8.22

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Q5

$$\underline{x}' = \begin{pmatrix} 3 & -18 \\ 2 & -9 \end{pmatrix} \underline{x}$$

let, $\underline{y} = \underline{k} e^{\lambda t} \neq 0$

$\therefore \underline{x}_1 = ?$

$\underline{x}_2 = ?$

$$\lambda I - A = 0$$

$$\Rightarrow \lambda = -3, -3$$

$\lambda = -3$

$$(A I - A)\, \underline{k}\, e^{-3t} = \underline{0}$$

$$\Rightarrow (-3I - A) \begin{vmatrix} k_1 \\ k_2 \end{vmatrix} = \begin{vmatrix} 0 \\ 0 \end{vmatrix}$$

$$\Rightarrow \left(-3 \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} - A \right) \begin{vmatrix} k_1 \\ k_2 \end{vmatrix} = \begin{vmatrix} 0 \\ 0 \end{vmatrix}$$

$$\Rightarrow \begin{pmatrix} -3 & 0 \\ 0 & -3 \end{pmatrix} - A \begin{vmatrix} k_1 \\ k_2 \end{vmatrix} = \begin{vmatrix} 0 \\ 0 \end{vmatrix}$$

$$\Rightarrow \left(\begin{pmatrix} -3 & 0 \\ 0 & -3 \end{pmatrix} - \begin{vmatrix} 3 & -18 \\ 2 & -9 \end{vmatrix} \right) \begin{vmatrix} k_1 \\ k_2 \end{vmatrix} = \begin{vmatrix} 0 \\ 0 \end{vmatrix}$$

$$\Rightarrow \begin{vmatrix} -6 & 18 \\ -2 & 6 \end{vmatrix} \begin{vmatrix} k_1 \\ k_2 \end{vmatrix} = \begin{vmatrix} 0 \\ 0 \end{vmatrix}$$

$$\Rightarrow -6k_1 + 18k_2 = 0$$

$$-2k_1 + 6k_2 = 0$$

$$\Rightarrow k_1 = 3k_2 \qquad \therefore k = \begin{vmatrix} 3 \\ 1 \end{vmatrix}$$

$$\boxed{X_1 = \begin{vmatrix} 3 \\ 1 \end{vmatrix} e^{-3t}}$$

$X_2 =$

$$(\lambda I - A)\, P = k$$

$$\Rightarrow \begin{vmatrix} -6 & 18 \\ -2 & 6 \end{vmatrix} \begin{vmatrix} P_1 \\ P_2 \end{vmatrix} = \begin{vmatrix} 3 \\ 1 \end{vmatrix}$$

$$\Rightarrow -6P_1 + 18P_2 = 3$$

$$\Rightarrow -2P_1 + 6P_2 = 1$$

$$\Rightarrow P_1 = \frac{1 - 6P_2}{-2}$$

if $P_2 = 1/6 \Rightarrow P_1 = 0$

or $P_2 = 0, \; P_1 = 1/2$

$$\Rightarrow P = \begin{vmatrix} 1/2 \\ 0 \end{vmatrix}$$

$$\Rightarrow X_2 = \underline{k} t e^{\lambda t} + \underline{P} e^{\lambda t}$$

$$= \begin{vmatrix} 3 \\ 1 \end{vmatrix} t e^{-3t} + \begin{vmatrix} 1/2 \\ 0 \end{vmatrix} e^{-3t}$$

$$X = C_1 X_1 + C_2 X_2$$

$$= C_1 \left[\begin{vmatrix} 3 \\ 1 \end{vmatrix} e^{-3t} \right] + C_2 \left[\begin{vmatrix} 3 \\ 1 \end{vmatrix} t e^{-3t} + \begin{vmatrix} 1/2 \\ 0 \end{vmatrix} e^{-3t} \right]$$

MID

$$\frac{dx}{dt} = 6x - y$$

$$\frac{dy}{dt} = 5x + 4y$$

$$A = \begin{vmatrix} 6 & -1 \\ 5 & 4 \end{vmatrix}$$

$$\lambda I - A = 0$$

$$\Rightarrow \begin{vmatrix} \lambda & 0 \\ 0 & \lambda \end{vmatrix} - \begin{vmatrix} 6 & -1 \\ 5 & 4 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} \lambda - 6 & +1 \\ -5 & \lambda - 4 \end{vmatrix} = 0$$

$$\Rightarrow (\lambda - 6)(\lambda - 4) - 5 = 0$$

$$\Rightarrow \lambda^2 - 6\lambda - 4\lambda + 24 + 5 = 0$$

$$\Rightarrow \lambda^2 - 10\lambda + 29 = 0$$

$$\Rightarrow \lambda = \frac{-(-10) \pm \sqrt{(-10)^2 - 4 \cdot 1 \cdot 29}}{2 \cdot 1}$$

$$= \frac{10 \pm \sqrt{100 - 116}}{2}$$

$$= \frac{10 \pm \sqrt{-16}}{2}$$

$$\Rightarrow 5 \pm \sqrt{-4}$$

$$\Rightarrow 5 \pm 2i$$

$$\therefore (\lambda I - A)\, k = 0$$

$$\Rightarrow \left(\begin{pmatrix} 5+2i & 0 \\ 0 & 5+2i \end{pmatrix} - \begin{vmatrix} 6 & -1 \\ 5 & 4 \end{vmatrix} \right) \begin{vmatrix} k_1 \\ k_2 \end{vmatrix} = \begin{vmatrix} 0 \\ 0 \end{vmatrix}$$

$$\Rightarrow \begin{pmatrix} 2i - 1 & 1 \\ -5 & 2i + 1 \end{pmatrix} \begin{vmatrix} k_1 \\ k_2 \end{vmatrix} = \begin{vmatrix} 0 \\ 0 \end{vmatrix}$$

$$\Rightarrow (2i - 1)k_1 + k_2 = 0$$

$$\Rightarrow 2ik_1 - k_1 + k_2 = 0$$

$$-5k_1 + (2i + 1)k_2 = 0$$

$$\Rightarrow -5k_1 + 2ik_2 + k_2 = 0$$

$$k_1 = 5$$

$$\Rightarrow k_2 = (1 - 2i)\, 5$$

$$\therefore k = 5 \begin{pmatrix} 1 \\ 1 - 2i \end{pmatrix}$$

$$X_1 = \begin{pmatrix} 1 \\ 1 - 2i \end{pmatrix} e^{(5+2i)t}$$

$$\begin{pmatrix} 5 - 2i - 6 & 1 \\ -5 & 5 - 2i - 4 \end{pmatrix} \begin{vmatrix} k_1 \\ k_2 \end{vmatrix} = \begin{vmatrix} 0 \\ 0 \end{vmatrix}$$

$$\Rightarrow \begin{pmatrix} -2i - 1 & 1 \\ -5 & -2i + 1 \end{pmatrix} \begin{vmatrix} k_1 \\ k_2 \end{vmatrix} = \begin{vmatrix} 0 \\ 0 \end{vmatrix}$$

$$\Rightarrow (-2i - 1)k_1 + k_2 = 0$$

$$\therefore k_1 = 5$$

$$k_2 = (2i + 1)\, 5$$

$$\Rightarrow k = 5 \begin{pmatrix} 1 \\ 2i + 1 \end{pmatrix}$$

$$\Rightarrow X_2 = \begin{vmatrix} 1 \\ 2i + 1 \end{vmatrix} e^{(5-2i)t}$$

[Removing i in complex but not needed for mid]