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MAT361
sec: 4

1/ $f(x) = c(1-x)^2 \qquad -1 < x < 1.$

We know,

(i) $\int_{-1}^{1} c\,(1-x)^2\,dx = 1.$

$\Rightarrow \int_{-1}^{1} c\,[1-2x+x^2]\,dx = 1.$

$\Rightarrow c\left[x - \cancel{2}\frac{x^2}{\cancel{2}} + \frac{x^3}{3}\right]_{-1}^{1} = 1.$

$\Rightarrow c\left[1 - 1 + 1/3 + 1 + 1 + 1/3\right] = 1.$

$\Rightarrow c = 3/8$ Answer

(ii) $F(1.5)$

$F(x) = \frac{3}{8}\int_{-1}^{x} (1-x)^2\,dx$

$= \frac{3}{8}\left[x - x^2 + x^3/3\right]_{-1}^{x}$

$= \frac{3}{8}\left[\left[x - x^2 + x^3/3\right] - \left[-1 - 1^2 - 1/3\right]\right]$

$\therefore F(x) = \frac{3}{8}\left[x - x^2 - \frac{x^3}{3} + 7/3\right]$

$\therefore F(1.5) = \frac{3}{8} \times \frac{11}{24}$

$= \frac{11}{64}$ Answer

(iii) $E(x^2) = \frac{3}{8}\int_{-1}^{1} x^2 (1-2x+x^2)\,dx$

$= \frac{3}{8} \int_{-1}^{1} (x^2 - 2x^3 + x^4)\,dx$

$= \frac{3}{8} \left[\frac{x^3}{3} - \frac{x^4}{2} + \frac{x^5}{5} \right]_{-1}^{1}$

$= \frac{3}{8} \times \frac{16}{15}$

$= 2/5$

$E(x) = \frac{3}{8}\int_{-1}^{1} x(1-2x+x^2)\,dx$

$= \frac{3}{8} \int_{-1}^{1} (x - 2x^2 + x^3)\,dx$

$= \frac{3}{8} \left[\frac{x^2}{2} - \frac{2x^3}{3} + \frac{x^4}{4} \right]_{-1}^{1}$

$= \frac{3}{8} \times (-4/3)$

$= -1/2$

$V(x) = E(x^2) - (E(x))^2$

$= \left(\frac{2}{5}\right) - (-1/2)^2$

$= \frac{2}{5} - 1/4$

$= \frac{3}{20}$ Answer.