Introduction to Communication Systems

Chapter 5
Frequency Modulation

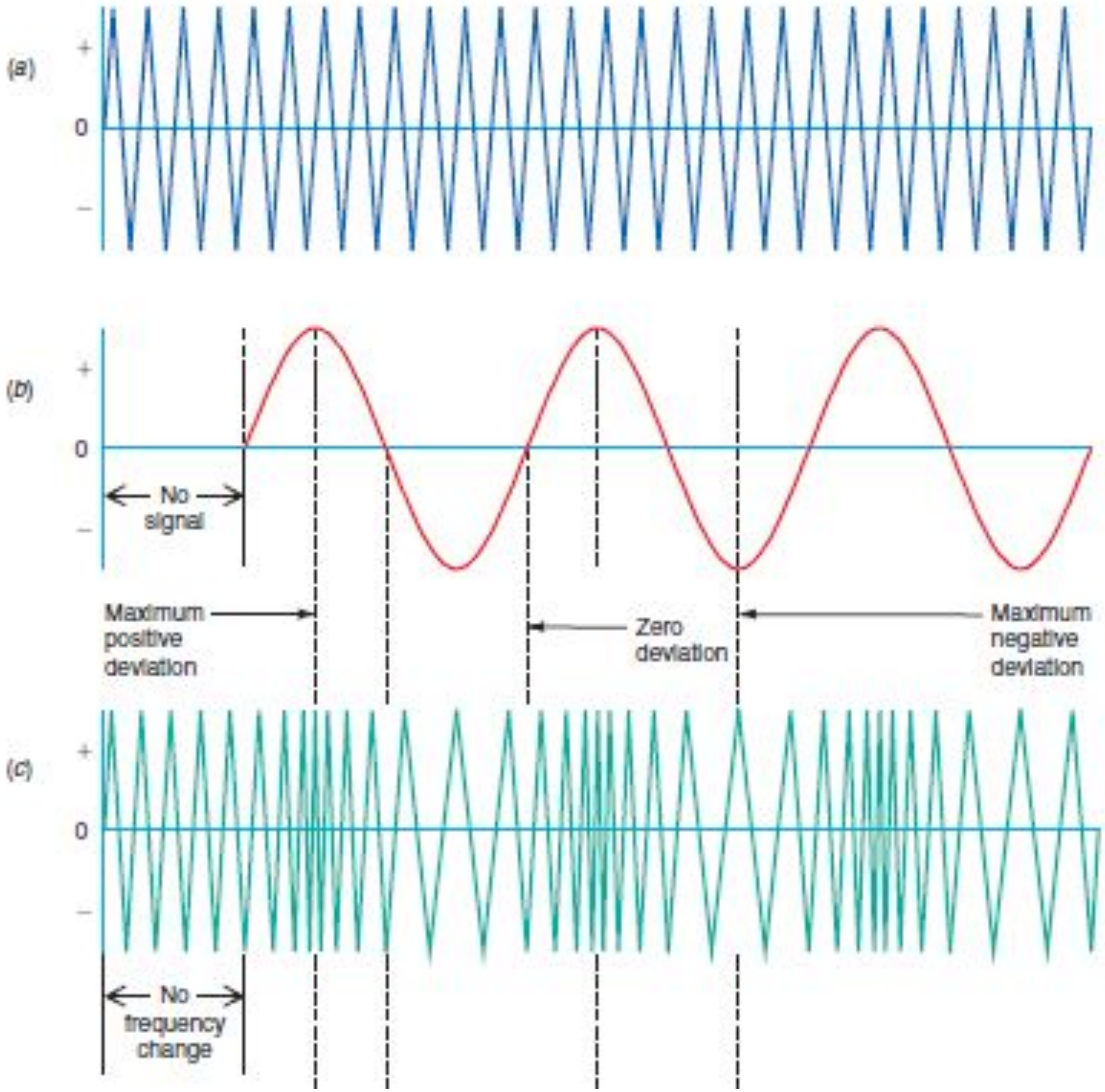
Dr. Saeed Mahmud Ullah
Professor
EEE, DU

Text Book

- Principles of Electronic Communication Systems
  - L. E. Frenzel
  - 4^{th} edition

Frequency Modulation

- In FM, the carrier amplitude remains constant and the carrier frequency is changed by the modulating signal. As the amplitude of the information signal varies, the carrier frequency shifts proportionately. As the modulating signal amplitude increases, the carrier frequency increases.
- The amount of change in carrier frequency produced by the modulating signal is known as the *frequency deviation* f_d. *Maximum* frequency deviation occurs at the maximum amplitude of the modulating signal.


(a)
+
0
−
(b)
+
0
−
No signal
Maximum positive deviation
Zero deviation
Maximum negative deviation
(c)
+
0
−
No frequency change

Example 5-1

A transmitter operates on a frequency of 915 MHz. The maximum FM deviation is ± 12.5 kHz. What are the maximum and minimum frequencies that occur during modulation?

$$915 \text{ MHz} = 915{,}000 \text{ kHz}$$

$$\text{Maximum deviation} = 915{,}000 + 12.5 = 915{,}012.5 \text{ kHz}$$

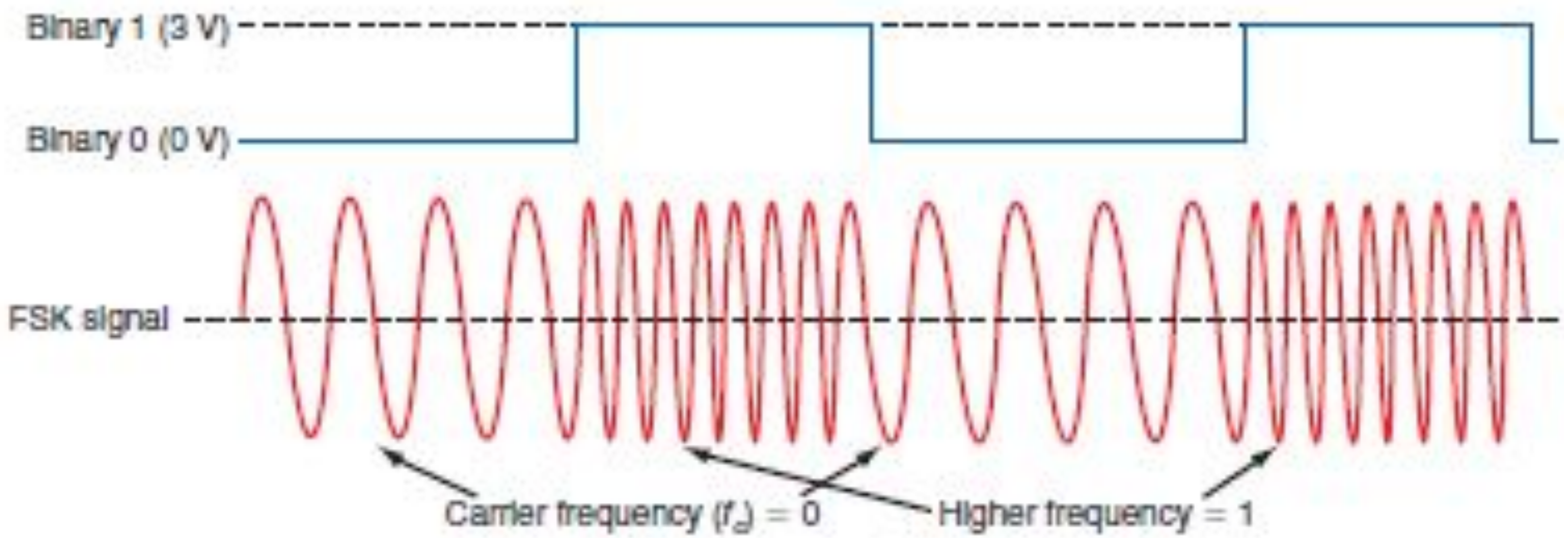
$$\text{Minimum deviation} = 915{,}000 - 12.5 = 914{,}987.5 \text{ kHz}$$

FSK (Frequency shift keying)

- When the modulating signal has only two amplitudes (pulse train), the carrier frequency changes according to the amplitude of the modulating signal.
- For example, when the modulating signal is a binary 0, the carrier frequency is the center frequency value. When the modulating signal is a binary 1, the carrier frequency abruptly changes to a higher frequency level.
- The amount of the shift depends on the amplitude of the binary signal. This kind of modulation, called *frequency-shift keying (FSK),*
- This is widely used in the transmission of binary data in Bluetooth headsets, wireless speakers, and many forms of industrial wireless.

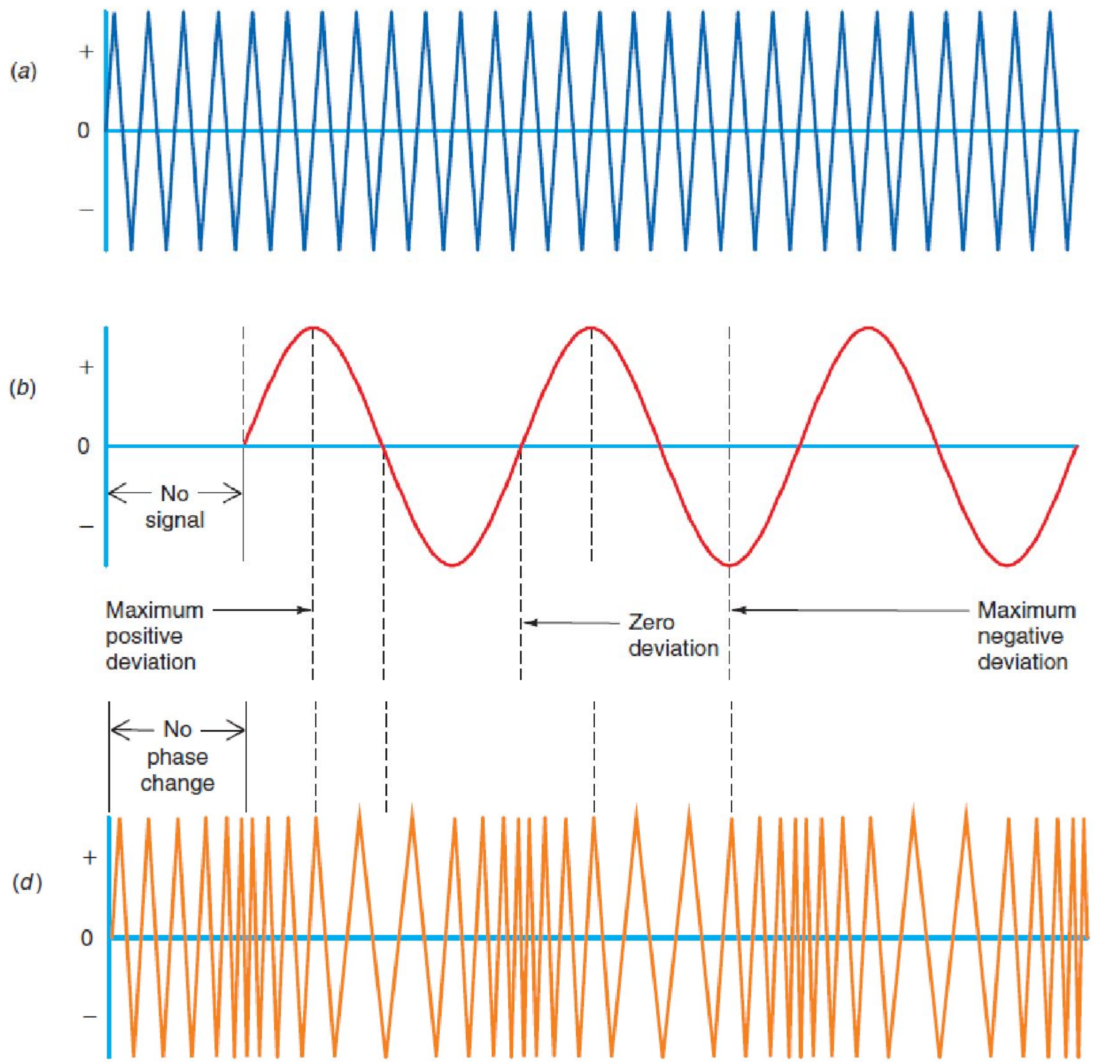
FSK (Frequency shift keying)

Figure 5-2 Frequency-modulating of a carrier with binary data produces FSK.



Phase modulation

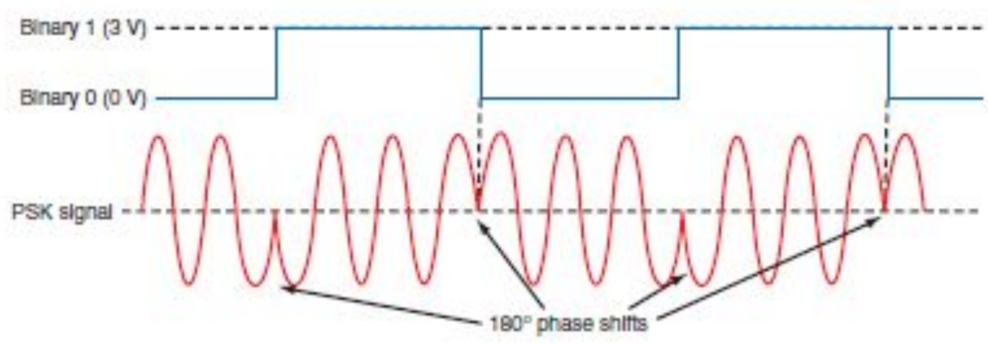
- When the amount of phase shift of a constant-frequency carrier is varied in accordance with a modulating signal, the resulting output is a *phase modulation (PM)* signal.
- Imagine a modulator circuit whose basic function is to produce a *phase shift,* i.e., a time separation between two sine waves of the same frequency. Assume that a phase shifter can be built that will cause the amount of phase shift to vary with the amplitude of the modulating signal. The greater the amplitude of the modulating signal, the greater the phase shift.


(a)
+
0
–
(b)
+
0
–
No
signal
Maximum
positive
deviation
Zero
deviation
Maximum
negative
deviation
No
phase
change
(d)
+
0
–

PSK (Phase shift keying)

- When the binary modulating signal is 0 V, or binary 0, the PM signal is simply the carrier frequency.
- When a binary 1 voltage level occurs, the modulator, which is a phase shifter, simply changes the phase of the carrier, not its frequency.
- In Fig. 5-6 the phase shift is 180°. Each time the signal changes from 0 to 1 or 1 to 0, there is a 180° phase shift. The PM signal is still the carrier frequency, but the phase has been changed with respect to the original carrier with a binary 0 input.
- The process of phase-modulating a carrier with binary data is called *phase-shift keying (PSK)* or *binary phase-shift keying (BPSK).*

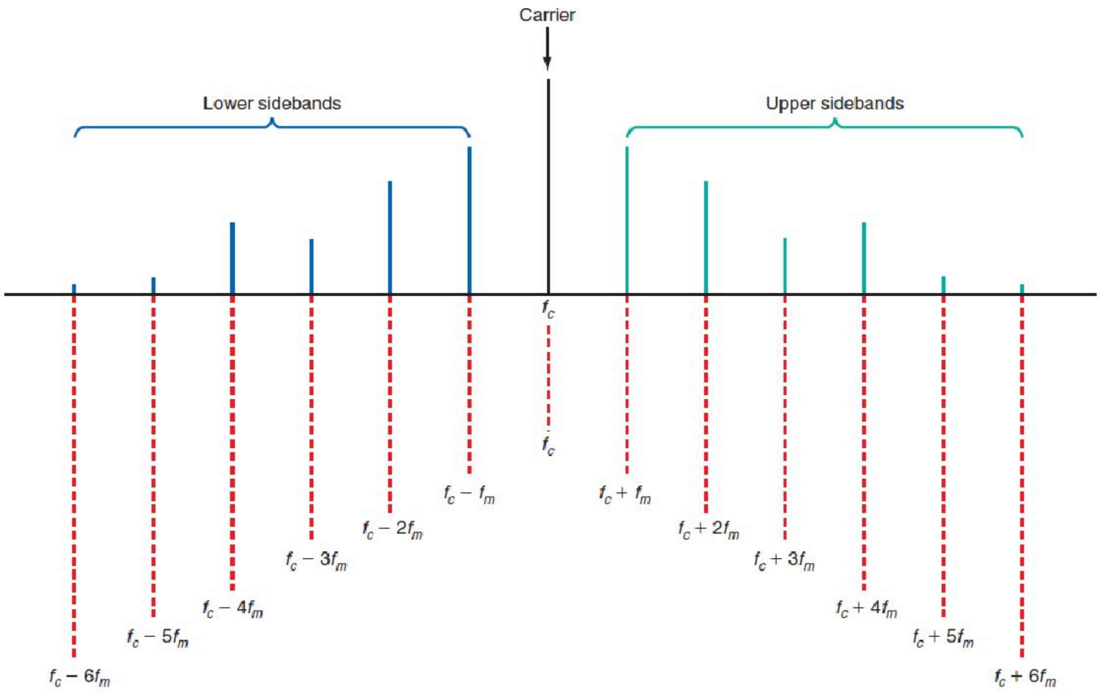
Figure 5-6 Phase modulation of a carrier by binary data produces PSK.



Sideband of FM (Ideal)

- FM process produces an infinite number of upper and lower sidebands and, therefore, a theoretically infinitely large bandwidth.
- However, in practice, only those sidebands with the largest amplitudes are significant in carrying the information. Typically any sideband whose amplitude is less than 1 percent of the unmodulated carrier is considered insignificant.

Figure 5-7 Frequency spectrum of an FM signal. Note that the carrier and sideband amplitudes shown are just examples. The amplitudes depend upon the modulation index m_f.



Modulation Index

- The ratio of the frequency deviation to the modulating frequency is known as the *modulation index* m_f :

$$m_f = \frac{f_d}{f_m}$$

where f_d is the frequency deviation and f_m is the modulating frequency

In most communication systems using FM, maximum limits are put on both the frequency deviation and the modulating frequency. For example, in **standard FM broadcasting**, the maximum permitted frequency deviation is **75 kHz** and the maximum permitted modulating frequency is **15 kHz**. This produces a modulation index of $m_f = 75/15 = 5$

Modulation Index

Example 5-2

What is the deviation ratio of TV sound if the maximum deviation is 25 kHz and the maximum modulating frequency is 15 kHz?

$$m_f = \frac{f_d}{f_m} = \frac{25}{15} = 1.667$$

Bessel Functions

Given the modulation index, the number and amplitudes of the significant sidebands can be determined by solving the basic equation of an FM signal. The FM equation, whose derivation is beyond the scope of this book, is $v_{FM} = V_c \sin[2\pi f_c t + m_f \sin(2\pi f_m t)]$, where v_{FM} is the instantaneous value of the FM signal and m_f is the modulation index. The term whose coefficient is m_f is the phase angle of the carrier. Note that this equation expresses the phase angle in terms of the sine wave modulating signal. This equation is solved with a complex mathematical process known as *Bessel functions*. It is not necessary to show this solution, but the result is as follows:

$$\begin{aligned} v_{FM} = V_c\{&J_0(\sin\omega_c t) + J_1[\sin(\omega_c + \omega_m)t - \sin(\omega_c - \omega_m)t] \\ &+ J_2[\sin(\omega_c + 2\omega_m)t + \sin(\omega_c - 2\omega_m)t] \\ &+ J_3[\sin(\omega_c + 3\omega_m)t - \sin(\omega_c - 3\omega_m)t] \\ &+ J_4[\sin(\omega_c + 4\omega_m)t + \sin(\omega_c - 4\omega_m)t] \\ &+ J_5[\sin\cdots] + \cdots\} \end{aligned}$$

where $\omega_c = 2\pi f_c =$ carrier frequency

$\omega_m = 2\pi f_m =$ modulating signal frequency

$V_c =$ peak value of unmodulated carrier

The amplitudes of the sidebands are determined by the J_n coefficients, which are, in turn, determined by the value of the modulation index. These amplitude coefficients are computed by using the expression

$$J_n(m_f) = \left(\frac{m_f}{2^n n!}\right)^n \left[1 - \frac{(m_f)^2}{2(2n+2)} + \frac{(m_f)^4}{2 \cdot 4(2n+2)(2n+4)} - \frac{(m_f)^6}{2 \cdot 4 \cdot 6(2n+2)(2n+4)(2n+6)} + \cdots\right]$$

where ! = factorial

n = sideband number (1, 2, 3, etc.)

n = 0 is the carrier

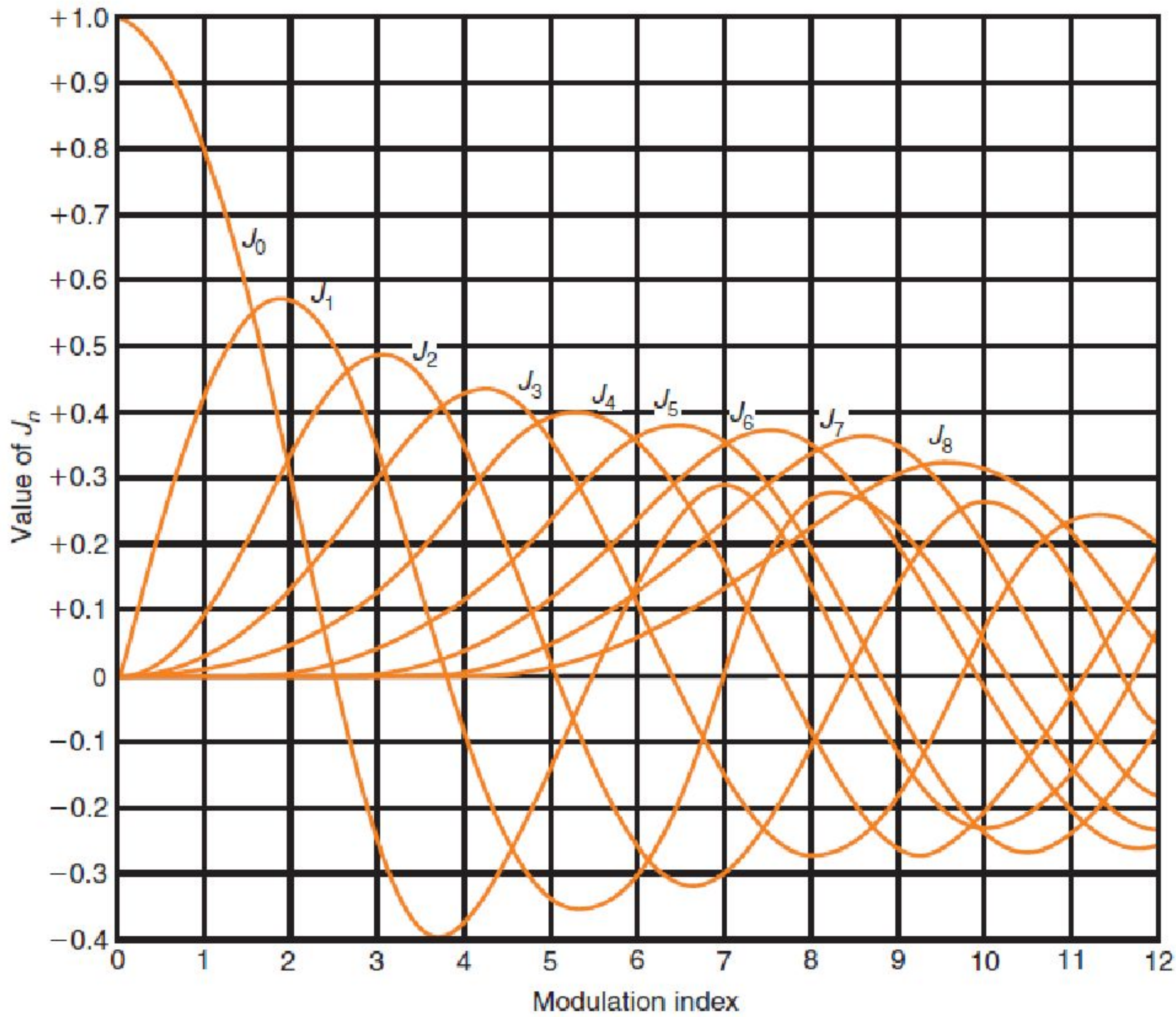
$m_f = \dfrac{f_d}{f_m}$ = frequency deviation

Bessel Function Table

Figure 5-8 Carrier and sideband amplitudes for different modulation indexes of FM signals based on the Bessel functions.

Modulation Index	Carrier	Sidebands (Pairs)															
		1st	2d	3d	4th	5th	6th	7th	8th	9th	10th	11th	12th	13th	14th	15th	16th
0.00	1.00	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
0.25	0.98	0.12	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
0.5	0.94	0.24	0.03	—	—	—	—	—	—	—	—	—	—	—	—	—	—
1.0	0.77	0.44	0.11	0.02	—	—	—	—	—	—	—	—	—	—	—	—	—
1.5	0.51	0.56	0.23	0.06	0.01	—	—	—	—	—	—	—	—	—	—	—	—
2.0	0.22	0.58	0.35	0.13	0.03	—	—	—	—	—	—	—	—	—	—	—	—
2.5	−0.05	0.50	0.45	0.22	0.07	0.02	—	—	—	—	—	—	—	—	—	—	—
3.0	−0.26	0.34	0.49	0.31	0.13	0.04	0.01	—	—	—	—	—	—	—	—	—	—
4.0	−0.40	−0.07	0.36	0.43	0.28	0.13	0.05	0.02	—	—	—	—	—	—	—	—	—
5.0	−0.18	−0.33	0.05	0.36	0.39	0.26	0.13	0.05	0.02	—	—	—	—	—	—	—	—
6.0	0.15	−0.28	−0.24	0.11	0.36	0.36	0.25	0.13	0.06	0.02	—	—	—	—	—	—	—
7.0	0.30	0.00	−0.30	−0.17	0.16	0.35	0.34	0.23	0.13	0.06	0.02	—	—	—	—	—	—
8.0	0.17	0.23	−0.11	−0.29	−0.10	0.19	0.34	0.32	0.22	0.13	0.06	0.03	—	—	—	—	—
9.0	−0.09	0.24	0.14	−0.18	−0.27	−0.06	0.20	0.33	0.30	0.21	0.12	0.06	0.03	0.01	—	—	—
10.0	−0.25	0.04	0.25	0.06	−0.22	−0.23	−0.01	0.22	0.31	0.29	0.20	0.12	0.06	0.03	0.01	—	—
12.0	−0.05	−0.22	−0.08	0.20	0.18	−0.07	−0.24	−0.17	0.05	0.23	0.30	0.27	0.20	0.12	0.07	0.03	0.01
15.0	−0.01	0.21	0.04	0.19	−0.12	0.13	0.21	0.03	−0.17	−0.22	−0.09	0.10	0.24	0.28	0.25	0.18	0.12

Figure 5-9 Plot of the Bessel function data from Fig. 5-8.

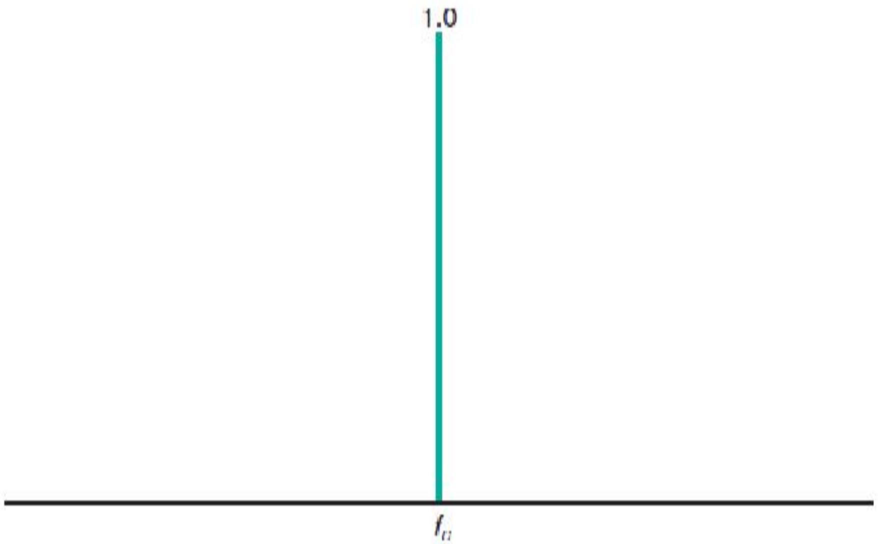


Example 5-3

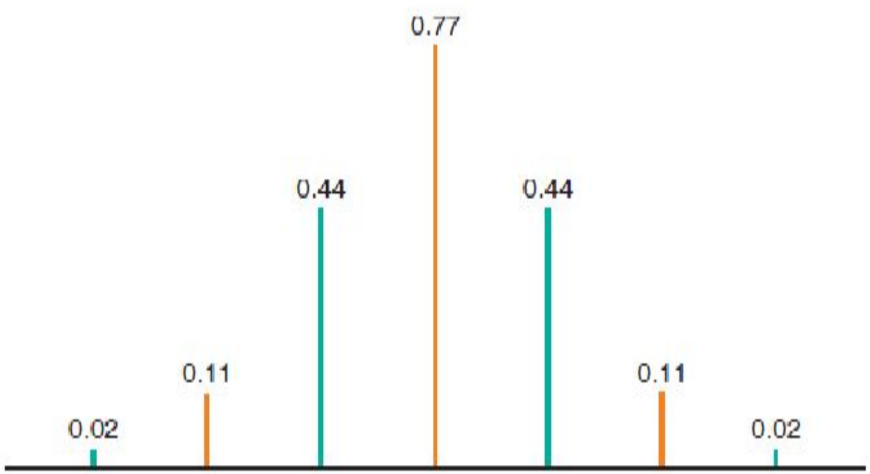
What is the maximum modulating frequency that can be used to achieve a modulation index of 2.2 with a deviation of 7.48 kHz?

$$f_m = \frac{f_d}{m_f} = \frac{7480}{2.2} = 3400 \text{ Hz} = 3.4 \text{ kHz}$$

Figure 5-10 Examples of FM signal spectra. (*a*) Modulation index of 0 (no modula
or sidebands). (*b*) Modulation index of 1. (*c*) Modulation index of 2.
(*d*) Modulation index of 0.25 (NBFM).



(*a*)



(*b*)

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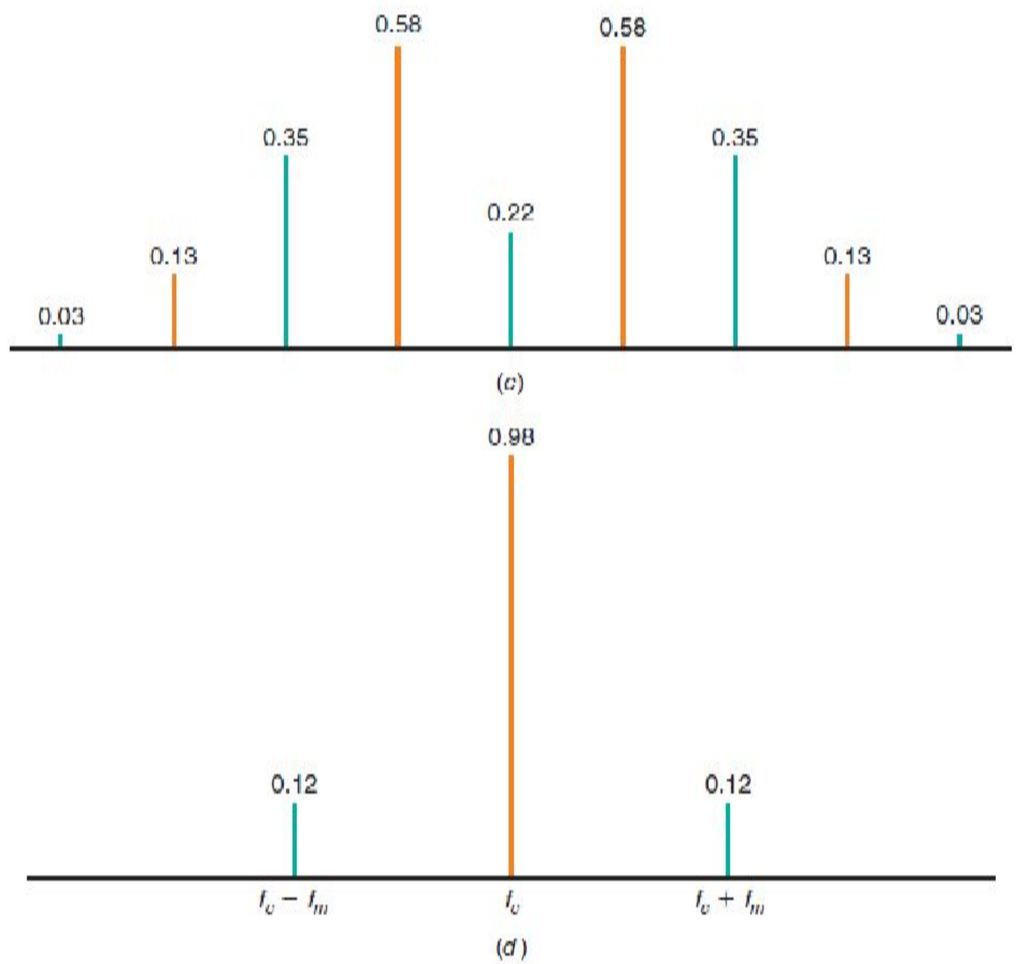

Example 5-4

State the amplitudes of the carrier and the first four sidebands of an FM signal with a modulation index of 4. (Use Figs. 5-8 and 5-9.)

$$J_0 = -0.4$$

$$J_1 = -0.07$$

$$J_2 = 0.36$$

$$J_3 = 0.43$$

$$J_4 = 0.28$$

FM bandwidth

The total bandwidth of an FM signal can be determined by knowing the modulation index and using Fig. 5-8. For example, assume that the highest modulating frequency of a signal is 3 kHz and the maximum deviation is 6 kHz. This gives a modulation index of $m_f = 6 \text{ kHz}/3 \text{ kHz} = 2$. Referring to Fig. 5-8, you can see that this produces four significant pairs of sidebands. The bandwidth can then be determined with the simple formula

$$\text{BW} = 2f_m N$$

where N is the number of significant sidebands in the signal. According to this formula, the bandwidth of our FM signal is

$$\text{BW} = 2(3 \text{ kHz})(4) = 24 \text{ kHz}$$

Carson's Rule

Another way to determine the bandwidth of an FM signal is to use *Carson's rule.* This rule recognizes only the power in the most significant sidebands with amplitudes greater than 2 percent of the carrier (0.02 or higher in Fig. 5-8). This rule is

$$\text{BW} = 2[f_{d(\max)} + f_{m(\max)}]$$

According to Carson's rule, the bandwidth of the FM signal in the previous example would be

$$\text{BW} = 2(6\text{ kHz} + 3\text{ kHz}) = 2(9\text{ kHz}) = 18\text{ kHz}$$

Example 5-5

What is the maximum bandwidth of an FM signal with a deviation of 30 kHz and a maximum modulating signal of 5 kHz as determined by (*a*) Fig. 5-8 and (*b*) Carson's rule?

a. $m_f = \dfrac{f_d}{f_m} = \dfrac{30 \text{ kHz}}{5 \text{ kHz}} = 6$

Fig. 5-8 shows nine significant sidebands spaced 5 kHz apart for $m_f = 6$.

$$\text{BW} = 2f_m N = 2(5 \text{ kHz})\, 9 = 90 \text{ kHz}$$

b. $\text{BW} = 2[f_{d\,(\text{max})} + f_{m\,(\text{max})}]$

$= 2(30 \text{ kHz} + 5 \text{ kHz})$

$= 2(35 \text{ kHz})$

$\text{BW} = 70 \text{ kHz}$