Ch-3 | Discrete Probability Distribution

3.1 Binomial Distribution

X = Discrete R.V.

The bernoulli Dis^n [Only two results]

Bernoulli R.V

$= \{\text{success, failure}\}$

$= \{1, 0\}$

$$\boxed{p(x) = \binom{n}{x} p^x (1-p)^{n-x} \; ; \; x = 0, 1, 2 \ldots n}$$

$p + q = 1$

$p(x = 1) = p$ (say)

$p(x = 0) = (1 - p)$

$E(x) = \sum x_i \, p(x_i) = 1 \times p + 0 \times (1-p) = p$

$V(x) = E(x^2) - (E(x))^2$

$= p - p^2$

$= p(1-p)$

Dice success chances.

(1) ~~Cha~~ If 6 success other failure

(2) Ch If even success or fail.

$X =$ Binomial R.V

$=$ Sum of Bernoulli R.Vs

$= \#$ of successes.

$P(X) = \binom{n}{x} p^x (1-p)^{n-x}$; $x = 0, 1, 2 \ldots n$

$E(X) = E(X_1 + X_2 + \cdots + X_n) = np$

[If $X_1 X_2 \cdot\cdot X_n$ are Bernoulli RVs]

$V(X) = V(X_1) + V(X_2) \cdot V(X_n)$

Then

$X = X_1 + X_2 + X_3 \cdot X$

$= np(1-p)$

* 20 coin toss — ~~12 heads~~

$X =$ 12 heads.

$P(X=12) = ? = \binom{20}{12}\left(\frac{1}{2}\right)^{12}\left(1-\frac{1}{2}\right)^{20-12}$

$P = P(H) = \frac{1}{2}$

$1-P = P(T) = 1-\frac{1}{2} = \frac{1}{2}$

$=$

* Batsmen

ball faced $n = 30$

Run score $= x = 15$ [

$E(x)$

$P = 0.85$

$V(x)$

$1-P = 0.15$

$P(15) = \binom{30}{15}(0.85)^{15}(0.15)^{15}$

$= (\cancel{0.087})\ 5.93 \times 10^{-6}$

Ex = $n = 50$

$P(A_1) = .10$

$P(\overline{A_1}) = 0.9$

$x = 7.$

$F(x) = nP$

$= 50 \times .10 = 5$

$V(x) = nP(1-P) = 4.5$

$P(x=7) = ?$ $\binom{50}{7}(.10)^7 (0.90)^{43}$

$= 0.11$

3.1] Bernoulli;

$X = \{1, 0\}$ $\quad X \sim Ber(P)$

$P(x) = \{P, 1-P\}$

$E(x) = P, \quad V(x) = P(1-P)$

$x = x_1 + x_2 + \dots + x_n$

$P(X=x) = \binom{n}{x} p^x (1-p)^{n-x}$

$X \sim Bin(n, p)$

$X \sim Bin(40, .10) \Rightarrow E(x) = 40 \times .10 = 4 \; ; V(x) =$

$P(X=3) = ?$

[No of success]

[Probab of success]

$$Y = X_1 + X_2 + \dots + X_n$$

$$E(X) = E(X_1 + X_2 + \dots + X_n)$$

$$= E(X_1) + E(X_2) + \dots E(X_n)$$

$$= np$$

$$V(Y) = V(X_1) + V(X_2) + \dots V(X_n)$$

$$= np(1-p)$$

3.2 Geometric & Negative Binomial Dist

$X =$ Geometric R.V

$=$ No. of trials to get 1st success.

$\Rightarrow$ No of proposals to get 1st wicket

$\Rightarrow$ No of proposals to get 1st girl.

$$P(X = x) = p(1-p)^{x-1}$$

$$E(X) = 1/p \text{ [No of trials]}$$

$$V(X) = \frac{1-p}{p^2}$$

$$X \sim Geom(p)$$

$F(x) =$ cdf

$$= P(X \leq x)$$

$$= \sum_{x=1}^{x} p(1-p)^{x-1}$$

$$= p + p(1-p) + \dots$$

$$= p(1-p)^{x-1}$$

* $X \sim G(\cdot 10)$

$\cancel{F(X) = 10}$ $P(X=4) = \cdot 10\,(1-\cdot 10)^{4-1}$

$\cancel{V(X) = 90}$ $= -72.9$

$E_X = 10$

$\cancel{p(1-p)}$

$p(1+(1-p)$

$+(1-p)^2$

$+(1-p)^{x-1}$

$= p\,\dfrac{1-(1-p)^x}{1-(1-p)}$

CDF

$= 1-(1-p)^x$

$F(\cancel{x}\leq 15)$

* $X : \underbrace{\sim}_{\text{Job faced 1st Job}} Gem(0{\cdot}08)$

$P(x=12) = \cdot 08\,(1-0{\cdot}08)^{12-1} = 0{\cdot}03$

$E(x) = 12.5 \approx 13$ [How many job faced]

$V(x) = 143.75$ [

$F(x\leq 15) = 1-(1-\cdot 10)^{15}$

$= 0.79$

⇒ Negative Binomial

X = Neg. Binomial R.V.

= No of ~~trials~~ to get r-th success!

$$P(x) = \binom{x-1}{r-1} p^r (1-p)^{x-r}$$

Success from rest no of trials.

$$X \sim \text{Neg B}(r, p)$$

$$X \sim \text{Neg B}(3, .10)$$

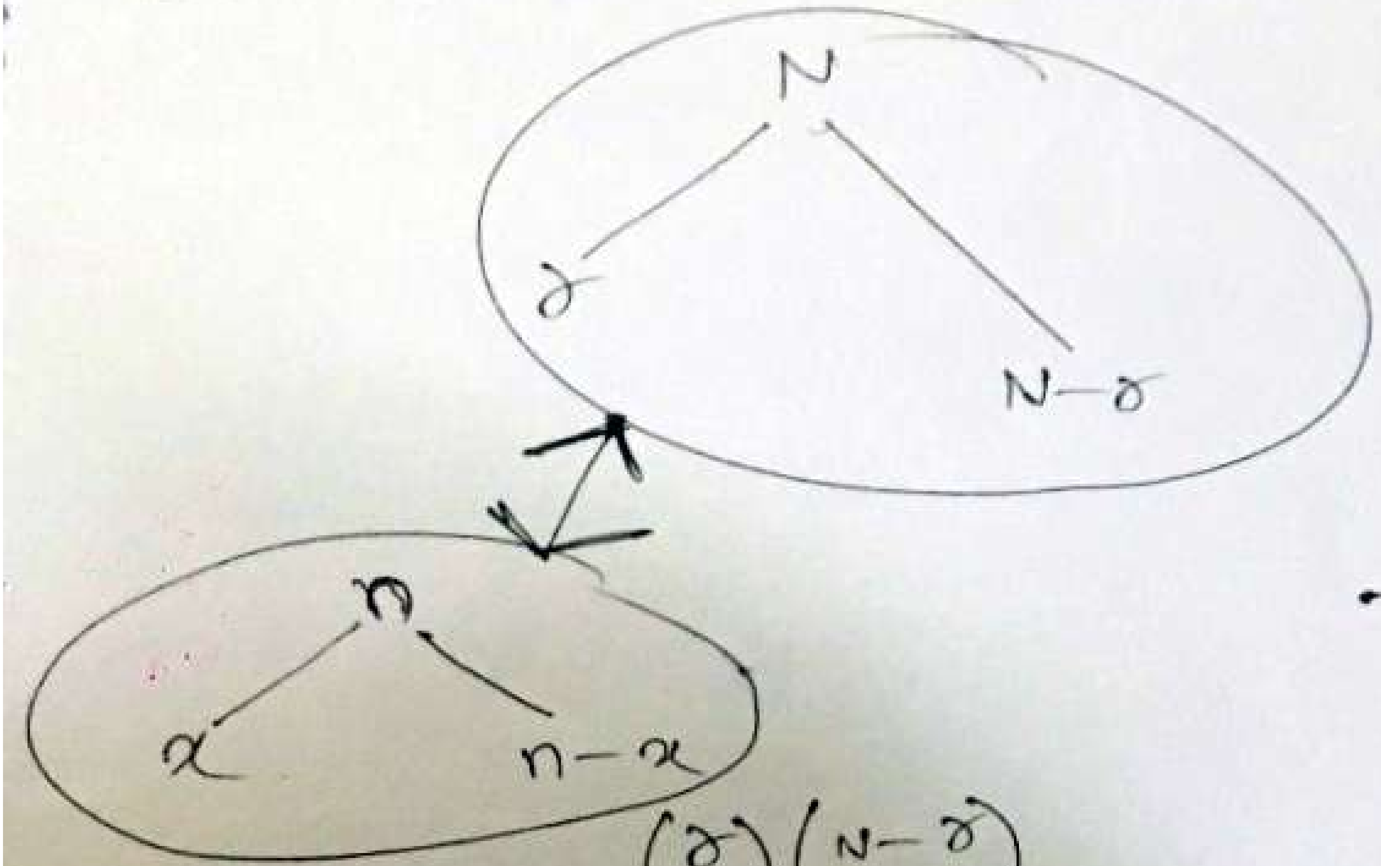
$$P(X=12) = \binom{12-1}{3-1} (.10)^3 (1-.10)^{12-3} = 0.021$$

[Probability]

$$E(x) = r/p$$

$$V(x) = \frac{r(1-p)}{p^2}$$

3.3 Hypergeometric Distribution



$$P(X=x) = \frac{\binom{r}{x}\binom{N-r}{n-x}}{\binom{N}{n}}$$

$N = 40 \quad (m = 35;\ f = 5)$

$n = 4 \quad (f = 1;\ m = 3)$

$P(X=1)$

$$P(X=1) = \frac{\binom{5}{1}\binom{35}{3}}{\binom{40}{4}}$$