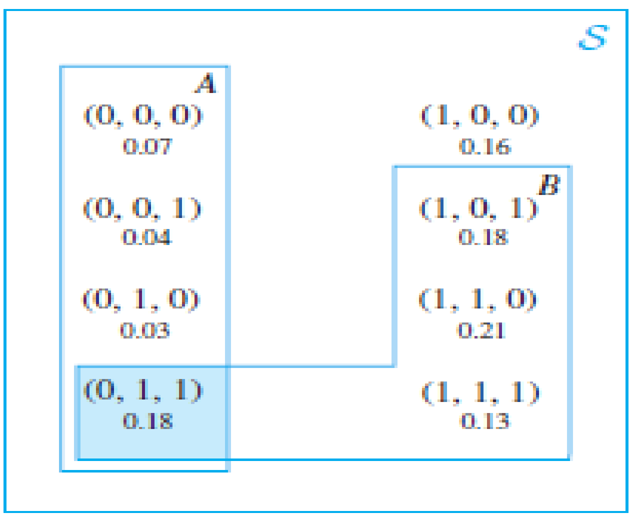
Conditional probability: Conditional probability is the probability of an event given that another event has already occurred. Conditional probability is denoted by $P(A|B) = P(A \cap B)/P(B)$.

Example: A manager supervises the operation of three power plants, plant X, plant Y and plant Z. At any given time, each of the three plants can be classified as either generating electricity (1) or not generating electricity (0). With the notation (0,1,0) used to represent the situation where plant Y is generating electricity but plants X and Z are both not generating electricity. Let A be the event that plant X is not generating electricity and B be the event that at least two out of the three plants are generating electricity.



If event B is known what is the probability of happening the event A?

Solution: we know, $P(A|B) = P(A \cap B)/P(B)$

$P(A \cap B) = P(0,1,1) = 0.18$ and $P(B) = P(1,0,1) + P(1,1,0) + P(1,1,1) + P(0,1,1) = 0.18 + 0.21 + 0.13 + 0.18 = 0.70$

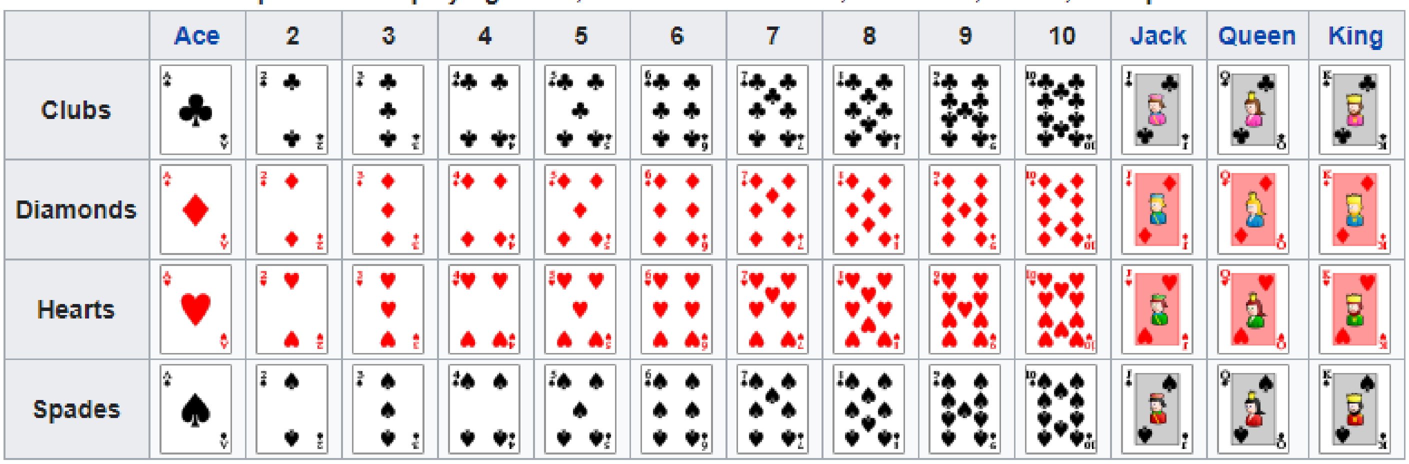
i.e $P(A|B) = 0.18/0.70 = 0.257$

$P(A) = 0.32$

Example: A card is drawn at random from a pack of cards. Calculate:

(a) *P(King*|card from red suit*)* = *?*

Example set of 52 playing cards; 13 of each suit clubs, diamonds, hearts, and spades



A= {K clubs, K diamonds, K hearts, K spades}
B = {13 diamonds, 13 hearts}
A ∩ B = {K diamonds, K hearts}
P(A|B) = P(A∩B)/P(B) = (2/52)/(26/52) = 2/26 = 1/13.

b) P(king|red picture card)=?
A= {K clubs, K diamonds, K hearts, K spades}
B = {J diamonds, Q diamonds, K diamonds, J hearts, Q hearts, K hearts}
A ∩ B = {K diamonds, K hearts}
P(A|B) = P(A∩B)/P(B) = (2/52)/(6/52) = 1/3

Classwork: ***P(*heart|card from black suit*)***
A = {13 hearts}
B = {13 clubs, 13 Spades}
A∩B = {}
P(A|B) = P(A∩B)/P(B) = 0/ (26/52) = 0.

Question: A car repair is either on time or late and either satisfactory or unsatisfactory. If a repair is made on time, then there is a probability of 0.85 that it is satisfactory. There is a probability of 0.77 that a repair will be made on time. What is the probability that a repair is made on time and is satisfactory?

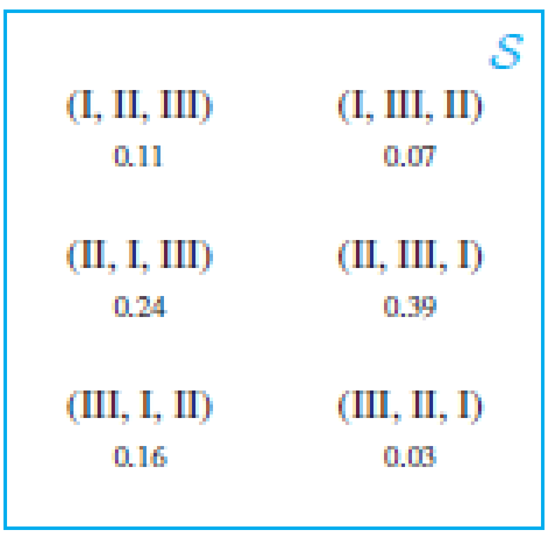
Solution: Let If a car repairment is Satisfactory then it is denoted by S,
If a car repairment is unsatisfactory then it is denoted by S'
If a car repairment is made on time then it is denoted by T
If a car repairment is made on late then it is denoted by T'

$P(S|T) = 0.85$, $P(T) = 0.77$ and $P(S \cap T) = ?$

We know, $P(S|T) = P(S \cap T) / P(T)$

Or, $P(S \cap T) = P(S|T)*P(T) = 0.85*0.77 = 0.6545$

Question: Three types of batteries are being tested, type I, type II, and type III. The outcome (I, II, III) denotes that the battery of type I fails first, the battery of type II next, and the battery of type III lasts the longest**.**



Calculate the probabilities a) A type I battery lasts longest conditional on a type II battery failing first
(b) A type I battery lasts longest conditional on a type II battery lasting the longest.

Solution: Let A denotes type I battery lasts longest and B denotes type II battery failing first.

A = {(II,III,I), (III,II,I)} and B = {(II,I,III),(II,III,I)}

We know, P(A|B) = P(A∩B)/P(B)

Here, A∩B = II,III,I

P(A∩B) = 0.39 and P(B) = 0. 24 + 0.39 = 0.63

So, P(A|B) = 0.39/0.63 = 0.62

Classwork: A type I battery lasts longest conditional on a type II battery not failing first