North South University

CSE 495: Introduction to Robotics

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Section: 1

Home Work 3

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Answer to question No-1

Given formula: $G(i,j) = \sum_{u=0}^{K-1} \sum_{v=0}^{l-1} F(u,v).\bar{I}(i+u, j+v)$

$$I = \begin{bmatrix} 7 & 4 & 1 \\ 8 & 5 & 2 \\ 9 & 6 & 3 \end{bmatrix}, \text{ with } \bar{I} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 7 & 4 & 1 & 0 \\ 0 & 8 & 5 & 2 & 0 \\ 0 & 9 & 6 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(a) $F = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

Now, $G(1,1) = F(0,0).\bar{I}(1,1) + F(0,1)\bar{I}(1,2) + F(0,2)\bar{I}(1,3) + F(1,0).\bar{I}(2,1) + \cancel{0} F(1,1)\bar{I}(2,2) + F(1,2)\bar{I}(2,3) + F(2,0).\bar{I}(3,1) + F(2,1)\bar{I}(3,2) + F(2,2)\bar{I}(3,3)$

$= 0.0 + 0.0 + 0.0 + 0.0 + 1.7 + 0.4 + 0.0 + 0.8 + 0.5$

$= 7$

$G(1,2) = F(0,0)\bar{I}(1,2) + F(0,1)\bar{I}(1,3) + F(0,2)\bar{I}(1,4) + F(1,0)\bar{I}(2,2) + F(1,1)\bar{I}(2,3) + F(1,2)\bar{I}(2,4) + F(2,0)\bar{I}(3,2) + F(2,1)\bar{I}(3,3) + F(2,2)\bar{I}(3,4)$

$= 0.0 + 0.0 + 0.0 + 0.7 + 1.4 + 0.1 + 0.8 + 0.5 + 0.2$

$= 4$

Instead of F_{11}, we get 0 for all other elements.

So, we can consider $F(1,1)$ only.

$G(1,3) = F(1,1)\,\bar{I}(2,4) = 1.1 = 1$

$G(2,1) = F(1,1)\,\bar{I}(3,2) = 1.8 = 8$

$G(2,2) = F(1,1)\,\bar{I}(3,3) = 1.5 = 5$

$G(2,3) = F(1,1)\,\bar{I}(3,4) = 1.2 = 2$

$G(3,1) = F(1,1)\,\bar{I}(4,2) = 1.9 = 9$

$G(3,2) = F(1,1)\,\bar{I}(4,3) = 1.6 = 6$

$G(3,3) = F(1,1)\,\bar{I}(4,4) = 1.3 = 3$

$$\therefore G = \begin{bmatrix} 7 & 4 & 1 \\ 8 & 5 & 2 \\ 9 & 6 & 3 \end{bmatrix}$$

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(b)

$$F = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Instead of F_{00}, we get 0 for all other elements.

So, we can consider ~~F_{00}~~ $F(0,0)$ only.

Now $G(1,1) = F(0,0)\, I(1,1) = 1.0 = 0$

$G(1,2) = F(0,0)\, I(1,2) = 1.0 = 0$

$G(1,3) = F(0,0)\, I(1,3) = 1.0 = 0$

$G(2,1) = F(0,0)\, I(2,1) = 1.0 = 0$

$G(2,2) = F(0,0)\, I(2,2) = 1.7 = 7$

$G(2,3) = F(0,0)\, I(2,3) = 1.4 = 4$

$G(3,1) = F(0,0)\, I(3,1) = 1.0 = 0$

$G(3,2) = F(0,0)\, I(3,2) = 1.8 = 8$

$G(3,3) = F(0,0)\, I(3,3) = 1.5 = 5$

$$\therefore G = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 7 & 4 \\ 0 & 8 & 5 \end{bmatrix}$$

(c) $F = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ -1 & -1 & -1 \end{bmatrix}$

Here $F_{00}, F_{01}, F_{02}, F_{20}, F_{21}, F_{22}$ are containing non-zero value, so will consider them.

$G(1,1) = F(0,0).\bar{I}(1,1) + F(0,1)\bar{I}(1,2) + F(0,2).\bar{I}(1,3) +$
$F(2,0).\bar{I}(3,1) + F(2,1)\bar{I}(3,2) + F(2,2)\bar{I}(3,3)$

$= 1.0 + 1.0 + 1.0 + (-1).0 + (-1).8 + (-1).5$

$= -8 - 5 = -13$

$G(1,2) = F(0,0).\bar{I}(1,2) + F(0,1).\bar{I}(1,3) + F(0,2).\bar{I}(1,4) +$
$F(2,0).\bar{I}(3,2) + F(2,1).\bar{I}(3,3) + F(2,2).\bar{I}(3,4)$

$= 1.0 + 1.0 + 1.0 + (-1).8 + (-1).5 + (-1).2$

$= -8 - 5 - 2 = -15$

$G(1,3) = F(0,0)\bar{I}(1,3) + F(0,1)\bar{I}(1,4) + F(0,2)\bar{I}(1,5) +$
$F(2,0)\bar{I}(3,3) + F(2,1)\bar{I}(3,4) + F(2,2)\bar{I}(2,5)$

$= 1.0 + 1.0 + 1.0 + (-1).5 + (-1).2 + (-1).0$

$= -5 - 2 = -7$

$$G(2,1) = F(0,0)\bar{I}(2,1) + F(0,1)\bar{I}(2,2) + F(0,2)\bar{I}(2,3) + F(2,0)\bar{I}(4,1) + F(2,1)\bar{I}(4,2) + F(2,2)\bar{I}(4,3)$$

$$= 1\cdot 0 + 1\cdot 7 + 1\cdot 4 + (-1)\cdot 0 + (-1)\cdot 9 + (-1)\cdot 6$$

$$= 0 + 7 + 4 - 0 - 9 - 6 = -4$$

$$G(2,2) = F(0,0)\bar{I}(2,2) + F(0,1)\bar{I}(2,3) + F(0,2)\bar{I}(2,4) + F(2,0)\bar{I}(4,2) + F(2,1)\bar{I}(4,3) + F(2,2)\bar{I}(4,4)$$

$$= 1\cdot 7 + 1\cdot 4 + 1\cdot 1 + (-1)\cdot 9 + (-1)\cdot 6 + (-1)\cdot 3$$

$$= 7 + 4 + 1 - 9 - 6 - 3 = -6$$

$$G(2,3) = F(0,0)\bar{I}(2,3) + F(0,1)\bar{I}(2,4) + F(0,2)\bar{I}(2,5) + F(2,0)\bar{I}(4,3) + F(2,1)\bar{I}(4,4) + F(2,2)\bar{I}(4,5)$$

$$= 1\cdot 4 + 1\cdot 1 + 1\cdot 0 + (-1)\cdot 6 + (-1)\cdot 3 + (-1)\cdot 0$$

$$= 4 + 1 + 0 - 6 - 3 - 0 = -4$$

$$G(3,1) = F(0,0)\bar{I}(3,1) + F(0,1)\bar{I}(3,2) + F(0,2)\bar{I}(3,3) + F(2,0)\bar{I}(5,1) + F(2,1)\bar{I}(5,2) + F(2,2)\bar{I}(5,3)$$

$$= 1\cdot 0 + 1\cdot 8 + 1\cdot 5 + (-1)\cdot 0 + (-1)\cdot 0 + (-1)\cdot 0$$

$$= 8 + 5$$

$$= 13$$

$$G(3,2) = F(0,0)\bar{I}(3,2) + F(0,1)\bar{I}(3,3) + F(0,2)\bar{I}(3,4) + F(2,0)\bar{I}(5,2) + F(2,1)\bar{I}(5,3) + F(2,2)\bar{I}(5,4)$$

$$= 1.8 + 1.5 + 1.2 + (-1).0 + (-1).0 + (-1).0$$

$$= 8 + 5 + 2 = 15$$

$$G(3,3) = F(0,0)\bar{I}(3,3) + F(0,1)\bar{I}(3,4) + F(0,2)\bar{I}(3,5) + F(2,0)\bar{I}(5,3) + F(2,1)\bar{I}(5,4) + F(2,2)\bar{I}(5,5)$$

$$= 1.5 + 1.2 + 1.0 + (-1).0 + (-1).0 + (-1).0$$

$$= 5 + 2 = 7$$

$$G = \begin{bmatrix} -13 & -15 & -7 \\ -4 & -6 & -4 \\ 13 & 15 & 7 \end{bmatrix}$$

The filter in part (c) is a horizontal edge detector. It moves vertically across the image, seeking areas where the intensity changes from the left to right that means the positive response or from right to left that means the negative response.

(d) $F' = \begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix}$

Here $F'_{00}, F'_{02}, F'_{10}, F'_{12}, F'_{20}, F'_{22}$ are containing non-zero value, so, we will consider them.

$$G(1,1) = F'(0,0)\,\bar{I}(1,1) + F'(0,2)\,\bar{I}(1,3) + F'(1,0)\,\bar{I}(2,1) +$$
$$F'(1,2)\,\bar{I}(2,3) + F'(2,0)\,\bar{I}(3,1) + F'(2,2)\,\bar{I}(3,3)$$

$$= (-1).0 + 1.0 + (-1).0 + 1.4 + (-1).0 + 1.5 = 9$$

By applying the same formula we can get,

$G(1,2) = (-1).0 + 1.0 + (-1).7 + (-1).8 + 1.1 + 1.2 = -12$

$G(1,3) = (-1).0 + 1.0 + (-1).4 + 1.0 + (-1).5 + 1.0 = -9$

$G(2,1) = (-1).0 + 1.4 + (-1).0 + 1.5 + (-1).0 + 1.6 = 15$

$G(2,2) = (-1).7 + 1.1 + (-1).8 + 1.2 + (-1).0 + 1.3 = -18$

$G(2,3) = (-1).4 + 1.0 + (-1).5 + 1.0 + (-1).6 + 1.0 = -15$

$G(3,1) = (-1).0 + 1.5 + (-1).0 + 1.6 + (-1).0 + 1.0 = 11$

$G(3,2) = (-1).8 + 1.2 + (-1).0 + 1.3 + (-1).0 + 1.0 = -12$

$G(3,3) = (-1).5 + 1.0 + (-1).6 + 1.0 + (-1).0 + 1.0 = -11$

$$G = \begin{bmatrix} 9 & -12 & -9 \\ 15 & -18 & -15 \\ 11 & -12 & -11 \end{bmatrix}$$

The filters $F = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ -1 & -1 & -1 \end{bmatrix}$ and $F' = \begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix}$

have different functionalities in edge detection.

F' focuses on detecting vertical changes in the image. It responds strongly to vertical edges, where there is a transition form dark to light or light to dark in a vertical direction.

F is intended to detect horizontal edges in the image. It reacts strongly to horizontal edges, where there is a shift from dark to light or light to dark in horizontal direction.

F' focuses on veretical edges and ~~R~~ F focuses on horizontal edges.

(e)

$$F = \frac{1}{16}\begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$

$G(1,1) = F(0,0)\bar{I}(1,1) + F(0,1)\bar{I}(1,2) + F(0,2)\bar{I}(1,3) +$
$\cancel{F(1,0)\bar{I}(2,2) + F(1,2)\bar{I}(2,3) + F(1,3)\bar{I}(2,4) +}$
$\cancel{F(2} \; F(1,0)\bar{I}(2,1) + F(1,1)\bar{I}(2,2) + F(1,2)\bar{I}(2,3) +$
$F(2,0)\bar{I}(3,1) + F(2,1)\bar{I}(3,2) + F(2,2)\bar{I}(3,3)$

$= \frac{1}{16}(1.0 + 2.0 + 1.0 + 2.0 + 4.7 + 2.4 + 1.0 + 2.8 + 1.5)$

$= \frac{1}{16} \times 57 = 3.5$

By applying the same foremula we get,

$G(1,2) = \frac{1}{16}(1.0 + 2.0 + 1.0 + 2.7 + 4.4 + 2.1 + 1.8 + 2.5 + 1.2)$

$= \frac{1}{16} \times 52 = 3.25$

$G(1,3) = \frac{1}{16}(1.0 + 2.0 + 1.0 + 2.4 + 4.1 + 2.0 + 1.5 + 2.2 + 1.0)$

$= \frac{1}{16} \times 21 = 1.31$

$G(2,1) = \frac{1}{16}(1.0 + 2.7 + 1.4 + 2.0 + 4.8 + 2.5 + 1.0 + 2.9 + 1.6)$

$= \frac{1}{16} \times 84 = 5.25$

$$G(2,2) = \frac{1}{16}(1.7 + 2.4 + 1.1 + 2.8 + 4.5 + 2.2 + 1.9 + 2.6 + 1.3)$$

$$= \frac{1}{16} \times 80 = 5$$

$$G(2,3) = \frac{1}{16}(1.4 + 2.1 + 1.0 + 2.5 + 4.2 + 2.0 + 1.6 + 2.3 + 1.0)$$

$$= \frac{1}{16} \times 36 = 2.25$$

$$G(3,1) = \frac{1}{16}(1.0 + 2.8 + 1.5 + 2.0 + 4.9 + 2.6 + 1.0 + 2.0 + 1.0)$$

$$= \frac{1}{16} \times 69 = 4.31$$

$$G(3,2) = \frac{1}{16}(1.8 + 2.5 + 1.2 + 2.9 + 4.6 + 2.3 + 10 + 2.0 + 1.0)$$

$$= \frac{1}{16} \times 68 = 4.25$$

$$G(3,3) = \frac{1}{16}(1.5 + 2.2 + 1.0 + 2.6 + 4.3 + 2.0 + 1.0 + 2.0 + 1.0)$$

$$= \frac{1}{16} \times 33 = 2.06$$

$$G = \begin{bmatrix} 3.5 & 3.25 & 1.31 \\ 5.25 & 5 & 2.25 \\ 4.31 & 4.25 & 2.06 \end{bmatrix}$$

The filter F is a smoothing filter. It serves as a smoothing or blurring tool for the image. By averaging each pixel with its neighboring pixel, the filter smooth the fit image. This filter can reduce the high frequency noise and it can also create a less detailed appearanc The normalization factor $\frac{1}{16}$ ensures that the smoothing is done in a way that preserves the overall brightness of the image.

(f) $F = \frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

Now,

$G(1,1) = F(0,0)\bar{I}(1,1) + F(0,1)\bar{I}(1,2) + F(0,2)\bar{I}(1,3) +$

$F(1,0)\bar{I}(2,1) + F(1,1)\bar{I}(2,2) + F(1,2)\bar{I}(2,3) +$

$F(2,0)\bar{I}(3,1) + F(2,1)\bar{I}(3,2) + F(2,2)\bar{I}(3,3)$

$= \frac{1}{9}(1.0 + 1.0 + 1.0 + 1.0 + 1.7 + 1.9 + 1.0 + 1.8 + 1.5)$

$= \frac{1}{9} \times 24 = 2.67$

By applying the same formula we can get,

$G(1,2) = \frac{1}{9}(1.0+1.0+1.0+1.7+1.4+1.1+1.8+1.5+1.2)$

$= \frac{1}{9} \times 27 = 3$

$G(1,3) = \frac{1}{9}(1.0+1.0+1.0+1.4+1.1+1.0+1.5+1.2+1.0)$

$= \frac{1}{9} \times 12 = 1.33$

$G(2,1) = \frac{1}{9} \times (1.0+1.7+1.4+1.0+1.8+1.5+1.0+1.9+1.6)$

$= \frac{1}{9} \times 39 = 4.33$

$G(2,2) = \frac{1}{9}(1.7+1.4+1.1+1.8+1.5+1.2+1.9+1.6+1.3)$

$= \frac{1}{9} \times 45 = 5$

$G(2,3) = \frac{1}{9}(1.4+1.1+1.0+1.5+1.2+1.0+1.6+1.3+1.0)$

$= \frac{1}{9} \times 21 = 2.33$

$G(3,1) = \frac{1}{9}(1.0+1.8+1.5+1.0+1.9+1.6+1.0+1.0+1.0)$

$= \frac{1}{9} \times 28 = 3.11$

$G(3,2) = \frac{1}{9}(1.8+1.5+1.2+1.9+1.6+1.3+1.0+1.0+1.0)$

$= \frac{1}{9} \times 33 = 3.67$

$G(3,3) = \frac{1}{9}(1.5+1.2+1.0+1.6+1.3+1.0+1.0+1.0+1.0)$

$= \frac{1}{9} \times 16 = 1.77$

$$G = \begin{bmatrix} 2.67 & 3 & 1.33 \\ 4.33 & 5 & 2.33 \\ 3.11 & 3.67 & 1.77 \end{bmatrix}$$

The filter in part(e) is Gaussian smoothing filter and the filter in part (f) is Average moving filter.

Gaussian smoothing filter is primarily used for smoothing or blurring image. Average moving filter is used for basic smoothing through pixel arranging. It simplifies the smoothing and less focus on preserving details

Answer to question NO-2

$\boxed{a}$

According to formula, the correlation defined as,

$$G(i,j) = \sum_{u=0}^{k-1} \sum_{v=0}^{l-1} F(u,v) \cdot \bar{I}(i+u, j+v)$$

where, F is the filter, $\bar{I} \in \mathbb{R}^{(m+k-1)\times(n+l-1)}$ is the original image, I, padded with zeros along it's edges.

Now, we can define filter F as a vector representation of f,

$$f = \text{Vector}(F)$$

Also, we can represt $\bar{I}(i,j)$ as $t(i,j)$ which is the vector representation of neighborhood patch of images.

$$t(i,j) = \text{vector}(\bar{I}(i-1 : i+2, j-1 : j+2))$$

we have to show that, we can write correlation as a vector dot product.

$$G(i,j) = f^T t_{ij}$$

As we apply vectorization which turns a matrix into a single column, we have to apply dot product.

For performing that dot product we need to use the neighborhood patch a, the dot product of a row vector and column vector can be expresed as a multiplication