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$$x = 2\cos t, \quad y = 2\sin t \quad \text{for } 0 \leq t \leq \frac{3\pi}{2}$$

Formula :

$$L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt$$

Ans:

Given,

$$x = 2\cos t.$$

$$\cancel{x = 2\cos t}$$

$$\Rightarrow \frac{dx}{dt} = \frac{d}{dt} (2\cos t).$$

$$\Rightarrow \frac{dx}{dt} = -2\sin t.$$

$$\Rightarrow \left(\frac{dx}{dt}\right)^2 = 4\sin^2 t$$

$$\begin{aligned} \therefore \left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 &= 4\sin^2 t + 4\cos^2 t \\ &= 4(\sin^2 t + \cos^2 t) = 4. \end{aligned}$$

$$y = 2\sin t$$

$$\Rightarrow \frac{dy}{dt} = \frac{d}{dt} (2\sin t).$$

$$\Rightarrow \frac{dy}{dt} = 2\cos t.$$

$$\Rightarrow \left(\frac{dy}{dt}\right)^2 = 4\cos^2 t.$$

Length of the curve:

$$L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt.$$

$$= \int_0^{\frac{3\pi}{2}} \sqrt{4} dt.$$

$$= \int_0^{\frac{3\pi}{2}} 2 dt.$$

$$= [2t]_0^{\frac{3\pi}{2}}.$$

$$= \left[2 \cdot \frac{3\pi}{2}\right] - [2 \cdot 0]$$

$$= 3\pi$$

(Ans).