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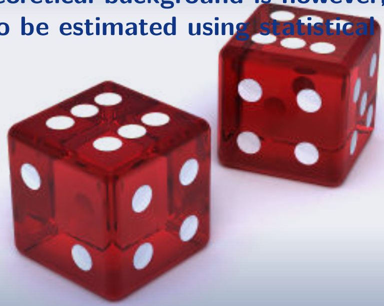

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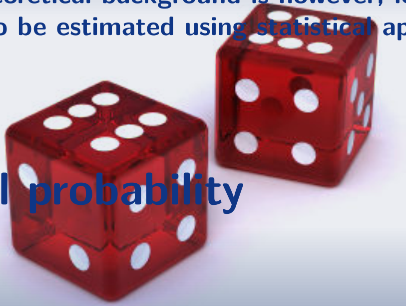

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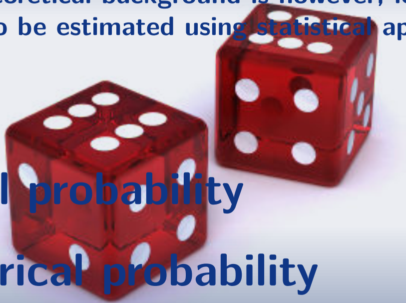

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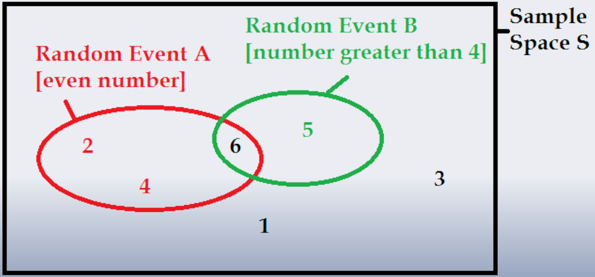

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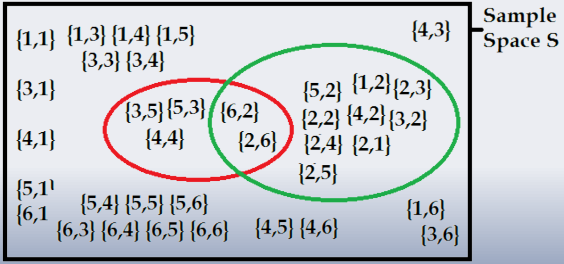

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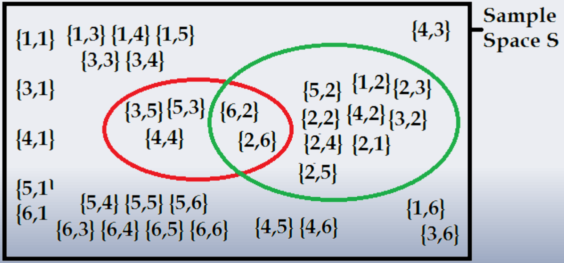

More complex events...

- Union of three random events A, B and C;

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\begin{aligned}
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- What about a multiple intersection of random events?

$$P(A \cap B \cap C) = ???$$

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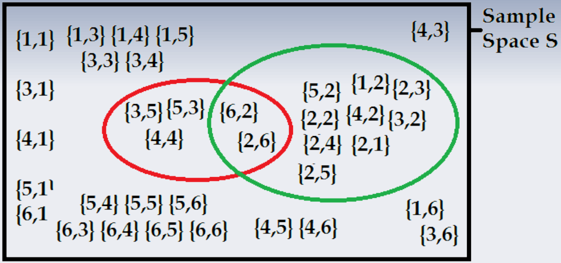
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{3,3} {3,4}
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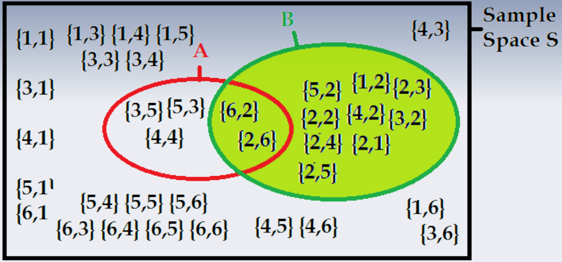

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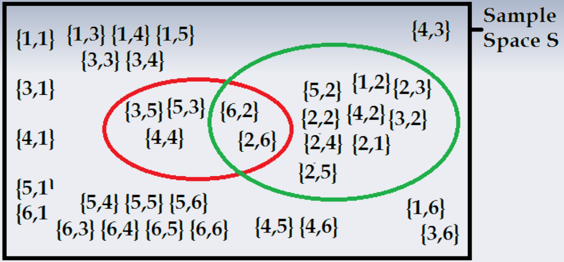

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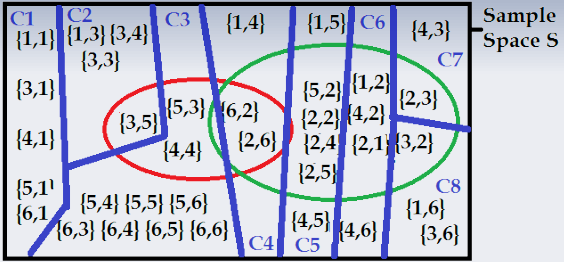

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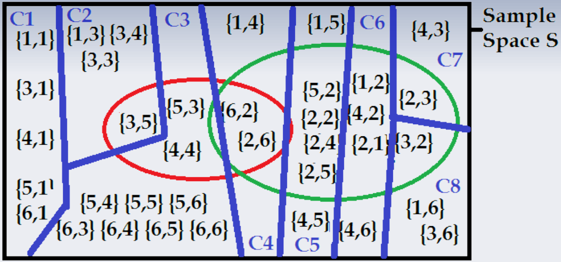

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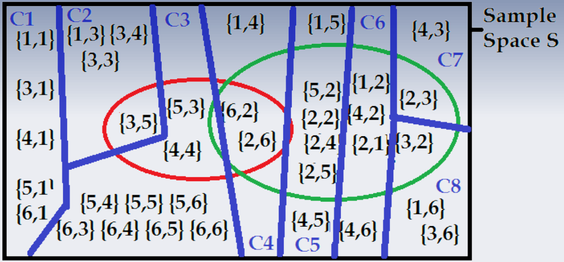

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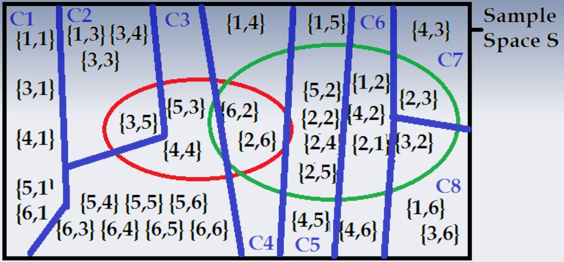

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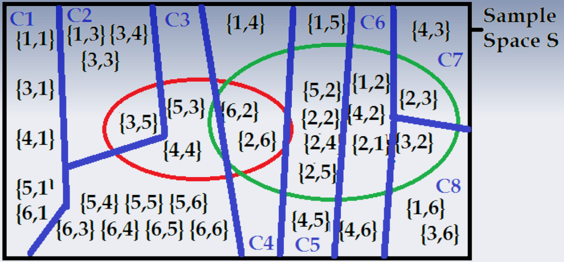

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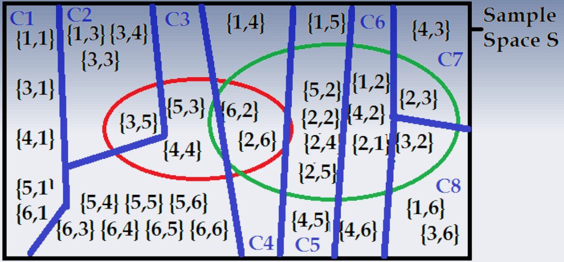

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$$P(A) = \sum_{i=1}^{8} P(A|C_i)P(C_i)$$

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  - out of boys, there is 10 % of them with long hairs;
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- very easy to prove (using The Law of Total Probability);
- very useful for practical examples;
- probability $P(A_i|B)$ is called a "posterior probability";
- probability $P(A_i)$ is called a "prior probability";

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- **All three statements above are equivalent and each of them directly implies the other two!**

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- We have two regular and symmetric dices - blue and green;
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$$\frac{1}{4} = P(A \cap B \cap C) \neq P(A).P(B).P(C) = \frac{1}{8}$$

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  - under the condition that each card is returned back?
  - under the condition that the card is not returned back?

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- The concept of geometrical probability was proposed to be used instead to avoid such problems...

What is useful to remind...

- **Permutations and combinations...**
  (drawing with replacement or without replacement)
- **Should we account for ordering or not?**
  (permutations or combinations should be used?)
- **What is the Multiplication Rule?**
  (how to use it together with permutations or combinations)

To be continued...

- Random Variable Definition;
- Discrete and Continuous Random Variables;
- Random Variable Characteristics;
- Chebyshev's Inequality;
- Joint and Marginal Distribution;
- Functions of Random Variables;