Classwork

A random variable X takes values between 4 and 6 with a probability density function $f(x) = \frac{1}{xln(1.5)}$ for $4 \leq x \leq 6$

(i) Check that the total area under the probability density function is equal to 1.

(ii) What is $P(4.5 \leq X \leq 5.5)$?

(iii) Calculate the cumulative distribution function.

Solution:

(i)

$$\int_4^6 \frac{1}{xln(1.5)}\,dx$$

$$= \frac{1}{ln(1.5)}\int_4^6 \frac{1}{x}\,dx$$

$$= \frac{1}{ln(1.5)}[lnx]_4^6$$

$$= \frac{1}{ln(1.5)}[ln6 - ln4]$$

$$= \frac{1}{ln(1.5)}ln\frac{6}{4}$$

$$= \frac{1}{ln(1.5)}ln(1.5)$$

$$= 1$$

(ii)

$$P(4.5 \leq x \leq 5.5) = \int_{4.5}^{5.5} \frac{1}{xln(1.5)}\,dx$$

$$= \frac{1}{ln(1.5)}\int_{4.5}^{5.5} \frac{1}{x}\,dx$$

$$= \frac{1}{ln(1.5)}[lnx]_{4.5}^{5.5}$$

$$= \frac{1}{ln(1.5)}[ln(5.5) - ln(4.5)]$$

$$= \frac{1}{ln(1.5)} ln \frac{5.5}{4.5}$$

$$= 0.49$$

(iii) Cumulative distribution function:

$$F(x) = \int_4^x \frac{1}{xln(1.5)} dx$$

$$= \frac{1}{ln(1.5)} \int_4^x \frac{1}{x} dx$$

$$= \frac{1}{ln(1.5)} [lnx]_4^x$$

$$= \frac{1}{ln(1.5)} (lnx - ln4)$$