# PHY 107
# Vector/Scalar

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June 9, 2018

# OUTLINE

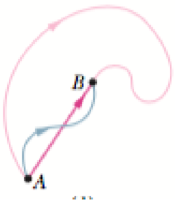
- Vector and Scalar
- Displacement Vector
- Adding vectors geometrically/Properties of vector addition
- Head to tail arrangement
- Components of vectors
- Unit vectors
- Adding Vectors by components
- Multiplication
- Scalar Product
- Vector Product

# Vector and Scalar

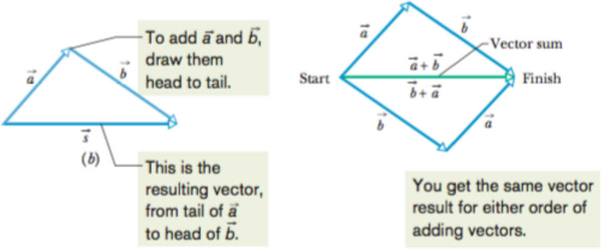
A **vector** is a direction in a space of some specific dimension
Vector quantity has both magnitude and direction e.g. velocity, displacement...Such quantity is represented by the use of an overhead arrow e.g. $\overrightarrow{v}$
**Scalar quantity** has magnitude only e.g. speed, temperature...

# Displacement Vector

It is a vector to denote the change in position of a particle.
It tells us NOTHING about the path taken by the particle.



# Adding vectors geometrically/Properties of vector addition




2 important properties of vector addition:

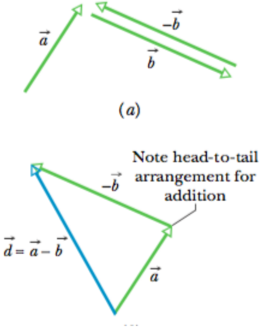
Commutative Law: the order of addition does NOT matter

$$\vec{a} + \vec{b} = \vec{b} + \vec{a}$$

Associative Law: In case of more than 2 vectors , we can group them in any order.

$$(\vec{a} + \vec{b}) + \vec{c} = \vec{a} + (\vec{b} + \vec{c})$$

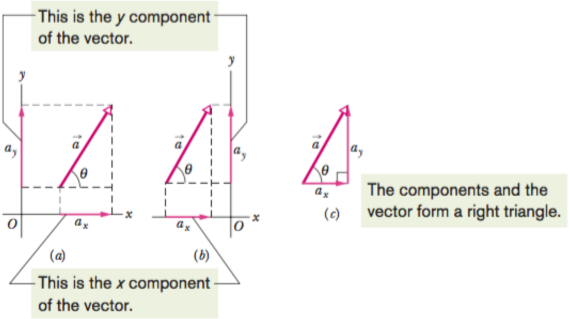
# Head to tail arrangement




Vectors can be added/subtracted, but they need to be of the same kind.

# Components of vectors

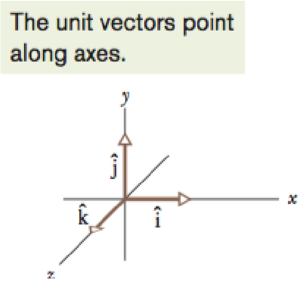
A component of a vector is the projection of the vector on an axis.

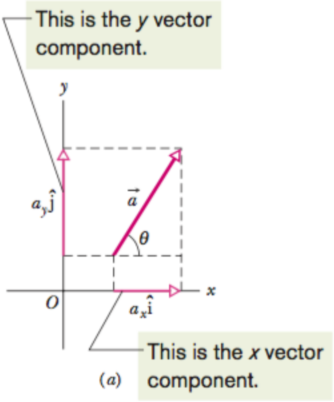



$a_x = |\overrightarrow{a}||\overrightarrow{b}|cos(\theta)$
$a_y = |\overrightarrow{a}||\overrightarrow{b}|sin(\theta)$
Given $a_x$ and $a_y$, can we compute $\overrightarrow{a}, \theta$?
$a = \sqrt{a_x^2 + a_y^2}$ , $tan(\theta) = \frac{a_y}{a_x}$

# Unit vectors

A unit vector is a vector of magnitude 1 and points in a particular direction



# Adding vectors by components

Let us say we have two vectors $\overrightarrow{a}$ and $\overrightarrow{b}$:
$\overrightarrow{a} = a_x\hat{i} + a_y\hat{j}$
$\overrightarrow{b} = b_x\hat{i} + b_y\hat{j}$
Find $\overrightarrow{r} = \overrightarrow{a} + \overrightarrow{b}$
$\overrightarrow{r} = (a_x + b_x)\hat{i} + (a_y + b_y)\hat{j}$

# Multiplication

**Multiplying a vector by a scalar**

$\overrightarrow{k} = s\overrightarrow{a}$

if s is +ve, then $\overrightarrow{k}$ has the same direction as $\overrightarrow{a}$

if s is -ve, then $\overrightarrow{k}$ has the opposite direction as $\overrightarrow{a}$

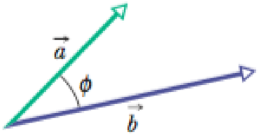
**Multiplying a vector by a vector**

SCALAR PRODUCT: gives you a scalar

VECTOR PRODUCT: gives you a vector

# Scalar Product

The scalar product of vectors $\vec{a}$ and $\vec{b}$ is:
$\vec{a}.\vec{b} = |\vec{a}||\vec{b}|\cos\phi$




Both $\phi$ and $(360 - \phi)$ would give the same scalar product

| $\phi$ | $\vec{a}.\vec{b}$ |
|---|---|
| 0 | ab (Max) |
| 90 | 0 |

# Scalar Product

Commutative Law applies to a scalar product

$\overrightarrow{a}.\overrightarrow{b} = \overrightarrow{b}.\overrightarrow{a}$

In UNIT vector notation (2D):

$\overrightarrow{a}.\overrightarrow{b} = (a_x\hat{i} + a_y\hat{j}).(b_x\hat{i} + b_y j) = a_x b_x + a_y b_y$

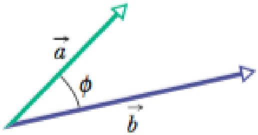
# Vector Product

The vector product of $\overrightarrow{a}$ and $\overrightarrow{b}$ ($\overrightarrow{a}$ X $\overrightarrow{b}$) gives a third vector $\overrightarrow{c}$ of magnitude

$c = ab\sin\phi$

$\phi$: smaller of the two angles between $\overrightarrow{a}$ and $\overrightarrow{b}$

since $sin(\phi) \neq sin(360 - \phi)$

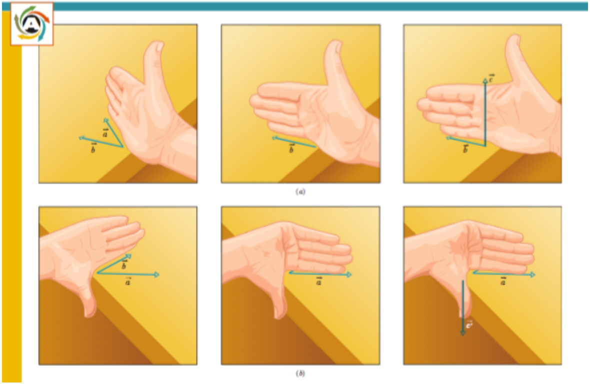



**Note that $\phi$ and $(360 - \phi)$ would give different vector products**

| $\phi$ | $\lvert\overrightarrow{a}\ X\ \overrightarrow{b}\rvert$ |
|---|---|
| 0 | 0 |
| 90 | ab |

# Vector Product

How to determine the direction of the third vector?
Right hand rule: Sweep your fingers (starting with the first vector) towards the second vector, then the thumb points to the third direction



$$(\vec{a} \ X \ \vec{b}) = -(\vec{b} \ X \ \vec{a})$$

# Vector Product

In UNIT vector notation (3D):

$\overrightarrow{a}\ X\ \overrightarrow{b} = (a_x\hat{i} + a_y\hat{j} + a_z\hat{k})\ X\ (b_x\hat{i} + b_y\hat{j} + b_z\hat{k})$
$= a_x b_x(\hat{i}\ X\ \hat{i}) + a_x b_y(\hat{i}\ X\ \hat{j}) + a_x b_z(\hat{i}\ X\ \hat{k}) + a_y b_x(\hat{j}\ X\ \hat{i}) + a_y b_y(\hat{j}\ X\ \hat{j}) + a_y b_z(\hat{j}\ X\ \hat{k}) + a_z b_x(\hat{k}\ X\ \hat{i}) + a_z b_y(\hat{k}\ X\ \hat{j}) + a_z b_z(\hat{k}\ X\ \hat{k})$
$=(a_y b_z - a_z b_y)\hat{i} + (a_z b_x - a_x b_z)\hat{j} + (a_x b_y - a_y b_x)\hat{k}$

# Reference

Fundamentals of Physics by Halliday and Resnick