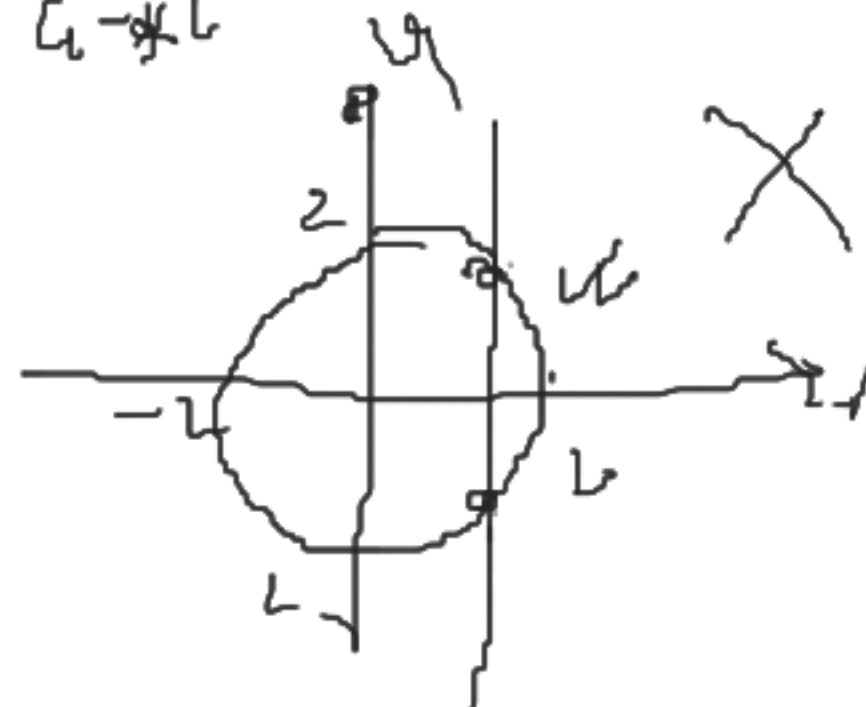


# Introduction, Functions and Families of Function

Q1 Functions:  $y$  depends on  $x$   $y = f(x)$

Ex 3  $x^2 + y^2 = 4$

$y = \pm \sqrt{4 - x^2}$



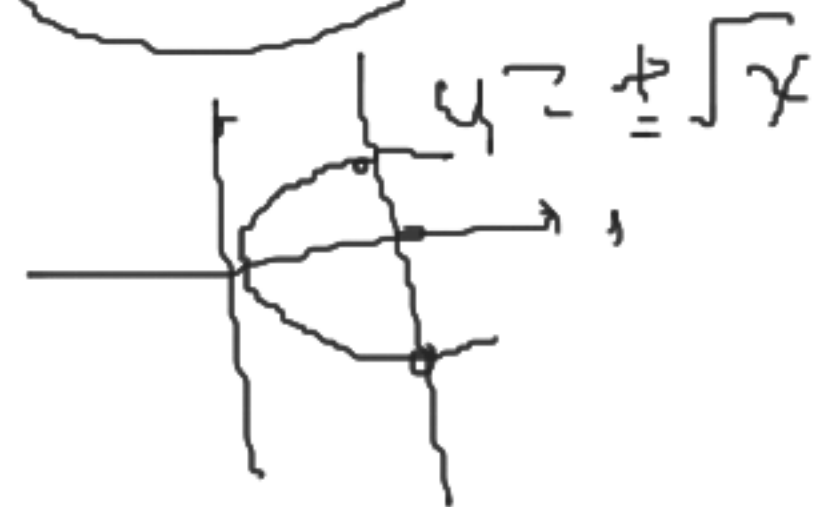
$y = 3x - 1$   $f(x) = 3x - 1$



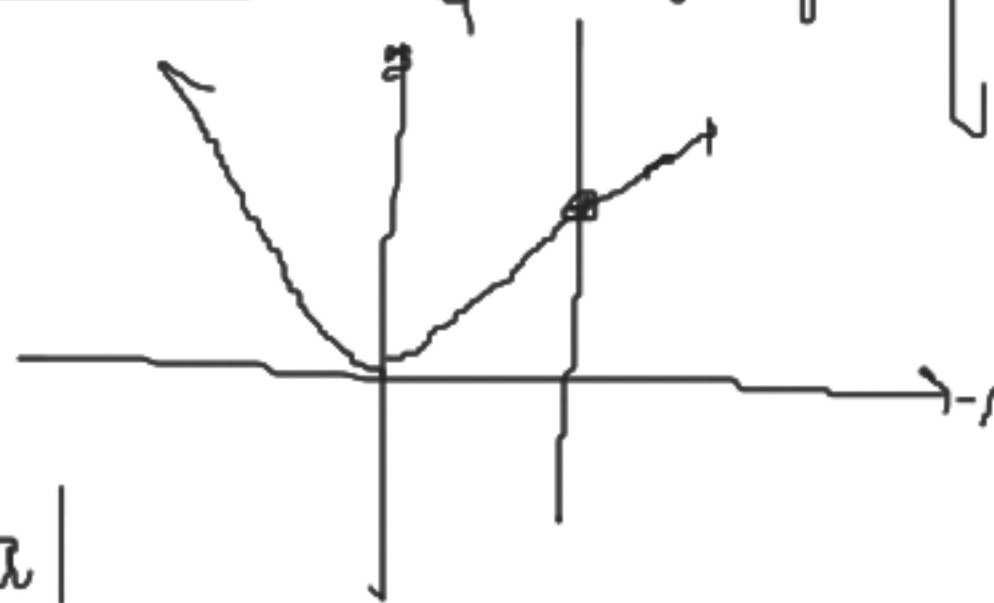
$y = x^2$

$y = x^3$

$y^2 = x$  X



The Absolute Value Function:



$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

Properties:

(i)  $|-a| = |a|$

(ii)  $|ab| = |a||b|$

(iii)  $\left|\frac{a}{b}\right| = \frac{|a|}{|b|}; b \neq 0$

(iv)  $|a+b| \leq |a| + |b|$

$\sqrt{x^2} = |x|$

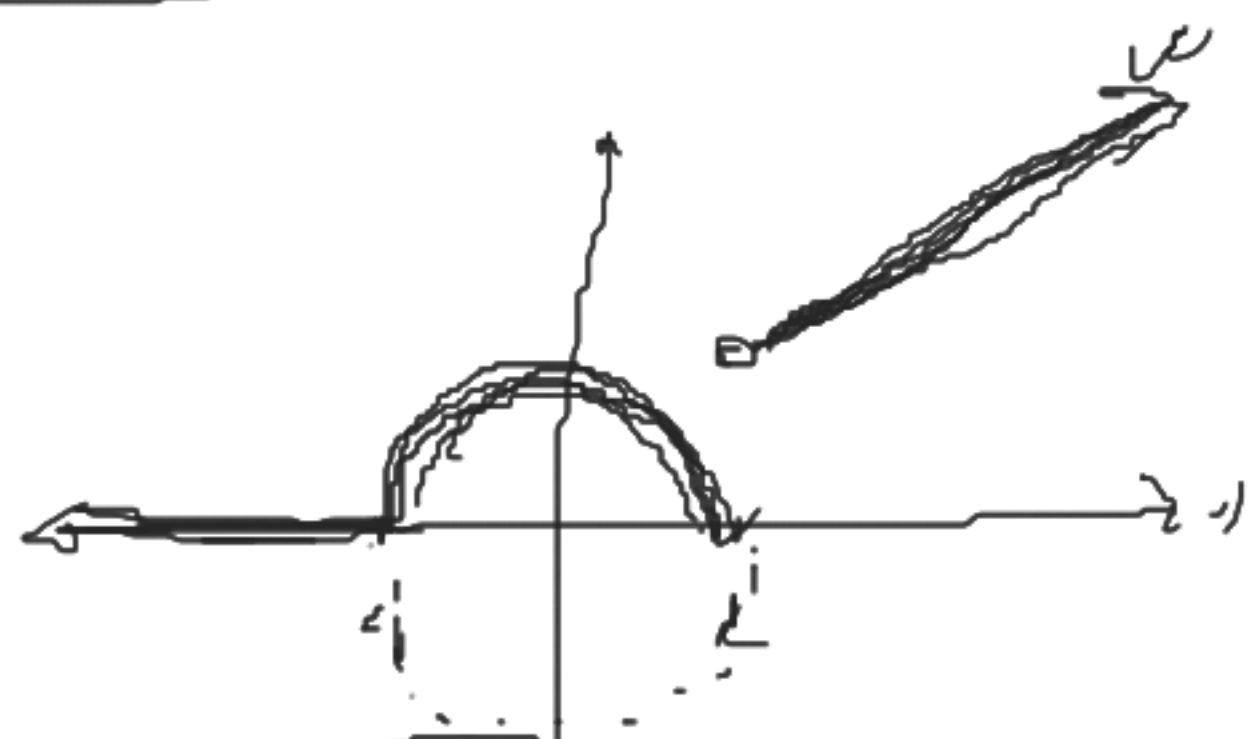
Ex.

$\sqrt{x^2} = |x|$

$\sqrt{(-2)^2} = -2$

$\sqrt{(-2)^2} = |-2| = 2$

# Piecewise-Defined Function:



$$f(x) = \begin{cases} 0 & , x \leq -1 \\ \sqrt{1-x^2} & , -1 \leq x \leq 1 \\ x & , x \geq 1 \end{cases}$$

$$\begin{aligned} f(x) &= \sqrt{1-x^2} \\ y &= \sqrt{1-x^2} & y^2 &= 1-x^2 \\ & & x^2 + y^2 &= 1 \end{aligned}$$

Domain and Range

$$f(x) = x^2$$

$$D: (-\infty, \infty)$$

$$D: \{x \mid x \text{ is any real number}\}$$

$$R: [0, \infty)$$

$$f(x) = \sqrt{x-1}$$

$$x-1 \geq 0 \Rightarrow x \geq 1$$

$$D: [1, \infty)$$

Ex 6: (a)  $f(x) = x^3$  (b)  $f(x) = \frac{1}{(x-1)(x-3)}$

Soln: (a)  $D: (-\infty, \infty)$

(b)  $(x-1)(x-3) \neq 0 \quad \therefore x \neq 1, x \neq 3$

$\therefore D: \{x: x \text{ is any real number except } 1 \text{ \& } 3\}$

$D: (-\infty, 1) \cup (1, 3) \cup (3, \infty)$

H.W. Ex 0.1:  $7-10, 15-18, 23-24, 27$

Q.2 New functions from old :

$$\frac{3}{2} = \boxed{1.5}$$
$$3 + 2 = \boxed{5}$$

Let  $\underline{f}$  and  $\underline{g}$

$$(f+g)(x) = f(x) + g(x)$$

$$(f-g)(x) = f(x) - g(x)$$

$$(fg)(x) = f(x) \cdot g(x)$$

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$$

$$\boxed{g(x) \neq 0}$$

$$(f+g)(x) = f(x) + g(x)$$

$$= \sqrt{x-2} + x^2$$

$$D: [2, \infty)$$

Ex.

$$f(x) = \sqrt{x-2},$$
$$D: [2, \infty)$$

$$g(x) = x^2$$

$$D: (-\infty, \infty)$$

$$f(x) = \sqrt{x-2}, \quad g(x) = x, \quad D: (-\infty, +\infty)$$

$$D: [2, \infty)$$

$$\frac{g(x)}{f(x)} = \left( \frac{g}{f} \right)(x) = \frac{g(x)}{\underline{f(x)}} = \frac{x}{\sqrt{x-2}}$$

$$[2, \infty) \cap (-\infty, +\infty) \left( \frac{g}{f} \right)(x) : D: [2, \infty)$$

$$= [2, \infty) \text{ and } f(x) \neq 0$$

$$\sqrt{x-2} \neq 0$$

$$x-2 \neq 0$$

$$\boxed{x \neq 2}$$

## Composition of Function :

$f \circ g$

$$\checkmark (f \circ g)(x) = f(\underline{g(x)})$$

$$(g \circ f)(x) = g(f(x))$$

Domain of  $f \circ g$  is consist of all  $x$  in the domain of  $g$  ✓

for which  $g(x)$  is the domain of  $f$

$$\checkmark \underline{g \circ f}$$

Ex 3 :  $f(x) = x^2 + 3$      $g(x) = \sqrt{x}$   
 (a)  $(f \circ g)(x) = ?$  and domain of  $f \circ g = ?$

Soln (a)  $(f \circ g)(x) = f(g(x)) = f(\sqrt{x}) = (\sqrt{x})^2 + 3 = \boxed{x+3}$

$f(x) = x^2 + 3$  : D:  $(-\infty, \infty)$   
 $g(x) = \sqrt{x}$  : D:  $\underline{\underline{[0, \infty)}}$

$\therefore$  Domain of  $f \circ g$  is  $[0, \infty)$  .  $x \geq 0$

(b)  $(g \circ f)(x) = g(f(x)) = g(\underline{x^2 + 3}) = \boxed{\sqrt{x^2 + 3}}$

D:  $\underline{\underline{(-\infty, +\infty)}}$



# Exponential Function:

Translation:

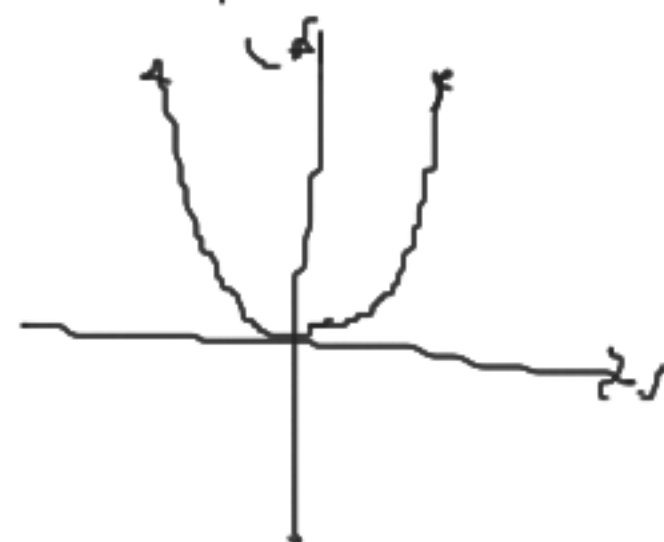
$$y = f(x)$$

$$y = \underline{f(x)} + c$$

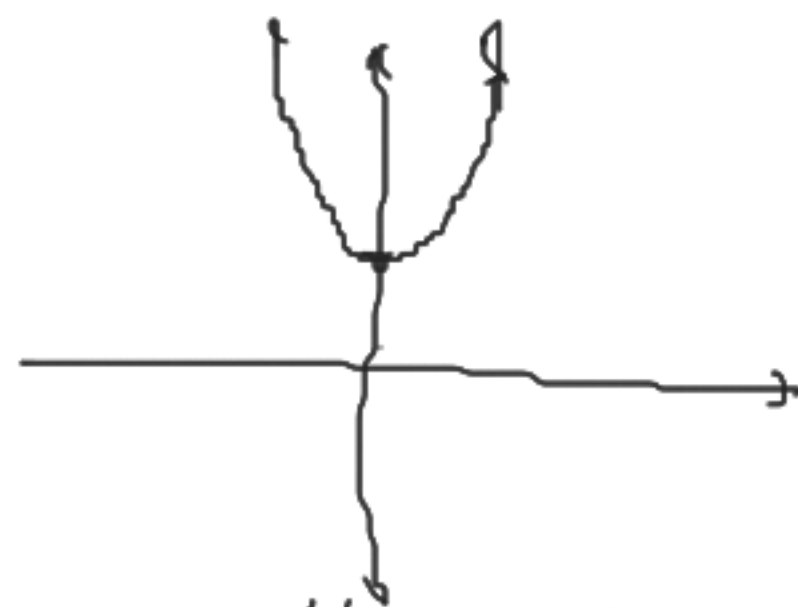
$$y = f(x) \pm 1$$

Ex:

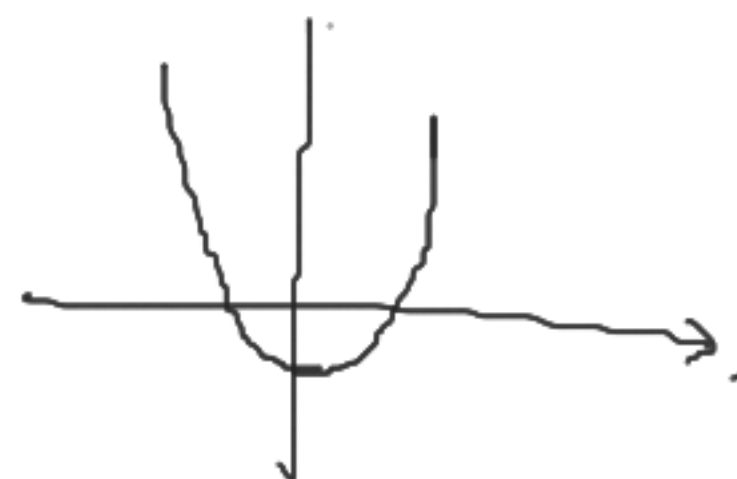
$$y = x^2$$



$$y = x^2 + 1$$



$$y = x^2 - 1$$

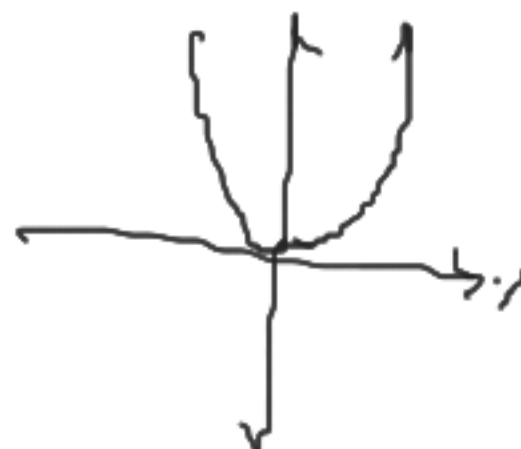


$$y = f(x),$$

$$y = f(x+1)$$

$$y = f(x-1)$$

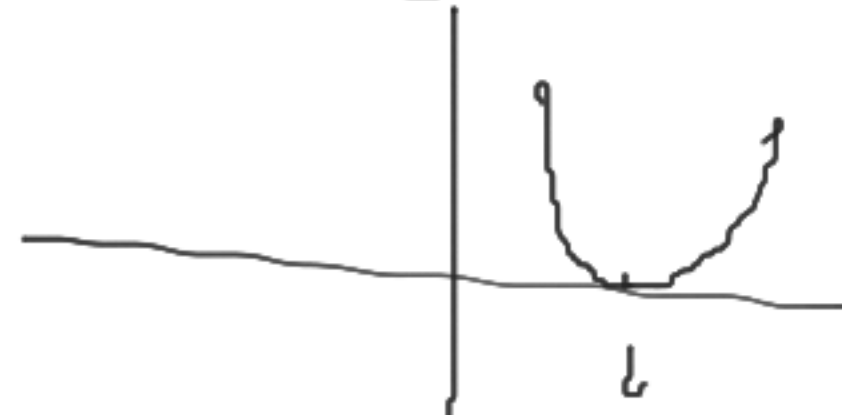
$$y = x^2$$



$$y = (x+1)^2$$



$$y = (x-1)^2$$



H.W. Ex 0.2 :

5-15, 27-32, 51-54, 59