15. Hypothesis testing

Hypothesis test for the mean (μ)

Case 1: X has a normal distribution **with known population variance** (σ^2)

Case 2: X has a normal distribution with unknown population variance (σ^2)

Case 3: X has a general distribution, but we have a large sample size ($n \geq 30$).

- Hypothesis testing has 4 steps –

  Step 1: Null hypothesis

  Alternative hypothesis

  Step 2: Test statistic: Test statistic will give a calculated value which will use to take decision either we accept or reject H_0.

  Step 3: Rejection region: If calculated value falls in the rejection region, we reject H_0 (null hypothesis).

  Step 4: Comment. (Since the calculated value falls in the rejection region, so we reject H_0 (null hypothesis) or since the calculated value does not fall in the rejection region, so we can not reject H_0 (null hypothesis)).

	Case 1	Case 2	Case 3
Null hypothesis (H_0) Alternative hypothesis (H_1)	$H_0: \mu = \mu_0$ $H_1: \mu > \mu_0$ $or, H_1: \mu < \mu_0$ $or, H_1: \mu \neq \mu_0$		
Test Statistic	$\frac{\bar{x}-\mu_0}{\frac{\sigma}{\sqrt{n}}} \sim N(0,1)$	$\frac{\bar{x}-\mu_0}{\sqrt{\frac{s^2}{n}}} \sim t_{(n-1)}$ $s^2 = \frac{\sum(x_i - \bar{x})^2}{n-1}$	σ^2 is known, the test statistics is $\frac{\bar{x}-\mu_0}{\sqrt{\frac{\sigma^2}{n}}} \sim N(0,1)$ When population variance (σ^2) is unknown, The test statistic is, $\frac{\bar{x}-\mu_0}{\sqrt{\frac{s^2}{n}}} \sim N(0,1)$
Rejection Region	μ > μ0: $[Z_\alpha, +\infty[$ μ < μ0: $]-\infty, -Z_\alpha]$ μ ≠ μ0: $]-\infty, -Z_{\frac{\alpha}{2}}] \cup [Z_{\frac{\alpha}{2}}, +\infty[$	μ > μ0: $[t_\alpha, +\infty[$ μ < μ0: $]-\infty, -t_\alpha]$ μ ≠ μ0: $]-\infty, -t_{\frac{\alpha}{2}}] \cup [t_{\frac{\alpha}{2}}, +\infty[$	$H_1: \mu > \mu_0$ The rejection region is $[Z_\alpha, +\infty[$ $H_1: \mu < \mu_0$ The rejection region is $]-\infty, -Z_\alpha]$ When $H_1: \mu \neq \mu_0$ The rejection region is $]-\infty, -Z_{\frac{\alpha}{2}}] \cup [Z_{\frac{\alpha}{2}}, +\infty[$

Comment	**Step 4:** Comment. (Since the calculated value falls in the rejection region, so we reject H_0 (null hypothesis) or since the calculated value does not fall in the rejection region, so we can not reject H_0 (null hypothesis)). And, H0 reject mean, H1 correct. H0 not reject mean H1 incorrect.

Here, $\bar{x}$ (Sample mean)

μ_0 (Given)

σ (Population standard deviation)

n (Sample size)

Lecture 16

P. A	P. B
befor	during
1 ✓	✓
2 ✓	✓
10 ✓	✓

Matched pairs t test:

Matched-pairs t test is used to test whether there is a significant mean difference between two sets of paired data.

Step1: $H_0 : \mu_D = 0$

$H_1 : \mu_D > 0$ or, $H_1 : \mu_D < 0$ or $H_1 : \mu_D \neq 0$

D is the difference between paired value from two data sets

$D = Y - X$

$\bar{D}$ is the sample mean difference between paired observation/Data.

S_D^2 is the variance of the difference.

n is the number of paired data

Step2: Test statistics $= \dfrac{\bar{D}}{\sqrt{\frac{S_D^2}{n}}} \sim t_{n-1}$.

Ex

ID	$D_i = Y_i - X_i$
1	$Y_1 - X_1$
2	$Y_2 - X_2$
⋮	
n	$Y_n - X_n$

$\therefore \bar{D} = \dfrac{(Y_1 - X_1) + (Y_2 - X_2) + \dots + (Y_n - X_n)}{n}$

$= \dfrac{D_1 + D_2 + \dots + D_n}{n}$

$$S_D^2 = \frac{\sum_{i=1}^{n} (D_i - \bar{D})^2}{n-1}$$

Step3: $H_1 : \mu_D > 0$

rejection region: $[t_\alpha, n-1, +\infty[$

$H_1 : \mu_D < 0$

rej... reg... $]-\infty, -t_{\alpha, n-1}]$

$H_1 : \mu_D \neq 0$

$]-\infty, -t_{\frac{\alpha}{2}, n-1}] \cup [t_{\frac{\alpha}{2}, n-1}, +\infty[$

Step4: comment.

Independent Sample t test:

also called unpaired sample t test helps us to compare means of two sets ~~(A & B)~~

M_1	M_2
R_A	R_B
A_1	B_1
A_2	B_2
⋮	⋮
A_{n_1}	B_{n_2}

Step1: $H_0 : \mu_1 = \mu_2 \rightarrow H_0 : \mu_1 - \mu_2 = 0$

$H_1 : \mu_1 < \mu_2 \rightarrow H_1 : \mu_1 - \mu_2 < 0$

$H_1 : \mu_1 > \mu_2 \rightarrow H_1 : \mu_1 - \mu_2 > 0$

$H_1 : \mu_1 \neq \mu_2 \rightarrow H_1 : \mu_1 - \mu_2 \neq 0$

Step2: Test statistics $= \dfrac{\bar{X} - \bar{Y}}{\sqrt{S_P^2 \left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} \sim t_{n_1 + n_2 - 2}$

$\bar{X} = \dfrac{A_1 + A_2 + \dots + A_{n_1}}{n_1}$

$\bar{Y} = \dfrac{B_1 + B_2 + \dots + B_{n_2}}{n_2}$.

where, $$S_P^2 = \frac{\sum_{i=1}^{n_1} (x_i - \bar{X})^2 + \sum_{j=1}^{n_2} (Y_i - \bar{Y})^2}{n_1 + n_2 - 2}$$

Step3:

$H_1 : \mu_1 - \mu_2 < 0$

rejec. region: $]-\infty, -t_\alpha]$

$H_1 : \mu_1 - \mu_2 > 0$

rej... region..: $[t_\alpha, \infty[$

$H_1 : \mu_1 - \mu_2 \neq 0$

The rejec... region: $]-\infty, -t_{\frac{\alpha}{2}}] \cup [t_{\frac{\alpha}{2}}, +\infty[$

Step4: Comment.

d

Matched Pairs t test:

Step 1: $H_0 : \mu_D = 0$

$H_1 : \mu_D > 0$

Or $H_1 : \mu_D < 0$

Or $H_! : \mu_D \neq 0$

Here, $\mu_D = \mu_y - \mu_x$

Step 2: Test statistic $= \frac{\bar{D}}{\sqrt{\frac{s^2{}_D}{n}}} \sim t_{n-1}$

Here, D is the difference between paired value from two data sets.

From the data, we get

Id (i)	Di = Yi - Xi
1	13
2	3
3	-1
4	9
5	7
6	7
7	6
8	4
9	-2
10	2

$\therefore \bar{D} = \frac{13+3+\cdots\ldots+2}{10} = 4.8$

$s^2{}_D = \frac{\sum_{i=1}^{10}(Di-\bar{D})^2}{n-1}$

$= \frac{(D_1-\bar{D})^2+(D_2-\bar{D})^2+\cdots+(D_{10}-\bar{D})^2}{10-1}$

Independent sample t test:

■ **Step 1:** $H_0 : \mu_1 = \mu_2$

$H_1 : \mu_1 < \mu_2$

or $\quad H_1 : \mu_1 > \mu_2$

or $\quad H_1 : \mu_1 \neq \mu_2$

$$\Downarrow$$

$H_0 : \mu_1 - \mu_2 = 0$

$H_1 : \mu_1 - \mu_2 < 0$

or $\quad H_1 : \mu_1 - \mu_2 > 0$

or $\quad H_1 : \mu_1 - \mu_2 \neq 0$

■ **Step 2:** Test statistic $= \dfrac{\bar{x}-\bar{y}}{\sqrt{s^2{}_p\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}} \sim t_{n_1+n_2-2}$

Where $s^2{}_p = \dfrac{\sum_{i=1}^{n_1}(x_i-\bar{x})^2+\sum_{j=1}^{n_2}(y_j-\bar{y})^2}{n_1+n_2-2}$

Step 3: If $H_1 : \mu_1 - \mu_2 < 0$

The rejection region: $]-\infty, -t_\alpha]$

If $H_1 : \mu_1 - \mu_2 > 0$

The rejection region: $[\, t_\alpha, +\infty[$

If $H_1 : \mu_1 - \mu_2 \neq 0$

The rejection region: $\left]-\infty, -t_{\frac{\alpha}{2}}\right] \cup \left[t_{\frac{\alpha}{2}}, +\infty\right[$

Step 4: Comment.

■ **Question:** In an experiment, we compare the result of treatment A and treatment B by seeing the survival time of mouses.

Treatment A: 17 19 15 18 21 18

Treatment B: 18 15 13 16 13

Investigate whether treatment B gives better result? Test this at 5% level of sig.

Solution: $H_0 : \mu_1 = \mu_2 \;\Rightarrow\; H_0 : \mu_1 - \mu_2 = 0$

$H_1 : \mu_1 < \mu_2 \;\Rightarrow H_1 : \mu_1 - \mu_2 < 0$

Test statistic $= \dfrac{\bar{x}-\bar{y}}{\sqrt{s^2{}_p\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}}$

Here $s^2{}_p = \dfrac{\sum_{i=1}^{6}(x_i-\bar{x})^2+\sum_{j=1}^{5}(y_j-\bar{y})^2}{n_1+n_2-2}$

And $\bar{x} = \dfrac{17+\cdots+18}{6} = 18$

$\bar{y} = \dfrac{18+\cdots+13}{5} = 15$

$s^2{}_p = \dfrac{(17-18)^2+(19-18)^2+\cdots+(18-18)^2+(18-15)^2+(15-15)^2\ldots+(13-15)^2}{6+5-2}$

$= 4.22$

$\therefore$ Test statistic $= \dfrac{18-15}{\sqrt{4.22\left(\frac{1}{6}+\frac{1}{5}\right)}}$

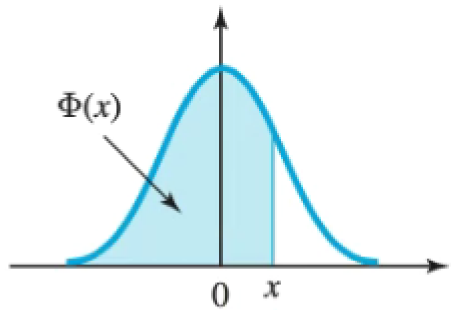
$= 2.41$

Rejection region: $]-\infty, -t_\alpha]$

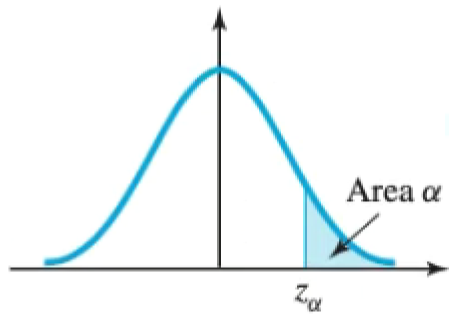
$=]-\infty, -1.833]$

Comment: Since the calculate value does not fall in the rejection region, so we can not reject H_0. That is, treatment B does not give better result.

Note:



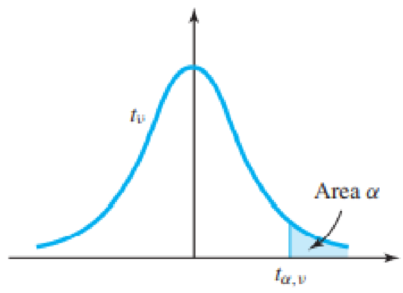
Critical Points



α	z_α
0.10	1.282
0.05	1.645
0.025	1.960
0.01	2.326
0.005	2.576

pto(for t alfa)

Table III: Critical Points of the t-Distribution



v=n-1

Degrees of freedom ν	α						
	0.10	0.05	0.025	0.01	0.005	0.001	0.0005
1	3.078	6.314	12.706	31.821	63.657	318.31	636.62
2	1.886	2.920	4.303	6.965	9.925	22.326	31.598
3	1.638	2.353	3.182	4.541	5.841	10.213	12.924
4	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	1.319	1.714	2.069	2.500	2.807	3.485	3.767
24	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	1.311	1.699	2.045	2.462	2.756	3.396	3.659
30	1.310	1.697	2.042	2.457	2.750	3.385	3.646
40	1.303	1.684	2.021	2.423	2.704	3.307	3.551
60	1.296	1.671	2.000	2.390	2.660	3.232	3.460
120	1.289	1.658	1.980	2.358	2.617	3.160	3.373
∞	1.282	1.645	1.960	2.326	2.576	3.090	3.291