Bernoulli distribution: a single trial is conducted which takes a binary outcome. The probability mass function is

$$P(X=x) = P^{x}(1-P)^{1-x} \qquad x=0,1$$

Expectation: E(x) = p

Variance: V(x) = p(1-p)

Ex: if you toss a coin one times. For example, Random variable X indicates the no of head.

Binomial distribution: n trails (n>1) are conducted which takes a binary outcome. The probability mass function is

$$P(X=x) = \binom{n}{x}P^{x}(1-P)^{n-x} \qquad x=0,1,2,..,n$$

Expectation: E(x) = np

Variance: V(x) = np(1-p)

Ex: if you toss a coin five times. For example, Random variable X indicates the no of tail.   n = 5 x = 0,1,2,3,4,5

Example: Suppose a milk factory has 20 containers and there is a probability of 0.261 that a milk container is underweight. a) What is the distribution of the number of underweight containers in a box? b) Calculate expected number of underweight cartons in a box and also calculate its variance. c) Calculate the probability that a box contains exactly seven underweight containers and also d) calculate the probability that a box contains no more than three underweight containers. e) Calculate the probability that a box contain at least two underweight containers.

Solution: a) Binomial distribution

$$P(X=x) = \binom{20}{x}P^{x}(1-p)^{20-x} \qquad x=0,1,2,..,20$$

b) E(x) = np = 20*0.261 = 5.22, V(x) = np(1-p) = 20*0.261*(1-0.261) = 3.857

Standard deviation = $\sqrt{3.857}$ = 1.96

c) P(X=7) = $\binom{20}{7}0.261^7(1-0.261)^{20-7}$ = 0.125

d)

$$P(X \le 3) = P(X=0) + P(X=1) + P(X=2) + P(X=3)$$
$$= \binom{20}{0} \times 0.261^0 \times 0.739^{20} + \binom{20}{1} \times 0.261^1 \times 0.739^{19}$$
$$+ \binom{20}{2} \times 0.261^2 \times 0.739^{18} + \binom{20}{3} \times 0.261^3 \times 0.739^{17}$$
$$= 0.0024 + 0.0167 + 0.0559 + 0.1185 = 0.1935$$

e) P(X≥2) = P(2) + P(3) + …. + P(20) = ?

We know that , total probability = 1

P(0) + P(1) + P(2) + P(3) + …. + P(20) = 1

⇒ P(2) + P(3) + …. + P(20) = 1- P(0)-P(1)

⇒ P(2) + P(3) + …. + P(20) = 1-0.0024 – 0.0167 = .9809

Poisson distribution

The Poisson distribution is used when a random variable counts the number of events that occur in an time interval. For example, 1) the number of telephone calls received by an operator within a certain time limit. 2) The number of patients arriving in an emergency room between 10 and 11 pm.

The probability mass function is P(X=x) = $\frac{e^{-\lambda}\lambda^x}{x!}$ x = 0,1,2,3…..

Expectation: E(x) = λ

Variance: V(x) = λ

Difference between binomial distribution and Poisson distribution:

1) In binomial distribution, number of trials are fixed. In poisson distribution, number of trials are infinite.

2) In binomial distribution, Variance ≤ Mean. In Poisson distribution, Mean = Variance.

3) Ex of binomial distribution: Coin tossing experiment. Ex of Poisson distribution: The number of patients arriving in an emergency room between 10 and 11 pm.

Example: Suppose that the number of errors in a piece of software has a parameter $\lambda = 3$. This parameter immediately implies that the expected number of errors is three and that the variance in the number of errors is also equal to three.

a) What is distribution of the number of errors in a piece of software.

b) Calculate the probability that a piece of software has no errors.

c) Calculate the probability that there are three or more errors in a piece of software.

Solution: a) The number of errors in a piece of software follows passion distribution

$$P(X=x) = \frac{e^{-3}3^x}{x!} \qquad x = 0,1,2,3\ldots$$

b) $P(X = 0) = \frac{e^{-3}3^0}{0!} = 0.05$

c) $P(X=3) + P(X=4) + \ldots = ?$

We know,

$P(X=0) + P(X=1) + P(X=2) + P(X=3) + P(X=4) + \ldots = 1$

$\Rightarrow P(X=3) + P(X=4) + \ldots = 1 - P(X=0) - P(X=1) - P(X=2)$

$\Rightarrow$ P(X=3) + P(X=4) + …=1-$\frac{e^{-3}3^0}{0!}$ - $\frac{e^{-3}3^1}{1!}$-$\frac{e^{-3}3^2}{2!}$

$\Rightarrow$ P(X=3) + P(X=4) + …= 0.577