ANSWERS TO ODD-NUMBERED EXERCISES

Exercise Set 0.1 (Page 12)

1. **(a)** $-2.9, -2.0, 2.35, 2.9$ **(b)** none **(c)** $y = 0$ **(d)** $-1.75 \le x \le 2.15$ **(e)** $y_{\max} = 2.8$ at $x = -2.6$; $y_{\min} = -2.2$ at $x = 1.2$

3. **(a)** yes **(b)** yes **(c)** no **(d)** no

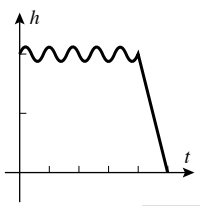
5. **(a)** 1999, about \$47,700 **(b)** 1993, \$41,600 **(c)** first year

7. **(a)** -2; 10; 10; 25; 4; $27t^2 - 2$ **(b)** 0; 4; -4; 6; $2\sqrt{2}$; $f(3t) = 1/(3t)$ for $t > 1$ and $f(3t) = 6t$ for $t \le 1$

9. **(a)** domain: $x \ne 3$; range: $y \ne 0$ **(b)** domain: $x \ne 0$; range: $\{-1, 1\}$ **(c)** domain: $x \le -\sqrt{3}$ or $x \ge \sqrt{3}$; range: $y \ge 0$
(d) domain: $-\infty < x < +\infty$; range: $y \ge 2$
(e) domain: $x \ne (2n + \frac{1}{2})\pi, n = 0, \pm 1, \pm 2, \ldots$; range: $y \ge \frac{1}{2}$
(f) domain: $-2 \le x < 2$ or $x > 2$; range: $0 \le y < 2$ or $y > 2$

11. **(a)** no; births and deaths **(b)** decreases for 8 hours, takes a jump upward, and repeats

13.



15. function; $y = \sqrt{25 - x^2}$

17. function; $y = \begin{cases} \sqrt{25 - x^2}, & -5 \le x \le 0 \\ -\sqrt{25 - x^2}, & 0 < x \le 5 \end{cases}$

19. False; for example, the graph of the function $f(x) = x^2 - 1$ crosses the x-axis at $x = \pm 1$.

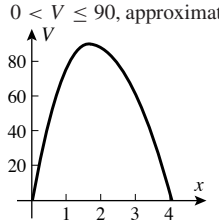
21. False; the range also includes 0.

23. **(a)** 2, 4 **(b)** none **(c)** $x \le 2$; $4 \le x$ **(d)** $y_{\min} = -1$; no maximum

25. $h = L(1 - \cos\theta)$

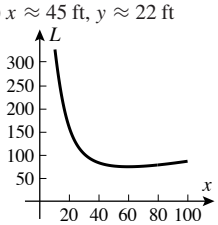
27. **(a)** $f(x) = \begin{cases} 2x + 1, & x < 0 \\ 4x + 1, & x \ge 0 \end{cases}$ **(b)** $g(x) = \begin{cases} 1 - 2x, & x < 0 \\ 1, & 0 \le x < 1 \\ 2x - 1, & x \ge 1 \end{cases}$

29. **(a)** $V = (8 - 2x)(15 - 2x)x$
(b) $0 < x < 4$
(c) $0 < V \le 90$, approximately



(d) V appears to be maximal for $x \approx 1.7$.

31. **(a)** $L = x + 2y$
(b) $L = x + 2000/x$
(c) $0 < x \le 100$
(d) $x \approx 45$ ft, $y \approx 22$ ft



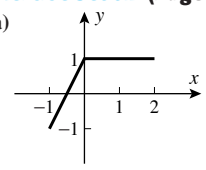
33. **(a)** $r \approx 3.4$, $h \approx 13.7$ **(b)** taller
(c) $r \approx 3.1$ cm, $h \approx 16.0$ cm, $C \approx 4.76$ cents

35. **(i)** $x = 1, -2$ **(ii)** $g(x) = x + 1$, all x

37. **(a)** 25°F **(b)** 13°F **(c)** 5°F **39.** 15°F

Exercise Set 0.2 (Page 24)

1. **(a)**



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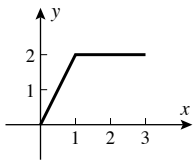
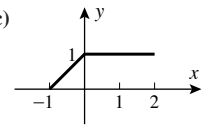


(c)



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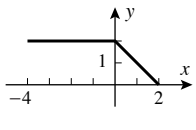
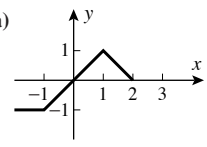


3. **(a)**



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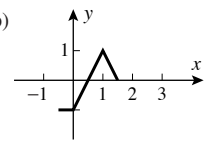

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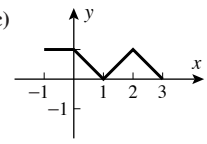
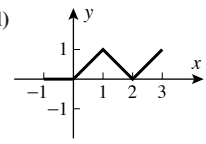


(d)



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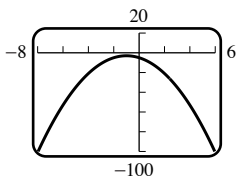

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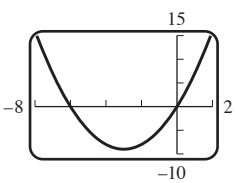

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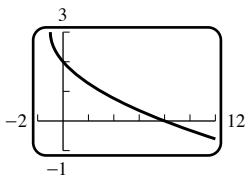

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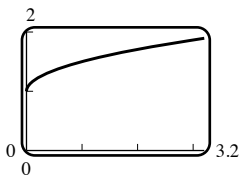
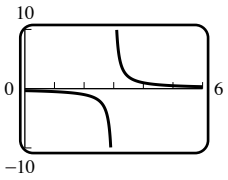
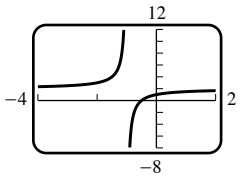
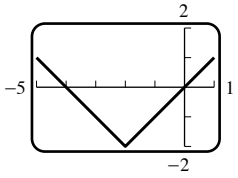
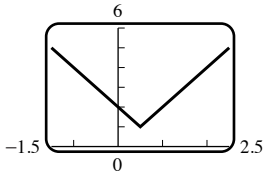
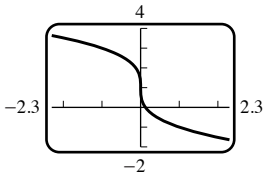


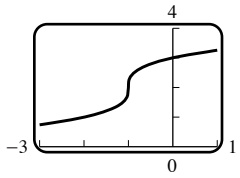
13. 

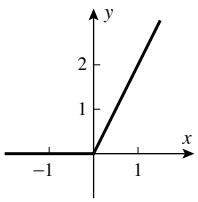
15. 

17. 

19. 

21. 

23. 

25. (a) 

(b) $y = \begin{cases} 0, & x \le 0 \\ 2x, & x > 0 \end{cases}$

27. $3\sqrt{x-1}, x \ge 1$; $\sqrt{x-1}, x \ge 1$; $2x-2, x \ge 1$; $2, x > 1$

29. (a) 3 (b) 9 (c) 2 (d) 2 (e) $\sqrt{2+h}$ (f) $(3+h)^3 + 1$

31. $1 - x, x \le 1$; $\sqrt{1-x^2}, |x| \le 1$

33. $\dfrac{1}{1-2x}, x \ne \dfrac{1}{2}, 1$; $-\dfrac{1}{2x} - \dfrac{1}{2}, x \ne 0, 1$ **35.** $x^{-6} + 1$

37. (a) $g(x) = \sqrt{x}, h(x) = x + 2$ (b) $g(x) = |x|, h(x) = x^2 - 3x + 5$

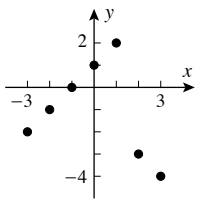
39. (a) $g(x) = x^2, h(x) = \sin x$ (b) $g(x) = 3/x, h(x) = 5 + \cos x$

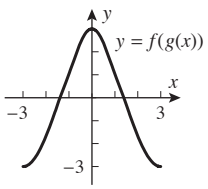
41. (a) $g(x) = x^3, h(x) = 1 + \sin(x^2)$
(b) $g(x) = \sqrt{x}, h(x) = 1 - \sqrt[3]{x}$

Responses to True–False questions may be abridged to save space.

43. True; see Definition 0.2.1.

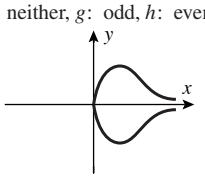
45. True; see Theorem 0.2.3 and the definition of even function that follows.

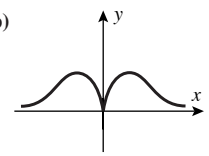
47. 

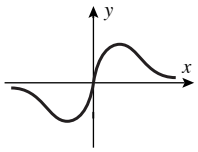
49. 

51. $\pm 1.5, \pm 2$ **53.** $6x + 3h, 3w + 3x$ **55.** $-\dfrac{1}{x(x+h)}, -\dfrac{1}{xw}$

57. f: neither, g: odd, h: even

59. (a) 

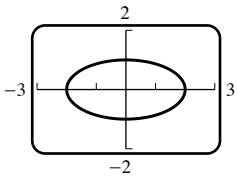
(b) 

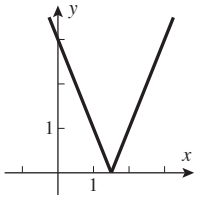
(c) 

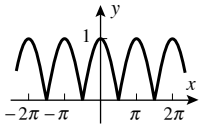
61. (a) even (b) odd

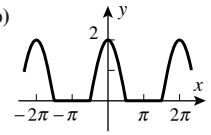
63. (a) even (b) odd (c) even (d) neither (e) odd (f) even

67. (a) y-axis
(b) origin
(c) x-axis, y-axis, origin

69. 

71. 

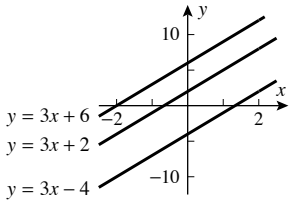
73. (a) 

(b) 

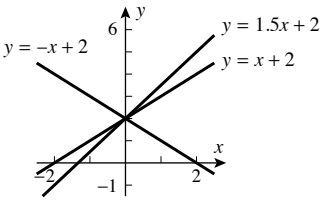
75. yes; $f(x) = x^k, g(x) = x^n$

Exercise Set 0.3 (Page 35)

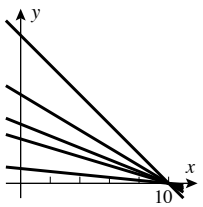
1. (a) $y = 3x + b$ (c)
(b) $y = 3x + 6$



3. (a) $y = mx + 2$ (c)
(b) $y = -x + 2$

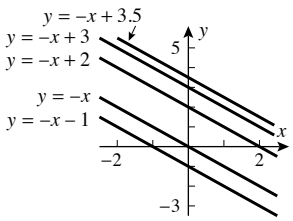


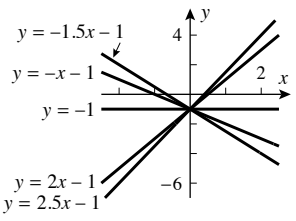
5. $y = \pm \dfrac{9 - x_0 x}{\sqrt{9 - x_0^2}}$

7. 

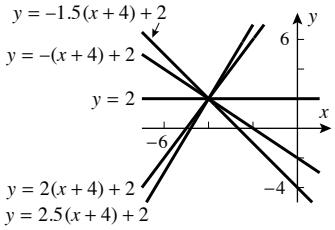
y-intercepts represent current value of item being depreciated.

9. (a) slope: -1 (b) y-intercept: $y = -1$

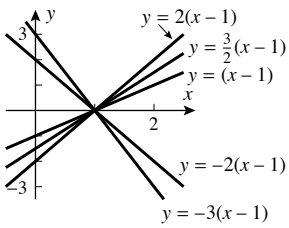




(c) pass through $(-4, 2)$

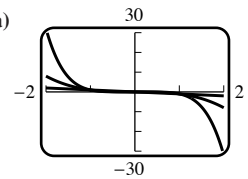


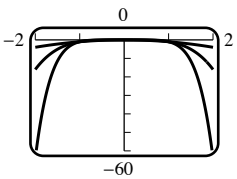
(d) x-intercept: $x = 1$



11. **(a)** VI **(b)** IV **(c)** III **(d)** V **(e)** I **(f)** II

13. **(a)**





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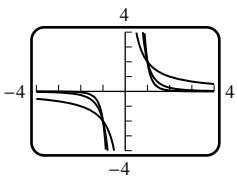
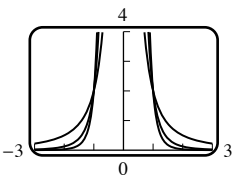
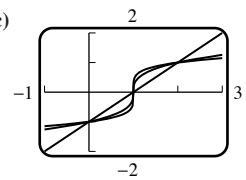
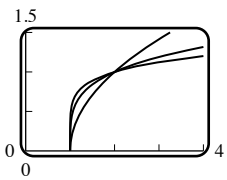


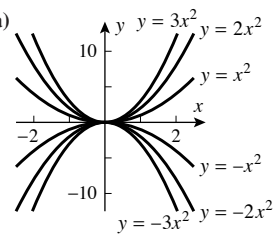


(c)





15. **(a)**



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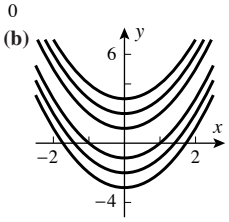

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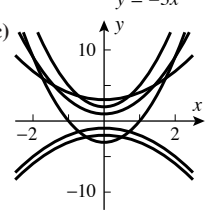
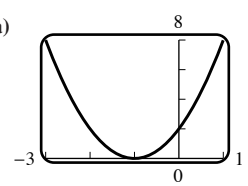
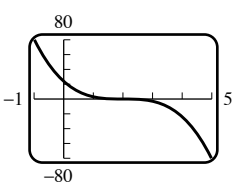


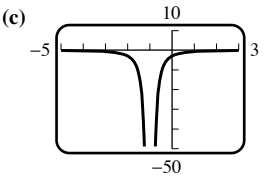
17. **(a)**



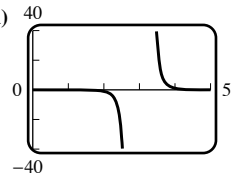
(b)



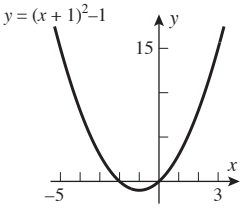
(c)



(d)



19.

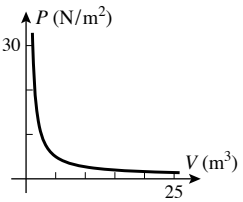


21. **(a)** newton-meters (N·m) **(b)** 20 N·m

(c)

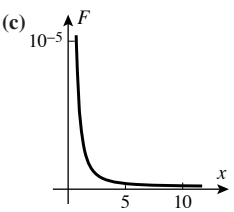
V (L)	0.25	0.5	1.0	1.5	2.0
P (N/m^2)	80×10^3	40×10^3	20×10^3	13.3×10^3	10×10^3

(d)



23. **(a)** $k = 0.000045$ N·m^2
(b) 0.000005 N
(d) The force becomes infinite; the force tends to zero.

(c)



Responses to True–False questions may be abridged to save space.

25. True; see Figure 0.3.2(b).

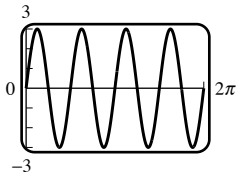
27. False; the constant of proportionality is $2 \cdot 6 = 12$.

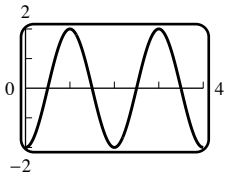
29. **(a)** II; $y = 1$, $x = -1, 2$ **(b)** I; $y = 0$, $x = -2, 3$ **(c)** IV; $y = 2$ **(d)** III; $y = 0$, $x = -2$

31. **(a)** $y = 3\sin(x/2)$ **(b)** $y = 4\cos 2x$ **(c)** $y = -5\sin 4x$

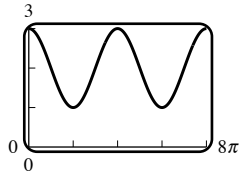
33. **(a)** $y = \sin[x + (\pi/2)]$ **(b)** $y = 3 + 3\sin(2x/9)$
(c) $y = 1 + 2\sin\left(2x - \dfrac{\pi}{2}\right)$

35. **(a)** amplitude $= 3$, period $= \pi/2$ **(b)** amplitude $= 2$, period $= 2$



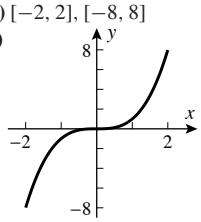


(c) amplitude $= 1$, period $= 4\pi$



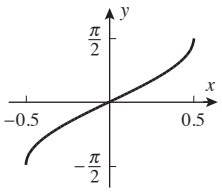
37. $x = \frac{5\sqrt{13}}{2}\sin\left(2\pi t + \tan^{-1}\frac{1}{2\sqrt{3}}\right)$

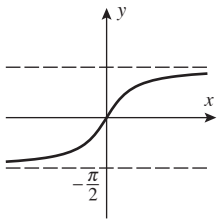
Exercise Set 0.4 (Page 48)

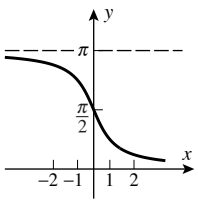
1. (a) yes (b) no (c) yes (d) no
3. (a) yes (b) yes (c) no (d) yes (e) no (f) no
5. (a) yes (b) no
7. (a) $8, -1, 0$
   (b) $[-2, 2], [-8, 8]$
   (c)


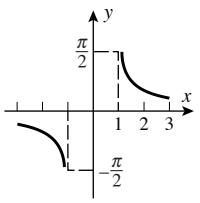
9. $\frac{1}{7}(x+6)$
11. $\sqrt[3]{(x+5)/3}$
13. $-\sqrt{3/x}$
15. $\begin{cases} (5/2)-x, & x > 1/2 \\ 1/x, & 0 < x \le 1/2 \end{cases}$
17. $x^{1/4} - 2$ for $x \ge 16$
19. $\frac{1}{2}(3 - x^2)$ for $x \le 0$
21. (a) $f^{-1}(x) = \dfrac{-b + \sqrt{b^2 - 4a(c-x)}}{2a}$
    (b) $f^{-1}(x) = \dfrac{-b - \sqrt{b^2 - 4a(c-x)}}{2a}$
23. (a) $y = (6.214 \times 10^{-4})x$ (b) $x = \dfrac{10^4}{6.214}y$
    (c) how many meters in y miles
25. (b) symmetric about the line $y = x$ 27. 10

Responses to True–False questions may be abridged to save space.

31. False; $f^{-1}(2) = 2$
33. True; see Theorem 0.4.3.
35. $\frac{4}{5}, \frac{3}{5}, \frac{3}{4}, \frac{5}{3}, \frac{5}{4}$
37. (a) $0 \le x \le \pi$ (b) $-1 \le x \le 1$ (c) $-\pi/2 < x < \pi/2$
    (d) $-\infty < x < +\infty$ 39. $\frac{24}{25}$
41. (a) $\dfrac{1}{\sqrt{1+x^2}}$ (b) $\dfrac{\sqrt{1-x^2}}{x}$ (c) $\dfrac{\sqrt{x^2-1}}{x}$ (d) $\dfrac{1}{\sqrt{x^2-1}}$
43. (a)


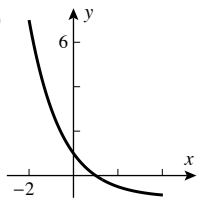
(b)


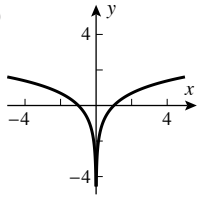
45. (a) $x = 2.76258$ rad (b) $\theta = 217.59°$
47. (a) 0.25545, error (b) $|x| \le \sin 1$
49. (a) $\cot^{-1}(x)$


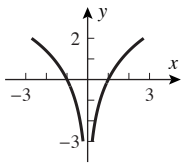
$\csc^{-1}(x)$


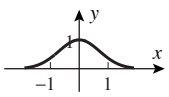
(b) $\cot^{-1} x$: all x, $0 < y < \pi$
$\csc^{-1} x$: $|x| \ge 1$, $0 < |y| < \pi/2$
51. (a) 55.0° (b) 33.6° (c) 25.8° 53. (a) 21.1 hours (b) 2.9 hours
55. 29°

Exercise Set 0.5 (Page 61)

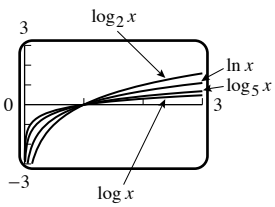
1. (a) -4 (b) 4 (c) $\frac{1}{4}$ 3. (a) 2.9690 (b) 0.0341
5. (a) 4 (b) -5 (c) 1 (d) $\frac{1}{2}$ 7. (a) 1.3655 (b) -0.3011
9. (a) $2r + \dfrac{s}{2} + \dfrac{t}{2}$ (b) $s - 3r - t$
11. (a) $1 + \log x + \frac{1}{2}\log(x-3)$ (b) $2\ln|x| + 3\ln\sin x - \frac{1}{2}\ln(x^2+1)$
13. $\log \frac{256}{3}$ 15. $\ln \dfrac{\sqrt[3]{x}(x+1)^2}{\cos x}$ 17. 0.01 19. e^2 21. 4
23. $\sqrt{3/2}$ 25. $-\dfrac{\ln 3}{2\ln 5}$ 27. $\frac{1}{3}\ln\frac{7}{2}$ 29. -2
31. (a) domain: $(-\infty, +\infty)$; range: $(-1, +\infty)$


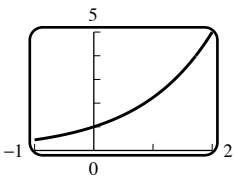
(b) domain: $x \ne 0$; range: $(-\infty, +\infty)$


33. (a) domain: $x \ne 0$; range: $(-\infty, +\infty)$


(b) domain: $(-\infty, +\infty)$; range: $(0, 1]$


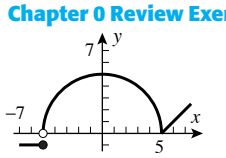
Responses to True–False questions may be abridged to save space.

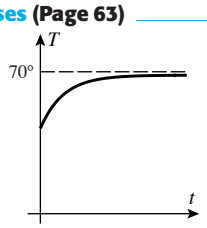
35. False; exponential functions have constant base and variable exponent.
37. True; $\ln x = \log_e x$ 39. 2.8777, -0.3174
41.


43. $x \approx 1.471, 7.857$
45. (a) no (d) $y = (\sqrt{5})^x$
    (b) $y = 2^{x/4}$
    (c) $y = 2^{-x}$


47. $\log\frac{1}{2} < 0$, so $3\log\frac{1}{2} < 2\log\frac{1}{2}$ 49. 201 days
51. (a) 7.4, basic (b) 4.2, acidic (c) 6.4, acidic (d) 5.9, acidic
53. (a) 140 dB, damage (b) 120 dB, damage (c) 80 dB, no damage
    (d) 75 dB, no damage
55. ≈ 200 57. (a) $\approx 5 \times 10^{16}$ J (b) ≈ 0.67

Chapter 0 Review Exercises (Page 63)

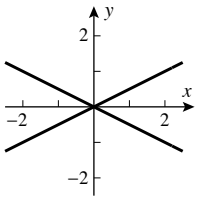
1.


3.


5. (a) $C = 5x^2 + (64/x)$ (b) $x > 0$

7. (a) $V = (6 - 2x)(5 - x)x$ ft^3
(b) $0 < x < 3$
(c) 3.57 ft × 3.79 ft × 1.21 ft

9.



11.

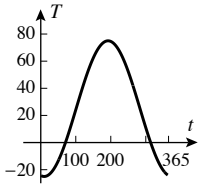
x	−4	−3	−2	−1	0	1	2	3	4
$f(x)$	0	−1	2	1	3	−2	−3	4	−4
$g(x)$	3	2	1	−3	−1	−4	4	−2	0
$(f \circ g)(x)$	4	−3	−2	−1	1	0	−4	2	3
$(g \circ f)(x)$	−1	−3	4	−4	−2	1	2	0	3

13. $0, -2$ **15.** $1/(2 - x^2), x \neq \pm 1, \pm\sqrt{2}$

17. (a) odd (b) even (c) neither (d) even

19. (a) circles of radius 1 centered on the parabola $y = x^2$
(b) parabolas congruent to $y = x^2$ that open up with vertices on the line $y = x/2$

21. (a)



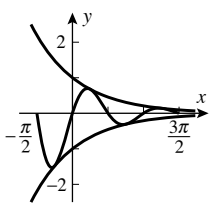
(b) January 11 (c) 122 days

23. A: $\left(-\frac{2}{3}\pi, 1 - \sqrt{3}\right)$; B: $\left(\frac{1}{3}\pi, 1 + \sqrt{3}\right)$; C: $\left(\frac{2}{3}\pi, 1 + \sqrt{3}\right)$; D: $\left(\frac{5}{3}\pi, 1 - \sqrt{3}\right)$

27. (a) $\frac{1}{2}(x + 1)^{1/3}$ (b) none (c) $\frac{1}{2}\ln(x - 1)$ (d) $\dfrac{x + 2}{x - 1}$
(e) $\dfrac{1}{2 + \sin^{-1} x}$ (f) $\tan\left(\dfrac{1}{3x} - \dfrac{1}{3}\right), x < -\dfrac{2}{3\pi - 2}$ or $x > \dfrac{2}{3\pi + 2}$

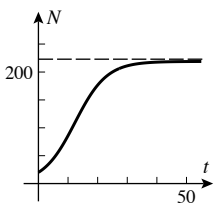
29. (a) $\frac{33}{65}$ (b) $\frac{56}{65}$ **31.** $\frac{10^{60}}{63360} \approx 1.6 \times 10^{55}$ miles **33.** $15x + 2$

35. (a)



(b) $-\frac{\pi}{2}, 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$; $-\frac{\pi}{4}, \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}$

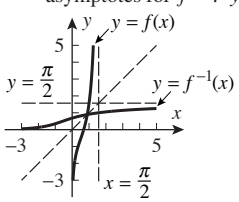
37. (a)



(b) about 10 years (c) 220 sheep

39. (b) 3.654, 332105.108

41. (a) f is increasing (b) asymptotes for f: $x = 0$ and $x = \pi/2$; asymptotes for f^{-1}: $y = 0$ (as $x \to -\infty$) and $y = \pi/2$ (as $x \to +\infty$)



Exercise Set 1.1 (Page 77)

1. (a) 3 (b) 3 (c) 3 (d) 3

3. (a) −1 (b) 3 (c) does not exist (d) 1

5. (a) 0 (b) 0 (c) 0 (d) 3 **7.** (a) $-\infty$ (b) $-\infty$ (c) $-\infty$ (d) 1

9. (a) $+\infty$ (b) $+\infty$ (c) 2 (d) 2 (e) $-\infty$ (f) the lines $x = -2$, $x = 0$, $x = 2$

11.

x	−0.01	−0.001	−0.0001	0.0001	0.001	0.01
$f(x)$	0.99502	0.99950	0.99995	1.00005	1.00050	1.00502

The limit appears to be 1.

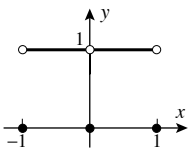
13. (a) $\frac{1}{3}$ (b) $+\infty$ (c) $-\infty$ **15.** (a) 3 (b) does not exist

Responses to True–False questions may be abridged to save space.

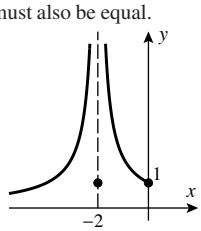
17. False; see Example 6.

19. False; the one-sided limits must also be equal.

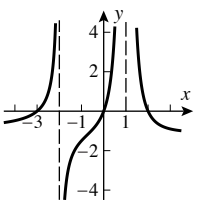
21.



23.



25.



27. $y = -2x - 1$

29. $y = 4x - 3$

31. (a) rest length (b) 0. As speed approaches c, length shrinks to zero.

Exercise Set 1.2 (Page 87)

1. (a) −6 (b) 13 (c) −8 (d) 16 (e) 2 (f) $-\frac{1}{2}$

3. 6 **5.** $\frac{3}{4}$ **7.** 4 **9.** $-\frac{4}{5}$ **11.** −3 **13.** $\frac{3}{2}$ **15.** $+\infty$

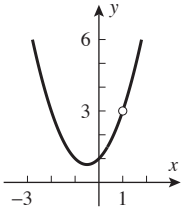
17. does not exist **19.** $-\infty$ **21.** $+\infty$ **23.** does not exist **25.** $+\infty$

27. $+\infty$ **29.** 6 **31.** (a) 2 (b) 2 (c) 2

Responses to True–False questions may be abridged to save space.

33. True; this is Theorem 1.2.2(a). **35.** False; see Example 9. **37.** $\frac{1}{4}$

39. (a) 3 (b)



41. (a) Theorem 1.2.2(a) does not apply.
(b) $\lim\limits_{x \to 0^+} \left(\dfrac{1}{x} - \dfrac{1}{x^2}\right) = \lim\limits_{x \to 0^+} \left(\dfrac{x - 1}{x^2}\right) = -\infty$ **43.** $a = 2$

45. The left and/or right limits could be $\pm\infty$; or the limit could exist and equal any preassigned real number.

Exercise Set 1.3 (Page 96)

1. (a) $-\infty$ (b) $+\infty$ **3.** (a) 0 (b) −1

5. (a) −12 (b) 21 (c) −15 (d) 25 (e) 2 (f) $-\frac{3}{5}$ (g) 0
(h) does not exist

7. (a)

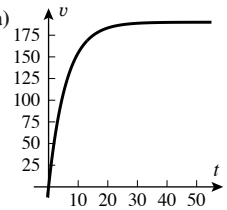
x	0.1	0.01	0.001	0.0001	0.00001	0.000001
$f(x)$	1.471128	1.560797	1.569796	1.570696	1.570786	1.570795

The limit appears to be $\pi/2$. (b) $\pi/2$

9. $-\infty$ **11.** $+\infty$ **13.** $\frac{3}{2}$ **15.** 0 **17.** 0 **19.** $-\infty$ **21.** $-\frac{1}{7}$
23. $\frac{-\sqrt[3]{5}}{2}$ **25.** $-\sqrt{5}$ **27.** $1/\sqrt{6}$ **29.** $\sqrt{3}$ **31.** 0 **33.** 1
35. 1 **37.** $-\infty$ **39.** e

Responses to True–False questions may be abridged to save space.

41. False; 1^∞ is an indeterminate form. The limit is e^2.
43. True; consider $f(x) = (\sin x)/x$.
45. $\lim_{t\to+\infty} n(t) = +\infty$; $\lim_{t\to+\infty} e(t) = c$ **47.** **(a)** $+\infty$ **(b)** -5
51. **(a)** no **(b)** yes; $\tan x$ and $\sec x$ at $x = n\pi + \pi/2$, and $\cot x$ and $\csc x$ at $x = n\pi$, $n = 0, \pm1, \pm2, \ldots$
55. $+\infty$ **57.** $+\infty$ **59.** 1 **61.** e
65. **(a)**

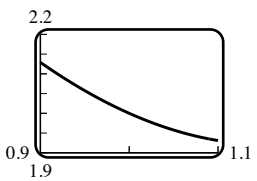


(b) $c = 190$
(c) It is the terminal velocity of the skydiver.

67. **(a)** e **(c)** e^a **69.** $x + 2$ **71.** $1 - x^2$ **73.** $\sin x$

Exercise Set 1.4 (Page 106)

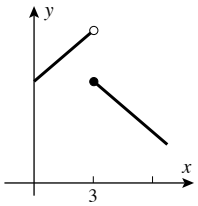
1. **(a)** $|x| < 0.1$ **(b)** $|x - 3| < 0.0025$ **(c)** $|x - 4| < 0.000125$
3. **(a)** $x_0 = 3.8025$, $x_1 = 4.2025$ **(b)** $\delta = 0.1975$
5. $\delta = 0.0442$ **7.** $\delta = 0.13$ **9.** $\delta = 0.05$ **11.** $\delta = 0.05$



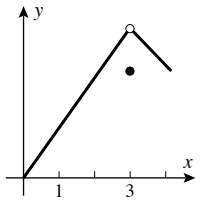
13. $\delta = \sqrt[3]{8.001} - 2 \approx 8.332986 \cdot 10^{-5}$ **15.** $\delta = 1/505 \approx 0.000198$
17. $\delta = 1$ **19.** $\delta = \frac{1}{3}\epsilon$ **21.** $\delta = \epsilon/2$ **23.** $\delta = \epsilon$ **25.** $\delta = \epsilon$
29. **(b)** 65 **(c)** $\epsilon/65$; 65; 65; $\epsilon/65$ **31.** $\delta = \min\left(1, \frac{1}{6}\epsilon\right)$
33. $\delta = \min\left(\frac{1}{2}, \frac{\epsilon}{2}\right)$ **35.** $\delta = 2\epsilon$
39. **(a)** $\sqrt{10}$ **(b)** 99 **(c)** -10 **(d)** -101
41. **(a)** $-\sqrt{\frac{1-\epsilon}{\epsilon}}$; $\sqrt{\frac{1-\epsilon}{\epsilon}}$ **(b)** $\sqrt{\frac{1-\epsilon}{\epsilon}}$ **(c)** $-\sqrt{\frac{1-\epsilon}{\epsilon}}$
43. 10 **45.** 999 **47.** -202 **49.** -57.5 **51.** $N = \dfrac{1}{\sqrt{\epsilon}}$
53. $N = -\dfrac{5}{2} - \dfrac{11}{2\epsilon}$ **55.** $N = (1 + 2/\epsilon)^2$
57. **(a)** $|x| < \frac{1}{10}$ **(b)** $|x - 1| < \frac{1}{1000}$
(c) $|x - 3| < \frac{1}{10\sqrt{10}}$ **(d)** $|x| < \frac{1}{10}$
59. $\delta = 1/\sqrt{M}$ **61.** $\delta = 1/M$ **63.** $\delta = 1/(-M)^{1/4}$ **65.** $\delta = \epsilon$
67. $\delta = \epsilon^2$ **69.** $\delta = \epsilon$ **71.** **(a)** $\delta = -1/M$ **(b)** $\delta = 1/M$
73. **(a)** $N = M - 1$ **(b)** $N = M - 1$
75. **(a)** 0.4 amps **(b)** about 0.39474 to 0.40541 amps **(c)** $3/(7.5 + \delta)$ to $3/(7.5 - \delta)$ **(d)** $\delta \approx 0.01870$ **(e)** current approaches $+\infty$

Exercise Set 1.5 (Page 118)

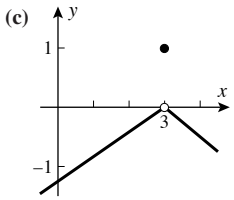
1. **(a)** not continuous, $x = 2$ **(b)** not continuous, $x = 2$
(c) not continuous, $x = 2$ **(d)** continuous **(e)** continuous
(f) continuous
3. **(a)** not continuous, $x = 1, 3$ **(b)** continuous
(c) not continuous, $x = 1$ **(d)** continuous
(e) not continuous, $x = 3$ **(f)** continuous
5. **(a)** no **(b)** no **(c)** no **(d)** yes **(e)** yes **(f)** no **(g)** yes
7. **(a)**



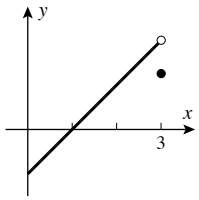
(b)



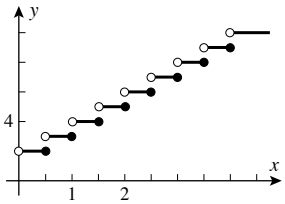
(c)



(d)



9. **(a)**

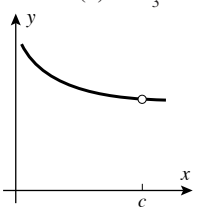


(b) One second could cost you one dollar.

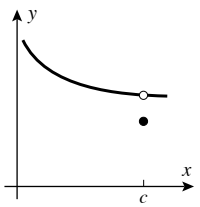
11. none **13.** none **15.** $-1/2, 0$ **17.** $-1, 0, 1$ **19.** none
21. none

Responses to True–False questions may be abridged to save space.

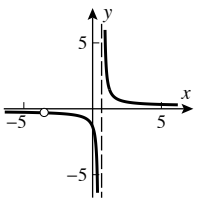
23. True; the composition of continuous functions is continuous.
25. False; let f and g be the functions in Exercise 6.
27. True; $f(x) = \sqrt{f(x)} \cdot \sqrt{f(x)}$
29. **(a)** $k = 5$ **(b)** $k = \frac{4}{3}$ **31.** $k = 4$, $m = 5/3$
33. **(a)**



(b)



35. **(a)** $x = 0$, not removable **(b)** $x = -3$, removable
(c) $x = 2$, removable; $x = -2$, not removable
37. **(a)** $x = \frac{1}{2}$, not removable; at $x = -3$, removable
(b) $(2x - 1)(x + 3)$



45. $f(x) = 1$ for $0 \le x < 1$, $f(x) = -1$ for $1 \le x \le 2$
49. $x = -1.25$, $x = 0.75$ **51.** $x = 2.24$

Exercise Set 1.6 (Page 125)

1. none **3.** $x = n\pi$, $n = 0, \pm1, \pm2, \ldots$
5. $x = n\pi$, $n = 0, \pm1, \pm2, \ldots$
7. $2n\pi + (\pi/6)$, $2n\pi + (5\pi/6)$, $n = 0, \pm1, \pm2, \ldots$
9. $\left[-\frac{1}{2}, \frac{1}{2}\right]$ **11.** $(0, 3)$ and $(3, +\infty)$ **13.** $(-\infty, -1]$ and $[1, +\infty)$
15. **(a)** $\sin x$, $x^3 + 7x + 1$ **(b)** $|x|$, $\sin x$ **(c)** x^3, $\cos x$, $x + 1$
17. 1 **19.** $-\pi/6$ **21.** 1 **23.** 3 **25.** $+\infty$ **27.** $\frac{7}{3}$
29. 0 **31.** 0 **33.** 1 **35.** 2 **37.** does not exist **39.** 0

41. **(a)**

x	4	4.5	4.9	5.1	5.5	6
$f(x)$	0.093497	0.100932	0.100842	0.098845	0.091319	0.076497

The limit appears to be $\frac{1}{10}$. **(b)** $\frac{1}{10}$

Responses to True–False questions may be abridged to save space.

43. True; use the Squeezing Theorem.

45. False; consider $f(x) = \tan^{-1} x$.

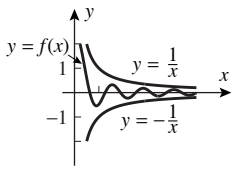
47. **(a)** Using degrees instead of radians **(b)** $\pi/180$

49. $k = \frac{1}{2}$ **51.** **(a)** 1 **(b)** 0 **(c)** 1

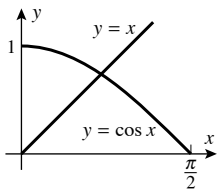
53. $-\pi$ **55.** $-\sqrt{2}$ **57.** 1 **59.** 5 **61.** $-|x| \le x\cos(50\pi/x) \le |x|$

63. $\lim_{x\to 0} \sin(1/x)$ does not exist

65. The limit is 0.

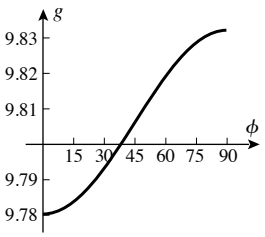


67. **(b)**



(c) 0.739

69. **(a)** Gravity is strongest at the poles and weakest at the equator.



Chapter 1 Review Exercises (Page 128)

1. **(a)** 1 **(b)** does not exist **(c)** does not exist **(d)** 1 **(e)** 3 **(f)** 0 **(g)** 0 **(h)** 2 **(i)** $\frac{1}{2}$

3. **(a)** 0.405 **5.** 1 **7.** $-3/2$ **9.** $32/3$

11. **(a)** $y = 0$ **(b)** none **(c)** $y = 2$ **13.** 1 **15.** $3 - k$ **17.** 0

19. e^{-3} **21.** \$2001.60, \$2009.66, \$2013.62, \$2013.75

23. **(a)** $2x/(x-1)$ is one example.

25. **(a)** $\lim_{x\to 2} f(x) = 5$ **(b)** $\delta = 0.0045$

27. **(a)** $\delta = 0.0025$ **(b)** $\delta = 0.0025$ **(c)** $\delta = 1/9000$ (Some larger values also work.)

31. **(a)** $-1, 1$ **(b)** none **(c)** $-3, 0$ **33.** no; not continuous at $x = 2$

35. Consider $f(x) = x$ for $x \neq 0$, $f(0) = 1$, $a = -1$, $b = 1$, $k = 0$.

Chapter 1 Making Connections (Page 130)

Where correct answers to a Making Connections exercise may vary, no answer is listed. Sample answers for these questions are available on the Book Companion Site.

4. **(a)** The circle through the origin with center $\left(0, \frac{1}{8}\right)$

(b) The circle through the origin with center $\left(0, \frac{1}{2}\right)$

(c) The circle does not exist.

(d) The circle through the origin with center $\left(0, \frac{1}{2}\right)$

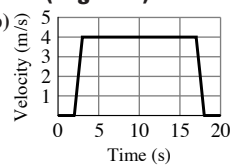
(e) The circle through $(0, 1)$ with center at the origin.

(f) The circle through the origin with center $\left(0, \dfrac{1}{2g(0)}\right)$

(g) The circle does not exist.

Exercise Set 2.1 (Page 140)

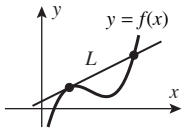
1. **(a)** 4 m/s **(b)**

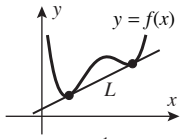


3. **(a)** 0 cm/s **(b)** $t = 0$, $t = 2$, and $t = 4.2$ **(c)** maximum: $t = 1$; minimum: $t = 3$ **(d)** -7.5 cm/s

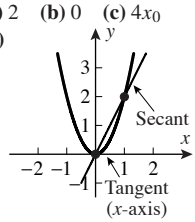
5. straight line with slope equal to the velocity

7. Answers may vary. **9.** Answers may vary.

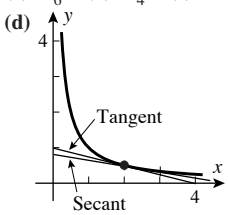




11. **(a)** 2 **(b)** 0 **(c)** $4x_0$ **(d)**



13. **(a)** $-\frac{1}{6}$ **(b)** $-\frac{1}{4}$ **(c)** $-1/x_0^2$ **(d)**



15. **(a)** $2x_0$ **(b)** -2 **17.** **(a)** $1 + \dfrac{1}{2\sqrt{x_0}}$ **(b)** $\frac{3}{2}$

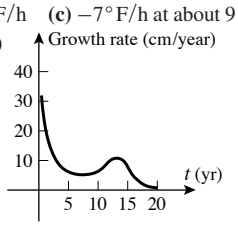
Responses to True–False questions may be abridged to save space.

19. True; set $h = x - 1$, so $x = 1 + h$ and $h \to 0$ is equivalent to $x \to 1$.

21. False; velocity is a ratio of change in position to change in time.

23. **(a)** 72°F at about 4:30 P.M. **(b)** 4°F/h **(c)** -7°F/h at about 9 P.M.

25. **(a)** first year
(b) 6 cm/year
(c) 10 cm/year at about age 14
(d)



27. **(a)** 19,200 ft **(b)** 480 ft/s **(c)** 66.94 ft/s **(d)** 1440 ft/s

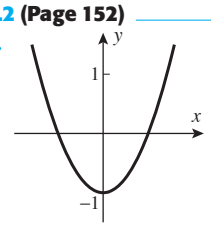
29. **(a)** 720 ft/min **(b)** 192 ft/min

Exercise Set 2.2 (Page 152)

1. $2, 0, -2, -1$

3. **(b)** 3 **(c)** 3

5.



7. $y = 5x - 16$

9. $4x$, $y = 4x - 2$

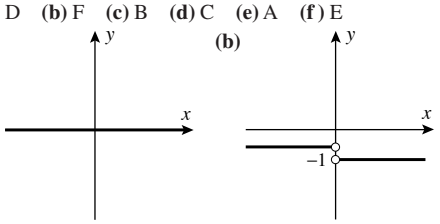
11. $3x^2$; $y = 0$

13. $\dfrac{1}{2\sqrt{x+1}}$; $y = \frac{1}{6}x + \frac{5}{3}$ **15.** $-1/x^2$ **17.** $2x - 1$

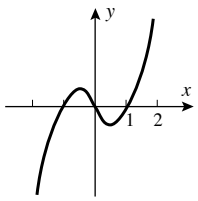
19. $-1/(2x^{3/2})$ **21.** $8t + 1$

23. **(a)** D **(b)** F **(c)** B **(d)** C **(e)** A **(f)** E

25. **(a)** **(b)**



(c)

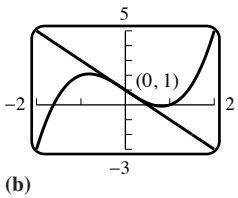


Responses to True–False questions may be abridged to save space.

27. False; $f'(a) = 0$ **29.** False; for example, $f(x) = |x|$

31. **(a)** $\sqrt{x}$, 1 **(b)** x^2, 3 **33.** -2

35. $y = -2x + 1$



37. **(b)**

w	1.5	1.1	1.01	1.001	1.0001	1.00001
$[f(w)-f(1)]/(w-1)$	1.6569	1.4355	1.3911	1.3868	1.3863	1.3863

39. **(a)** 0.04, 0.22, 0.88 **(b)** best: $\dfrac{f(2)-f(0)}{2-0}$; worst: $\dfrac{f(3)-f(1)}{3-1}$

41. **(a)** dollars per foot **(b)** the price per additional foot **(c)** positive **(d)** \$1000

43. **(a)** $F \approx 200$ lb, $dF/d\theta \approx 50$ lb/rad **(b)** $\mu = 0.25$

45. **(a)** $T \approx 115°$F, $dT/dt \approx -3.35°$F/min **(b)** $k = -0.084$

Exercise Set 2.3 (Page 161)

1. $28x^6$ **3.** $24x^7 + 2$ **5.** 0 **7.** $-\frac{1}{3}(7x^6 + 2)$

9. $-3x^{-4} - 7x^{-8}$ **11.** $24x^{-9} + (1/\sqrt{x})$

13. $f'(x) = ex^{e-1} - \dfrac{\sqrt{10}}{x^{(1+\sqrt{10})}}$

15. $12x(3x^2 + 1)$ **17.** 7 **19.** $2t - 1$ **21.** 15 **23.** -8 **25.** 0

27. 0 **29.** $32t$ **31.** $3\pi r^2$

Responses to True–False questions may be abridged to save space.

33. True; apply the difference and constant multiple rules.

35. False; $\dfrac{d}{dx}[4f(x) + x^3]\Big|_{x=2} = [4f'(x) + 3x^2]\Big|_{x=2} = 32$

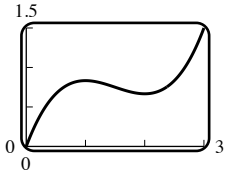
37. **(a)** $4\pi r^2$ **(b)** 100π **39.** $y = 5x + 17$

41. **(a)** $42x - 10$ **(b)** 24 **(c)** $2/x^3$ **(d)** $700x^3 - 96x$

43. **(a)** $-210x^{-8} + 60x^2$ **(b)** $-6x^{-4}$ **(c)** $6a$

45. **(a)** 0 **(b)** 112 **(c)** 360

49. $(1, \frac{5}{6})$ $(2, \frac{2}{3})$

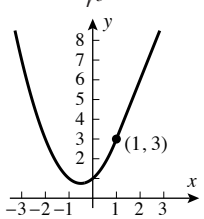


51. $y = 3x^2 - x - 2$ **53.** $x = \frac{1}{2}$

55. $(2 + \sqrt{3}, -6 - 4\sqrt{3})$, $(2 - \sqrt{3}, -6 + 4\sqrt{3})$

57. $-2x_0$ **61.** $-\dfrac{2GmM}{r^3}$ **63.** $f'(x) > 0$ for all $x \neq 0$

65. yes, 3



67. not differentiable at $x = 1$ **69.** **(a)** $x = \frac{2}{3}$ **(b)** $x = \pm 2$ **71.** **(b)** yes

73. **(a)** $n(n-1)(n-2)\cdots 1$ **(b)** 0 **(c)** $a_n n(n-1)(n-2)\cdots 1$

79. $-12/(2x+1)^3$ **81.** $-2/(x+1)^3$

Exercise Set 2.4 (Page 168)

1. $4x + 1$ **3.** $4x^3$ **5.** $18x^2 - \frac{3}{2}x + 12$

7. $-15x^{-2} - 14x^{-3} + 48x^{-4} + 32x^{-5}$ **9.** $3x^2$

11. $\dfrac{-3x^2 - 8x + 3}{(x^2+1)^2}$ **13.** $\dfrac{3x^2 - 8x}{(3x-4)^2}$ **15.** $\dfrac{x^{3/2} + 10x^{1/2} + 4 - 3x^{-1/2}}{(x+3)^2}$

17. $2(1 + x^{-1})(x^{-3} + 7) + (2x + 1)(-x^{-2})(x^{-3} + 7) + (2x + 1)(1 + x^{-1})(-3x^{-4})$ **19.** $3(7x^6 + 2)(x^7 + 2x - 3)^2$ **21.** $\frac{7}{16}$

23. -29 **25.** 0 **27.** **(a)** $-\frac{37}{4}$ **(b)** $-\frac{23}{16}$

29. **(a)** 10 **(b)** 19 **(c)** 9 **(d)** -1 **31.** $-2 \pm \sqrt{3}$ **33.** none **35.** -2

39. $F''(x) = xf''(x) + 2f'(x)$

41. $R'(120) = 1800$; increasing the price by Δp dollars increases revenue by approximately $1800\Delta p$ dollars.

43. $f'(x) = -nx^{-n-1}$

Exercise Set 2.5 (Page 172)

1. $-4\sin x + 2\cos x$ **3.** $4x^2 \sin x - 8x\cos x$

5. $(1 + 5\sin x - 5\cos x)/(5 + \sin x)^2$ **7.** $\sec x \tan x - \sqrt{2}\sec^2 x$

9. $-4\csc x \cot x + \csc^2 x$ **11.** $\sec^3 x + \sec x \tan^2 x$ **13.** $-\dfrac{\csc x}{1 + \csc x}$

15. 0 **17.** $\dfrac{1}{(1 + x\tan x)^2}$ **19.** $-x\cos x - 2\sin x$

21. $-x\sin x + 5\cos x$ **23.** $-4\sin x \cos x$

25. **(a)** $y = x$ **(b)** $y = 2x - (\pi/2) + 1$ **(c)** $y = 2x + (\pi/2) - 1$

29. **(a)** $x = \pm\pi/2, \pm 3\pi/2$ **(b)** $x = -3\pi/2, \pi/2$ **(c)** no horizontal tangent line **(d)** $x = \pm 2\pi, \pm\pi, 0$

31. 0.087 ft/deg **33.** 1.75 m/deg

Responses to True–False questions may be abridged to save space.

35. False; by the product rule, $g'(x) = f(x)\cos x + f'(x)\sin x$.

37. True; $f(x) = (\sin x)/(\cos x) = \tan x$, so $f'(x) = \sec^2 x$.

39. $-\cos x$ **41.** 3, 7, 11, . . .

43. **(a)** all x **(b)** all x **(c)** $x \neq (\pi/2) + n\pi$, $n = 0, \pm 1, \pm 2, \ldots$ **(d)** $x \neq n\pi$, $n = 0, \pm 1, \pm 2, \ldots$ **(e)** $x \neq (\pi/2) + n\pi$, $n = 0, \pm 1, \pm 2, \ldots$ **(f)** $x \neq n\pi$, $n = 0, \pm 1, \pm 2, \ldots$ **(g)** $x \neq (2n+1)\pi$, $n = 0, \pm 1, \pm 2, \ldots$ **(h)** $x \neq n\pi/2$, $n = 0, \pm 1, \pm 2, \ldots$ **(i)** all x

Exercise Set 2.6 (Page 178)

1. 6 **3.** **(a)** $(2x-3)^5$, $10(2x-3)^4$ **(b)** $2x^5 - 3$, $10x^4$

5. **(a)** -7 **(b)** -8 **7.** $37(x^3 + 2x)^{36}(3x^2 + 2)$

9. $-2\left(x^3 - \dfrac{7}{x}\right)^{-3}\left(3x^2 + \dfrac{7}{x^2}\right)$ **11.** $\dfrac{24(1-3x)}{(3x^2 - 2x + 1)^4}$

13. $\dfrac{3}{4\sqrt{3x}\sqrt{4 + \sqrt{3x}}}$ **15.** $-\dfrac{2}{x^3}\cos\left(\dfrac{1}{x^2}\right)$ **17.** $-20\cos^4 x \sin x$

19. $-\dfrac{3}{\sqrt{x}}\cos(3\sqrt{x})\sin(3\sqrt{x})$ **21.** $28x^6 \sec^2(x^7)\tan(x^7)$

23. $-\dfrac{5\sin(5x)}{2\sqrt{\cos(5x)}}$

25. $-3[x + \csc(x^3 + 3)]^{-4}[1 - 3x^2\csc(x^3 + 3)\cot(x^3 + 3)]$

27. $10x^3 \sin 5x \cos 5x + 3x^2 \sin^2 5x$

29. $-x^3 \sec\left(\dfrac{1}{x}\right)\tan\left(\dfrac{1}{x}\right) + 5x^4 \sec\left(\dfrac{1}{x}\right)$

31. $\sin(\cos x)\sin x$ **33.** $-6\cos^2(\sin 2x)\sin(\sin 2x)\cos 2x$

35. $35(5x + 8)^6(1 - \sqrt{x})^6 - \dfrac{3}{\sqrt{x}}(5x + 8)^7(1 - \sqrt{x})^5$

37. $\dfrac{33(x-5)^2}{(2x+1)^4}$ **39.** $-\dfrac{2(2x+3)^2(52x^2 + 96x + 3)}{(4x^2 - 1)^9}$

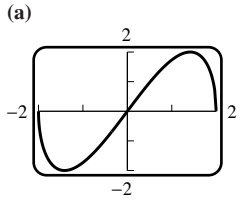
41. $5[x\sin 2x + \tan^4(x^7)]^4[2x\cos 2x + \sin 2x + 28x^6 \tan^3(x^7)\sec^2(x^7)]$

43. $y = -x$ **45.** $y = -1$ **47.** $y = 8\sqrt{\pi}x - 8\pi$ **49.** $y = \frac{7}{2}x - \frac{3}{2}$

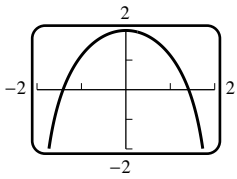
51. $-25x\cos(5x) - 10\sin(5x) - 2\cos(2x)$ **53.** $4(1 - x)^{-3}$

55. $3\cot^2\theta\csc^2\theta$ **57.** $\pi(b - a)\sin 2\pi\omega$

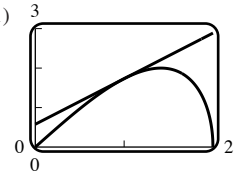
59. **(a)**



(c) $\dfrac{4-2x^2}{\sqrt{4-x^2}}$



(d) $y-\sqrt{3}=\frac{2}{\sqrt{3}}(x-1)$



Responses to True–False questions may be abridged to save space.

61. False; by the chain rule, $\dfrac{d}{dx}[\sqrt{y}]=\dfrac{1}{2\sqrt{y}}\cdot\dfrac{dy}{dx}=\dfrac{f'(x)}{2\sqrt{f(x)}}$.

63. False; by the chain rule, $dy/dx=(-\sin[g(x)])\cdot g'(x)$.

65. **(c)** $f=1/T$ **(d)** amplitude $=0.6$ cm, $T=2\pi/15$ seconds per oscillation, $f=15/(2\pi)$ oscillations per second

67. $\frac{7}{24}\sqrt{6}$ **69.** **(a)** 10 lb/in^2, -2 lb/in^2/mi **(b)** -0.6 lb/in^2/s

71. $\begin{cases}\cos x, & 0<x<\pi\\ -\cos x, & -\pi<x<0\end{cases}$

73. **(c)** $-\dfrac{1}{x}\cos\dfrac{1}{x}+\sin\dfrac{1}{x}$ **(d)** limit as x goes to 0 does not exist

75. **(a)** 21 **(b)** -36 **77.** $1/(2x)$ **79.** $\frac{2}{3}x$ **83.** $f'(g(h(x)))g'(h(x))h'(x)$

Chapter 2 Review Exercises (Page 181)

3. **(a)** $2x$ **(b)** 4 **5.** 58.75 ft/s **7.** **(a)** 13 mi/h **(b)** 7 mi/h

9. **(a)** $-2/\sqrt{9-4x}$ **(b)** $1/(x+1)^2$

11. **(a)** $x=-2,-1,1,3$ **(b)** $(-\infty,-2),(-1,1),(3,+\infty)$ **(c)** $(-2,-1),(1,3)$ **(d)** 4

13. **(a)** 78 million people per year **(b)** 1.3% per year

15. **(a)** $x^2\cos x+2x\sin x$ **(c)** $4x\cos x+(2-x^2)\sin x$

17. **(a)** $(6x^2+8x-17)/(3x+2)^2$ **(c)** $118/(3x+2)^3$

19. **(a)** 2000 gal/min **(b)** 2500 gal/min **21.** **(a)** 3.6 **(b)** -0.777778

23. $f(1)=0$, $f'(1)=5$ **25.** $y=-16x$, $y=-145x/4$

29. **(a)** $8x^7-\dfrac{3}{2\sqrt{x}}-15x^{-4}$ **(b)** $(2x+1)^{100}(1030x^2+10x-1414)$

31. **(a)** $\dfrac{(x-1)(15x+1)}{2\sqrt{3x+1}}$ **(b)** $-3(3x+1)^2(3x+2)/x^7$

33. $x=-\frac{7}{2},-\frac{1}{2},2$ **35.** $y=\pm 2x$

37. $x=n\pi\pm(\pi/4)$, $n=0,\pm1,\pm2,\ldots$ **39.** $y=-3x+(1+9\pi/4)$

41. **(a)** $40\sqrt{3}$ **(b)** 7500

Chapter 2 Making Connections (Page 184)

Where correct answers to a Making Connections exercise may vary, no answer is listed. Sample answers for these questions are available on the Book Companion Site.

2. **(c)** $k=2$ **(d)** $h'(x)=0$

3. **(b)** $f'\cdot g\cdot h\cdot k+f\cdot g'\cdot h\cdot k+f\cdot g\cdot h'\cdot k+f\cdot g\cdot h\cdot k'$

4. **(c)** $\dfrac{f'\cdot g\cdot h-f\cdot g'\cdot h+f\cdot g\cdot h'}{g^2}$

Exercise Set 3.1 (Page 190)

1. **(a)** $(6x^2-y-1)/x$ **(b)** $4x-2/x^2$ **3.** $-\dfrac{x}{y}$ **5.** $\dfrac{1-2xy-3y^3}{x^2+9xy^2}$

7. $\dfrac{-y^{3/2}}{x^{3/2}}$ **9.** $\dfrac{1-2xy^2\cos(x^2y^2)}{2x^2y\cos(x^2y^2)}$

11. $\dfrac{1-3y^2\tan^2(xy^2+y)\sec^2(xy^2+y)}{3(2xy+1)\tan^2(xy^2+y)\sec^2(xy^2+y)}$ **13.** $-\dfrac{8}{9y^3}$ **15.** $\dfrac{2y}{x^2}$

17. $\dfrac{\sin y}{(1+\cos y)^3}$ **19.** $-1/\sqrt{3},1/\sqrt{3}$

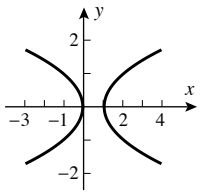
Responses to True–False questions may be abridged to save space.

21. False; the graph of f need only coincide with a portion of the graph of the equation in x and y.

23. False; the equation is equivalent to $x^2=y^2$ and $y=|x|$ satisfies this equation.

25. $-15^{-3/4}\approx-0.1312$ **27.** $-\frac{9}{13}$ **29.** $\dfrac{2t^3+3a^2}{2a^3-6at}$ **31.** $-\dfrac{b^2\lambda}{a^2\omega}$

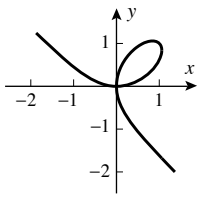
35. **(a)**



(c) $x=-y^2$ or $x=y^2+1$

37. $a=\frac{1}{4}$, $b=\frac{5}{4}$

41. **(a)**



(c) $2\sqrt[3]{2}/3$ **43.** $-\dfrac{2y^3+3t^2y}{(6ty^2+t^3)\cos t}$

Exercise Set 3.2 (Page 195)

1. $1/x$ **3.** $1/(1+x)$ **5.** $2x/(x^2-1)$ **7.** $\dfrac{1-x^2}{x(1+x^2)}$ **9.** $2/x$

11. $\dfrac{1}{2x\sqrt{\ln x}}$ **13.** $1+\ln x$ **15.** $2x\log_2(3-2x)-\dfrac{2x^2}{(\ln 2)(3-2x)}$

17. $\dfrac{2x(1+\log x)-x/(\ln 10)}{(1+\log x)^2}$ **19.** $1/(x\ln x)$ **21.** $2\csc 2x$

23. $-\dfrac{1}{x}\sin(\ln x)$ **25.** $2\cot x/(\ln 10)$ **27.** $\dfrac{3}{x-1}+\dfrac{8x}{x^2+1}$

29. $-\tan x+\dfrac{3x}{4-3x^2}$

Responses to True–False questions may be abridged to save space.

31. True; $\lim\limits_{x\to 0^+}\dfrac{1}{x}=+\infty$ **33.** True; $1/x$ is an odd function.

35. $x\sqrt[3]{1+x^2}\left[\dfrac{1}{x}+\dfrac{2x}{3(1+x^2)}\right]$

37. $\dfrac{(x^2-8)^{1/3}\sqrt{x^3+1}}{x^6-7x+5}\left[\dfrac{2x}{3(x^2-8)}+\dfrac{3x^2}{2(x^3+1)}-\dfrac{6x^5-7}{x^6-7x+5}\right]$

39. **(a)** $-\dfrac{1}{x(\ln x)^2}$ **(b)** $-\dfrac{\ln 2}{x(\ln x)^2}$

41. $y=ex-2$ **43.** $y=-x/e$ **45.** **(a)** $y=x/e$ **47.** $A(w)=w/2$

51. $f(x)=\ln(x+1)$ **53.** **(a)** 3 **(b)** -5 **55.** **(a)** 0 **(b)** $\sqrt{2}$

Exercise Set 3.3 (Page 201)

1. **(b)** $\frac{1}{9}$ **3.** $-2/x^2$ **5.** **(a)** no **(b)** yes **(c)** yes **(d)** yes

7. $\dfrac{1}{15y^2+1}$ **9.** $\dfrac{1}{10y^4+3y^2}$ **13.** $f(x)+g(x)$, $f(g(x))$

15. $7e^{7x}$ **17.** $x^2e^x(x+3)$ **19.** $\dfrac{4}{(e^x+e^{-x})^2}$

21. $(x\sec^2 x+\tan x)e^{x\tan x}$ **23.** $(1-3e^{3x})e^{x-e^{3x}}$

25. $\dfrac{x-1}{e^x-x}$ **27.** $2^x\ln 2$ **29.** $\pi^{\sin x}(\ln\pi)\cos x$

31. $(x^3-2x)^{\ln x}\left[\dfrac{3x^2-2}{x^3-2x}\ln x+\dfrac{1}{x}\ln(x^3-2x)\right]$

33. $(\ln x)^{\tan x}\left[\dfrac{\tan x}{x\ln x}+(\sec^2 x)\ln(\ln x)\right]$

35. $(\ln x)^{\ln x}\left[\dfrac{\ln(\ln x)}{x}+\dfrac{1}{x}\right]$ 37. $(x^3+x^2-4x+1)e^x$

39. $((\ln 3)x^2+2x+(\ln 3)\sqrt{x}+1/(2\sqrt{x}))3^x$

41. $(\ln 4)(3\cos x-e^x)4^{3\sin x-e^x}$ 43. $3/\sqrt{1-9x^2}$

45. $-\dfrac{1}{|x|\sqrt{x^2-1}}$ 47. $3x^2/(1+x^6)$ 49. $-\dfrac{\sec^2 x}{\tan^2 x}=-\csc^2 x$

51. $\dfrac{e^x}{|x|\sqrt{x^2-1}}+e^x\sec^{-1}x$ 53. 0 55. 0 57. $-\dfrac{1}{2\sqrt{x}(1+x)}$

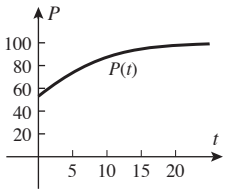
Responses to True–False questions may be abridged to save space.

59. False; consider $y=Ae^x$. 61. True; use the chain rule.

65. $\dfrac{(3x^2+\tan^{-1}y)(1+y^2)}{(1+y^2)e^y-x}$ 67. (b) $1-(\sqrt{3}/3)$

69. (b) $y=(88x-89)/7$

75. (a)



(b) $P(t)\to 100\%$
(c) $P'(t)\to 0$

77. 3 79. $\ln 10$ 81. 12π 83. $9.8t$; if the fluid offers no resistance, then the speed will increase at a constant rate of 9.8 m/s.

Exercise Set 3.4 (Page 208)

1. (a) 6 (b) $-\frac{1}{3}$ 3. (a) -2 (b) $6\sqrt{5}$

5. (b) $A=x^2$ (c) $\dfrac{dA}{dt}=2x\dfrac{dx}{dt}$ (d) 12 ft^2/ min

7. (a) $\dfrac{dV}{dt}=\pi\left(r^2\dfrac{dh}{dt}+2rh\dfrac{dr}{dt}\right)$ (b) -20π in^3/s; decreasing

9. (a) $\dfrac{d\theta}{dt}=\dfrac{\cos^2\theta}{x^2}\left(x\dfrac{dy}{dt}-y\dfrac{dx}{dt}\right)$ (b) $-\frac{5}{16}$ rad/s; decreasing

11. $\dfrac{4\pi}{15}$ in^2/ min 13. $\dfrac{1}{\sqrt{\pi}}$ mi/h 15. 4860π cm^3/ min 17. $\frac{5}{6}$ ft/s

19. $\dfrac{125}{\sqrt{61}}$ ft/s 21. 704 ft/s

23. (a) 500 mi, 1716 mi (b) 1354 mi; 27.7 mi/min

25. $\dfrac{9}{20\pi}$ ft/min 27. 125π ft^3/min 29. 250 mi/h

31. $\dfrac{36\sqrt{69}}{25}$ ft/min 33. $\dfrac{8\pi}{5}$ km/s 35. $600\sqrt{7}$ mi/h

37. (a) $-\frac{60}{7}$ units per second (b) falling 39. -4 units per second

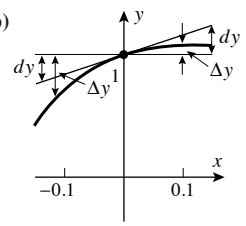
41. $x=\pm\sqrt{\dfrac{-5+\sqrt{33}}{2}}$ 43. 4.5 cm/s; away 47. $\dfrac{20}{9\pi}$ cm/s

Exercise Set 3.5 (Page 217)

1. (a) $f(x)\approx 1+3(x-1)$ (b) $f(1+\Delta x)\approx 1+3\Delta x$ (c) 1.06

3. (a) $1+\frac{1}{2}x$, 0.95, 1.05 17. $|x|<1.692$

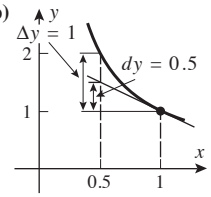
(b)



19. $|x|<0.3158$ 21. (a) 0.0174533 (b) $x_0=45°$ (c) 0.694765

23. 83.16 25. 8.0625 27. 8.9944 29. 0.1 31. 0.8573

33. 0.780398 37. (a) 0.5, 1 (b)



39. $3x^2\,dx$, $3x^2\Delta x+3x(\Delta x)^2+(\Delta x)^3$

41. $(2x-2)\,dx$, $2x\Delta x+(\Delta x)^2-2\Delta x$

43. (a) $(12x^2-14x)\,dx$ (b) $(-x\sin x+\cos x)\,dx$

45. (a) $\dfrac{2-3x}{2\sqrt{1-x}}\,dx$ (b) $-17(1+x)^{-18}\,dx$

Responses to True–False questions may be abridged to save space.

47. False; $dy=(dy/dx)\,dx$ 49. False; consider any linear function.

51. 0.0225 53. 0.0048 55. (a) ± 2 ft^2 (b) side: $\pm 1\%$; area: $\pm 2\%$

57. (a) opposite: ± 0.151 in; adjacent: ± 0.087 in
(b) opposite: $\pm 3.0\%$; adjacent: $\pm 1.0\%$

59. $\pm 10\%$ 61. ± 0.017 cm^2 63. $\pm 6\%$ 65. $\pm 0.5\%$ 67. $15\pi/2$ cm^3

69. (a) $\alpha=1.5\times 10^{-5}/°\text{C}$ (b) 180.1 cm long

Exercise Set 3.6 (Page 226)

1. (a) $\frac{2}{3}$ (b) $\frac{2}{3}$

Responses to True–False questions may be abridged to save space.

3. True; the expression $(\ln x)/x$ is undefined if $x\le 0$.

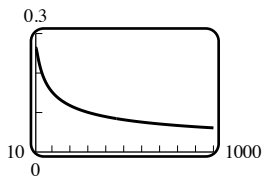
5. False; applying L'Hôpital's rule repeatedly shows that the limit is 0.

7. 1 9. 1 11. -1 13. 0 15. $-\infty$ 17. 0 19. 2 21. 0

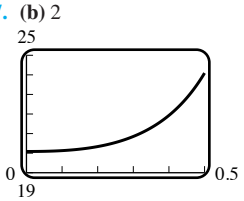
23. π 25. $-\frac{5}{3}$ 27. e^{-3} 29. e^2 31. $e^{2/\pi}$ 33. 0 35. $\frac{1}{2}$

37. $+\infty$ 39. 1 41. 1 43. 1 45. 1 47. (b) 2

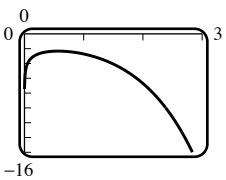
49. 0

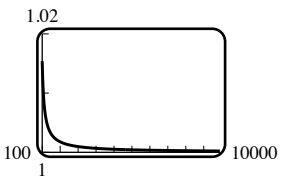


51. e^3



53. no horizontal asymptote 55. $y=1$





57. (a) 0 (b) $+\infty$ (c) 0 (d) $-\infty$ (e) $+\infty$ (f) $-\infty$ 59. 1

61. does not exist 63. Vt/L 67. (a) no (b) Both limits equal 0.

69. does not exist

Chapter 3 Review Exercises (Page 228)

1. (a) $\dfrac{2-3x^2-y}{x}$ (b) $-\dfrac{1}{x^2}-2x$ 3. $-\dfrac{y^2}{x^2}$

5. $\dfrac{y\sec(xy)\tan(xy)}{1-x\sec(xy)\tan(xy)}$ 7. $-\dfrac{21}{16y^3}$

9. $2/(2-\pi)$ 13. $(\sqrt[3]{4}/3,\ \sqrt[3]{2}/3)$

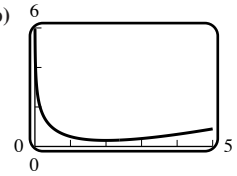
15. $\dfrac{1}{x+1}+\dfrac{2}{x+2}-\dfrac{3}{x+3}-\dfrac{4}{x+4}$ 17. $\dfrac{1}{x}$ 19. $\dfrac{1}{3x(\ln x+1)^{2/3}}$

21. $\dfrac{1}{(\ln 10)x\ln x}$ 23. $\dfrac{3}{2x}+\dfrac{2x^3}{1+x^4}$ 25. $2x$ 27. $e^{\sqrt{x}}(2+\sqrt{x})$

29. $\dfrac{2}{\pi(1+4x^2)}$ 31. $e^x x^{(e^x)}\left(\ln x+\dfrac{1}{x}\right)$ 33. $\dfrac{1}{|2x+1|\sqrt{x^2+x}}$

35. $\dfrac{x^3}{\sqrt{x^2+1}}\left(\dfrac{3}{x}-\dfrac{x}{x^2+1}\right)$

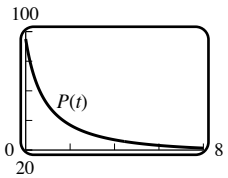
37. **(b)**



(d) curve must have a horizontal tangent line between $x = 1$ and $x = e$ **(e)** $x = 2$

39. e^2 **41.** $e^{1/e}$ **43.** No; for example, $f(x) = x^3$. **45.** $\left(\frac{1}{3}, e\right)$

51. **(a)**



(b) $P(t) \to 19$ **(c)** $P'(t) \to 0$

53. $+\infty, +\infty$: yes; $+\infty, -\infty$: no; $-\infty, +\infty$: no; $-\infty, -\infty$: yes

55. $+\infty$ **57.** $\frac{1}{9}$ **59.** 500π m^2/min

61. **(a)** $-0.5, 1, 0.5$ **(b)** $\pi/4, 1, \pi/2$ **(c)** $3, -1.0$

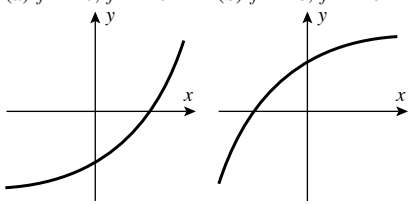
63. **(a)** between 139.48 m and 144.55 m **(c)** $|d\phi| \le 0.98°$

Chapter 3 Making Connections (Page 230)

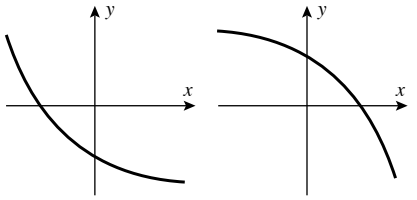
Answers are provided in the Student Solutions Manual.

Exercise Set 4.1 (Page 241)

1. **(a)** $f' > 0, f'' > 0$ **(b)** $f' > 0, f'' < 0$



(c) $f' < 0, f'' > 0$ **(d)** $f' < 0, f'' < 0$



3. A: $dy/dx < 0, d^2y/dx^2 > 0$; B: $dy/dx > 0, d^2y/dx^2 < 0$; C: $dy/dx < 0, d^2y/dx^2 < 0$ **5.** $x = -1, 0, 1, 2$

7. **(a)** $[4, 6]$ **(b)** $[1, 4], [6, 7]$ **(c)** $(1, 2), (3, 5)$ **(d)** $(2, 3), (5, 7)$ **(e)** $x = 2, 3, 5$

9. **(a)** $[1, 3]$ **(b)** $(-\infty, 1], [3, +\infty)$ **(c)** $(-\infty, 2), (4, +\infty)$ **(d)** $(2, 4)$ **(e)** $x = 2, 4$

Responses to True–False questions may be abridged to save space.

11. True; see definition of decreasing: $f(x_1) > f(x_2)$ whenever $0 \le x_1 < x_2 \le 2$.

13. False; for example, $f(x) = (x - 1)^3$ is increasing on $[0, 2]$ and $f'(1) = 0$.

15. **(a)** $[3/2, +\infty)$ **(b)** $(-\infty, 3/2]$ **(c)** $(-\infty, +\infty)$ **(d)** none **(e)** none

17. **(a)** $(-\infty, +\infty)$ **(b)** none **(c)** $(-1/2, +\infty)$ **(d)** $(-\infty, -1/2)$ **(e)** $-1/2$

19. **(a)** $[1, +\infty)$ **(b)** $(-\infty, 1]$ **(c)** $(-\infty, 0), \left(\frac{2}{3}, +\infty\right)$ **(d)** $\left(0, \frac{2}{3}\right)$ **(e)** $0, \frac{2}{3}$

21. **(a)** $\left[\dfrac{3-\sqrt{5}}{2}, \dfrac{3+\sqrt{5}}{2}\right]$ **(b)** $\left(-\infty, \dfrac{3-\sqrt{5}}{2}\right], \left[\dfrac{3+\sqrt{5}}{2}, +\infty\right)$ **(c)** $\left(0, \dfrac{4-\sqrt{6}}{2}\right), \left(\dfrac{4+\sqrt{6}}{2}, +\infty\right)$ **(d)** $(-\infty, 0), \left(\dfrac{4-\sqrt{6}}{2}, \dfrac{4+\sqrt{6}}{2}\right)$ **(e)** $0, \dfrac{4 \pm \sqrt{6}}{2}$

23. **(a)** $[-1/2, +\infty)$ **(b)** $(-\infty, -1/2]$ **(c)** $(-2, 1)$ **(d)** $(-\infty, -2), (1, +\infty)$ **(e)** $-2, 1$

25. **(a)** $[-1, 0], [1, +\infty)$ **(b)** $(-\infty, -1], [0, 1]$ **(c)** $(-\infty, 0), (0, +\infty)$ **(d)** none **(e)** none

27. **(a)** $(-\infty, 0]$ **(b)** $[0, +\infty)$ **(c)** $(-\infty, -1), (1, +\infty)$ **(d)** $(-1, 1)$ **(e)** $-1, 1$

29. **(a)** $[0, +\infty)$ **(b)** $(-\infty, 0]$ **(c)** $(-2, 2)$ **(d)** $(-\infty, -2), (2, +\infty)$ **(e)** $-2, 2$

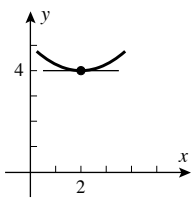
31. **(a)** $[0, +\infty)$ **(b)** $(-\infty, 0]$ **(c)** $\left(-\sqrt{\dfrac{1+\sqrt{7}}{3}}, \sqrt{\dfrac{1+\sqrt{7}}{3}}\right)$ **(d)** $\left(-\infty, -\sqrt{\dfrac{1+\sqrt{7}}{3}}\right), \left(\sqrt{\dfrac{1+\sqrt{7}}{3}}, +\infty\right)$ **(e)** $\pm\sqrt{\dfrac{1+\sqrt{7}}{3}}$

33. increasing: $[-\pi/4, 3\pi/4]$; decreasing: $[-\pi, -\pi/4], [3\pi/4, \pi]$; concave up: $(-3\pi/4, \pi/4)$; concave down: $(-\pi, -3\pi/4), (\pi/4, \pi)$; inflection points: $-3\pi/4, \pi/4$

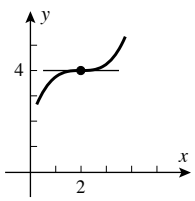
35. increasing: none; decreasing: $(-\pi, \pi)$; concave up: $(-\pi, 0)$; concave down: $(0, \pi)$; inflection point: 0

37. increasing: $[-\pi, -3\pi/4], [-\pi/4, \pi/4], [3\pi/4, \pi]$; decreasing: $[-3\pi/4, -\pi/4], [\pi/4, 3\pi/4]$; concave up: $(-\pi/2, 0), (\pi/2, \pi)$; concave down: $(-\pi, -\pi/2), (0, \pi/2)$; inflection points: $0, \pm\pi/2$

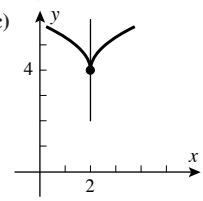
39. **(a)**



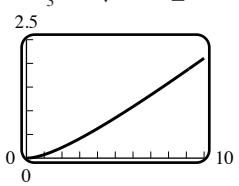
(b)

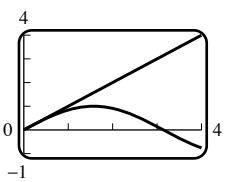


(c)

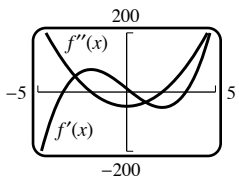


41. $1 + \frac{1}{3}x - \sqrt[3]{1+x} \ge 0$ if $x > 0$ **43.** $x \ge \sin x$





47.

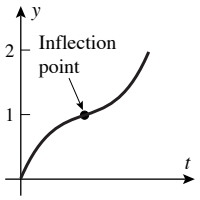


points of inflection at $x = -2, 2$; concave up on $[-5, -2], [2, 5]$; concave down on $[-2, 2]$; increasing on $[-3.5829, 0.2513]$ and $[3.3316, 5]$; decreasing on $[-5, -3.5829]$, $[0.2513, 3.3316]$

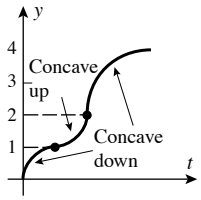
49. $-2.464202, 0.662597, 2.701605$ **53.** **(a)** true **(b)** false

57. **(c)** inflection point $(1, 0)$; concave up on $(1, +\infty)$; concave down on $(-\infty, 1)$

63.

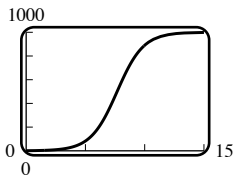


65.



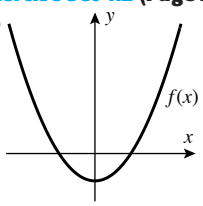
67. (a) $\dfrac{LAk}{(1+A)^2}$ (c) $\dfrac{1}{k}\ln A$

69. the eighth day

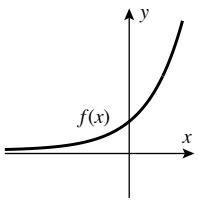


Exercise Set 4.2 (Page 252)

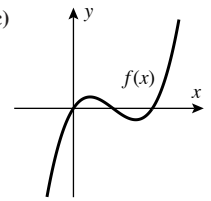
1. (a)



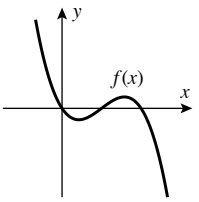
(b)



(c)



(d)



5. (b) nothing (c) f has a relative minimum at $x = 1$, g has no relative extremum at $x = 1$.

7. critical: $0, \pm\sqrt{2}$; stationary: $0, \pm\sqrt{2}$

9. critical: $-3, 1$; stationary: $-3, 1$ **11.** critical: $0, \pm 5$; stationary: 0

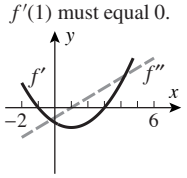
13. critical: $n\pi/2$ for every integer n; stationary: $n\pi + \pi/2$ for every integer n

Responses to True–False questions may be abridged to save space.

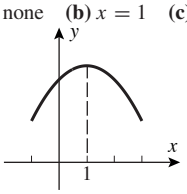
15. False; for example, $f(x) = (x-1)^2(x-1.5)$ has a relative maximum at $x = 1$, but $f(2) = 0.5 > 0 = f(1)$.

17. False; to apply the second derivative test (Theorem 4.2.4) at $x = 1$, $f'(1)$ must equal 0.

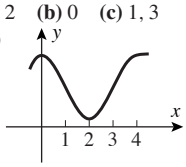
19.



21. (a) none (b) $x = 1$ (c) none (d)



23. (a) 2 (b) 0 (c) 1, 3 (d)



25. 0 (neither), $\sqrt[3]{5}$ (min) **27.** -2 (min), $2/3$ (max) **29.** 0 (min)

31. -1 (min), 1 (max) **33.** relative maximum at $(4/3, 19/3)$

35. relative maximum at $(\pi/4, 1)$; relative minimum at $(3\pi/4, -1)$

37. relative maximum at $(1, 1)$; relative minima at $(0, 0)$, $(2, 0)$

39. relative maximum at $(-1, 0)$; relative minimum at $(-3/5, -108/3125)$

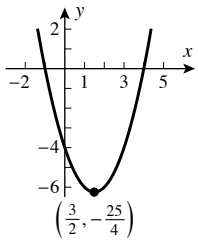
41. relative maximum at $(-1, 1)$; relative minimum at $(0, 0)$

43. no relative extrema **45.** relative minimum at $(0, \ln 2)$

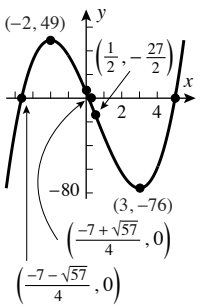
47. relative minimum at $(-\ln 2, -1/4)$

49. relative maximum at $(3/2, 9/4)$; relative minima at $(0, 0)$, $(3, 0)$

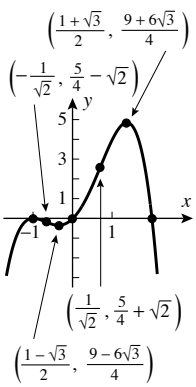
51. intercepts: $(0, -4)$, $(-1, 0)$, $(4, 0)$; stationary point: $(3/2, -25/4)$ (min); inflection points: none



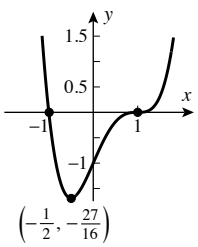
53. intercepts: $(0, 5)$, $\left(\dfrac{-7 \pm \sqrt{57}}{4}, 0\right)$, $(5, 0)$; stationary points: $(-2, 49)$ (max), $(3, -76)$ (min); inflection point: $(1/2, -27/2)$



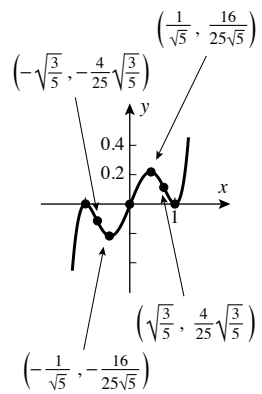
55. intercepts: $(-1, 0)$, $(0, 0)$, $(2, 0)$; stationary points: $(-1, 0)$ (max), $\left(\dfrac{1-\sqrt{3}}{2}, \dfrac{9-6\sqrt{3}}{4}\right)$ (min), $\left(\dfrac{1+\sqrt{3}}{2}, \dfrac{9+6\sqrt{3}}{4}\right)$ (max); inflection points: $\left(-\dfrac{1}{\sqrt{2}}, \dfrac{5}{4} - \sqrt{2}\right)$, $\left(\dfrac{1}{\sqrt{2}}, \dfrac{5}{4} + \sqrt{2}\right)$,



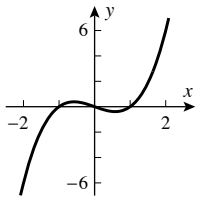
57. intercepts: $(0, -1)$, $(-1, 0)$, $(1, 0)$; stationary points: $(-1/2, -27/16)$ (min), $(1, 0)$ (neither); inflection points: $(0, -1)$, $(1, 0)$



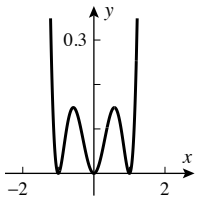
59. intercepts: $(-1, 0)$, $(0, 0)$, $(1, 0)$; stationary points: $(-1, 0)$ (max), $\left(-\dfrac{1}{\sqrt{5}}, -\dfrac{16}{25\sqrt{5}}\right)$ (min), $\left(\dfrac{1}{\sqrt{5}}, \dfrac{16}{25\sqrt{5}}\right)$ (max), $(1, 0)$ (min); inflection points: $\left(-\sqrt{\dfrac{3}{5}}, -\dfrac{4}{25}\sqrt{\dfrac{3}{5}}\right)$, $(0, 0)$, $\left(\sqrt{\dfrac{3}{5}}, \dfrac{4}{25}\sqrt{\dfrac{3}{5}}\right)$



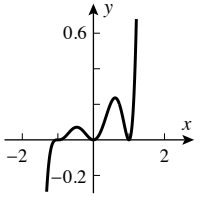
61. (a)



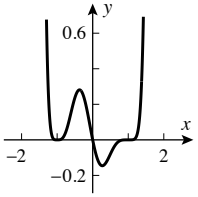
(b)



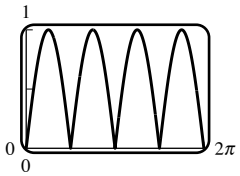
(c)



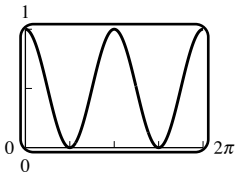
(d)



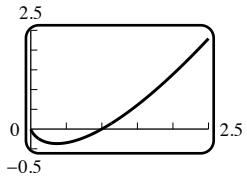
63. relative min of 0 at $x = \pi/2, \pi, 3\pi/2$; relative max of 1 at $x = \pi/4, 3\pi/4, 5\pi/4, 7\pi/4$



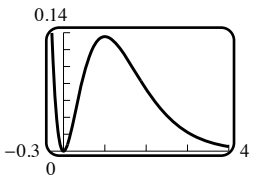
65. relative min of 0 at $x = \pi/2, 3\pi/2$; relative max of 1 at $x = \pi$



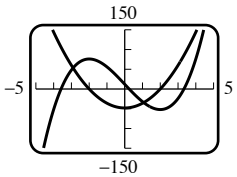
67. relative min of $-1/e$ at $x = 1/e$



69. relative min of 0 at $x = 0$; relative max of $1/e^2$ at $x = 1$



71. relative minima at $x = -3.58, 3.33$; relative max at $x = 0.25$

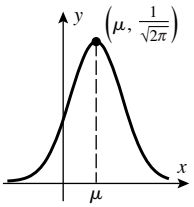


73. relative maximum at $x \approx -0.272$; relative minimum at $x \approx 0.224$

75. relative maximum at $x = 0$; relative minima at $x \approx \pm 0.618$

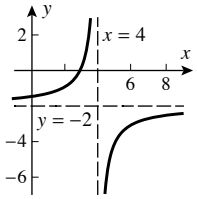
77. **(a)** 54 **(b)** 9

79. **(b)**

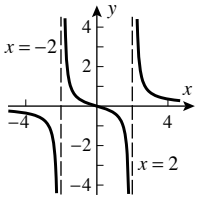


Exercise Set 4.3 (Page 264)

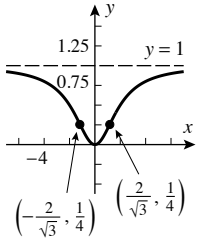
1. stationary points: none;
inflection points: none;
asymptotes: $x = 4$, $y = -2$;
asymptote crossings: none



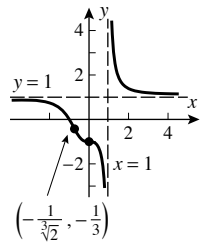
3. stationary points: none;
inflection point: (0, 0);
asymptotes: $x = \pm 2$, $y = 0$;
asymptote crossings: (0, 0)



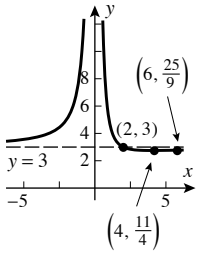
5. stationary point: (0, 0);
inflection points: $\left(\pm\frac{2}{\sqrt{3}}, \frac{1}{4}\right)$;
asymptote: $y = 1$;
asymptote crossings: none



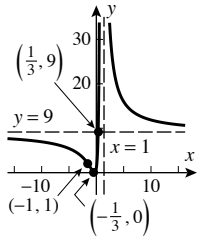
7. stationary point: (0, −1);
inflection points: (0, −1), $\left(-\frac{1}{\sqrt[3]{2}}, -\frac{1}{3}\right)$;
asymptotes: $x = 1$, $y = 1$;
asymptote crossings: none



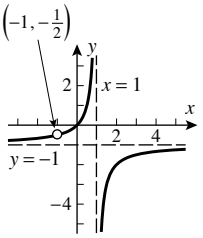
9. stationary point: (4, 11/4);
inflection point: (6, 25/9);
asymptotes: $x = 0$, $y = 3$;
asymptote crossing: (2, 3)



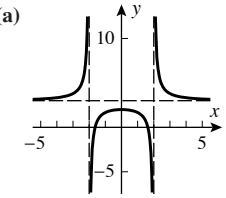
11. stationary point: (−1/3, 0);
inflection point: (−1, 1);
asymptotes: $x = 1$, $y = 9$;
asymptote crossing: (1/3, 9)



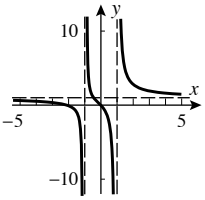
13. stationary points: none;
inflection points: none;
asymptotes: $x = 1$, $y = -1$;
asymptote crossings: none



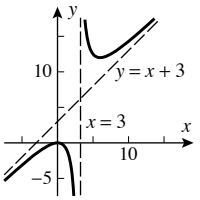
15. **(a)**



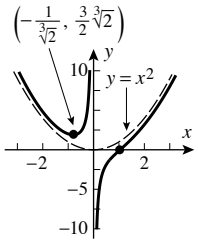
(b)



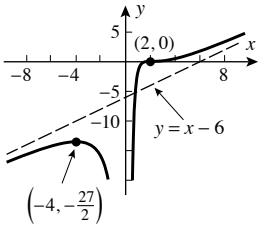
17.



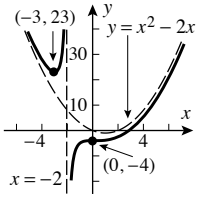
19. stationary point: $\left(-\frac{1}{\sqrt[3]{2}}, \frac{3}{2}\sqrt[3]{2}\right)$;
inflection point: $(1, 0)$;
asymptotes: $y = x^2$, $x = 0$;
asymptote crossings: none



21. stationary points: $(-4, -27/2)$, $(2, 0)$;
inflection point: $(2, 0)$;
asymptotes: $x = 0$, $y = x - 6$;
asymptote crossing: $(2/3, -16/3)$



23. stationary points: $(-3, 23)$, $(0, -4)$;
inflection point: $(0, -4)$;
asymptotes: $x = -2$, $y = x^2 - 2x$;
asymptote crossings: none



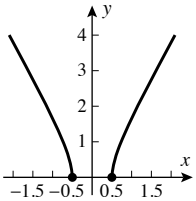
25. **(a)** VI **(b)** I **(c)** III **(d)** V **(e)** IV **(f)** II

Responses to True–False questions may be abridged to save space.

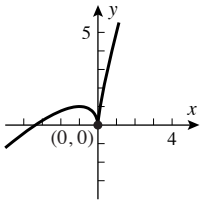
27. True; if $\deg P > \deg Q$, then $f(x)$ is unbounded as $x \to \pm\infty$; if $\deg P < \deg Q$, then $f(x) \to 0$ as $x \to \pm\infty$.

29. False; for example, $f(x) = (x-1)^{1/3}$ is continuous (with vertical tangent line) at $x = 1$, but $f'(x) = \dfrac{1}{3(x-1)^{2/3}}$ has a vertical asymptote at $x = 1$.

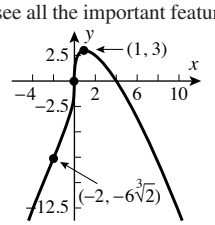
31. critical points: $(\pm 1/2, 0)$;
inflection points: none

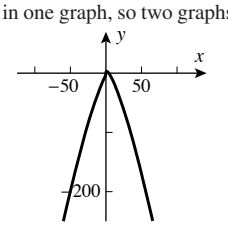


33. critical points: $(-1, 1)$, $(0, 0)$;
inflection points: none

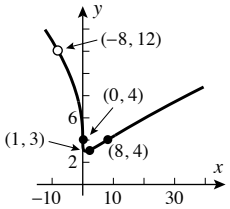


35. critical points: $(0, 0)$, $(1, 3)$; inflection points: $(0, 0)$, $(-2, -6\sqrt[3]{2})$.
It's hard to see all the important features in one graph, so two graphs are shown:

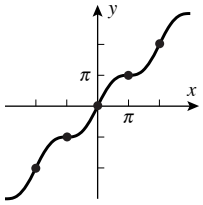




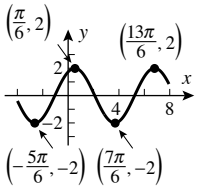
37. critical points: $(0, 4)$, $(1, 3)$;
inflection points: $(0, 4)$, $(8, 4)$



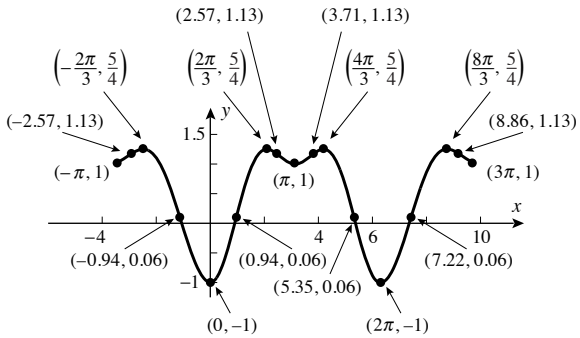
39. extrema: none;
inflection points: $x = \pi n$ for integers n



41. minima: $x = 7\pi/6 + 2\pi n$ for integers n;
maxima: $x = \pi/6 + 2\pi n$ for integers n;
inflection points: $x = 2\pi/3 + \pi n$ for integers n

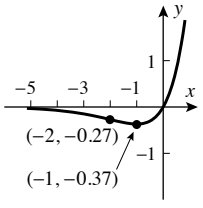


43. relative minima: 1 at $x = \pi$; -1 at $x = 0, 2\pi$;
relative maxima: $5/4$ at $x = -2\pi/3, 2\pi/3, 4\pi/3, 8\pi/3$;
inflection points where $\cos x = \dfrac{-1 \pm \sqrt{33}}{8}$: $(-2.57, 1.13)$, $(-0.94, 0.06)$, $(0.94, 0.06)$, $(2.57, 1.13)$, $(3.71, 1.13)$, $(5.35, 0.06)$, $(7.22, 0.06)$, $(8.86, 1.13)$



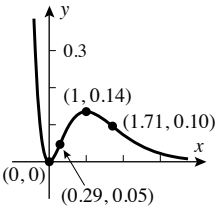
45. **(a)** $+\infty$, 0

(b)



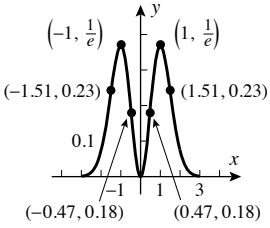
47. **(a)** 0, $+\infty$

(b)

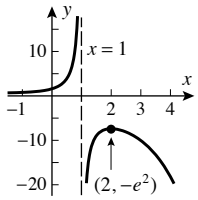


49. **(a)** 0, 0

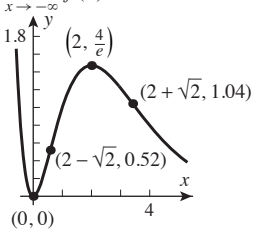
(b) relative max $= 1/e$ at $x = \pm 1$; relative min $= 0$ at $x = 0$;
inflection points where $x = \pm\sqrt{\dfrac{5 \pm \sqrt{17}}{4}}$:
about $(\pm 0.47, 0.18)$, $(\pm 1.51, 0.23)$; asymptote: $y = 0$



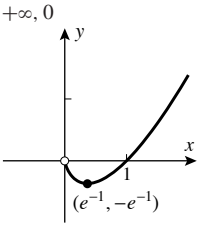
51. **(a)** $-\infty, 0$
(b) relative max $= -e^2$ at $x = 2$;
no relative min;
no inflection points;
asymptotes: $y = 0, x = 1$



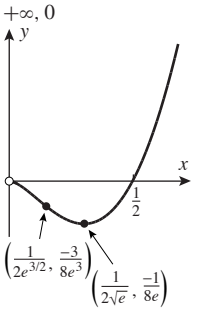
53. **(a)** $0, +\infty$
(b) critical points at $x = 0.2$;
relative min at $x = 0$,
relative max at $x = 2$;
points of inflection at $x = 2 \pm \sqrt{2}$;
horizontal asymptote $y = 0$ as $x \to +\infty$
$\lim_{x \to -\infty} f(x) = +\infty$



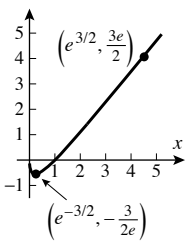
55. **(a)** $+\infty, 0$
(b)

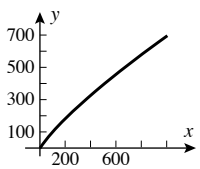


57. **(a)** $+\infty, 0$
(b)

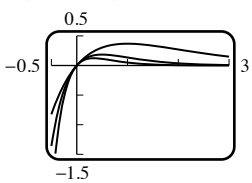


59. **(a)** $+\infty, 0$
(b) no relative max; relative min $= -\dfrac{3}{2e}$ at $x = e^{-3/2}$;
inflection point: $(e^{3/2}, 3e/2)$;
no asymptotes. It's hard to see all the important features in one graph, so two graphs are shown:



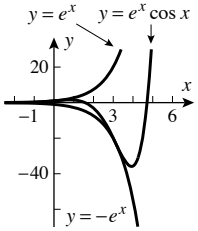


61. **(a)**

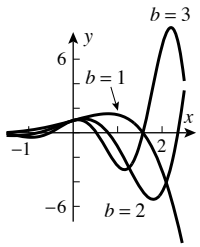


(b) relative max at $x = 1/b$;
inflection point at $x = 2/b$

63. **(a)** does not exist, 0
(b) $y = e^x$ and $y = e^x \cos x$ intersect for $x = 2\pi n$, and $y = -e^x$ and $y = e^x \cos x$ intersect for $x = 2\pi n + \pi$, for all integers n.

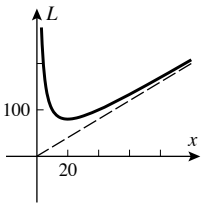


(c) Graphs for $a = 1$:

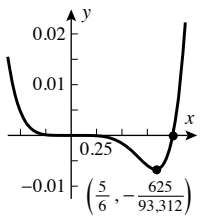


65. **(a)** $x = 1, 2.5, 3, 4$ **(b)** $(-\infty, 1], [2.5, 3]$
(c) relative max at $x = 1, 3$;
relative min at $x = 2.5$ **(d)** $x \approx 0.6, 1.9, 4$

67.



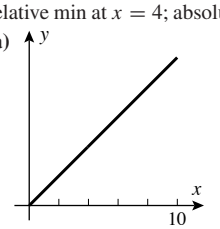
69. Graph misses zeros at $x = 0, 1$ and min at $x = 5/6$.



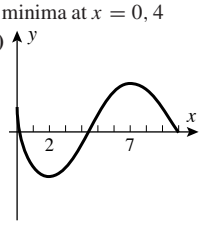
Exercise Set 4.4 (Page 272)

1. relative maxima at $x = 2, 6$; absolute max at $x = 6$;
relative min at $x = 4$; absolute minima at $x = 0, 4$

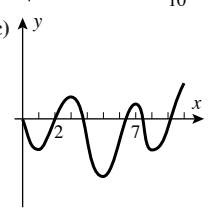
3. **(a)**



(b)



(c)

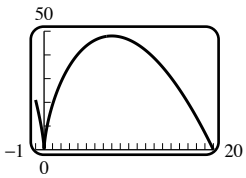


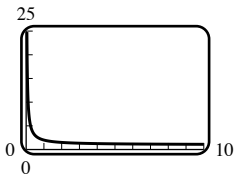
7. max $= 2$ at $x = 1, 2$;
min $= 1$ at $x = 3/2$
9. max $= 8$ at $x = 4$;
min $= -1$ at $x = 1$
11. maximum value $3/\sqrt{5}$ at $x = 1$;
minimum value $-3/\sqrt{5}$ at $x = -1$
13. max $= \sqrt{2} - \pi/4$ at $x = -\pi/4$;
min $= \pi/3 - \sqrt{3}$ at $x = \pi/3$
15. maximum value 17 at $x = -5$; minimum value 1 at $x = -3$.

Responses to True–False questions may be abridged to save space.

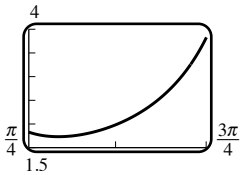
17. True; see the Extreme-Value Theorem (4.4.2).
19. True; see Theorem 4.4.3.
21. no maximum; min $= -9/4$ at $x = 1/2$
23. maximum value $f(1) = 1$; no minimum
25. no maximum or minimum
27. max $= -2 - 2\sqrt{2}$ at $x = -1 - \sqrt{2}$; no minimum
29. no maximum; min $= 0$ at $x = 0, 2$

31. maximum value 48 at $x = 8$; minimum value 0 at $x = 0, 20$ **33.** no maximum or minimum

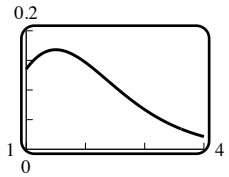




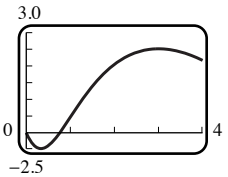
35. max $= 2\sqrt{2} + 1$ at $x = 3\pi/4$; min $= \sqrt{3}$ at $x = \pi/3$



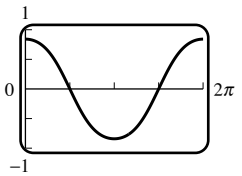
37. maximum value $\frac{27}{8}e^{-3}$ at $x = \frac{3}{2}$; minimum value $64/e^8$ at $x = 4$



39. max $= 5\ \ln 10 - 9$ at $x = 3$; min $= 5\ \ln(10/9) - 1$ at $x = 1/3$



41. maximum value $\sin(1) \approx 0.84147$; minimum value $-\sin(1) \approx -0.84147$



43. maximum value 2; minimum value $-\frac{1}{4}$

45. max $= 3$ at $x = 2n\pi$; min $= -3/2$ at $x = \pm 2\pi/3 + 2n\pi$ for any integer n **49.** 2, at $x = 1$

53. maximum $y = 4$ at $t = \pi, 3\pi$; minimum $y = 0$ at $t = 0, 2\pi$

Exercise Set 4.5 (Page 283)

1. **(a)** 1 **(b)** $\frac{1}{2}$ **3.** 500 ft parallel to stream, 250 ft perpendicular

5. 500 ft (\$3 fencing) × 750 ft (\$2 fencing) **7.** 5 in × $\frac{12}{5}$ in

9. $10\sqrt{2} \times 10\sqrt{2}$ **11.** 80 ft (\$1 fencing), 40 ft (\$2 fencing)

15. maximum area is 108 when $x = 2$

17. maximum area is 144 when $x = 2$

19. 11,664 in^3 **21.** $\frac{200}{27}$ ft^3 **23.** base 10 cm square, height 20 cm

25. ends $\sqrt[3]{3V/4}$ units square, length $\frac{4}{3}\sqrt[3]{3V/4}$

27. height $= 2R/\sqrt{3}$, radius $= \sqrt{2/3}R$

31. height = radius $= \sqrt[3]{500/\pi}$ cm **33.** $L/12$ by $L/12$ by $L/12$

35. height $= L/\sqrt{3}$, radius $= \sqrt{2/3}L$

37. height $= 2\sqrt[3]{75/\pi}$ cm, radius $= \sqrt{2}\sqrt[3]{75/\pi}$ cm

39. height $= 4R$, radius $= \sqrt{2}R$

41. $R(x) = 225x - 0.25x^2$; $R'(x) = 225 - 0.5x$; 450 tons

43. **(a)** 7000 units **(b)** yes **(c)** \$15 **45.** 13,722 lb **47.** $3\sqrt{3}$

49. height $= r/\sqrt{2}$ **51.** $1/\sqrt{5}$ **53.** $\left(\sqrt{2}, \frac{1}{2}\right)$ **55.** $\left(-1/\sqrt{3}, \frac{3}{4}\right)$

57. **(a)** π mi **(b)** $2\sin^{-1}(1/4)$ mi **59.** $4(1 + 2^{2/3})^{3/2}$ ft

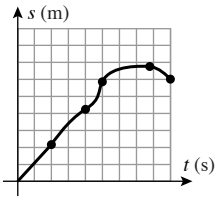
61. 30 cm from the weaker source **63.** $\sqrt{24} = 2\sqrt{6}$ ft

Exercise Set 4.6 (Page 294)

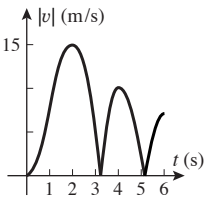
1. **(a)** positive, negative, slowing down
(b) positive, positive, speeding up
(c) negative, positive, slowing down

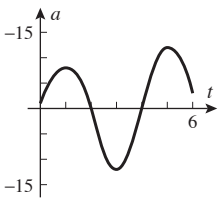
3. **(a)** left
(b) negative
(c) speeding up
(d) slowing down

5.



7.





Responses to True–False questions may be abridged to save space.

9. False; a particle has positive velocity when its position versus time graph is increasing; if that positive velocity is decreasing, the particle would be slowing down.

11. False; acceleration is the derivative of velocity (with respect to time); speed is the absolute value of velocity.

13. **(a)** 6.2 ft/s^2 **(b)** $t = 0$ s

15. **(a)**

t	s	v	a
1	0.71	0.56	−0.44
2	1	0	−0.62
3	0.71	−0.56	−0.44
4	0	−0.79	0
5	−0.71	−0.56	0.44

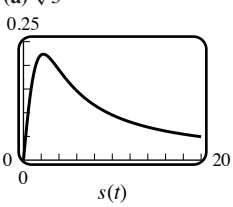
(b) stopped at $t = 2$;
moving right at $t = 1$;
moving left at $t = 3, 4, 5$
(c) speeding up at $t = 3$;
slowing down at $t = 1, 5$;
neither at $t = 2, 4$

17. **(a)** $v(t) = 3t^2 - 6t$, $a(t) = 6t - 6$
(b) $s(1) = -2$ ft, $v(1) = -3$ ft/s, $|v(1)| = 3$ ft/s, $a(1) = 0$ ft/s^2
(c) $t = 0, 2$ s **(d)** speeding up for $0 < t < 1$ and $2 < t$, slowing down for $1 < t < 2$ **(e)** 58 ft

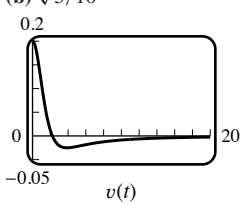
19. **(a)** $v(t) = 3\pi \sin(\pi t/3)$, $a(t) = \pi^2 \cos(\pi t/3)$ **(b)** $s(1) = 9/2$ ft, $v(1) =$ speed $= 3\sqrt{3}\pi/2$ ft/s, $a(1) = \pi^2/2$ ft/s^2 **(c)** $t = 0$ s, 3 s
(d) speeding up: $0 < t < 1.5, 3 < t < 4.5$;
slowing down: $1.5 < t < 3, 4.5 < t < 5$ **(e)** 31.5 ft

21. **(a)** $v(t) = -\frac{1}{3}(t^2 - 6t + 8)e^{-t/3}$, $a(t) = \frac{1}{9}(t^2 - 12t + 26)e^{-t/3}$
(b) $s(1) = 9e^{-1/3}$ ft, $v(1) = -e^{-1/3}$ ft/s, speed $= e^{-1/3}$ ft/s, $a(1) = \frac{5}{3}e^{-1/3}$ ft/s^2 **(c)** $t = 2$ s, 4 s
(d) speeding up: $2 < t < 6 - \sqrt{10}, 4 < t < 6 + \sqrt{10}$; slowing down: $0 < t < 2, 6 - \sqrt{10} < t < 4, 6 + \sqrt{10} < t$
(e) $8 - 24e^{-2/3} + 48e^{-4/3} - 33e^{-5/3}$

23. **(a)** $\sqrt{5}$ **(b)** $\sqrt{5}/10$

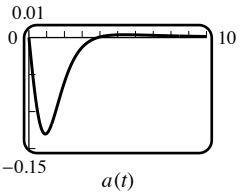


$s(t)$

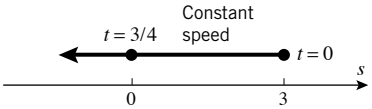


$v(t)$

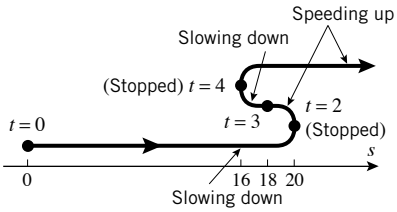
(c) speeding up for $\sqrt{5} < t < \sqrt{15}$; slowing down for $0 < t < \sqrt{5}$ and $\sqrt{15} < t$



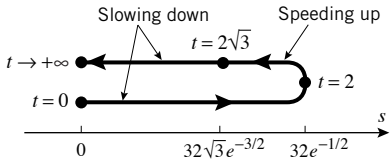
25.



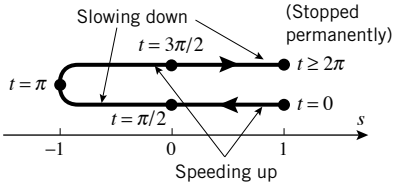
27.



29.



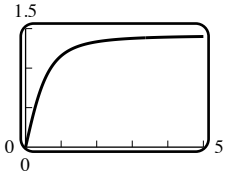
31.



33. **(a)** 12 ft/s **(b)** $t = 2.2$ s, $s = -24.2$ ft

35. **(a)** $t = 2 \pm 1/\sqrt{3}$, $s = \ln 2$, $v = \pm\sqrt{3}$ **(b)** $t = 2$, $s = 0$, $a = 6$

37. **(a)**



(b) $\sqrt{2}$

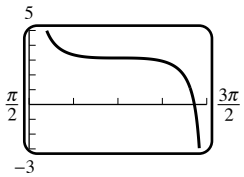
39. **(b)** $\frac{2}{3}$ unit **(c)** $0 \le t < 1$ and $t > 2$

Exercise Set 4.7 (Page 300)

1. 1.414213562 **3.** 1.817120593 **5.** $x \approx 1.76929$

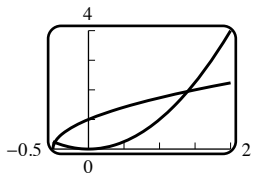
7. $x \approx 1.224439550$ **9.** $x \approx -1.24962$ **11.** $x \approx 1.02987$

13. $x \approx 4.493409458$



15. $x \approx 0.68233$

17. -0.474626618, 1.395336994 **19.** $x \approx 0.58853$ or 3.09636



Responses to True–False questions may be abridged to save space.

21. True; $x = x_{n+1}$ is the x-intercept of the tangent line to $y = f(x)$ at $x = x_n$.

23. False; for example, if $f(x) = x(x-3)^2$, Newton's Method fails (analogous to Figure 4.7.4) with $x_1 = 1$ and approximates the root $x = 3$ for $x_1 > 1$.

25. **(b)** 3.162277660 **27.** -4.098859132

29. $x = -1$ or $x \approx 0.17951$ **31.** (0.589754512, 0.347810385)

33. **(b)** $\theta \approx 2.99156$ rad or 171° **35.** -1.220744085, 0.724491959

37. $i = 0.053362$ or 5.33% **39.** **(a)** The values do not converge.

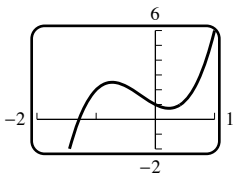
Exercise Set 4.8 (Page 308)

1. $c = 4$ **3.** $c = \pi$ **5.** $c = 1$ **7.** $c = -\sqrt{5}$

9. **(a)** $[-2, 1]$

(b) $c \approx -1.29$

(c) -1.2885843



Responses to True–False questions may be abridged to save space.

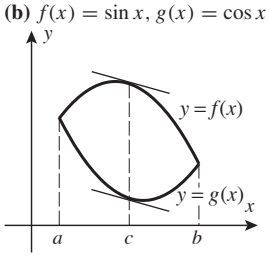
11. False; Rolle's Theorem requires the additional hypothesis that f is differentiable on (a, b) and $f(a) = f(b) = 0$; see Example 2.

13. False; the Constant Difference Theorem applies to two functions with equal derivatives on an interval to conclude that the functions differ by a constant on the interval.

15. **(b)** $\tan x$ is not continuous on $[0, \pi]$. **25.** $f(x) = xe^x - e^x + 2$

35. **(b)** $f(x) = \sin x$, $g(x) = \cos x$

37.



41. $a = 6$, $b = -3$

Chapter 4 Review Exercises (Page 310)

1. **(a)** $f(x_1) < f(x_2)$; $f(x_1) > f(x_2)$; $f(x_1) = f(x_2)$

(b) $f' > 0$; $f' < 0$; $f' = 0$

3. **(a)** $\left[\frac{5}{2}, +\infty\right)$ **(b)** $\left(-\infty, \frac{5}{2}\right]$ **(c)** $(-\infty, +\infty)$ **(d)** none **(e)** none

5. **(a)** $[0, +\infty)$ **(b)** $(-\infty, 0]$ **(c)** $(-\sqrt{2/3}, \sqrt{2/3})$

(d) $(-\infty, -\sqrt{2/3})$, $(\sqrt{2/3}, +\infty)$ **(e)** $-\sqrt{2/3}$, $\sqrt{2/3}$

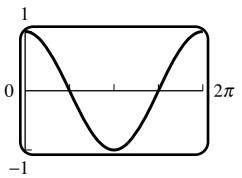
7. **(a)** $[-1, +\infty)$ **(b)** $(-\infty, -1]$ **(c)** $(-\infty, 0)$, $(2, +\infty)$

(d) $(0, 2)$ **(e)** $0, 2$

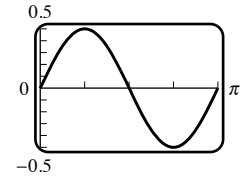
9. **(a)** $(-\infty, 0]$ **(b)** $[0, +\infty)$ **(c)** $(-\infty, -1/\sqrt{2})$, $(1/\sqrt{2}, +\infty)$

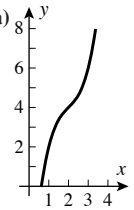
(d) $(-1/\sqrt{2}, 1/\sqrt{2})$ **(e)** $\pm 1/\sqrt{2}$

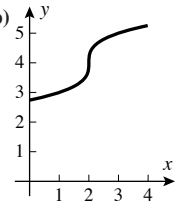
11. increasing on $[\pi, 2\pi]$;
decreasing on $[0, \pi]$;
concave up on $(\pi/2, 3\pi/2)$;
concave down on $(0, \pi/2)$, $(3\pi/2, 2\pi)$;
inflection points: $(\pi/2, 0)$, $(3\pi/2, 0)$

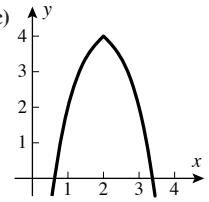


13. increasing on $[0, \pi/4]$, $[3\pi/4, \pi]$;
decreasing on $[\pi/4, 3\pi/4]$;
concave up on $(\pi/2, \pi)$;
concave down on $(0, \pi/2)$;
inflection point: $(\pi/2, 0)$



15. **(a)** 

(b) 

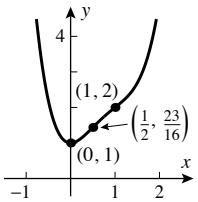
(c) 

17. $-\dfrac{b}{2a} \le 0$ **19.** $x = -1$ **21.** **(a)** at an inflection point

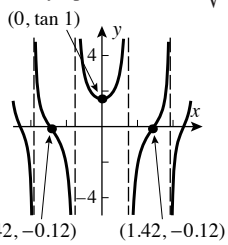
25. **(a)** $x = \pm\sqrt{2}$ (stationary points) **(b)** $x = 0$ (stationary point)

27. **(a)** relative max at $x = 1$, relative min at $x = 7$, neither at $x = 0$
(b) relative max at $x = \pi/2,\ 3\pi/2$; relative min at $x = 7\pi/6,\ 11\pi/6$
(c) relative max at $x = 5$

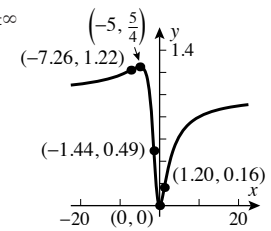
29. $\lim\limits_{x\to-\infty} f(x) = +\infty$, $\lim\limits_{x\to+\infty} f(x) = +\infty$;
relative min at $x = 0$;
points of inflection at $x = \frac{1}{2},\ 1$;
no asymptotes



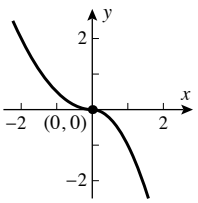
31. $\lim\limits_{x\to\pm\infty} f(x)$ does not exist; critical point at $x = 0$; relative min at $x = 0$; point of inflection when $1 + 4x^2\tan(x^2 + 1) = 0$;
vertical asymptotes at $x = \pm\sqrt{\pi\left(n + \frac{1}{2}\right) - 1},\ n = 0, 1, 2, \ldots$



33. critical points at $x = -5,\ 0$; relative max at $x = -5$, relative min at $x = 0$; points of inflection at $x \approx -7.26,\ -1.44,\ 1.20$; horizontal asymptote $y = 1$ for $x \to \pm\infty$

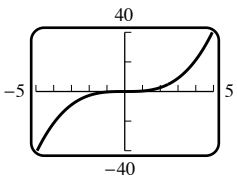


35. $\lim\limits_{x\to-\infty} f(x) = +\infty$, $\lim\limits_{x\to+\infty} f(x) = -\infty$;
critical point at $x = 0$;
no extrema;
inflection point at $x = 0$
(f changes concavity);
no asymptotes



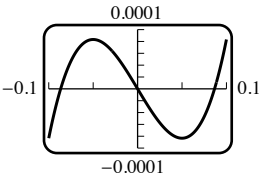
37. no relative extrema **39.** relative min of 0 at $x = 0$

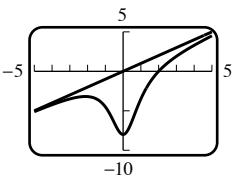
41. relative min of 0 at $x = 0$ **43.** relative min of 0 at $x = 0$

45. **(a)** 

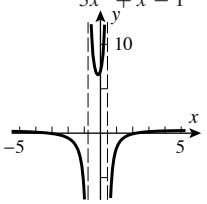
(b) relative max at $x = -\frac{1}{20}$; relative min at $x = \frac{1}{20}$

(c) The finer details can be seen when graphing over a much smaller x-window.



47. **(a)** 

49. $f(x) = \dfrac{x^2 + x - 7}{3x^2 + x - 1},\ x \ne \dfrac{1}{2}$

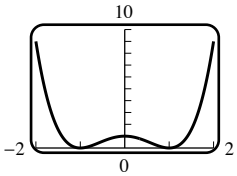


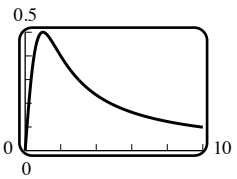
horizontal asymptote $y = 1/3$
vertical asymptotes at
$x = (-1 \pm \sqrt{13})/6$

53. **(a)** true **(b)** false

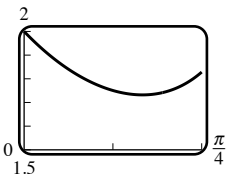
55. **(a)** no max; min $= -13/4$ at $x = 3/2$ **(b)** no max or min
(c) no max; min $m = e^2/4$ at $x = 2$
(d) no max; min $m = e^{-1/e}$ at $x = 1/e$

57. **(a)** minimum value 0 for $x = \pm 1$; no maximum **(b)** max $= 1/2$ at $x = 1$; min $= 0$ at $x = 0$

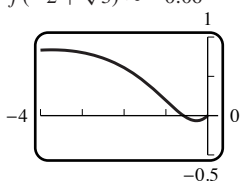


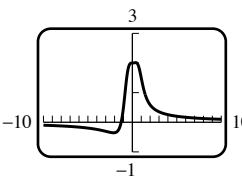


(c) maximum value 2 at $x = 0$; minimum value $\sqrt{3}$ at $x = \pi/6$



(d) maximum value $f(-2 - \sqrt{3}) \approx 0.84$; minimum value $f(-2 + \sqrt{3}) \approx -0.06$



59. **(a)** 

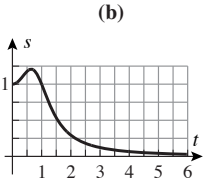
(b) minimum: $(-2.111985, -0.355116)$; maximum: $(0.372591, 2.012931)$

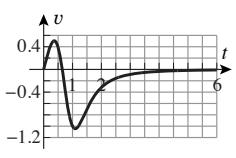
61. width $= 4\sqrt{2}$, height $= 3\sqrt{2}$ **63.** 2 in square

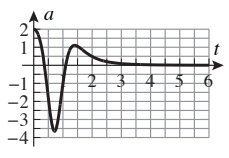
65. **(a)** yes **(b)** yes

67. **(a)** $v = -2\dfrac{t(t^4 + 2t^2 - 1)}{(t^4 + 1)^2},\ a = 2\dfrac{3t^8 + 10t^6 - 12t^4 - 6t^2 + 1}{(t^4 + 1)^3}$

(b)





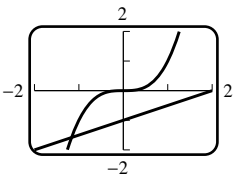


(c) $t \approx 0.64$, $s \approx 1.2$ **(d)** $0 \le t \le 0.64$

(e) speeding up when $0 \le t < 0.36$ and $0.64 < t < 1.1$, otherwise slowing down **(f)** maximum speed ≈ 1.05 when $t \approx 1.10$

69. $x \approx -2.11491,\ 0.25410,\ 1.86081$

71. $x \approx -1.165373043$



73. 249×10^6 km

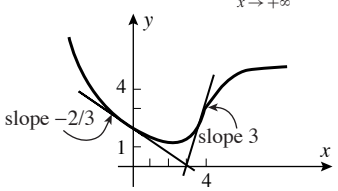
75. **(a)** yes, $c = 0$ **(b)** no **(c)** yes, $c = \sqrt{\pi/2}$

77. use Rolle's Theorem

Chapter 4 Making Connections (Page 314)

Where correct answers to a Making Connections exercise may vary, no answer is listed. Sample answers for these questions are available on the Book Companion Site.

1. **(a)** no zeros **(b)** one **(c)** $\lim_{x \to +\infty} g'(x) = 0$



2. **(a)** $(-2.2, 4),\ (2, 1.2),\ (4.2, 3)$
(b) critical numbers at $x = -5.1, -2, 0.2, 2$;
local min at $x = -5.1, 2$; local max at $x = -2$;
no extrema at $x = 0.2$; $f''(1) \approx -1.2$

3. $x = -4, 5$ 4. **(d)** $f(c) = 0$

6. **(a)** route (i): 10 s; route (iv): 10 s
(b) $2 \le x \le 5$; $\dfrac{4\sqrt{10}}{2.1} + \dfrac{5}{0.7} \approx 13.166$ s
(c) $0 \le x \le 2$; 10 s
(d) route (i) or (iv); 10 s

Exercise Set 5.1 (Page 321)

1.

n	2	5	10	50	100
A_n	0.853553	0.749739	0.710509	0.676095	0.671463

3.

n	2	5	10	50	100
A_n	1.57080	1.93376	1.98352	1.99935	1.99984

5.

n	2	5	10	50	100
A_n	0.583333	0.645635	0.668771	0.688172	0.690653

7.

n	2	5	10	50	100
A_n	0.433013	0.659262	0.726130	0.774567	0.780106

9.

n	2	5	10	50	100
A_n	3.71828	2.85174	2.59327	2.39772	2.37398

11.

n	2	5	10	50	100
A_n	1.04720	0.75089	0.65781	0.58730	0.57894

13. $3(x-1)$ 15. $x(x+2)$ 17. $(x+3)(x-1)$

Responses to True–False questions may be abridged to save space.

19. False; the limit would be the area of the circle 4π.

21. True; this is the basis of the antiderivative method.

23. area $= A(6) - A(3)$ 27. $f(x) = 2x$; $a = 2$

Exercise Set 5.2 (Page 330)

1. **(a)** $\displaystyle\int \frac{x}{\sqrt{1+x^2}}dx = \sqrt{1+x^2} + C$
(b) $\displaystyle\int (x+1)e^x dx = xe^x + C$

5. $\dfrac{d}{dx}\left[\sqrt{x^3+5}\right] = \dfrac{3x^2}{2\sqrt{x^3+5}}$, so $\displaystyle\int \frac{3x^2}{2\sqrt{x^3+5}}dx = \sqrt{x^3+5} + C$.

7. $\dfrac{d}{dx}[\sin(2\sqrt{x})] = \dfrac{\cos(2\sqrt{x})}{\sqrt{x}}$, so $\displaystyle\int \frac{\cos(2\sqrt{x})}{\sqrt{x}}dx = \sin(2\sqrt{x}) + C$.

9. **(a)** $(x^9/9) + C$ **(b)** $\frac{7}{12}x^{12/7} + C$ **(c)** $\frac{2}{9}x^{9/2} + C$

11. $\dfrac{5}{2}x^2 - \dfrac{1}{6x^4} + C$ 13. $-\dfrac{1}{2}x^{-2} - \dfrac{12}{5}x^{5/4} + \dfrac{8}{3}x^3 + C$

15. $(x^2/2) + (x^5/5) + C$ 17. $3x^{4/3} - \frac{12}{7}x^{7/3} + \frac{3}{10}x^{10/3} + C$

19. $\dfrac{x^2}{2} - \dfrac{2}{x} + \dfrac{1}{3x^3} + C$ 21. $2\ln|x| + 3e^x + C$

23. $-3\cos x - 2\tan x + C$ 25. $\tan x + \sec x + C$

27. $\tan\theta + C$ 29. $\sec x + C$ 31. $\theta - \cos\theta + C$

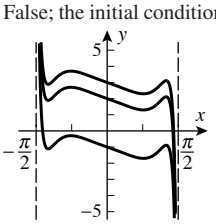
33. $\frac{1}{2}\sin^{-1}x - 3\tan^{-1}x + C$ 35. $\tan x - \sec x + C$

Responses to True–False questions may be abridged to save space.

37. True; this is Equations (1) and (2).

39. False; the initial condition is not satisfied since $y(0) = 2$.

41.



43. **(a)** $y(x) = \frac{3}{4}x^{4/3} + \frac{5}{4}$
(b) $y = -\cos t + t + 1 - \pi/3$
(c) $y(x) = \frac{2}{3}x^{3/2} + 2x^{1/2} - \frac{8}{3}$

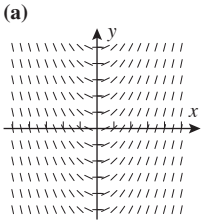
45. **(a)** $y = 4e^x - 3$ **(b)** $y = \ln|t| + 5$

47. $s(t) = 16t^2 + 20$ 49. $s(t) = 2t^{3/2} - 15$

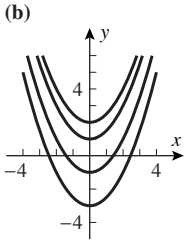
51. $f(x) = \frac{4}{15}x^{5/2} + C_1x + C_2$ 53. $y = x^2 + x - 6$

55. $f(x) = \cos x + 1$ 57. $y = x^3 - 6x + 7$

59. **(a)**

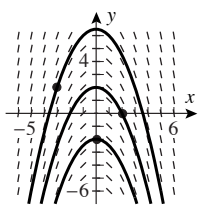


(b)

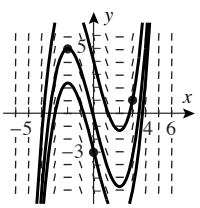


(c) $f(x) = \dfrac{x^2}{2} - 1$

61. (b)



63. (c)



67. **(b)** $\pi/2$ 69. $\tan x - x + C$

71. **(a)** $\frac{1}{2}(x - \sin x) + C$ **(b)** $\frac{1}{2}(x + \sin x) + C$

73. $v = \dfrac{1087}{\sqrt{273}}T^{1/2}$ ft/s

Exercise Set 5.3 (Page 338)

1. **(a)** $\dfrac{(x^2+1)^{24}}{24} + C$ **(b)** $-\dfrac{\cos^4 x}{4} + C$

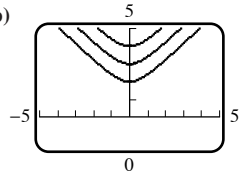
3. **(a)** $\frac{1}{4}\tan(4x+1) + C$ **(b)** $\frac{1}{6}(1+2y^2)^{3/2} + C$

5. **(a)** $-\frac{1}{2}\cot^2 x + C$ **(b)** $\frac{1}{10}(1+\sin t)^{10} + C$

7. **(a)** $\frac{2}{7}(1+x)^{7/2} - \frac{4}{5}(1+x)^{5/2} + \frac{2}{3}(1+x)^{3/2} + C$
(b) $-\cot(\sin x) + C$

9. **(a)** $\ln|\ln x| + C$ **(b)** $-\frac{1}{5}e^{-5x} + C$

11. (a) $\frac{1}{3}\tan^{-1}(x^3)+C$ (b) $\sin^{-1}(\ln x)+C$
15. $\frac{1}{40}(4x-3)^{10}+C$ **17.** $-\frac{1}{7}\cos 7x+C$ **19.** $\frac{1}{4}\sec 4x+C$
21. $\frac{1}{2}e^{2x}+C$ **23.** $\frac{1}{2}\sin^{-1}(2x)+C$ **25.** $\frac{1}{21}(7t^2+12)^{3/2}+C$
27. $\dfrac{3}{2(1-2x)^2}+C$ **29.** $-\dfrac{1}{40(5x^4+2)^2}+C$ **31.** $e^{\sin x}+C$
33. $-\frac{1}{6}e^{-2x^3}+C$ **35.** $\tan^{-1}e^x+C$ **37.** $\frac{1}{5}\cos(5/x)+C$
39. $-\frac{1}{15}\cos^5 3t+C$ **41.** $\frac{1}{2}\tan(x^2)+C$ **43.** $-\frac{1}{6}(2-\sin 4\theta)^{3/2}+C$
45. $\sin^{-1}(\tan x)+C$ **47.** $\frac{1}{6}\sec^3 2x+C$ **49.** $-e^{-x}+C$
51. $-e^{-2\sqrt{x}}+C$ **53.** $\frac{1}{6}(2y+1)^{3/2}-\frac{1}{2}(2y+1)^{1/2}+C$
55. $-\frac{1}{2}\cos 2\theta+\frac{1}{6}\cos^3 2\theta+C$ **57.** $t+\ln|t|+C$
59. $\int[\ln(e^x)+\ln(e^{-x})]\,dx=C$
61. (a) $\sin^{-1}\left(\frac{1}{3}x\right)+C$ (b) $\frac{1}{\sqrt{5}}\tan^{-1}\left(\frac{x}{\sqrt{5}}\right)+C$
(c) $\dfrac{1}{\sqrt{\pi}}\sec^{-1}\left(\dfrac{x}{\sqrt{\pi}}\right)+C$
63. $\dfrac{1}{b}\dfrac{(a+bx)^{n+1}}{n+1}+C$ **65.** $\dfrac{1}{b(n+1)}\sin^{n+1}(a+bx)+C$
67. (a) $\frac{1}{2}\sin^2 x+C_1;\ -\frac{1}{2}\cos^2 x+C_2$ (b) They differ by a constant.
69. $\frac{2}{15}(5x+1)^{3/2}-\frac{158}{15}$ **71.** $y=-\frac{1}{2}e^{2t}+\frac{13}{2}$
73. (a) $\sqrt{x^2+1}+C$ **75.** $f(x)=\frac{2}{9}(3x+1)^{3/2}+\frac{7}{9}$
77. 1280
(b)



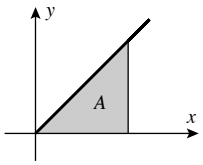
Exercise Set 5.4 (Page 350)

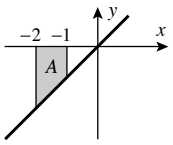
1. (a) 36 (b) 55 (c) 40 (d) 6 (e) 11 (f) 0 **3.** $\sum_{k=1}^{10} k$ **5.** $\sum_{k=1}^{10} 2k$
7. $\sum_{k=1}^{6}(-1)^{k+1}(2k-1)$ **9.** (a) $\sum_{k=1}^{50} 2k$ (b) $\sum_{k=1}^{50}(2k-1)$ **11.** 5050
13. 2870 **15.** 214,365 **17.** $\frac{3}{2}(n+1)$ **19.** $\frac{1}{4}(n-1)^2$
Responses to True–False questions may be abridged to save space.
21. True; by parts (a) and (c) of Theorem 5.4.2.
23. False; consider $[a,b]=[-1,0]$.
25. (a) $\left(2+\dfrac{3}{n}\right)^4\cdot\dfrac{3}{n}, \left(2+\dfrac{6}{n}\right)^4\cdot\dfrac{3}{n}, \left(2+\dfrac{9}{n}\right)^4\cdot\dfrac{3}{n}, \left(2+\dfrac{3(n-1)}{n}\right)^4\cdot\dfrac{3}{n}, (2+3)^4\cdot\dfrac{3}{n}$ (b) $\sum_{k=0}^{n-1}\left(2+k\cdot\dfrac{3}{n}\right)^4\dfrac{3}{n}$
27. (a) 46 (b) 52 (c) 58 **29.** (a) $\frac{\pi}{4}$ (b) 0 (c) $-\frac{\pi}{4}$
31. (a) 0.7188, 0.7058, 0.6982 (b) 0.6928, 0.6931, 0.6931
(c) 0.6688, 0.6808, 0.6882
33. (a) 4.8841, 5.1156, 5.2488 (b) 5.3471, 5.3384, 5.3346
(c) 5.6841, 5.5156, 5.4088
35. $\frac{15}{4}$ **37.** 18 **39.** 320 **41.** $\frac{15}{4}$ **43.** 18 **45.** 16 **47.** $\frac{1}{3}$ **49.** 0
51. $\frac{2}{3}$ **53.** (b) $\frac{1}{4}(b^4-a^4)$
55. $\dfrac{n^2+2n}{4}$ if n is even; $\dfrac{(n+1)^2}{4}$ if n is odd **57.** $3^{17}-3^4$ **59.** $-\frac{399}{400}$
61. (b) $\frac{1}{2}$ **65.** (a) yes (b) yes

Exercise Set 5.5 (Page 360)

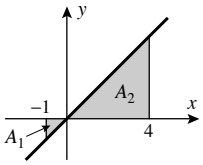
1. (a) $\frac{71}{6}$ (b) 2 **3.** (a) $-\frac{117}{16}$ (b) 3 **5.** $\int_{-1}^{2} x^2\,dx$
7. $\int_{-3}^{3} 4x(1-3x)\,dx$
9. (a) $\lim_{\max\Delta x_k\to 0}\sum_{k=1}^{n} 2x_k^*\Delta x_k;\ a=1,b=2$
(b) $\lim_{\max\Delta x_k\to 0}\sum_{k=1}^{n}\dfrac{x_k^*}{x_k^*+1}\Delta x_k;\ a=0,b=1$

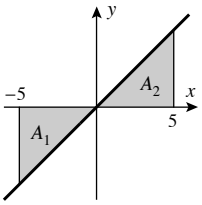
13. (a) $A=\frac{9}{2}$ (b) $-A=-\frac{3}{2}$



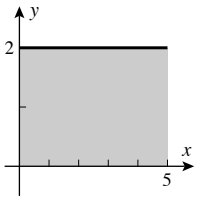


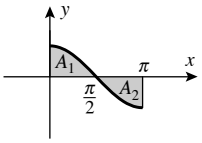
(c) $-A_1+A_2=\frac{15}{2}$ (d) $-A_1+A_2=0$



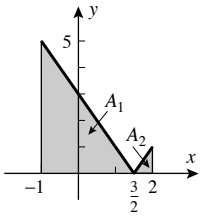


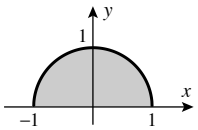
15. (a) $A=10$ (b) $A_1-A_2=0$ by symmetry





(c) $A_1+A_2=\frac{13}{2}$ (d) $\pi/2$

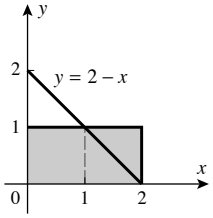


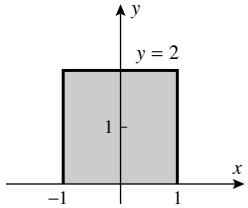


17. (a) 2 (b) 4 (c) 10 (d) 10
19. (a) 0.8 (b) −2.6 (c) −1.8 (d) −0.3
21. -1 **23.** 3 **25.** -4 **27.** $(1+\pi)/2$
Responses to True–False questions may be abridged to save space.
29. False; see Theorem 5.5.8(a).
31. False; consider $f(x)=x-2$ on $[0,3]$.
33. (a) negative (b) positive **37.** $\frac{25}{2}\pi$ **39.** $\frac{5}{2}$
45. (a) integrable (b) integrable (c) not integrable (d) integrable

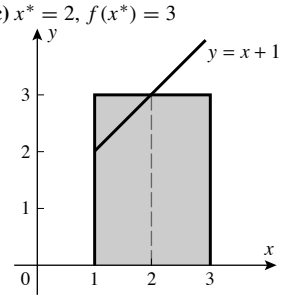
Exercise Set 5.6 (Page 373)

1. (a) $\int_0^2(2-x)\,dx=2$ (b) $\int_{-1}^{1}2\,dx=4$ (c) $\int_1^3(x+1)\,dx=6$
3. (a) $x^*=f(x^*)=1$ (b) x^* is any point in $[-1,1]$, $f(x^*)=2$





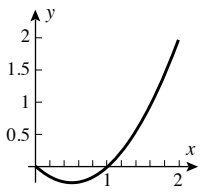
(c) $x^* = 2$, $f(x^*) = 3$

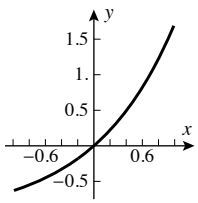


5. $\frac{65}{4}$ **7.** 14 **9.** $\frac{3}{2}$ **11.** **(a)** $\frac{4}{3}$ **(b)** -7 **13.** 48 **15.** 3 **17.** $\frac{844}{5}$
19. 0 **21.** $\sqrt{2}$ **23.** $5e^3 - 10$ **25.** $\pi/4$ **27.** $\pi/12$ **29.** -12
31. **(a)** 5/2 **(b)** $2 - \frac{\sqrt{2}}{2}$ **33.** **(a)** $e + (1/e) - 2$ **(b)** 1
35. **(a)** $\frac{17}{6}$ **(b)** $F(x) = \begin{cases} \dfrac{x^2}{2}, & x \le 1 \\ \dfrac{x^3}{3} + \dfrac{1}{6}, & x > 1 \end{cases}$

Responses to True–False questions may be abridged to save space.

37. False; since $|x|$ is continuous, it has an antiderivative.
39. True; by the Fundamental Theorem of Calculus.
41. 0.6659; $\frac{2}{3}$ **43.** 3.1060; $2 \tan 1$ **45.** 12 **47.** $\frac{9}{2}$
49. area $= 1$ **51.** area $= e + e^{-1} - 2$



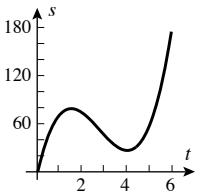


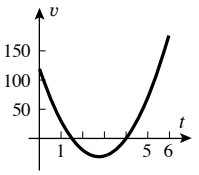
53. **(b)** degree mode, 0.93
55. **(a)** change in height from age 0 to age 10 years; inches
(b) change in radius from time $t = 1$ s to time $t = 2$ s; centimeters
(c) difference between speed of sound at 100°F and at 32°F; feet per second **(d)** change in position from time t_1 to time t_2; centimeters
57. **(a)** $3x^2 - 3$ **59.** **(a)** $\sin(x^2)$ **(b)** $e^{\sqrt{x}}$ **61.** $-x \sec x$
63. **(a)** 0 **(b)** 5 **(c)** $\frac{4}{5}$
65. **(a)** $x = 3$ **(b)** increasing on $[3, +\infty)$, decreasing on $(-\infty, 3]$
(c) concave up on $(-1, 7)$, concave down on $(-\infty, -1)$ and $(7, +\infty)$
67. **(a)** $(0, +\infty)$ **(b)** $x = 1$
69. **(a)** 120 gal **(b)** 420 gal **(c)** 2076.36 gal **71.** 1

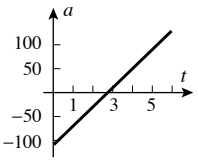
Exercise Set 5.7 (Page 382)

1. **(a)** displacement $= 3$; distance $= 3$
(b) displacement $= -3$; distance $= 3$
(c) displacement $= -\frac{1}{2}$; distance $= \frac{3}{2}$
(d) displacement $= \frac{3}{2}$; distance $= 2$
3. **(a)** 35.3 m/s **(b)** 51.4 m/s **5.** **(a)** $t^3 - t^2 + 1$ **(b)** $4t + 3 - \frac{1}{3}\sin 3t$
7. **(a)** $\frac{3}{2}t^2 + t - 4$ **(b)** $t + 1 - \ln t$
9. **(a)** displacement $= 1$ m; distance $= 1$ m
(b) displacement $= -1$ m; distance $= 3$ m
11. **(a)** displacement $= \frac{9}{4}$ m; distance $= \frac{11}{4}$ m
(b) displacement $= 2\sqrt{3} - 6$ m; distance $= 6 - 2\sqrt{3}$ m
13. 4, 13/3 **15.** 296/27, 296/27
17. **(a)** $s = 2/\pi$, $v = 1$, $|v| = 1$, $a = 0$
(b) $s = \frac{1}{2}$, $v = -\frac{3}{2}$, $|v| = \frac{3}{2}$, $a = -3$ **19.** $t \approx 1.27$ s

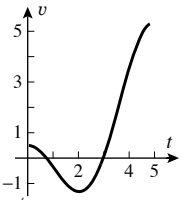
21.



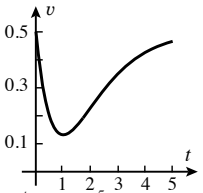




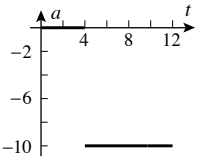
Responses to True–False questions may be abridged to save space.
23. True; if $a(t) = a_0$, then $v(t) = a_0 t + v_0$.
25. False; consider $v(t) = \sin t$ on $[0, 2\pi]$.
27. **(a)**



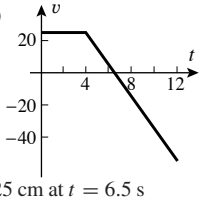
(b) $5/2 - \sin(5) + 5\cos(5)$
29. **(a)**



(b) $3/2 + 6e^{-5}$
31. **(a)**



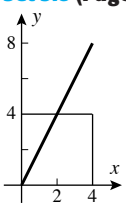
(b)



(c) 120 cm, -20 cm **(d)** 131.25 cm at $t = 6.5$ s
33. **(a)** $-\frac{121}{5}$ ft/s^2 **(b)** $\frac{70}{33}$ s **(c)** $\frac{60}{11}$ s **35.** 280 m **37.** 50 s, 5000 ft
39. **(a)** 16 ft/s, -48 ft/s **(b)** 196 ft **(c)** 112 ft/s
41. **(a)** 1 s **(b)** $\frac{1}{2}$ s **43.** **(a)** 6.122 s **(b)** 183.7 m **(c)** 6.122 s **(d)** 60 m/s
45. **(a)** 5 s **(b)** 272.5 m **(c)** 10 s **(d)** -49 m/s
(e) 12.46 s **(f)** 73.1 m/s

Exercise Set 5.8 (Page 388)

1. **(a)** 4 **(c)**



(b) 2
3. 6
5. $2/\pi$
7. $\dfrac{1}{e-1}$
9. $\dfrac{\pi}{12(\sqrt{3}-1)}$
11. $\dfrac{1 - e^{-8}}{8}$ **13.** **(a)** 5.28 **(b)** 4.305 **(c)** 4 **15.** **(a)** $-\frac{1}{6}$ **(b)** $\frac{1}{2}$

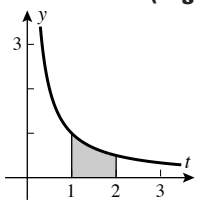
Responses to True–False questions may be abridged to save space.
19. False; let $g(x) = \cos x$; $f(x) = 0$ on $[0, 3\pi/2]$.
21. True; see Theorem 5.5.4(*b*).
23. **(a)** $\frac{263}{4}$ **(b)** 31 **25.** 1404π lb **27.** 97 cars/min **31.** 27

Exercise Set 5.9 (Page 393)

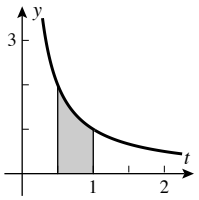
1. **(a)** $\frac{1}{2}\int_1^5 u^3\,du$ **(b)** $\frac{3}{2}\int_9^{25}\sqrt{u}\,du$ **(c)** $\dfrac{1}{\pi}\displaystyle\int_{-\pi/2}^{\pi/2}\cos u\,du$
(d) $\int_1^2 (u+1)u^5\,du$ **3.** **(a)** $\frac{1}{2}\int_{-1}^{1} e^u\,du$ **(b)** $\int_1^2 u\,du$
5. 10 **7.** 0 **9.** $\frac{1192}{15}$ **11.** $8 - (4\sqrt{2})$ **13.** $-\frac{1}{48}$ **15.** $\ln\frac{21}{13}$
17. $\pi/6$ **19.** $25\pi/6$ **21.** $\pi/8$ **23.** $2/\pi$ m **25.** 6 **27.** $\pi/18$
29. 1/21 **31.** 2 **33.** $\frac{2}{3}(\sqrt{10} - 2\sqrt{2})$ **35.** $2(\sqrt{7} - \sqrt{3})$ **37.** 1
39. 0 **41.** $(\sqrt{3} - 1)/3$ **43.** $\frac{106}{405}$ **45.** $(\ln 3)/2$ **47.** $\pi/(6\sqrt{3})$
49. $\pi/9$ **51.** **(a)** $\frac{23}{4480}$ **53.** **(a)** $\frac{5}{3}$ **(b)** $\frac{5}{3}$ **(c)** $-\frac{1}{2}$ **57.** Method 2
59. Method 1 **61.** $\approx 48{,}233{,}500{,}000$ **63.** **(a)** 0.45 **(b)** 0.461
65. $(\ln 7)/2$ **67.** **(a)** $2/\pi$ **71.** **(b)** $\frac{3}{2}$ **(c)** $\pi/4$

Exercise Set 5.10 (Page 406)

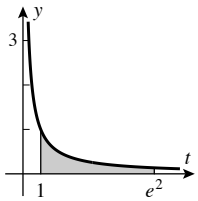
1. (a)



(b)



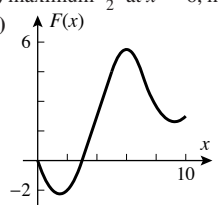
(c)



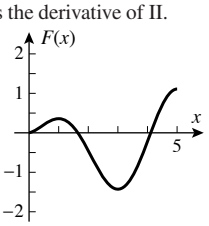
3. (a) 7 (b) -5 (c) -3 (d) 6
5. 1.603210678; magnitude of error is < 0.0063
7. (a) $x^{-1}, x > 0$ (b) $x^2, x \neq 0$ (c) $-x^2, -\infty < x < +\infty$ (d) $-x, -\infty < x < +\infty$ (e) $x^3, x > 0$ (f) $\ln x + x, x > 0$ (g) $x - \sqrt[3]{x}, -\infty < x < +\infty$ (h) $e^x/x, x > 0$
9. (a) $e^{\pi \ln 3}$ (b) $e^{\sqrt{2}\ln 2}$ 11. (a) $\sqrt{e}$ (b) e^2 13. $x^2 - x$
15. (a) $3/x$ (b) 1 17. (a) 0 (b) 0 (c) 1

Responses to True–False questions may be abridged to save space.

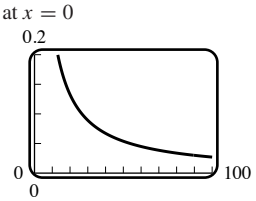
19. True; both equal $-\ln a$.
21. False; the integrand is unbounded on $[-1, e]$ and thus the integrand is undefined.
23. (a) $2x^3\sqrt{1+x^2}$ (b) $-\frac{2}{3}(x^2+1)^{3/2} + \frac{2}{5}(x^2+1)^{5/2} - \frac{4\sqrt{2}}{15}$
25. (a) $-\cos(x^3)$ (b) $-\tan^2 x$ 27. $-3\dfrac{3x-1}{9x^2+1} + 2x\dfrac{x^2-1}{x^4+1}$
29. (a) $3x^2\sin^2(x^3) - 2x\sin^2(x^2)$ (b) $\dfrac{2}{1-x^2}$
31. (a) $F(0) = 0, F(3) = 0, F(5) = 6, F(7) = 6, F(10) = 3$
(b) increasing on $\left[\frac{3}{2}, 6\right]$ and $\left[\frac{37}{4}, 10\right]$, decreasing on $\left[0, \frac{3}{2}\right]$ and $\left[6, \frac{37}{4}\right]$
(c) maximum $\frac{15}{2}$ at $x = 6$, minimum $-\frac{9}{4}$ at $x = \frac{3}{2}$
(d)



33. $F(x) = \begin{cases} (1-x^2)/2, & x < 0 \\ (1+x^2)/2, & x \geq 0 \end{cases}$ 35. $y(x) = x^2 + \ln x + 1$
37. $y(x) = \tan x + \cos x - (\sqrt{2}/2)$
39. $P(x) = P_0 + \int_0^x r(t)\,dt$ individuals 41. I is the derivative of II.
43. (a) $t = 3$ (b) $t = 1, 5$ (c) $t = 5$ (d) $t = 3$
(e) F is concave up on $\left(0, \frac{1}{2}\right)$ and $(2, 4)$, concave down on $\left(\frac{1}{2}, 2\right)$ and $(4, 5)$.
(f)

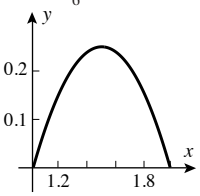


45. (a) relative maxima at $x = \pm\sqrt{4k+1}, k = 0, 1, \ldots$; relative minima at $x = \pm\sqrt{4k-1}, k = 1, 2, \ldots$
(b) $x = \pm\sqrt{2k}, k = 1, 2, \ldots$, and at $x = 0$
47. $f(x) = 2e^{2x}, a = \ln 2$ 49. 0.06



Chapter 5 Review Exercises (Page 408)

1. $-\dfrac{1}{4x^2} + \dfrac{8}{3}x^{3/2} + C$ 3. $-4\cos x + 2\sin x + C$
5. $3x^{1/3} - 5e^x + C$ 7. $\tan^{-1} x + 2\sin^{-1} x + C$
9. (a) $y(x) = 2\sqrt{x} - \frac{2}{3}x^{3/2} - \frac{4}{3}$ (b) $y(x) = \sin x - 5e^x + 5$
(c) $y(x) = \frac{5}{4} + \frac{3}{4}x^{4/3}$ (d) $y(x) = \frac{1}{2}e^{x^2} - \frac{1}{2}$
13. $\frac{1}{2}\sec^{-1}(x^2 - 1) + C$ 15. $\frac{1}{3}\sqrt{5 + 2\sin 3x} + C$
17. $-\dfrac{1}{3a}\dfrac{1}{ax^3 + b} + C$
19. (a) $\sum_{k=0}^{14}(k+4)(k+1)$ (b) $\sum_{k=5}^{19}(k-1)(k-4)$
21. $\frac{32}{3}$ 23. 0.35122, 0.42054, 0.38650
27. (a) $\frac{3}{4}$ (b) $-\frac{3}{2}$ (c) $-\frac{35}{4}$ (d) -2 (e) not enough information (f) not enough information
29. (a) $2 + (\pi/2)$ (b) $\dfrac{1}{3}(10^{3/2} - 1) - \dfrac{9\pi}{4}$ (c) $\pi/8$
31. 48 33. $\frac{2}{3}$ 35. $\frac{3}{2} - \sec 1$ 37. $\frac{5}{2}$ 39. $\frac{52}{3}$ 41. $e^3 - e$
43. area $= \frac{1}{6}$



45. $\frac{22}{3}$
47. (a) $x^3 + 1$
49. e^{x^2}
51. $|x - 1|$
53. $\dfrac{\cos x}{1 + \sin^3 x}$
57. (b) $\dfrac{\pi}{2}$; $\tan^{-1} x + \tan^{-1}\left(\dfrac{1}{x}\right) = \dfrac{\pi}{2}$
59. (a) $F(x)$ is 0 if $x = 1$, positive if $x > 1$, and negative if $x < 1$.
(b) $F(x)$ is 0 if $x = -1$, positive if $-1 < x \leq 2$, and negative if $-2 \leq x < -1$.
61. (a) $\frac{4}{3}$ (b) $e - 1$ 63. $\frac{3}{10}$ 67. $\frac{1}{4}t^4 - \frac{2}{3}t^3 + t + 1$
69. $t^2 - 3t + 7$ 71. 12 m, 20 m 73. $\frac{1}{3}$ m, $\frac{10}{3} - 2\sqrt{2}$ m
75. displacement $= -6$ m; distance $= \frac{13}{2}$ m
77. (a) 2.2 s (b) 387.2 ft 79. $v_0/2$ ft/s 81. $\frac{121}{5}$ 83. $\frac{2}{3}$ 85. 0
87. $2 - 2/\sqrt{e}$ 89. (a) e^2 (b) $e^{1/3}$

Chapter 5 Making Connections (Page 412)

Where correct answers to a Making Connections exercise may vary, no answer is listed. Sample answers for these questions are available on the Book Companion Site.

1. (b) $b^2 - a^2$ 2. 16/3 3. 12
4. (a) the sum for f is m times that for g
(b) $m\displaystyle\int_0^1 g(x)\,dx = \int_0^m f(x)\,du$
5. (a) they are equal (b) $\displaystyle\int_2^3 g(x)\,dx = \int_4^9 f(u)\,du$

Exercise Set 6.1 (Page 419)

1. 9/2 3. 1 5. (a) 4/3 (b) 4/3 7. 49/192 9. 1/2 11. $\sqrt{2}$
13. $\frac{1}{2}$ 15. $\pi - 1$ 17. 24 19. 37/12 21. $4\sqrt{2}$ 23. $\frac{1}{2}$
25. $\ln 2 - \frac{1}{2}$

Responses to True–False questions may be abridged to save space.

27. True; use area Formula (1) with $f(x) = g(x) + c$.
29. True; the integrand must assume both positive and negative values. By the Intermediate-Value Theorem, the integrand must be equal to 0 somewhere in $[a, b]$.
31. $k \approx 0.9973$ 33. 9152/105 35. $9/\sqrt[3]{4}$
37. (a) 4/3 (b) $m = 2 - \sqrt[3]{4}$ 39. 1.180898334
41. 0.4814, 2.3639, 1.1897 43. 2.54270

45. racer 1's lead over racer 2 at time $t = 0$
47. the increase in population from 1960 to 2010 **49.** $a^2/6$

Exercise Set 6.2 (Page 428)

1. 8π **3.** $13\pi/6$ **5.** $(1-\sqrt{2}/2)\pi$ **7.** 8π **9.** $32/5$ **11.** $256\pi/3$ **13.** $2048\pi/15$ **15.** 4π **17.** $\pi^2/4$ **19.** $3/5$ **21.** 2π **23.** $72\pi/5$ **25.** $\frac{\pi}{2}(e^2-1)$

Responses to True–False questions may be abridged to save space.

27. False; see the solids associated with Exercises 9 and 10.
29. False; see Example 2 where the cross-sectional area is a linear function of x.
31. $4\pi ab^2/3$ **33.** π **35.** $\int_a^b \pi[f(x)-k]^2\,dx$ **37.** **(b)** $40\pi/3$ **39.** $648\pi/5$ **41.** $\pi/2$ **43.** $\pi/15$ **45.** $40{,}000\pi$ ft^3 **47.** $1/30$ **49.** **(a)** $2\pi/3$ **(b)** $16/3$ **(c)** $4\sqrt{3}/3$ **51.** 0.710172176 **53.** π **57.** **(b)** left ≈ 11.157; right ≈ 11.771; $V \approx$ average $= 11.464$ cm^3

59. $V = \begin{cases} 3\pi h^2, & 0 \le h < 2 \\ \frac{1}{3}\pi(12h^2 - h^3 - 4), & 2 \le h \le 4 \end{cases}$ **61.** $\frac{2}{3}r^3\tan\theta$ **63.** $16r^3/3$

Exercise Set 6.3 (Page 436)

1. $15\pi/2$ **3.** $\pi/3$ **5.** $2\pi/5$ **7.** 4π **9.** $20\pi/3$ **11.** $\pi\ln 2$ **13.** $\pi/2$ **15.** $\pi/5$

Responses to True–False questions may be abridged to save space.

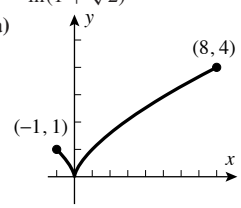
17. True; this is a restatement of Formula (1).
19. True; see Formula (2).
21. $2\pi e^2$ **23.** 1.73680 **25.** **(a)** $7\pi/30$ **(b)** easier
27. **(a)** $\int_0^1 2\pi(1-x)x\,dx$ **(b)** $\int_0^1 2\pi(1+y)(1-y)\,dy$
29. $7\pi/4$ **31.** $\pi r^2 h/3$ **33.** $V = \frac{4}{3}\pi(L/2)^3$ **35.** $b = 1$

Exercise Set 6.4 (Page 441)

1. $L = \sqrt{5}$ **3.** $(85\sqrt{85}-8)/243$ **5.** $\frac{1}{27}(80\sqrt{10}-13\sqrt{13})$ **7.** $\frac{17}{6}$

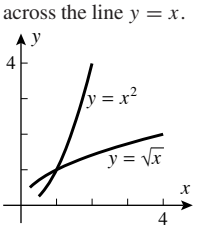
Responses to True–False questions may be abridged to save space.

9. False; f' is undefined at the endpoints ± 1.
11. True; if $f(x) = mx + c$ over $[a, b]$, then $L = \sqrt{1+m^2}(b-a)$, which is equal to the given sum.
13. $L = \ln(1+\sqrt{2})$
15. **(a)**



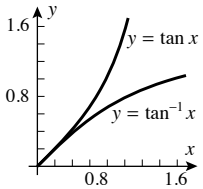
(b) dy/dx does not exist at $x = 0$.
(c) $L = (13\sqrt{13} + 80\sqrt{10} - 16)/27$

17. **(a)** They are mirror images across the line $y = x$.



(b) $\int_{1/2}^{2}\sqrt{1+4x^2}\,dx$, $\int_{1/4}^{4}\sqrt{1+\frac{1}{4x}}\,dx$, $x = \sqrt{u}$ transforms the first integral into the second.
(c) $\int_{1/4}^{4}\sqrt{1+\frac{1}{4y}}\,dy$, $\int_{1/2}^{2}\sqrt{1+4y^2}\,dy$
(d) 4.0724, 4.0716
(e) The first: Both are underestimates of the arc length, so the larger one is more accurate.
(f) 4.0724, 4.0662 **(g)** 4.0729

19. **(a)** They are mirror images across the line $y = x$.



(b) $\int_0^{\pi/3}\sqrt{1+\sec^4 x}\,dx$, $\int_0^{\sqrt{3}}\sqrt{1+\frac{1}{(1+x^2)^2}}\,dx$, $x = \tan^{-1}u$ transforms the first integral into the second.
(c) $\int_0^{\sqrt{3}}\sqrt{1+\frac{1}{(1+y^2)^2}}\,dy$, $\int_0^{\pi/3}\sqrt{1+\sec^4 y}\,dy$
(d) 2.0566, 2.0567
(e) The second: Both are underestimates of the arc length, so the larger one is more accurate. **(f)** 2.0509, 2.0571 **(g)** 2.0570
23. 4354 ft **25.** 196.31 yards **27.** $(2\sqrt{2}-1)/3$
29. π **31.** $L = \sqrt{2}(e^{\pi/2}-1)$ **33.** **(b)** 9.69 **(c)** 5.16 cm

Exercise Set 6.5 (Page 447)

1. $35\pi\sqrt{2}$ **3.** 8π **5.** $40\pi\sqrt{82}$ **7.** 24π **9.** $16\pi/9$ **11.** $16,911\pi/1024$ **13.** $S \approx 14.42$ **15.** $S \approx 22.94$

Responses to True–False questions may be abridged to save space.

17. True; use Formula (1) with $r_1 = 0, r_2 = r, l = \sqrt{r^2+h^2}$.
19. True; the sum telescopes to the surface area of a cylinder.
21. 14.39 **23.** $S = \int_a^b 2\pi[f(x)+k]\sqrt{1+[f'(x)]^2}\,dx$
33. $\frac{8}{3}\pi(17\sqrt{17}-1)$ **35.** $\frac{\pi}{24}(17\sqrt{17}-1)$

Exercise Set 6.6 (Page 456)

1. 7.5 ft·lb **3.** $d = 7/4$ **5.** 100 ft·lb **7.** 160 J **9.** 20 lb/ft

Responses to True–False questions may be abridged to save space.

11. False; the work done is the same.
13. True; joules **15.** $47{,}385\pi$ ft·lb **17.** 261,600 J
19. **(a)** 926,640 ft·lb **(b)** hp of motor $= 0.468$ **21.** 75,000 ft·lb
23. 120,000 ft·tons **25.** **(a)** $2{,}400{,}000{,}000/x^2$ lb
(b) $(9.6\times 10^{10})/(x+4000)^2$ lb **(c)** 2.5344×10^{10} ft·lb
27. $v_f = 100$ m/s
29. **(a)** decrease of 4.5×10^{14} J **(b)** ≈ 0.107 **(c)** ≈ 8.24 bombs

Exercise Set 6.7 (Page 465)

1. **(a)** positive: m_2 is at the fulcrum, so it can be ignored; masses m_1 and m_3 are equidistant from position 5, but $m_1 < m_3$, so the beam will rotate clockwise. **(b)** The fulcrum should be placed $\frac{50}{7}$ units to the right of m_1.
3. $\left(\frac{1}{2},\frac{1}{2}\right)$ **5.** $\left(1,\frac{1}{2}\right)$ **7.** $\left(\frac{2}{3},\frac{1}{3}\right)$ **9.** $\left(\frac{5}{14},\frac{38}{35}\right)$ **11.** $\left(\frac{2}{3},\frac{1}{3}\right)$ **13.** $\left(-\frac{1}{2},4\right)$ **15.** $\left(\frac{1}{2},\frac{8}{5}\right)$ **17.** $\left(\frac{9}{20},\frac{9}{20}\right)$ **19.** $\left(\frac{49}{48},\frac{7}{3}-\ln 2\right)$ **23.** $\frac{4}{3}$; $\left(\frac{3}{5},\frac{3}{8}\right)$ **25.** 3; $\left(0,\frac{2}{3}\right)$ **27.** 8; $\left(\frac{\pi}{2},\frac{\pi}{8}\right)$
29. $\ln 4 - 1$; $\left(\frac{4\ln 4-3}{4\ln 4-4}, \frac{(\ln 2)^2+1-\ln 4}{\ln 4-1}\right)$

Responses to True–False questions may be abridged to save space.

31. True; use symmetry. **33.** True; use symmetry.
35. $\left(\frac{2a}{3},0\right)$ **37.** $(\bar{x},\bar{y}) = \left(0,\frac{(a+2b)c}{3(a+b)}\right)$
41. $2\pi^2 abk$ **43.** $(a/3, b/3)$

Exercise Set 6.8 (Page 472)

1. **(a)** $F = 31{,}200$ lb; $P = 312$ lb/ft^2
(b) $F = 2{,}452{,}500$ N; $P = 98.1$ kPa
3. 499.2 lb **5.** 8.175×10^5 N **7.** 1,098,720 N **9.** yes
11. $\rho a^3/\sqrt{2}$ lb

Responses to True–False questions may be abridged to save space.

13. True; this is a consequence of inequalities (4).

15. False; by Equation (7) the force can be arbitrarily large for a fixed volume of water.
17. 61,748 lb 19. 9.81×10^9 N 21. **(b)** $80\rho_0$ lb/min

Exercise Set 6.9 (Page 482)

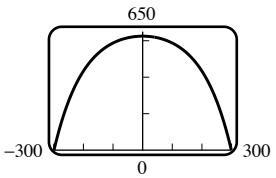
1. **(a)** ≈ 10.0179 **(b)** ≈ 3.7622 **(c)** $15/17 \approx 0.8824$ **(d)** ≈ -1.4436 **(e)** ≈ 1.7627 **(f)** ≈ 0.9730
3. **(a)** $\frac{4}{3}$ **(b)** $\frac{5}{4}$ **(c)** $\frac{312}{313}$ **(d)** $-\frac{63}{16}$
5.

	$\sinh x_0$	$\cosh x_0$	$\tanh x_0$	$\coth x_0$	$\operatorname{sech} x_0$	$\operatorname{csch} x_0$
(a)	2	$\sqrt{5}$	$2/\sqrt{5}$	$\sqrt{5}/2$	$1/\sqrt{5}$	1/2
(b)	3/4	5/4	3/5	5/3	4/5	4/3
(c)	4/3	5/3	4/5	5/4	3/5	3/4

9. $4\cosh(4x-8)$ 11. $-\frac{1}{x}\operatorname{csch}^2(\ln x)$
13. $\frac{1}{x^2}\operatorname{csch}\left(\frac{1}{x}\right)\coth\left(\frac{1}{x}\right)$ 15. $\frac{2+5\cosh(5x)\sinh(5x)}{\sqrt{4x+\cosh^2(5x)}}$
17. $x^{5/2}\tanh(\sqrt{x})\operatorname{sech}^2(\sqrt{x}) + 3x^2\tanh^2(\sqrt{x})$
19. $\frac{1}{\sqrt{9+x^2}}$ 21. $\frac{1}{(\cosh^{-1}x)\sqrt{x^2-1}}$ 23. $-\frac{(\tanh^{-1}x)^{-2}}{1-x^2}$
25. $\frac{\sinh x}{|\sinh x|} = \begin{cases} 1, & x>0 \\ -1, & x<0 \end{cases}$ 27. $-\frac{e^x}{2x\sqrt{1-x}} + e^x\operatorname{sech}^{-1}x$
29. $\frac{1}{7}\sinh^7 x + C$ 31. $\frac{2}{3}(\tanh x)^{3/2} + C$ 33. $\ln(\cosh x) + C$
35. 37/375 37. $\frac{1}{3}\sinh^{-1}3x + C$ 39. $-\operatorname{sech}^{-1}(e^x) + C$
41. $-\operatorname{csch}^{-1}|2x| + C$ 43. $\frac{1}{2}\ln 3$

Responses to True–False questions may be abridged to save space.

45. True; see Figure 6.9.1 47. True; $f(x) = \sinh x$
49. 16/9 51. 5π 53. $\frac{3}{4}$
55. **(a)** $+\infty$ **(b)** $-\infty$ **(c)** 1 **(d)** -1 **(e)** $+\infty$ **(f)** $+\infty$
63. $|u|<1$: $\tanh^{-1}u + C$; $|u|>1$: $\tanh^{-1}(1/u) + C$
65. **(a)** $\ln 2$ **(b)** 1/2 71. 405.9 ft
73. **(a)**



(b) 1480.2798 ft **(c)** ± 283.6249 ft **(d)** 82°

75. **(b)** 14.44 m **(c)** $15\ln 3 \approx 16.48$ m

Chapter 6 Review Exercises (Page 485)

7. **(a)** $\int_a^b (f(x)-g(x))\,dx + \int_b^c (g(x)-f(x))\,dx + \int_c^d (f(x)-g(x))\,dx$ **(b)** 11/4
9. $4352\pi/105$ 11. $3/2 + \ln 4$ 13. 9 15. $\frac{\pi}{6}\left(65^{3/2} - 37^{3/2}\right)$
17. $3\sqrt{3}$ 19. **(a)** $W = \frac{1}{16}$ J **(b)** 5 m 21. $\left(\frac{8}{5}, 0\right)$
23. **(a)** $F = \int_0^1 \rho x 3\,dx$ N **(b)** $F = \int_1^4 \rho(1+x)2x\,dx$ lb/ft^2
(c) $F = \int_{-10}^{0} 9810|y|2\sqrt{\frac{125}{8}(y+10)}\,dy$ N

Chapter 6 Making Connections (Page 487)

Where correct answers to a Making Connections exercise may vary, no answer is listed. Sample answers for these questions are available on the Book Companion Site.

1. **(a)** πA_1 **(b)** $a = \frac{A_1}{2A_2}$ 2. 1,010,807 ft·lb 3. $\int_0^a 2\pi r f(r)\,dr$

Exercise Set 7.1 (Page 491)

1. $-2(x-2)^4 + C$ 3. $\frac{1}{2}\tan(x^2) + C$ 5. $-\frac{1}{3}\ln(2+\cos 3x) + C$
7. $\cosh(e^x) + C$ 9. $e^{\tan x} + C$ 11. $-\frac{1}{30}\cos^6 5x + C$
13. $\ln(e^x + \sqrt{e^{2x}+4}) + C$ 15. $2e^{\sqrt{x-1}} + C$ 17. $2\sinh\sqrt{x} + C$
19. $-\frac{2}{\ln 3}3^{-\sqrt{x}} + C$ 21. $\frac{1}{2}\coth\frac{2}{x} + C$ 23. $-\frac{1}{4}\ln\left|\frac{2+e^{-x}}{2-e^{-x}}\right| + C$
25. $\sin^{-1}(e^x) + C$ 27. $-\frac{1}{2}\cos(x^2) + C$ 29. $-\frac{1}{\ln 16}4^{-x^2} + C$
31. **(a)** $\frac{1}{2}\sin^2 x + C$ **(b)** $-\frac{1}{4}\cos 2x + C$
33. **(b)** $\ln\left|\tan\frac{x}{2}\right| + C$ **(c)** $\ln\left|\cot\left(\frac{\pi}{4} - \frac{x}{2}\right)\right| + C$

Exercise Set 7.2 (Page 498)

1. $-e^{-2x}\left(\frac{x}{2} + \frac{1}{4}\right) + C$ 3. $x^2e^x - 2xe^x + 2e^x + C$
5. $-\frac{1}{3}x\cos 3x + \frac{1}{9}\sin 3x + C$ 7. $x^2\sin x + 2x\cos x - 2\sin x + C$
9. $\frac{x^2}{2}\ln x - \frac{x^2}{4} + C$ 11. $x(\ln x)^2 - 2x\ln x + 2x + C$
13. $x\ln(3x-2) - x - \frac{2}{3}\ln(3x-2) + C$ 15. $x\sin^{-1}x + \sqrt{1-x^2} + C$
17. $x\tan^{-1}(3x) - \frac{1}{6}\ln(1+9x^2) + C$ 19. $\frac{1}{2}e^x(\sin x - \cos x) + C$
21. $(x/2)[\sin(\ln x) - \cos(\ln x)] + C$ 23. $x\tan x + \ln|\cos x| + C$
25. $\frac{1}{2}x^2e^{x^2} - \frac{1}{2}e^{x^2} + C$ 27. $\frac{1}{4}(3e^4+1)$ 29. $(2e^3+1)/9$
31. $3\ln 3 - 2$ 33. $\frac{5\pi}{6} - \sqrt{3} + 1$ 35. $-\pi/2$
37. $\frac{1}{3}\left(2\sqrt{3}\pi - \frac{\pi}{2} - 2 + \ln 2\right)$

Responses to True–False questions may be abridged to save space.

39. True; see the subsection "Guidelines for Integration by Parts."
41. False; e^x isn't a factor of the integrand.
43. $2(\sqrt{x}-1)e^{\sqrt{x}} + C$ 47. $-(3x^2+5x+7)e^{-x} + C$
49. $(4x^3-6x)\sin 2x - (2x^4-6x^2+3)\cos 2x + C$
51. $\frac{e^{ax}}{a^2+b^2}(a\sin bx - b\cos bx) + C$ 53. **(a)** $\frac{1}{2}\sin^2 x + C$
55. **(a)** $A = 1$ **(b)** $V = \pi(e-2)$ 57. $V = 2\pi^2$ 59. $\pi^3 - 6\pi$
61. **(a)** $-\frac{1}{4}\sin^3 x\cos x - \frac{3}{8}\sin x\cos x + \frac{3}{8}x + C$ **(b)** 8/15
65. **(a)** $\frac{1}{3}\tan^3 x - \tan x + x + C$ **(b)** $\frac{1}{3}\sec^2 x\tan x + \frac{2}{3}\tan x + C$
(c) $x^3e^x - 3x^2e^x + 6xe^x - 6e^x + C$
69. $(x+1)\ln(x+1) - x + C$ 71. $\frac{1}{2}(x^2+1)\tan^{-1}x - \frac{1}{2}x + C$

Exercise Set 7.3 (Page 506)

1. $-\frac{1}{4}\cos^4 x + C$ 3. $\frac{\theta}{2} - \frac{1}{20}\sin 10\theta + C$
5. $\frac{1}{3a}\cos^3 a\theta - \cos a\theta + C$ 7. $\frac{1}{2a}\sin^2 ax + C$
9. $\frac{1}{3}\sin^3 t - \frac{1}{5}\sin^5 t + C$ 11. $\frac{1}{8}x - \frac{1}{32}\sin 4x + C$
13. $-\frac{1}{10}\cos 5x + \frac{1}{2}\cos x + C$ 15. $-\frac{1}{3}\cos(3x/2) - \cos(x/2) + C$
17. 2/3 19. 0 21. 7/24 23. $\frac{1}{2}\tan(2x-1) + C$
25. $\ln|\cos(e^{-x})| + C$ 27. $\frac{1}{4}\ln|\sec 4x + \tan 4x| + C$
29. $\frac{1}{3}\tan^3 x + C$ 31. $\frac{1}{16}\sec^4 4x + C$ 33. $\frac{1}{7}\sec^7 x - \frac{1}{5}\sec^5 x + C$
35. $\frac{1}{4}\sec^3 x\tan x - \frac{5}{8}\sec x\tan x + \frac{3}{8}\ln|\sec x + \tan x| + C$
37. $\frac{1}{3}\sec^3 t + C$ 39. $\tan x + \frac{1}{3}\tan^3 x + C$
41. $\frac{1}{8}\tan^2 4x + \frac{1}{4}\ln|\cos 4x| + C$ 43. $\frac{2}{3}\tan^{3/2}x + \frac{2}{7}\tan^{7/2}x + C$
45. $\frac{1}{2} - \frac{\pi}{8}$ 47. $-\frac{1}{2} + \ln 2$ 49. $-\frac{1}{5}\csc^5 x + \frac{1}{3}\csc^3 x + C$
51. $-\frac{1}{2}\csc^2 x - \ln|\sin x| + C$

Responses to True–False questions may be abridged to save space.

53. True; $\int \sin^5 x\cos^8 x\,dx = \int \sin x(1-\cos^2 x)^2\cos^8 x\,dx = -\int (1-u^2)^2u^8\,du = -\int (u^8 - 2u^{10} + u^{12})\,du$
55. False; use this identity to help evaluate integrals of the form $\int \sin mx\cos nx\,dx$.
59. $L = \ln(\sqrt{2}+1)$ 61. $V = \pi/2$

67. $-\dfrac{1}{\sqrt{a^2+b^2}}\ln\left[\dfrac{\sqrt{a^2+b^2}+a\cos x-b\sin x}{a\sin x+b\cos x}\right]+C$

69. **(a)** $\frac{2}{3}$ **(b)** $3\pi/16$ **(c)** $\frac{8}{15}$ **(d)** $5\pi/32$

Exercise Set 7.4 (Page 513)

1. $2\sin^{-1}(x/2)+\frac{1}{2}x\sqrt{4-x^2}+C$ 3. $8\sin^{-1}\left(\dfrac{x}{4}\right)-\dfrac{x\sqrt{16-x^2}}{2}+C$

5. $\frac{1}{16}\tan^{-1}(x/2)+\dfrac{x}{8(4+x^2)}+C$ 7. $\sqrt{x^2-9}-3\sec^{-1}(x/3)+C$

9. $-(x^2+2)\sqrt{1-x^2}+C$ 11. $\dfrac{\sqrt{9x^2-4}}{4x}+C$ 13. $\dfrac{x}{\sqrt{1-x^2}}+C$

15. $\ln|\sqrt{x^2-9}+x|+C$ 17. $\dfrac{-x}{9\sqrt{4x^2-9}}+C$

19. $\frac{1}{2}\sin^{-1}(e^x)+\frac{1}{2}e^x\sqrt{1-e^{2x}}+C$ 21. $2/3$ 23. $(\sqrt{3}-\sqrt{2})/2$

25. $\dfrac{10\sqrt{3}+18}{243}$

Responses to True–False questions may be abridged to save space.

27. True; with the restriction $-\pi/2\le\theta\le\pi/2$, this substitution gives $\sqrt{a^2-x^2}=a\cos\theta$ and $dx=a\cos\theta\,d\theta$.

29. False; use the substitution $x=a\sec\theta$ with $0\le\theta<\pi/2$ $(x\ge a)$ or $\pi/2\le\theta<\pi$ $(x\le -a)$.

31. $\frac{1}{2}\ln(x^2+4)+C$

33. $L=\sqrt{5}-\sqrt{2}+\ln\dfrac{2+2\sqrt{2}}{1+\sqrt{5}}$ 35. $S=\dfrac{\pi}{32}\left[18\sqrt{5}-\ln(2+\sqrt{5})\right]$

37. $\tan^{-1}(x-2)+C$ 39. $\sin^{-1}\left(\dfrac{x-1}{2}\right)+C$

41. $\ln(x-3+\sqrt{(x-3)^2+1})+C$

43. $2\sin^{-1}\left(\dfrac{x+1}{2}\right)+\dfrac{1}{2}(x+1)\sqrt{3-2x-x^2}+C$

45. $\dfrac{1}{\sqrt{10}}\tan^{-1}\sqrt{\frac{2}{5}}(x+1)+C$ 47. $\pi/6$

49. $u=\sin^2 x$, $\frac{1}{2}\displaystyle\int\sqrt{1-u^2}du$
$=\frac{1}{4}[\sin^2 x\sqrt{1-\sin^4 x}+\sin^{-1}(\sin^2 x)]+C$

51. **(a)** $\sinh^{-1}(x/3)+C$ **(b)** $\ln\left(\dfrac{\sqrt{x^2+9}}{3}+\dfrac{x}{3}\right)+C$

Exercise Set 7.5 (Page 521)

1. $\dfrac{A}{x-3}+\dfrac{B}{x+4}$ 3. $\dfrac{A}{x}+\dfrac{B}{x^2}+\dfrac{C}{x-1}$

5. $\dfrac{A}{x}+\dfrac{B}{x^2}+\dfrac{C}{x^3}+\dfrac{Dx+E}{x^2+2}$ 7. $\dfrac{Ax+B}{x^2+5}+\dfrac{Cx+D}{(x^2+5)^2}$

9. $\dfrac{1}{5}\ln\left|\dfrac{x-4}{x+1}\right|+C$ 11. $\frac{5}{2}\ln|2x-1|+3\ln|x+4|+C$

13. $\ln\left|\dfrac{x(x+3)^2}{x-3}\right|+C$ 15. $\dfrac{x^2}{2}-3x+\ln|x+3|+C$

17. $3x+12\ln|x-2|-\dfrac{2}{x-2}+C$

19. $\ln|x^2-3x-10|+C$

21. $x+\dfrac{x^3}{3}+\ln\left|\dfrac{(x-1)^2(x+1)}{x^2}\right|+C$

23. $3\ln|x|-\ln|x-1|-\dfrac{5}{x-1}+C$

25. $\dfrac{2}{x-3}+\ln|x-3|+\ln|x+1|+C$

27. $\dfrac{2}{x+1}-\dfrac{1}{2(x+1)^2}+\ln|x+1|+C$

29. $-\frac{7}{34}\ln|4x-1|+\frac{6}{17}\ln(x^2+1)+\frac{3}{17}\tan^{-1}x+C$

31. $3\tan^{-1}x+\frac{1}{2}\ln(x^2+3)+C$

33. $\dfrac{x^2}{2}-2x+\dfrac{1}{2}\ln(x^2+1)+C$

Responses to True–False questions may be abridged to save space.

35. True; partial fractions rewrites proper rational functions $P(x)/Q(x)$ as a sum of terms of the form $\dfrac{A}{(Bx+C)^k}$ and/or $\dfrac{Dx+E}{(Fx^2+Gx+H)^k}$.

37. True; $\dfrac{2x+3}{x^2}=\dfrac{2x}{x^2}+\dfrac{3}{x^2}=\dfrac{2}{x}+\dfrac{3}{x^2}$.

39. $\dfrac{1}{6}\ln\left(\dfrac{1-\sin\theta}{5+\sin\theta}\right)+C$ 41. $e^x-2\tan^{-1}\left(\frac{1}{2}e^x\right)+C$

43. $V=\pi\left(\frac{19}{5}-\frac{9}{4}\ln 5\right)$ 45. $\dfrac{1}{\sqrt{2}}\tan^{-1}\left(\dfrac{x+1}{\sqrt{2}}\right)+\dfrac{1}{x^2+2x+3}+C$

47. $\frac{1}{8}\ln|x-1|-\frac{1}{5}\ln|x-2|+\frac{1}{12}\ln|x-3|-\frac{1}{120}\ln|x+3|+C$

Exercise Set 7.6 (Page 531)

1. Formula (60): $\frac{4}{3}x+\frac{4}{9}\ln|3x-1|+C$

3. Formula (65): $\dfrac{1}{5}\ln\left|\dfrac{x}{5+2x}\right|+C$

5. Formula (102): $\frac{1}{5}(x-1)(2x+3)^{3/2}+C$

7. Formula (108): $\dfrac{1}{2}\ln\left|\dfrac{\sqrt{4-3x}-2}{\sqrt{4-3x}+2}\right|+C$

9. Formula (69): $\dfrac{1}{8}\ln\left|\dfrac{x+4}{x-4}\right|+C$

11. Formula (73): $\dfrac{x}{2}\sqrt{x^2-3}-\dfrac{3}{2}\ln|x+\sqrt{x^2-3}|+C$

13. Formula (95): $\dfrac{x}{2}\sqrt{x^2+4}-2\ln(x+\sqrt{x^2+4})+C$

15. Formula (74): $\dfrac{x}{2}\sqrt{9-x^2}+\dfrac{9}{2}\sin^{-1}\dfrac{x}{3}+C$

17. Formula (79): $\sqrt{4-x^2}-2\ln\left|\dfrac{2+\sqrt{4-x^2}}{x}\right|+C$

19. Formula (38): $-\dfrac{\sin 7x}{14}+\dfrac{1}{2}\sin x+C$

21. Formula (50): $\dfrac{x^4}{16}[4\ln x-1]+C$

23. Formula (42): $\dfrac{e^{-2x}}{13}[-2\sin(3x)-3\cos(3x)]+C$

25. Formula (62): $\dfrac{1}{2}\displaystyle\int\frac{u\,du}{(4-3u)^2}=\frac{1}{18}\left[\frac{4}{4-3e^{2x}}+\ln|4-3e^{2x}|\right]+C$

27. Formula (68): $\dfrac{2}{3}\displaystyle\int\frac{du}{u^2+4}=\frac{1}{3}\tan^{-1}\frac{3\sqrt{x}}{2}+C$

29. Formula (76): $\dfrac{1}{2}\displaystyle\int\frac{du}{\sqrt{u^2-9}}=\frac{1}{2}\ln|2x+\sqrt{4x^2-9}|+C$

31. Formula (81): $\dfrac{1}{4}\displaystyle\int\frac{u^2}{\sqrt{2-u^2}}du=-\frac{1}{4}x^2\sqrt{2-4x^4}$
$+\dfrac{1}{4}\sin^{-1}\left(\sqrt{2}x^2\right)+C$

33. Formula (26): $\displaystyle\int\sin^2 u\,du=\frac{1}{2}\ln x-\frac{1}{4}\sin(2\ln x)+C$

35. Formula (51): $\dfrac{1}{4}\displaystyle\int ue^u\,du=\frac{1}{4}(-2x-1)e^{-2x}+C$

37. $u=\sin 3x$, Formula (67): $\dfrac{1}{3}\displaystyle\int\frac{du}{u(u+1)^2}$
$=\dfrac{1}{3}\left(\dfrac{1}{\sin 3x+1}+\ln\left|\dfrac{\sin 3x}{\sin 3x+1}\right|\right)+C$

39. $u=4x^2$, Formula (70): $\dfrac{1}{8}\displaystyle\int\frac{du}{u^2-1}=\frac{1}{16}\ln\left|\frac{4x^2-1}{4x^2+1}\right|+C$

41. $u=2e^x$, Formula (74): $\dfrac{1}{2}\displaystyle\int\sqrt{3-u^2}\,du=\frac{1}{2}e^x\sqrt{3-4e^{2x}}$
$+\dfrac{3}{4}\sin^{-1}\left(\dfrac{2e^x}{\sqrt{3}}\right)+C$

43. $u=3x$, Formula (112): $\dfrac{1}{3}\displaystyle\int\sqrt{\frac{5}{3}u-u^2}\,du=\frac{18x-5}{36}\sqrt{5x-9x^2}$
$+\dfrac{25}{216}\sin^{-1}\left(\dfrac{18x-5}{5}\right)+C$

45. $u = 2x$, Formula (44): $\int u \sin u\, du = \sin 2x - 2x\cos 2x + C$

47. $u = -\sqrt{x}$, Formula (51): $2\int ue^u\, du = -2(\sqrt{x}+1)e^{-\sqrt{x}} + C$

49. $x^2 + 6x - 7 = (x+3)^2 - 16$, $u = x+3$, Formula (70):
$\displaystyle\int \frac{du}{u^2-16} = \frac{1}{8}\ln\left|\frac{x-1}{x+7}\right| + C$

51. $x^2 - 4x - 5 = (x-2)^2 - 9$, $u = x-2$, Formula (77):
$\displaystyle\int \frac{u+2}{\sqrt{9-u^2}}\,du = -\sqrt{5+4x-x^2} + 2\sin^{-1}\left(\frac{x-2}{3}\right) + C$

53. $u = \sqrt{x-2}$, $\frac{2}{5}(x-2)^{5/2} + \frac{4}{3}(x-2)^{3/2} + C$

55. $u = \sqrt{x^3+1}$,
$\displaystyle\frac{2}{3}\int u^2(u^2-1)\,du = \frac{2}{15}(x^3+1)^{5/2} - \frac{2}{9}(x^3+1)^{3/2} + C$

57. $u = x^{1/3}$, $\displaystyle\int \frac{3u^2}{u^3-u}\,du = \frac{3}{2}\ln|x^{2/3}-1| + C$

59. $u = x^{1/4}$, $\displaystyle 4\int \frac{1}{u(1-u)}\,du = 4\ln\frac{x^{1/4}}{|1-x^{1/4}|} + C$

61. $u = x^{1/6}$,
$\displaystyle 6\int \frac{u^3}{u-1}\,du = 2x^{1/2} + 3x^{1/3} + 6x^{1/6} + 6\ln|x^{1/6}-1| + C$

63. $u = \sqrt{1+x^2}$, $\displaystyle\int (u^2-1)\,du = \frac{1}{3}(1+x^2)^{3/2} - (1+x^2)^{1/2} + C$

65. $\displaystyle\int \frac{1}{1+\frac{2u}{1+u^2}+\frac{1-u^2}{1+u^2}}\cdot\frac{2}{1+u^2}\,du = \int\frac{1}{u+1}\,du$
$= \ln|\tan(x/2)+1| + C$

67. $\displaystyle\int \frac{d\theta}{1-\cos\theta} = \int \frac{1}{u^2}\,du = -\cot(\theta/2) + C$

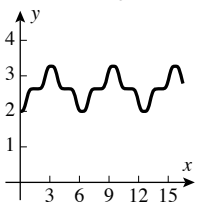
69. $\displaystyle\int \frac{1}{\frac{2u}{1+u^2}+\frac{2u}{1+u^2}\cdot\frac{1+u^2}{1-u^2}}\cdot\frac{2}{1+u^2}\,du = \int\frac{1-u^2}{2u}\,du$
$\displaystyle = \frac{1}{2}\ln|\tan(x/2)| - \frac{1}{4}\tan^2(x/2) + C$

71. $x = \dfrac{4e^2}{1+e^2}$ 73. $A = 6 + \frac{25}{2}\sin^{-1}\frac{4}{5}$ 75. $A = \frac{1}{40}\ln 9$

77. $V = \pi(\pi-2)$ 79. $V = 2\pi(1-4e^{-3})$

81. $L = \sqrt{65} + \frac{1}{8}\ln(8+\sqrt{65})$ 83. $S = 2\pi\left[\sqrt{2} + \ln(1+\sqrt{2})\right]$

85.



91. $\frac{1}{31}\cos^{31}x\sin^{31}x + C$

93. $-\frac{1}{9}\ln|1+x^{-9}| + C$

Exercise Set 7.7 (Page 544)

1. $\displaystyle\int_0^3 \sqrt{x+1}\,dx = \frac{14}{3} \approx 4.66667$
(a) $M_{10} = 4.66760$; $|E_M| \approx 0.000933996$
(b) $T_{10} = 4.66480$; $|E_T| \approx 0.00187099$
(c) $S_{20} = 4.66667$; $|E_S| \approx 9.98365 \times 10^{-7}$

3. $\displaystyle\int_0^{\pi/2} \cos x\,dx = 1$
(a) $M_{10} = 1.00103$; $|E_M| \approx 0.00102882$
(b) $T_{10} = 0.997943$; $|E_T| \approx 0.00205701$
(c) $S_{20} = 1.00000$; $|E_S| \approx 2.11547 \times 10^{-7}$

5. $\displaystyle\int_1^3 e^{-2x}\,dx = \frac{-1+e^4}{2e^6} \approx 0.0664283$
(a) $M_{10} = 0.0659875$; $|E_M| \approx 0.000440797$
(b) $T_{10} = 0.0673116$; $|E_T| \approx 0.000883357$
(c) $S_{20} = 0.0664289$; $|E_S| \approx 5.87673 \times 10^{-7}$

7. (a) $|E_M| \le \dfrac{9}{3200} = 0.0028125$
(b) $|E_T| \le \dfrac{9}{1600} = 0.005625$
(c) $|E_S| \le \dfrac{81}{10{,}240{,}000} \approx 7.91016 \times 10^{-6}$

9. (a) $|E_M| \le \dfrac{\pi^3}{19{,}200} \approx 0.00161491$
(b) $|E_T| \le \dfrac{\pi^3}{9600} \approx 0.00322982$
(c) $|E_S| \le \dfrac{\pi^5}{921{,}600{,}000} \approx 3.32053 \times 10^{-7}$

11. (a) $|E_M| \le \dfrac{1}{75e^2} \approx 0.00180447$
(b) $|E_T| \le \dfrac{2}{75e^2} \approx 0.00360894$
(c) $|E_S| \le \dfrac{1}{56{,}250e^2} \approx 2.40596 \times 10^{-6}$

13. (a) $n = 24$ (b) $n = 34$ (c) $n = 8$

15. (a) $n = 13$ (b) $n = 18$ (c) $n = 4$

17. (a) $n = 43$ (b) $n = 61$ (c) $n = 8$

Responses to True–False questions may be abridged to save space.

19. False; T_n is the average of L_n and R_n.

21. False; $S_{50} = \frac{2}{3}M_{25} + \frac{1}{3}T_{25}$

23. $g(x) = \frac{1}{24}x^2 - \frac{3}{8}x + \frac{13}{12}$

25. $S_{10} = 1.49367$; $\displaystyle\int_{-1}^{1} e^{-x^2}\,dx \approx 1.49365$

27. $S_{10} = 3.80678$; $\displaystyle\int_{-1}^{2} x\sqrt{1+x^3}\,dx \approx 3.80554$

29. $S_{10} = 0.904524$; $\displaystyle\int_0^1 \cos x^2\,dx \approx 0.904524$

31. (a) $M_{10} = 3.14243$; error $E_M \approx -0.000833331$
(b) $T_{10} = 3.13993$; error $E_T \approx 0.00166666$
(c) $S_{20} = 3.14159$; error $E_S \approx 6.20008 \times 10^{-10}$

33. $S_{14} = 0.693147984$, $|E_S| \approx 0.000000803 = 8.03 \times 10^{-7}$

35. $n = 116$ 39. 3.82019 41. 3975 ft 43. 37.9 mi 45. 9.3 L

47. (a) $\max|f''(x)| \approx 3.844880$ (b) $n = 18$ (c) 0.904741

49. (a) The maximum value of $|f^{(4)}(x)|$ is approximately 12.4282.
(b) $n = 6$ (c) $S_6 = 0.983347$

Exercise Set 7.8 (Page 554)

1. (a) improper; infinite discontinuity at $x = 3$ (b) not improper
(c) improper; infinite discontinuity at $x = 0$
(d) improper; infinite interval of integration
(e) improper; infinite interval of integration and infinite discontinuity at $x = 1$ (f) not improper

3. $\frac{1}{2}$ 5. $\ln 2$ 7. $\frac{1}{2}$ 9. $-\frac{1}{4}$ 11. $\frac{1}{3}$ 13. divergent 15. 0

17. divergent 19. divergent 21. $\pi/2$ 23. 1 25. divergent

27. $\frac{9}{2}$ 29. divergent 31. $\pi/2$

Responses to True–False questions may be abridged to save space.

33. True; see Theorem 7.8.2 with $p = \frac{4}{3} > 1$.

35. False; the integrand $\dfrac{1}{x(x-3)}$ is continuous on $[1, 2]$.

37. 2 39. 2 41. $\frac{1}{2}$

43. (a) 2.726585 (b) 2.804364 (c) 0.219384 (d) 0.504067 45. 12

47. -1 49. $\frac{1}{3}$ 51. (a) $V = \pi/2$ (b) $S = \pi[\sqrt{2} + \ln(1+\sqrt{2})]$

53. (b) $1/e$ (c) It is convergent. 55. $V = \pi$

59. $\dfrac{2\pi NI}{kr}\left(1 - \dfrac{a}{\sqrt{r^2+a^2}}\right)$ 61. Method 1

63. (b) 2.4×10^7 mi·lb 65. (a) $\dfrac{1}{s^2}$ (b) $\dfrac{2}{s^3}$ (c) $\dfrac{e^{-3s}}{s}$

69. (a) 1.047 73. 1.809

Chapter 7 Review Exercises (Page 557)

1. $\frac{2}{27}(4+9x)^{3/2}+C$ 3. $-\frac{2}{3}\cos^{3/2}\theta+C$ 5. $\frac{1}{6}\tan^3(x^2)+C$
7. **(a)** $2\sin^{-1}(\sqrt{x/2})+C$; $-2\sin^{-1}(\sqrt{2-x}/\sqrt{2})+C$; $\sin^{-1}(x-1)+C$
9. $-xe^{-x}-e^{-x}+C$ 11. $x\ln(2x+3)-x+\frac{3}{2}\ln(2x+3)+C$
13. $(4x^4-12x^2+6)\sin(2x)+(8x^3-12x)\cos(2x)+C$
15. $\frac{1}{2}\theta-\frac{1}{20}\sin 10\theta+C$ 17. $-\frac{1}{6}\cos 3x+\frac{1}{2}\cos x+C$
19. $-\frac{1}{8}\sin^3(2x)\cos 2x-\frac{3}{16}\cos 2x\sin 2x+\frac{3}{8}x+C$
21. $\frac{9}{2}\sin^{-1}(x/3)-\frac{1}{2}x\sqrt{9-x^2}+C$ 23. $\ln|x+\sqrt{x^2-1}|+C$
25. $\dfrac{x\sqrt{x^2+9}}{2}-\dfrac{9\ln(|\sqrt{x^2+9}+x|)}{2}+C$ 27. $\frac{1}{5}\ln\left|\dfrac{x-1}{x+4}\right|+C$
29. $\frac{1}{2}x^2-2x+6\ln|x+2|+C$
31. $\ln|x+2|+\dfrac{4}{x+2}-\dfrac{2}{(x+2)^2}+C$
33. **(a)** $\ln\dfrac{\sqrt{x^2-1}}{|x|}+C,\ |x|>1$ **(b)** $\ln\dfrac{\sqrt{1-x^2}}{|x|}+C,\ 0<|x|<1$
(c) $-\ln|x|+\frac{1}{2}\ln|x^2-1|+C,\ x\neq 0,\pm 1$
35. Formula (40): $-\dfrac{\cos 16x}{32}+\dfrac{\cos 2x}{4}+C$
37. Formula (113): $\frac{1}{24}(8x^2-2x-3)\sqrt{x-x^2}+\frac{1}{16}\sin^{-1}(2x-1)+C$
39. Formula (28): $\frac{1}{2}\tan 2x-x+C$
41. $\displaystyle\int_1^3\frac{1}{\sqrt{x+1}}=4-2\sqrt{2}\approx 1.17157$
(a) $M_{10}=1.17138$; $|E_M|\approx 0.000190169$
(b) $T_{10}=1.17195$; $|E_T|\approx 0.000380588$
(c) $S_{20}=1.17157$; $|E_S|\approx 8.35151\times 10^{-8}$
43. **(a)** $|E_M|\le\dfrac{1}{1600\sqrt{2}}\approx 0.000441942$
(b) $|E_T|\le\dfrac{1}{800\sqrt{2}}\approx 0.000883883$
(c) $|E_S|\le\dfrac{7}{15{,}360{,}000\sqrt{2}}\approx 3.22249\times 10^{-7}$
45. **(a)** $n=22$ **(b)** $n=30$ **(c)** $n=6$ 47. 1 49. 6
51. e^{-1} 53. $a=\pi/2$ 55. $\dfrac{x}{3\sqrt{3+x^2}}+C$ 57. $\frac{5}{12}-\frac{1}{2}\ln 2$
59. $\frac{1}{6}\sin^3 2x-\frac{1}{10}\sin^5 2x+C$
61. $\frac{2}{13}e^{2x}\cos 3x+\frac{3}{13}e^{2x}\sin 3x+C$
63. $-\frac{1}{6}\ln|x-1|+\frac{1}{15}\ln|x+2|+\frac{1}{10}\ln|x-3|+C$
65. $4-\pi$ 67. $\ln\dfrac{\sqrt{e^x+1}-1}{\sqrt{e^x+1}+1}+C$ 69. $\dfrac{\pi}{12}+\dfrac{\sqrt{3}}{2}-1$
71. $\sqrt{x^2+2x+2}+2\ln(\sqrt{x^2+2x+2}+x+1)+C$ 73. $\dfrac{1}{2(a^2+1)}$

Chapter 7 Making Connections (Page 559)

Where correct answers to a Making Connections exercise may vary, no answer is listed. Sample answers for these questions are available on the Book Companion Site.

3. **(a)** $\Gamma(1)=1$ **(c)** $\Gamma(2)=1,\Gamma(3)=2,\Gamma(4)=6$
5. **(b)** 1.37078 seconds

Exercise Set 8.1 (Page 566)

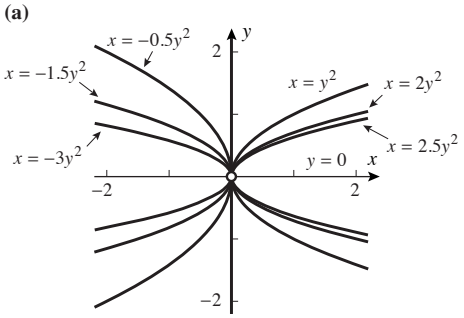
3. **(a)** first order **(b)** second order

Responses to True–False questions may be abridged to save space.

5. False; only first-order derivatives appear.
7. True; it is third order.
15. $y(x)=e^{-2x}-2e^x$ 17. $y(x)=2e^{2x}-2xe^{2x}$
19. $y(x)=\sin 2x+\cos 2x$ 21. $y(x)=-2x^2+2x+3$
23. $y(x)=2/(3-2x)$ 25. $y(x)=2/x^2$
27. **(a)** $\dfrac{dy}{dt}=ky^2,\ y(0)=y_0\ (k>0)$
(b) $\dfrac{dy}{dt}=-ky^2,\ y(0)=y_0\ (k>0)$
29. **(a)** $\dfrac{ds}{dt}=\dfrac{1}{2}s$ **(b)** $\dfrac{d^2s}{dt^2}=2\dfrac{ds}{dt}$ 33. **(b)** $L/2$

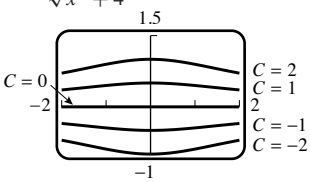
Exercise Set 8.2 (Page 575)

1. $y=Cx$ 3. $y=Ce^{-\sqrt{1+x^2}}-1,\ C\neq 0$
5. $2\ln|y|+y^2=e^x+C$ 7. $y=\ln(\sec x+C)$
9. $y=\dfrac{1}{1-C(\csc x-\cot x)},\ y=0$
11. $y^2+\sin y=x^3+\pi^2$ 13. $y^2-2y=t^2+t+3$
15. **(a)**

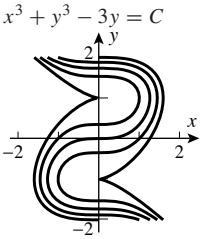


(b) $y^2=x/2$

17. $y=\dfrac{C}{\sqrt{x^2+4}}$

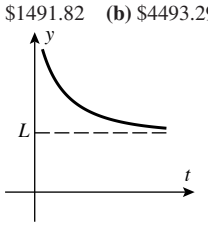


19. $x^3+y^3-3y=C$

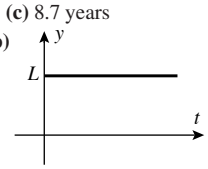


Responses to True–False questions may be abridged to save space.

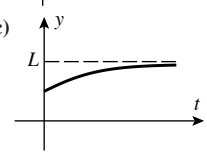
21. True; since $\dfrac{1}{f(y)}\,dy=dx$. 23. True; since $\left(\frac{1}{2}\right)^5 32=1$.
27. $y=\ln\left(\dfrac{x^2}{2}-1\right)$
29. **(a)** $y'(t)=y(t)/50,\ y(0)=10{,}000$ **(b)** $y(t)=10{,}000e^{t/50}$
(c) $50\ln 2\approx 34.66$ hr **(d)** $50\ln(4.5)\approx 75.20$ hr
31. **(a)** $\dfrac{dy}{dt}=-ky,\ k\approx 0.1810$ **(b)** $y=5.0\times 10^7 e^{-0.181t}$
(c) $\approx 219{,}000$ atoms **(d)** 12.72 days
33. $50\ln(100)\approx 230.26$ days 35. 3.30 days
39. **(b)** 70 years **(c)** 20 years **(d)** 7%
43. **(a)** no **(b)** same, $r\%$ 45. **(b)** $\ln(2)/\ln(5/4)\approx 3.106$ hr
47. **(a)** \$1491.82 **(b)** \$4493.29 **(c)** 8.7 years
51. **(a)**



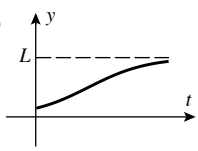
(b)



(c)



(d)

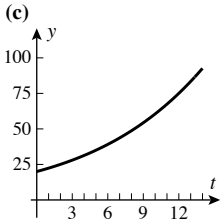


53. $y_0\approx 2,\ L\approx 8,\ k\approx 0.5493$
55. **(a)** $y_0=5$ **(b)** $L=12$ **(c)** $k=1$ **(d)** $t=0.3365$
(e) $\dfrac{dy}{dt}=\dfrac{1}{12}y(12-y),\ y(0)=5$
57. **(a)** $y=\dfrac{1000}{1+49e^{-0.115t}}$

(b)

t	0	1	2	3	4	5	6	7
$y(t)$	20	22	25	28	31	35	39	44
t	8	9	10	11	12	13	14	
$y(t)$	49	54	61	67	75	83	93	

(c)



59. **(a)** $T = 21 + 74e^{-kt}$ **(b)** 6.22 min

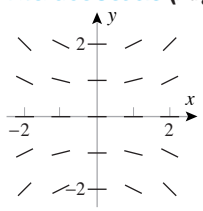
61. **(a)** $v = c \ln \dfrac{m_0}{m_0 - kt} - gt$ **(b)** 3044 m/s

63. **(a)** $h \approx (2 - 0.003979t)^2$ **(b)** 8.4 min

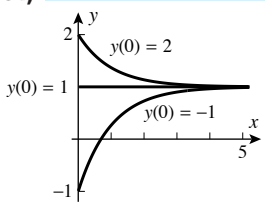
65. **(a)** $v = 128/(4t + 1)$, $x = 32 \ln(4t + 1)$

Exercise Set 8.3 (Page 584)

1.



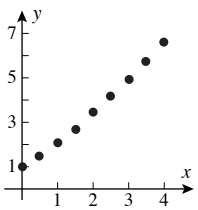
3.



5. $y \to 1$ as $x \to +\infty$

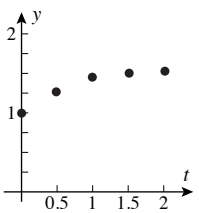
7.

n	0	1	2	3	4	5	6	7	8
x_n	0	0.5	1	1.5	2	2.5	3	3.5	4
y_n	1.00	1.50	2.07	2.71	3.41	4.16	4.96	5.82	6.72



9.

n	0	1	2	3	4
t_n	0.00	0.50	1.00	1.50	2.00
y_n	1.00	1.27	1.42	1.49	1.53



11. 0.62

Responses to True–False questions may be abridged to save space.

13. True; the derivative is positive.

15. True; $y = y_0$ is a solution if y_0 is a root of p.

17. **(b)** $y(1/2) = \sqrt{3}/2$ 19. **(b)** The x-intercept is $\ln 2$.

23. **(a)** $y' = \dfrac{2xy - y^3}{3xy^2 - x^2}$ **(c)** $xy^3 - x^2y = 2$

25. **(b)** $\lim\limits_{n \to +\infty} y_n = \lim\limits_{n \to +\infty} \left(\dfrac{n+1}{n}\right)^n = e$

Exercise Set 8.4 (Page 592)

1. $y = e^{-3x} + Ce^{-4x}$ 3. $y = e^{-x} \sin(e^x) + Ce^{-x}$ 5. $y = \dfrac{C}{\sqrt{x^2 + 1}}$

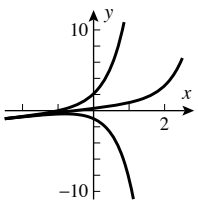
7. $y = \dfrac{x}{2} + \dfrac{3}{2x}$ 9. $y = 4e^{x^2} - 1$

Responses to True–False questions may be abridged to save space.

11. False; $y = x^2$ is a solution to $dy/dx = 2x$, but $y + y = 2x^2$ is not.

13. True; it will approach the concentration of the entering fluid.

15.



17. $\lim\limits_{x \to +\infty} y = \begin{cases} +\infty & \text{if } y_0 \geq 1/4 \\ -\infty, & \text{if } y_0 < 1/4 \end{cases}$

19. **(a)**

n	0	1	2	3	4	5
x_n	0	0.2	0.4	0.6	0.8	1.0
y_n	1	1.20	1.48	1.86	2.35	2.98

(b) $y = -(x + 1) + 2e^x$

x_n	0	0.2	0.4	0.6	0.8	1.0
$y(x_n)$	1	1.24	1.58	2.04	2.65	3.44
Absolute error	0	0.04	0.10	0.19	0.30	0.46
Percentage error	0	3	6	9	11	13

21. **(a)** $200 - 175e^{-t/25}$ oz **(b)** 136 oz 23. 25 lb

27. **(a)** $I(t) = 2 - 2e^{-2t}$ A **(b)** $I(t) \to 2$ A

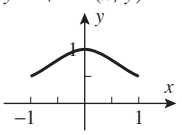
Chapter 8 Review Exercises (Page 594)

1. **(a)** linear **(b)** both **(c)** separable **(d)** neither

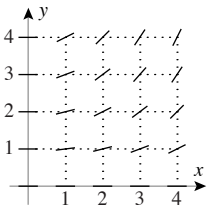
3. $y = \tan(x^3/3 + C)$ 5. $\ln|y| + y^2/2 = e^x + C$ and $y = 0$

7. $y^{-4} + 4 \ln(x/y) = 1$

9.

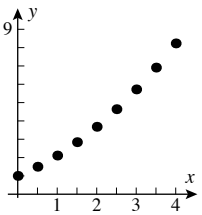


11.



13.

n	0	1	2	3	4	5	6	7	8
x_n	0	0.5	1	1.5	2	2.5	3	3.5	4
y_n	1	1.50	2.11	2.84	3.68	4.64	5.72	6.91	8.23



15. $y(1) \approx 1.00$

n	0	1	2	3	4	5
t_n	0	0.2	0.4	0.6	0.8	1.0
y_n	1.00	1.20	1.26	1.10	0.94	1.00

17. **(a)** $y \approx 2e^{0.1386t}$ **(b)** $y = 5e^{0.015t}$ **(c)** $y \approx 0.5995e^{0.5117t}$ **(d)** $y \approx 0.8706e^{0.1386t}$ 19. $y = e^{-2x} + Ce^{-3x}$

21. $y = -1 + 4e^{x^2/2}$ 23. $y = 2 \operatorname{sech} x + \frac{1}{2}(x \operatorname{sech} x + \sinh x)$

25. about 646 oz

Chapter 8 Making Connections (Page 595)

Where correct answers to a Making Connections exercise may vary, no answer is listed. Sample answers for these questions are available on the Book Companion Site.

1. **(b)** $y = 2 - 3e^{-2x}$

3. **(a)** $du/dx = (f(u) - u)/x$ **(b)** $x^2 - 2xy - y^2 = C$

Exercise Set 9.1 (Page 605)

1. **(a)** $\dfrac{1}{3^{n-1}}$ **(b)** $\dfrac{(-1)^{n-1}}{3^{n-1}}$ **(c)** $\dfrac{2n-1}{2n}$ **(d)** $\dfrac{n^2}{\pi^{1/(n+1)}}$

3. **(a)** 2, 0, 2, 0 **(b)** 1, −1, 1, −1 **(c)** $2(1+(-1)^n)$; $2+2\cos n\pi$

5. **(a)** The limit doesn't exist due to repeated oscillation between −1 and 1. **(b)** −1; 1; −1; 1; −1 **(c)** no

7. $\frac{1}{3}, \frac{2}{4}, \frac{3}{5}, \frac{4}{6}, \frac{5}{7}$; converges, $\lim\limits_{n\to+\infty} \dfrac{n}{n+2} = 1$

9. 2, 2, 2, 2, 2; converges, $\lim\limits_{n\to+\infty} 2 = 2$

11. $\dfrac{\ln 1}{1}, \dfrac{\ln 2}{2}, \dfrac{\ln 3}{3}, \dfrac{\ln 4}{4}, \dfrac{\ln 5}{5}$; converges, $\lim\limits_{n\to+\infty} \dfrac{\ln n}{n} = 0$

13. 0, 2, 0, 2, 0; diverges **15.** $-1, \frac{16}{9}, -\frac{54}{28}, \frac{128}{65}, -\frac{250}{126}$; diverges

17. $\frac{6}{2}, \frac{12}{8}, \frac{20}{18}, \frac{30}{32}, \frac{42}{50}$; converges, $\lim\limits_{n\to+\infty} \dfrac{1}{2}\left(1+\dfrac{1}{n}\right)\left(1+\dfrac{2}{n}\right) = \dfrac{1}{2}$

19. $e^{-1}, 4e^{-2}, 9e^{-3}, 16e^{-4}, 25e^{-5}$; converges, $\lim\limits_{n\to+\infty} n^2e^{-n} = 0$

21. $2, \left(\dfrac{5}{3}\right)^2, \left(\dfrac{6}{4}\right)^3, \left(\dfrac{7}{5}\right)^4, \left(\dfrac{8}{6}\right)^5$; converges, $\lim\limits_{n\to+\infty} \left[\dfrac{n+3}{n+1}\right]^n = e^2$

23. $\left\{\dfrac{2n-1}{2n}\right\}_{n=1}^{+\infty}$; converges, $\lim\limits_{n\to+\infty} \dfrac{2n-1}{2n} = 1$

25. $\left\{(-1)^{n+1}\dfrac{1}{3^n}\right\}_{n=1}^{+\infty}$; converges, $\lim\limits_{n\to+\infty} (-1)^{n+1}\dfrac{1}{3^n} = 0$

27. $\left\{(-1)^{n+1}\left(\dfrac{1}{n}-\dfrac{1}{n+1}\right)\right\}_{n=1}^{+\infty}$;

converges, $\lim\limits_{n\to+\infty} (-1)^{n+1}\left(\dfrac{1}{n}-\dfrac{1}{n+1}\right) = 0$

29. $\{\sqrt{n+1}-\sqrt{n+2}\}_{n=1}^{+\infty}$; converges, $\lim\limits_{n\to+\infty} (\sqrt{n+1}-\sqrt{n+2}) = 0$

Responses to True–False questions may be abridged to save space.

31. True; a sequence is a function whose domain is a set of integers.

33. False; for example, $\{(-1)^{n+1}\}$ diverges with terms that oscillate between 1 and −1. **35.** The limit is 0.

37. for example, $\{(-1)^n\}_{n=1}^{+\infty}$ and $\{\sin(\pi n/2)+1/n\}_{n=1}^{+\infty}$

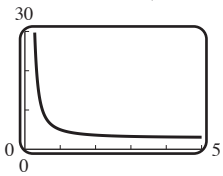
39. **(a)** 1, 2, 1, 4, 1, 6 **(b)** $a_n = \begin{cases} n, & n \text{ odd} \\ 1/2^n, & n \text{ even} \end{cases}$

(c) $a_n = \begin{cases} 1/n, & n \text{ odd} \\ 1/(n+1); & n \text{ even} \end{cases}$

(d) (a) diverges; (b) diverges; (c) $\lim\limits_{n\to+\infty} a_n = 0$

43. **(a)** $(0.5)^{2n}$ **(c)** $\lim\limits_{n\to+\infty} a_n = 0$ **(d)** $-1 \le a_0 \le 1$

45. **(a)**



(b) $\lim\limits_{n\to+\infty} (2^n+3^n)^{1/n} = 3$

49. **(a)** $N = 4$ **(b)** $N = 10$ **(c)** $N = 1000$

Exercise Set 9.2 (Page 613)

1. strictly decreasing **3.** strictly increasing **5.** strictly decreasing

7. strictly increasing **9.** strictly decreasing **11.** strictly increasing

Responses to True–False questions may be abridged to save space.

13. True; $a_{n+1} - a_n > 0$ for all n is equivalent to $a_1 < a_2 < a_3 < \cdots < a_n < \cdots$.

15. False; for example, $\{(-1)^{n+1}\} = \{1, -1, 1, -1, \ldots\}$ is bounded but diverges.

17. strictly increasing **19.** strictly increasing

21. eventually strictly increasing **23.** eventually strictly increasing

25. Yes; the limit lies in the interval [1, 2].

27. **(a)** $\sqrt{2}, \sqrt{2+\sqrt{2}}, \sqrt{2+\sqrt{2+\sqrt{2}}}$ **(e)** $L = 2$

29. **(a)** 60, $\frac{1500}{7} \approx 214.3$, $\frac{3750}{13} \approx 288.5$, $\frac{75{,}000}{251} \approx 298.8$ **(d)** $L = 300$

Exercise Set 9.3 (Page 621)

1. **(a)** $2, \frac{12}{5}, \frac{62}{25}, \frac{312}{125}; \frac{5}{2}\left(1-\left(\frac{1}{5}\right)^n\right)$; $\lim\limits_{n\to+\infty} s_n = \frac{5}{2}$ (converges)

(b) $\frac{1}{4}, \frac{3}{4}, \frac{7}{4}, \frac{15}{4}; -\frac{1}{4}(1-2^n)$; $\lim\limits_{n\to+\infty} s_n = +\infty$ (diverges)

(c) $\frac{1}{6}, \frac{1}{4}, \frac{3}{10}, \frac{1}{3}; \frac{1}{2} - \dfrac{1}{n+2}$; $\lim\limits_{n\to+\infty} s_n = \dfrac{1}{2}$ (converges)

3. $\frac{4}{7}$ **5.** 6 **7.** $\frac{1}{3}$ **9.** $\frac{1}{6}$ **11.** diverges **13.** $\frac{448}{3}$

15. **(a)** Exercise 5 **(b)** Exercise 3 **(c)** Exercise 7 **(d)** Exercise 9

Responses to True–False questions may be abridged to save space.

17. False; an infinite series converges if its sequence of *partial sums* converges.

19. True; the sequence of partial sums $\{s_n\}$ for the harmonic series satisfies $s_{2^n} > \dfrac{n+1}{2}$, so this series diverges.

21. 1 **23.** $\frac{532}{99}$ **27.** 70 m

29. **(a)** $S_n = -\ln(n+1)$; $\lim\limits_{n\to+\infty} S_n = -\infty$ (diverges)

(b) $S_n = \sum\limits_{k=2}^{n+1}\left[\ln\dfrac{k-1}{k} - \ln\dfrac{k}{k+1}\right]$, $\lim\limits_{n\to+\infty} S_n = -\ln 2$

31. **(a)** converges for $|x| < 1$; $S = \dfrac{x}{1+x^2}$

(b) converges for $|x| > 2$; $S = \dfrac{1}{x^2-2x}$

(c) converges for $x > 0$; $S = \dfrac{1}{e^x-1}$

33. $a_n = \dfrac{1}{2^{n-1}}a_1 + \dfrac{1}{2^{n-1}} + \dfrac{1}{2^{n-2}} + \cdots + \dfrac{1}{2}$, $\lim\limits_{n\to+\infty} a_n = 1$

Exercise Set 9.4 (Page 629)

1. **(a)** $\frac{4}{3}$ **(b)** $-\frac{3}{4}$

3. **(a)** $p = 3$, converges **(b)** $p = \frac{1}{2}$, diverges

(c) $p = 1$, diverges **(d)** $p = \frac{2}{3}$, diverges

5. **(a)** diverges **(b)** diverges **(c)** diverges **(d)** no information

7. **(a)** diverges **(b)** converges

9. diverges **11.** diverges **13.** diverges **15.** diverges **17.** diverges

19. converges **21.** diverges **23.** converges **25.** converges for $p > 1$

29. **(a)** diverges **(b)** diverges

Responses to True–False questions may be abridged to save space.

31. False; for example, $\sum\limits_{k=0}^{\infty} 2^{-k}$ converges to 2, but $\sum \dfrac{1}{2^{-k}} = \sum 2^k$ diverges.

33. True; see Theorem 9.4.4.

35. **(a)** $(\pi^2/2) - (\pi^4/90)$ **(b)** $(\pi^2/6) - (5/4)$ **(c)** $\pi^4/90$

37. **(d)** $\frac{1}{11} < \frac{1}{6}\pi^2 - s_{10} < \frac{1}{10}$

39. **(a)** $\displaystyle\int_n^{+\infty} \frac{1}{x^4}\,dx = \frac{1}{3n^3}$; apply Exercise 36(b) **(b)** $n = 6$

(c) $\dfrac{\pi^4}{90} \approx 1.08238$

41. converges

Exercise Set 9.5 (Page 636)

1. **(a)** converges **(b)** diverges **5.** converges **7.** converges

9. diverges **11.** converges **13.** inconclusive **15.** diverges

17. diverges **19.** converges

Responses to True–False questions may be abridged to save space.

21. False; the limit comparison test uses a limit of the quotient of corresponding terms taken from two different sequences.

23. True; use the limit comparison test with the convergent series $\sum(1/k^2)$.

25. converges **27.** converges **29.** converges **31.** converges

33. diverges **35.** converges **37.** diverges **39.** converges

41. converges **43.** diverges **45.** converges **47.** converges

49. converges

51. $u_k = \dfrac{k!}{1\cdot 3\cdot 5\cdots(2k-1)}$; $\rho = \lim\limits_{k\to+\infty}\dfrac{k+1}{2k+1} = \dfrac{1}{2}$; converges
53. **(a)** converges **(b)** diverges **55.** **(a)** converges

Exercise Set 9.6 (Page 646)

3. diverges **5.** converges **7.** converges absolutely **9.** diverges
11. converges absolutely **13.** conditionally convergent **15.** divergent
17. conditionally convergent **19.** conditionally convergent
21. divergent **23.** conditionally convergent **25.** converges absolutely
27. converges absolutely

Responses to True–False questions may be abridged to save space.

29. False; an alternating series has terms that alternate between positive and negative.
31. True; if a series converges but diverges absolutely, then it converges conditionally.
33. |error| < 0.125 **35.** |error| < 0.1 **37.** $n = 9999$
39. $n = 39{,}999$ **41.** |error| < 0.00074; $s_{10} \approx 0.4995$; $S = 0.5$
43. 0.84 **45.** 0.41 **47.** **(c)** $n = 50$
49. **(a)** If $a_k = \dfrac{(-1)^k}{\sqrt{k}}$, then $\sum a_k$ converges and $\sum a_k^2$ diverges.
If $a_k = \dfrac{(-1)^k}{k}$, then $\sum a_k$ converges and $\sum a_k^2$ also converges.
(b) If $a_k = \dfrac{1}{k}$, then $\sum a_k^2$ converges and $\sum a_k$ diverges. If $a_k = \dfrac{1}{k^2}$, then $\sum a_k^2$ converges and $\sum a_k$ also converges.

Exercise Set 9.7 (Page 657)

1. **(a)** $1 - x + \frac{1}{2}x^2$, $1 - x$ **(b)** $1 - \frac{1}{2}x^2$, 1
3. **(a)** $1 + \frac{1}{2}(x-1) - \frac{1}{8}(x-1)^2$ **(b)** 1.04875 **5.** 1.80397443
7. $p_0(x) = 1$, $p_1(x) = 1 - x$, $p_2(x) = 1 - x + \frac{1}{2}x^2$,
$p_3(x) = 1 - x + \dfrac{1}{2}x^2 - \dfrac{1}{3!}x^3$,
$p_4(x) = 1 - x + \dfrac{1}{2}x^2 - \dfrac{1}{3!}x^3 + \dfrac{1}{4!}x^4$; $\displaystyle\sum_{k=0}^{n}\frac{(-1)^k}{k!}x^k$

9. $p_0(x) = 1$, $p_1(x) = 1$, $p_2(x) = 1 - \dfrac{\pi^2}{2!}x^2$, $p_3(x) = 1 - \dfrac{\pi^2}{2!}x^2$,
$p_4(x) = 1 - \dfrac{\pi^2}{2!}x^2 + \dfrac{\pi^4}{4!}x^4$; $\displaystyle\sum_{k=0}^{\lfloor n/2\rfloor}\frac{(-1)^k\pi^{2k}}{(2k)!}x^{2k}$ (See Exercise 74 of Section 0.2.)

11. $p_0(x) = 0$, $p_1(x) = x$, $p_2(x) = x - \frac{1}{2}x^2$, $p_3(x) = x - \frac{1}{2}x^2 + \frac{1}{3}x^3$,
$p_4(x) = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4$; $\displaystyle\sum_{k=1}^{n}\frac{(-1)^{k+1}}{k}x^k$

13. $p_0(x) = 1$, $p_1(x) = 1$, $p_2(x) = 1 + \dfrac{x^2}{2}$,
$p_3(x) = 1 + \dfrac{x^2}{2}$, $p_4(x) = 1 + \dfrac{x^2}{2} + \dfrac{x^4}{4!}$; $\displaystyle\sum_{k=0}^{\lfloor n/2\rfloor}\frac{1}{(2k)!}x^{2k}$ (See Exercise 74 of Section 0.2.)

15. $p_0(x) = 0$, $p_1(x) = 0$, $p_2(x) = x^2$, $p_3(x) = x^2$,
$p_4(x) = x^2 - \frac{1}{6}x^4$; $\displaystyle\sum_{k=0}^{\lfloor n/2\rfloor-1}\frac{(-1)^k}{(2k+1)!}x^{2k+2}$ (See Exercise 74 of Section 0.2.)

17. $p_0(x) = e$, $p_1(x) = e + e(x-1)$,
$p_2(x) = e + e(x-1) + \dfrac{e}{2}(x-1)^2$,
$p_3(x) = e + e(x-1) + \dfrac{e}{2}(x-1)^2 + \dfrac{e}{3!}(x-1)^3$,
$p_4(x) = e + e(x-1) + \dfrac{e}{2}(x-1)^2 + \dfrac{e}{3!}(x-1)^3 + \dfrac{e}{4!}(x-1)^4$;
$\displaystyle\sum_{k=0}^{n}\frac{e}{k!}(x-1)^k$

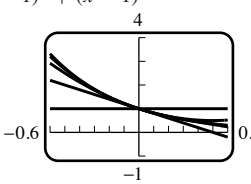
19. $p_0(x) = -1$, $p_1(x) = -1 - (x+1)$,
$p_2(x) = -1 - (x+1) - (x+1)^2$,
$p_3(x) = -1 - (x+1) - (x+1)^2 - (x+1)^3$,
$p_4(x) = -1 - (x+1) - (x+1)^2 - (x+1)^3 - (x+1)^4$;
$\displaystyle\sum_{k=0}^{n}(-1)(x+1)^k$

21. $p_0(x) = p_1(x) = 1$, $p_2(x) = p_3(x) = 1 - \dfrac{\pi^2}{2}\left(x - \dfrac{1}{2}\right)^2$,
$p_4(x) = 1 - \dfrac{\pi^2}{2}\left(x - \dfrac{1}{2}\right)^2 + \dfrac{\pi^4}{4!}\left(x - \dfrac{1}{2}\right)^4$;
$\displaystyle\sum_{k=0}^{\lfloor n/2\rfloor}\frac{(-1)^k\pi^{2k}}{(2k)!}\left(x - \frac{1}{2}\right)^{2k}$ (See Exercise 74 of Section 0.2.)

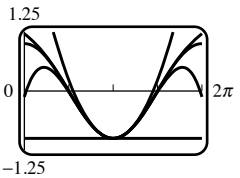
23. $p_0(x) = 0$, $p_1(x) = (x-1)$, $p_2(x) = (x-1) - \frac{1}{2}(x-1)^2$,
$p_3(x) = (x-1) - \frac{1}{2}(x-1)^2 + \frac{1}{3}(x-1)^3$,
$p_4(x) = (x-1) - \frac{1}{2}(x-1)^2 + \frac{1}{3}(x-1)^3 - \frac{1}{4}(x-1)^4$;
$\displaystyle\sum_{k=1}^{n}\frac{(-1)^{k-1}}{k}(x-1)^k$

25. **(a)** $p_3(x) = 1 + 2x - x^2 + x^3$
(b) $p_3(x) = 1 + 2(x-1) - (x-1)^2 + (x-1)^3$

27. $p_0(x) = 1$, $p_1(x) = 1 - 2x$,
$p_2(x) = 1 - 2x + 2x^2$,
$p_3(x) = 1 - 2x + 2x^2 - \frac{4}{3}x^3$

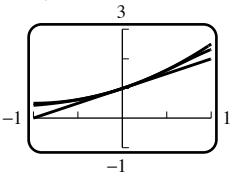


29. $p_0(x) = -1$, $p_2(x) = -1 + \frac{1}{2}(x-\pi)^2$,
$p_4(x) = -1 + \frac{1}{2}(x-\pi)^2 - \frac{1}{24}(x-\pi)^4$,
$p_6(x) = -1 + \frac{1}{2}(x-\pi)^2 - \frac{1}{24}(x-\pi)^4 + \frac{1}{720}(x-\pi)^6$



Responses to True–False questions may be abridged to save space.

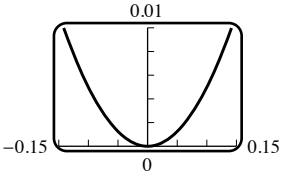
31. True; $y = f(x_0) + f'(x_0)(x - x_0)$ is the first-degree Taylor polynomial for f about $x = x_0$.
33. False; $p_6^{(4)}(x_0) = f^{(4)}(x_0)$ **35.** 1.64870 **37.** IV
39. **(a)**



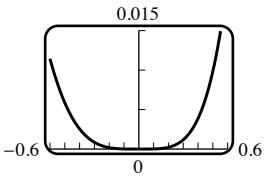
(b)

x	−1.000	−0.750	−0.500	−0.250	0.000	0.250	0.500	0.750	1.000
$f(x)$	0.431	0.506	0.619	0.781	1.000	1.281	1.615	1.977	2.320
$p_1(x)$	0.000	0.250	0.500	0.750	1.000	1.250	1.500	1.750	2.000
$p_2(x)$	0.500	0.531	0.625	0.781	1.000	1.281	1.625	2.031	2.500

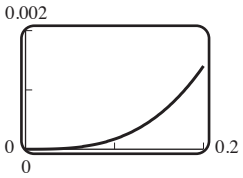
(c) $|e^{\sin x} - (1+x)| < 0.01$ for $-0.14 < x < 0.14$



(d) $\left|e^{\sin x} - \left(1 + x + \dfrac{x^2}{2}\right)\right| < 0.01$ for $-0.50 < x < 0.50$



41. **(a)** $[0, 0.137]$ **(b)**



43. $(-0.569, 0.569)$ **45.** $(-0.281, 0.281)$

Exercise Set 9.8 (Page 667)

1. $\sum_{k=0}^{\infty} \frac{(-1)^k}{k!}x^k$ **3.** $\sum_{k=0}^{\infty} \frac{(-1)^k\pi^{2k}}{(2k)!}x^{2k}$ **5.** $\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k}x^k$

7. $\sum_{k=0}^{\infty} \frac{1}{(2k)!}x^{2k}$ **9.** $\sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)!}x^{2k+2}$ **11.** $\sum_{k=0}^{\infty} \frac{e}{k!}(x-1)^k$

13. $\sum_{k=0}^{\infty}(-1)(x+1)^k$ **15.** $\sum_{k=0}^{\infty} \frac{(-1)^k\pi^{2k}}{(2k)!}\left(x-\frac{1}{2}\right)^{2k}$

17. $\sum_{k=1}^{\infty} \frac{(-1)^{k-1}}{k}(x-1)^k$ **19.** $-1 < x < 1$; $\dfrac{1}{1+x}$

21. $1 < x < 3$; $\dfrac{1}{3-x}$ **23.** **(a)** $-2 < x < 2$ **(b)** $f(0) = 1$; $f(1) = \frac{2}{3}$

Responses to True–False questions may be abridged to save space.

25. True; see Theorem 9.8.2(*c*).

27. True; the polynomial *is* the Maclaurin series and converges for all x.

29. $R = 1$; $[-1, 1)$ **31.** $R = +\infty$; $(-\infty, +\infty)$ **33.** $R = \frac{1}{5}$; $\left[-\frac{1}{5}, \frac{1}{5}\right]$

35. $R = 1$; $[-1, 1]$ **37.** $R = 1$; $(-1, 1]$ **39.** $R = +\infty$; $(-\infty, +\infty)$

41. $R = 1$; $[-1, 1]$ **43.** $R = \frac{4}{3}$; $\left(-\frac{19}{3}, -\frac{11}{3}\right)$ **45.** $R = 1$; $(-2, 0]$

47. $R = 1$; $[-2, 0]$ **49.** $R = +\infty$; $(-\infty, +\infty)$ **51.** $(-\infty, +\infty)$

57. radius $= R$ **63.** **(a)** $n = 5$; $s_5 \approx 1.1026$ **(b)** $\zeta(3.7) \approx 1.10629$

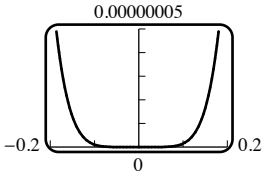
Exercise Set 9.9 (Page 676)

3. 0.069756 **5.** 0.99500 **7.** 0.99619 **9.** 0.5208

11. **(a)** $\sum_{k=1}^{\infty} 2\dfrac{(1/9)^{2k-1}}{2k-1}$ **(b)** 0.223

13. **(a)** 0.4635; 0.3218 **(b)** 3.1412 **(c)** no

15. **(a)** error $\le 9 \times 10^{-8}$ **(b)**



17. **(a)** $\sum_{k=0}^{\infty}(-1)^k x^k$ **(b)** $1 + \dfrac{x}{3} + \sum_{k=2}^{\infty}(-1)^{k-1}\dfrac{2\cdot 5\cdots(3k-4)}{3^k k!}x^k$

(c) $\sum_{k=0}^{\infty}(-1)^k \dfrac{(k+2)(k+1)}{2}x^k$ **23.** 23.406%

Exercise Set 9.10 (Page 686)

1. **(a)** $1 - x + x^2 - \cdots + (-1)^k x^k + \cdots$; $R = 1$

(b) $1 + x^2 + x^4 + \cdots + x^{2k} + \cdots$; $R = 1$

(c) $1 + 2x + 4x^2 + \cdots + 2^k x^k + \cdots$; $R = \frac{1}{2}$

(d) $\dfrac{1}{2} + \dfrac{1}{2^2}x + \dfrac{1}{2^3}x^2 + \cdots + \dfrac{1}{2^{k+1}}x^k + \cdots$; $R = 2$

3. **(a)** $(2+x)^{-1/2} = \dfrac{1}{2^{1/2}} - \dfrac{1}{2^{5/2}}x + \dfrac{1\cdot 3}{2^{9/2}\cdot 2!}x^2 - \dfrac{1\cdot 3\cdot 5}{2^{13/2}\cdot 3!}x^3 + \cdots$

(b) $(1-x^2)^{-2} = 1 + 2x^2 + 3x^4 + 4x^6 + \cdots$

5. **(a)** $2x - \dfrac{2^3}{3!}x^3 + \dfrac{2^5}{5!}x^5 - \dfrac{2^7}{7!}x^7 + \cdots$; $R = +\infty$

(b) $1 - 2x + 2x^2 - \dfrac{4}{3}x^3 + \cdots$; $R = +\infty$

(c) $1 + x^2 + \dfrac{1}{2!}x^4 + \dfrac{1}{3!}x^6 + \cdots$; $R = +\infty$

(d) $x^2 - \dfrac{\pi^2}{2}x^4 + \dfrac{\pi^4}{4!}x^6 - \dfrac{\pi^6}{6!}x^8 + \cdots$; $R = +\infty$

7. **(a)** $x^2 - 3x^3 + 9x^4 - 27x^5 + \cdots$; $R = \frac{1}{3}$

(b) $2x^2 + \dfrac{2^3}{3!}x^4 + \dfrac{2^5}{5!}x^6 + \dfrac{2^7}{7!}x^8 + \cdots$; $R = +\infty$

(c) $x - \dfrac{3}{2}x^3 + \dfrac{3}{8}x^5 + \dfrac{1}{16}x^7 + \cdots$; $R = 1$

9. **(a)** $x^2 - \dfrac{2^3}{4!}x^4 + \dfrac{2^5}{6!}x^6 - \dfrac{2^7}{8!}x^8 + \cdots$

(b) $12x^3 - 6x^6 + 4x^9 - 3x^{12} + \cdots$

11. **(a)** $1 - (x-1) + (x-1)^2 - \cdots + (-1)^k(x-1)^k + \cdots$ **(b)** $(0,2)$

13. **(a)** $x + x^2 + \dfrac{x^3}{3} - \dfrac{x^5}{30} + \cdots$ **(b)** $x - \dfrac{x^3}{24} + \dfrac{x^4}{24} - \dfrac{71}{1920}x^5 + \cdots$

15. **(a)** $1 + \frac{1}{2}x^2 + \frac{5}{24}x^4 + \frac{61}{720}x^6 + \cdots$ **(b)** $x - x^2 + \frac{1}{3}x^3 - \frac{1}{30}x^5 + \cdots$

19. $2 - 4x + 2x^2 - 4x^3 + 2x^4 + \cdots$

25. $[-1, 1]$; $[-1, 1)$ **27.** **(a)** $\sum_{k=0}^{\infty} x^{2k+1}$ **(b)** $f^{(5)}(0) = 5!$, $f^{(6)}(0) = 0$

(c) $f^{(n)}(0) = n!c_n = \begin{cases} n! & \text{if } n \text{ odd} \\ 0 & \text{if } n \text{ even} \end{cases}$

29. **(a)** 1 **(b)** $-\frac{1}{3}$ **31.** 0.3103 **33.** 0.200

35. **(a)** $\sum_{k=0}^{\infty} \dfrac{x^{4k}}{k!}$; $R = +\infty$ **37.** **(a)** 3/4 **(b)** $\ln(4/3)$

39. **(a)** $x - \frac{1}{6}x^3 + \frac{3}{40}x^5 - \frac{5}{112}x^7 + \cdots$

(b) $x + \sum_{k=1}^{\infty}(-1)^k \dfrac{1\cdot 3\cdot 5\cdots(2k-1)}{2^k k!(2k+1)}x^{2k+1}$ **(c)** $R = 1$

41. **(a)** $y(t) = y_0 \sum_{k=0}^{\infty} \dfrac{(-1)^k(0.000121)^k t^k}{k!}$ **(c)** $0.9998790073\,y_0$

43. $2\pi\sqrt{\dfrac{L}{g}}\left(1 + \dfrac{k^2}{4} + \dfrac{9k^4}{64}\right)$

Chapter 9 Review Exercises (Page 689)

9. **(a)** true **(b)** sometimes false **(c)** sometimes false
(d) true **(e)** sometimes false **(f)** sometimes false
(g) false **(h)** sometimes false **(i)** true
(j) true **(k)** sometimes false **(l)** sometimes false

11. **(a)** $\left\{\dfrac{n+2}{(n+1)^2-n^2}\right\}_{n=1}^{+\infty}$; converges, $\lim\limits_{n\to+\infty} \dfrac{n+2}{(n+1)^2-n^2} = \dfrac{1}{2}$

(b) $\left\{(-1)^{n+1}\dfrac{n}{2n+1}\right\}_{n=1}^{+\infty}$; diverges

15. **(a)** converges **(b)** converges **17.** **(a)** converges **(b)** diverges

19. **(a)** diverges **(b)** converges **21.** $\dfrac{1}{4\cdot 5^{99}}$

23. **(a)** 2 **(b)** diverges **(c)** 3/4 **(d)** $\pi/4$ **25.** $p > 1$

29. **(a)** $p_0(x) = 1$, $p_1(x) = 1 - 7x$, $p_2(x) = 1 - 7x + 5x^2$,
$p_3(x) = 1 - 7x + 5x^2 + 4x^3$, $p_4(x) = 1 - 7x + 5x^2 + 4x^3$

33. **(a)** $e^2 - 1$ **(b)** 0 **(c)** $\cos e$ **(d)** $\frac{1}{3}$

37. **(a)** $x - \frac{2}{3}x^3 + \frac{2}{15}x^5 - \frac{4}{315}x^7$ **(b)** $x - \frac{2}{3}x^3 + \frac{2}{15}x^5 - \frac{4}{315}x^7$

Chapter 9 Making Connections (Page 691)

Where correct answers to a Making Connections exercise may vary, no answer is listed. Sample answers for these questions are available on the Book Companion Site.

1. **(a)** $\dfrac{a\sin\theta}{1-\cos\theta}$ **(b)** $a\csc\theta$ **(c)** $a\cot\theta$

2. **(a)** $A = 1$, $B = -2$ **(b)** $s_n = 2 - \dfrac{2^{n+1}}{3^{n+1} - 2^{n+1}}$; 2

4. **(a)** $124.58 < d < 124.77$ **(b)** $1243 < s < 1424$

6. **(b)** $v(t) \approx v_0 - \left(\dfrac{cv_0}{m} + g\right)t + \dfrac{c^2}{2m^2}\left(v_0 + \dfrac{mg}{c}\right)t^2$

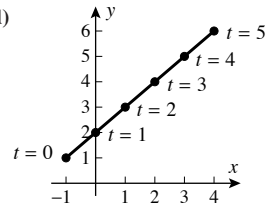
Exercise Set 10.1 (Page 700)

1. (a) $y = x + 2\ (-1 \le x \le 4)$

(c)

t	0	1	2	3	4	5
x	−1	0	1	2	3	4
y	1	2	3	4	5	6

(d)



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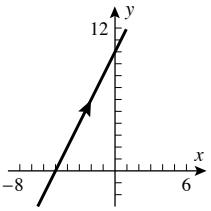
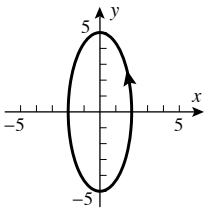
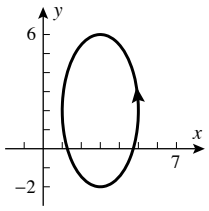


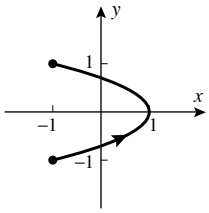
5.



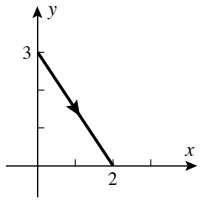
7.



9.



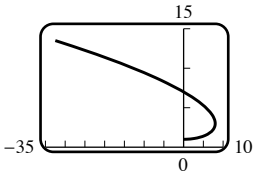
11.



13. $x = 5\cos t,\ y = -5\sin t\ (0 \le t \le 2\pi)$ **15.** $x = 2,\ y = t$

17. $x = t^2,\ y = t\ (-1 \le t \le 1)$

19. (a)

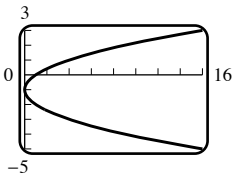


(b)

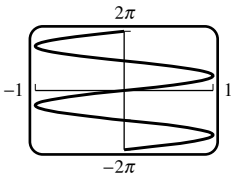
t	0	1	2	3	4	5
x	0	5.5	8	4.5	−8	−32.5
y	1	1.5	3	5.5	9	13.5

(c) $t = 0, 2\sqrt{3}$ (d) $0 < t < 2\sqrt{2}$ (e) $t = 2$

21. (a)



(b)

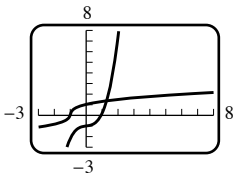


23. (a) IV (b) II (c) V (d) VI (e) III (f) I **25.** (b) $\frac{1}{2}$ (c) $\frac{3}{4}$

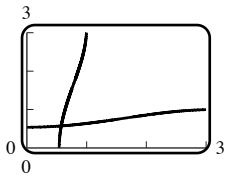
27. (b) from (x_0, y_0) to (x_1, y_1)

(c) $x = 3 - 2(t-1),\ y = -1 + 5(t-1)$

29.



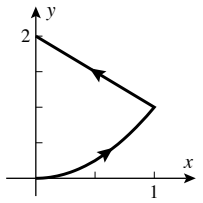
31.



Responses to True–False questions may be abridged to save space.

33. False; $x = \sin t$, $y = \cos^2 t$ describe only the portion of the parabola $y = 1 - x^2$ with $-1 \le x \le 1$.

35. True; $\dfrac{dy}{dx} = \dfrac{dy/dt}{dx/dt} = \dfrac{12t^3 - 6t^2}{x'(t)}$ **37.**



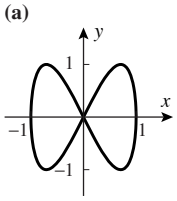
39. (a) $x = 4\cos t,\ y = 3\sin t$ (b) $x = -1 + 4\cos t,\ y = 2 + 3\sin t$

41. $-4, 4$ **43.** both are positive **45.** $4, 4$ **47.** $2/\sqrt{3}, -1/(3\sqrt{3})$

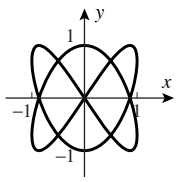
49. $\sqrt{3}, 4$ **51.** $y = -e^{-2}x + 2e^{-1}$

53. (a) $0, \pi, 2\pi$ (b) $\pi/2, 3\pi/2$

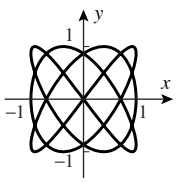
55. (a) (b) $y = -2x,\ y = 2x$



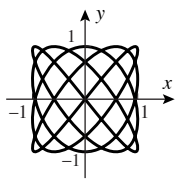
$a = 1, b = 2$



$a = 2, b = 3$

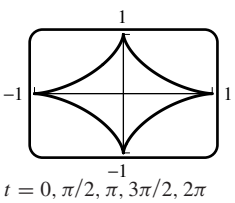


$a = 3, b = 4$



$a = 4, b = 5$

57. $y = 2x - 8,\ y = -2x + 8$ **59.**



$t = 0, \pi/2, \pi, 3\pi/2, 2\pi$

61. (a) $\dfrac{dy}{dx} = \dfrac{3\sin t}{1 - 3\cos t}$ (b) $\theta \approx -0.4345$

63. (a) ellipses with fixed center, varying shapes and sizes

(b) ellipses with varying center, fixed shape, size, and orientation

(c) circles of radius 1 with centers on line $y = x - 1$

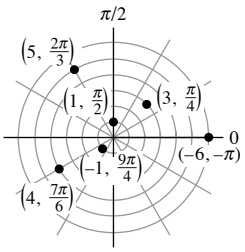
65. $\frac{1}{3}(5\sqrt{5} - 8)$ **67.** 3π **69.** $\frac{\sqrt{10}}{2}(e^2 - e^{-2})$

73. (b) $x = \cos t + \cos 2t,\ y = \sin t + \sin 2t$ (c) yes

75. $S = 49\pi$ **77.** $S = \sqrt{2}\pi$

Exercise Set 10.2 (Page 716)

1.



3. **(a)** $(3\sqrt{3}, 3)$ **(b)** $(-7/2, 7\sqrt{3}/2)$ **(c)** $(3\sqrt{3}, 3)$ **(d)** $(0, 0)$ **(e)** $(-7\sqrt{3}/2, 7/2)$ **(f)** $(-5, 0)$

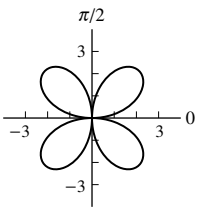
5. **(a)** $(5, \pi), (5, -\pi)$ **(b)** $(4, 11\pi/6), (4, -\pi/6)$ **(c)** $(2, 3\pi/2), (2, -\pi/2)$ **(d)** $(8\sqrt{2}, 5\pi/4), (8\sqrt{2}, -3\pi/4)$ **(e)** $(6, 2\pi/3), (6, -4\pi/3)$ **(f)** $(\sqrt{2}, \pi/4), (\sqrt{2}, -7\pi/4)$

7. **(a)** (5, 0.92730) **(b)** (10, −0.92730) **(c)** (1.27155, 2.47582)

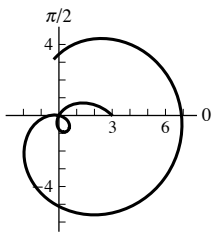
9. **(a)** circle **(b)** line **(c)** circle **(d)** line

11. **(a)** $r = 3\sec\theta$ **(b)** $r = \sqrt{7}$ **(c)** $r = -6\sin\theta$ **(d)** $r^2\cos\theta\sin\theta = 4/9$

13.



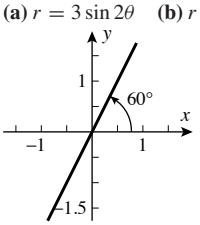
15.



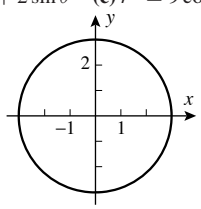
17. **(a)** $r = 5$ **(b)** $r = 6\cos\theta$ **(c)** $r = 1 - \cos\theta$

19. **(a)** $r = 3\sin 2\theta$ **(b)** $r = 3 + 2\sin\theta$ **(c)** $r^2 = 9\cos 2\theta$

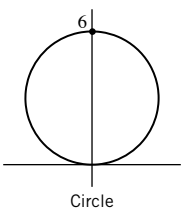
21.



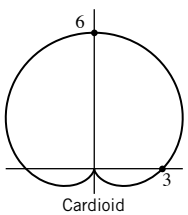
23.



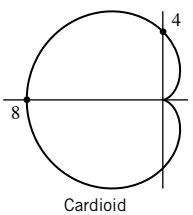
25.



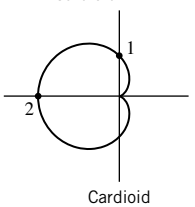
27.



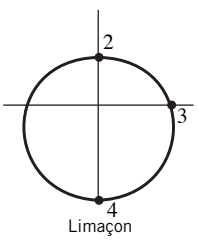
29.



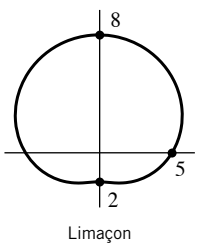
31.



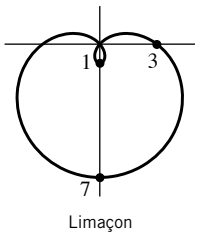
33.



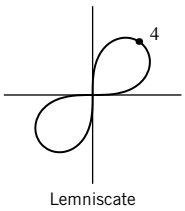
35.



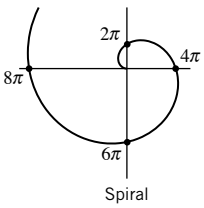
37.



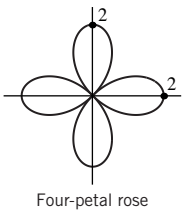
39.



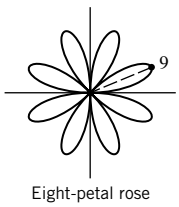
41.



43.



45.

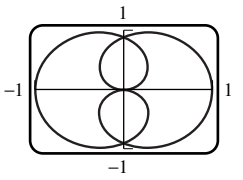


Responses to True–False questions may be abridged to save space.

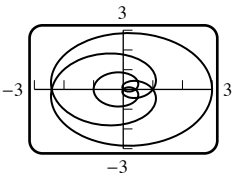
47. True; $\left(-1, \frac{\pi}{3}\right)$ describes the same point as $\left(1, \frac{\pi}{3} + \pi\right)$, which describes the same point as $\left(1, \frac{\pi}{3} + \pi - 2\pi\right) = \left(1, -\frac{2\pi}{3}\right)$.

49. False; $-1 < \sin 2\theta < 0$ for $\pi/2 < \theta < \pi$, so this portion of the graph is in the fourth quadrant.

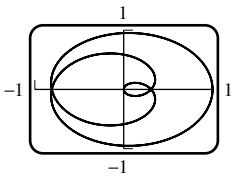
51. $0 \le \theta \le 4\pi$



53. $0 \le \theta \le 8\pi$

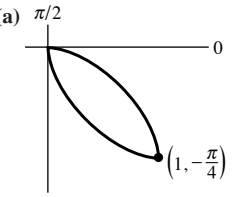


55. $0 \le \theta \le 5\pi$

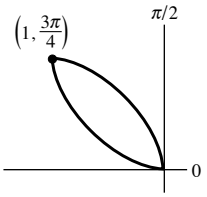


57. **(a)** $-4\pi < \theta < 4\pi$

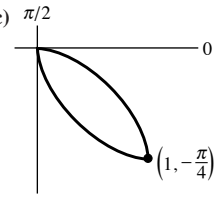
61. **(a)**



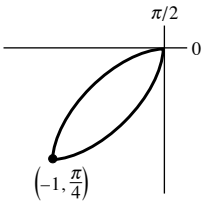
(b)

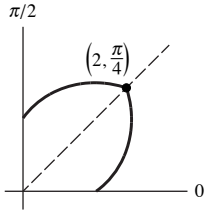


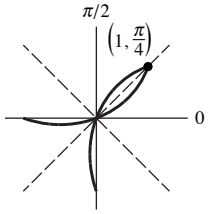
(c)



(d)



63. (a) 

(b) 

67. (a) $r = 1 + \frac{\sqrt{2}}{2}(\cos\theta + \sin\theta)$ (b) $r = 1 + \sin\theta$
(c) $r = 1 - \cos\theta$ (d) $r = 1 - \frac{\sqrt{2}}{2}(\cos\theta + \sin\theta)$

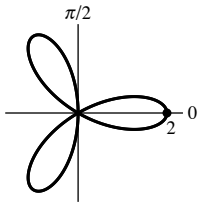
69. $(3/2, \pi/3)$ **73.** $\sqrt{2}$

Exercise Set 10.3 (Page 726)

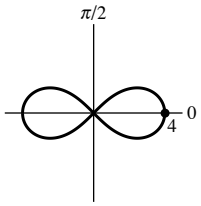
1. $\sqrt{3}$ **3.** $\dfrac{\tan 2 - 2}{2\tan 2 + 1}$ **5.** $1/2$ **7.** $1, 0, -1$

9. horizontal: $(3a/2, \pi/3)$, $(0, \pi)$, $(3a/2, 5\pi/3)$;
vertical: $(2a, 0)$, $(a/2, 2\pi/3)$, $(a/2, 4\pi/3)$

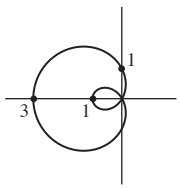
11. $(0, 0)$, $(\sqrt{2}/4, \pi/4)$, $(\sqrt{2}/4, 3\pi/4)$

13. 

$\theta = \pi/2, \pm\pi/6$

15. 

$\theta = \pm\pi/4$

17. 

$\theta = \pm\pi/3$

19. $L = 2\pi a$ **21.** $L = 8a$

23. (b) ≈ 2.42

(c)

n	2	3	4	5	6	7
L	2.42211	2.22748	2.14461	2.10100	2.07501	2.05816

n	8	9	10	11	12	13	14
L	2.04656	2.03821	2.03199	2.02721	2.02346	2.02046	2.01802

n	15	16	17	18	19	20
L	2.01600	2.01431	2.01288	2.01167	2.01062	2.00971

25. (a) $\displaystyle\int_{\pi/2}^{\pi} \tfrac{1}{2}(1-\cos\theta)^2 d\theta$ (b) $\displaystyle\int_0^{\pi/2} 2\cos^2\theta\, d\theta$
(c) $\displaystyle\int_0^{\pi/2} \tfrac{1}{2}\sin^2 2\theta\, d\theta$ (d) $\displaystyle\int_0^{2\pi} \tfrac{1}{2}\theta^2\, d\theta$
(e) $\displaystyle\int_{-\pi/2}^{\pi/2} \tfrac{1}{2}(1-\sin\theta)^2\, d\theta$ (f) $\displaystyle\int_0^{\pi/4} \cos^2 2\theta\, d\theta$

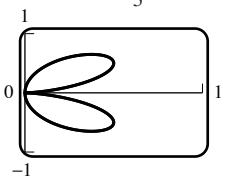
27. (a) πa^2 (b) πa^2 **29.** 6π **31.** 4π **33.** $\pi - 3\sqrt{3}/2$
35. $\pi/2 - \frac{1}{4}$ **37.** $10\pi/3 - 4\sqrt{3}$ **39.** π **41.** $9\sqrt{3}/2 - \pi$
43. $(\pi + 3\sqrt{3})/4$ **45.** $\pi - 2$

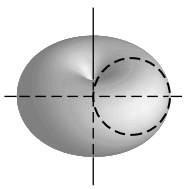
Responses to True–False questions may be abridged to save space.

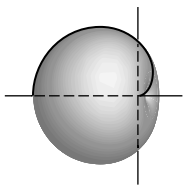
47. True; apply Theorem 10.3.1: $\cos\dfrac{\theta}{2}\Big|_{\theta=3\pi} = 0$ and $\dfrac{dr}{d\theta}\Big|_{\theta=3\pi} = \dfrac{1}{2} \neq 0$, so the line $\theta = 3\pi$ (the x-axis) is tangent to the curve at the origin.

49. False; the area of the sector is $\dfrac{\theta}{2\pi}\cdot\pi r^2 = \dfrac{1}{2}\theta r^2$.

51. (b) a^2 (c) $2\sqrt{3} - \dfrac{2\pi}{3}$ **53.** $8\pi^3 a^2$

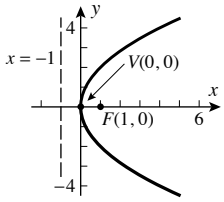
59. $\pi/16$ 

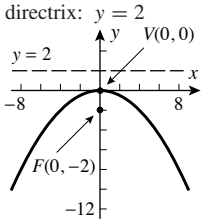
65. π^2 

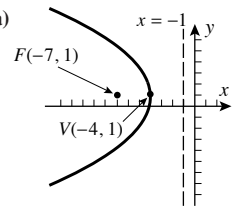
67. $32\pi/5$ 

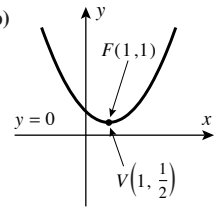
Exercise Set 10.4 (Page 744)

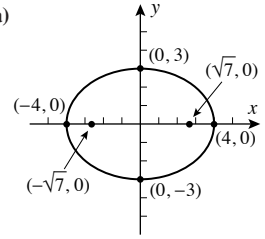
1. (a) $x = y^2$ (b) $-3y = x^2$ (c) $\dfrac{x^2}{9} + \dfrac{y^2}{4} = 1$ (d) $\dfrac{x^2}{4} + \dfrac{y^2}{9} = 1$
(e) $y^2 - x^2 = 1$ (f) $\dfrac{x^2}{4} - \dfrac{y^2}{4} = 1$

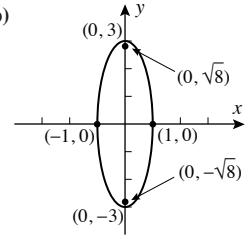
3. (a) focus: $(1, 0)$;
vertex: $(0, 0)$;
directrix: $x = -1$


(b) focus: $(0, -2)$;
vertex: $(0, 0)$;
directrix: $y = 2$


5. (a) 

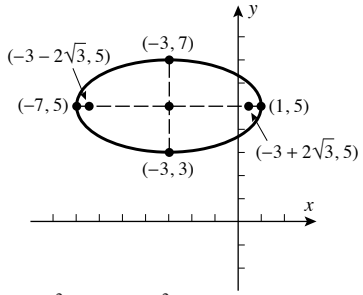
(b) 

7. (a) 

(b) 

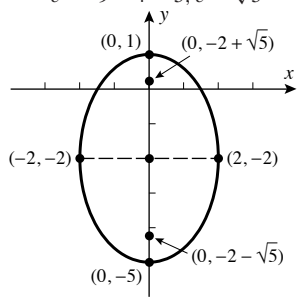
9. **(a)** $\dfrac{(x+3)^2}{16}+\dfrac{(y-5)^2}{4}=1$

$c^2=16-4=12,\ c=2\sqrt{3}$

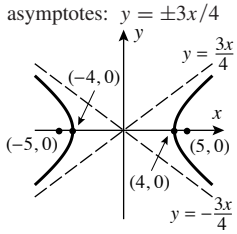


(b) $\dfrac{x^2}{4}+\dfrac{(y+2)^2}{9}=1$

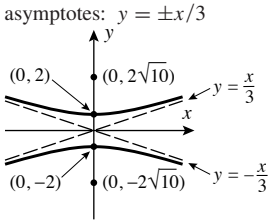
$c^2=9-4=5,\ c=\sqrt{5}$



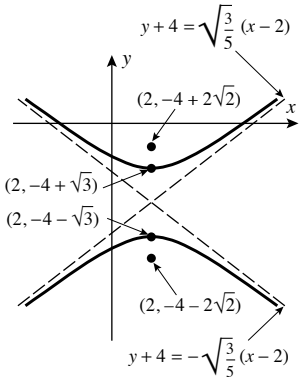
11. **(a)** vertices: $(\pm4, 0)$; foci: $(\pm5, 0)$; asymptotes: $y=\pm3x/4$



(b) vertices: $(0, \pm2)$; foci: $(0, \pm2\sqrt{10})$; asymptotes: $y=\pm x/3$

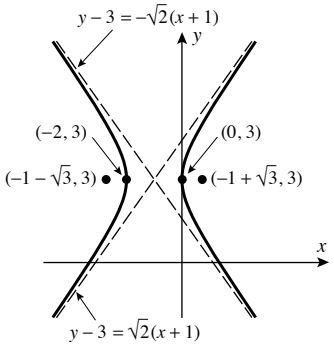


13. **(a)** $c^2=3+5=8,\ c=2\sqrt{2}$



(b) $\dfrac{(x+1)^2}{1}-\dfrac{(y-3)^2}{2}=1$

$c^2=1+2=3,\ c=\sqrt{3}$



15. **(a)** $y^2=12x$ **(b)** $x^2=-y$ **17.** $y^2=2(x-1)$

19. **(a)** $\dfrac{x^2}{9}+\dfrac{y^2}{4}=1$ **(b)** $\dfrac{x^2}{16}+\dfrac{y^2}{25}=1$

21. **(a)** $\dfrac{x^2}{81/8}+\dfrac{y^2}{36}=1$ **(b)** $\dfrac{(x+1)^2}{4}+\dfrac{(y-2)^2}{5}=1$

23. **(a)** $\dfrac{x^2}{4}-\dfrac{y^2}{5}=1$ **(b)** $\dfrac{y^2}{4}-\dfrac{x^2}{9}=1$

25. **(a)** $\dfrac{y^2}{9}-\dfrac{x^2}{16}=1,\ \dfrac{x^2}{16}-\dfrac{y^2}{9}=1$ **(b)** $\dfrac{x^2}{9/5}-\dfrac{y^2}{36/5}=1$

Responses to True–False questions may be abridged to save space.

27. False; the description matches a parabola.

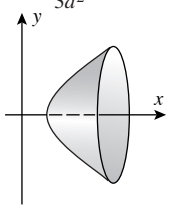
29. False; the distance from the parabola's focus to its directrix is $2p$; see Figure 10.4.6.

31. **(a)** 16 ft **(b)** $8\sqrt{3}$ ft **35.** $\frac{1}{16}$ ft

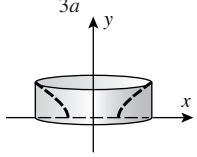
39. $\frac{1}{32}(x-4)^2+\frac{1}{36}(y-3)^2=1$ **41.** 96

45. $L=D\sqrt{1+p^2},\ T=\frac{1}{2}pD$ **47.** (64.612, 200)

49. **(a)** $V=\dfrac{\pi b^2}{3a^2}(b^2-2a^2)\sqrt{a^2+b^2}+\dfrac{2}{3}ab^2\pi$



(b) $V=\dfrac{2b^4}{3a}\pi$



55. $k=\pm4$; $(2, 1)$, $(-2, -1)$

57. $\left(\pm\dfrac{3\sqrt{13}}{2}, -9\right)$

59. **(a)** $(x-1)^2-5(y+1)^2=5$, hyperbola

(b) $x^2-3(y+1)^2=0,\ x=\pm\sqrt{3}(y+1)$, two lines

(c) $4(x+2)^2+8(y+1)^2=4$, ellipse

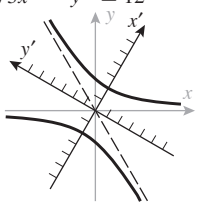
(d) $3(x+2)^2+(y+1)^2=0$, the point $(-2, -1)$ (degenerate case)

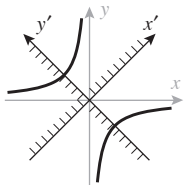
(e) $(x+4)^2+2y=2$, parabola

(f) $5(x+4)^2+2y=-14$, parabola

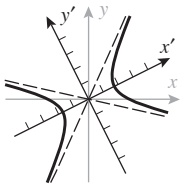
Exercise Set 10.5 (Page 753)

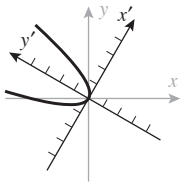
1. (a) $x' = -1 + 3\sqrt{3},\ y' = 3 + \sqrt{3}$ **3.** $y'^2 - x'^2 = 18$, hyperbola
(b) $3x'^2 - y'^2 = 12$
(c)



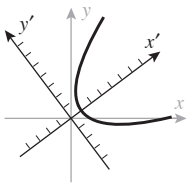


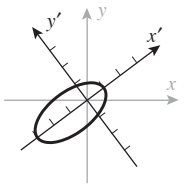
5. $\frac{1}{3}x'^2 - \frac{1}{2}y'^2 = 1$, hyperbola **7.** $y' = x'^2$, parabola





9. $y'^2 = 4(x' - 1)$, parabola **11.** $\frac{1}{4}(x' + 1)^2 + y'^2 = 1$, ellipse

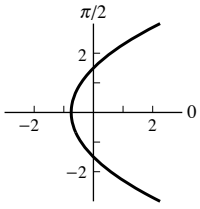


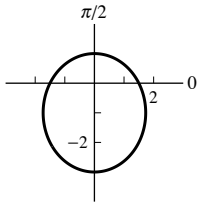


13. $x^2 + xy + y^2 = 3$
19. vertex: $(0, 0)$; focus: $(-1/\sqrt{2}, 1/\sqrt{2})$; directrix: $y = x - \sqrt{2}$
21. vertex: $(4/5, 3/5)$; focus: $(8/5, 6/5)$; directrix: $4x + 3y = 0$
23. foci: $\pm(4\sqrt{7}/5, 3\sqrt{7}/5)$; vertices: $\pm(16/5, 12/5)$;
ends: $\pm(-9/5, 12/5)$
25. foci: $(1 - \sqrt{5}/2, -\sqrt{3} + \sqrt{15}/2), (1 + \sqrt{5}/2, -\sqrt{3} - \sqrt{15}/2)$;
vertices: $(-1/2, \sqrt{3}/2), (5/2, -5\sqrt{3}/2)$;
ends: $(1 + \sqrt{3}, 1 - \sqrt{3}), (1 - \sqrt{3}, -1 - \sqrt{3})$
27. foci: $\pm(\sqrt{15}, \sqrt{5})$; vertices: $\pm(2\sqrt{3}, 2)$;
asymptotes: $y = \dfrac{5\sqrt{3} \pm 8}{11}x$
29. foci: $\left(-\dfrac{4}{\sqrt{5}} \pm 2\sqrt{\dfrac{13}{5}}, \dfrac{8}{\sqrt{5}} \pm \sqrt{\dfrac{13}{5}}\right)$;
vertices: $(2/\sqrt{5}, 11/\sqrt{5}), (-2\sqrt{5}, \sqrt{5})$;
asymptotes: $y = 7x/4 + 3\sqrt{5},\ y = -x/8 + 3\sqrt{5}/2$

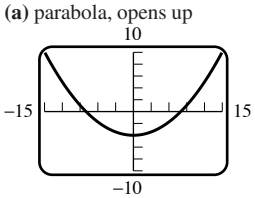
Exercise Set 10.6 (Page 761)

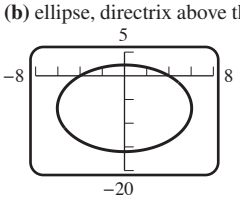
1. (a) $e = 1,\ d = \frac{3}{2}$ (b) $e = \frac{1}{2},\ d = 3$





3. (a) parabola, opens up (b) ellipse, directrix above the pole

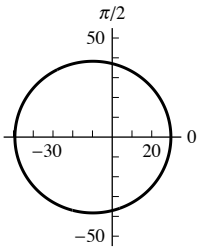




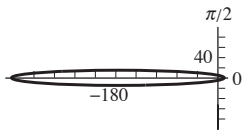
5. (a) $r = \dfrac{6}{4 + 3\cos\theta}$ (b) $r = \dfrac{1}{1 + \cos\theta}$ (c) $r = \dfrac{12}{3 + 4\sin\theta}$
7. (a) $r_0 = 2,\ r_1 = 6;\ \frac{1}{12}x^2 + \frac{1}{16}(y + 2)^2 = 1$
(b) $r_0 = \frac{1}{3},\ r_1 = 1;\ \frac{9}{4}\left(x - \frac{1}{3}\right)^2 + 3y^2 = 1$
9. (a) $r_0 = 1,\ r_1 = 3;\ (y - 2)^2 - \dfrac{x^2}{3} = 1$
(b) $r_0 = 1,\ r_1 = 5;\ \dfrac{(x + 3)^2}{4} - \dfrac{y^2}{5} = 1$
11. (a) $r = \dfrac{12}{2 + \cos\theta}$ (b) $r = \dfrac{64}{25 - 15\sin\theta}$
13. $r = \dfrac{5\sqrt{2} + 5}{1 + \sqrt{2}\cos\theta}$ or $r = \dfrac{5\sqrt{2} - 5}{1 + \sqrt{2}\cos\theta}$

Responses to True–False questions may be abridged to save space.

19. True; the eccentricity e of an ellipse satisfies $0 < e < 1$ (Theorem 10.6.1).
21. False; eccentricity correlates to the "flatness" of an ellipse, which is independent of the distance between its foci.
23. (a) $T \approx 248$ yr
(b) $r_0 = 29.6645$ AU $\approx 4{,}449{,}675{,}000$ km,
$r_1 = 49.3355$ AU $\approx 7{,}400{,}325{,}000$ km
(c) $r \approx \dfrac{37.05}{1 + 0.249\cos\theta}$ AU (d)



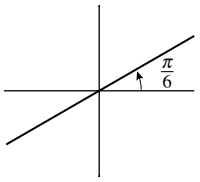
25. (a) $a \approx 178.26$ AU
(b) $r_0 \approx 0.8735$ AU,
$r_1 \approx 355.64$ AU
(c) $r \approx \dfrac{1.74}{1 + 0.9951\cos\theta}$ AU (d)



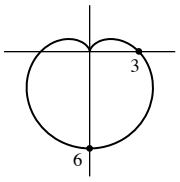
27. 563 km, 4286 km

Chapter 10 Review Exercises (Page 763)

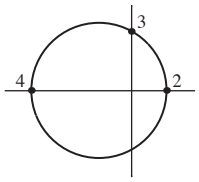
1. $x = \sqrt{2}\cos t,\ y = -\sqrt{2}\sin t\ (0 \le t \le 3\pi/2)$ **3.** (a) $-1/4,\ 1/4$
5. (a) $t = \pi/2 + n\pi$ for $n = 0, \pm1, \ldots$ (b) $t = n\pi$ for $n = 0, \pm1, \ldots$
7. (a) $(-4\sqrt{2}, -4\sqrt{2})$ (b) $(7/\sqrt{2}, -7/\sqrt{2})$ (c) $(4\sqrt{2}, 4\sqrt{2})$
(d) $(5, 0)$ (e) $(0, -2)$ (f) $(0, 0)$
9. (a) $(5, 0.6435)$ (b) $(\sqrt{29}, 5.0929)$ (c) $(1.2716, 0.6658)$
11. (a) parabola (b) hyperbola (c) line (d) circle
13. **15.** **17.**



Line



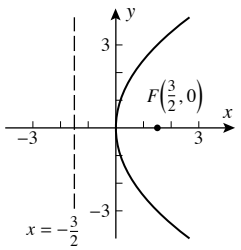
Cardioid



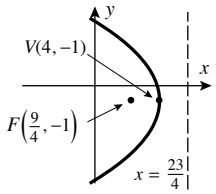
Limaçon

19. (a) $-2,\ 1/4$ (b) $-3\sqrt{3}/4,\ 3\sqrt{3}/4$
21. (a) The top is traced from right to left as t goes from 0 to π. The bottom is traced from right to left as t goes from π to 2π, except for the loop, which is traced counterclockwise as t goes from $\pi + \sin^{-1}(1/4)$ to $2\pi - \sin^{-1}(1/4)$. (b) $y = 1$
(c) horizontal: $t = \pi/2, 3\pi/2$; vertical: $t = \pi + \sin^{-1}(1/\sqrt[3]{4})$, $2\pi - \sin^{-1}(1/\sqrt[3]{4})$
(d) $r = 4 + \csc\theta,\ \theta = \pi + \sin^{-1}(1/4),\ \theta = 2\pi - \sin^{-1}(1/4)$

23. $A = 6\pi$ **25.** $A = \dfrac{5\pi}{12} - \dfrac{\sqrt{3}}{2}$ **27.**

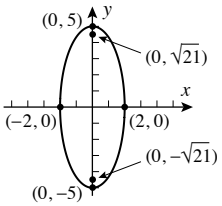


29.



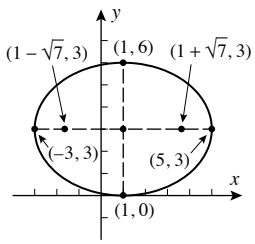
focus: $(9/4, -1)$;
vertex: $(4, -1)$;
directrix: $x = 23/4$

31.

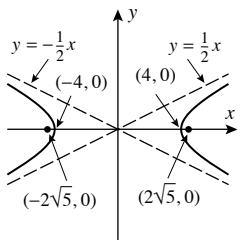


foci: $(0, \pm\sqrt{21})$;
vertices: $(0, \pm 5)$;
ends: $(\pm 2, 0)$

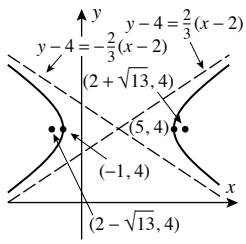
33.



35.



37.



39. $x^2 = -16y$ **41.** $y^2 - x^2 = 9$

43. **(b)** $x = \dfrac{v_0^2}{g}\sin\alpha\cos\alpha$; $y = y_0 + \dfrac{v_0^2\sin^2\alpha}{2g}$

45. $\theta = \pi/4$; $5(y')^2 - (x')^2 = 6$; hyperbola

47. $\theta = \tan^{-1}(1/2)$; $y' = (x')^2$; parabola

49. **(a)** (i) ellipse; (ii) right; (iii) 1 **(b)** (i) hyperbola; (ii) left; (iii) 1/3 **(c)** (i) parabola; (ii) above; (iii) 1/3 **(d)** (i) parabola; (ii) below; (iii) 3

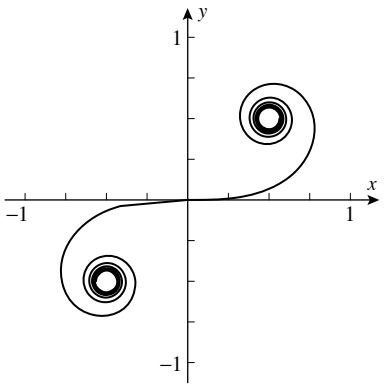
51. **(a)** $\dfrac{(x+3)^2}{25} + \dfrac{(y-2)^2}{9} = 1$ **(b)** $(x+2)^2 = -8y$
(c) $\dfrac{(y-5)^2}{4} - 16(x+1)^2 = 1$

53. 15.86543959

Chapter 10 Making Connections (Page 766)

Where correct answers to a Making Connections exercise may vary, no answer is listed. Sample answers for these questions are available on the Book Companion Site.

1. **(a)**



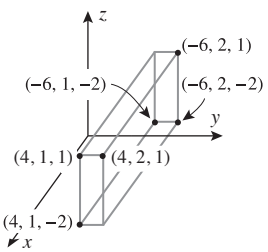
(c) $L = \displaystyle\int_{-1}^{1}\left[\cos^2\left(\frac{\pi t^2}{2}\right) + \sin^2\left(\frac{\pi t^2}{2}\right)\right]dt = 2$

2. **(a)** P: $(b\cos t, b\sin t)$;
Q: $(a\cos t, a\sin t)$;
R: $(a\cos t, b\sin t)$

Exercise Set 11.1 (Page 771)

1. **(a)** (0, 0, 0), (3, 0, 0), (3, 5, 0), (0, 5, 0), (0, 0, 4), (3, 0, 4), (3, 5, 4), (0, 5, 4)
(b) (0, 1, 0), (4, 1, 0), (4, 6, 0), (0, 6, 0), (0, 1, −2), (4, 1, −2), (4, 6, −2), (0, 6, −2)

3. (4, 2, −2), (4, 2, 1), (4, 1, 1), (4, 1, −2), (−6, 1, 1), (−6, 2, 1), (−6, 2, −2), (−6, 1, −2)



5. **(a)** point **(b)** line parallel to the y-axis
(c) plane parallel to the yz-plane

9. radius $\sqrt{74}$, center $(2, 1, -4)$ **11.** **(b)** (2, 1, 6) **(c)** area 49

13. **(a)** $(x-7)^2 + (y-1)^2 + (z-1)^2 = 16$
(b) $(x-1)^2 + y^2 + (z+1)^2 = 16$
(c) $(x+1)^2 + (y-3)^2 + (z-2)^2 = 14$
(d) $\left(x + \frac{1}{2}\right)^2 + (y-2)^2 + (z-2)^2 = \frac{5}{4}$

15. $(x-2)^2 + (y+1)^2 + (z+3)^2 = r^2$;
(a) $r^2 = 9$ **(b)** $r^2 = 1$ **(c)** $r^2 = 4$

Responses to True–False questions may be abridged to save space.

19. False; see Figure 11.1.6.

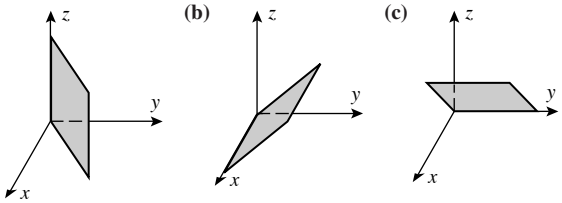
21. True; see Figure 11.1.3.

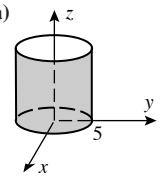
23. sphere, center $(-5, -2, -1)$, radius 7

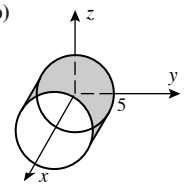
25. sphere; center $\left(\frac{1}{2}, \frac{3}{4}, -\frac{5}{4}\right)$, radius $\dfrac{3\sqrt{6}}{4}$

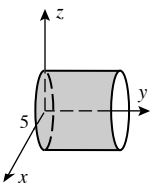
27. no graph

29. **(a)** **(b)** **(c)**

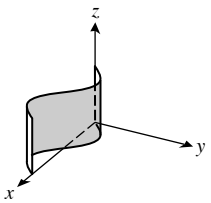


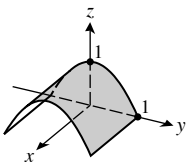
31. **(a)** 

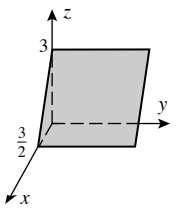
(b) 

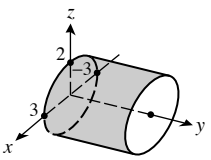
(c) 

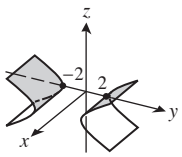
33. **(a)** $-2y + z = 0$ **(b)** $-2x + z = 0$ **(c)** $(x-1)^2 + (y-1)^2 = 1$ **(d)** $(x-1)^2 + (z-1)^2 = 1$

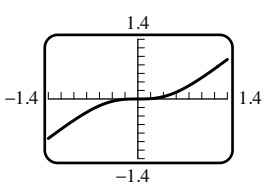
35. 

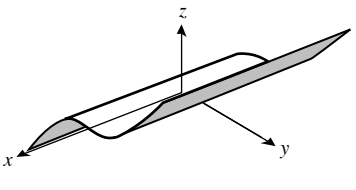
37. 

39. 

41. 

43. 

45. 

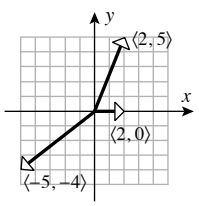


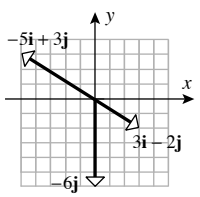
47. largest distance, $3 + \sqrt{6}$; smallest distance, $3 - \sqrt{6}$

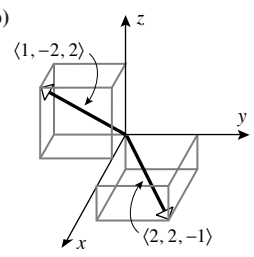
49. all points outside the circular cylinder $(y+3)^2 + (z-2)^2 = 16$

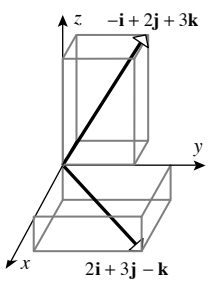
51. $r = (2 - \sqrt{3})R$ **53.** **(b)** $y^2 + z^2 = e^{2x}$

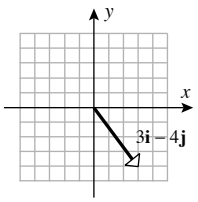
Exercise Set 11.2 (Page 782)

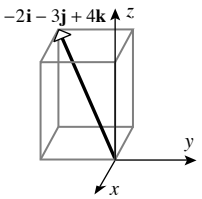
1. **(a-c)** 

(d-f) 

3. **(a,b)** 

(c,d) 

5. **(a)** $\langle 3, -4 \rangle$ 

(b) $\langle -2, -3, 4 \rangle$ 

7. **(a)** $\langle -1, 3 \rangle$ **(b)** $\langle -7, 2 \rangle$ **(c)** $\langle -3, 6, 1 \rangle$

9. **(a)** $(4, -4)$ **(b)** $(8, -1, -3)$

11. **(a)** $-\mathbf{i} + 4\mathbf{j} - 2\mathbf{k}$ **(b)** $18\mathbf{i} + 12\mathbf{j} - 6\mathbf{k}$ **(c)** $-\mathbf{i} - 5\mathbf{j} - 2\mathbf{k}$ **(d)** $40\mathbf{i} - 4\mathbf{j} - 4\mathbf{k}$ **(e)** $-2\mathbf{i} - 16\mathbf{j} - 18\mathbf{k}$ **(f)** $-\mathbf{i} + 13\mathbf{j} - 2\mathbf{k}$

13. **(a)** $\sqrt{2}$ **(b)** $5\sqrt{2}$ **(c)** $\sqrt{21}$ **(d)** $\sqrt{14}$

15. **(a)** $2\sqrt{3}$ **(b)** $\sqrt{14} + \sqrt{2}$ **(c)** $2\sqrt{14} + 2\sqrt{2}$ **(d)** $2\sqrt{37}$ **(e)** $(1/\sqrt{6})\mathbf{i} + (1/\sqrt{6})\mathbf{j} - (2/\sqrt{6})\mathbf{k}$ **(f)** 1

Responses to True–False questions may be abridged to save space.

17. False; $\|\mathbf{i} + \mathbf{j}\| = \sqrt{2} \neq 1 + 1 = 2$

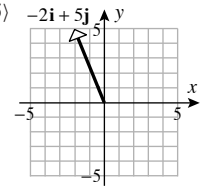
19. True; one in the same direction and one in the opposite direction.

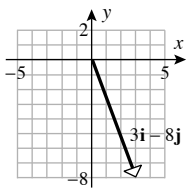
21. **(a)** $(-1/\sqrt{17})\mathbf{i} + (4/\sqrt{17})\mathbf{j}$ **(b)** $(-3\mathbf{i} + 2\mathbf{j} - \mathbf{k})/\sqrt{14}$ **(c)** $(4\mathbf{i} + \mathbf{j} - \mathbf{k})/(3\sqrt{2})$

23. **(a)** $\langle -\frac{3}{2}, 2 \rangle$ **(b)** $\dfrac{1}{\sqrt{5}}\langle 7, 0, -6 \rangle$

25. **(a)** $\langle 3\sqrt{2}/2, 3\sqrt{2}/2 \rangle$ **(b)** $\langle 0, 2 \rangle$ **(c)** $\langle -5/2, 5\sqrt{3}/2 \rangle$ **(d)** $\langle -1, 0 \rangle$

27. $\langle (\sqrt{3} - \sqrt{2})/2, (1 + \sqrt{2})/2 \rangle$

29. **(a)** $\langle -2, 5 \rangle$ 

(b) $\langle 3, -8 \rangle$ 

31. $\langle -\frac{2}{3}, 1 \rangle$ **33.** $\mathbf{u} = \frac{5}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} + \frac{1}{7}\mathbf{k}$, $\mathbf{v} = \frac{8}{7}\mathbf{i} - \frac{1}{7}\mathbf{j} - \frac{4}{7}\mathbf{k}$

35. $\sqrt{5}$, 3 **37.** **(a)** $\pm\frac{5}{3}$ **(b)** 3

39. **(a)** $\langle 1/\sqrt{10}, 3/\sqrt{10} \rangle$, $\langle -1/\sqrt{10}, -3/\sqrt{10} \rangle$ **(b)** $\langle 1/\sqrt{2}, -1/\sqrt{2} \rangle$, $\langle -1/\sqrt{2}, 1/\sqrt{2} \rangle$ **(c)** $\pm\dfrac{1}{\sqrt{26}}\langle 5, 1 \rangle$

41. **(a)** the circle of radius 1 about the origin
(b) the closed disk of radius 1 about the origin
(c) all points outside the closed disk of radius 1 about the origin

43. **(a)** the (hollow) sphere of radius 1 about the origin
(b) the closed ball of radius 1 about the origin
(c) all points outside the closed ball of radius 1 about the origin

45. magnitude $= 30\sqrt{5}$ lb, $\theta \approx 26.57°$

47. magnitude ≈ 207.06 N, $\theta = 45°$

49. magnitude ≈ 94.995 N, $\theta \approx 28.28°$

51. magnitude ≈ 9.165 lb, angle $\approx -70.890°$

53. ≈ 183.02 lb, 224.13 lb **55.** $450\sqrt{2} + 150\sqrt{6}$ lb, $300 + 300\sqrt{3}$ lb

57. **(a)** $c_1 = -2$, $c_2 = 1$

Exercise Set 11.3 (Page 792)

1. **(a)** -10; $\cos\theta = -1/\sqrt{5}$ **(b)** -3; $\cos\theta = -3/\sqrt{58}$ **(c)** 0; $\cos\theta = 0$ **(d)** -20; $\cos\theta = -20/(3\sqrt{70})$

3. **(a)** obtuse **(b)** acute **(c)** obtuse **(d)** orthogonal

5. $\sqrt{2}/2, 0, -\sqrt{2}/2, -1, -\sqrt{2}/2, 0, \sqrt{2}/2$

7. **(a)** vertex B **(b)** $82°, 60°, 38°$ **13.** $r = 7/5$

15. **(a)** $\alpha = \beta \approx 55°$, $\gamma \approx 125°$ **(b)** $\alpha \approx 48°$, $\beta \approx 132°$, $\gamma \approx 71°$

19. **(a)** $\approx 35°$ **(b)** $90°$

21. $64°, 41°, 60°$ **23.** $71°, 61°, 36°$

25. (a) $\left\langle \frac{2}{3}, \frac{4}{3}, \frac{4}{3} \right\rangle, \left\langle \frac{4}{3}, -\frac{7}{3}, \frac{5}{3} \right\rangle$
(b) $\left\langle -\frac{74}{49}, -\frac{111}{49}, \frac{222}{49} \right\rangle, \left\langle \frac{270}{49}, \frac{62}{49}, \frac{121}{49} \right\rangle$

27. (a) $\langle 1, 1 \rangle + \langle -4, 4 \rangle$ (b) $\left\langle 0, -\frac{8}{5}, \frac{4}{5} \right\rangle + \left\langle -2, \frac{13}{5}, \frac{26}{5} \right\rangle$
(c) $\mathbf{v} = \langle 1, 4, 1 \rangle$ is orthogonal to $\mathbf{b}$.

Responses to True–False questions may be abridged to save space.

29. True; $\mathbf{v} + \mathbf{w} = \mathbf{0}$ implies $0 = \mathbf{v} \cdot (\mathbf{v} + \mathbf{w}) = \|\mathbf{v}\|^2 \neq 0$, a contradiction.

31. True; see Equation (12). **33.** $\sqrt{564/29}$ **35.** 169.8 N

37. 375 ft·lb **39.** $-5\sqrt{3}$ J **47.** (a) 40° (b) $x \approx -0.682328$

Exercise Set 11.4 (Page 803)

1. (a) $-\mathbf{j} + \mathbf{k}$ **3.** $\langle 7, 10, 9 \rangle$ **5.** $\langle -4, -6, -3 \rangle$

7. (a) $\langle -20, -67, -9 \rangle$ (b) $\langle -78, 52, -26 \rangle$
(c) $\langle 0, -56, -392 \rangle$ (d) $\langle 0, 56, 392 \rangle$

9. $\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0$ **11.** $\pm\frac{1}{\sqrt{6}}\langle 2, 1, 1 \rangle$

Responses to True–False questions may be abridged to save space.

13. True; see Theorem 11.4.5(*c*).

15. False; let $\mathbf{v} = \mathbf{u} = \mathbf{i}$ and let $\mathbf{w} = 2\mathbf{i}$.

17. $\sqrt{59}$ **19.** $\sqrt{374}/2$

21. 80 **23.** -3 **25.** 16 **27.** (a) yes (b) yes (c) no

29. (a) 9 (b) $\sqrt{122}$ (c) $\sin^{-1}\left(\frac{9}{14}\right)$

31. (a) $2\sqrt{141/29}$ (b) $6/\sqrt{5}$ **33.** $\frac{2}{3}$ **37.** $\theta = \pi/4$

39. (a) $10\sqrt{2}$ lb·ft, direction of rotation about P is counterclockwise looking along $\overrightarrow{PQ} \times \mathbf{F} = -10\mathbf{i} + 10\mathbf{k}$ toward its initial point
(b) 10 lb·ft, direction of rotation about P is counterclockwise looking along $-10\mathbf{i}$ toward its initial point
(c) 0 lb·ft, no rotation about P

41. ≈ 36.19 N·m **45.** $-8\mathbf{i} - 20\mathbf{j} + 2\mathbf{k}$, $-8\mathbf{i} - 8\mathbf{k}$ **49.** 1.887850

Exercise Set 11.5 (Page 810)

1. (a) $L_1: x = 1, y = t$, $L_2: x = t, y = 1$, $L_3: x = t, y = t$
(b) $L_1: x = 1, y = 1, z = t$, $L_2: x = t, y = 1, z = 1$, $L_3: x = 1, y = t, z = 1$, $L_4: x = t, y = t, z = t$

3. (a) $x = 3 + 2t, y = -2 + 3t$; line segment: $0 \le t \le 1$
(b) $x = 5 - 3t, y = -2 + 6t, z = 1 + t$; line segment: $0 \le t \le 1$

5. (a) $x = 2 + t, y = -3 - 4t$ (b) $x = t, y = -t, z = 1 + t$

7. (a) $P(2, -1)$, $\mathbf{v} = 4\mathbf{i} - \mathbf{j}$ (b) $P(-1, 2, 4)$, $\mathbf{v} = 5\mathbf{i} + 7\mathbf{j} - 8\mathbf{k}$

9. (a) $\langle -3, 4 \rangle + t\langle 1, 5 \rangle$; $-3\mathbf{i} + 4\mathbf{j} + t(\mathbf{i} + 5\mathbf{j})$
(b) $\langle 2, -3, 0 \rangle + t\langle -1, 5, 1 \rangle$; $2\mathbf{i} - 3\mathbf{j} + t(-\mathbf{i} + 5\mathbf{j} + \mathbf{k})$

Responses to True–False questions may be abridged to save space.

11. False; the lines could be skew.

13. False; see part (b) of the solution to Example 3.

15. $x = -5 + 2t, y = 2 - 3t$ **17.** $x = 3 + 4t, y = -4 + 3t$

19. $x = -1 + 3t, y = 2 - 4t, z = 4 + t$

21. $x = -2 + 2t, y = -t, z = 5 + 2t$

23. (a) $x = 7$ (b) $y = \frac{7}{3}$ (c) $x = \dfrac{-1 \pm \sqrt{85}}{6}, y = \dfrac{43 \mp \sqrt{85}}{18}$

25. $(-2, 10, 0)$; $(-2, 0, -5)$; the line does not intersect the yz-plane.

27. $(0, 4, -2)$, $(4, 0, 6)$ **29.** $(1, -1, 2)$ **33.** The lines are parallel.

35. The points do not lie on the same line.

39. $\langle x, y \rangle = \langle -1, 2 \rangle + t\langle 1, 1 \rangle$

41. the point $1/n$ of the way from $(-2, 0)$ to $(1, 3)$

43. the line segment joining the points $(1, 0)$ and $(-3, 6)$

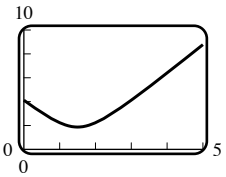
45. $(5, 2)$ **47.** $2\sqrt{5}$ **49.** distance $= \sqrt{35/6}$

51. (a) $x = x_0 + (x_1 - x_0)t, y = y_0 + (y_1 - y_0)t, z = z_0 + (z_1 - z_0)t$
(b) $x = x_1 + at, y = y_1 + bt, z = z_1 + ct$

53. (b) $\langle x, y, z \rangle = \langle 1 + 2t, -3 + 4t, 5 + t \rangle$

55. (b) 84° (c) $x = 7 + t, y = -1, z = -2 + t$

57. $x = t, y = 2 + t, z = 1 - t$

59. (a) $\sqrt{17}$ cm (b) 

(d) $\sqrt{14}/2$ cm

Exercise Set 11.6 (Page 819)

1. $x = 3, y = 4, z = 5$ **3.** $x + 4y + 2z = 28$ **5.** $z = 0$

7. $x - y = 0$ **9.** $y + z = 1$ **11.** $2y - z = 1$

13. (a) parallel (b) perpendicular (c) neither

15. (a) parallel (b) neither (c) perpendicular

17. (a) point of intersection is $\left(\frac{5}{2}, \frac{5}{2}, \frac{5}{2}\right)$ (b) no intersection

19. 35°

Responses to True–False questions may be abridged to save space.

21. True; each will be the negative of the other.

23. True; the direction vector of L must be orthogonal to both normal vectors.

25. $4x - 2y + 7z = 0$ **27.** $4x - 13y + 21z = -14$

29. $x + y - 3z = 6$ **31.** $x + 5y + 3z = -6$

33. $x + 2y + 4z = \frac{29}{2}$ **35.** $x = 5 - 2t, y = 5t, z = -2 + 11t$

37. $7x + y + 9z = 25$ **39.** yes

41. $x = -\frac{11}{7} - 23t, y = -\frac{12}{7} + t, z = -7t$

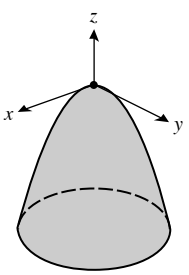
43. $\frac{5}{3}$ **45.** $5/\sqrt{54}$ **47.** $25/\sqrt{126}$

49. $(x - 2)^2 + (y - 1)^2 + (z + 3)^2 = \frac{121}{14}$ **51.** $5/\sqrt{12}$

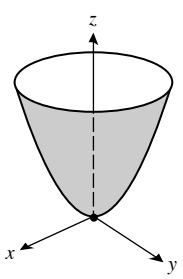
Exercise Set 11.7 (Page 830)

1. (a) elliptic paraboloid, $a = 2, b = 3$
(b) hyperbolic paraboloid, $a = 1, b = 5$
(c) hyperboloid of one sheet, $a = b = c = 4$
(d) circular cone, $a = b = 1$ (e) elliptic paraboloid, $a = 2, b = 1$
(f) hyperboloid of two sheets, $a = b = c = 1$

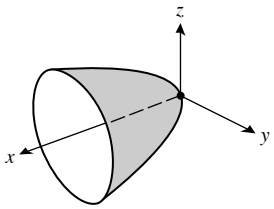
3. (a) $-z = x^2 + y^2$, circular paraboloid opening down the negative z-axis



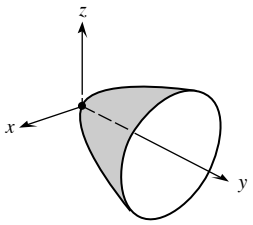
(b) $z = x^2 + y^2$, circular paraboloid, no change
(c) $z = x^2 + y^2$, circular paraboloid, no change
(d) $z = x^2 + y^2$, circular paraboloid, no change



(e) $x = y^2 + z^2$, circular paraboloid opening along the positive x-axis

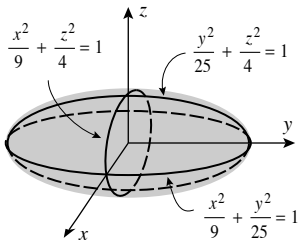


(f) $y = x^2 + z^2$, circular paraboloid opening along the positive y-axis

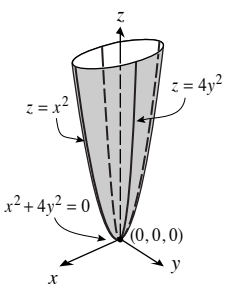


5. **(a)** hyperboloid of one sheet, axis is y-axis
(b) hyperboloid of two sheets separated by yz-plane
(c) elliptic paraboloid opening along the positive x-axis
(d) elliptic cone with x-axis as axis
(e) hyperbolic paraboloid straddling the x-axis
(f) paraboloid opening along the negative y-axis

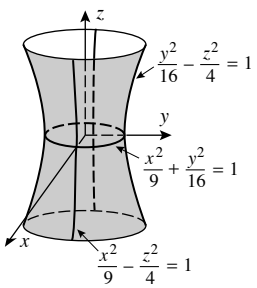
7. **(a)** $x = 0$: $\dfrac{y^2}{25} + \dfrac{z^2}{4} = 1$;
$y = 0$: $\dfrac{x^2}{9} + \dfrac{z^2}{4} = 1$;
$z = 0$: $\dfrac{x^2}{9} + \dfrac{y^2}{25} = 1$



(b) $x = 0$: $z = 4y^2$;
$y = 0$: $z = x^2$;
$z = 0$: $x = y = 0$



(c) $x = 0$: $\dfrac{y^2}{16} - \dfrac{z^2}{4} = 1$;
$y = 0$: $\dfrac{x^2}{9} - \dfrac{z^2}{4} = 1$;
$z = 0$: $\dfrac{x^2}{9} + \dfrac{y^2}{16} = 1$



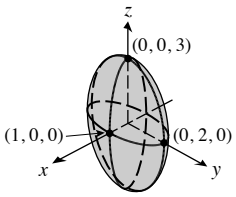
9. **(a)** $4x^2 + z^2 = 3$; ellipse **(b)** $y^2 + z^2 = 3$; circle
(c) $y^2 + z^2 = 20$; circle **(d)** $9x^2 - y^2 = 20$; hyperbola
(e) $z = 9x^2 + 16$; parabola **(f)** $9x^2 + 4y^2 = 4$; ellipse

Responses to True–False questions may be abridged to save space.

11. False; "quadric" refers to second powers.

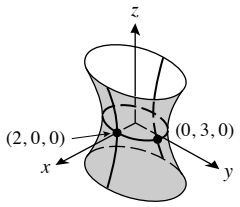
13. False; none of the cross sections need be circles.

15.



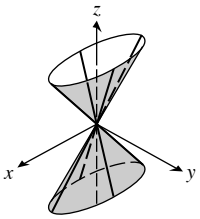
Ellipsoid

17.



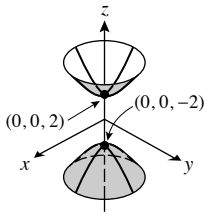
Hyperboloid of one sheet

19.



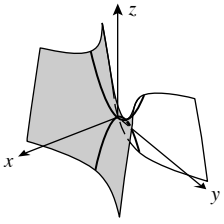
Elliptic cone

21.



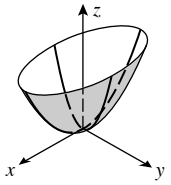
Hyperboloid of two sheets

23.



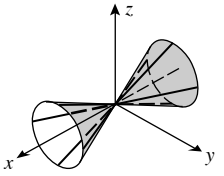
Hyperbolic paraboloid

25.



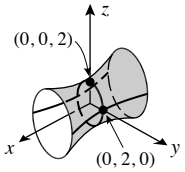
Elliptic paraboloid

27.



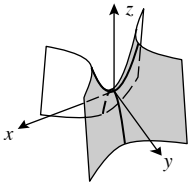
Circular cone

29.



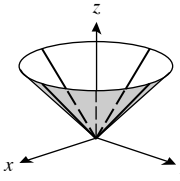
Hyperboloid of one sheet

31.

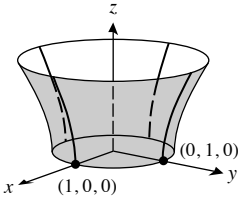


Hyperbolic paraboloid

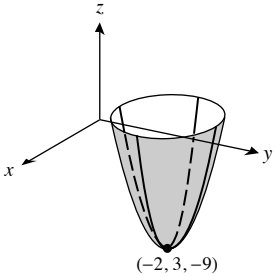
33.



35.

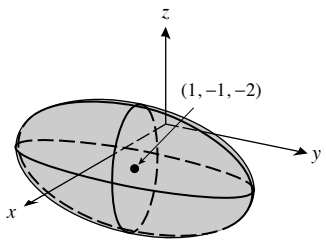


37.



Circular paraboloid

39.



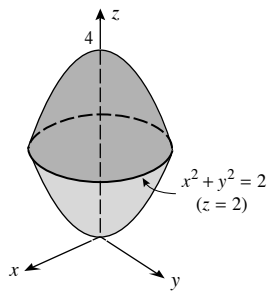
Ellipsoid

41. **(a)** $\frac{x^2}{9}+\frac{y^2}{4}=1$ **(b)** $6, 4$ **(c)** $(\pm\sqrt{5}, 0, \sqrt{2})$
(d) The focal axis is parallel to the x-axis.

43. **(a)** $\frac{y^2}{4}-\frac{x^2}{4}=1$ **(b)** $(0, \pm 2, 4)$ **(c)** $(0, \pm 2\sqrt{2}, 4)$
(d) The focal axis is parallel to the y-axis.

45. **(a)** $z+4=y^2$ **(b)** $(2, 0, -4)$ **(c)** $\left(2, 0, -\frac{15}{4}\right)$
(d) The focal axis is parallel to the z-axis.

47. circle of radius $\sqrt{2}$ in the plane $z=2$, centered at $(0, 0, 2)$



49. $y=4(x^2+z^2)$ **51.** $z=(x^2+y^2)/4$ (circular paraboloid)

Exercise Set 11.8 (Page 837)

1. **(a)** $(8, \pi/6, -4)$ **(b)** $(5\sqrt{2}, 3\pi/4, 6)$
(c) $(2, \pi/2, 0)$ **(d)** $(8, 5\pi/3, 6)$

3. **(a)** $(2\sqrt{3}, 2, 3)$ **(b)** $(-4\sqrt{2}, 4\sqrt{2}, -2)$
(c) $(5, 0, 4)$ **(d)** $(-7, 0, -9)$

5. **(a)** $(2\sqrt{2}, \pi/3, 3\pi/4)$ **(b)** $(2, 7\pi/4, \pi/4)$
(c) $(6, \pi/2, \pi/3)$ **(d)** $(10, 5\pi/6, \pi/2)$

7. **(a)** $(5\sqrt{6}/4, 5\sqrt{2}/4, 5\sqrt{2}/2)$ **(b)** $(7, 0, 0)$
(c) $(0, 0, 1)$ **(d)** $(0, -2, 0)$

9. **(a)** $(2\sqrt{3}, \pi/6, \pi/6)$ **(b)** $(\sqrt{2}, \pi/4, 3\pi/4)$
(c) $(2, 3\pi/4, \pi/2)$ **(d)** $(4\sqrt{3}, 1, 2\pi/3)$

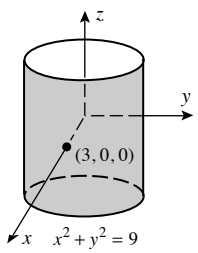
11. **(a)** $(5\sqrt{3}/2, \pi/4, -5/2)$ **(b)** $(0, 7\pi/6, -1)$
(c) $(0, 0, 3)$ **(d)** $(4, \pi/6, 0)$

Responses to True–False questions may be abridged to save space.

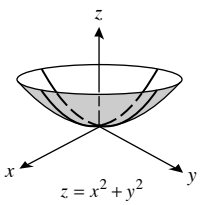
15. True; see Figure 11.8.1*b*.

17. True; see Figures 11.8.3 and 11.8.4.

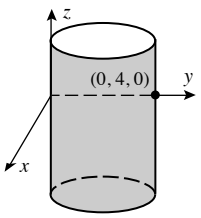
19.



21.

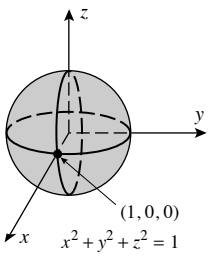


23.



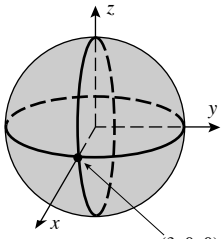
$x^2+(y-2)^2=4$

25.



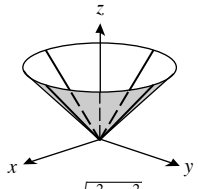
$x^2+y^2+z^2=1$

27.



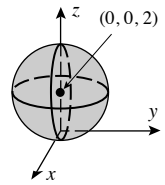
$x^2+y^2+z^2=9$

29.



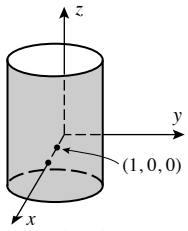
$z=\sqrt{x^2+y^2}$

31.



$x^2+y^2+(z-2)^2=4$

33.



$(x-1)^2+y^2=1$

35. **(a)** $z=3$ **(b)** $\rho=3\sec\phi$ **37.** **(a)** $z=3r^2$ **(b)** $\rho=\frac{1}{3}\csc\phi\cot\phi$

39. **(a)** $r=2$ **(b)** $\rho=2\csc\phi$ **41.** **(a)** $r^2+z^2=9$ **(b)** $\rho=3$

43. **(a)** $2r\cos\theta+3r\sin\theta+4z=1$
(b) $2\rho\sin\phi\cos\theta+3\rho\sin\phi\sin\theta+4\rho\cos\phi=1$

45. **(a)** $r^2\cos^2\theta=16-z^2$ **(b)** $\rho^2(1-\sin^2\phi\sin^2\theta)=16$

47. all points on or above the paraboloid $z=x^2+y^2$ that are also on or below the plane $z=4$

49. all points on or between concentric spheres of radii 1 and 3 centered at the origin

51. spherical: $(4000, \pi/6, \pi/6)$; rectangular: $(1000\sqrt{3}, 1000, 2000\sqrt{3})$

53. **(a)** $(10, \pi/2, 1)$ **(b)** $(0, 10, 1)$ **(c)** $(\sqrt{101}, \pi/2, \tan^{-1}10)$

Chapter 11 Review Exercises (Page 838)

3. **(b)** $-1/2, \pm\sqrt{3}/2$ **(d)** true

5. $(x+3)^2+(y-5)^2+(z+4)^2=r^2$;
(a) $r^2=16$ **(b)** $r^2=25$ **(c)** $r^2=9$

7. $(7, 5)$

9. **(a)** $-\frac{3}{4}$ **(b)** $\frac{1}{7}$ **(c)** $(48\pm25\sqrt{3})/11$ **(d)** $c=\frac{4}{3}$

13. 13 ft·lb **15.** **(a)** $\sqrt{26}/2$ **(b)** $\sqrt{26}/3$

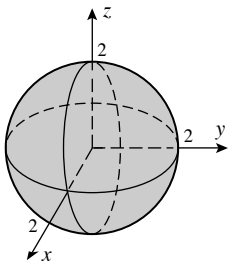
17. **(a)** 29 **(b)** $\frac{29}{\sqrt{65}}$ **19.** $x=4+t, y=1-t, z=2$

21. $x+5y-z-2=0$ **23.** $a_1a_2+b_1b_2+c_1c_2=0$

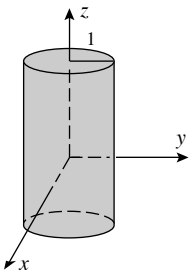
25. **(a)** hyperboloid of one sheet **(b)** sphere **(c)** circular cone

27. **(a)** $z=x^2-y^2$ **(b)** $xz=1$

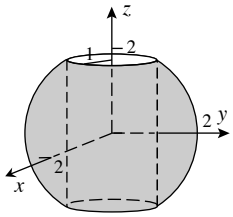
29. (a)



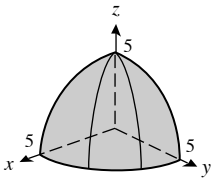
(b)



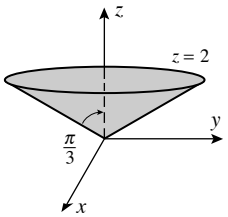
(c)



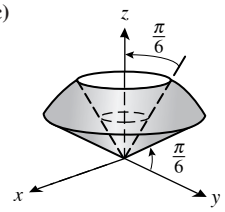
31. (a)



(b)



(c)



▶ Chapter 11 Making Connections (Page 840)

Answers are provided in the Student Solutions Manual.

▶ Exercise Set 12.1 (Page 845)

1. $(-\infty, +\infty)$; $\mathbf{r}(\pi) = -\mathbf{i} - 3\pi\mathbf{j}$ **3.** $[2, +\infty)$; $\mathbf{r}(3) = -\mathbf{i} - \ln 3\mathbf{j} + \mathbf{k}$

5. $\mathbf{r} = 3\cos t\,\mathbf{i} + (t + \sin t)\mathbf{j}$ **7.** $x = 3t^2$, $y = -2$

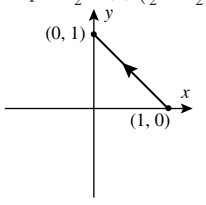
9. the line in 2-space through $(3, 0)$ with direction vector $\mathbf{a} = -2\mathbf{i} + 5\mathbf{j}$

11. the line in 3-space through the point $(0, -3, 1)$ and parallel to the vector $2\mathbf{i} + 3\mathbf{k}$

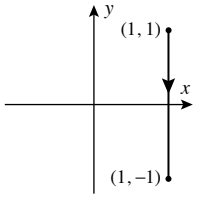
13. an ellipse centered at $(0, 0, 1)$ in the plane $z = 1$

15. **(a)** slope $-\frac{3}{2}$ **(b)** $\left(\frac{5}{2}, 0, \frac{3}{2}\right)$

17. (a)

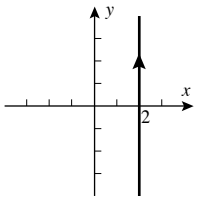


(b)

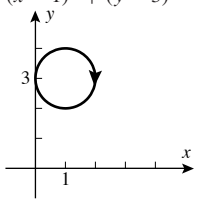


19. $\mathbf{r} = (1 - t)(3\mathbf{i} + 4\mathbf{j})$, $0 \le t \le 1$

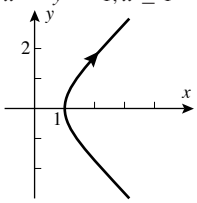
21. $x = 2$



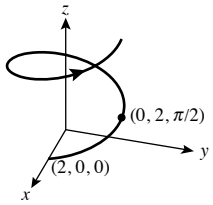
23. $(x - 1)^2 + (y - 3)^2 = 1$



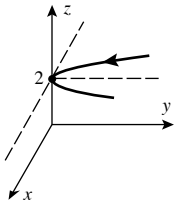
25. $x^2 - y^2 = 1, x \ge 1$



27.



29.

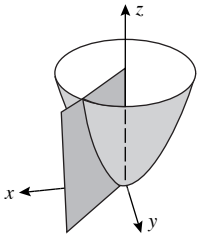


Responses to True–False questions may be abridged to save space.

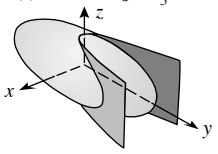
31. False; the natural domain of a vector-valued function is the *intersection* of the domains of its component functions.

33. True; $\mathbf{r}(t) = (1 - t)\mathbf{r}_0 + t\mathbf{r}_1 (0 \le t \le 1)$ represents the line segment in 3-space that is traced from $\mathbf{r}_0$ to $\mathbf{r}_1$.

35. $x = t$, $y = t$, $z = 2t^2$



37. $\mathbf{r}(t) = t\mathbf{i} + t^2\mathbf{j} \pm \frac{1}{3}\sqrt{81 - 9t^2 - t^4}\,\mathbf{k}$ **43.** $c = 3/(2\pi)$



47. **(a)** III, since the curve is a subset of the plane $y = -x$

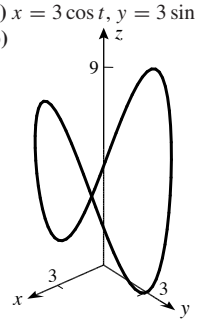
(b) IV, since only x is periodic in t and y, z increase without bound

(c) II, since all three components are periodic in t

(d) I, since the projection onto the yz-plane is a circle and the curve increases without bound in the x-direction

49. **(a)** $x = 3\cos t,\ y = 3\sin t,\ z = 9\cos^2 t$

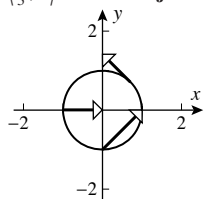
(b)



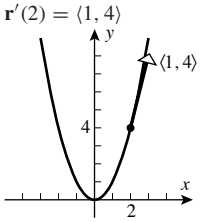
Exercise Set 12.2 (Page 856)

1. $\langle \frac{1}{3}, 0 \rangle$ **3.** $2\mathbf{i} - 3\mathbf{j} + 4\mathbf{k}$ **5.** **(a)** continuous **(b)** not continuous

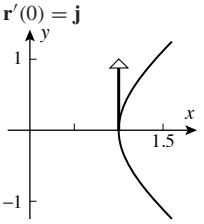
7.



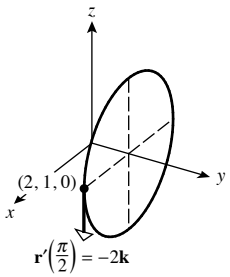
9. $(\sin t)\mathbf{j}$ **11.** $\mathbf{r}'(2) = \langle 1, 4 \rangle$



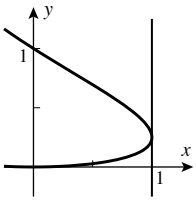
13. $\mathbf{r}'(0) = \mathbf{j}$



15. $\mathbf{r}'(\pi/2) = -2\mathbf{k}$



17.



19. $x = 1 + 2t,\ y = 2 - t$

21. $x = 1 - \sqrt{3}\pi t,\ y = \sqrt{3} + \pi t,\ z = 1 + 3t$

23. $\mathbf{r} = (-\mathbf{i} + 2\mathbf{j}) + t\left(2\mathbf{i} + \frac{3}{4}\mathbf{j}\right)$

25. $\mathbf{r} = (4\mathbf{i} + \mathbf{j}) + t(-4\mathbf{i} + \mathbf{j} + 4\mathbf{k})$

27. **(a)** $\mathbf{i} - \mathbf{j} + \mathbf{k}$ **(b)** $-\mathbf{i} + \mathbf{k}$ **(c)** 0

29. $7t^6;\ 18t^5\mathbf{i} - 10t^4\mathbf{j}$

31. $3t\mathbf{i} + 2t^2\mathbf{j} + \mathbf{C}$

33. $\langle te^t - e^t,\ t\ln t - t \rangle + \mathbf{C}$ **35.** $\mathbf{j}$ **37.** $(5\sqrt{5} - 1)/3$

39. $\frac{52}{3}\mathbf{i} + 4\mathbf{j}$

Responses to True–False questions may be abridged to save space.

41. False; for example, $\mathbf{r}(t) = \langle t, |t| \rangle$ is continuous at $t = 0$, but the specified limit doesn't exist at $t = 0$.

43. True; see the definition of $\int_a^b \mathbf{r}(t)\,dt$.

45. $(t^2 + 1)\mathbf{i} + (t^3 - 1)\mathbf{j}$

47. $y(t) = \left(\frac{1}{2}t^2 + 2\right)\mathbf{i} + (e^t - 1)\mathbf{j}$

49. **(a)** $(-2, 4, 6)$ and $(1, 1, -3)$ **(b)** $76°,\ 71°$ **51.** $68°$

Exercise Set 12.3 (Page 866)

1. smooth **3.** not smooth, $\mathbf{r}'(1) = \mathbf{0}$ **5.** $L = \frac{3}{2}$ **7.** $L = e - e^{-1}$

9. $L = 28$ **11.** $L = 2\pi\sqrt{10}$ **13.** $\mathbf{r}'(\tau) = 4\mathbf{i} + 8(4\tau + 1)\mathbf{j}$

15. $\mathbf{r}'(\tau) = 2\tau e^{\tau^2}\mathbf{i} - 8\tau e^{-\tau^2}\mathbf{j}$

Responses to True–False questions may be abridged to save space.

17. False; $\int_a^b \|\mathbf{r}'(t)\|\,dt$ is a scalar that represents the arc length of the curve in 2-space traced by $\mathbf{r}(t)$ from $t = a$ to $t = b$ (Theorem 12.3.1).

19. False; $\mathbf{r}'$ isn't defined at the point corresponding to the origin.

21. **(a)** $x = \frac{s}{\sqrt{2}},\ y = \frac{s}{\sqrt{2}}$ **(b)** $x = y = z = \frac{s}{\sqrt{3}}$

23. **(a)** $x = 1 + \frac{s}{3},\ y = 3 - \frac{2s}{3},\ z = 4 + \frac{2s}{3}$ **(b)** $\left\langle \frac{28}{3}, -\frac{41}{3}, \frac{62}{3} \right\rangle$

25. $x = 3 + \cos s,\ y = 2 + \sin s,\ 0 \le s \le 2\pi$

27. $x = \frac{1}{3}[(3s + 1)^{2/3} - 1]^{3/2},\ y = \frac{1}{2}[(3s + 1)^{2/3} - 1],\ s \ge 0$

29. $x = \left(\frac{s}{\sqrt{2}} + 1\right)\cos\left[\ln\left(\frac{s}{\sqrt{2}} + 1\right)\right]$,

$y = \left(\frac{s}{\sqrt{2}} + 1\right)\sin\left[\ln\left(\frac{s}{\sqrt{2}} + 1\right)\right]$,

$0 \le s \le \sqrt{2}(e^{\pi/2} - 1)$

33. $x = 2a\cos^{-1}[1 - s/(4a)] - 2a(1 - [1 - s/(4a)]^2)^{1/2}(2[1 - s/(4a)]^2 - 1)$,

$y = \dfrac{s(8a - s)}{8a}$ for $0 \le s \le 8a$

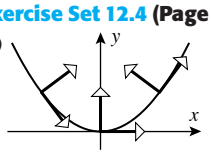
35. **(a)** $9/2$ **(b)** $9 - 2\sqrt{6}$ **37.** **(a)** $\sqrt{3}(1 - e^{-2})$ **(b)** $4\sqrt{5}$

39. **(a)** $g(\tau) = \pi(\tau)$ **(b)** $g(\tau) = \pi(1 - \tau)$ **41.** 44 in

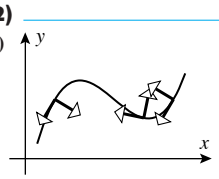
43. **(a)** $2t + \dfrac{1}{t}$ **(b)** $2t + \dfrac{1}{t}$ **(c)** $8 + \ln 3$

Exercise Set 12.4 (Page 872)

1. **(a)**



(b)



5. $\mathbf{T}(1) = \frac{2}{\sqrt{5}}\mathbf{i} + \frac{1}{\sqrt{5}}\mathbf{j},\ \mathbf{N}(1) = \frac{1}{\sqrt{5}}\mathbf{i} - \frac{2}{\sqrt{5}}\mathbf{j}$

7. $\mathbf{T}\left(\frac{\pi}{3}\right) = -\frac{\sqrt{3}}{2}\mathbf{i} + \frac{1}{2}\mathbf{j},\ \mathbf{N}\left(\frac{\pi}{3}\right) = -\frac{1}{2}\mathbf{i} - \frac{\sqrt{3}}{2}\mathbf{j}$

9. $\mathbf{T}\left(\frac{\pi}{2}\right) = -\frac{4}{\sqrt{17}}\mathbf{i} + \frac{1}{\sqrt{17}}\mathbf{k},\ \mathbf{N}\left(\frac{\pi}{2}\right) = -\mathbf{j}$

11. $\mathbf{T}(0) = \frac{1}{\sqrt{3}}\mathbf{i} + \frac{1}{\sqrt{3}}\mathbf{j} + \frac{1}{\sqrt{3}}\mathbf{k},\ \mathbf{N}(0) = -\frac{1}{\sqrt{2}}\mathbf{i} + \frac{1}{\sqrt{2}}\mathbf{j}$

13. $x = s,\ y = 1$ **15.** $\mathbf{B} = \frac{4}{5}\cos t\,\mathbf{i} - \frac{4}{5}\sin t\,\mathbf{j} - \frac{3}{5}\mathbf{k}$ **17.** $\mathbf{B} = -\mathbf{k}$

19. $\mathbf{T}\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}(-\mathbf{i} + \mathbf{j}),\ \mathbf{N}\left(\frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2}(\mathbf{i} + \mathbf{j})$,
$\mathbf{B}\left(\frac{\pi}{4}\right) = \mathbf{k}$; rectifying: $x + y = \sqrt{2}$; osculating: $z = 1$;
normal: $-x + y = 0$

Responses to True–False questions may be abridged to save space.

21. False; $\mathbf{T}(t)$ points in the direction of increasing parameter but may not be orthogonal to $\mathbf{r}(t)$. For example, if $\mathbf{r}(t) = \langle t, t \rangle$, then $\mathbf{T}(t) = \langle 1/\sqrt{2}, 1/\sqrt{2} \rangle$ is parallel to $\mathbf{r}(t)$.

23. True; $\mathbf{T}(s) = \mathbf{r}'(s)$, the unit tangent vector, and $\mathbf{N}(s) = \dfrac{\mathbf{r}''(s)}{\|\mathbf{r}''(s)\|}$, the unit normal vector, are orthogonal, so $\mathbf{r}'(s)$ and $\mathbf{r}''(s)$ are orthogonal.

Exercise Set 12.5 (Page 879)

1. $\kappa \approx 2$ **3.** **(a)** I is the curvature of II. **(b)** I is the curvature of II.

5. $\dfrac{6}{|t|(4 + 9t^2)^{3/2}}$ **7.** $\dfrac{12e^{2t}}{(9e^{6t} + e^{-2t})^{3/2}}$ **9.** $\frac{4}{17}$ **11.** $\dfrac{1}{2\cosh^2 t}$

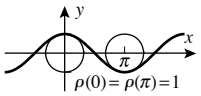
13. $\kappa = \frac{2}{5},\ \rho = \frac{5}{2}$ **15.** $\kappa = \dfrac{\sqrt{2}}{3},\ \rho = \dfrac{3\sqrt{2}}{2}$ **17.** $\kappa = \frac{1}{4}$

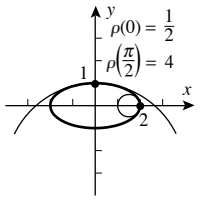
Responses to True–False questions may be abridged to save space.

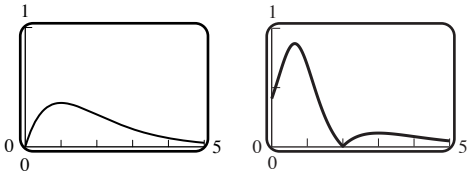
19. True; see Example 1: a circle of radius a has constant curvature $1/a$.

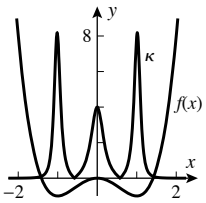
21. False; see Definition 12.5.1: the curvature of the graph of $\mathbf{r}(s)$ is $\|\mathbf{r}''(s)\|$, the length of $\mathbf{r}''(s)$.

25. 1 **27.** $\dfrac{e^{-1}}{(1 + e^{-2})^{3/2}}$ **29.** $\frac{96}{125}$ **31.** $\dfrac{1}{\sqrt{2}}$

33. (a) 

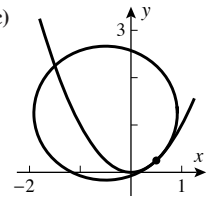
(b) 

35. 

37. (a) $\kappa = \dfrac{|12x^2 - 4|}{[1 + (4x^3 - 4x)^2]^{3/2}}$
(b) 

(c) $\rho = \frac{1}{4}$ for $x = 0$ and $\rho = \frac{1}{8}$ when $x = \pm 1$

41. $\dfrac{3}{2\sqrt{2}}$ **43.** $\frac{2}{3}$ **45.** $\rho = 2|p|$ **47.** $(3, 0), (-3, 0)$

51. (b) $\rho = \sqrt{2}$ (c) 

55. $a = \dfrac{1}{2r}$

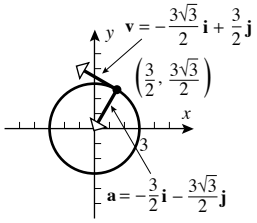
63. $\tau = \dfrac{2}{(t^2 + 2)^2}$

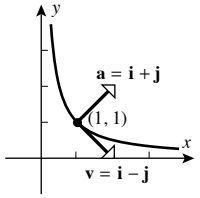
65. $\tau = -\dfrac{\sqrt{2}}{(e^t + e^{-t})^2}$

Exercise Set 12.6 (Page 891)

1. $\mathbf{v}(t) = -3\sin t\,\mathbf{i} + 3\cos t\,\mathbf{j}$
$\mathbf{a}(t) = -3\cos t\,\mathbf{i} - 3\sin t\,\mathbf{j}$
$\|\mathbf{v}(t)\| = 3$

3. $\mathbf{v}(t) = e^t\mathbf{i} - e^{-t}\mathbf{j}$
$\mathbf{a}(t) = e^t\mathbf{i} + e^{-t}\mathbf{j}$
$\|\mathbf{v}(t)\| = \sqrt{e^{2t} + e^{-2t}}$

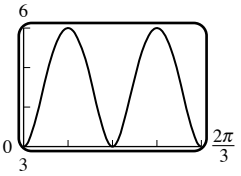




5. $\mathbf{v} = \mathbf{i} + \mathbf{j} + \mathbf{k}$, $\|\mathbf{v}\| = \sqrt{3}$, $\mathbf{a} = \mathbf{j} + 2\mathbf{k}$

7. $\mathbf{v} = -\sqrt{2}\mathbf{i} + \sqrt{2}\mathbf{j} + \mathbf{k}$, $\|\mathbf{v}\| = \sqrt{5}$, $\mathbf{a} = -\sqrt{2}\mathbf{i} - \sqrt{2}\mathbf{j}$

13. minimum speed $3\sqrt{2}$ when $\mathbf{r} = 24\mathbf{i} + 8\mathbf{j}$

15. (a) 

(b) maximum speed = 6, minimum speed = 3
(d) The maximum speed first occurs when $t = \pi/6$.

17. $\mathbf{v}(t) = (1 - \sin t)\mathbf{i} + (\cos t - 1)\mathbf{j}$;
$\mathbf{r}(t) = (t + \cos t - 1)\mathbf{i} + (\sin t - t + 1)\mathbf{j}$

19. $\mathbf{v}(t) = (1 - \cos t)\mathbf{i} + \sin t\,\mathbf{j} + e^t\mathbf{k}$;
$\mathbf{r}(t) = (t - \sin t - 1)\mathbf{i} + (1 - \cos t)\mathbf{j} + e^t\mathbf{k}$

21. 15° **23.** (a) $0.7\mathbf{i} + 2.7\mathbf{j} - 3.4\mathbf{k}$ (b) $\mathbf{r}_0 = -0.7\mathbf{i} - 2.9\mathbf{j} + 4.8\mathbf{k}$

25. $\Delta\mathbf{r} = 8\mathbf{i} + \frac{26}{3}\mathbf{j}$, $s = (13\sqrt{13} - 5\sqrt{5})/3$

27. $\Delta\mathbf{r} = 2\mathbf{i} - \frac{2}{3}\mathbf{j} + \sqrt{2}\ln 3\mathbf{k}$; $s = \frac{8}{3}$

31. (a) $a_T = 0$, $a_N = \sqrt{2}$ (b) $a_T\mathbf{T} = \mathbf{0}$, $a_N\mathbf{N} = \mathbf{i} + \mathbf{j}$ (c) $1/\sqrt{2}$

33. (a) $a_T = 2\sqrt{5}$, $a_N = 2\sqrt{5}$ (b) $a_T\mathbf{T} = 2\mathbf{i} + 4\mathbf{j}$, $a_N\mathbf{N} = 4\mathbf{i} - 2\mathbf{j}$
(c) $2/\sqrt{5}$

35. (a) $a_T = -7/\sqrt{6}$, $a_N = \sqrt{53/6}$
(b) $a_T\mathbf{T} = -\frac{7}{6}(\mathbf{i} - 2\mathbf{j} + \mathbf{k})$, $a_N\mathbf{N} = \frac{13}{6}\mathbf{i} + \frac{5}{3}\mathbf{j} + \frac{7}{6}\mathbf{k}$ (c) $\dfrac{\sqrt{53}}{6\sqrt{6}}$

37. $a_T = -3$, $a_N = 2$, $\mathbf{T} = -\mathbf{j}$, $\mathbf{N} = \mathbf{i}$ **39.** $-3/2$

41. $a_N = 8.41 \times 10^{10}$ km/s^2

43. $a_N = 18/(1 + 4x^2)^{3/2}$ **45.** $a_N = 0$

Responses to True–False questions may be abridged to save space.

47. True; the velocity and unit tangent vectors have the same direction, so are parallel.

49. False; in this case the velocity and acceleration vectors will be parallel, but they may have opposite direction.

53. ≈ 257.20 N

55. $40\sqrt{3}$ ft **57.** 800 ft/s **59.** 15° or 75° **61.** (c) ≈ 14.942 ft

63. (a) $\rho \approx 176.78$ m (b) $\frac{125}{4}$ m

65. (b) R is maximum when $\alpha = 45°$, maximum value v_0^2/g

67. (a) 2.62 s (b) 181.5 ft

69. (a) $v_0 \approx 83$ ft/s, $\alpha \approx 8°$ (b) 268.76 ft

Exercise Set 12.7 (Page 901)

7. 7.75 km/s **9.** 10.88 km/s

11. (a) minimum distance = 220,680 mi, maximum distance = 246,960 mi (b) 27.5 days

13. (a) 17,224 mi/h (b) $e \approx 0.071$, apogee altitude = 819 mi

Chapter 12 Review Exercises (Page 902)

3. the circle of radius 3 in the xy-plane, with center at the origin

5. a parabola in the plane $x = -2$, vertex at $(-2, 0, -1)$, opening upward

11. $x = 1 + t$, $y = -t$, $z = t$ **13.** $(\sin t)\mathbf{i} - (\cos t)\mathbf{j} + \mathbf{C}$

15. $y(t) = \left(\frac{1}{3}t^3 + 1\right)\mathbf{i} + (t^2 + 1)\mathbf{j}$ **17.** 15/4

19. $\mathbf{r}(s) = \dfrac{s - 3}{3}\mathbf{i} + \dfrac{12 - 2s}{3}\mathbf{j} + \dfrac{9 + 2s}{3}\mathbf{k}$ **25.** 3/5 **27.** 0

29. (a) speed (b) distance traveled
(c) distance of the particle from the origin

33. (a) $\mathbf{r}(t) = \left(\frac{1}{6}t^4 + t\right)\mathbf{i} + \left(\frac{1}{2}t^2 + 2t\right)\mathbf{j} - \left(\frac{1}{4}\cos 2t + t - \frac{1}{4}\right)\mathbf{k}$
(b) 3.475 **35.** 10.65 km/s **37.** 24.78 ft

Chapter 12 Making Connections (Page 904)

Where correct answers to a Making Connections exercise may vary, no answer is listed. Sample answers for these questions are available on the Book Companion Site.

1. (c) (i) $\mathbf{N} = \frac{1}{\sqrt{5}}\mathbf{i} - \frac{2}{\sqrt{5}}\mathbf{j}$ (ii) $\mathbf{N} = -\mathbf{j}$

2. (b) (i) $\mathbf{N} = -\sin t\,\mathbf{i} - \cos t\,\mathbf{j}$
(ii) $\mathbf{N} = \dfrac{-(4t + 18t^3)\mathbf{i} + (2 - 18t^4)\mathbf{j} + (6t + 12t^3)\mathbf{k}}{2\sqrt{81t^8 + 117t^6 + 54t^4 + 13t^2 + 1}}$

3. (c) $\kappa(s) \to +\infty$, so the spiral winds ever tighter.

4. semicircle: 53.479 ft; quarter-circle: 60.976 ft; point: 64.001 ft

Exercise Set 13.1 (Page 914)

1. (a) 5 (b) 3 (c) 1 (d) -2 (e) $9a^3 + 1$ (f) $a^3b^2 - a^2b^3 + 1$

3. (a) $x^2 - y^2 + 3$ (b) $3x^3y^4 + 3$ **5.** $x^3e^{x^3(3y+1)}$

7. (a) $t^2 + 3t^{10}$ (b) 0 (c) 3076

9. (a) 2.5 mg/L (b) $C(100, t) = 20(e^{-0.2t} - e^{-t})$
(c) $C(x, 1) = 0.2x(e^{-0.2} - e^{-1}) \approx 0.09x$

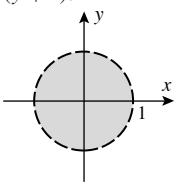
11. (a) WCI = 17.8°F (b) WCI = 22.6°F **13.** (a) 30°F (b) 22.5°F

15. (a) 66% (b) 73.5% (c) 60.6%

17. (a) 19 (b) -9 (c) 3 (d) $a^6 + 3$ (e) $-t^8 + 3$ (f) $(a + b)(a - b)^2b^3 + 3$

19. $(y+1)e^{x^2(y+1)z^2}$ **21.** **(a)** $80\sqrt{\pi}$ **(b)** $n(n+1)/2$

23.



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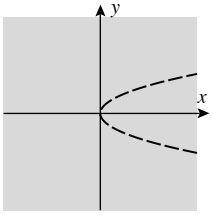


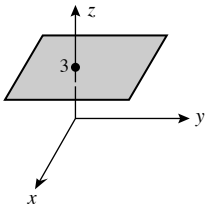
27. **(a)** all points above or on the line $y = -2$ **(b)** all points on or within the sphere $x^2 + y^2 + z^2 = 25$ **(c)** all points in 3-space

Responses to True–False questions may be abridged to save space.

29. True; the interval [0, 1] is the intersection of the domains of $\sin^{-1} t$ and $\sqrt{t}$.

31. False; the natural domain is an infinite solid cylinder.

33.



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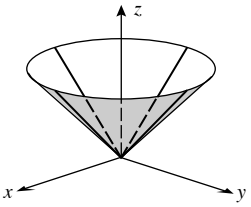

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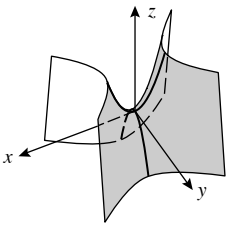

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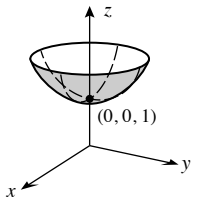

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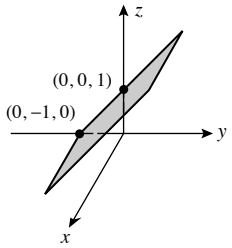


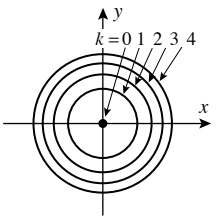
43. **(a)** hyperbolas **(b)** parabolas **(c)** noncircular ellipses **(d)** lines

45. **(a)** \$130 **(b)** \$275

47. **(a)** $1 - x^2 - y^2$ **(b)** $\sqrt{x^2 + y^2}$ **(c)** $x^2 + y^2$

49. **(a)** A **(b)** B **(c)** increase **(d)** decrease **(e)** increase **(f)** decrease

51.



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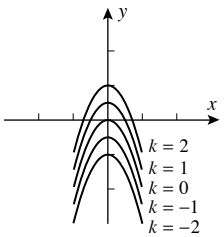

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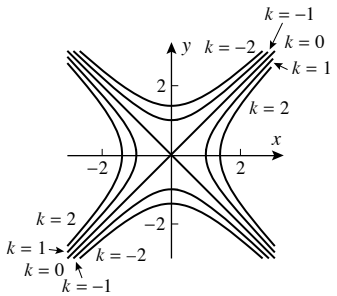

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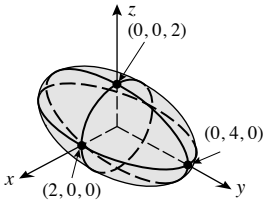
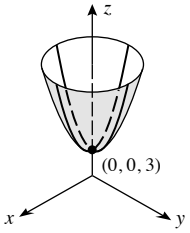


59.



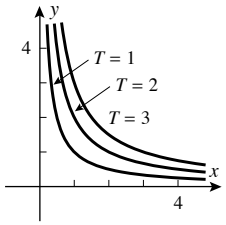
61. concentric spheres, common center at (2, 0, 0)

63. concentric cylinders, common axis the y-axis

65. **(a)** $x^2 - 2x^3 + 3xy = 0$ **(b)** $x^2 - 2x^3 + 3xy = 0$ **(c)** $x^2 - 2x^3 + 3xy = -18$

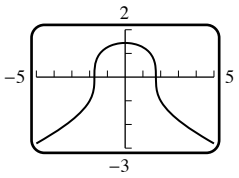
67. **(a)** $x^2 + y^2 - z = 5$ **(b)** $x^2 + y^2 - z = -2$ **(c)** $x^2 + y^2 - z = 0$

69. **(a)**

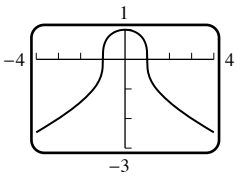


(b) the path $xy = 4$

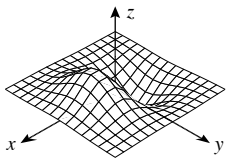
71. **(a)**



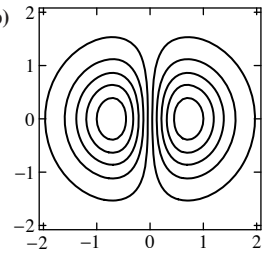
(b)



73. **(a)**



(b)



75. **(a)** The graph of g is the graph of f shifted one unit in the positive x-direction.

(b) The graph of g is the graph of f shifted one unit up the z-axis.

(c) The graph of g is the graph of f shifted one unit down the y-axis and then inverted with respect to the plane $z = 0$.

▶ Exercise Set 13.2 (Page 925)

1. 35 **3.** −8 **5.** 0

7. **(a)** along $x = 0$ limit does not exist

(b) along $x = 0$ limit does not exist

9. 1 **11.** 0 **13.** 0 **15.** limit does not exist **17.** $\frac{8}{3}$ **19.** 0

21. limit does not exist **23.** 0 **25.** 0 **27.** 0

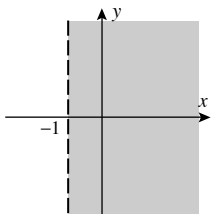
Responses to True–False questions may be abridged to save space.

29. True; by the definition of open set.

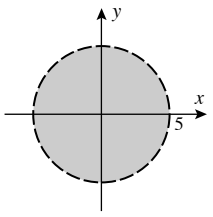
31. False; let $f(x, y) = \begin{cases} 1, & x \le 0 \\ -1, & x > 0 \end{cases}$ and let $g(x, y) = -f(x, y)$.

33. **(a)** no **(d)** no; yes **37.** $-\pi/2$ **39.** no

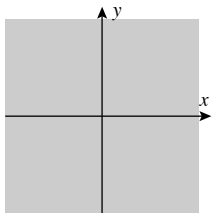
41.



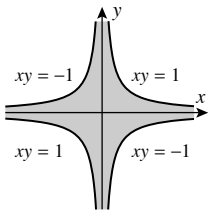
43.



45.



47.



49. all of 3-space **51.** all points not on the cylinder $x^2 + z^2 = 1$

Exercise Set 13.3 (Page 936)

1. **(a)** $9x^2y^2$ **(b)** $6x^3y$ **(c)** $9y^2$ **(d)** $9x^2$ **(e)** $6y$ **(f)** $6x^3$ **(g)** 36 **(h)** 12

3. $18xy - 15x^4y,\ 9x^2 - 3x^5$

5. $(16x + 40)(x^2 + 5x - 2y)^7,\ -16(x^2 + 5x - 2y)^7$

7. $-\dfrac{7}{q}e^{-7p/q},\ \dfrac{7p}{q^2}e^{-7p/q}$

9. $(15x^2y + 7y^2)\cos(5x^3y + 7xy^2),\ (5x^3 + 14xy)\cos(5x^3y + 7xy^2)$

11. **(a)** $\frac{3}{8}$ **(b)** $\frac{1}{4}$ **13.** **(a)** $-4\cos 7$ **(b)** $2\cos 7$

15. $\partial z/\partial x = -4;\ \partial z/\partial y = \frac{1}{2}$ **17.** **(a)** 4.9 **(b)** 1.2

19. $z = f(x, y)$ has II as its graph, f_x has I as its graph, and f_y has III as its graph.

Responses to True–False questions may be abridged to save space.

21. True; on $y = 2$, $f(x, 2) = c$ is a constant function of x.

23. True; z must be a linear function of x and y.

25. $8xy^3e^{x^2y^3},\ 12x^2y^2e^{x^2y^3}$

27. $x^3/(y^{3/5} + x) + 3x^2\ln(1 + xy^{-3/5}),\ -\frac{3}{5}x^4/(y^{8/5} + xy)$

29. $-\dfrac{y(x^2 - y^2)}{(x^2 + y^2)^2},\ \dfrac{x(x^2 - y^2)}{(x^2 + y^2)^2}$

31. $(3/2)x^2y(5x^2 - 7)(3x^5y - 7x^3y)^{-1/2}$
$(1/2)x^3(3x^2 - 7)(3x^5y - 7x^3y)^{-1/2}$

33. $\dfrac{y^{-1/2}}{y^2 + x^2},\ -\dfrac{xy^{-3/2}}{y^2 + x^2} - \dfrac{3}{2}y^{-5/2}\tan^{-1}\left(\dfrac{x}{y}\right)$

35. $-\frac{4}{3}y^2\sec^2 x(y^2\tan x)^{-7/3},\ -\frac{8}{3}y\tan x(y^2\tan x)^{-7/3}$

37. $-6, -21$ **39.** $1/\sqrt{17}, 8/\sqrt{17}$

41. **(a)** $2xy^4z^3 + y$ **(b)** $4x^2y^3z^3 + x$ **(c)** $3x^2y^4z^2 + 2z$
(d) $2y^4z^3 + y$ **(e)** $32z^3 + 1$ **(f)** 438

43. $2z/x,\ z/y,\ \ln(x^2y\cos z) - z\tan z$

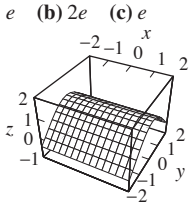
45. $-y^2z^3/(1 + x^2y^4z^6),\ -2xyz^3/(1 + x^2y^4z^6),\ -3xy^2z^2/(1 + x^2y^4z^6)$

47. $yze^z\cos(xz),\ e^z\sin(xz),\ ye^z(\sin(xz) + x\cos(xz))$

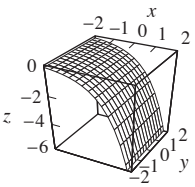
49. $x/\sqrt{x^2 + y^2 + z^2},\ y/\sqrt{x^2 + y^2 + z^2},\ z/\sqrt{x^2 + y^2 + z^2}$

51. **(a)** e **(b)** $2e$ **(c)** e

53. **(a)**



(b)



55. 4 **57.** -2

59. **(a)** $\partial V/\partial r = 2\pi rh$ **(b)** $\partial V/\partial h = \pi r^2$ **(c)** 48π **(d)** 64π

61. **(a)** $\dfrac{1}{5}\dfrac{\text{lb}}{\text{in}^2\cdot\text{K}}$ **(b)** $-\dfrac{25}{8}\dfrac{\text{in}^5}{\text{lb}}$

63. **(a)** $\dfrac{\partial V}{\partial l} = 6$ **(b)** $\dfrac{\partial V}{\partial w} = 15$ **(c)** $\dfrac{\partial V}{\partial h} = 10$

67. **(a)** $\pm\sqrt{6}/4$ **69.** $-x/z, -y/z$

71. $-\dfrac{2x + yz^2\cos(xyz)}{xyz\cos(xyz) + \sin(xyz)};\ -\dfrac{xz^2\cos(xyz)}{xyz\cos(xyz) + \sin(xyz)}$

73. $-x/w, -y/w, -z/w$

75. $-\dfrac{yzw\cos(xyz)}{2w + \sin(xyz)}, -\dfrac{xzw\cos(xyz)}{2w + \sin(xyz)}, -\dfrac{xyw\cos(xyz)}{2w + \sin(xyz)}$

77. $e^{x^2}, -e^{y^2}$

79. $f_x(x, y) = 2xy^3\sin(x^6y^9),\ f_y(x, y) = 3x^2y^2\sin(x^6y^9)$

81. **(a)** $-\dfrac{\cos y}{4\sqrt{x^3}}$ **(b)** $-\sqrt{x}\cos y$ **(c)** $-\dfrac{1}{2\sqrt{x}}\sin y$ **(d)** $-\dfrac{1}{2\sqrt{x}}\sin y$

83. **(a)** $6\cos(3x^2 + 6y^2) - 36x^2\sin(3x^2 + 6y^2)$
(b) $12\cos(3x^2 + 6y^2) - 144y^2\sin(3x^2 + 6y^2)$
(c) $-72xy\sin(3x^2 + 6y^2)$ **(d)** $-72xy\sin(3x^2 + 6y^2)$

85. $-32y^3$ **87.** $-e^x\sin y$ **89.** $\dfrac{20}{(4x - 5y)^2}$ **91.** $\dfrac{2(x - y)}{(x + y)^3}$

93. **(a)** $\dfrac{\partial^3 f}{\partial x^3}$ **(b)** $\dfrac{\partial^3 f}{\partial y^2\partial x}$ **(c)** $\dfrac{\partial^4 f}{\partial x^2\partial y^2}$ **(d)** $\dfrac{\partial^4 f}{\partial y^3\partial x}$

95. **(a)** $30xy^4 - 4$ **(b)** $60x^2y^3$ **(c)** $60x^3y^2$

97. **(a)** -30 **(b)** -125 **(c)** 150

99. **(a)** $15x^2y^4z^7 + 2y$ **(b)** $35x^3y^4z^6 + 3y^2$ **(c)** $21x^2y^5z^6$
(d) $42x^3y^5z^5$ **(e)** $140x^3y^3z^6 + 6y$ **(f)** $30xy^4z^7$ **(g)** $105x^2y^4z^6$
(h) $210xy^4z^6$

107. $\dfrac{\partial f}{\partial v} = 8vw^3x^4y^5,\ \dfrac{\partial f}{\partial w} = 12v^2w^2x^4y^5,\ \dfrac{\partial f}{\partial x} = 16v^2w^3x^3y^5,$
$\dfrac{\partial f}{\partial y} = 20v^2w^3x^4y^4$

109. $\dfrac{\partial f}{\partial v_1} = \dfrac{2v_1}{v_3^2 + v_4^2},\ \dfrac{\partial f}{\partial v_2} = \dfrac{-2v_2}{v_3^2 + v_4^2},\ \dfrac{\partial f}{\partial v_3} = \dfrac{-2v_3(v_1^2 - v_2^2)}{(v_3^2 + v_4^2)^2},$
$\dfrac{\partial f}{\partial v_4} = \dfrac{-2v_4(v_1^2 - v_2^2)}{(v_3^2 + v_4^2)^2}$

111. **(a)** 0 **(b)** 0 **(c)** 0 **(d)** 0 **(e)** $2(1 + yw)e^{yw}\sin z\cos z$
(f) $2xw(2 + yw)e^{yw}\sin z\cos z$

113. $-i\sin(x_1 + 2x_2 + \cdots + nx_n)$

115. **(a)** xy-plane, $12x^2 + 6x$ **(b)** $y \neq 0, -3x^2/y^2$

117. $f_x(2, -1) = 11,\ f_y(2, -1) = -8$

119. **(b)** does not exist if $y \neq 0$ and $x = -y$

Exercise Set 13.4 (Page 947)

1. 5.04 **3.** 4.14 **9.** $dz = 7\,dx - 2\,dy$ **11.** $dz = 3x^2y^2\,dx + 2x^3y\,dy$

13. $dz = \dfrac{y}{1 + x^2y^2}\,dx + \dfrac{x}{1 + x^2y^2}\,dy$ **15.** $dw = 8\,dx - 3\,dy + 4\,dz$

17. $dw = 3x^2y^2z\,dx + 2x^3yz\,dy + x^3y^2\,dz$

19. $dw = \dfrac{yz}{1 + x^2y^2z^2}\,dx + \dfrac{xz}{1 + x^2y^2z^2}\,dy + \dfrac{xy}{1 + x^2y^2z^2}\,dz$

21. $df = 0.10,\ \Delta f = 0.1009$ **23.** $df = 0.03,\ \Delta f \approx 0.029412$

25. $df = 0.96,\ \Delta f \approx 0.97929$

Responses to True–False questions may be abridged to save space.

27. False; see the discussion at the beginning of this section.

29. True; see Theorems 13.4.3 and 13.4.4.

31. The increase in the area of the rectangle is given by the sum of the areas of the three small rectangles, and the total differential is given by the sum of the areas of the upper left and lower right rectangles.

33. **(a)** $L = \frac{1}{5} - \frac{4}{125}(x - 4) - \frac{3}{125}(y - 3)$ **(b)** 0.000176603

35. **(a)** $L = 0$ **(b)** 0.0024

37. **(a)** $L = 6 + 6(x - 1) + 3(y - 2) + 2(z - 3)$ **(b)** -0.00481

39. **(a)** $L = e + e(x - 1) - e(y + 1) - e(z + 1)$ **(b)** 0.01554

45. 0.5 **47.** 1, 1, -1, 2 **49.** $(-1, 1)$ **51.** $(1, 0, 1)$ **53.** 8%

55. $r\%$ **57.** 0.3%

59. **(a)** $(r + s)\%$ **(b)** $(r + s)\%$ **(c)** $(2r + 3s)\%$ **(d)** $\left(3r + \dfrac{s}{2}\right)\%$

61. $\approx 39\ \text{ft}^2$

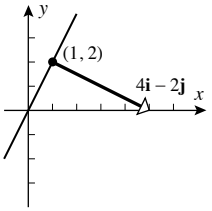
Exercise Set 13.5 (Page 956)

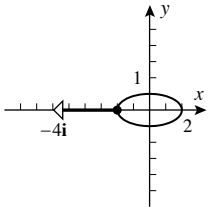
1. $42t^{13}$ 3. $3t^{-2}\sin(1/t)$ 5. $-\frac{10}{3}t^{7/3}e^{1-t^{10/3}}$ 7. $\dfrac{dw}{dt} = 165t^{32}$
9. $-2t\cos t^2$ 11. 3264 13. 0
17. $24u^2v^2 - 16uv^3 - 2v + 3,\ 16u^3v - 24u^2v^2 - 2u - 3$
19. $-\dfrac{2\sin u}{3\sin v}, -\dfrac{2\cos u\cos v}{3\sin^2 v}$ 21. $e^u, 0$
23. $3r^2\sin\theta\cos^2\theta - 4r^3\sin^3\theta\cos\theta,$
$-2r^3\sin^2\theta\cos\theta + r^4\sin^4\theta + r^3\cos^3\theta - 3r^4\sin^2\theta\cos^2\theta$
25. $\dfrac{x^2+y^2}{4x^2y^3}, \dfrac{y^2-3x^2}{4xy^4}$ 27. $\dfrac{\partial z}{\partial r} = \dfrac{2r\cos^2\theta}{r^2\cos^2\theta+1}, \dfrac{\partial z}{\partial\theta} = \dfrac{-2r^2\cos\theta\sin\theta}{r^2\cos^2\theta+1}$
29. $\dfrac{dw}{d\rho} = 2\rho(4\sin^2\phi + \cos^2\phi), \dfrac{\partial w}{\partial\phi} = 6\rho^2\sin\phi\cos\phi, \dfrac{dw}{d\theta} = 0$
31. $-\pi$ 33. $\sqrt{3}e^{\sqrt{3}}, (2-4\sqrt{3})e^{\sqrt{3}}$ 35. -0.779 rad/s

Responses to True–False questions may be abridged to save space.

37. False; the symbols ∂z and ∂x have no individual meaning.
39. False; consider $z = xy, x = t, y = t$.
41. $-\dfrac{2xy^3}{3x^2y^2 - \sin y}$
43. $-\dfrac{ye^{xy}}{xe^{xy} + ye^y + e^y}$ 45. $\dfrac{2x+yz}{6yz-xy}, \dfrac{xz-3z^2}{6yz-xy}$
47. $\dfrac{ye^x}{15\cos 3z + 3}, \dfrac{e^x}{15\cos 3z + 3}$
61. $\dfrac{\partial w}{\partial\rho} = (\sin\phi\cos\theta)\dfrac{\partial w}{\partial x} + (\sin\phi\sin\theta)\dfrac{\partial w}{\partial y} + (\cos\phi)\dfrac{\partial w}{\partial z},$
$\dfrac{\partial w}{\partial\phi} = (\rho\cos\phi\cos\theta)\dfrac{\partial w}{\partial x} + (\rho\cos\phi\sin\theta)\dfrac{\partial w}{\partial y} - (\rho\sin\phi)\dfrac{\partial w}{\partial z},$
$\dfrac{\partial w}{\partial\theta} = -(\rho\sin\phi\sin\theta)\dfrac{\partial w}{\partial x} + (\rho\sin\phi\cos\theta)\dfrac{\partial w}{\partial y}$
65. (a) $\dfrac{dw}{dt} = \sum_{i=1}^{4}\dfrac{\partial w}{\partial x_i}\dfrac{dx_i}{dt}$ (b) $\dfrac{\partial w}{\partial v_j} = \sum_{i=1}^{4}\dfrac{\partial w}{\partial x_i}\dfrac{\partial x_i}{\partial v_j}, j = 1, 2, 3$

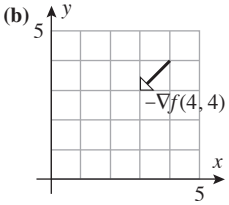
Exercise Set 13.6 (Page 968)

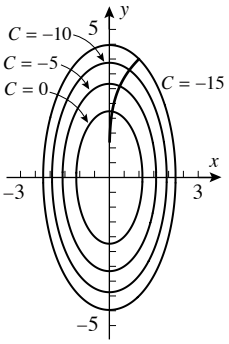
1. $6\sqrt{2}$ 3. $-3/\sqrt{10}$ 5. -320 7. $-314/741$ 9. 0 11. $-8\sqrt{2}$
13. $\sqrt{2}/4$ 15. $5/\sqrt{3}$ 17. $-8/63$ 19. $1/2 + \sqrt{3}/8$ 21. $2\sqrt{2}$
23. $1/\sqrt{5}$ 25. $-\frac{3}{2}e$ 27. $3/\sqrt{11}$ 29. (a) 5 (b) 10 (c) $-5\sqrt{5}$
31. III 33. $\cos(7y^2 - 7xy)(-7y\mathbf{i} + (14y - 7x)\mathbf{j})$
35. $\left(\dfrac{-84y}{(6x-7y)^2}\right)\mathbf{i} + \left(\dfrac{84x}{(6x-7y)^2}\right)\mathbf{j}$ 37. $-9x^8\mathbf{i} - 3y^2\mathbf{j} + 12z^{11}\mathbf{k}$
39. $\nabla w = \dfrac{x}{x^2+y^2+z^2}\mathbf{i} + \dfrac{y}{x^2+y^2+z^2}\mathbf{j} + \dfrac{z}{x^2+y^2+z^2}\mathbf{k}$
41. $40\mathbf{i} + 32\mathbf{j}$ 43. $-36\mathbf{i} - 12\mathbf{j}$ 45. $4(\mathbf{i} + \mathbf{j} + \mathbf{k})$
47. 

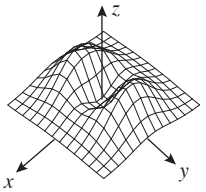
49. 

51. $\pm(-4\mathbf{i} + \mathbf{j})/\sqrt{17}$ 53. $\mathbf{u} = (3\mathbf{i} - 2\mathbf{j})/\sqrt{13}, \|\nabla f(-1, 1)\| = 4\sqrt{13}$
55. $\mathbf{u} = (4\mathbf{i} - 3\mathbf{j})/5, \|\nabla f(4, -3)\| = 1$ 57. $\dfrac{1}{\sqrt{2}}(\mathbf{i} - \mathbf{j}), 3\sqrt{2}$
59. $\dfrac{1}{\sqrt{2}}(-\mathbf{i} + \mathbf{j}), \dfrac{1}{\sqrt{2}}$
61. $\mathbf{u} = -(\mathbf{i} + 3\mathbf{j})/\sqrt{10}, -\|\nabla f(-1, -3)\| = -2\sqrt{10}$
63. $\mathbf{u} = (3\mathbf{i} - \mathbf{j})/\sqrt{10}, -\|\nabla f(\pi/6, \pi/4)\| = -\sqrt{5}$
65. $(\mathbf{i} - 11\mathbf{j} + 12\mathbf{k})/\sqrt{266}, -\sqrt{266}$

Responses to True–False questions may be abridged to save space.

67. False; they are equal. 69. False; let $\mathbf{u} = \mathbf{i}$ and let $f(x, y) = y$.
71. $8/\sqrt{29}$
73. (a) $\approx 1/\sqrt{2}$
(b) 

75. $9x^2 + y^2 = 9$
77. $36/\sqrt{17}$
79. (a) $2e^{-\pi/2}\mathbf{i}$
81. $-\frac{5}{3}(2\mathbf{i} - \mathbf{j} - 2\mathbf{k})$
87. $x(t) = e^{-8t}, y(t) = 4e^{-2t}$
89. 

91. (a) 

(c) $\nabla f = [2x - 2x(x^2 + 3y^2)]e^{-(x^2+y^2)}\mathbf{i} + [6y - 2y(x^2 + 3y^2)]e^{-(x^2+y^2)}\mathbf{j}$
(d) $x = y = 0$ or $x = 0, y = \pm 1$ or $x = \pm 1, y = 0$

Exercise Set 13.7 (Page 975)

1. (a) $x + y + 2z = 6$ (b) $x = 2 + t, y = 2 + t, z = 1 + 2t$
(c) 35.26°
3. tangent plane: $3x - 4z = -25$;
normal line: $x = -3 + (3t/4), y = 0, z = 4 - t$
5. tangent plane: $9x - 4y - 10z = -76$;
normal line: $x = -4 + 9t, y = 5 - 4t, z = 2 - 10t$
7. tangent plane: $48x - 14y - z = 64$;
normal line: $x = 1 + 48t, y = -2 - 14t, z = 12 - t$
9. tangent plane: $x - y - z = 0$;
normal line: $x = 1 + t, y = -t, z = 1 - t$
11. tangent plane: $3y - z = -1$;
normal line: $x = \pi/6, y = 3t, z = 1 - t$
13. (a) all points on the x-axis or y-axis (b) $(0, -2, -4)$
15. $\left(\frac{1}{2}, -2, -\frac{3}{4}\right)$ 17. (a) $(-2, 1, 5), (0, 3, 9)$ (b) $\dfrac{4}{3\sqrt{14}}, \dfrac{4}{\sqrt{222}}$

Responses to True–False questions may be abridged to save space.

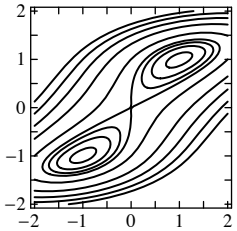
19. False; they need only be parallel.
21. True; see Formula (15) of Section 13.4.
23. $\pm\dfrac{1}{\sqrt{227}}(\mathbf{i} - \mathbf{j} - 15\mathbf{k})$ 27. $(1, 2/3, 2/3), (-1, -2/3, -2/3)$
29. $x = 1 + 8t, y = -1 + 5t, z = 2 + 6t$
31. $x = 3 + 4t, y = -3 - 4t, z = 4 - 3t$

Exercise Set 13.8 (Page 985)

1. (a) minimum at $(2, -1)$, no maxima
(b) maximum at $(0, 0)$, no minima (c) no maxima or minima
3. minimum at $(3, -2)$, no maxima 5. relative minimum at $(0, 0)$
7. relative minimum at $(0, 0)$; saddle points at $(\pm 2, 1)$
9. saddle point at $(1, -2)$ 11. relative minimum at $(2, -1)$
13. relative minima at $(-1, -1)$ and $(1, 1)$ 15. saddle point at $(0, 0)$

17. no critical points **19.** relative maximum at $(-1, 0)$

21. saddle point at $(0, 0)$; relative minima at $(1, 1)$ and $(-1, -1)$



Responses to True–False questions may be abridged to save space.

23. False; let $f(x, y) = y$.

25. True; this follows from Theorem 13.8.6.

27. **(b)** relative minimum at $(0, 0)$

31. absolute maximum 0, absolute minimum -12

33. absolute maximum 3, absolute minimum -1

35. absolute maximum $\frac{33}{4}$, absolute minimum $-\frac{1}{4}$

37. 16, 16, 16

39. maximum at $(1, 2, 2)$

41. $2a/\sqrt{3}, 2a/\sqrt{3}, 2a/\sqrt{3}$

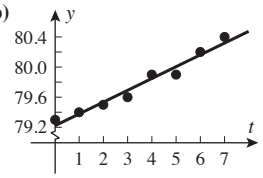
43. length and width 2 ft, height 4 ft

45. **(a)** $x = 0$: minimum -3, maximum 0; $x = 1$: minimum 3, maximum $13/3$; $y = 0$: minimum 0, maximum 4; $y = 1$: minimum -3, maximum 3
(b) $y = x$: minimum 0, maximum 3; $y = 1 - x$: maximum 4, minimum -3
(c) minimum -3, maximum $13/3$

47. length and width $\sqrt[3]{2V}$, height $\sqrt[3]{2V}/2$ **51.** $y = \frac{3}{4}x + \frac{19}{12}$

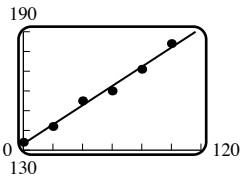
53. $y = 0.5x + 0.8$

55. **(a)** $y = 79.22 + 0.1571t$ **(b)**



(c) about 81.6 years

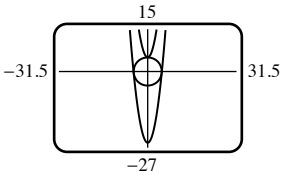
57. **(a)** $P = \dfrac{2798}{21} + \dfrac{171}{350}T$ **(b)**



(c) $T \approx -272.7096°\text{C}$

Exercise Set 13.9 (Page 996)

1. **(a)** 4 **3.** **(a)**



(c) maximum $\frac{101}{4}$, minimum -5

5. maximum $\sqrt{2}$ at $(-\sqrt{2}, -1)$ and $(\sqrt{2}, 1)$, minimum $-\sqrt{2}$ at $(-\sqrt{2}, 1)$ and $(\sqrt{2}, -1)$

7. maximum $\sqrt{2}$ at $(1/\sqrt{2}, 0)$, minimum $-\sqrt{2}$ at $(-1/\sqrt{2}, 0)$

9. maximum 6 at $\left(\frac{4}{3}, \frac{2}{3}, -\frac{4}{3}\right)$, minimum -6 at $\left(-\frac{4}{3}, -\frac{2}{3}, \frac{4}{3}\right)$

11. maximum is $1/(3\sqrt{3})$ at $(1/\sqrt{3}, 1/\sqrt{3}, 1/\sqrt{3})$, $(1/\sqrt{3}, -1/\sqrt{3}, -1/\sqrt{3})$, $(-1/\sqrt{3}, 1/\sqrt{3}, -1/\sqrt{3})$, and $(-1/\sqrt{3}, -1/\sqrt{3}, 1/\sqrt{3})$; minimum is $-1/(3\sqrt{3})$ at $(1/\sqrt{3}, 1/\sqrt{3}, -1/\sqrt{3})$, $(1/\sqrt{3}, -1/\sqrt{3}, 1/\sqrt{3})$, $(-1/\sqrt{3}, 1/\sqrt{3}, 1/\sqrt{3})$, and $(-1/\sqrt{3}, -1/\sqrt{3}, -1/\sqrt{3})$

Responses to True–False questions may be abridged to save space.

13. False; a Lagrange multiplier is a scalar.

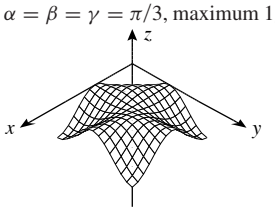
15. False; we must solve three equations in three unknowns.

17. $\left(\frac{3}{10}, -\frac{3}{5}\right)$ **19.** $\left(\frac{1}{6}, \frac{1}{3}, \frac{1}{6}\right)$

21. $(3, 6)$ is closest and $(-3, -6)$ is farthest **23.** $5(\mathbf{i} + \mathbf{j} + \mathbf{k})/\sqrt{3}$

25. 9, 9, 9 **27.** $(\pm\sqrt{5}, 0, 0)$ **29.** length and width 2 ft, height 4 ft

33. **(a)** $\alpha = \beta = \gamma = \pi/3$, maximum $1/8$
(b)



Chapter 13 Review Exercises (Page 997)

1. **(a)** xy **(b)** $e^{r+s}\ln(rs)$

5. **(a)** not defined on line $y = x$ **(b)** not continuous

9. **(a)** 12 Pa/min **(b)** 240 Pa/min

15. df (the differential of f) is an approximation for Δf (the change in f)

17. $dV = -0.06667\ \text{m}^3$; $\Delta V = -0.07267\ \text{m}^3$ **19.** 2

21. $\dfrac{-f_y^2 f_{xx} + 2f_x f_y f_{xy} - f_x^2 f_{yy}}{f_y^3}$ **25.** $\frac{7}{2} + \frac{4}{5}\ln 2$ **27.** $-7/\sqrt{5}$

29. $(0, 0, 2)$, $(1, 1, 1)$, $(-1, -1, 1)$ **31.** $\left(-\frac{1}{3}, -\frac{1}{2}, 2\right)$

33. relative minimum at $(15, -8)$

35. saddle point at $(0, 0)$, relative minimum at $(3, 9)$

37. absolute maximum of 4 at $(\pm 1, \pm 2)$, absolute minimum of 0 at $(\pm\sqrt{2}, 0)$ and $(0, \pm 2\sqrt{2})$

39. $I_1 : I_2 : I_3 = \dfrac{1}{R_1} : \dfrac{1}{R_2} : \dfrac{1}{R_3}$

41. **(a)** $\partial P/\partial L = c\alpha L^{\alpha-1}K^{\beta}$, $\partial P/\partial K = c\beta L^{\alpha}K^{\beta-1}$

Chapter 13 Making Connections (Page 999)

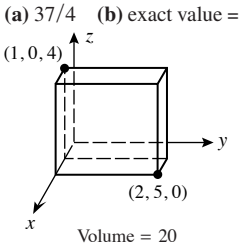
Answers are provided in the Student Solutions Manual.

Exercise Set 14.1 (Page 1007)

1. 7 **3.** 2 **5.** 2 **7.** 3 **9.** $1 - \ln 2$ **11.** $\dfrac{1 - \ln 2}{2}$ **13.** 0 **15.** $\frac{1}{3}$

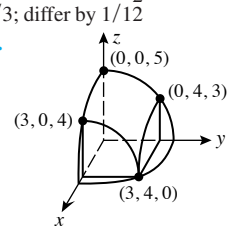
17. **(a)** $37/4$ **(b)** exact value $= 28/3$; differ by $1/12$

19.



Volume = 20

21.



Responses to True–False questions may be abridged to save space.

23. False; ΔA_k is the area of such a rectangular region.

25. False; $\displaystyle\iint_R f(x, y)\,dA = \int_1^5 \int_2^4 f(x, y)\,dy\,dx$.

29. 19 **31.** 8 **33.** $\dfrac{1}{3\pi}$ **35.** 48 **37.** $1 - \dfrac{2}{\pi}$ **39.** $\frac{14}{3}\,°\text{C}$

41. 1.381737122 **43.** first integral equals $\frac{1}{2}$, second equals $-\frac{1}{2}$; no

Exercise Set 14.2 (Page 1015)

1. $\frac{1}{40}$ **3.** 9 **5.** $\frac{\pi}{2}$ **7.** $\frac{1}{12}$

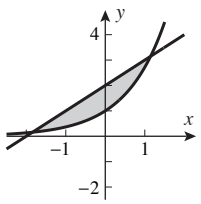
9. (a) $\int_0^2 \int_0^{x^2} f(x,y)\,dy\,dx$ (b) $\int_0^4 \int_{\sqrt{y}}^2 f(x,y)\,dx\,dy$

11. (a) $\int_1^2 \int_{-2x+5}^3 f(x,y)\,dy\,dx + \int_2^4 \int_1^3 f(x,y)\,dy\,dx + \int_4^5 \int_{2x-7}^3 f(x,y)\,dy\,dx$ (b) $\int_1^3 \int_{(5-y)/2}^{(y+7)/2} f(x,y)\,dx\,dy$

13. (a) $\frac{16}{3}$ (b) 38 **15.** 576 **17.** 0 **19.** $\frac{\sqrt{17}-1}{2}$ **21.** $\frac{50}{3}$

23. $-\frac{7}{60}$ **25.** $\frac{1-\cos 8}{3}$

27. (a)



(b) $(-1.8414, 0.1586)$, $(1.1462, 3.1462)$
(c) -0.4044
(d) -0.4044

29. $\sqrt{2}-1$ **31.** 32

Responses to True–False questions may be abridged to save space.

33. False; $\int_0^1 \int_{x^2}^{2x} f(x,y)\,dy\,dx$ integrates $f(x,y)$ over the region between the graphs of $y = x^2$ and $y = 2x$ for $0 \le x \le 1$ and results in a number, but $\int_{x^2}^{2x} \int_0^1 f(x,y)\,dx\,dy$ produces an expression involving x.

35. False; although R is symmetric across the x-axis, the integrand may not be.

37. 12 **39.** 27π **41.** 170 **43.** $\frac{27\pi}{2}$ **45.** $\frac{\pi}{2}$

47. $\int_0^{\sqrt{2}} \int_{y^2}^2 f(x,y)\,dx\,dy$ **49.** $\int_1^{e^2} \int_{\ln x}^2 f(x,y)\,dy\,dx$

51. $\int_0^{\pi/2} \int_0^{\sin x} f(x,y)\,dy\,dx$ **53.** $\frac{1-e^{-16}}{8}$ **55.** $\frac{e^8-1}{3}$

57. (a) 0 (b) $\tan 1$ **59.** 0 **61.** $\frac{\pi}{2} - \ln 2$ **63.** $\frac{2}{3}$°C **65.** 0.676089

Exercise Set 14.3 (Page 1024)

1. $\frac{1}{6}$ **3.** $\frac{2}{9}a^3$ **5.** 0 **7.** $\frac{3\pi}{2}$ **9.** $\frac{\pi}{16}$ **11.** $\int_{\pi/6}^{5\pi/6} \int_2^{4\sin\theta} f(r,\theta) r\,dr\,d\theta$

13. $8\int_0^{\pi/2} \int_1^3 r\sqrt{9-r^2}\,dr\,d\theta$ **15.** $2\int_0^{\pi/2} \int_0^{\cos\theta} (1-r^2) r\,dr\,d\theta$

17. $\frac{64\sqrt{2}}{3}\pi$ **19.** $\frac{5\pi}{32}$ **21.** $\frac{27\pi}{16}$ **23.** $(1-\cos 9)\pi$ **25.** $\frac{\pi}{8}\ln 5$ **27.** $\frac{\pi}{8}$

29. $\frac{16}{9}$ **31.** $\frac{\pi}{2}\left(1 - \frac{1}{\sqrt{1+a^2}}\right)$ **33.** $\frac{\pi}{4}(\sqrt{5}-1)$

Responses to True–False questions may be abridged to save space.

35. True; the disk is given in polar coordinates by $0 \le r \le 2, 0 \le \theta \le 2\pi$.

37. False; the integrand is missing a factor of r:

$$\iint_R f(r,\theta)\,dA = \int_0^{\pi/2} \int_1^2 f(r,\theta) r\,dr\,d\theta.$$

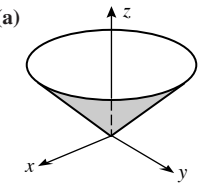
39. $\pi a^2 h$ **41.** $\frac{1}{5} + \frac{\pi}{2}$

43. (a) $\frac{4}{3}\pi a^2 c$ (b) $\approx 1.0831682 \times 10^{21}$ m^3 **45.** $2a^2$

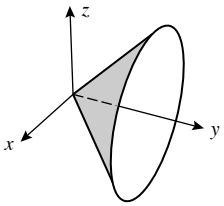
Exercise Set 14.4 (Page 1036)

1. 6π **3.** $\frac{\sqrt{5}}{6}$ **5.** $\sqrt{2}\pi$ **7.** $\frac{(10\sqrt{10}-1)\pi}{18}$ **9.** 8π

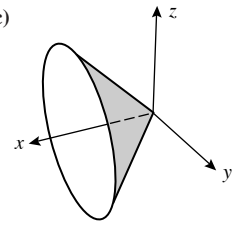
11. (a)



(b)



(c)



13. (a) $x = u, y = v, z = \frac{5}{2} + \frac{3}{2}u - 2v$ (b) $x = u, y = v, z = u^2$

15. (a) $x = \sqrt{5}\cos u, y = \sqrt{5}\sin u, z = v; 0 \le u \le 2\pi, 0 \le v \le 1$
(b) $x = 2\cos u, y = v, z = 2\sin u; 0 \le u \le 2\pi, 1 \le v \le 3$

17. $x = u, y = \sin u \cos v, z = \sin u \sin v$

19. $x = r\cos\theta, y = r\sin\theta, z = \frac{1}{1+r^2}$

21. $x = r\cos\theta, y = r\sin\theta, z = 2r^2\cos\theta\sin\theta$

23. $x = r\cos\theta, y = r\sin\theta, z = \sqrt{9-r^2}; r \le \sqrt{5}$

25. $x = \frac{1}{2}\rho\cos\theta, y = \frac{1}{2}\rho\sin\theta, z = \frac{\sqrt{3}}{2}\rho$ **27.** $z = x - 2y$; a plane

29. $(x/3)^2 + (y/2)^2 = 1; 2 \le z \le 4$; part of an elliptic cylinder

31. $(x/3)^2 + (y/4)^2 = z^2; 0 \le z \le 1$; part of an elliptic cone

33. (a) $x = r\cos\theta, y = r\sin\theta, z = r, 0 \le r \le 2$;
$x = u, y = v, z = \sqrt{u^2+v^2}, 0 \le u^2 + v^2 \le 4$

35. (a) $0 \le u \le 3, 0 \le v \le \pi$ (b) $0 \le u \le 4, -\pi/2 \le v \le \pi/2$

37. (a) $0 \le \phi \le \pi/2, 0 \le \theta \le 2\pi$ (b) $0 \le \phi \le \pi, 0 \le \theta \le \pi$

39. $2x + 4y - z = 5$ **41.** $z = 0$ **43.** $x - y + \frac{\sqrt{2}}{2}z = \frac{\pi\sqrt{2}}{8}$

45. $\frac{(17\sqrt{17} - 5\sqrt{5})\pi}{6}$

Responses to True–False questions may be abridged to save space.

47. False; the surface area is $S = \iint_R \sqrt{[f_x(x,y)]^2 + [f_y(x,y)]^2 + 1}\,dA$.

49. True; see the discussion preceding Definition 14.4.1.

51. $4\pi a^2$ **55.** $4\pi^2 ab$ **57.** 9.099

59. $(x/a)^2 + (y/b)^2 + (z/c)^2 = 1$; ellipsoid

61. $(x/a)^2 + (y/b)^2 - (z/c)^2 = -1$; hyperboloid of two sheets

Exercise Set 14.5 (Page 1045)

1. 8 **3.** $\frac{47}{3}$ **5.** $\frac{81}{5}$ **7.** $\frac{128}{15}$ **9.** $\pi(\pi-3)/2$ **11.** $\frac{1}{6}$ **13.** 9.425

15. 4 **17.** $\frac{256}{15}$

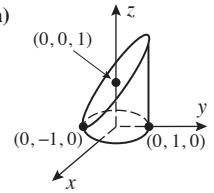
19. (a) $\int_{-1}^1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \int_{4x^2+y^2}^{4-3y^2} f(x,y,z)\,dz\,dy\,dx$

(b) $\int_{-1}^1 \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} \int_{4x^2+y^2}^{4-3y^2} f(x,y,z)\,dz\,dx\,dy$

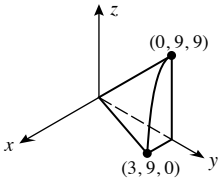
21. $4\int_0^1 \int_0^{\sqrt{1-x^2}} \int_{4x^2+y^2}^{4-3y^2} dz\,dy\,dx$

23. $2\int_{-3}^3 \int_0^{\frac{1}{3}\sqrt{9-x^2}} \int_0^{x+3} dz\,dy\,dx$

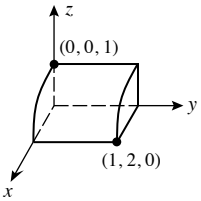
25. (a)



(b)



(c)



Responses to True–False questions may be abridged to save space.

27. True; apply Fubini's Theorem (Theorem 14.5.1).

29. False;

$$\iiint_G f(x,y,z)\,dV = \int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} f(x,y,z)\,dz\,dy\,dx.$$

33. $\frac{3}{4}$ **35.** 3.291

37. (a) $\displaystyle\int_0^a \int_0^{b(1-x/a)} \int_0^{c(1-x/a-y/b)} dz\,dy\,dx$ is one example.

39. (a) $\displaystyle\int_0^2 \int_0^{\sqrt{4-x^2}} \int_0^5 f(x,y,z)\,dz\,dy\,dx$

(b) $\displaystyle\int_0^9 \int_0^{3-\sqrt{x}} \int_y^{3-\sqrt{x}} f(x,y,z)\,dz\,dy\,dx$

(c) $\displaystyle\int_0^2 \int_0^{4-x^2} \int_y^{8-y} f(x,y,z)\,dz\,dy\,dx$

Exercise Set 14.6 (Page 1056)

1. $\frac{\pi}{4}$ **3.** $\frac{\pi}{16}$

5. The region is bounded by the xy-plane and the upper half of a sphere of radius 1 centered at the origin; $f(r,\theta,z) = z$.

7. The region is the portion of the first octant inside a sphere of radius 1 centered at the origin; $f(\rho,\theta,\phi) = \rho\cos\phi$.

9. $\frac{81\pi}{2}$ **11.** $\frac{152}{3}\pi$ **13.** $\frac{64\pi}{3}$ **15.** $\frac{11\pi a^3}{3}$ **17.** $\frac{\pi a^6}{48}$

19. $\frac{32(2\sqrt{2}-1)\pi}{15}$

Responses to True–False questions may be abridged to save space.

21. False; the factor r^2 should be r [Formula (6)]:

$$\iiint_G f(x,y,z)\,dV = \iiint_{\substack{\text{appropriate}\\ \text{limits}}} f(r\cos\theta, r\sin\theta, z)\,r\,dz\,dr\,d\theta.$$

23. True; G is the spherical wedge bounded by the spheres $\rho = 1$ and $\rho = 3$, the half-planes $\theta = 0$ and $\theta = 2\pi$, and above the cone $\phi = \pi/4$, so

$$(\text{volume of } G) = \iiint_G dV = \int_0^{\pi/4} \int_0^{2\pi} \int_1^3 \rho^2 \sin\phi\,d\rho\,d\theta\,d\phi.$$

25. (a) $\frac{5}{2}(-8+3\ln 3)\ln(\sqrt{5}-2)$ (b) $f(x,y,z) = \dfrac{y^3}{x^3\sqrt{1+z^2}}$; G is the cylindrical wedge $1 \le r \le 4$, $\dfrac{\pi}{6} \le \theta \le \dfrac{\pi}{3}$, $-2 \le z \le 2$

27. $\frac{4\pi a^3}{3}$ **29.** $\frac{2(\sqrt{3}-1)\pi}{3}$

Exercise Set 14.7 (Page 1068)

1. -17 **3.** $\cos(u-v)$ **5.** $x = \frac{2}{9}u + \frac{5}{9}v$, $y = -\frac{1}{9}u + \frac{2}{9}v$; $\frac{1}{9}$

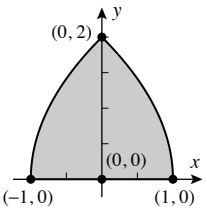
7. $x = \dfrac{\sqrt{u+v}}{\sqrt{2}}$, $y = \dfrac{\sqrt{v-u}}{\sqrt{2}}$; $\dfrac{1}{4\sqrt{v^2-u^2}}$ **9.** 5 **11.** $\dfrac{1}{v}$

Responses to True–False questions may be abridged to save space.

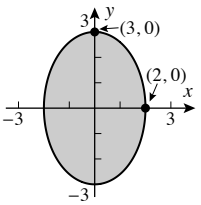
13. False; $|\partial(x,y)/\partial(u,v)| = \|\partial\mathbf{r}/\partial u \times \partial\mathbf{r}/\partial v\|$; evaluating this at (u_0, v_0) gives the area of the indicated parallelogram.

15. False; $\partial(x,y)/\partial(r,\theta) = r$.

17.



19.



21. $\frac{3}{2}\ln 3$ **23.** $1 - \frac{1}{2}\sin 2$ **25.** 96π **27.** $\dfrac{\pi}{24}(1-\cos 1)$ **29.** $\frac{192}{5}\pi$

31. $u = \begin{cases} \cot^{-1}(x/y), & y \ne 0 \\ 0, & y = 0 \text{ and } x > 0 \\ \pi, & y = 0 \text{ and } x < 0 \end{cases}$

$v = \sqrt{x^2+y^2}$; other answers possible

33. $u = (3/7)x - (2/7)y$, $v = (-1/7)x + (3/7)y$; other answers possible

35. $\frac{1}{4}\ln\frac{5}{2}$

37. $\frac{1}{2}\left[\ln(\sqrt{2}+1) - \frac{\pi}{4}\right]$ **39.** $\frac{35}{256}$ **41.** $2\ln 3$ **45.** $21/8$

Exercise Set 14.8 (Page 1077)

1. $M = \frac{13}{20}$, center of gravity $\left(\frac{190}{273}, \frac{6}{13}\right)$

3. $M = a^4/8$, center of gravity $(8a/15, 8a/15)$

5. $\left(\frac{1}{2}, \frac{1}{2}\right)$ **7.** $\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right)$

Responses to True–False questions may be abridged to save space.

9. True; recall this from Section 6.7.

11. False; the center of gravity of the lamina is $(\bar{x}, \bar{y}) = (M_y/M, M_x/M)$, where M_y and M_x are the lamina's first moments about the y- and x-axes, respectively, and M is the mass of the lamina.

15. $\left(\dfrac{128}{105\pi}, \dfrac{128}{105\pi}\right)$ **17.** $\left(\dfrac{4a}{3\pi}, 0\right)$ **19.** $\left(\frac{1}{4}, \frac{1}{4}, \frac{1}{4}\right)$ **21.** $\left(\frac{1}{2}, 0, \frac{3}{5}\right)$

23. $(3a/8, 3a/8, 3a/8)$

25. $M = a^4/2$, center of gravity $(a/3, a/2, a/2)$

27. $M = \frac{1}{6}$, center of gravity $\left(0, \frac{16}{35}, \frac{1}{2}\right)$ **29.** (a) $\left(\frac{5}{8}, \frac{5}{8}\right)$ (b) $\left(\frac{2}{3}, \frac{1}{2}\right)$

31. (1.177406, 0.353554, 0.231557)

33. $\dfrac{27\pi}{4}$ **35.** $\pi k a^4$ **37.** $\left(0, 0, \frac{7}{16\sqrt{2}-14}\right)$ **39.** $\left(\frac{4}{3}, 0, \frac{10}{9}\right)$

41. $(3a/8, 3a/8, 3a/8)$ **43.** $(2-\sqrt{2})\pi/4$ **45.** $(0, 0, 8/15)$

47. $(0, 195/152, 0)$ **51.** $\frac{1}{2}\delta\pi a^4 h$ **53.** $\frac{1}{2}\delta\pi h(a_2^4 - a_1^4)$ **57.** $2\pi^2 abk$

59. $(a/3, b/3)$

Chapter 14 Review Exercises (Page 1081)

3. (a) $\displaystyle\iint_R dA$ (b) $\displaystyle\iiint_G dV$ (c) $\displaystyle\iint_R \sqrt{1+\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2}\,dA$

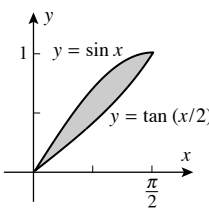
5. $\displaystyle\int_0^1 \int_{1-\sqrt{1-y^2}}^{1+\sqrt{1-y^2}} f(x,y)\,dx\,dy$

7. (a) $a=2, b=1, c=1, d=2$ or $a=1, b=2, c=2, d=1$ (b) 3

9. $-\dfrac{1}{\sqrt{2}\pi}$

11. $\displaystyle\int_0^1 \int_{2y}^2 e^x e^y\,dx\,dy$

13.



15. $\frac{1}{3}(1-\cos 64)$ **17.** a^2 **19.** $\frac{3}{2}$ **21.** 32π

23. (a) $\displaystyle\int_0^{2\pi} \int_0^{\pi/3} \int_0^a \rho^4 \sin^3\phi\,d\rho\,d\phi\,d\theta$

(b) $\displaystyle\int_0^{2\pi} \int_0^{\sqrt{3}a/2} \int_{r/\sqrt{3}}^{\sqrt{a^2-r^2}} r^3\,dz\,dr\,d\theta$

(c) $\displaystyle\int_{-\sqrt{3}a/2}^{\sqrt{3}a/2}\int_{-\sqrt{(3a^2/4)-x^2}}^{\sqrt{(3a^2/4)-x^2}}\int_{\sqrt{x^2+y^2}/\sqrt{3}}^{\sqrt{a^2-x^2-y^2}}(x^2+y^2)\,dz\,dy\,dx$

25. $\dfrac{\pi a^3}{9}$ 27. $\dfrac{1}{24}(26^{3/2}-10^{3/2})\approx 4.20632$ 29. $2x+4y-z=5$

33. **(a)** $\dfrac{1}{2(u+w)}$ **(b)** $\frac{1}{2}(7\ln 7-5\ln 5-3\ln 3)$ 35. $\left(\frac{8}{5},0\right)$

37. $(0,0,h/4)$

Chapter 14 Making Connections (Page 1082)

Where correct answers to a Making Connections exercise may vary, no answer is listed. Sample answers for these questions are available on the Book Companion Site.

1. **(b)** $\dfrac{\pi}{4}$ 3. **(a)** 1.173108605 **(b)** 1.173108605

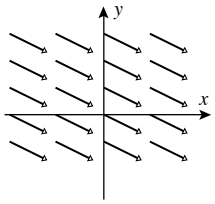
4. **(a)** the sphere $0\le x^2+y^2+z^2\le 1$ **(b)** 4.934802202 **(c)** $\pi^2/2$

5. **(b)** 4.4506 6. $\frac{4}{35}\pi a^3$

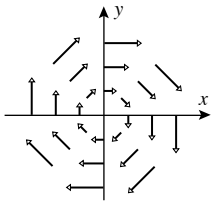
Exercise Set 15.1 (Page 1092)

1. **(a)** III **(b)** IV 3. **(a)** true **(b)** true **(c)** true

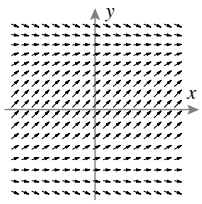
5.



7.



9.



Responses to True–False questions may be abridged to save space.

11. False; the vector field has a nonzero **k**-component.

13. True; this is the curl of **F**.

15. **(a)** all x, y **(b)** all x, y 17. $\operatorname{div}\mathbf{F}=2x+y$, $\operatorname{curl}\mathbf{F}=z\mathbf{i}$

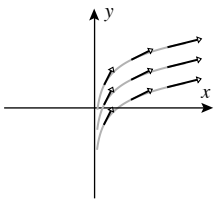
19. $\operatorname{div}\mathbf{F}=0$, $\operatorname{curl}\mathbf{F}=(40x^2z^4-12xy^3)\mathbf{i}+(14y^3z+3y^4)\mathbf{j}-(16xz^5+21y^2z^2)\mathbf{k}$

21. $\operatorname{div}\mathbf{F}=\dfrac{2}{\sqrt{x^2+y^2+z^2}}$, $\operatorname{curl}\mathbf{F}=0$ 23. $4x$ 25. 0

27. $(1+y)\mathbf{i}+x\mathbf{j}$

29. $\nabla\cdot(k\mathbf{F})=k\nabla\cdot\mathbf{F}$, $\nabla\cdot(\mathbf{F}+\mathbf{G})=\nabla\cdot\mathbf{F}+\nabla\cdot\mathbf{G}$, $\nabla\cdot(\phi\mathbf{F})=\phi\nabla\cdot\mathbf{F}+\nabla\phi\cdot\mathbf{F}$, $\nabla\cdot(\nabla\times\mathbf{F})=0$ 47. **(b)** $x^2+y^2=K$

49. $\dfrac{dy}{dx}=\dfrac{1}{x}$, $y=\ln x+K$



Exercise Set 15.2 (Page 1108)

1. **(a)** 1 **(b)** 0 3. 16 7. 8 9. -28

11. **(a)** $\frac{4\sqrt{2}-2}{3}$ **(b)** 1 **(c)** $\frac{2}{3}$

13. **(a)** 3 **(b)** 3 **(c)** 3 **(d)** 3

Responses to True–False questions may be abridged to save space.

15. False; line integrals of functions are independent of the orientation of the curve. 17. True; this is Equation (26).

19. 2 21. $\frac{13}{20}$ 23. $1-\pi$ 25. 3 27. $-1-(\pi/4)$ 29. $1-e^3$

31. **(a)** $\dfrac{63\sqrt{17}}{64}+\frac{1}{4}\ln(4+\sqrt{17})-\frac{1}{8}\ln\dfrac{\sqrt{17}+1}{\sqrt{17}-1}-\frac{1}{4}\ln(\sqrt{2}+1)+\frac{1}{8}\ln\dfrac{\sqrt{2}+1}{\sqrt{2}-1}$ **(b)** $\dfrac{\pi^3}{24}+\dfrac{e^{\pi/2}}{5}+\dfrac{\pi}{4}+\dfrac{6}{5}$

33. **(a)** -1 **(b)** -2 35. $\frac{5}{2}$ 37. 0 39. $1-e^{-1}$ 41. $6\sqrt{3}$

43. $5k\tan^{-1}3$ 45. $\frac{3}{5}$ 47. $\frac{27}{28}$ 49. $\frac{3}{4}$ 51. $\dfrac{17\sqrt{17}-1}{4}$

53. **(b)** $S=\displaystyle\int_C z(t)\,dt$ **(c)** 4π 55. $\lambda=-12$

Exercise Set 15.3 (Page 1120)

1. conservative, $\phi=\dfrac{x^2}{2}+\dfrac{y^2}{2}+K$ 3. not conservative

5. conservative, $\phi=x\cos y+y\sin x+K$

9. -6 11. $9e^2$ 13. 32 15. $W=-\frac{1}{2}$ 17. $W=1-e^{-1}$

Responses to True–False questions may be abridged to save space.

19. False; the integral must be 0 for *all* closed curves C.

21. True; if $\nabla\phi$ is constant, then ϕ must be a linear function.

23. $\ln 2-1$ 25. ≈ -0.307 27. no 33. $h(x)=Ce^x$

35. **(a)** $W=-\dfrac{1}{\sqrt{14}}+\dfrac{1}{\sqrt{6}}$ **(b)** $W=-\dfrac{1}{\sqrt{14}}+\dfrac{1}{\sqrt{6}}$ **(c)** $W=0$

Exercise Set 15.4 (Page 1127)

1. 0 3. 0 5. 0 7. 8π 9. -4 11. -1 13. 0

Responses to True–False questions may be abridged to save space.

15. False; Green's Theorem applies to closed curves.

17. True; the integral is the area of the region bounded by C.

19. **(a)** ≈ -3.550999378 **(b)** ≈ -0.269616482 21. $\frac{3}{8}a^2\pi$ 23. $\frac{1}{2}abt_0$

27. Formula (1) of Section 6.1 29. $\frac{250}{3}$ 31. $-3\pi a^2$ 33. $\left(\frac{8}{15},\frac{8}{21}\right)$

35. $\left(0,\dfrac{4a}{3\pi}\right)$ 37. the circle $x^2+y^2=1$ 39. 69

Exercise Set 15.5 (Page 1136)

1. $\frac{15}{2}\pi\sqrt{2}$ 3. $\dfrac{\pi}{4}$ 5. $-\dfrac{\sqrt{2}}{2}$ 7. 9

Responses to True–False questions may be abridged to save space.

9. True; this follows from the definition.

11. False; the integral is the total mass of the lamina.

13. **(b)** $2\pi\left[1-\sqrt{1-r^2}+\dfrac{r^2}{2}\right]\to 3\pi$ as $r\to 1^-$

(c) $\mathbf{r}(\phi,\theta)=\sin\phi\cos\theta\mathbf{i}+\sin\phi\sin\theta\mathbf{j}+\cos\phi\mathbf{k}$, $0\le\theta\le 2\pi$, $0\le\phi\le\pi/2$;

$$\iint(1+z)\,dS=\int_0^{2\pi}\int_0^{\pi/2}(1+\cos\phi)\sin\phi\,d\phi\,d\theta=3\pi$$

17. **(c)** $4\pi/3$

19. **(a)** $\dfrac{\sqrt{29}}{16}\displaystyle\int_0^6\int_0^{(12-2x)/3}xy(12-2x-3y)\,dy\,dx$

(b) $\dfrac{\sqrt{29}}{4}\displaystyle\int_0^3\int_0^{(12-4z)/3}yz(12-3y-4z)\,dy\,dz$

(c) $\dfrac{\sqrt{29}}{9}\displaystyle\int_0^3\int_0^{6-2z}xz(12-2x-4z)\,dx\,dz$

21. $\dfrac{18\sqrt{29}}{5}$

23. $\displaystyle\int_0^4\int_1^2 y^3z\sqrt{4y^2+1}\,dy\,dz$; $\dfrac{1}{2}\displaystyle\int_0^4\int_1^4 xz\sqrt{1+4x}\,dx\,dz$

25. $\dfrac{391\sqrt{17}}{15}-\dfrac{5\sqrt{5}}{3}$ 27. $\frac{4}{3}\pi\delta_0$ 29. $\frac{1}{4}(37\sqrt{37}-1)$ 31. $M=\delta_0 S$

33. $(0,0,149/65)$ 35. $\dfrac{93}{\sqrt{10}}$ 37. $\dfrac{\pi}{4}$ 39. 57.895751

Exercise Set 15.6 (Page 1146)

1. **(a)** zero **(b)** zero **(c)** positive **(d)** negative **(e)** zero **(f)** zero

3. -80 5. 30 7. 200π 9. 4 11. 2π 13. $\dfrac{14\pi}{3}$ 15. 0

17. 18π 19. $\frac{4}{9}$ 21. **(a)** 8 **(b)** 24 **(c)** 0

Responses to True–False questions may be abridged to save space.

23. False; the Möbius strip has no orientation.

25. False; the net volume can be zero because as much fluid passes through the surface in the negative direction as in the positive direction.

27. -3π 29. **(a)** 0 m^3/s **(b)** 0 kg/s 31. **(b)** 32/3

33. **(a)** $4\pi a^{k+3}$ **(b)** $k = -3$ 35. $a = 2, 3$

Exercise Set 15.7 (Page 1157)

1. 3 3. $\frac{4\pi}{3}$

Responses to True–False questions may be abridged to save space.

5. False; it equates a surface integral and a triple integral.

7. True; see subsection entitled Sources and Sinks.

9. 12 11. $3\pi a^2$ 13. 180π 15. $\frac{192\pi}{5}$ 17. $\frac{\pi}{2}$ 19. $\frac{4608}{35}$

21. 135π 23. **(a)** $\mathbf{F} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ **(b)** $\mathbf{F} = -x\mathbf{i} - y\mathbf{j} - z\mathbf{k}$

33. no sources or sinks

35. sources at all points except the origin, no sinks 37. $\frac{7\pi}{4}$

Exercise Set 15.8 (Page 1164)

1. $\frac{3}{2}$ 3. 0 5. 2π 7. 16π 9. 0 11. πa^2

Responses to True–False questions may be abridged to save space.

13. True; see Theorem 15.8.1. 15. False; the circulation is $\int_C \mathbf{F} \cdot \mathbf{T}\, ds$.

17. **(a)** $\frac{3}{2}$ **(b)** -1 **(c)** $-\frac{1}{\sqrt{2}}\mathbf{j} - \frac{1}{\sqrt{2}}\mathbf{k}$ 23. $-\frac{5\pi}{4}$

Chapter 15 Review Exercises (Page 1166)

3. $\dfrac{1-x}{\sqrt{(1-x)^2+(2-y)^2}}\mathbf{i} + \dfrac{2-y}{\sqrt{(1-x)^2+(2-y)^2}}\mathbf{j}$ 5. $\mathbf{i} + \mathbf{j} + \mathbf{k}$

7. **(a)** $\displaystyle\int_a^b \left[f(x(t), y(t))\frac{dx}{dt} + g(x(t), y(t))\frac{dy}{dt} \right] dt$

(b) $\displaystyle\int_a^b f(x(t), y(t))\sqrt{x'(t)^2 + y'(t)^2}\, dt$

11. 0 13. $-7/2$ 17. **(a)** $h(x) = Cx^{-3/2}$ **(b)** $g(y) = C/y^3$

21. $A = \dfrac{1}{2}\displaystyle\int_\alpha^\beta r^2\, d\theta$

23. $\displaystyle\iint_R f(x(u, v), y(u, v), z(u, v))\|r_u \times r_v\|\, du\, dv$ 25. yes 27. 2π

31. -8π 35. **(a)** conservative **(b)** not conservative

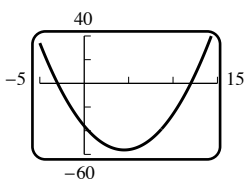
Chapter 15 Making Connections (Page 1168)

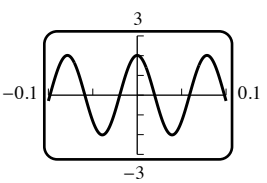
Answers are provided in the Student Solutions Manual.

Appendix A (Page A11)

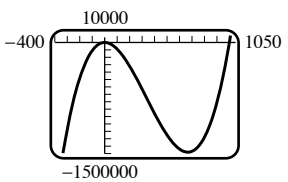
1. (e) 3. (b), (c) 5. $[-3, 3] \times [0, 5]$

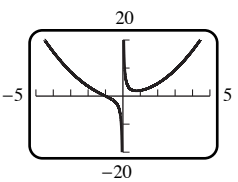
7. $[-5, 14] \times [-60, 40]$ 9. $[-0.1, 0.1] \times [-3, 3]$





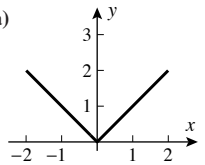
11. $[-400, 1050] \times [-1500000, 10000]$ 13. $[-2, 2] \times [-20, 20]$



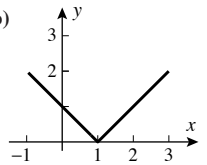


17. **(a)** $f(x) = \sqrt{16 - x^2}$ **(b)** $f(x) = -\sqrt{16 - x^2}$ **(e)** no

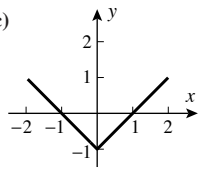
19. **(a)**



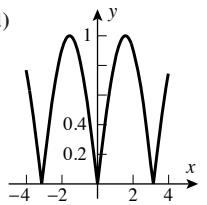
(b)



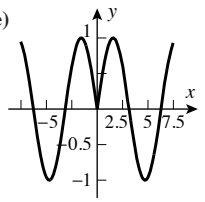
(c)



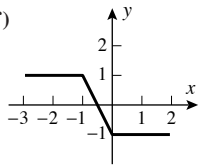
(d)



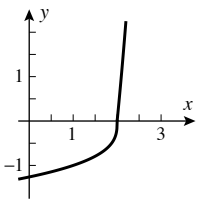
(e)



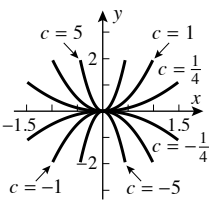
(f)



21.

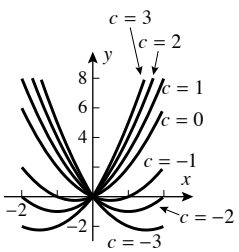


25. **(a)**



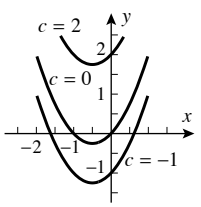
The graph is stretched in the vertical direction, and reflected across the x-axis if $c < 0$.

(b)



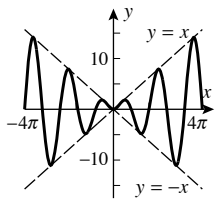
The graph is translated so its vertex is on the parabola $y = -x^2$.

(c)

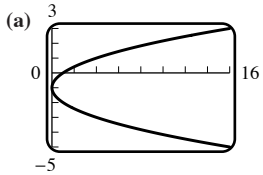


The graph is translated vertically.

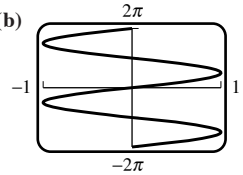
27.



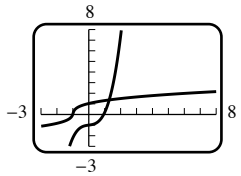
29. **(a)**



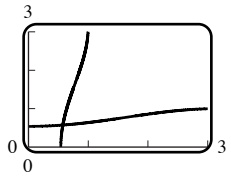
(b)



31.



33.



35. **(a)** $x = 4\cos t,\ y = 3\sin t$ **(b)** $x = -1 + 4\cos t,\ y = 2 + 3\sin t$

Appendix B (Page A23)

1. **(a)** $\frac{5}{12}\pi$ **(b)** $\frac{13}{6}\pi$ **(c)** $\frac{1}{9}\pi$ **(d)** $\frac{23}{30}\pi$

3. **(a)** $12°$ **(b)** $(270/\pi)°$ **(c)** $288°$ **(d)** $540°$

5.

	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\csc\theta$	$\sec\theta$	$\cot\theta$
(a)	$\sqrt{21}/5$	$2/5$	$\sqrt{21}/2$	$5/\sqrt{21}$	$5/2$	$2/\sqrt{21}$
(b)	$3/4$	$\sqrt{7}/4$	$3/\sqrt{7}$	$4/3$	$4/\sqrt{7}$	$\sqrt{7}/3$
(c)	$3/\sqrt{10}$	$1/\sqrt{10}$	3	$\sqrt{10}/3$	$\sqrt{10}$	$1/3$

7. $\sin\theta = 3/\sqrt{10},\ \cos\theta = 1/\sqrt{10}$ **9.** $\tan\theta = \sqrt{21}/2,\ \csc\theta = 5/\sqrt{21}$

11. 1.8

13.

	θ	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\csc\theta$	$\sec\theta$	$\cot\theta$
(a)	$225°$	$-1/\sqrt{2}$	$-1/\sqrt{2}$	1	$-\sqrt{2}$	$-\sqrt{2}$	1
(b)	$-210°$	$1/2$	$-\sqrt{3}/2$	$-1/\sqrt{3}$	2	$-2/\sqrt{3}$	$-\sqrt{3}$
(c)	$5\pi/3$	$-\sqrt{3}/2$	$1/2$	$-\sqrt{3}$	$-2/\sqrt{3}$	2	$-1/\sqrt{3}$
(d)	$-3\pi/2$	1	0	—	1	—	0

15.

	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\csc\theta$	$\sec\theta$	$\cot\theta$
(a)	$4/5$	$3/5$	$4/3$	$5/4$	$5/3$	$3/4$
(b)	$-4/5$	$3/5$	$-4/3$	$-5/4$	$5/3$	$-3/4$
(c)	$1/2$	$-\sqrt{3}/2$	$-1/\sqrt{3}$	2	$-2/\sqrt{3}$	$-\sqrt{3}$
(d)	$-1/2$	$\sqrt{3}/2$	$-1/\sqrt{3}$	-2	$2/\sqrt{3}$	$-\sqrt{3}$
(e)	$1/\sqrt{2}$	$1/\sqrt{2}$	1	$\sqrt{2}$	$\sqrt{2}$	1
(f)	$1/\sqrt{2}$	$-1/\sqrt{2}$	-1	$\sqrt{2}$	$-\sqrt{2}$	-1

17. **(a)** 1.2679 **(b)** 3.5753

19.

	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\csc\theta$	$\sec\theta$	$\cot\theta$
(a)	$a/3$	$\sqrt{9-a^2}/3$	$a/\sqrt{9-a^2}$	$3/a$	$3/\sqrt{9-a^2}$	$\sqrt{9-a^2}/a$
(b)	$a/\sqrt{a^2+25}$	$5/\sqrt{a^2+25}$	$a/5$	$\sqrt{a^2+25}/a$	$\sqrt{a^2+25}/5$	$5/a$
(c)	$\sqrt{a^2-1}/a$	$1/a$	$\sqrt{a^2-1}$	$a/\sqrt{a^2-1}$	a	$1/\sqrt{a^2-1}$

21. **(a)** $3\pi/4 \pm n\pi,\ n = 0, 1, 2, \ldots$
(b) $\pi/3 \pm 2n\pi$ and $5\pi/3 \pm 2n\pi,\ n = 0, 1, 2, \ldots$

23. **(a)** $\pi/6 \pm n\pi,\ n = 0, 1, 2, \ldots$
(b) $4\pi/3 \pm 2n\pi$ and $5\pi/3 \pm 2n\pi,\ n = 0, 1, 2, \ldots$

25. **(a)** $3\pi/4 \pm n\pi,\ n = 0, 1, 2, \ldots$
(b) $\pi/6 \pm n\pi,\ n = 0, 1, 2, \ldots$

27. **(a)** $\pi/3 \pm 2n\pi$ and $2\pi/3 \pm 2n\pi,\ n = 0, 1, 2, \ldots$
(b) $\pi/6 \pm 2n\pi$ and $11\pi/6 \pm 2n\pi,\ n = 0, 1, 2, \ldots$

29. $\sin\theta = 2/5,\ \cos\theta = -\sqrt{21}/5,\ \tan\theta = -2/\sqrt{21},$
$\csc\theta = 5/2,\ \sec\theta = -5/\sqrt{21},\ \cot\theta = -\sqrt{21}/2$

31. **(a)** $\theta = \pm n\pi,\ n = 0, 1, 2, \ldots$ **(b)** $\theta = \pi/2 \pm n\pi,\ n = 0, 1, 2, \ldots$
(c) $\theta = \pm n\pi,\ n = 0, 1, 2, \ldots$ **(d)** $\theta = \pm n\pi,\ n = 0, 1, 2, \ldots$
(e) $\theta = \pi/2 \pm n\pi,\ n = 0, 1, 2, \ldots$ **(f)** $\theta = \pm n\pi,\ n = 0, 1, 2, \ldots$

33. **(a)** $2\pi/3$ cm **(b)** $10\pi/3$ cm **35.** $\frac{2}{5}$

37. **(a)** $\dfrac{2\pi - \theta}{2\pi}R$ **(b)** $\dfrac{\sqrt{4\pi\theta - \theta^2}}{2\pi}R$ **39.** $\frac{21}{4}\sqrt{3}$ **41.** 9.2 ft

43. $h = d(\tan\beta - \tan\alpha)$ **45.** **(a)** $4\sqrt{5}/9$ **(b)** $-\frac{1}{9}$

47. $\sin 3\theta = 3\sin\theta\cos^2\theta - \sin^3\theta,\ \cos 3\theta = \cos^3\theta - 3\sin^2\theta\cos\theta$

61. **(a)** $\cos\theta$ **(b)** $-\sin\theta$ **(c)** $-\cos\theta$ **(d)** $\sin\theta$

69. **(a)** $153°$ **(b)** $45°$ **(c)** $117°$ **(d)** $89°$ **71.** **(a)** $60°$ **(b)** $117°$

Appendix C (Page A32)

1. **(a)** $q(x) = x^2 + 4x + 2,\ r(x) = -11x + 6$
(b) $q(x) = 2x^2 + 4,\ r(x) = 9$
(c) $q(x) = x^3 - x^2 + 2x - 2,\ r(x) = 2x + 1$

3. **(a)** $q(x) = 3x^2 + 6x + 8,\ r(x) = 15$
(b) $q(x) = x^3 - 5x^2 + 20x - 100,\ r(x) = 504$
(c) $q(x) = x^4 + x^3 + x^2 + x + 1,\ r(x) = 0$

5.

x	0	1	−3	7
$p(x)$	−4	−3	101	5001

7. **(a)** $q(x) = x^2 + 6x + 13,\ r = 20$ **(b)** $q(x) = x^2 + 3x - 2,\ r = -4$

9. **(a)** $\pm1, \pm2, \pm3, \pm4, \pm6, \pm8, \pm12, \pm24$
(b) $\pm1, \pm2, \pm5, \pm10, \pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{5}{3}, \pm\frac{10}{3}$ **(c)** $\pm1, \pm17$

11. $(x+1)(x-1)(x-2)$ **13.** $(x+3)^3(x+1)$

15. $(x+3)(x+2)(x+1)^2(x-3)$ **17.** -3 **19.** $-2, -\frac{2}{3}, -1 \pm \sqrt{3}$

21. $-2, 2, 3$ **23.** $2, 5$ **25.** 7 cm

INDEX

RATIONAL FUNCTIONS CONTAINING POWERS OF $a + bu$ IN THE DENOMINATOR

60. $\displaystyle\int \frac{u\,du}{a+bu} = \frac{1}{b^2}[bu - a\ln|a+bu|] + C$

61. $\displaystyle\int \frac{u^2\,du}{a+bu} = \frac{1}{b^3}\left[\frac{1}{2}(a+bu)^2 - 2a(a+bu) + a^2\ln|a+bu|\right] + C$

62. $\displaystyle\int \frac{u\,du}{(a+bu)^2} = \frac{1}{b^2}\left[\frac{a}{a+bu} + \ln|a+bu|\right] + C$

63. $\displaystyle\int \frac{u^2\,du}{(a+bu)^2} = \frac{1}{b^3}\left[bu - \frac{a^2}{a+bu} - 2a\ln|a+bu|\right] + C$

64. $\displaystyle\int \frac{u\,du}{(a+bu)^3} = \frac{1}{b^2}\left[\frac{a}{2(a+bu)^2} - \frac{1}{a+bu}\right] + C$

65. $\displaystyle\int \frac{du}{u(a+bu)} = \frac{1}{a}\ln\left|\frac{u}{a+bu}\right| + C$

66. $\displaystyle\int \frac{du}{u^2(a+bu)} = -\frac{1}{au} + \frac{b}{a^2}\ln\left|\frac{a+bu}{u}\right| + C$

67. $\displaystyle\int \frac{du}{u(a+bu)^2} = \frac{1}{a(a+bu)} + \frac{1}{a^2}\ln\left|\frac{u}{a+bu}\right| + C$

RATIONAL FUNCTIONS CONTAINING $a^2 \pm u^2$ IN THE DENOMINATOR ($a > 0$)

68. $\displaystyle\int \frac{du}{a^2+u^2} = \frac{1}{a}\tan^{-1}\frac{u}{a} + C$

69. $\displaystyle\int \frac{du}{a^2-u^2} = \frac{1}{2a}\ln\left|\frac{u+a}{u-a}\right| + C$

70. $\displaystyle\int \frac{du}{u^2-a^2} = \frac{1}{2a}\ln\left|\frac{u-a}{u+a}\right| + C$

71. $\displaystyle\int \frac{bu+c}{a^2+u^2}\,du = \frac{b}{2}\ln(a^2+u^2) + \frac{c}{a}\tan^{-1}\frac{u}{a} + C$

INTEGRALS OF $\sqrt{a^2+u^2}$, $\sqrt{a^2-u^2}$, $\sqrt{u^2-a^2}$ AND THEIR RECIPROCALS ($a > 0$)

72. $\displaystyle\int \sqrt{u^2+a^2}\,du = \frac{u}{2}\sqrt{u^2+a^2} + \frac{a^2}{2}\ln(u+\sqrt{u^2+a^2}) + C$

73. $\displaystyle\int \sqrt{u^2-a^2}\,du = \frac{u}{2}\sqrt{u^2-a^2} - \frac{a^2}{2}\ln|u+\sqrt{u^2-a^2}| + C$

74. $\displaystyle\int \sqrt{a^2-u^2}\,du = \frac{u}{2}\sqrt{a^2-u^2} + \frac{a^2}{2}\sin^{-1}\frac{u}{a} + C$

75. $\displaystyle\int \frac{du}{\sqrt{u^2+a^2}} = \ln(u+\sqrt{u^2+a^2}) + C$

76. $\displaystyle\int \frac{du}{\sqrt{u^2-a^2}} = \ln|u+\sqrt{u^2-a^2}| + C$

77. $\displaystyle\int \frac{du}{\sqrt{a^2-u^2}} = \sin^{-1}\frac{u}{a} + C$

POWERS OF u MULTIPLYING OR DIVIDING $\sqrt{a^2-u^2}$ OR ITS RECIPROCAL

78. $\displaystyle\int u^2\sqrt{a^2-u^2}\,du = \frac{u}{8}(2u^2-a^2)\sqrt{a^2-u^2} + \frac{a^4}{8}\sin^{-1}\frac{u}{a} + C$

79. $\displaystyle\int \frac{\sqrt{a^2-u^2}\,du}{u} = \sqrt{a^2-u^2} - a\ln\left|\frac{a+\sqrt{a^2-u^2}}{u}\right| + C$

80. $\displaystyle\int \frac{\sqrt{a^2-u^2}\,du}{u^2} = -\frac{\sqrt{a^2-u^2}}{u} - \sin^{-1}\frac{u}{a} + C$

81. $\displaystyle\int \frac{u^2\,du}{\sqrt{a^2-u^2}} = -\frac{u}{2}\sqrt{a^2-u^2} + \frac{a^2}{2}\sin^{-1}\frac{u}{a} + C$

82. $\displaystyle\int \frac{du}{u\sqrt{a^2-u^2}} = -\frac{1}{a}\ln\left|\frac{a+\sqrt{a^2-u^2}}{u}\right| + C$

83. $\displaystyle\int \frac{du}{u^2\sqrt{a^2-u^2}} = -\frac{\sqrt{a^2-u^2}}{a^2u} + C$

POWERS OF u MULTIPLYING OR DIVIDING $\sqrt{u^2 \pm a^2}$ OR THEIR RECIPROCALS

84. $\displaystyle\int u\sqrt{u^2+a^2}\,du = \frac{1}{3}(u^2+a^2)^{3/2} + C$

85. $\displaystyle\int u\sqrt{u^2-a^2}\,du = \frac{1}{3}(u^2-a^2)^{3/2} + C$

86. $\displaystyle\int \frac{du}{u\sqrt{u^2+a^2}} = -\frac{1}{a}\ln\left|\frac{a+\sqrt{u^2+a^2}}{u}\right| + C$

87. $\displaystyle\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\sec^{-1}\left|\frac{u}{a}\right| + C$

88. $\displaystyle\int \frac{\sqrt{u^2-a^2}\,du}{u} = \sqrt{u^2-a^2} - a\sec^{-1}\left|\frac{u}{a}\right| + C$

89. $\displaystyle\int \frac{\sqrt{u^2+a^2}\,du}{u} = \sqrt{u^2+a^2} - a\ln\left|\frac{a+\sqrt{u^2+a^2}}{u}\right| + C$

90. $\displaystyle\int \frac{du}{u^2\sqrt{u^2\pm a^2}} = \mp\frac{\sqrt{u^2\pm a^2}}{a^2u} + C$

91. $\displaystyle\int u^2\sqrt{u^2+a^2}\,du = \frac{u}{8}(2u^2+a^2)\sqrt{u^2+a^2} - \frac{a^4}{8}\ln(u+\sqrt{u^2+a^2}) + C$

92. $\displaystyle\int u^2\sqrt{u^2-a^2}\,du = \frac{u}{8}(2u^2-a^2)\sqrt{u^2-a^2} - \frac{a^4}{8}\ln|u+\sqrt{u^2-a^2}| + C$

93. $\displaystyle\int \frac{\sqrt{u^2+a^2}}{u^2}\,du = -\frac{\sqrt{u^2+a^2}}{u} + \ln(u+\sqrt{u^2+a^2}) + C$

94. $\displaystyle\int \frac{\sqrt{u^2-a^2}}{u^2}\,du = -\frac{\sqrt{u^2-a^2}}{u} + \ln|u+\sqrt{u^2-a^2}| + C$

95. $\displaystyle\int \frac{u^2}{\sqrt{u^2+a^2}}\,du = \frac{u}{2}\sqrt{u^2+a^2} - \frac{a^2}{2}\ln(u+\sqrt{u^2+a^2}) + C$

96. $\displaystyle\int \frac{u^2}{\sqrt{u^2-a^2}}\,du = \frac{u}{2}\sqrt{u^2-a^2} + \frac{a^2}{2}\ln|u+\sqrt{u^2-a^2}| + C$

INTEGRALS CONTAINING $(a^2+u^2)^{3/2}$, $(a^2-u^2)^{3/2}$, $(u^2-a^2)^{3/2}$ ($a > 0$)

97. $\displaystyle\int \frac{du}{(a^2-u^2)^{3/2}} = \frac{u}{a^2\sqrt{a^2-u^2}} + C$

98. $\displaystyle\int \frac{du}{(u^2\pm a^2)^{3/2}} = \pm\frac{u}{a^2\sqrt{u^2\pm a^2}} + C$

99. $\displaystyle\int (a^2-u^2)^{3/2}\,du = -\frac{u}{8}(2u^2-5a^2)\sqrt{a^2-u^2} + \frac{3a^4}{8}\sin^{-1}\frac{u}{a} + C$

100. $\displaystyle\int (u^2+a^2)^{3/2}\,du = \frac{u}{8}(2u^2+5a^2)\sqrt{u^2+a^2} + \frac{3a^4}{8}\ln(u+\sqrt{u^2+a^2}) + C$

101. $\displaystyle\int (u^2-a^2)^{3/2}\,du = \frac{u}{8}(2u^2-5a^2)\sqrt{u^2-a^2} + \frac{3a^4}{8}\ln|u+\sqrt{u^2-a^2}| + C$

POWERS OF u MULTIPLYING OR DIVIDING $\sqrt{a+bu}$ OR ITS RECIPROCAL

102. $\displaystyle\int u\sqrt{a+bu}\,du = \frac{2}{15b^2}(3bu-2a)(a+bu)^{3/2}+C$

103. $\displaystyle\int u^2\sqrt{a+bu}\,du = \frac{2}{105b^3}(15b^2u^2-12abu+8a^2)(a+bu)^{3/2}+C$

104. $\displaystyle\int u^n\sqrt{a+bu}\,du = \frac{2u^n(a+bu)^{3/2}}{b(2n+3)} - \frac{2an}{b(2n+3)}\int u^{n-1}\sqrt{a+bu}\,du$

105. $\displaystyle\int \frac{u\,du}{\sqrt{a+bu}} = \frac{2}{3b^2}(bu-2a)\sqrt{a+bu}+C$

106. $\displaystyle\int \frac{u^2\,du}{\sqrt{a+bu}} = \frac{2}{15b^3}(3b^2u^2-4abu+8a^2)\sqrt{a+bu}+C$

107. $\displaystyle\int \frac{u^n\,du}{\sqrt{a+bu}} = \frac{2u^n\sqrt{a+bu}}{b(2n+1)} - \frac{2an}{b(2n+1)}\int \frac{u^{n-1}\,du}{\sqrt{a+bu}}$

108. $\displaystyle\int \frac{du}{u\sqrt{a+bu}} = \begin{cases} \dfrac{1}{\sqrt{a}}\ln\left|\dfrac{\sqrt{a+bu}-\sqrt{a}}{\sqrt{a+bu}+\sqrt{a}}\right|+C & (a>0) \\ \dfrac{2}{\sqrt{-a}}\tan^{-1}\sqrt{\dfrac{a+bu}{-a}}+C & (a<0) \end{cases}$

109. $\displaystyle\int \frac{du}{u^n\sqrt{a+bu}} = -\frac{\sqrt{a+bu}}{a(n-1)u^{n-1}} - \frac{b(2n-3)}{2a(n-1)}\int \frac{du}{u^{n-1}\sqrt{a+bu}}$

110. $\displaystyle\int \frac{\sqrt{a+bu}\,du}{u} = 2\sqrt{a+bu} + a\int \frac{du}{u\sqrt{a+bu}}$

111. $\displaystyle\int \frac{\sqrt{a+bu}\,du}{u^n} = -\frac{(a+bu)^{3/2}}{a(n-1)u^{n-1}} - \frac{b(2n-5)}{2a(n-1)}\int \frac{\sqrt{a+bu}\,du}{u^{n-1}}$

POWERS OF u MULTIPLYING OR DIVIDING $\sqrt{2au-u^2}$ OR ITS RECIPROCAL

112. $\displaystyle\int \sqrt{2au-u^2}\,du = \frac{u-a}{2}\sqrt{2au-u^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{u-a}{a}\right)+C$

113. $\displaystyle\int u\sqrt{2au-u^2}\,du = \frac{2u^2-au-3a^2}{6}\sqrt{2au-u^2} + \frac{a^3}{2}\sin^{-1}\left(\frac{u-a}{a}\right)+C$

114. $\displaystyle\int \frac{\sqrt{2au-u^2}\,du}{u} = \sqrt{2au-u^2} + a\sin^{-1}\left(\frac{u-a}{a}\right)+C$

115. $\displaystyle\int \frac{\sqrt{2au-u^2}\,du}{u^2} = -\frac{2\sqrt{2au-u^2}}{u} - \sin^{-1}\left(\frac{u-a}{a}\right)+C$

116. $\displaystyle\int \frac{du}{\sqrt{2au-u^2}} = \sin^{-1}\left(\frac{u-a}{a}\right)+C$

117. $\displaystyle\int \frac{du}{u\sqrt{2au-u^2}} = -\frac{\sqrt{2au-u^2}}{au}+C$

118. $\displaystyle\int \frac{u\,du}{\sqrt{2au-u^2}} = -\sqrt{2au-u^2} + a\sin^{-1}\left(\frac{u-a}{a}\right)+C$

119. $\displaystyle\int \frac{u^2\,du}{\sqrt{2au-u^2}} = -\frac{(u+3a)}{2}\sqrt{2au-u^2} + \frac{3a^2}{2}\sin^{-1}\left(\frac{u-a}{a}\right)+C$

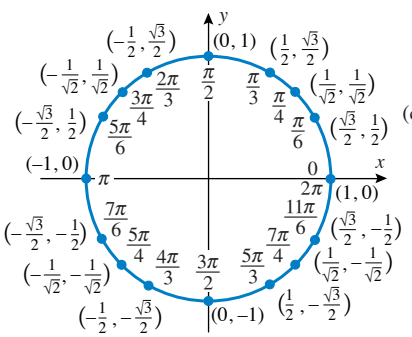
INTEGRALS CONTAINING $(2au-u^2)^{3/2}$

120. $\displaystyle\int \frac{du}{(2au-u^2)^{3/2}} = \frac{u-a}{a^2\sqrt{2au-u^2}}+C$

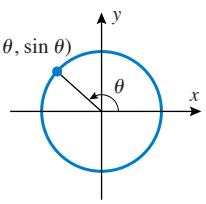
121. $\displaystyle\int \frac{u\,du}{(2au-u^2)^{3/2}} = \frac{u}{a\sqrt{2au-u^2}}+C$

THE WALLIS FORMULA

122. $\displaystyle\int_0^{\pi/2}\sin^n u\,du = \int_0^{\pi/2}\cos^n u\,du = \frac{1\cdot3\cdot5\cdot\cdots\cdot(n-1)}{2\cdot4\cdot6\cdot\cdots\cdot n}\cdot\frac{\pi}{2}$ (n an even integer and $n\ge 2$) or $\dfrac{2\cdot4\cdot6\cdot\cdots\cdot(n-1)}{3\cdot5\cdot7\cdot\cdots\cdot n}$ (n an odd integer and $n\ge 3$)



TRIGONOMETRY REVIEW



PYTHAGOREAN IDENTITIES

$\sin^2\theta+\cos^2\theta=1$ $\quad\tan^2\theta+1=\sec^2\theta$ $\quad1+\cot^2\theta=\csc^2\theta$

SIGN IDENTITIES

$\sin(-\theta)=-\sin\theta$ $\quad\cos(-\theta)=\cos\theta$ $\quad\tan(-\theta)=-\tan\theta$

$\csc(-\theta)=-\csc\theta$ $\quad\sec(-\theta)=\sec\theta$ $\quad\cot(-\theta)=-\cot\theta$

COMPLEMENT IDENTITIES

$\sin\left(\frac{\pi}{2}-\theta\right)=\cos\theta$ $\quad\cos\left(\frac{\pi}{2}-\theta\right)=\sin\theta$ $\quad\tan\left(\frac{\pi}{2}-\theta\right)=\cot\theta$

$\csc\left(\frac{\pi}{2}-\theta\right)=\sec\theta$ $\quad\sec\left(\frac{\pi}{2}-\theta\right)=\csc\theta$ $\quad\cot\left(\frac{\pi}{2}-\theta\right)=\tan\theta$

SUPPLEMENT IDENTITIES

$\sin(\pi-\theta)=\sin\theta$ $\quad\cos(\pi-\theta)=-\cos\theta$ $\quad\tan(\pi-\theta)=-\tan\theta$

$\csc(\pi-\theta)=\csc\theta$ $\quad\sec(\pi-\theta)=-\sec\theta$ $\quad\cot(\pi-\theta)=-\cot\theta$

$\sin(\pi+\theta)=-\sin\theta$ $\quad\cos(\pi+\theta)=-\cos\theta$ $\quad\tan(\pi+\theta)=\tan\theta$

$\csc(\pi+\theta)=-\csc\theta$ $\quad\sec(\pi+\theta)=-\sec\theta$ $\quad\cot(\pi+\theta)=\cot\theta$

ADDITION FORMULAS

$\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$

$\sin(\alpha-\beta)=\sin\alpha\cos\beta-\cos\alpha\sin\beta$

$\tan(\alpha+\beta)=\dfrac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$

$\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$

$\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta$

$\tan(\alpha-\beta)=\dfrac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}$

DOUBLE-ANGLE FORMULAS

$\sin2\alpha=2\sin\alpha\cos\alpha$

$\cos2\alpha=\cos^2\alpha-\sin^2\alpha$

$\cos2\alpha=2\cos^2\alpha-1$

$\cos2\alpha=1-2\sin^2\alpha$

HALF-ANGLE FORMULAS

$\sin^2\frac{\alpha}{2}=\dfrac{1-\cos\alpha}{2}$ $\qquad\cos^2\frac{\alpha}{2}=\dfrac{1+\cos\alpha}{2}$