Classwork

Suppose that the random variable *X* measures the thickness of the paint in millimeters at a randomly chosen point on a randomly chosen car panel, and that *X* takes values between 0.125 and 0.5 mm with a probability density function of

$$f(x) = \frac{512}{93}(0.5 - (x - 0.25)^2)$$
$$\text{for } 0.125 \le x \le 0.5$$

What is the expected or average paint thickness?

Solution: Given,

$$f(x) = \frac{512}{93}(0.5 - (x - 0.25)2) \qquad 0.125 \le x \le 0.5$$

$$E(x) = \int_{0.125}^{0.5} x\, f(x)\, dx$$

$$= \int_{0.125}^{0.5} x \frac{512}{93}(0.5 - (x - 0.25)^2)\, dx$$

$$= \frac{512}{93}\int_{0.125}^{0.5} x(0.5 - (x^2 - 0.5x + 0.0625))\, dx$$

$$= \frac{512}{93}\int_{0.125}^{0.5} (0.5x - (x^3 - 0.5x^2 + 0.0625x))\, dx$$

$$= \frac{512}{93}\int_{0.125}^{0.5} [-x^3 + 0.5x^2 + 0.4375x]\, dx$$

$$= \frac{512}{93}\left[-\frac{x^4}{4} + 0.5\frac{x^3}{3} + 0.4375\frac{x^2}{2}\right]_{0.125}^{0.5}$$

$$= \frac{512}{93}\left[-\frac{(0.5)^4}{4} + 0.5\frac{(0.5)^3}{3} + 0.4375\frac{(0.5)^2}{2} - \left[-\frac{(0.125)^4}{4} + 0.5\frac{(0.125)^3}{3} + 0.4375\frac{(0.125)^2}{2}\right]\right]$$

$$= \frac{512}{93}\left[\frac{23}{384} - \frac{181}{49152}\right]$$

$$= 0.3$$