

SET-3

1.  $f(x, y) = xy$  &  $0 < x < 1, 0 < y < 2$

$$\begin{aligned} \text{cdf} = F(x, y) &= \int_0^x \int_0^y xy \, dx \, dy \\ &= \int_0^2 \left[ \frac{x^2}{2} \right]_0^x dy \\ &= \left[ \frac{1^2}{2} - 0 \right] \end{aligned}$$

$$F(x) = \frac{\int_0^1}{0.5}$$

$P(0.5 < x, y < 1)$

$$= \int_0^1 \int_{0.5}^1 xy \, dx \, dy$$

$$= \int_0^1 \left[ \frac{x^2}{2} \right]_{0.5}^1 dy$$

$$= \int_0^1 \left( \frac{1}{2} - \frac{1}{8} \right) y \, dy$$

$$= \frac{3}{8} \left[ \frac{y^2}{2} \right]_0^1$$

$$= \frac{3}{8} \cdot \frac{1}{2} = 3/16 \text{ Answer}$$

covariance,  $\text{cov}(x, y) = E(xy) - E(x) \cdot E(y)$

$$E(x) = \int_0^1 xy$$

$$g(x) = \int_0^2 xy \, dy$$

$$= x \left[ \frac{y^2}{2} \right]_0^2 = x \cdot \frac{2^2}{2} = 2x$$

$$\begin{aligned}
 h(y) &= \int_0^1 uy \, du \\
 &= y \left[ \frac{u^2}{2} \right]_0^1 \\
 &= y \cdot \frac{1}{2} = \frac{1}{2} y.
 \end{aligned}$$

$$\begin{aligned}
 E(u) &= \int_0^1 u g(u) \, du \\
 &= \int_0^1 2u^2 \, du \\
 &= 2 \left[ \frac{u^3}{3} \right]_0^1 \\
 &= 2 \cdot \left[ \frac{1}{3} - 0 \right] \\
 &= \frac{2}{3}
 \end{aligned}$$

$$\begin{aligned}
 E(y) &= \int_0^2 y h(y) \, dy \\
 &= \int_0^2 \frac{1}{2} y^2 \, dy \\
 &= \frac{1}{2} \left[ \frac{y^3}{3} \right]_0^2 \\
 &= \frac{1}{2} \left[ \frac{8}{3} \right] \\
 &= \frac{4}{3}
 \end{aligned}$$

$$\begin{aligned}
 E(uy) &= \int_0^1 \int_0^1 uy \cdot uy \, du \, dy \\
 &= \int_0^1 \left[ \frac{u^3}{3} \right]_0^1 y^2 \, dy = \int_0^1 \frac{1}{3} y^2 \, dy \\
 &= \frac{1}{3} \left[ \frac{y^3}{3} \right]_0^1 = \frac{1}{3} \times \frac{8}{3} = \frac{8}{9}.
 \end{aligned}$$

covariance,  $\text{cov}(x, y) = E(xy) - E(x)E(y)$

$$= \frac{8}{9} - \frac{2}{3} \times \frac{4}{3}$$

$$= \frac{8}{9} - \frac{8}{9}$$

$$= 0.$$

There are no relation between two ~~var~~ variable  $x$  and  $y$ . Because  $\text{cov} = 0$  as well as  $\text{corr} = 0$ .