PHY 107

HW1 Solution

November 16, 2020

NOTE Solving techniques for each problem are shown and directions provided to get familiar with some of the notions discussed in class. Some answers are approximate.

Problem 1

Length (m), time(s), mass (kg), temperature (K), current (A)

Derived quantity is a quantity formed from a combination of the base units. Examples: density (kg/m^3), volume (m^3).

Pico

Density is mass/unit volume.

Density of water in kg/m^3 is 1000

Problem 2

(1) $1\ ft\ =\ 0.305\ m$
Squaring both sides yields $1\ ft^2\ =\ 0.305^2\ m^2$
Area of the lot $= 100\ ft\ \times\ 150\ ft\ =\ 15000\ ft^2 = 15000\ \times\ 0.305^2\ m^2 = 1395\ m^2$

(2) $[a] = m/s^2 = L/T^2$
$[r] = m = L$
$[v] = m/s = L/T^1$

$a \propto r^n v^m$
$L/T^2 \propto L^n (L/T)^m$
$LT^{-2} \propto L^{n+m}T^{-m}$

Equating the exponents results in:
$n + m = 1; -2 = -m$
$m = 2, n = -1$
The relationship in this scenario is $a \propto r^{-1}v^2$

Problem 3

$$x_{mean} = \frac{1+3+8}{3} = 4$$

$$\sigma = \sqrt{\frac{\sum_{i=1}^{N}(x_i - x_{mean})^2}{N-1}}$$

$$\sigma = \sqrt{\frac{\sum_{i=1}^{3}(x_i - x_{mean})^2}{3-1}}$$

$$\sigma = \sqrt{\frac{(x_1 - x_{mean})^2 + (x_2 - x_{mean})^2 + (x_3 - x_{mean})^2}{2}}$$

$$\sigma = \sqrt{\frac{(1-4)^2+(3-4)^2+(8-4)^2}{2}}$$
$$\sigma = \sqrt{13}$$

Problem 4

a) $3\hat{i}$ - 2 $\hat{j}$ + $5\hat{k}$
b) $5\hat{i}$ - 4 $\hat{j}$ - 3 $\hat{k}$
c) $11\hat{i}$ - 9 $\hat{j}$ -10 $\hat{k}$
d) 4(-1) + (-3)(1) +1(4)=-3
e) $(a_yb_z - b_ya_z)\hat{i} + (a_zb_x - b_za_x)\hat{j} + (a_xb_y - b_xa_y)\hat{k}$
$= (-3(4) - 1(1))\hat{i} + (1(-1) - 4(4))\hat{j} + (4(1) - (-1)(-3))\hat{k}$
$= -13\hat{i} - 17\hat{j} + \hat{k}$

Problem 5

(i) $\overrightarrow{a} + \overrightarrow{b} = -4\hat{i} - 6\hat{j}$
$|\overrightarrow{a} + \overrightarrow{b}| = \sqrt{(-4)^2 + (-6)^2}$
Plot the vector $\overrightarrow{a} + \overrightarrow{b}$ and you will see that the direction angle is $\tan^{-1}(\frac{-6}{-4})$ from the negative x-axis (moving counterclockwise).
(ii) x component: $10\cos(30)$
y component: $10\sin(30)$

Problem 6

This problem is about finding the vector $\overrightarrow{c}$. Let $\overrightarrow{c}$ be $c_1\hat{i} + c_2\hat{j}$.
$\overrightarrow{c} \perp \overrightarrow{a}$ implies $\overrightarrow{c}.\overrightarrow{a} = 0 \rightarrow 5c_1 - 6.5c_2 = 0$
$\overrightarrow{c}.\overrightarrow{b} = 15 \rightarrow -3.5c_1 + 7c_2 = 15$
Solve these two equations simultaneously to find the two unknowns c_1, c_2

Problem 7

(1)$\overrightarrow{A} = |A|\cos(70)\hat{i} + |A|\sin(70)\hat{j} = 1.23\hat{i} + 3.38\hat{j}$
$\overrightarrow{B} = |B|\cos(30+180)\hat{i} + |B|\sin(30+180)\hat{j} = -2.08\hat{i} - 1.2\hat{j}$

(2) $\overrightarrow{C} = 3\overrightarrow{A} - 4\overrightarrow{B} = 12.01\hat{i} + 14.94\hat{j}$

(3)Magnitude= $\sqrt{(12.01)^2 + (14.94)^2} = 19.16$
Direction Angle:
Plot the vector $\overrightarrow{C}$ and you will notice that the direction angle is $\tan^{-1}(\frac{14.94}{12.01})$ from the positive x-axis (moving counterclockwise).

Problem 8

$$\overrightarrow{D}.\overrightarrow{G} = 2(3) + (-4)(4) + 1(10) = 6 - 16 + 10 = 0$$