Artificial Intelligence

CSE 440/EEE 333/ETE333

Chapter 8
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Limitations of Propositional Logic

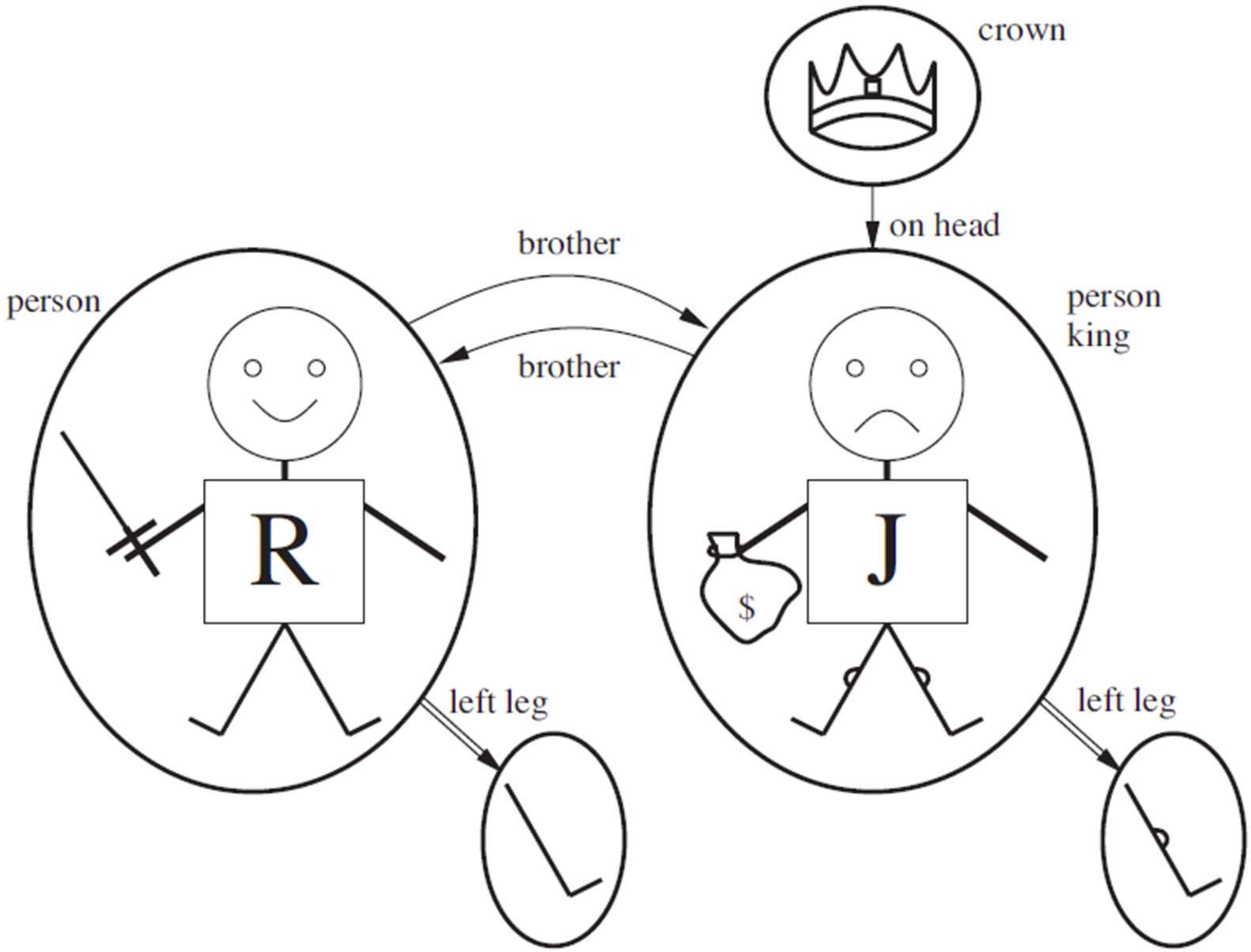
- In the 4x4 wumpus world, how can we say that pits cause breezes in adjacent squares?
  - We need 16 different rules like this:

    $B_{12} \leftrightarrow (P_{11} \vee P_{22} \vee P_{13})$
- How can we say that adding 1 to an even number produces an odd number?
  - We need infinite symbols and infinite rules.
  - A symbol O_1 for “1 is odd”, a symbol E_2 for “2 is even”, …
- What do these limitations buy us?
  - Simple syntax: just symbols and connectives.
  - Inference algorithms (like TT-Entails) that are horribly slow (exponential time), but at least terminate in finite time.

First-Order Logic

- In first-order logic, we have a richer language, that can explicitly represent:
  - Objects (called **constants**).
    - John, Mary, house backpack, Arlington, Texas…
  - Relations (also called **predicates**). These are boolean functions (they can only evaluate to true or false).
    - Siblings(John, Mary)
    - >(100, 5)
    - Red(laptop551)
    - Team(John, Mary, Sue, Jim)
  - Functions (One value for a given input)
    - Capital(Texas)
    - Mother(John)
    - 25 + 12 (here, + is a function).

Relationships Models for first order logic



First-Order-Logic Syntax

$$
\begin{array}{rcl}
\mathit{Sentence} & \rightarrow & \mathit{AtomicSentence} \mid \mathit{ComplexSentence} \\
\mathit{AtomicSentence} & \rightarrow & \mathit{Predicate} \mid \mathit{Predicate}(\mathit{Term}, \ldots) \mid \mathit{Term} = \mathit{Term} \\
\mathit{ComplexSentence} & \rightarrow & (\mathit{Sentence}) \mid [\mathit{Sentence}] \\
& \mid & \neg\ \mathit{Sentence} \\
& \mid & \mathit{Sentence} \wedge \mathit{Sentence} \\
& \mid & \mathit{Sentence} \vee \mathit{Sentence} \\
& \mid & \mathit{Sentence} \Rightarrow \mathit{Sentence} \\
& \mid & \mathit{Sentence} \Leftrightarrow \mathit{Sentence} \\
& \mid & \mathit{Quantifier}\ \mathit{Variable}, \ldots\ \mathit{Sentence} \\
\mathit{Term} & \rightarrow & \mathit{Function}(\mathit{Term}, \ldots) \\
& \mid & \mathit{Constant} \\
& \mid & \mathit{Variable} \\
\mathit{Quantifier} & \rightarrow & \forall \mid \exists \\
\mathit{Constant} & \rightarrow & A \mid X_1 \mid \mathit{John} \mid \ldots \\
\mathit{Variable} & \rightarrow & a \mid x \mid s \mid \ldots \\
\mathit{Predicate} & \rightarrow & \mathit{True} \mid \mathit{False} \mid \mathit{After} \mid \mathit{Loves} \mid \mathit{Raining} \mid \ldots \\
\mathit{Function} & \rightarrow & \mathit{Mother} \mid \mathit{Left\ leg} \mid \ldots \\
\text{Operator Precedence}^{3} & : & \neg, \wedge, \vee, \Rightarrow, \leftrightarrow
\end{array}
$$

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[3]Otherwise the grammar is ambiguous

First-Order-Logic More on Syntax

- Three kinds of symbols
  - Constant: objects
  - Predicate: relations
  - Function: funcions (i.e. can return values other than truth values)
- Predicate and Function have **arity**.
- Symbols have an interpretation
- Terms: *LeftLeg*(*John*)
- Atomic Sentences state facts: *Brother*(*Richard*, *John*)
- Complex Sentence:

  $Brother(R, J) \land Brother(J, R)$ or

  $\neg King(Richard) \rightarrow King(John)$
- Universal Quantifiers: $\forall x\ King(x) \rightarrow Person(x)$
- Existential Quantifiers: $\exists x\ Crown(x) \land OnHead(x, John)$

Variables and Quantifiers

- Variables can only be used together with quantifiers.
- Quantifiers need variables in order to be used.
- Examples:
  - $\forall x, y\ Brother(x, y) \rightarrow Sibling(x, y)$
  - $\exists x\ 2*5 + x = 18$

Examples

- For the wumpus world, to say that "pits cause breezes in adjacent squares" using propositional logic, we need 16 rules like this:

$$B_{12} \leftrightarrow (P_{11} \vee P_{22} \vee P_{13})$$

- In first-order logic, how can we say the same thing?

Examples

- For the wumpus world, to say that "pits cause breezes in adjacent squares" using propositional logic, we need 16 rules like this:

$$B_{12} \leftrightarrow (P_{11} \vee P_{22} \vee P_{13})$$

- In first-order logic, how can we say the same thing?

$$\forall x_1, y_1\ Breeze(x_1, y_1) \leftrightarrow \exists x_2, y_2\ Pit(x_2, y_2) \wedge Adjacent(x_1, y_1, x_2, y_2)$$

Examples

- For the wumpus world, to say that "there is only one monster" using propositional logic, we need 16 rules like this:

$$M_{23} \rightarrow \neg(M_{11} \vee M_{12} \vee M_{13} \ldots)$$

- In first-order logic, how can we say the same thing?

$$\forall x_1, y_1 \;\; Monster(x_1, y_1) \rightarrow$$
$$\forall x_2, y_2 \;\; Monster(x_2, y_2) \rightarrow (x_1, y_1) = (x_2, y_2)$$

First-Order-Logic Try this

- What is the interpretation for:
  - $King(Richard) \lor King(John)$
  - $\neg Brother(LeftLeg(Richard), John)$
  - $\forall x\ \forall y\ Brother(x, y) \rightarrow Sibling(x, y)$
  - $In(Paris, France) \land In(Marseilles, France)$
  - $\forall c\ Country(c) \land Border(c, Ecuador) \rightarrow In(c, SouthAmerica)$
  - $\exists c\ Country(c) \land Border(c, Spain) \land Border(c, Italy)$

First-Order-Logic More Facts

- **Richard has only two brothers, John and Geoffrey:**

$Brother(John, Richard) \land Brother(Geoffrey, Richard) \land$

$(John \neq Geoffrey) \land$

$\forall x\ Brother(x, Richard) \rightarrow (x = John \lor x = Geoffrey)$

- **No Region in South America borders any region in Europe**

$\forall c, d\ In(c, SouthAmerica) \land In(d, Europe) \rightarrow \neg Border(c, d)$

- **No two adjacent countries have the same map color**

$\forall x, y\ Country(x) \land Country(y) \land Border(x, y) \rightarrow$

$\neg(Color(x) = Color(y)) \land \neg(x = y)$

Assertions and Queries in FOL ASK and TELL

- $TELL(KB, King(John))$
- $TELL(KB, \forall x\ King(x) \rightarrow Person(x))$

- $ASK(KB, King(John))$ return True
- $ASK(KB, \exists x\ Person(x))$ return True

- $ASKVARS(KB, Person(x))$ yields $\{x/John, x/Richard\}$, a binding list

First Order Logic Kinship

“The son of my father is my brother”,

“One’s grandmother is the mother of one’s parent”; etc.

- Domain: People.
- Unary predicates: *Male*; *Female*
- Relations:

  Parent, Sibling, Brother, Sister, Child, Daughter, Son, Spouse, Wife, Husband, Grandparent, Grandchild, Cousin, Aunt, Uncle
- Functions: *Mother*, *Father*

First Order Logic Kinship

"One's mother is one's female parent"

$\forall m, c \; Mother(c) = m \leftrightarrow Female(m) \wedge Parent(m, c)$

"A sibling is another child of one's parents"

$\forall x, y \; Sibling(x, y) \leftrightarrow x \neq y \wedge \exists p \; Parent(p, x) \wedge Parent(p, y)$

"Wendy is female"

Female(wendy)

First Order Logic Wumpus: Encoding complex rules

Can encode:

- Raw percepts:

$$\forall t, s, g, m, c\ Percept([s, b, Glitter, m, c], t) \leftrightarrow Glitter(t)$$

- Reflex actions: $\forall t\ Glitter(t) \rightarrow BestAction(Grab, t)$

Instead of encoding stuff like:

- $Adjacent(Square_{1,2}, Square_{1,1})$
- $Adjacent(Square_{3,4}, Square_{4,4})$

Encode:

$$\forall x, y, a, b\ Adjacent([x, y], [a, b]) \leftrightarrow$$

$$(x = a \wedge (y = b-1 \vee y = b+1)) \vee (y = b \wedge (x = a-1 \vee x = a+1))$$

Creating a Knowledge Base

- Identify the Task
- Assemble the relevant knowledge
- Decide on a vocabulary of predicates, functions and constants
- Encode general knowledge about the domain (rules)
- Encode a description of the problem
- Pose queries to the inference procedure and get answers
- Debug the KB

Inference in First Order Logic

Given $\forall x\ King(x) \wedge Greedy(x) \rightarrow Evil(x)$

One can infer

- $King(John) \wedge Greedy(John) \rightarrow Evil(John)$
- $King(Richard) \wedge Greedy(Richard) \rightarrow Evil(Richard)$
- $King(Father(John)) \wedge Greedy(Father(John)) \rightarrow Evil(Father(John))$

Inference in First Order Logic

- Universal Instantiation (in a ∀ rule, substitute all symbols)
- Existential Instantiation (in a ∃ rule, substitute one symbol, use the rule and discard)
- Skolem Constants is a new variable that represents a new inference.

Inference in First Order Logic

Suppose KB:

- $\forall x\ King(x) \wedge Greedy(x) \rightarrow Evil(x)$
- $King(John)$
- $Greedy(John)$
- $Brother(Richard, John)$

Apply UI using $\{x/John\}$ and $\{x/Richard\}$

- $King(John) \wedge Greedy(John) \rightarrow Evil(John)$
- $King(Richard) \wedge Greedy(Richard) \rightarrow Evil(Richard)$

And discard the Universally quantified sentence. We can get the KB to be propositions.

Inference in First Order Logic

Suppose KB:

- $\forall x\ King(x) \land Greedy(x) \rightarrow Evil(x)$
- $King(John)$
- $\exists y\ Greedy(y)$

Apply UI using $\{x/John\}$ and $\{x/Richard\}$

Inference Generalized Modus Ponens

for atomic sentences p_i, p_i' and q, where there is a substitution θ such that $SUBST(\theta, p_i') = SUBST(\theta, p_i)$, for all i

$$\frac{p_1', p_2', \ldots, p_n', (p_1 \wedge p_2 \wedge \ldots \wedge p_n \Rightarrow q)}{SUBST(\theta, q)}$$

$$\begin{array}{ll}
p_1' = King(John) & p_1 = King(x) \\
p_2' = Greedy(y) & p_2 = Greedy(x) \\
\theta = \{x/John, y/John\} & q = Evil(x) \\
SUBST(\theta, q) & .
\end{array}$$

Inference Unification

UNIFY(*p*, *q*) = θ Where *SUBST*(θ, *p*) = *SUBST*(θ, *q*)

For example:

- We ask *ASKVARS*(*Knows*(*John*, *x*)) (Whom does John know?)
- *UNIFY*(*Knows*(*John*, *x*), *Knows*(*John*, *Jane*)) = {*x*/*Jane*}
- *UNIFY*(*Knows*(*John*, *x*), *Knows*(*y*, *Bill*)) = {*y*/*John*, *x*/*Bill*}
- *UNIFY*(*Knows*(*John*, *x*), *Knows*(*y*, *Mother*(*y*))) = {*y*/*John*, *x*/*Mother*(*John*)}

Inference Putting it all together

"The Law says that it is a crime for an American to sell weapons to hostile nations. The country Nono, and enemy of America, has some missiles , and all of its missiles were sold to it by Colonel West, who is American"

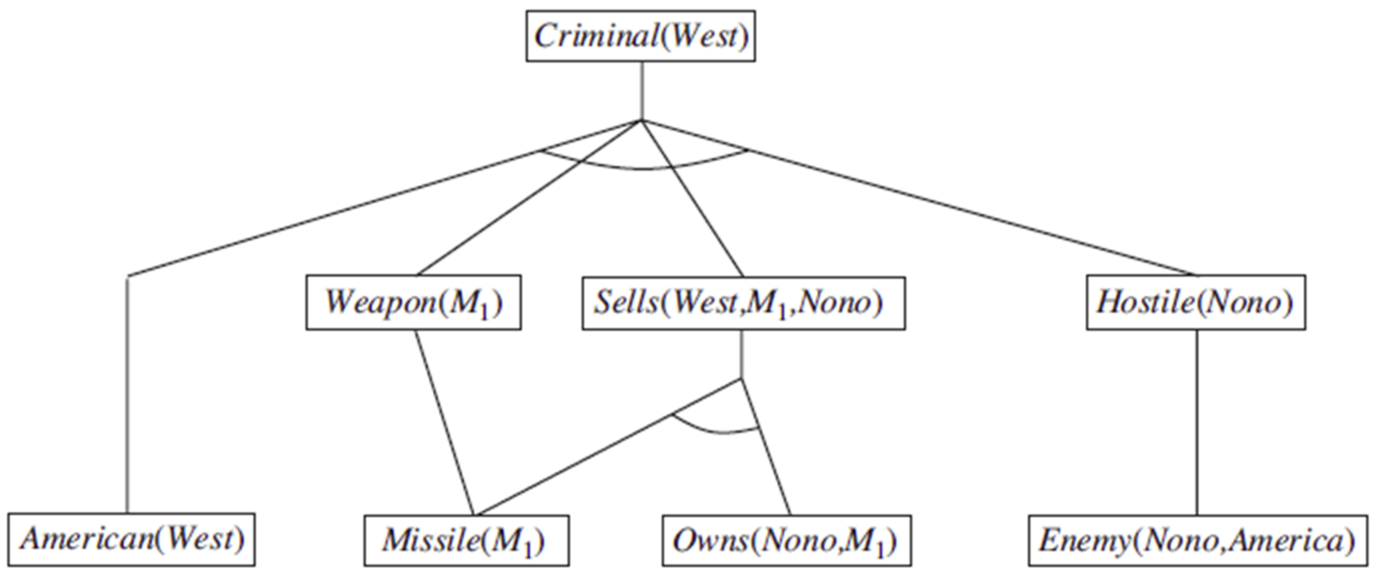
Prove that Colonel West is a Criminal

Inference Putting it all together

"The Law says that it is a crime for an American to sell weapons to hostile nations. The country Nono, and enemy of America, has some missiles , and all of its missiles were sold to it by Colonel West, who is American"

- R1:
  $American(x) \wedge Weapon(y) \wedge Sells(x, y, z) \wedge Hostile(z) \rightarrow Criminal(x)$
- R2: $Owns(Nono, M_1)$ Nono has some missiles
- R3: $Missile(M_1)$
- R4: $Missile(x) \rightarrow Weapon(x)$ A missile is a weapon
- R5: $Missile(x) \wedge Owns(Nono, x) \rightarrow Sells(West, x, Nono)$ All missiles sold by west
- R6: $Enemy(x, America) \rightarrow Hostile(x)$ Enemies of America are hostile
- R7: $American(West)$ West is american
- R8: $Enemy(Nono, America)$

Inference Graph



Inference Putting it all together

Iteration 1:

- R5 satisfied with $\{x/M_1\}$ and R9: *Sells*(*West*, M_1, *Nono*) is added
- R4 satisfied with $\{x/M_1\}$ and R10: *Weapon*(M_1) is added
- R6 satisfied with $\{x/Nono\}$ and R11: *Hostile*(*Nono*) is added

Iteration 2:

- R1 is satisfied with $\{x/West, y/M1, z/Nono\}$ and *Criminal*(*West*) is added.

Inference Discussion

- Once we have facts that evaluate to T or F
- We can apply Forward Chaining, Backwards Chaining and Resolution
- The key is to understand Unification
- Very similar to Logical agents.