

$$1) \quad x \frac{dy}{dx} = x^2 y$$

$$\frac{dy}{dx} = xy$$

x	0	1	2	3
y	0	1	2	3
$\frac{dy}{dx}$	0	1	4	9

$$\textcircled{2} \quad \left. \begin{aligned} \frac{\partial M}{\partial y} &= \frac{\partial N}{\partial x} \\ \text{equation: } M(x, y) dx + N(x, y) dy &= 0 \end{aligned} \right\} \text{Condition}$$

now,

$$(2y \sin x \cos x - y + 2y^2 e^{xy^2}) dx - (x - \sin^2 x - 4xy e^{xy^2}) dy = 0$$

$$M(x, y) = 2y \sin x \cos x - y + 2y^2 e^{xy^2}$$

$$N(x, y) = -(x - \sin^2 x - 4xy e^{xy^2})$$

$$\therefore \frac{\partial M}{\partial y} = 2 \sin x \cos x - 1 + 2(y^2 e^{xy^2} \cdot 2y + e^{xy^2} \cdot 2y)$$

$$\frac{\partial M}{\partial y} = 2 \sin x \cos x - 1 + 4y^3 e^{xy^2} x + 4y e^{x^2}$$

$$\frac{\partial N}{\partial x} = -1 + \sin 2x + 4y e^{y^2 x} + 4y^3 x e^{xy^2}$$

$$\boxed{\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}}$$

equation is
exact.

There exists a solution $f(x, y) = C$ such that

$$\frac{\partial f}{\partial x} = M(x, y) = 2y \sin x \cos x - y + 2y^2 e^{xy^2}$$

$$\int \frac{\partial f}{\partial x} dx = \int (y \sin 2x - y + 2y^2 e^{xy^2}) dx$$

$$f(x, y) = \frac{-1}{2} \cos(2x) \cdot y - xy + 2e^{xy^2} + g(y)$$
$$= -\frac{1}{2} y (1 - 2\sin^2 x) - xy + 2e^{xy^2} + g(y)$$

$$\rightarrow f(x, y) = -\frac{1}{2} y + y \sin^2 x - xy + 2e^{xy^2} + g(y)$$

$$\text{and } \frac{\partial f}{\partial y} = N(x, y) = -x + \sin^2 x + 4xy e^{xy^2}$$

$$\text{so, } -\frac{1}{2} + \sin^2 x - x + 4xy e^{xy^2} + g'(y) = -x$$
$$+ \sin^2 x + 4xy e^{xy^2}$$

$$\text{or, } \int g'(y) dy = \int \frac{1}{2} dy$$

$$\therefore g(y) = \frac{y}{2}$$

$$\text{so, } y \sin^2 x - xy + 2e^{xy^2} = C$$

