NORTH SOUTH UNIVERSITY

Committed to the Highest Standards of Academic Excellence
School of Engineering & Physical Sciences
Department of Mathematics & Physics

MAT 350 (Engineering Mathematics) - Section: 06
Assignment:03 Semester: Summer 2022
Deadline: 17/07/2022

Questions:

Solve each differential equation by variation of parameters

1. $y'' + y = \cos^2 x$
2. $y'' + y = \sec\theta \tan\theta$
3. $y'' + y = \tan x$
4. $y'' - y = \sinh 2x$
5. $y'' - 2y' + y = \dfrac{e^x}{1+x^2}$
6. $2y'' + 2y' + y = 4\sqrt{x}$
7. Solve each differential equation by variation of parameters, subject to the initial conditions , y(0)=1,y'(0)=0.

$$y'' + 2y' - 8y = 2e^{-2x} - e^{-x}$$

8. $y'' + 3y' + 2y = \sin e^x$
9. Solve each differential equation by variation of parameters, subject to the initial conditions , y(0)=1,y'(0)=0.

$$2y'' + y' - y = x + 1$$

Solve the following differential equations and justify your answers by using Inverse operator method:

10. $\dfrac{d^2y}{dx^2} + 4y = x^3$
11. $\dfrac{d^2y}{dx^2} - 2\dfrac{dy}{dx} + 2y = e^{-2x}$
12. $(D^2 - 4D + 4)y = e^{2x}$ given that $y(0) = 1$ and $y'(0) = 5$

13. $\frac{d^2y}{dx^2}+2\frac{dy}{dx}+y=\cos 2x,$

14. $(D^2-1)(D^2-9)y=\cos 2x,$

Spring-Mass System: Free Undamped motion

15. A 20 kg mass is attached to a spring. If the frequency of simple harmonic motion is $\frac{2}{\pi}$ cycles/s, what is the spring constant k? What is the frequency of simple harmonic motion if the original mass is replaced with 80 kg mass?
16. A force of 400 newtons stretches a spring 2 meters. A mass of 50 kilograms is attached to the end of the spring and is initially released from the equilibrium position with an upward velocity of 10m/s. Find the equation of motion.
17. Another spring whose constant is 20N/m is suspended from the same rigid support but parallel to the spring/mass system in the previous question. A mass of 20 kilograms is attached to the second spring, and both masses are initially released from the equilibrium position with an upward velocity of 10m/s.
    a. Which mass exhibits the greater amplitude of motion?
    b. which mass is moving faster at $t=\frac{\pi}{4}s?\ At\frac{\pi}{2}\ s?$
    c. At what times are the two masses in the same position? Where are the masses at these times? In which directions are the masses moving?

Spring-Mass System: Free Damped motion

18. A 1 kg mass is attached to spring whose constant is 16N/m, and the entire system is then submerged in a liquid that imparts a damping force numerically equal to 10 times the instantaneous velocity. Determine the equations of motion if
    a. the mass is initially released from rest from a point 1 meter below the equilibrium position, and then
    b. the mass is initially released from a point 1 meter below the equilibrium position with an upward velocity of 12m/s.
19. A 4 foot spring measures 8 feet long after a mass weighing 8 pounds is attached to it. The medium through which the mass moves offers a damping force numerically equal to $\sqrt{2}$ times the instantaneous velocity. Find the equation of motion if the mass is initially released from the equilibrium position with a downward velocity of 5 ft/s. Find the time at which the mass attains its extreme displacement from the equilibrium position. What is the position of the mass at this instant?
20. A mass weighing 4 pounds is attached to a spring whose constant is 2 lb/ft. The medium offers a damping force that is numerically equal to the instantaneous velocity. The mass is initially released from a point 1 foot above the equilibrium position with a downward velocity of 8 ft/s. Determine the time at which the mass

passes through the equilibrium position. Find the time at which the mass attains its extreme displacement from the equilibrium position. What is the position of the mass at this instant?