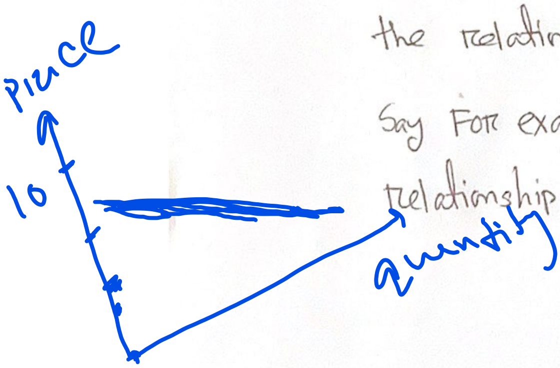
Mat 116

Lecture: 3

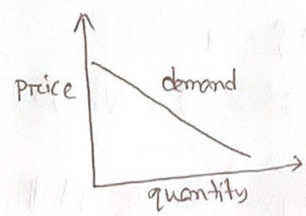
Graphs of equation in two variables; Intercepts; symmetry

The graph of an equation in two variables x and y consists of the set of points in the xy plane whose coordinates (x, y) satisfy the equation.

Graphs play an important role in helping us to visualize the relationships that exist between two variables.



Say for example, the demand curve/line represents the relationship between the price and the demand of the quantity



Determine whether a point is on the graph of an Equation:

Determine if the following points are on the graph of the equation $2x - y = 6$

(a) $(2, 3)$ (b) $(2, -2)$

$4 - 3$
$= 1 \neq 6$

Soln: (a) For the point $(2, 3)$, check to see if $x = 2$ and $y = 3$ satisfies the equation $2x - y = 6$

If we substitute $x = 2$ and $y = 3$ on the left hand side of the eqn $2x - y = 6$ we get

$$2(2) - 3 = 4 - 3 = 1 \neq 6$$

So the point $(2, 3)$ is not on the graph of eqn $2x - y = 6$

because it doesnot satisfy the equation.

(b) $(2,-2)$. Here $x = 2$, $y = -2$

$2x - y = (2)(2) - (-2) = 4 + 2 = 6$ which satisfies the given equation.

So the point $(2,-2)$ lies on the eqn of graph of the eqn $2x - y = 6$.

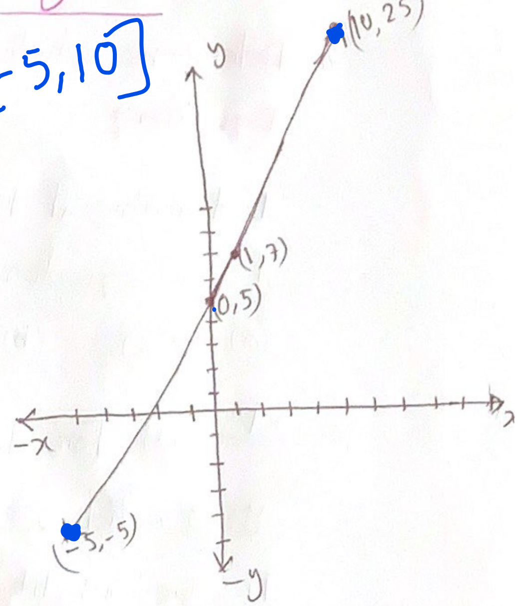
Exercise:

1. Equation: $y = x^4 - \sqrt{x}$
   Points: $(0,0)$; $(1,1)$; $(-1,0)$
2. Equation: $x^2 + y^2 = 4$
   Points: $(0,2)$; $(-2,2)$; $(\sqrt{2}, \sqrt{2})$

Graph an Equation by Plotting points:

Example: $y = 2x + 5$ $\quad x \in [-5, 10]$

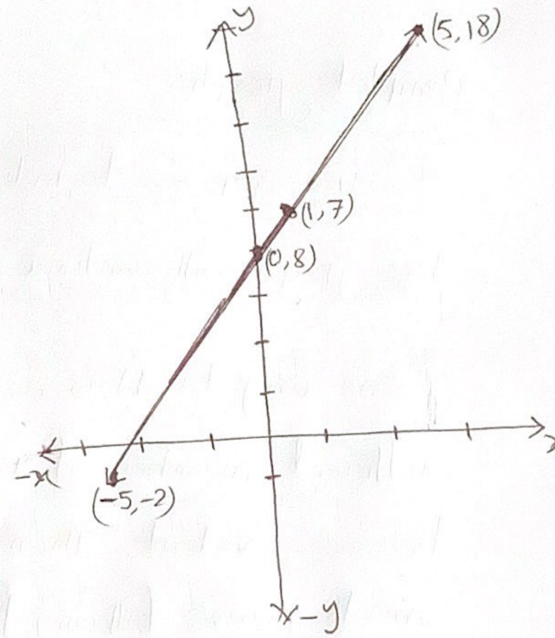
If	then	Points on graph
$x=0$	$y = 0+5 = 5$	$(0,5)$
$x=1$	$y = 2+5 = 7$	$(1,7)$
$x=-5$	$y = -10+5 = -5$	$(-5,-5)$
$x=10$	$y = 20+5 = 25$	$(10,25)$



Example : 2

$y = 2x + 8$

If	then	Points on graph
$x=0$	$y=0+8=8$	$(0,8)$
$x=1$	$y=2+8=10$	$(1,10)$
$x=-5$	$y=-10+8=-2$	$(-5,-2)$
$x=5$	$y=10+8=18$	$(5,18)$



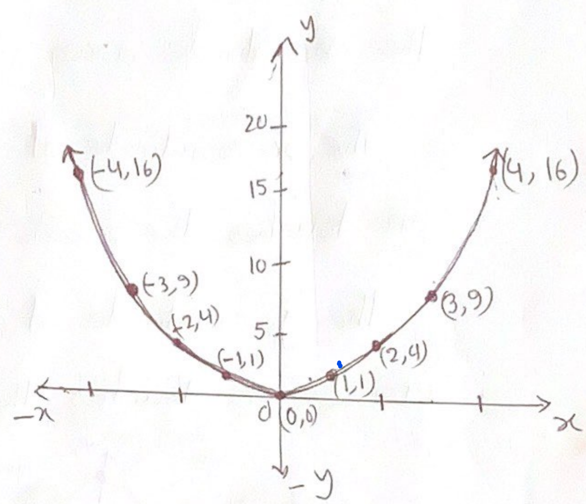
Exercise:

i) $y = 3x - 9$ ii) $y = x^2 - 9$ iii) $y = 3x + 2$

Example: 3

Graph the equation $y = x^2$

x	$y=x^2$	(x,y)
-4	$y=16$	$(-4,16)$
-3	$y=9$	$(-3,9)$
-2	$y=4$	$(-2,4)$
-1	$y=1$	$(-1,1)$
0	$y=0$	$(0,0)$
1	$y=1$	$(1,1)$
2	$y=4$	$(2,4)$
3	$y=9$	$(3,9)$
4	$y=16$	$(4,16)$



Exercise: Find the intercepts and graph the eqn.

i) $y = x^2 - 9$ ii) $9x^2 + 4y = 36$

Complete graph:

Use arrows to indicate that the pattern shows in the graph will continue.

One way to obtain a complete graph is to plot a sufficient number of points on the graph until a pattern becomes evident. Then these points are connected with a smooth curve following the suggested pattern. But how many points are sufficient?

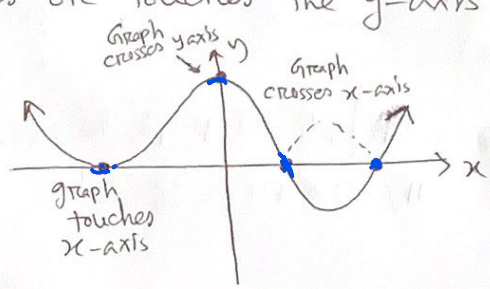
Two techniques can reduce the number of points required to graph an equation involve finding intercepts and checking for symmetry.

Intercepts:

The points, if any, at which a graph crosses or touches the coordinate axes are called the intercepts.

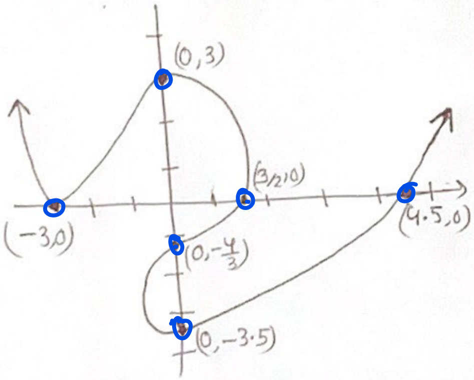
The x-coordinate of a point at which the graph crosses or touches the x-axis is called an x-intercept.

The y-coordinate of a point at which the graph crosses or touches the y-axis is called the y-intercept.



Example:

Find the intercepts of the graph. what are its x-intercepts? what are y-intercepts?



The points on the graph are

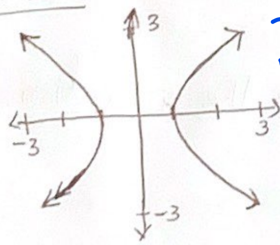
$(-3,0), (0,3), \left(\frac{3}{2},0\right), \left(0,-\frac{4}{3}\right), (0,-3.5)$
$(4.5,0)$

The x intercepts are $-3, \frac{3}{2}, 4.5$

The y " " $-3.5, -\frac{4}{3}, 3$

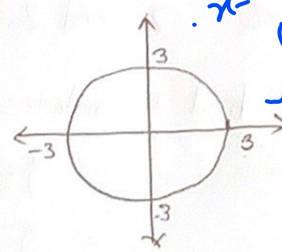
Exercise:

1)



no y-intercept
x-intercept
$(-1,0),(1,0)$

11)



. x-intercept
$(-3,0),(3,0)$
y-intercept
$(0,3),(0,-3)$

Find intercepts from an equation:

Procedure:

1. Find x-intercepts, let $y=0$ in the eqn and solve for x.

11) Find y-intercepts, let $x=0$ in the eqn and solve for y.

Examples

Find the x intercepts and y intercepts of the graph $y=x^2-4$.. The graph $y=x^2-4$

Example:

Solution:

To find the x-intercepts, let $y=0$ in the given equation.

So we get

$$x^2-4=0$$

$$\Rightarrow (x+2)(x-2)=0$$

$$\therefore x+2=0 \quad \text{or} \quad x-2=0$$

$$\Rightarrow x=-2 \quad \text{or} \quad x=2$$

The eq^n has two solutions, -2 and 2. The x-intercepts are -2 and 2.

To find y-intercepts, let $x=0$ in the equation.

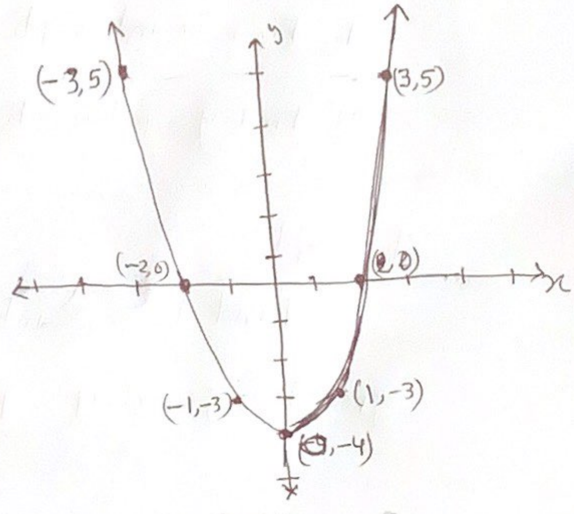
$$y=x^2-4$$

$$\Rightarrow y=0-4=-4$$

The y-intercept is -4.

Since $x^2 \geq 0$ for all x, we deduce from the eq^n $y=x^2-4$ that $y \geq -4$ for all x.

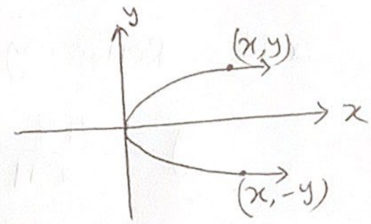
x	$y=x^2-4$	(x,y)
-3	$y=5$	$(-3,5)$
-1	-3	$(-1,-3)$
1	-3	$(1,-3)$
3	5	$(3,5)$



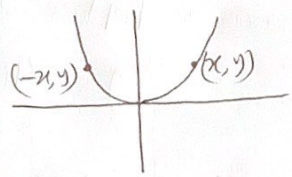
$*\ y=x^2-1$

Test an equation for symmetry:

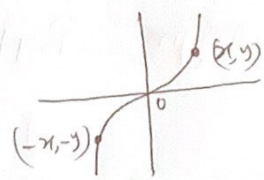
A graph is said to be symmetric with respect to the x-axis, if for every point (x, y) on the graph, the point $(x, -y)$ is also on the graph.



A graph is said to be symmetric with respect to the y-axis if, for every point (x, y) on the graph, the point $(-x, y)$ is also on the graph.



A graph is said to be symmetric with respect to the origin if, for every point (x, y) on the graph, the point $(-x, -y)$ is also on the graph.



Procedures:

x-axis → Replace y by $-y$, if an equivalent eq^n results then the graph of the eq^n is symmetric with respect to x-axis

y-axis → Replace x by $-x$, if an equivalent eq^n results, then the graph of the eq^n is symmetric with respect to y-axis

Origin → Replace x by $-x$ and y by $-y$.

Example:

Test $y = \frac{4x^2}{x^2+1}$ for symmetry

Solution:

x axis : Replace y by $-y$.

$$-y = \frac{4x^2}{x^2+1}$$

$\Rightarrow y = -\frac{4x^2}{x^2+1}$ which is not equivalent to $y = \frac{4x^2}{x^2+1}$,

So the graph of this eqn is not symmetric about x-axis

y axis : Replace x by $-x$.

$y = \frac{4(-x)^2}{(-x)^2+1} = \frac{4x^2}{x^2+1}$ which is equivalent to

$y = \frac{4x^2}{x^2+1}$. So the graph of the given eqn is

symmetric about y-axis.

origin : Replace x by $-x$ and y by $-y$.

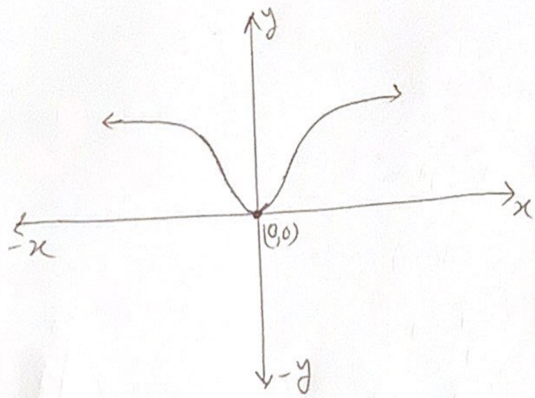
$$-y = \frac{4(-x)^2}{(-x)^2+1}$$

$$\Rightarrow -y = \frac{4x^2}{x^2+1}$$

$\Rightarrow y = -\frac{4x^2}{x^2+1}$ which is not equivalent to

the given equation. Thus the graph of $y = \frac{4x^2}{x^2+1}$

is not symmetric about origin.



Example:

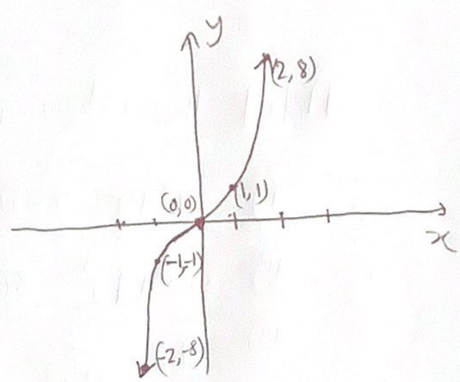
$$y = x^3$$

x-axis $\Rightarrow -y = x^3 \Rightarrow y = -x^3 \neq$ original eq^n.

y-axis $\Rightarrow y = (-x)^3 \Rightarrow y = -x^3 \neq$ " "

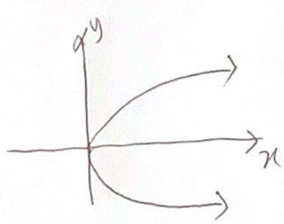
origin $\rightarrow -y = (-x)^3 \Rightarrow y = x^3$ equivalent to original eq^n. Symmetry about origin

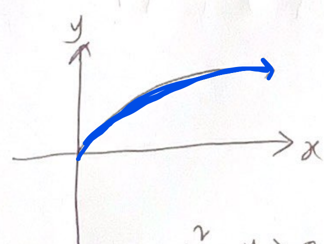
x	$y = x^3$	(x, y)
0	0	(0,0)
1	1	(1,1)
2	8	(2,8)
3	27	(3,27)



Example: $x = y^2$

$y = \pm\sqrt{x}$, symmetric about x-axis

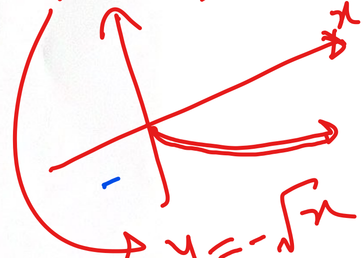




$x = y^2, \ y \geq 0$

$y = \sqrt{x}$

$x = y^2, \ y \leq 0$



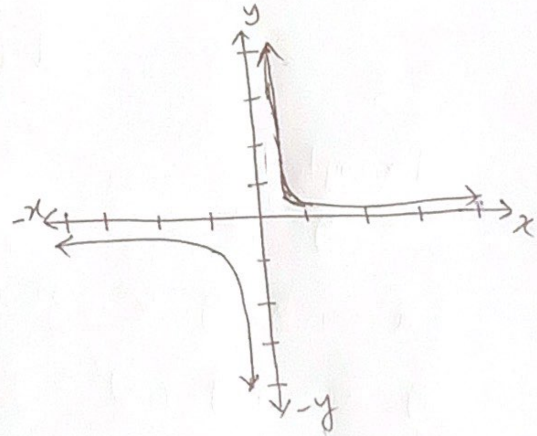
$y = -\sqrt{x}$

Example:

$y = \frac{1}{x}$

Symmetric about origin.

x	$y=\frac{1}{x}$	(x,y)
$\frac{1}{10}$	10	$(\frac{1}{10}, 10)$
$\frac{1}{3}$	3	$(\frac{1}{3}, 3)$
$\frac{1}{2}$	2	$(\frac{1}{2}, 2)$
1	1	$(1,1)$
2	$\frac{1}{2}$	$(2, \frac{1}{2})$
3	$\frac{1}{3}$	$(3, \frac{1}{3})$
10	$\frac{1}{10}$	$(10, \frac{1}{10})$



☐ list the intercepts and test for symmetry

i) $x^2+y-9=0$

ii) $9x^2+4y^2=36$

iii) $y=\frac{3x}{x^2+9}$

☐ If $(3,b)$ is a point on the graph of $y=4x+1$, then what is b?

$b = 12+1$

$\therefore \boxed{b=13}$