NAME : ASIF RAHMAN

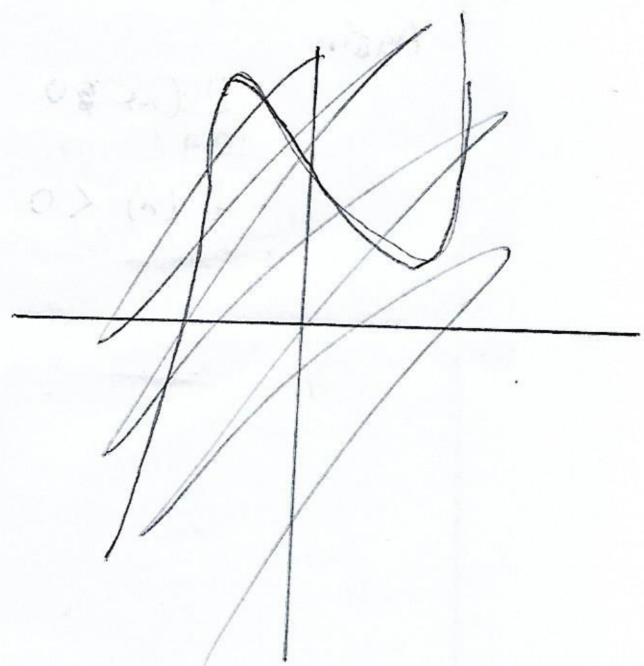
ID : 2012691642

COURSE : MAT120

Answer to the Question no-1

Given function,

$$f(x) = x^3 - 3x + 3$$



first derivative,

$$f'(x) = 3x^2 - 3$$

Second derivative,

$$f''(x) = 6x$$

Let $f''(x) = 0$

or, $6x = 0$

$\therefore x = 0$

For critical value $f'(x) = 0$,

So, $f'(x) = 0$

or, $3x^2 - 3 = 0$

$\therefore x = \pm 1$

Critical points are

$x = -1, +1$

$(-\infty, \alpha) = (-\alpha, -1] \cup [-1, 1] \cup [1, \alpha)$

$(-\alpha, 1]$: $f'(0) = -3 < 0$. So, $f(x)$ is decreasing

~~at~~ over interval $(-\alpha, -1]$

$[-1, 0]$: $f'(-0.5) = -2.25 < 0$. Here $f(x)$ is decreasing over interval $[-1, 0]$

$[0, 1]$: $f'(0.5) = -2.25 < 0$. Here $f(x)$ is decreasing over interval $[1, 0]$

So, $f(x)$ is decreasing over the whole interval $(-\alpha, \alpha)$

Inflection point

$f''(x) = 6x$

We know, for inflection point $f''(x) = 0$.

Here $f''(x)$ is defined everywhere.

$f''(x) = 0$

$6x = 0$

$\therefore x = 0$

Inflection point is at $x = 0$

$(-\alpha, \alpha) = (-\alpha, 0] \cup [0, \infty)$

$(-\alpha, 0]$: $x = -1$; $f''(-1) < 0$. Hence the graph is concave down in the interval $[0, \pi]$

$(0, \alpha)$: $x = 1$; $f''(1) > 0$. Hence the graph is concave up in the interval.

Maxima & Minima

$f'(x) = x^3 - 3x + 3$ | $f''(x) = 6x$

$f'(x) = 3x^2 - 3$

For critical value $f'(x) = 3x^2 - 3 = 0$

$\Rightarrow 3x^2 = 3$

$\therefore x = \pm 1$

i.e. $x = +1$
$x = -1$

Here, at $x = -1$,

$$f''(-1) = -6 < 0$$

So, local maxima exists at $x = -1$

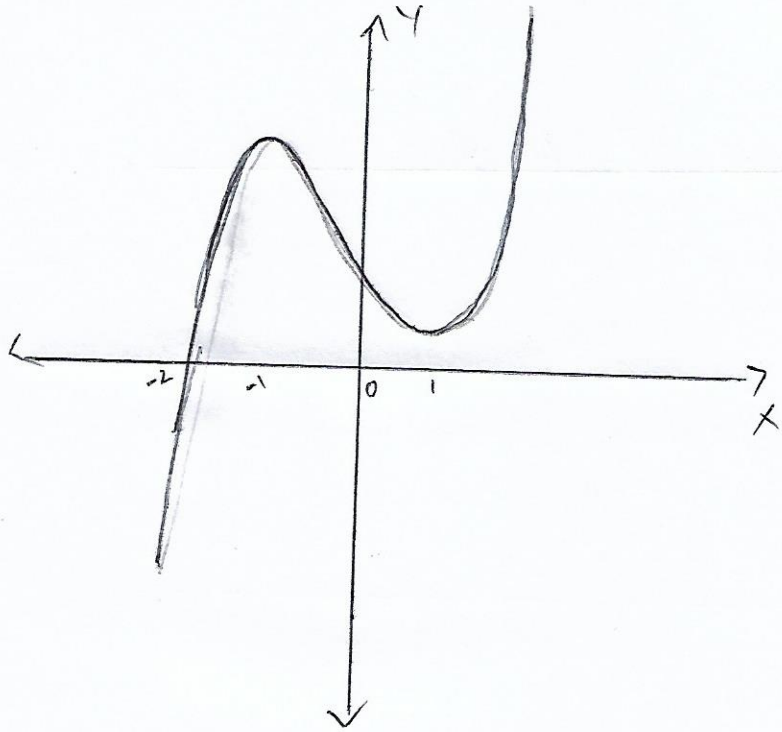
and the value is $f(-1) = 5$

At $x = 1$

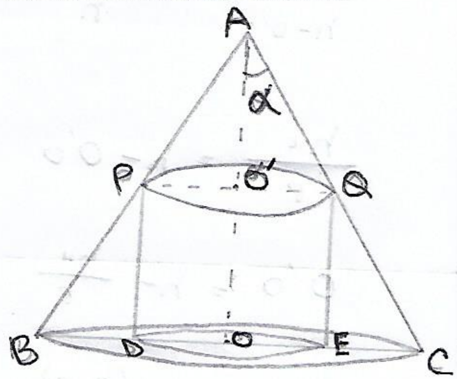
$$f''(1) = 6 > 0$$

So, local minima exists at $x = 1$ and the

value is $f(1) = 1$


Y
-2
-1
0
1
X

Answer to the Question no-2



Let $OC = r$ be the radius of cone and OA is the height.

$\angle OAQ = \alpha$ is the semi-vertical angle of cone.

Let $OE = x$ be the radius of cylinder.

Height of cylinder $= OO'$

In $\Delta QAO'Q$,

$$\tan\alpha = \frac{O'Q}{AO'}$$

$$= \frac{OE}{OA - O'O}$$

$$= \frac{x}{h - O'O} \quad \dots\dots (i)$$

In ΔAOC,

$$\tan\alpha = \frac{OC}{OA}$$

$$= \frac{r}{h} \quad \text{——} \quad (ii)$$

From equation ① & ②,

$$\frac{x}{h - O'O} = \frac{r}{h}$$

$$\frac{hx}{r} = h - O'O$$

$$O'O = h - \frac{hx}{r}$$

$$O'O = \frac{h(r-x)}{r}$$

Now,

Curved surface Area of cylinder = $2\pi \times radius \times height$

$$\Rightarrow S = 2\pi \times x \times O'O$$

$$= 2\pi x \frac{h(r-x)}{r} \quad [\text{replacing } O'O]$$

$$= \frac{2\pi h}{r}(rx - x^2)$$

$$= k(rx - x^2) \quad \left[k = \frac{2\pi h}{r} \text{ ; which is constant}\right]$$

So,

$$S'(x) = \frac{d}{dx}(k(rx - x^2))$$

$$= \frac{d}{dx}(rx - x^2) \cdot k$$

$$= k\ (r - 2x)$$

Putting $S' = 0$

$$0 = k(r - 2x)$$

$$r - 2x = 0$$

$$\therefore x = \frac{r}{2}$$

Now,

$S''(x)$ at $x = \frac{r}{2}$,

$$S'' = \frac{d}{dx}(k(r - 2x))$$

$$= k \frac{d}{dx}\underline{(r-2x)}$$

$$= k(0-2)$$

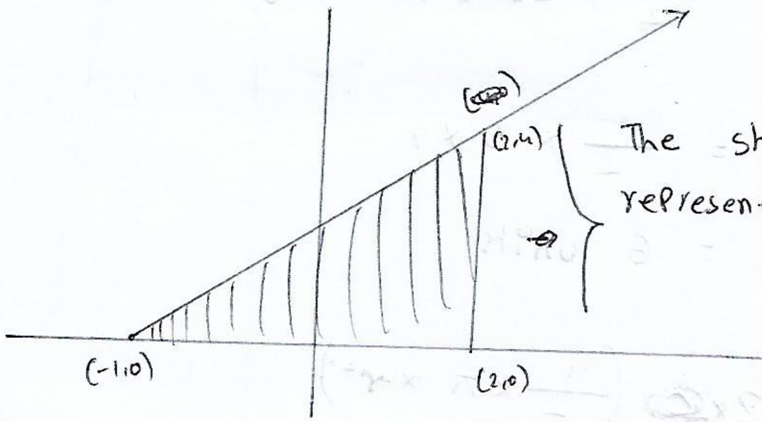
$$= -2k$$

So, $S'' = -2k$

Hence, at $x = \frac{r}{2}$ which is the maxima of s (Proved)

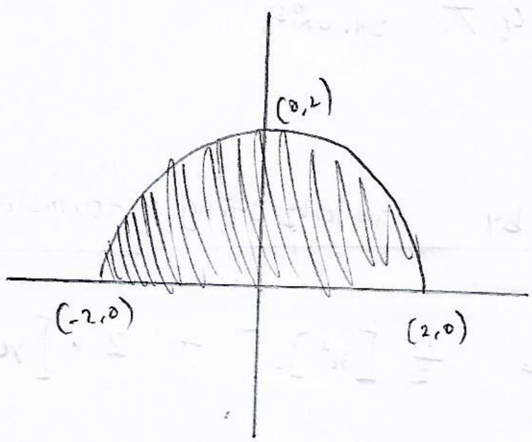
Answer to the Question no- 4

a) Answer:



The shaded area represents $\int_{-1}^{2} (x+2)\, dx$

b) Answer:



the shaded area represents $\int_{-2}^{2} \sqrt{4-x^2}\, dx$

Interogation by geometric formula

a) $\frac{1}{2} \times \text{base} \times \text{height}$

$= \frac{1}{2} \times 3 \times 4$

$= 6$ units

b) $2 \times \left(\frac{1}{2} \times \pi \times r^2\right)$

$= 4\pi$ sq. units

Interogation by interogating formula

a) $\int_{-1}^{2} (x+2)\, dx = \frac{1}{2}\left[x^2\right]_{-1}^{2} + 2 \times \left[x\right]_{-1}^{2}$

$= 6$ sq. units

b) $2 \times \int_{-2}^{2} \sqrt{4-x^2}\, dx = 2 \cdot 2\pi$

$= 4\pi$ sq. units

Answer to the Question no- 5

Given $v(t) = t^2 - 2t$ m/s and interval $0 \le t \le 4$

i) Answer:

$$\text{Displacement} = \int_0^4 v(t)\,dt$$

$$= \int_0^4 (t^2 - 2t)\,dt$$

$$= \left[\frac{t^3}{3} - t^2\right]_0^4$$

$$= 5.33$$

So, the particle will be at same position as it was at $t = 0$ and will be at a displacement of 5.33 at $t = 4$

ii) Answer:

The velocity can be written as $v(t) = t^2 - 2t$

Or, $v(t) = t(t-2)$

$$= \int_0^4 [v(t)]\, dt$$

$$= \int_0^2 -v(t)\, dt + \int_2^4 v(t)\, dt$$

$$= \int_0^2 -(t^2 - 2t)\, dt + \int_2^4 (t^2 - 2t)\, dt$$

$$= -\left[\frac{3}{3} - (+2)\right]_0^2 + \left[\frac{3}{3} - (+2)\right]_2^4$$

$$= \frac{4}{3} + \left[\frac{16}{3} + \frac{4}{3}\right]$$

$$= \frac{4}{3} + \frac{20}{3}$$

$$= 8\ m$$

So, the distance travelled is 8m.