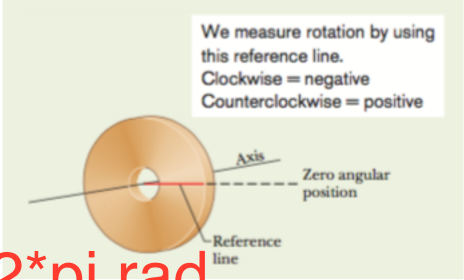
EXAMPLE Constant angular acceleration, grindstone

A grindstone rotates at constant angular acceleration $\alpha = 0.35 rad/s^2$. At time t = 0, it has an angular velocity of $\omega_0 = -4.6 rad/s$ and a reference line on it is horizontal, at the angular position $\theta_0 = 0$.

(a) At what time after t = 0 is the reference line at the angular position $\theta = 5.0\ rev$?

(b) Describe the grindstone's rotation between t = 0 and t = 32 s.

(c) At what time t does the grindstone momentarily stop?



1 rev= 2*pi rad

Rotation with constant angular acceleration

Solution

a)$\theta - \theta_0 = \omega_0 t + 0.5\alpha t^2$

b) The wheel slows in its rotation in the negative direction, stops, and then reverses to rotate in the positive direction. After the reference line comes back through its initial orientation of $\theta = 0$, the wheel turns an additional 5.0 rev by time t = 32 s.

c)$\omega = \omega_0 + \alpha t$

Relation between linear and angular quantities

A point in a rigid rotating body, at a perpendicular distance r from the rotation axis, moves in a circle with radius r. If the body rotates through an angle θ, the point moves along an arc with length s

Position

$s = r\theta$

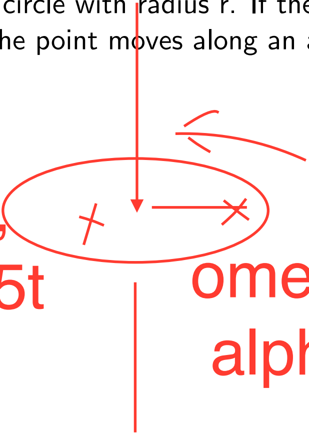
Speed

$v = \omega r$

Acceleration

$a_t = \alpha r$

$a_r = \omega^2 r$



r=2m,
theta=5t

omega=5
alpha=0

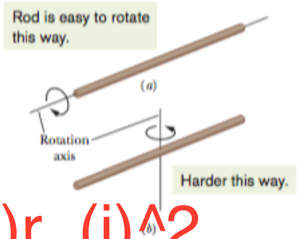
Kinetic Energy of rotation K=0.5*I*omega^2

Treat the table saw (and any other rotating rigid body) as a collection of particles with different speeds.

$K = \sum 0.5 m_i v_i^2 = \sum 0.5 m_i (\omega r_i)^2 = 0.5(\sum m_i r_i^2)\omega^2$

The quantity in parentheses: the mass of the rotating body is distributed about its axis of rotation.

Rotational inertia I of the body is a constant for a particular rigid body and a particular rotation axis.

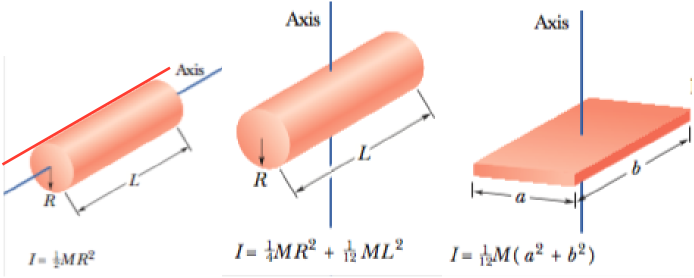


I=sum m_(i)r_(i)^2

The mass is distributed much closer to the rotation axis in the first rotation.

Calculating the rotational inertia

$I = \int r^2 dm$ (rotational inertia, continuous body)
Parallel axis theorem: $I = I_{com} + Mh^2$
I_{com} of the body about a parallel axis that extends through the body's center of mass. Let h be the perpendicular distance between the given axis and the axis through the center of mass (remember these two axes must be parallel).

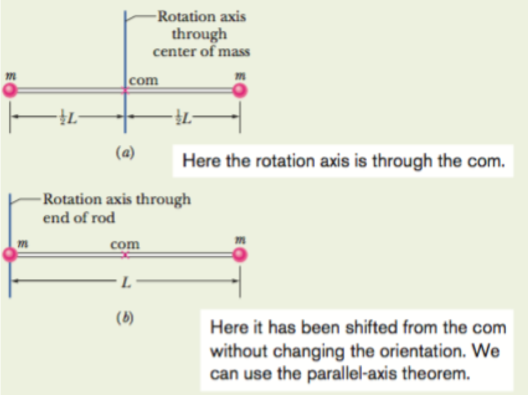


Rotational Inertia

EXAMPLE Rotational inertia of a two-particle system

The figure below shows a rigid body consisting of two particles of mass m connected by a rod of length L and negligible mass.

a) What is the rotational inertia I_{com} about an axis through the center of mass, perpendicular to the rod as shown? b) What is the rotational inertia I of the body about an axis through the left end of the rod and parallel to the first axis (Fig. 10-13b)?

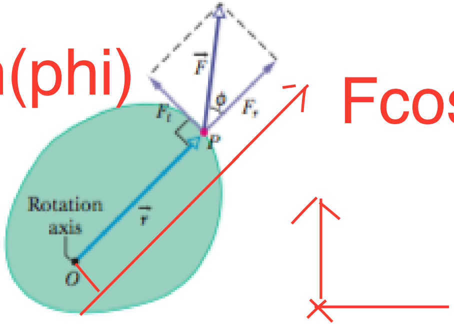


Torque

$$\tau = \underline{r(Fsin(\phi))}$$

Fsin(phi)

Fcos(phi)



Think about the components of the applied force
More to come in the next chapter

Moment=force*perp distance

Newton's Second Law in Angular form

The rotational analog of Newton's Second Law is:

$\tau_{net} = I\alpha$

τ_{net}: net torque acting on a particle or rigid body

I: rotational inertia of the particle or body about the rotation axis

α: resulting angular acceleration about that axis

Problems of importance

Check the book (edition: Extended 9th)
The rotational variables: 4, 15
Relating the Linear and Angular Variables: 19
Kinetic energy of rotation: 33
Calculating the rotational inertia: 35, 37, 41
Torque: 45

Reference

Fundamentals of Physics by Halliday and Resnik