$\tan\theta = \frac{50}{s}$

$\Rightarrow s = \frac{50}{\tan\theta} = 50\cot\theta$

$\frac{ds}{d\theta} = \frac{d}{d\theta}(50\cot\theta)$

$\Rightarrow \frac{ds}{d\theta} = -50\operatorname{cosec}^2\theta$

when $\theta = 45^\circ$, then $\frac{ds}{d\theta} = -50\operatorname{cosec}^2\frac{\pi}{4} = -50(\sqrt{2})^2 = -100$ ft/radian

$= -100 \times \frac{\pi}{180}$ ft/degree.

$y = \frac{\sin x \sec x}{1 + x\tan x}$

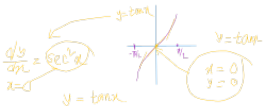
$\sin x \sec x = \sin x \frac{1}{\cos x} = \frac{\sin x}{\cos x} = \tan x$

$\frac{dy}{dx} = \frac{(1+x\tan x)\frac{d}{dx}(\tan x) - \tan x\left(0 + x\frac{d}{dx}(\tan x)\right)}{(1+x\tan x)^2}$

$= \frac{(1+x\tan x)\sec^2 x - \tan x(0 + x\sec^2 x)}{(1+x\tan x)^2}$

$= \frac{\sec^2 x + x\tan x\sec^2 x - x\tan x\sec^2 x}{(1+x\tan x)^2}$

$= \frac{1}{(1+x\tan x)^2}$




$y = \tan x$

$\frac{dy}{dx} = \frac{d}{dx}(\tan x) = \sec^2 x$

The slope of the tangent line to $y = \tan x$ at $x = 0$ is given by

$\left.\frac{dy}{dx}\right|_{x=0} = \sec^2 0 = 1$

So, the equation of the tangent line is

$y - y_1 = m(x - x_1)$

$\Rightarrow y - 0 = 1(x - 0)$

$\Rightarrow y = x$



**Chain rule of Differentiation:**

$y = f(x)$ $\quad x = g(t)$

$\frac{dy}{dx}$ $\quad \frac{dx}{dt}$

$\frac{dy}{dt} = \frac{dy}{dx}\frac{dx}{dt}$ → Chain Rule

$y = \cos(x^3)$ $\quad \frac{dy}{dx} = ?$

$y = \cos z$ $\quad z = x^3$

$y = \cos(x^3)$

$\therefore \frac{dy}{dx} = \frac{d}{dx}(\cos(x^3))$

$= -\sin(x^3)\frac{d}{dx}(x^3)$

$= -3x^2\sin(x^3)$

$f(x) = \left(x^2 - \frac{7}{x}\right)^{-2}$

$\frac{d}{dx}(f(x)) = \frac{d}{dx}\left(x^2 - \frac{7}{x}\right)^{-2}$

$\Rightarrow f'(x) = (-2)\left(x^2 - \frac{7}{x}\right)^{-2-1}\frac{d}{dx}\left(x^2 - \frac{7}{x}\right)$

$= -2\left(x^2 - \frac{7}{x}\right)^{-3}\left(2x + \frac{7}{x^2}\right)$

$f(x) = \cos^3\left(\frac{x}{x+1}\right)$

$= \left(\cos\left(\frac{x}{x+1}\right)\right)^3$

$\frac{d}{dx}(f(x)) = \frac{d}{dx}\left(\cos\left(\frac{x}{x+1}\right)\right)^3$

$\Rightarrow f'(x) = 3\left(\cos\left(\frac{x}{x+1}\right)\right)^{3-1}\frac{d}{dx}\left(\cos\left(\frac{x}{x+1}\right)\right)$

$= 3\cos^2\left(\frac{x}{x+1}\right)\left(-\sin\left(\frac{x}{x+1}\right)\right)\frac{d}{dx}\left(\frac{x}{x+1}\right)$

$= -3\cos^2\left(\frac{x}{x+1}\right)\sin\left(\frac{x}{x+1}\right)\frac{(x+1)\frac{d}{dx}(x) - x\frac{d}{dx}(x+1)}{(x+1)^2}$

$= -3\cos^2\left(\frac{x}{x+1}\right)\sin\left(\frac{x}{x+1}\right)\frac{x+1-x}{(x+1)^2}$

$= -\frac{3}{(x+1)^2}\cos^2\left(\frac{x}{x+1}\right)\sin\left(\frac{x}{x+1}\right).$

**Implicit Differentiation:**

$\frac{dy}{dx}$

$x^2 + y^2 = 3xy$

$y = f(x)$

$x^2 + y^2 = 3xy$

$\frac{d}{dx}(x^2 + y^2) = \frac{d}{dx}(3xy)$

$\Rightarrow 2x + 2y\frac{dy}{dx} = 3\frac{d}{dx}(xy)$

$\Rightarrow 2x + 2y\frac{dy}{dx} = 3x\frac{dy}{dx} + 3y\frac{d}{dx}(x)$

$\Rightarrow 2x + 2y\frac{dy}{dx} = 3x\frac{dy}{dx} + 3y$

$\Rightarrow \frac{dy}{dx}(2y - 3x) = 3y - 2x$

$\Rightarrow \frac{dy}{dx} = \frac{3y - 2x}{2y - 3x}$

$\frac{dy}{dx}$

$x\cos y = y$

$\frac{d}{dx}(x\cos y) = \frac{dy}{dx}$

$\Rightarrow x\frac{d}{dx}(\cos y) + \cos y\frac{d}{dx}(x) = \frac{dy}{dx}$

$\Rightarrow x(-\sin y)\frac{dy}{dx} + \cos y = \frac{dy}{dx}$

$\Rightarrow \frac{dy}{dx}(1 + x\sin y) = \cos y$

$\Rightarrow \frac{dy}{dx} = \frac{\cos y}{1 + x\sin y}$