Quartiles: Quartiles are the three cut points that will divide a dataset into four equal-sized groups.

1) 1^{st} quartile/lower quartile

2) 2^{nd} quartile/Median

3) 3^{rd} quartile/Upper quartile

1) 1^{st} quartile: 1^{st} quartile (Q_1) is known as lower quartile and 25% of the data lies below this point.

For 1^{st} quartile, F(x) = 0.25

2) 2^{nd} quartile: 2^{nd} quartile (Q_2) is the median of a data set and 50% of the data lies below this point.

For median, F(x) = 0.5

3) 3^{rd} quartile: 3^{rd} quartile **(**Q_3) is known as upper quartile and 75% of the data lies below this point.

For 3^{rd} quartile, F(x) = 0.75

Interquartile range: Interquartile range is the difference between upper quartile and lower quartile. IQR = Q_3- Q_1

Example: Suppose that the diameter of a metal cylinder has a probability density function $f(x) = 1.5 - 6(x - 50.0)^2$ for $49.5 \leq x \leq 50.5$

i) Show that total area under the probability density function = 1 or Prove that this is a valid PDF.

ii) Calculate The probability that a metal cylinder has a diameter between 49.8 and 50.1 mm.

iii) Find cumulative distribution function.

iv) Calculated the expected diameter of a metal cylinder

v) Find 1st, 2nd and 3rd quartile of a metal cylinder diameter.

vi) Find interquartile range

Solution: 1st quartile: F(x) = 0.25

$\Rightarrow 1.5*x - 2(x-50)^3 - 74.5 = 0.25$ [from solution iii]

$\Rightarrow 1.5x - 2(x^3 - 150x^2 + 7500x - 125000) - 74.5 - 0.25 = 0$

$\Rightarrow 1.5x - 2x^3 + 300x^2 - 15000x + 250000 - 74.75 = 0$

$\Rightarrow -2x^3 + 300x^2 - 14998.5x + 249925.25 = 0$

X= 50.9 or 49.8 or 49.2

Since $49.5 \leq x \leq 50.5$

So 1st quartile = 49.8

(Note: Mode set up⇒5:EQN⇒4⇒(-2)⇒=⇒(300)⇒=⇒(-14998.5)⇒=⇒249925.25⇒=)

2nd quartile: F(x) = 0.50

$\Rightarrow 1.5*x - 2(x-50)^3 - 74.5 - 0.50 = 0$

$\Rightarrow 1.5x - 2(x^3 - 150x^2 + 7500x - 125000) - 74.5 - 0.50 = 0$

$\Rightarrow -2x^3 + 300x^2 - 14998.5x + 249925 = 0$

X= 49.1 or 50.8 or 50

Since $49.5 \leq x \leq 50.5$

So 2nd quartile = 50

3rd quartile: F(x) = 0.75

$\Rightarrow 1.5*x - 2(x-50)^3 - 74.5 - 0.75 = 0$

$\Rightarrow 1.5x - 2(x^3 - 150x^2 + 7500x - 125000) - 74.5 - 0.75 = 0$

$\Rightarrow -2x^3 + 300x^2 - 14998.5x + 249924.75 = 0$

X = 49 or 50.7 or 50.2

Since $49.5 \leq x \leq 50.5$

So 3rd quartile = 50.2

Interquartile range = 3rd quartile – 1st quartile = 50.2 – 49.8 = 0.4