Binomial Distribution Problems

(1) A company owns 400 laptops. Each laptop has an 8% probability of not working. You randomly select 20 laptops for your salespeople.
(a) What is the likelihood that 5 will be broken? (b) What is the likelihood that they will all work?
(c) What is the likelihood that they will all be broken?

(2) A study indicates that 4% of American teenagers have tattoos. You randomly sample 30 teenagers. What is the likelihood that exactly 3 will have a tattoo?

(3) An XYZ cell phone is made from 55 components. Each component has a .002 probability of being defective. What is the probability that an XYZ cell phone will not work perfectly?

(4) The ABC Company manufactures toy robots. About 1 toy robot per 100 does not work. You purchase 35 ABC toy robots. What is the probability that exactly 4 do not work?

(5) The LMB Company manufactures tires. They claim that only .007 of LMB tires are defective. What is the probability of finding 2 defective tires in a random sample of 50 LMB tires?

(6) An HDTV is made from 100 components. Each component has a .005 probability of being defective. What is the probability that an HDTV will not work perfectly?

Binomial Distribution SOLUTIONS

(1) (a) ${}^{20}C_5\,(.08)^5\,(.92)^{15} = .0145$ (b) ${}^{20}C_0\,(.08)^0(.92)^{20} = .1887$

(c) ${}^{20}C_{20}\,(.08)^{20}(.92)^0 = .0000000000000000000001$

(2) ${}^{30}C_3\,(.04)^3\,(.96)^{27} = .0863$

(3) Probability that it will work (0 defective components) ${}^{55}C_0\,(.002)^0\,(.998)^{55} = .896$
Probability that it will not work perfectly is 1 - .896 = .104 or 10.4%

(4) ${}^{35}C_4\,(.01)^4\,(.99)^{31} = .00038$

(5) ${}^{50}C_2\,(.007)^2\,(.993)^{48} = .0428$

(6) Probability that it will work (0 defective components) ${}^{100}C_0\,(.005)^0\,(.995)^{100} = .606$

Probability that it will not work perfectly is 1 - .606 = .394 or 39.40%

Poisson Distribution Problems

1. A life insurance salesman sells on the average 3 life insurance policies per week. Use Poisson's law to calculate the probability that in a given week he will sell

a. Some policies
b. 2 or more policies but less than 5 policies.
c. Assuming that there are 5 working days per week, what is the probability that in a given day he will sell one policy?

2. If electricity power failures occur according to a Poisson distribution with an average of 3 failures every twenty weeks, calculate the probability that there will not be more than one failure during a particular week.

3. A small life insurance company has determined that on the average it receives 6 death claims per day. Find the probability that the company receives at least seven death claims on a randomly selected day.

4. The number of traffic accidents that occurs on a particular stretch of road during a month follows a Poisson distribution with a mean of 9.4. Find the probability that less than two accidents will occur on this stretch of road during a randomly selected month.

Poisson Distribution SOLUTIONS

1. $m = 3$

(a) "Some policies" means "1 or more policies": $P(X > 0) = 1 - P(x_0)$

$$P(X = x_0) = \frac{3^0 e^{-3}}{0!} = 0.04.9787$$

$$P(X \geq 1) = 0.95021$$

(b) The probability of selling 2 or more, but less than 5 policies is:

$$P(2 \leq X < 5) = \frac{3^2 e^{-3}}{2!} + \frac{3^3 e^{-3}}{3!} + \frac{3^4 e^{-3}}{4!} = 0.61611$$

(c) Average number of policies sold per day: $\frac{3}{5} = 0.6$

So on a given day, $P(x) = \frac{0.6^1 e^{-0.6}}{1!} = 0.32929$

2. The average number of failures per week is: $m = 3/20 = 0.15$

"Not more than one failure" means we need to include the probabilities for "0 failures" plus "1 failure".

$$P(X = x_0) + P(X = x_1) = \frac{0.15^0 e^{-0.15}}{0!} + \frac{0.15^1 e^{-0.15}}{1!} + \frac{0.15^2 e^{-0.15}}{2!} = 0.98981$$

3. $P(x \geq 7) = 1 - P(x \leq 6) = 0.393697$

4. $P(x < 2) = P(x = 0) + P(x = 1) = 0.000860$

Normal Distribution Problems

1. X is a normally normally distributed variable with mean $\mu = 30$ and standard deviation $\sigma = 4$. Find

a) $P(x < 40)$
b) $P(x > 21)$
c) $P(30 < x < 35)$

2. A radar unit is used to measure speeds of cars on a motorway. The speeds are normally distributed with a mean of 90 km/hr and a standard deviation of 10 km/hr. What is the probability that a car picked at random is travelling at more than 100 km/hr?

3. For a certain type of computers, the length of time bewteen charges of the battery is normally distributed with a mean of 50 hours and a standard deviation of 15 hours. John owns one of these computers and wants to know the probability that the length of time will be between 50 and 70 hours.

4. Entry to a certain University is determined by a national test. The scores on this test are normally distributed with a mean of 500 and a standard deviation of 100. Tom wants to be admitted to this university and he knows that he must score better than at least 70% of the students who took the test. Tom takes the test and scores 585. Will he be admitted to this university?

5. The length of similar components produced by a company are approximated by a normal distribution model with a mean of 5 cm and a standard deviation of 0.02 cm. If a component is chosen at random

a) what is the probability that the length of this component is between 4.98 and 5.02 cm?
b) what is the probability that the length of this component is between 4.96 and 5.04 cm?

6. The length of life of an instrument produced by a machine has a normal ditribution with a mean of 12 months and standard deviation of 2 months. Find the probability that an instrument produced by this machine will last

a) less than 7 months.
b) between 7 and 12 months.

7. The time taken to assemble a car in a certain plant is a random variable having a normal distribution of 20 hours and a standard deviation of 2 hours. What is the probability that a car can be assembled at this plant in a period of time

a) less than 19.5 hours?
b) between 20 and 22 hours?

8. A large group of students took a test in Physics and the final grades have a mean of 70 and a standard deviation of 10. If we can approximate the distribution of these grades by a normal distribution, what percent of the students

a) scored higher than 80?

b) should pass the test (grades$\geq$60)?
c) should fail the test (grades<60)?

9. The annual salaries of employees in a large company are approximately normally distributed with a mean of \$50,000 and a standard deviation of \$20,000.

a) What percent of people earn less than \$40,000?
b) What percent of people earn between \$45,000 and \$65,000?
c) What percent of people earn more than \$70,000?

Normal Distribution SOLUTIONS

1. a) For $x = 40$, the z-value $z = (40 - 30) / 4 = 2.5$

Hence $P(x < 40) = P(z < 2.5) =$ [area to the left of 2.5] $= 0.9938$

b) For $x = 21$, $z = (21 - 30) / 4 = -2.25$

Hence $P(x > 21) = P(z > -2.25) =$ [total area] - [area to the left of -2.25]

$= 1 - 0.0122 = 0.9878$

c) For $x = 30$, $z = (30 - 30) / 4 = 0$ and for $x = 35$, $z = (35 - 30) / 4 = 1.25$

Hence $P(30 < x < 35) = P(0 < z < 1.25) =$ [area to the left of $z = 1.25$] - [area to the left of 0]

$= 0.8944 - 0.5 = 0.3944$

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2. Let x be the random variable that represents the speed of cars. x has $\mu = 90$ and $\sigma = 10$. We have to find the probability that x is higher than 100 or $P(x > 100)$

For $x = 100$, $z = (100 - 90) / 10 = 1$

$P(x > 90) = P(z >, 1) =$ [total area] - [area to the left of $z = 1$]

$= 1 - 0.8413 = 0.1587$

The probability that a car selected at a random has a speed greater than 100 km/hr is equal to 0.1587

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3. Let x be the random variable that represents the length of time. It has a mean of 50 and a standard deviation of 15. We have to find the probability that x is between 50 and 70 or $P(50< x < 70)$

For $x = 50$, $z = (50 - 50) / 15 = 0$

For $x = 70$, $z = (70 - 50) / 15 = 1.33$ (rounded to 2 decimal places)

$P(50< x < 70) = P(0< z < 1.33) =$ [area to the left of $z = 1.33$] - [area to the left of $z = 0$]

$= 0.9082 - 0.5 = 0.4082$

The probability that John's computer has a length of time between 50 and 70 hours is equal to 0.4082.

4. Let x be the random variable that represents the scores. x is normally ditsributed with a mean of 500 and a standard deviation of 100. The total area under the normal curve represents the total number of students who took the test. If we multiply the values of the areas under the curve by 100, we obtain percentages.

For $x = 585$, $z = (585 - 500) / 100 = 0.85$

The proportion P of students who scored below 585 is given by

P = [area to the left of $z = 0.85$] = 0.8023 = 80.23%

Tom scored better than 80.23% of the students who took the test and he will be admitted to this University.

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5. a) $P(4.98 < x < 5.02) = P(-1 < z < 1)$

$= 0.6826$

b) $P(4.96 < x < 5.04) = P(-2 < z < 2)$

$= 0.9544$

6. a) $P(x < 7) = P(z < -2.5)$

$= 0.0062$

b) $P(7 < x < 12) = P(-2.5 < z < 0)$

$= 0.4938$

7. a) $P(x < 19.5) = P(z < -0.25)$

$= 0.4013$

b) $P(20 < x < 22) = P(0 < z < 1)$

$= 0.3413$

8. a) For $x = 80$, $z = 1$

Area to the right (higher than) $z = 1$ is equal to 0.1586 = 15.87% scored more that 80.

b) For $x = 60$, $z = -1$

Area to the right of $z = -1$ is equal to 0.8413 = 84.13% should pass the test.

c)100% - 84.13% = 15.87% should fail the test.

a) For x = 40000, z = -0.5

Area to the left (less than) of z = -0.5 is equal to 0.3085 = 30.85% earn less than $40,000.

b) For x = 45000 , z = -0.25 and for x = 65000, z = 0.75

Area between z = -0.25 and z = 0.75 is equal to 0.3720 = 37.20 earn between $45,000 and $65,000.

c)For x = 70000, z = 1

Area to the right (higher) of z = 1 is equal to 0.1586 = 15.86% earn more than $70,000.