Introduction to Robotics (Fall 2022)

MAE 345 / ECE 345 / COS 346 / MAE 549

Assignment 2

Due Wed, Sept. 28th by midnight

Instructions: Collaboration on the assignment is permitted (encouraged). But, you must turn in your own solutions for Part I of this assignment (theory). You will complete Part II (hardware implementation) as a team. Instructions for submission via Gradescope are at the end of this pdf.

Part I: Theory

Problem 1 (MAE 345). MAE 345 students only (MAE 549 students: please skip this and do the problem marked "Problem 1 (MAE 549)").

(15 pts) Suppose we define a Linear Quadratic Regulator (LQR) problem using Q and R matrices (as discussed in class) for some linear system $\dot{\bar{x}} = A\bar{x} + B\bar{u}$. Let K be the resulting gain matrix (i.e., the matrix that defines the controller). Now suppose we define $Q' \triangleq 2Q$ and $R' \triangleq 2R$. Consider the LQR problem defined by Q' and R' (for the same linear system $\dot{\bar{x}} = A\bar{x} + B\bar{u}$). Denote the resulting gain matrix by K'. Which of the following is true? Please provide an explanation of your answer.

(a) $K' = K$,
(b) $K' = 2K$,
(c) $K' = K/2$,
(d) None of the above.

Problem 1 (MAE 549). MAE 549 students only (MAE 345 students are welcome to attempt this, but we won't grade it).

Consider a Linear Quadratic Regular (LQR) problem defined by cost matrices Q and R (as discussed in class) for a linear system $\dot{\bar{x}} = A\bar{x} + B\bar{u}$ with $\bar{x} \in \mathbb{R}^n$ and $\bar{u} \in \mathbb{R}^m$. Let S be the (positive definite matrix) solution to the corresponding Riccatti equation. Define $V(\bar{x}) = \bar{x}^T S \bar{x}$.

(a) (2 pts) Consider the set $X = \{\bar{x} \in \mathbb{R}^n \mid V(\bar{x}) \leq 1\}$. What shape does this set have?

(b) (8 pts) Now suppose we define $Q' \triangleq 2Q$ and $R' \triangleq 2R$. Consider the LQR problem defined by Q' and R' (for the same linear system). Let S' be the solution to the corresponding Riccatti equation and define $V'(\bar{x}) = \bar{x}^T S' \bar{x}$. Let $X' = \{\bar{x} \in \mathbb{R}^n \mid V'(\bar{x}) \leq 1\}$. How does the volume of the set X' compare with the volume of the set X (i.e., is it bigger, smaller, the same size)? Explain your answer.

(c) (5 pts) Let K be the gain matrix corresponding to the original LQR problem (given by Q and R) and let K' be the gain matrix for the problem given by Q' and R'. Which of the following is true? Please provide an explanation of your answer.

(a) $K' = K$,
(b) $K' = 2K$,
(c) $K' = K/2$,
(d) None of the above.

Problem 2 (both MAE 345 and MAE 549 students should complete this).

(20 pts) Consider a linear system: $\dot{\bar{x}} = A\bar{x}$, where $\bar{x} \in \mathbb{R}^n$ is the state of the system (there is no control input here). Now, suppose that this system is *locally* asymptotically stable (to the origin). Prove that this implies *global* asymptotic stability. [Hint: You can make use of the fact that the solution to a linear differential equation is given by $\bar{x}(0)e^{At}$, where $\bar{x}(0)$ is the initial state of the system. Also, recall the definitions of local and global asymptotic stability from Lecture 4.]
[Note: this is not true in general for nonlinear systems. This is a special property of linear systems!]

Part II: Hardware Implementation (both MAE 345 and 549 students should complete all portions of this)

You will now implement an LQR controller to make our Crazyflie drone hover! **You will work in teams to complete this. Please get started on things early!**

Go to the github repository for the course:
https://github.com/Princeton-Introduction-to-Robotics/F2022
and follow the instructions on the README to download the new assignment. You should see `Lab2.ipynb`. Open this file up with jupyter notebook and complete all portions of Lab 2. Only the portion on "Running the Quadrotor" needs to be completed in G105 (EQuad) or ACEE012 (Andlinger). We recommend completing all other portions before you go the netted areas.

Lab safety (please read). We take safety very seriously. Even though the drones are small, they still pose risks. Please be careful at all times when working with the drones. Specifically:

- Never work alone in the lab (you should be working with your team).
- Please wear safety glasses at all times when testing out the drones. Safety glasses are available in G105 and ACEE012.
- Please stand outside the netted area when flying the drone.

Replacement parts. We have provided you a number of replacement parts for the drones (propellers and arms); we also have some parts available in the labs. You will probably break a few propellers, so these will be useful! **Please make sure to put the propellers on the drone in the correct orientation.**
(See: https://www.bitcraze.io/documentation/tutorials/getting-started-with-crazyflie-2-x/#attach-the-propellers for details on the correct orientation.)

Submission instructions

Part I: Theory Submission You can submit your written work for Part I (Theory) in Gradescope to "HW2: Theory".

Part II: Coding Submission Please create a zip file containing your completed Jupyter Notebook (Lab2.ipynb), a video of your drone hovering (please ensure the file is < 100MB), and the file `quad_data/quad_traj.npz`. Submit the zip file on Gradescope to "HW2: Coding".