North South University
Department of Mathematics and Physics
Assignment (Midterm Examination), Spring 2021
Course: MAT 361(Probability and Statistics)
Due: 19.04.2021 11:59 PM
Marks: 20

Answer any five questions.

Q1. A computer-consulting firm presently has bids out on three projects. Let $A_i = \text{awarded project } i$, for $i = 1,2,3$.

Suppose that $P(A_1) = .22$, $P(A_2) = .25$, $P(A_3) = .28$, $P(A_1 \cap A_2) = .11$, $P(A_1 \cap A_3) = .05$, $P(A_2 \cap A_3) = .07$, $P(A_1 \cap A_2 \cap A_3) = .01$.

A. Express in words each of the following events, and compute the probability of each event.

a. $A_1 \cup A_2$,

b. $A_1' \cap A_2' \cap A_3'$,

c. $A_1' \cap A_2' \cap A_3$, and

B. Are the three events pairwise independent? Find the conditional probability, $P(A_3 / \overline{A}_1)$ and interpret the probability.

Q2. Two safety inspectors inspect a new building and assign it a "safety score" of 1, 2, 3, or 4. Suppose that the random variable X is the score assigned by the first inspector and the random variable Y is the score assigned by the second inspector, and that they have a joint probability mass function given in the following table:

X \ Y	1	2	3	4
1	0.09	0.02	0.2	0.02
2	0.01	0.1	0.1	0.03
3	0.04	0.01	0.04	0.2
4	0.01	0.02	0.1	0.01

(a) What is the probability that both inspectors assign different safety score?

(b) What is the probability that the first inspector assigns a higher safety score than the second inspector?

(c) Are the scores assigned by the two inspectors independent of each other? Would you expect them to be independent? How would you interpret the situation if they were independent?

(d) If the first inspector assigns a score of 2, what is the conditional variance of the score assigned by the second inspector?

(e) What is the correlation between the scores assigned by the two inspectors? If you are responsible for training the safety inspectors to perform proper safety evaluations of buildings, what correlation value would you like there to be between the scores of two safety inspectors?

Q3. Suppose that X and Y are random variables denoting the fraction of a day that a request for merchandise occurs and the receipt of shipment occurs, respectively. The joint probability density function is

$$f(x,y)=\begin{cases}1, & 0\le x\le 1,\ 0\le y\le 1,\\ 0, & \text{otherwise.}\end{cases}$$

a) What is the probability that both the request for merchandise and receipt of an order occur during the first half of the day?

b) What is the probability that a request for merchandise occurs after its receipt? Before its receipt.

c) Determine the correlation coefficient between X and Y. Comment on the correlation coefficient.

d) Determine the CDF, F(x,y) of the above probability function.

e) Determine P(0<x<.4, 0.5<y<1).

Q4. **A.** When circuit boards used in the manufacture of compact disc players are tested, the long run percentage of defectives is 5%. Let $X =$ the number of defective boards in a random sample of size $n=25,$ stating the underlying assumptions, answer the following questions:

i) Determine $P(X\le 2)$,

ii) What is the probability that none of the 25 boards are defective?

iii) Calculate the expected value and standard deviation of X.

B. Suppose the number X of tornadoes observed in a particular region during a one-year period has a Poisson distribution with $\lambda = 8$.

i. Compute $P(X \leq 5)$,

ii. How many tornadoes can be expected to be observed during the 1-month period? What is the standard deviation of the number of observed tornadoes during 6 months period?

Q5. The amount of sugar (X) contained in 2-kg packets is actually normally distributed with a mean of $\mu = 2.03$ kg and a standard deviation of $\sigma = 0.014$ kg.

~~(a)~~ What proportion of sugar packets are underweight?

~~(b)~~ If an alternative package-filling machine is used for which the weights of the packets are normally distributed with a mean of $\mu = 2.05$ kg and a standard deviation of $\sigma = 0.016$ kg, does this result in an increase or a decrease in the proportion of underweight packets?

~~(c)~~ What is the probability that the amount of sugar of a randomly selected bearing is between 1 and 2 SDs from its mean value?

~~(d)~~ What is the value of c for which there is a 95% probability that a packet weight is within the interval [2.03 − c, 2.03 + c]?

(e) What is the value of c for which there is a 95% probability that a packet weight is within the interval [2.5 − c, 2.5 + c]?

Q6. 4.1.2 A new battery supposedly with a charge of 1.5 volts actually has a voltage with a uniform distribution **between 1.43 and 1.60 volts**. If a box contains 500 batteries,

a) What are the expectation and variance of the number of batteries in the box with a voltage less than 1.5 volts?

b) Use the binomial distribution to calculate the probability that at least 140 batteries in the box with a voltage less than 1.5 volts. (Use excel if necessary).

c) Also, use the Poisson distribution to make an approximate calculation of the probability that at least 140 batteries in the box with a voltage less than 1.5 volts.