Assignment
(instead of midterm)

MAT 361
Probability and Statistics

Section 4

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North South University

Name	: A.S.M. Samiul Islam
Student ID	: 1921826642
Email Address	: Samiul.islam03@northsouth.edu

Mid Assignment

1) 2 marbles, 1 red(R) and 1 blue(B)

possible outcomes throug first experiment. that takes 1 marbel from the box and replace it are,

drawing 1 red marble, then blue marble (RB)
drawing 1 blue marble, then red marble (BR),
drawing 1 marble, then same marble (RR, BB).

$\therefore$ Sample Space, $S = \{RR, RB, BR, BB\}$

Again in the second experiment is same but no replacing. the first marble. Then outputs are, drawing 1 marble, then another one.

$\therefore$ Sample space, $S = \{RB, BR\}$

2)

a) given, $f(x) = cxe^{-\frac{x}{2}}; x>0$

We know, $\int_{0^+}^{\infty} f(x)\,dx = 1$

$\Rightarrow \int_{0^+}^{\infty} cxe^{-x/2}\,dx = 1$

$\Rightarrow 4c\left[-e^{-\frac{x}{2}}\left[\frac{x}{2}+1\right]\right]_{x=0^+}^{\infty} = 1$

$\Rightarrow c\left[-e^{-x/2}(2x+4)\right]_{x=0^+}^{\infty} = 1$

$\Rightarrow c\left[\frac{-(2x+4)}{e^{x/2}}\right]_{x=0^+}^{\infty} = 1$

$\Rightarrow c\left[-0-\left(\frac{-(0+4)}{1}\right)\right] = 1$

$\Rightarrow c\left[-(-4)\right] = 1 \Rightarrow 4c = 1$

$\therefore c = 1/4$ Answer

Let,

$z = \frac{-x}{2}$ $\therefore x = -2z$
$dx = -dz$

$I = \int cxe^{-x/2}\,dx$
$= \int c(-2z)\,e^{z}(-2dz)$
$= 4c\int z\,e^{z}\,dz$

Let $u = z$ $\quad dv = e^{z}$
$du = dz$ $\quad v = e^{z}$

$I_1 = ze^{z} - \int e^{z}dz$
$= ze^{z} - e^{z}$

$\therefore I = 4c(ze^{z} - e^{z})$
$= 4c\,e^{z}[z-1]$
$= 4c\left[e^{\cancel{z}}\left(-\frac{x}{2}-1\right)\right]$

2)(b) cumulative distribution function of x,

$$CDF = F(x) = \int_{0^+}^{x} f(x)\,dx$$

$$= \int_0^x \frac{1}{4} x e^{-x/2}\,dx$$

$$= \frac{1}{4}\left[\frac{-2x}{e^{x/2}} - \frac{4}{e^{x/2}}\right]_{x=0}^{x}$$

$$= \frac{1}{4}\left[-2xe^{-x/2} - 4e^{-x/2} - 0 - (-4)\right]$$

$$\therefore CDF = 1 - \frac{xe^{-x/2}}{2} - e^{-x/2}$$

Answer

3) Given, $f(x,y) = x + y$. ; $0<x<c$, & $0<y<1$.

(a) $$\int_0^1\int_0^c f(x,y)\,dx\,dy = 1$$

$$\Rightarrow \int_0^1\int_0^c [x+y]\,dx\,dy = 1$$

$$\Rightarrow \int_0^1 \left[\frac{x^2}{2} + xy\right]_{x=0}^{c} dy = 1$$

$$\Rightarrow \int_0^1 \left[\frac{c^2}{2} + cy - 0 - 0\right] dy = 1$$

$$\Rightarrow \int_0^1 \left[\frac{c^2}{2} + cy\right] dy = 1$$

$$\Rightarrow \left[\frac{c^2}{2}y + c\frac{y^2}{2}\right]_{y=0}^{1} = 1$$

$$\Rightarrow \left[\frac{c^2 \cdot 1}{2} + c\frac{1^2}{2}\right] = 1$$

$$\Rightarrow \frac{c^2 + c}{2} = 1$$

$$\Rightarrow c^2 + c - 2 = 0$$ [$\because 0<x<c$, c must be positive]

& $c = 1, -2$ [not possible] $\therefore c = 1$ Answer

(b) $g(x) = \int_0^1 f(x,y)\, dy$

$= \int_0^1 (x+y)\, dy$

$= \left[xy + \frac{y^2}{2} \right]_{y=0}^{1}$

$= \left[x \cdot 1 + \frac{1^2}{2} - 0 - 0 \right]$

$\therefore g(x) = x + \frac{1}{2}$

$h(y) = \int_0^1 f(x,y)\, dx$

$= \int_0^1 (x+y)\, dx$

$= \left[\frac{x^2}{2} + xy \right]_{x=0}^{1}$

$= \left[\frac{1^2}{2} + 1 \cdot y - 0 - 0 \right]$

$\therefore h(y) = \left[\frac{1}{2} + y \right]$

$\therefore$ marginal probability density function's are,

$g(x) = x + \frac{1}{2}$

$h(y) = y + \frac{1}{2}$ Answer

(c) if the random variables x and y are, independent, then,

$g(x) \cdot h(y) = f(x,y) = x+y$

$g(x) \cdot h(y) = (x + \frac{1}{2})(y + \frac{1}{2})$

$= xy + \frac{x}{2} + \frac{y}{2} + \frac{1}{4}$

$= xy + \frac{1}{2}(x+y) + \frac{1}{4} = \frac{1}{4}(4xy + 2(x+y) + 1)$

$\therefore g(x) \cdot h(y) \neq f(x,y)$

So the random variables x and y are not independent.

(d) If $y = 0.5$

The conditional probability density function of x,

$$f(x|y) = \frac{f(x,y)}{h(y)}$$

$$= f(x|y=0.05) = \frac{f(x, y=.05)}{h(y=.05)}$$

$$= \frac{x+0.5}{0.5+\frac{1}{2}}$$

$$= \frac{x+0.5}{0.5+0.5} = \frac{x+0.5}{1}$$

$$= x+0.5$$

$$= x+\frac{1}{2}$$

Answer

4| (a) Marginal probability mass function of x,

$$P(x=i) = \sum_{j=0}^{3} P_{ij} = P_{i0} + P_{i1} + P_{i2} + P_{i3}$$

$$\therefore P(X=1) = \sum_{j=0}^{3} P_{1j} = 0.10 + 0.15 + 0 + 0.05 = 0.30$$

$$P(X=2) = \sum_{j=0}^{3} P_{2j} = 0.20 + 0.05 + 0.05 + 0.20 = 0.50$$

$$P(X=3) = \sum_{j=0}^{3} P_{3j} = 0.05 + 0 + 0.10 + 0.05 = 0.20$$

Similarly, Marginal probability mass function of y,

$$P(y=j) = \sum_{i=1}^{3} P_{ij} = P_{1j} + P_{2j} + P_{3j}$$

$$P(Y=0) = \sum_{i=1}^{3} P_{i0} = 0.10 + 0.20 + 0.05 = 0.35$$

$$P(y=1) = \sum_{i=1}^{3} P_{i1} = 0.15 + 0.05 + 0 = 0.20$$

$$P(y=2) = \sum_{i=1}^{3} P_{i2} = 0 + 0.05 + 0.10 = 0.15$$

$$P(y=3) = \sum_{i=1}^{3} P_{i3} = 0.05 + 0.20 + 0.05 = 0.3$$

Answer

(b) $$P(x \mid y=1) = \frac{P(x, y=1)}{P(y=1)}$$

$$P(y=1) = 0.20$$

$$P(X=1 \mid y=1) = \frac{P(X=1, y=1)}{P(y=1)} = \frac{0.15}{0.20} = 0.75$$

$$P(X=2 \mid y=1) = \frac{P(X=2, y=1)}{P(y=1)} = \frac{0.05}{0.20} = 0.25$$

$$P(X=3 \mid y=1) = \frac{P(X=3, y=1)}{P(y=1)} = \frac{0}{0.20} = 0$$

Answer

(c) $E(X|y=1) = \sum_{i=1}^{3} i\,P(X|y=1)$

$= \{1 \times P(X=1|y=1)\} + (2 \times 0.25) + (3 \times 0)$

$= (1 \times 0.75) + 0.5$

$= 1.25$ Answer

(d) $E(X|y=1) = 1.25$

$E((X|y=1)^2) = \sum_{i=1}^{3} i^2\,P(X|y=1)$

$= (1^2 \times 0.75) + (2^2 \times 0.25) + (3^2 \times 0)$

$= 0.75 + 1$

$= 1.75.$

$V(X|y=1) = E((X|y=1)^2) - (E(X|y=1))^2$

$= 1.75 - (1.25)^2$

$= 1.75 - 1.5625$

$= 0.1875$ Answer

(e) $E(xy) = \sum_{i=1}^{3} \sum_{j=0}^{3} ij\,P_{ij}$

$= (1 \times 0 \times 0.10) + (1 \times 1 \times 0.15) + (1 \times 2 \times 0) + (1 \times 3 \times 0.05)$
$+ (2 \times 0 \times 0.20) + (2 \times 1 \times 0.05) + (2 \times 2 \times 0.05)$
$+ (2 \times 3 \times 0.20) + (3 \times 0 \times 0.05) + (3 \times 1 \times 0) + (3 \times 2 \times 0.10)$
$+ (3 \times 3 \times 0.05)$

$= 0 + 0.15 + 0 + 0.15 + 0 + 0.1 + 0.2 + 1.2 + 0 + 0 + 0.6 + 0.45$

$= 2.85$ Answer