Function

Relation:

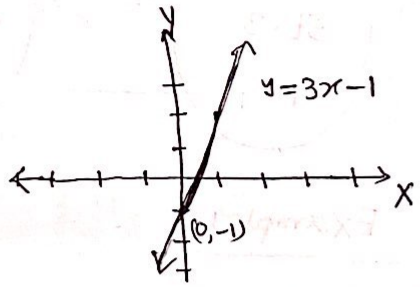
A relation is a correspondence between two sets. When the value of one variable is related to the value of a second variable then we have a relation.

For example, if x and y are two elements in these sets and if a relation exists between x and y, then we say that x corresponds to y or y depends on x.

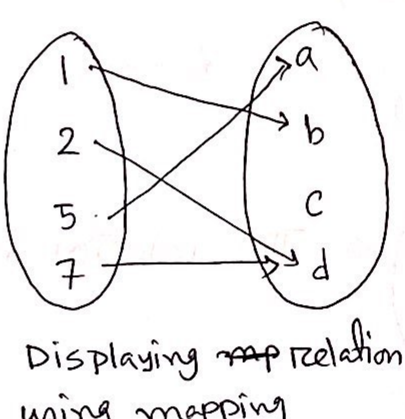
We can express relations between two sets by using equations or graph or mapping or ordered pairs.

Express by equations:

For example $y=3x-1$ shows a relation between x and y. Here x serves as input and y serves as output.



Express by mapping:



Displaying ~~map~~ relation using mapping

ordered pairs:

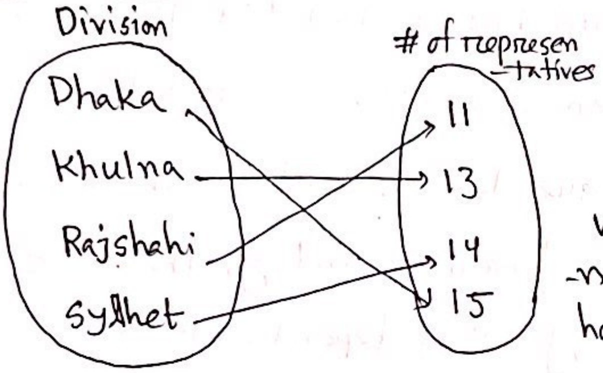
$(1,b), (2,d), (5,a), (7,d)$

Displaying relation as an ordered pairs.

Function:

A Function is a special type of relation.

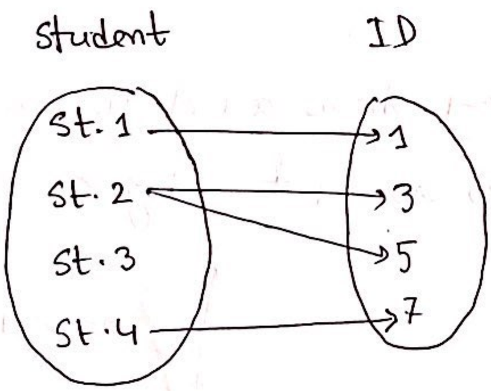
Example: 1



In this example the relationship is represented by mapping. We can see Dhaka has 15 representatives, Khulna has 13, Rajshahi has 14 and sylhet has 15.

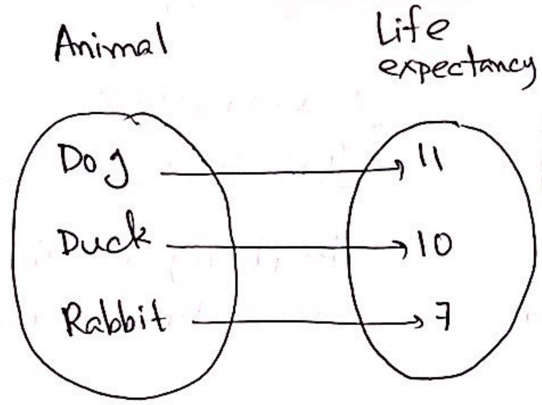
So for each division we have unique number of representatives. So this relation is a function.

Example : 2



Not a function. Because st. 2 has two ID. So you cannot assign a single ID for that student. St. 3 doesn't have any ID.

Example : 3



This relation is a function.

Thus a **function** from X into Y is a relation that associates with each element of X exactly one element of Y.

The set X is called **domain** of the function.

The set of all images of the elements in the domain is called the **range** of the function.

Determine whether a relation represents a function

(a) $\{(1,4), (2,5), (3,6), (4,7)\}$

(b) $\{(1,4), (2,4), (3,5), (6,10)\}$

(c) $\{(-3,9), (-2,4), (0,0), (1,1), (-3,8)\}$

Solution:

(a) This relation is a function because there are no ordered pairs with the same first element and different second elements. The domain of this function $\{1, 2, 3, 4\}$ and range is $\{4, 5, 6, 7\}$.

(b) This relation is also a function.

Domain = $\{1, 2, 3, 6\}$ Range = $\{4, 5, 10\}$

(c) This relation is not a function, because there are two ordered pairs $(-3,9)$ and $(-3,8)$ that have the same first element and different second elements.

Determine whether an equation is a function:

i) Determine if the equation $y=2x-5$ defines y as a function of x.

Solution: $y=2x-5$ is a function because for any input x, we will have only one output y.

for example, if $x=1$, then $y=2.1-5=-3$.

if $x=3$, then $y=2.3-5=1$.

ii) Determine if $x^2+y^2=1$ defines y as a function of x

Solution: Given $x^2+y^2=1$

$$\Rightarrow y^2=1-x^2$$

$$\Rightarrow y=\pm\sqrt{1-x^2}$$

For values of x between -1 and 1, two values of y result. If $x=0$, $y=\pm 1$. This means that $x^2+y^2=1$ does not define a function.

Example: Find values of function:

$f=2x^2-3x$, Evaluate

(a) $f(3)$ (b) $f(x)+f(3)$ (c) $3f(x)$ (d) $f(-x)$

(e) $f(3x)$ f) $f(x+3)$ (g) $\dfrac{f(x+h)-f(x)}{h}$ $h\neq 0$

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Sol^n: (a) $f(x) = 2x^2 - 3x$

$\therefore f(3) = 2.(3)^2 - 3(3) = 18 - 9 = 9$

(b) $f(x) + f(3) = (2x^2 - 3x) + 9 = 2x^2 - 3x + 9$

(f) $f(x+3) = 2(x+3)^2 - 3(x+3)$

$= 2(x^2 + 6x + 9) - 3x - 9$

$= 2x^2 + 12x + 18 - 3x - 9$

$= 2x^2 + 9x + 9$

Find Domain of Function:

The domain of f is the largest set of real numbers for which the value of $f(x)$ is a real number.

Rules:

1. If the equation has a denominator, exclude any numbers that give a zero denominator.

2. If the equation has a radical of even index, exclude any numbers that cause the expression inside the radical to be negative.

Find domain of each of the following functions:

(a) $f(x) = x^2 + 5x$ (b) $g(x) = \dfrac{3x}{x^2 - 4}$

(c) $h(t) = \sqrt{4 - 3t}$ (d) $F(x) = \dfrac{\sqrt{3x+12}}{x-5}$

(a) $f(x) = x^2 + 5x$

This function is valid or defined for all values of x.
So domain is the set of all real numbers.

$D = (-\infty, \infty)$

(b) $f(x) = \dfrac{3x}{x^2-4}$

This function has a denominator x^2-4 and the function is not defined when $x^2-4=0$ i.e the function is not defined at $x=2$ or $x=-2$.

Thus domain of g is $\{x \mid x \neq -2, x = +2\}$

(c) $h(t) = \sqrt{4-3t}$

This function is defined when $\sqrt{4-3t}$ is defined. i.e

when $4-3t \geq 0$

$\Rightarrow -3t \geq -4 \Rightarrow t \leq \frac{4}{3}$

The domain of h is $\{t \mid t \leq \frac{4}{3}\}$ or the interval $(-\infty, \frac{4}{3}]$.

(d) $F(x) = \dfrac{\sqrt{3x+12}}{x-5}$

This function requires $3x+12 \geq 0$ and $x-5 \neq 0$

i.e $x \geq -4$ and $x \neq 5$

combining these two restrictions, domain of F is $\{x \mid x \geq -4, x \neq 5\}$.

Properties of function:

i) $(f+g)(x) = f(x) + g(x)$

ii) $(f-g)(x) = f(x) - g(x)$

iii) $(f * g)(x) = f(x) * g(x)$

iv) $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$

Example:

$f(x) = \frac{1}{x+2} \qquad g(x) = \frac{x}{x-1}$

(a) $(f+g)(x)$ (b) $(f-g)(x)$ (c) $(f\,g)(x)$ (d) $\left(\frac{f}{g}\right)(x)$

Soln:

$$(f+g)(x) = f(x) + g(x)$$
$$= \frac{1}{x+2} + \frac{x}{x-1}$$
$$= \frac{x-1+x(x+2)}{(x+2)(x-1)} = \frac{x-1+x^2+2x}{(x+2)(x-1)}$$
$$= \frac{x^2+3x-1}{(x+2)(x-1)}$$

The domain of $f(x)$ is $\{x \mid x \neq -2\}$ and $g(x)$ is $\{x \mid x \neq 1\}$

Therefore the domain of $f+g$ is $\{x \mid x \neq -2, x \neq 1\}$

(d) $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{1/x+2}{x/x-1} = \frac{1}{x+2}\,\frac{x-1}{x}$

$$= \frac{x-1}{x(x+2)}$$

The domain of $\frac{f}{g}$ is $\{x \mid x \neq -2, x \neq 0, x \neq 1\}$

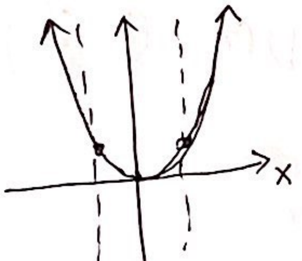
The graph of function:

Identify the graph of a function:

Vertical line test:

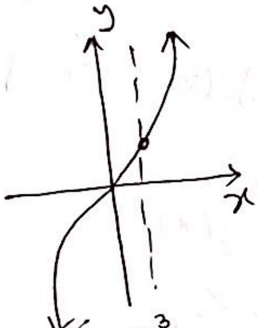
A set of points in the xy-plane is the graph of function if and only if every vertical line intersects the graph in at most one point.

For example:



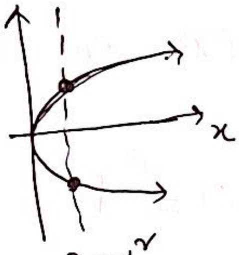
$y = x^2$

function



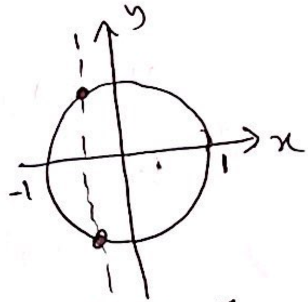
$y = x^3$

function



$x = y^2$

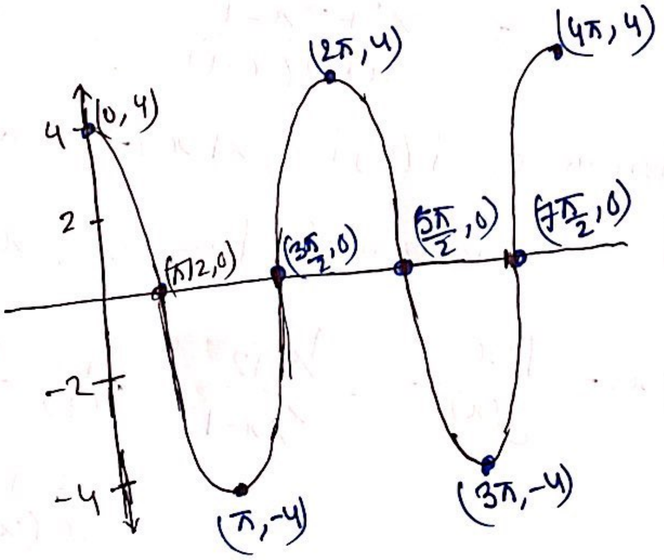
not a function



$x^2 + y^2 = 1$

not a function

Example:



a). What are $f(0)$, $f(3\pi/2)$ and $f(3\pi)$

Ans: $f(0) = 4$, $f\left(\frac{3\pi}{2}\right) = 0$, $f(3\pi) = -4$

(b) what is the domain of f?

Ans: The domain of f is $\{x \mid 0 \leq x \leq 4\pi\}$ or $[0, 4\pi]$.

(c) what is the range of f

Ans: The range of f is $\{y \mid -4 \leq y \leq 4\}$ or $[-4, 4]$

(d) List the intercepts points:

Ans: The intercepts points are

$(0,4)$, $\left(\frac{\pi}{2}, 0\right)$, $\left(\frac{3\pi}{2}, 0\right)$, $\left(\frac{5\pi}{2}, 0\right)$ and $\left(\frac{7\pi}{2}, 0\right)$

(e) How many times does the line $y=2$ intersect the graph?

Ans: four times

(f) for what values of x does $f(x) = -4$?

Ans: we have $f(x) = -4$ when $x = \pi$ and $x = 3\pi$

(g) for what values of x is $f(x) > 0$?

Ans: To determine where $f(x) > 0$, we have to determine x-values from 0 to 4π for which the y-coordinate is positive. This occurs on

$\left[0, \frac{\pi}{2}\right) \cup \left(\frac{3\pi}{2}, \frac{5\pi}{2}\right) \cup \left(\frac{7\pi}{2}, 4\pi\right]$.

Example: $f(x) = \frac{x+1}{x+2}$

(a) find domain of f?

Ans: The domain of f is $\{x \mid x \neq -2\}$

(b) Is the point $\left(1, \frac{1}{2}\right)$ on the graph of f?

Ans: when $x=1$ then

$$f(1) = \frac{1+1}{1+2} = \frac{2}{3}$$

The point $\left(1, \frac{2}{3}\right)$ is on the graph of f, the point $\left(1, \frac{1}{2}\right)$ is not.

(c) If $x=2$, what is $f(x)$? What point is on the graph of f?

Ans: When $x=2$ then $f(2) = \frac{2+1}{2+2} = \frac{3}{4}$

The point $\left(2, \frac{3}{4}\right)$ is on the graph of f.

(d) If $f(x)=2$, what is x? what point is on the graph of f?

Ans: $f(x) = 2$

$\Rightarrow \frac{x+1}{x+2} = 2$

$\Rightarrow x+1 = 2(x+2)$

$\Rightarrow x+1 = 2x+4$

$\Rightarrow 2x - x = 1-4$ $\Rightarrow x = -3$

The point $(-3,2)$ is on the graph of f.

(e) what are the x-intercepts of the graph? what points are on the graph of f?

Ans: To find x-intercept we have to put $f(x)=0$.

$\therefore f(x) = \frac{x+1}{x+2}$

$\Rightarrow \frac{x+1}{x+2} = 0 \Rightarrow x+1=0$

$\Rightarrow x=-1$

So -1 is the only x-intercept.

Since $f(-1)=0$, the point $(-1,0)$ is on the graph of f.

Exercise:

Section 2.1: 15, 17, 19, 24, 31, 34, 36, 39, 45, 46, 47-62, 63, 66, 71, 73.

Section 2.2: 9, 11-22, 24, 27.