



Assignment 4

MAT 361 Probability and Statistics

Section 4

Spring 2021
North South University

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Assignment-41) Probability, $p = 0.09$

Archer shoots 9 arrows.

$$\therefore n = 9$$

Binomial Distribution, $P(X=x) = \binom{9}{x} p^x (1-p)^{9-x}$

a) Exactly two arrows score bull's eyes,

$$P(X=2) = \binom{9}{2} 0.09^2 (1-0.09)^{9-2}$$

$$= 36 \times 0.09^2 \times 0.91^7$$

$$= 0.15069$$

$$= 0.1507 \text{ Answer}$$

b) At least two arrows score bull's-eyes,

$$P(X \geq 2) = P(X=2) + P(X=3) + \dots + P(X=9)$$

$$= 1 - P(X=0) - P(X=1)$$

$$= 1 - \binom{9}{0} 0.09^0 (1-0.09)^{9-0} - \binom{9}{1} \cdot 0.09^1 \cdot 0.91^{9-1}$$

$$= 1 - 0.4279 - 0.3809$$

$$= 0.191166 \text{ Answer}$$

c) Expected number of bull's-eyes scored,

$$E(X) = np$$

$$= 9 \times 0.09$$

$$= 0.81 \text{ Answer}$$

d) Variance, $V(X) = np(1-p)$

$$= 9 \times 0.09 \times (1-0.09) = 0.7371 \text{ Answer}$$

$$\text{Standard deviation} = \sqrt{V(X)} = \sqrt{0.7371} = 0.8585$$

Answer

2) A company receives 60% of its orders over the internet.

\therefore Probability, $p = 0.6$

18 independently placed order, $\therefore n = 18$.

It is binomial distribution, $p(x=k) = \binom{18}{k} 0.6^k (1-0.6)^{18-k}$

(a) between eight and ten of the orders are received,

Probability = $p(x=8) + p(x=9) + p(x=10)$

$$= \binom{18}{8} \times 0.6^8 \times (1-0.6)^{18-8} + \binom{18}{9} \times 0.6^9 \times 0.4^{18-9} + \binom{18}{10} \times 0.6^{10} \times 0.4^{18-10}$$

$$= 0.0771 + 0.1284 + 0.1734$$

$$= 0.3789 \text{ Answer}$$

(b) not more than four of the orders are received over the internet,

\therefore Probability, = $p(x=0) + p(x=1) + p(x=2) + p(x=3) + p(x=4)$

$$= \binom{18}{0} \times 0.6^0 \times 0.4^{18-0} + \binom{18}{1} \times 0.6^1 \times 0.4^{18-1} + \binom{18}{2} \times 0.6^2 \times 0.4^{18-2} \\ + \binom{18}{3} \times 0.6^3 \times 0.4^{18-3} + \binom{18}{4} \times 0.6^4 \times 0.4^{18-4}$$

$$= 6.87 \times 10^{-8} + 1.855 \times 10^{-6} + 2.3657 \times 10^{-5} + 1.8925 \times 10^{-4} \\ + 1.06455 \times 10^{-3}$$

$$= 1.2794 \times 10^{-3}$$

$$= 1.28 \times 10^{-3}$$

$$= 0.00128 \text{ Answer}$$

3) parameter, $p = 0.09$

(a) Considering as geometric distribution,

$$p(x=k) = (1-p)^{k-1} p$$

$$\therefore p(x=4) = (1-0.09)^{4-1} \times 0.09$$

$$= 0.91^3 \times 0.09$$

$$= 0.0678 \text{ Answer}$$

(b) 3rd bull's-eye is scored with the tenth arrow.

$$r = 3, n = 10 \text{ \& } p = 0.09$$

$$P(X=x) = \binom{x-1}{r-1} (1-p)^{(x-r)} p^r$$

$$\begin{aligned} \therefore P(10) &= \binom{10-1}{3-1} (1-0.09)^{(10-3)} 0.09^3 \\ &= \binom{9}{2} (0.91)^7 \cdot 0.09^3 \\ &= 0.0136 \text{ Answer} \end{aligned}$$

(c) Expected number of arrows before the first bull's eye scored,

$$\begin{aligned} E(x) &= \frac{1}{p} = \frac{1}{0.09} \\ &= 11.11 \text{ Answer} \end{aligned}$$

(d) Expected number of arrows shot before the third bull's eye is scored,

$$\begin{aligned} E(x) &= \frac{n(1-p)}{p} & E(x) &= \frac{r}{p} \\ &= \frac{3(1-p)}{p} & &= \frac{3}{0.09} \\ & & &= 33.33 \end{aligned}$$

4) parameter, $\lambda = 2.4$, no crack.

$$P(X=x) = \frac{e^{-\lambda} \lambda^x}{x!}$$

$$\begin{aligned} P(X=0) &= \frac{e^{-2.4} \lambda^0}{0!} \\ &= \frac{1}{1 \cdot e^{2.4}} = 0.0907 \text{ Answer} \end{aligned}$$

four or more check,

$$P(X \geq 4) = P(X=4) + P(X=5) + \dots$$

$$= 1 - P(X=3) - P(X=2) - P(X=1) - P(X=0)$$

$$= 1 - \frac{e^{-2.4} \times 2.4^3}{3!} - \frac{e^{-2.4} \times 2.4^2}{2!} - \frac{e^{-2.4} \times 2.4^1}{1!} - \frac{e^{-2.4} \times 2.4^0}{0!}$$

$$= 1 - 0.209 - 0.2613 - 0.2177 - 0.0907$$

$$= 0.2213$$

Answer

$$\lambda = 2.4$$

5] Given, $\mu = 3.00 \text{ mm}$

standard deviation of $\sigma = 0.12 \text{ mm}$

\therefore Variance $= \sigma^2$

(i) $P(X > 3.2)$

$$= P(3.2 < X < \infty)$$

$$= P\left(\frac{3.2 - \mu}{\sigma} < \frac{X - \mu}{\sigma} < \frac{\infty - \mu}{\sigma}\right)$$

$$= P\left(\frac{3.2 - 3}{0.12} < \frac{X - 3}{0.12} < \infty\right)$$

$$= F(\infty) - F(1.66\bar{3})$$

$$= 1 - F(1.67)$$

$$= 1 - 0.9525$$

$$= 0.0475 \text{ Answer}$$

(ii) ~~$P(X < 3)$~~ $P(X < 2.7)$

$$= P(-\infty < X < 2.7) = P\left(\frac{-\infty - 3}{0.12} < \frac{X - 3}{0.12} < \frac{2.7 - 3}{0.12}\right)$$

$$= P(-\infty < \frac{X - 3}{0.12} < -2.5)$$

$$= F(-2.5)$$

$$= 0.0062 \text{ Answer}$$