

$$\int u v dx = u \int v dx - \int \left[\frac{d}{dx}(u) \int v dx \right] dx \quad \text{LIATE}$$

$$\int \sin^n x dx = -\frac{1}{n} \sin^{n-1} x \cos x + \frac{n-1}{n} \int \sin^{n-2} x dx$$

$$\int \cos^n x dx = \frac{1}{n} \cos^{n-1} x \sin x + \frac{n-1}{n} \int \cos^{n-2} x dx$$

$$\int \tan^n x dx = \frac{\tan^{n-1} x}{n-1} - \int \tan^{n-2} x dx$$

$$\int \sec^n x dx = \frac{\sec^{n-2} x \tan x}{n-1} + \frac{n-2}{n-1} \int \sec^{n-2} x dx$$

$$\int \cot^n x dx = -\frac{\cot^{n-1} x}{n-1} - \int \cot^{n-2} x dx$$

$$\int \operatorname{cosec}^n x dx = -\frac{\operatorname{cosec}^{n-2} x \cot x}{n-1} + \frac{n-2}{n-1} \int \operatorname{cosec}^{n-2} x dx$$

$$\int \tan x dx = \ln |\sec x|$$

$$\int \cot x dx = \ln |\sin x|$$

$$\int \sec x dx = \ln |\sec x + \tan x|$$

$$A = \int_a^b [f(x) - g(x)] dx$$

$$V = \pi \int_a^b [f(x)^2 - g(x)^2] dx$$

$$L = \int_a^b \sqrt{1 + f'(x)^2} dx$$

$$\frac{d}{dx} \tan^{-1} x \quad \int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a}$$

$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1} \frac{x}{a}$$

$$\int \frac{dx}{x \sqrt{x^2 - a^2}} = \frac{1}{a} \sec^{-1} \frac{x}{a}$$

$$\int \sin^n x \, dx = \frac{-\sin^{n-1} x \cos x}{n} + \frac{n-1}{n} \int \sin^{n-2} x \, dx$$

$$\int \cos^n x \, dx = \frac{\cos^{n-1} x \sin x}{n} + \frac{n-1}{n} \int \cos^{n-2} x \, dx$$

$$\int \tan^n x \, dx = \frac{\tan^{n-1} x}{n-1} - \int \tan^{n-2} x \, dx$$

$$\int \sec^n x \, dx = \frac{\sec^{n-2} x \tan x}{n-1} + \frac{n-2}{n-1} \int \sec^{n-2} x \, dx$$

$$\text{Cor} \quad e^x = \frac{e^x + e^{-x}}{2} + \frac{e^x - e^{-x}}{2} = \cosh x + \sinh x \quad \cosh^2 x - \sinh^2 x = 1$$

$$\frac{d}{dx} (\sin x) = \cos x$$

$$\frac{d}{dx} (\sinh x) = \cosh x$$

$$\frac{d}{dx} (\cos x) = -\sin x$$

$$\frac{d}{dx} (\cosh x) = \sinh x$$

$$\frac{d}{dx} (\tan x) = \sec^2 x$$

$$\frac{d}{dx} (\tanh x) = \operatorname{sech}^2 x$$

$$\frac{d}{dx} (\cot x) = -\operatorname{cosec}^2 x$$

$$\frac{d}{dx} (\coth x) = -\operatorname{cosec}^2 h x$$

$$\frac{d}{dx} (\sec x) = \sec x \tan x$$

$$\frac{d}{dx} (\operatorname{sech} x) = -\operatorname{sech} x \tanh x$$

$$\frac{d}{dx} (\operatorname{cosec} x) = -\operatorname{cosec} x \cot x$$

$$\frac{d}{dx} (\operatorname{cosech} x) = -\operatorname{cosech} x \coth x$$

$$A = \int_a^b [f(x) - g(x)] dx$$

$$V = \pi \int_a^b [f(x)]^2 - [g(x)]^2 dx$$

$$L = \int_a^b \sqrt{1 + [f'(x)]^2} dx$$

$$\frac{d}{dx} (\sin^{-1} ax) = \frac{1}{\sqrt{a^2 - x^2}}$$

$$\frac{d}{dx} (\sinh^{-1} ax) = \frac{1}{\sqrt{a^2 + x^2}}$$

$$\frac{d}{dx} (\cos^{-1} ax) = \frac{-1}{\sqrt{a^2 - x^2}}$$

$$\frac{d}{dx} (\cosh^{-1} ax) = \frac{1}{\sqrt{x^2 - a^2}}$$

$$\frac{d}{dx} (\tan^{-1} ax) = \frac{1}{a^2 + x^2}$$

$$\frac{d}{dx} (\tanh^{-1} ax) = \frac{1}{a^2 - x^2}, x < a$$

$$\frac{d}{dx} (\cot^{-1} ax) = \frac{-1}{a^2 + x^2}$$

$$\frac{d}{dx} (\coth^{-1} ax) = \frac{1}{a^2 - x^2}, x > a$$

$$\frac{d}{dx} (\operatorname{cosec}^{-1} ax) = \frac{1}{x \sqrt{x^2 - a^2}}$$

$$\frac{d}{dx} (\operatorname{sech}^{-1} ax) = \frac{-1}{x \sqrt{a^2 - x^2}}$$

$$\frac{d}{dx} (\operatorname{cosech}^{-1} ax) = \frac{-1}{x \sqrt{x^2 - a^2}}$$

$$\frac{d}{dx} (\operatorname{coth}^{-1} ax) = \frac{-1}{x \sqrt{a^2 + x^2}}$$

$$A = \int_a^b [f(x) - g(x)] dx = \int_c^d [f(y) - g(y)] dy$$

Arc length, $L = \int_a^b \sqrt{1 + [f'(x)]^2} dx = \int_c^d \sqrt{1 + [g'(y)]^2} dy$

Area of a surface of a revolution,

$$S = 2\pi r l$$

$$S = \int_a^b 2\pi f(x) \sqrt{1 + [f'(x)]^2} dx, \text{ revolved about } x \text{ axis}$$

$$S = \int_c^d 2\pi g(y) \sqrt{1 + [g'(y)]^2} dy, \text{ revolved about } y \text{ axis}$$

$$\text{Volume, } V = \pi \int_a^b [f(x)^2 - g(x)^2] dx = \pi \int_c^d [f(y)^2 - g(y)^2] dy$$

(revolved about x axis) (revolved about y axis)

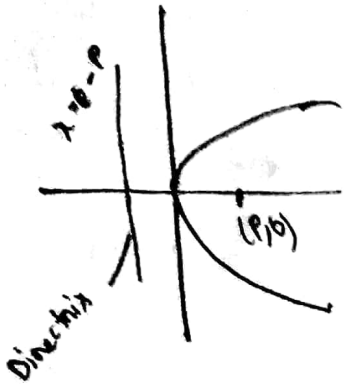
Cylindrical shell,

$$V = \int_a^b 2\pi x f(x) dx, \text{ revolved about } y \text{ axis}$$

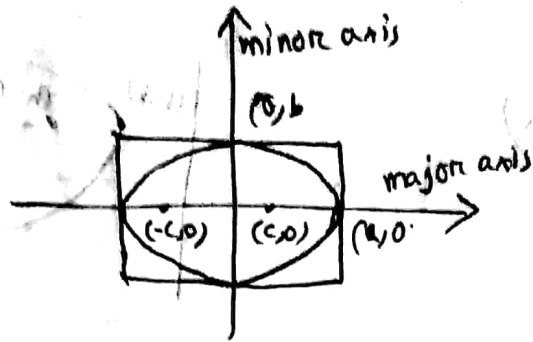
$$V = \int_c^d 2\pi y g(y) dy, \text{ revolved about } x \text{ axis}$$

Parabola

$$y^2 = 4px$$



Ellipse



Focus (c,0)

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$a^2 = b^2 + c^2$$

a = major axis

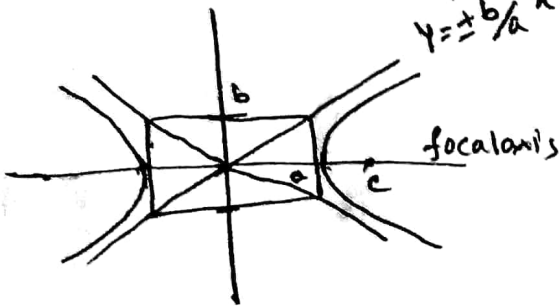
Hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 0$$

$$y = \pm \frac{b}{a}x$$

$$\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$$



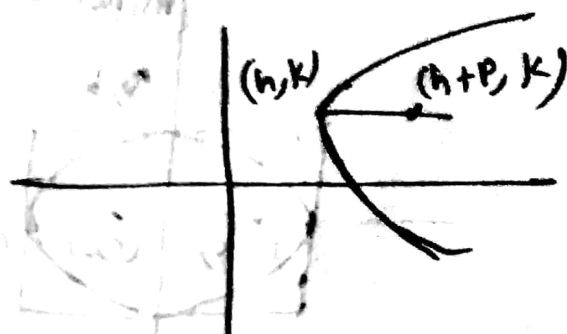
$$c^2 = a^2 + b^2$$

if x is positive, then x has a focal axis

if y " " , then y " " " "

Parabola with vertex (h, k)

$$(y-k)^2 = 4p(x-h)$$



Ellipse with vertex (h, k)

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

Hyperbola with vertex (h, k)

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

$$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$$

Conic sections in polar Co-ordinates

$$\text{eccentricity, } e = \frac{\text{distance to the focus}}{\text{distance to the directrix}}$$

for parabola, $e = 1$, directrix, $x = -p$

for ellipse, $0 < e < 1$ directrix, $x = \frac{a^2}{c}$

for hyperbola, $e > 1$, directrix, $x = \frac{a^2}{c}$

$$r = \frac{ed}{1 + e \cos \theta}, \quad d = \text{distance to the directrix}$$

directrix is right for $+e \cos \theta$



directrix is left for $-e \cos \theta$



$$r = \frac{ed}{1 + e \sin \theta}$$

directrix is above for $+e \sin \theta$



directrix is below for $-e \sin \theta$



For ellipse, $r_0 = \frac{ed}{1+e\cos\theta}$ $\theta=0$, $r_0 = \frac{ed}{1+e}$

$\theta=\pi$, $r_1 = \frac{ed}{1+e\cos\pi} = \frac{ed}{1-e}$

$a = \frac{1}{2}(r_1 + r_0)$ $b = \sqrt{r_1 r_0}$

$c = \frac{1}{2}(r_1 - r_0)$

Slope, $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$

For horizontal line, $\frac{dy}{dx} = 0$ i.e. $\frac{dy}{dt} = 0$ and $\frac{dx}{dt} \neq 0$

for vertical line, $\frac{dy}{dx} = \infty$, i.e. $\frac{dx}{dt} = 0$ and $\frac{dy}{dt} \neq 0$

$$\frac{d^2 y}{dx^2} = \frac{dv}{dr} (y')$$

$$= \frac{\frac{dy}{dx}}{\frac{dx}{dt}} \quad \left[y = \frac{dy}{dx} \right]$$

Circumference / Arc length, $L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$

$(r, \theta + \pi)$ and $(-r, \theta)$ are the polar co-ordinates of the same point

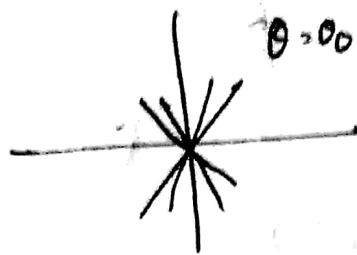
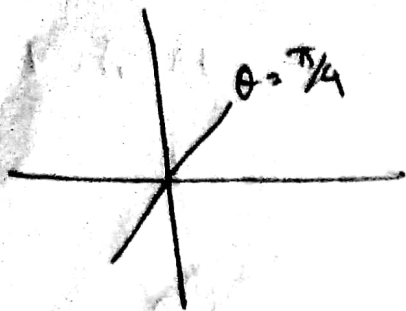
$$(r, \theta + 2n\pi) = (r, \theta - 2n\pi)$$

If θ is replace with $\pi - \theta$, the function is symmetric with y axis

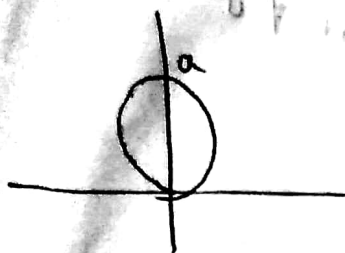
$\pi \quad 0 \quad " \quad " \quad " \quad 0 + \pi$

on n i " " - r) " " " " " & origin

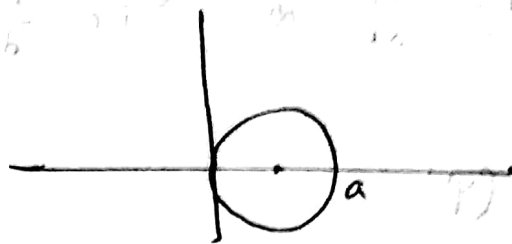
It " " " - 0, " " " " " X axis



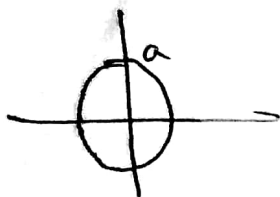
$$r = a \sin \theta$$



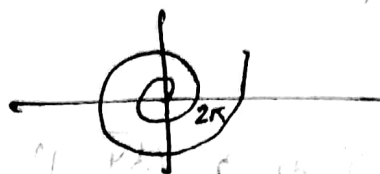
$$r = a \cos \theta$$



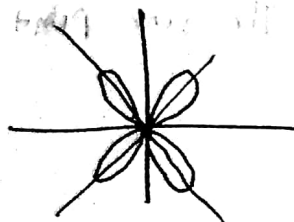
$$r = a$$



$$r = 0$$



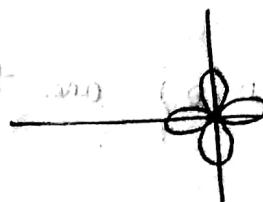
$$r = a \sin n\theta$$



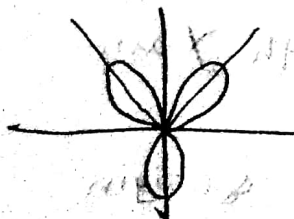
$$r = a \sin 2\theta$$

$$2 \times 2 = 4$$

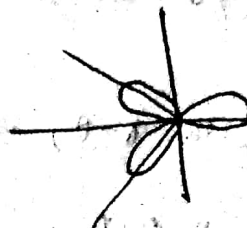
$$r = a \cos n\theta$$



$$r = a \cos 2\theta$$

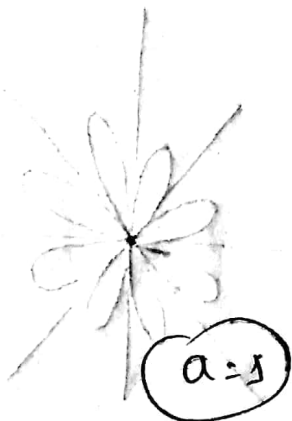
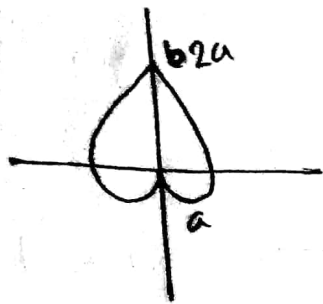


$$r = a \sin 3\theta$$

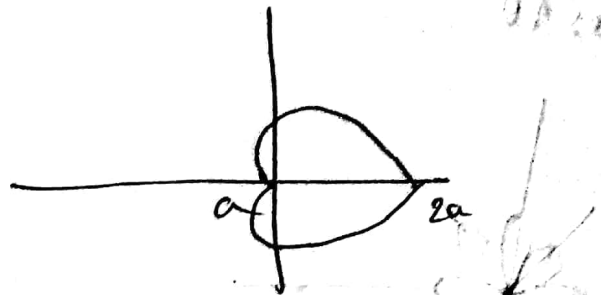


$$r = a \cos 3\theta$$

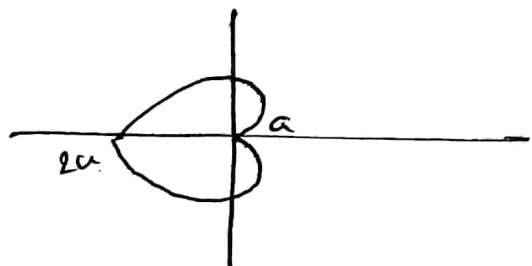
$$r = a + b \sin \theta$$



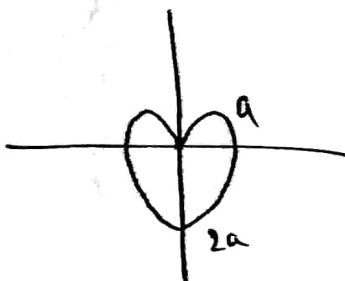
$$r = a + a \cos \theta$$



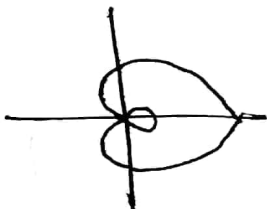
$$r = a - a \cos \theta$$



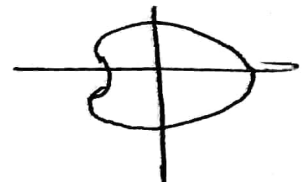
$$r = a - b \sin \theta$$



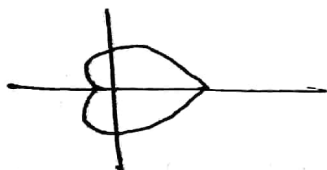
$$\frac{a}{b} < 1$$



$$\frac{a}{b} \geq 2$$



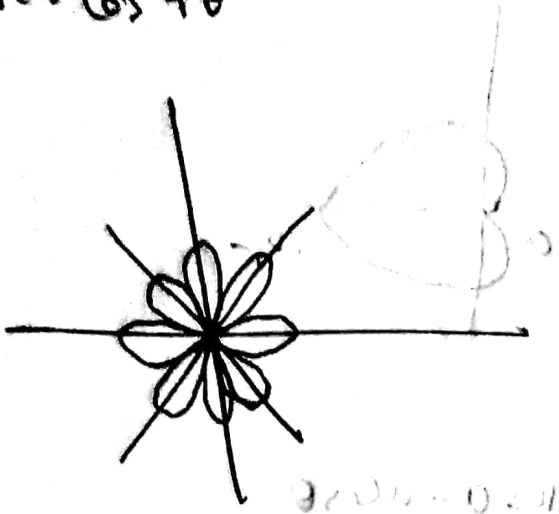
$$1 \leq \frac{a}{b} \leq 2$$



$$\text{Area, } A = \int_{\alpha}^{\beta} \frac{1}{2} r^2 d\theta$$

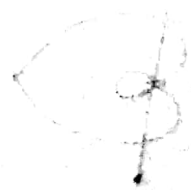
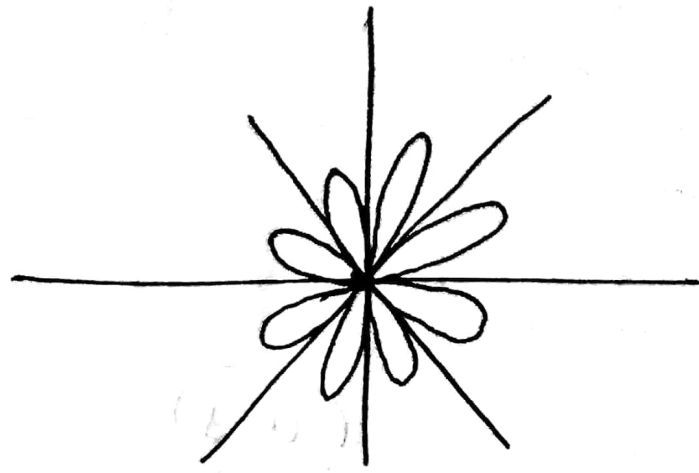
0(0)010.01

$$r = 6 \sin 4\theta$$



0(0)010.01

$$r = \sin 4\theta$$



0(0)010.01

= u'

$$\frac{x}{(x-1)(x+2)} = \frac{A}{x-1} + \frac{B}{x+2}$$

$$\frac{x}{(x-1)^2(x+2)} = \frac{A}{x+2} + \frac{B}{x-1} + \frac{C}{(x-1)^2}$$

$$\frac{x}{(x^2+1)(x^2+4)} = \frac{Ax+B}{x^2+1} + \frac{Cx+D}{x^2+4}$$

$$\frac{x}{(x+3)(x^2+4)^2} = \frac{A}{x+3} + \frac{Bx+C}{x^2+4} + \frac{Dx+E}{(x^2+4)^2}$$