Q1) a) method of underdmined coefficients.

let, $y = e^{mx} \neq 0$

$y' = me^{mx}$

$y'' = m^2 e^{mx}$

$y'' + 4y' + 4y = 2x + 6 \quad \ldots (1)$

substituting y, y', y'' into (1),

$m^2 e^{mx} + 4me^{mx} + 4e^{mx} = 0$

$(m^2 + 4m + 4)\, e^{mx} = 0$

now, $m^2 + 4m + 4 = 0$

$(m+2)^2 = 0$

so, $m = -2, -2$

$\therefore\ y_c = C_1 e^{-2x} + C_2 x e^{-2x} \quad \ldots (11)$

now, $y_1 = e^{-2x}$ & $y_2 = xe^{-2x}$

then, $y_p = Ax + B$

$y_p' = A$

$y_p'' = 0$

again subtituting y_p, y_p', y_p'' into (i)

$0 + 4A + 4(Ax+B) = 2x+6$ ..... (iii)

equating coefficient of x, $4A = 2$

$A = \frac{1}{2}$

" " " ~~const,~~ constant,

$4A + 4B = 6$

or $4(\frac{1}{2}) + 4B = 6$

$\therefore B = 1$

$\therefore y_p = Ax + B = \frac{x}{2} + 1$

$y = y_c + y_p$

$\therefore y = C_1 e^{-2x} + C_2 x e^{-2x} + \frac{x}{2} + 1$

(Ans)

b) variation of parameter.

$$y_p = A_x\, y_1 + B_x\, y_2$$

<u>Wronskian:</u>

$$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = \begin{vmatrix} e^{-2x} & xe^{-2x} \\ -2e^{-2x} & -2xe^{-2x}+e^{-2x} \end{vmatrix} = e^{-4x} \neq 0$$

(linearly independent)

$$W_1 = \begin{vmatrix} 0 & xe^{-2x} \\ 2x+6 & -2xe^{-2x}+e^{-2x} \end{vmatrix} = -2x^2e^{-2x} - 6xe^{-2x}$$

$$W_2 = \begin{vmatrix} e^{-2x} & 0 \\ -2e^{-2x} & 2x+6 \end{vmatrix} = 2xe^{-2x} + 6e^{-2x}$$

note

* use tabular form to find xe^{2x} & x^2e^{2x}

$\int xe^{2x} = \frac{(2x-1)e^{2x}}{4}$

$\int x^2e^{2x} = \frac{(2x^2-2x+1)e^{2x}}{4}$

* or use calculator

$$A(x) = \int \frac{W_1}{W}\,dx$$

$$= \int \left(-2x^2e^{2x} - 6xe^{2x}\right)dx$$

$$= -(x^2+2x-1)e^{2x}$$

$$B(x) = \int \frac{W_2}{W}\,dx$$

$$= \int \left(2xe^{2x} + 6e^{2x}\right)dx$$

$$= \frac{(2x+5)e^{2x}}{2}$$

$$y_p = \left\{-(x^2+2x-1)\cancel{e^{2x}}\right\}\cancel{e^{-2x}} + \left\{\frac{(2x+5)\cancel{e^{2x}}}{2}\right\}x\cancel{e^{-2x}}$$

$$= -\cancel{x^2} - 2x + 1 + \cancel{x^2} + \frac{5x}{2}$$

$$= 1 + \frac{x}{2}$$

Comment: Whatever way we use it yields the same result. The first way is easier in my opinion.