Lecture-26

* We have Sensor data and we want to perform "Fitting" to fit a model to it.

* We have an equation $\longrightarrow$ $\boxed{y = Ax}$, where x is unknown &

$\boxed{x \in R^{n \times 1}}$ $\longrightarrow$ x is a Column Vector

Solution $\longrightarrow$ $\boxed{x = A^{+} y}$,

where A^{+} is called the "Pseudo Inverse".

y is observations.

& $\boxed{y \in R^{m \times 1}}$ $\longrightarrow$ y is also a Column Vector.

A is model &

$\boxed{A \in R^{m \times n}}$ $\longrightarrow$ A has m rows & n columns.

Assumption $\longrightarrow$ A is full rank $\longrightarrow$ It has linearly independant Vectors

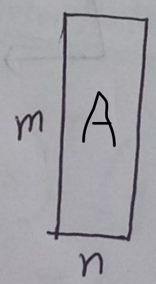
* We have 3 cases:—

1) $m = n \Rightarrow \boxed{A^{+} = A^{-1}}$

(Square Matrix full rank)

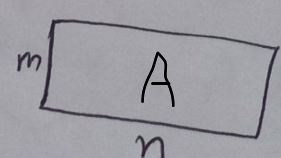
2) $m > n \Rightarrow \boxed{A^{+} = (A^{T}A)^{-1} A^{T}}$

(Least Square Solution) $\longrightarrow$



3) $m < n \Rightarrow \boxed{A^{+} = A^{T}(AA^{T})^{-1}}$

(Minimum Norm Solution) $\longrightarrow$



* Number-2 is the usual case because we're always collecting data.

Example:

$$\underset{(\text{input})}{} \xrightarrow{x} \boxed{\quad\quad} \longrightarrow \underset{(\text{output})}{y}$$

Suppose someone gives you the input-output data below. Your task is to fit a model to it.

Index i	Input x	Output y
1	2	3
2	1	2
3	3	4
4	7	8
5	5	3

↓

No. of datas are needed to fit a model.

Solution:— You do a scatter plot & decide to fit a line.

$\boxed{y = mx + b}$ ⟶ We need to know m & b as x & y are already given.

$$y = \begin{bmatrix} 3 \\ 2 \\ 4 \\ 8 \\ 3 \end{bmatrix}, \quad x = \begin{bmatrix} 2 \\ 1 \\ 3 \\ 7 \\ 5 \end{bmatrix} \longrightarrow \begin{bmatrix} 3 \\ 2 \\ 4 \\ 8 \\ 3 \end{bmatrix} = m \begin{bmatrix} 2 \\ 1 \\ 3 \\ 7 \\ 5 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} b$$

b is a Scalar Value.

* As b is a Scalar ~~Vector~~, hence we need to put a vector.

$$\underbrace{\begin{bmatrix} 3 \\ 2 \\ 4 \\ 8 \\ 3 \end{bmatrix}}_{y} = \underbrace{\begin{bmatrix} 2 & 1 \\ 1 & 1 \\ 3 & 1 \\ 7 & 1 \\ 5 & 1 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} m \\ b \end{bmatrix}}_{x}$$

→ It's written in the form $\boxed{y = Ax}$.

$$\Rightarrow \begin{bmatrix} m \\ b \end{bmatrix} = (A^T A)^{-1} A^T \begin{bmatrix} 3 \\ 2 \\ 4 \\ 8 \\ 3 \end{bmatrix}$$

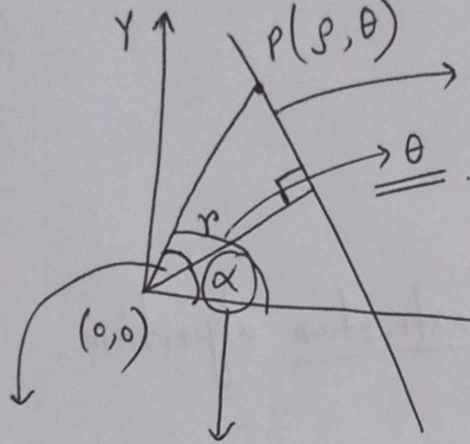
→ This is the solution.

Polar Co-ordinate datas are present (ρ, θ) in Lidar points.

1) Segmentation → Which points belong to which line?

2) After a group of points, we will do the "Line Fitting".

} → 2 conditions to create ~~a~~ ~~map~~ mapping.



Find out the equation of this line.

Here, r is the perpendicular distance from origin (0,0) to the line.

$\theta - \alpha$: The angle of r with x-axis.

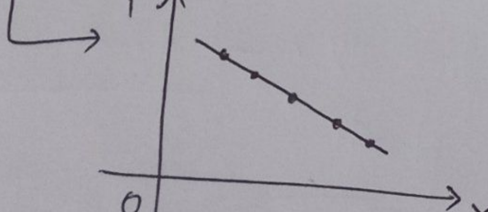
* In Polar Co-ordinates (r, α), are the line parameters.

* From Trigonometric Functions, we get:—

$$\cos(\theta - \alpha) = \frac{r}{\rho}$$

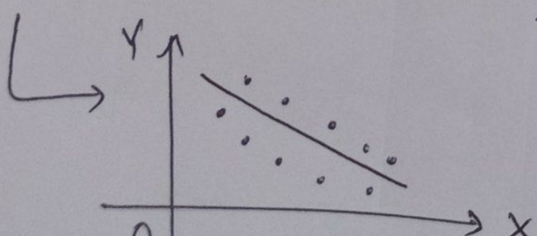
$$\therefore \boxed{\rho \cos(\theta - \alpha) = r}$$ → Equation of Line in Polar Co-ordinates.

* If $\boxed{e = 0}$, then all the points are on the line.



$\boxed{e = y - Ax}$ → Equation of "Error"

* If $\boxed{e \neq 0}$, then all the points are NOT on the line.



3 algorithms for "Line Segmentation".

1) Split & Merge algorithm. → Most popular line extraction algorithm.

↳ * Connect 1st & last data.

* From each point, calculate the perpendicular distance.

* Start splitting from the maximum distance.

↳ Fix a threshold for maximum distance.

* After the maximum distance is reached, then stop splitting.

* Then merge the points together.