Basic Data Structures: Trees

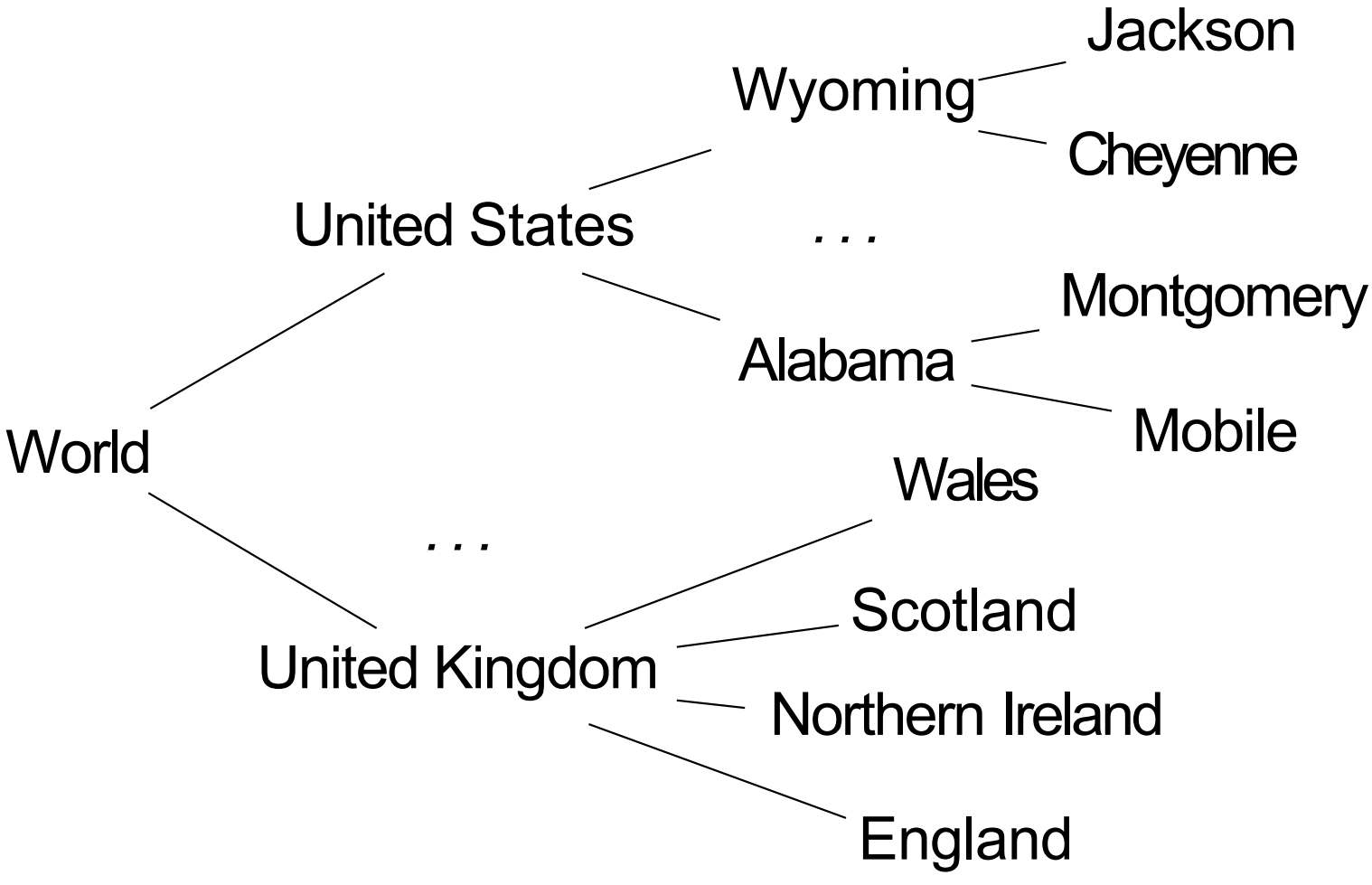
References

See the chapter 10.4 in [CLRS] Thomas H. Cormen, Charles E. Leiserson, Ronald L. Rivest, Clifford Stein. Introduction to Algorithms (3rd Edition). MIT Press and McGraw-Hill. 2009.

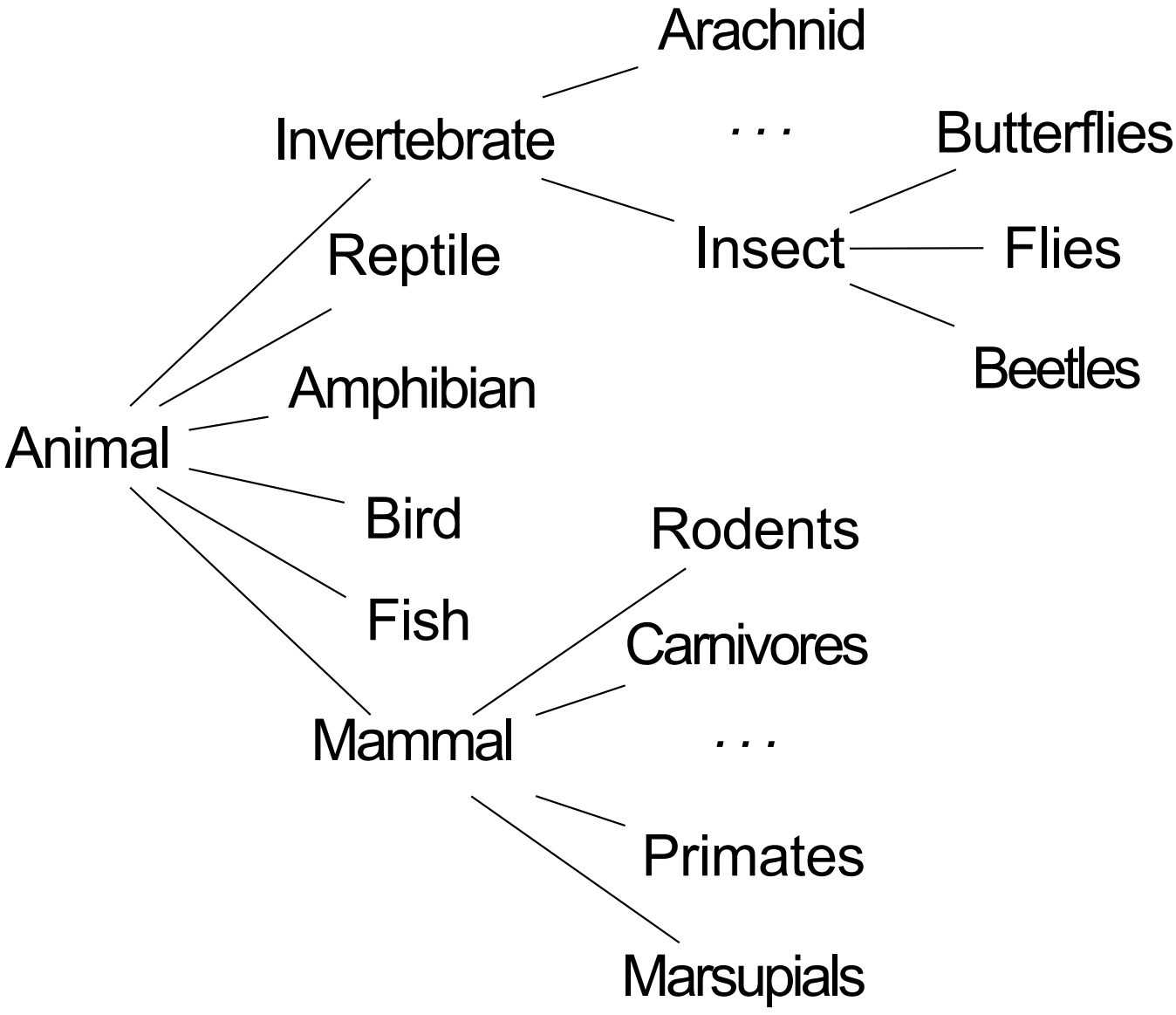
Data Structures

Data Structures and Algorithms

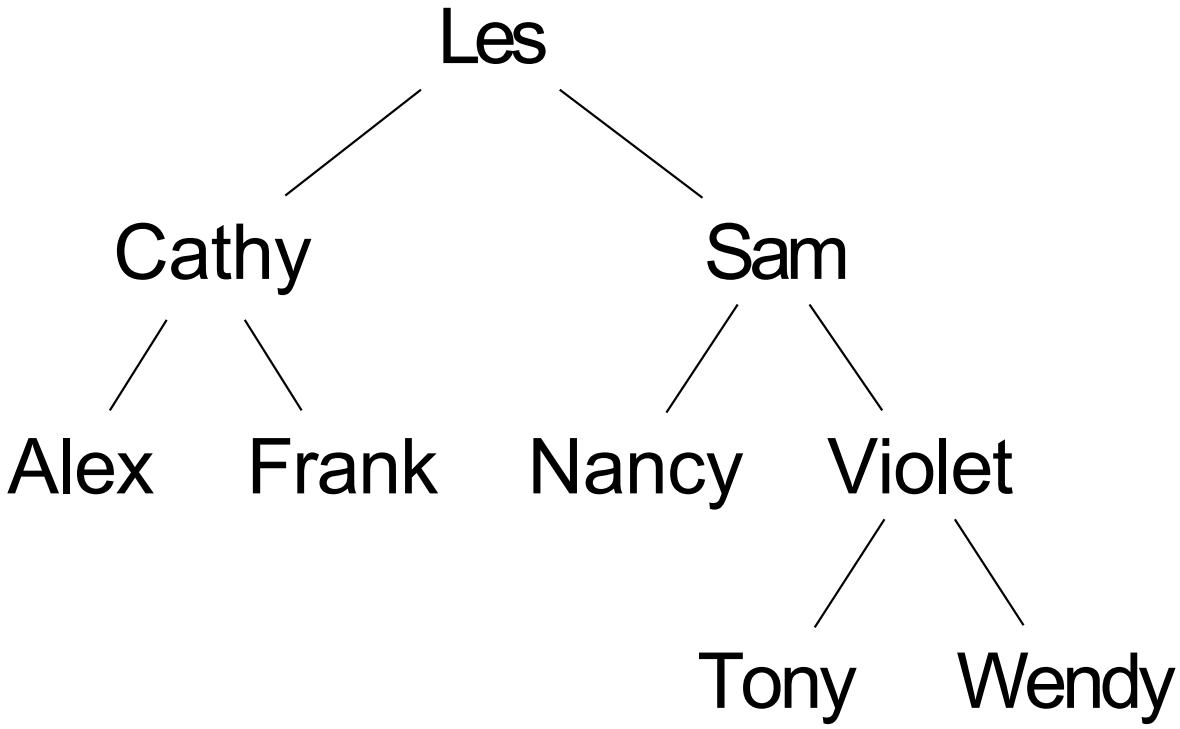
Geography Hierarchy



Animal Kingdom (partial)



Binary Search Tree



Definition

A Tree is:

- empty, or
- a node with:
  - a key, and
  - a list of child trees.

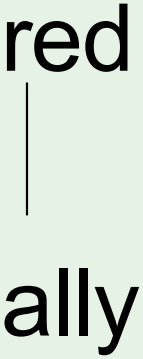
Simple Tree

Empty tree:

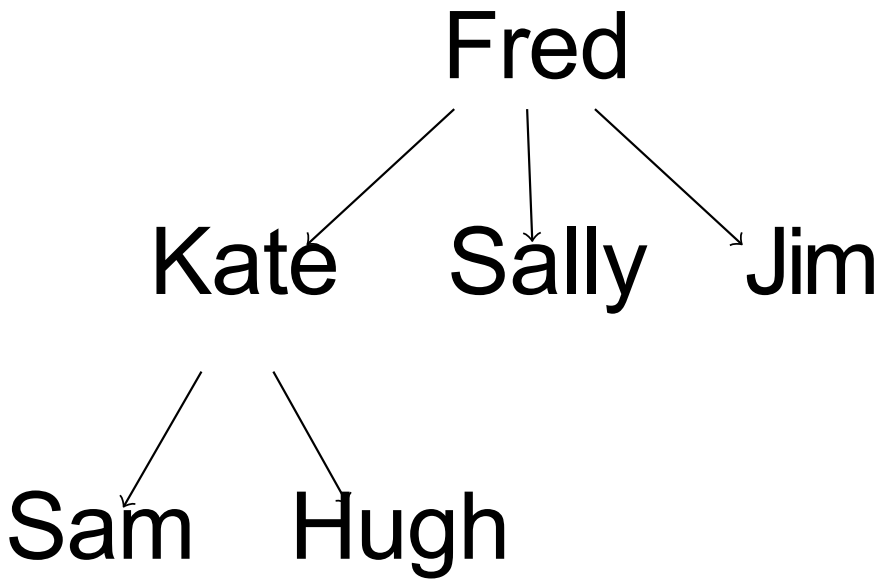
Tree with one node:

Fred

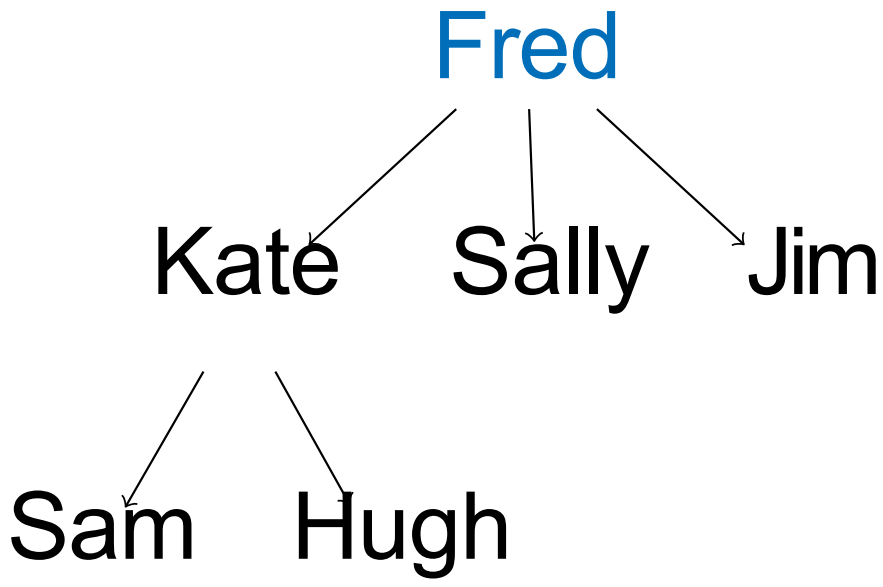
Tree with two nodes:



Terminology



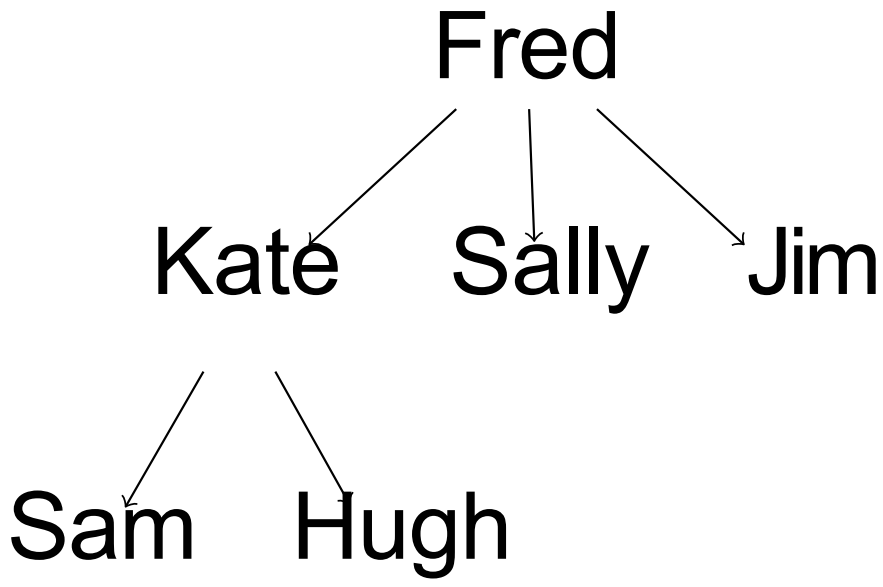
Terminology



***Root*:**

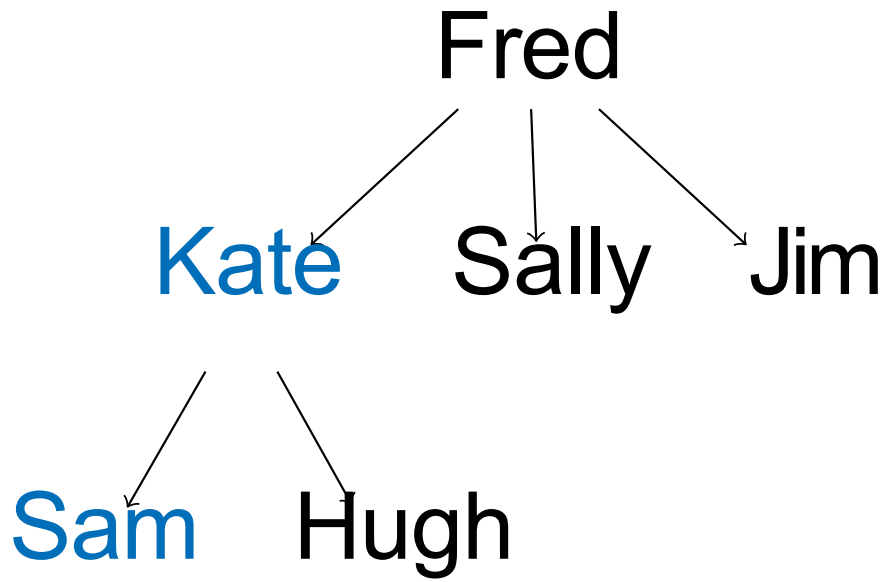
top node in the tree

Terminology



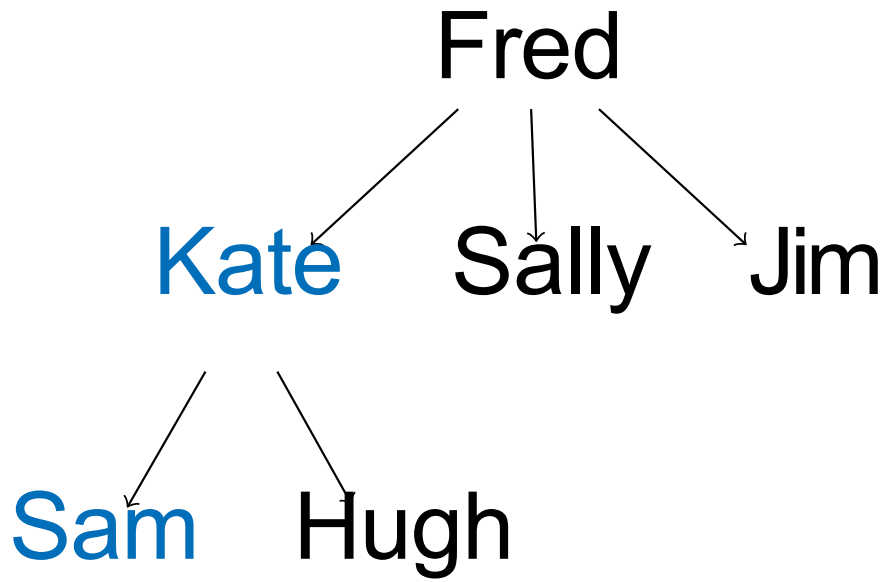
A *child* has a line down directly from a *parent*

Terminology



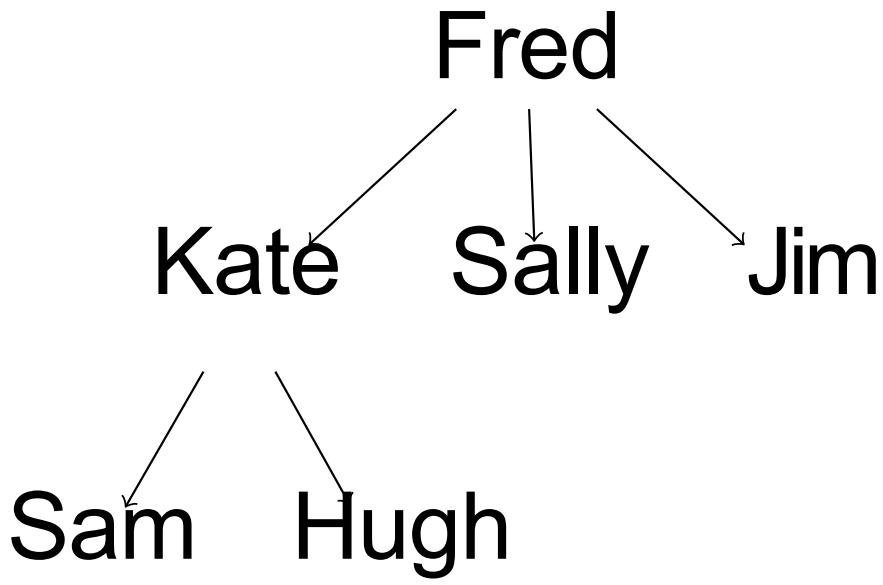
Kate is a *parent* of Sam

Terminology



Sam is a *child* of Kate

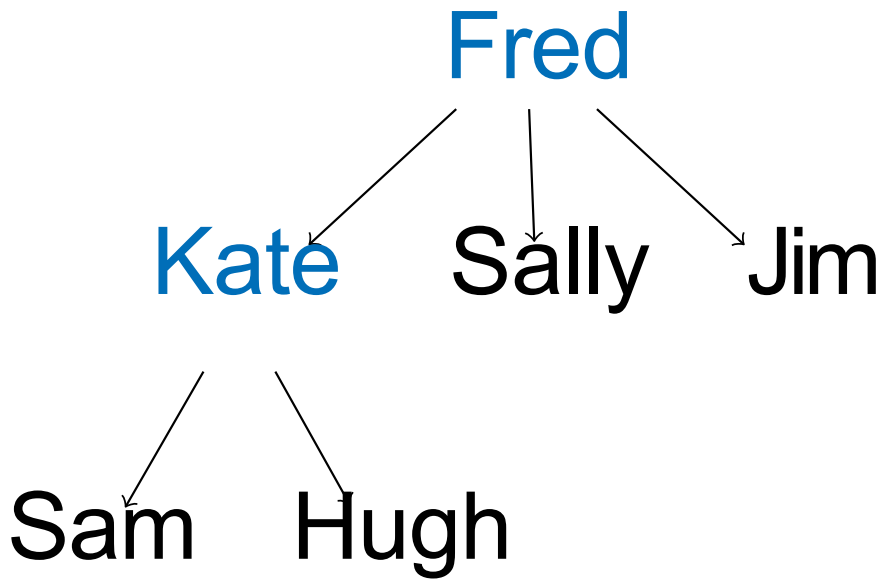
Terminology



***Ancestor*:**

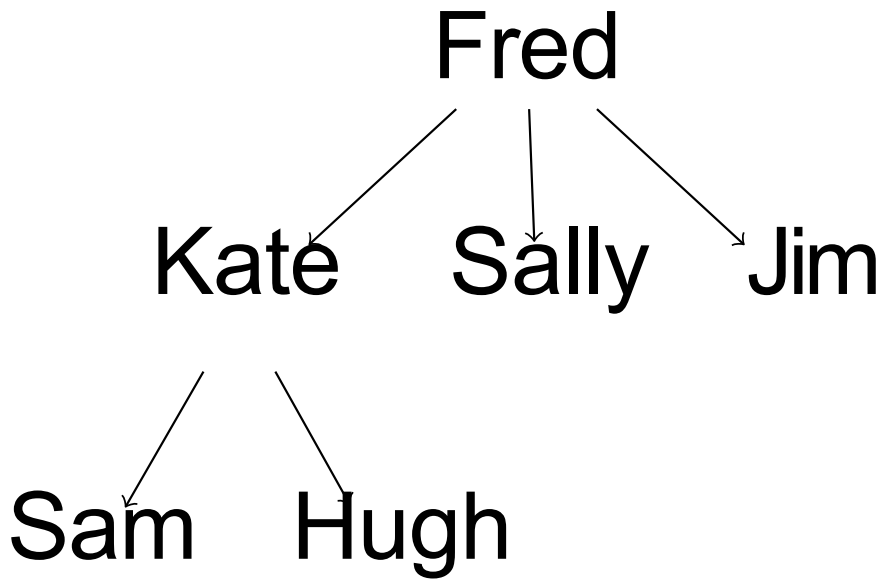
parent, or parent of parent, etc.

Terminology



***Ancestors* of Sam**

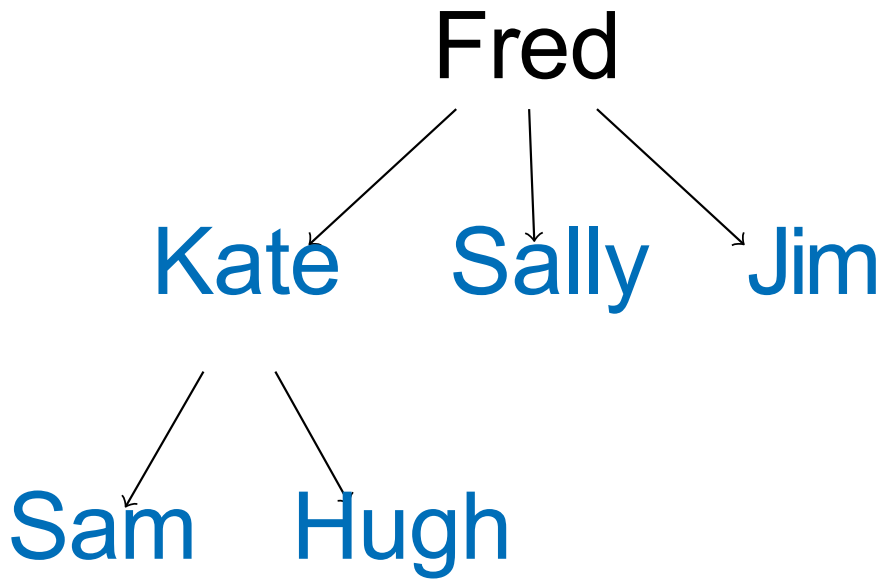
Terminology



***Descendant*:**

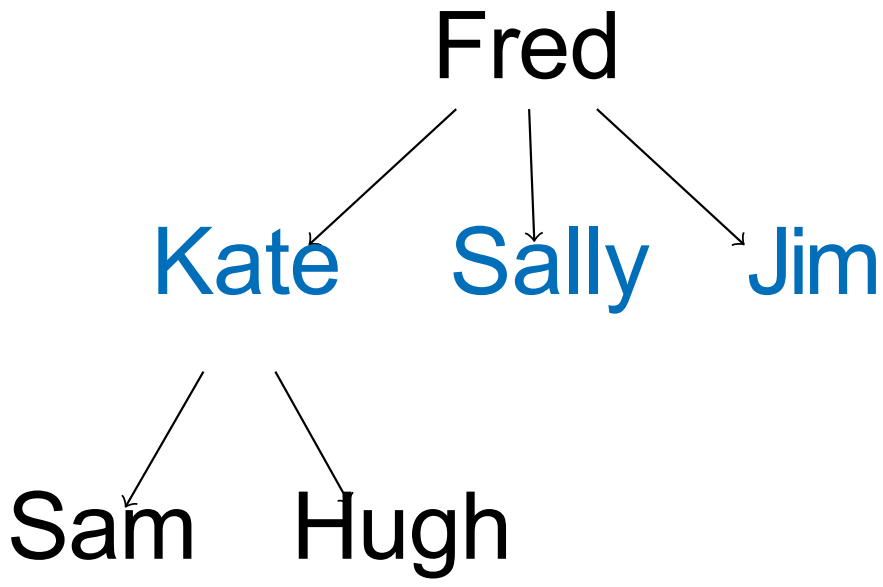
child, or child of child, etc.

Terminology



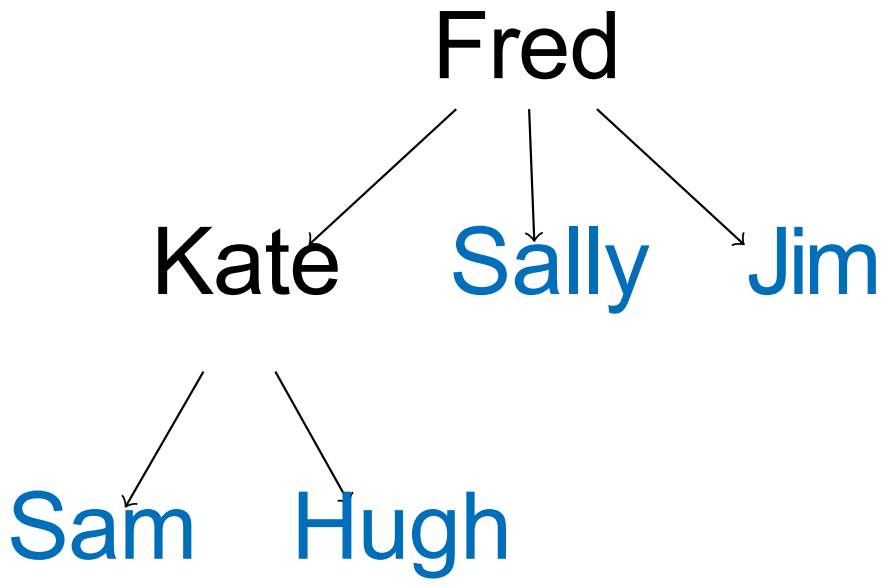
***Descendants* of Fred**

Terminology



***Sibling*:**
sharing the same parent

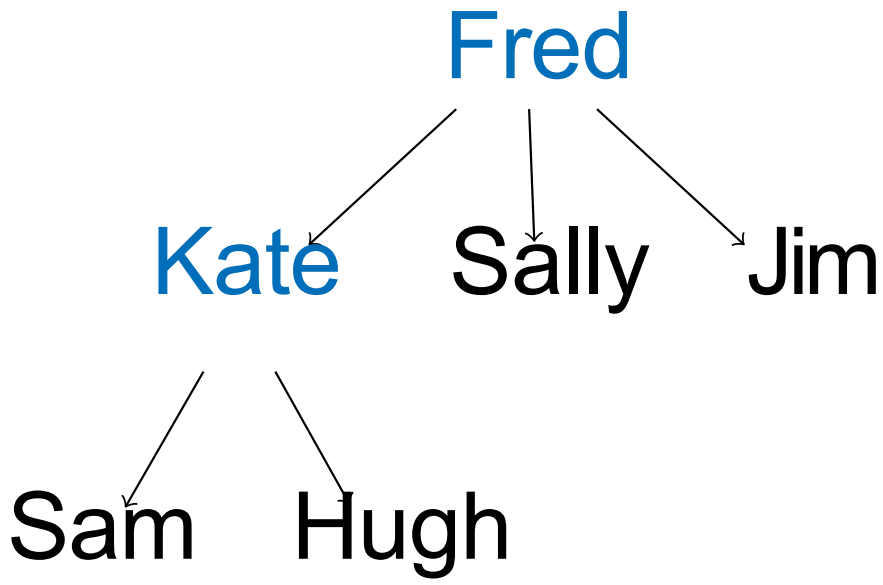
Terminology



Leaf **:**

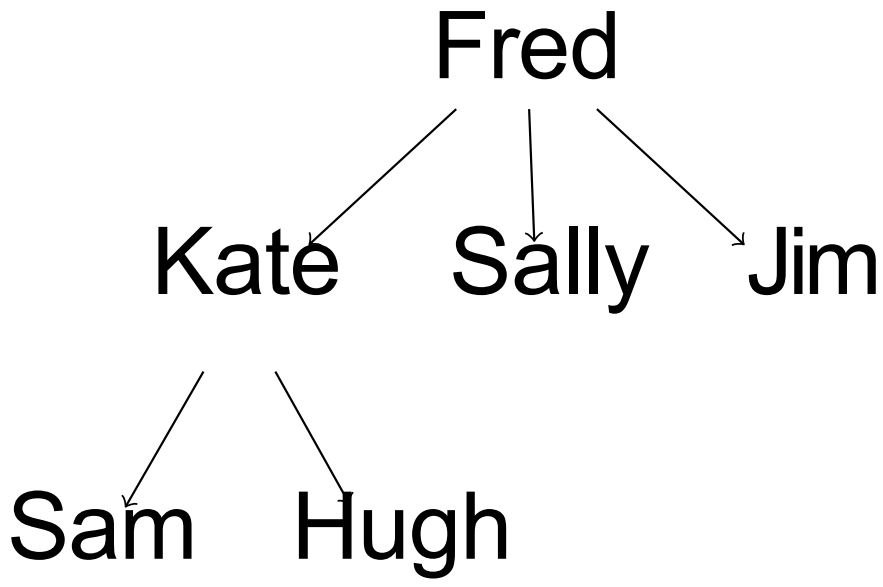
node with no children

Terminology



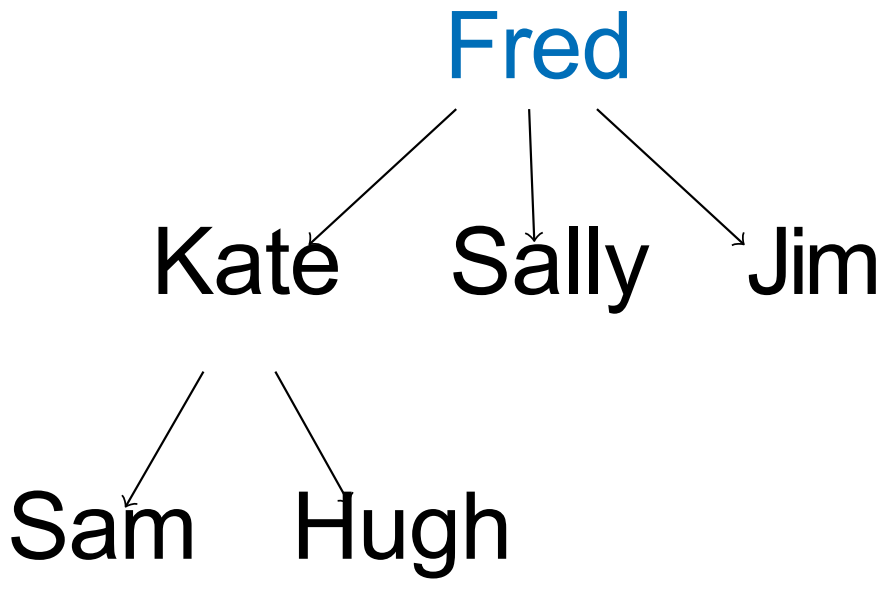
Interior node (non-leaf)

Terminology



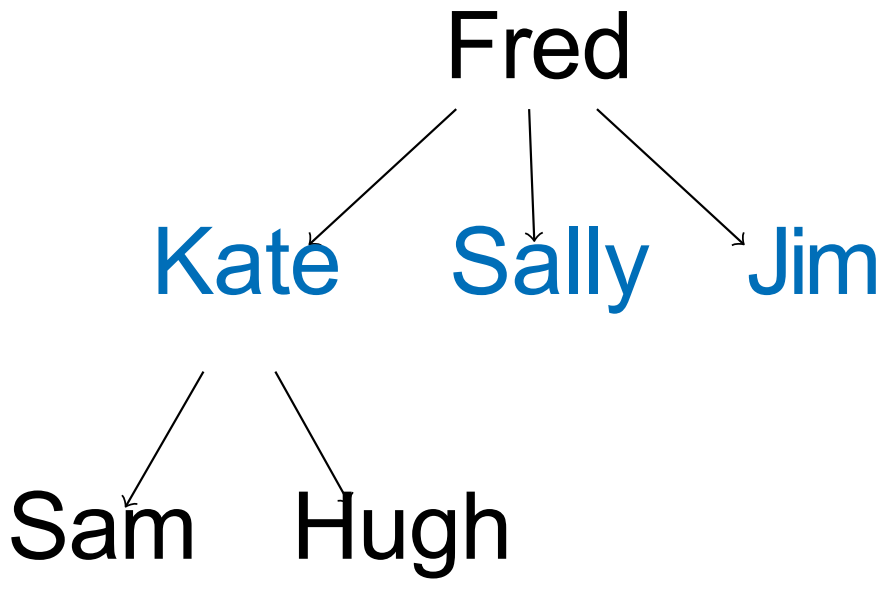
***Level*: num edges between root and node**

Terminology



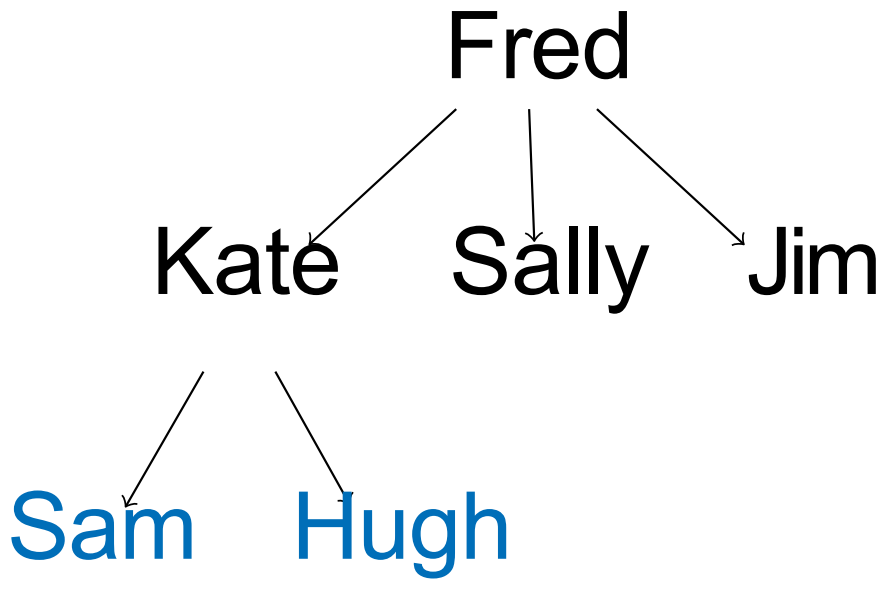
Level 0

Terminology



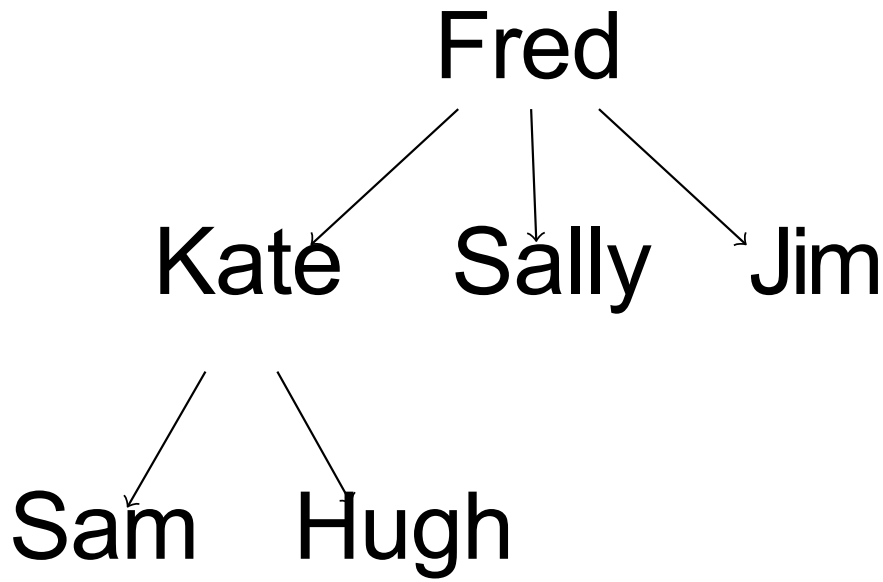
Level 1

Terminology



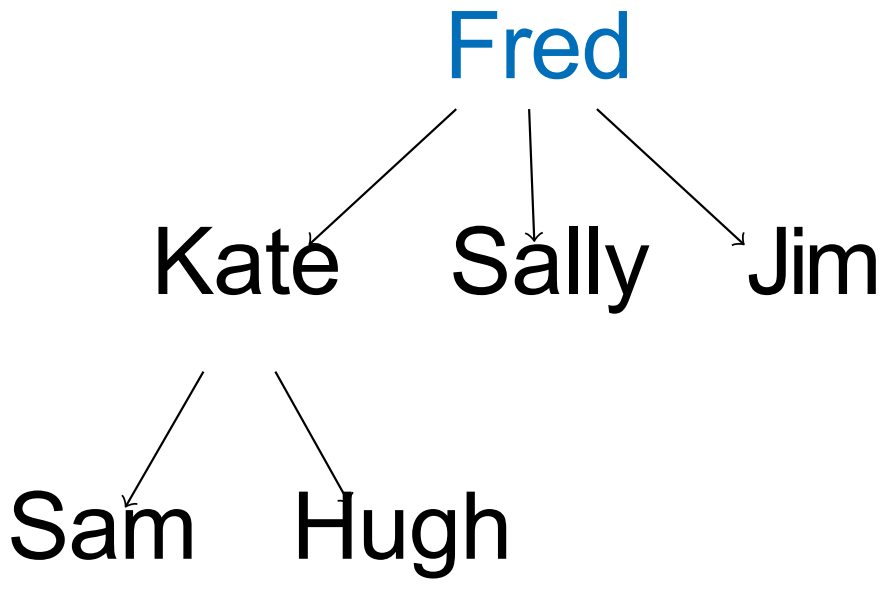
Level 2

Terminology



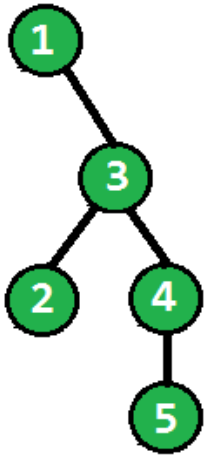
The height of a tree (also known as depth) is the maximum distance between the root node of the tree and the leaf node of the tree. It can also be defined as the number of edges from the root node to the leaf node. The root node is at level 0. Therefore, if there is only one node, i.e., the root node, the height of the tree is 0.

Terminology

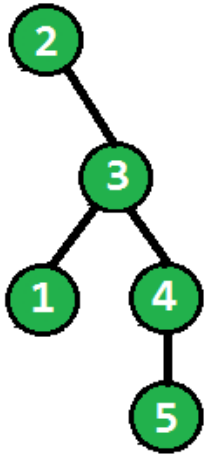


Height 2

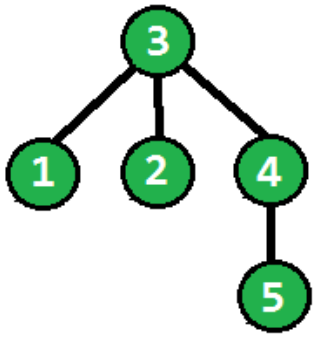
Question



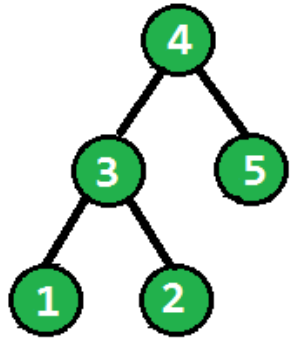
Root - 1
Height - 3



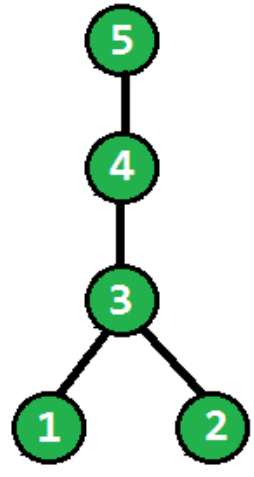
Root - 2
Height - 3



Root - 3
Height - 2

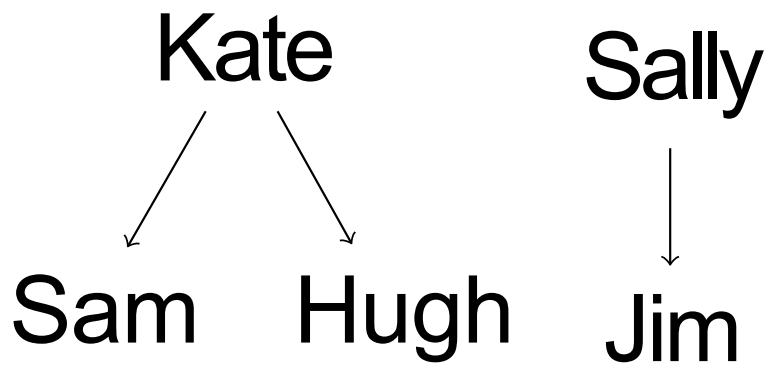


Root - 4
Height - 2



Root - 5
Height - 3

Terminology



***Forest*:**

collection of trees

Node contains:

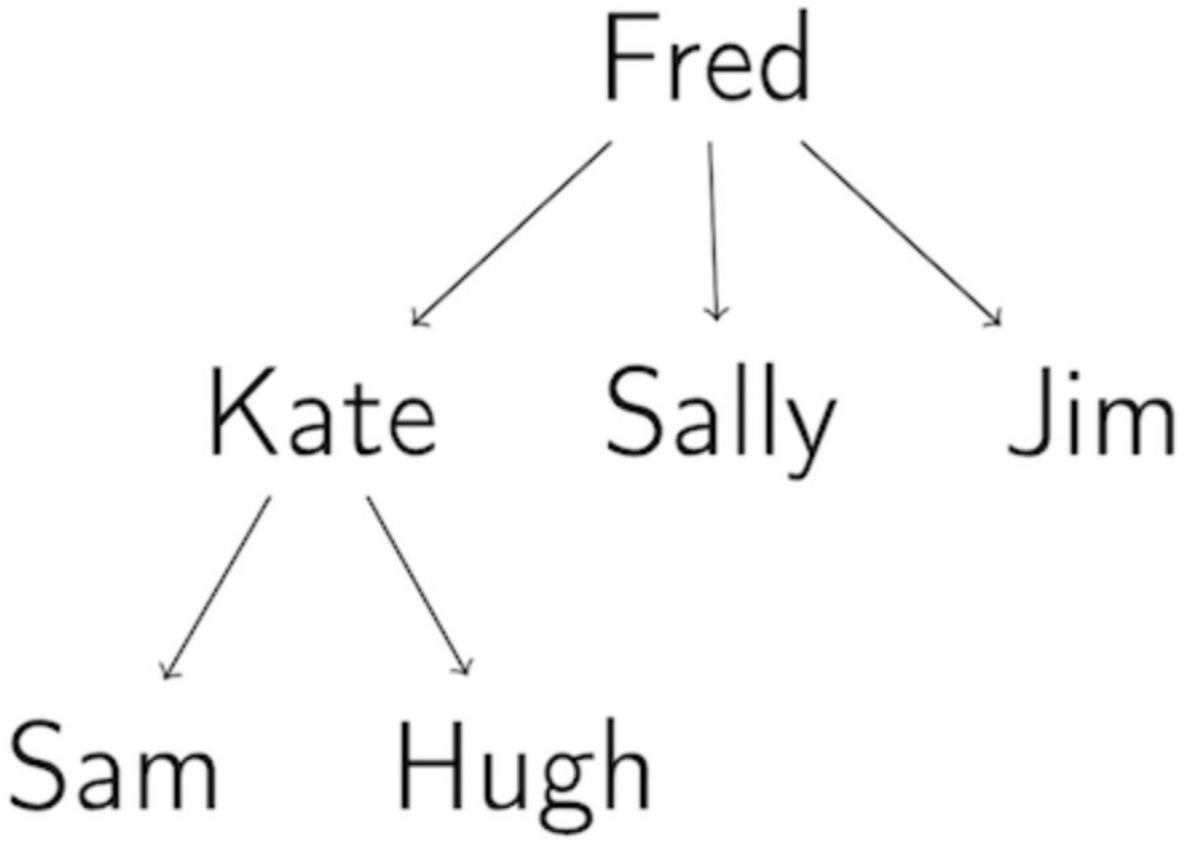
- `key`
- `children`: list of children nodes

For binary tree, node contains:

- `key`
- `left`
- `right`

Question

Is Hugh a child of Fred?



Walking a Tree

Often we want to visit the nodes of a tree in a particular order.

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Often we want to visit the nodes of a tree in a particular order.

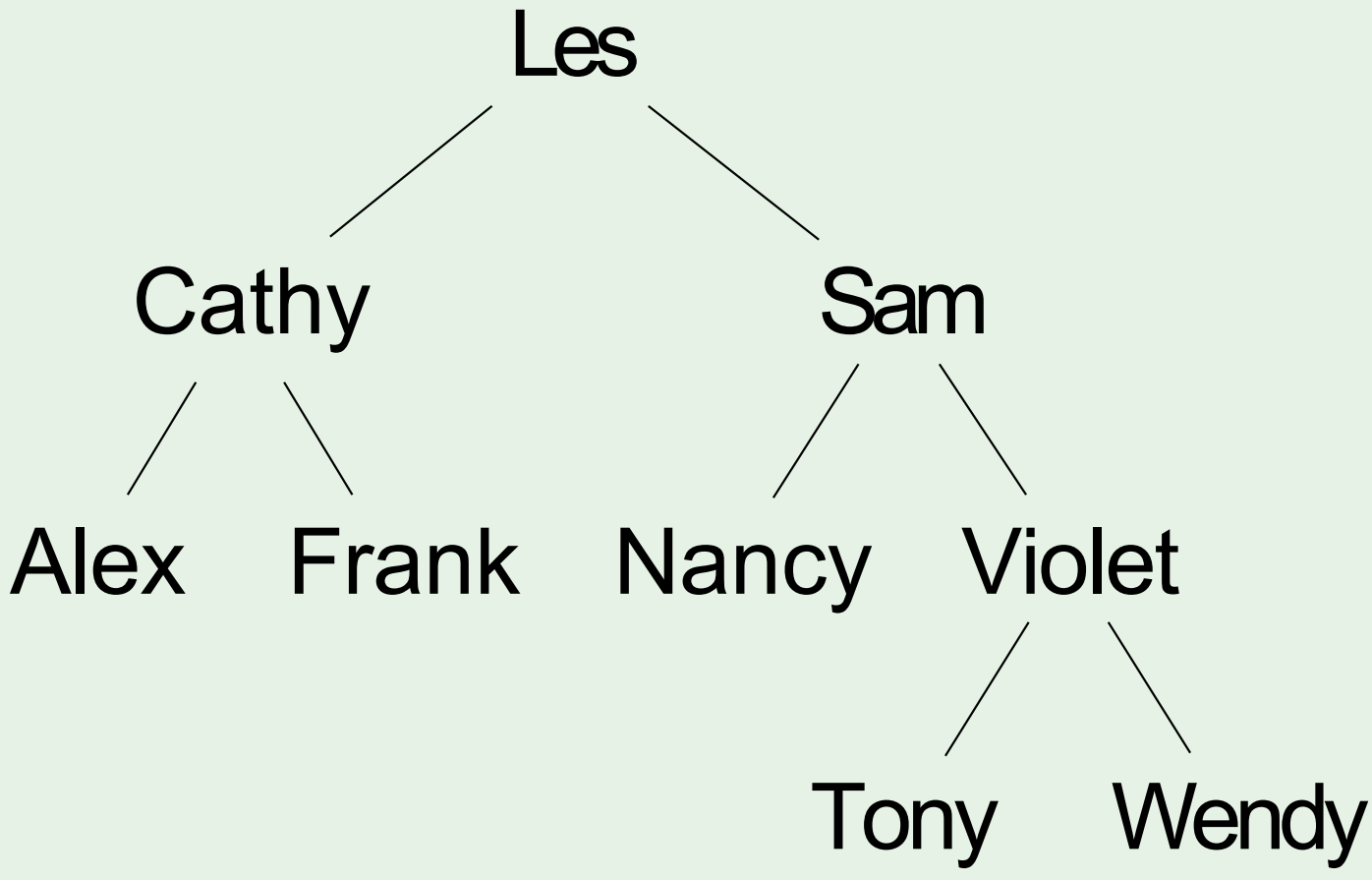
For example, print the nodes of the tree.

- Depth-first: We completely traverse one sub-tree before exploring a sibling sub-tree.
- Breadth-first: We traverse all nodes at one level before progressing to the next level.

Depth-first

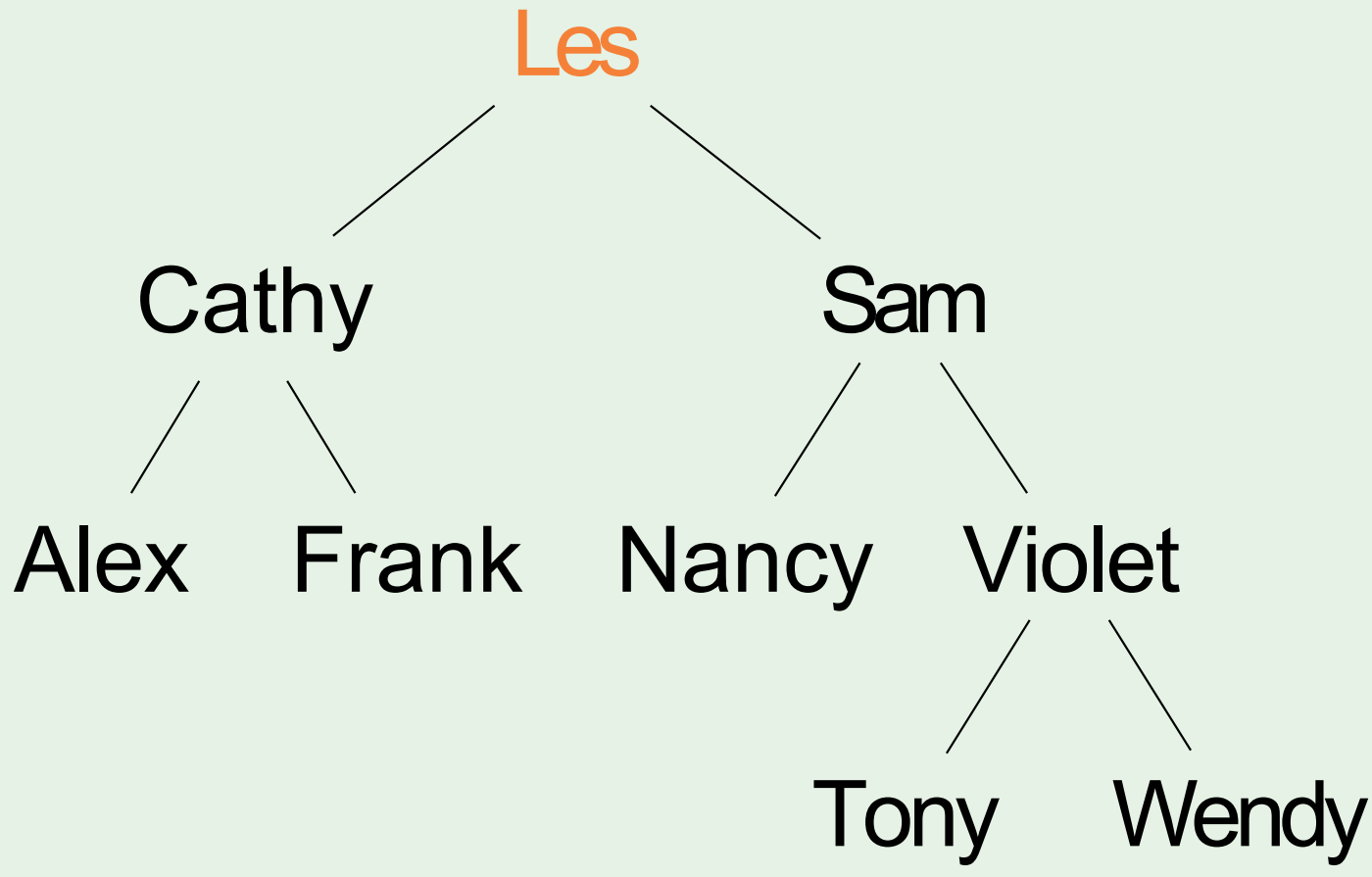
```
InOrderTraversal(tree)
if tree = nil :
   return
InOrderTraversal(tree.left)
Print(tree.key )
InOrderTraversal(tree.right)
```

InOrderTraversal



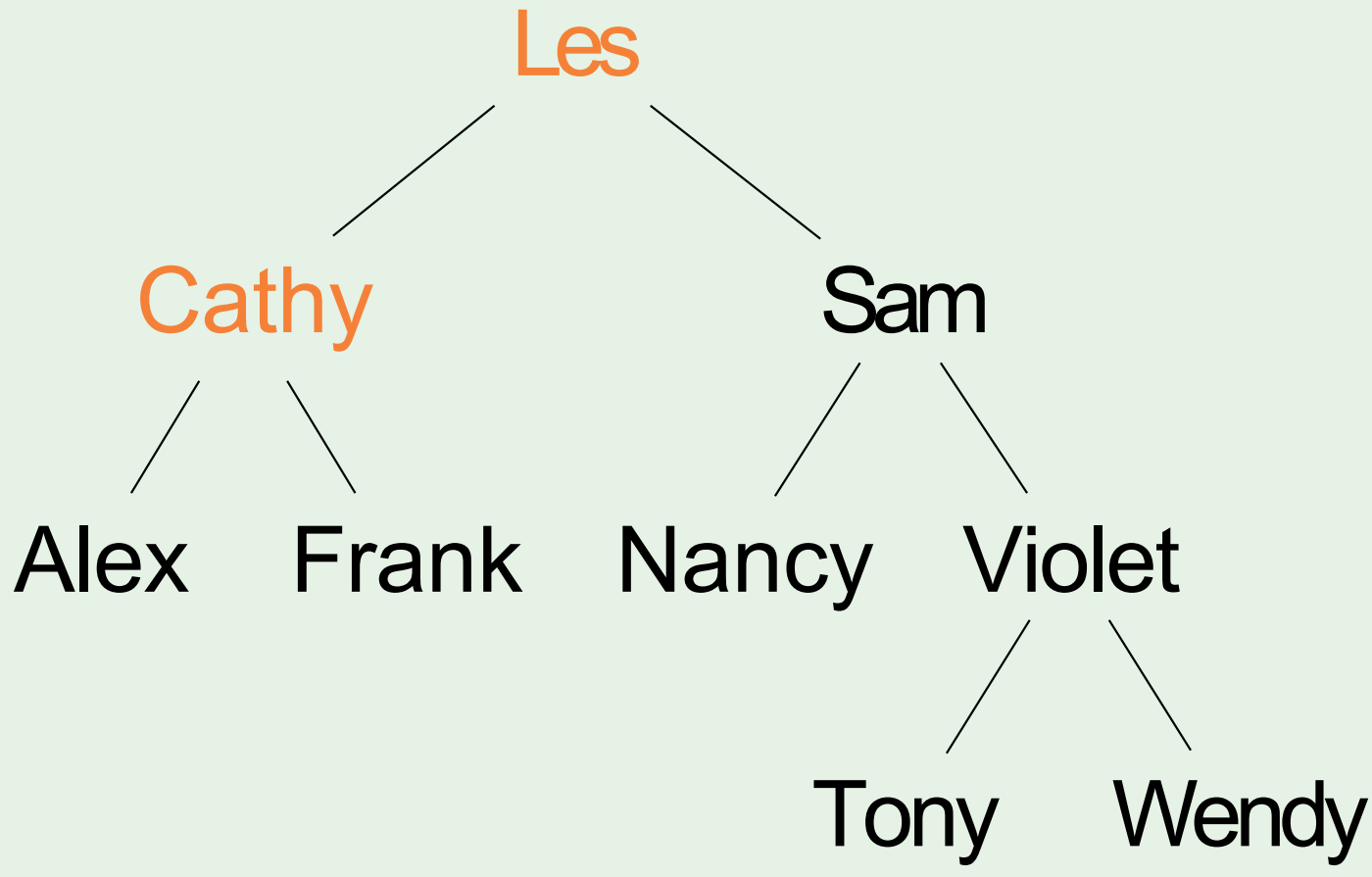
Output:

InOrderTraversal



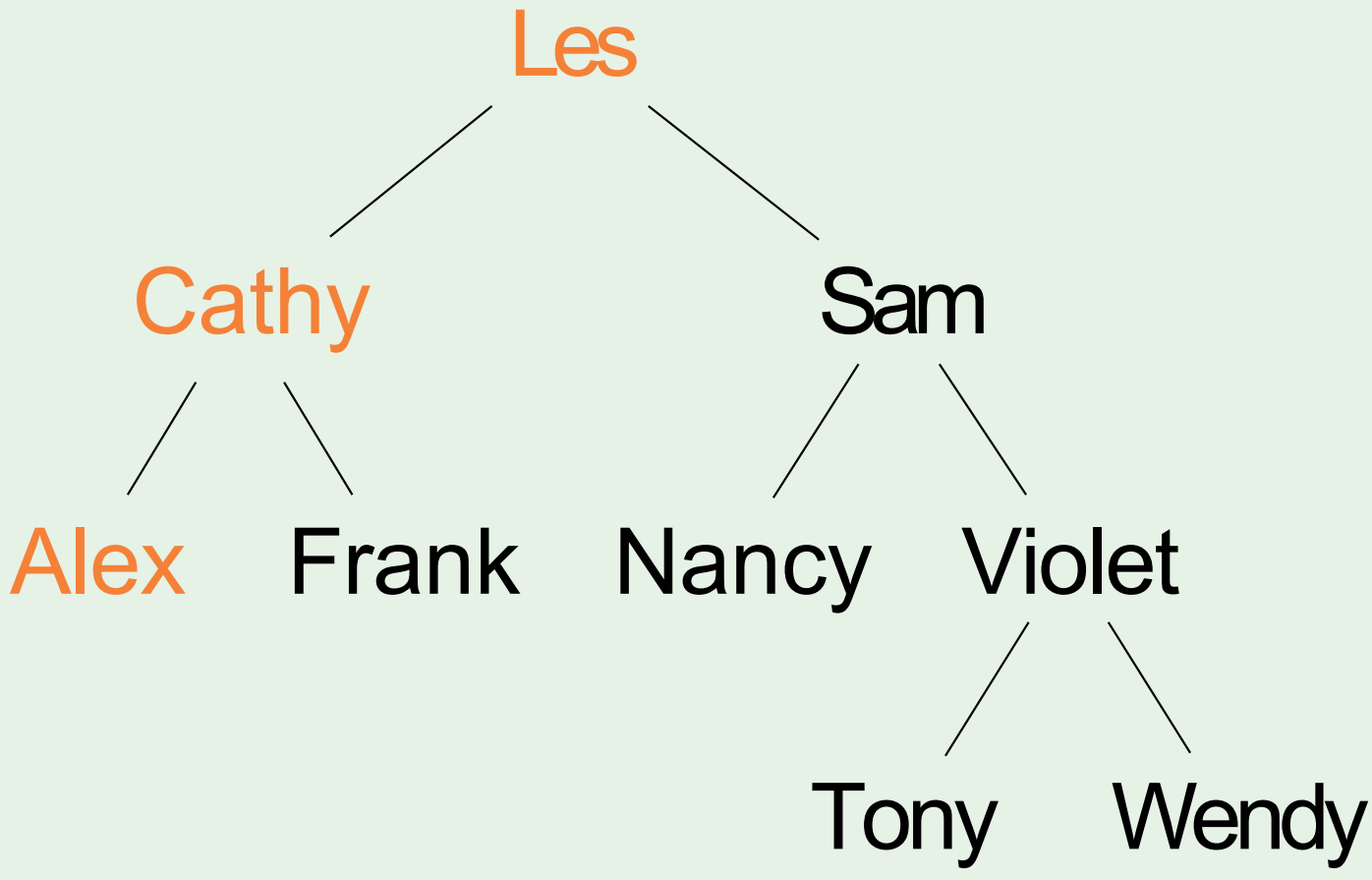
Output:

InOrderTraversal



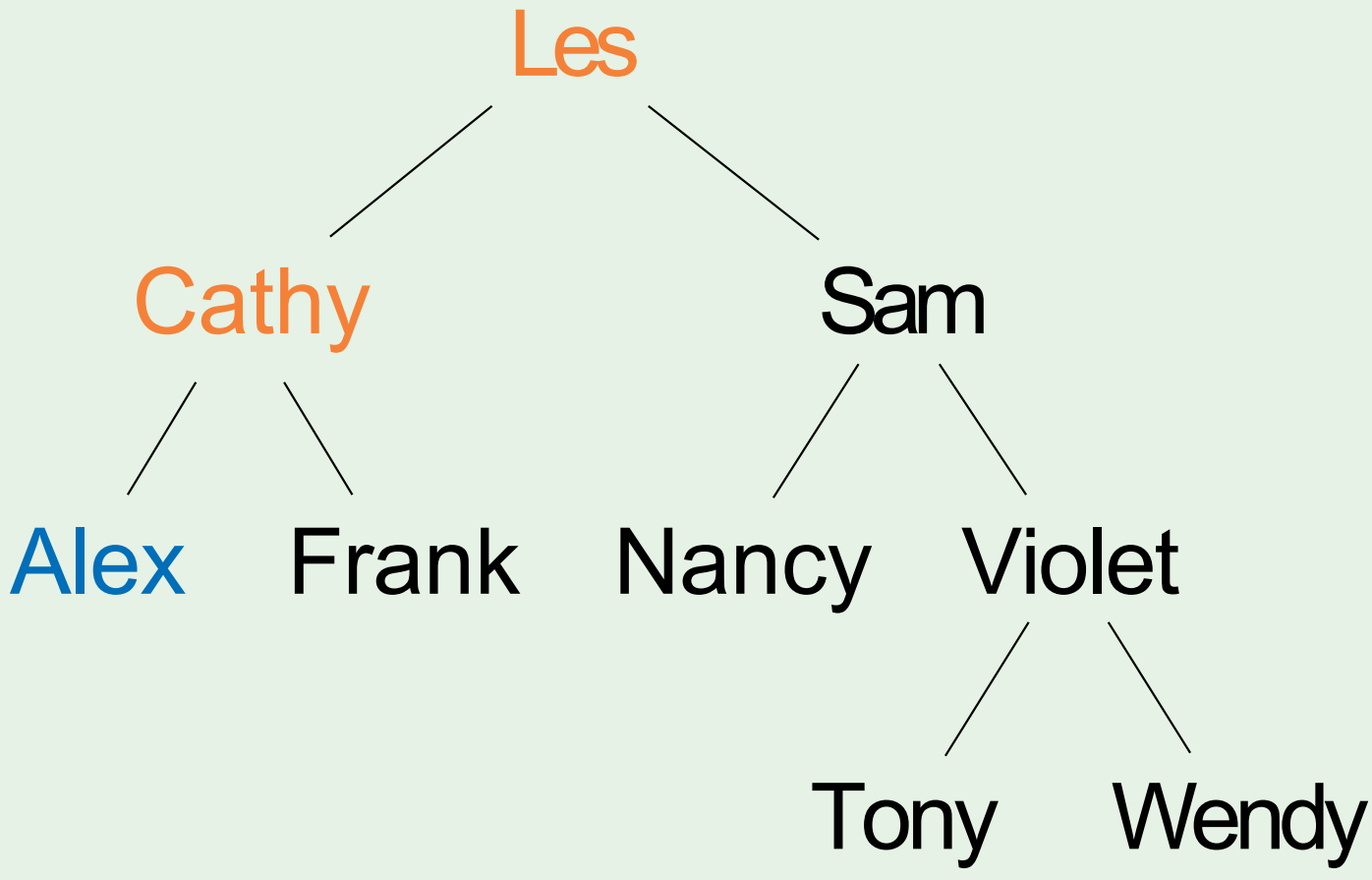
Output:

InOrderTraversal



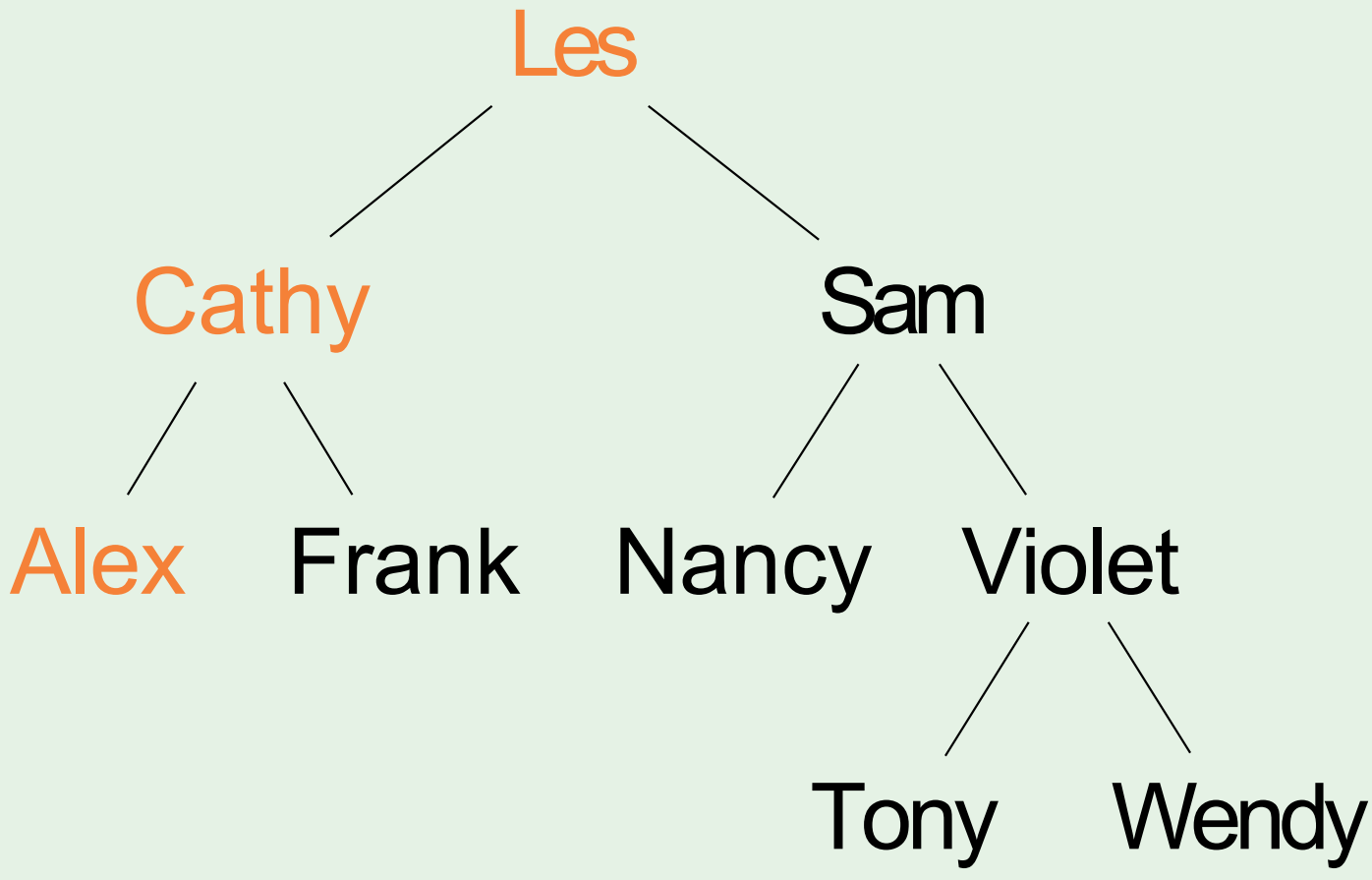
Output:

InOrderTraversal



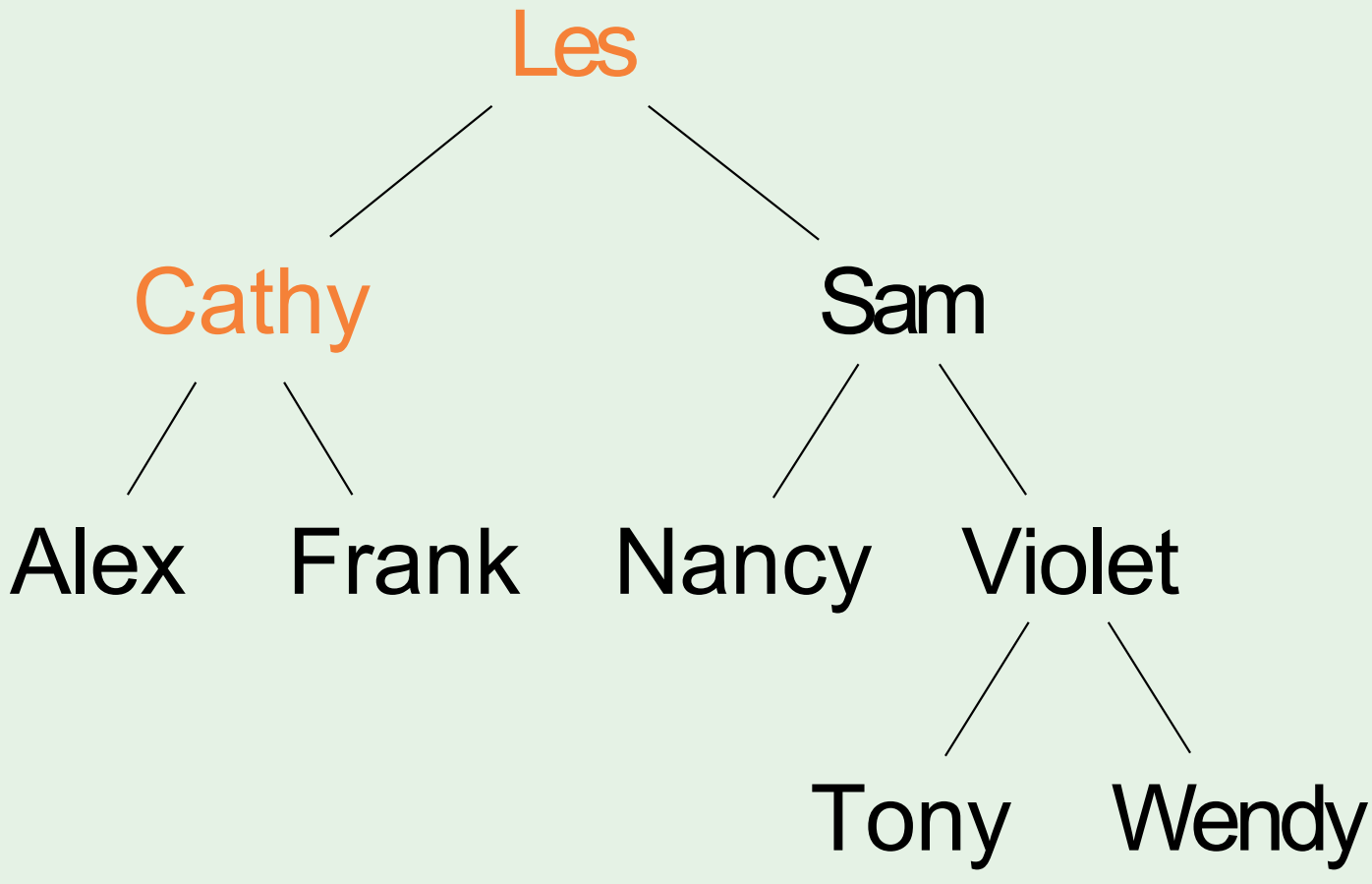
Output: Alex

InOrderTraversal



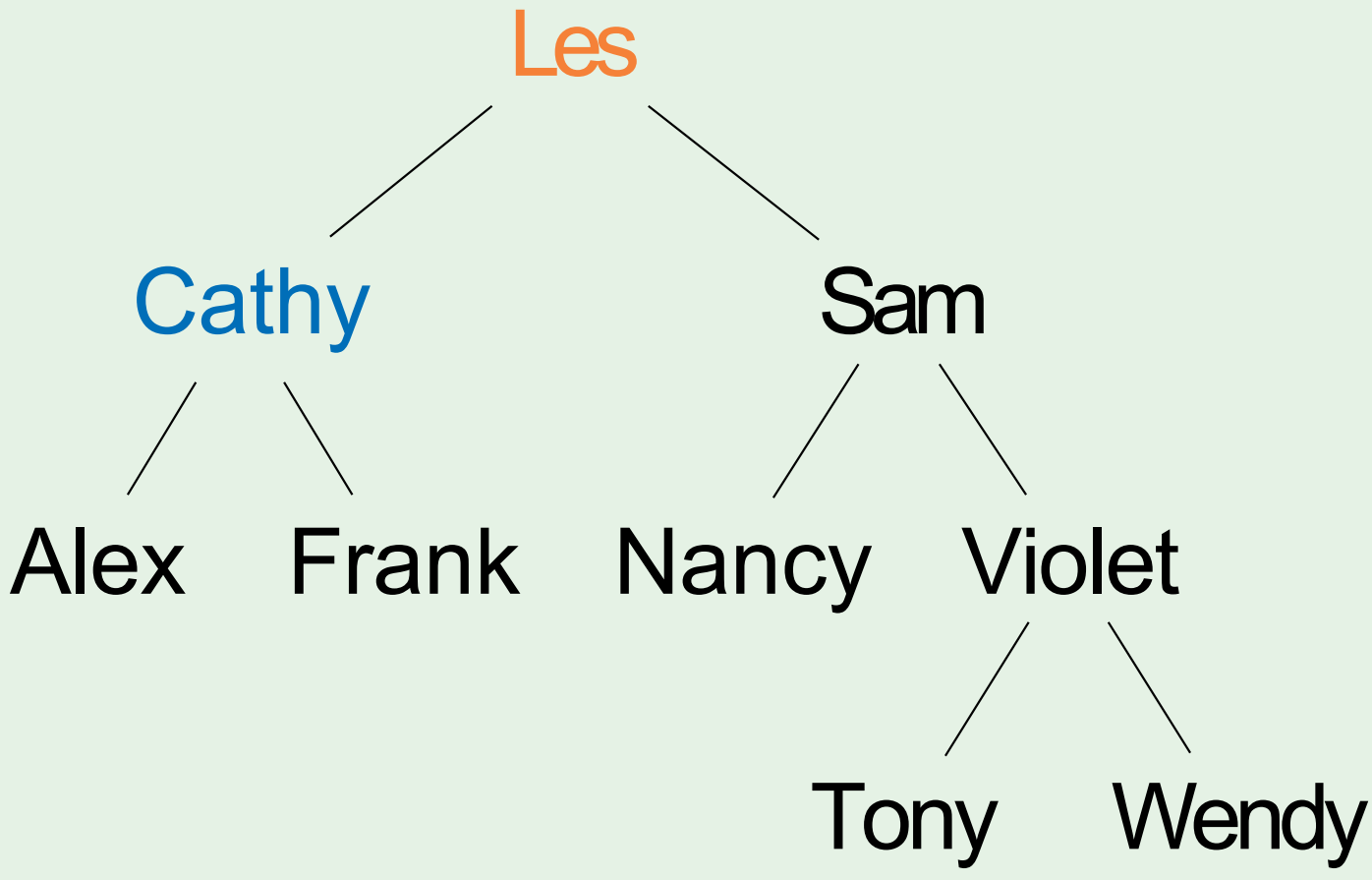
Output: Alex

InOrderTraversal



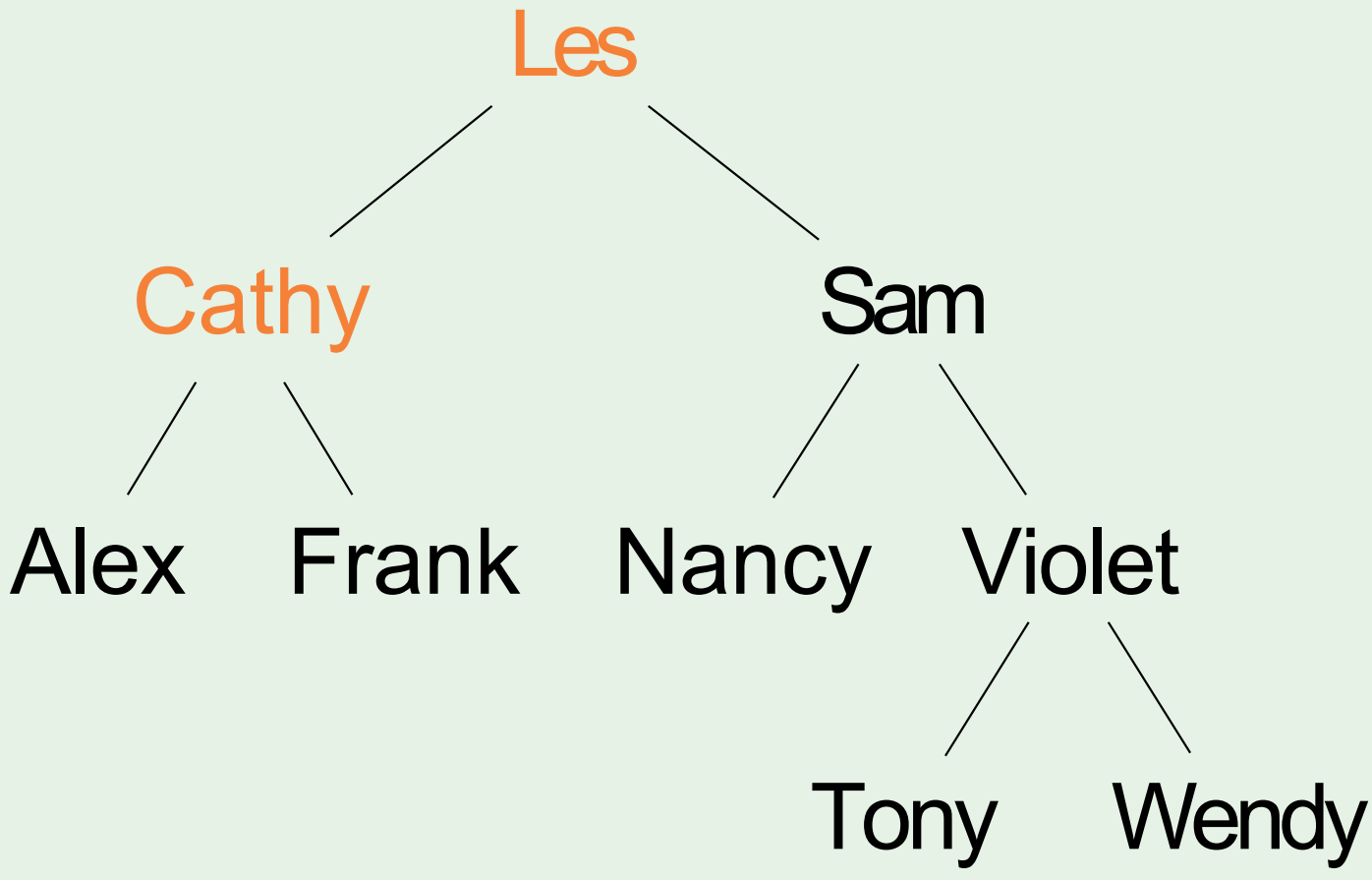
Output: Alex

InOrderTraversal



Output: Alex Cathy

InOrderTraversal



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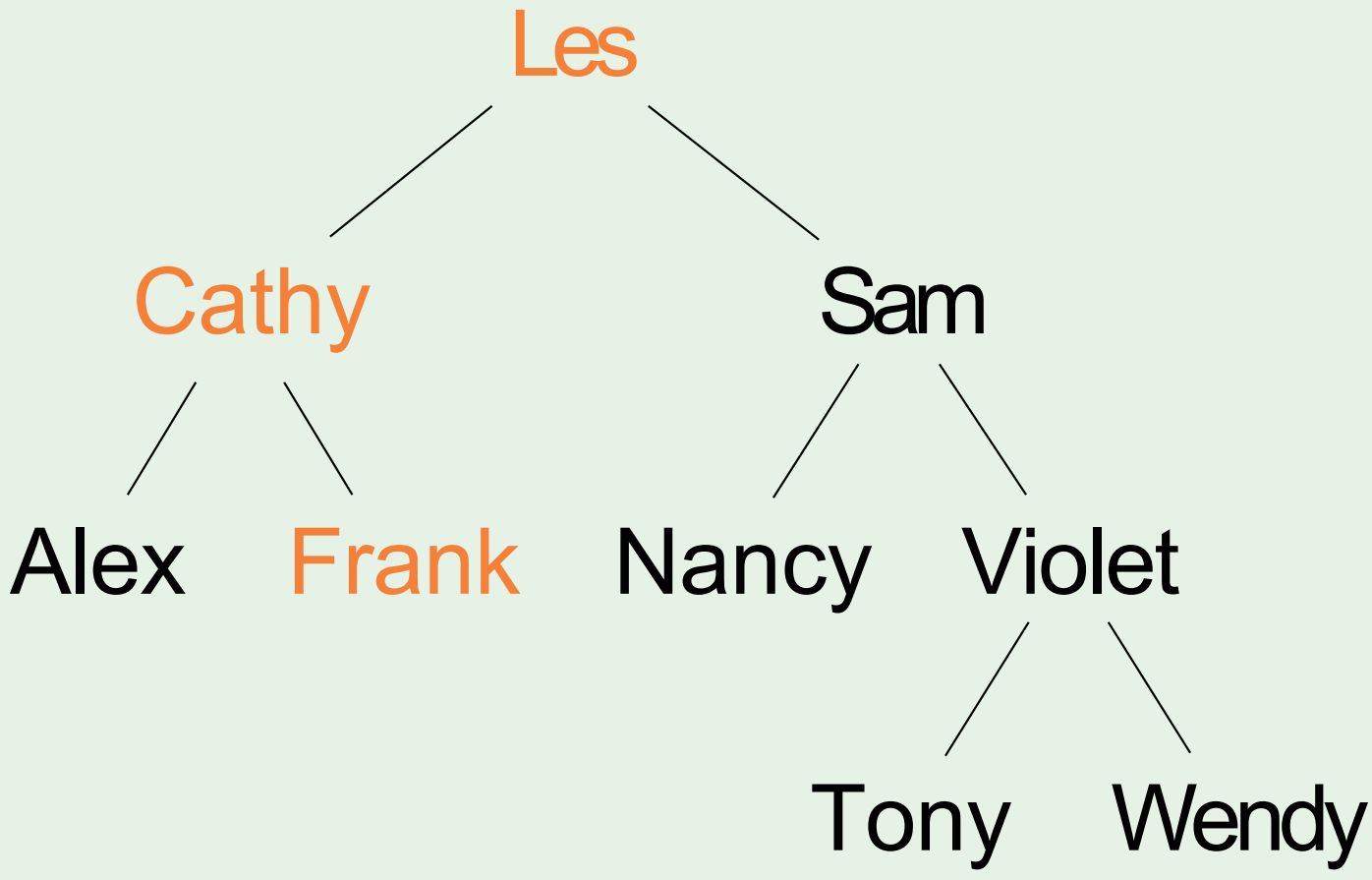

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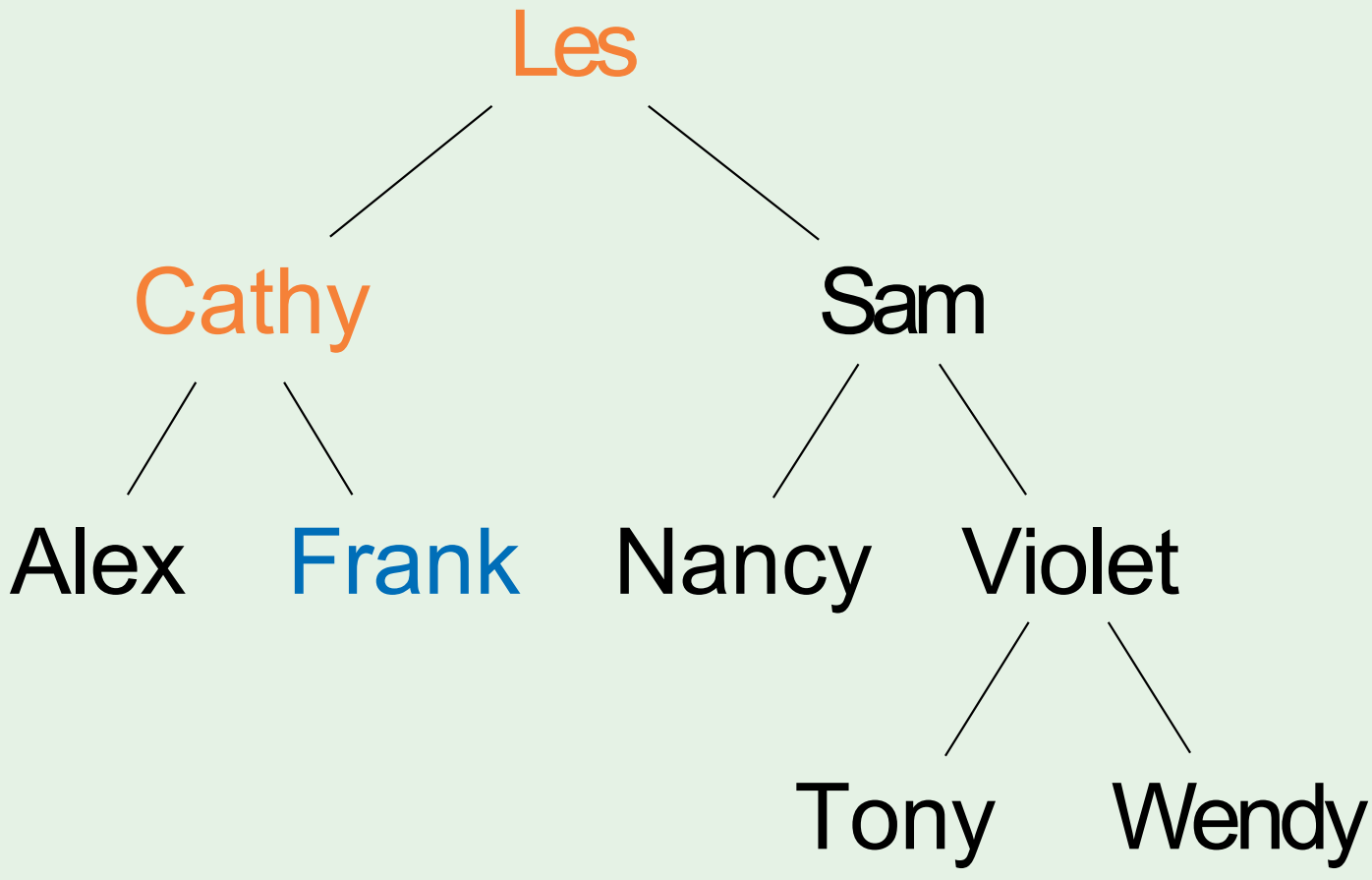
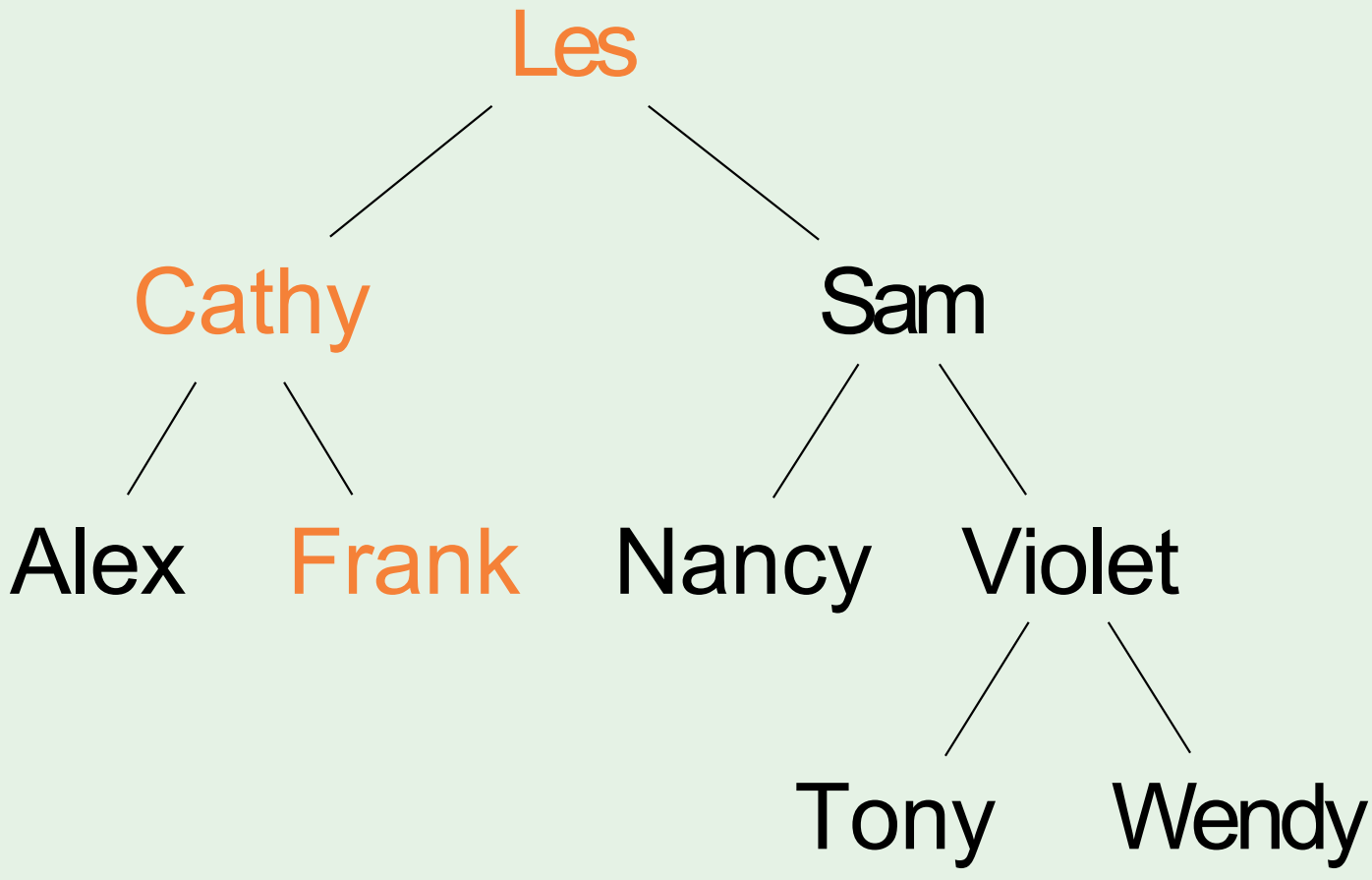


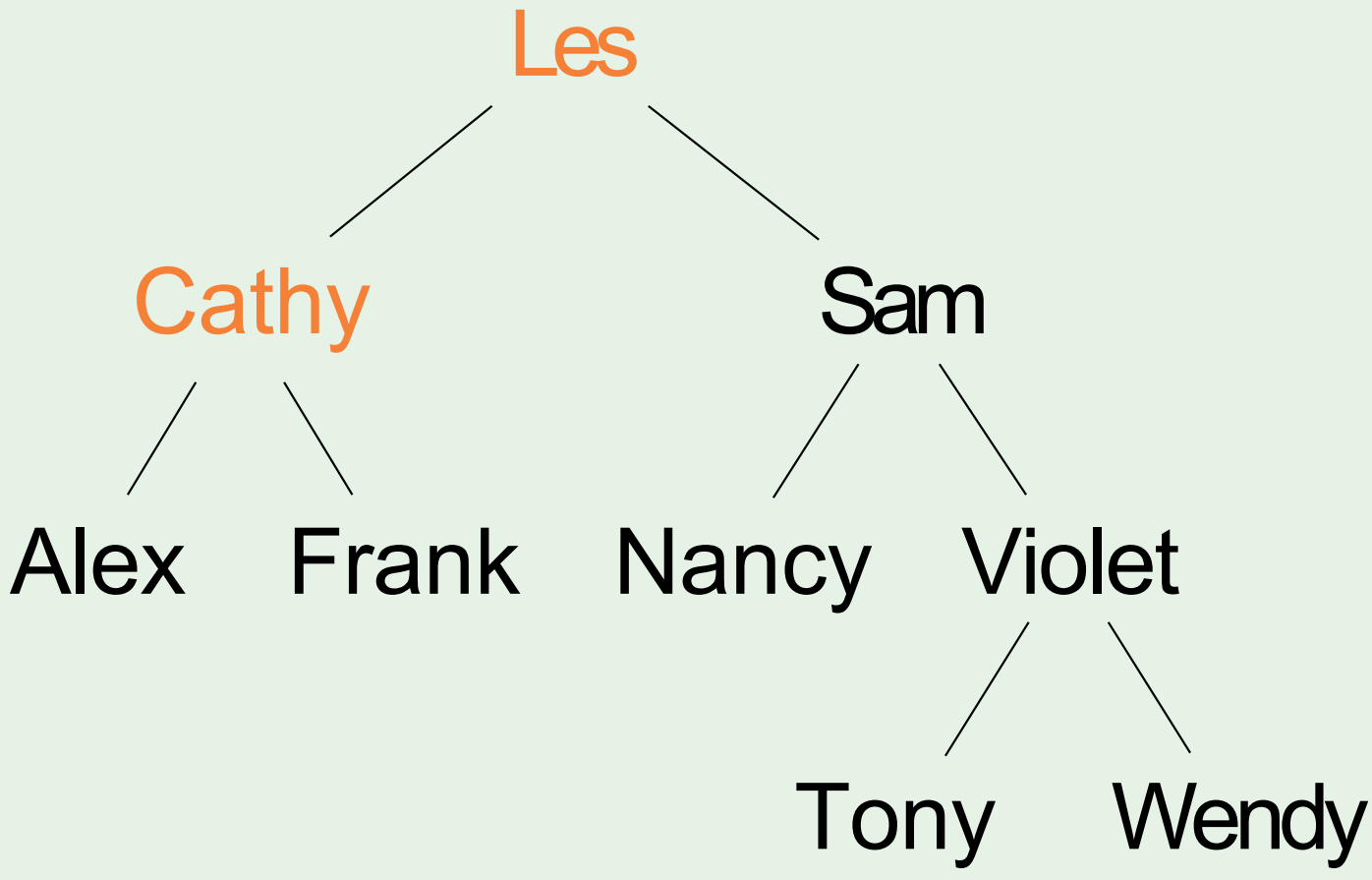
Output: Alex Cathy Frank

InOrderTraversal



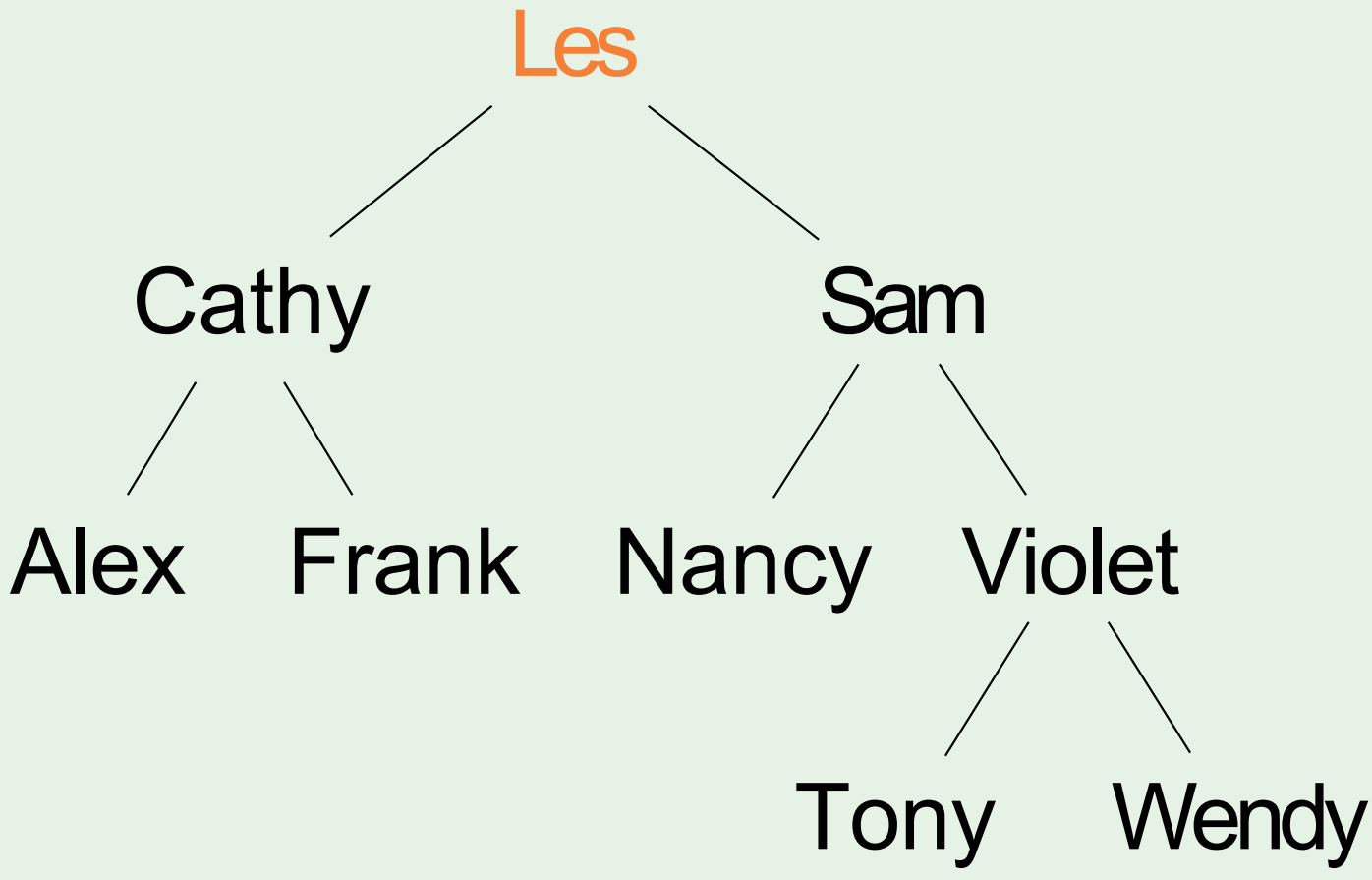
Output: Alex Cathy Frank

InOrderTraversal



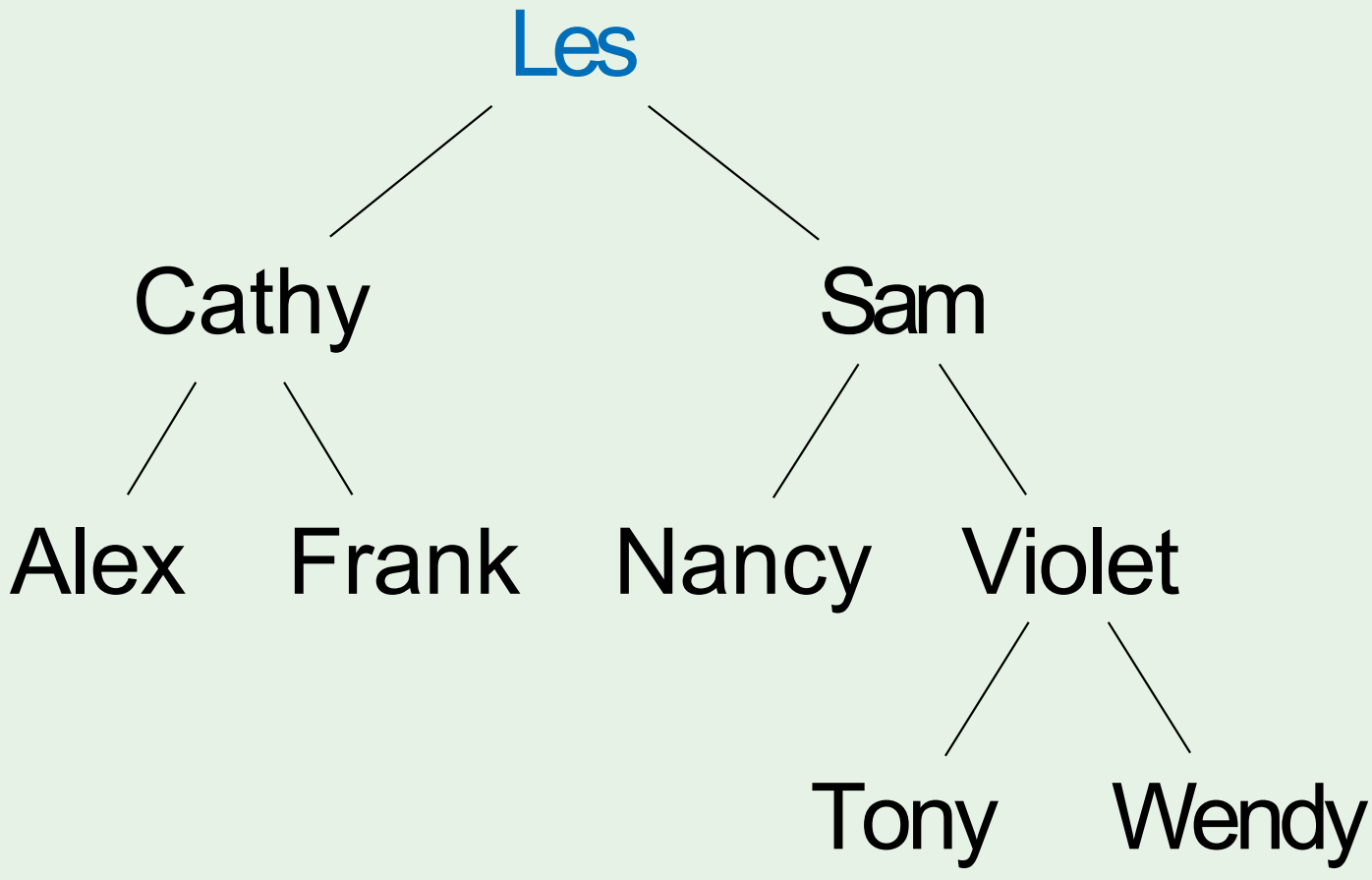
Output: Alex Cathy Frank

InOrderTraversal



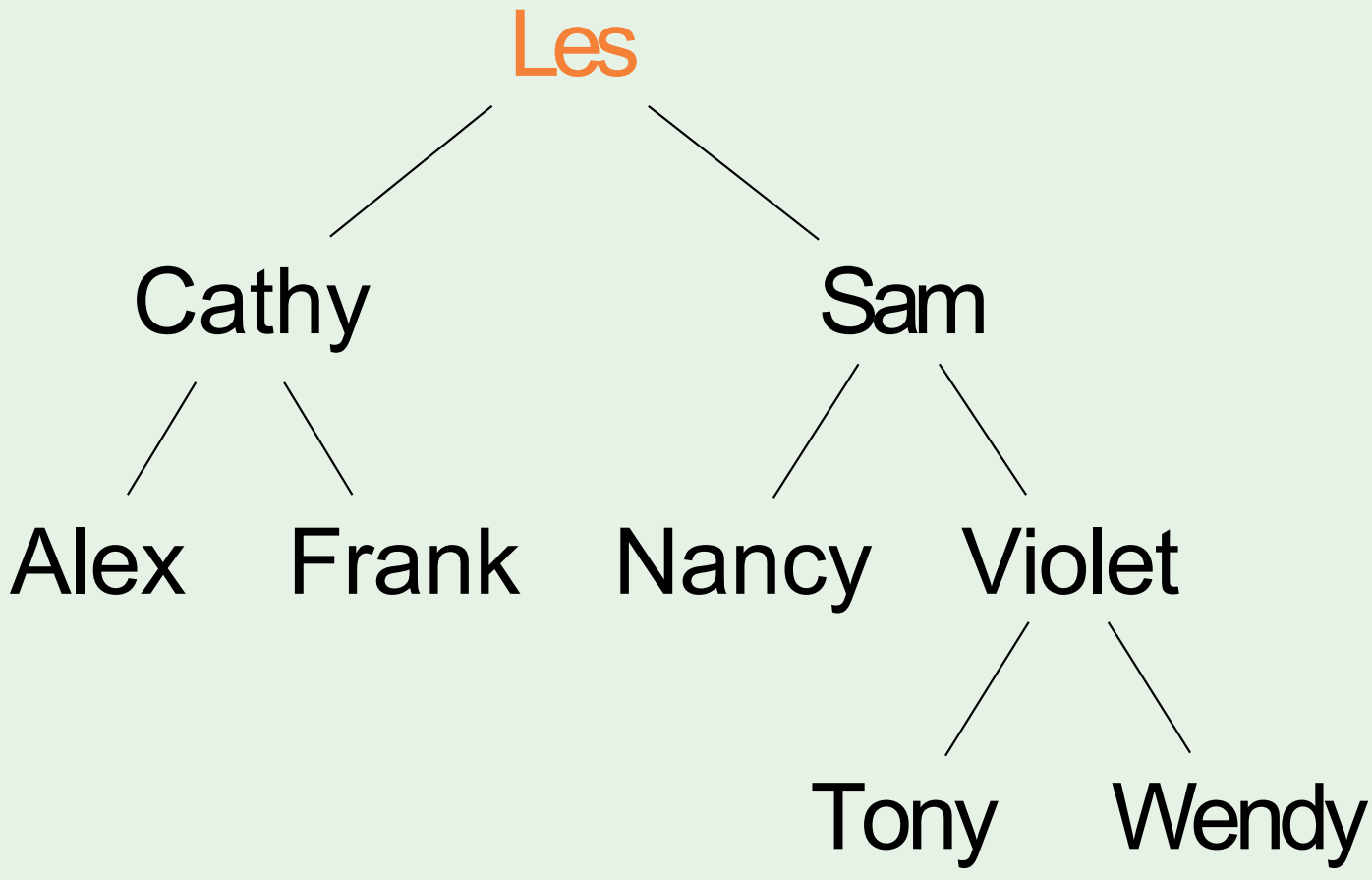
Output: Alex Cathy Frank

InOrderTraversal



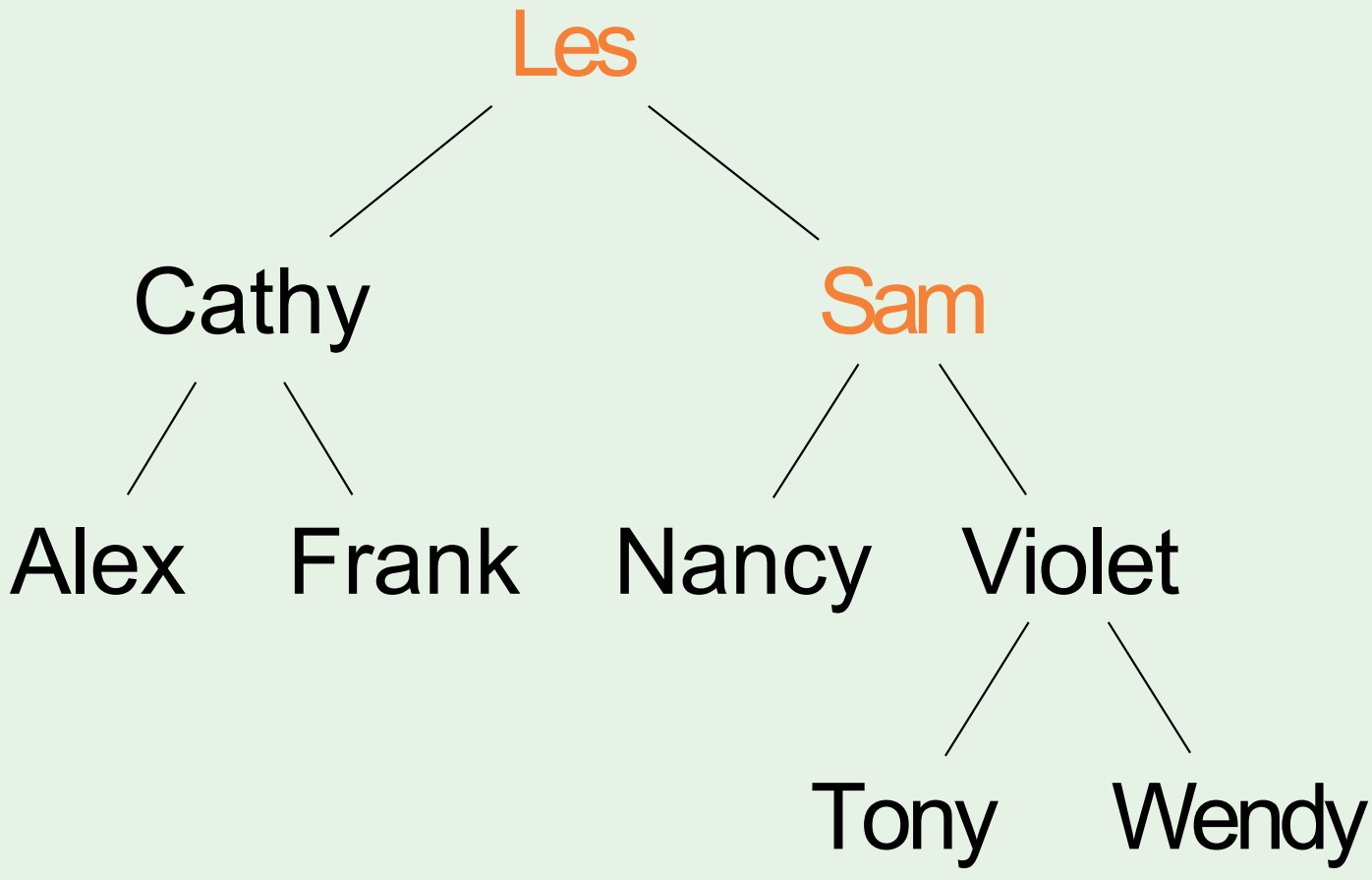
Output: Alex Cathy Frank Les

InOrderTraversal



Output: Alex Cathy Frank Les

InOrderTraversal



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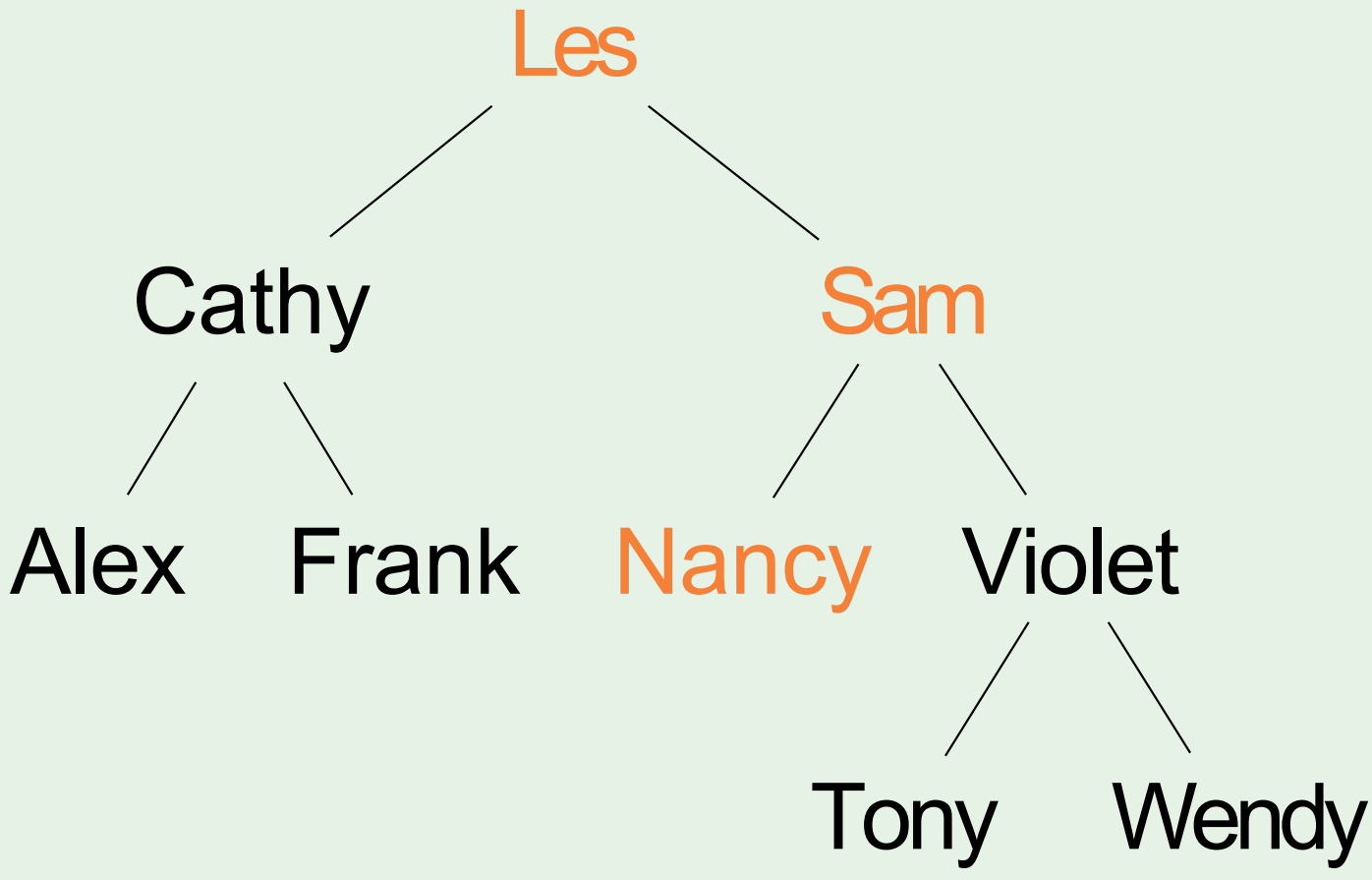
InOrderTraversal



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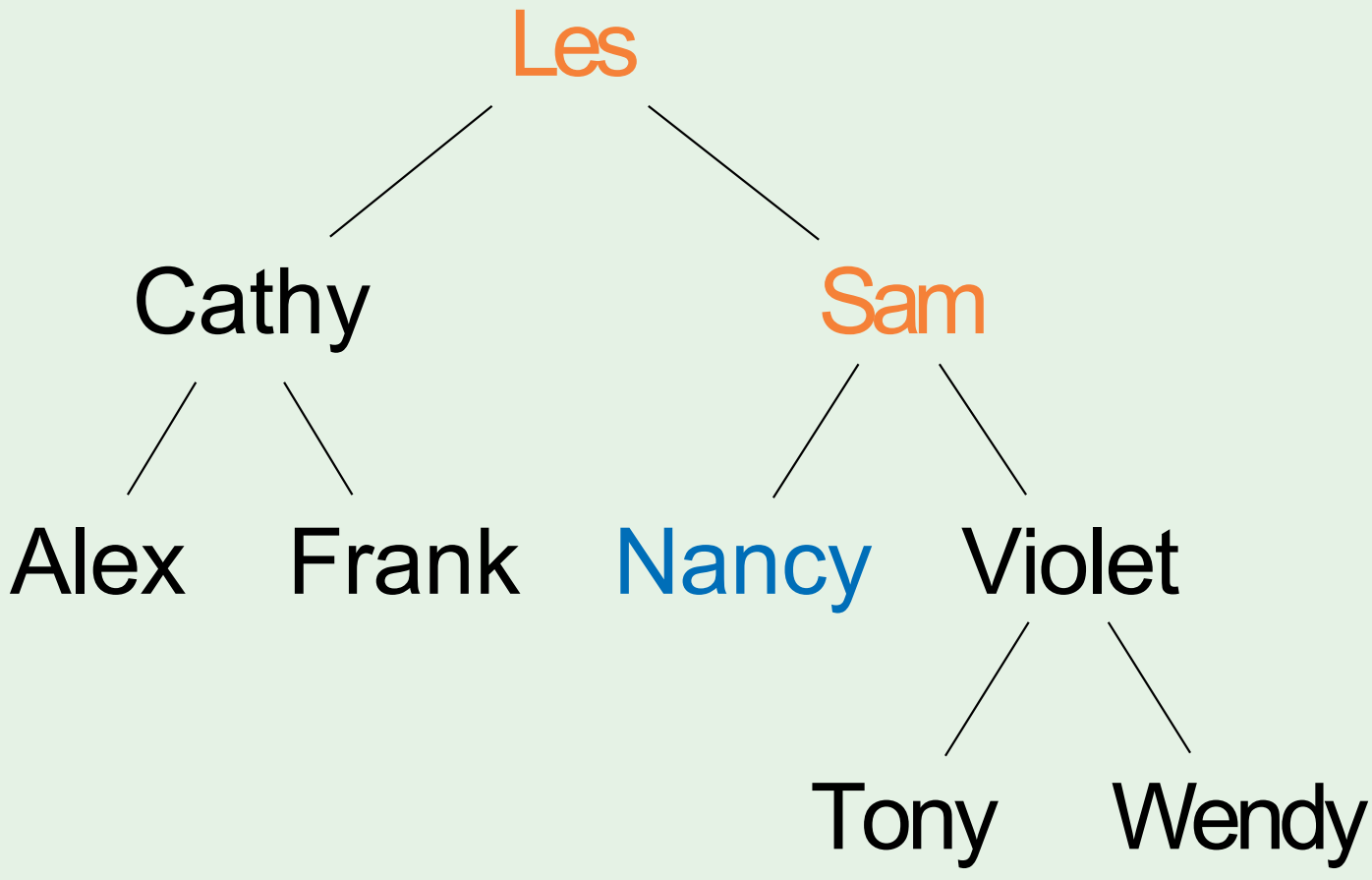
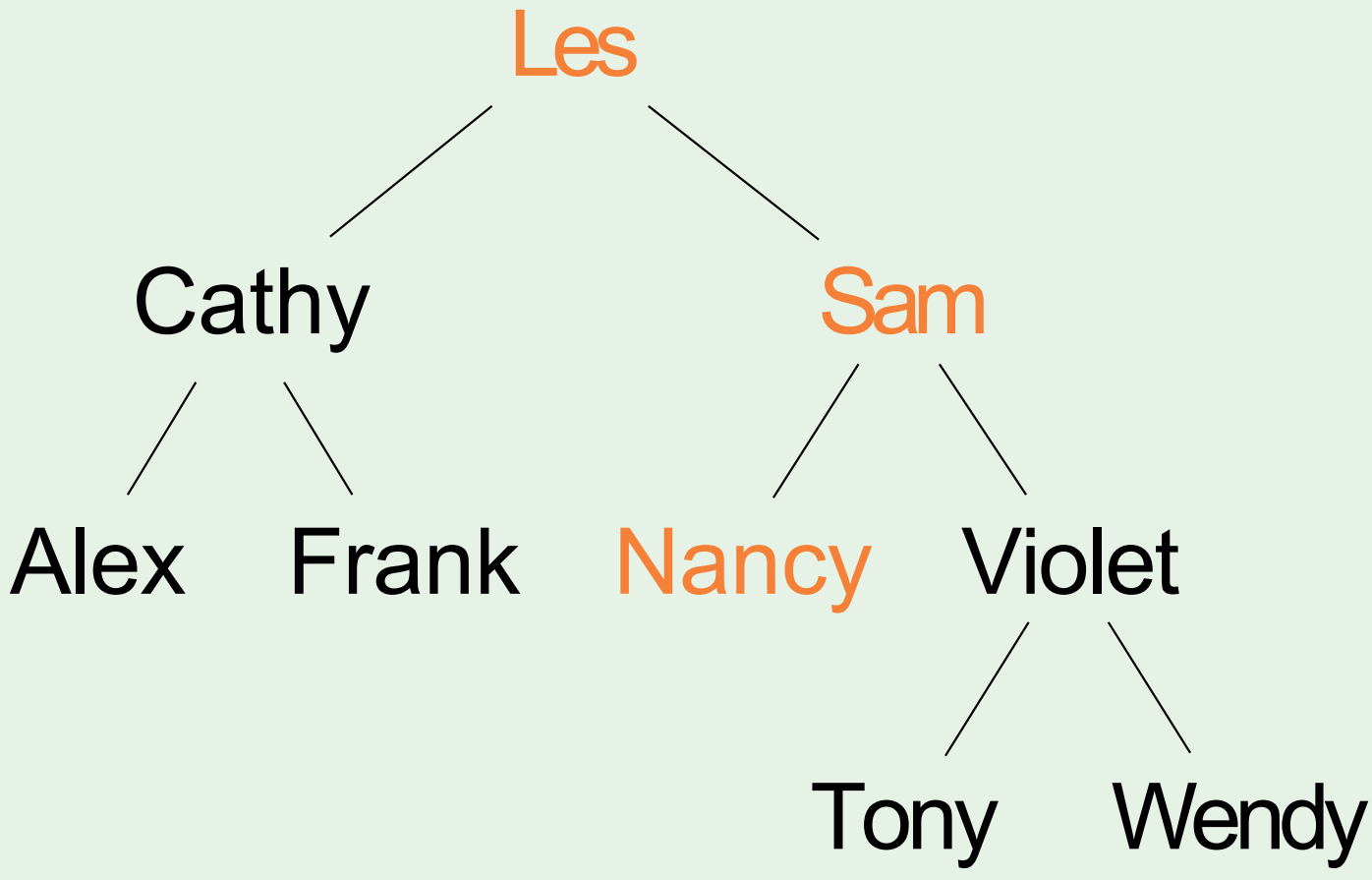
InOrderTraversal



Output: Alex Cathy Frank Les Nancy

InOrderTraversal



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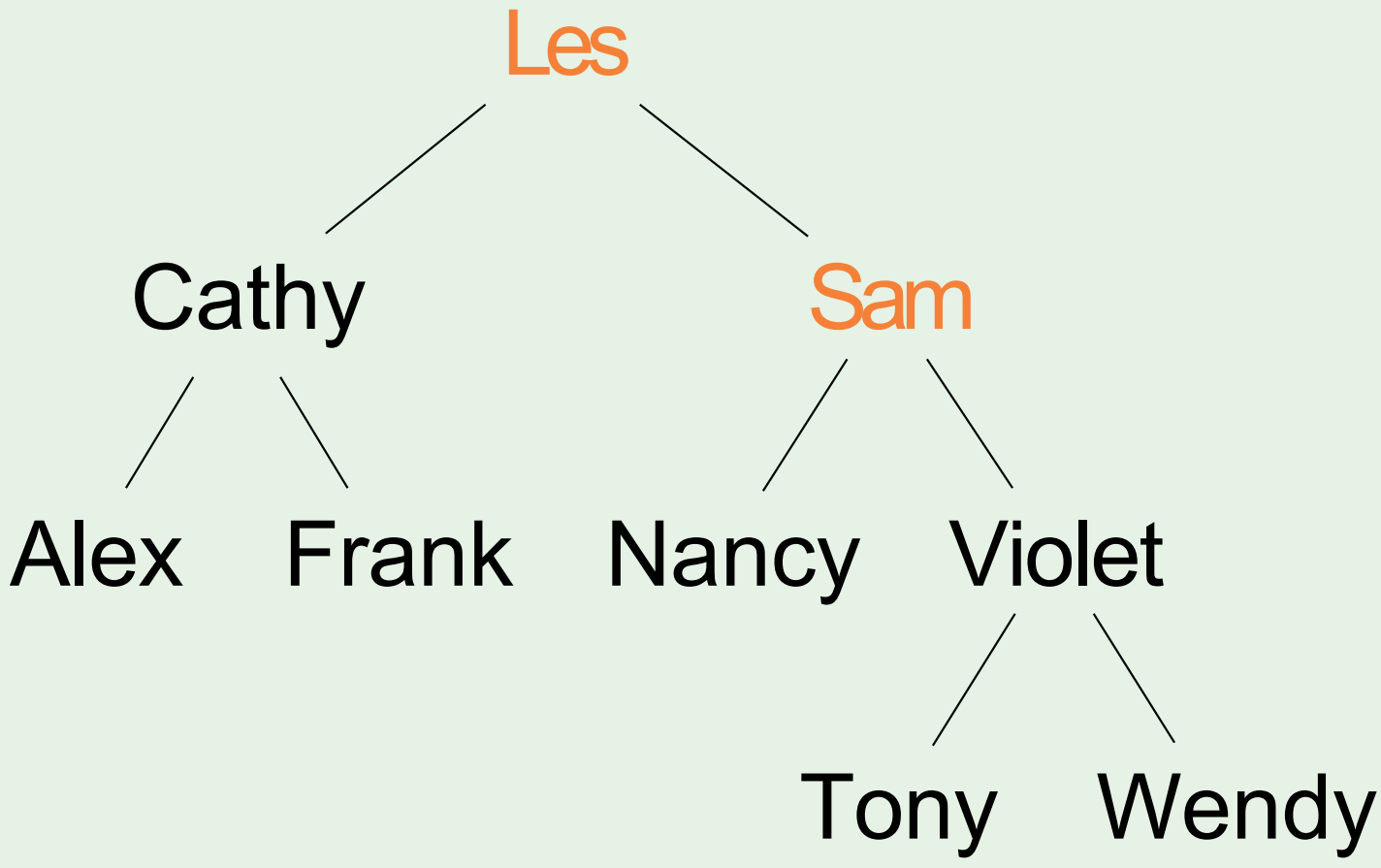
InOrderTraversal



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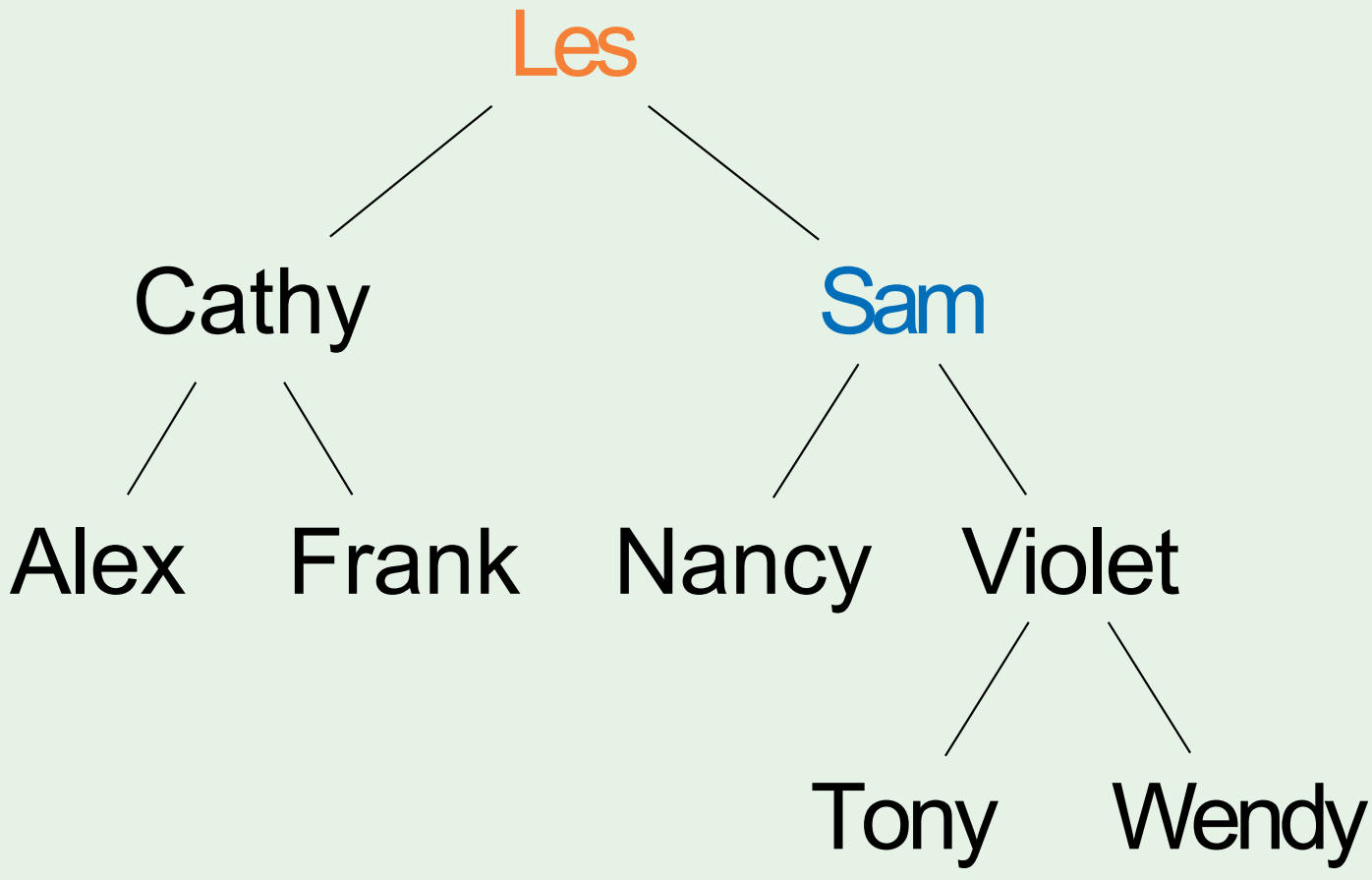
InOrderTraversal



Output: Alex Cathy Frank Les Nancy Sam

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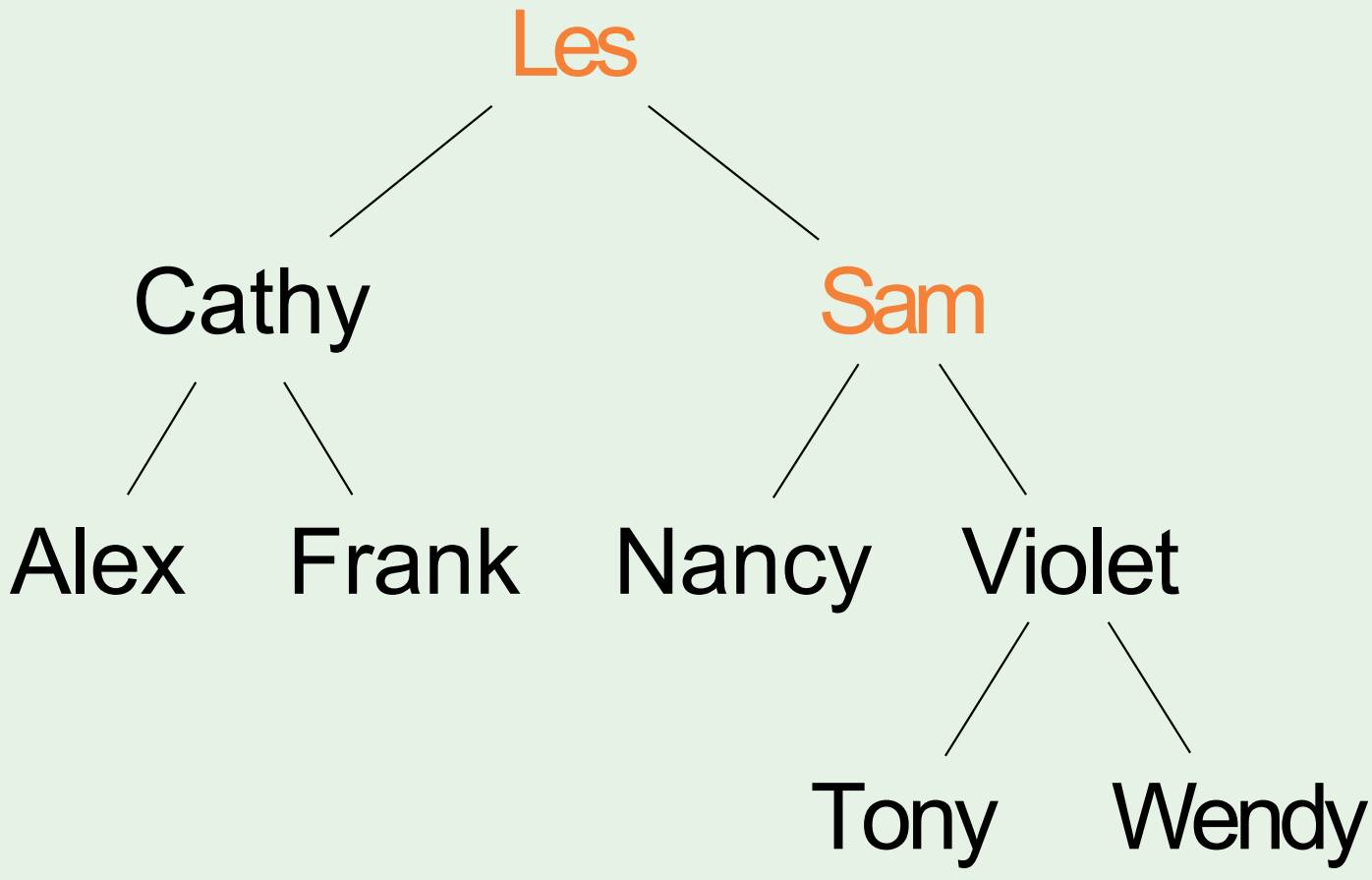

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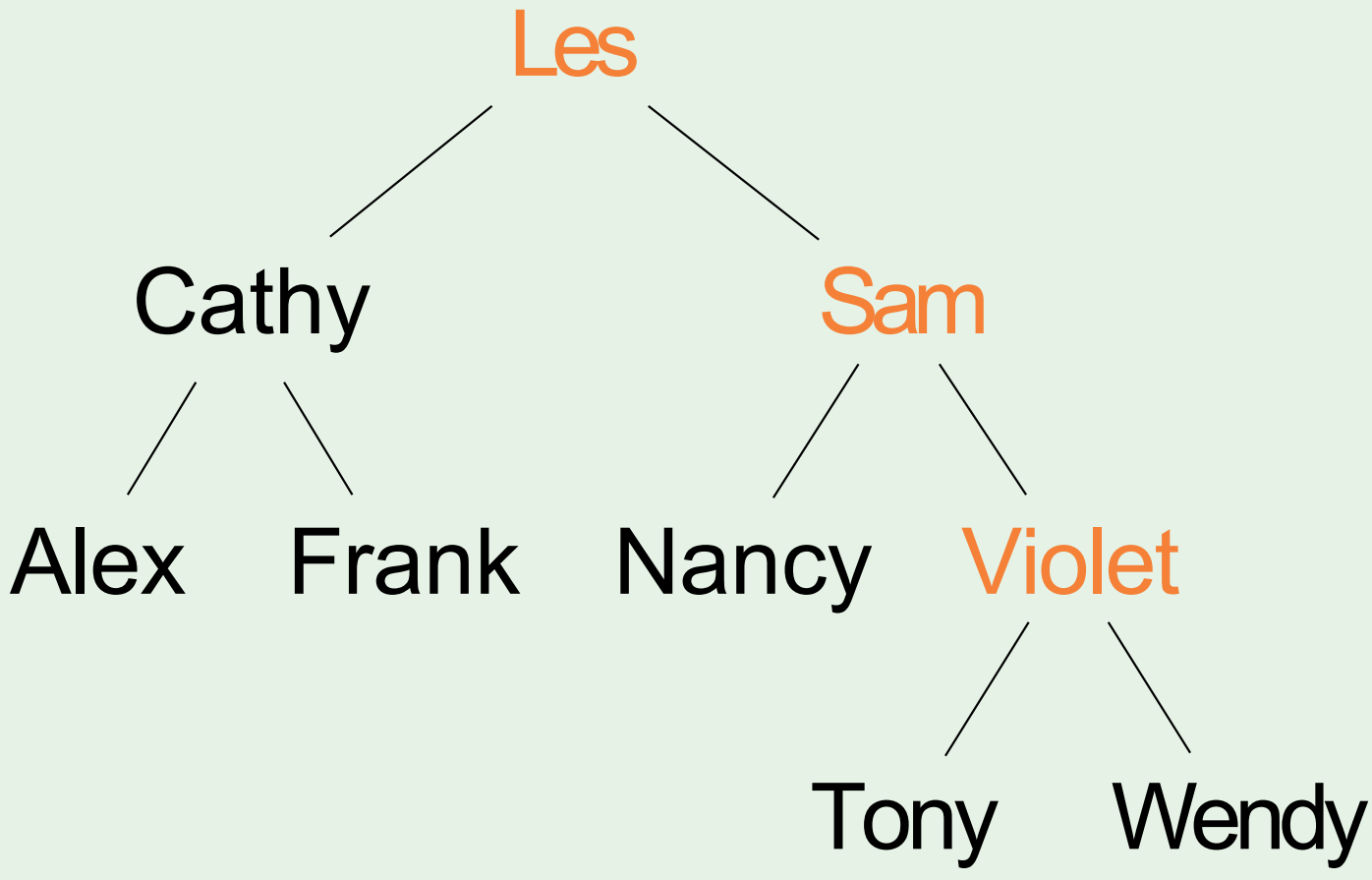
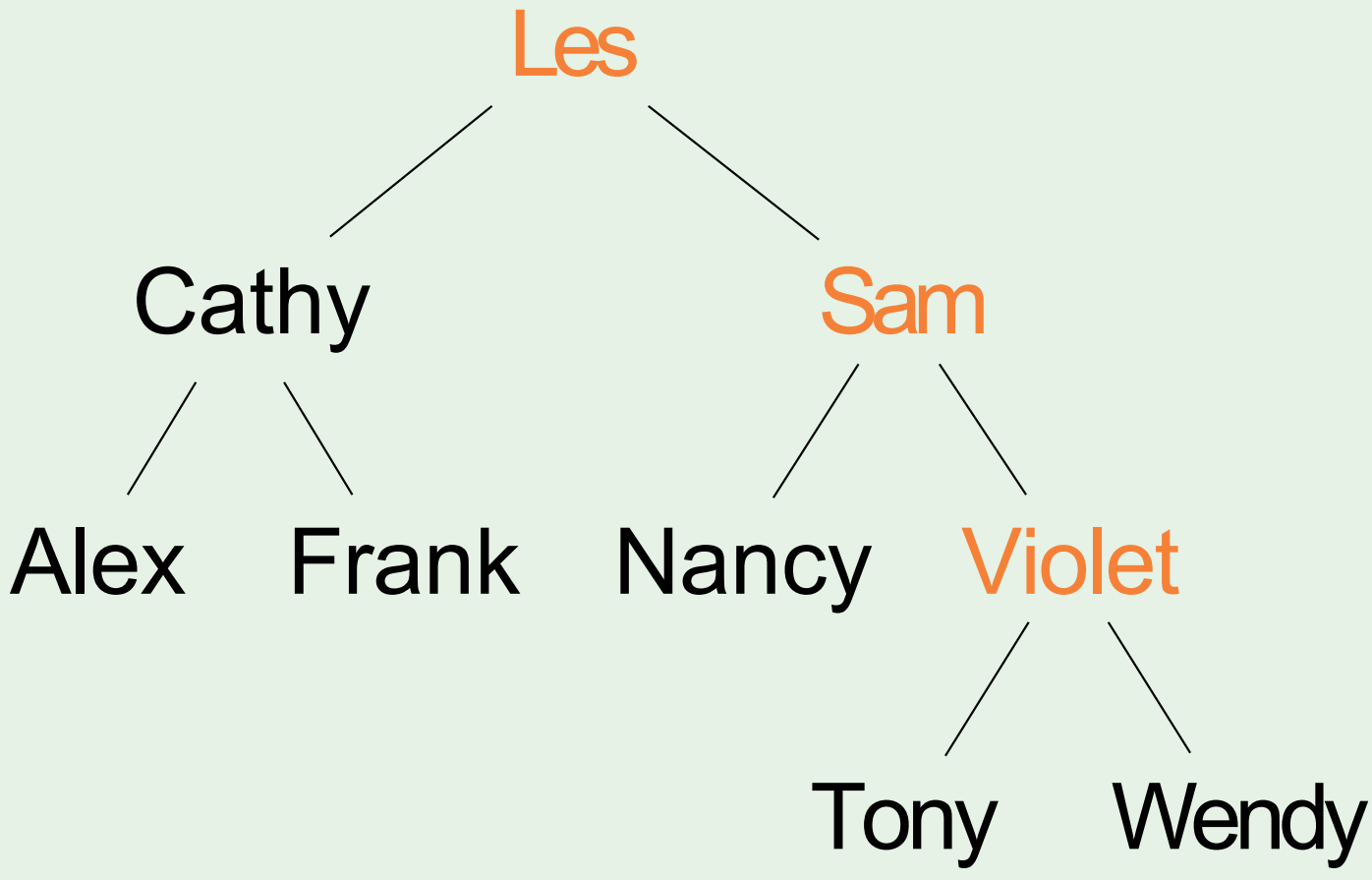


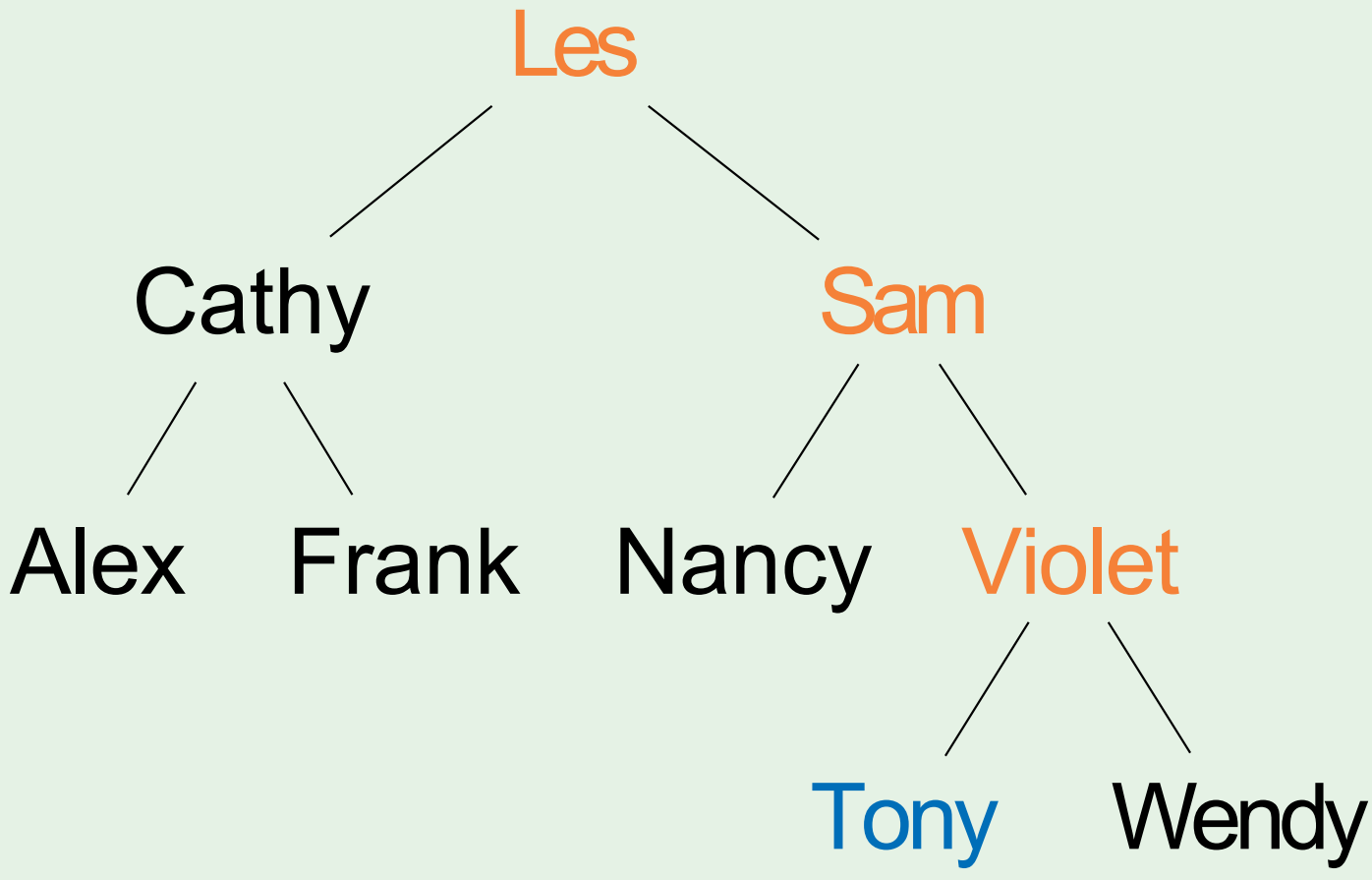
Output: Alex Cathy Frank Les Nancy Sam

InOrderTraversal



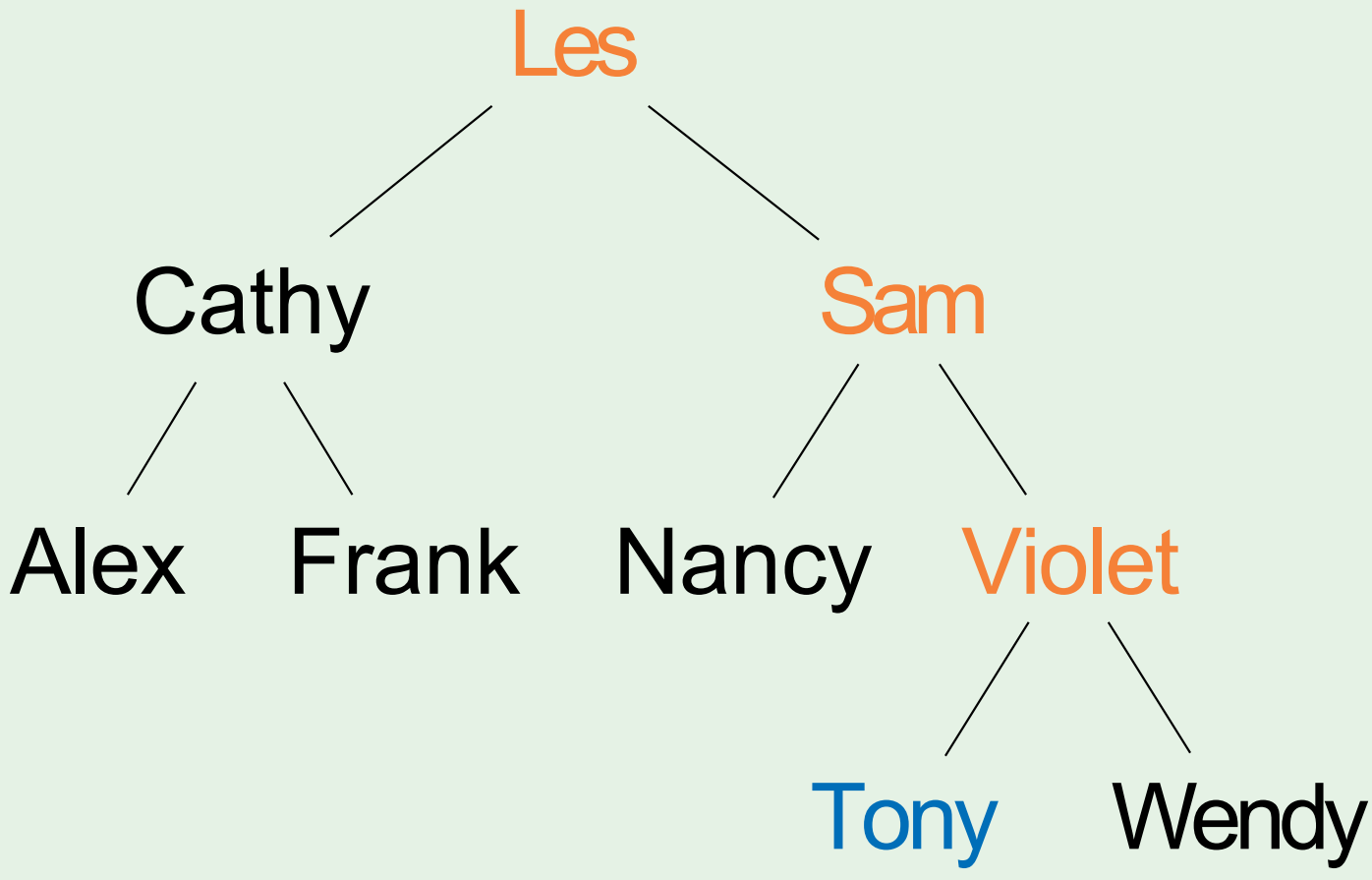
Output: Alex Cathy Frank Les Nancy Sam

InOrderTraversal



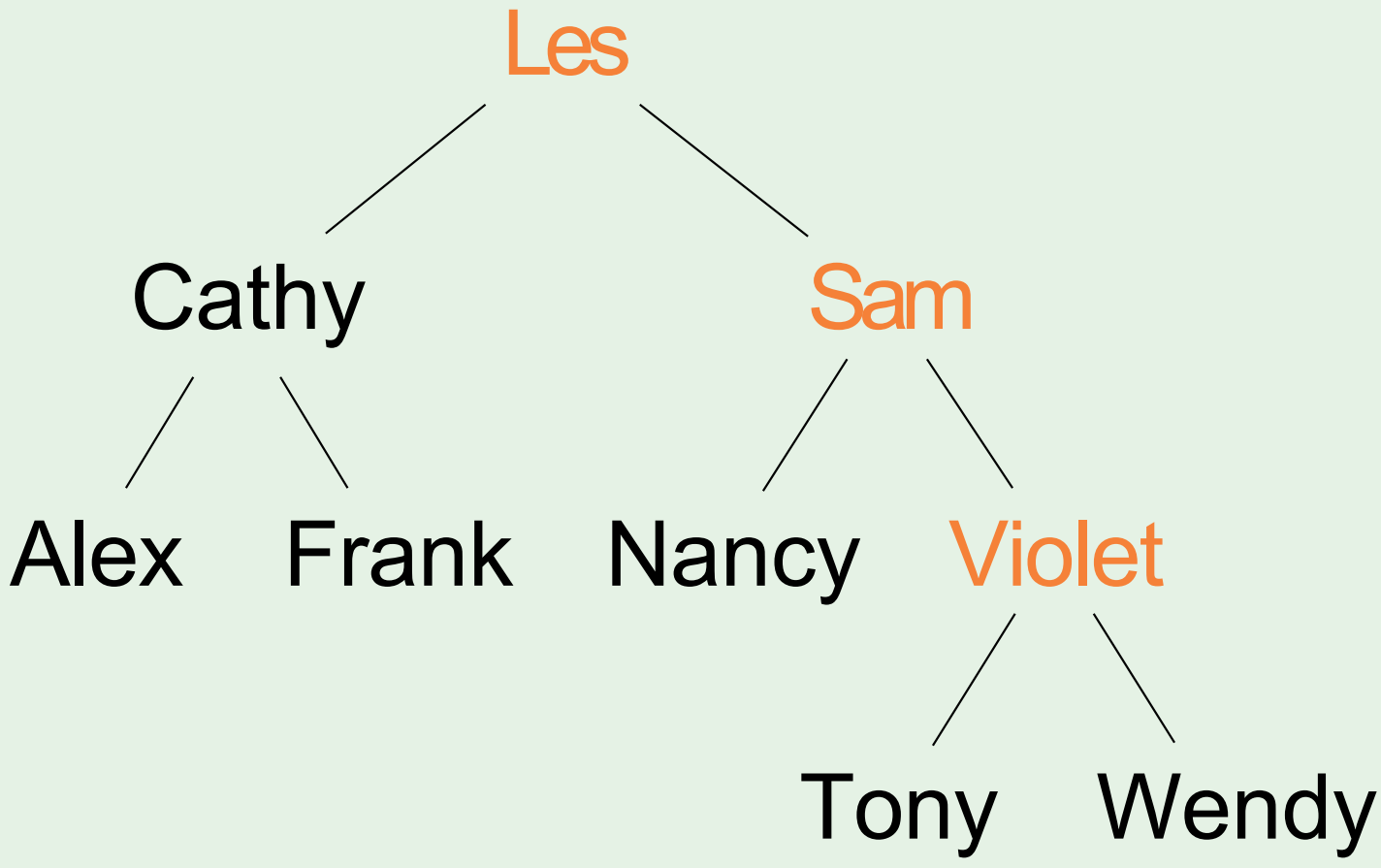
Output: Alex Cathy Frank Les Nancy Sam Tony

InOrderTraversal



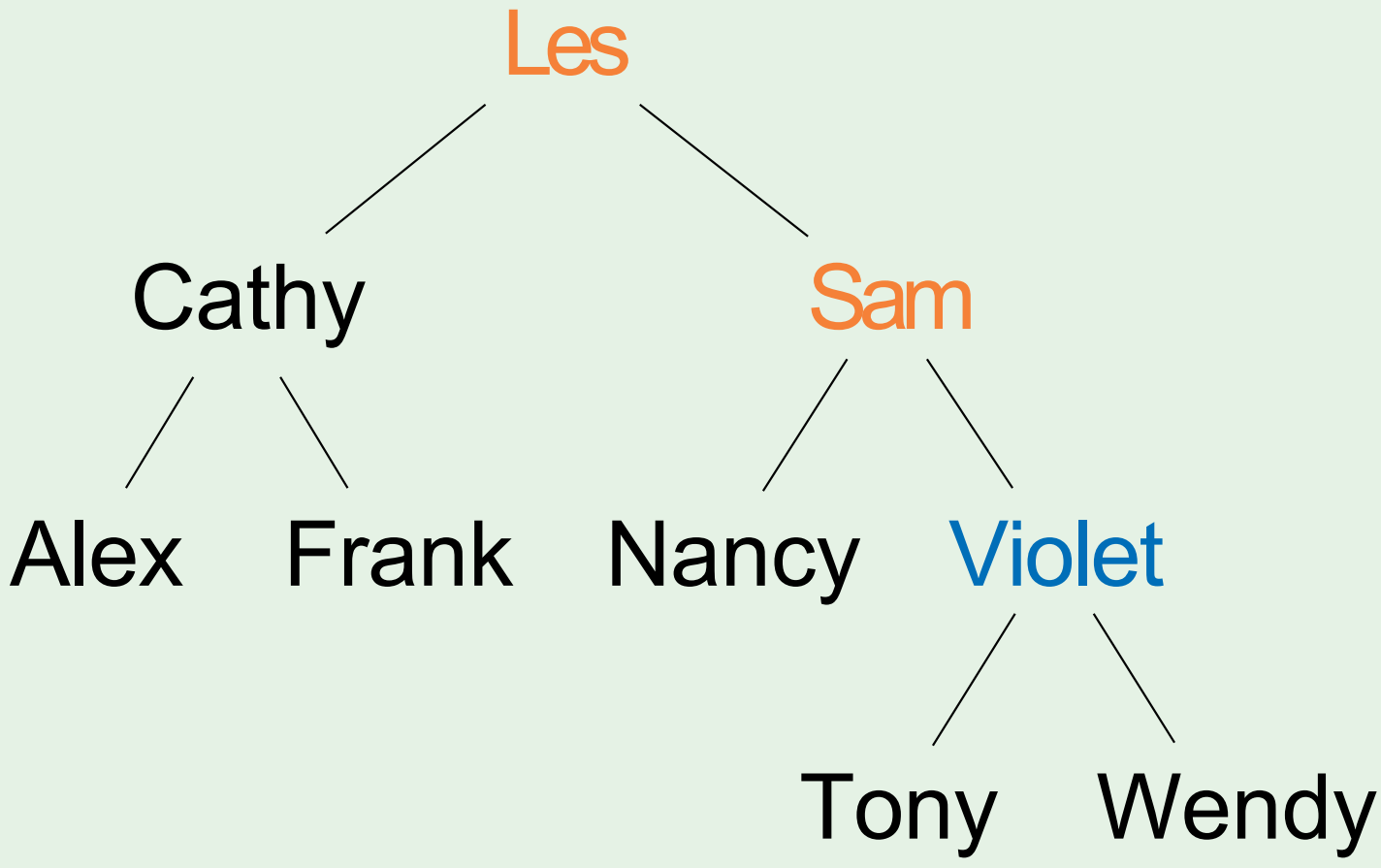
Output: Alex Cathy Frank Les Nancy Sam Tony

InOrderTraversal



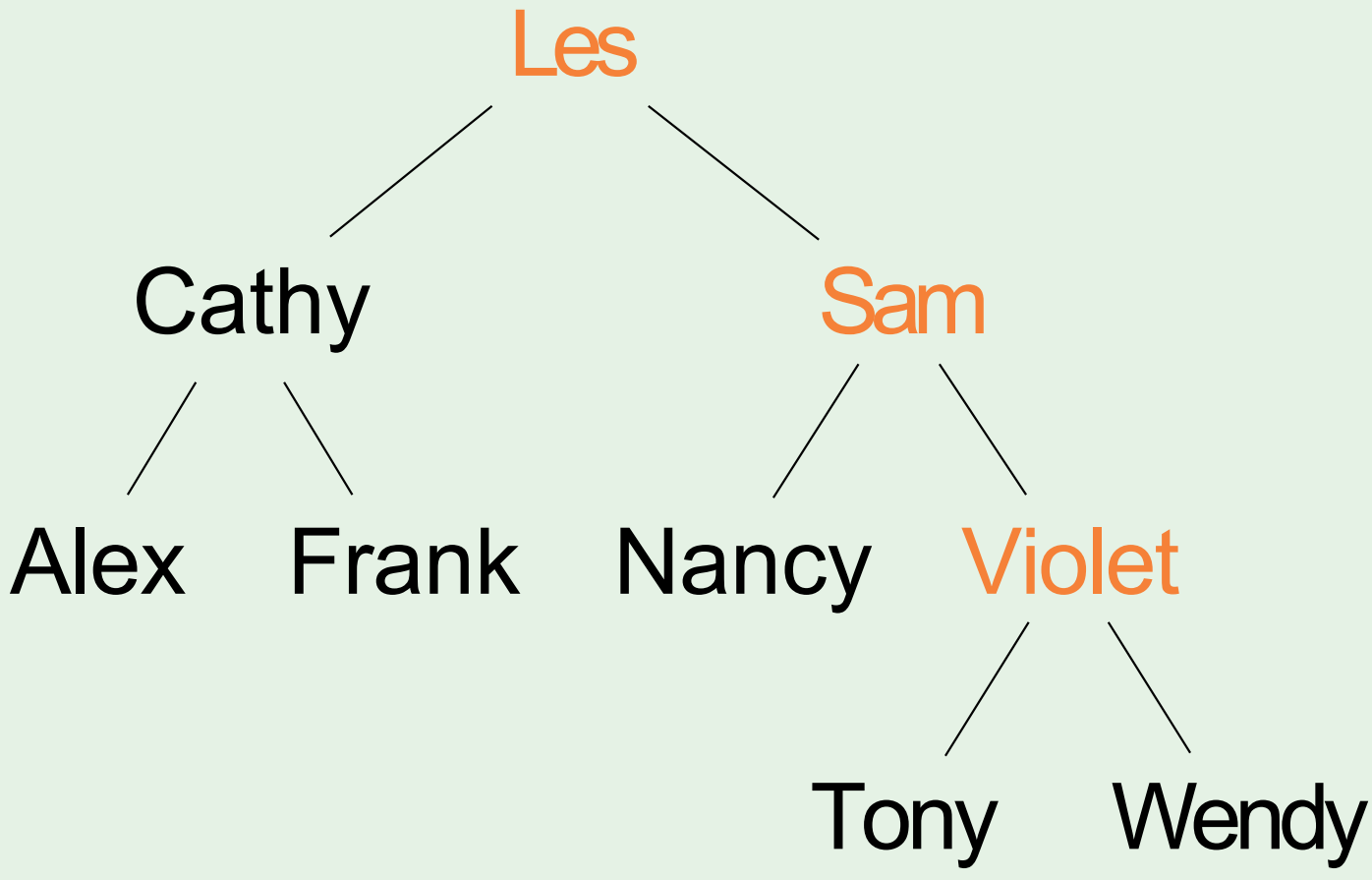
Output: Alex Cathy Frank Les Nancy Sam Tony

InOrderTraversal



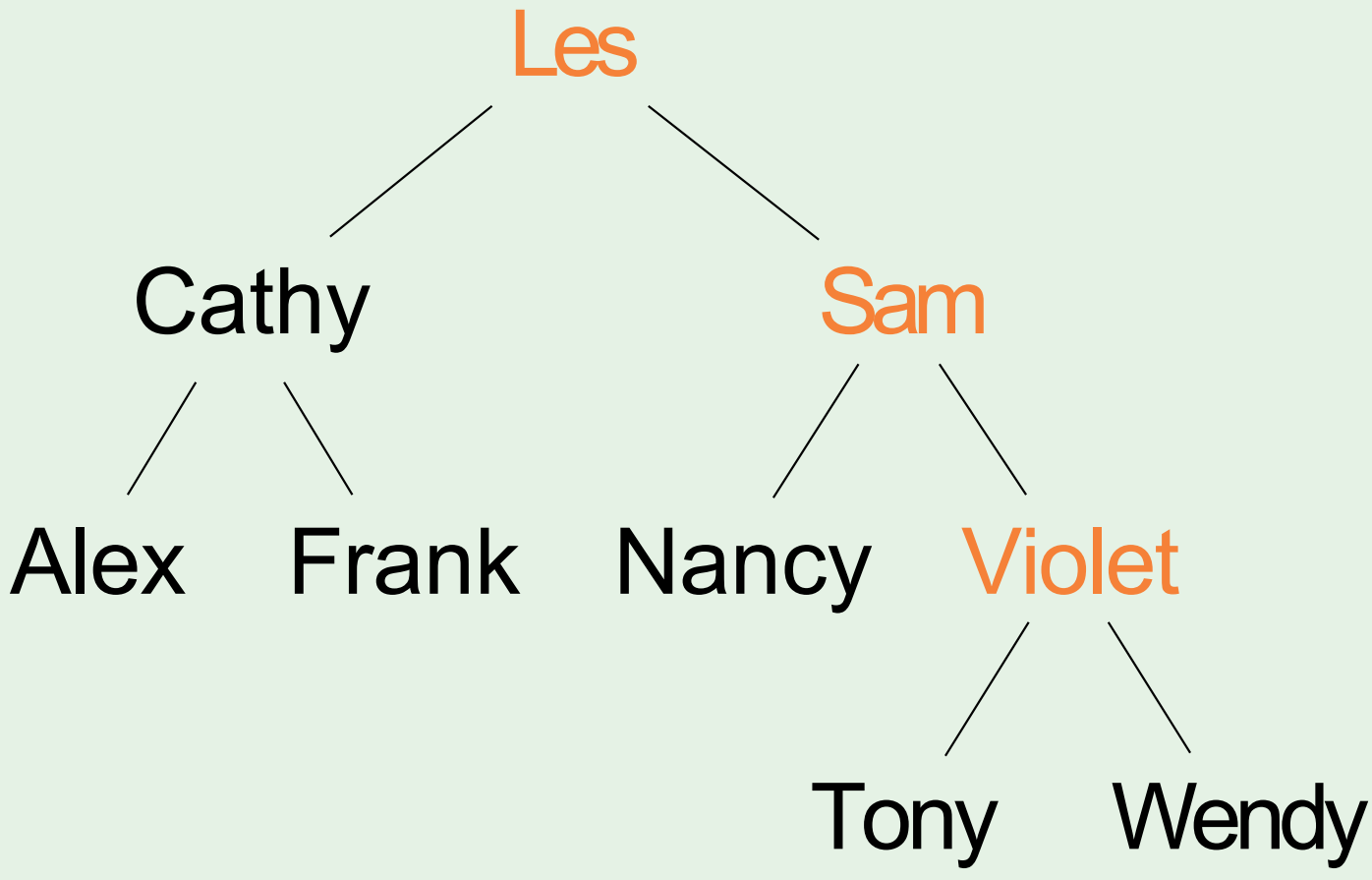
Output: Alex Cathy Frank Les Nancy Sam Tony Violet

InOrderTraversal



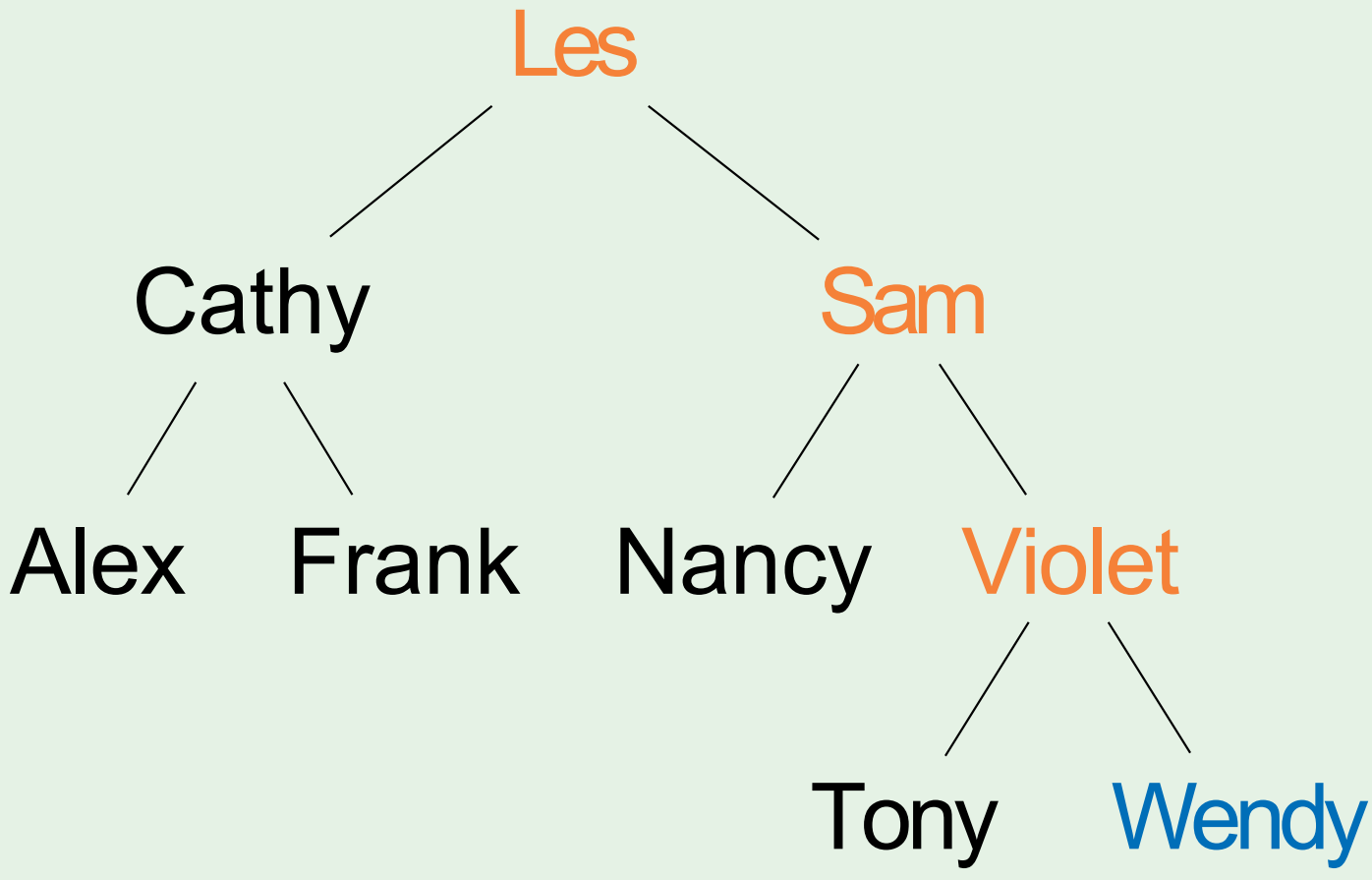
Output: Alex Cathy Frank Les Nancy Sam Tony Violet

InOrderTraversal



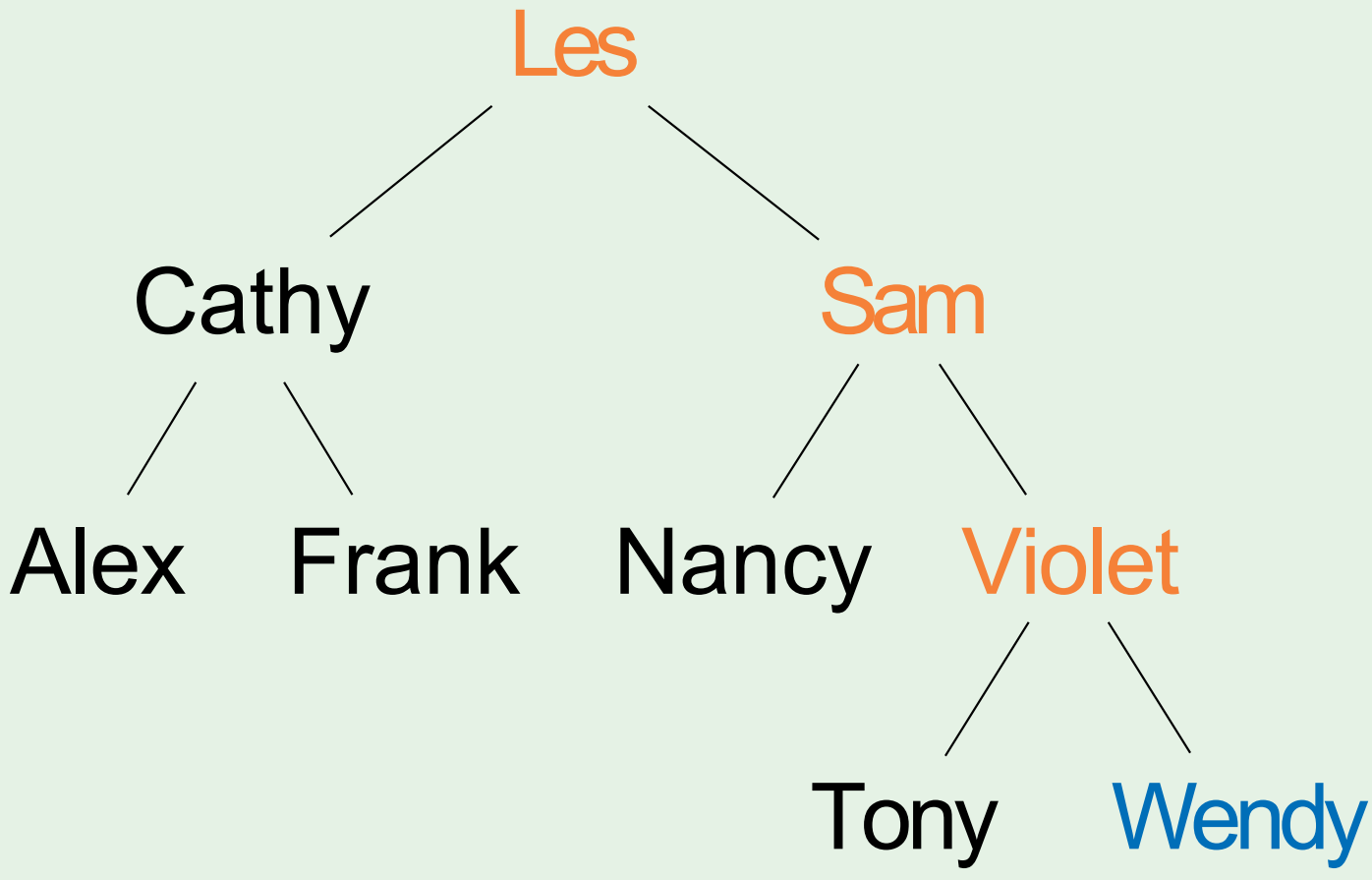
Output: Alex Cathy Frank Les Nancy Sam Tony Violet

InOrderTraversal



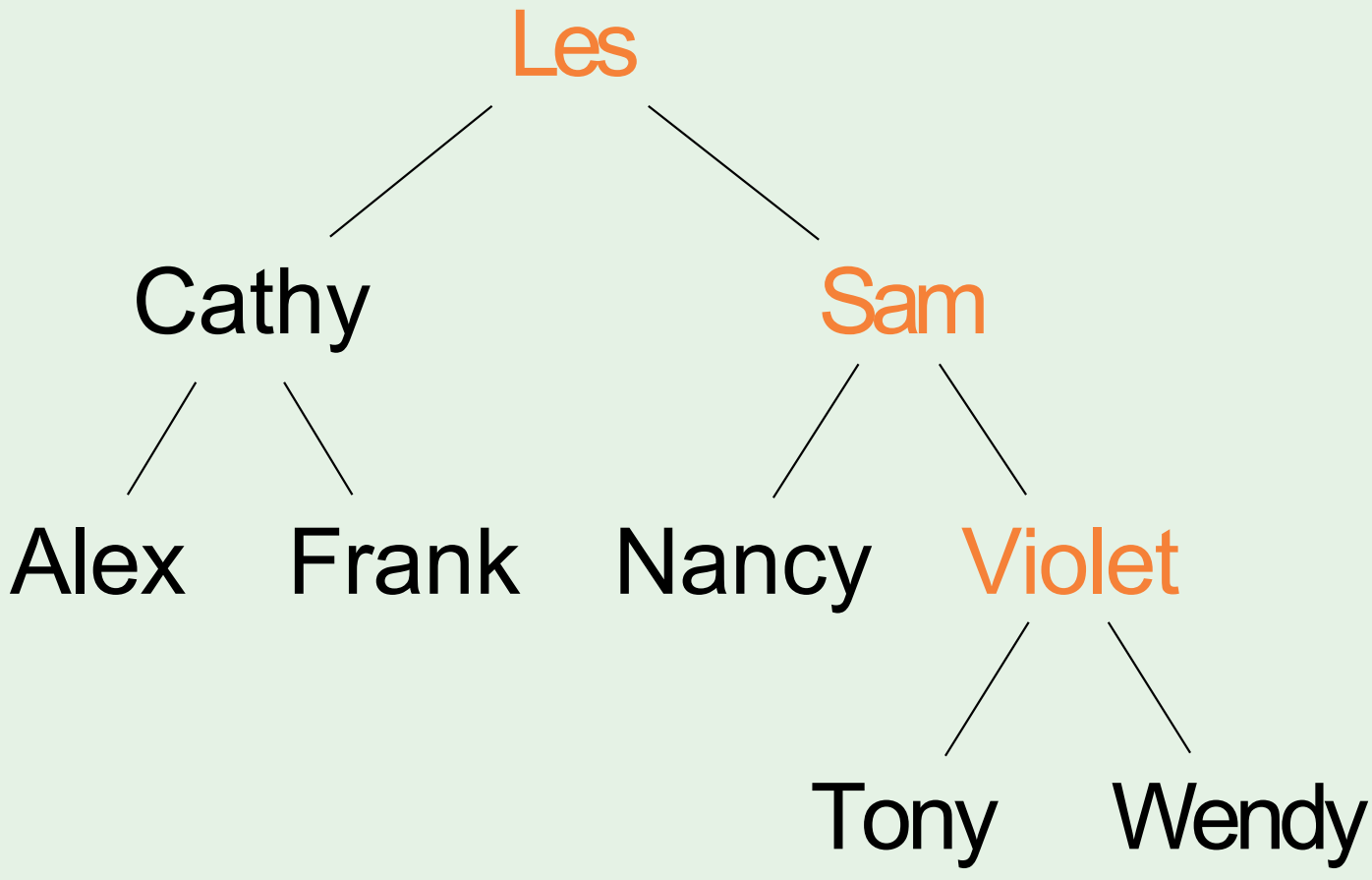
Output: Alex Cathy Frank Les Nancy Sam Tony Violet Wendy

InOrderTraversal



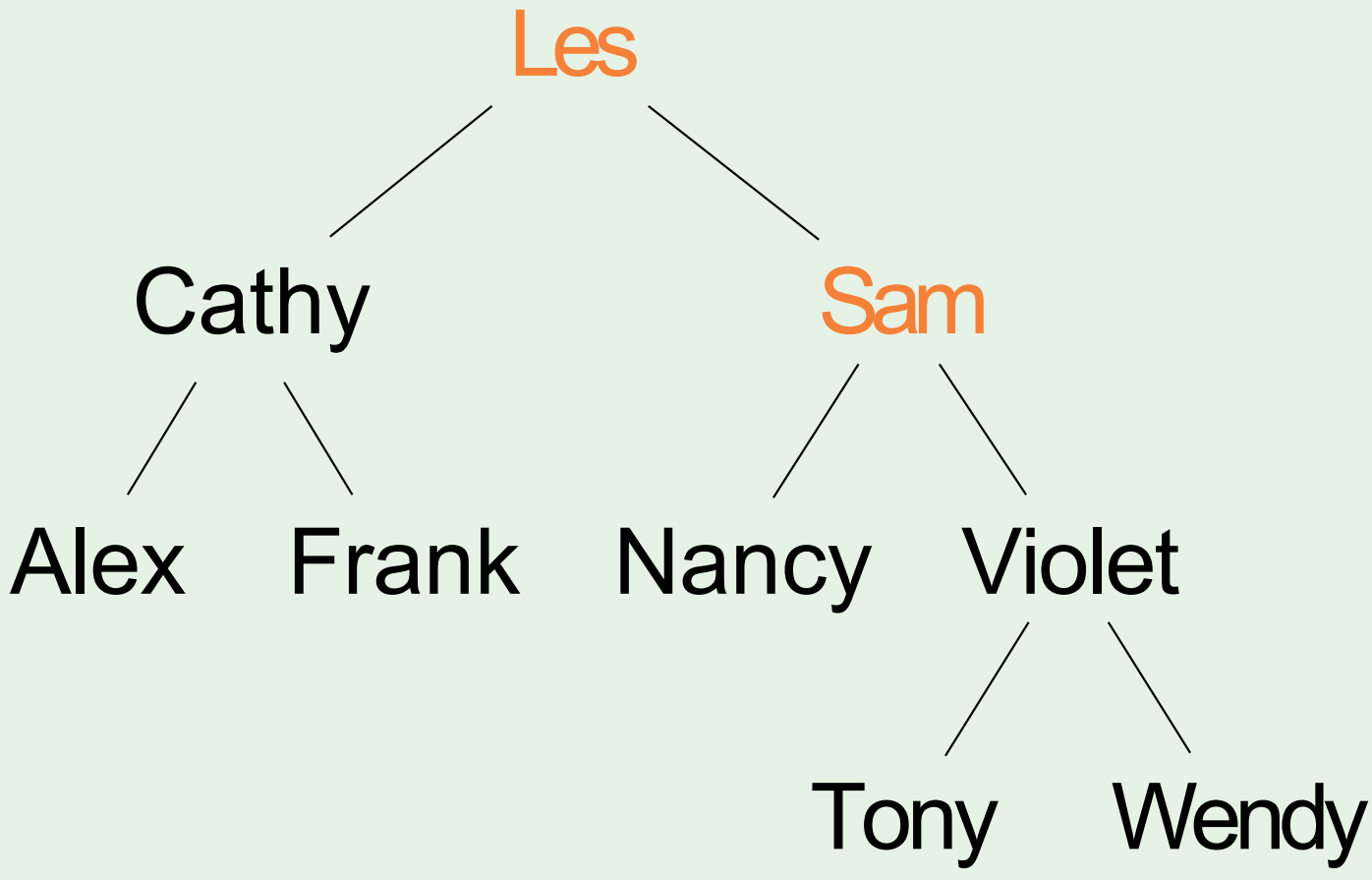
Output: Alex Cathy Frank Les Nancy Sam Tony Violet Wendy

InOrderTraversal



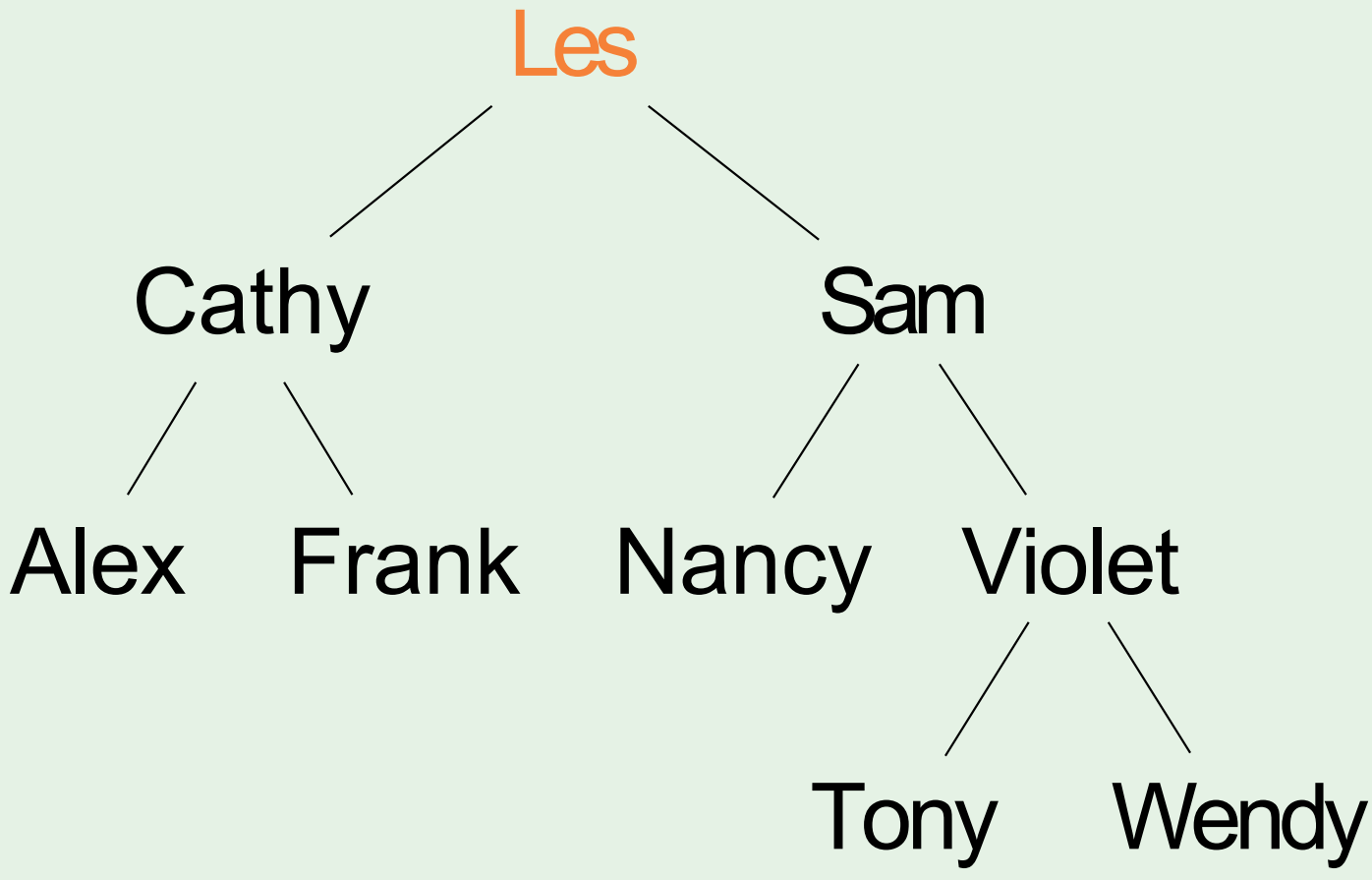
Output: Alex Cathy Frank Les Nancy Sam Tony Violet Wendy

InOrderTraversal



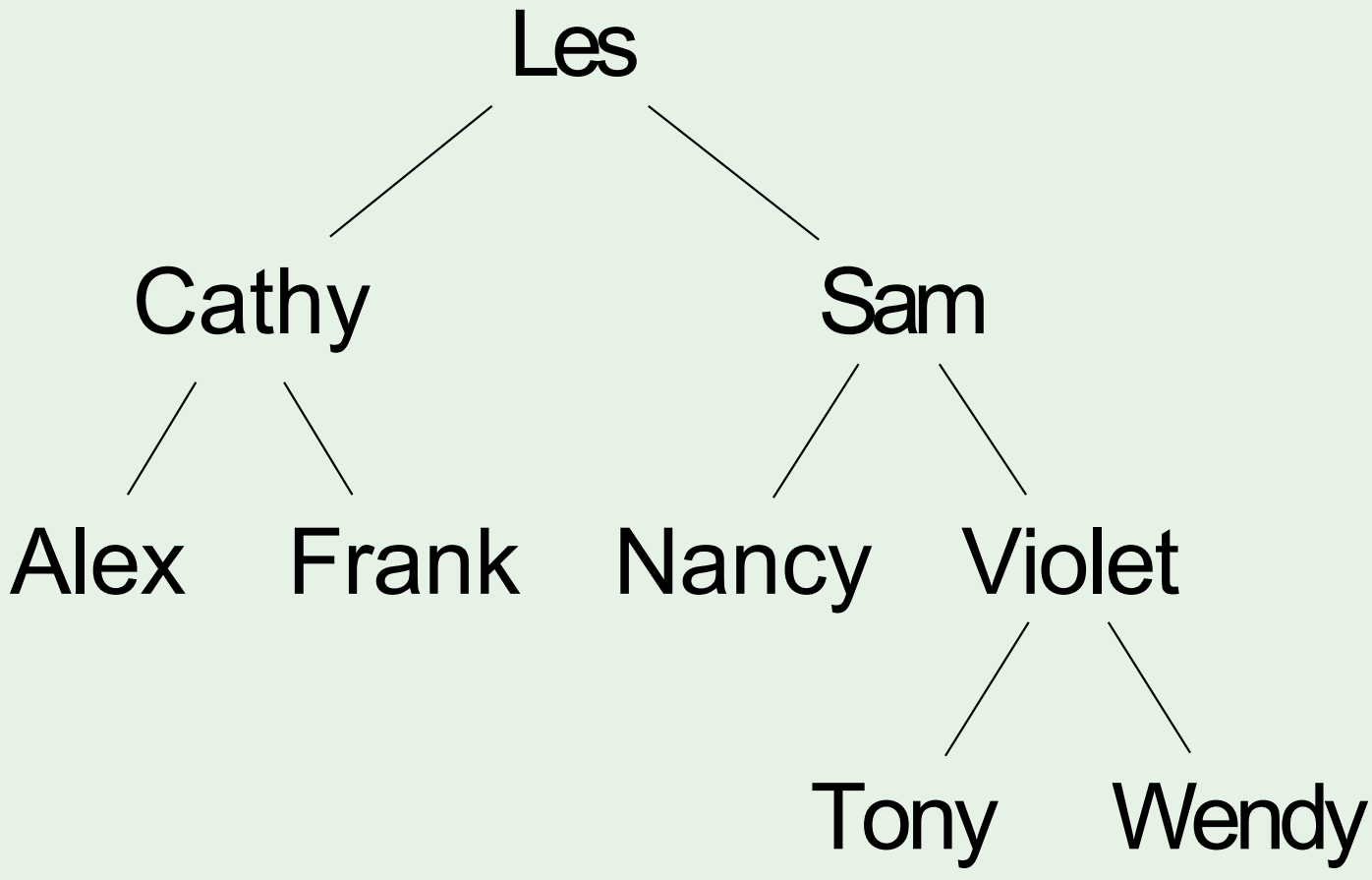
Output: Alex Cathy Frank Les Nancy Sam Tony Violet Wendy

InOrderTraversal



Output: Alex Cathy Frank Les Nancy Sam Tony Violet Wendy

InOrderTraversal

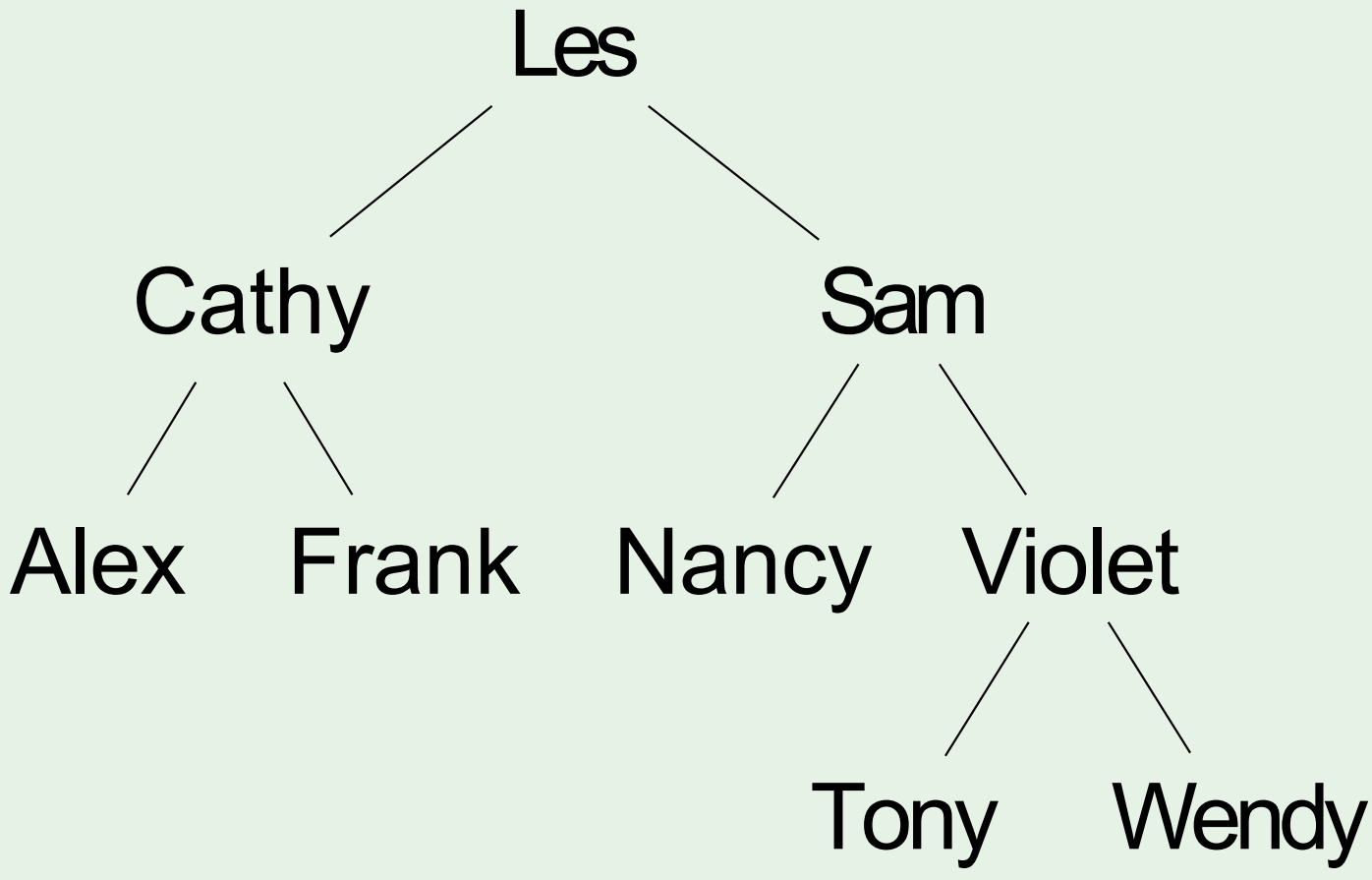


Output: Alex Cathy Frank Les Nancy Sam Tony Violet Wendy

Depth-first

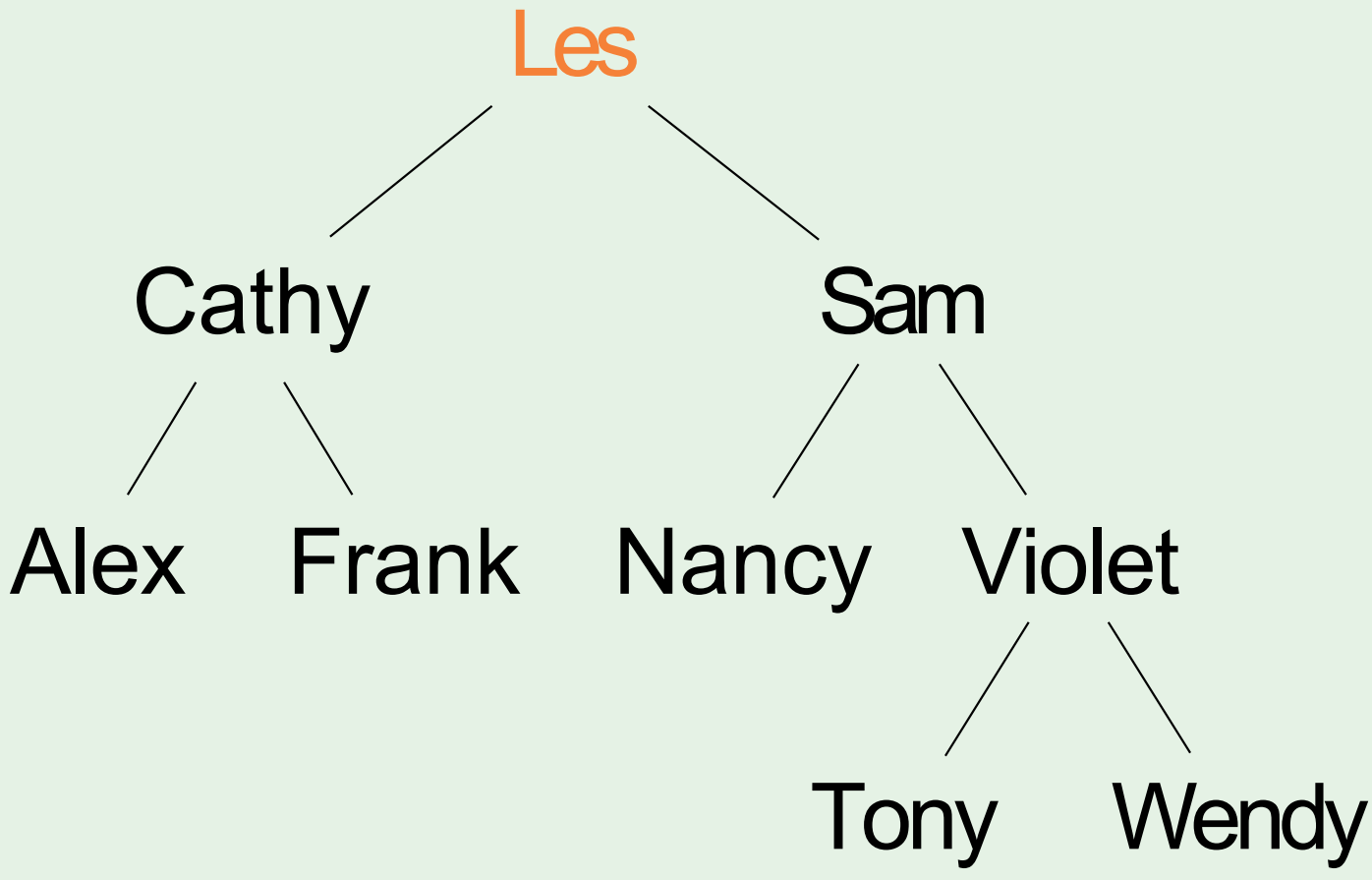
```
PreOrderTraversal(tree)
if tree = nil :
   return
Print(tree.key )
PreOrderTraversal(tree.left)
PreOrderTraversal(tree.right)
```

PreOrderTraversal



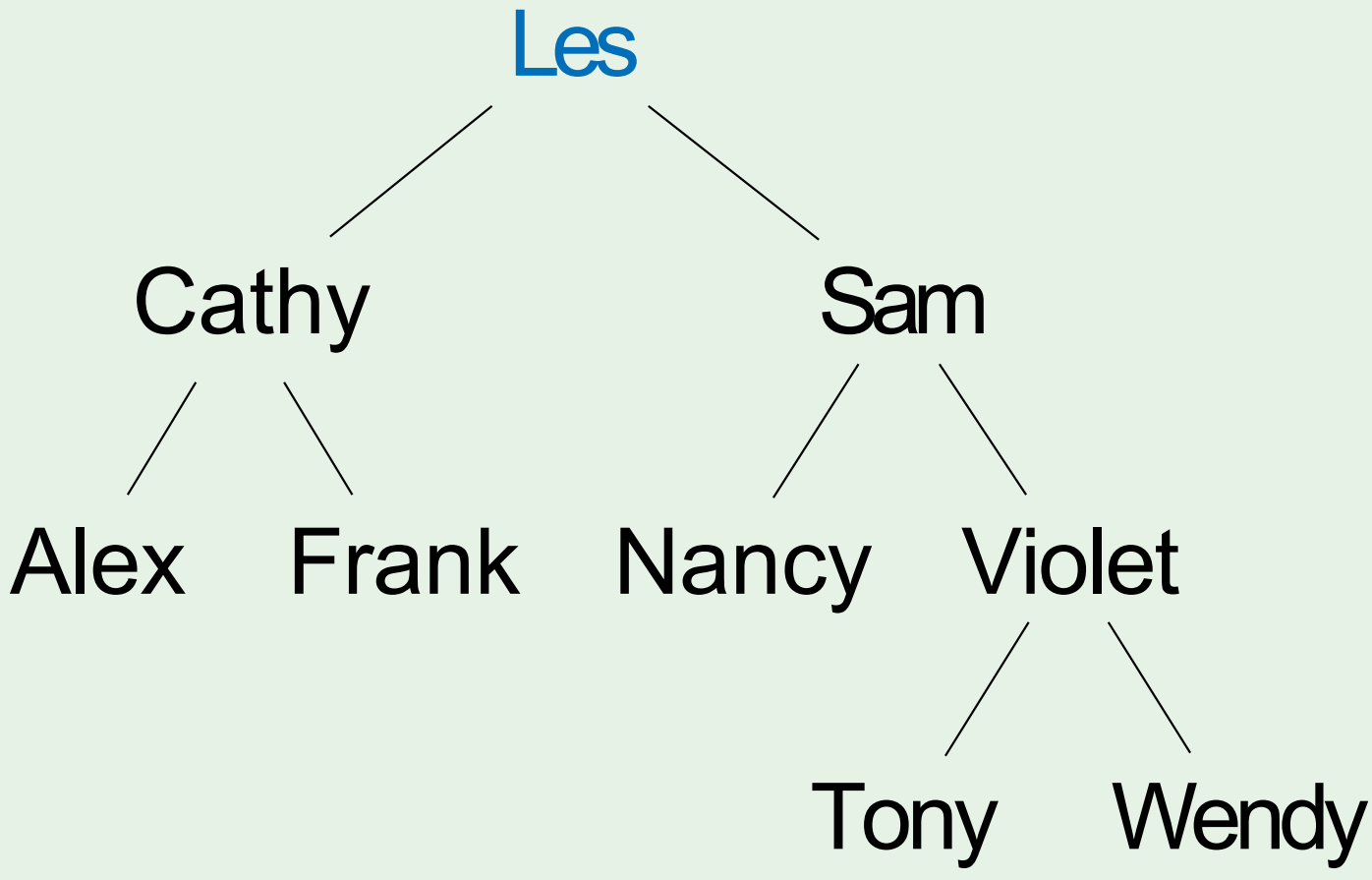
Output:

PreOrderTraversal



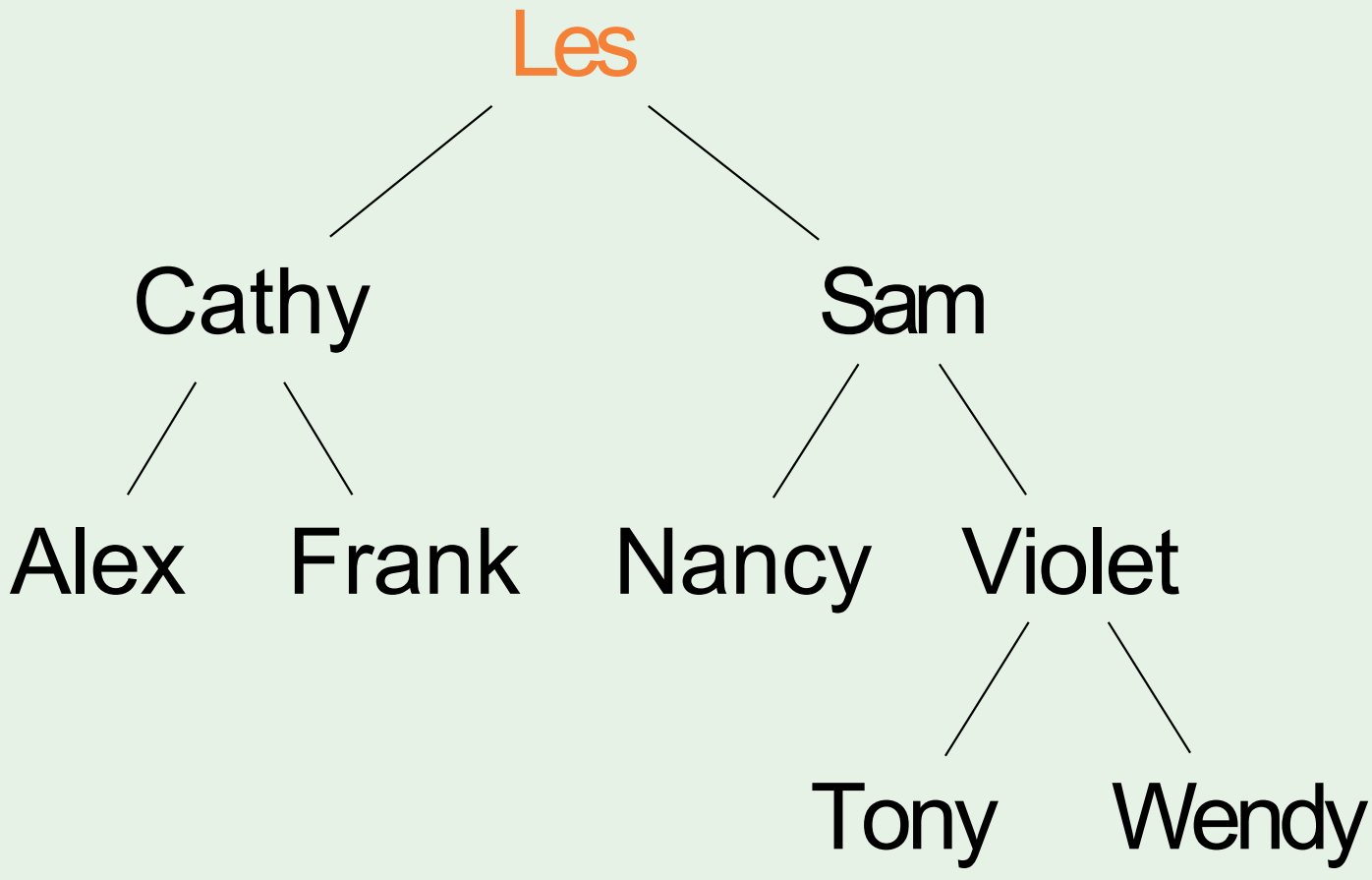
Output:

PreOrderTraversal



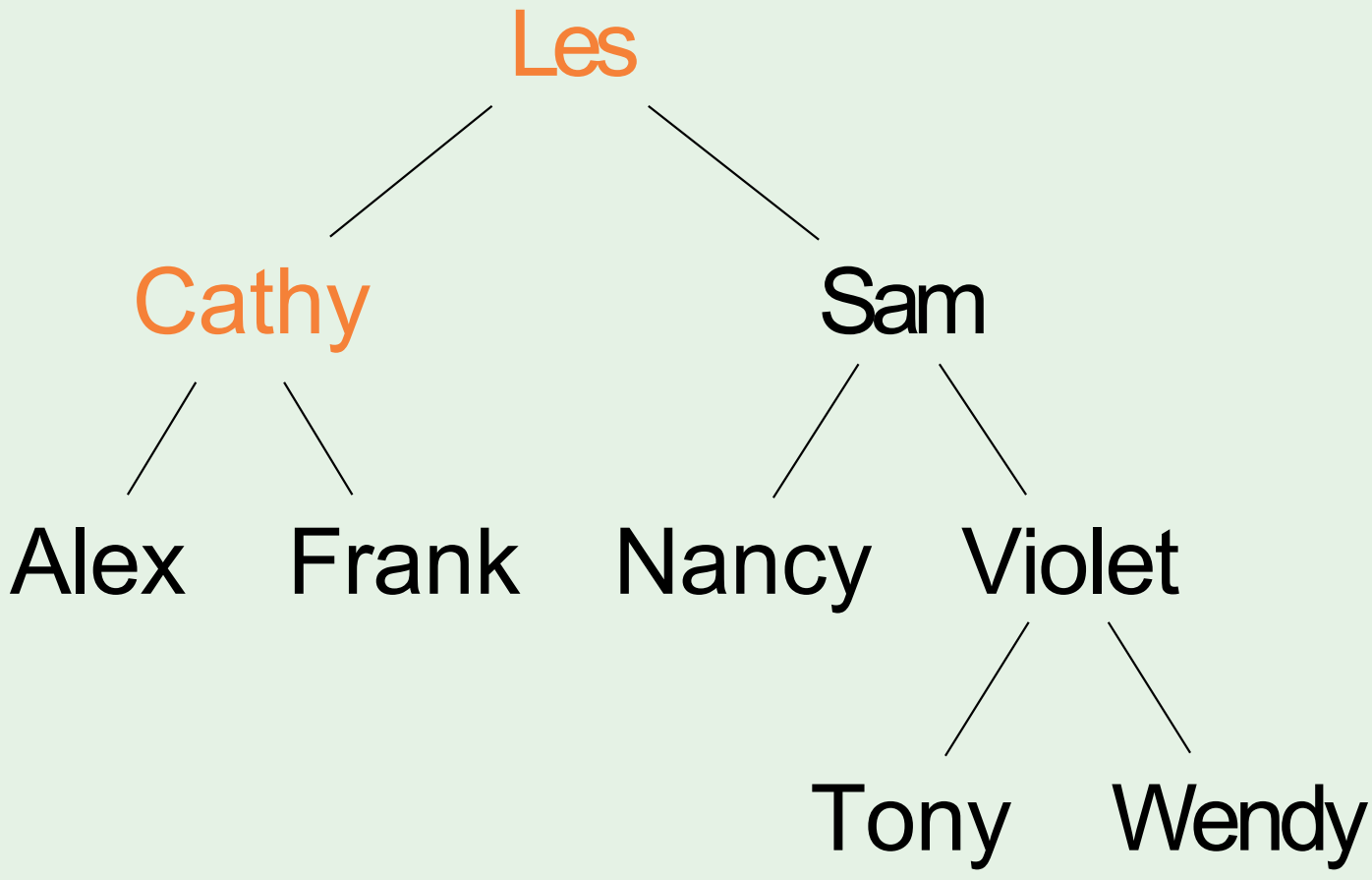
Output: Les

PreOrderTraversal



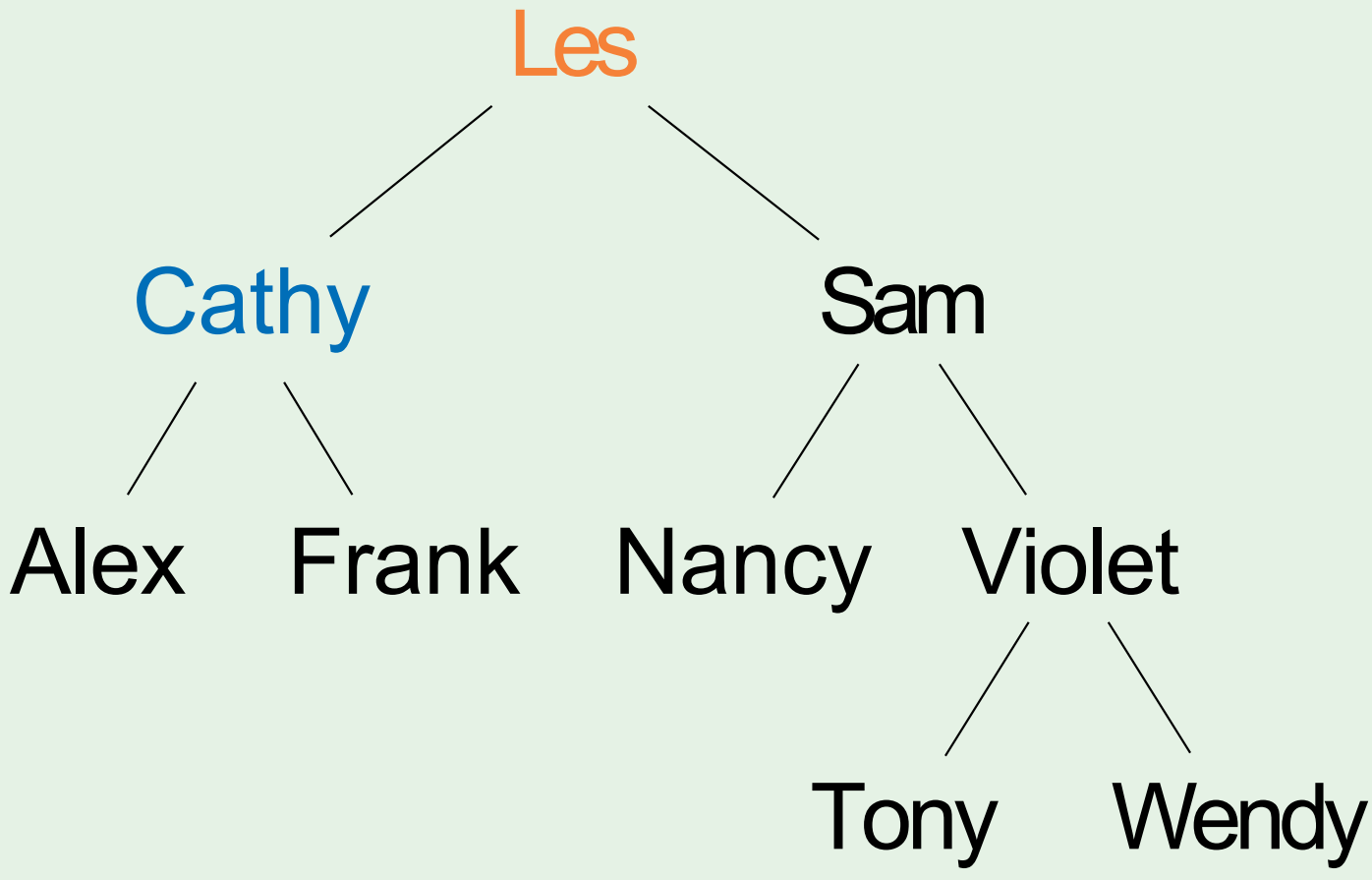
Output: Les

PreOrderTraversal



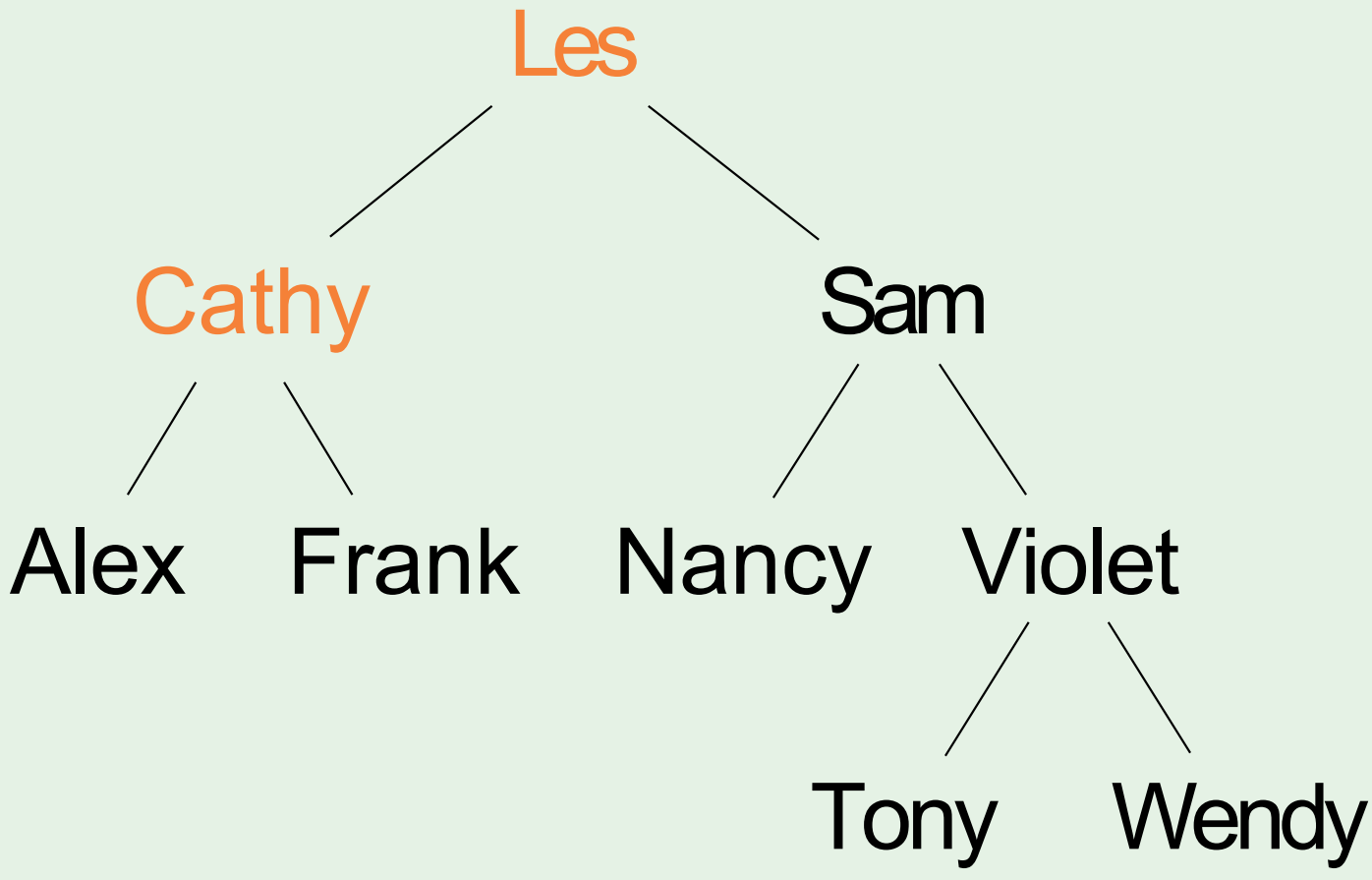
Output: Les

PreOrderTraversal



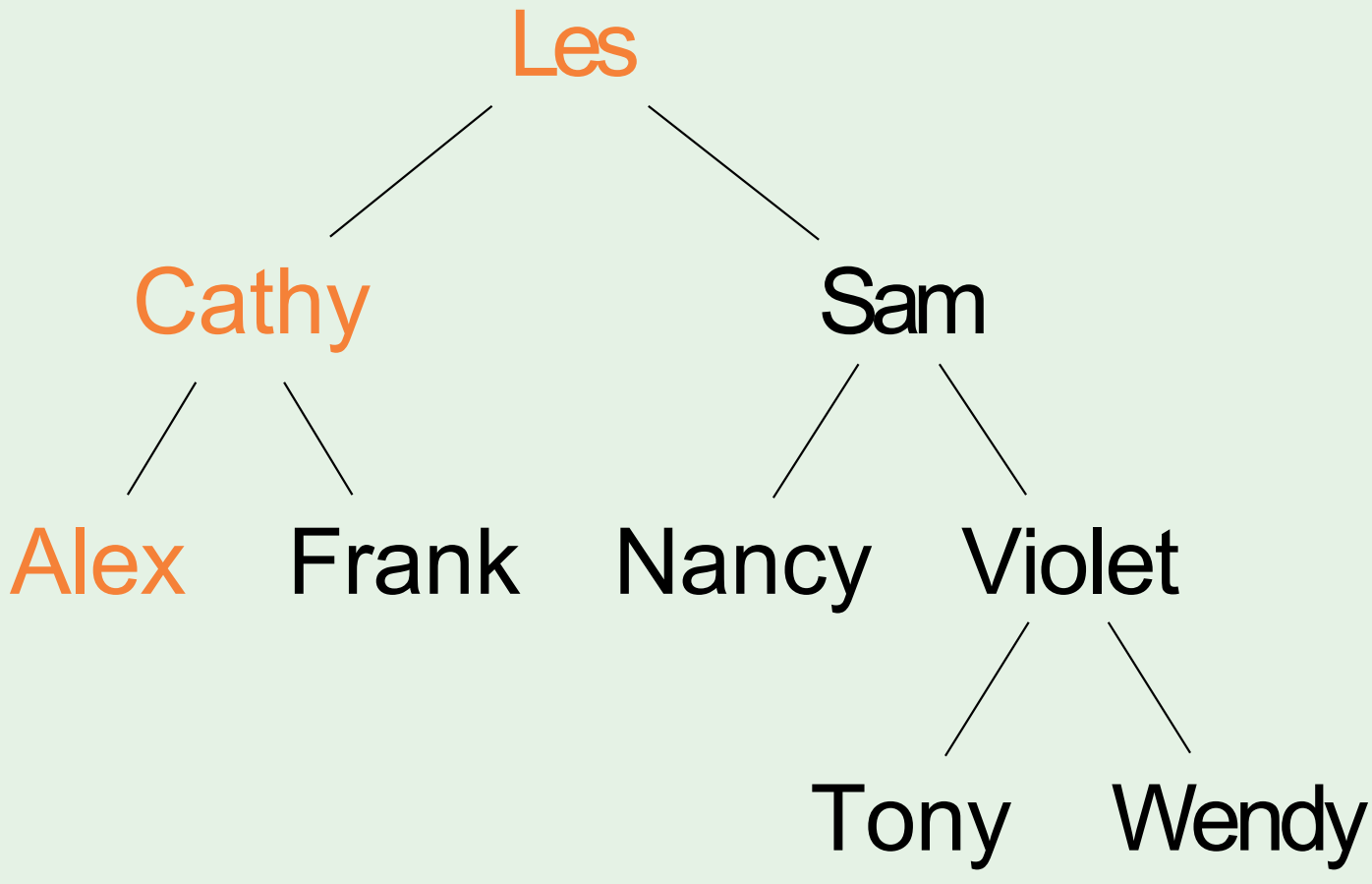
Output: Les Cathy

PreOrderTraversal



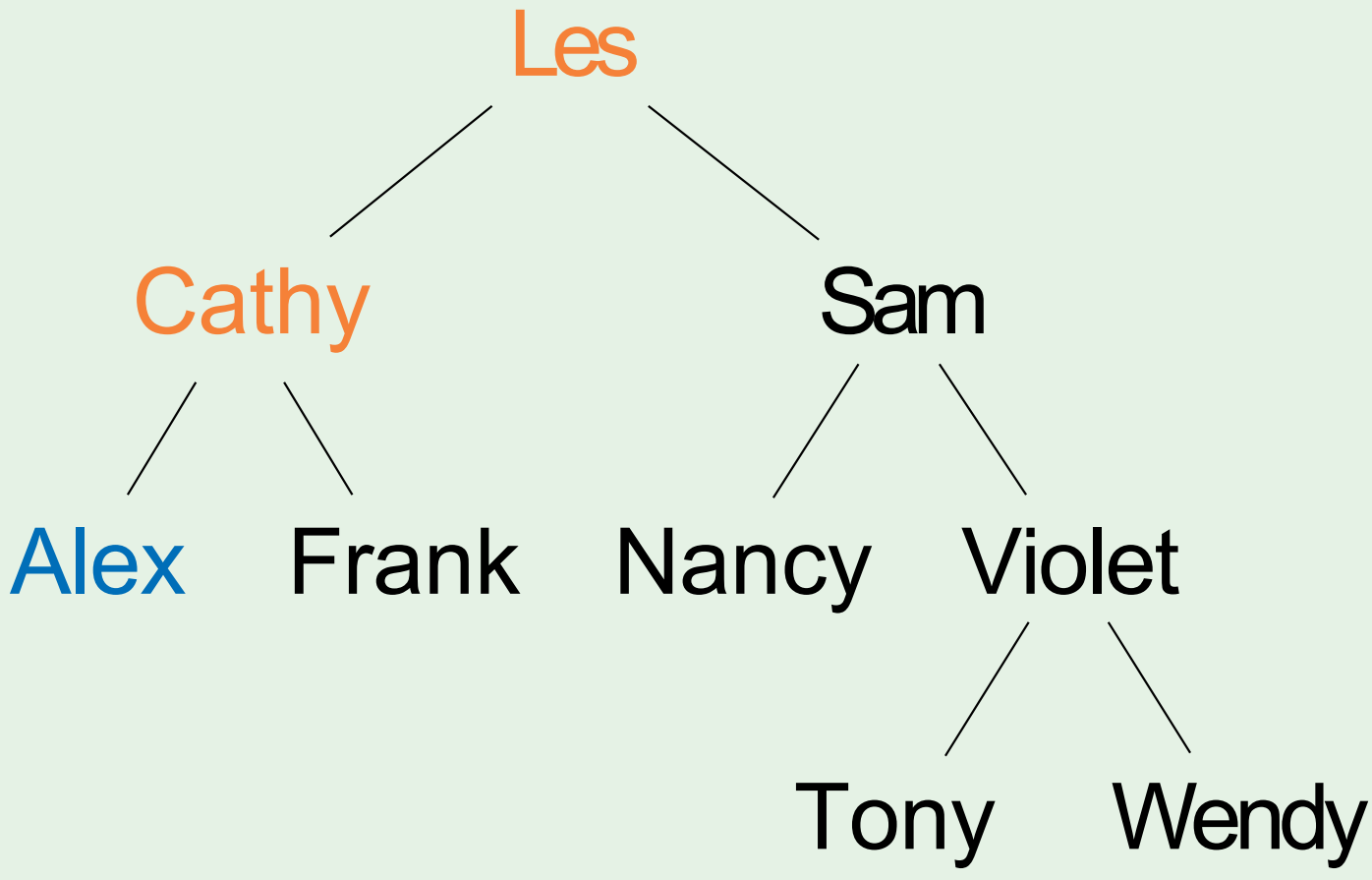
Output: Les Cathy

PreOrderTraversal



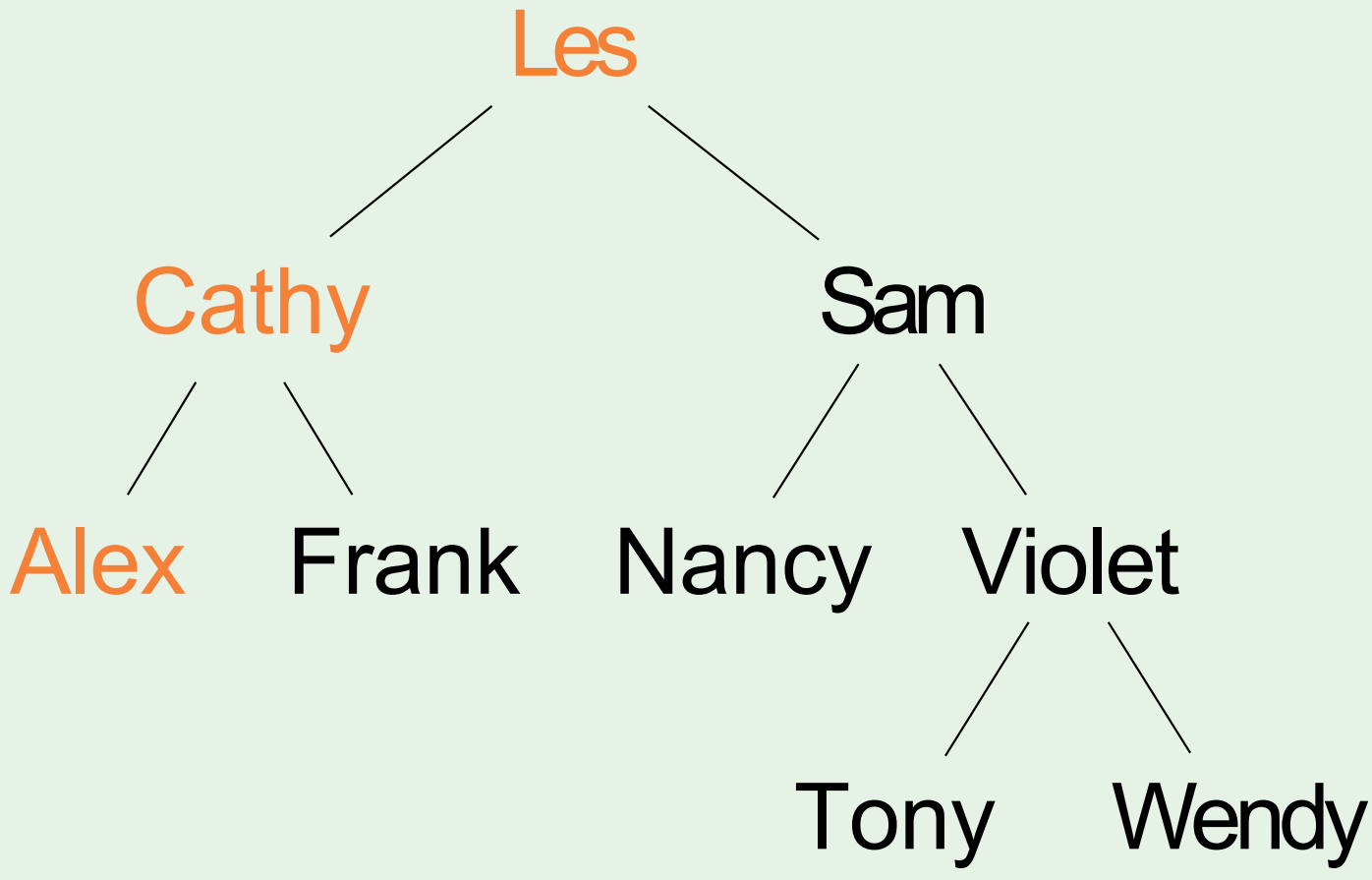
Output: Les Cathy

PreOrderTraversal



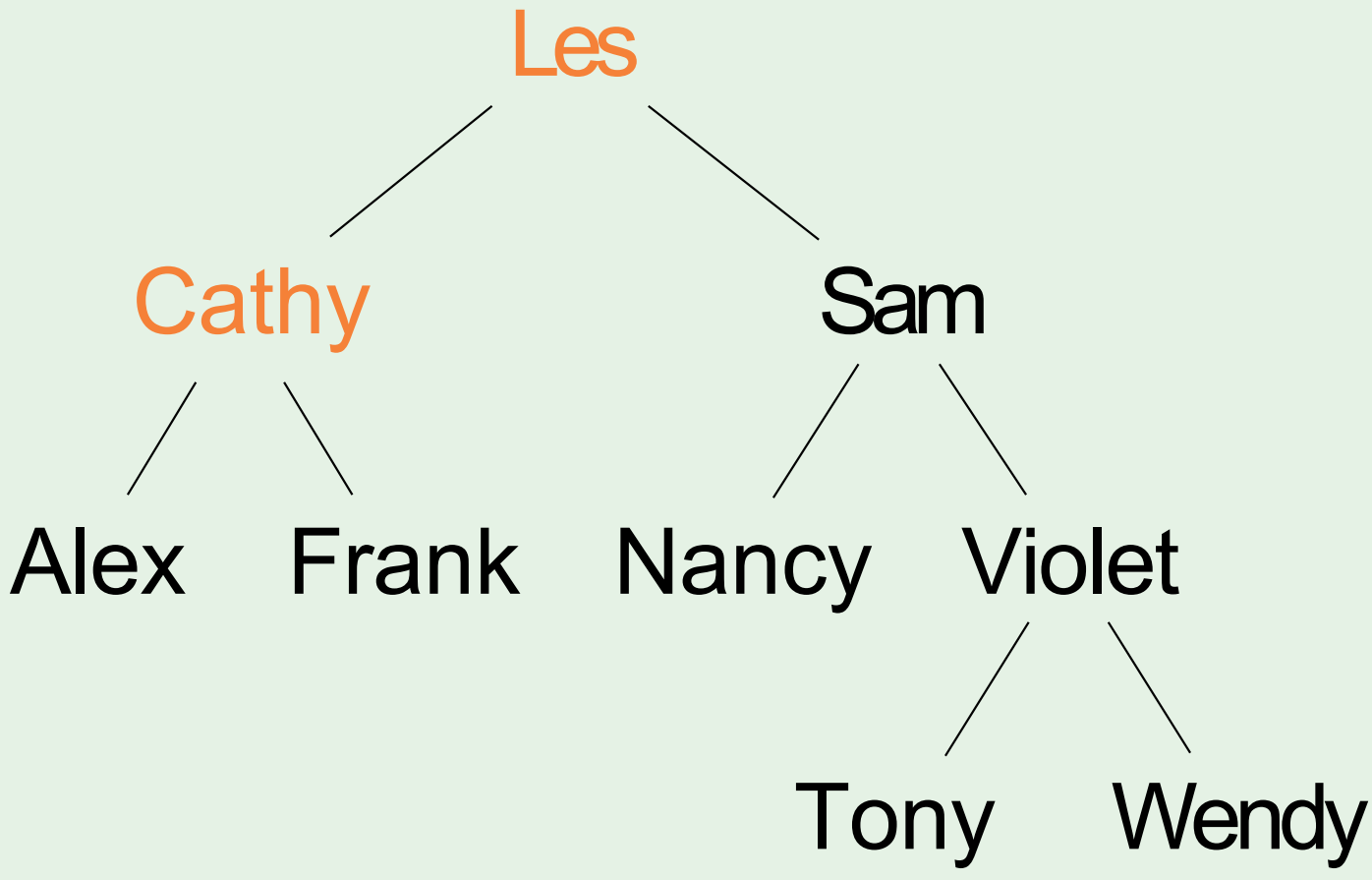
Output: Les Cathy Alex

PreOrderTraversal



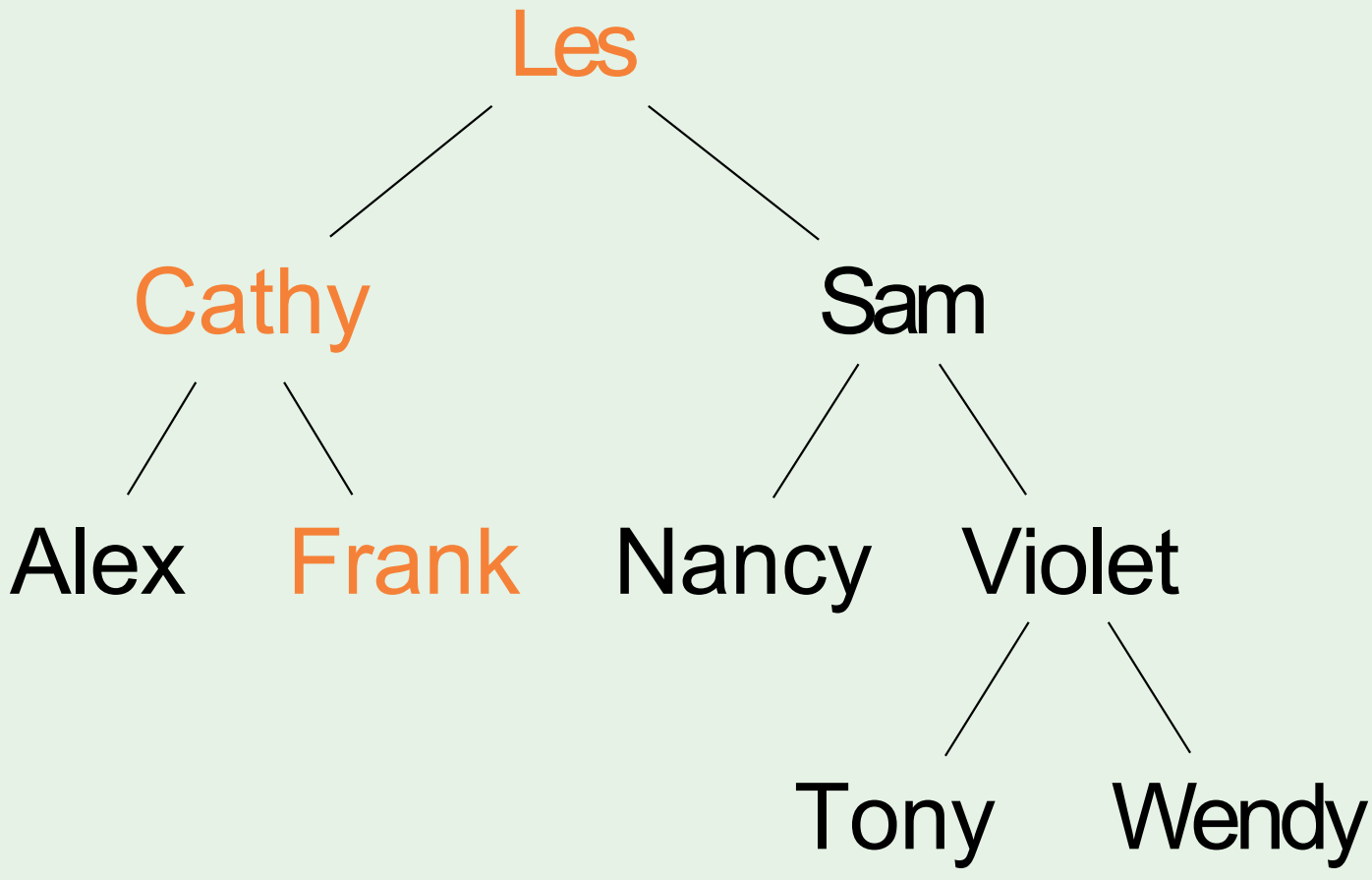
Output: Les Cathy Alex

PreOrderTraversal



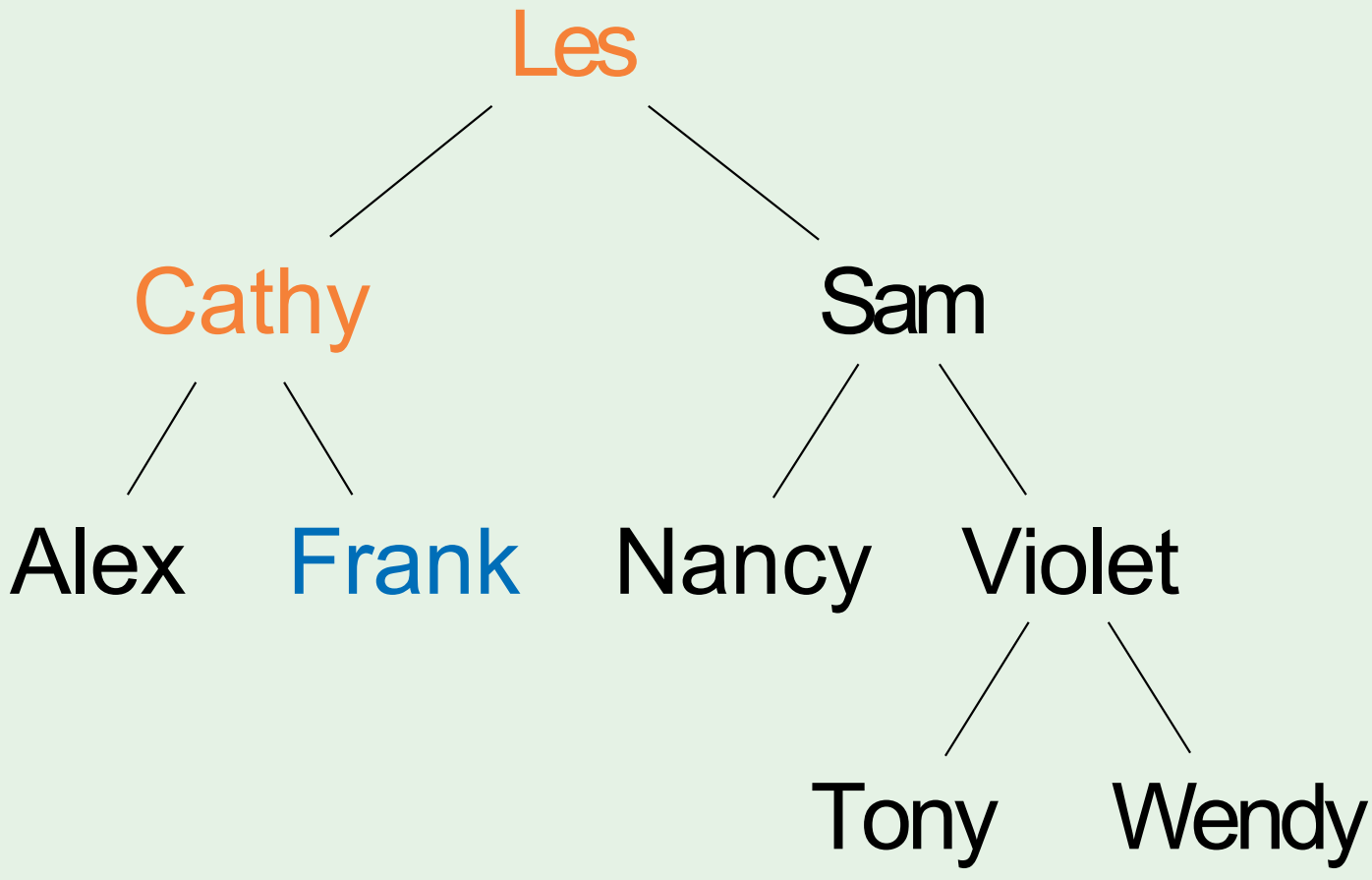
Output: Les Cathy Alex

PreOrderTraversal



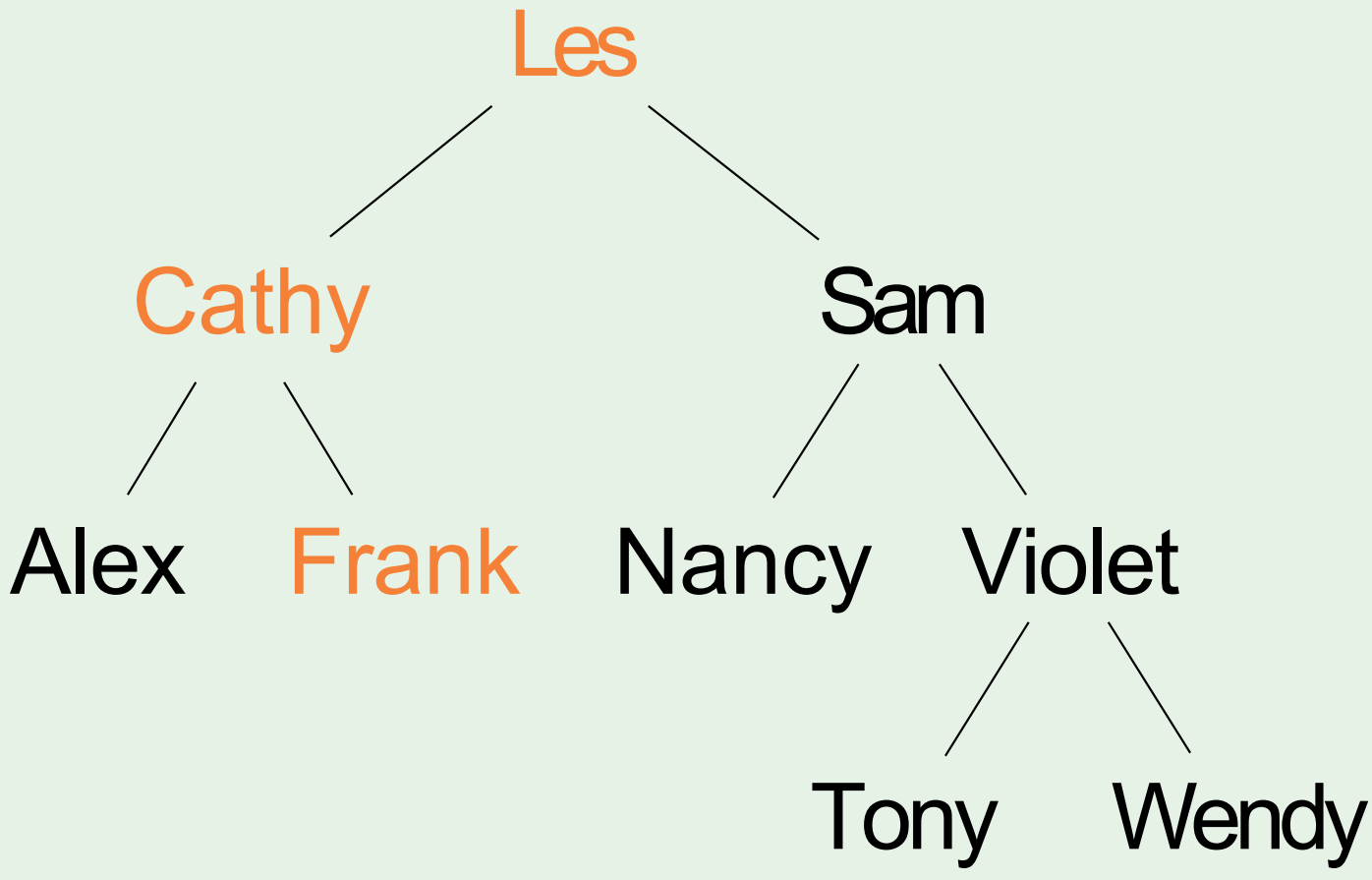
Output: Les Cathy Alex

PreOrderTraversal



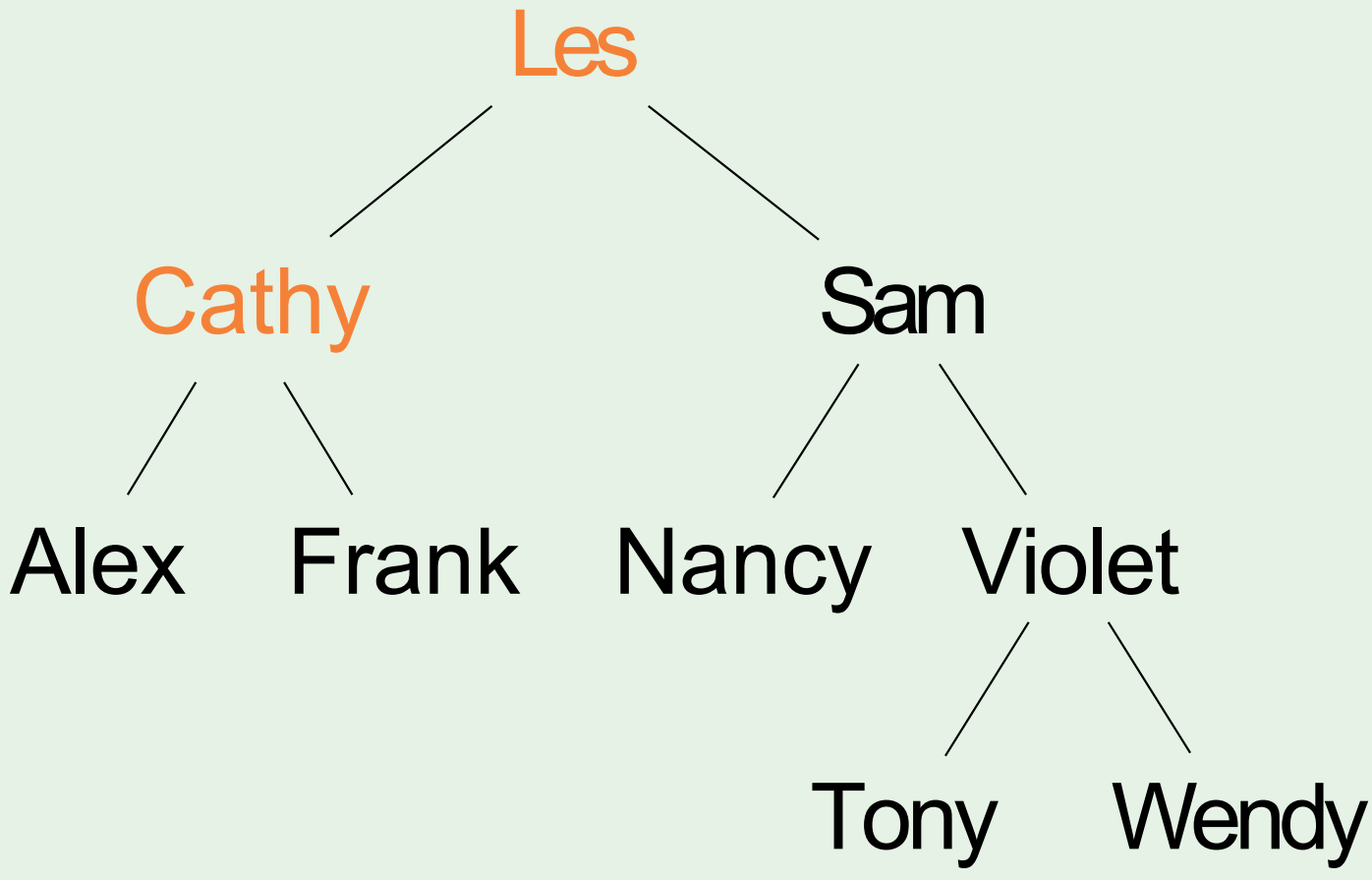
Output: Les Cathy Alex Frank

PreOrderTraversal



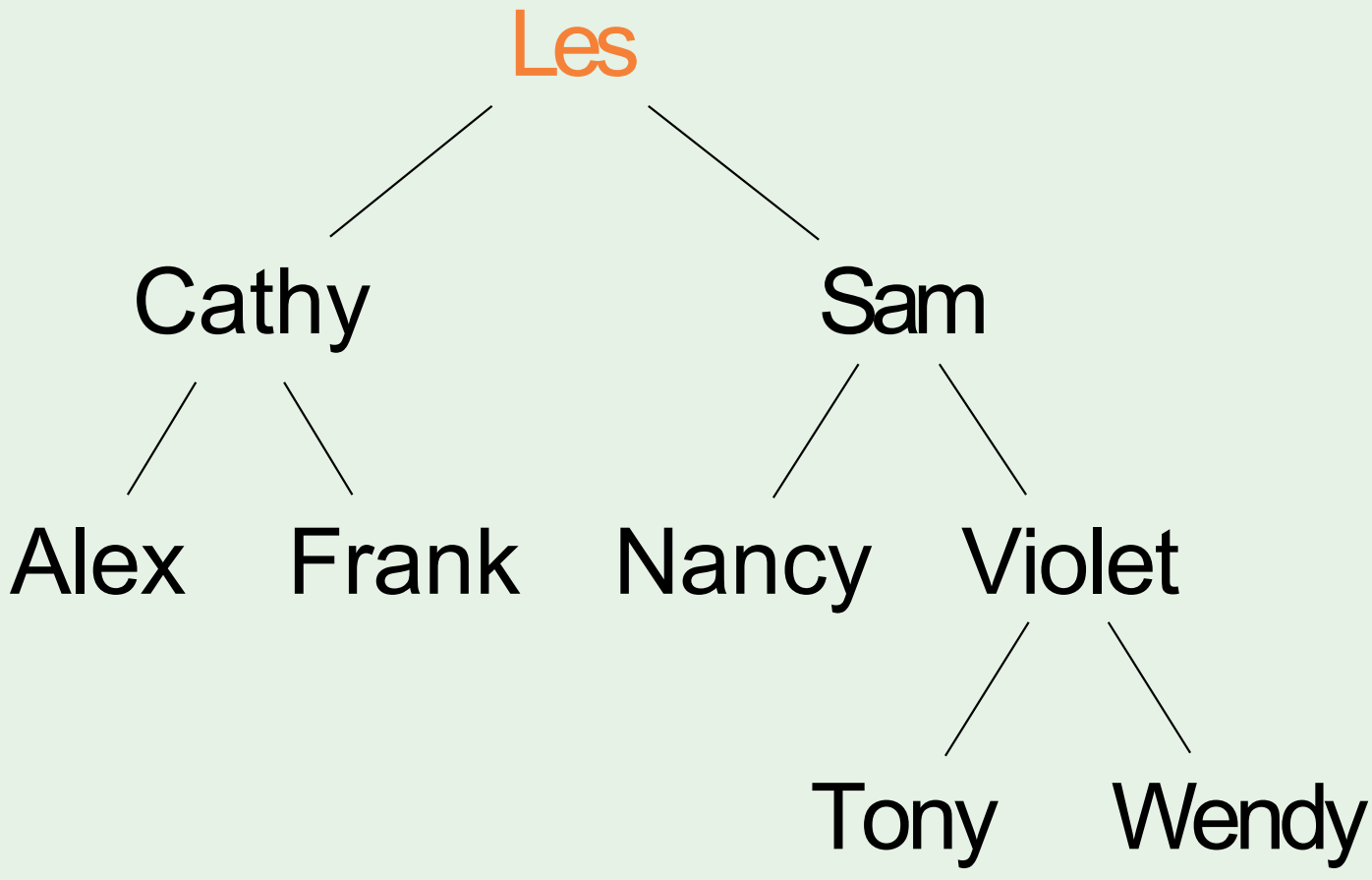
Output: Les Cathy Alex Frank

PreOrderTraversal



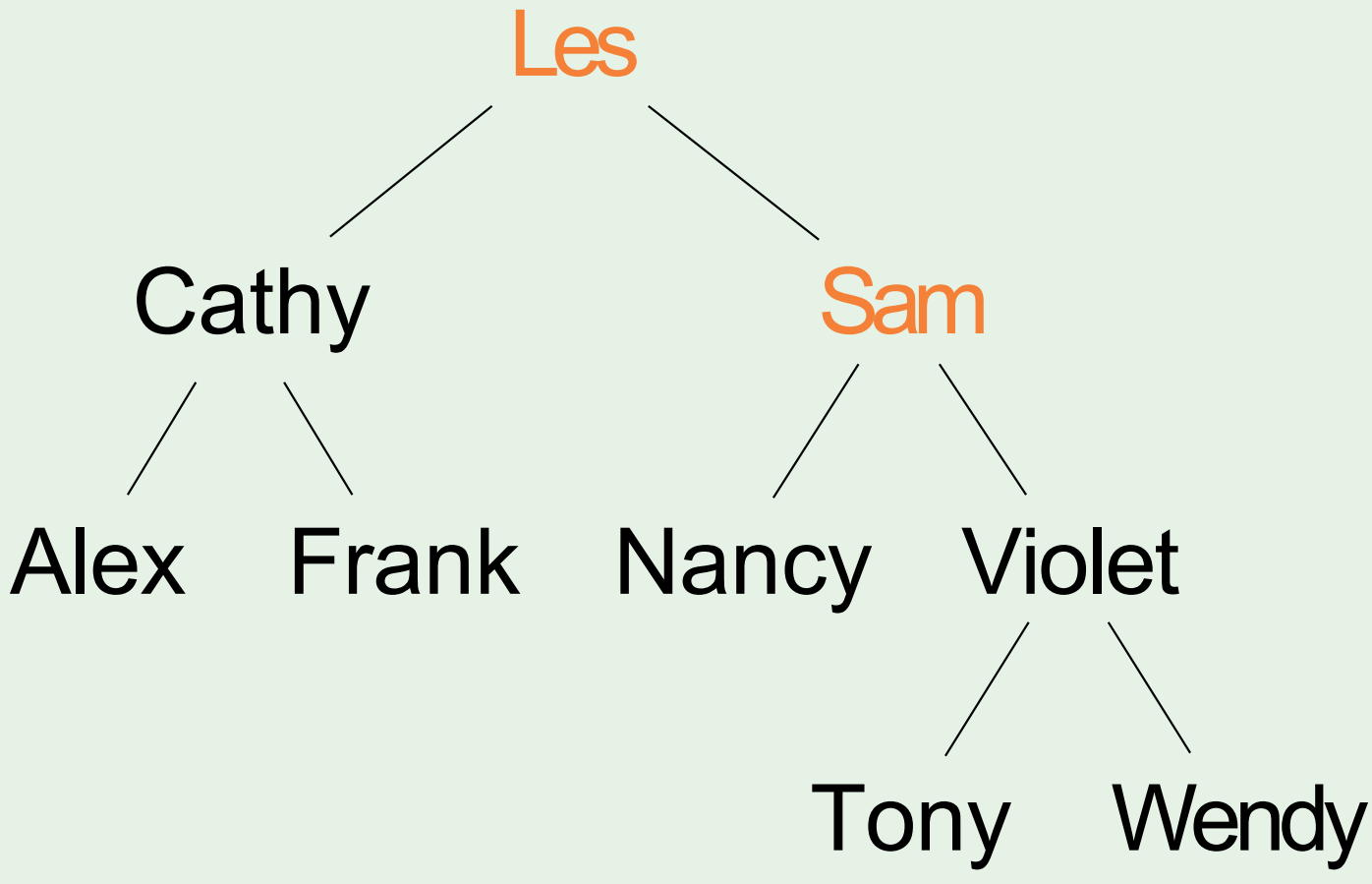
Output: Les Cathy Alex Frank

PreOrderTraversal



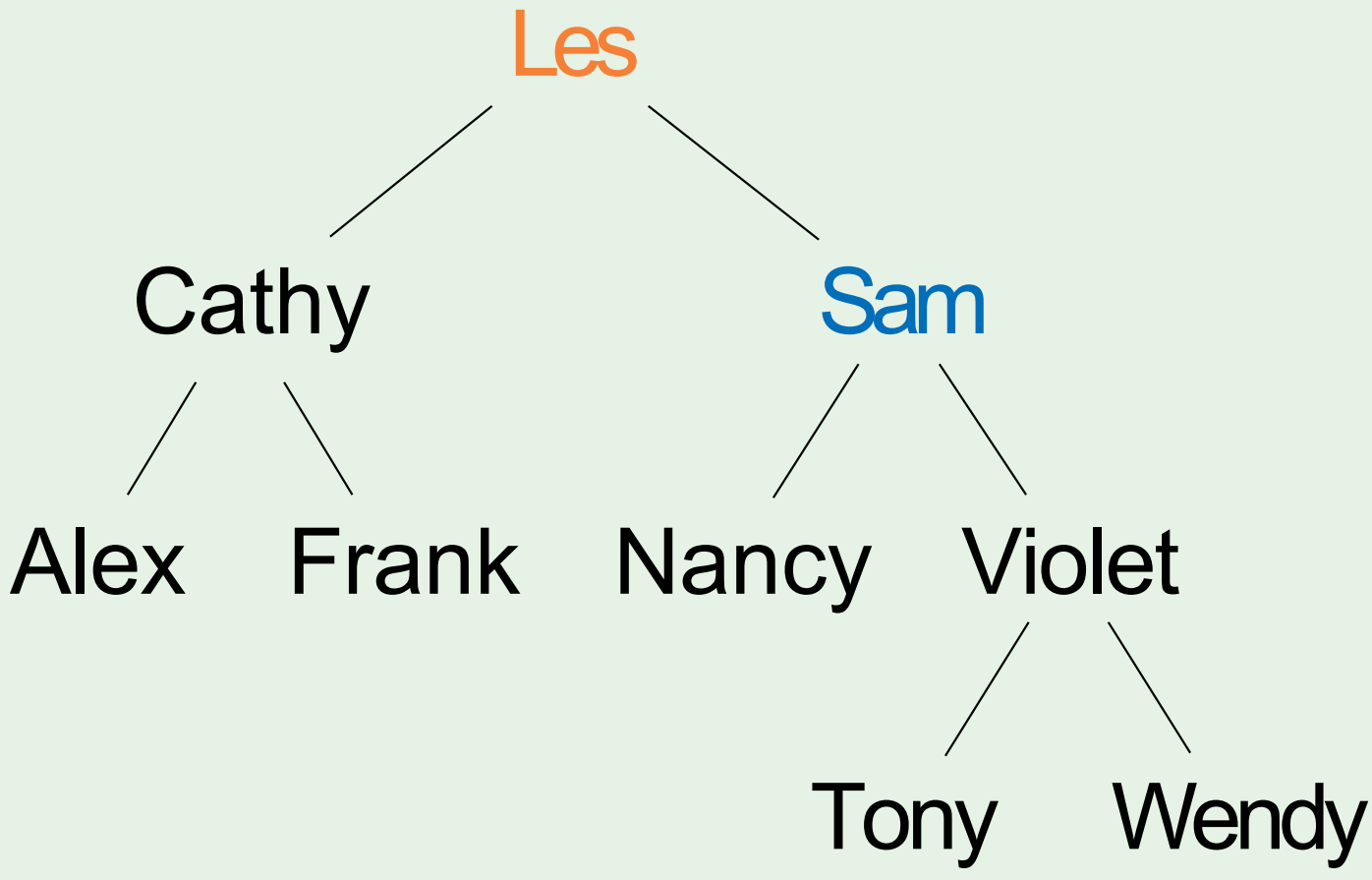
Output: Les Cathy Alex Frank

PreOrderTraversal



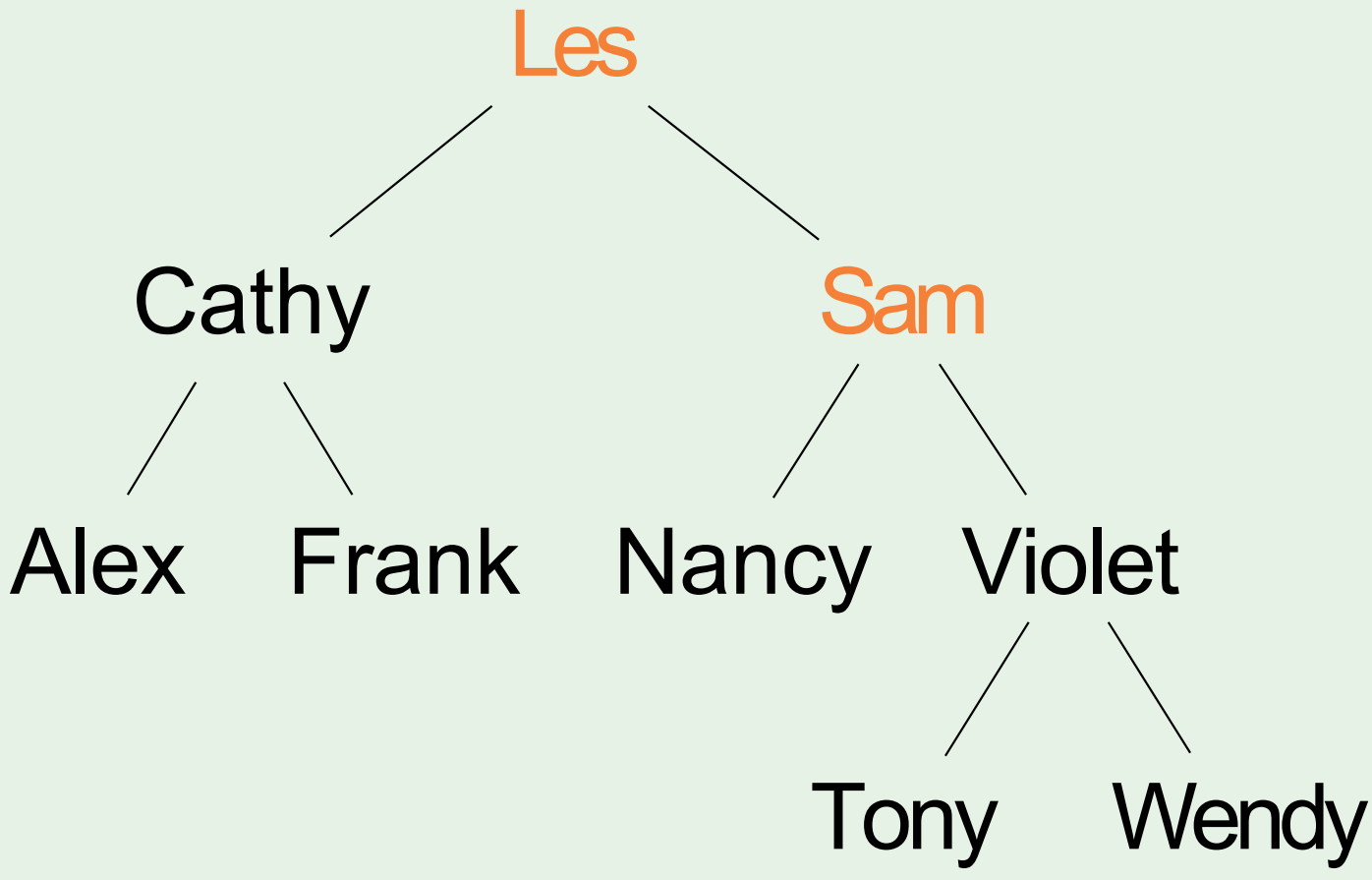
Output: Les Cathy Alex Frank

PreOrderTraversal



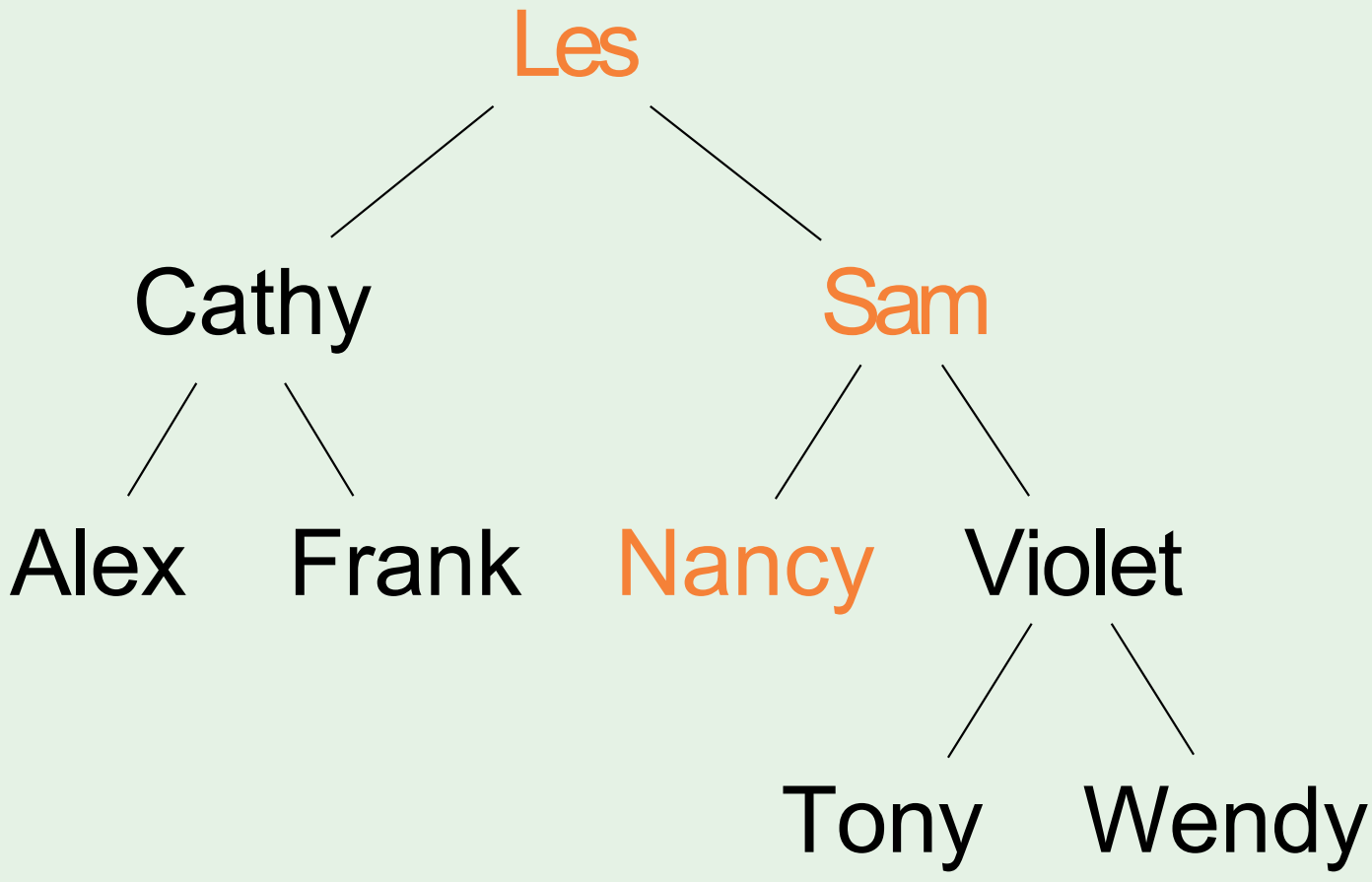
Output: Les Cathy Alex Frank Sam

PreOrderTraversal



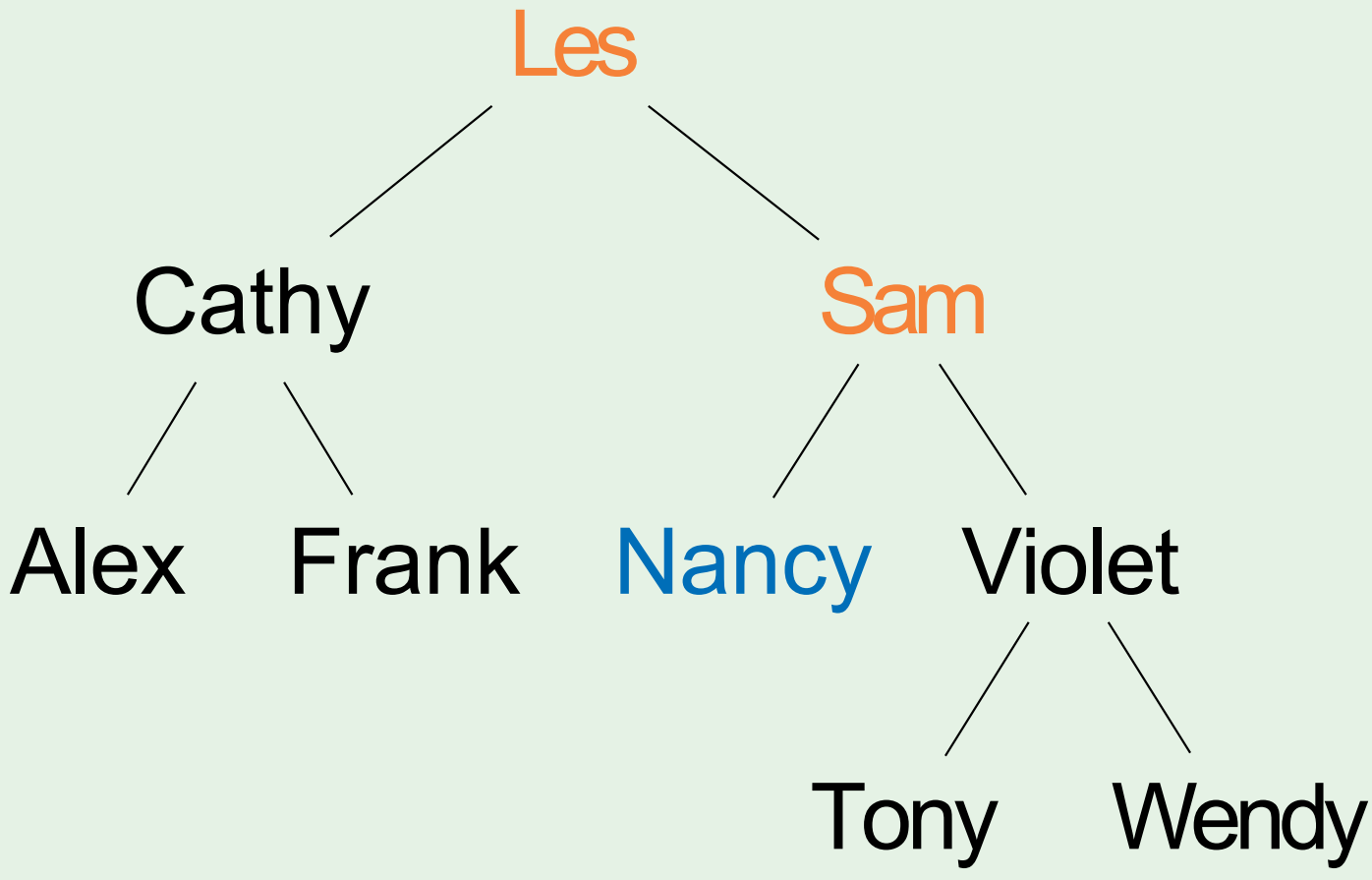
Output: Les Cathy Alex Frank Sam

PreOrderTraversal



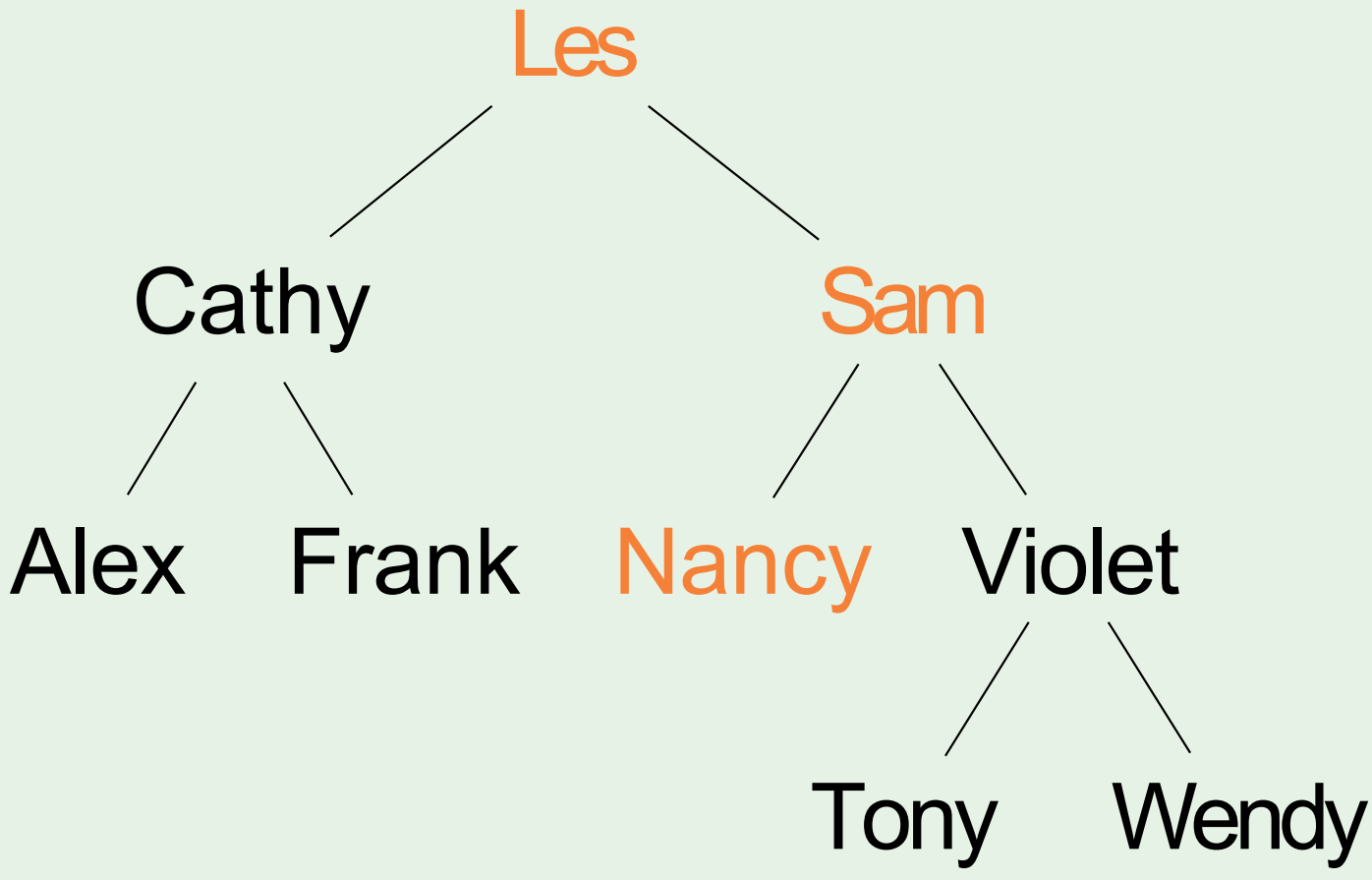
Output: Les Cathy Alex Frank Sam

PreOrderTraversal



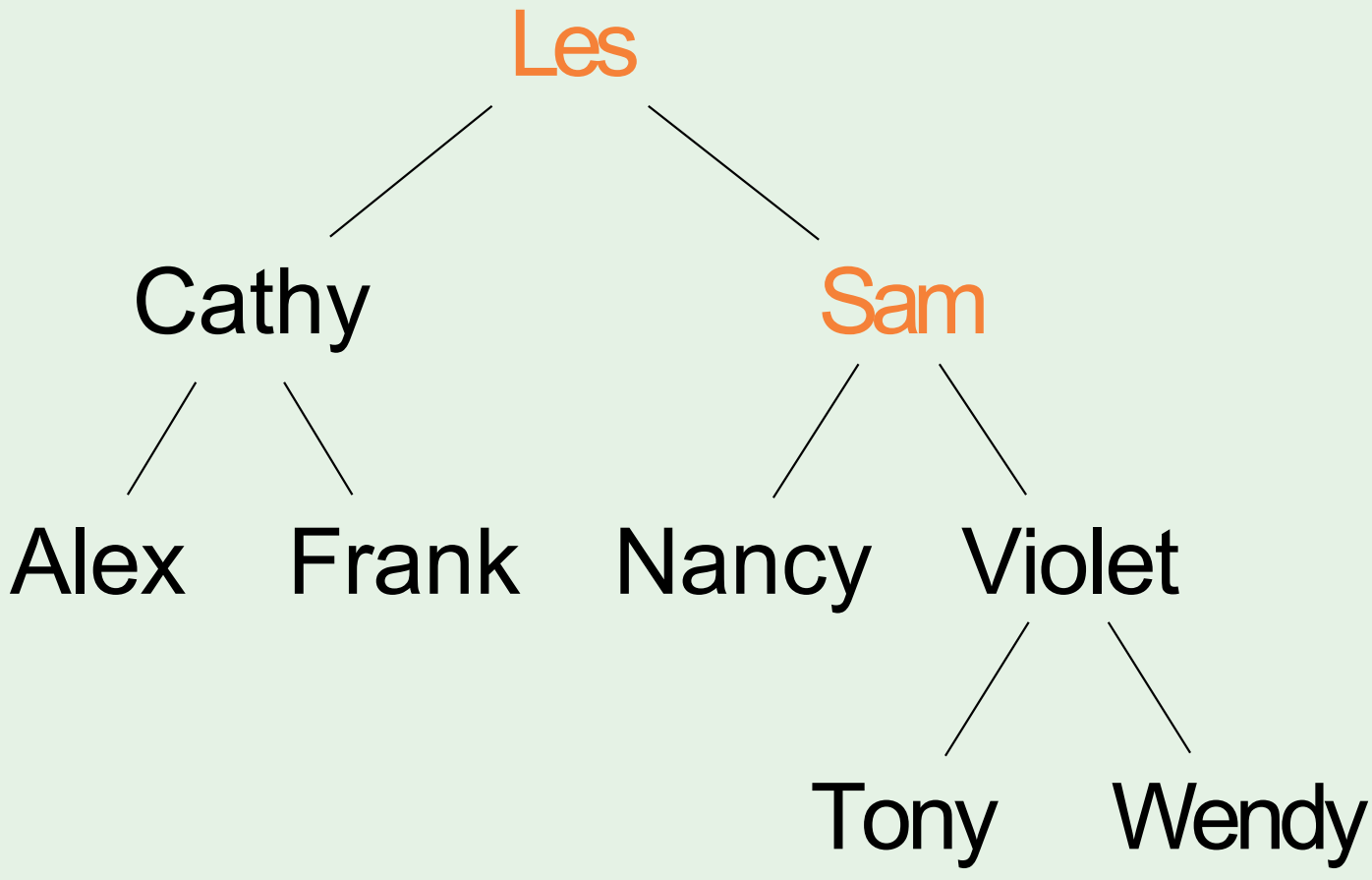
Output: Les Cathy Alex Frank Sam Nancy

PreOrderTraversal



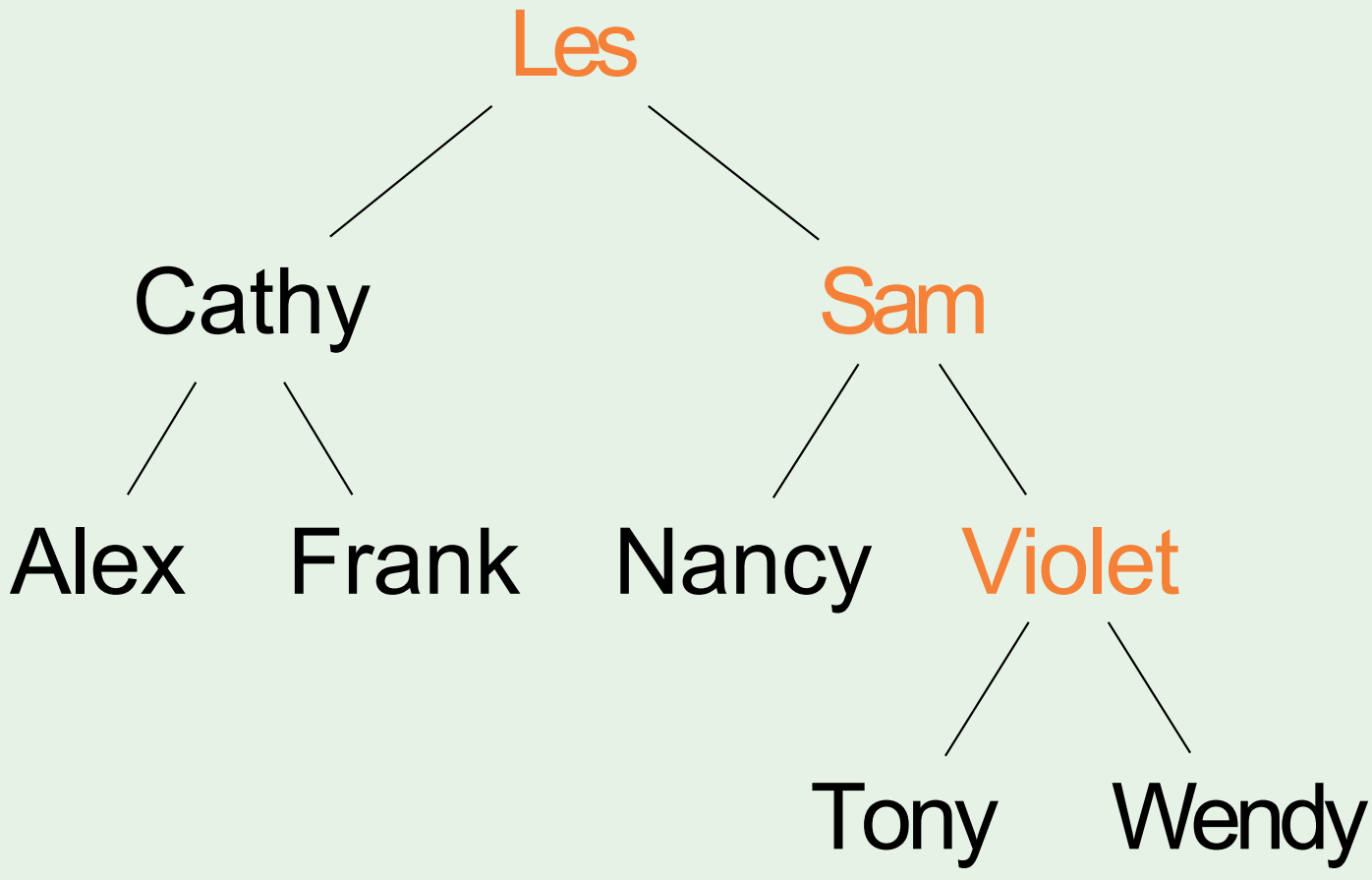
Output: Les Cathy Alex Frank Sam Nancy

PreOrderTraversal



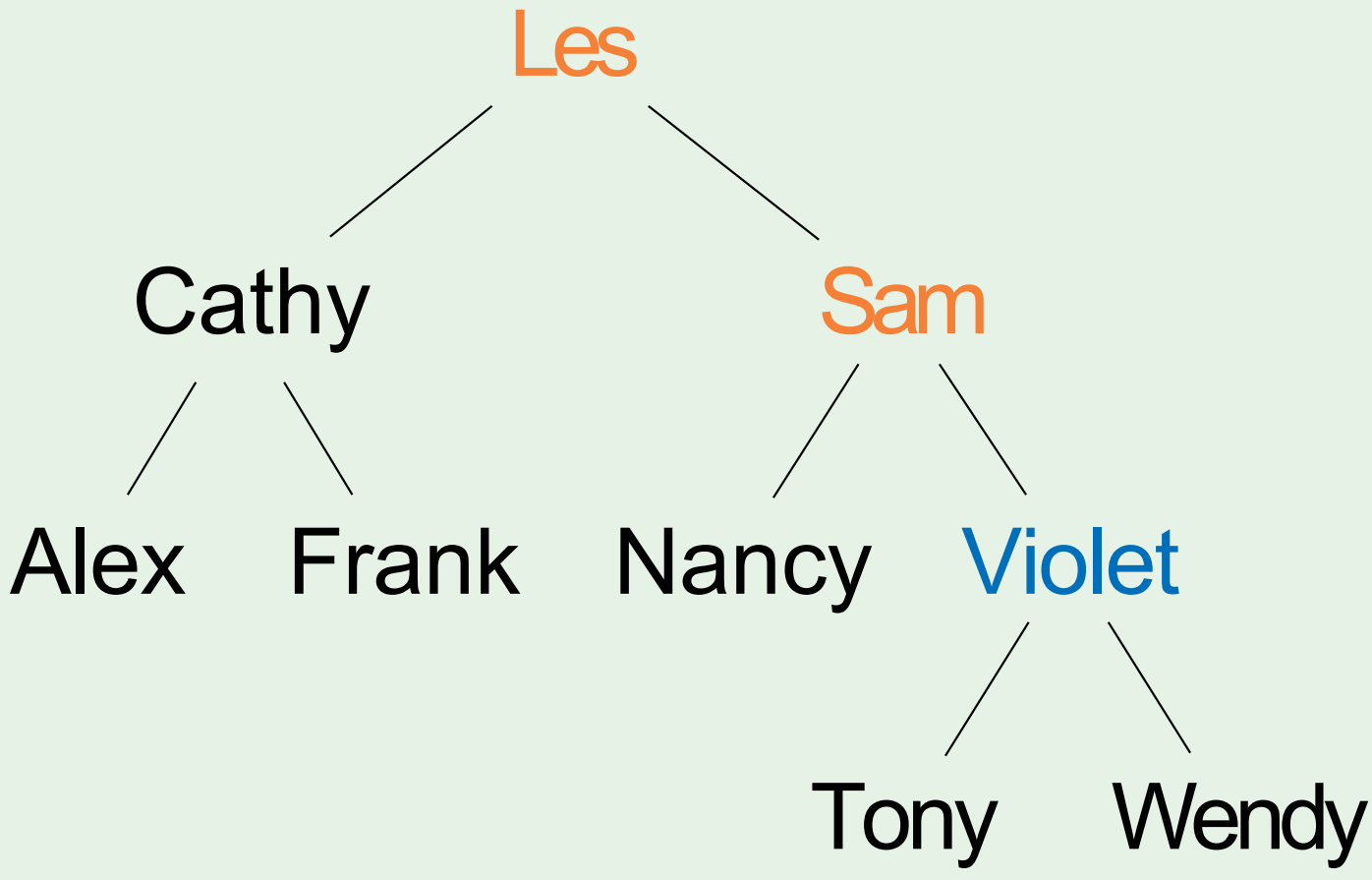
Output: Les Cathy Alex Frank Sam Nancy

PreOrderTraversal



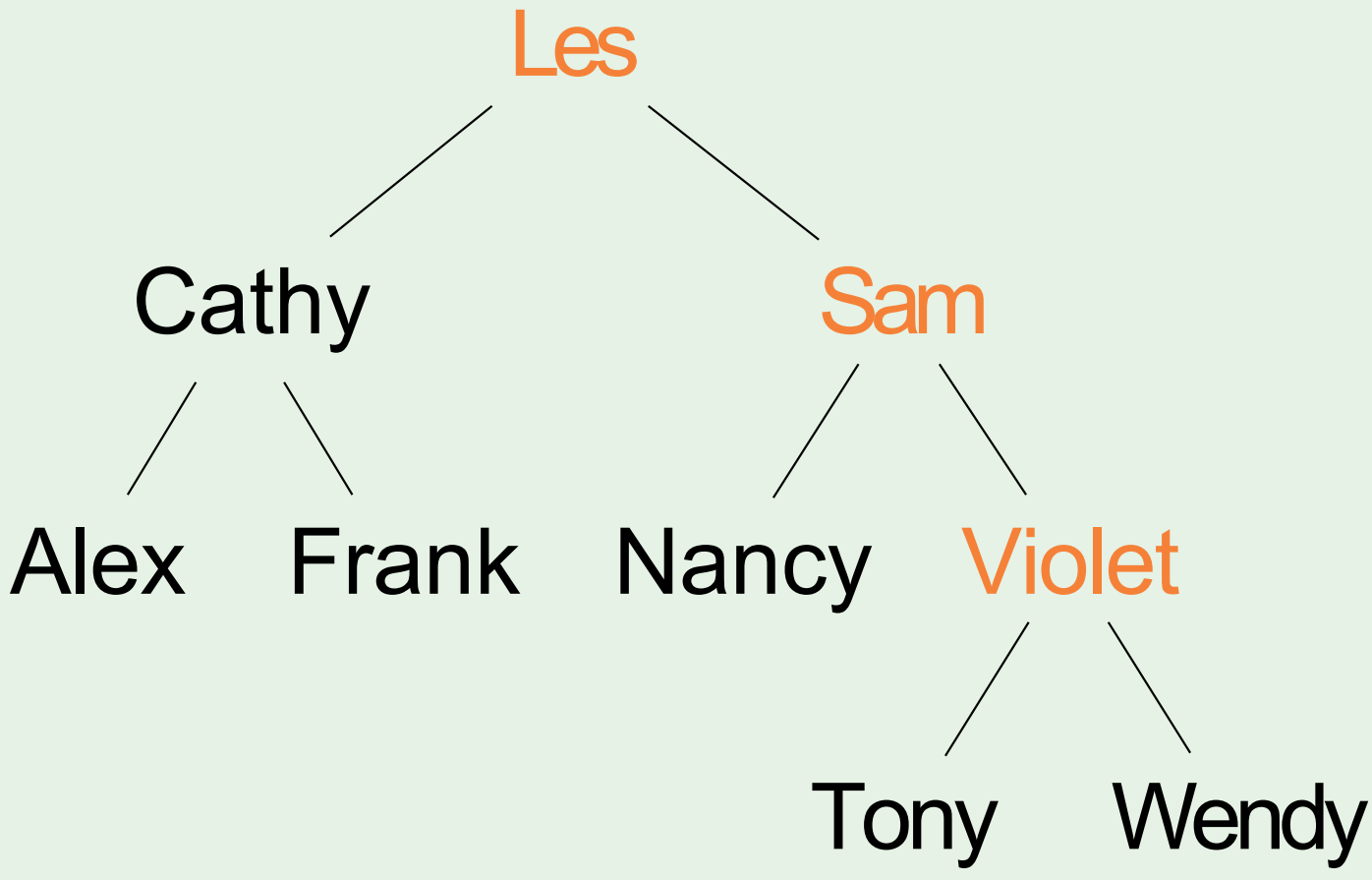
Output: Les Cathy Alex Frank Sam Nancy

PreOrderTraversal



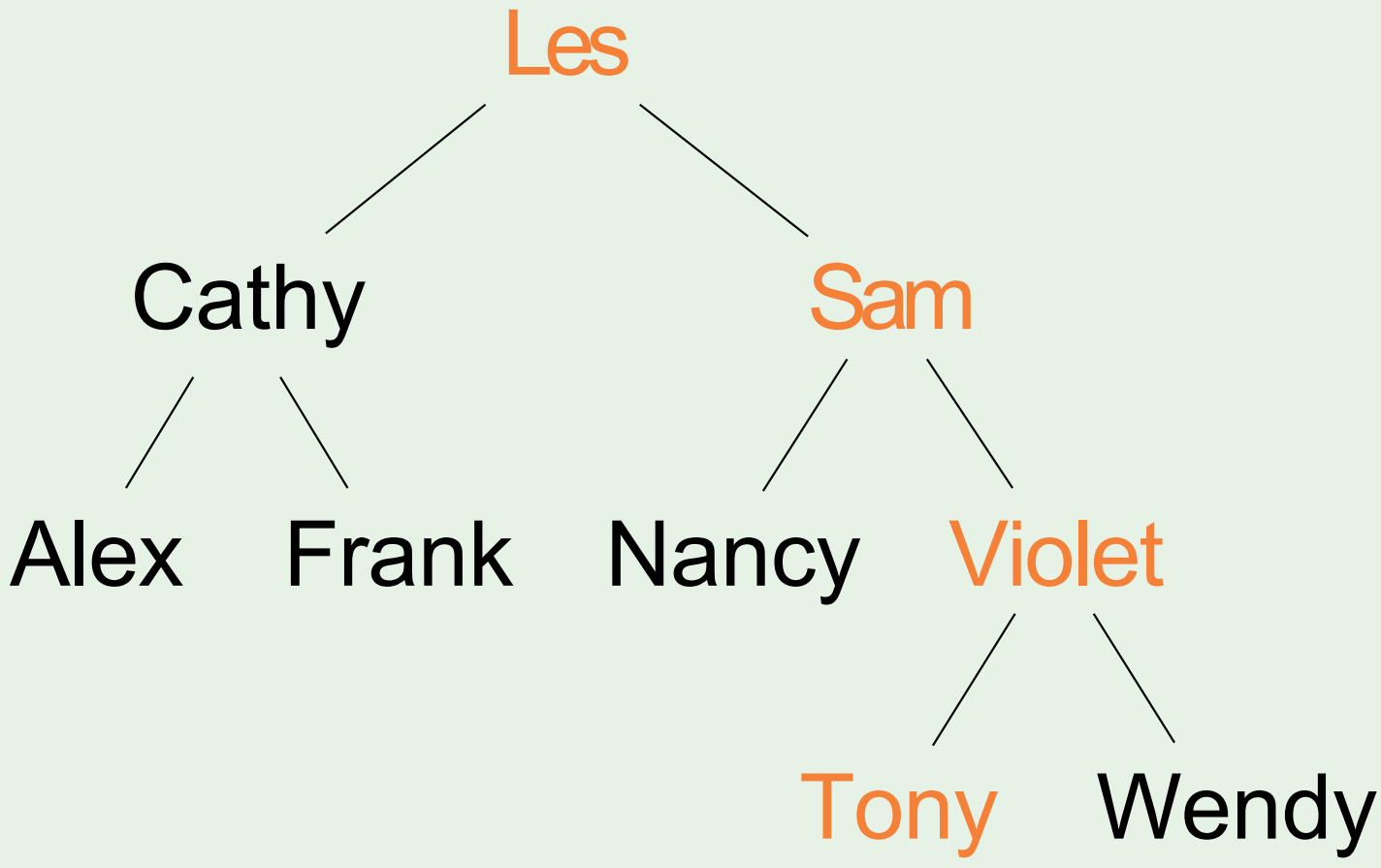
Output: Les Cathy Alex Frank Sam Nancy Violet

PreOrderTraversal



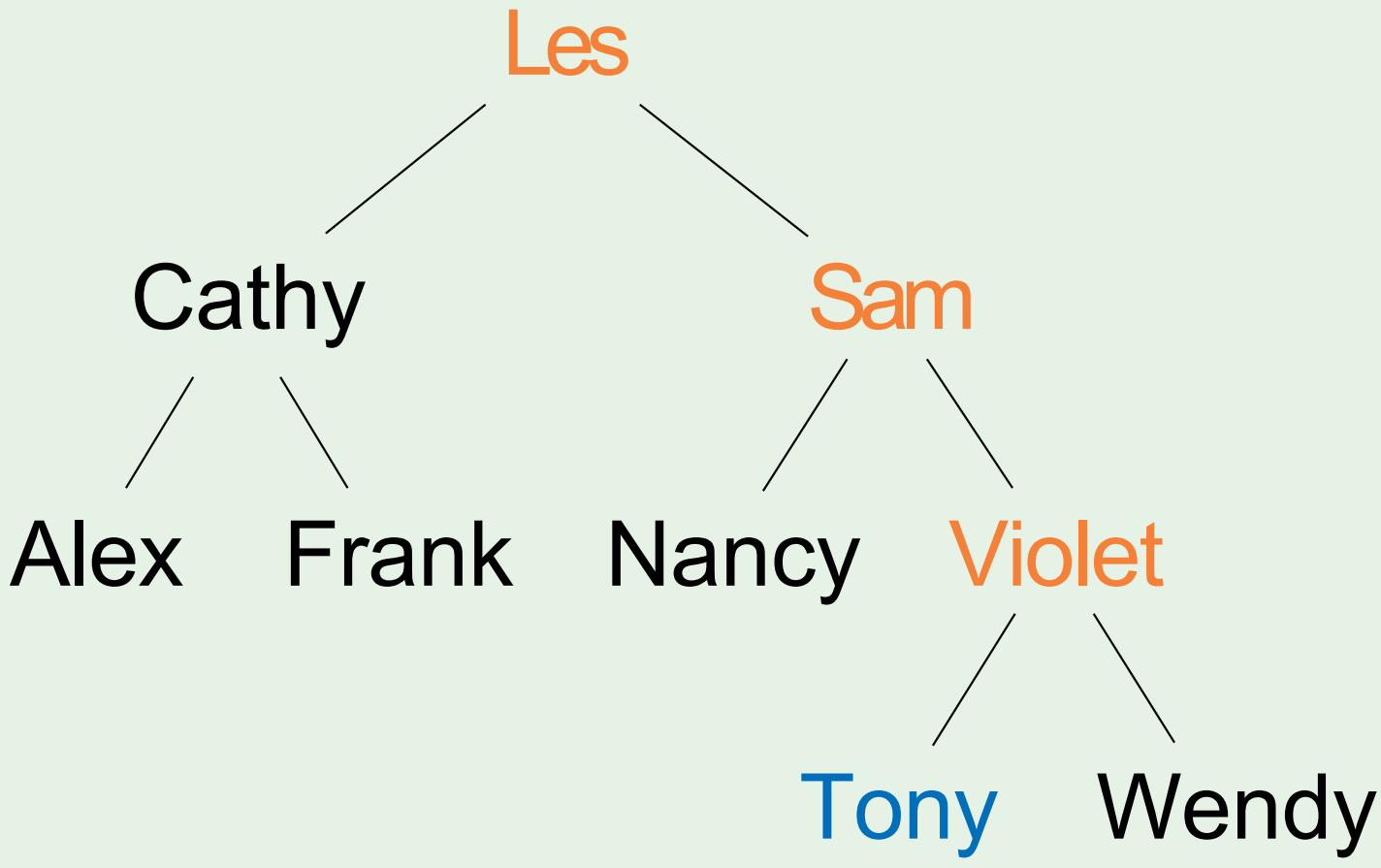
Output: Les Cathy Alex Frank Sam Nancy Violet

PreOrderTraversal



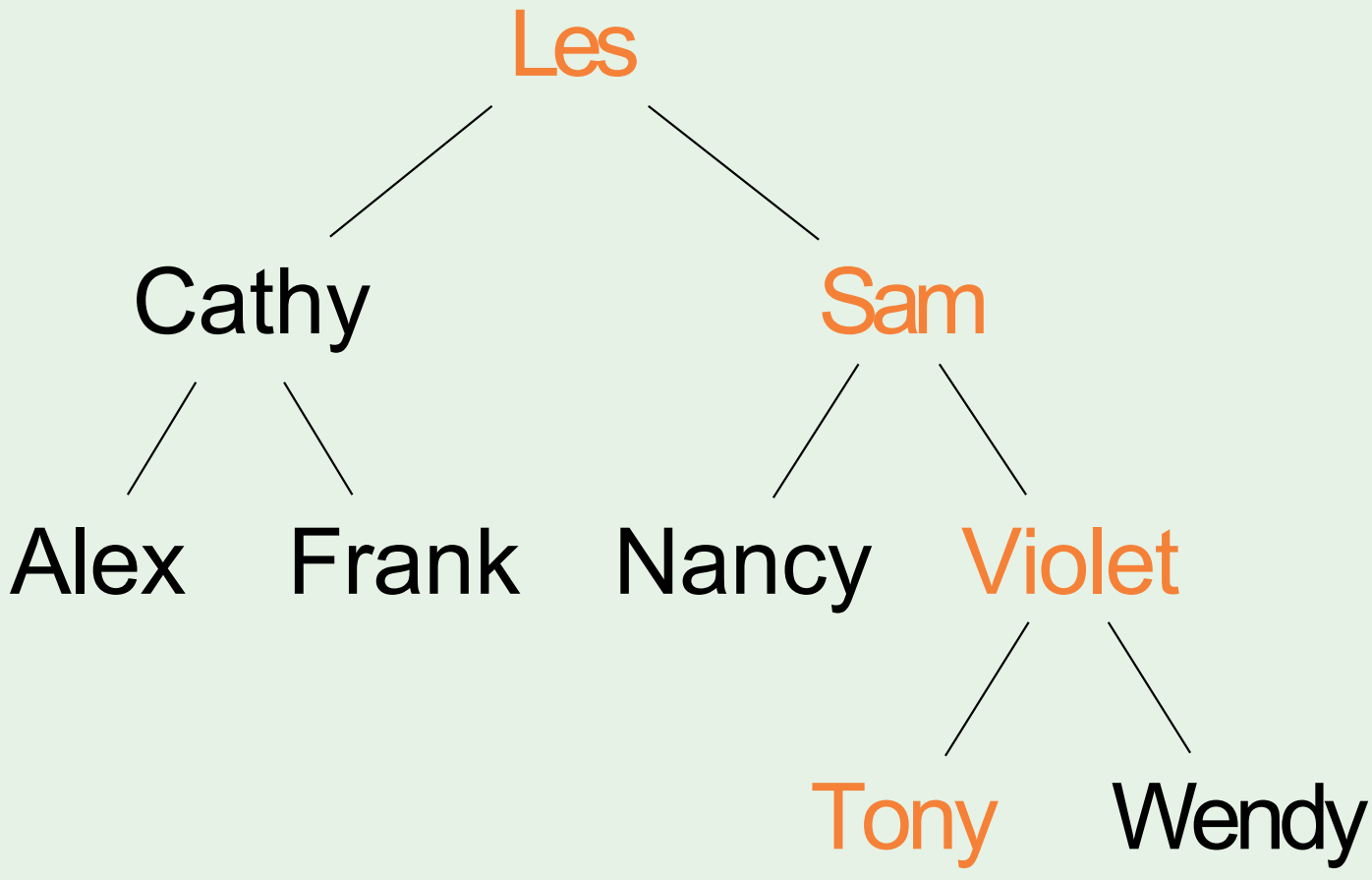
Output: Les Cathy Alex Frank Sam Nancy Violet

PreOrderTraversal



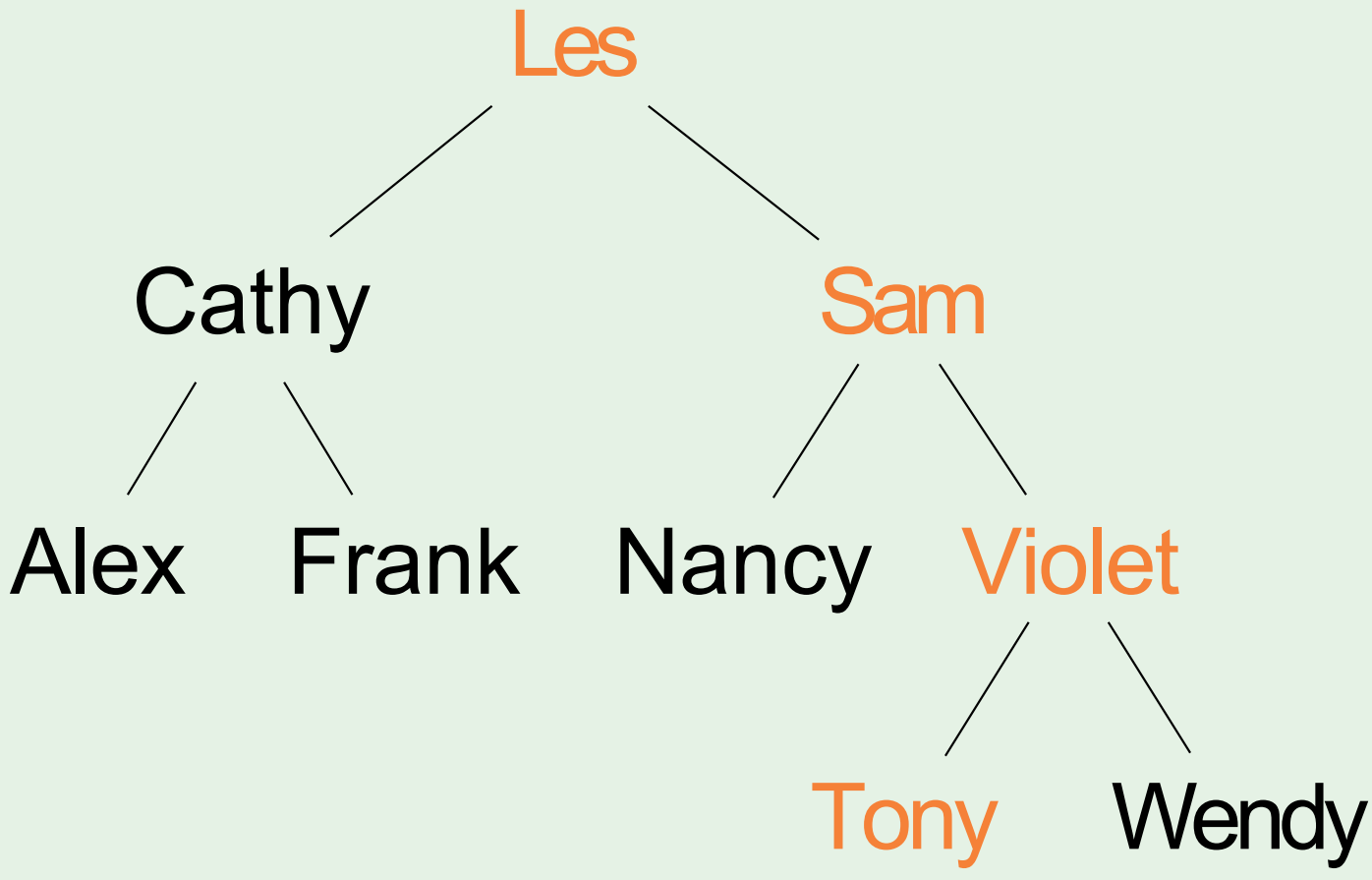
Output: Les Cathy Alex Frank Sam Nancy Violet Tony

PreOrderTraversal



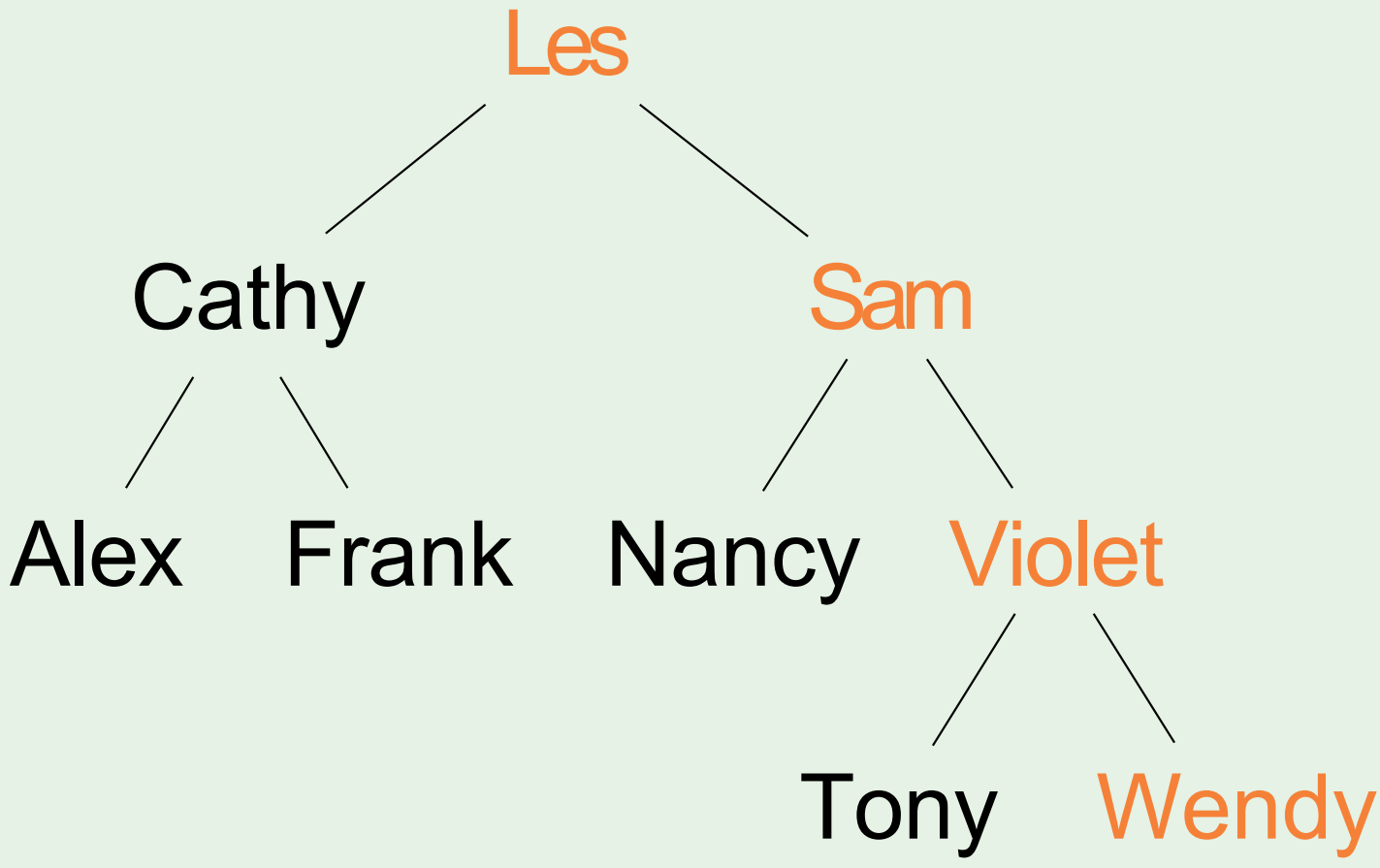
Output: Les Cathy Alex Frank Sam Nancy Violet Tony

PreOrderTraversal



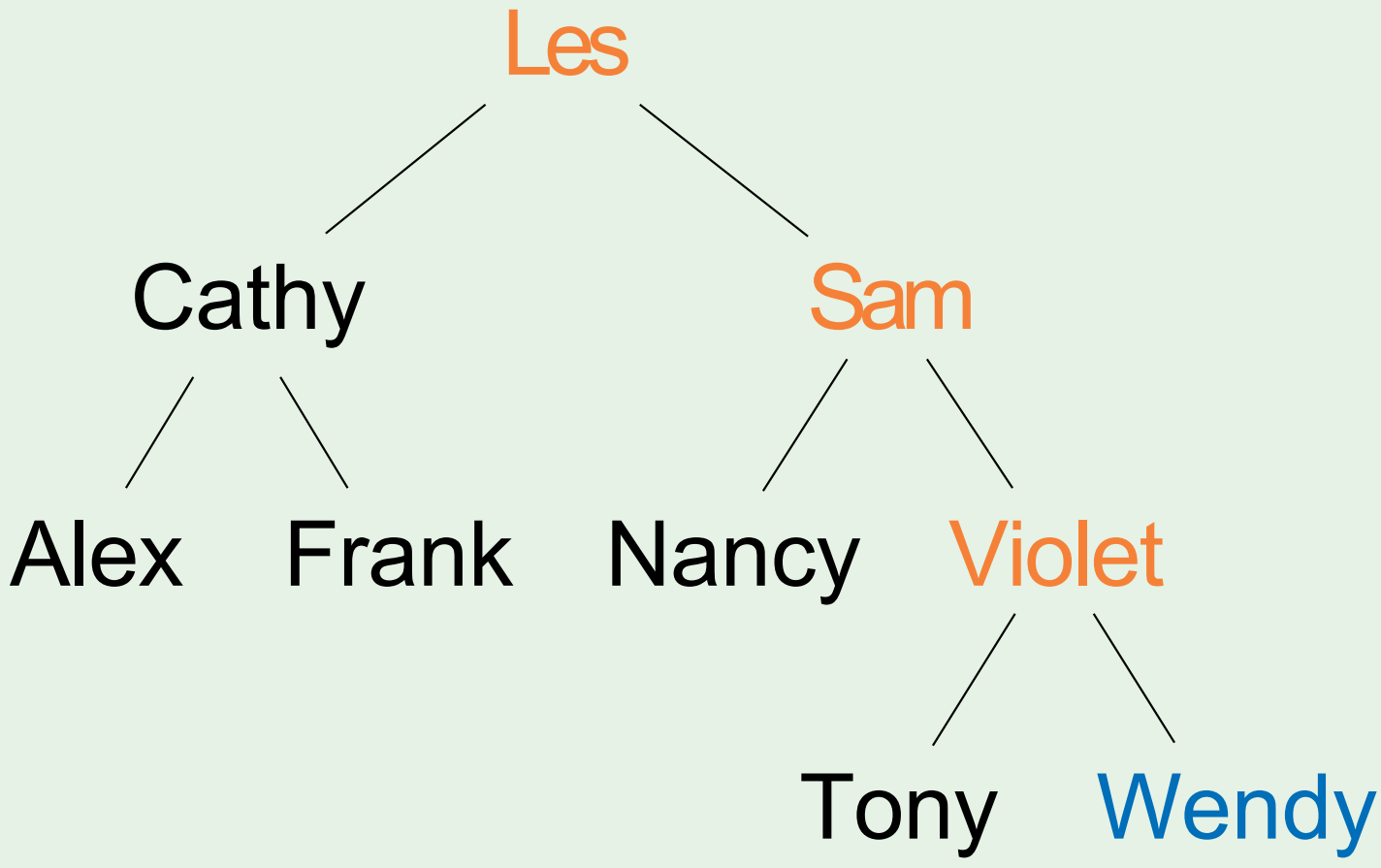
Output: Les Cathy Alex Frank Sam Nancy Violet Tony

PreOrderTraversal



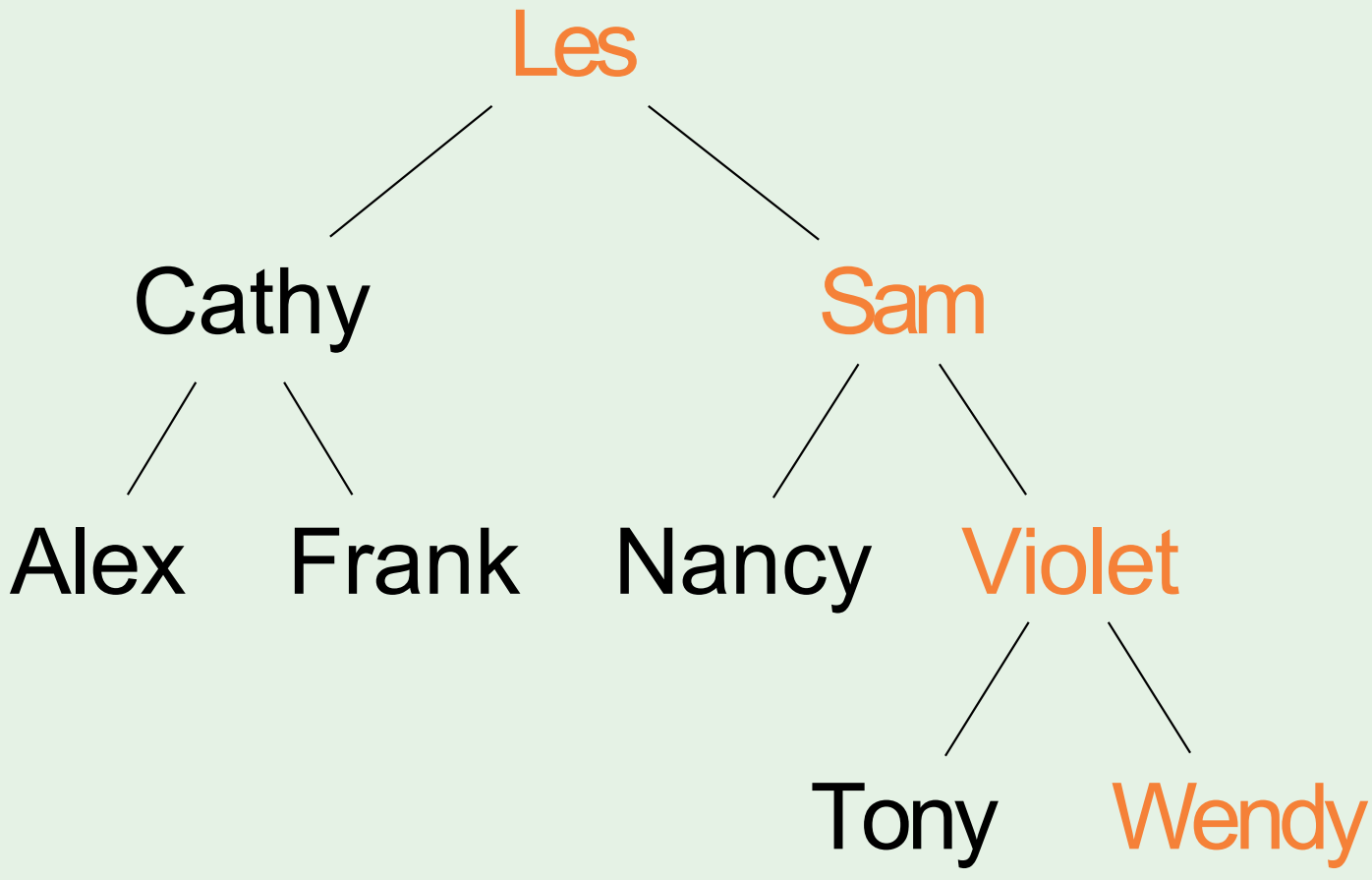
Output: Les Cathy Alex Frank Sam Nancy Violet Tony

PreOrderTraversal



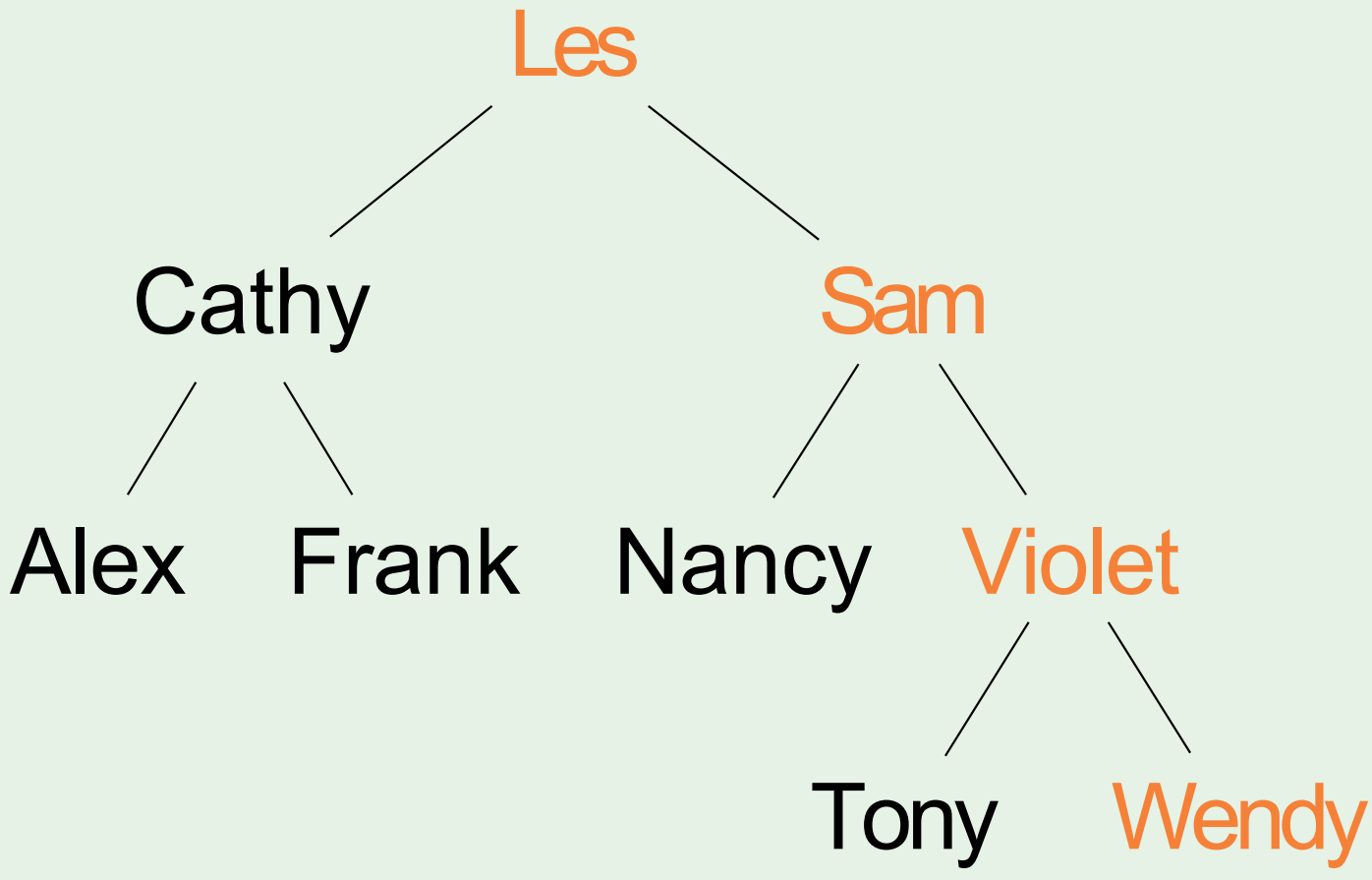
Output: Les Cathy Alex Frank Sam Nancy Violet Tony Wendy

PreOrderTraversal



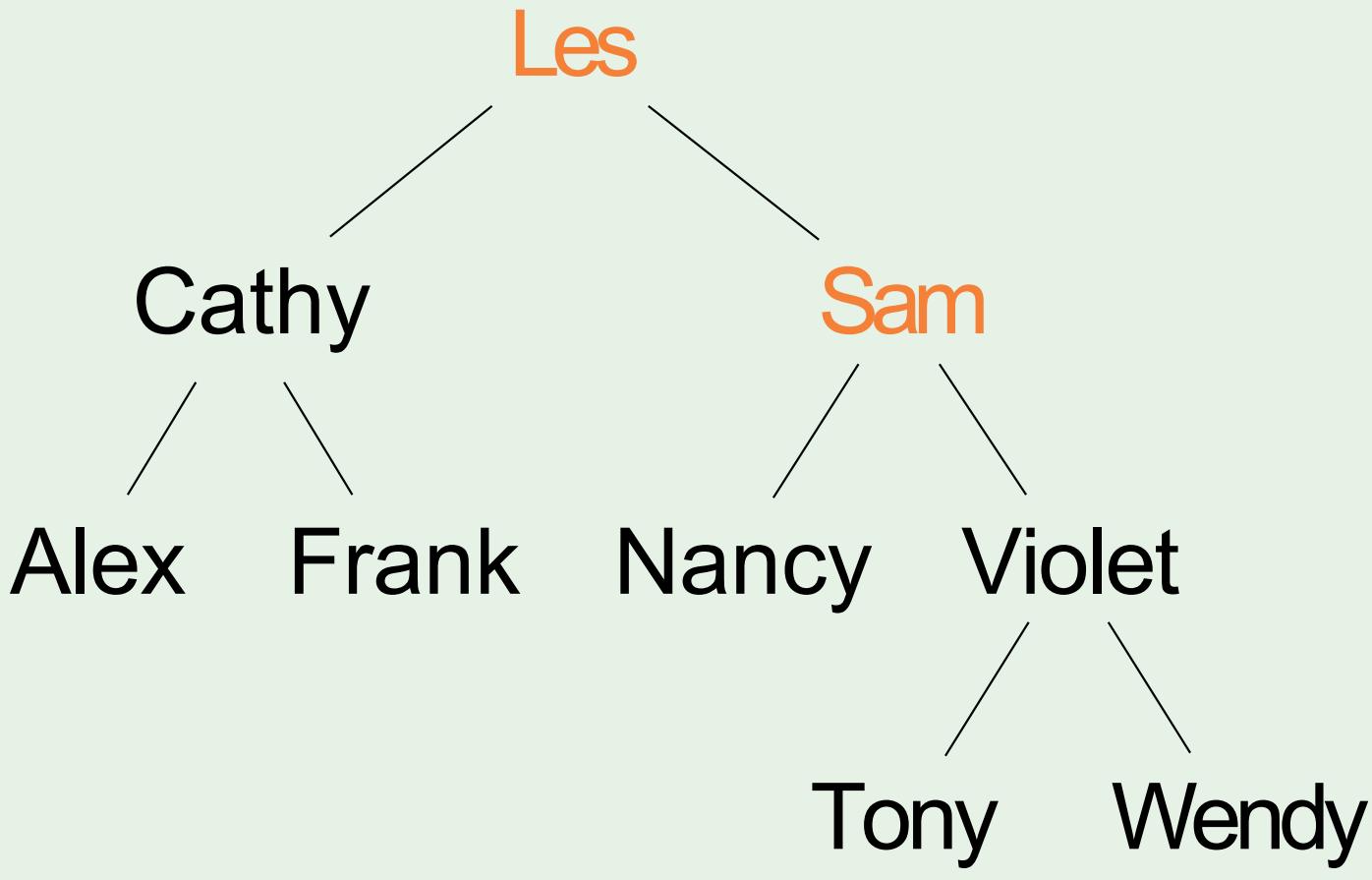
Output: Les Cathy Alex Frank Sam Nancy Violet Tony Wendy

PreOrderTraversal



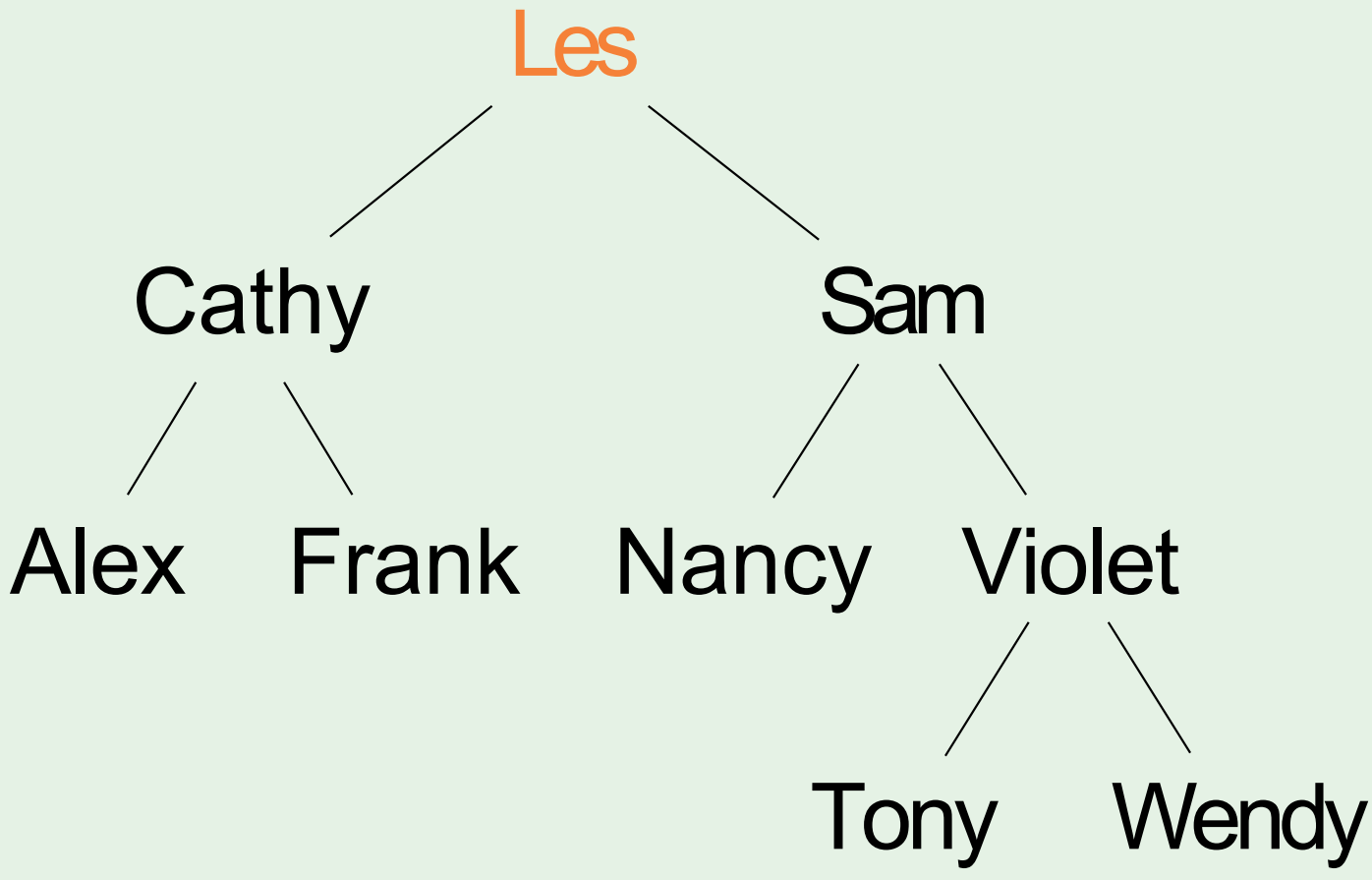
Output: Les Cathy Alex Frank Sam Nancy Violet Tony Wendy

PreOrderTraversal



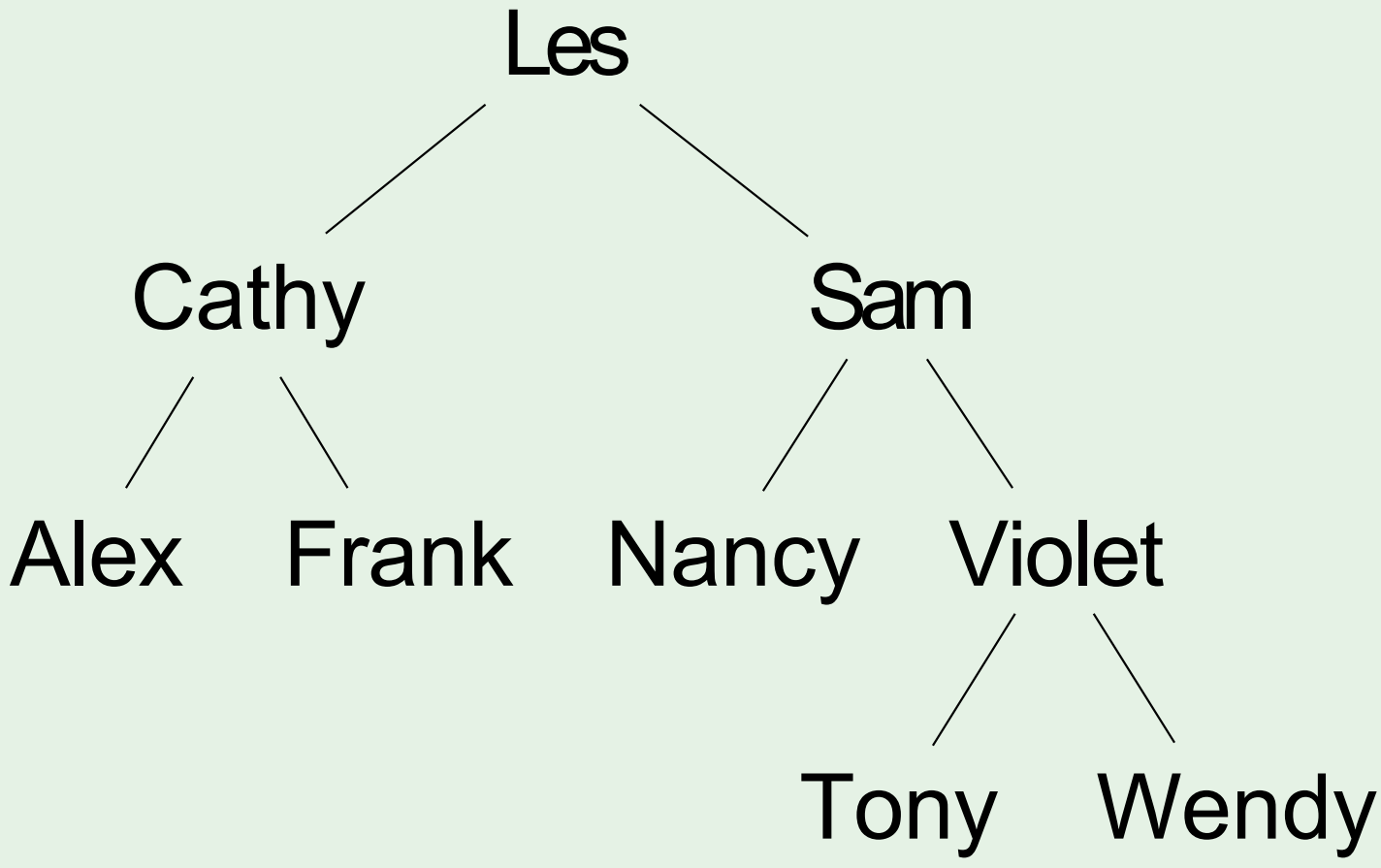
Output: Les Cathy Alex Frank Sam Nancy Violet Tony Wendy

PreOrderTraversal



Output: Les Cathy Alex Frank Sam Nancy Violet Tony Wendy

PreOrderTraversal

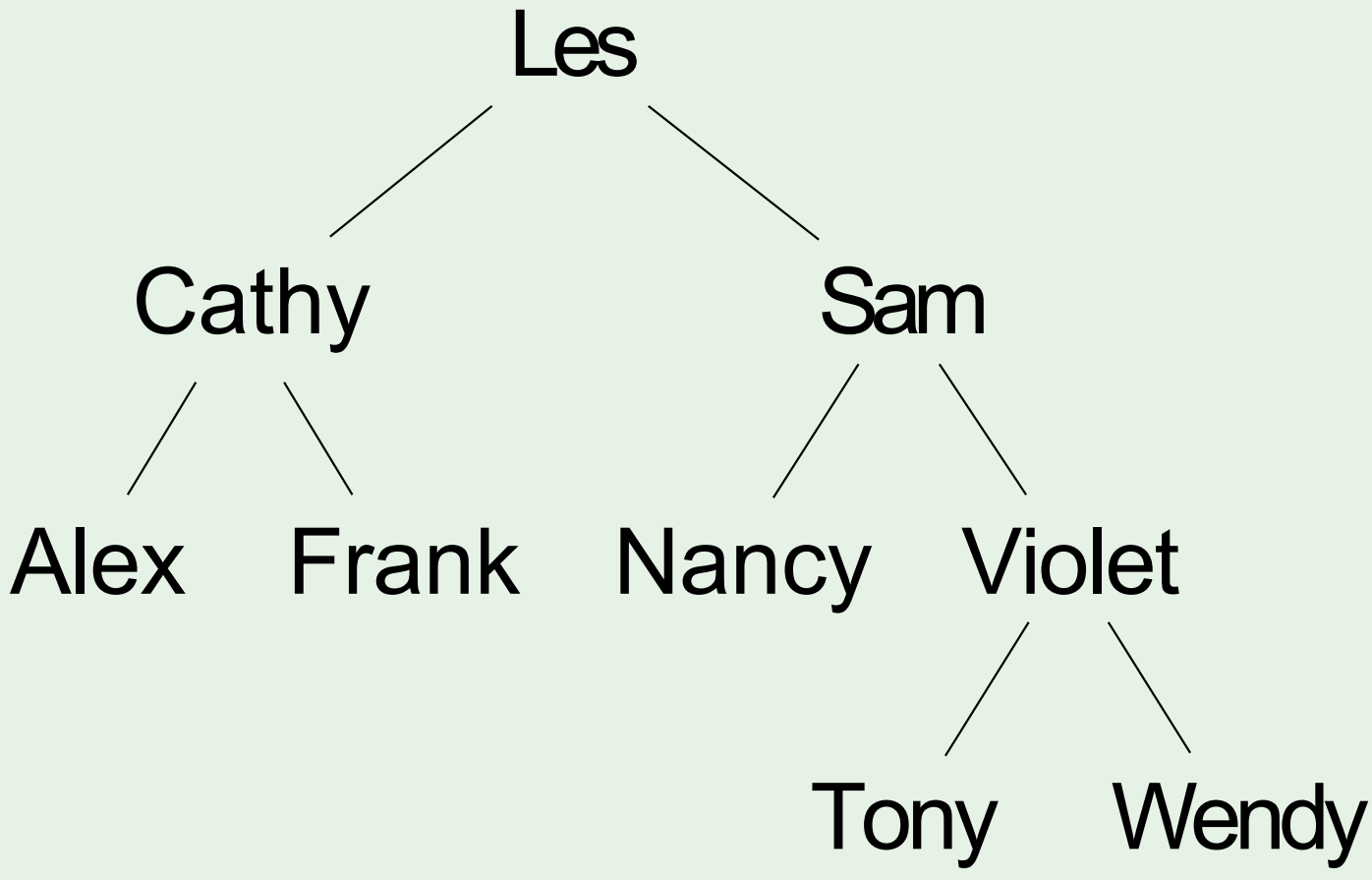


Output: Les Cathy Alex Frank Sam Nancy Violet Tony Wendy

Depth-first

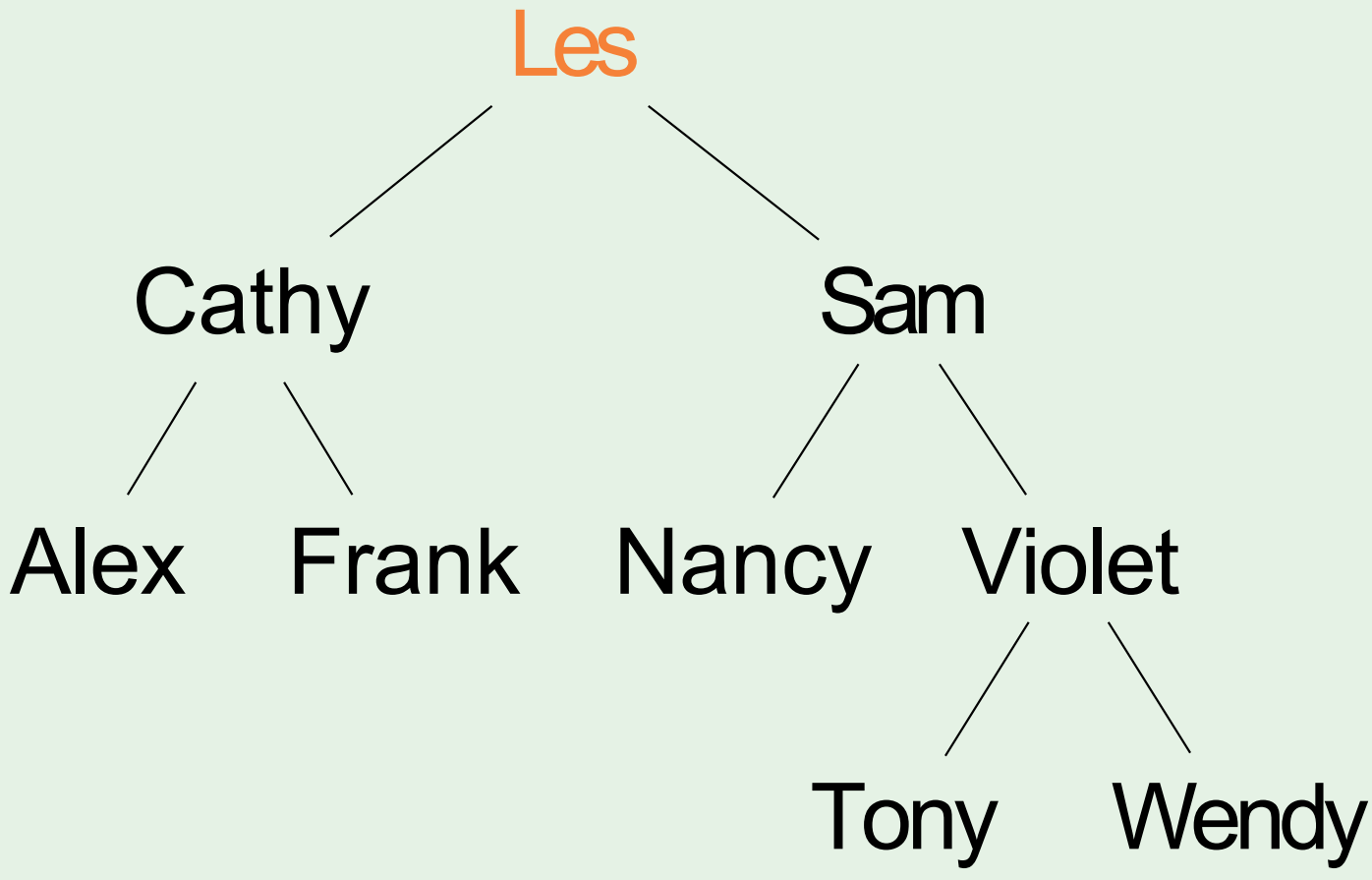
```
PostOrderTraversal(tree)
if tree = nil :
   return
PostOrderTraversal(tree.left)
PostOrderTraversal(tree.right)
Print(tree.key )
```

PostOrderTraversal



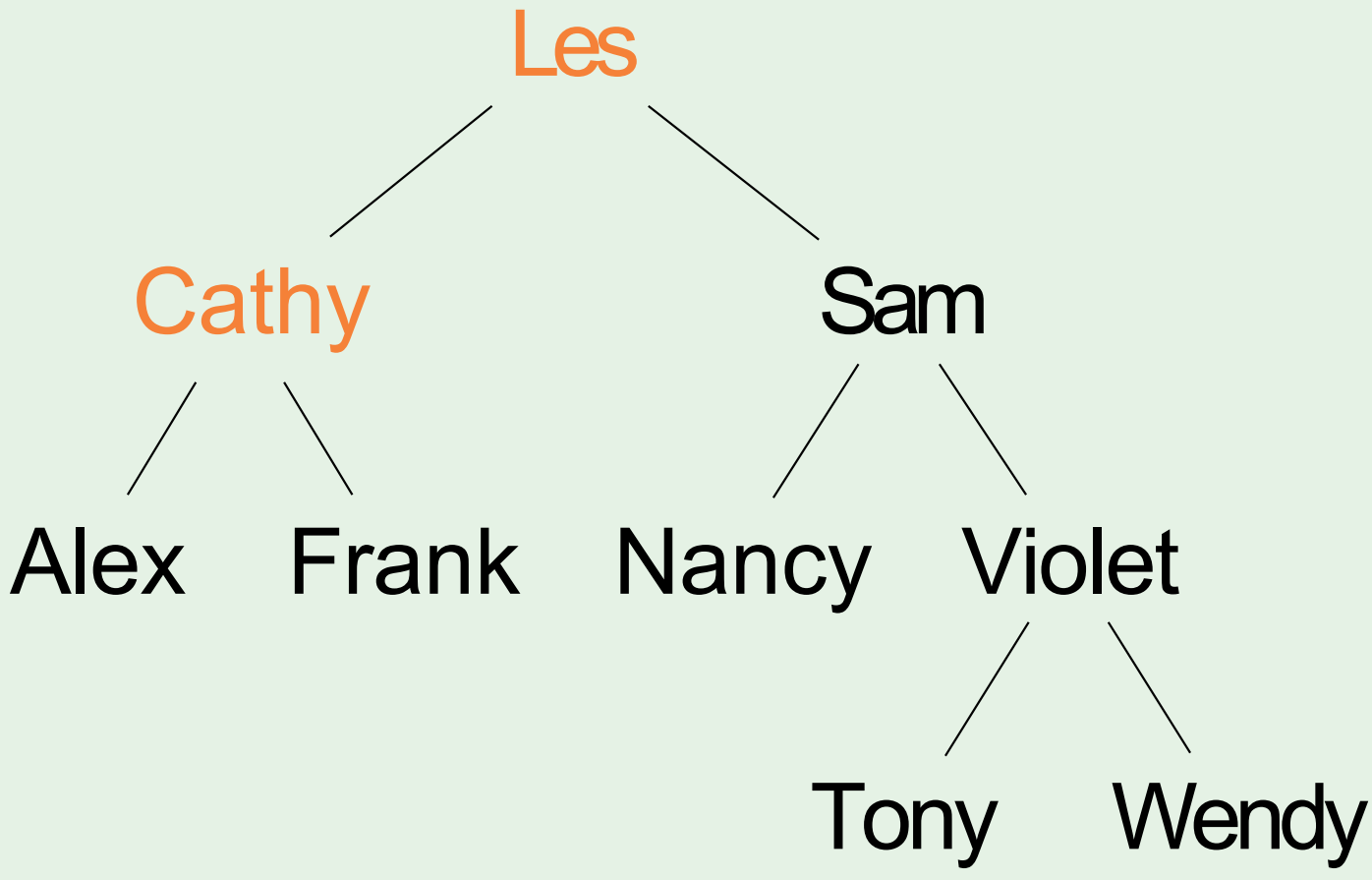
Output:

PostOrderTraversal



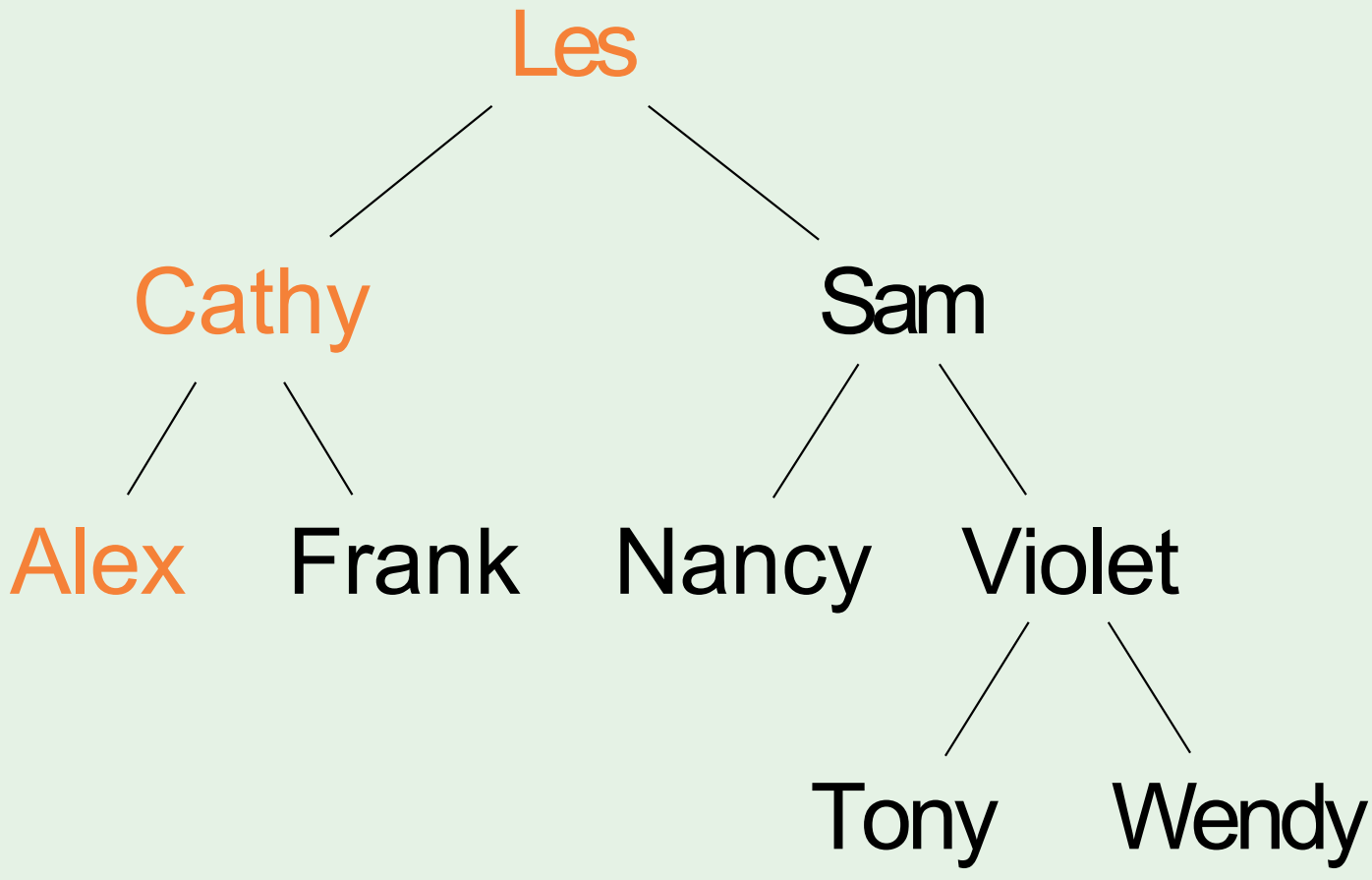
Output:

PostOrderTraversal



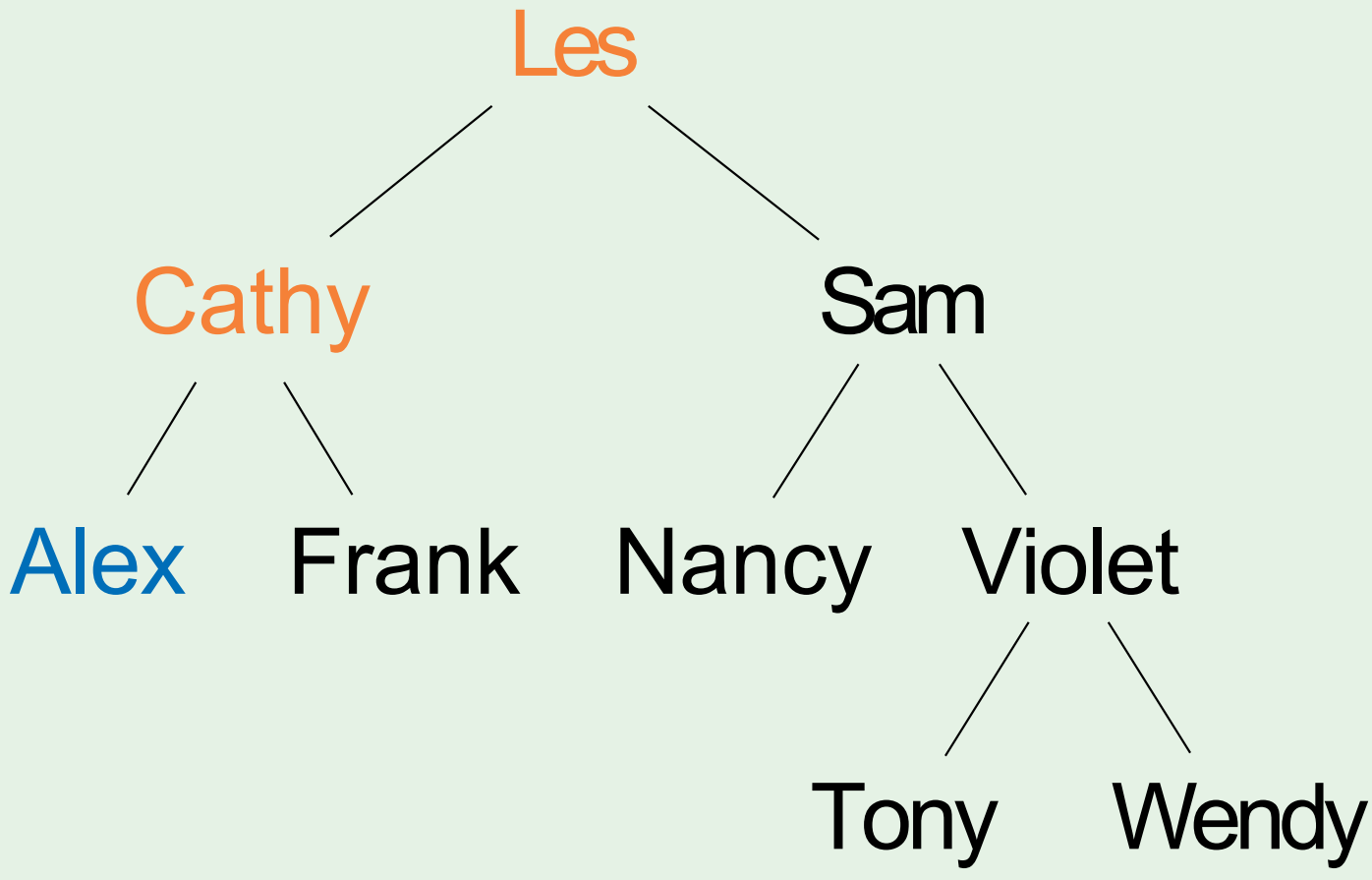
Output:

PostOrderTraversal



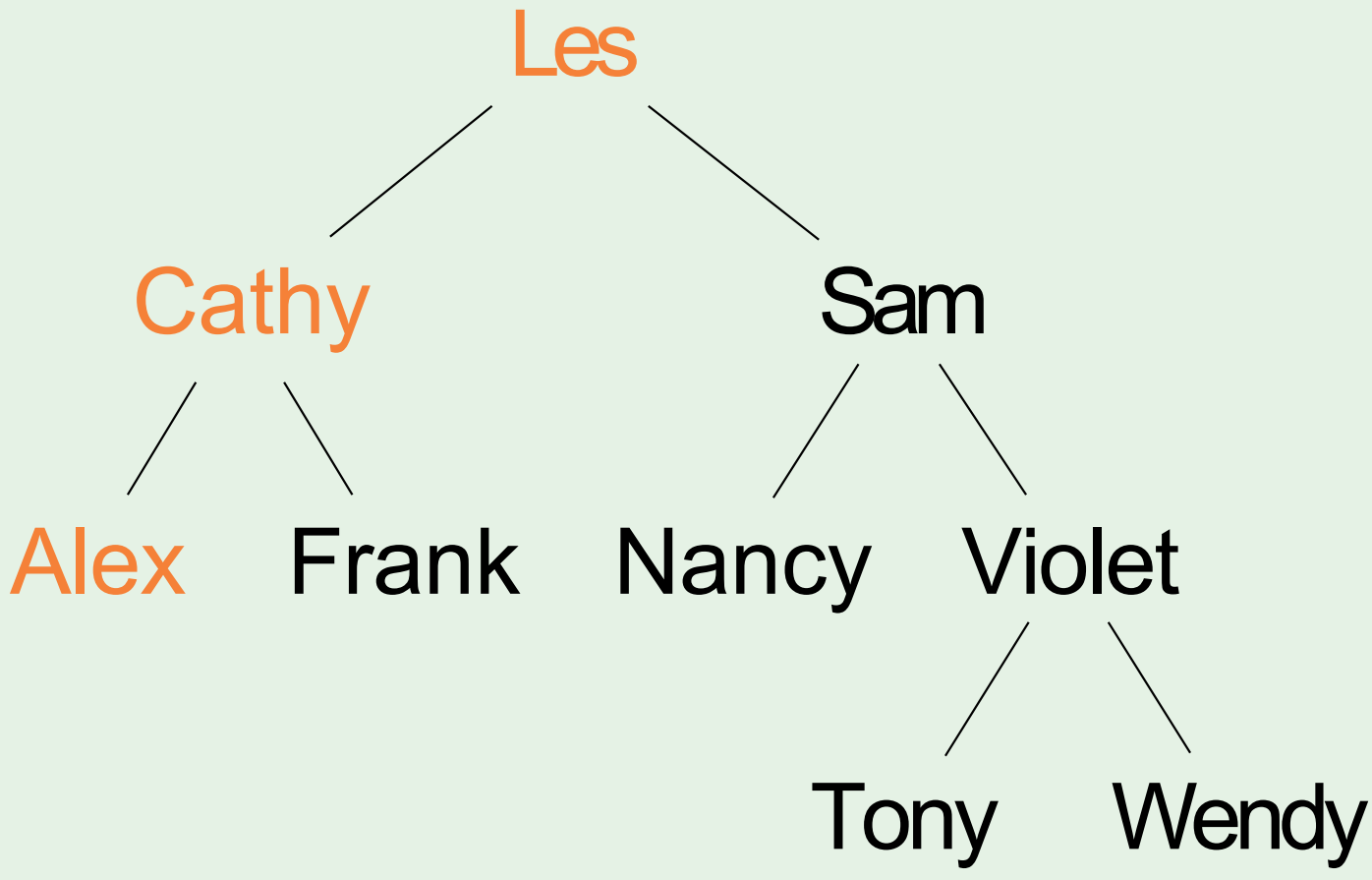
Output:

PostOrderTraversal



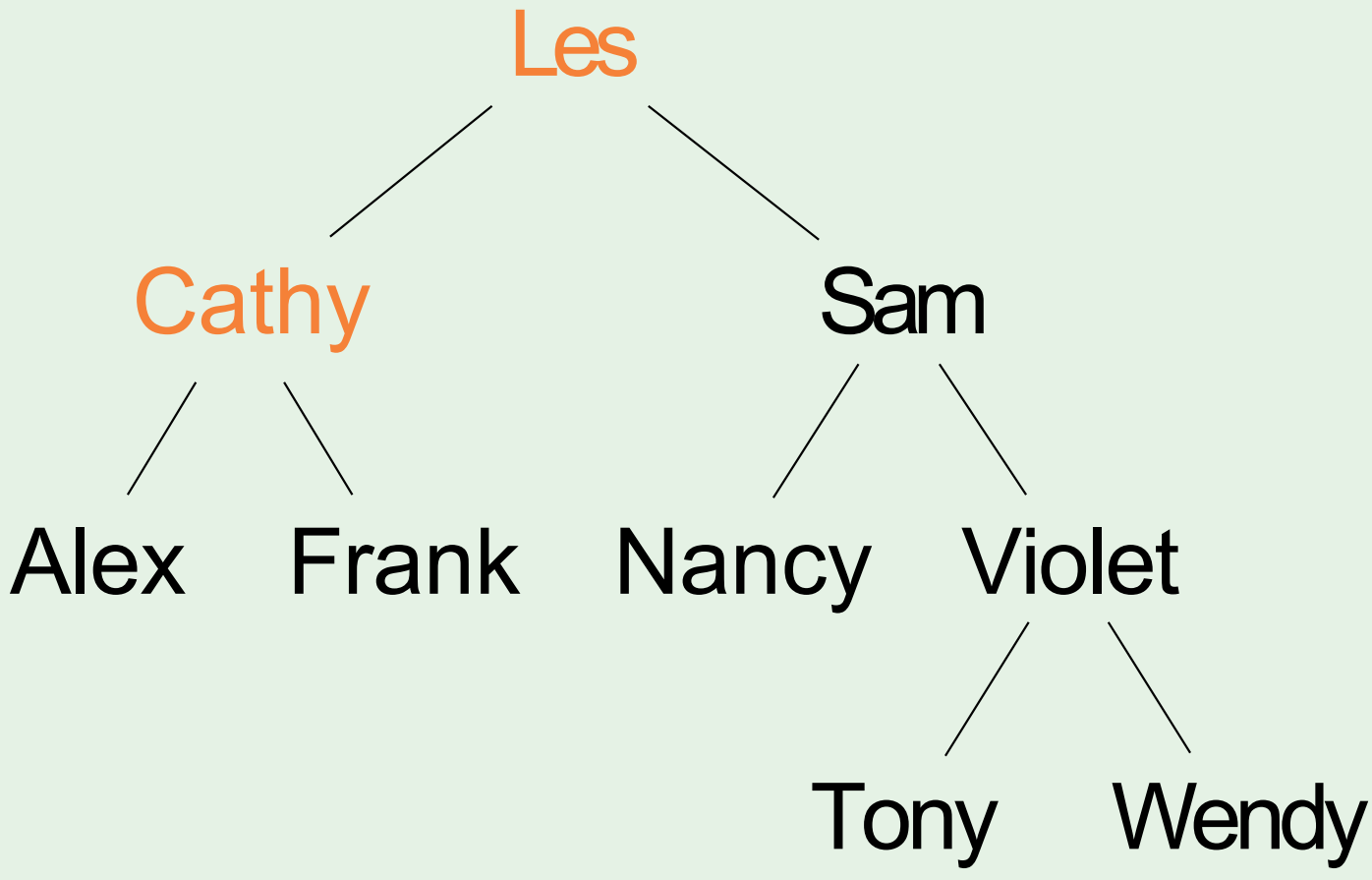
Output: Alex

PostOrderTraversal



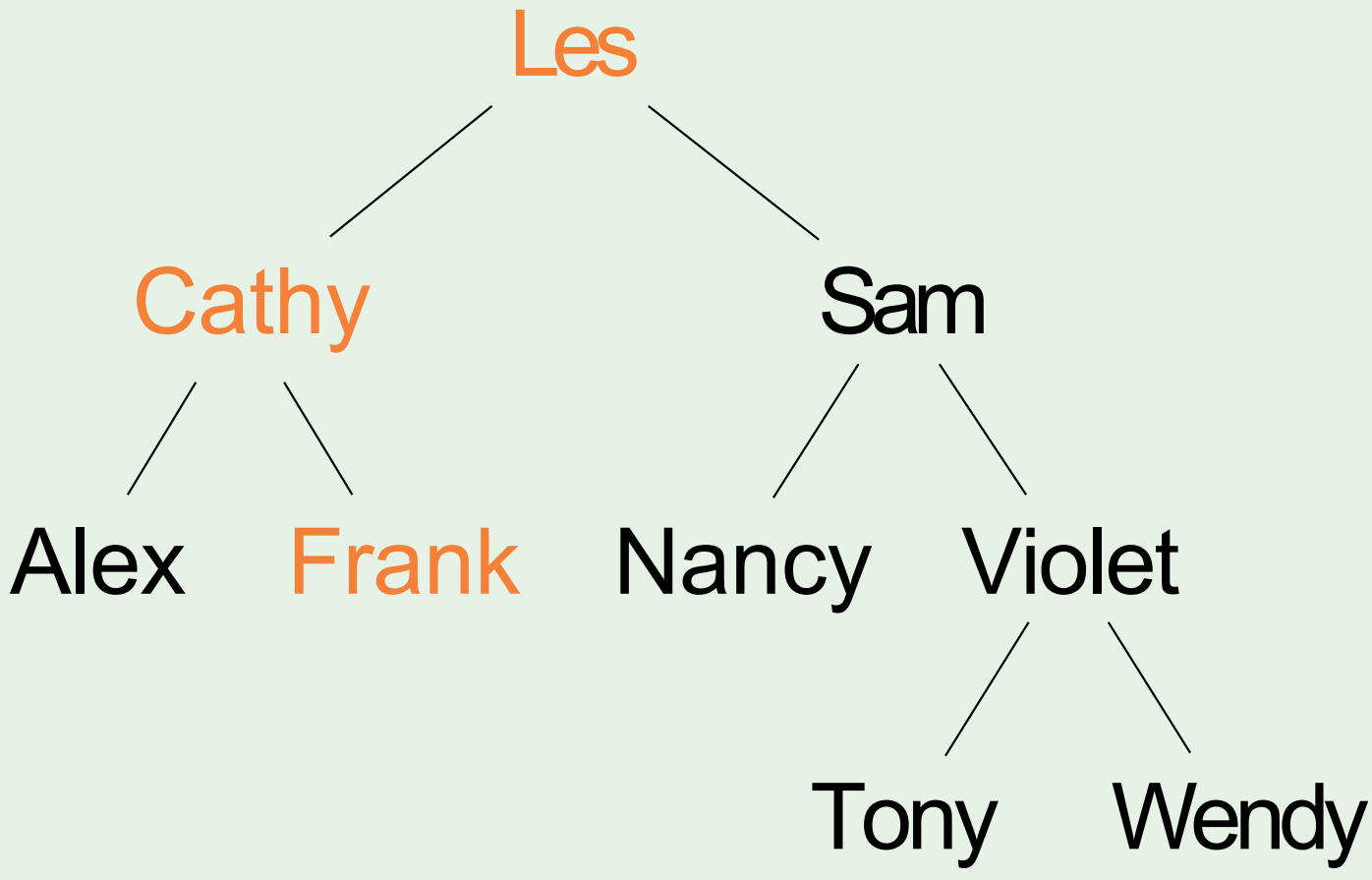
Output: Alex

PostOrderTraversal



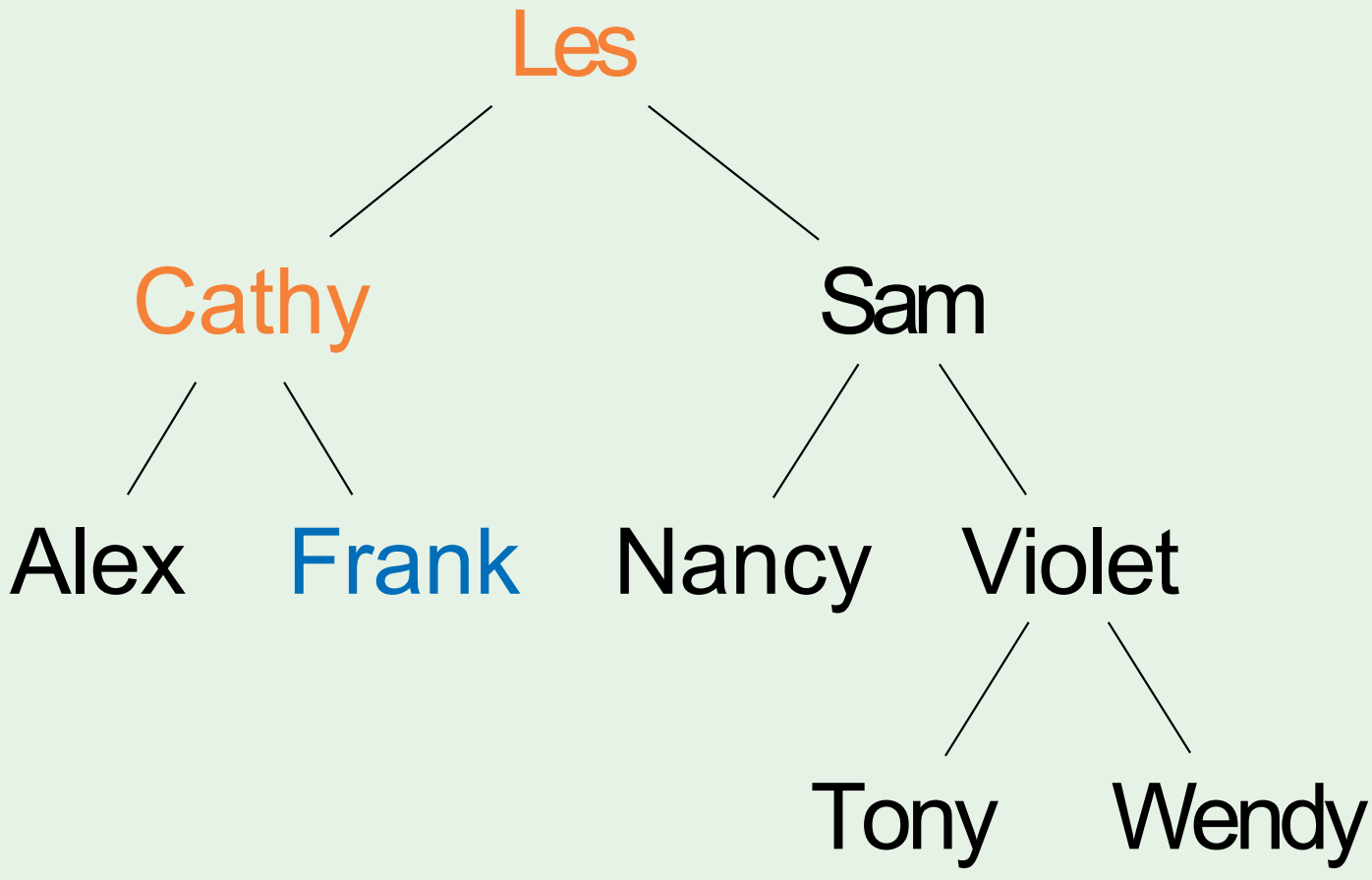
Output: Alex

PostOrderTraversal



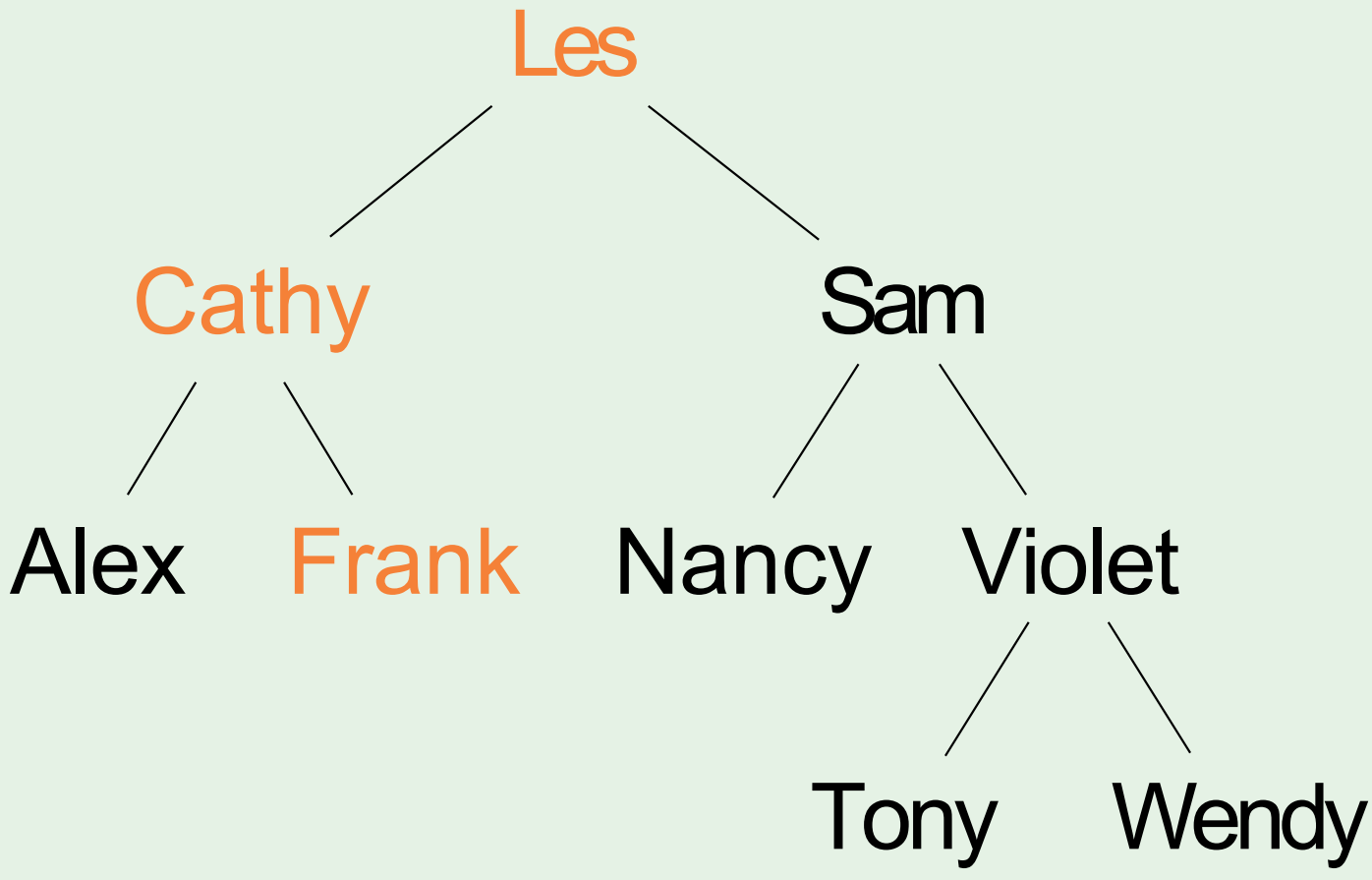
Output: Alex

PostOrderTraversal



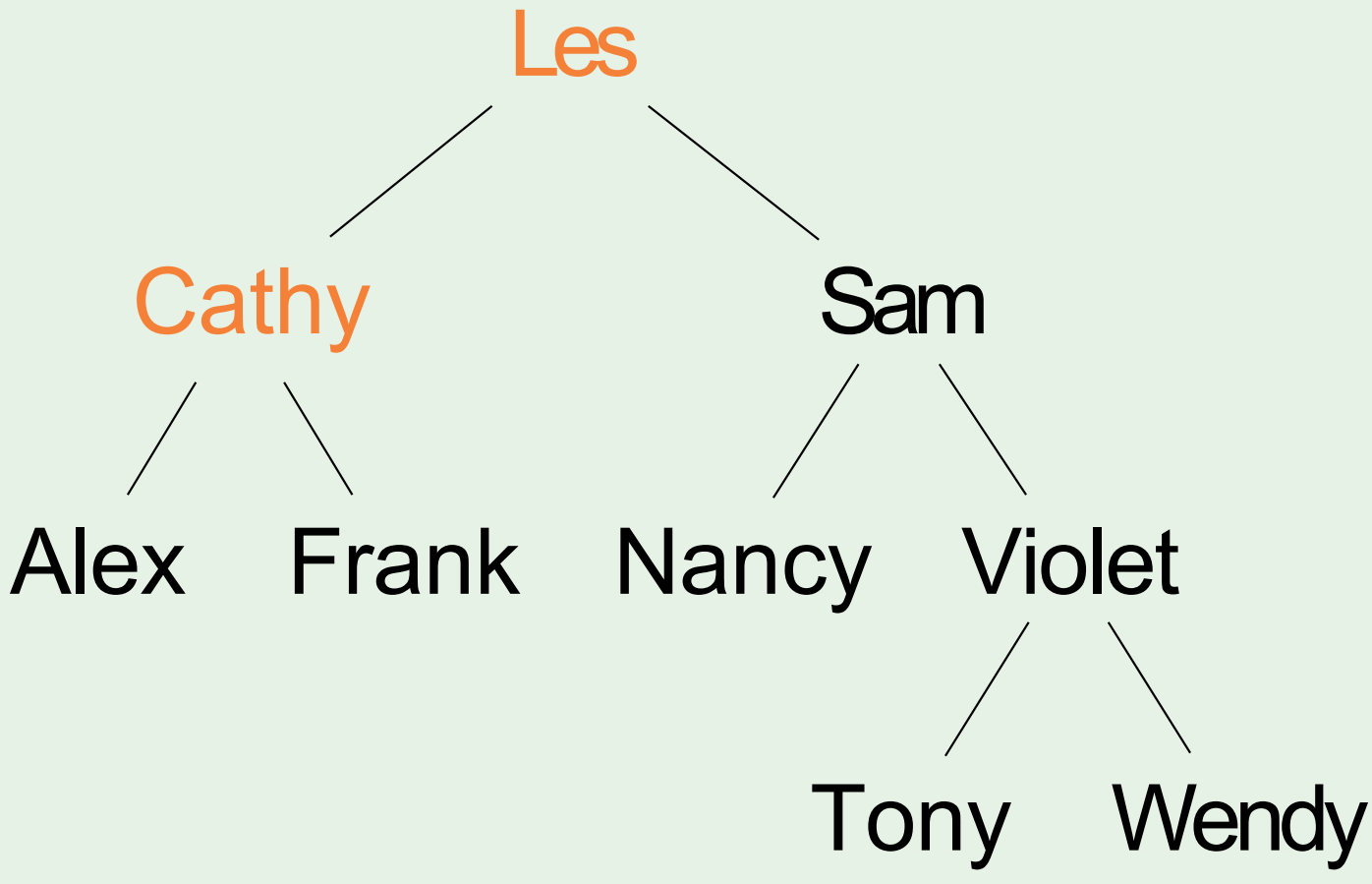
Output: Alex Frank

PostOrderTraversal



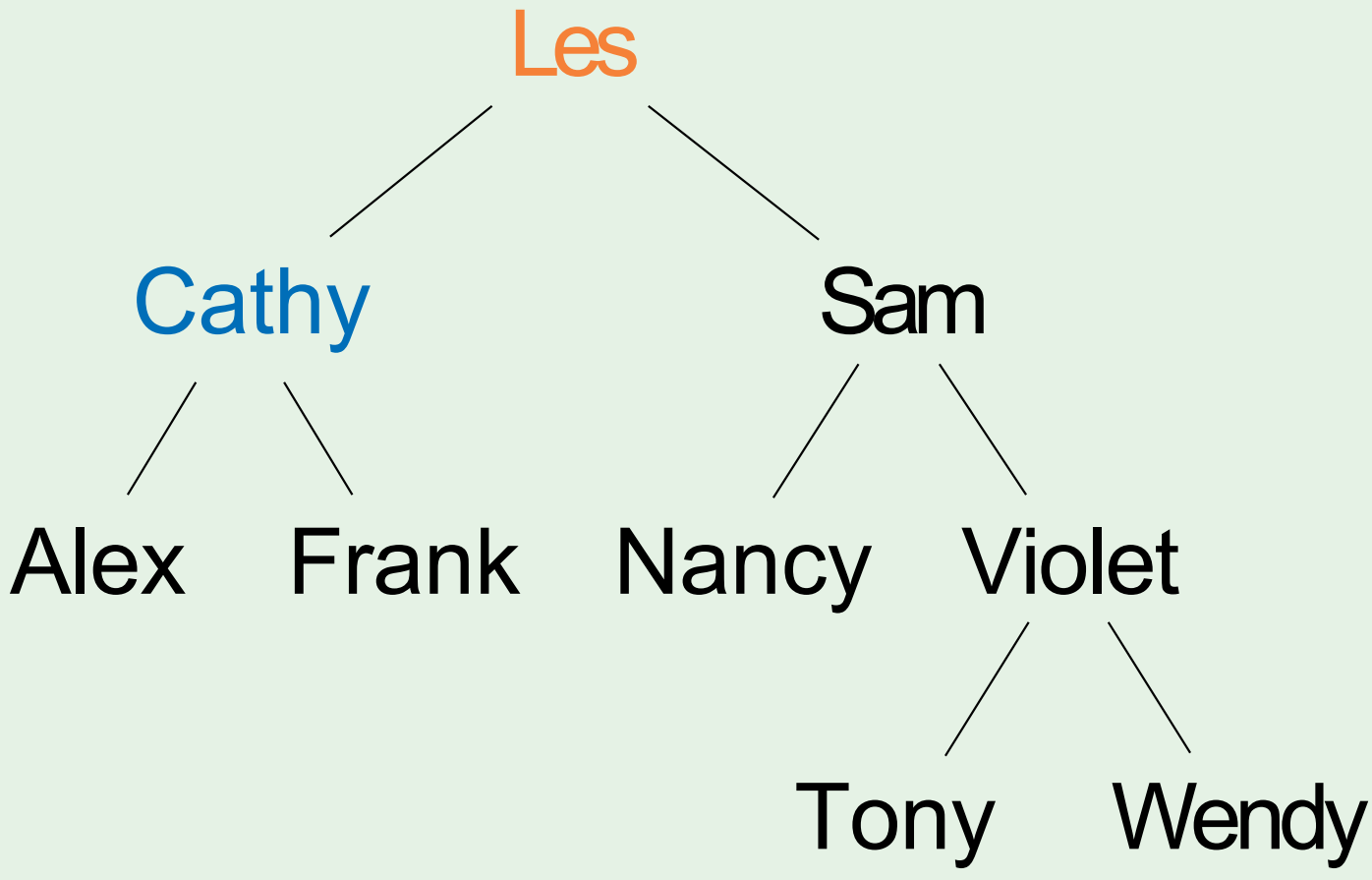
Output: Alex Frank

PostOrderTraversal



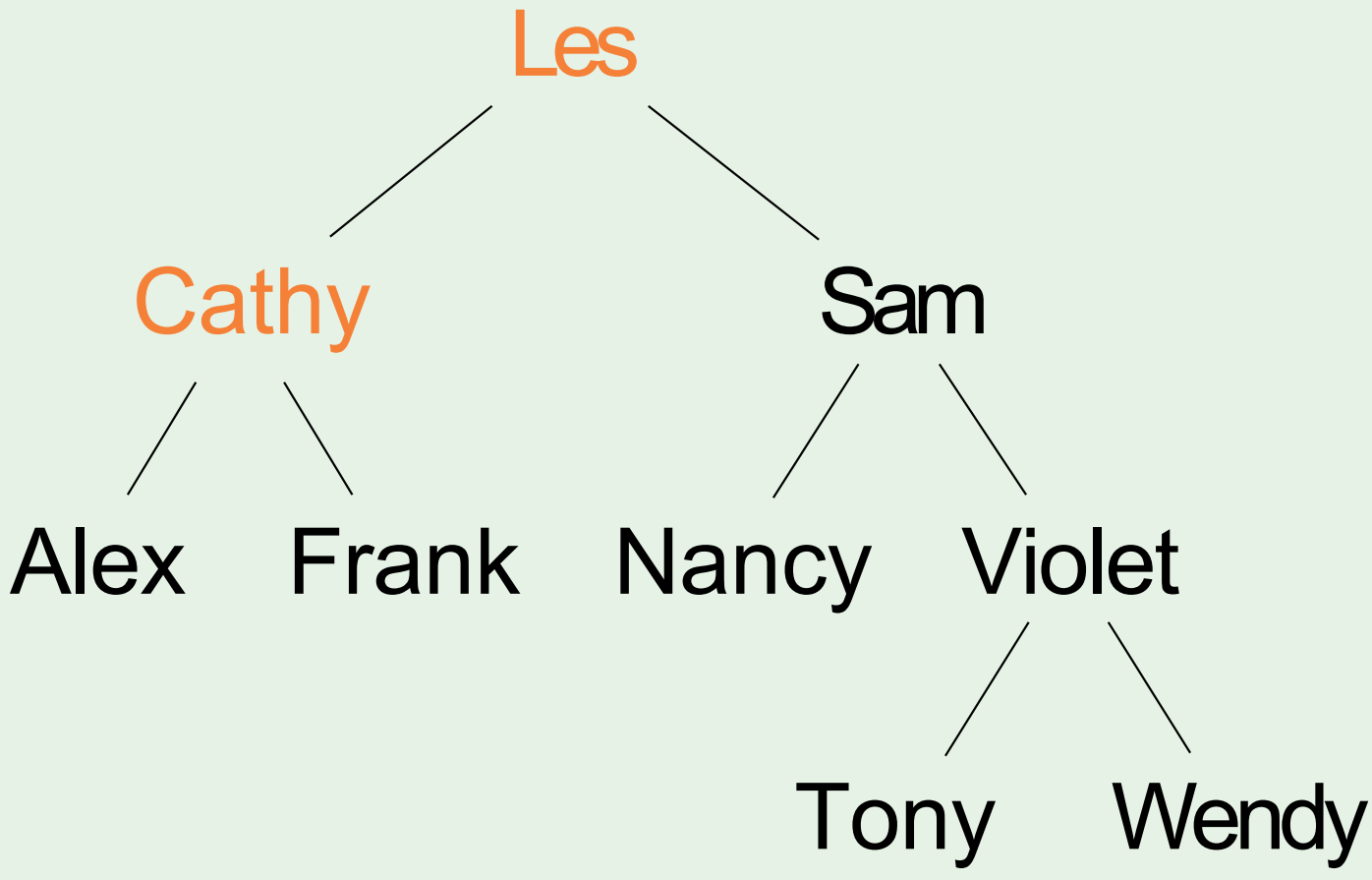
Output: Alex Frank

PostOrderTraversal



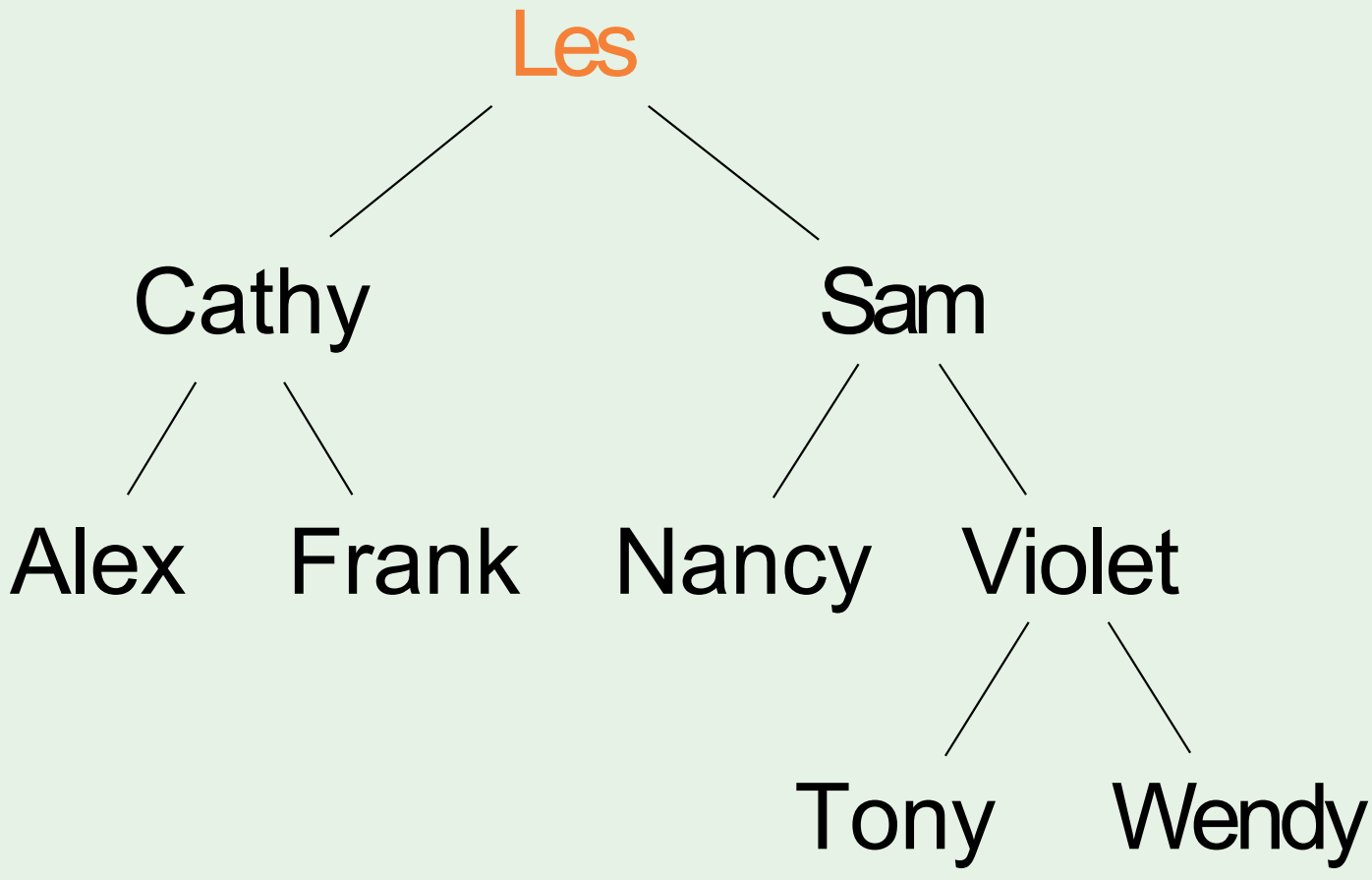
Output: Alex Frank Cathy

PostOrderTraversal



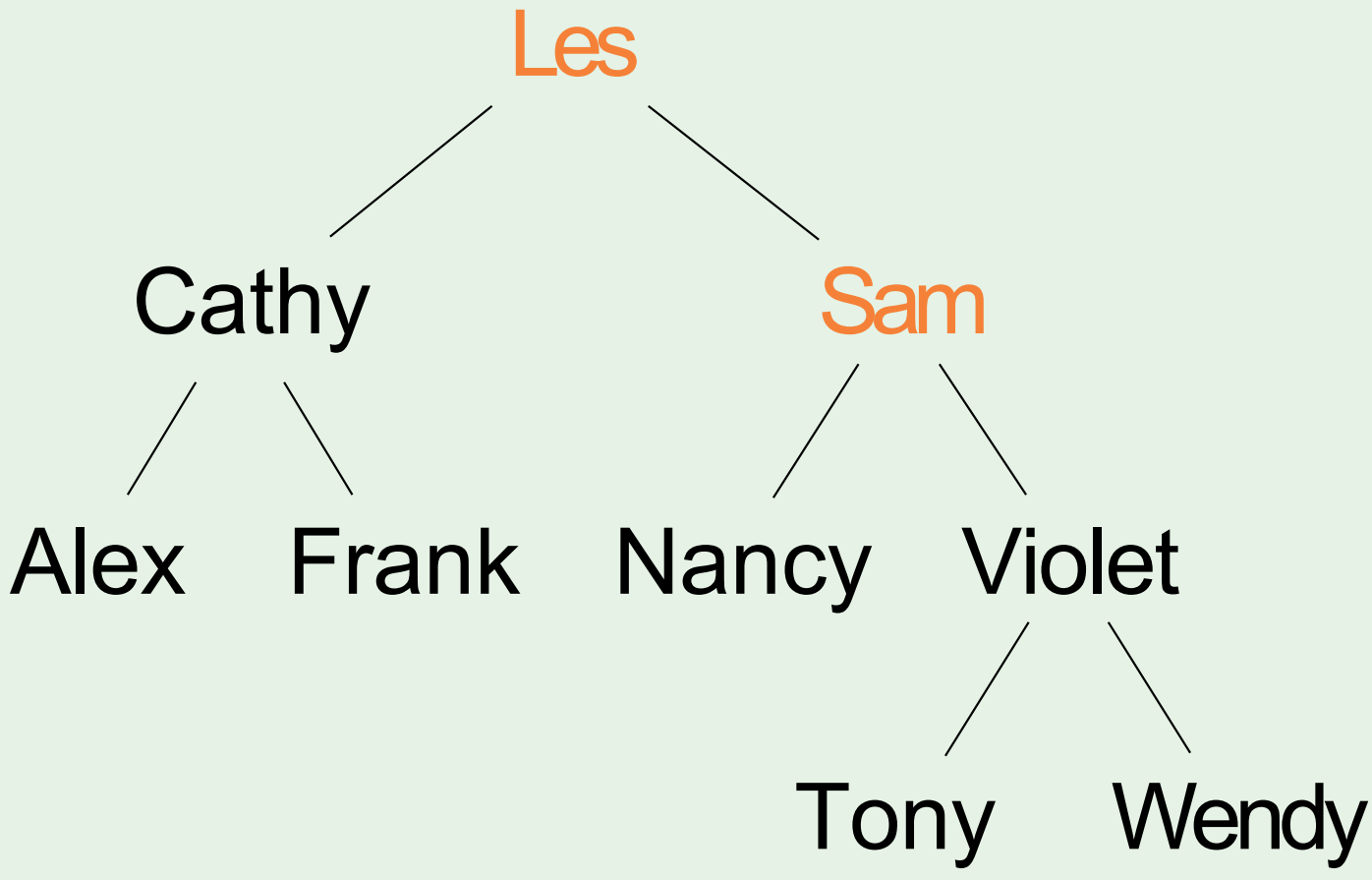
Output: Alex Frank Cathy

PostOrderTraversal



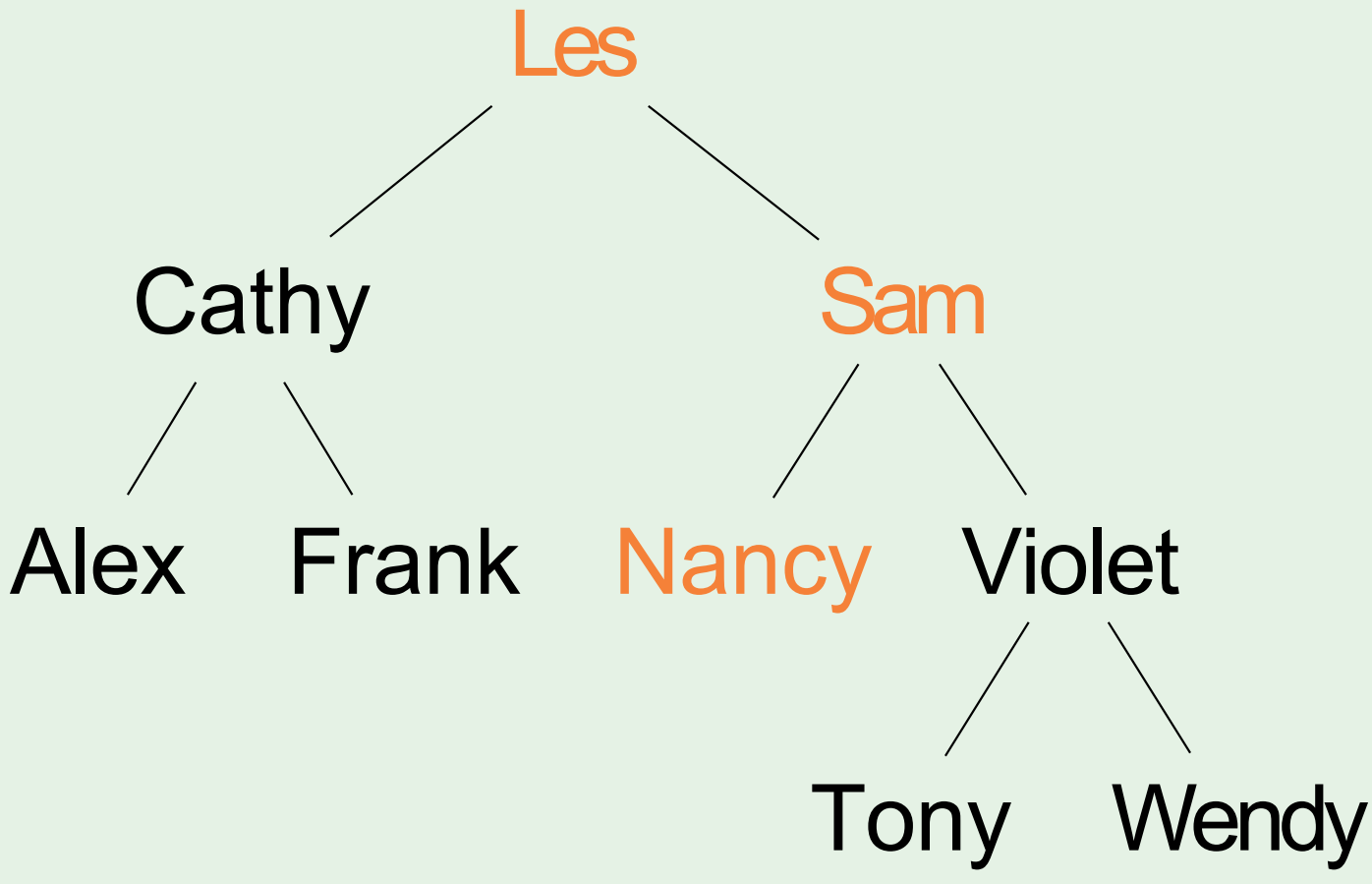
Output: Alex Frank Cathy

PostOrderTraversal



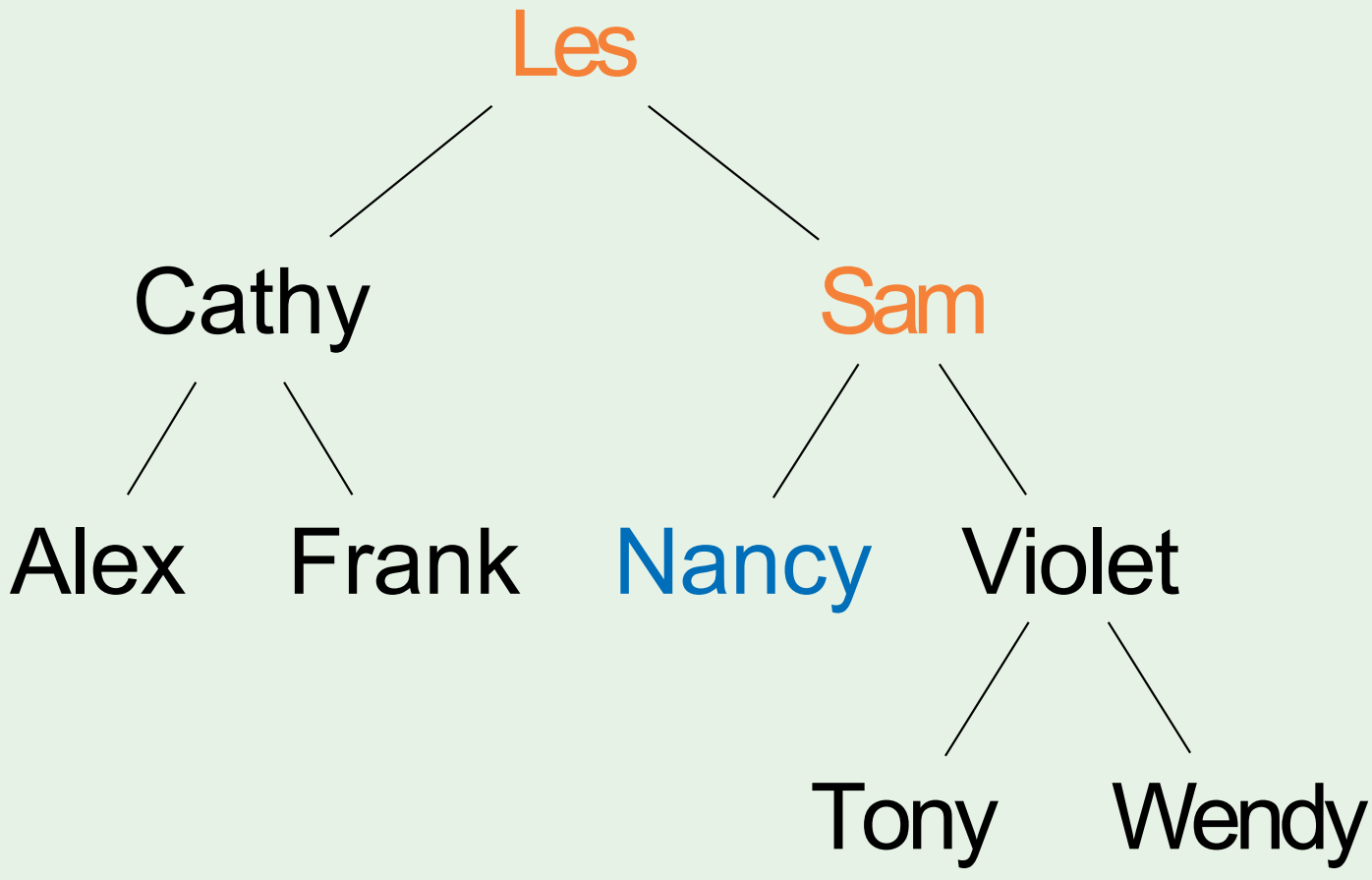
Output: Alex Frank Cathy

PostOrderTraversal



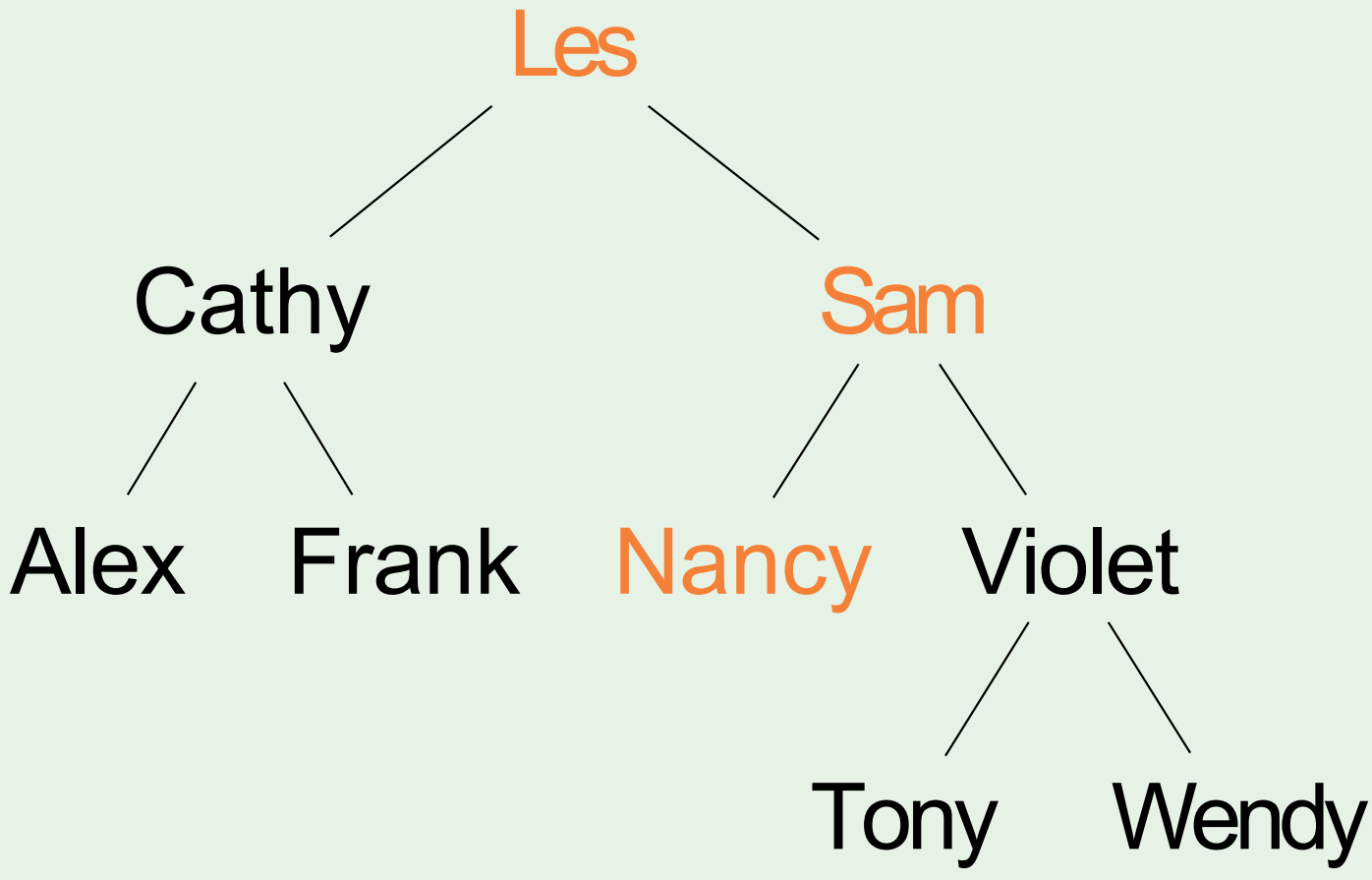
Output: Alex Frank Cathy

PostOrderTraversal



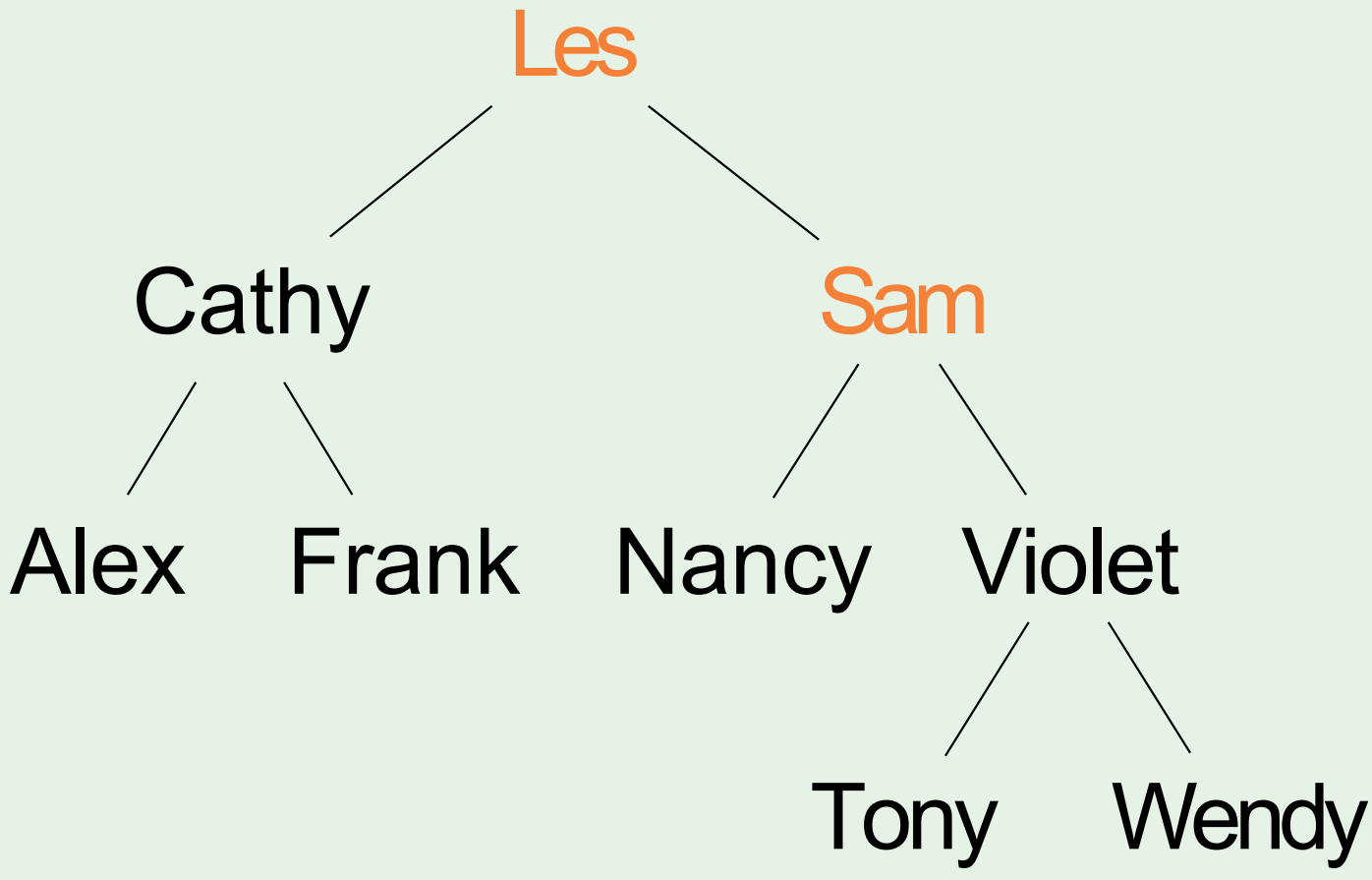
Output: Alex Frank Cathy Nancy

PostOrderTraversal



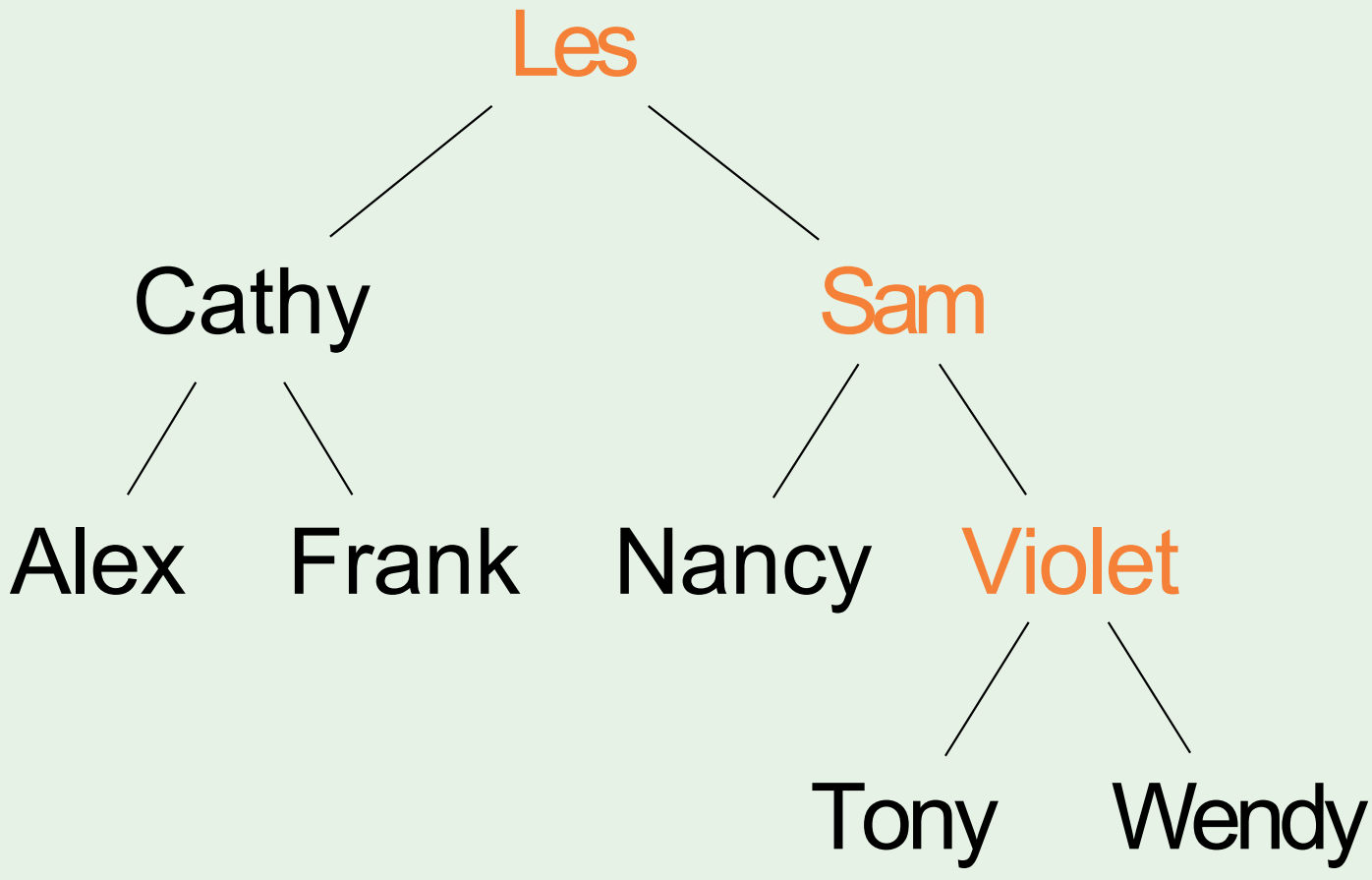
Output: Alex Frank Cathy Nancy

PostOrderTraversal



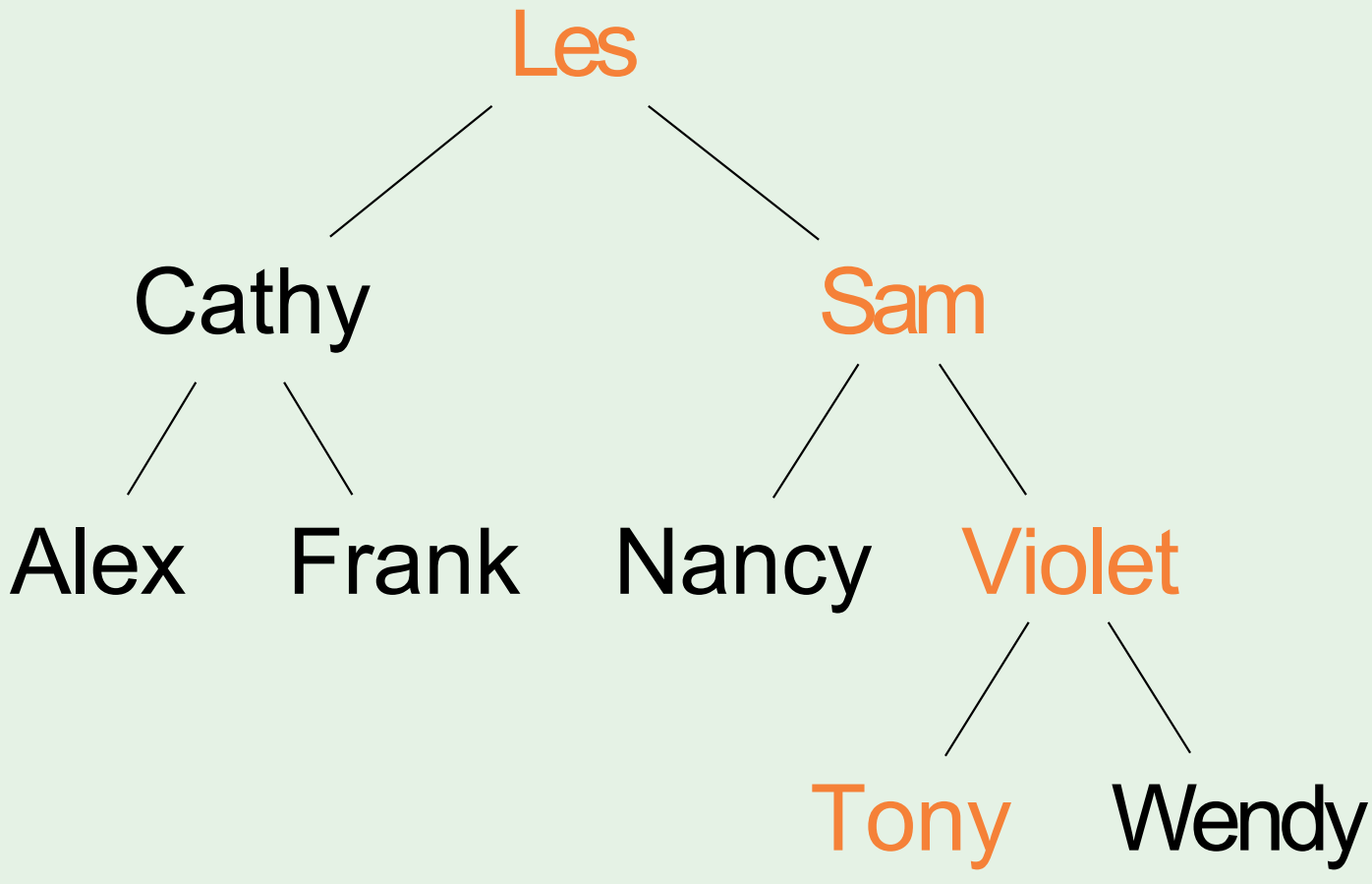
Output: Alex Frank Cathy Nancy

PostOrderTraversal



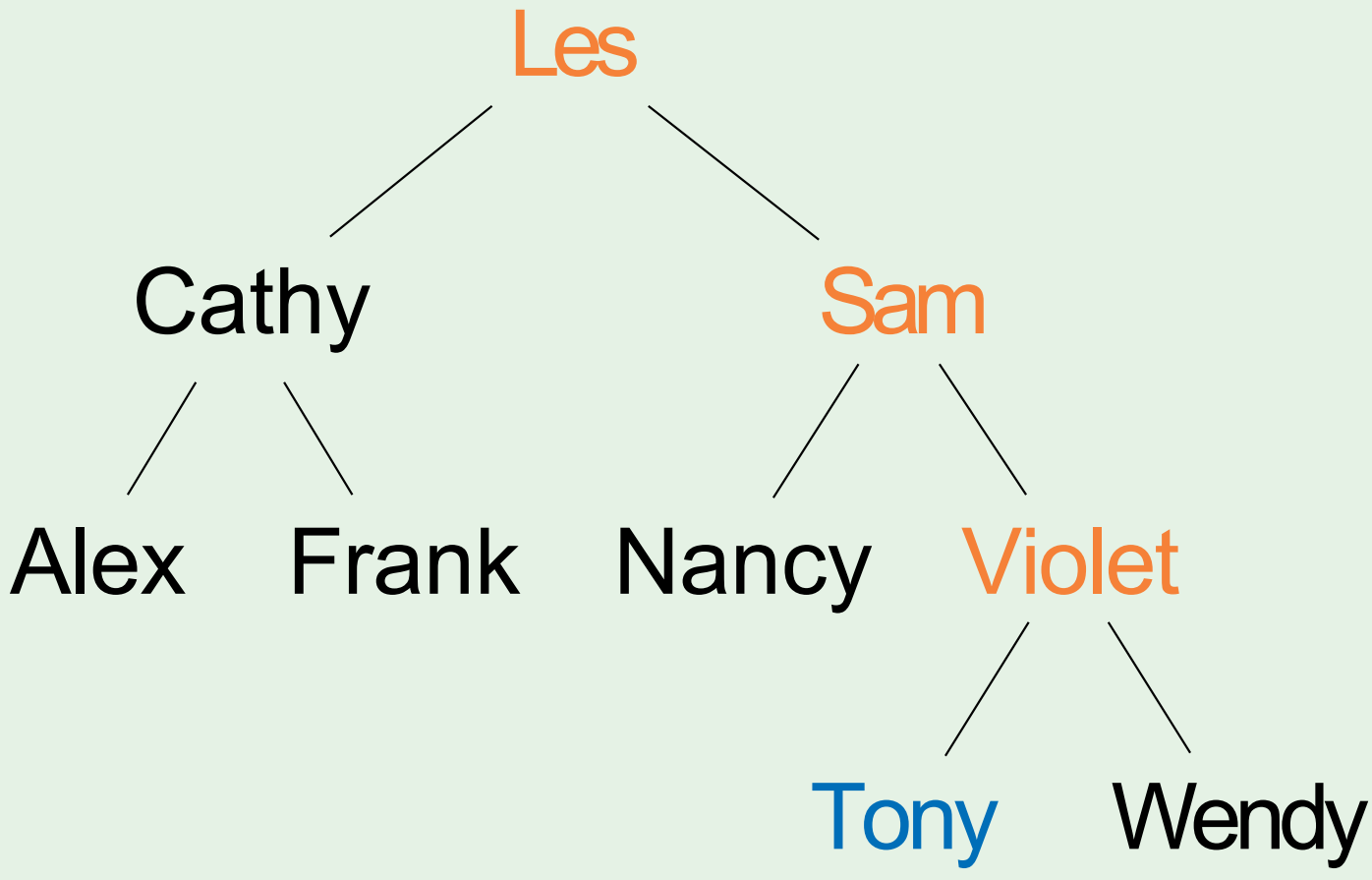
Output: Alex Frank Cathy Nancy

PostOrderTraversal



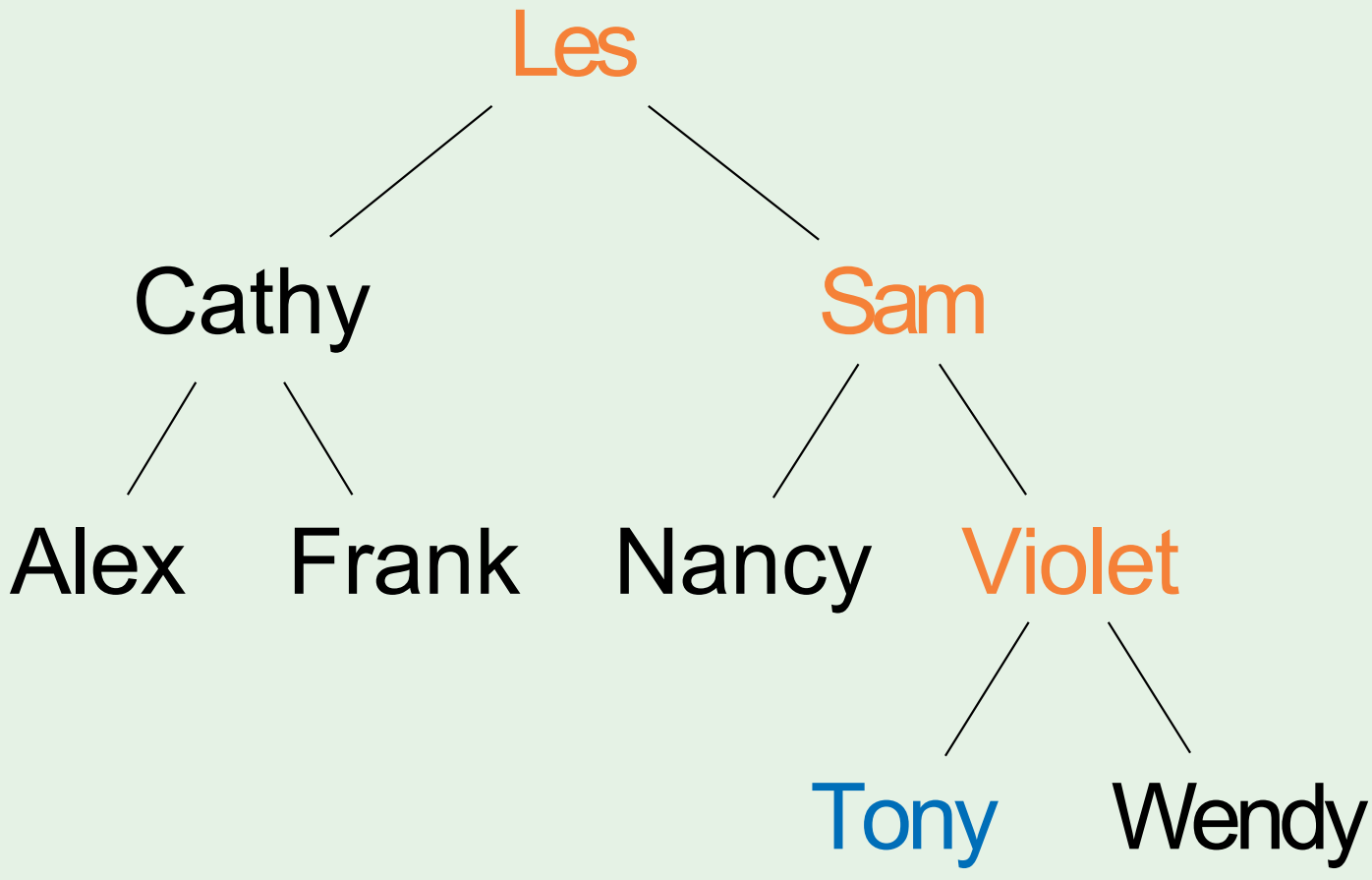
Output: Alex Frank Cathy Nancy

PostOrderTraversal



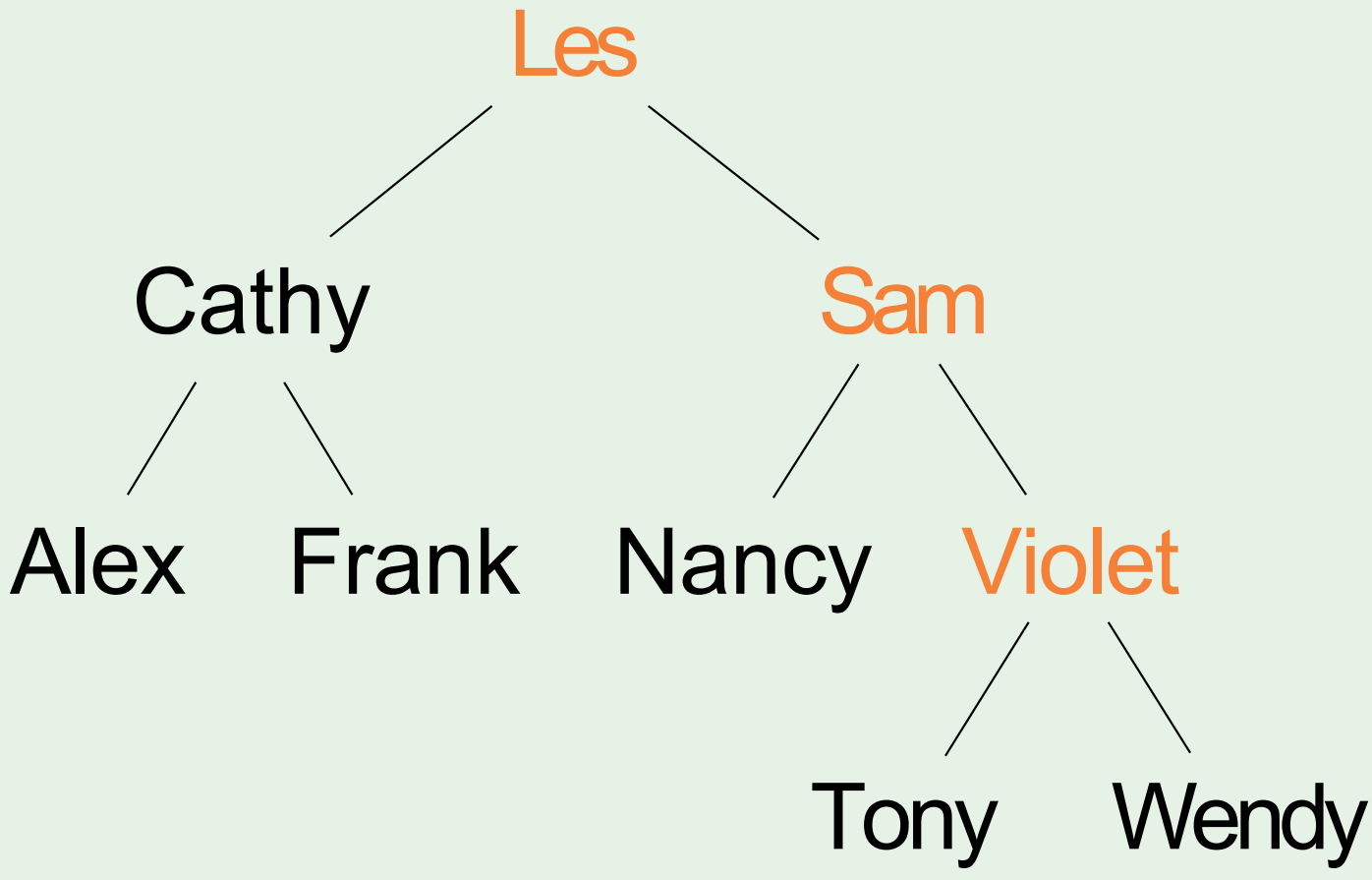
Output: Alex Frank Cathy Nancy Tony

PostOrderTraversal



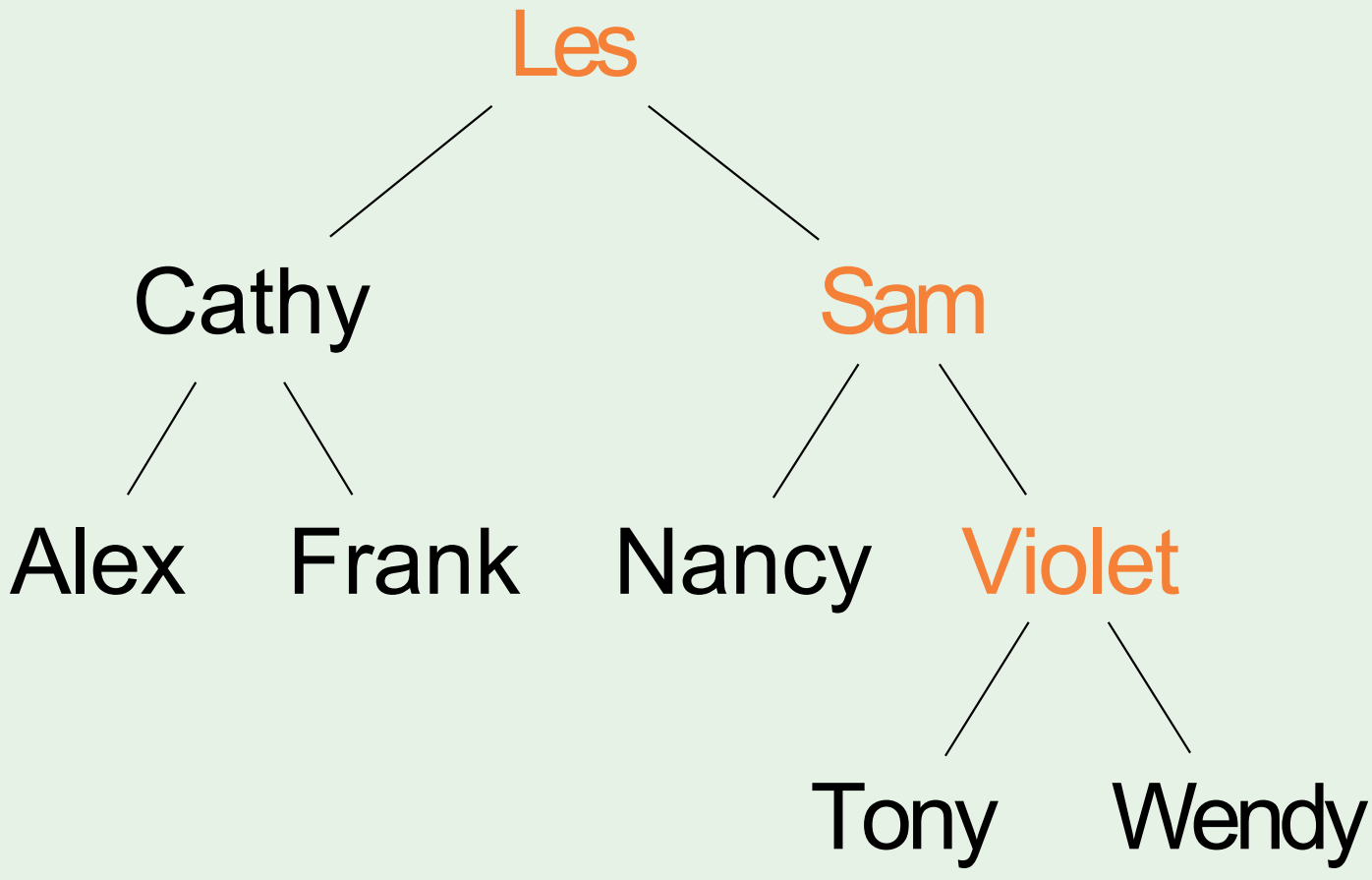
Output: Alex Frank Cathy Nancy Tony

PostOrderTraversal



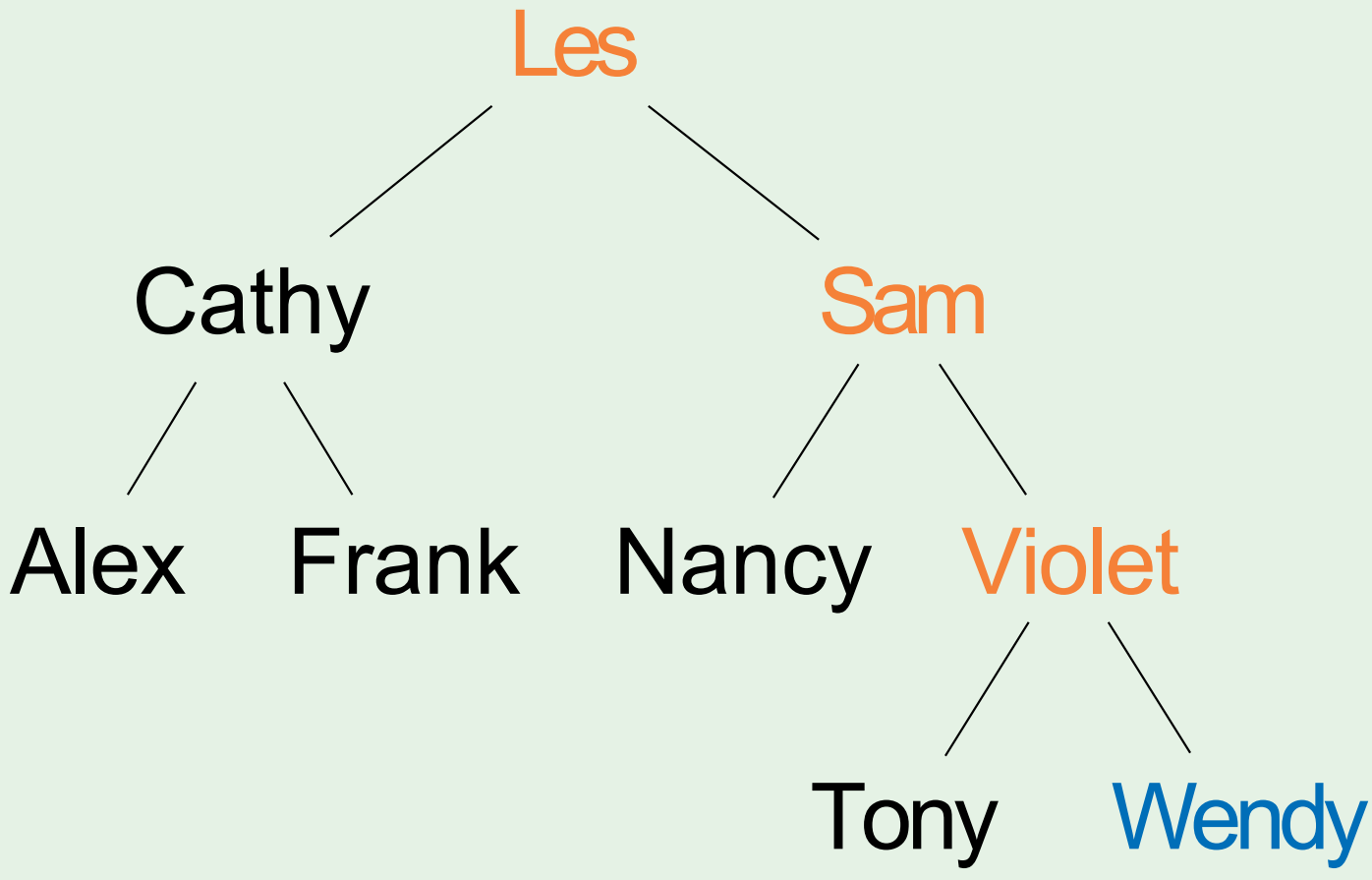
Output: Alex Frank Cathy Nancy Tony

PostOrderTraversal



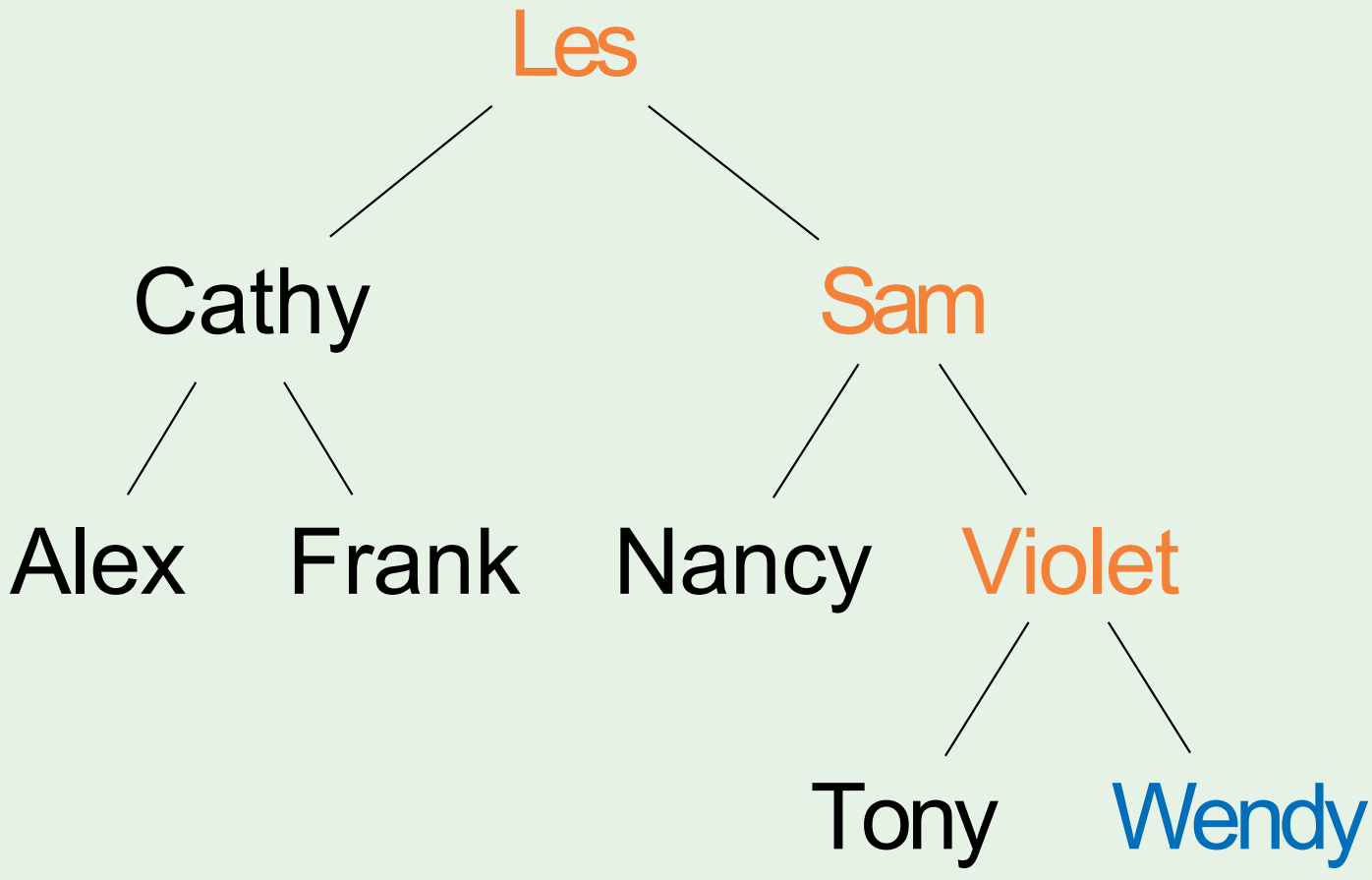
Output: Alex Frank Cathy Nancy Tony

PostOrderTraversal



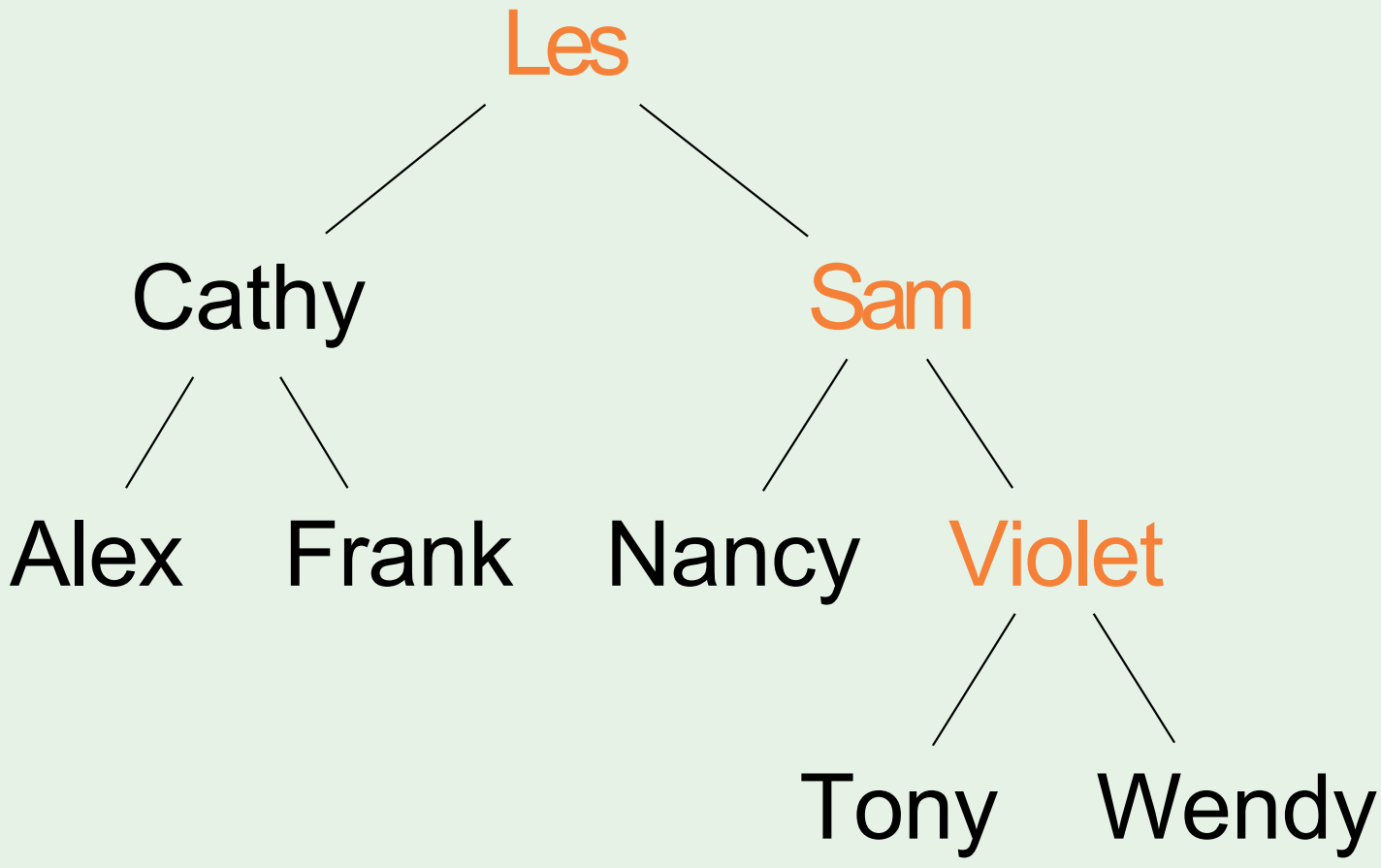
Output: Alex Frank Cathy Nancy Tony Wendy

PostOrderTraversal



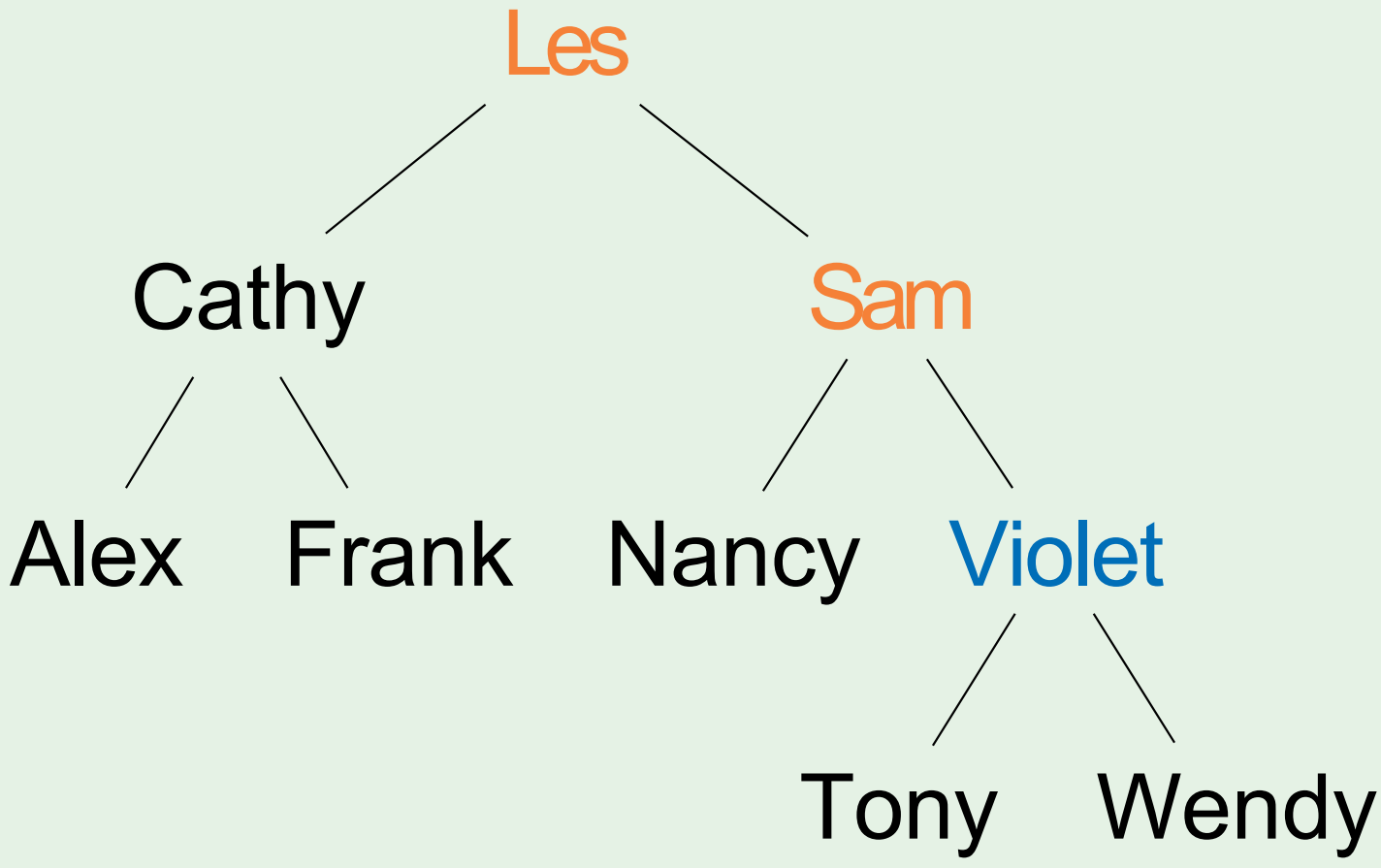
Output: Alex Frank Cathy Nancy Tony Wendy

PostOrderTraversal



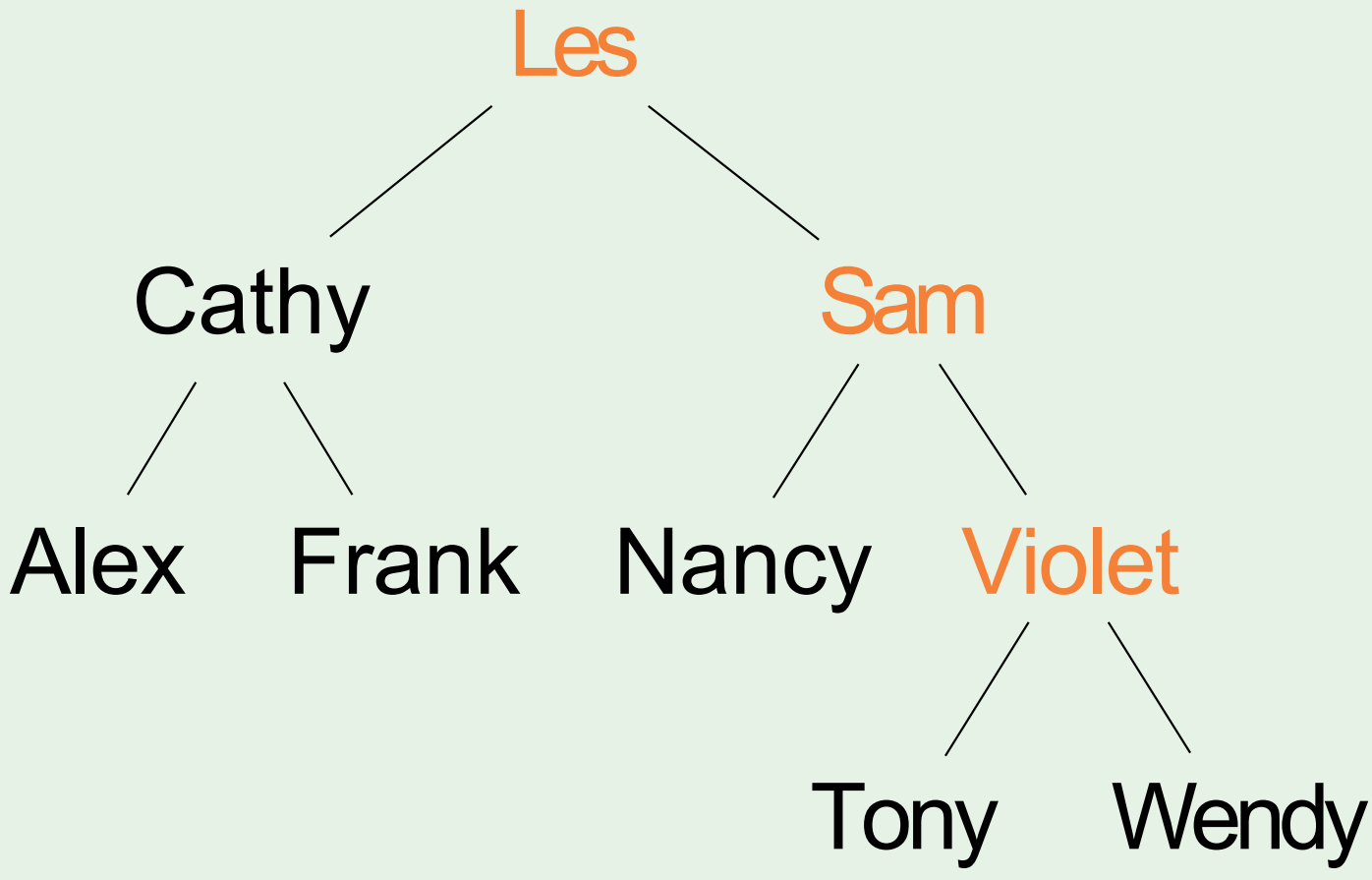
Output: Alex Frank Cathy Nancy Tony Wendy

PostOrderTraversal



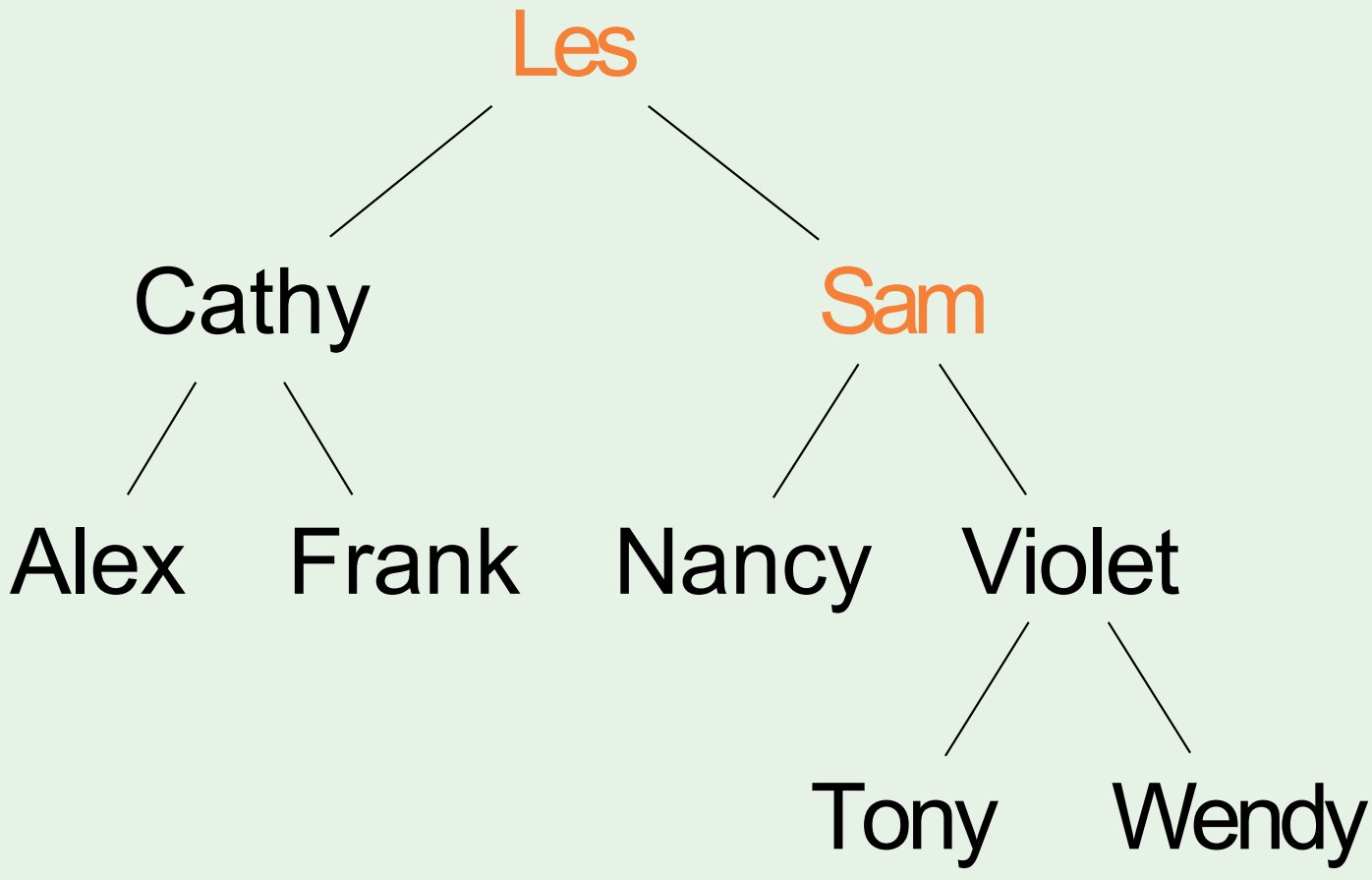
Output: Alex Frank Cathy Nancy Tony Wendy Violet

PostOrderTraversal



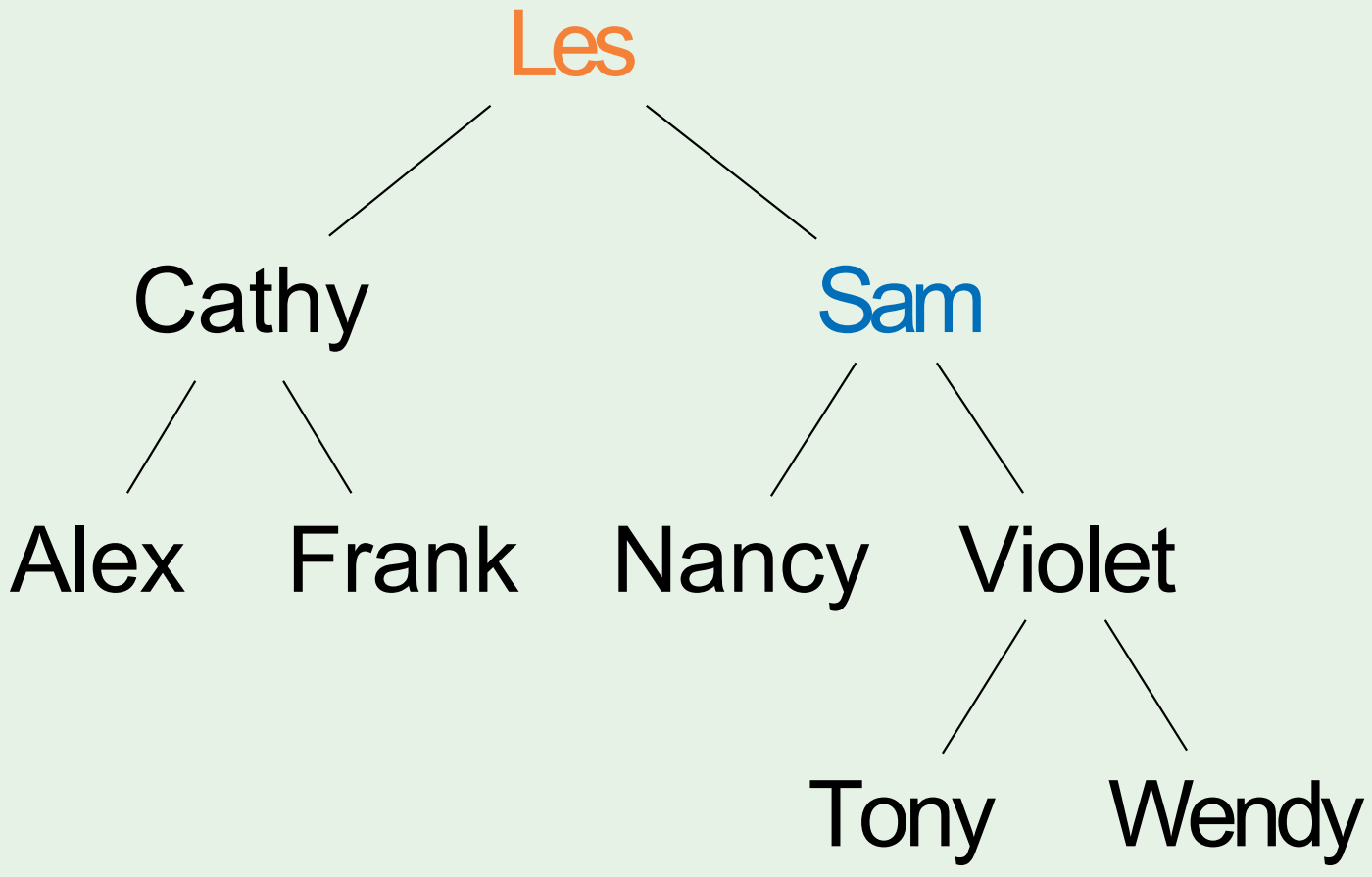
Output: Alex Frank Cathy Nancy Tony Wendy Violet

PostOrderTraversal



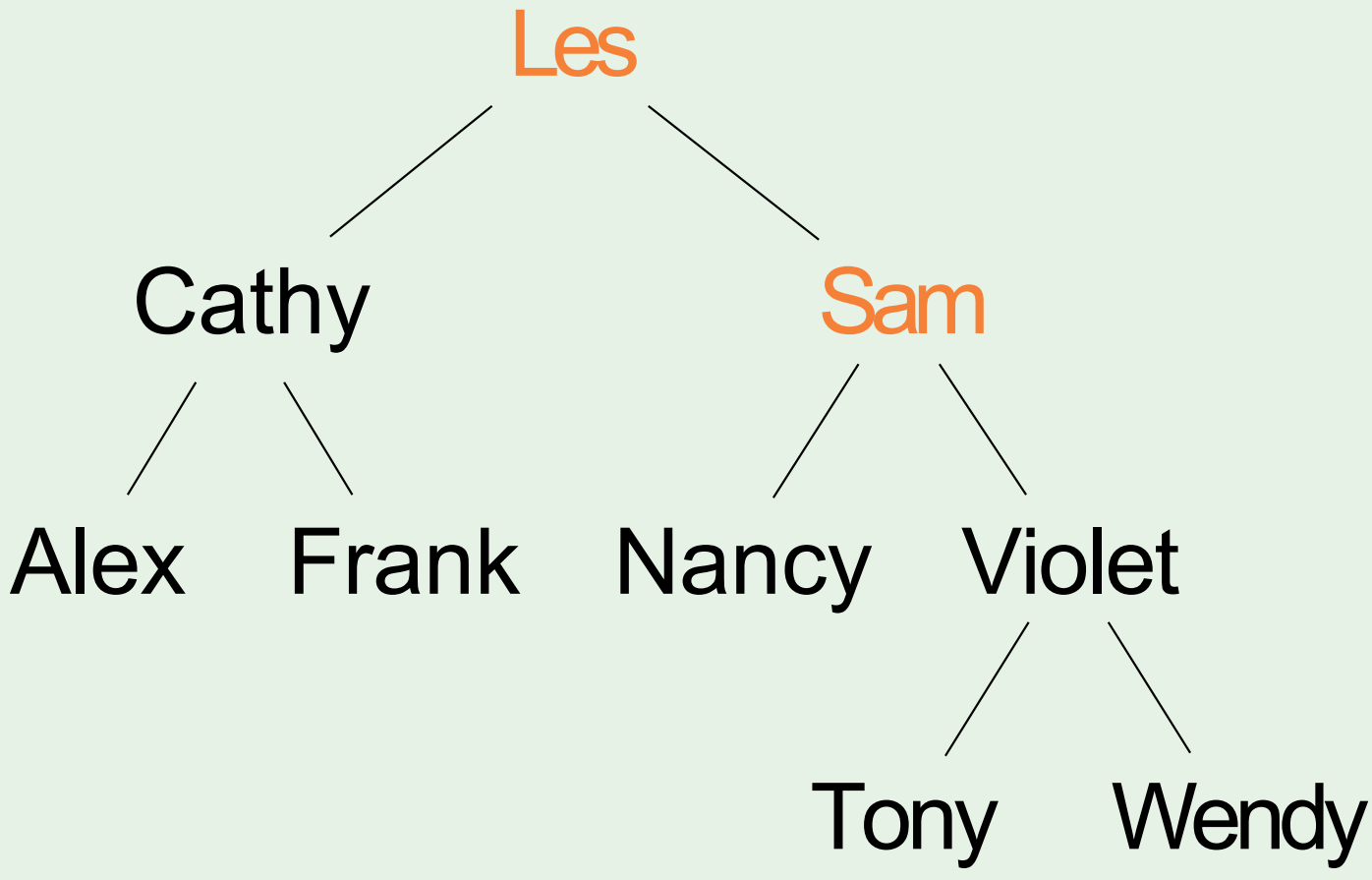
Output: Alex Frank Cathy Nancy Tony Wendy Violet

PostOrderTraversal



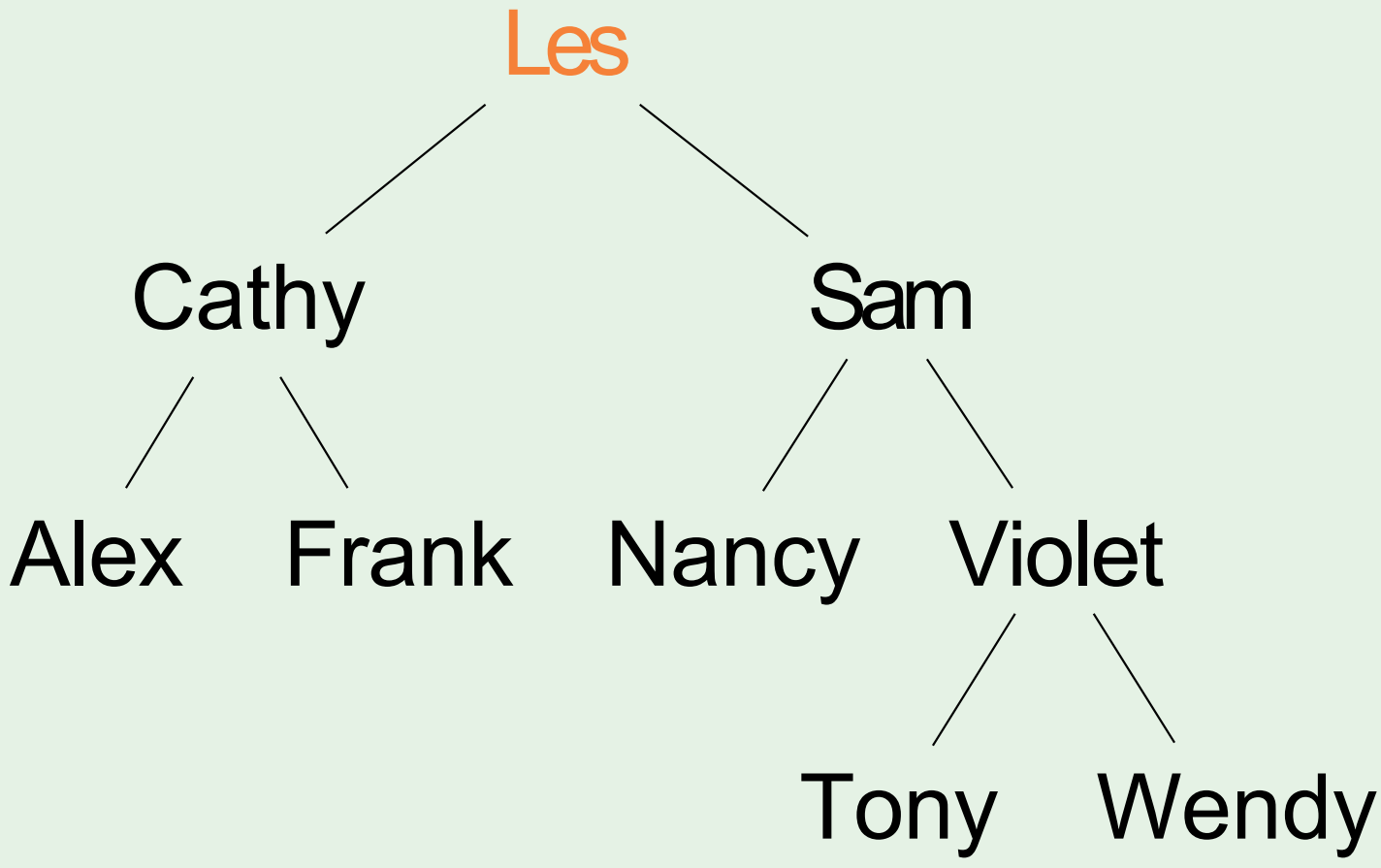
Output: Alex Frank Cathy Nancy Tony Wendy Violet Sam

PostOrderTraversal



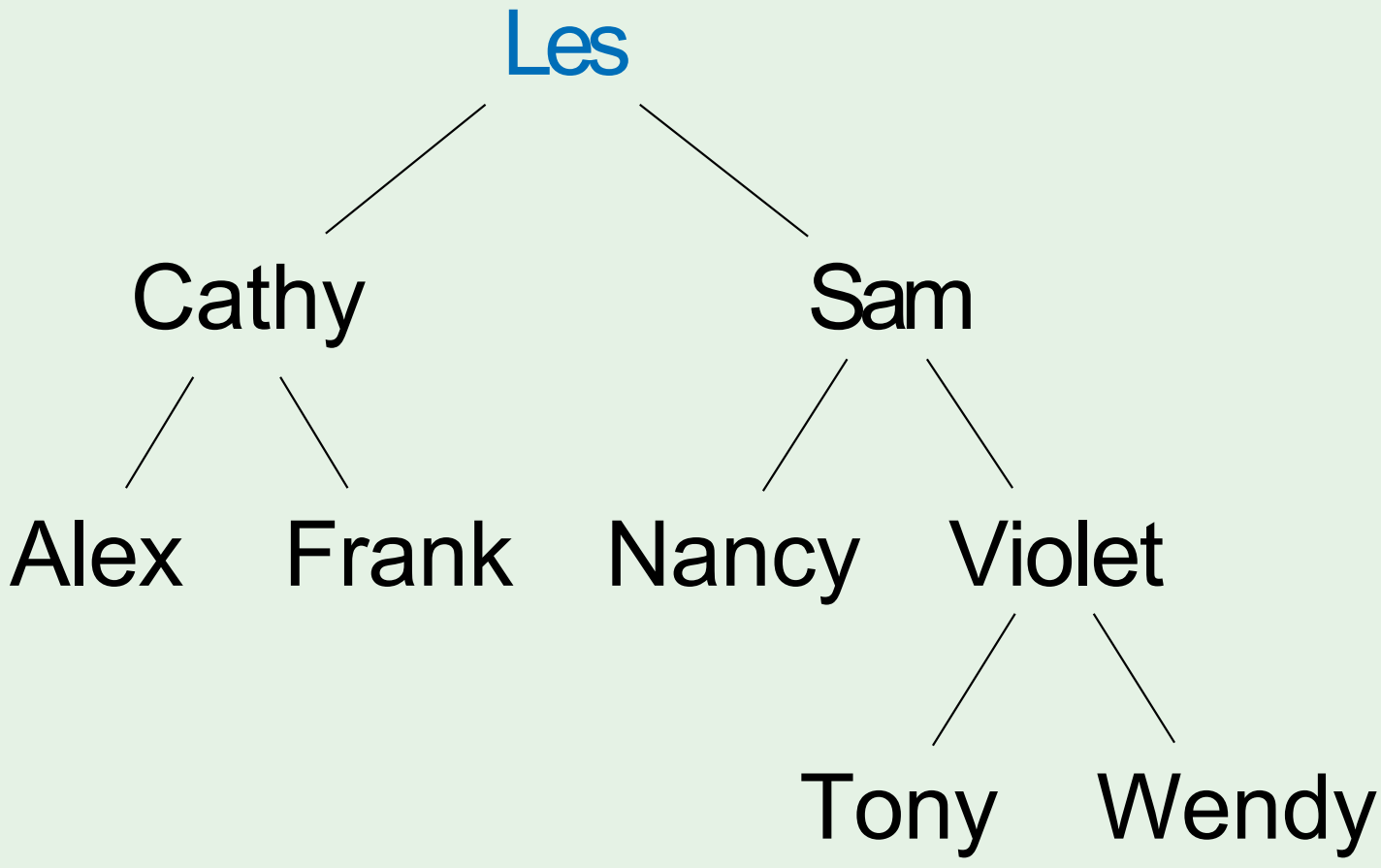
Output: Alex Frank Cathy Nancy Tony Wendy Violet Sam

PostOrderTraversal



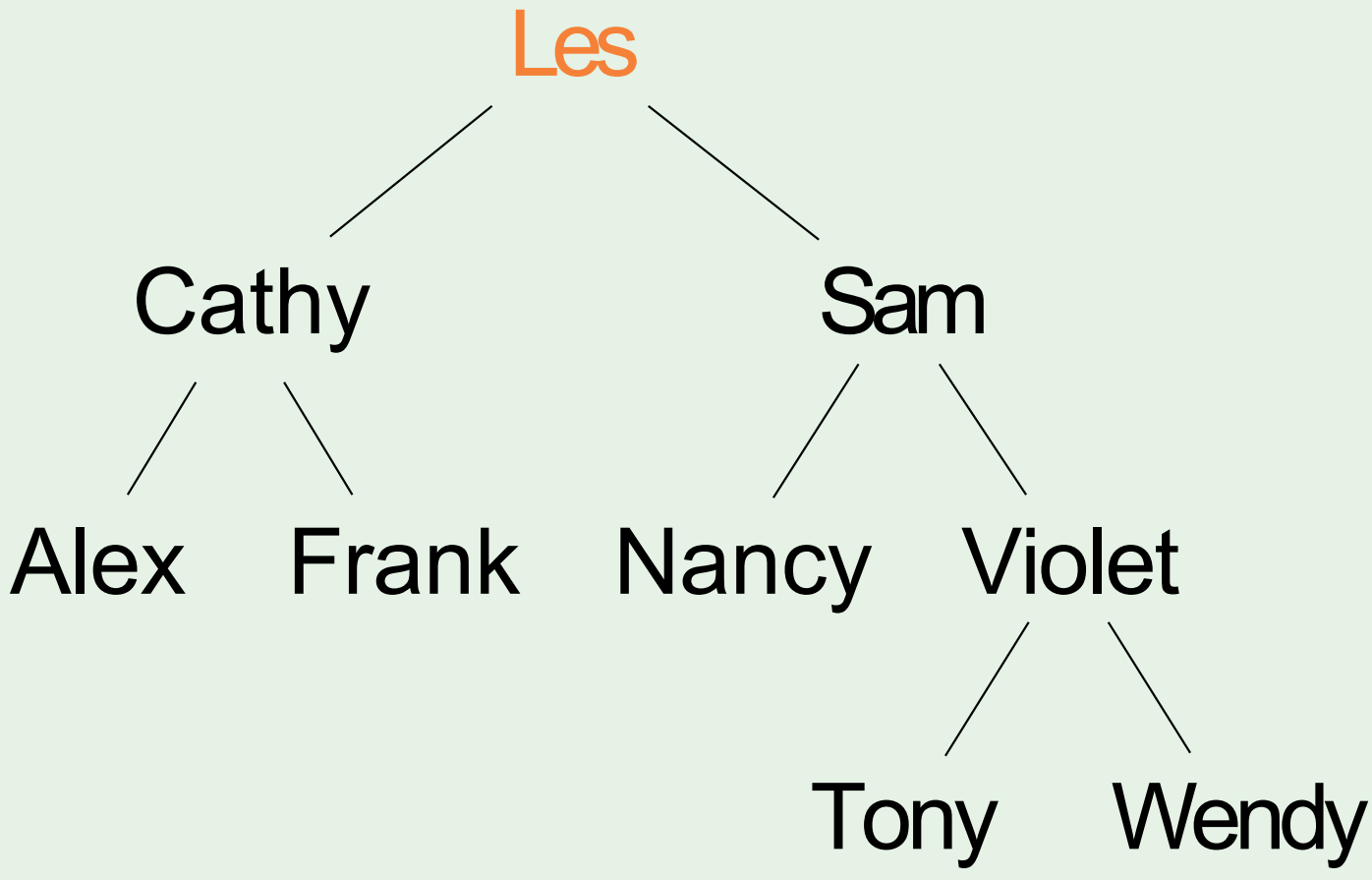
Output: Alex Frank Cathy Nancy Tony Wendy Violet Sam

PostOrderTraversal



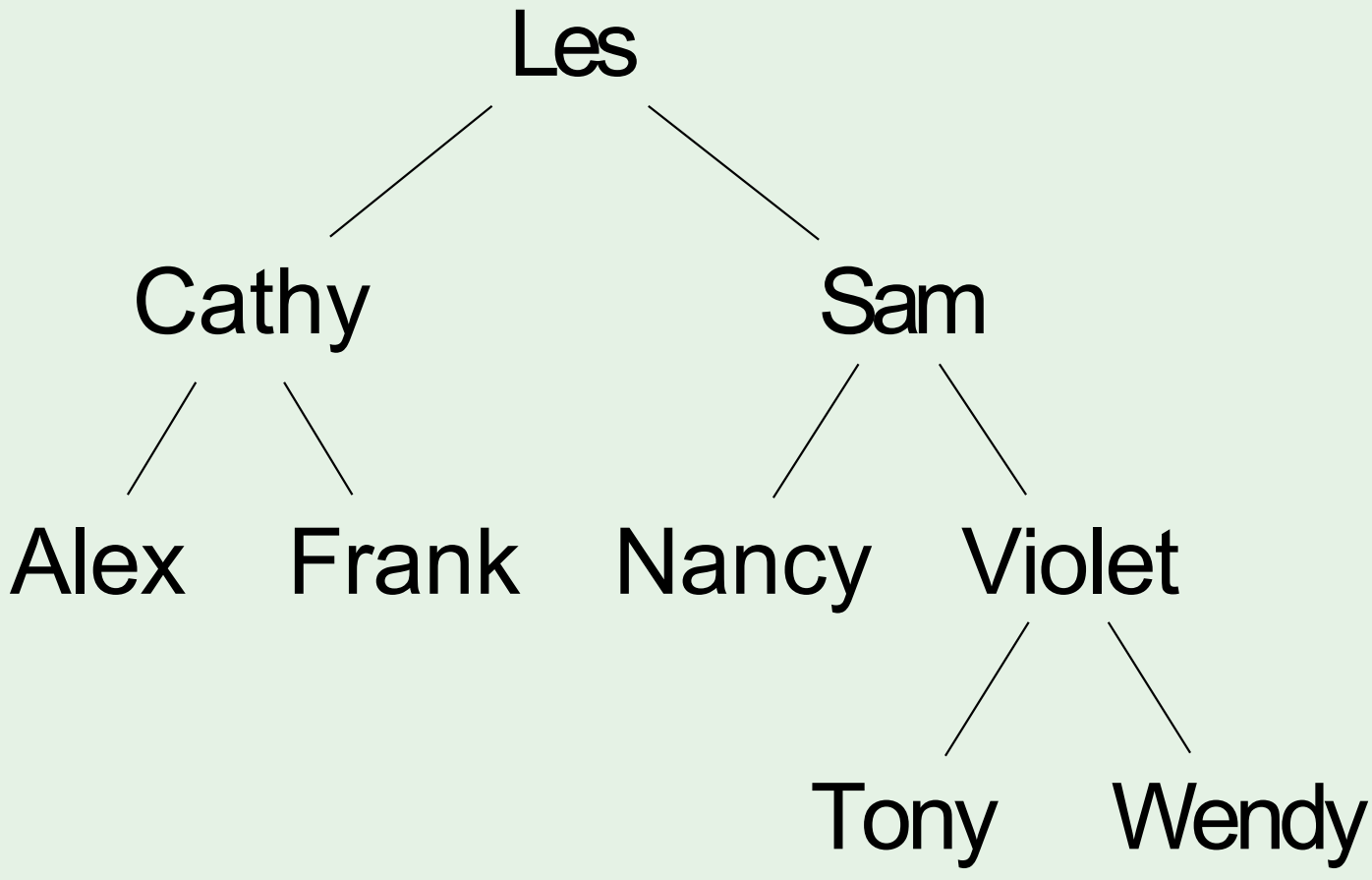
Output: Alex Frank Cathy Nancy Tony Wendy Violet Sam Les

PostOrderTraversal



Output: Alex Frank Cathy Nancy Tony Wendy Violet Sam Les

PostOrderTraversal



Output: Alex Frank Cathy Nancy Tony Wendy Violet Sam Les

Breadth-first

```
LevelTraversal(tree)
if tree = nil :  return

Queue q
q.Enqueue(tree)
```

Breadth-first

```
LevelTraversal(tree)
if tree = nil :  return

Queue q
q.Enqueue(tree)
while not q.Empty() :
    node ← q.Dequeue()
```

Breadth-first

```
LevelTraversal(tree)
if tree = nil :  return

Queue q
q.Enqueue(tree)
while not q.Empty() :
    node ← q.Dequeue()
    Print(node)
```

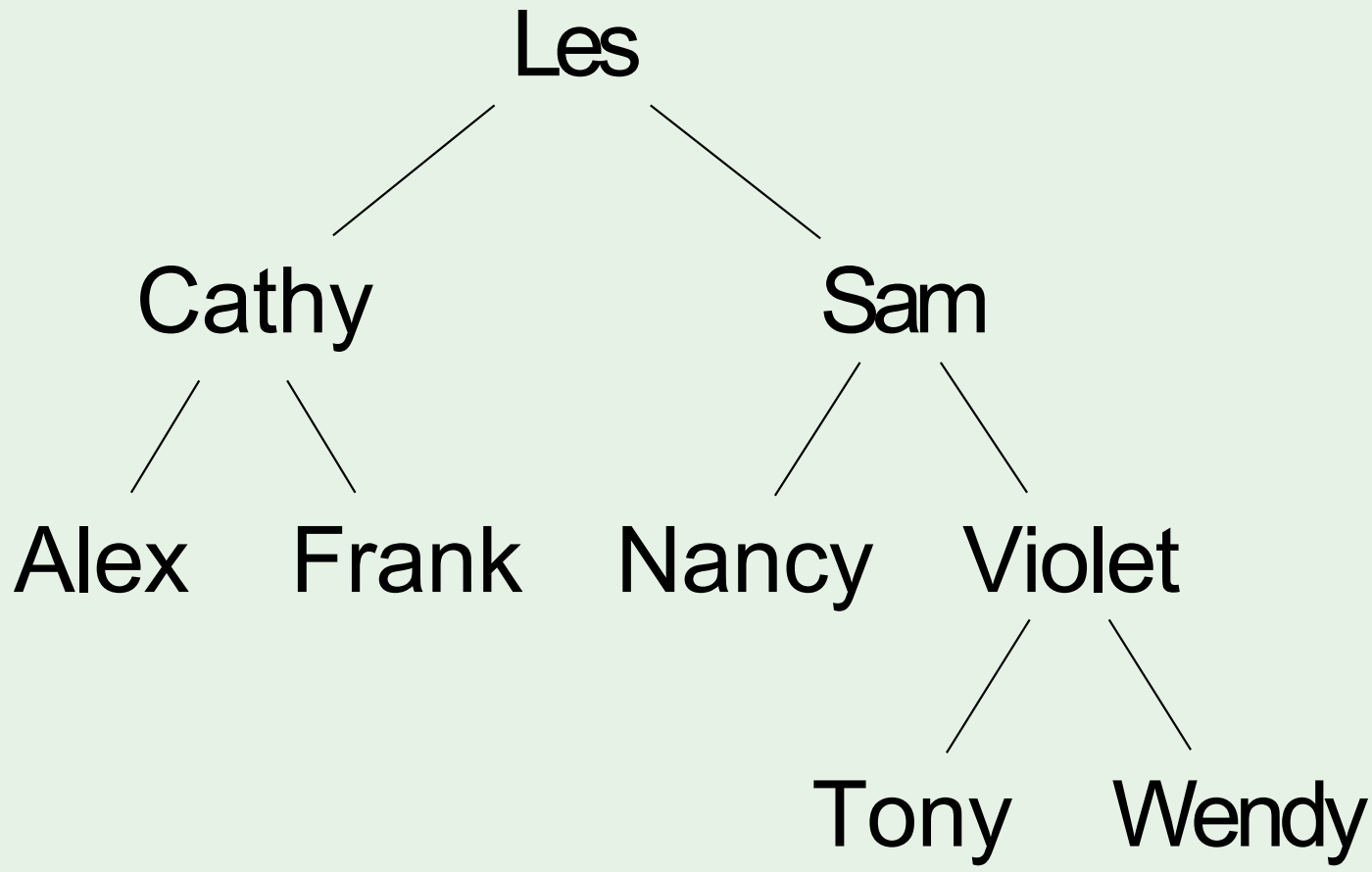
Breadth-first

```
LevelTraversal(tree)
if tree = nil : return

Queue q
q.Enqueue(tree)
while not q.Empty():
    node ← q.Dequeue()
    Print(node)
    if node.left ≠ nil :
        q.Enqueue(node.left)
```

Breadth-first

```
LevelTraversal(tree)
if tree = nil : return

Queue q
q.Enqueue(tree)
while not q.Empty():
    node ← q.Dequeue()
    Print(node)
    if node.left ≠ nil :
        q.Enqueue(node.left)
    if node.right ≠ nil :
        q.Enqueue(node.right)
```

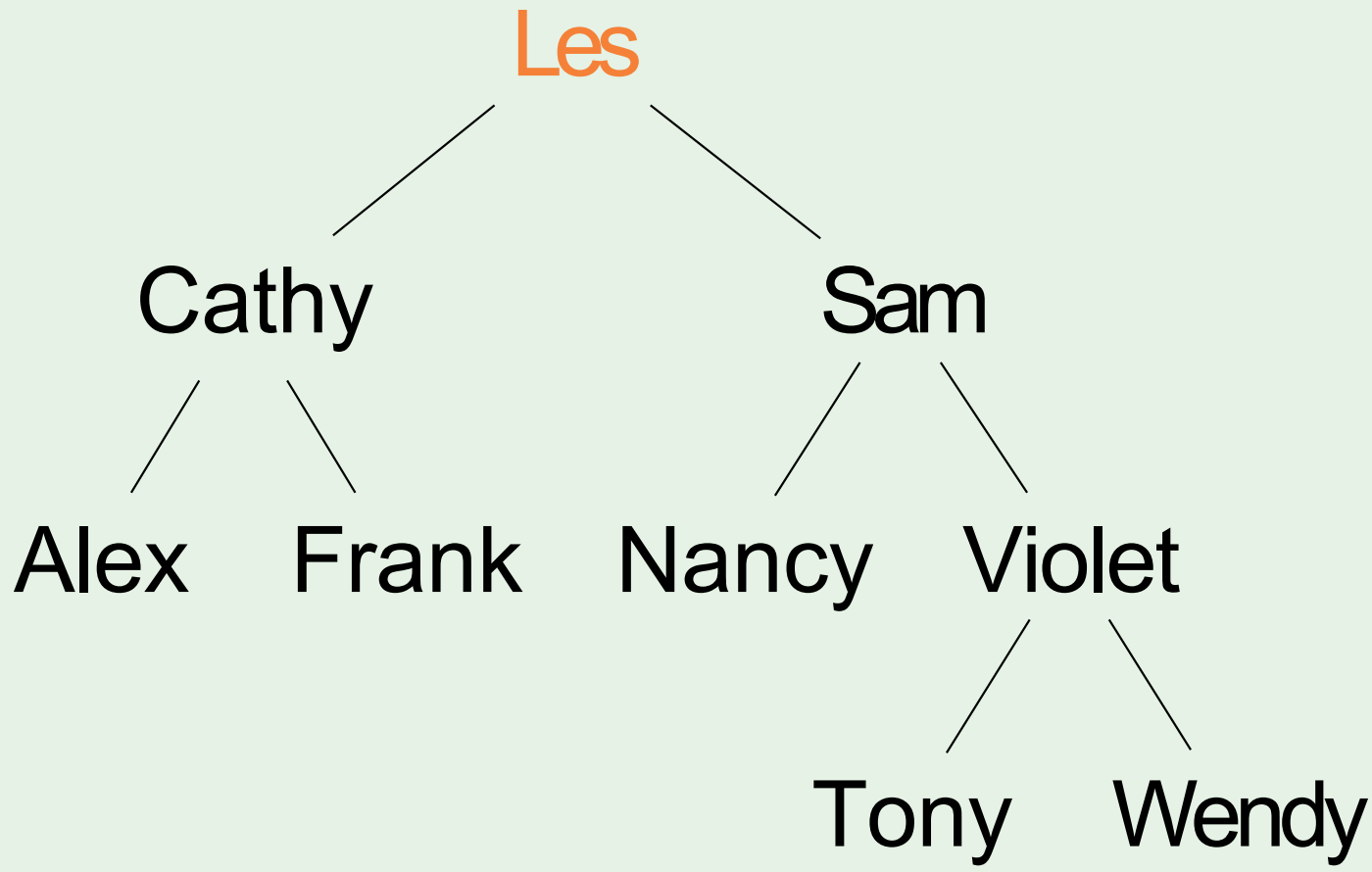
LevelTraversal



Output:

Queue: Les

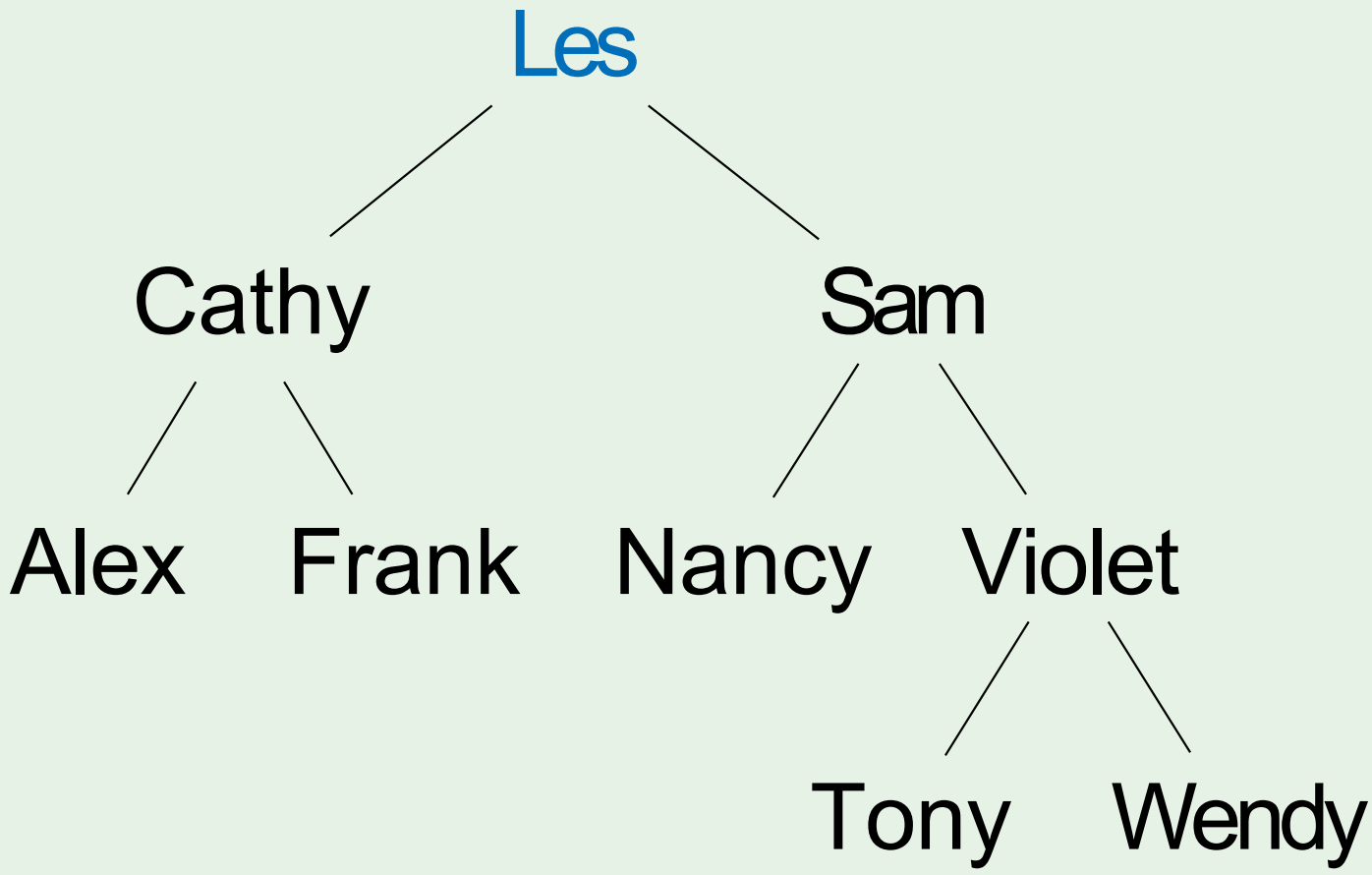
LevelTraversal



Output:

Queue:

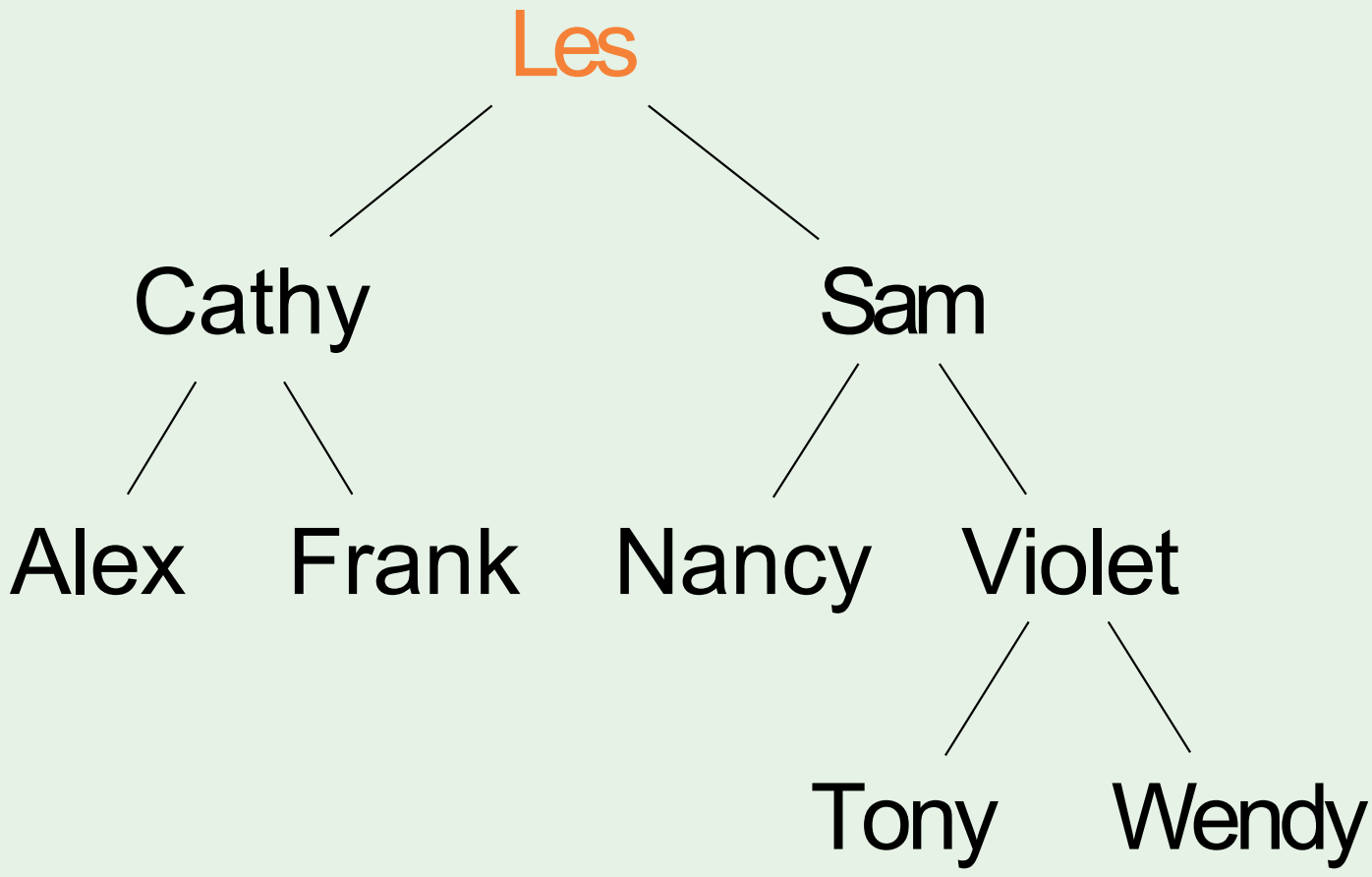
LevelTraversal



Output: Les

Queue:

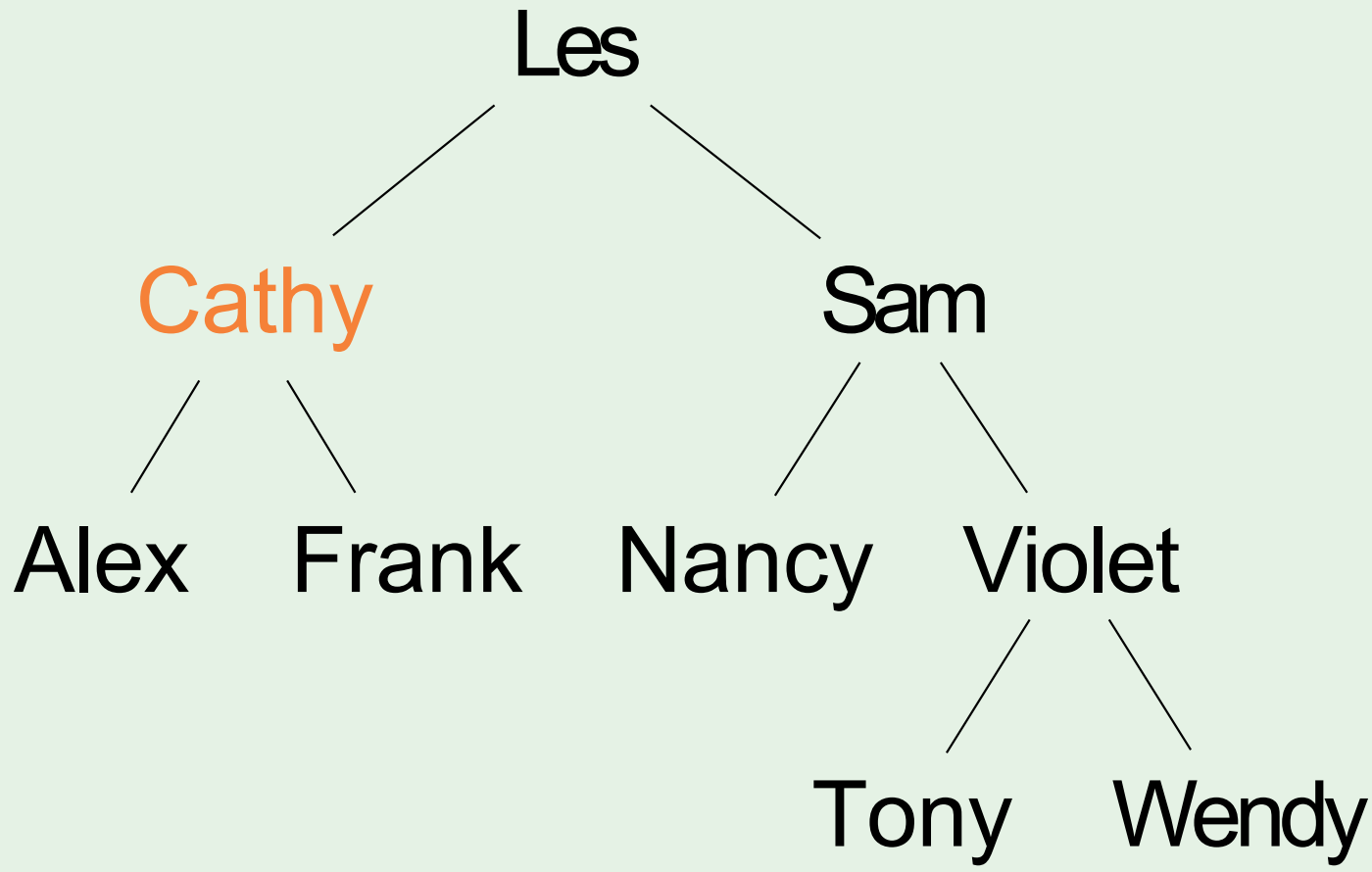
LevelTraversal



Output:
Les

Queue: Cathy, Sam

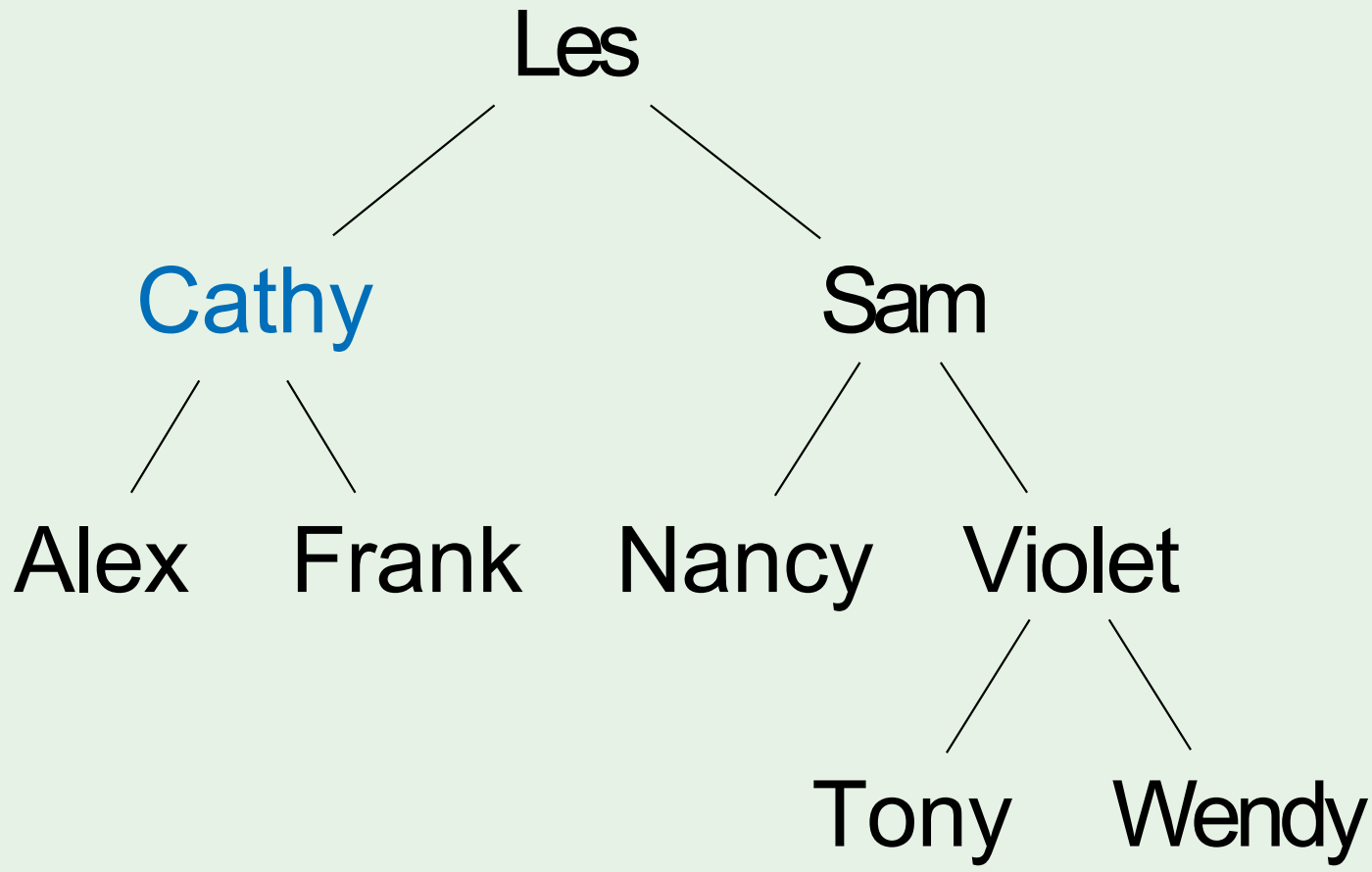
LevelTraversal



Output: Les

Queue: Sam

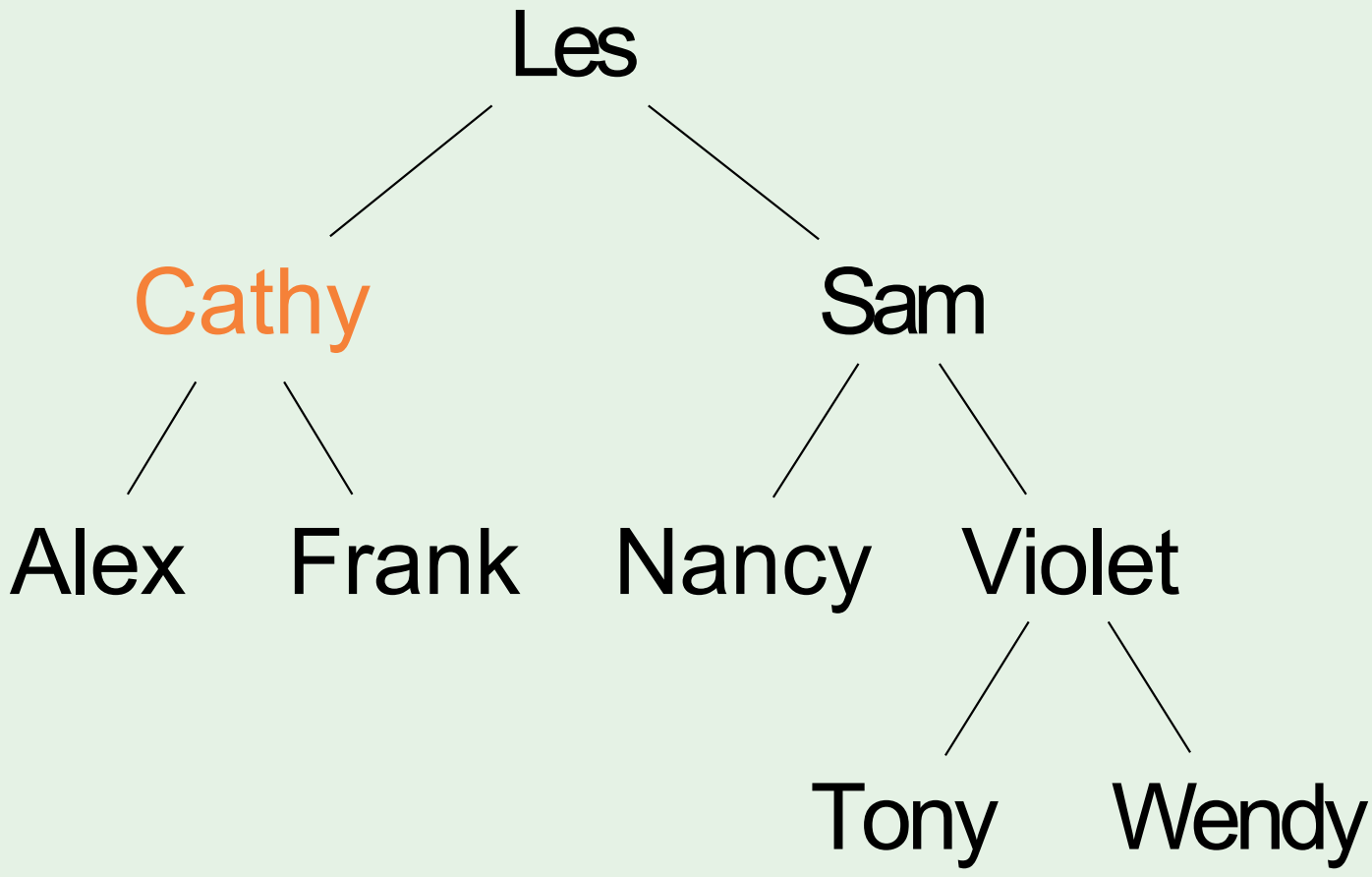
LevelTraversal



Output: Les Cathy

Queue: Sam

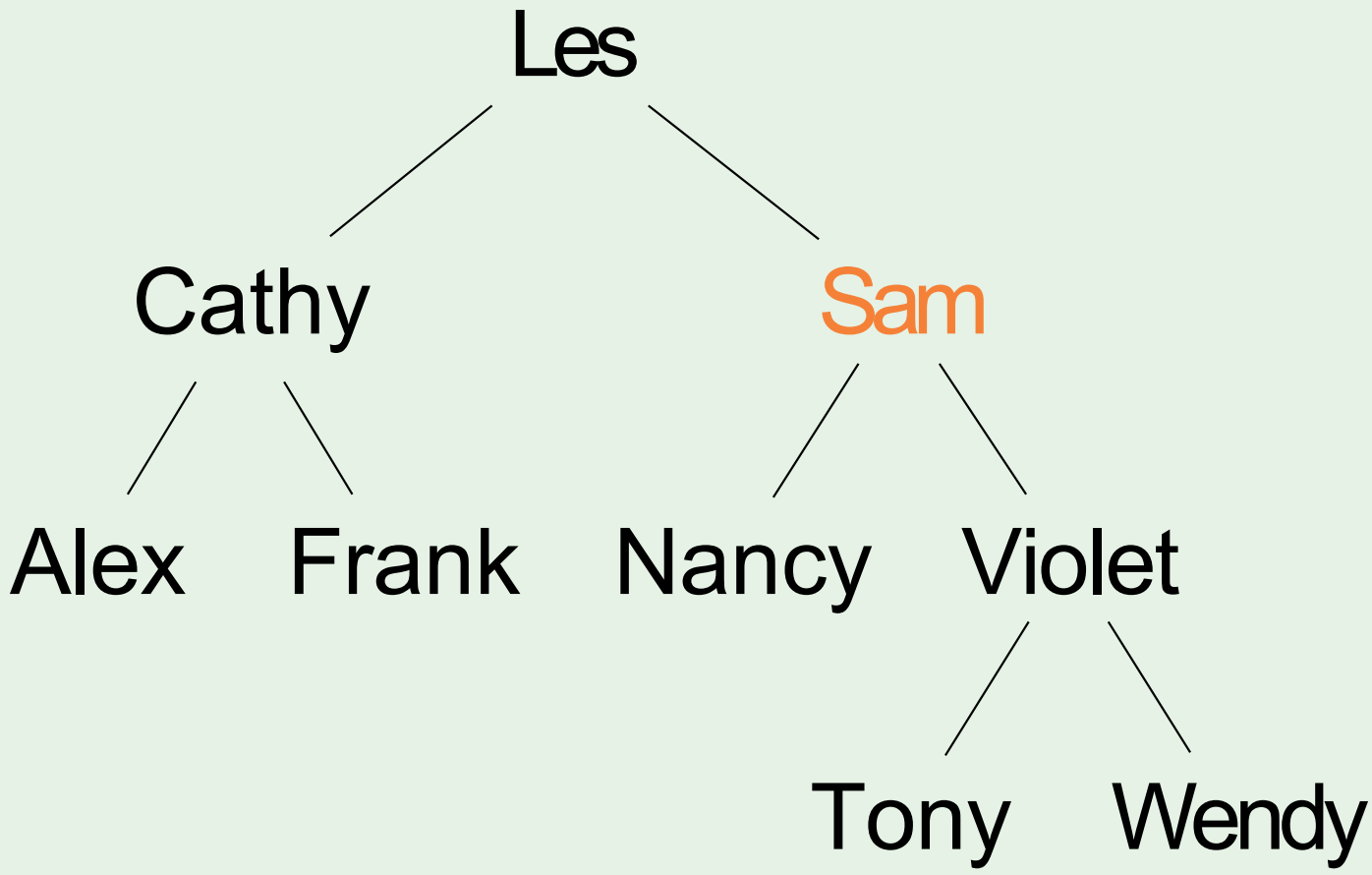
LevelTraversal



Output: Les Cathy

Queue: Sam,Alex, Frank

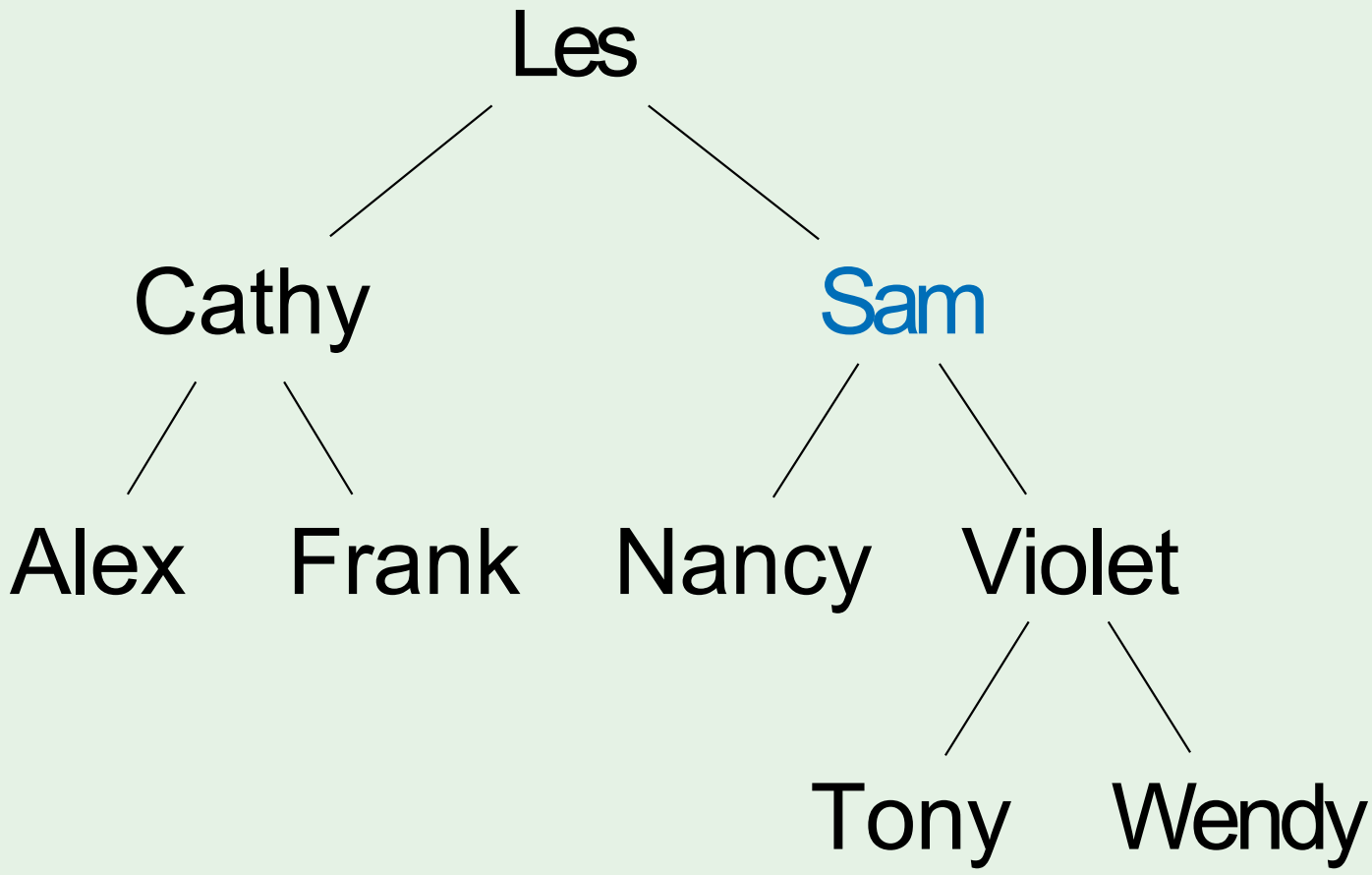
LevelTraversal



Output: Les Cathy

Queue: Alex, Frank

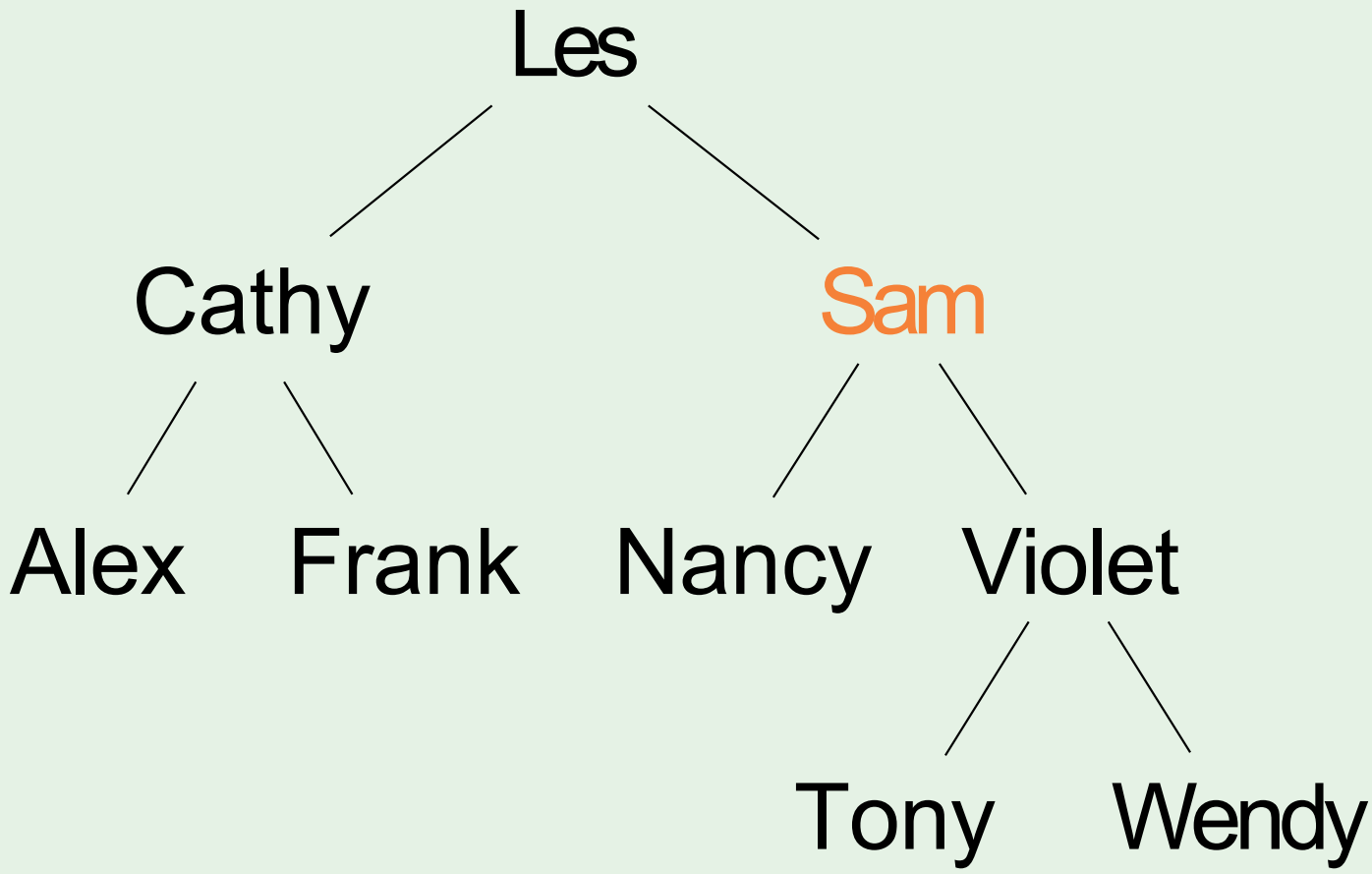
LevelTraversal



Output: Les Cathy Sam

Queue: Alex, Frank

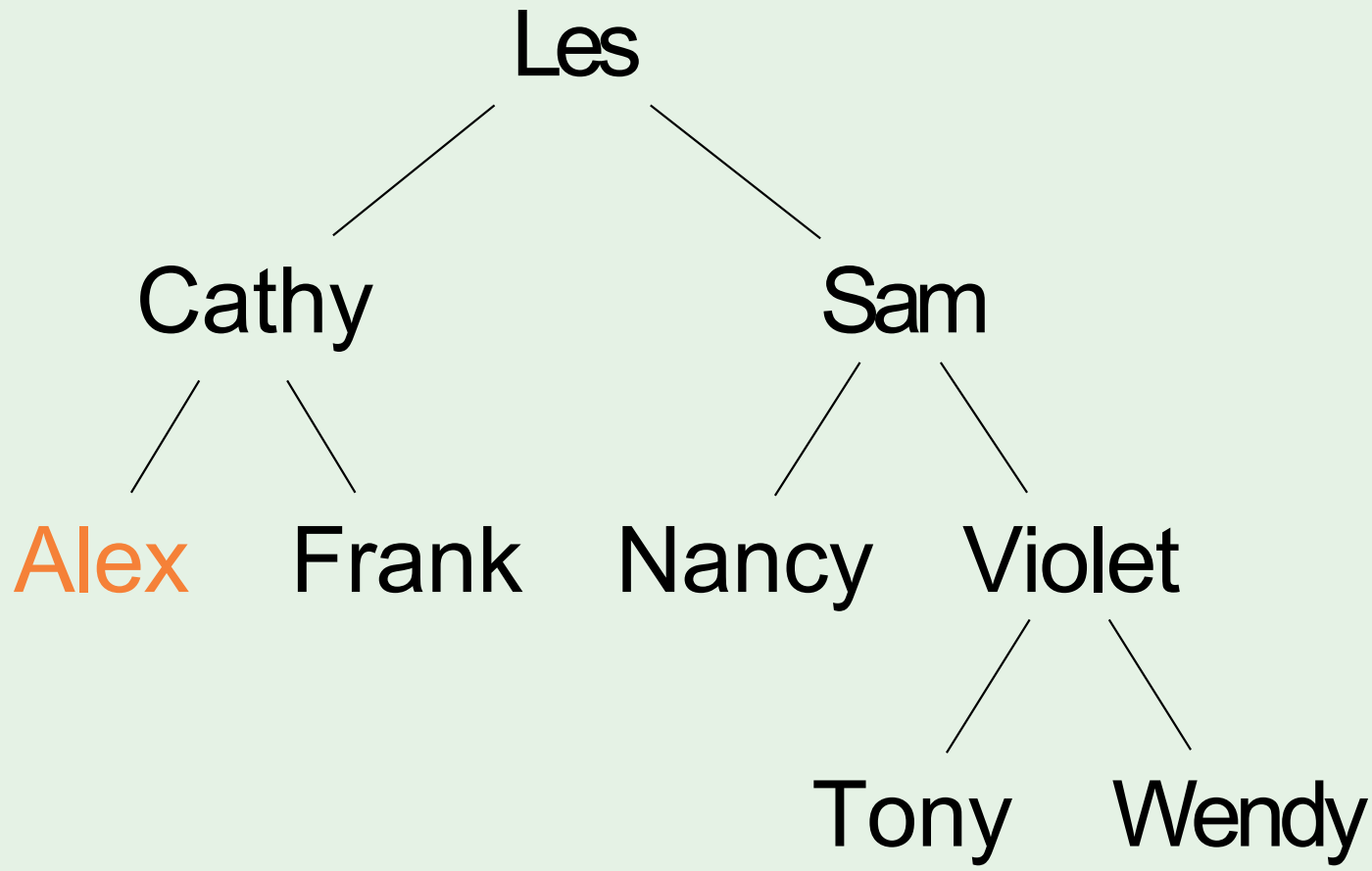
LevelTraversal



Output: Les Cathy Sam

Queue: Alex, Frank, Nancy,

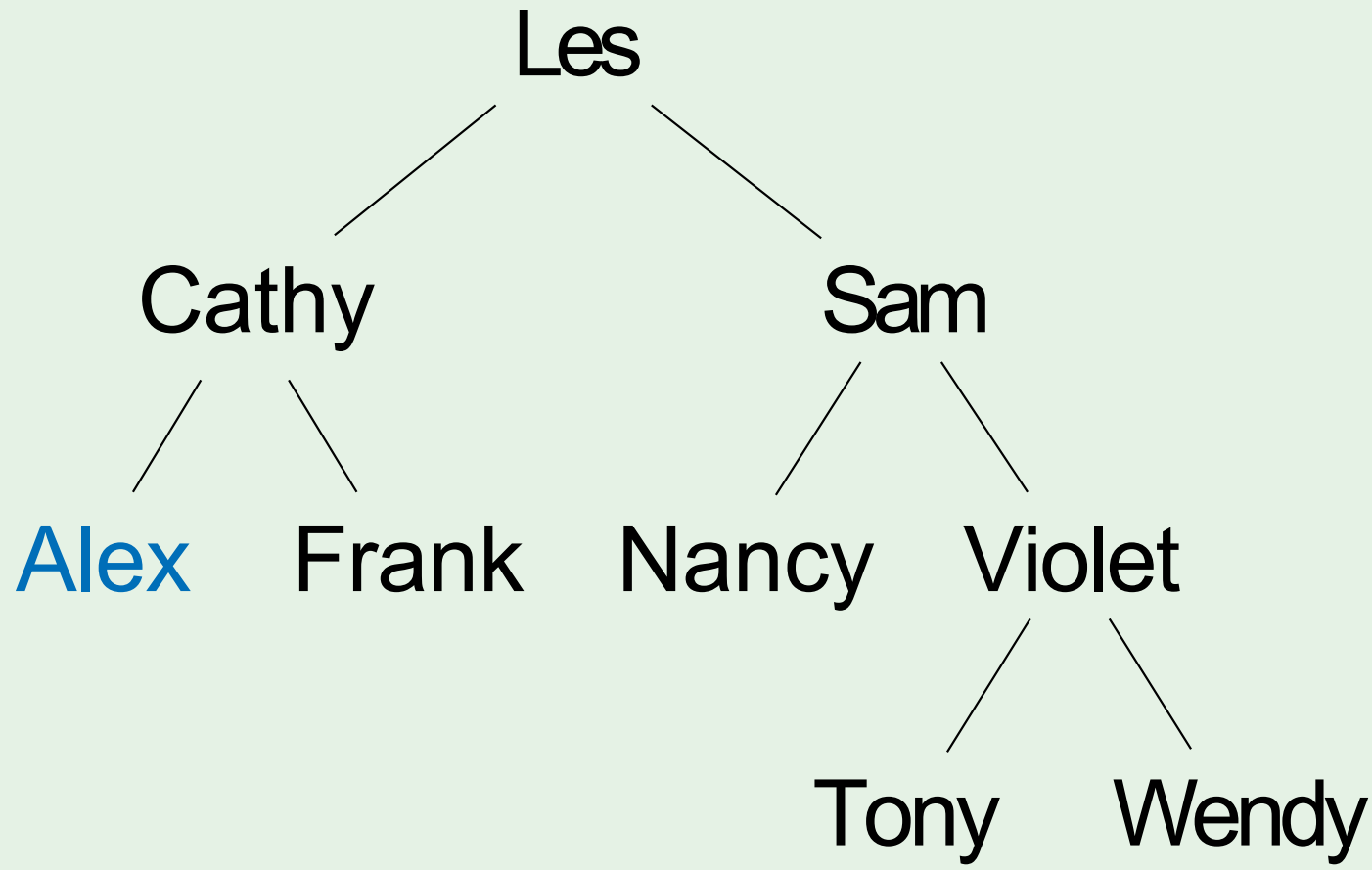
LevelTraversal



Output: Les Cathy Sam

Queue: Frank, Nancy,

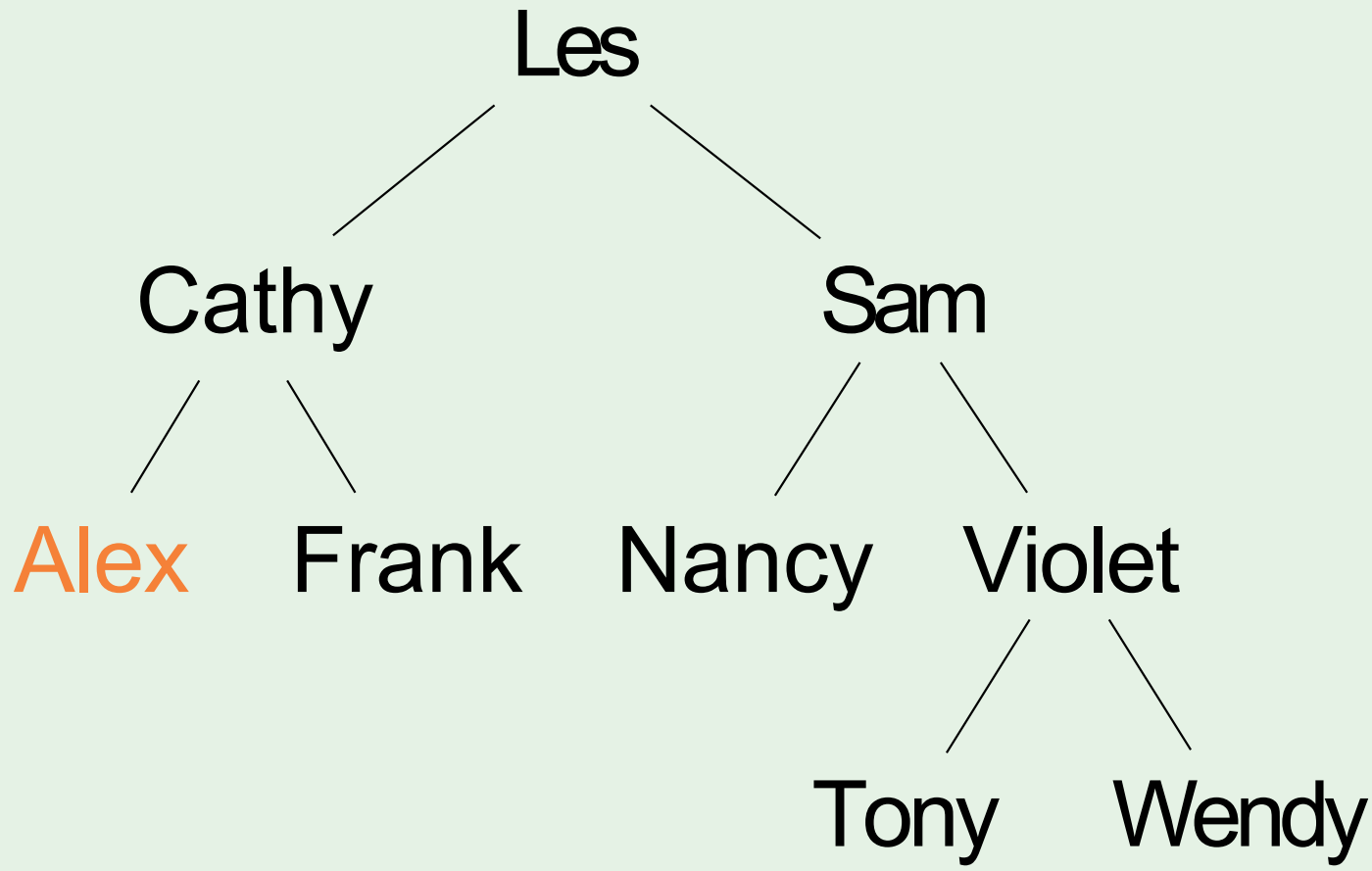
LevelTraversal



Output: Les Cathy Sam Alex

Queue: Frank, Nancy,

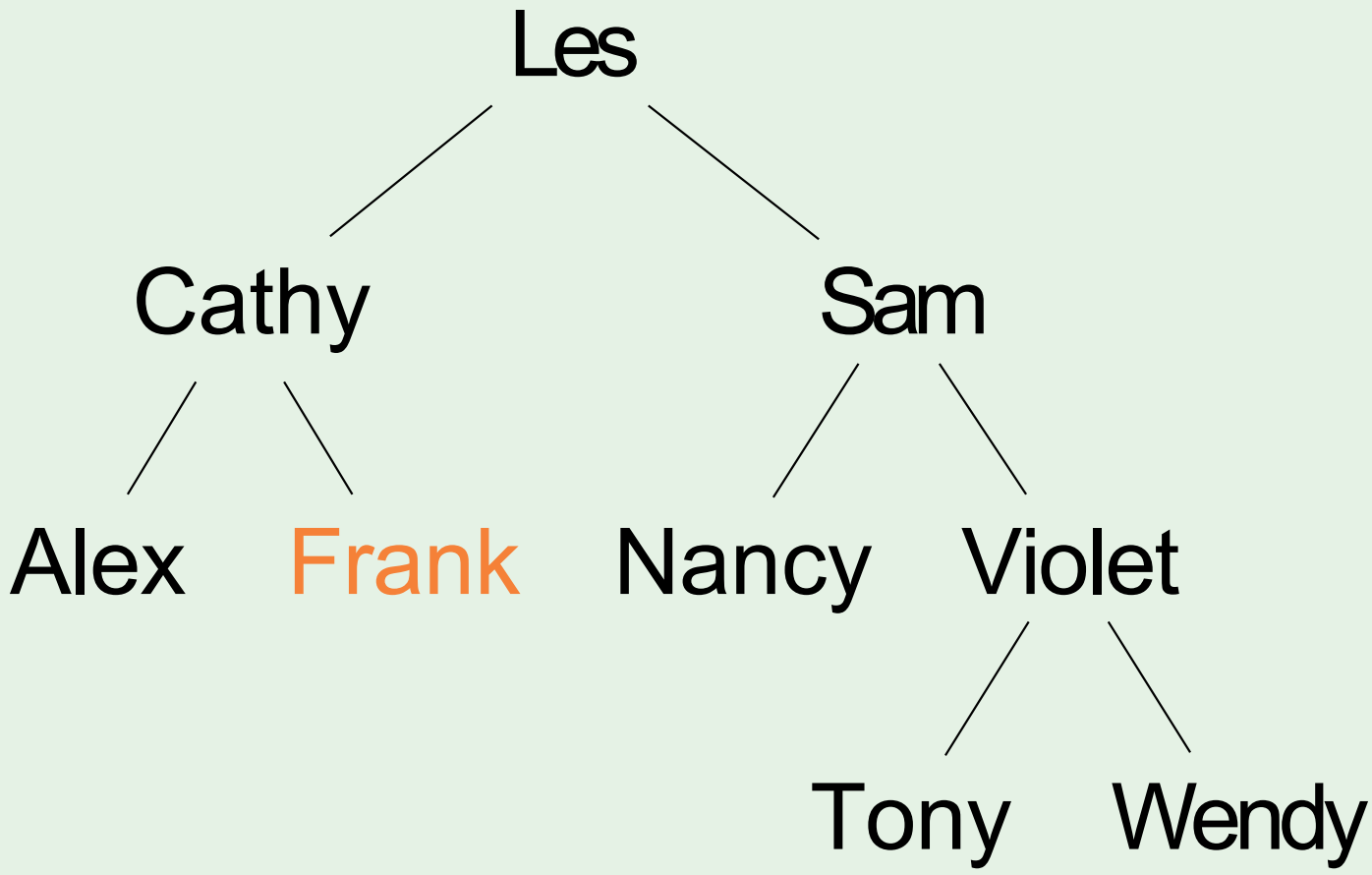
LevelTraversal



Output: Les Cathy Sam Alex

Queue: Frank, Nancy,

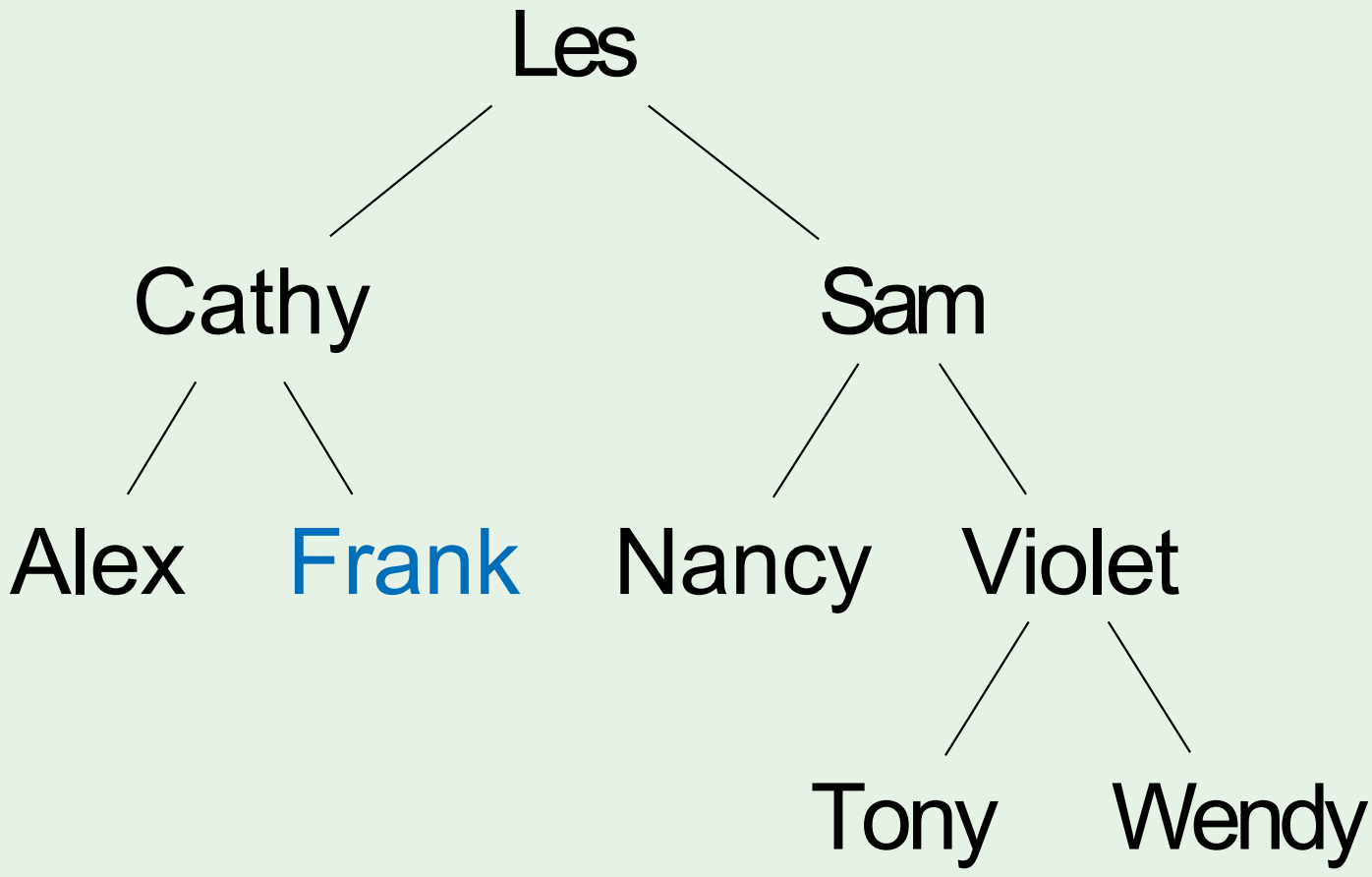
LevelTraversal



Output: Les Cathy Sam Alex

Queue: Nancy, Violet

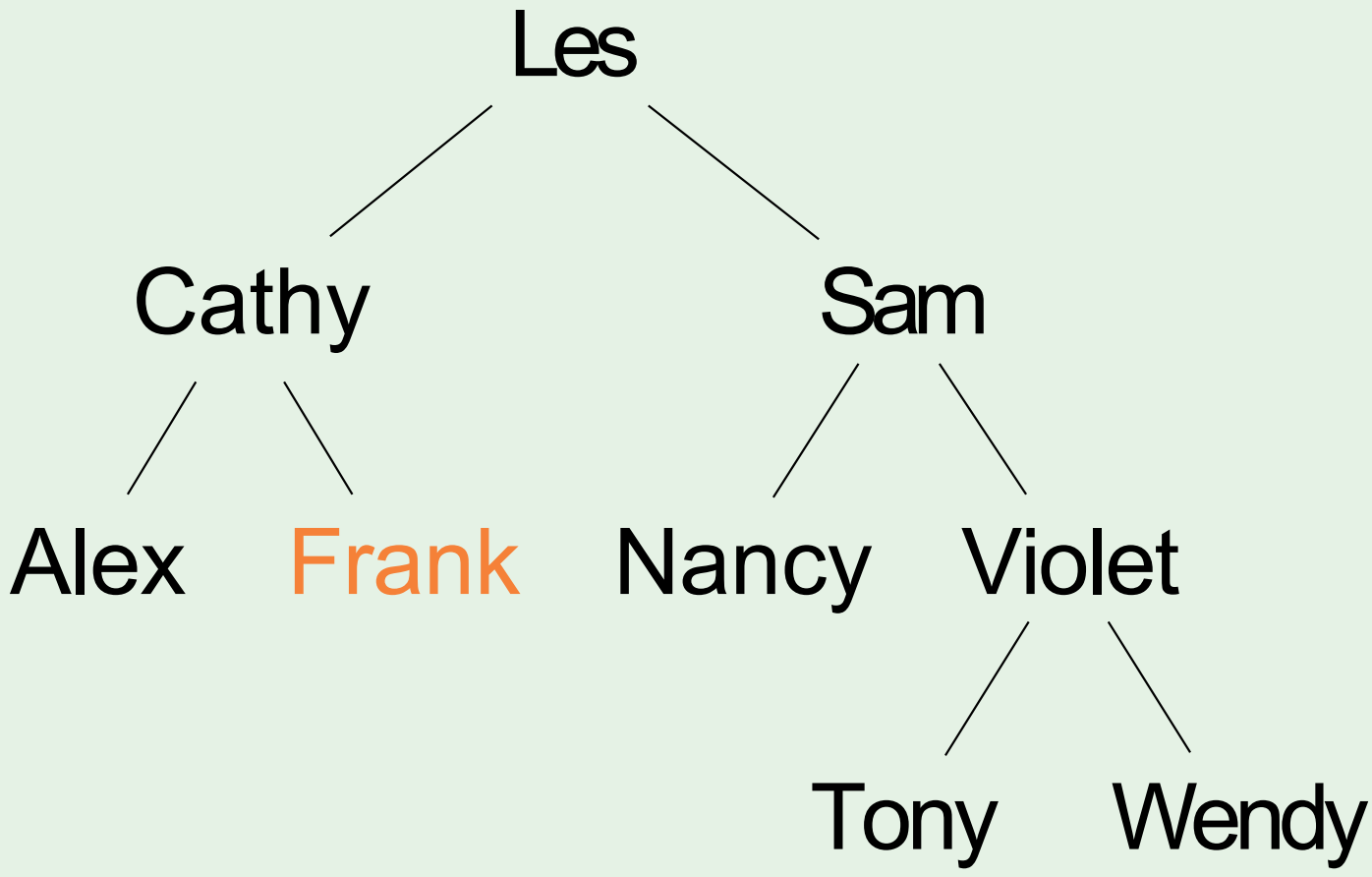
LevelTraversal



Output: Les Cathy Sam Alex Frank

Queue: Nancy, Violet

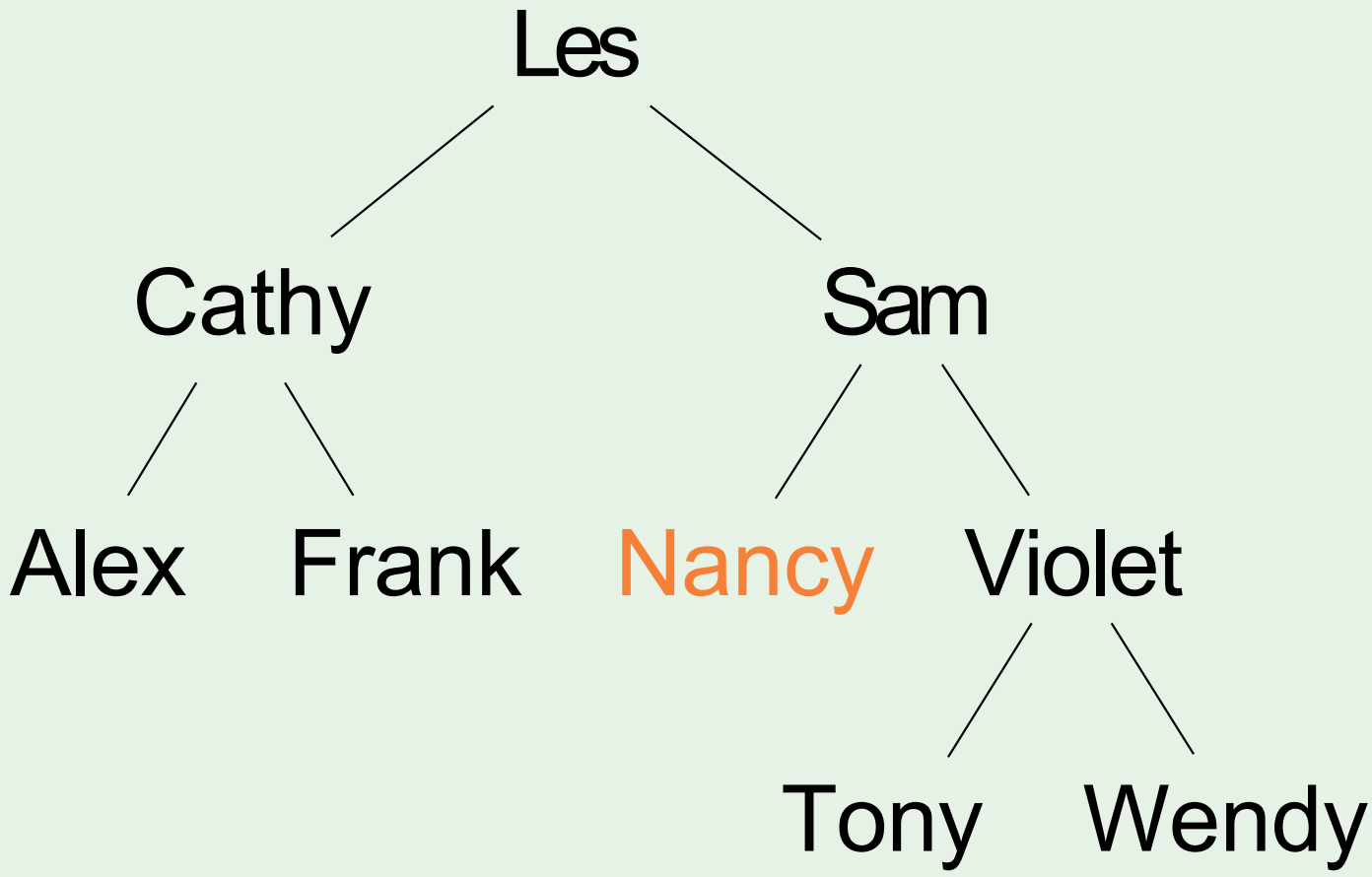
LevelTraversal



Output: Les Cathy Sam Alex Frank

Queue: Nancy, Violet

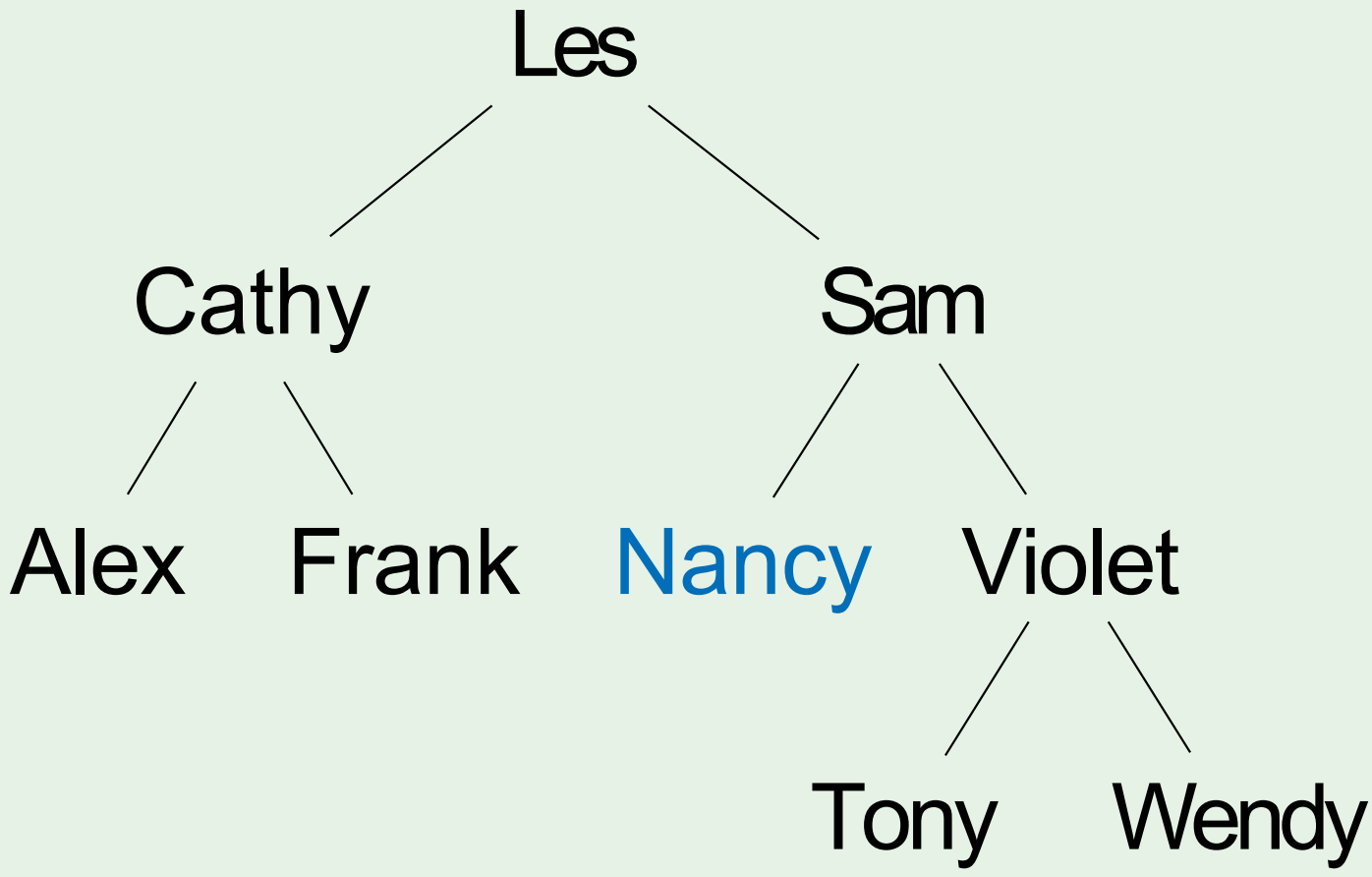
LevelTraversal



Output: Les Cathy Sam Alex Frank

Queue: Violet

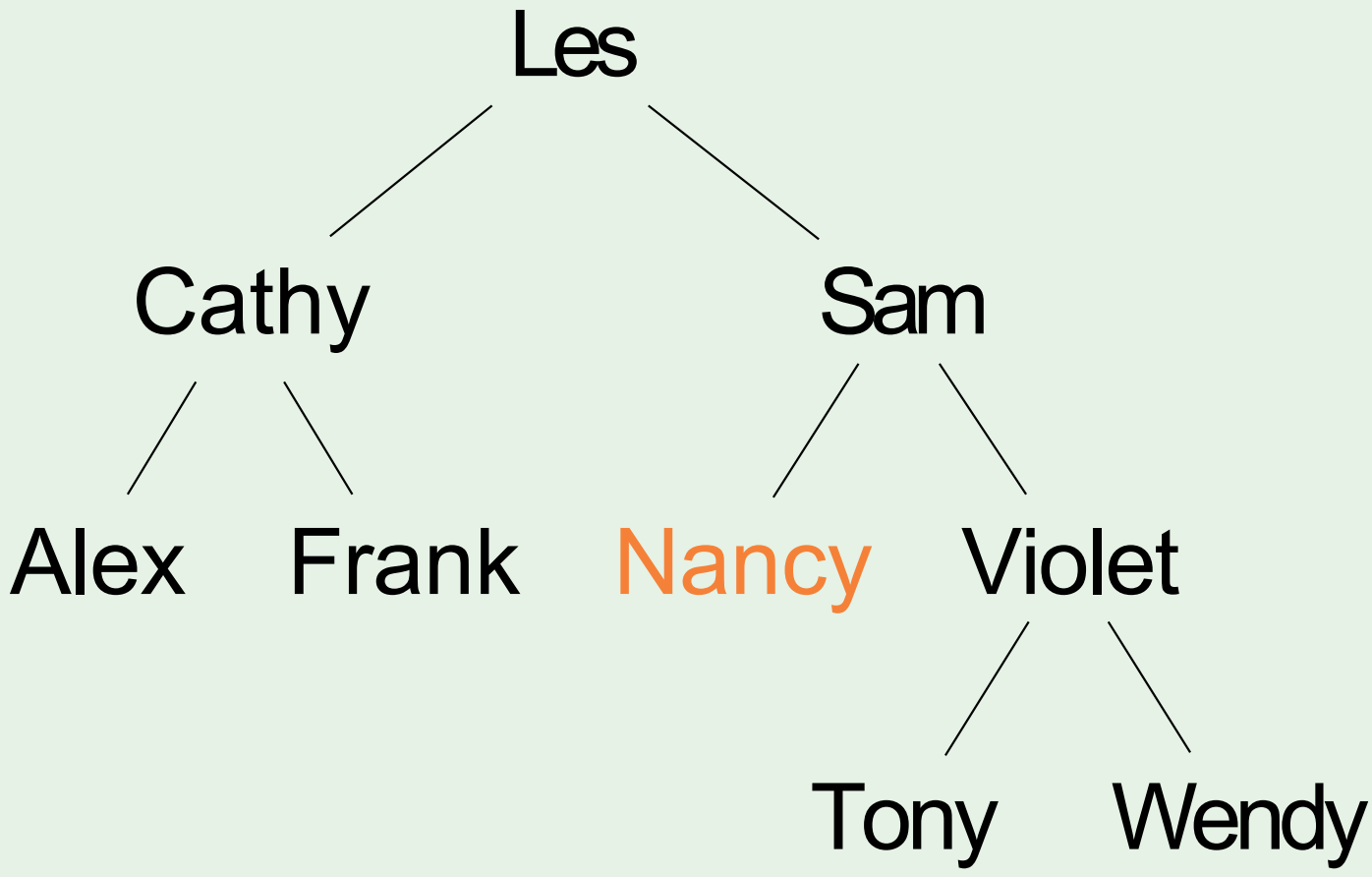
LevelTraversal



Output: Les Cathy Sam Alex Frank Nancy

Queue: Violet

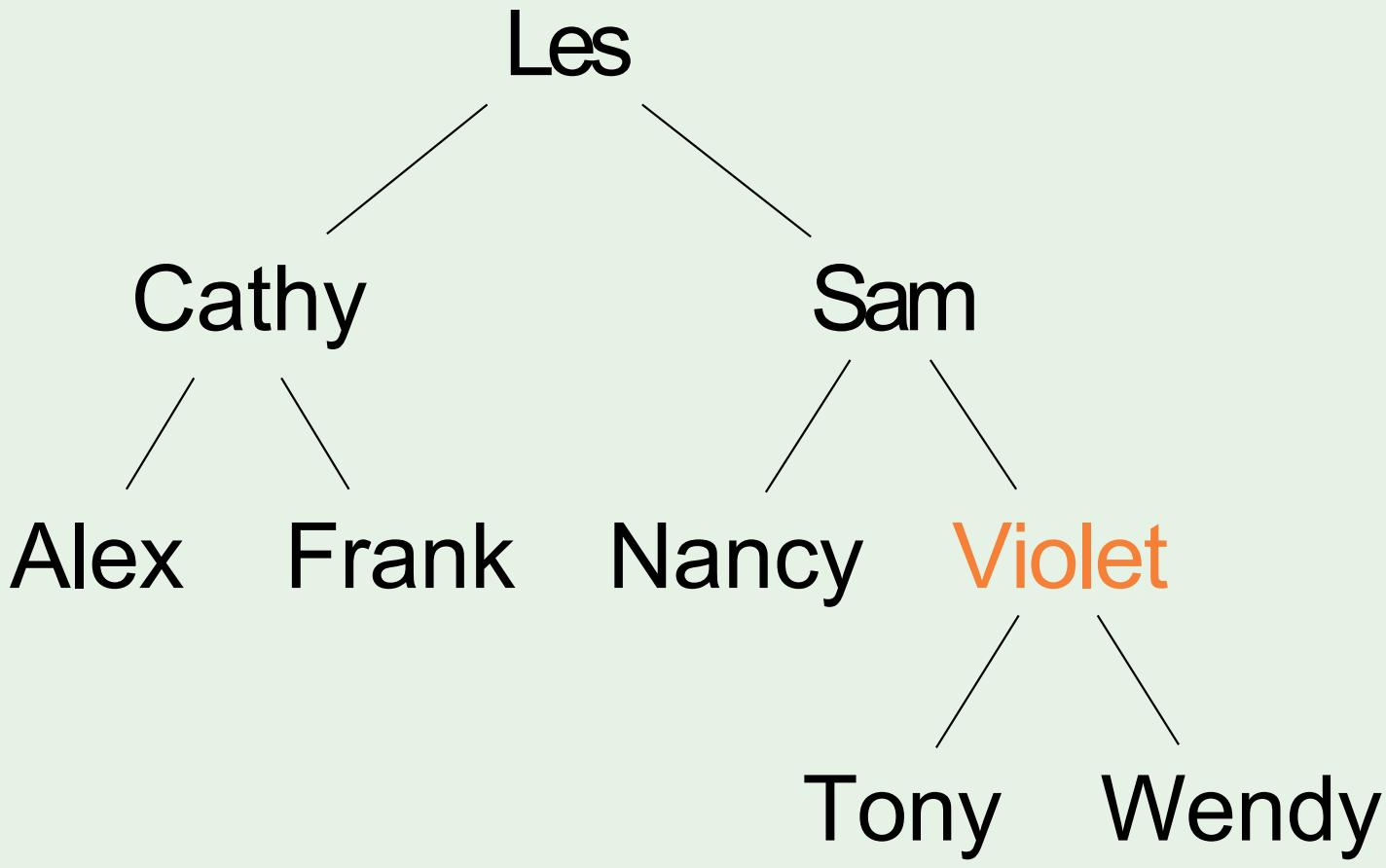
LevelTraversal



Output: Les Cathy Sam Alex Frank Nancy

Queue: Violet

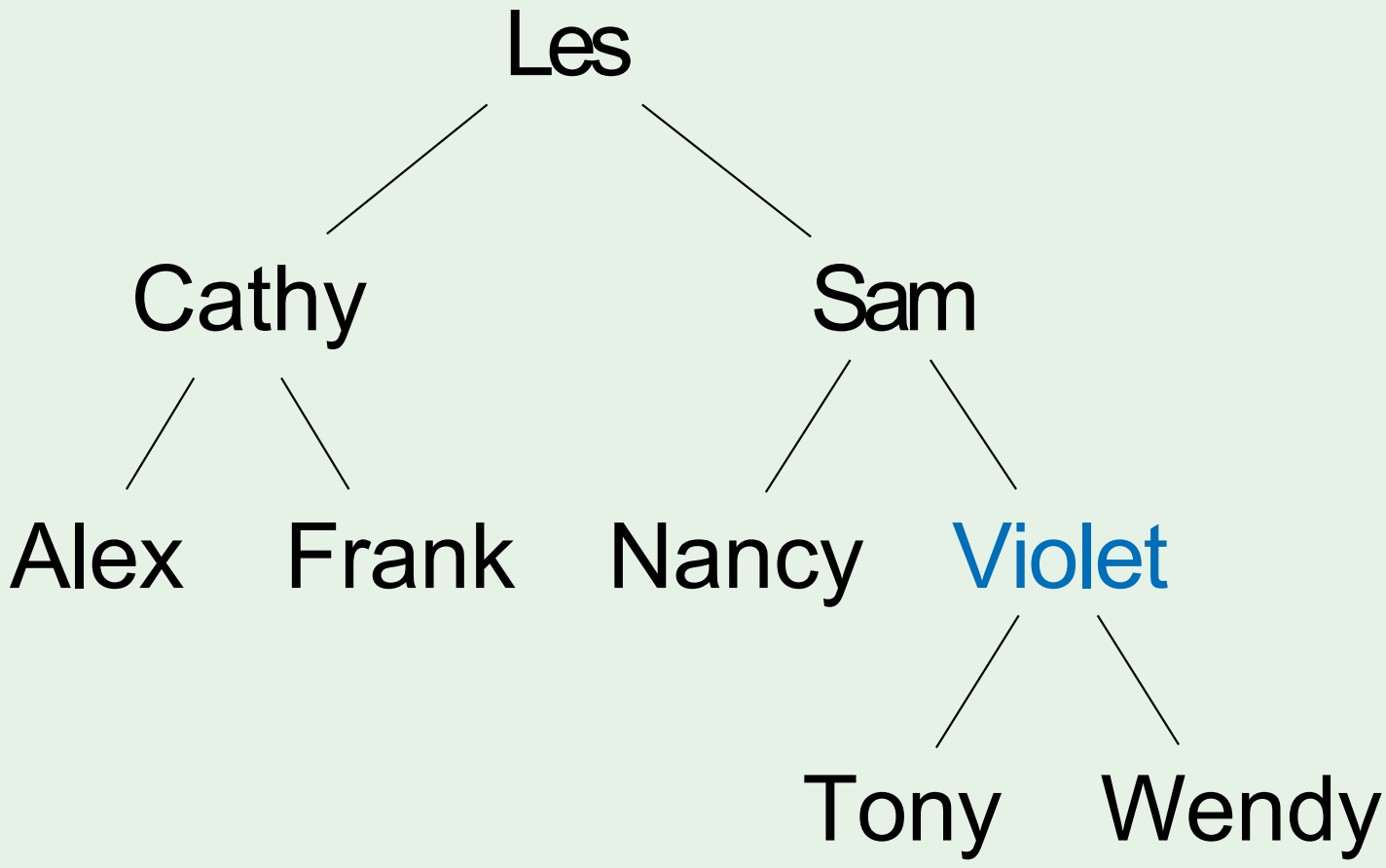
LevelTraversal



Output: Les Cathy Sam Alex Frank Nancy

Queue:

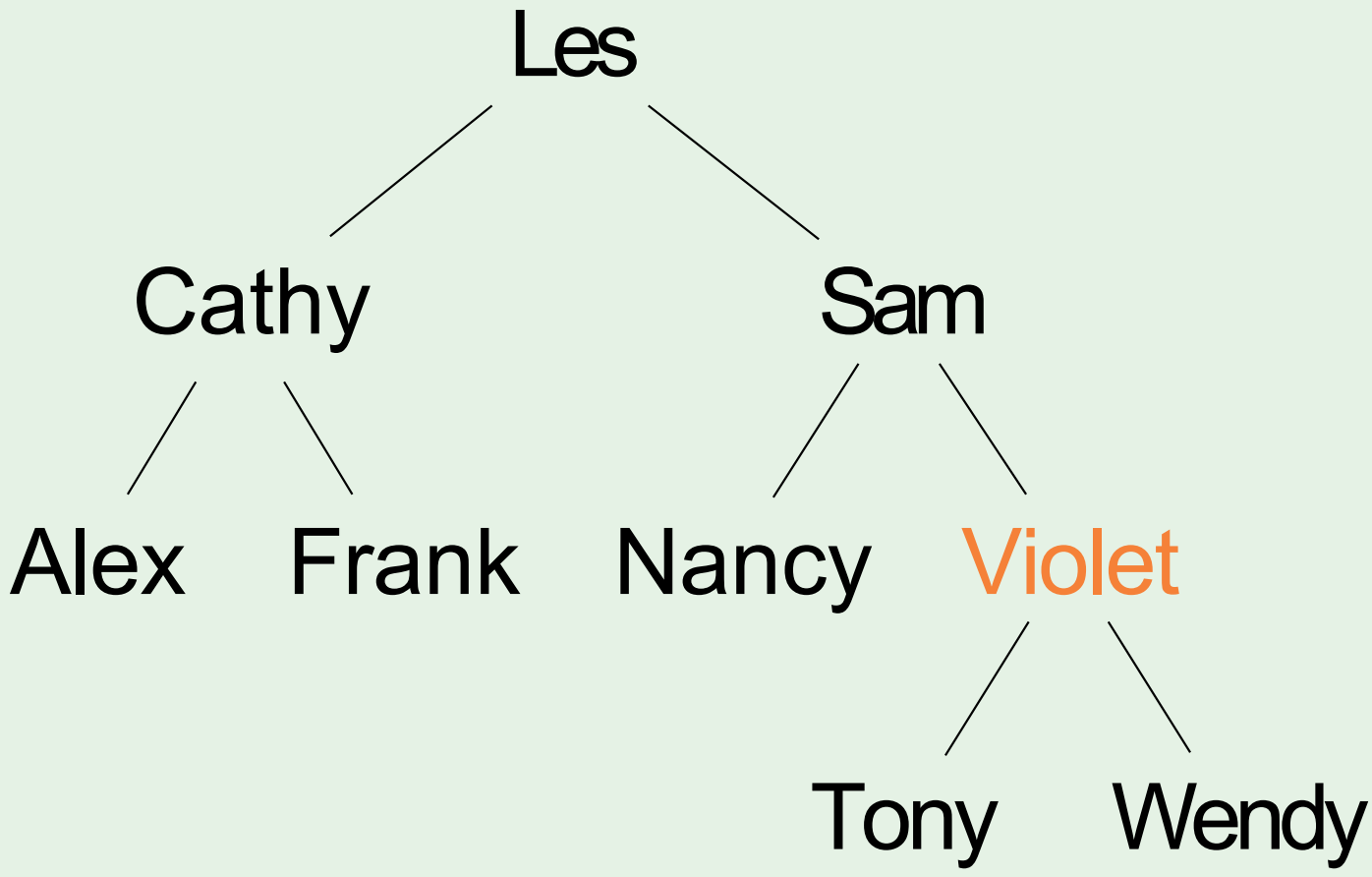
LevelTraversal



Output: Les Cathy Sam Alex Frank Nancy Violet

Queue:

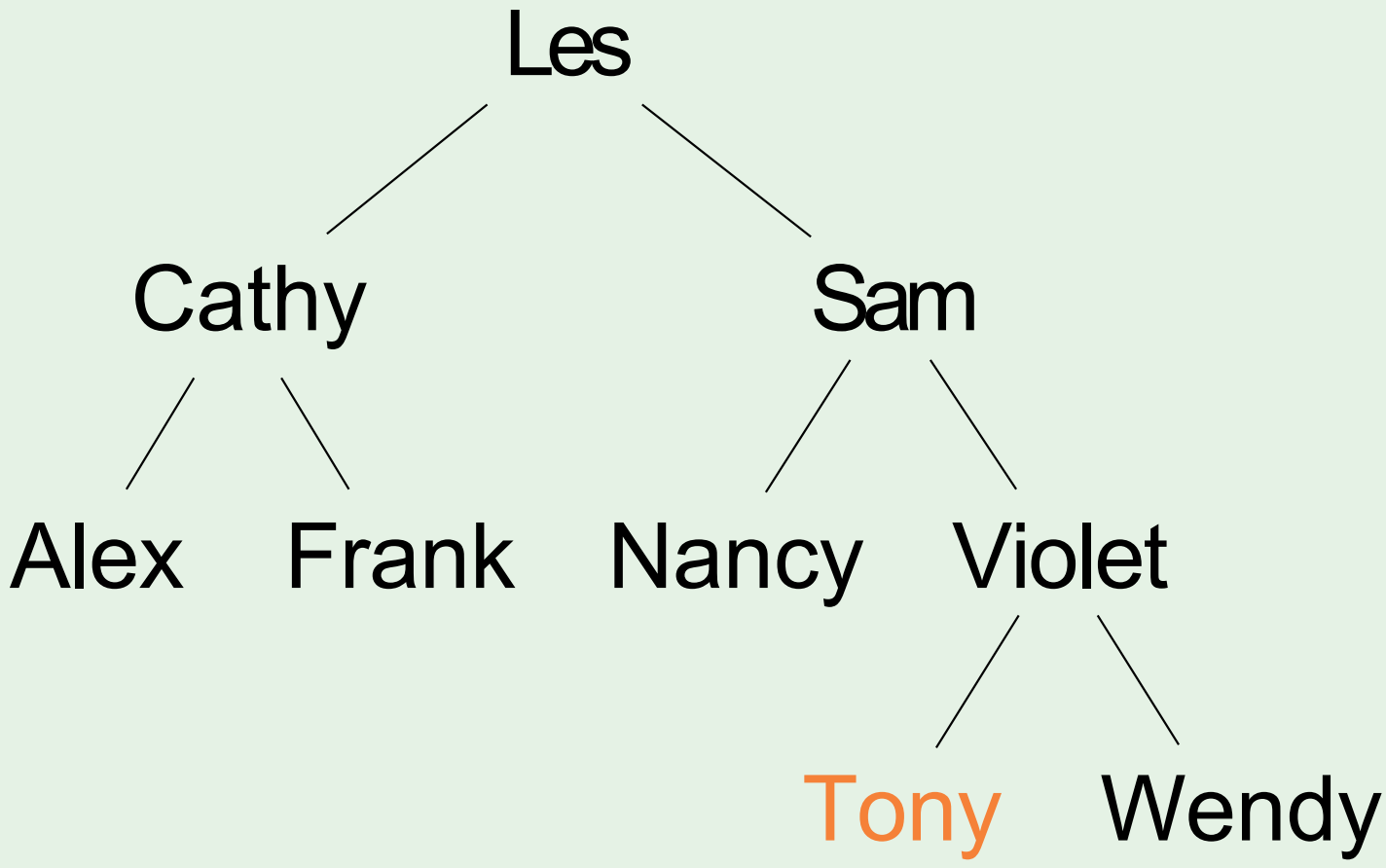
LevelTraversal



Output: Les Cathy Sam Alex Frank Nancy Violet

Queue: Tony Wendy

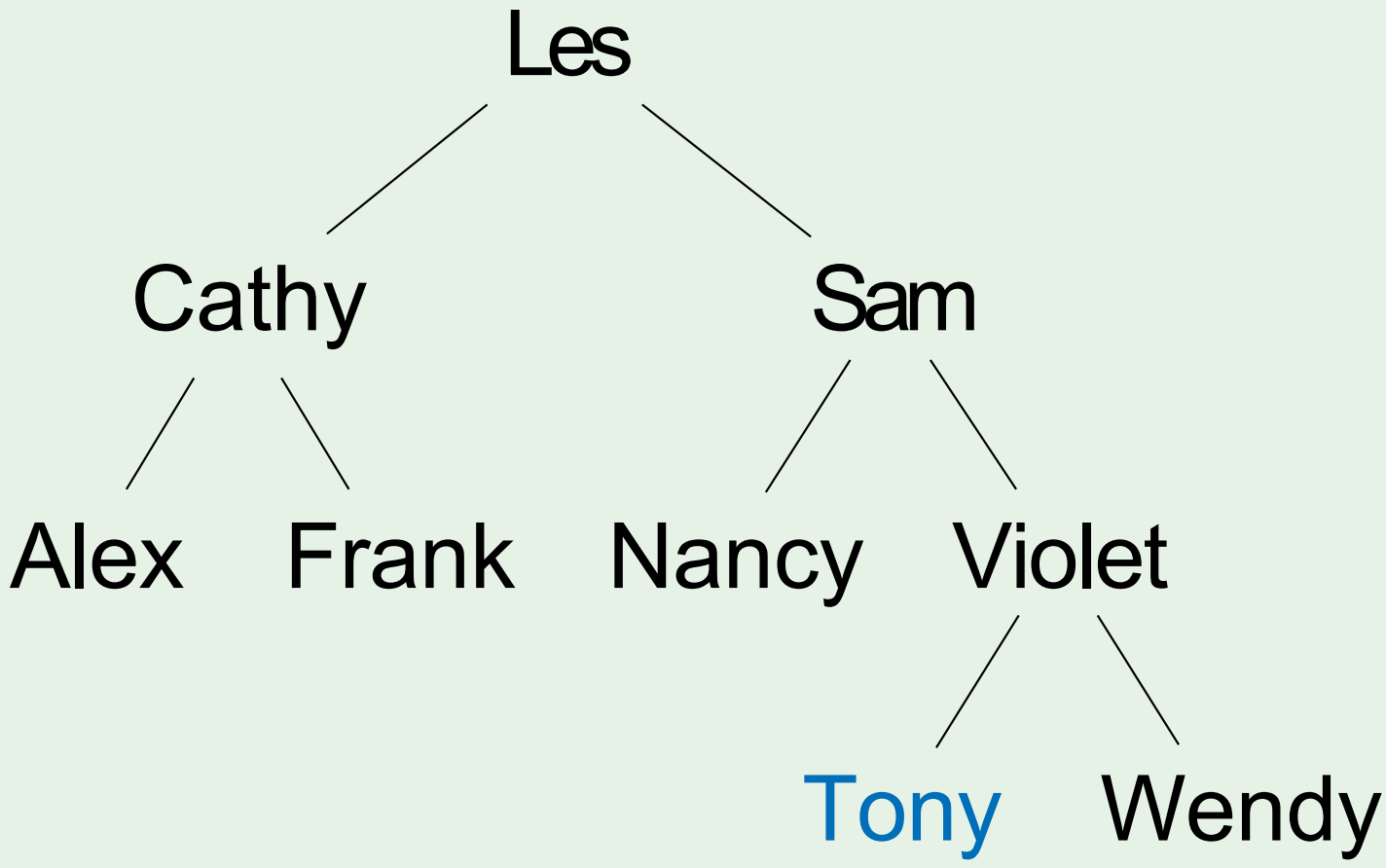
LevelTraversal



Output: Les Cathy Sam Alex Frank Nancy Violet

Queue: Wendy

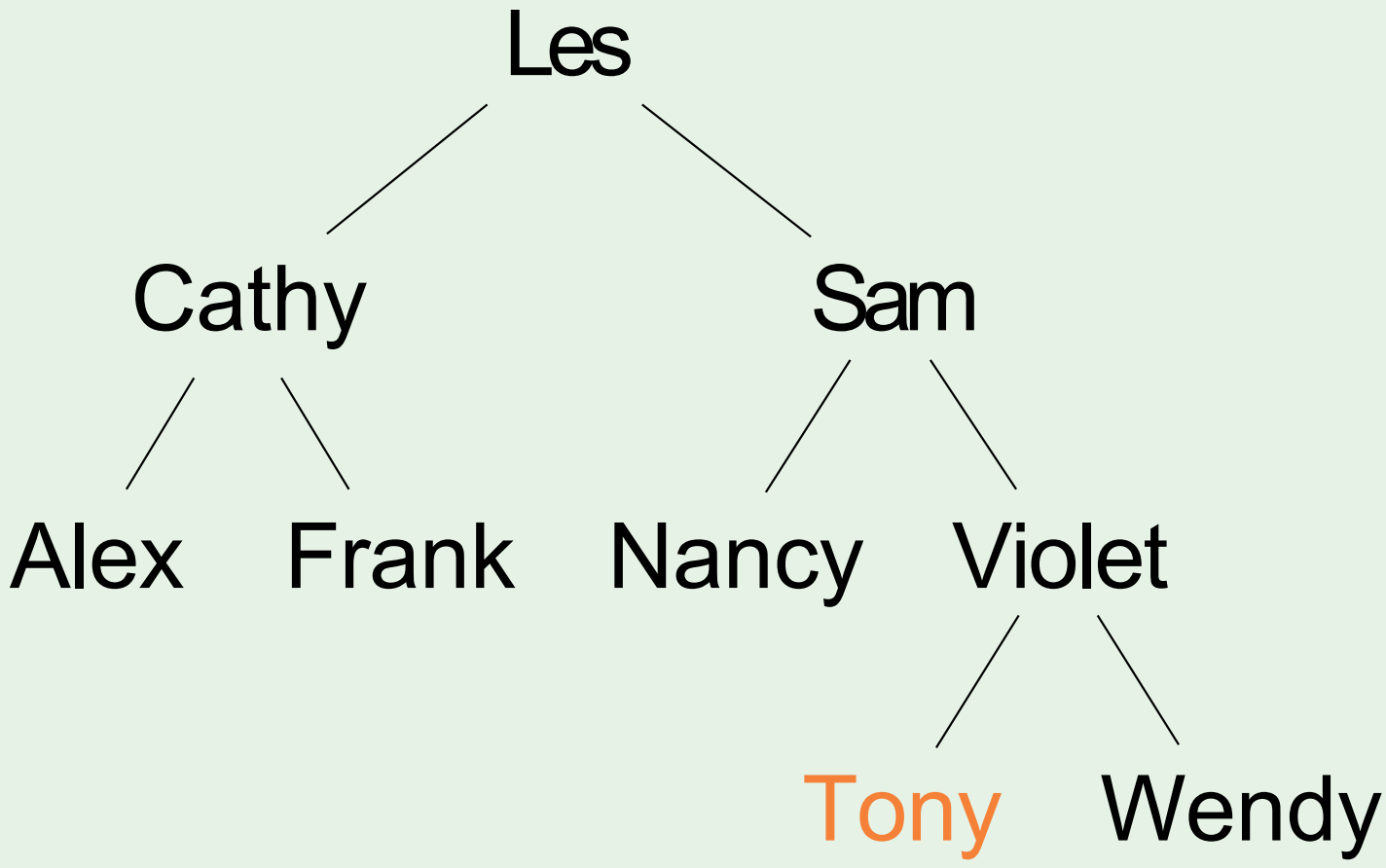
LevelTraversal



Output: Les Cathy Sam Alex Frank Nancy Violet Tony

Queue: Wendy

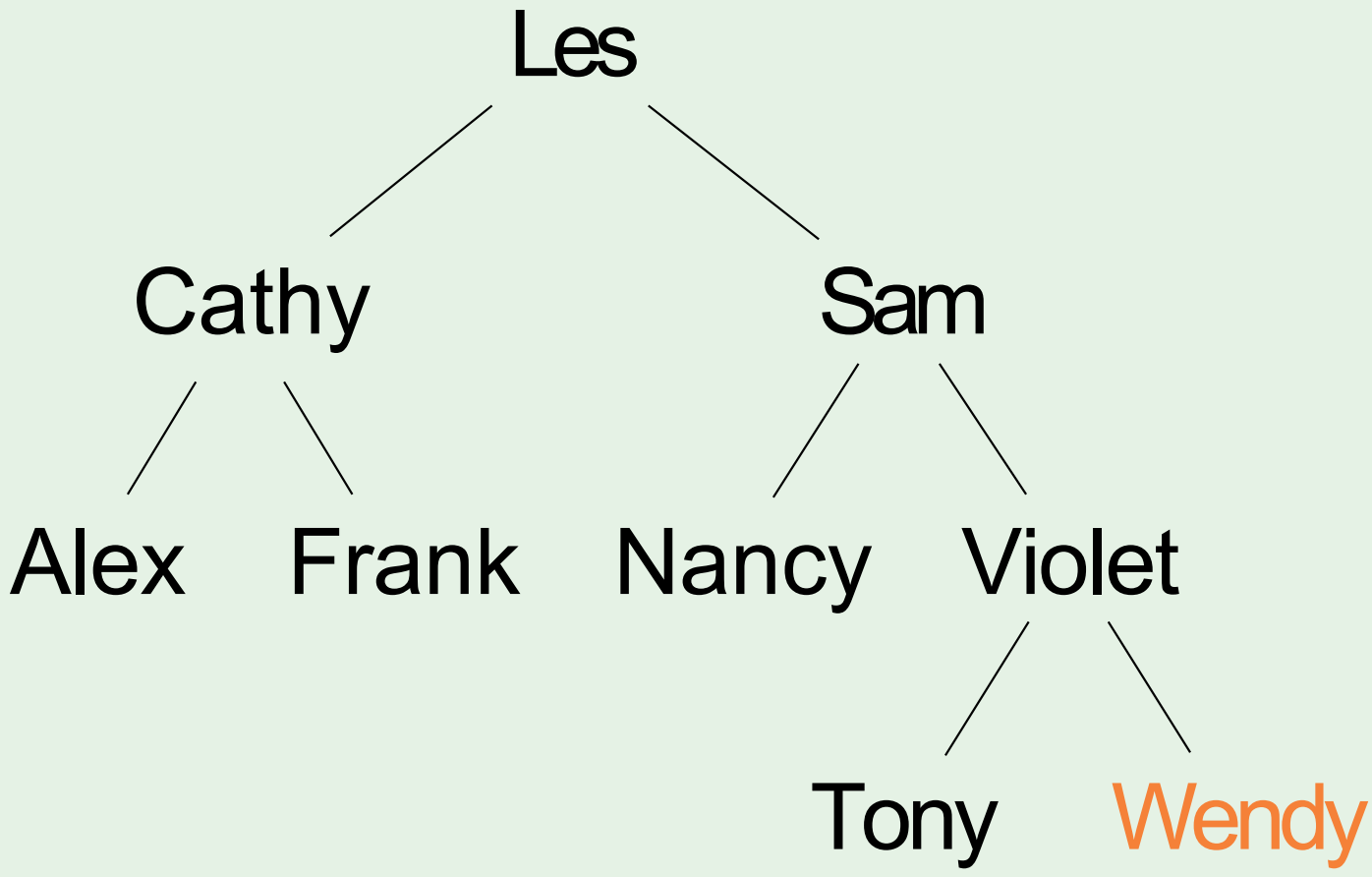
LevelTraversal



Output: Les Cathy Sam Alex Frank Nancy Violet Tony

Queue: Wendy

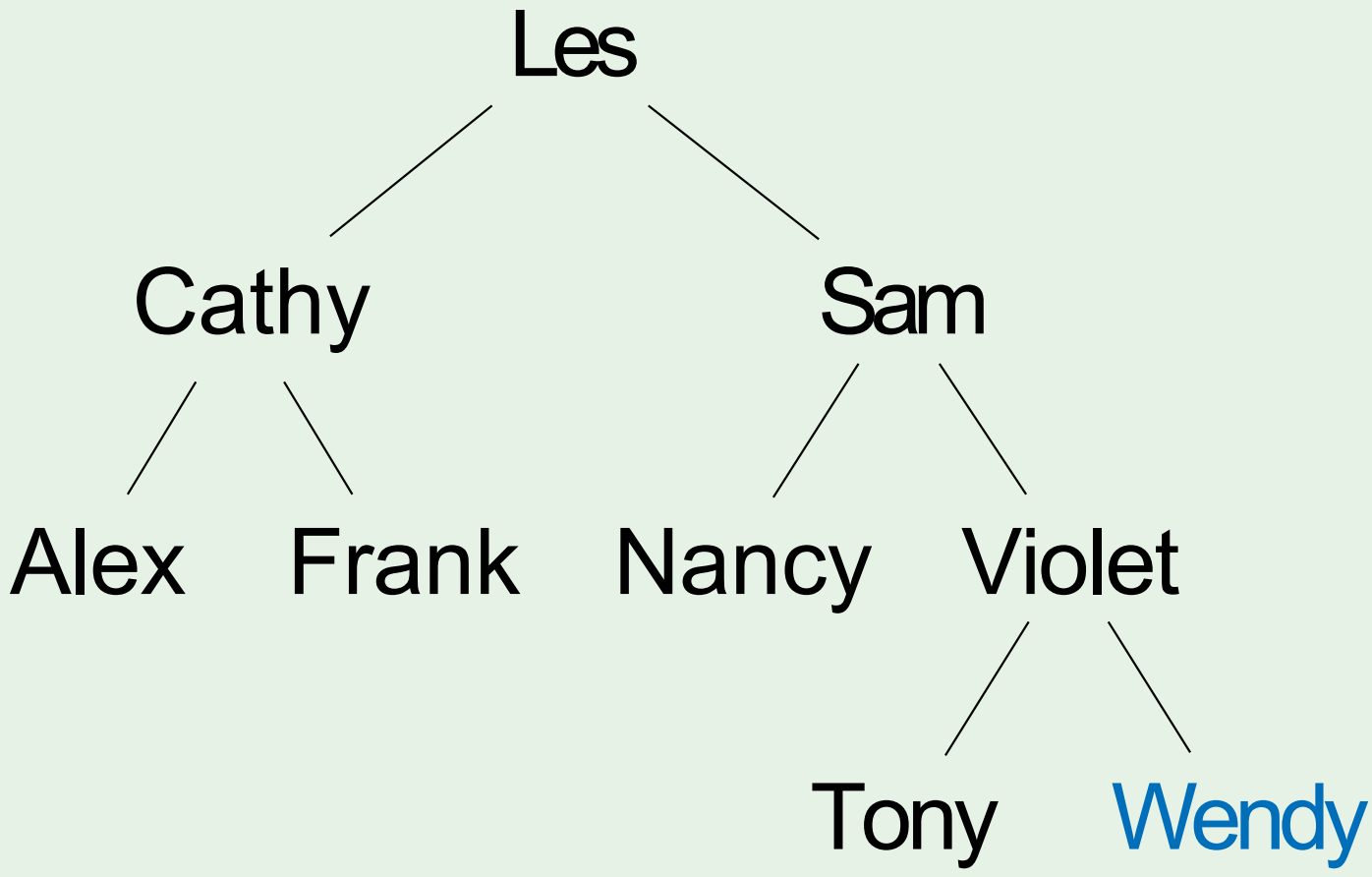
LevelTraversal



Output: Les Cathy Sam Alex Frank Nancy

Violet Tony

LevelTraversal



Output: Les Cathy Sam Alex Frank Nancy Violet Tony Wendy

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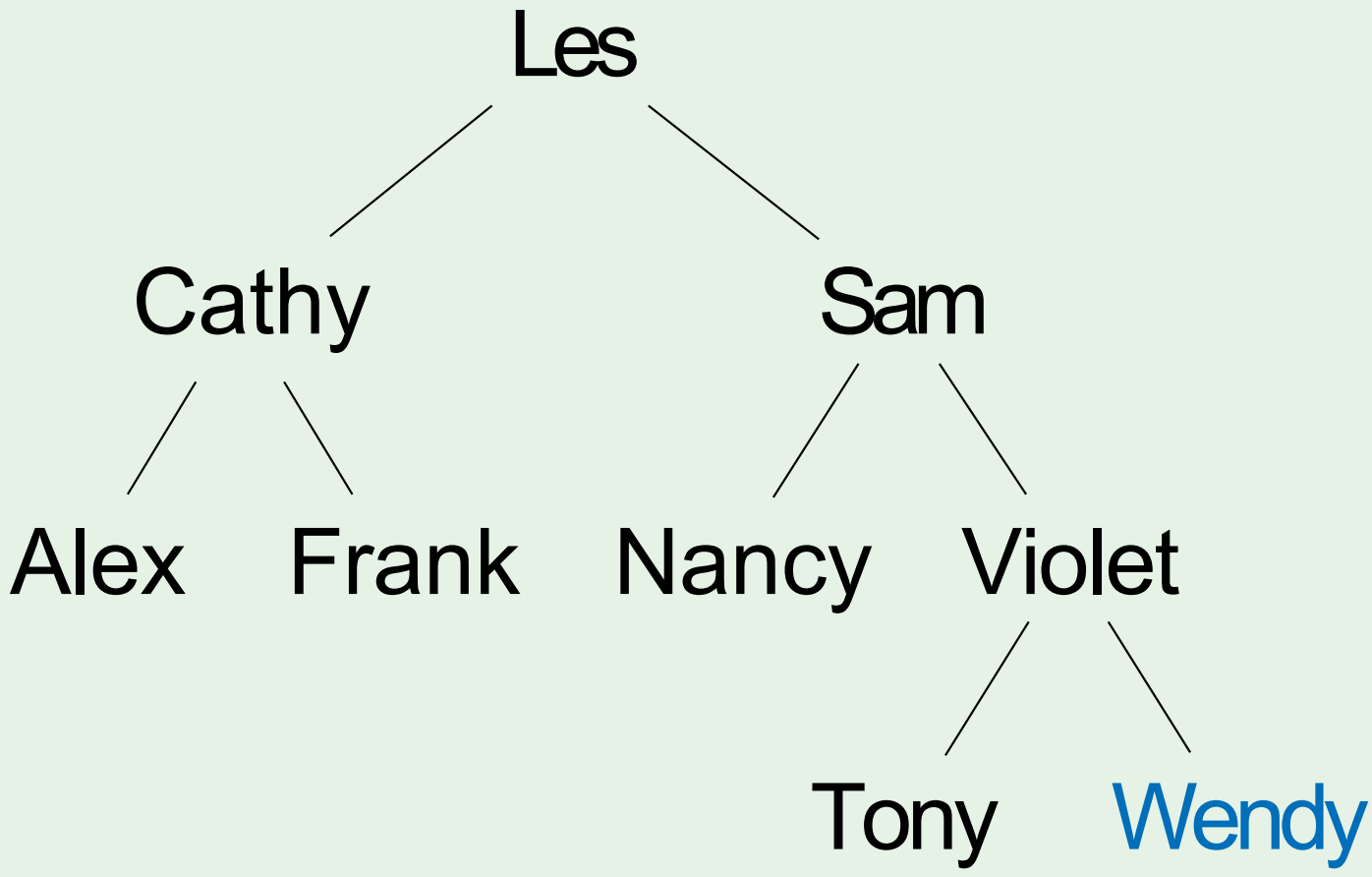
LevelTraversal



Output: Les Cathy Sam Alex Frank Nancy Violet Tony Wendy

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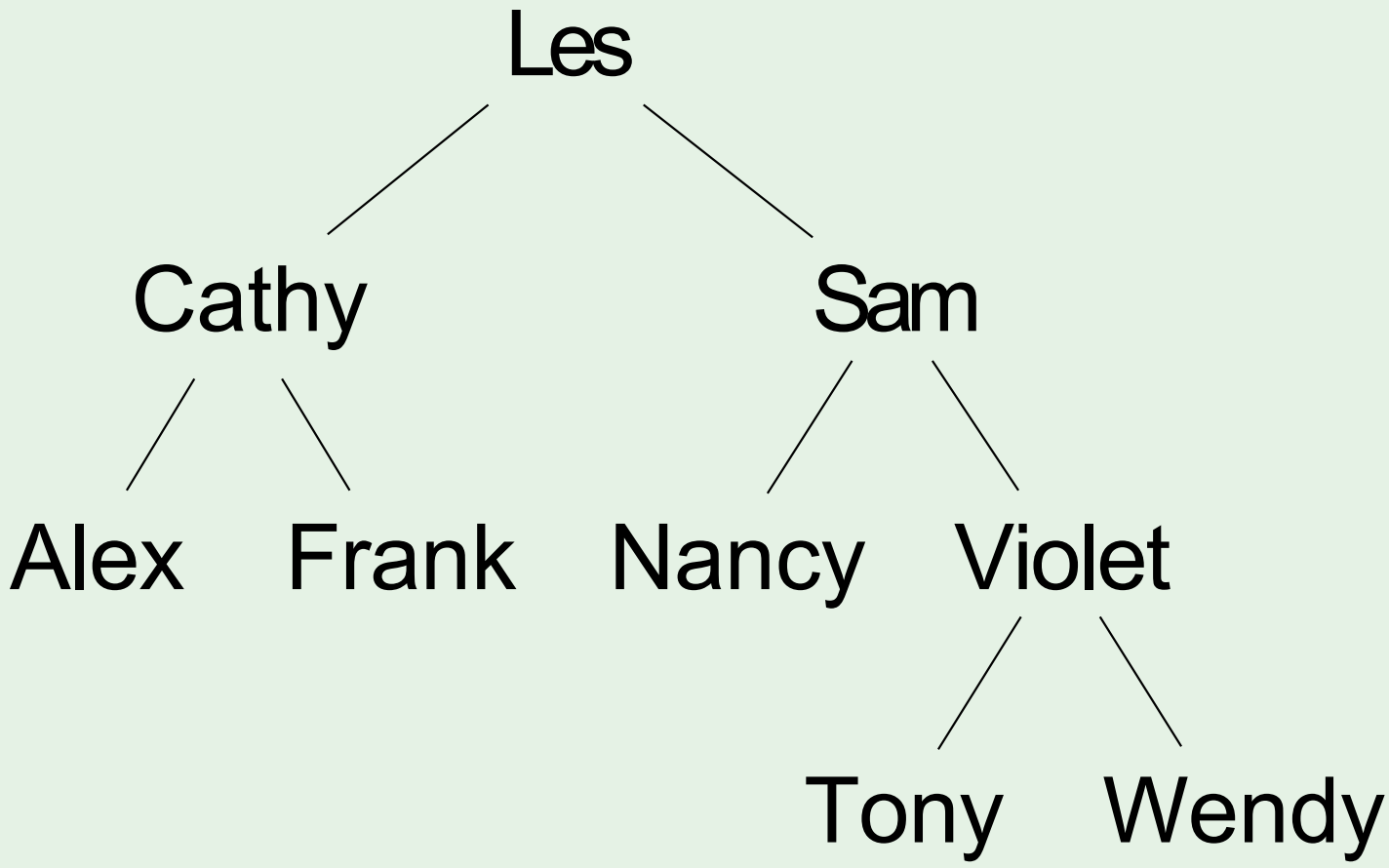
LevelTraversal



Output: Les Cathy Sam Alex Frank Nancy Violet Tony Wendy

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- In Computer Science, trees grow down!

For Tree-traversal quiz