Probability

Dr. Md. Israt Rayhan
Professor
Institute of Statistical Research and Training
(ISRT)
University of Dhaka
Email: israt@isrt.ac.bd

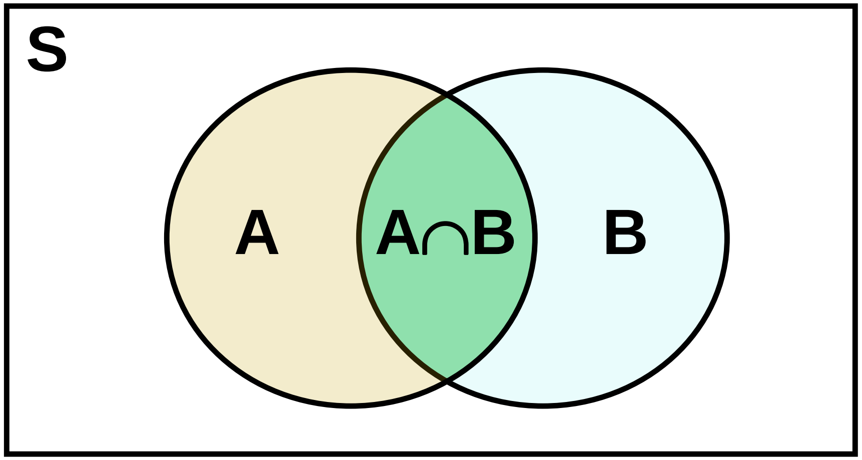
Important Terms

- Random Experiment – a process leading to an uncertain outcome
- Basic Outcome – a possible outcome of a random experiment
- Sample Space – the collection of all possible outcomes of a random experiment
- Event – any subset of basic outcomes from the sample space

Important Terms

(continued)

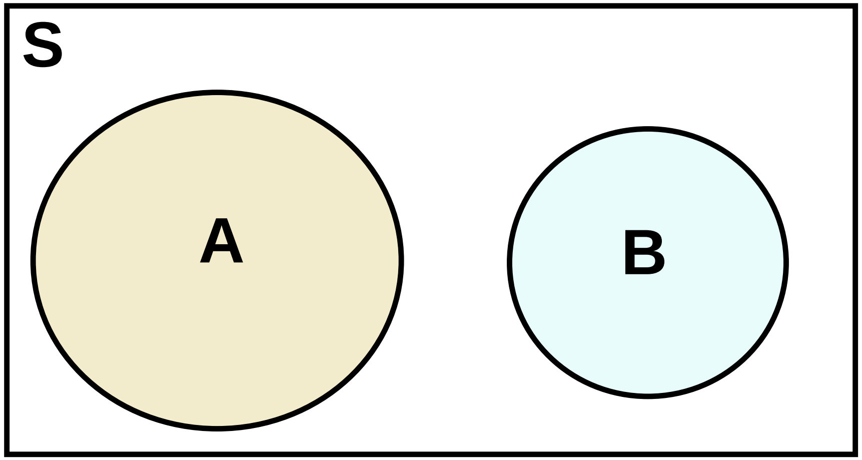
- Intersection of Events – If A and B are two events in a sample space S, then the intersection, A ∩ B, is the set of all outcomes in S that belong to both A and B



Important Terms

(continued)

- A and B are Mutually Exclusive Events if they have no basic outcomes in common
  - i.e., the set A ∩ B is empty

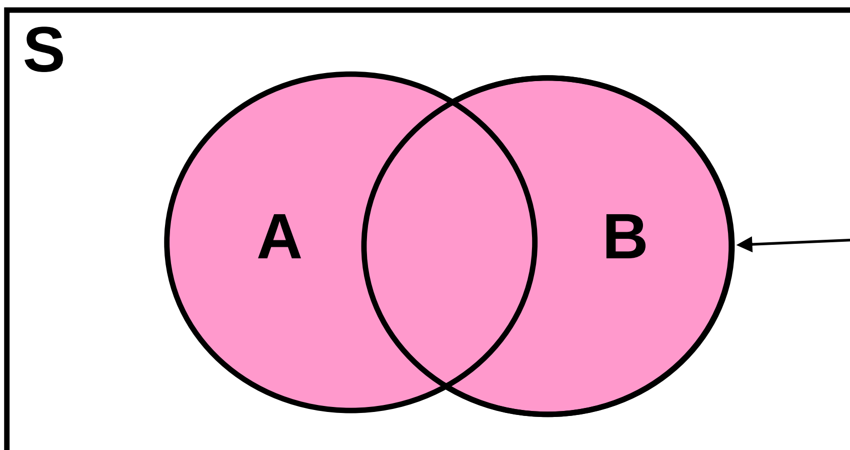


Important Terms

(continued)

- Union of Events – If A and B are two events in a sample space S, then the union, A ∪ B, is the set of all outcomes in S that belong to either

  A or B



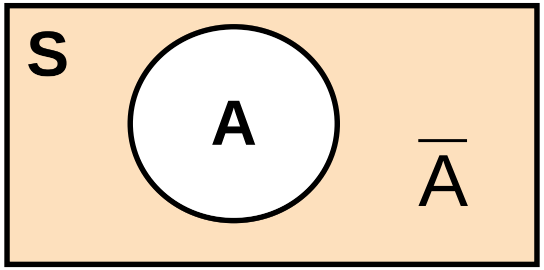
The entire shaded area represents **A** ∪ **B**

Important Terms

(continued)

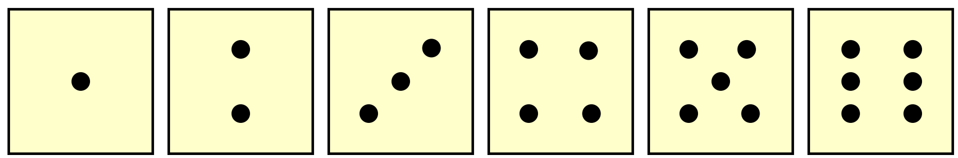
- Events E_1, E_2, … E_k are Collectively Exhaustive events if $E_1 \cup E_2 \cup \ldots \cup E_k = S$
  - i.e., the events completely cover the sample space

- The Complement of an event A is the set of all basic outcomes in the sample space that do not belong to A. The complement is denoted $\overline{A}$



Examples

Let the Sample Space be the collection of all possible outcomes of rolling one die:



S = [1, 2, 3, 4, 5, 6]

Let A be the event “Number rolled is even”

Let B be the event “Number rolled is at least 4”

Then

A = [2, 4, 6]   and   B = [4, 5, 6]

Examples

(continued)

S = [1, 2, 3, 4, 5, 6] | A = [2, 4, 6] | B = [4, 5, 6]

Complements:

$$\overline{A} = [1, 3, 5] \qquad \overline{B} = [1, 2, 3]$$

Intersections:

$$A \cap B = [4, 6] \qquad \overline{A} \cap B = [5]$$

Unions:

$$A \cup B = [2, 4, 5, 6]$$

$$A \cup \overline{A} = [1, 2, 3, 4, 5, 6] = S$$

Examples

(continued)

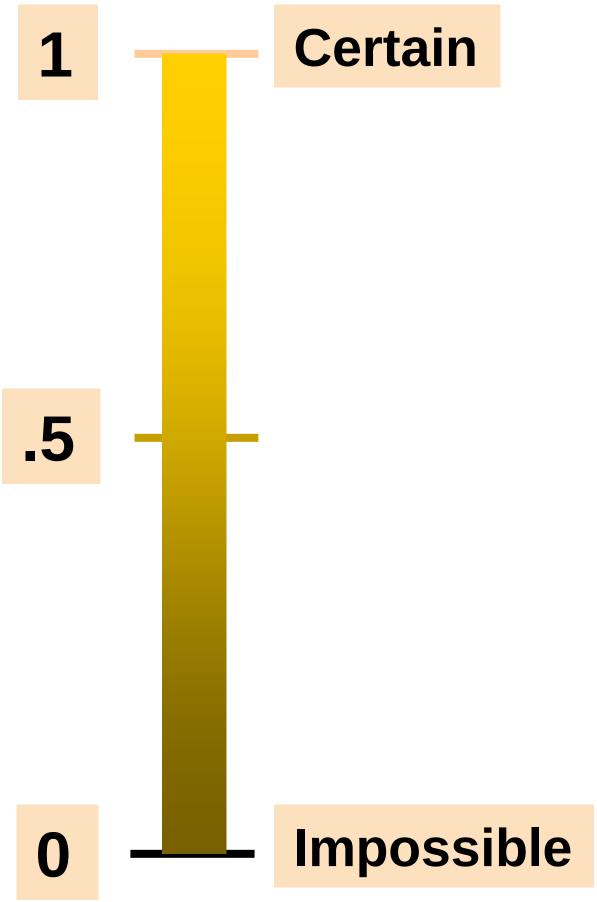
S = [1, 2, 3, 4, 5, 6]	A = [2, 4, 6]	B = [4, 5, 6]

- Mutually exclusive:
  - A and B are not mutually exclusive
    - The outcomes 4 and 6 are common to both
- Collectively exhaustive:
  - A and B are not collectively exhaustive
    - $A \cup B$ does not contain 1 or 3

Probability

- Probability – the chance that an uncertain event will occur (always between 0 and 1)

$0 \le P(A) \le 1$ For any event A



Assessing Probability

Classical probability

$$\text{probability of event } A = \frac{N_A}{N} = \frac{\text{number of outcomes that satisfy the event}}{\text{total number of outcomes in the sample space}}$$

- Assumes all outcomes in the sample space are equally likely to occur

Probability Rules

- The Complement rule:

$$\boxed{P(\overline{A}) = 1 - P(A)} \qquad \text{i.e., } P(A) + P(\overline{A}) = 1$$

- The Addition rule:
  - The probability of the union of two events is

$$\boxed{P(A \cup B) = P(A) + P(B) - P(A \cap B)}$$

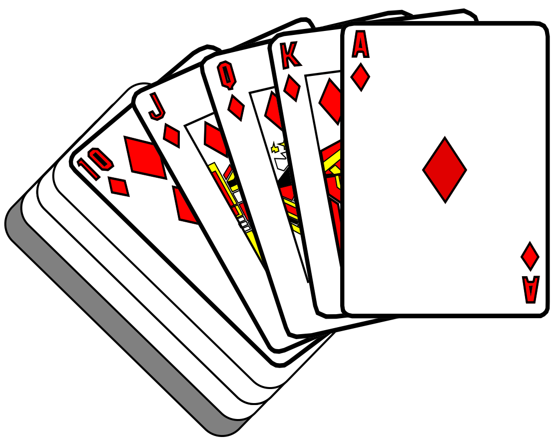
Addition Rule Example

Consider a standard deck of 52 cards, with four suits:

♥ ♣ ♦ ♠

Let event A = card is an Ace

Let event B = card is from a red suit



Addition Rule Example

(continued)

$$P(\text{Red} \cup \text{Ace}) = P(\text{Red}) + P(\text{Ace}) - P(\text{Red} \cap \text{Ace})$$

$$= 26/52 + 4/52 - \boxed{2/52} = 28/52$$

Don't count the two red aces twice!

Type	Color		Total
	Red	Black	
Ace	2	2	4
Non-Ace	24	24	48
Total	26	26	52

Conditional Probability

- A conditional probability is the probability of one event, given that another event has occurred:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

The conditional probability of A given that B has occurred

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

The conditional probability of B given that A has occurred

Conditional Probability Example

- Of the cars on a used car lot, 70% have air conditioning (AC) and 40% have a CD player (CD). 20% of the cars have both.

- What is the probability that a car has a CD player, given that it has AC ?

i.e., we want to find $P(CD \mid AC)$

Conditional Probability Example

(continued)

- Of the cars on a used car lot, **70%** have air conditioning (AC) and **40%** have a CD player (CD).
  20% of the cars have both.

	CD	No CD	Total
AC	**.2**	**.5**	**.7**
No AC	**.2**	**.1**	**.3**
Total	**.4**	**.6**	**1.0**

$$P(CD|AC) = \frac{P(CD \cap AC)}{P(AC)} = \frac{.2}{.7} = .2857$$

Statistical Independence

- Two events are statistically independent if and only if:

$$\boxed{P(A \cap B) = P(A)P(B)}$$

  - Events A and B are independent when the probability of one event is not affected by the other event

- If A and B are independent, then

$$\boxed{P(A \mid B) = P(A)} \quad \text{if } P(B)>0$$

$$\boxed{P(B \mid A) = P(B)} \quad \text{if } P(A)>0$$

Statistical Independence Example

- Of the cars on a used car lot, **70%** have air conditioning (AC) and **40%** have a CD player (CD).
  20% of the cars have both.

	CD	No CD	Total
AC	.2	.5	.7
No AC	.2	.1	.3
Total	.4	.6	1.0

- Are the events AC and CD statistically independent?

Statistical Independence Example

(continued)

	CD	No CD	Total
AC	**.2**	**.5**	**.7**
No AC	**.2**	**.1**	**.3**
Total	**.4**	**.6**	**1.0**

$P(AC \cap CD) = 0.2$

$$\left.\begin{array}{l} P(AC) = 0.7 \\ P(CD) = 0.4 \end{array}\right\} P(AC)P(CD) = (0.7)(0.4) = 0.28$$

$P(AC \cap CD) = 0.2 \quad \neq \quad P(AC)P(CD) = 0.28$

So the two events are not statistically independent

Odds

- The odds in favor of a particular event are given by the ratio of the probability of the event divided by the probability of its complement
- The odds in favor of A are

$$\boxed{\text{odds} = \frac{P(A)}{1-P(A)} = \frac{P(A)}{P(\overline{A})}}$$

Odds: Example

- Calculate the probability of winning if the odds of winning are 3 to 1:

$$\boxed{\text{odds} = \frac{3}{1} = \frac{P(A)}{1\text{-}P(A)}}$$

  - Now multiply both sides by 1 – P(A) and solve for P(A):

$$3 \text{ x } (1- P(A)) = P(A)$$
$$3 – 3P(A) = P(A)$$
$$3 = 4P(A)$$
$$\boxed{P(A) = 0.75}$$

Thank you