OneNote

Home Insert Draw View Tell me

Share

Text Mode | Lasso Select | Insert Space | Eraser | Pen | Marker | Highlighter | Ink Color | 0.25 mm | 0.35 mm | 0.5 mm | 0.7 mm | 1 mm

Saturday, October 5, 2024 7:05 PM

Show the planar quadrotor is differentially flat

for $z = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$

states: $x,\ v_x,\ y,\ v_y,\ \phi,\ \omega$

ϕ angular displacement

$\omega = \dot{\phi}$ angular velocity

$\boxed{x = z_1}$, $\boxed{y = z_2}$

$\boxed{v_x = \dot{x} = \dot{z}_1}$, $\boxed{v_y = \dot{y} = \dot{z}_2}$

$\dot{v}_x = \ddot{z}_1 \qquad \dot{v}_y = \ddot{z}_2$

SS

OneNote

Home Insert Draw View Tell me

Share

New Window | Switch Background | Immersive Reader | Page Versions | Deleted Notes | Zoom Out | Zoom In | 50% | Reset Zoom | Page Width | Password Protection | Paper Color | Paper Style | Hide Page Title | Hide Authors | Translate | Check Accessibility

Saturday, October 5, 2024 7:52 PM

$$\ddot{z}_1 = -\frac{(T_1+T_2)\sin\phi}{m} \quad \cdots (1)$$

$$\ddot{z}_2 = \frac{(T_1+T_2)\cos\phi}{m} - g$$

$$\ddot{z}_2 + g = \frac{(T_1+T_2)\cos\phi}{m} \quad \cdots (2)$$

$$-\tan\phi = \frac{\ddot{z}_1}{\ddot{z}_2+g}$$

$$\Rightarrow \boxed{\phi = \tan^{-1}\left(-\frac{\ddot{z}_1}{\ddot{z}_2+g}\right)}$$

$$\omega = \dot{\phi}$$

$$\omega = \dot{\phi} = \frac{1}{1+\left(-\frac{\ddot{z}_1}{\ddot{z}_2+g}\right)^2}\frac{d}{dt}\left(-\frac{\ddot{z}_1}{\ddot{z}_2+g}\right)$$

$$\frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2}\frac{dx}{dx}$$

$$1$$

So, all states are functions of $z, \dot{z}, \ddot{z}, \dddot{z}$

SS

Aside

z is function of time $z(t)$

$$\frac{d}{dt}\left(\frac{\dot z_2}{\dot z_1}\right) = \dot z_2 \frac{d}{dt}\left(\frac{1}{\dot z_1}\right) + \frac{1}{\dot z_1}\frac{d}{dt}\left(\dot z_2\right)$$

$$= \dot z_2 \frac{d}{dt}\left(\dot z_1^{-1}\right) + \frac{1}{\dot z_1}\ddot z_2$$

$$= \dot z_2 (-1)\, \dot z_1^{-2} + \frac{\ddot z_2}{\dot z_1}$$

$$= \frac{-\dot z_2}{\dot z_1^2} + \frac{\ddot z_2}{\dot z_1}$$

$$= \mu(\dot z, \ddot z)$$

$$\boxed{\frac{d}{dt}\left(t^3\right) = 3t^2}$$

OneNote

Home Insert Draw View Tell me

Share

Text Mode Lasso Select Insert Space Eraser Pen Marker Highlighter Ink Color 0.25 mm 0.35 mm 0.5 mm 0.7 mm 1 mm

Saturday, October 5, 2024 8:16 PM

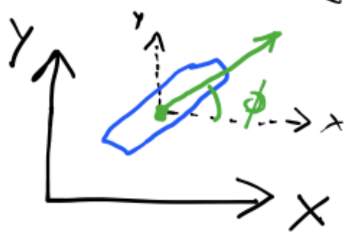
Input $\longrightarrow$ $\boxed{\dot{x} = f(x,u)}$ $\longrightarrow$ output

u x

Inputs (u) cause the states (x) to change

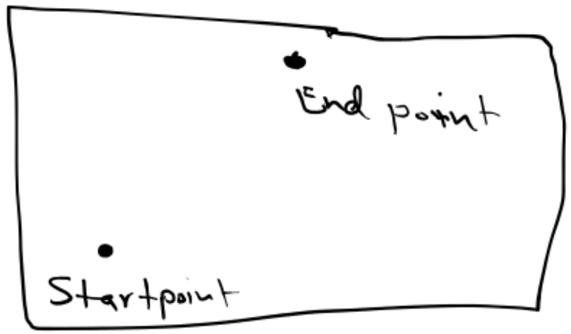
Acceleration $\longrightarrow$ $\boxed{\text{CAR}}$ $\longrightarrow$ distance covered

States (x) are properties of the system



$$V = V_x \hat{x} + V_y \hat{y}$$

$$P = x \hat{x} + y \hat{y}$$



SS

Saturday, October 5, 2024 8:27 PM

For a differentially flat system, we can write the the trajectory as a linear combination of basis functions.

$$z = \sum_{i=1}^{n} a_i \psi_i(t)$$

Here ψ_i's are basis functions & a_i's are constant weights.

For $n=4$

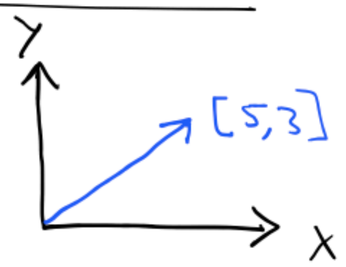
$$z = a_1 \psi_1(t) + a_2 \psi_2(t) + a_3 \psi_3(t) + a_4 \psi_4(t)$$

OneNote

Home Insert Draw View Tell me Share

Example: basis vectors

2 dimensions: R^2

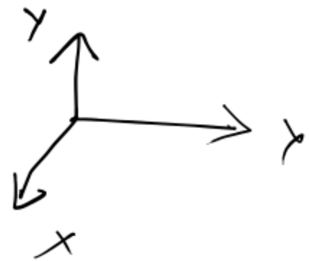


In 2 dimensions the basis vectors are $\begin{bmatrix}1\\0\end{bmatrix}$ & $\begin{bmatrix}0\\1\end{bmatrix}$

Any vector in R^2 can be written as a linear combination of $\begin{bmatrix}1\\0\end{bmatrix}$ & $\begin{bmatrix}0\\1\end{bmatrix}$

$$\begin{bmatrix}5\\3\end{bmatrix} = 5\begin{bmatrix}1\\0\end{bmatrix} + 3\begin{bmatrix}0\\1\end{bmatrix}$$

3 dimensions: R^3



Basis vectors in R^3 $\begin{bmatrix}1\\0\\0\end{bmatrix}, \begin{bmatrix}0\\1\\0\end{bmatrix}, \begin{bmatrix}0\\0\\1\end{bmatrix}$

$$\begin{bmatrix}3\\7\\8\end{bmatrix} = 3\begin{bmatrix}1\\0\\0\end{bmatrix} + 7\begin{bmatrix}0\\1\\0\end{bmatrix} + 8\begin{bmatrix}0\\0\\1\end{bmatrix}$$

SS