Ans 1(a)

$\dot{x}_1 = u_1$

or $\int \dot{x}_1 \, dt = \int u_1 dt + c$

or $x_1 = u_1$

$\dot{x}_2 = u_2$

or $\int \dot{x}_2 \, dt = \int u_2 dt + c$

or $x_2 =$

$\dot{x}_3 = x_2 u_1$

or $\int \dot{x}_3 \, dt = \int x_2 u_1 dt + c$

$=$

Expressing u_1, x_2 & u_2 in x we get,

$u_1 = \dot{x}_1$, $x_2 = \frac{\dot{x}_3}{u_1}$, $u_2 = \dot{x}_2$

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Step-01: State Variable

$x_1(t) = a_{10} + a_{11}t + a_{12}t^2 + a_{13}t^3$

$x_1(t) = a_{10} + a_{11}t + a_{12}t^2 + a_{13}t^3$

Step-02: Setup Equation

Initial Condition

$x_1(0) = a_{10} = 1$

$x_2(0) = a_{20} = 1$

$x_3(0) = a_{30} = \beta$

$\dot{x}_1(0) = a_{11} = 1$

Final Condition

$x_1(T) = a_{10} + a_{11}T + a_{12}T^2 + a_{13}T^3 = 1$

$x_2(T) = a_{20} + a_{21}T + a_{22}T^2 + a_{23}T^3 = 1$

$x_3(T) = a_{30} + a_{31}T + a_{32}T^2 + a_{33}T^3 = 1$

$\dot{x}_2(T) = a_{21} + 2a_{22}T + 3a_{23}T^2 = 1$

Step-03: Construct & Solve Matrix Equation

For $x_1(t)$:

$a_{12} + a_{13}T^3 = 1$ $\qquad\qquad a_{10} = 1$

$\qquad\qquad\qquad\qquad\qquad a_{11} = 1$

$\therefore a_{12} = 0$ & $a_{13} = 0$

For $x_2(t)$:

$a_{20} = 1$

$a_{21}T + a_{22}T^2 + a_{23}T^3 = 0$ [From $x_2(T) = 1$]

$a_{21} + 2a_{22}T + 3a_{23}T^2 = 0$ [From $\dot{x}_2(T) = 1$]

$$\begin{bmatrix} T & T^2 & T^3 \\ 1 & 2T & 3T^2 \end{bmatrix} \begin{pmatrix} a_{21} \\ a_{22} \\ a_{23} \end{pmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$ Matrix

For $x_3(t)$:

$a_{30} = 1$

$a_{31}T + a_{32}T^2 + a_{33}T^3 = 6$ [From $x_3(T) = 1$]

This simplifies to

$a_{31} = 0,\ a_{32} = 0,\ a_{33} = 0$

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Answer

Step-01: Formulate the Problem

$$x_1(t) = a_{10} + a_{11}\psi_2(t) + a_{12}\psi_3(t) + a_{13}\psi_4(t) + a_{14}\psi_5(t) + a_{15}\psi_6(t)$$

where a_{ij} are the coefficients for $\psi_j(t)$ and $i = 1, 2, 3$.

Step-02: Setting up the Equation

For $x_1(t)$:

At $t = 0$

follow later format

$$x_1(0) = a_{10} + a_{11}\psi_2(0) + a_{12}\psi_3(T) + a_{13}\psi_4(T) + a_{14}\psi_5(T) + a_{15}\psi_6(T)$$

For given condition in Ques

$1 = a_{10} + 0 + 0 + 0 + 0 + 0$

or, $a_{10} = 1$

At $t = T$,

$$x_1(T) = a_{10} + a_{11}\psi_2(T) + a_{12}\psi_3(T) + a_{13}\psi_4(T) + a_{14}\psi_5(T) + a_{15}\psi_6(T)$$

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or, $a_{10} + a_{11}T + a_{12}T^2 + a_{13}T^3 + a_{14}T^4 + a_{15}T^5 = 1$

For $x_2(t)$:

At $t=0$:

$x_2(0) = a_{20} + a_{21}\psi_2(0) + a_{22}\psi_3(0)$

$+ a_{23}\psi_4(0) + a_{24}\psi_5(0) +$

$+ a_{25}\psi_6 = a_{20}$

or, As Per Given Conditions,

$x_1(T) =$ write in full

$= 1$

At $t=T$:

$x_2(T) = a_{20} + a_{21}\psi_2(T)$

$+ a_{22}\psi_3(T) + a_{23}\psi_4(T)$

$+ a_{24}\psi_5(T) + a_{25}\psi_6(T).$

As per the conditions
write conditions

$a_{20} + a_{21}T + a_{22}T^2 + a_{23}T^3$

$+ a_{24}T^4 + a_{25}T^5 = 1$

$\dot{x}_2(T) = a_{21} + a_{22}2T + 3a_{23}T^2 + 4a_{24}T^3 + 5a_{25}T^4 = x_2(T) = 1$

For $r_3(t)$

$t=0$:

$r_3(0) = a_{30} + a_{31}\psi_2(0) + a_{32}\psi_3(0) + a_{33}\psi_4(0) + a_{34}\psi_5(0) + a_{35}\psi_6(0) = r_{30}$

or, $r_3 = a_{30} + 0 + 0 + 0 + 0 + 0$

or, $1 = a_{30}$

$\therefore a_{30} = 1$

Step-03: ~~Matrix~~ Linear Eqn & Matrix

① Linear Eqn:

② Matrix

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & T & T^2 & T^3 & T^4 & T^5 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & T & T^2 & T^3 & T^4 \\ 0 & 0 & 1 & 2T & 3T^2 & 4T^3 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & T & T^2 & T^3 & T^4 & T^5 \end{bmatrix} \begin{bmatrix} a_{11} \\ a_{11} \\ a_{12} \\ a_{13} \\ a_{14} \\ a_{15} \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

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2(a)

$$x(t) = \int v(t) \cos(\theta(t))\, dt$$

$$y(t) = \int v(t) \cdot \sin(\theta(t))\, dt$$

Given,

$$\dot{x}(t) = v(t) \cos\theta(t)$$

$$\dot{y}(t) = v(t) \sin(\theta(t))$$

$$\dot{v}(t) = a(t)$$

$$\dot{\theta}(t) = \omega(t)$$

Step-01:

Given

$$\dot{x}(t) = v(t) \cos(\theta(t))$$

$$\text{or, } \int \dot{x}(t)\, dt = \int v(t) \cos(\theta(t))\, dt$$

$$\text{or, } x(t)\, dt = \int v(t) \cos(\theta(t))\, dt$$

$$\dot{y}(t) = v(t) \sin\theta(t)$$

$$\text{or, } \int \dot{y}(t) = \int v(t) \sin(\theta(t))\ldots$$

$$\text{or, } y(t) = v(t) \sin(\theta(t))$$

Step-02:

$$\dot{x}(t) = \frac{dx}{dt} = \frac{d}{dt} \int \{v(t) \cos(\theta(t))\} dt$$

$$= v(t) \cos(\theta(t))$$

$$\text{or, } v(t) = \frac{\dot{x}(t)}{\cos(\theta(t))} \quad (1)$$

$$\dot{y}(t) = \frac{dy}{dt} = \frac{d}{dt} \int v(t) \sin(\theta\ldots$$

$$= v(t) \sin(\theta(t))$$

$$\text{or, } v(t) = \frac{\dot{y}(t)}{\sin(\theta(t))}$$

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Step 03: From (i) & (ii),

$$\frac{\dot{x}(t)}{\cos(\theta(t))} = \frac{\dot{y}(t)}{\sin(\theta(t))}$$

or, $\tan(\theta(t)) = \frac{\dot{y}(t)}{\dot{x}(t)}$

or, $\theta(t) = \tan^{-1}\left(\frac{\dot{y}(t)}{\dot{x}(t)}\right)$

Step-04:

$$\dot{x}(t) = \frac{d}{dt}\int v(t)\cos(\theta(t))\,dt$$

$$= \frac{d}{dt}\int \frac{\dot{y}(t)}{\sin(\theta(t))}\cos(\theta(t))\,dt$$

$$= \frac{d}{dt}\int\left(\frac{\dot{y}(t)}{1+\left(\frac{\dot{y}(t)}{\dot{x}(t)}\right)^2}\right)dt$$

$$= \frac{d}{dt}\int\sqrt{\dot{x}^2(t)+\dot{y}(t)}\,dt$$

$$= \frac{\dot{x}(t)\ddot{x}(t)+\dot{y}(t)\ddot{y}(t)}{\sqrt{(\dot{x})^2+\dot{y}^2(t)}}$$

$$\dot{y}(t) = \frac{d}{dt}\int(v(t)\sin(\theta(t))\,dt$$

$$= \frac{d}{dt}\int\left(\frac{\dot{y}(t)\sin(\theta(t))}{\sin(\theta(t))}\right)dt$$

$$= \frac{d}{dt}\int\dot{y}(t)\,dt$$

$$= \dot{y}(t)$$

So, $\dot{x}(t)$ & $\dot{y}(t)$ are expressed sole in terms of $x(t)$ and $y(t)$

~~Sub~~ In pythagoras theorem

$$v(t) = \sqrt{\dot{x}^2(t) + \dot{y}^2(t)}$$

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2b

Step-01: Define the Trajectory Polynomials

$x(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3$

$y(t) = b_0 + b_1 t + b_2 t^2 + b_3 t^3$

$V(t) = c_0 + c_1 t$

$\theta(t) = d_0 + d_1 t$

Step-02: Apply Initial & Final Conditions

① For $x(t)$ and $y(t)$:

$x(0)=0$	$y(0)=0$,
$x(15)=5$	$y(15)=5$
$x'(0)=0$	$y'(0)=0$
$x'(15)=0$	$y'(15)=0$

ⅱ For $V(t)$ & $\theta(t)$:

$V(0) = 0.5$, $V(15) = 0.5$

$\theta(0) = -\frac{\pi}{2}$, $\theta(15) = -\frac{\pi}{2}$

Step-03: Solve the coefficient for Matrix

Formulating the Matrix for $x(t)$:

① $a_0 + a_1 \cdot 0 + a_2 \cdot (0)^2 + a_3 (0)^3 = 0$

or, $a_0 = 0$

② $a_0 + a_1 \cdot 15 + a_2 (15)^2 + a_3 (15)^3 = 5$

or, $0 + 15 a_1 +$ $= 5$

or,

(3) $a_1 + 2a_2 0 + 3a_3 0 = 0$

$\Rightarrow a_1 = 0$

(4) $a_1 + 2a_2(15) + 3a_3(15)^2 = 0$

or $a_1 + 30a_2 +$

~~we~~ From the system we can conclude to Matrix.

$255a_2 + 3375 = 5$

$30a_2 + 675a_3 = 0$

$$\begin{bmatrix} 225 & 3375 \\ 30 & 675 \end{bmatrix} \begin{bmatrix} a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \end{bmatrix}$$

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2(a)

Given,

$$\begin{pmatrix} \ddot{x}(t) \\ \ddot{y}(t) \end{pmatrix} = \begin{bmatrix} \cos\theta(t) & -v(t)\sin\theta(t) \\ \sin\theta(t) & v(t)\cos\theta(t) \end{bmatrix} \begin{bmatrix} a(t) \\ \omega(t) \end{bmatrix}$$

or, $$\begin{pmatrix} a(t) \\ \omega(t) \end{pmatrix} = \begin{bmatrix} \cos\theta_d & -v_d(t)\sin\theta_d(t) \\ \sin\theta_d & v_d(t)\cos\theta_d(t) \end{bmatrix} \begin{bmatrix} \ddot{x}_d(t) \\ \ddot{y}_d(t) \end{bmatrix}$$