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Answer to question No: 1.

(a)
$$\frac{xy^3}{1+y^2} = \frac{8}{5}.$$

$$\Rightarrow 5xy^3 = 8+8y^2$$

$$\Rightarrow 5\frac{d}{dt}(xy^3) = \frac{d}{dt}(8+8y^2).$$

$$\Rightarrow 5\left[x\frac{d}{dt}y^3 + y^3\frac{dx}{dt}\right] = \frac{d}{dt}(8) + 8\left(\frac{d}{dt}y^2\right).$$

$$\Rightarrow 5\left[x(3y^2)\frac{dy}{dt} + y^3\frac{dx}{dt}\right] = 0 + 8(2y)\frac{dy}{dt}$$

$$\Rightarrow 15xy^2\frac{dy}{dt} + 5y^3\frac{dx}{dt} = 16y.\frac{dy}{dt}.$$

$$\Rightarrow 15xy\frac{dy}{dt} + 5y^2\frac{dx}{dt} = 16\frac{dy}{dt}.$$

Here,
$$\frac{dx}{dt} = 6 \text{ unit/sec} \quad ; x = 1, y = 2.$$

$$\therefore 15\times1\times2\frac{dy}{dt} + 5(2)^2 6 = 16\times\frac{dy}{dt}.$$

$$\Rightarrow 3.\frac{dy}{dt} - 16\frac{dy}{dt} = -120.$$

$$\Rightarrow 14\frac{dy}{dt} = -120.$$

$$\Rightarrow \frac{dy}{dt} = -\frac{120}{14} = -\frac{60}{7}$$

(b) Particle is fallin at that instant.

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Answer to question NO: 2.

Given, $y = x^3 - 3x + 2$ ...... ①

Here, ~~x intercepts:~~

$x^3 - 3x + 2 = (x+2)(x-1)^2$

$\therefore$ x intercepts $x = -2, x = 1$.

At ①, for $x = 0$,

$y = 2$, which is y intercept.

Here,

$$\lim_{x \to +\infty} (x^3 - 3x + 2) = \lim_{x \to +\infty} x^3 = +\infty$$

$$\lim_{x \to -\infty} (x^3 - 3x + 2) = \lim_{x \to -\infty} x^3 = -\infty.$$

So, graph increases without bound since $x \to +\infty$.

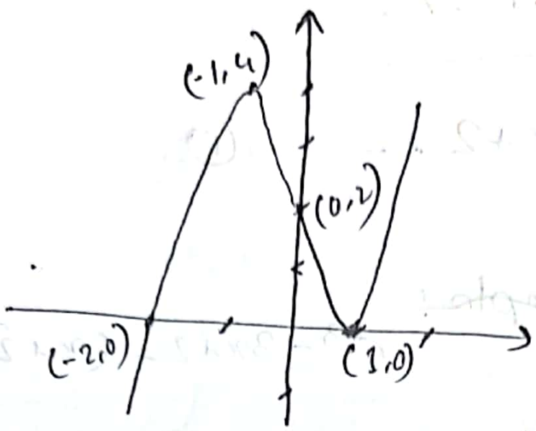
And, graph decreas without bound since $x \to -\infty$.

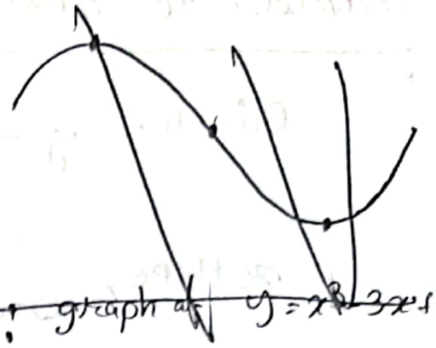
Again,

$$\frac{dy}{dx} = 3x^2 - 3 = 3(x^2 - 1^2)$$

$$= 3(x+1)(x-1).$$

$$\therefore \frac{d^2y}{dx^2} = \frac{d}{dx}(3x^2 - 3) = 6x.$$





~~fig: graph of $y = x^3 - 3x + 2$~~

As, $\frac{dy}{dx}$ changes from + to - at $x = -1$, So relative maximum is at that point.

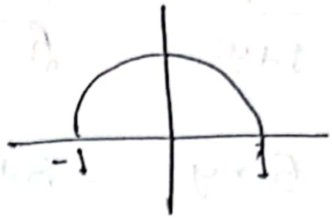
And, as it changes from - to + at $x = 1$, So, relative minimum would be there.

Again, sign of $\frac{d^2y}{dx^2}$ changes from - to + at $x = 0$, So, inflection point is there.

Answer to question NO: 3.

(a) $f(x) = \sqrt{1-x^2} \quad [-1, 1]$.

It is a semi circle of $r = 1$.



$\therefore$ Area of R $= \frac{1}{2} \pi r^2$

$= \frac{1}{2} \pi (1)^2 = \frac{\pi}{2}$

(b) Here,

$\Delta x = \frac{b-a}{6} = \frac{1}{3}$.

$\therefore$ end ~~points~~ points are $-1, -2/3, -1/3, 0, 1/3, 2/3, 1$.

Area of $R = \sum f(x_i) \Delta x$.

$= \frac{1}{2} \left[f(-1) + f(-2/3) + f(-1/3) + f(0) + f(1/3) + f(2/3) \right]$.

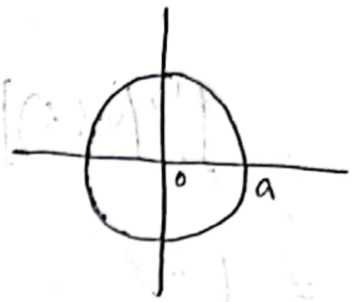
$= \frac{1}{3} \left(0 + \frac{\sqrt{5}}{3} + \frac{2\sqrt{2}}{3} + \frac{2\sqrt{2}}{3} + \frac{\sqrt{9}}{3} + \frac{1}{3} \right)$

$= 1.45877669388$.

$\therefore$ Error $= 0.1120196377$

Answer to question NO:4.

Given, equation of circle:



$$x^2+y^2=a^2$$

$$\Rightarrow y^2 = a^2-x^2.$$

$$\therefore y = \sqrt{a^2-x^2}$$

Here, total area of the circle would be the area covered by the circle in firsth quadrant multiplied by 4 times.

So,

$$\text{Area} = 4\int_0^a y\,dx$$

$$= 4\int_0^a \sqrt{a^2-x^2}\,dx$$

$$= 4\left[\frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}x/a\right]_0^a$$

$$= 4\left[0+\frac{a^2}{2}\sin^{-1}\frac{a}{a} - \left(0+\frac{a^2}{2}\sin^{-1}\frac{0}{a}\right)\right]$$

$$= 4\left[\frac{a^2}{2}\sin^{-1}1 - 0\right].$$

$$= 2a^2.\frac{\pi}{2} = \pi a^2 \text{ unit}^2.$$

Answer to question NO! 5.

Here,

$v(t) = t^2 - 4t$ m/s & interval $0 \le t \le 10$.

(1)

Here,

$$\text{Displacement} = \int_0^{10} v(t)\,dt.$$

$$= \int_0^{10} (t^2 - 4t)\,dt.$$

$$= \left[\frac{t^3}{3} - 4 \cdot \frac{t^2}{2}\right]_0^{10}$$

$$= \left[\frac{10^3}{3} - 2(10)^2\right]$$

$$= \cancel{233.33}\ 133.33$$

So, at $t=0$ particle would be at initial position & at $t=10$ particle would be at a displacement of ~~233.33m~~ 133.33

(11) Velocity can be written as $v(t) = t^2 - 4t$. or $v(t) = t(t-4)$.

$t=0$; $t=4$.

$\cancel{\int_0^{10} [v(t)] dt}$

$\cancel{= \int_0^2}$

$$S_1 = \int_0^4 (t^2 - 4t)\,dt$$

$$= \left[\frac{t^3}{3} - 4\frac{t^2}{2}\right]_0^4 = -\frac{32}{3} \text{ m}$$

$$S_2 = \int_4^{10} (t^2 - 4t)\,dt = \left[t^3/3 - 4\,t^2/2\right]_4^{10}$$

$= 144$ m.

$\therefore$ Distance $= -\left(\frac{-32}{3}\right) + 144 = \frac{32}{3} + 144$.

$= 154.67$ m.