Final Assignment

Department of Electrical & Computer Engineering

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Submitted By

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Course: Theory of Electromagnetics (EEE361)

Set Number: 06

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The statement:

CODE OF HONOR PLEDGE

I pledge on my honor that I have not given or received any unauthorized assistance on this assignment.

Signature: Mohammed Mahmudur Rahman
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1. An expression for an electric field is given below

$$E = 20Cos(\omega t - 2x - 3z)a_y \text{ V/m}$$

Is incident on a dielectric slab (Z ≥ 0) 5.5With $\mu_r = 1.0$ and $\varepsilon_r = 2.5$ Find:

   a. The polarization of the wave
   b. The angle of incidence
   c. The reflected E and H field
   d. The transmitted E and H field

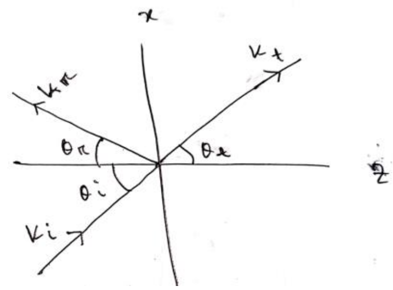
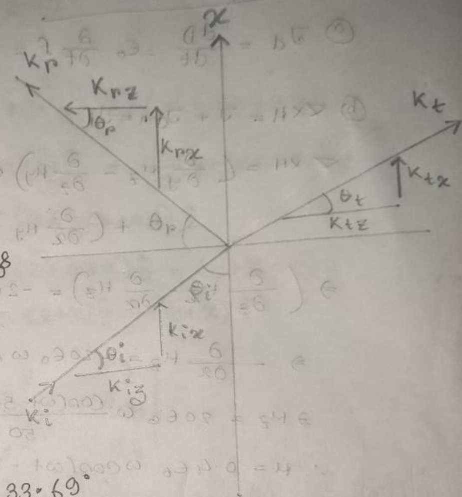


Fig: Problem 1

Answer to question NO:1.



ⓐ $K_i = 2a_x + 3a_z$

$= \sqrt{2^2+3^2}$

$= 3.60$

$K_i = \omega\sqrt{\mu_0\epsilon_0}$

$\Rightarrow 3.6 = \frac{\omega}{c}$

$\therefore \omega = 3.6 \times c = 3.6 \times 3 \times 10^8$

ⓑ $\tan\theta_i = \frac{k_x}{k_z}$

$= 2/3$.

$\therefore \theta_i = \tan^{-1}(0.667) = 33.69°$

ⓒ $E_r = E_{r_0}\cos(\omega t - k_r . r)\, a_y$.

$\Rightarrow K_r = K_{rx}\, a_x - K_{rz}\, a_z$.

$\Rightarrow K_{rx} = K_r \sin\theta_r \quad K_{rz} = k_r \cos\theta_r$.

$\therefore K_r = 2a_x - 3a_z$

As per snells law,

$$\sin\theta_t = \frac{n_1}{n_2}\sin\theta_i$$

$$= \frac{c\sqrt{\mu_1\epsilon_1}}{c\sqrt{\mu_2\epsilon_2}}\sin\theta_i$$

$$= \frac{\sin 33.69}{\sqrt{1.0}} = 0.554.$$

$\therefore \theta_t = \sin^{-1}(0.554) = 33.65$.

Here, $\eta_1 - \eta_0 = 120\pi$.
$= 377$.

$\therefore \eta_2 = \frac{377}{\sqrt{2.5}} = 238.4$.

$\therefore T_1 = \frac{238.4\cos 33.69 - 377\cos 33.65}{238.4\cos 33.69 + 377\cos 33.65}$.

$E_{r_0} = T_1 E_{i_0} = \cancel{0} - 0.209 \times 20$.
$= -4.185$.

$\therefore E_r = -4.185\cos(108 \times 10^7 t - 2x + 3z)a_y$ V/m.

(d) $E_r = E_{t_0}\cos(\omega t - K_r r)a_y$

$K_r = \beta_2 = \omega\sqrt{\mu_2 \epsilon_2} = \omega/c \sqrt{\mu_{r_2}\epsilon_{r_2}}$.

$= \frac{108 \times 10^7}{3 \times 10^8} \times \sqrt{1 \times 2.5} = 5.692$. #

$T_\perp = \frac{E_{t_0}}{E_{i_0}}$

$= \frac{2\eta_2 \cos\theta_i}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_i} = \frac{2 \times 238.4\cos 33.69}{238.4\cos 33.69 + 377\cos 33.65}$

$= 0.7746$

$E_{t_0} = T_\perp E_{i_0}$
$= 0.7746 \times 20 = 15.492$.

2. Suppose **E** fields and **H** fields are:

$$E = E_0 e^{j(k.r-\omega t)}$$
$$H = H_0 e^{j(k.r-\omega t)}$$

$Where$ k = $k_x a_x + k_y a_y + k_z a_z$ and r = $x a_x + y a_y + z a_z$

Show that $\nabla \times E = -\partial B/\partial t$ can be expressed as K × E = $\mu\omega H$ and deduce $a_k \times a_E = a_H$

For the same fields:

Show that Maxwell's equation in a source-free region can be written as

$$k.E = 0$$
$$k.h=0$$
$$k \times E = \mu\omega H$$
$$k \times H = -\mu\omega H$$

From these equations deduce $a_k \times a_E = a_H$ and $a_k \times a_H = -a_E$

Answer to question No: 2.

\# Let, A is a uniform vector & $\phi(r)$ is a scalar.

$$\nabla \times E(\phi A) = \left(\frac{\partial}{\partial x} a_x + \frac{\partial}{\partial y} a_y + \frac{\partial}{\partial z} a_z\right) \times E_0 e^{j(k_x x + k_y y + k_z z - \omega t)}$$

$$= j(k_x a_x + k_y a_y + k_z a_z) e^{j(k_x x + k_y y + k_z z - \omega t)} \times E_0$$

$$= jk \times E_0 e^{j(kx \ldots)} = jk \times E$$

Again, $-\frac{\partial B}{\partial t} = j\omega\mu H$, so, $\nabla \times E = -\frac{\partial B}{\partial t}$ become $k \times E = \omega\mu H$.

$\therefore a_k \times a_E = a_H$

\# $\nabla \cdot E = \left(\frac{\partial}{\partial x} a_x + \frac{\partial}{\partial y} a_y + \frac{\partial}{\partial z} a_z\right) \cdot E_0 e^{j(k_x x + k_y y + k_z z - \omega t)}$

$$= j(k_x a_x + k_y a_y + k_z a_z) e^{j(k_x a_x + k_y y + k_z z - \omega t)} \cdot E_0$$

$$\Rightarrow jk \cdot E_0 e^{j(k_x a_x + k_y y + k_z z - \omega t)} = jk \cdot E = 0.$$

$\Rightarrow k \cdot E = 0.$

Similarly,

$\nabla \cdot H = jk \cdot H = 0$

$\Rightarrow k \cdot H = 0.$

And, $\nabla \times H = \frac{\partial D}{\partial t} \Rightarrow k \times H = -\epsilon\omega E.$

From, $k \times E = \mu\omega H$, $a_x \times a_k = a_H$.

and, from, $k \times H = -\epsilon\omega E$, $a_k \times a_H = -a_E$.

3. a. let $F_1 = x^2\hat{z}$ and $F_2 = x\hat{x} + y\hat{y} + z\hat{z}$ calculate the divergence and curl of F_1 and F_2. Which one can be written as the gradient of a scalar? Find a scalar potential that does the job.

   Which one can be written as the curl of a vector? Find a suitable vector potential.

   b. Show that $F_3 = yz\hat{x} + zx\hat{y} + xy\hat{z}$ can be written both as the gradient of a scalar and as the curl of a vector. Find scalar and vector potentials for this function.

Answer to question NO: 3.

ⓐ $F_1 = x^2 \hat{z}$, $F_2 = x\hat{x} + y\hat{y} + z\hat{z}$

$\vec{\nabla} \cdot \vec{F_1} = \frac{\partial}{\partial x}(x^2 \hat{z}) = 2x$.

$\vec{\nabla} \cdot \vec{F_2} = \frac{\partial}{\partial x}(x\hat{x}) + \frac{\partial}{\partial y}(y\hat{y}) + \frac{\partial}{\partial z}(z\hat{z})$.

$= 1 + 1 + 1$.

$= 3$.

$$\text{curl}(F_1) = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 0 & 0 & x^2 \end{vmatrix}$$

$= [\partial/\partial y (x^2) - 0]\,\hat{x}$

$= x^2 \hat{x}$

$$\text{curl}(F_2) = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x & y & z \end{vmatrix}$$

$= \hat{x}\left(\partial/\partial y (z) - \frac{\partial}{\partial z}(y)\right) - \hat{y}\left[\frac{\partial}{\partial x}(z) - \frac{\partial}{\partial z}(x)\right]$

$+ \hat{z}\left[\frac{\partial}{\partial x}(y) - \frac{\partial}{\partial y}(x)\right]$.

$= 0 - 0 + 0 = 0$

Determining Gradient of F_1 :

$\vec{\nabla} \cdot F = \left(\frac{\partial}{\partial x}\hat{x} + \frac{\partial}{\partial y}\hat{y} + \frac{\partial}{\partial y}\hat{z}\right)(0 + 0 + x^2)$.

$= x^2 (\hat{z})$.

So, Divergence of F_1 & F_2 is the gradient of a scalar.

3(b) Here,

$$F_3 = yz\hat{x} + zx\hat{y} + xy\hat{z}.$$

Gradient of F_3,

$$\vec{\nabla} \cdot F_3 = \left(\frac{\partial}{\partial x}\hat{x} + \frac{\partial}{\partial y}\hat{y} + \frac{\partial}{\partial z}\hat{z}\right)(yz + zx + xy)$$

$$= \frac{\partial}{\partial x}(yz + zx + xy)\hat{x} + \frac{\partial}{\partial y}(yz + zx + xy)\hat{y} + \frac{\partial}{\partial z}(yz + zx + xy)\hat{z}.$$

$$= (0 + z + y)\hat{x} + (z + 0 + x)\hat{y} + (y + x + 0)\hat{z}$$

$$= (z+y)\hat{x} + (z+x)\hat{y} + (y+x)\hat{z}$$

Curl of F_3

$$= \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ yz & zx & xy \end{vmatrix}$$

$$= \left[\frac{\partial}{\partial y}(xy) - \frac{\partial}{\partial z}(zx)\right]\hat{x} - \left[\frac{\partial}{\partial x}(xy) - \frac{\partial}{\partial z}(yz)\right]\hat{y} + \left[\frac{\partial}{\partial x}(zx) - \frac{\partial}{\partial y}(yz)\right]\hat{z}$$

$$= (x - x)\hat{x} - (y - y)\hat{y} + (z - z)\hat{z} = 0.$$

$$\text{Div}(F_3),\ \vec{\nabla} \cdot F_3 = \frac{\partial}{\partial x}(yz) + \frac{\partial}{\partial y}(xz) + \frac{\partial}{\partial z}(xy).$$

$$= 0$$

4. For time varying fields: Find which if the following equations are not satisfy Maxwell's Equation. Also state why the expression/s don't satisfy Maxwell's Equation? (Show Calculation)
   a. $\nabla \cdot J + \frac{\partial \rho_v}{\partial t} = 0$
   b. $\nabla \cdot D = \rho_V$
   c. $\nabla \cdot E = -\frac{\partial B}{\partial t}$
   d. $\oint H \cdot dl = \int (\sigma E + \varepsilon \frac{\partial E}{\partial t}) \cdot dS$
   e. $\oint B \cdot dS = 0$

Answer to question No:4.

Here, Equation (ii) with (b) is: $\nabla \cdot D = \rho_v$.

which is a differential form of Gauss' electric law. So, we can say this a proved equation for Electromagnetism according to Maxwell's equation.

And, equation (e) is $\oint B \cdot ds = 0$.

But, as per Gauss' magnetic law $\oint \vec{B} \cdot \vec{ds} = 0$ ~~in~~ [integral form].

And, according to ampere's law,

$$\oint B \cdot \vec{ds} = \mu_0 \epsilon_0 \cdot \frac{\partial \phi_E}{\partial t} + \mu_0 I.$$

So, equation (e) is not a possible equation of electromagnetism according to Maxwell's equation.

Provided equation (c) is $\nabla \cdot \vec{E} = -\frac{\partial B}{\partial t}$

This equation also doesn't a possible equation of electromagnetism as the differentia form of Maxwell-Farady's equation is $\nabla \times \vec{E} = -\frac{\partial B}{\partial t}$.

Equation (d) is $\oint H \cdot dl = \int (\sigma E + \epsilon \cdot \frac{\partial E}{\partial t}) \cdot ds$

But, as per maxwell's equation: $\oint H \cdot dl = \int J \cdot ds + \frac{\partial}{\partial t} \int D \cdot ds$.

where, J = current density & D = displacement flux density.

As, $\epsilon \cdot \frac{\partial E}{\partial t} \cdot ds \neq \frac{\partial}{\partial t} \int D \cdot ds$, So, this also not a possible equation of electromagnetism.

And, finally equation (a) is $\nabla \cdot J + \frac{\partial \rho_v}{\partial t} = 0$

As per electromagnetic theory, the continuity equation is an emperical law expressing (local) charge conservation. Mathematically, it is an automatic consequence of Maxwell's equations.

5. In free space $E = 20Cos(\omega t - 50x)$ a_y V/m, Find:
   a. J_d
   b. H
   c. ω

Answer to question No: 5.

(a) $J_d = \frac{\partial D}{\partial t} = \epsilon_0 \frac{\partial}{\partial t} E = -20\,\epsilon_0\,\omega \sin(\omega t - 50x)\,a_y$ A/m

(b) $\nabla \times H = J + J_d = J_d$.

$\nabla \times H = \left(\frac{\partial}{\partial y} H_z - \frac{\partial}{\partial z} H_y\right) a_x + \left(\frac{\partial}{\partial z} H_x - \frac{\partial}{\partial x} H_z\right) a_y$
$+ \left(\frac{\partial}{\partial x} H_y - \partial/\partial y\, H_x\right) a_z$

$\Rightarrow \left(\frac{\partial}{\partial z} H_x - \frac{\partial}{\partial x} H_z\right) = -20\epsilon_0 \omega \sin(\omega t - 50x)$.

$\Rightarrow -\frac{\partial}{\partial x} H_z = -20\epsilon_0\,\omega \sin(\omega t - 50x)$.

$\Rightarrow H_z = 20\epsilon_0\,\omega \frac{\cos(\omega t - 50x)}{50} = 0.4\,\epsilon_0 \omega \cos(\omega t - 50x)$

$\therefore H = 0.4\,\epsilon_0\,\omega \cos(\omega t - 50x)\,a_z$.

(c) ~~we~~ As per law,

$\omega = \frac{E}{\hbar} = \frac{20}{6.626\times10^{-34}}$ $[\hbar = 6.626\times10^{-34}]$

$= 3.018\times10^{34}$.

$\therefore \omega = 3.018\times10^{34} \cos(\omega t - 50x)\,a_y$. V/m.