Priority Queues: Introduction

Data Structures

Data Structures and Algorithms

Outline

1. Overview
2. Naive Implementations

Queue

A queue is an abstract data type supporting the following main operations:

- `PushBack`(*e*) adds an element to the back of the queue;
- `PopFront`() extracts an element from the front of the queue.

Priority Queue (Informally)

A priority queue is a generalization of a queue where each element is assigned a priority and elements come out in order by priority.

Priority Queues: Typical Use Case

Scheduling jobs

- Want to process jobs one by one in order of decreasing priority. While the current job is processed, new jobs may arrive.

Priority Queues: Typical Use Case

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Priority Queues: Typical Use Case

Scheduling jobs

- Want to process jobs one by one in order of decreasing priority. While the current job is processed, new jobs may arrive.
- To add a job to the set of scheduled jobs, call `Insert`(*job*).
- To process a job with the highest priority, get it by calling `ExtractMax`().

Priority Queue (Formally)

Definition

Priority queue is an abstract data type supporting the following main operations:

- `Insert`(*p*) adds a new element with priority *p*
- `ExtractMax`() extracts an element with maximum priority

Example

Contents:

Queries:

Example

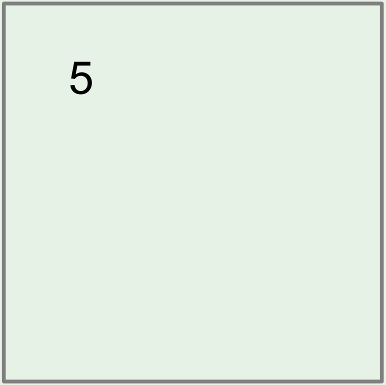
Contents:

Queries:

`Insert`(5)

Example

Contents:

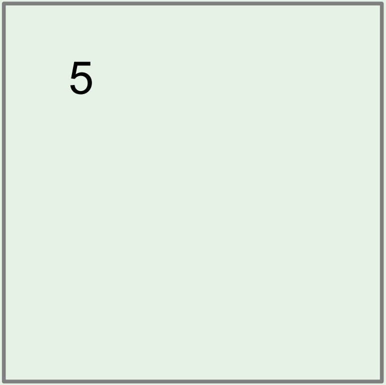


5

Queries:

Example

Contents:

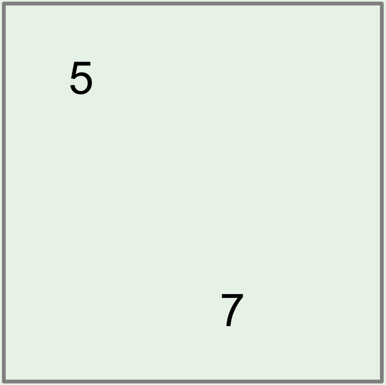


Queries:

`Insert`(7)

Example

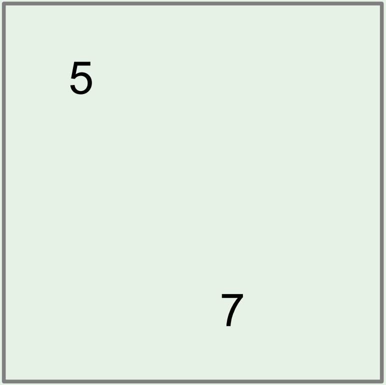
Contents:



Queries:

Example

Contents:



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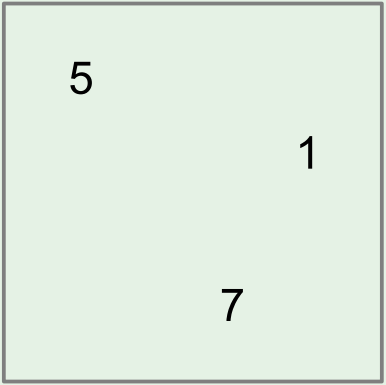
7

Queries:

`Insert`(1)

Example

Contents:



5

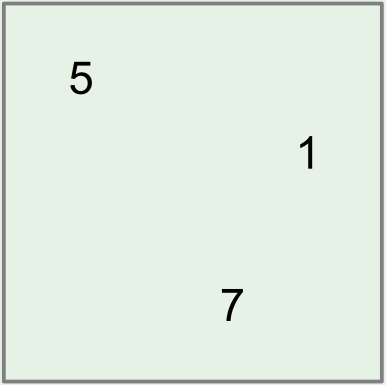
1

7

Queries:

Example

Contents:

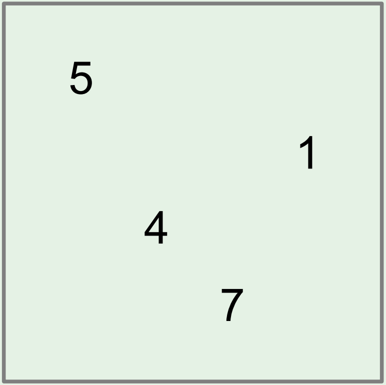


Queries:

`Insert`(4)

Example

Contents:



5

1

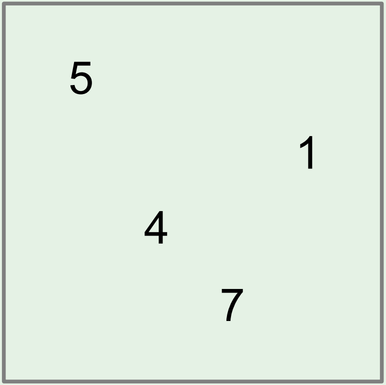
4

7

Queries:

Example

Contents:



5

1

4

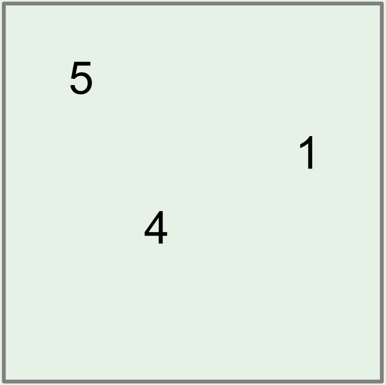
7

Queries:

`ExtractMax`() → 7

Example

Contents:



5

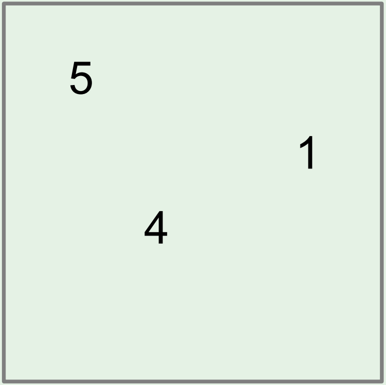
1

4

Queries:

Example

Contents:



5

1

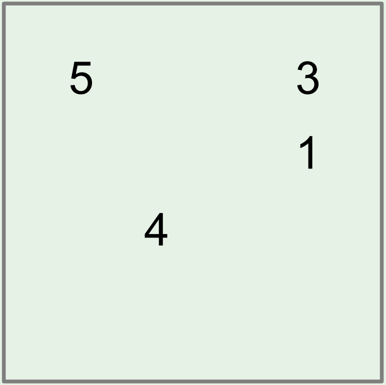
4

Queries:

`Insert`(3)

Example

Contents:



5 3

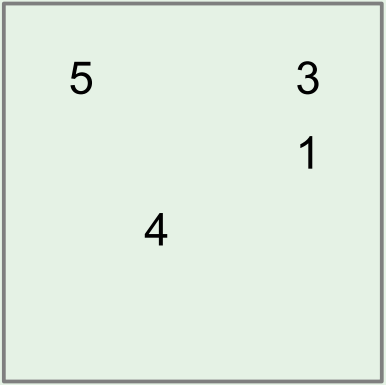
1

4

Queries:

Example

Contents:



5

3

1

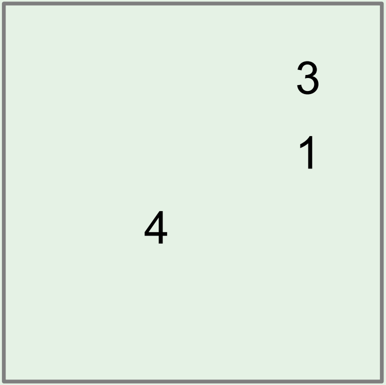
4

Queries:

`ExtractMax`() → 5

Example

Contents:



3

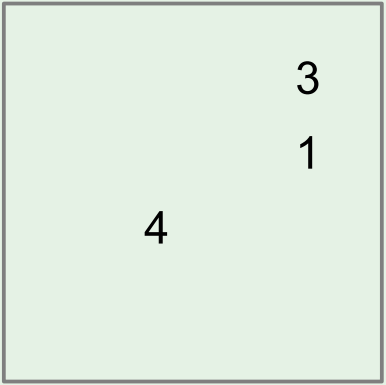
1

4

Queries:

Example

Contents:



3

1

4

Queries:

`ExtractMax`() → 4

Example

Contents:

3

1

Queries:

Question

What will be the output of the following program? (As an answer, provide a sequence of integers separated by spaces.)

```
create an empty priority queue
Insert(18)
Insert(12)
Insert(14)
print(ExtractMax())
print(ExtractMax())
Insert(15)
print(ExtractMax())
Insert(10)
print(ExtractMax())
print(ExtractMax())
```

Additional Operations

- `Remove`(*it*) removes an element pointed by an iterator *it*
- `GetMax`() returns an element with maximum priority (without changing the set of elements)
- `ChangePriority`(*it*, *p*) changes the priority of an element pointed by *it* to *p*

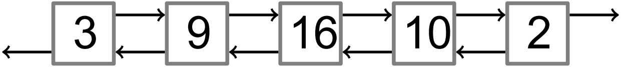
Outline

1. Overview

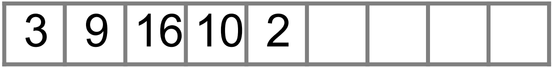
2. Implementations

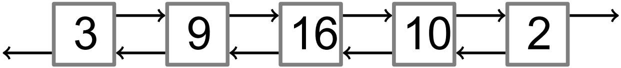
Unsorted Array/List

3	9	16	10	2				



Unsorted Array/List

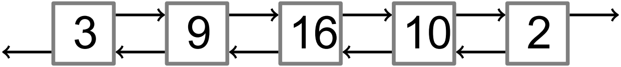




- `Insert`(*e*)
  - add *e* to the end
  - running time: $O(1)$

Unsorted Array/List

3	9	16	10	2				



- `Insert`(*e*)
    - add *e* to the end
    - running time: $O(1)$
- `ExtractMax`()
    - scan the array/list
    - running time: $O(n)$

Sorted Array

2	3	9	10	16				

Sorted Array

2	3	9	10	16				

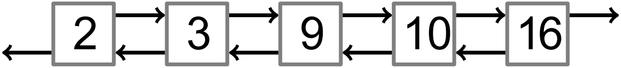
- `ExtractMax`()
  - extract the last element
  - running time: *O*(1)

Sorted Array

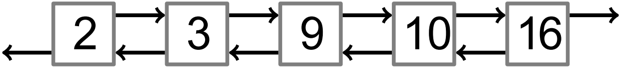
2	3	9	10	16				

- `ExtractMax`()
    - extract the last element
    - running time: $O(1)$
- `Insert`(e)
    - find a position for e ($O(\log n)$ by using binary search), shift all elements to the right of it by 1 ($O(n)$), insert e ($O(1)$)
    - running time: $O(n)$

Sorted List

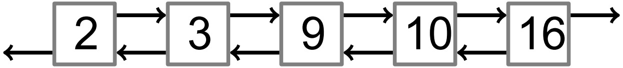


Sorted List



- `ExtractMax`()
  - extract the last element
  - running time: $O(1)$

Sorted List



- `ExtractMax`()
  - extract the last element
  - running time: $O(1)$
- `Insert`(e)
  - find a position for e ($O(n)$; note: cannot use binary search), insert e ($O(1)$)
  - running time: $O(n)$

Question

Assume that you know in advance that in your application there will be

n calls to Insert and *n* calls to ExtractMax. Which of the following two implementations of the priority queue is preferable in this case? Explain your response.

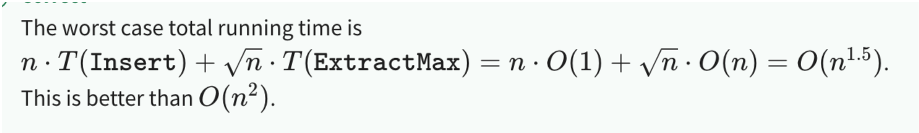
❑ Array

❑ Sorted array

Question

Assume that you know in advance that in your application there will be
n calls to Insert and *n* calls to ExtractMax. Which of the following two implementations of the priority queue is preferable in this case? Explain your response.

□ Array



The worst case total running time is

$$n \cdot T(\texttt{Insert}) + \sqrt{n} \cdot T(\texttt{ExtractMax}) = n \cdot O(1) + \sqrt{n} \cdot O(n) = O(n^{1.5}).$$

This is better than $O(n^2)$.

□ Sorted array

Summary

	`Insert`	`ExtractMax`
Unsorted array/list	$O(1)$	$O(n)$
Sorted array/list	$O(n)$	$O(1)$

Summary

	`Insert`	`ExtractMax`
Unsorted array/list	$O(1)$	$O(n)$
Sorted array/list	$O(n)$	$O(1)$
Binary heap	$O(\log n)$	$O(\log n)$