PHY 107

HW2 Solution

November 8, 2018

NOTE Solving techniques for each problem are shown and directions provided to get familiar with some of the notions discussed in class. Some answers are approximate.

Problem 1

a)$x = 12t^2 - 2t^3$
$x = 12(3)^2 - 2(3)^3 = \ 54\ m$

b)$v = \frac{dx}{dt} = 24t - 6t^2$
$v(3) = 24(3) - 6(3)^2 = \ 18\ m/s$

c) $a = 24 - 12t$
$a = 24 - 12(3) = \ -12\ m/s^2$

d,e) $\frac{dx}{dt} = 0 \ \rightarrow \ 24t \ - \ 6t^2 \ = \ 6t(4-t) \ = \ 0 \rightarrow \ t \ = \ 0, \ 4\ s$
$x(0) = 0, \ x(4) \ = \ 64$
The maximum positive coordinate is 64 m.

Problem 2

This problem has a journey in 2 segments. Find a quantity for each segment and add them up.
Segment 1:
$u_1 = 0\ , v_1 = 20, \ a_1 = 2$
$v_1 = u_1 + a_1 t_1 \rightarrow t_1 = \ 10\ s$

Segment 2:
$u_2 = 20\ , v_2 = 0, \ a_2 = -1$
$v_2 = u_2 + a_2 t_2 \rightarrow t_2 = 20\ s$

Time elapsed from start to stop $= t_1 \ + \ t_2 \ = \ 30\ s$
Segment 1:
$S_1 = u_1 t_1 + \frac{1}{2} a_1 t_1^2 \rightarrow S_1 \ = \ 100\ m$

Segment 2:
$S_2 = u_2 t_2 + \frac{1}{2} a_2 t_2^2 \rightarrow S_2 \ = \ 20(20) \ + \ 0.5(-1)(20)^2 \ = \ 200\ m$
Distance traveled from start to stop: $S_1 + S_2 \ = \ 300\ m$

Problem 3

Part 1
a)$S \ = \ 50\ m\ , v \ = \ 0\ m/s, \ a \ = \ -g\ m/s^2$
$v^2 = u^2 + 2aS \rightarrow u = \ 31.32\ m/s$
b) $v = u + at$
$0 = 31.32 \ - \ gt \rightarrow t \ = \ 3.19\ s$
Therefore, the particle is in the air for $2 \times 3.19 = 6.38\ s$
Part 2
$v = \int a\ dt$
$v = \int 5t\ dt = \frac{5t^2}{2} \ + \ C$
Use the information given in the question:
$v(2) = 17;$

$\frac{5(2)^2}{2} + C = 17 \to C = 7$
$v(4) = \frac{5(4)^2}{2} + 7 = \ 47\ m/s$

Problem 4

$\overrightarrow{v} = \frac{d\overrightarrow{r}}{dt} = 8t\hat{j} + \hat{k}$
$\overrightarrow{a} = \frac{d\overrightarrow{v}}{dt} = 8\hat{j}$

Problem 5

The motion is almost horizontal at the table edge, hence, the ball has a horizontal speed, but no vertical speed at the beginning of this 2D motion.
a) Consider vertical motion equation to find the time in the air: $S = ut + \frac{1}{2}at^2$
$-1.2 \ = \ 0(t) + \frac{1}{2}(-g)t^2 \to t = \ 0.495\ s$
b) Horizontal displacement = Horizontal velocity $\times$ time
$1.52 \ = v_H \times t \to v_H = \frac{1.52}{0.495} = \ 3.07\ m/s$

Problem 6

The motion is almost horizontal when the particle just comes out of the gun, hence, the particle has a horizontal speed, but no vertical speed at the beginning of this 2D motion.
a) Consider vertical motion equation to find the time in the air: $S = ut + \frac{1}{2}at^2$
$-45 \ = \ 0(t) + \frac{1}{2}(-g)t^2 \to t = \ 3.03\ s$
b) Horizontal displacement = Horizontal velocity $\times$ time
$S_H \ = \ v_H \ \times \ t \ = \ 250 \ \times \ t = 250 \times 3.03 = \ 757.23\ m$

Problem 7

a)To find the maximum height, we need to compute how high the ball goes above the height of 9.1m.

$$v^2 = u^2 + 2as_2$$
$$0 = 6.1^2 + 2(-g)s_2$$
$$H_{max} \ = \ 9.1 \ + s_2 \ \ = 9.1 + 1.89 = \ 10.99\ m$$

b) The question wants us to find the horizontal displacement from the point of launch to the point the ball hits the ground. In order to do so, there is a need to find the initial vertical velocity (vertical velocity at the time of the launch). The maximum height can be used to find the initial vertical velocity:
Let's consider the motion of the particle from the point of launch to the max height:

$$u = ?, v = 0, S = H_{max}$$
$$v^2 = u^2 + 2(-g)S \to u \ = \ 14.7\ m/s$$

Now that we know what the initial vertical velocity is, we can find the time of the whole journey (from the point of launch to the point the ball hits the ground):
If the particle leaves the ground and then comes back to the ground, then the vertical displacement is essentially zero

$$S = ut + \frac{1}{2}(-g)t^2$$
$$0 \ = \ ut \ - \ 0.5gt^2 \to \ t \ = \ \frac{2u}{g}$$
$$t = \frac{2(14.7)}{9.81} = 3\ s$$

Horizontal velocity can be obtained from the velocity vector given for height 9.1 m. Horizontal velocity is always constant, $v_H \ = \ 7.6\ m/s$
Horizontal displacement = Horizontal velocity $\times$ time $\ = \ 7.6 \times 3 = 22.78\ m$