Course Name : Physics – I
Course # PHY 107

Notes-6 : Newton's laws - Part Two

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Topics to be studied

- Review: Newton's laws
- Types of forces and their properties
- Frictional forces: Coefficient of frictions
- Static and Kinetic frictional forces
- Free-body diagrams
- Examples

Review:

- Newton's 1^{st} and 2^{nd} laws can be expressed mathematically as

$$\sum \vec{F} = m\vec{a} \,,$$

  where $\sum \vec{F}$ is the cause of motion and $\vec{a}$ is the effect. The mass m is the measure of the object which is an inherent property of the object. m is a scaler.
- Any object must obey either the 1^{st} or the 2^{nd} law.
- The 1^{st} can also equivalently expressed as:

$$\begin{array}{ll} 1) & \text{The object is in equilibrium.} \\ 2) & \vec{v} = \text{constant.} \\ 3) & \vec{a} = 0. \\ 4) & \sum \vec{F} \equiv \vec{F}_{\text{tot}} = 0. \end{array}$$

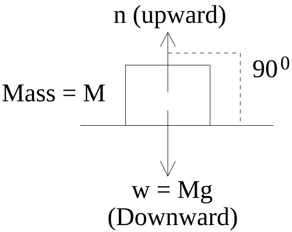
- The 3^{rd} law defined the interaction between the objects. If an object applies a force $\vec{F}_{12}$ (called the 'Action'), there exists an opposite force $\vec{F}_{21}$ (Called the 'Reaction') such that $\vec{F}_{12} = -\vec{F}_{21}$.

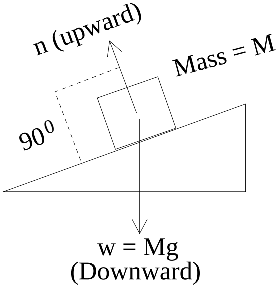
Type of forces:

- In this course we will mainly deal with five different types of forces:
- Mechanical Force $\vec{F}$: This is a force whose magnitude and direction can be completely controlled by ourselves. As for example, 'Push', 'Pull', etc.

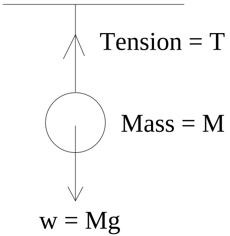
- Weight w: This is a force whose magnitude and direction are fixed by the gravitational force. We can not change either the magnitude or the direction. The magnitude is given by $w = mg$, where m is the mass of the object, and g is the magnitude of the acceleration due to gravity. This force is always directed to the center of the Planet. Therefore, unless the object has been changed, the weight of a particular object is fixed both in magnitude and direction.

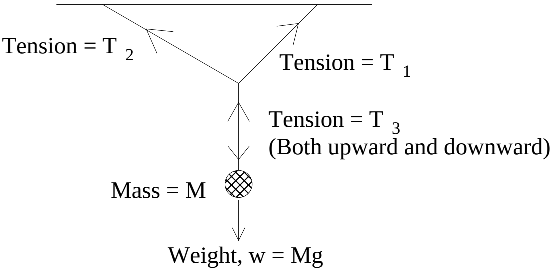
- Normal force $\vec{n}$. This is a reaction force. So the magnitude is fixed by the Newton's $3^{\rm rd}$ law, but the direction is always perpendicular to the plane of a surface (that is, the normal direction).





- Tension $\vec{T}$. This is also a reaction force. It's direction is always along the string which is tied or attached to the object.





- Frictional force $\vec{f}$. This is a intermolecular force, and it is determined by various intermolecular bonding, like, van der Wall force, cohesive force, adhesive force, viscosity, surface tension, etc. It's magnitude is determined empirically, but it is always direction parallel to the plane of the surface and opposite to the direction of motion.
- These five force will be considered in mechanics. Not any other forces, like electric charge, nuclear forces, etc.
- In the following several slides, we will develop the types of frictions, how to classify them, how to formulate them, etc.

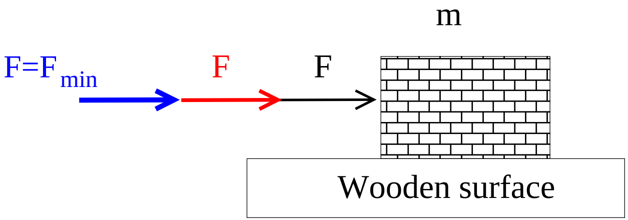
Frictional forces:

- This force intermolecular, and hence it would not be possible to derive its governing equations without the use of Quantum Mechanics. Instead, the formula to compute the frictional forces will be derived empirically.
- There are two types of frictional forces: the static frictional force $f_{\rm s}$ and the kinetic frictional force $f_{\rm k}$.
- The static friction requires that both $v = 0$ and $a = 0$. That is, the object in question must remain at rest.
- The kinetic friction requires that $v \neq 0$, but the acceleration may or may not be zero. If $a = 0$, then v=constant, and the $1^{\rm st}$ will be applied, and if $a \neq 0$, the $2^{\rm nd}$ will be applied.

Static Friction:

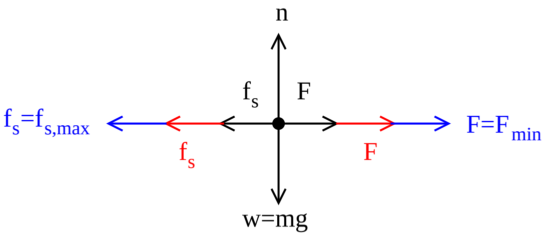
Here we will try to understand the factors that govern the static frictional forces. It is a very common experience that an object at rest on a horizontal rough surface requires a minimum force to start moving. Obviously the net force on the object is be zero, because it is at rest until starts moving.

- Consider the following scenario as shown in the adjacent figure.
- Until the object starts moving, the net force on it is zero, i.e. $\sum F = 0$, or $F = f_s$.
- When $F = F_{\min}$, the object is about to move. This is the force required to start moving.



Static Friction: Continues

- The adjacent figure shows the fee-body diagram.



- There are four forces acting. Since the object is at rest, we must have: $\sum \vec{F} = 0$.
- The y-component gives: $n = w = mg$. The x-component gives: $F = f_s$ until the object starts moving. Note that as F increases, f_s also increases.
- When $F = F_{\min}$, $f_s = f_{s,\max}$. This is the threshold value, and any applied force more than $F_{\min}$ will make the object move and become kinetic friction.
- The minimum force required to start moving is the same as the maximum frictional force that prevents moving !!! $\therefore$ $F_{\min} = f_{s,\max}$.

Maximum Static Frictional Force:

Now we will compute the maximum static frictional force $f_{\mathrm{s,max}}$. To find out the factors that determine the maximum frictional force, we consider three cases:

- Case-1: Forces applied on the same masses, but sitting on different surfaces.
- Case-2: Forces applied on different masses, but sitting on the same surface.
- Case-3: Forces applied on same masses and these are at rest on the same surface, but one masses is pushed down.

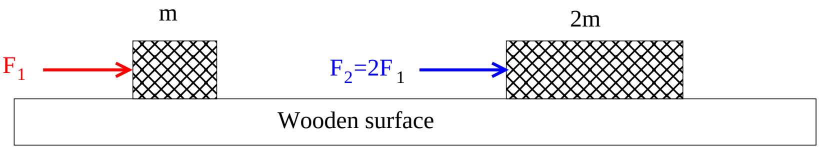
Case-1:

The figure below shows the forces applied. Forces applied on the same masses, but sitting on different surfaces: One is wooden surface and the other is concrete surface.

- Clearly, the force required to move is higher when the surface is concrete, *i.e.* $F_w < F_c$.
- This observation implies that the rougher the surface stronger the maximum static frictional force. Therefore, it may be concluded that $f_{\mathrm{s,max}} \propto \mu_{\mathrm{s}}$, where μ_{s} is called the coefficient of static friction. When $\mu_{\mathrm{s}} = 0$, it is the perfectly smooth surface (absolutely no friction), and $\mu_{\mathrm{s}} \to \infty$ as the roughness increases. Actually it means that the intermolecular forces are getting stronger.

Case-2:

The figure below shows the forces applied. Forces applied on different masses, but sitting on the same surface. One mass is double than the other mass.



- Clearly, the force required to move is higher when the mass is greater.
- This observation shows that more mass requires more applied force to move. But more mass means more weight and also more normal force. Note that weight and normal may not be equal always.
- To distinguish whether it is the wight or normal that determines the frictional force, we consider the Case-3.

Case-3:

Forces applied on same masses on the same surface, but one mass is pushed down.

- Clearly, the force required to move is higher when the mass is pushed down.
- This observation shows the normal force is the determining factor, not the weight. Here weight is same for both masses. But the mass that is pushed down has more normal force.

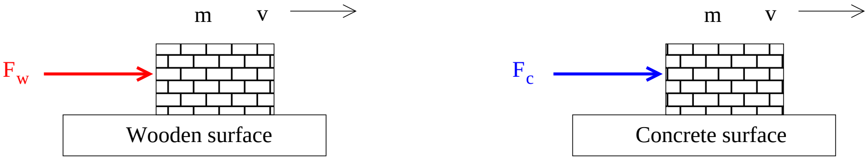
The above analysis clearly shows that the maximum static frictional force depends on roughness of the surface and the normal force on the mass, *i.e.* $f_{\mathrm{s,max}} \propto \mu_{\mathrm{s}}$ and $f_{\mathrm{s,max}} \propto n$. Finally, we can write:

$$\boxed{f_{\mathrm{s,max}} = \mu_{\mathrm{s}} n}.$$

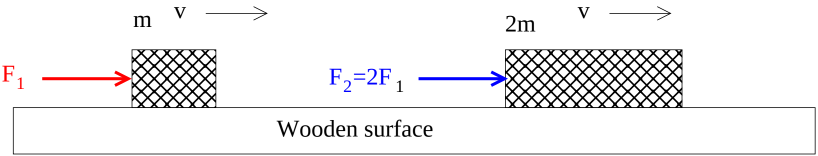
Kinetic Friction:

The analysis to understand the determining factors for kinetic friction are exactly the same as the static frictional case, except the fact that the mass is now moving. Consider the following diagrams:

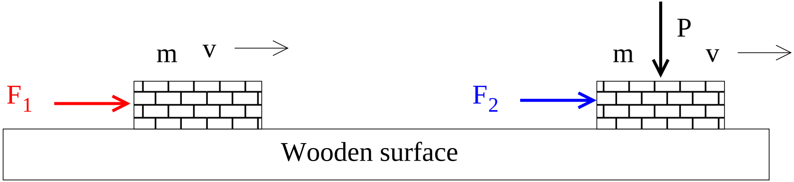
- Case - 1 : Here $F_w < F_c$.



- Case - 2 : Here either $F \propto w$ or $F \propto n$.



- Case - 3 : Finally $F \propto n$.



- The above analysis clearly shows that

$$\boxed{f_{\mathrm{k}} = \mu_{\mathrm{k}} n},$$

where μ_{k} is called the coefficient of the kinetic friction.

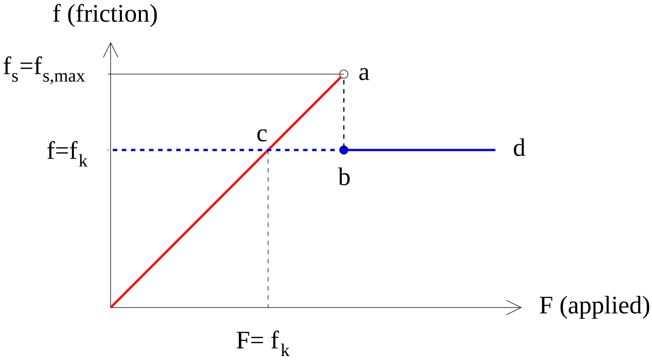
- It is also very well known that it is always easier to keep moving rather than to start moving. In simple terms, it means that more force is required to start moving, and less force is needed to keep it moving.
- Mathematically, this implies that:

$$f_{\mathrm{s,max}} > f_{\mathrm{k}} \implies \mu_{\mathrm{s}} > \mu_{\mathrm{k}} .$$

Frictional Forces: Short Summary

The detail analysis of the frictional forces clearly indicates that:

- Fapplied $= F_{\rm s}$, $f_{\rm s,max} = \mu_{\rm s} n$, $f_k = \mu_{\rm k} n$ and $\mu_{\rm s} > \mu_{\rm k}$.
- Graphically the behavior can be depicted in the adjacent diagram. On the red color line $1^{\rm st}$ law is applied.
- At any point on the blue color line, the net force is $\sum F = F - f_{\rm k}$. So only at point c, $F = f_{\rm k}$ ($1^{\rm st}$ law).

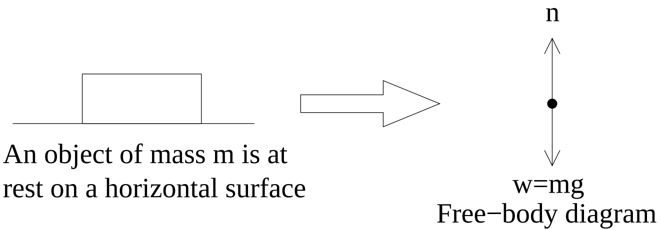


The Free-Body Diagram

- A free body diagram is a vector diagram of all forces and accelerations acting on a system or body..
- All forces are represented by arrows. Each arrow represent the force, both in magnitude and direction. Note that the magnitude requires the scaled diagram which very often is ignored, but the direction needs to be accurate. So, usually schematic diagrams are drawn.
- All vectors are drawn such a way that they are directed away from the object (more precisely the center of mass), or from the action point.
- When all the forces in a free-body diagram are added by triangular or an other method, it must obey either the Newton's 1^{st} or 2^{nd} law.
- From the free-body diagram, each force can be expressed in Cartesian form by choosing a suitable coordinate axes.

Example-1:

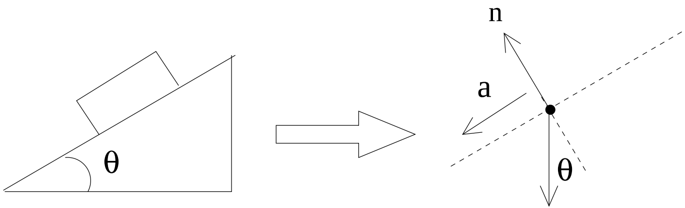
- An object on a horizontal surface:



- The second figure is the vector diagram of all forces acting on the object on the horizintal surface.
- Only two forces acting on. Since the object is at rest, the net force must be zero to satisfy Newton's 1$^{\text{st}}$ law. Choosing the conventional axes, we can write:
$\sum \vec{F} = 0 \quad \Longrightarrow \quad \sum F_x = 0 \quad \text{and} \quad \sum F_y = n - w = 0. \quad \therefore \quad \boxed{n = w = mg}.$

Example-2:

- An object on an inclined surface:



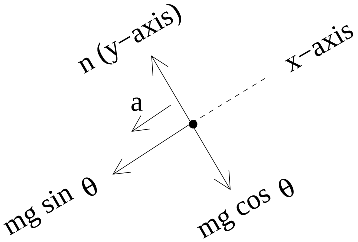
An object of mass m on a frictionless inclined surface

Free−body diagram

- Here the object is not in Equilibrium, because the net force is not zero.
- This is obvious from the fact that the surface is frictionless, and hence nothing can stop it from sliding down.
- The free-body diagram clearly shows the directions of the forces, and they are not opposite to each other as in the previous example, and so does not cancel out.
- Since the net force is not zero, the object must be obeying Newton's 2^{nd} law, and it will be sliding down with acceleration which is also indicated in the diagram.

Example-2 (continue):

- Components of forces:



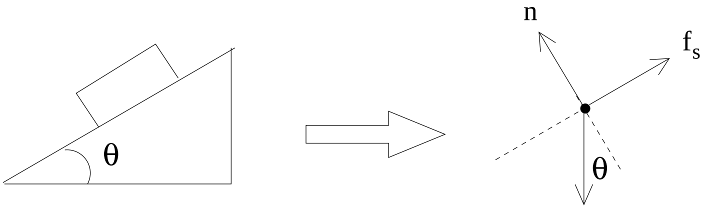
The diagram clearly shows all vectors accordingly. In vector form, we have. $\sum \vec{F} = m\vec{a}$. In components, we find:

  - The y-component of the net force is zero: $\sum F_y \equiv n - mg\cos\theta = 0$.
  - The x-component of the net force is not zero: $\sum F_x \equiv -mg\sin\theta = ma_y$. $\therefore\ a_x = -g\sin\theta$.

- To write down the Newton's 2^{nd} law in components form in the simplest form, we have chose the x-axis along up the inclined surface and the y-axis is perpendicular to it.
- Note that it is a fundamental principle that physical properties CAN NOT depend on how the coordinates are chosen. It must be same in any coordinate system. That's why we are choosing the coordinates such that the algebraic expressions become simple.

Example-3:

- An object at rest on an inclined surface:



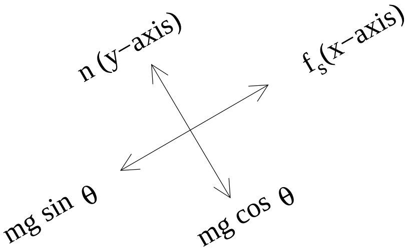
An object of mass m at rest on a frictional inclined surface

Free−body diagram

- Here the object is in Equilibrium, because it is at rest, and so net force is zero.
- Since the surface is frictional, there are three forces acting on th eobject, namely, th eweight, normal force and the frictional force.
- The free-body diagram clearly shows the directions of the forces, and they are not opposite to each other, but cancels out in components.

Example-3 (continue):

- Since the net force is zero, the object must be obeying Newton's 1^{st} law, and it will remain at rest on the inclined surface.
- Components of forces:



The diagram clearly shows all vectors accordingly. In vector form, we have. $\sum \vec{F} = 0$. In components, we find:

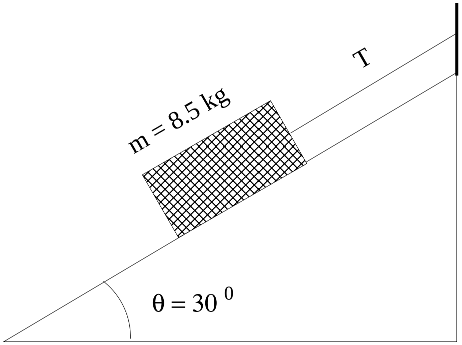
  - The y-component of the net force is zero: $\sum F_y \equiv n - mg\cos\theta = 0$ (same as in the example-2).
  - The x-component of the net force is also zero: $\sum F_x \equiv f_\mathrm{s} - mg\sin\theta = 0$.
    $\therefore \ \dfrac{f_\mathrm{s}}{n} = \tan\theta$.

- Note that if the static friction is maximum at an angle $\theta = \theta_\mathrm{s}$, then we can write:

$$\tan\theta_\mathrm{s} = \frac{f_{\mathrm{s,max}}}{n} = \frac{\mu_\mathrm{s} n}{n} \quad \Rightarrow \quad \therefore \ \boxed{\mu_\mathrm{s} = \tan\theta_\mathrm{s}} \, .$$

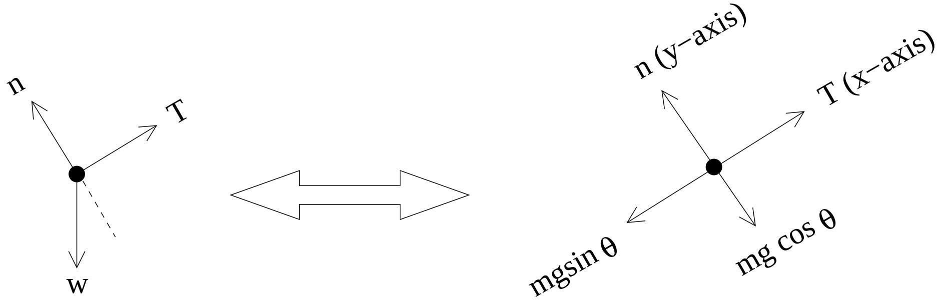
Problem # 5.17:

The adjacent figure shows a box of mass $8.5\,\mathrm{kg}$ is at rest on an inclined surface. The mass is tied by a string as shown. the angle of inclination is 30°. Find: the tension T, the normal force n, the acceleration of the object when the string is cut.



Solution # 5.17

The free body diagram including the friction is shown below.



Here, the x- and y-axes are chosen up the inclined surface and perpendicular to it for convenience.

Since the object is at rest, it must be obeying the 1$^{\text{st}}$ law. Therefore, we find:

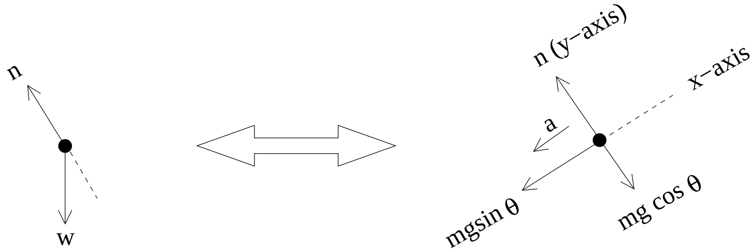
- The x-component gives:

$$\begin{aligned} \sum F_x &= T - mg\sin\theta = 0. \\ \therefore \quad T &= mg\sin\theta = (120)(9.80)(\sin 30^\circ)\mathrm{N} \ , \\ &= 40.4\,\mathrm{N} \ . \end{aligned}$$

- The y-component gives:

$$\begin{aligned} \sum F_y &= n - mg\cos\theta = 0 \ , \\ \therefore \quad n &= mg\cos\theta = (120)(9.80)(\cos 30^\circ)\mathrm{N} \ , \\ &= 69.9\,\mathrm{N} \ . \end{aligned}$$

- The figure below shows the free-body diagram after the string has been cut. In this case, $T = 0$. Since the surface is frictionless, and the object will slide down with acceleration according to the 2$^{\mathrm{nd}}$ law.

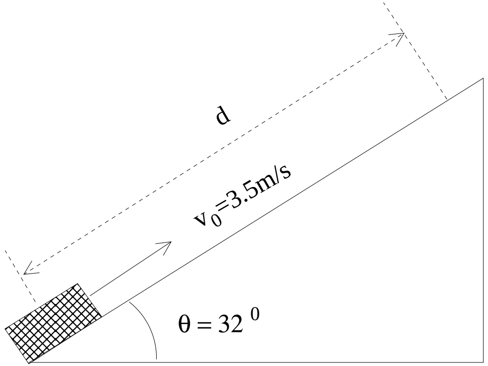


- The y-component remains exactly the same as in the previous case as it is evident from the fee-body diagram. Hence, $n = mg\cos\theta = 69.9\,\mathrm{N}$.
- The x-component becomes, by the 2$^{\mathrm{nd}}$ law,

$$\begin{aligned}\sum F_x &= -mg\sin\theta = ma\,, \\ \therefore\quad a &= -g\sin\theta = -(9.80)(\sin 30^\circ)\mathrm{m/s^2} = -4.9\,\mathrm{m/s^2}\,.\end{aligned}$$

Problem # 5.31:

A block is projected up the frictionless inclined plane with $v_0 = 3.5\,\mathrm{m/s}$. The angle of inclonation is $\theta = 32^\circ$. (a) How far up does it go? (b) How long does it take to go there? (c) What is the speed when it gets back to the bottom?



Solution:

- The free-body diagram is exactly the same as in the previous example, $a = -g \sin\theta = 9.80 \sin 32^\circ \mathrm{m/s^2} = -5.19\,\mathrm{m/s^2}$.
- Since the velocity is positive it is slowing down up the incline. Therefore, using the 4$^{\mathrm{th}}$ equation of motion, we find:

$$v_x^2 = v_0^2 + 2a(x - x_0) \quad \Longrightarrow 0 = (3.5)^2 - 2(9.8)d \ . \qquad \therefore \ d = 1.18\,\mathrm{m} \ .$$

- To find the time to go up, we use the 2$^{\mathrm{nd}}$ equation of motion, and find:

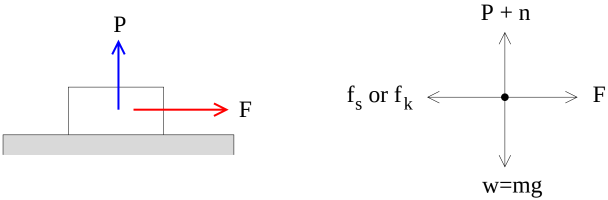
$$v_x = v_0 + at \quad \Longrightarrow \ 0 = (3.5) - 5.19t \ . \qquad \therefore \ t = 0.67\,\mathrm{sec} \ .$$

- In this case, $v_i = 0$, NOT the final velocity. So, the 4$^{\mathrm{th}}$ equation of motion gives:

$$\begin{aligned} v_f^2 &= v_i^2 + 2a(x - x_0) = 0 + 2(-g \sin\theta)(-d) \ , \\ &= 2(9.8 \sin 32^\circ)(1.18)\mathrm{m^2/s^2} \ . \qquad \therefore \ v_f = 3.5\,\mathrm{m/s} \ . \end{aligned}$$

Problem # 6.5:

A 2.5 kg block is initially at rest on a horizontal surface. A horizontal force $\vec{F}$ of magnitude 6.0 N and a vertical force $\vec{P}$ are then applied to the block (see the Figure below).



The coefficients of friction for the block and surface are $\mu_\mathrm{s} = 0.40$ and $\mu_\mathrm{k} = 0.25$. Determine the magnitude of the frictional force acting on the block if the magnitude of $\vec{P}$ is (a) 8.0 N, (b) 10 N, and (c) 12 N.

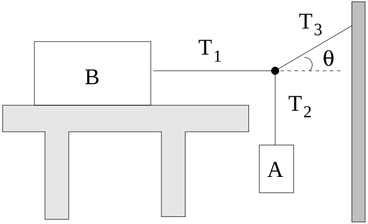
Solution # 6.5:

The frictional force may be static or kinetic depending on whether the force F is less or greater than $f_{\mathrm{s,max}}$, because the block was at rest initially. If $F < f_{\mathrm{s,max}}$, then the block is at rest, and we need to find f_{s}. And if $F > f_{\mathrm{s,max}}$, then the block is moving, and hence we need to find f_{k}.

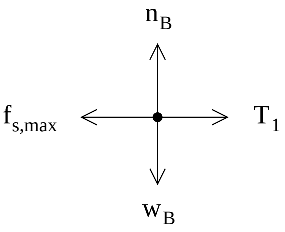
- (a) For $P = 8.0\,\mathrm{N}$,, the normal force is $n = mg - p = [2.5 \times 9.8 - 8]\mathrm{N} = 16.5\,\mathrm{N}$. Hence the maximum frictional force is $f_{\mathrm{s,max}} = \mu_{\mathrm{s}} n = 0.40 \times 16.5\,\mathrm{N} = 6.6\,\mathrm{N}$. Clearly, $F < f_{\mathrm{s,max}}$, and so the block is at rest. Therefore, $f_{\mathrm{s}} = F = 6.0\,\mathrm{N}$.
- (b) For $P = 10\,\mathrm{N}$,, the normal force is $n = mg - p = [2.5 \times 9.8 - 10]\mathrm{N} = 14.5\,\mathrm{N}$. Hence the maximum frictional force is $f_{\mathrm{s,max}} = \mu_{\mathrm{s}} n = 0.40 \times 14.5\,\mathrm{N} = 5.8\,\mathrm{N}$. Clearly, $F > f_{\mathrm{s,max}}$, and so the block is moving. Therefore, $f_{\mathrm{k}} = \mu_{\mathrm{k}} n = 0.25 \times 14.5\,\mathrm{N} = 3.6\,\mathrm{N}$.
- (c) For $P = 12\,\mathrm{N}$,, the normal force is $n = mg - p = [2.5 \times 9.8 - 12]\mathrm{N} = 12.5\,\mathrm{N}$. Hence the maximum frictional force is $f_{\mathrm{s,max}} = \mu_{\mathrm{s}} n = 0.40 \times 12.5\,\mathrm{N} = 5.0\,\mathrm{N}$. Clearly, $F > f_{\mathrm{s,max}}$, and so the block is moving. Therefore, $f_{\mathrm{k}} = \mu_{\mathrm{k}} n = 0.25 \times 12.5\,\mathrm{N} = 3.1\,\mathrm{N}$.

Problem # 6.25:

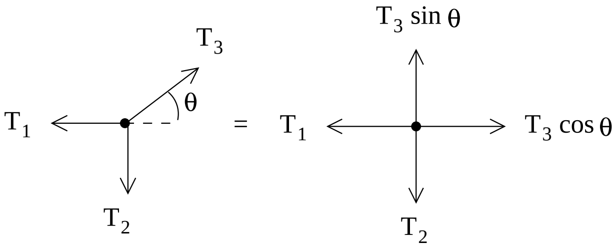
Block B in the adjacent Figure weighs 711 N. The coefficient of static friction between block and table is 0.25; angle angle θ is 30°; assume that the cord between B and the knot is horizontal. Find the maximum weight of block A for which the system will be stationary.



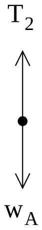
The free-body diagrams are:



For block B



At the junction of three strings



For block A

Solution # 6.25:

The free-body diagrams clearly shows the components of forces acting on the blocks and at the junction. From the free-body diagrams, it is clear that as w_{A} increases, tension T_2 also increases which in turn increases the tension T_1. Since T_1 is the applied force in block B, and the block is at rest, we have, $T_1 = f_{\mathrm{s}}$. Hence, if w_{A}, f_{s} also increases until f_{s} equals $f_{\mathrm{s,max}}$.

- The free-body diagram for block B gives:

$$n_{\mathrm{B}} = w_{\mathrm{B}} \text{ and } T_1 = f_{\mathrm{s,max}} = \mu_{\mathrm{s}} n_{\mathrm{B}} = \mu_{\mathrm{s}} w_{\mathrm{B}} \ .$$

- The free-body diagram at the junction gives:

$$T_1 = T_3 \cos\theta \text{ and } T_2 = T_3 \sin\theta. \quad \therefore \ T_2 = T_1 \tan\theta \ .$$

- From the free-body diagram for block A, we obtain,

$$w_A = T_2 = T_1 \tan\theta = \mu_{\mathrm{s}} w_{\mathrm{B}} \tan\theta = (0.25)(711)(\tan 30^\circ)\mathrm{N} = 102.6\,\mathrm{N} \ .$$