

PHY 107

Gravitation

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OUTLINE

- ▶ Newton's law of gravitation
- ▶ Superposition
- ▶ Potential Energy

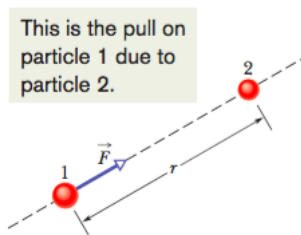
Newton's Law of Gravitation

Gravitation: Tendency of bodies to move toward one another

Newton's law of gravitation: Every particle attracts any other particle with a gravitational force of magnitude

$$F = G \frac{m_1 m_2}{r^2}$$

$$G = 6.67 \times 10^{-11} \text{ Nm}^2/\text{kg}^2$$



Principle of Superposition

Given a group of particles, we find the net (or resultant) gravitational force on any one of them from the others by using the principle of superposition.

For n interacting particles, we can write the principle of superposition for the gravitational forces on particle 1 as:

$$\vec{F}_{1,net} = \vec{F}_{12} + \vec{F}_{13} + \vec{F}_{14} + \dots \vec{F}_{1n}$$

Newton's Law of Gravitation

Example Figure shows an arrangement of three particles, particle 1 of mass $m_1 = 6.0$ kg and particles 2 and 3 of mass $m_2 = 4.0$ kg, and distance $a = 2.0$ cm. What is the net gravitational force $F_{1,net}$ on particle 1 due to the other particles?

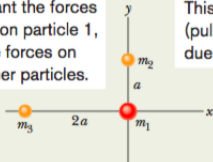
Solution:

$$F_{12} = \frac{Gm_1m_2}{a^2}$$

$$F_{13} = \frac{Gm_1m_3}{(2a)^2}$$

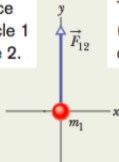
Newton's Law of Gravitation

We want the forces (pulls) on particle 1, *not* the forces on the other particles.



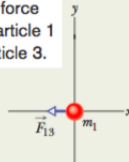
(a)

This is the force (pull) on particle 1 due to particle 2.

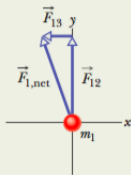


(b)

This is the force (pull) on particle 1 due to particle 3.

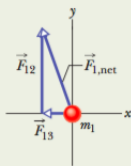


(c)



(d)

This is one way to show the net force on particle 1. Note the head-to-tail arrangement.



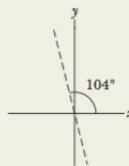
(e)

This is another way, also a head-to-tail arrangement.



(f)

A calculator's inverse tangent can give this for the angle.



(g)

But this is the correct angle.

Gravitational Potential Energy

Let's broaden our view...

$U=0$ for $r = \infty$

Potential Energy is negative for any finite separation and becomes progressively more negative as the particles move closer together.

GPE of the two-particle system:

$$U = -\frac{GMm}{r} \text{ (gravitational potential energy)}$$

(Check the book for proof)

GPE: a property of the two particle system (rather than of either particle alone).

What would be the GPE for a three particle system?

Potential Energy and Force: $F = -\frac{dU}{dr}$

Escape speed

Consider a projectile of mass m leaving the surface of a planet with escape speed v :

$$v = \sqrt{\frac{2GM}{R}} \text{ (Can be derived by COE)}$$

M : mass of the planet

R : radius of the planet

Example

Asteroid falling from space, mechanical energy

An asteroid, headed directly toward Earth, has a speed of 12 km/s relative to the planet when the asteroid is 10 Earth radii from Earth's center. Neglecting the effects of Earth's atmosphere on the asteroid, find the asteroid's speed v_f when it reaches Earth's surface.

Solution:

$$\text{COE: } K_f + U_f = K_i + U_i$$
$$0.5mv_f^2 - \frac{GMm}{R_E} = 0.5mv_i^2 - \frac{GMm}{10R_E}$$

We solve for v_f

Problems of importance

Reference book (Extended 9th edition)

Newton's Law of Gravitation: 3

Gravitation and Principle of Superposition: 6

Reference

Fundamentals of Physics by Halliday and Resnik