Exercise 7.4

37. $\displaystyle\int \frac{dx}{x^2-4x+5}$

$$= \int \frac{dx}{(x-2)^2+1}$$

Put, $u = x-2 \Rightarrow du = dx$ So, $\displaystyle\int \frac{du}{u^2+1} = \tan^{-1} u + C = \tan^{-1}(x+2) + C$

38. $\displaystyle\int \frac{dx}{\sqrt{2x-x^2}}$

$$= \int \frac{dx}{\sqrt{1-(x-1)^2}}$$

Put, $u = x-1 \Rightarrow du = dx$ So, $\displaystyle\int \frac{du}{\sqrt{1-u^2}} = \sin^{-1} u + C = \sin^{-1}(x-1) + C$

39. $\displaystyle\int \frac{dx}{\sqrt{3+2x-x^2}}$

$$= \int \frac{dx}{\sqrt{4-(x-1)^2}}$$

Put, $u = x-1 \Rightarrow du = dx$ So, $\displaystyle\int \frac{du}{\sqrt{2^2-u^2}} = \sin^{-1}\frac{u}{2} + C = \sin^{-1}\frac{x-1}{2} + C$

40. $\displaystyle\int \frac{dx}{16x^2+16x+5}$

$$= \int \frac{dx}{(4x+2)^2+1}$$

Put, $u = 4x+2 \Rightarrow du = 4dx$ So, $\displaystyle\frac{1}{4}\int \frac{du}{u^2+1} = \frac{1}{4}\tan^{-1} u + C$

$$= \frac{1}{4}\tan^{-1}(4x+2) + C$$

41. $\int \frac{dx}{\sqrt{x^2-6x+10}}$

$= \int \frac{dx}{\sqrt{(x-3)^2+1}} = \ln\left(x-3+\sqrt{(x-3)^2+1}\right)+C$

42. $\int \frac{x}{x^2+2x+2}\,dx$

$= \int \frac{x\,dx}{(x+1)^2+1}$

Put, $u = x+1 \Rightarrow du = dx$

So, $\int \frac{u-1}{u^2+1}\,du = \int \frac{u}{u^2+1}\,du - \int \frac{du}{u^2+1}$

$= \frac{1}{2}\ln(u^2+1) - \tan^{-1}u + c$

$= \frac{1}{2}\ln(x^2+2x+2) - \tan^{-1}(x+1) + C$

43. $\int \sqrt{3-2x-x^2}\,dx$

$= \int \sqrt{4-(x+1)^2}\,dx$

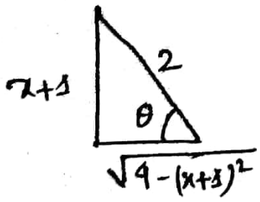
Put, $x+1 = 2\sin\theta \Rightarrow dx = 2\cos\theta\,d\theta$

$\int \sqrt{2^2-4\sin^2\theta} \times 2\cos\theta\,d\theta$

$= \int 4\cos^2\theta\,d\theta = 4 \times \frac{1}{2}\cos\theta\cdot\sin\theta + \frac{1}{2}\theta \times 4 + c$

$= \sin\theta\cdot\cos\theta + 2\theta + C$

$$= 2 \times \frac{\sqrt{4-(x+1)^2}}{2} \times \frac{x+1}{2} + 2\sin^{-1}\frac{x+1}{2} + C$$



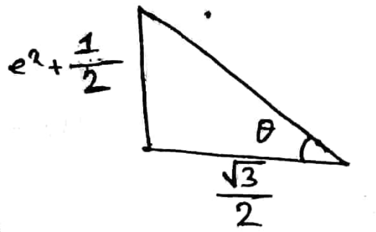
$$= \frac{1}{2}(x+1)\sqrt{3-2x-x^2} + 2\sin^{-1}\frac{x+1}{2} + C$$

44. $\displaystyle\int \frac{e^x}{\sqrt{1+e^x+e^{2x}}}\,dx$

$$= \int \frac{e^x\,dx}{\sqrt{\left(e^x+\frac{1}{2}\right)^2+\frac{3}{4}}}$$

Put, $e^x + \frac{1}{2} = \frac{\sqrt{3}}{2}\tan\theta \Rightarrow e^x\,dx = \frac{\sqrt{3}}{2}\sec^2\theta\,d\theta$

So, $\displaystyle\int \frac{\sqrt{3}}{2} \times \frac{\sec^2\theta\,d\theta}{\sqrt{\frac{3}{4}(\tan^2\theta+1)}}$



$$= \int \sec\theta\,d\theta = \ln|\sec\theta+\tan\theta| + C$$

$$= \ln\left|\frac{2\sqrt{e^{2x}+e^x+1}}{\sqrt{3}} + \frac{2e^x+1}{\sqrt{3}}\right| + C$$

45. $\displaystyle\int \frac{dx}{2x^2+4x+7}$

$$= \int \frac{dx}{2(x+1)^2+5}$$

$$= \frac{1}{2}\int \frac{dx}{(x+1)^2+5/2}$$

Put, $x+1 = u \Rightarrow du = dx$

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So, $\frac{1}{2}\int \frac{du}{u^2+5/2}$

$$= \frac{1}{2} \times \frac{\sqrt{2}}{\sqrt{5}} \tan^{-1} \frac{u}{\sqrt{\frac{5}{2}}} + C$$

$$= \frac{1}{\sqrt{10}} \tan^{-1} \frac{\sqrt{2}(x+1)}{\sqrt{5}} + C$$

96. $\int \frac{2x+3}{4x^2+4x+5} dx$

$$= \int \frac{2x+3}{4\left(x+\frac{1}{2}\right)^2+4} dx$$

Put, $u = x + \frac{1}{2}$ $\Rightarrow du = dx$

$$\int \frac{2u+2}{4u^2+4} du = \frac{1}{2}\int \frac{u}{u^2+1} du + \frac{1}{2}\int \frac{du}{u^2+1}$$

$$= \frac{1}{2} \times \frac{1}{2} \ln(u^2+1) + \frac{1}{2} \tan^{-1} u + C$$

$$= \frac{1}{4} \ln\left(x^2+x+\frac{5}{4}\right) + \frac{1}{2} \tan^{-1}\left(x+\frac{1}{2}\right) + C$$

97. $\int_1^2 \frac{dx}{\sqrt{4x-x^2}}$

$$= \int_1^2 \frac{dx}{\sqrt{4-(x-2)^2}} = \left[\sin^{-1} \frac{x-2}{2}\right]_1^2 = \frac{\pi}{6}$$

98. $\int_0^4 \sqrt{4x-x^2}\, dx$

$= \int_0^4 \sqrt{4-(x-2)^2}\, dx$

Put, $x-2 = 2\sin\theta \Rightarrow dx = 2\cos\theta\, d\theta$

$x=4,\ \theta = \frac{\pi}{2}$ $\quad x = 0,\ \theta = -\frac{\pi}{2}$

$\int_{-\pi/2}^{\pi/2} 2\cos\theta \times 2\cos\theta\, d\theta$

$= 4 \int_{-\pi/2}^{\pi/2} \cos^2\theta\, d\theta$

$= \left[4 \times \frac{1}{2} \cos\theta \sin\theta + 4 \times \frac{1}{2}\theta\right]_{-\pi/2}^{\pi/2}$

$= 2\pi$