EEE363

Electrical Machines

Lecture # 21

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Rotor current frequency

$$\boxed{n_s = \frac{120 f_e}{P}}$$

$$\boxed{n_s - n_m = \frac{120 f_r}{P}}$$

f_r is rotor current frequency

f_e is supply frequency

$$\frac{f_r}{f_e} = \frac{n_s - n_m}{n_s} = \mathrm{s} \quad \Rightarrow \quad f_r = s.f_e$$

Problem # 1

A 208-V, 10-hp, four-pole, 60-Hz, Y-connected induction motor has a full-load slip of 5 percent.

(a) *What is the synchronous speed of this motor?*

(b) *What is the rotor speed of this motor at the rated load?*

(c) *What is the rotor frequency of this motor at the rated load?*

(d) *What is the shaft torque of this motor at the rated load?*

(a) $$n_{sync} = \frac{120 f_e}{P} = \frac{120(60\text{ Hz})}{4\text{ poles}} = 1800\text{ r/min}$$

(b) The rotor speed of the motor is given by

$$n_m = (1 - s)n_{sync} = (1 - 0.95)(1800\text{ r/min})$$
$$= 1710\text{ r/min}$$

(c) The rotor current frequency of this motor is given by

$$f_r = sf_e = (0.05)(60\text{ Hz}) = 3\text{ Hz}$$

(d) The shaft load torque is given by

$$\tau_{load} = \frac{P_{out}}{\omega_m}$$
$$= \frac{(10\text{ hp})(746\text{ W/hp})}{(1710\text{ r/min})(2\pi\text{ rad/r})(1\text{ min}/60\text{ s})}$$
$$= 41.7\text{ N}\bullet\text{m}$$

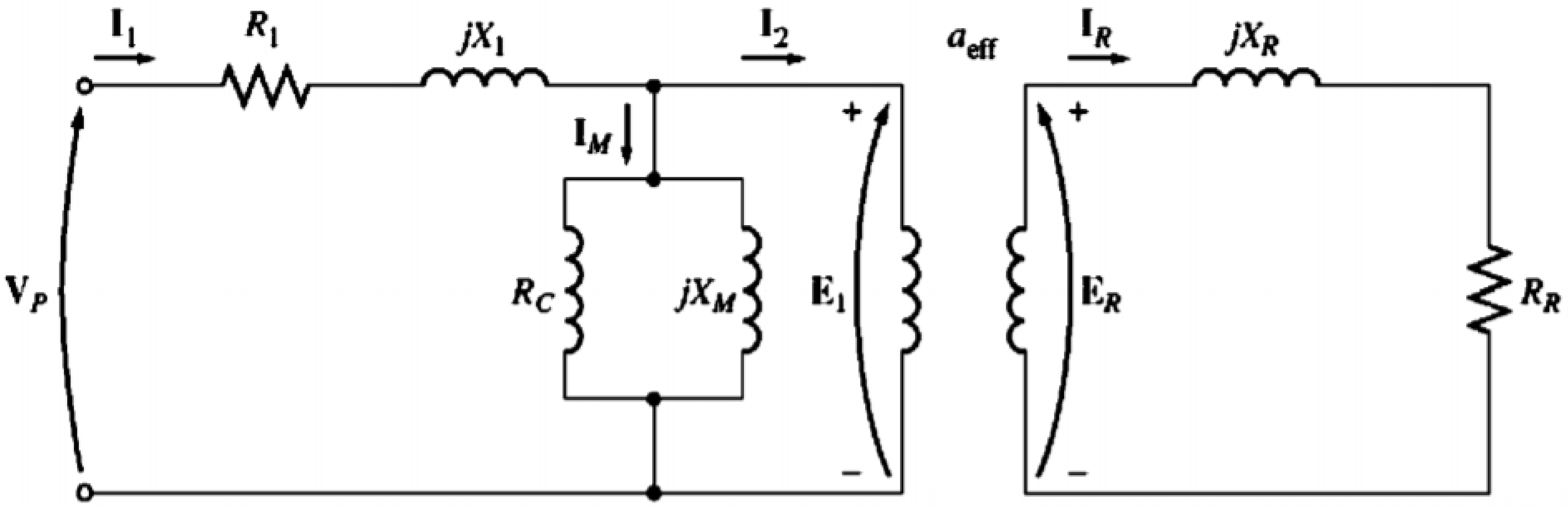
Types

1. Wound rotor induction motor

2. Cage rotor induction motor

Equivalent circuit

- An induction motor relies for its operation on the induction of voltages and currents in its rotor circuit from the stator circuit (transformer action).
- The higher reluctance caused by the air gap means that a higher magnetizing current is required to obtain a given flux level.
- Therefore, the magnetizing reactance X_M in the equivalent circuit will have a much smaller value than it would in an ordinary transformer.

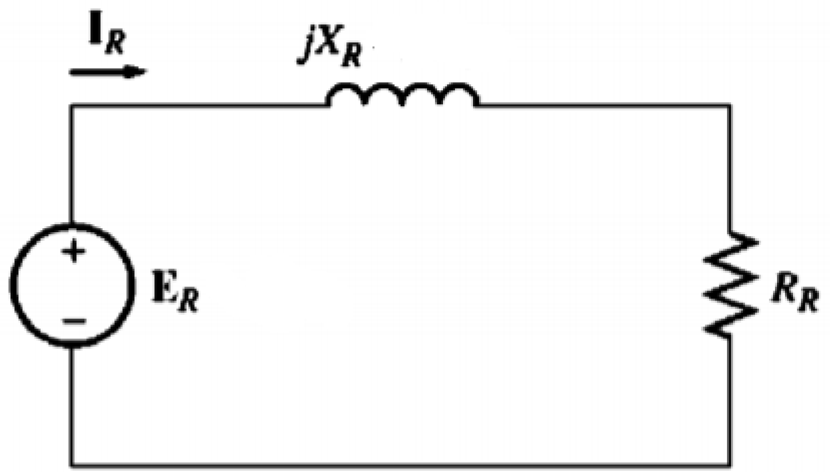


Rotor circuit model

- ✓ An induction motor equivalent circuit differs from a transformer equivalent circuit primarily in the effects of varying rotor frequency on the rotor voltage E_R and the rotor impedances R_R and jX_R .

- ✓ The greater the relative speed between rotor and stator magnetic field, the greater the rotor voltage and frequency.

- ✓ The largest relative motion occurs when the rotor is stationary (Locked or Blocked rotor condition). Rotor induced emf under this condition is E_{RO}.



$$E_R = s.E_{RO}$$

$$X_R = \omega_r L_R = (s.\omega_e)L_R = s.X_{RO}$$

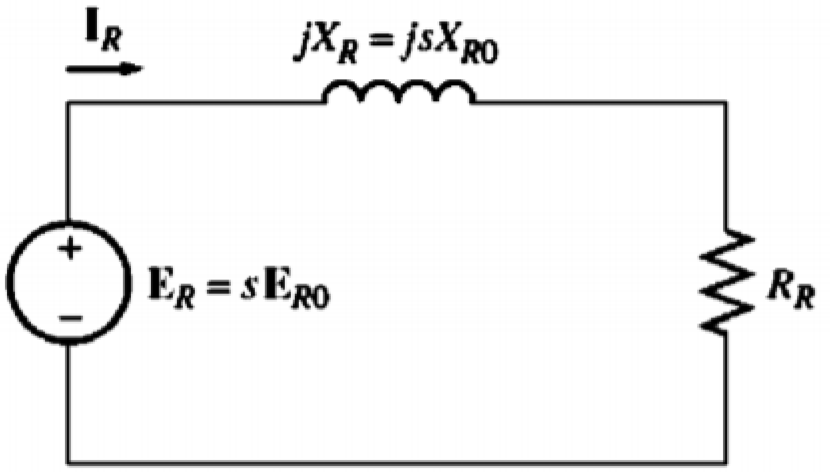
$$\mathbf{I}_R = \frac{\mathbf{E}_R}{R_R + jX_R}$$

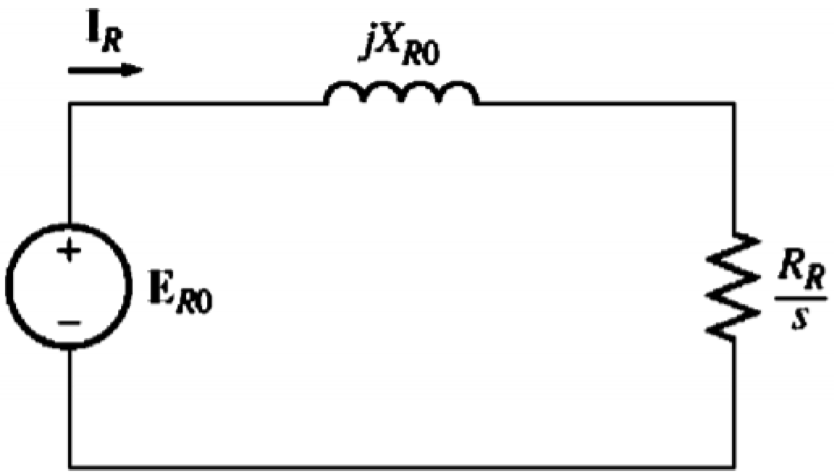
$$\frac{E_{RO}}{E_R} = \frac{n_s}{n_s - n_m} = \frac{1}{s}$$

Rotor circuit model

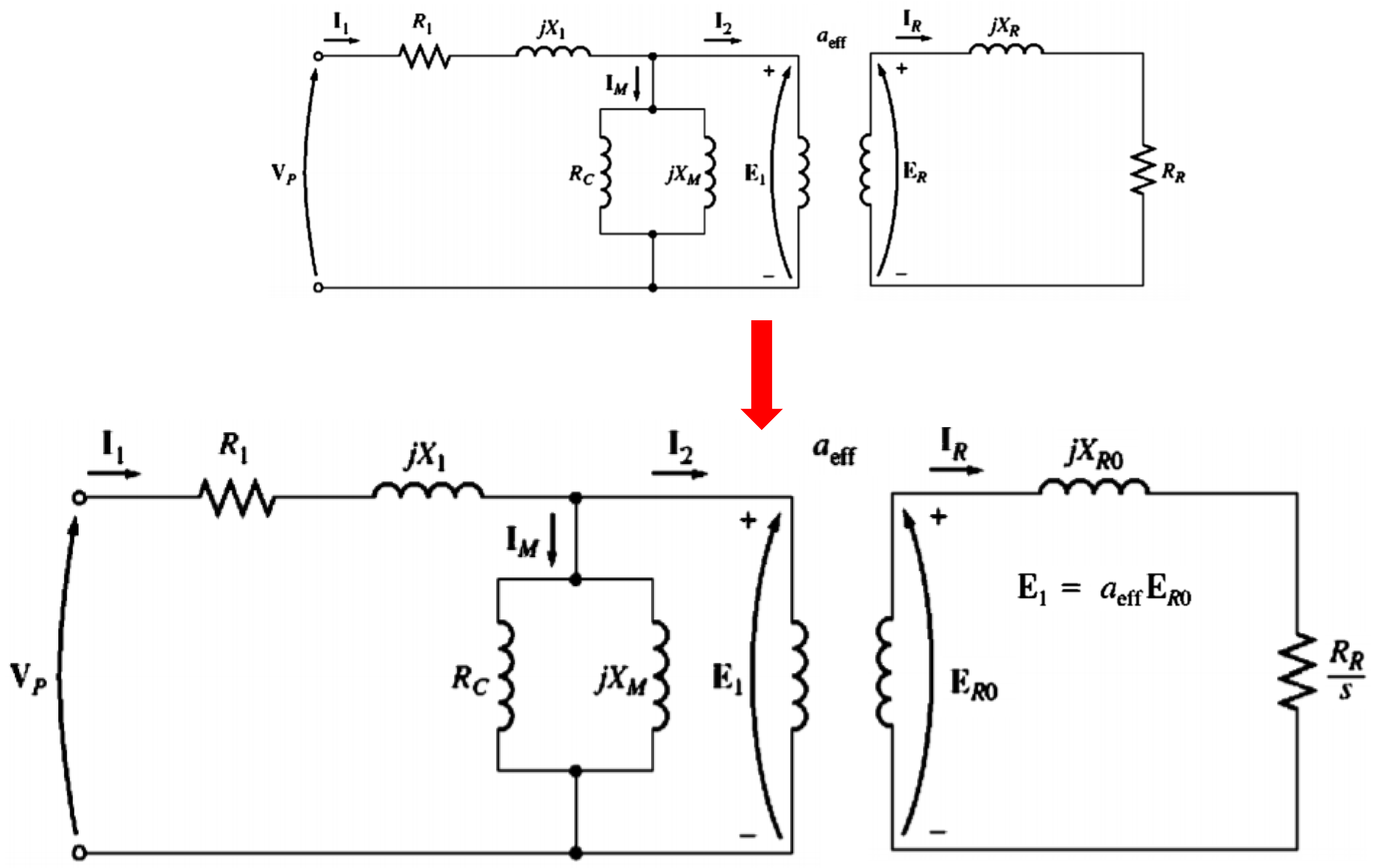
$$\mathbf{I}_R = \frac{\mathbf{E}_R}{R_R + jX_R}$$

$$I_R = \frac{s.E_{RO}}{R_R + js.X_{RO}} = \frac{E_{RO}}{\left({}^{R_R}/_{s}\right) + jX_{RO}}$$

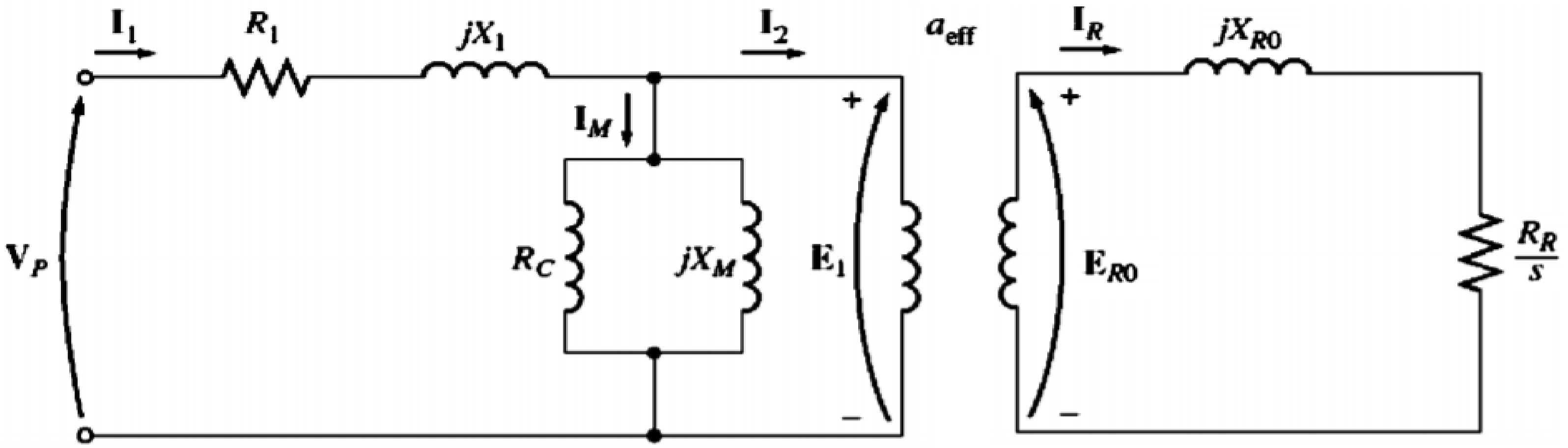


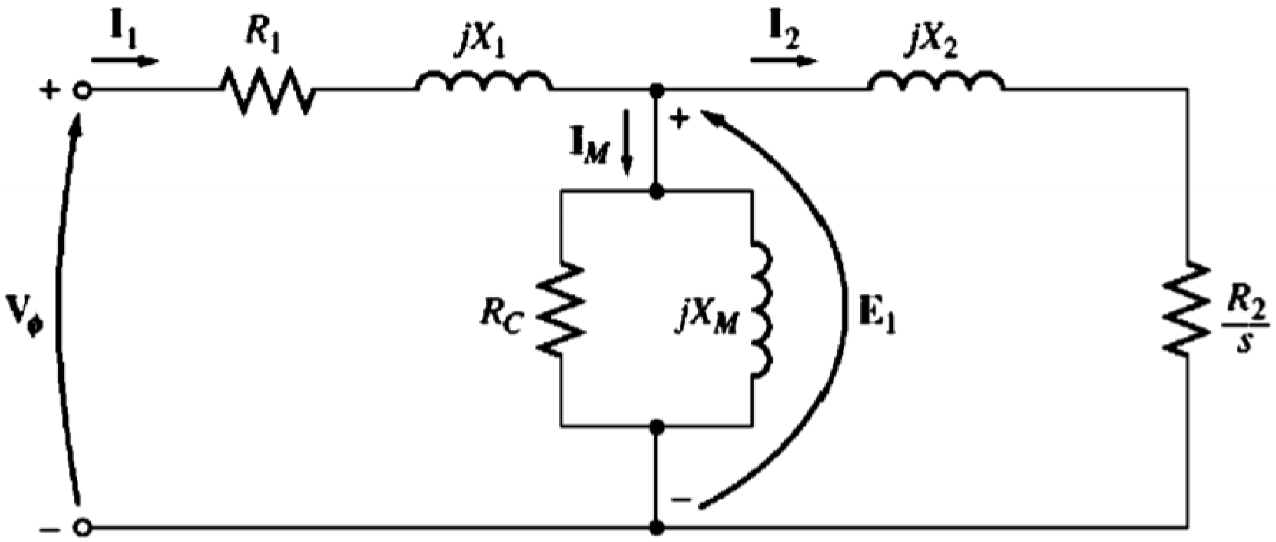


Final equivalent circuit



Final equivalent circuit



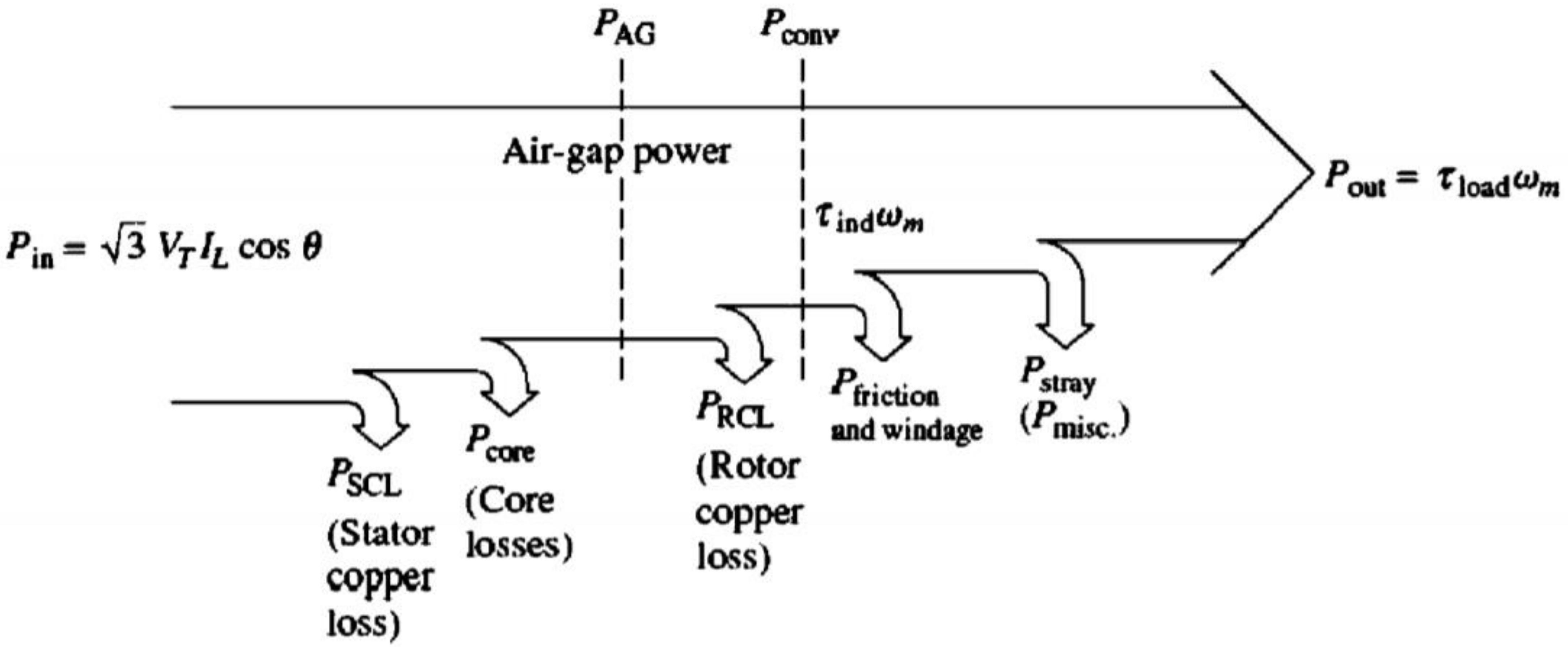


$$\mathbf{I}_2 = \frac{\mathbf{I}_R}{a_{\text{eff}}}$$

$$R_2 = a_{\text{eff}}^2 R_R$$

$$X_2 = a_{\text{eff}}^2 X_{R0}$$

Power flow diagram



Power and Torque in an induction motor

$$\mathbf{I}_1 = \frac{\mathbf{V}_\phi}{Z_{eq}}$$

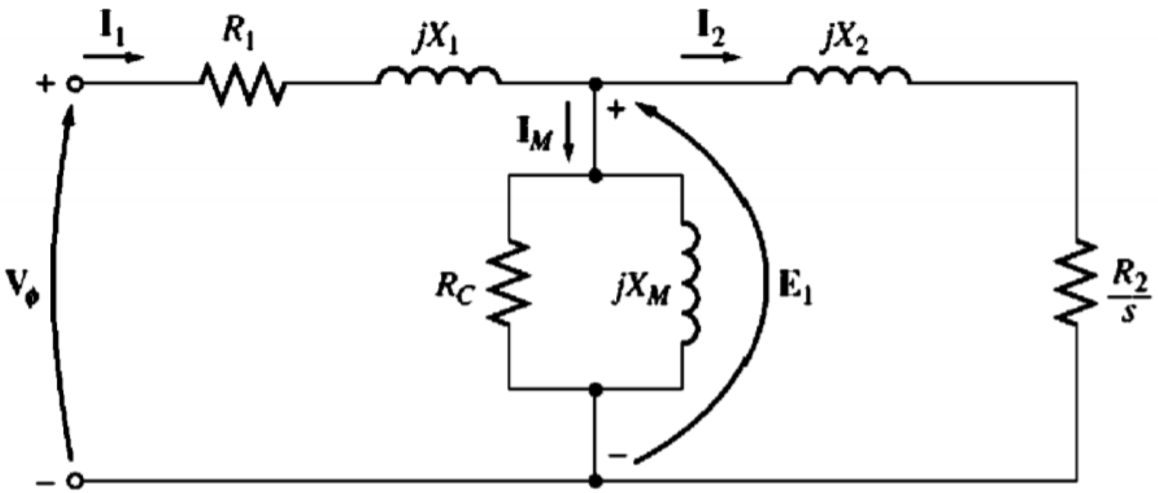
Stator Cu loss $\boxed{P_{SCL} = 3I_1^2 R_1}$

$$P_{core} = \frac{3E_1^2}{R_C}$$

Air Gap Power $\boxed{P_{AG} = P_{in} - P_{SCL} - P_{core}}$

Can also be expressed as $\boxed{P_{AG} = 3I_2^2 \frac{R_2}{s}}$

Rotor Cu loss $\boxed{P_{RCL} = 3I_2^2 R_2}$



Power and Torque in an induction motor

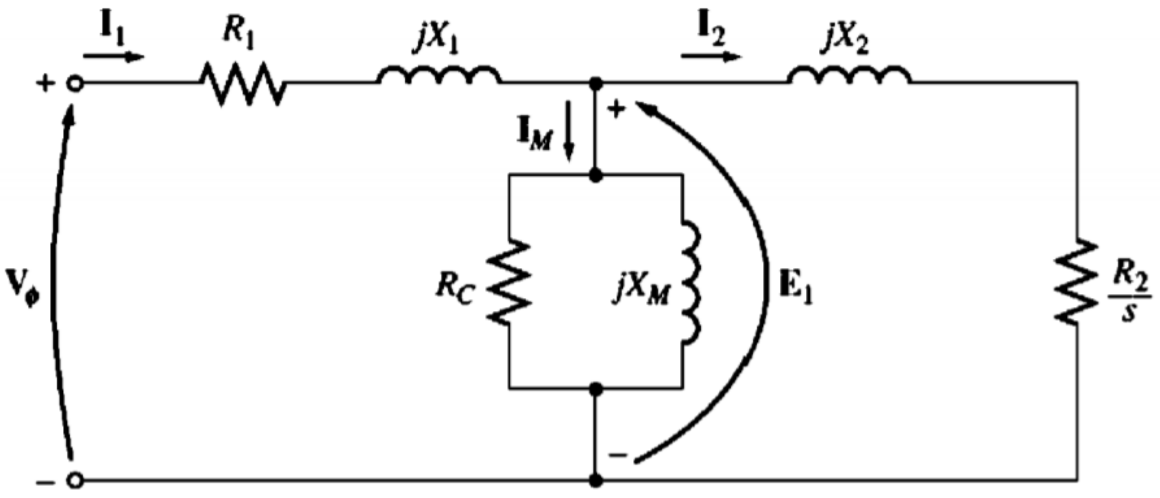
Converted power

$$P_{conv} = P_{AG} - P_{RCL}$$

$$= 3I_2^2\frac{R_2}{s} - 3I_2^2 R_2$$

$$= 3I_2^2 R_2\left(\frac{1}{s} - 1\right)$$

$$\boxed{P_{conv} = 3I_2^2 R_2\left(\frac{1 - s}{s}\right)}$$



$$\boxed{P_{AG} = 3I_2^2\frac{R_2}{s}}$$

$$\boxed{P_{RCL} = 3I_2^2 R_2}$$

$$P_{RCL} = sP_{AG}$$

$$P_{conv} = P_{AG} - P_{RCL}$$

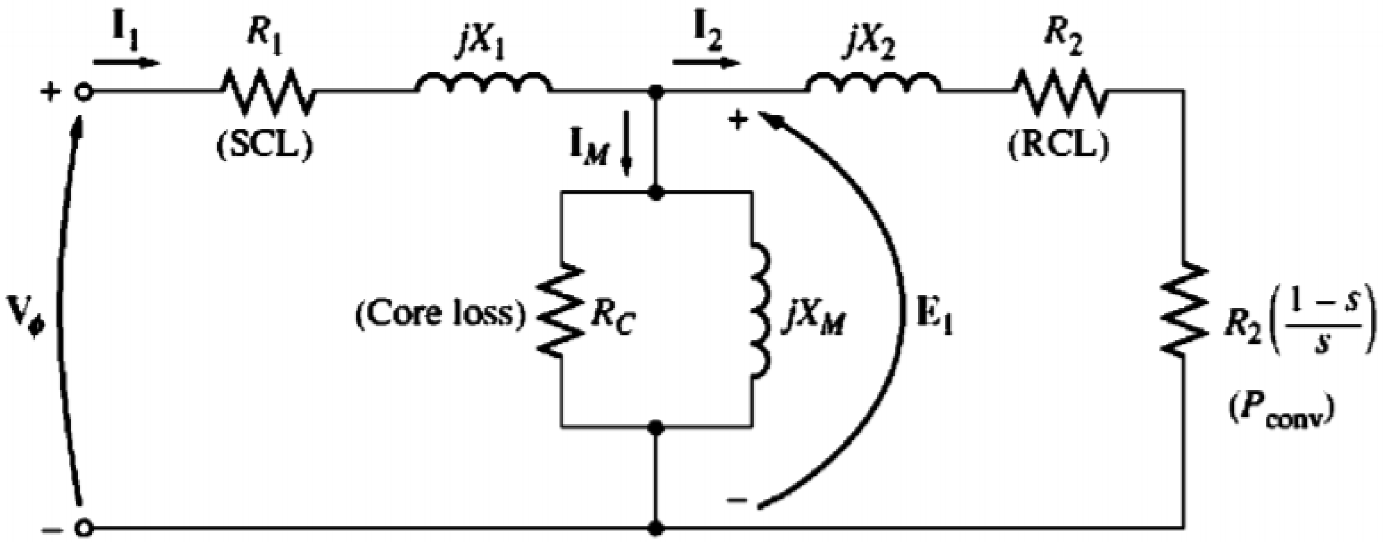
$$= P_{AG} - sP_{AG}$$

$$\boxed{P_{conv} = (1 - s)P_{AG}}$$

Power and Torque in an induction motor

$$\tau_{ind} = \frac{P_{conv}}{\omega_m} = \frac{(1-s)P_{AG}}{(1-s)\omega_{sync}} = \frac{P_{AG}}{\omega_{sync}}$$

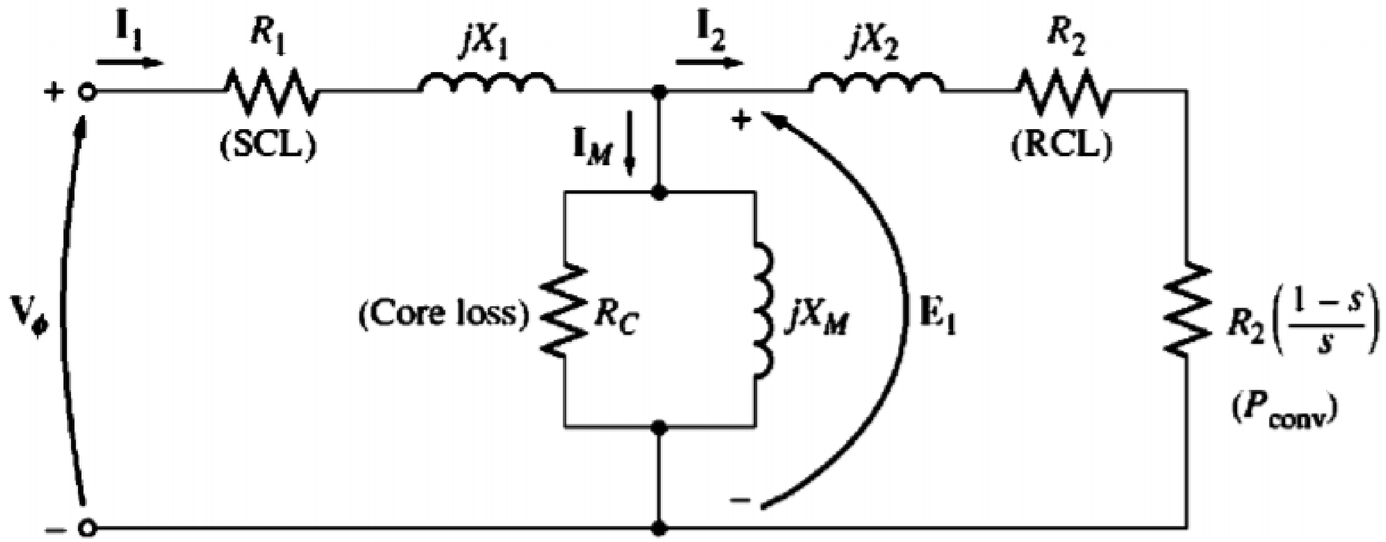
$$\boxed{P_{conv} = 3I_2^2 R_2\left(\frac{1-s}{s}\right)}$$



Power and Torque in an induction motor

$$\tau_{ind} = \frac{P_{conv}}{\omega_m} = \frac{(1-s)P_{AG}}{(1-s)\omega_{sync}} = \frac{P_{AG}}{\omega_{sync}}$$

$$\boxed{P_{conv} = 3I_2^2 R_2\left(\frac{1-s}{s}\right)}$$



Problem # 2

A 460-V. 25-hp. 60 Hz. four-pole. Y-connected induction motor has the following impedances in ohms per phase referred to the stator circuit:

$$R_1 = 0.641\ \Omega \qquad R_2 = 0.332\ \Omega \qquad X_M = 26.3\ \Omega$$
$$X_1 = 1.106\ \Omega \qquad X_2 = 0.464\ \Omega$$

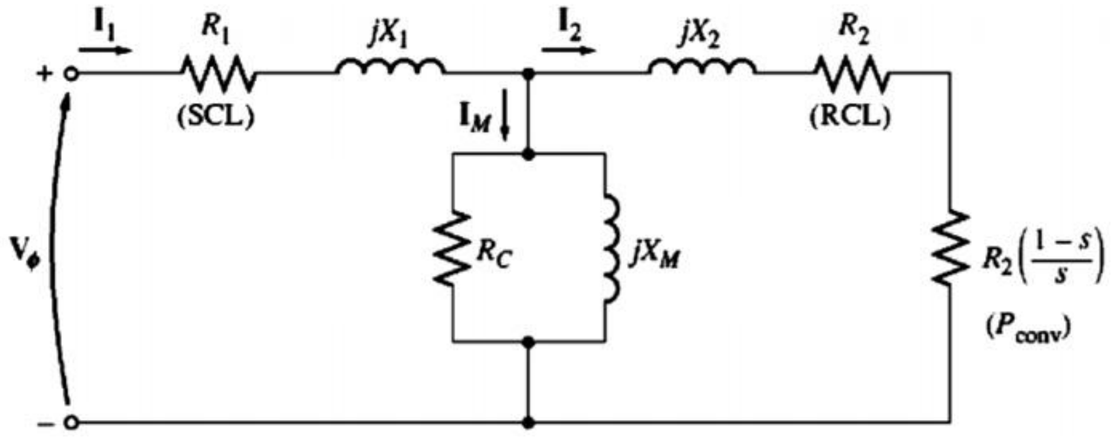
The total rotational losses are 1100 W and are assumed to be constant. The core loss is lumped in with the rotational losses. For a rotor slip of 2.2 percent at the rated voltage and rated frequency. find the motor's

(a) Speed (b) Stator current (c) Power factor (d) P_{conv} and P_{out} (e) τ_{ind} and τ_{load} (f) Efficiency

$$n_{sync} = \frac{120 f_e}{P} = \frac{120(60\text{ Hz})}{4\text{ poles}} = 1800\text{ r/min}$$

$$n_m = (1 - s)n_{sync} = (1 - 0.022)(1800\text{ r/min}) = 1760\text{ r/min}$$

$$\omega_m = (1 - s)\omega_{sync} = (1 - 0.022)(188.5\text{ rad/s}) = 184.4\text{ rad/s}$$

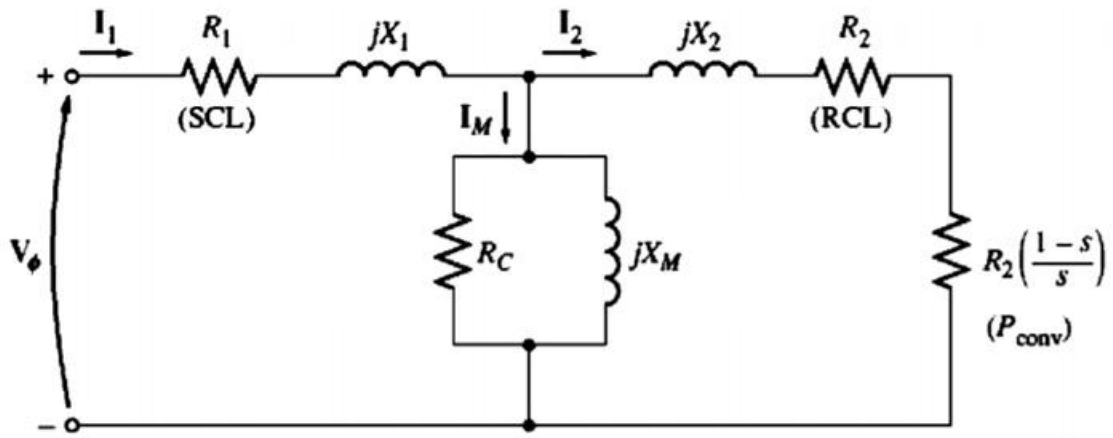


Problem # 2

$$Z_2 = \frac{R_2}{s} + jX_2 \quad = \frac{0.332}{0.022} + j0.464$$

$$= 15.09 + j0.464\ \Omega$$

$$= 15.10\angle 1.76^\circ\ \Omega$$



$$Z_f = \frac{1}{1/jX_M + 1/Z_2} \quad = \frac{1}{-j0.038 + 0.0662\angle -1.76^\circ}$$

$$= \frac{1}{0.0773\angle -31.1^\circ} = 12.94\angle 31.1^\circ\ \Omega$$

$$Z_{\text{tot}} = Z_{\text{stat}} + Z_f \quad = 0.641 + j1.106 + 12.94\angle 31.1^\circ\ \Omega$$

$$= 11.72 + j7.79 = 14.07\angle 33.6^\circ\ \Omega$$

$$\mathbf{I}_1 = \frac{\mathbf{V}_\phi}{Z_{\text{tot}}} \quad = \frac{266\angle 0^\circ\ \text{V}}{14.07\angle 33.6^\circ\ \Omega} \quad = \quad 18.88\angle -33.6^\circ\ \text{A}$$

Problem # 2

The motor power factor is $\mathrm{PF} = \cos 33.6° = 0.833$ lagging

The input power to this motor is $P_{\text{in}} = \sqrt{3} V_T I_L \cos\theta = \sqrt{3}(460\ \text{V})(18.88\ \text{A})(0.833) = 12{,}530\ \text{W}$

The stator copper losses in this machine are $P_{\text{SCL}} = 3I_1^2 R_1 = 3(18.88\ \text{A})^2(0.641\ \Omega) = 685\ \text{W}$

The air-gap power is given by $P_{\text{AG}} = P_{\text{in}} - P_{\text{SCL}} = 12{,}530\ \text{W} - 685\ \text{W} = 11{,}845\ \text{W}$

The power converted is $P_{\text{conv}} = (1 - s)P_{\text{AG}} = (1 - 0.022)(11{,}845\ \text{W}) = 11{,}585\ \text{W}$

The power P_{out} is given by $P_{\text{out}} = P_{\text{conv}} - P_{\text{rot}} = 11{,}585\ \text{W} - 1100\ \text{W} = 10{,}485\ \text{W}$

$$= 10{,}485\ \text{W}\left(\frac{1\ \text{hp}}{746\ \text{W}}\right) = 14.1\ \text{hp}$$

Problem # 2

The induced torque is given by $$\tau_{\text{ind}} = \frac{P_{\text{AG}}}{\omega_{\text{sync}}} = \frac{11{,}845 \text{ W}}{188.5 \text{ rad/s}} = 62.8 \text{ N} \bullet \text{m}$$

The output torque is given by $$\tau_{\text{load}} = \frac{P_{\text{out}}}{\omega_m} = \frac{10{,}485 \text{ W}}{184.4 \text{ rad/s}} = 56.9 \text{ N} \bullet \text{m}$$

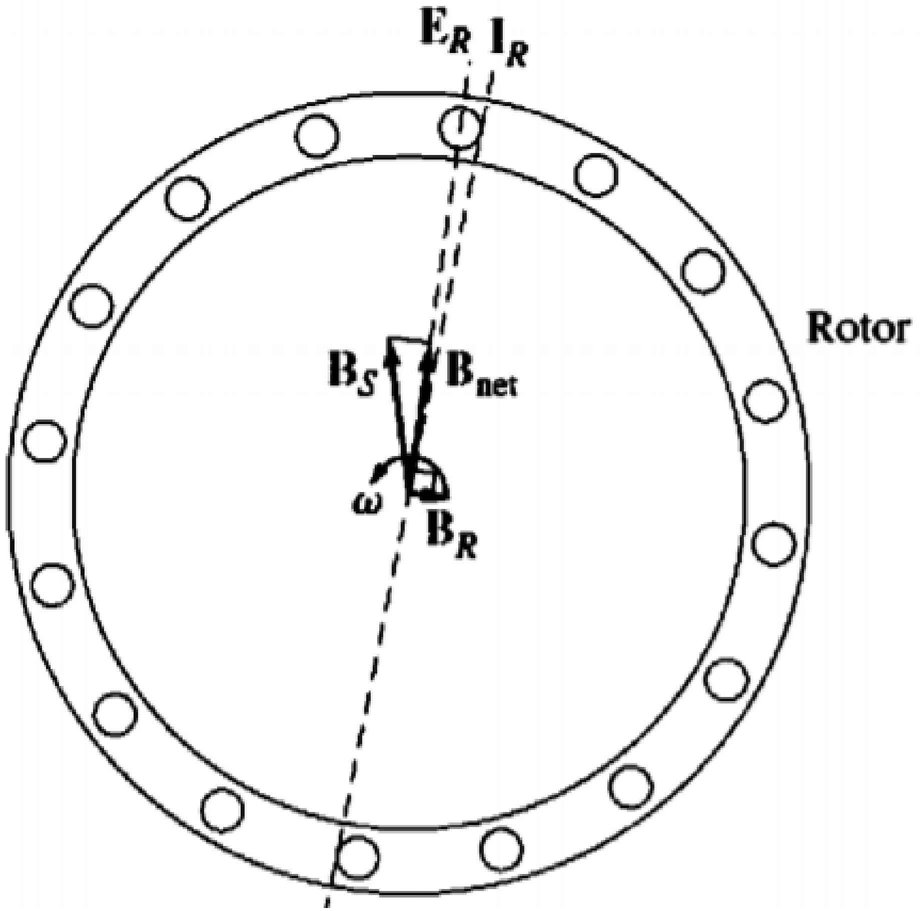
The motor's efficiency at this operating condition is $$\eta = \frac{P_{\text{out}}}{P_{\text{in}}} \times 100\%$$

$$= \frac{10{,}485 \text{ W}}{12{,}530 \text{ W}} \times 100\% = 83.7\%$$

Torque-Speed characteristics

Motor under light load or no-load condition

- At no load, the rotor rotates almost at synchronous speed and hence slip is very small.
- Thus E_R is small, So is I_R.
- Small I_R produces small B_R.
- Since s is small, rotor frequency $(f_r = s.f_e)$ is also very small.
- Thus $X_R \ll R_R$ and therefore, I_R is almost in phase with E_R.

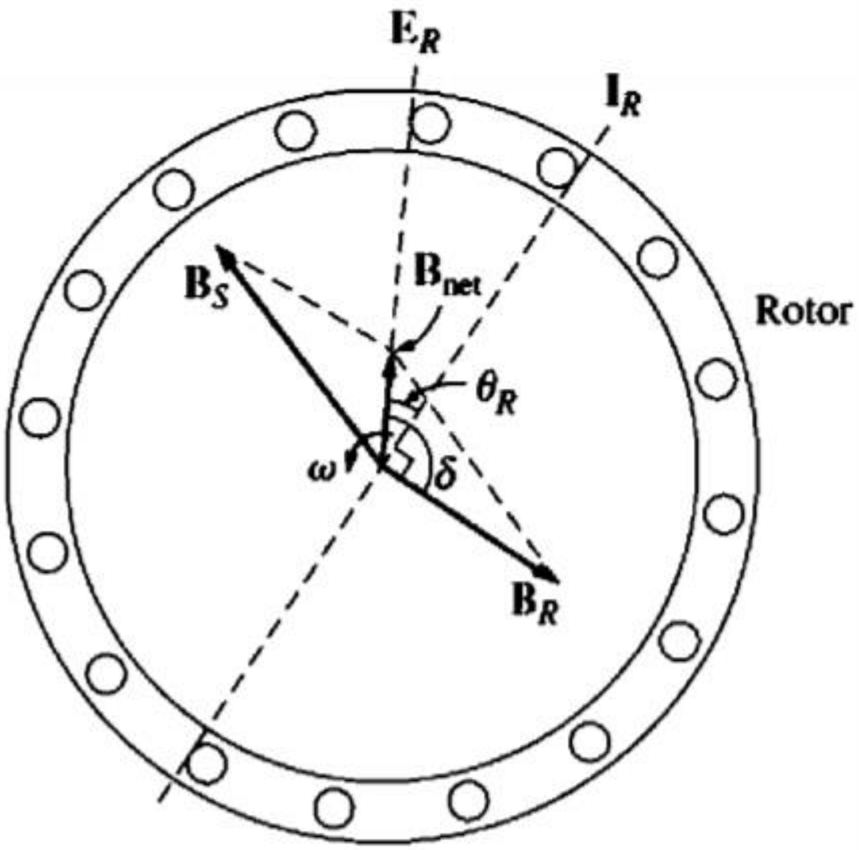


Torque-Speed characteristics

Motor on load

$$I_R = \frac{E_{RO}}{\left({R_R}/{s}\right) + jX_{RO}}$$

- Speed drops and hence slip increases.
- Thus E_R increases, So does I_R
- Large I_R produces larger B_R.
- Rotor frequency ($f_r = s.f_e$) now increases.
- X_R now becomes significant



Note that δ here is greater than 90°

$$\delta = 90 + \theta_R$$

Torque-Speed characteristics

$$\boldsymbol{\tau_{ind}} = k\ \mathbf{B_R} \times \mathbf{B_{net}}$$

$$\tau_{ind} = kB_R B_{net}\ sin\delta$$

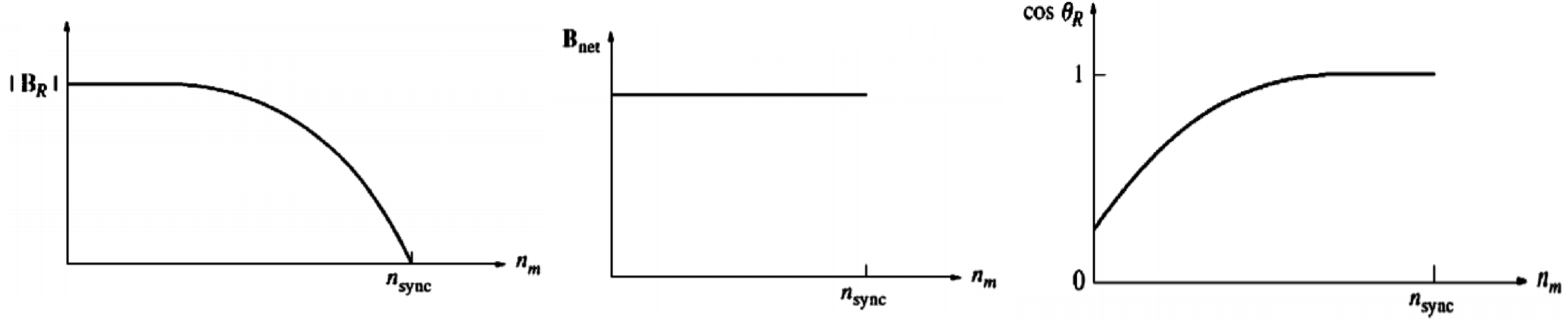
$$= kB_R B_{net}\sin(90 + \theta_R)$$

$$\tau_{ind} = kB_R B_{net}\cos\theta_R$$

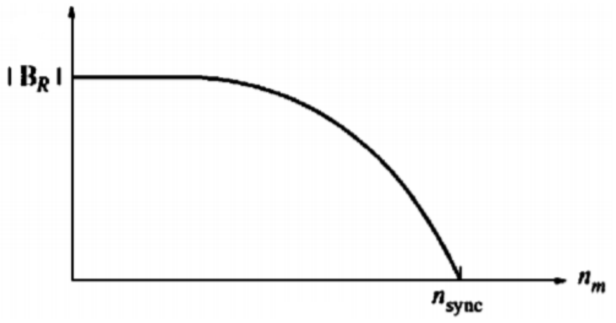
Rotor PF angle

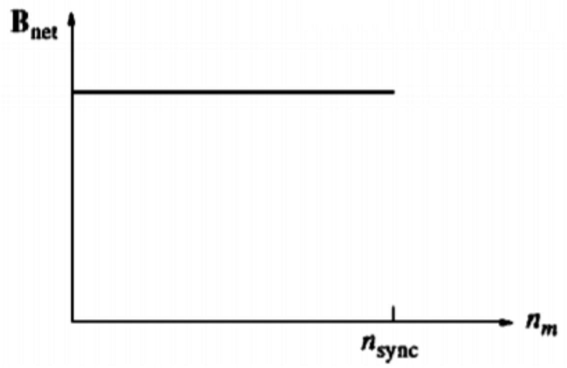
$$\theta_R = tan^{-}\left(\frac{X_R}{R_R}\right) = tan^{-}\left(\frac{s.X_{RO}}{R_R}\right)$$

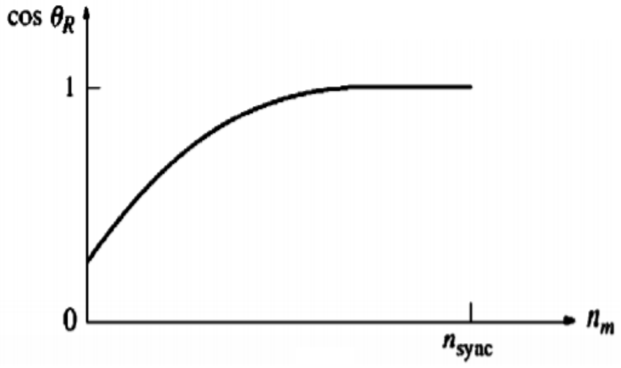
The net magnetic field B_{net} is proportional to E_1 and therefore is approx. const.

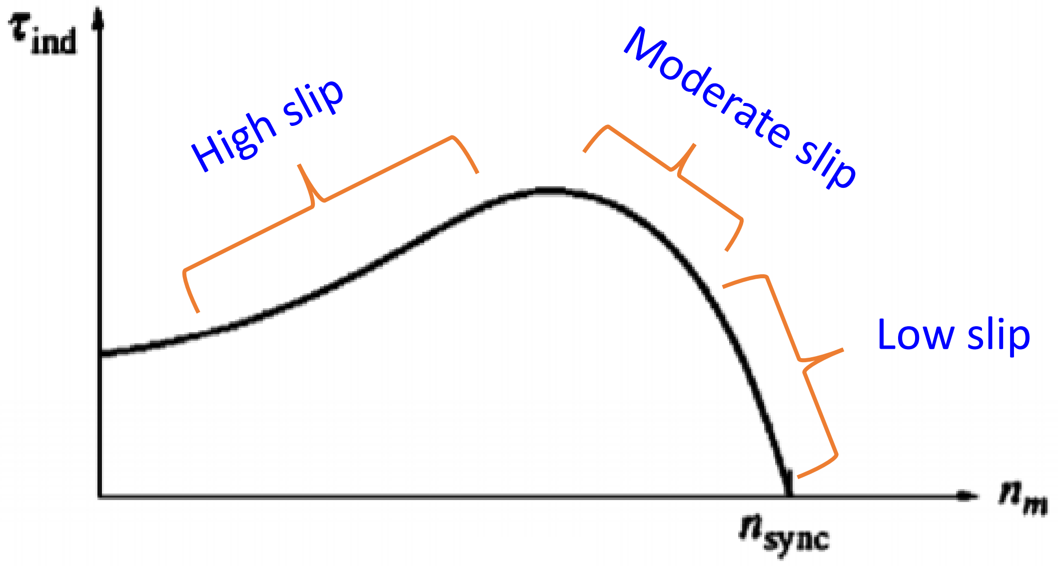


Torque-Speed characteristics









Torque-Speed characteristics

- ✓ In the low-slip region, the motor slip increases approximately linearly with increased load.
- ✓ In the moderate slip region, the rotor frequency is higher than before, and the rotor reactance is on the same order of magnitude as the rotor resistance. The pullout torque of the motor occurs at the point where, for an incremental increase in load, the increase in the rotor current is exactly balanced by the decrease in the rotor power factor.
- ✓ In the high-slip region, the induced torque actually decreases with increased load, since the increase in rotor current is completely overshadowed by the decrease in rotor power factor.
- ✓ For a typical induction motor, the *pullout torque* will be 200 to 250 percent of the rated full load torque of the machine, and the *starting torque* will be 150 percent or so of the full load torque