Poisson distribution: The Poisson distribution is used when a random variable counts the number of events. For example, 1) the number of telephone calls received by an operator within a certain time limit. 2) The number of patients arriving in an emergency room between 10 and 11 pm.

The probability mass function is P(X=x) = $\frac{e^{-\lambda}\lambda^{x}}{x!}$ x = 0,1,2,3…..

Expectation: E(x) = λ

Variance: V(x) = λ

Difference between binomial distribution and Poisson distribution:

1) In binomial distribution, number of trials are fixed. In poisson distribution, number of trials are infinite.

2) In binomial distribution, Variance $\leq$ Mean. In Poisson distribution, Mean = Variance.

For binomial distribution, Expectation: E(x) = np

Variance: V(x) = np(1-p) $[0 \leq p \leq 1, n > 1]$

Mean = variance when p = 0

$\Rightarrow$n.0 = n.0.(1-0)

$\Rightarrow$ 0 = 0.

Mean (n) >variance (0) when p = 1

because when p=1, mean = n.1 = n (n>1) and variance = n.1.(1-1) = 0

Mean > variance [0<P<1]

np > np(1-p)

EX: 50>50(1-p) here, (0<(1-p)<1)

$\Rightarrow$50>50 (1-0.2) if p=.2 $\Rightarrow$50>50.0.8 $\Rightarrow$50>40.

3) Ex of binomial distribution: Coin tossing experiment. Ex of Poisson distribution: The number of patients arriving in an emergency room between 10 and 11 pm.

Example: Suppose that the number of errors in a piece of software has a parameter $\lambda = 3$. This parameter immediately implies that the expected number of errors is three and that the variance in the number of errors is also equal to three.

a) What is distribution of the number of errors in a piece of software.

b) Calculate the probability that a piece of software has no errors.

c) Calculate the probability that there are three or more errors in a piece of software.

Solution: a) The number of errors in a piece of software follows Poission distribution

$$P(X=x) = \frac{e^{-3}3^x}{x!} \qquad x = 0,1,2,3\ldots$$

b) $P(X = 0) = \frac{e^{-3}3^0}{0!} = 0.05$

c) $P(X\geq 3) = P(X=3) + P(X=4) + \ldots = ?$

We know,

$P(X=0) + P(X=1) + P(X=2) + \mathbf{P(X=3) + P(X=4) + \ldots =}1$

$\Rightarrow P(X=3) + P(X=4) + \ldots = 1-P(X=0)-P(X=1)-P(X=2)$

$\Rightarrow P(X=3) + P(X=4) + \ldots =1-\frac{e^{-3}3^0}{0!} - \frac{e^{-3}3^1}{1!}-\frac{e^{-3}3^2}{2!}$

$\Rightarrow P(X=3) + P(X=4) + \ldots = 0.577.$

Geometric distribution: The binomial distribution is the distribution of the number of successes occurring in a fixed number of trials n, it is sometimes of interest to count instead the number of trials performed until **the first success occurs**. Such a random variable is said to have a geometric distribution.

The probability mass function is $P(X=x) = (1-p)^{(x-1)}p \quad x=1,2,3\ldots.$

Expectation $E(X) = \frac{1}{p}$

Variance $V(X) = \frac{1-p}{p^2}$.

Example: Suppose that a company wishes to hire **one new workers** and that each applicant interviewed has a probability of 0.6 of being found acceptable.

1) What is the distribution of the total number of applicants that the company needs to interview?

2) Calculate the probability that exactly six applicants need to be interviewed.

3) Calculate the probability that the company allows up to/at most six applicants to be interviewed.

4) Calculate the probability that at least six applicants need to be interviewed.

5) Calculate the expected number of interviews.

Solution: 1) The total number of applicants that the company needs to interview follows geometric distribution.

The probability mass function is $P(X=x) = (1-0.6)^{(x-1)}0.6 \quad x=1,2,3\ldots.$

2) $P(X=6) = (1-0.6)^{(6-1)}0.6 =$

3) $P(X\leq 6) = P(X=1) + P(X=2) + \ldots + P(X=6) = \ldots.$

4) $P(X\geq 6) = P(X=6) + P(X=7) + \ldots.=?$

We know, $P(X=1)+\ldots.+P(X=5) + \mathbf{P(X=6)+ P(X=7) + \ldots}=1$

$\Rightarrow \mathbf{P(X=6)+ P(X=7) + \ldots}= 1 - P(X=1) \ldots.-P(X=5) =$

5) E(X) = $\frac{1}{p} = \frac{1}{0.6} =$

Negative binomial distribution: The binomial distribution is the distribution of the number of successes occurring in a fixed number of trials n, it is sometimes of interest to count instead the number of trials performed until **the rth (r>1) success occurs**. Such a random variable is said to have a negative binomial distribution.

The p.m.f is P(X=x) = $\binom{x-1}{r-1}(1-p)^{(x-r)}p^r$ x = r,r+1,r+2….

Expectation E(X) = $\frac{r}{p}$

Variance V(X) = $\frac{r(1-p)}{p^2}$

Example: Suppose that a company wishes to hire **three new workers** and that each applicant interviewed has a probability of 0.6 of being found acceptable.
1) What is the distribution of the total number of applicants that the company needs to interview?
2) Calculate the probability that exactly six applicants need to be interviewed.
3) Calculate the probability that the company allows up to/at most six applicants to be interviewed.
4) Calculate the probability that at least six applicants need to be interviewed.
5) Calculate the expected number of interviews.

Solution: 1) The total number of applicants that the company needs to interview follows negative binomial distribution.

The p.m.f is $P(X=x) = \binom{x-1}{3-1}(1-0.6)^{(x-3)}(0.6)^3 \quad x = 3,4,5,6\ldots.$

2) $P(X=6) = \binom{6-1}{3-1}(1-0.6)^{(6-3)}(0.6)^3$ \

$= \binom{5}{2}(1-0.6)^3(0.6)^3$

$= 0.138$

3) $P(X\leq 6) = P(X=3) + P(X=4) + P(X=5) + P(X=6) =$………….

4) $P(X \geq 6) = P(X=6) + P(X=7) + \ldots =?$

We know, P(X=3)+….+P(X=5) + **P(X=6)+ P(X=7) + …**=1

⇒ **P(X=6)+ P(X=7) + …**= 1 – P(X=3) ….-P(X=5)=

5) $E(X) = \frac{3}{0.6} = 5$