PHY 107
Vector/Scalar

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OUTLINE

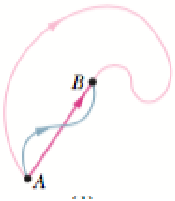
- Vector and Scalar
- Displacement Vector
- Adding vectors geometrically/Properties of vector addition
- Head to tail arrangement
- Components of vectors
- Unit vectors
- Adding Vectors by components
- Multiplication
- Scalar Product
- Vector Product

Vector and Scalar

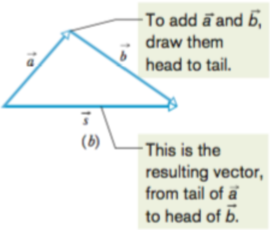
A **vector** is a direction in a space of some specific dimension
Vector quantity has both magnitude and direction e.g. velocity, displacement...Such quantity is represented by the use of an overhead arrow e.g. $\overrightarrow{v}$
Scalar quantity has magnitude only e.g. speed, temperature...

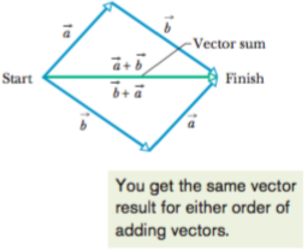
Displacement Vector

It is a vector to denote the change in position of a particle.
It tells us NOTHING about the path taken by the particle.



Adding vectors geometrically/Properties of vector addition





2 important properties of vector addition:

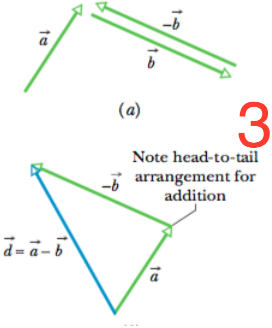
Commutative Law: the order of addition does NOT matter

$\vec{a} + \vec{b} = \vec{b} + \vec{a}$

Associative Law: In case of more than 2 vectors , we can group them in any order.

$(\vec{a} + \vec{b}) + \vec{c} = \vec{a} + (\vec{b} + \vec{c})$

Head to tail arrangement



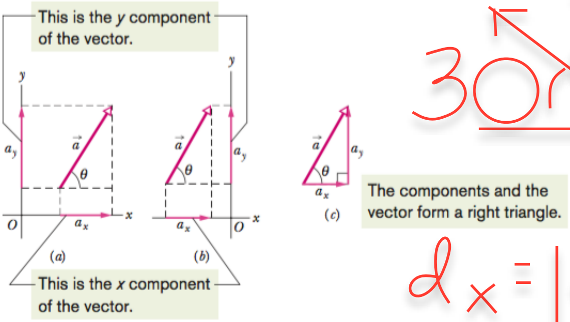
3-2=3+(-2)

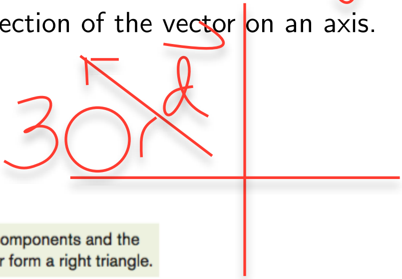
Vectors can be added/subtracted, but they need to be of the same kind.

Components of vectors

$\vec{d} = d_x \hat{i} + d_y \hat{j}$

A component of a vector is the projection of the vector on an axis.





$d_x = |d| \cos$

$d_y = |d| \sin$

$a_x = |\vec{a}| cos(\theta)$
$a_y = |\vec{a}| sin(\theta)$
Given a_x and a_y, can we compute $\vec{a}, \theta$?
$a = \sqrt{a_x^2 + a_y^2}$, $tan(\theta) = \frac{a_y}{a_x}$

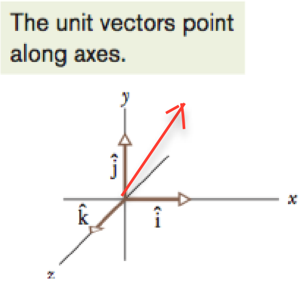
a=3i+4j

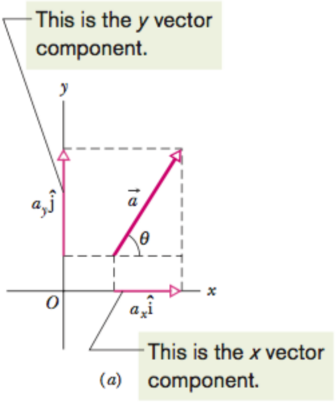
|a|=sqrt(3^(2)+4^(2))

Unit vectors

A unit vector is a vector of magnitude 1 and points in a particular direction

$$\vec{a}=3\hat{i}+4\hat{j}$$





Adding vectors by components

Let us say we have two vectors $\overrightarrow{a}$ and $\overrightarrow{b}$:
$\overrightarrow{a} = a_x\hat{i} + a_y\hat{j}$
$\overrightarrow{b} = b_x\hat{i} + b_y\hat{j}$
Find $\overrightarrow{r} = \overrightarrow{a} + \overrightarrow{b}$
$\overrightarrow{r} = (a_x + b_x)\hat{i} + (a_y + b_y)\hat{j}$

a=2i + 4j; b=3i + 7j; a+b=5i + 11j

Multiplication

$$\vec{k} = -2\vec{a}$$

Multiplying a vector by a scalar

$\vec{k} = s\vec{a}$

if s is +ve, then $\vec{k}$ has the same direction as $\vec{a}$

if s is -ve, then $\vec{k}$ has the opposite direction as $\vec{a}$

Multiplying a vector by a vector

SCALAR PRODUCT: gives you a scalar

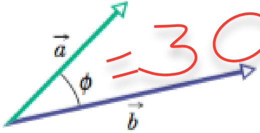
VECTOR PRODUCT: gives you a vector

$$W = \vec{F} \cdot \vec{d}$$

$$\vec{\tau} = \vec{r} \times \vec{F}$$

Scalar Product

The scalar product of vectors $\vec{a}$ and $\vec{b}$ is:
$\vec{a}\,.\,\vec{b} = |\vec{a}||\vec{b}|\cos\phi$



$= 30^\circ$

Both ϕ and $(360 - \phi)$ would give the same scalar product

ϕ	$\vec{a}\,.\,\vec{b}$
0	ab (Max)
90	0

$\vec{a} \cdot \vec{b} = 0$

90 deg means orthogonal

Scalar Product

a=3i+4j; b=2i-5j; a.b=?

Commutative Law applies to a scalar product

$\overrightarrow{a}.\overrightarrow{b} = \overrightarrow{b}.\overrightarrow{a}$

In UNIT vector notation (2D):

$\overrightarrow{a}.\overrightarrow{b} = (a_x\hat{i} + a_y\hat{j}).(b_x\hat{i} + b_y j) = a_x b_x + a_y b_y$

a.b = 3(2) + 4(-5) = 6 + -20 = -14

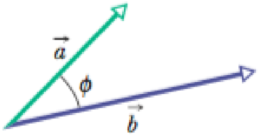
Vector Product

The vector product of $\overrightarrow{a}$ and $\overrightarrow{b}$ ($\overrightarrow{a}$ X $\overrightarrow{b}$) gives a third vector $\overrightarrow{c}$ of magnitude

c=absinϕ

ϕ: smaller of the two angles between $\overrightarrow{a}$ and $\overrightarrow{b}$

since $sin(\phi) \neq sin(360 - \phi)$



Note that ϕ and $(360 - \phi)$ would give different vector products

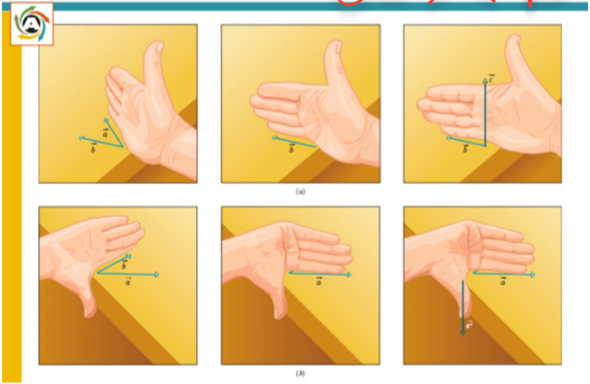
ϕ	$\lvert\overrightarrow{a} X \overrightarrow{b}\rvert$
0	0
90	ab

Vector Product

How to determine the direction of the third vector?

Right hand rule: Sweep your fingers (starting with the first vector) towards the second vector, then the thumb points to the third direction

$a \times b$



$$(\vec{a} \; X \; \vec{b}) = -(\vec{b} \; X \; \vec{a})$$

Vector Product

In UNIT vector notation (3D):

$$\vec{a}\ X\ \vec{b}\ = (a_x\hat{i} + a_y\hat{j} + a_z\hat{k})\ X\ (b_x\hat{i} + b_y\hat{j} + b_z\hat{k})$$
$$= a_x b_x(\hat{i}\ X\ \hat{i}) + a_x b_y(\hat{i}\ X\ \hat{j}) + a_x b_z(\hat{i}\ X\ \hat{k}) + a_y b_x(\hat{j}\ X\ \hat{i}) + a_y b_y(\hat{j}\ X\ \hat{j}) + a_y b_z(\hat{j}\ X\ \hat{k}) + a_z b_x(\hat{k}\ X\ \hat{i}) + a_z b_y(\hat{k}\ X\ \hat{j}) + a_z b_z(\hat{k}\ X\ \hat{k})$$
$$=(a_y b_z - a_z b_y)\hat{i} + (a_z b_x - a_x b_z)\hat{j} + (a_x b_y - a_y b_x)\hat{k}$$

a=2i-3j+4k;

b=-i+5j+8k; aXb= _i + _j + _k

Reference

Fundamentals of Physics by Halliday and Resnick