Artificial Intelligence

Fall 2017
CSE 440

Solving Problems by Searching
(Chapter 03)

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Informed Search Strategies

- Uninformed Search
  - in principle find solutions to any state space problem
  - they are typically too inefficient to do so in practice.
- Informed Search
  - Uses problem specific knowledge beyond the definition.
  - More efficient than uninformed search.

Best-First Search

- Instance of general Tree-Search or Graph-Search algorithm.
- Idea: use an **evaluation function** $f(n)$ for each node-
  - Cost estimate
  - Node with lowest evaluation expanded first
- Implementation is identical to Uniform-Cost Search - except for the use of f instead of g to order the priority queue
- Include a component of f a **heuristic function** $\boldsymbol{h(n)}$

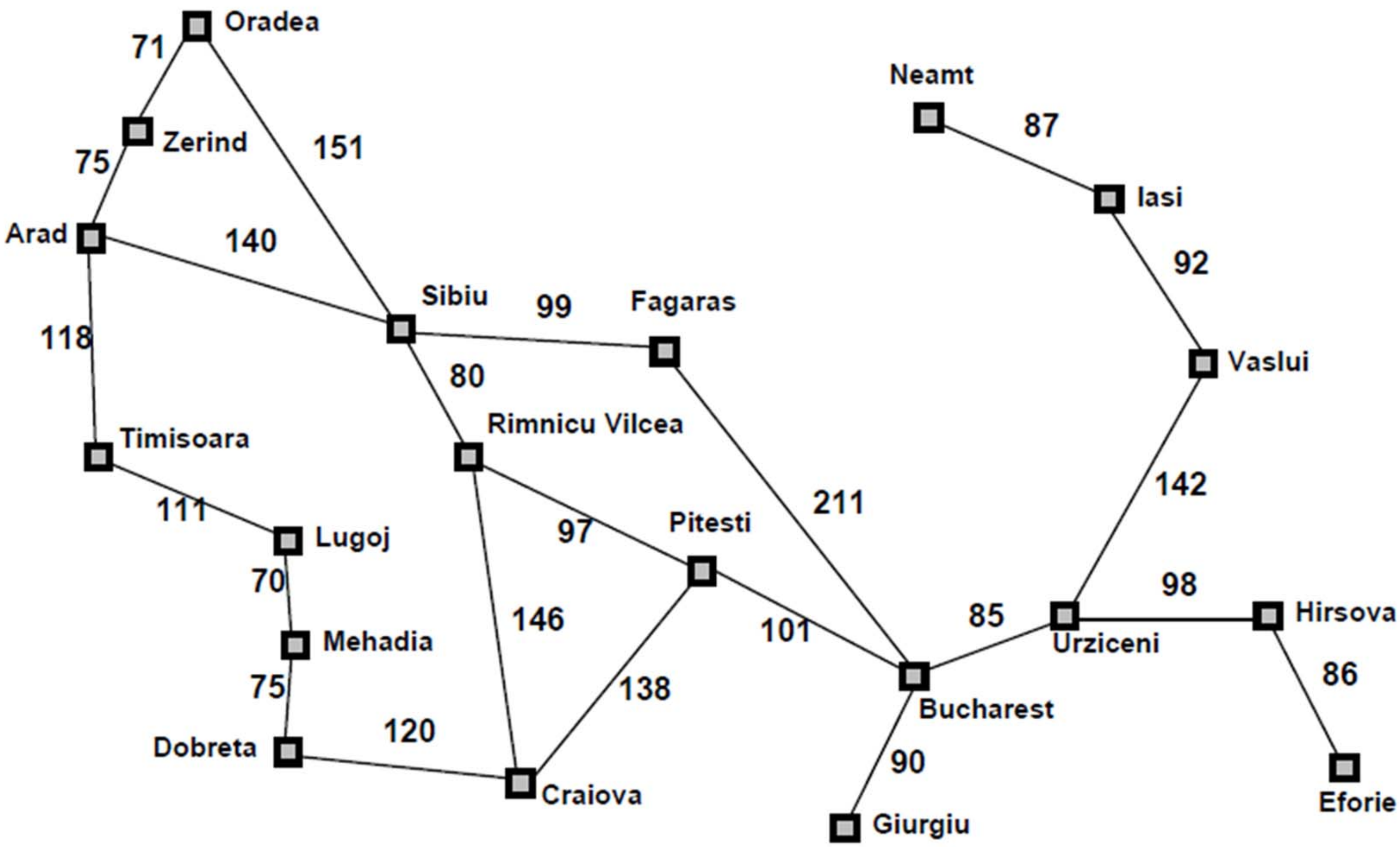
- Special Cases:
  - Greedy Best-First Search
  - A^* Search

What are heuristics?

- Heuristic: problem-specific knowledge that reduces expected search effort.
  - In blind search techniques, such knowledge can be encoded only via state space and operator representation.

- Informed search uses a heuristic evaluation function $h(n)$ that denotes the relative desirability of expanding a node/state.
  - often include some estimate of the cost to reach the nearest goal state from the current state.

Romania with step costs in km



Straight−line distance to Bucharest

Arad	366
Bucharest	0
Craiova	160
Dobreta	242
Eforie	161
Fagaras	178
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Hirsova	151
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Greedy best-first search

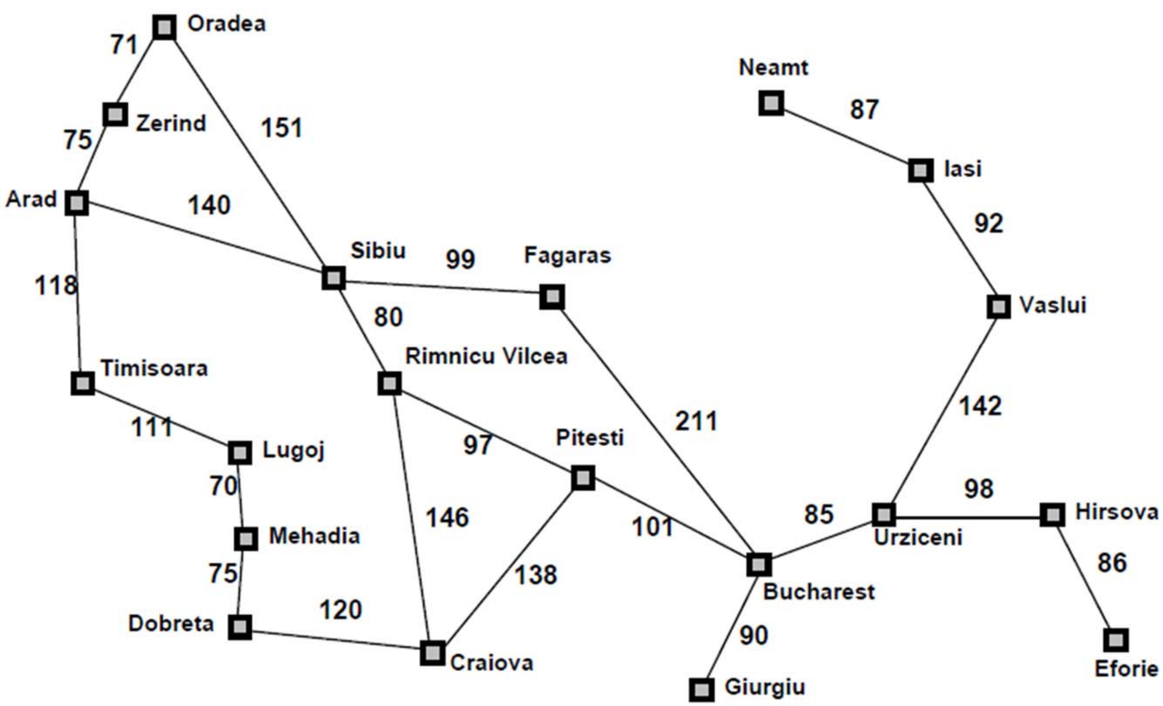
- Evaluation function $f(n) = h(n)$ (heuristic)

  = estimate of cost from n to *goal*

- e.g., $h_{SLD}(n)$ = straight-line distance from n to Bucharest

- Greedy best-first search expands the node that appears to be closest to goal

Greedy best-first search example



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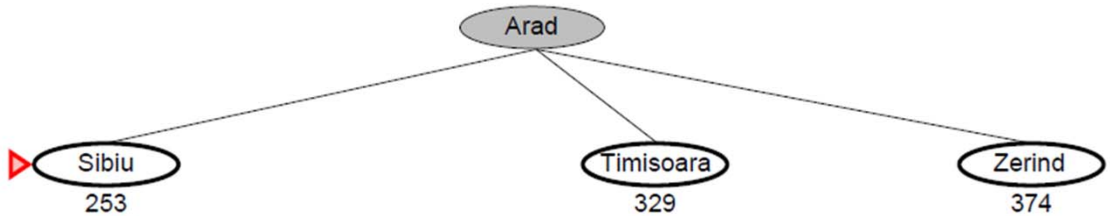

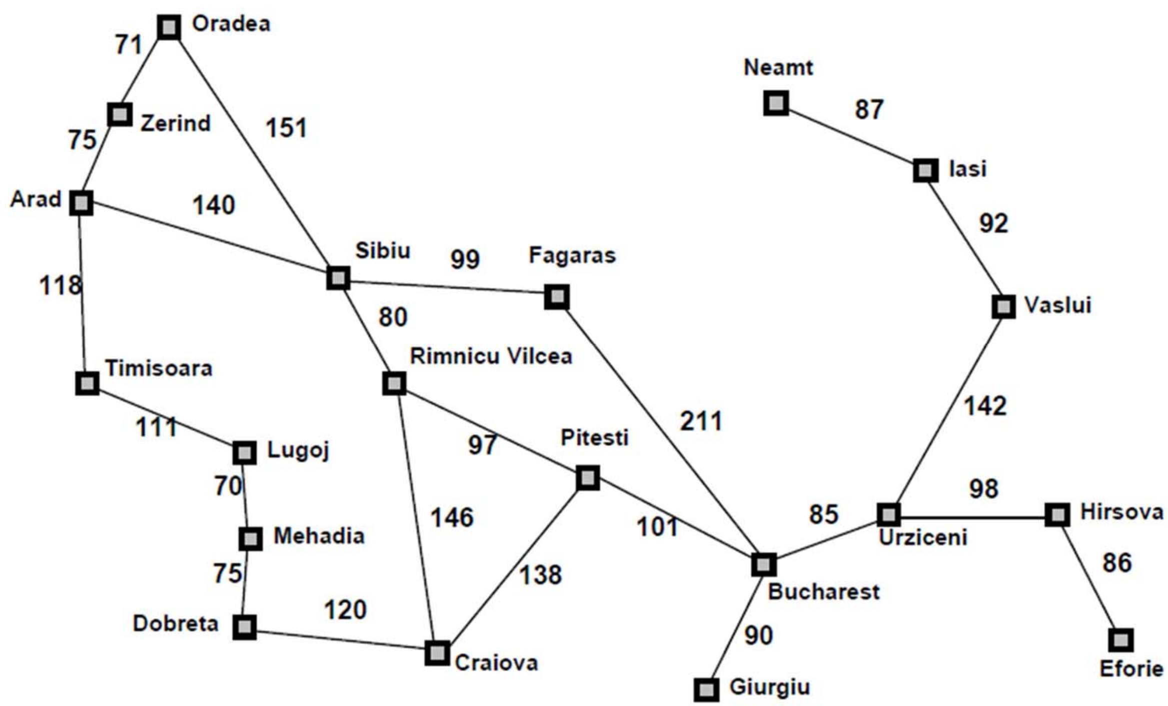

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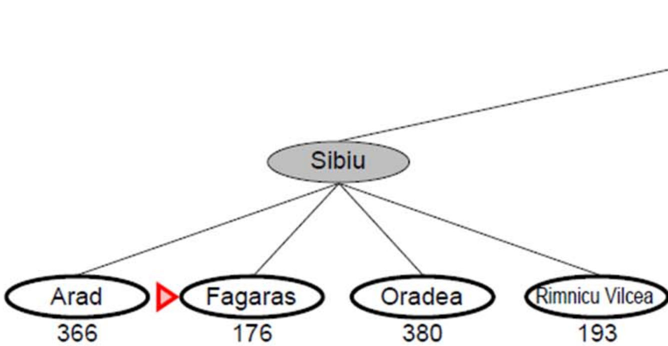

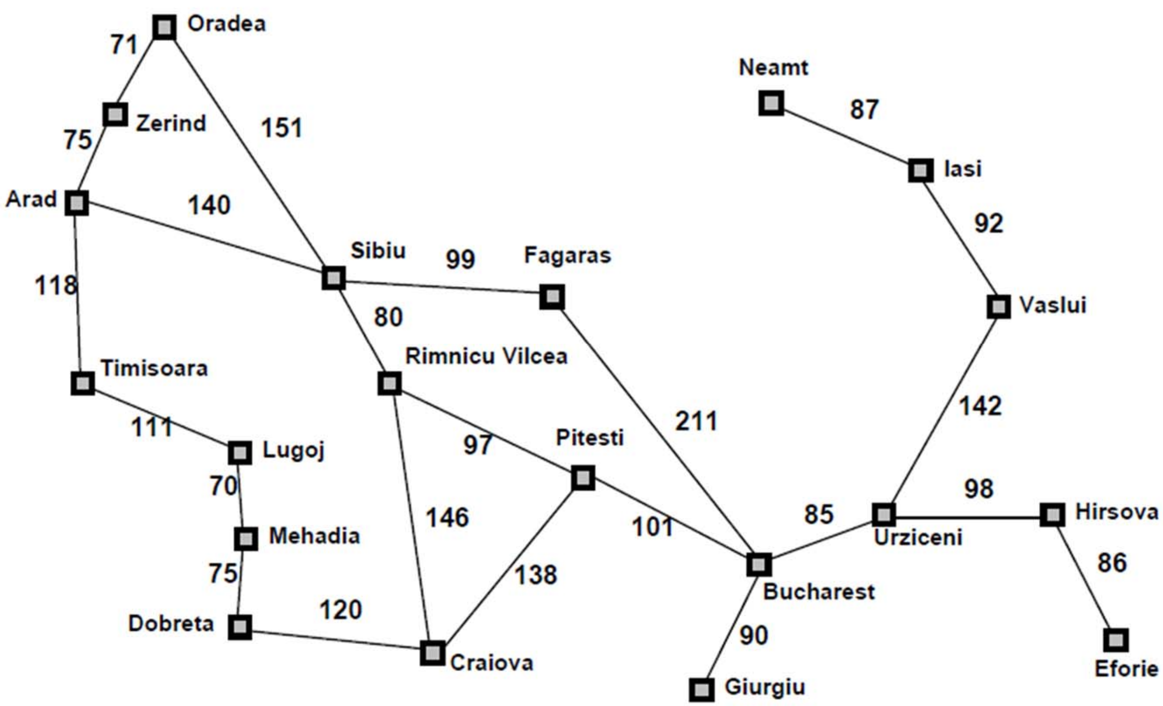

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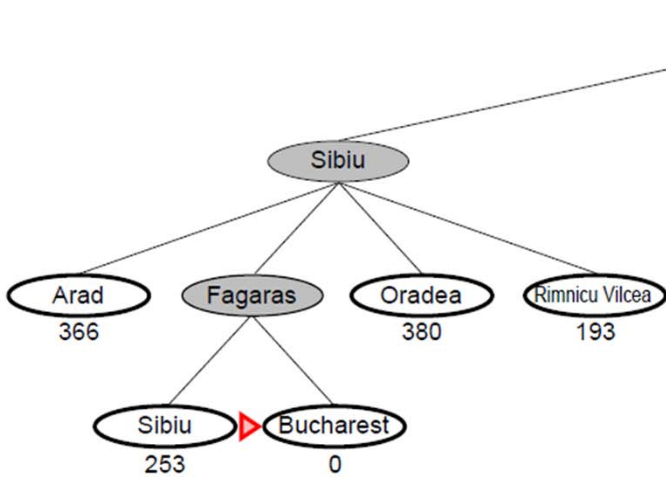

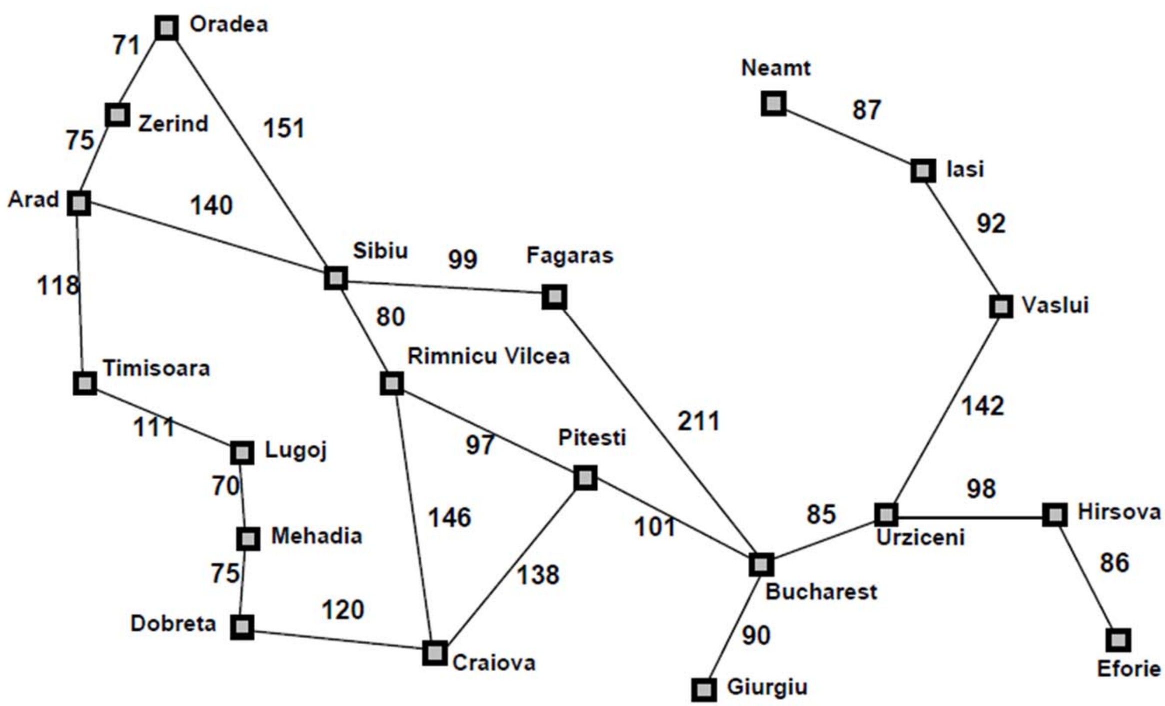

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Properties of greedy best-first search

- Complete? No – can get stuck in loops, e.g., Iasi → Neamt → Iasi → Neamt →

- Optimal? No

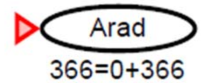
- Time? $O(b^m)$, but a good heuristic can give dramatic improvement

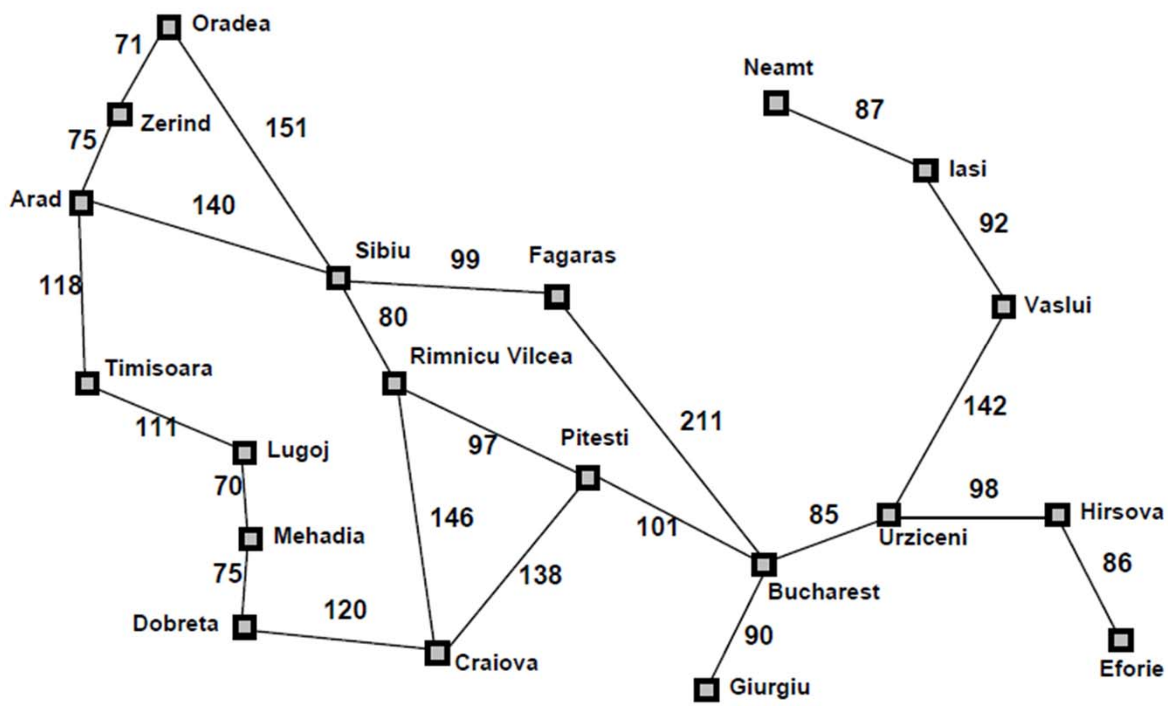
- Space? $O(b^m)$ - keeps all nodes in memory

A* search

- Idea: avoid expanding paths that are already expensive
- Evaluation function $f(n) = g(n) + h(n)$
- $g(n)$ = cost so far to reach n
- $h(n)$ = estimated cost from n to goal
- $f(n)$ = estimated total cost of path through n to goal
- Best First search has $f(n)=h(n)$
- Uniform Cost search has $f(n)=g(n)$

A* search example





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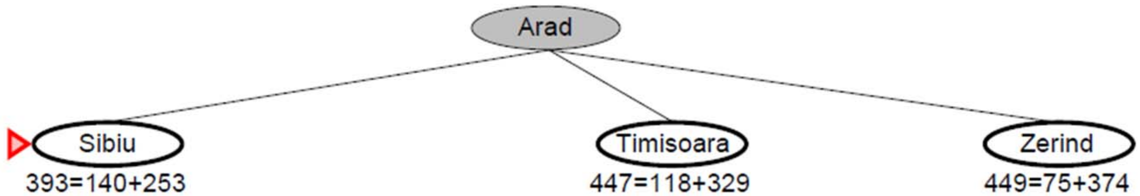

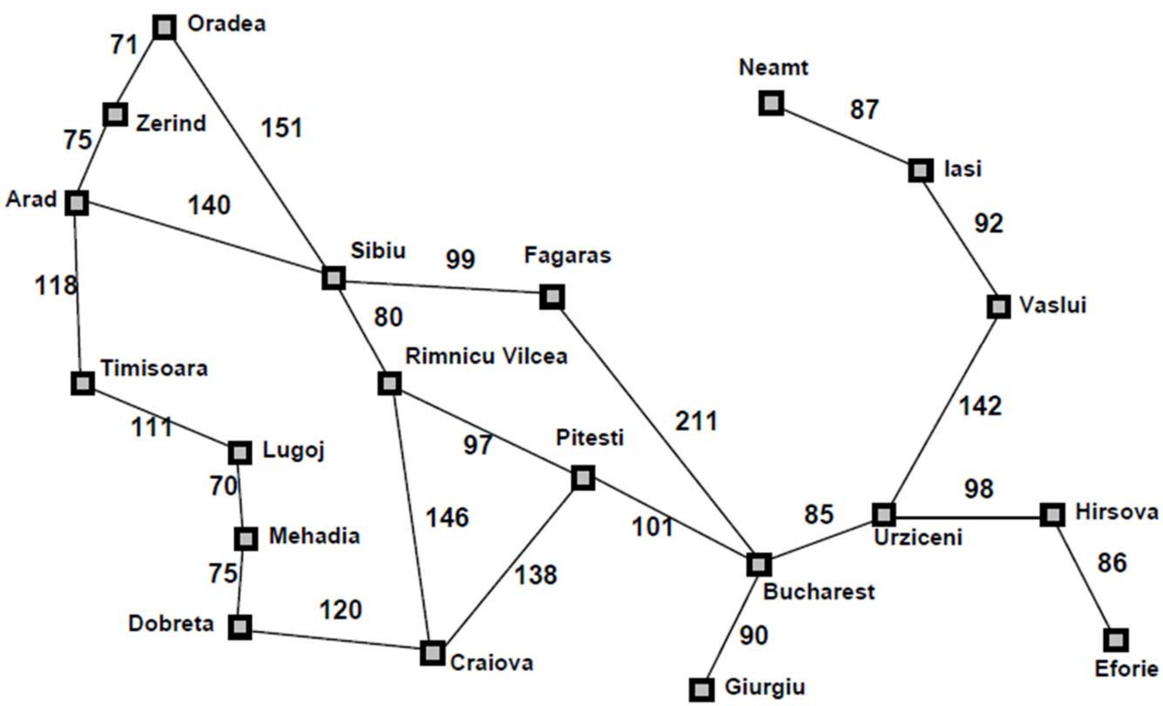

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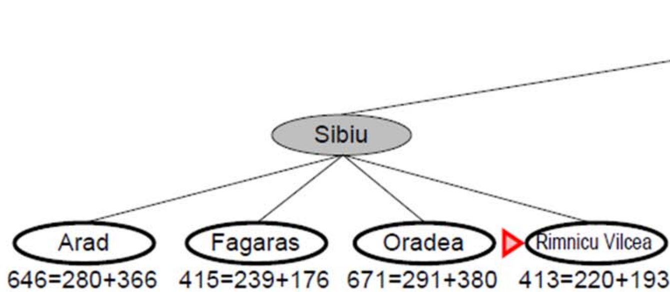

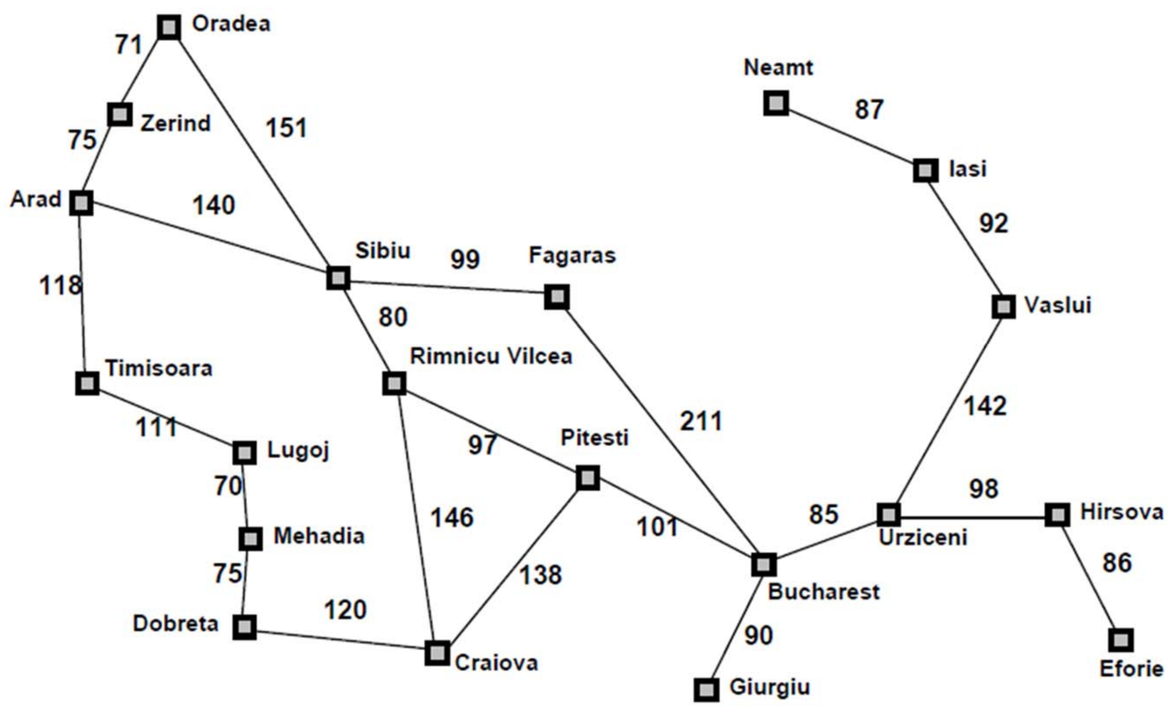

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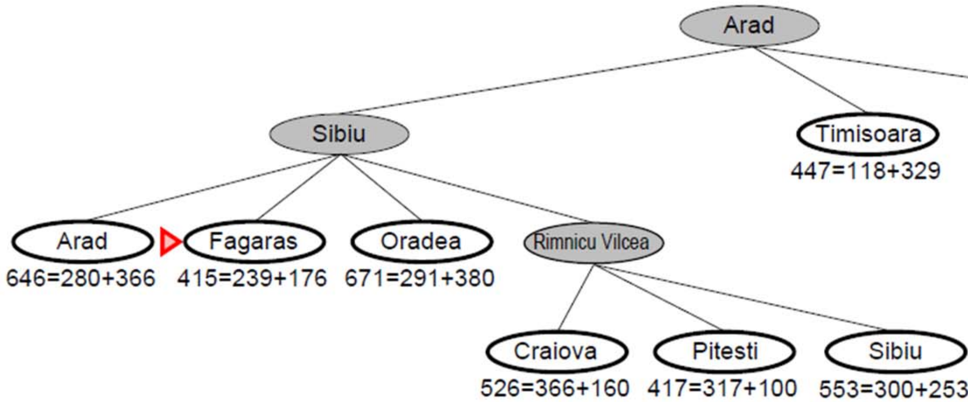

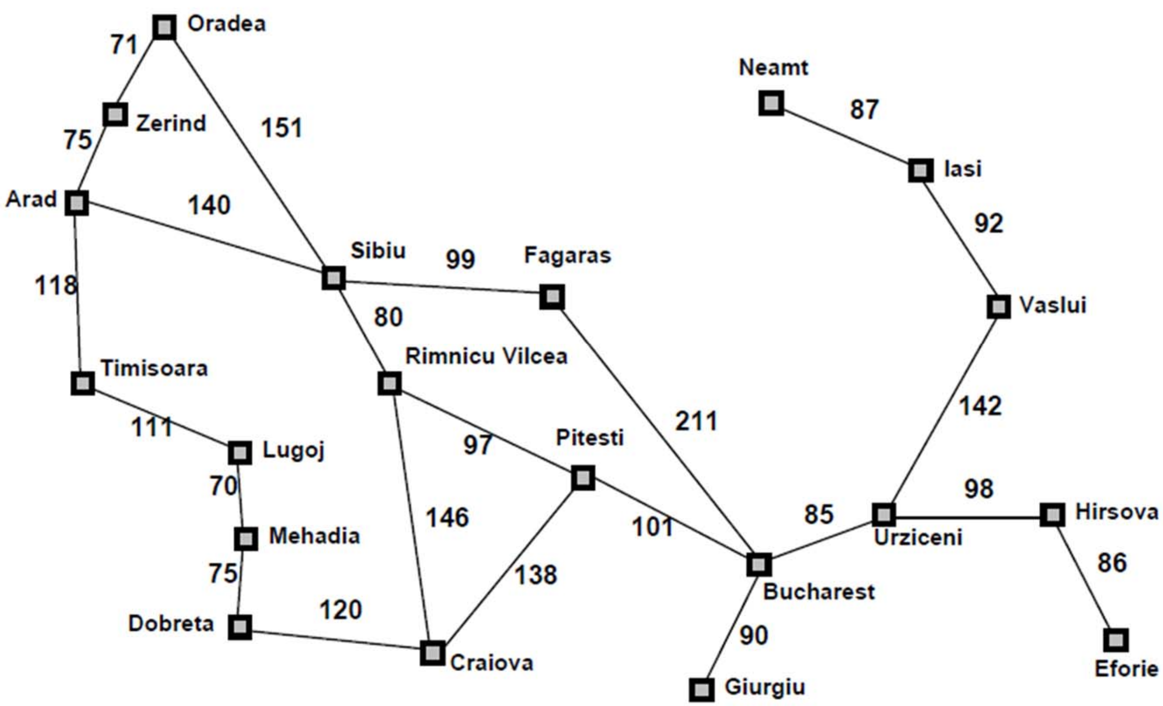

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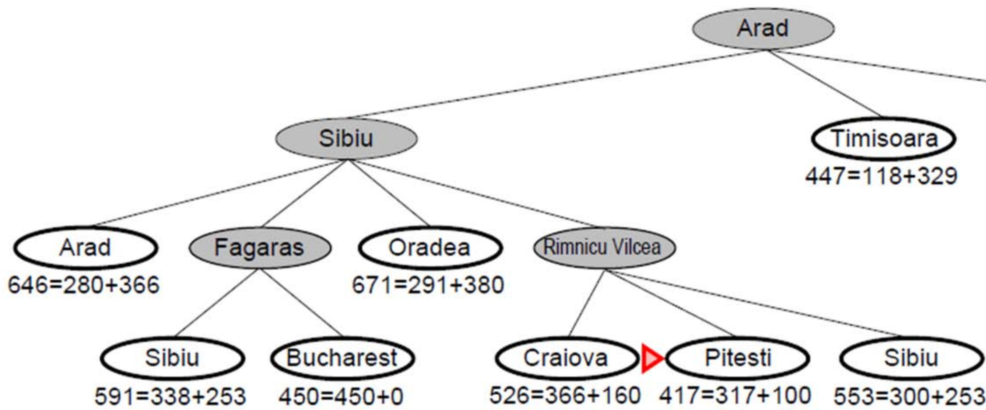

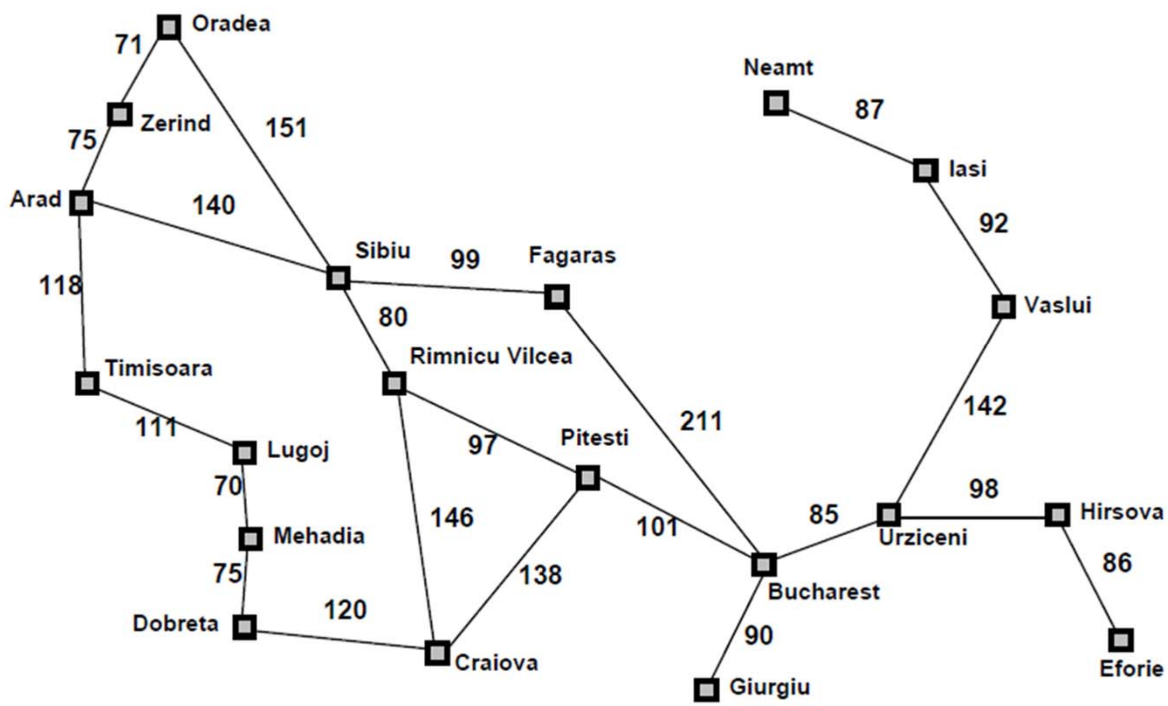

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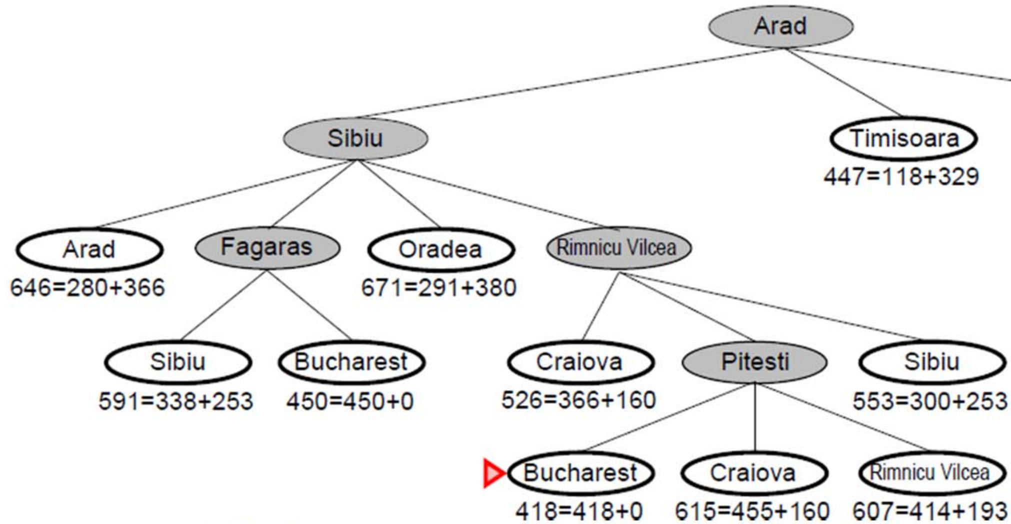

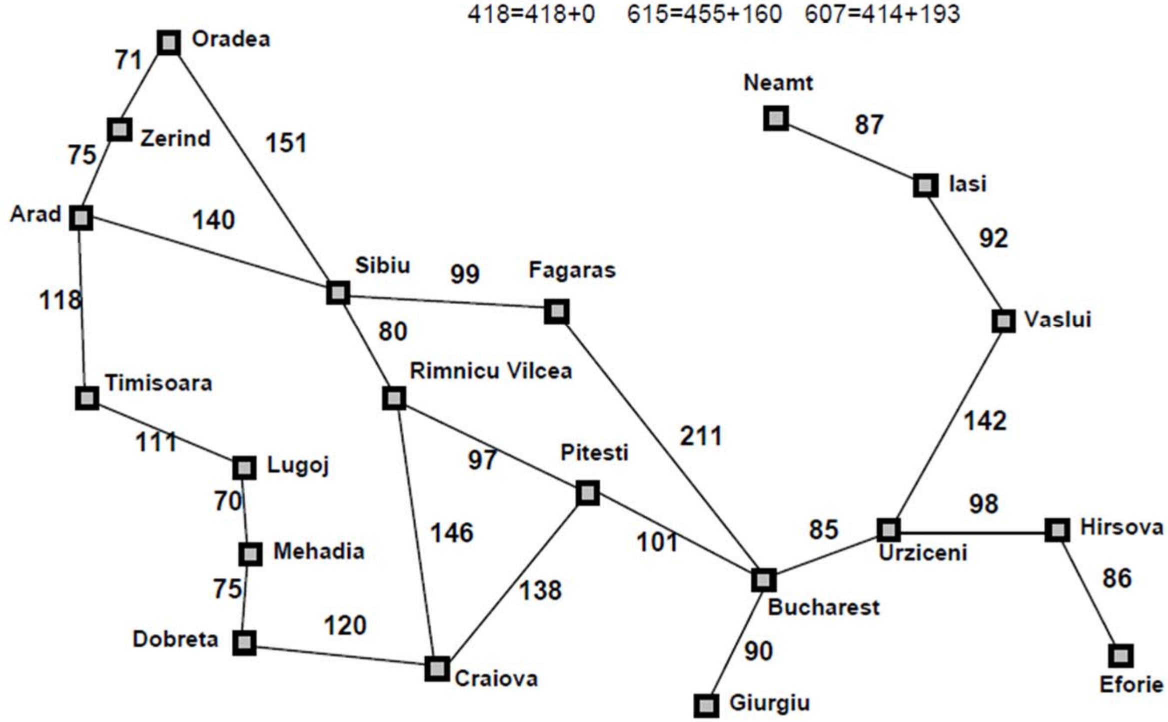

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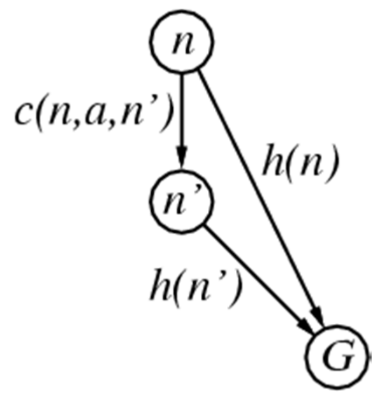
Admissible heuristics

- A heuristic $h(n)$ is admissible if for every node n, $h(n) \leq h^*(n)$, where $h^*(n)$ is the true cost to reach the goal state from n.
- An admissible heuristic never overestimates the cost to reach the goal, i.e., it is optimistic
- Example: $h_{SLD}(n)$ (never overestimates the actual road distance)

Consistent heuristics

- A heuristic is consistent if for every node *n*, every successor *n'* of *n* generated by any action *a*,

$$\boxed{h(n) \leq c(n, a, n') + h(n')}$$



- If *h* is consistent, we have

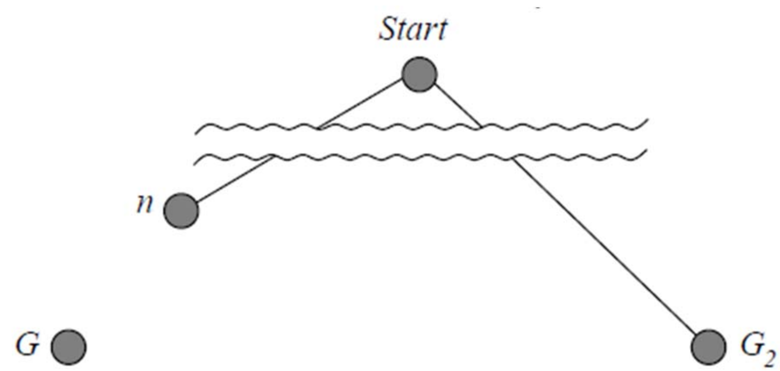
$$
\begin{aligned}
f(n') &= g(n') + h(n') && \text{(by def.)} \\
&= g(n) + c(n, a, n') + h(n') && (g(n') = g(n) + c(n, a, n')) \\
&\geq g(n) + h(n) = f(n) && \text{(consistency)} \\
f(n') &\geq f(n)
\end{aligned}
$$

It's the triangle inequality !

- i.e., *f(n)* is non-decreasing along any path.

Optimality of A* (proof)

- Suppose some suboptimal goal G_2 has been generated and is in the fringe. Let n be an unexpanded node in the fringe such that n is on a shortest path to an optimal goal G.



We want to prove: $f(n) < f(G_2)$
(then A* will prefer n over G_2)

- $f(G_2) = g(G_2)$ since $h(G_2) = 0$
- $f(G) = g(G)$ since $h(G) = 0$
- $g(G_2) > g(G)$ since G_2 is suboptimal

- $f(G_2) > f(G)$ from above
- $h(n) \leq h^*(n)$ since h is admissible (*under*-estimate)
- $g(n) + h(n) \leq g(n) + h^*(n)$ from above
- $f(n) \leq f(G)$ since $g(n) + h(n) = f(n)$ & $g(n) + h^*(n) = f(G)$
- $f(n) < f(G_2)$

Properties of A* search

- <u>Complete?</u> Yes (unless there are infinitely many nodes with $f \leq f(G)$, i.e. step-cost > ε)
- <u>Time?</u> Exponential b^d
- <u>Optimal?</u> Yes - cannot expand f_i+1 until f_i is finished
- <u>Space?</u> Keeps all nodes in memory