Lab 7: Measurement and Calculation of Power in AC circuits

A. Objectives

- Study the concept of instantaneous power and average power.
- Measure instantaneous and average power.
- Study the concept of Real power, Reactive Power, Apparent Power, Power Factor, Complex Power, Power Triangle and Power Factor Improvement

B. Background

Instantaneous Power:

Power is the rate of doing work or expending energy. The electrical unit of energy is the watt (W) and one watt is the rate of expending energy at the rate of one joule per second. In DC circuit the power dissipated in a resistive circuit is given by:

$$P = UI = I^2R = \frac{U^2}{R} \qquad (1)$$

where : P = power (W); U = potential difference (PD) (V); I = current (A); and R = resistance (Ω)

In AC circuits the instantaneous values of voltage, current and therefore power are constantly changing. However, at any instant we can still say that:

$$P(t)=v(t)\, i(t) \qquad (2)$$

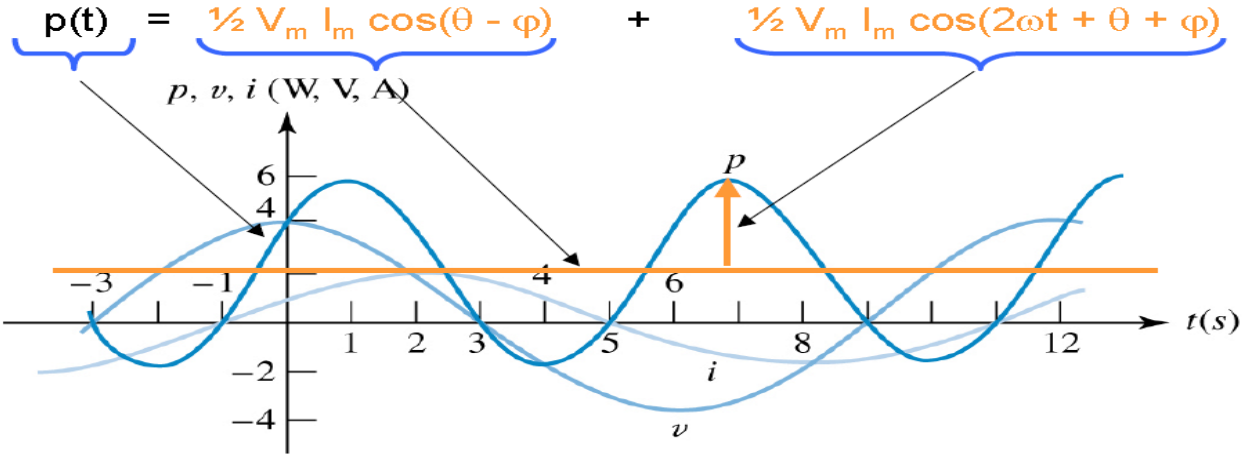
where: p(t) = instantaneous power (W); v(t) = instantaneous voltage (V) and i(t) = instantaneous current (A)

The RMS values (U, I and P) can be easily used in AC circuits with only resistance however, matters are more complicated when capacitance and inductance are involved. Remember that the RMS values are defined so that a current of RMS 1A AC will produce the same heating effect in a resistor as 1A DC.

Now, if $v(t) = V_m \cos(\omega t+\theta)$ and $i(t) = I_m \cos(\omega t +\phi)$, then
$p(t) = V_m I_m \cos(\omega t+\theta) \cos(\omega t +\phi) = ½ V_m I_m \cos(\theta - \phi) + ½ V_m I_m \cos(2\omega t + \theta + \phi)$…………………………..(3)

Average Power

In equation (3), p(t) has two parts: first part "$½ V_m I_m \cos(\theta - \phi)$" is time independent or constant. This portion is defined as the average power or real power. Average AC power can be thought as the DC equivalent of effective power.
For purely resistive circuits, current and voltage are in phase, so the average power is "$½ V_m I_m$" or "$Vm_{rms} Im_{rms}$". But in purely reactive circuit the voltage and current has phase angle difference of ± 90° and the net average power consumed by the reactive element is zero. This is because the power is received and returned back in cycles. The following figure shows the relationship of instantaneous power, average power and reactive power of an ac load.



True (real/active), Reactive and Apparent Power

Reactive loads such as inductors and capacitors dissipate zero power, yet the fact that they drop voltage and draw current gives the deceptive impression that they actually do dissipate power. This “phantom power” is called reactive power, and it is measured in a unit called Volt-Amps-Reactive (VAR), rather than watts. The mathematical symbol for reactive power is (unfortunately) the capital letter Q. The actual amount of power being used, or dissipated, in a circuit is called true power, and it is measured in watts (symbolized by the capital letter P, as always). The combination of reactive power and true power is called apparent power, and it is the product of a circuit’s voltage and current, without reference to phase angle. Apparent power is measured in the unit of Volt-Amps (VA) and is symbolized by the capital letter S.

As a rule, true power is a function of a circuit’s dissipative elements, usually resistances (R). Reactive power is a function of a circuit’s reactance (X). Apparent power is a function of a circuit’s total impedance (Z). Since we’re dealing with scalar quantities for power calculation, any complex starting quantities such as voltage, current, and impedance must be represented by their polar magnitudes, not by real or imaginary rectangular components. For instance, if I’m calculating true power from current and resistance, I must use the polar magnitude for current, and not merely the “real” or “imaginary” portion of the current. If I’m calculating apparent power from voltage and impedance, both of these formerly complex quantities must be reduced to their polar magnitudes for the scalar arithmetic.

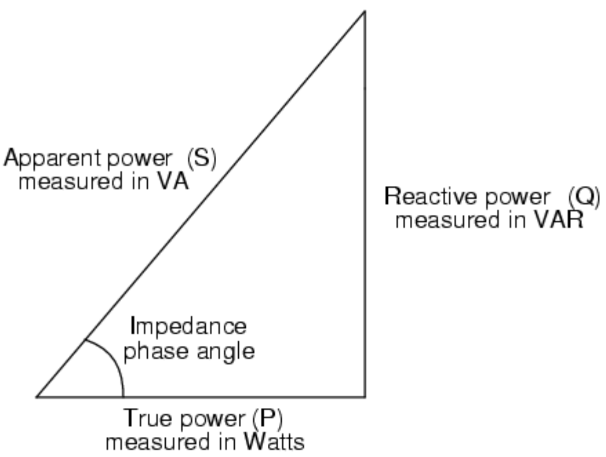
There are several power equations relating the three types of power to resistance, reactance, and impedance (all using scalar quantities):

$$True\ Power, P = I^2R = \frac{E^2}{R}\ (measured\ in\ units\ of\ Watts)$$
$$Reactive\ Power, Q = I^2X = \frac{E^2}{X}(measured\ in\ units\ of\ VAR)$$
$$Apparent\ Power, S = I^2Z = \frac{E^2}{Z} = IE(measured\ in\ uinits\ of\ VA)$$

Power Triangle

True, reactive, and apparent power relate to one another in trigonometric form, which is called the power triangle (as show below):

The "Power Triangle"



Power Factor

The angle of a “power triangle” graphically indicates the ratio between the amount of dissipated (or consumed) power and the amount of absorbed/returned power. It also happens to be the same angle as that of the circuit’s impedance in polar form. When expressed as a fraction, this ratio between true power and apparent power is called the power factor for the circuit. Because true power and apparent power form the adjacent and hypotenuse sides of a right triangle, respectively, the power factor ratio is also equal to the cosine of that phase angle:

$$Power\ Factor, pf = cos\theta = \frac{P}{S}(unitless)$$

For the purely resistive circuit, the power factor is 1 (perfect), because the reactive power equals zero. Here, the power triangle would look like a horizontal line, because the opposite (reactive power) side would have zero length.

For the purely inductive circuit, the power factor is zero, because true power equals zero. Here, the power triangle would look like a vertical line, because the adjacent (true power) side would have zero length.

The same could be said for a purely capacitive circuit. If there are no dissipative (resistive) components in the circuit, then the true power must be equal to zero, making any power in the circuit purely reactive. The power triangle for a purely capacitive circuit would again be a vertical line (pointing down instead of up as it was for the purely inductive circuit).

If a circuit is predominantly inductive, we say that its power factor is lagging (because the current wave for the circuit lags behind the applied voltage wave). Conversely, if a circuit is predominantly capacitive, we say that its power factor is leading.

Power Factor Improvement

Power factor is an important aspect to consider in an AC circuit, because any power factor less than 1 means that the circuit's wiring has to carry more current than what would be necessary with zero reactance in the circuit to deliver the same amount of (true) power to the resistive load. Thus poor power factor makes for an inefficient power delivery system.

Poor power factor can be corrected, paradoxically, by adding another load to the circuit drawing an equal and opposite amount of reactive power, to cancel out the effects of the load's inductive reactance. Inductive reactance can only be canceled by capacitive reactance. The effect of these two opposing reactances in parallel is to bring the circuit's total impedance equal to its total resistance (to make the impedance phase angle equal, or at least closer, to zero).

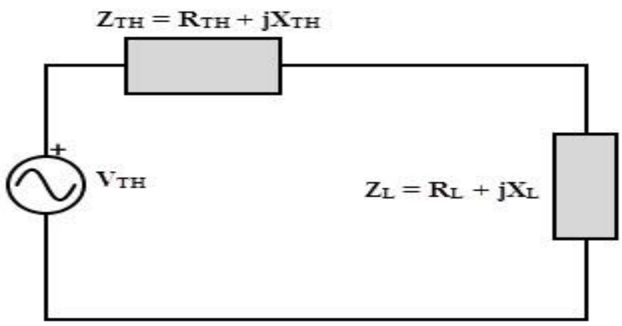
If the (uncorrected) reactive power (inductive) is known, the correct capacitor size (which should be connected directly parallel to the load) to produce the same quantity of (capacitive) reactive power can be calculated by using the power formulas:

$$C = \frac{1}{2\pi f X_C}$$

Maximum Power Transfer Theorem

In an active AC network consisting of source with internal impedance Z_S which is connected to a load Z_L, the maximum power transfer occurs from source to load when the load impedance is equal to the complex conjugate of source impedance Z_S.

Let's consider the below circuit consisting of Thevenin's voltage source with series Thevenin's equivalent resistance (which are actually replacing the complex part of the circuit) connected across the complex load.



From the above figure, Let $Z_L = R_L + jX_L$ and $Z_{TH} = R_{TH} + jX_{TH}$ then the current through the circuit is given as,

$$I = \frac{V_{TH}}{Z_{TH} + Z_L}$$

By substituting above given impedances, we get

$$I = \frac{V_{TH}}{R_{TH} + jX_{TH} + R_L + jX_L}$$

$$I = \frac{V_{TH}}{(R_L + R_{TH}) + j(X_L + X_{TH})}$$

Power delivered to the load, $P_L = I^2 R_L$

$$P_L = \frac{V_{TH}^2 \times R_L}{(R_L + R_{TH})^2 + (Z_L + Z_{TH})^2}$$

For power to be maximized, the above equation must be differentiated with respect to XL and equates it to zero. Then we get:

$$X_L + X_{TH} = 0$$

$$X_L = -X_{TH}$$

Substituting the X_L in power equation, we get

$$P_L = \frac{V_{TH}^2 \times R_L}{(R_L + R_{TH})^2}$$

Again taking derivative of the above equation and equating it to zero,

we get $\quad R_L + R_{TH} = 2R_L$

$$R_L = R_{TH}$$

Therefore, in AC circuits, if $X_L = -X_{TH}$ and $R_L = R_{TH}$, maximum power transfer takes place from source to load. This implies that maximum power transfer occurs when the impedance of the load is complex conjugate of the source impedance, i.e., $Z_L = Z^*_{TH}$.

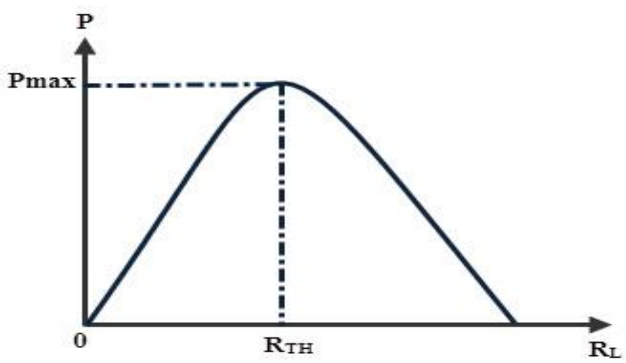
The maximum power,

$$P_{Max} = \frac{V_{TH^2}}{4R_{TH}}$$

Or

$$P_{Max} = \frac{V_{TH^2}}{4R_L}$$

For a given values the Thevenin's voltage and Thevenin's resistance, the variation of power delivered to the load with varying load resistance is shown in the figure.



Experiment 1: Instantaneous and Average Power

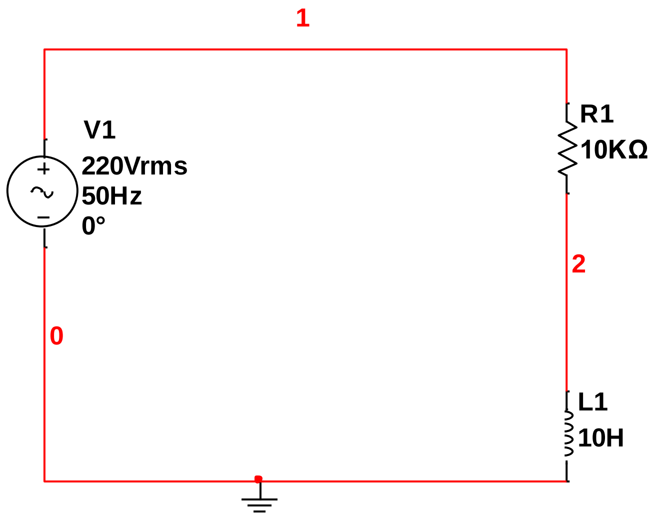


Fig.1: *Inductive load in a series circuit*

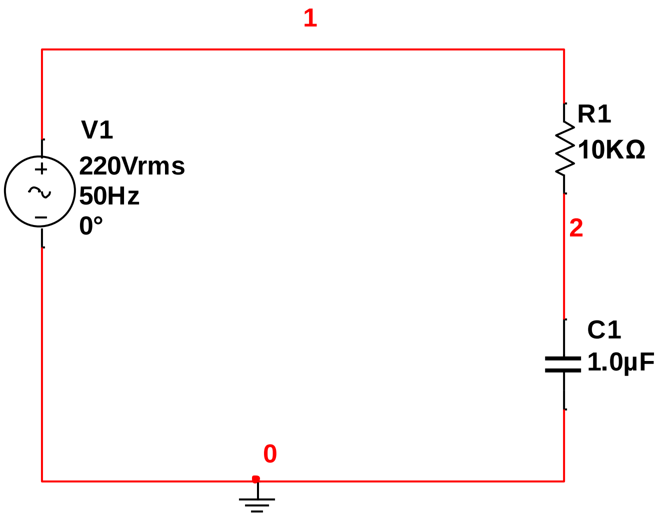


Fig.2: *Capacitive load in a series circuit*

Procedure

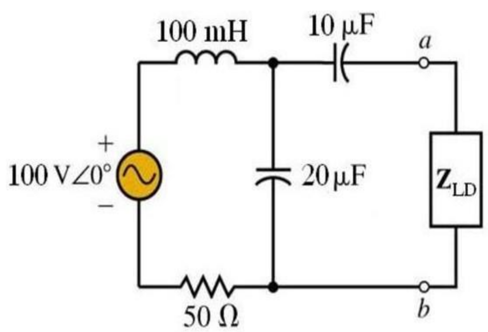
1. Construct the circuit of Fig.1 in MULTISIM and conduct a transient analysis. Using cursors, acquire the information needed to fill up table 1.1,1.2 & 1.3
2. Construct the circuit of Fig.2 in MULTISIM and conduct a transient analysis. Using cursors, acquire the information needed to fill up table 1.4,1.5 & 1.6

Simulation

1. Attach the transient analysis output graphs with your lab report.

Questions

1. How much average power is consumed by the inductor and the capacitor?
2. What is the effect of the inductor and capacitor on the instantaneous power of the R1?
3. Determine the Load Z_{LD} that will allow maximum power to be delivered to the load for the following circuit, if the frequency is 192.241 Hz. What should be the maximum power at the load?
   Construct the final circuit in MULTISIM and measure the power at the load. Is your result similar to the theoretical maximum power that you found? Attach the simulation screenshots in your lab report.



Data Sheet: Lab 7

Date:	Points:
Remarks:	

Signature of the Instructor

Student Information

Section:	Group:	Status:

Table 1.1:

	0°	**90°**	**180°**	**270°**
$v_S(t)$				
$v_{Rload}(t)$				
$v_{Lload}(t)$				
$i_S(t)$				
$P_{Rload}(t)$				
$P_{Lload}(t)$				

Table 1.2:

	Experimental peak
V_S	
V_{Rload}	
V_{Lload}	
$I_S=V_{Rload}/R_{load}$	
Δt	
$\Theta=\Delta t\times f\times 360$	
Δt_L	
Φ	

Table 1.3:

	Theoretical	Experimental	% Difference
V_S (rms)			
V_{Rload} (rms)			
V_{Lload} (rms)			
V_{load} (rms)			
I_S (rms)			
$P_{Rload}=V_{Rload}\times I_S\times\cos\Theta$			
$P_{Lload}=V_{Lload}\times I_S\times\cos\Theta$			
$P_{load}=V_{load}\times I_S\times\cos\Theta$			
$P_{total}=P_{Rload}+P_{Lload}$			
$P_S=V_S\times I_S\times\cos\Theta$			

Table 1.4:

	0°	**90°**	**180°**	**270°**
$v_S(t)$				
$v_{Rload}(t)$				
$V_{Cload}(t)$				
$i_S(t)$				
$P_{Rload}(t)$				
$P_{Cload}(t)$				

Table 1.5:

	Experimental peak
V_S	
V_{Rload}	
V_{Cload}	
$I_S=V_{Rload}/R_{load}$	
Δt	
$\Theta=\Delta t\times f\times 360$	
Δt_C	
Φ	

Table 1.6:

	Theoretical	Experimental	% Difference
V_S (rms)			
V_{Rload} (rms)			
V_{Cload} (rms)			
V_{load} (rms)			
I_S (rms)			
$P_{Rload}=V_{Rload}\times I_S\times\cos\Theta$			
$P_{Cload}=V_{Cload}\times I_S\times\cos\Theta$			
$P_{load}=V_{load}\times I_S\times\cos\Theta$			
$P_{total}=P_{Rload}+P_{Cload}$			
$P_S=V_S\times I_S\times\cos\Theta$			