Library Functions:

i) Constant function:

$f(x) = b$; b is a real number.

Properties:

Domain : set of all real numbers

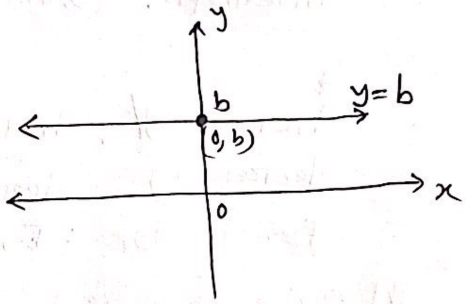
Range : A single number b.

Graph : Horizontal line

Intercepts: X-intercept → N/A

Y-intercept = b

Function type : Even Function



ii) Identity Function:

$f(x) = x$

Domain: Set of all real numbers

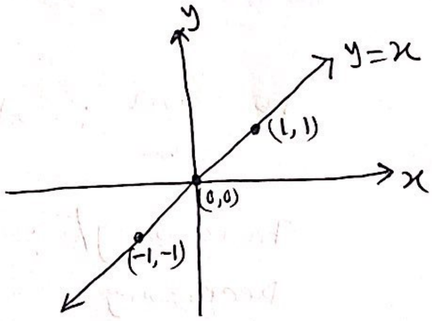
Range: Set of all real numbers

Intercepts: X-intercept = 0

Y-intercept = 0

Function type : odd function

Increasing or decreasing : Increasing over its domain



III) Square Function:

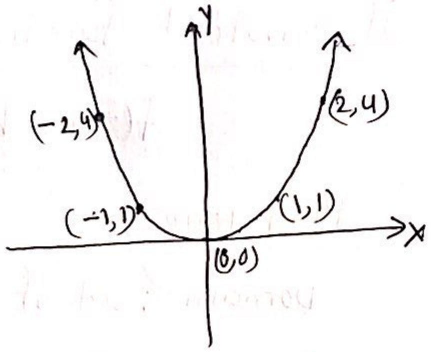
$$f(x) = x^2$$

Domain : set of all real numbers

Range : set of non-negative real numbers.

Graph : Parabola.

Intercepts : $(0,0)$; x-intercept $= 0$
y- " $= 0$



Increasing/ decreasing : Increasing on the interval $(0, \infty)$
decreasing on the interval $(-\infty, 0)$

Function type : Even function

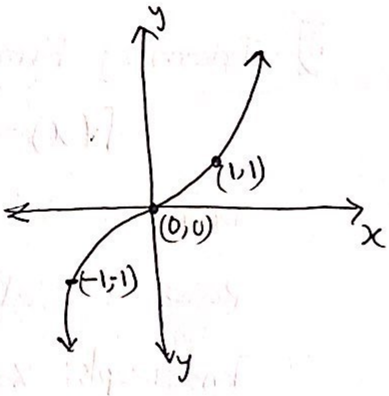
IV) Cube function:

$$f(x) = x^3$$

Domain : set of all real numbers.

Range : set of all real numbers.

Intercepts : $(0,0)$, x-intercept $= 0$
y - intercept $= 0$



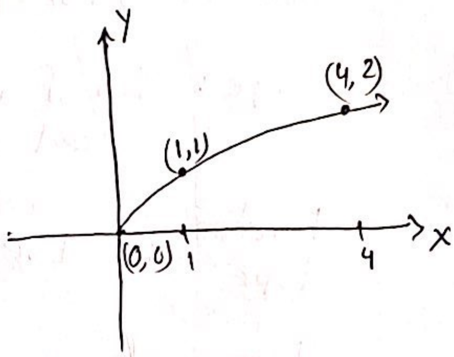
Function type : odd function

Increasing/ Decreasing : Increasing on the interval $(-\infty, \infty)$

VI) Square root Function:

$$f(x) = \sqrt{x}$$

Domain : set of nonnegative real numbers.



Range : set of non-negative real numbers.

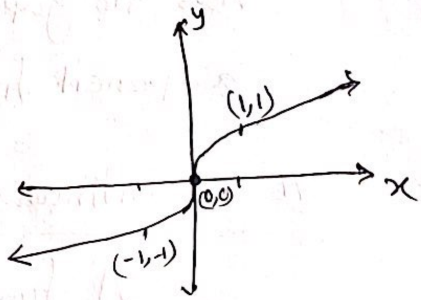
Intercepts : (0,0); x-intercept = 0
y " = 0

Function type : neither odd nor even.

Increasing/decreasing : Increasing on the interval $(0, \infty)$

VII) Cube root Function:

$$f(x) = \sqrt[3]{x}$$



Domain : set of all real numbers

Range : set of all real numbers.

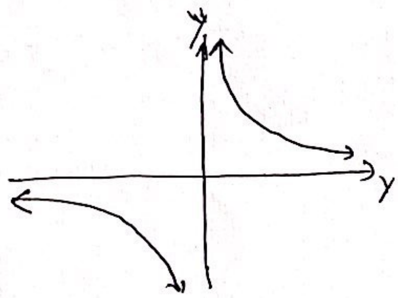
Intercepts : (0,0); x-intercept = 0
y- " = 0

Function type : odd function

Increasing/decreasing : Increasing on the interval $(-\infty, \infty)$

VIII) Reciprocal function:

$$f(x) = \frac{1}{x}$$



Domain : set of all nonzero real numbers

Range : set of all nonzero real numbers

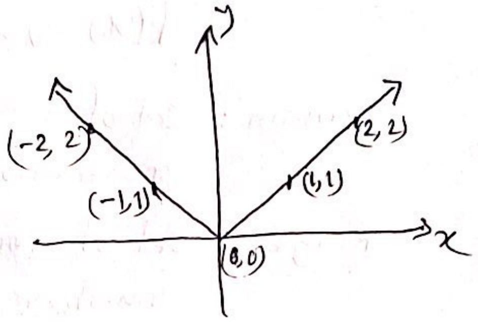
Intercepts : NO

Function type : odd function

Increasing/decreasing : decreasing on the intervals $(-\infty, 0)$ and $(0, \infty)$

VIII) Absolute value function:

$$f(x) = |x|$$



Domain: Set of all real numbers.

Range: set of nonnegative real numbers

Function type: Even function.

Increasing/decreasing: increasing on the interval $(0, \infty)$
decreasing on the interval $(-\infty, 0)$

continuous function:

A function is said to be continuous if its graph has no gaps or holes and can be drawn without lifting a pencil from the paper.

Discontinuous function:

A function is said to be discontinuous if its graph has gaps or holes so that its graph cannot be drawn without lifting a pencil from the paper.

Piecewise Function:

When a function is defined by different equations on different parts of its domain, then that function is called piecewise function.

$$f(x) = |x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

Example:

The function is defined as

$$f(x) = \begin{cases} -2x+1 & \text{if } -3 \leq x < 1 \\ 2 & \text{if } x = 1 \\ x^2 & \text{if } x > 1 \end{cases}$$

(a) Find $f(-2)$, $f(1)$, $f(2)$.

Sol^n: $f(-2)$, observe that when $x = -2$, the equation for f is given by $f(x) = -2x+1$. So

$$f(-2) = -2(-2)+1 = 5$$

When $x = 1$, the equation for f is $f(x) = 2$

i.e $\quad f(1) = 2$

When $x = 2$, the equation for f is $f(x) = x^2$

$$\therefore f(2) = 2^2 = 4$$

b) Determine the domain of f.

Soln: f is defined for all x greater than or equal to -3, so the domain of f is $\{x \mid x \geq -3\}$ or $[-3, \infty)$.

c) Locate any intercepts.

Soln: To find y-intercepts, let's put $x=0$ in the function $y = f(x) = -2x+1$. Because only $f(x) = -2x+1$ is the function where we get y intercepts.

$$f(0) = -2(0)+1 \Rightarrow f(0) = 1$$

So y-intercepts $= 1$

To find x-intercepts we have to put $f(x) = 0$

$\therefore f(x) = 0$

$-2x+1 = 0$

$\Rightarrow -2x = -1$

$\Rightarrow x = \frac{1}{2}$

$f(x) = 0$

$2 = 0$

no solution

$f(x) = 0$

$\Rightarrow x^2 = 0$

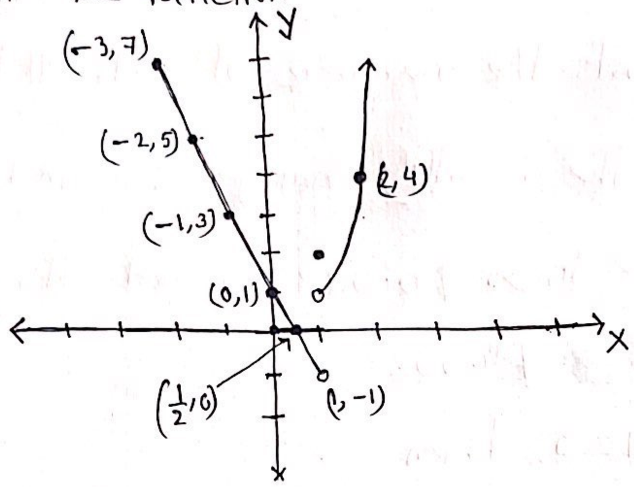
$\Rightarrow x = 0$

But $x=0$ doesnot satisfy the condition $x > 1$

So the only x-intercept is $\frac{1}{2}$

The intercepts are $(0, 1)$ and $(\frac{1}{2}, 0)$.

d) Graph the function.



e) Use the graph to find the range of f.

Sol^n: From the graph, we conclude that the range of f is $\{y \mid y > -1\}$ or the interval $(-1, \infty)$

f) The function f is not continuous because there is a jump in the graph at $x = 1$.

Application:

In the summer of 2009, Duke energy supplied electricity to residents of ohio for a monthly customer charge of \$4.50 plus 4.2354¢ per kilowatt-hour for the first 1000 kWhr supplied in the month and 5.3622¢ per kWhr for all usage over 1000 kWhr in the month.

If C is the monthly charge for x kWhr, develop a model relating the monthly charge and kilowatt hours used. Express C as a function of x.

Solution:

Let x represents the number of kilowatt hours used.

If $0 \le x \le 1000$, the monthly charge C can be found by multiplying x times $\$0.042345$ and adding the monthly customer charge of $\$4.50$.

So if $0 \le x \le 1000$, then

$$C(x) = 0.042345x + 4.50$$

For $x > 1000$, the charge is $0.042345(1000) + 4.50 + 0.053622(x-1000)$.

i.e if $x > 1000$ then

$$C(x) = 0.042345(1000) + 4.50 + 0.053622(x-1000)$$

$$= 46.845 + 0.053622(x-1000)$$

$$= 0.053622x - 6.777$$

Thus finally,

$$C(x) = \begin{cases} 0.042345x + 4.50 & \text{if } 0 \le x \le 1000. \\ 0.053622x - 6.77 & \text{if } x > 1000 \end{cases}$$

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