$\lim_{x\to+\infty} f(x) =$

$\lim_{x\to-\infty} f(x) =$

$\lim_{x\to+\infty} \frac{1}{x} = 0$

$\lim_{x\to-\infty} \frac{1}{x} = 0$

$\lim_{x\to+\infty} \frac{3x^2-6x+5}{4x^2+2x-11} = \lim_{x\to+\infty} \frac{3x^2}{4x^2} = \lim_{x\to+\infty} \frac{3}{4} = \frac{3}{4}$

$\lim_{x\to-\infty} \frac{x-2}{x^2+2x+1} = \lim_{x\to-\infty} \frac{x}{x^2} = \lim_{x\to-\infty} \frac{1}{x} = 0$ ($\frac{1}{x}$)

$\lim_{x\to+\infty} \frac{\sqrt{x^2+2}}{3x-6} = \lim_{x\to+\infty} \frac{\frac{\sqrt{x^2+2}}{x^2}}{\frac{3x-6}{\sqrt{x^2}}} = \lim_{x\to+\infty} \frac{\sqrt{1+\frac{2}{x^2}}}{3-\frac{6}{x}} = \frac{\sqrt{1+\frac{2}{\infty}}}{3-\frac{6}{\infty}} = \frac{\sqrt{1+0}}{3-0} = \frac{1}{3}$

$\frac{\sqrt{x^2+2}}{3x-6} = \frac{\frac{\sqrt{x^2+2}}{x^2}}{\frac{3x-6}{\sqrt{x^2}}} = \frac{\sqrt{x^2+2}}{\sqrt{x^2}} \times \frac{\sqrt{x^2}}{3x-6} = \frac{\sqrt{x^2+2}}{3x-6}$

$\lim_{x\to-\infty} \frac{\sqrt{5x^2-2}}{x+3} = \lim_{x\to-\infty} \frac{\frac{\sqrt{5x^2-2}}{|x|}}{\frac{x+3}{|x|}} = \lim_{x\to-\infty} \frac{\sqrt{5-\frac{2}{x^2}}}{-1-\frac{3}{x}}$ ($\sqrt{x^2}=|x|=-x$, $x\to-\infty$)

$= \frac{\sqrt{5-0}}{-1+\frac{3}{\infty}} = \frac{\sqrt{5}}{-1} = -\sqrt{5}$

$\sqrt{x^2} = x$; $\sqrt{(-x)^2} = -x$

$|x| = \begin{cases} x; & x \geq 0 \\ -x; & x < 0 \end{cases}$

$\lim_{x\to-\infty} \frac{\sqrt{3x^2+5}}{2x-7} = \lim_{x\to\infty} \frac{\frac{\sqrt{3x^2+5}}{|x|}}{\frac{2x-7}{|x|}}$

$\lim_{x\to-\infty} \frac{\sqrt{3x^2+5}}{\sqrt{x^2}}{\frac{2x-7}{\sqrt{x^2}}}$

$\lim_{x\to\infty} \sqrt{x^2-3x}-x = \lim_{x\to\infty} \frac{(\sqrt{x^2-3x}-x)(\sqrt{x^2-3x}+x)}{\sqrt{x^2-3x}+x}$

$= \lim_{x\to\infty} \frac{(\sqrt{x^2-3x})^2 - x^2}{\sqrt{x^2-3x}+x}$

$= \lim_{x\to\infty} \frac{x^2-3x-x^2}{\sqrt{x^2-3x}+x} = \lim_{x\to\infty} \frac{-3x}{\sqrt{x^2-3x}+x}$

$= \lim_{x\to\infty} \frac{\frac{-3x}{|x|}}{\frac{\sqrt{x^2-3x}+x}{|x|}} = \lim_{x\to\infty} \frac{\frac{-3x}{x}}{\frac{\sqrt{x^2-3x}}{\sqrt{x^2}}+\frac{x}{x}}$

$= \lim_{x\to\infty} \frac{-3}{1+\sqrt{1-\frac{3}{x}}} = \frac{-3}{1+\sqrt{1-\frac{3}{\infty}}} = -3$

$= \frac{-3}{1+\sqrt{1-0}} = -\frac{3}{2}$

$\lim_{x\to+\infty} 1+e^x = \lim_{x\to+\infty} e^x(1+e^{-x})$

$= e^{\infty}(1+e^{-\infty})$

$= e^{\infty}(1+0) = \infty$

$y = f(x)$

$|x-a| < \delta$

$|f(x)-L| < \varepsilon$

$\lim_{x\to a} f(x) = L$

$|f(x)-L| < \varepsilon$ where $|x-a| < \delta$

$\lim_{x\to 3} x^2 = 9$

$|x^2-9| < \varepsilon$ if $|x-3| < \delta$

$|x+3||x-3| < \varepsilon$

$\Rightarrow |x+3||x-3| < 7|x-3|$

$< 7\delta$

$|x^2-9| < 7\delta$ if $|x-3| < \delta$

$|x+3||x-3| < 7|x-3|$

$|x+3| < 7$

$\Rightarrow |x+3||x-3| < 7|x-3|$