NORTH SOUTH UNIVERSITY

DEPARTMENT OF MATHEMATICS & PHYSICS

ASSIGNMENT # 01

Spring 2022

Total Marks: 10

Course: MAT-120, Section - 11

Due: 26 February 2022 (Before 06: 00 pm)

Important Notes: ***1.*** *You have to answer all the questions serially.*
2. *You must prepare it by your hand writing, for submission please use pdf only.*
3. *Write page number and ID at the top of each page of your assignment.*
4. *You have to solve the assignment with honesty and integrity.*
5. *Submit the assignment soon once you complete it.*
6. *You should not share your solutions with others.*
7. *Use **A4** size papers.*
8. *Assignment will not be **accepted** after the due date.*
9. *Your score will be **zero** for any copy or plagiarism and*
10. *Must submit it through **CANVAS**.*

Question 1: (*a*) Find a formula for $f^{-1}(x)$, and state the domain and range of the function f^{-1}:

(i) $f(x) = \dfrac{1}{x^2-1};$ $x > 1$

(ii) $f(x) = \begin{cases} \log(x-1); & x \geq 2 \\ x-2; & x < 2 \end{cases}$

(*b*) (*i*) Show that $f(x) = \dfrac{3-x}{1-x}$ is its own inverse. (*ii*) What does the result in part (*i*) tell you about the graph of *f* ?.

(*c*) Prove that the following identities involving inverse trigonometric functions for $-1 \leq x \leq 1$:

$$\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$$
$$\cos(\sin^{-1}x) = \sqrt{1-x^2}$$
$$\sin(\cos^{-1}x) = \sqrt{1-x^2}$$
$$\tan(\sin^{-1}x) = \frac{x}{\sqrt{1-x^2}}$$

Question 2: (*a*) Solve for *x*

(i) $\log_{10}(2+x) = 4$

(ii) $\ln\left(\frac{1}{x}\right) + \ln(2x^3) = \ln 3$

(iii) $\ln 4x - 3\ln(x^2) = \ln 2$

(iv) $e^{-2x} - 3e^{-x} = -2$

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(b) Show that for $x > 0$ and $k \neq 0$ the equations

$$x^k = e^x \quad and \quad \frac{\ln x}{x} = \frac{1}{k}$$

have the same solutions.

(c) Let $f(x) = \frac{ax+b}{cx+d}$. What conditions on $a, b, c,$ and d guarantee that f^{-1} exists? Find $f^{-1}(x)$.

Question 3: (a) Find, if exist. If not, explain why?

$$\lim_{x \to 2} \frac{1-x}{(x+2)(x-2)}$$

(b) Find the following limit

$$\lim_{x \to 0} \left(\frac{1}{x} - \frac{2}{x^2 + 2x} \right)$$

(c) Let

$$f(x) = \begin{cases} e^x + 1; & x \geq 0 \\ 0; & -2 < x < 0 \\ x + 1; & x \leq -2 \end{cases}$$

- **(i)** Find $\lim_{x \to -3} f(x)$
- **(ii)** Find $\lim_{x \to -2} f(x)$
- **(iii)** Find $\lim_{x \to 0} f(x)$
- **(iv)** Find $\lim_{x \to 1} f(x)$
- **(v)** Sketch the graph of $y = f(x)$

Question 4: (a) Find values of x, if any, at which f is not continuous

$$f(x) = \begin{cases} \frac{3}{x-1}, & x \neq 1 \\ 3, & x = 1 \end{cases}$$

(b) Find values of the constants k and m, if possible, that will make the function f continuous everywhere

$$f(x) = \begin{cases} x^2 + 5, & x > 2 \\ m(x+1) + k, & -1 < x \leq 2 \\ 2x^3 + x + 7, & x \leq -1 \end{cases}$$