Assignment 4

MAT 361

Probability and Statistics

Section 4

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North South University

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Assignment-4

1) probability, $p = 0.09$

Archer shoots 9 arrows.

$\therefore n = 9$

Binomial Distribution, $P(X=x) = \binom{9}{x} p^x (1-p)^{9-x}$

a) Exactly two arrows score bull's eyes,

$$P(X=2) = \binom{9}{2} 0.09^2 (1-0.09)^{9-2}$$
$$= 36 \times 0.09^2 \times 0.91^7$$
$$= 0.15069$$
$$= 0.1507 \text{ Answer}$$

b) At least two arrows score bull's-eyes,

$$P(X \geq 2) = P(x=2) + P(x=3) + \ldots + P(x=9)$$
$$= 1 - P(x=0) - P(x=1)$$
$$= 1 - \binom{9}{0} 0.09^0 (1-0.09)^{9-0} - \binom{9}{1} 0.09^1 \cdot 0.91^{9-1}$$
$$= 1 - 0.381 - 0.43$$
$$= 1 - 0.4279 - 0.3809$$
$$= 0.191166$$
Answer

c) Expected number of bull's-eyes scored,

$$E(X) = np$$
$$= 9 \times 0.09$$
$$= 0.81 \text{ Answer}$$

d) Variance, $V(X) = np(1-p)$

$= 9 \times 0.09 \times (1-0.09) = 0.7371$ Answer

Standard deviation $= \sqrt{V(x)} = \sqrt{0.7371} = 0.8585$ Answer

2) A company recieves 60% of its orders over the internet.

$\therefore$ probability, $p = 0.6$

18 independently placed order. $\therefore n = 18$.

It is binomial distribution, $p(x=x) = \binom{18}{x} 0.6^{x} (1-0.6)^{18-x}$

(a) between eight and ten of the orders are recieved,

Probability $= p(x=8) + p(x=9) + p(x=10)$

$= \binom{18}{8} \times 0.6^{8} \times (1-0.6)^{18-8} + \binom{18}{9} \times 0.6^{9} \times 0.4^{18-9} + \binom{18}{10} 0.6^{10} 0.4^{18-10}$

$= 0.0771 + 0.1284 + 0.1734$

$= 0.3789$ Answer

(b) nor more than four of the orders are received over the internet,

$\therefore$ Probability, $= p(x=0) + p(x=1) + p(x=2) + p(x=3) + p(x=4)$

$= \binom{18}{0} 0.6^{0} \times 0.4^{18-0} + \binom{18}{1} \times 0.6^{1} \times 0.4^{18-1} + \binom{18}{2} \times 0.6^{2} \times 0.4^{18-2}$

$+ \binom{18}{3} 0.6^{3} \times 0.4^{18-3} + \binom{18}{4} \times 0.6^{4} \times 0.4^{18-4}$

$= 6.87 \times 10^{-8} + 1.855 \times 10^{-6} + 2.3657 \times 10^{-5} + 1.8925 \times 10^{-4}$

$+ 1.06455 \times 10^{-3}$

$= 1.2794 \times 10^{-3}$

$= 1.28 \times 10^{-3}$

$= 0.00128$ Answer

3] parameter, $p = 0.09$

(a) Considering as geometric distribution,

$p(x=x) = (1-p)^{(x-1)} p$

$\therefore p(x=4) = (1-0.09)^{4-1} \times 0.09$

$= 0.91^{3} \times 0.09$

$= 0.0678$

Answer

(b) 3rd bull's-eye is scored with the tenth arrow.

$r = 3,\ n = 10$ & $p = 0.09$

$$P(X = x) = \binom{x-1}{r-1} (1-p)^{(x-r)} p^r$$

$$\therefore P(10) = \binom{10-1}{3-1} (1-0.09)^{(10-3)} 0.09^3$$

$$= \binom{9}{2} (0.91)^7 \cdot 0.09^3$$

$$= 0.0136 \quad \text{Answer}$$

(c) Expected number of arrows before the first bull's eye scored,

$$E(x) = \frac{1}{p} = \frac{1}{0.09}$$

$$= 11.11 \quad \text{Answer}$$

(d) Expected number of arrows shot before the third bull's eye is scored,

~~$E(x) = \frac{r(1-p)}{p^2}$~~ $\quad E(x) = \frac{r}{p}$

~~$= 3(1-$~~ $\quad = \frac{3}{0.09}$

$= 33.33$

4) parameter, $\lambda = 2.4$, no crack.

$$P(X = x) = \frac{e^{-\lambda} \lambda^x}{x!}$$

$$P(X = 0) = \frac{e^{-2.4} \cdot \lambda^0}{0!}$$

$$= \frac{1}{1 \cdot e^{2.4}} = 0.0907 \quad \text{Answer}$$

four or more crack, $\lambda = 2.4$

$P(X \geq 4) = 1 - P(X=4) + P(X=5) + \dots$

$= 1 - P(X=3) - P(X=2) - P(X=1) - P(X=0)$

$= 1 - \frac{e^{-2.4} \times 2.4^3}{3!} - \frac{e^{-2.4} \times 2.4^2}{2!} - \frac{e^{-2.4} \times 2.4^1}{1!} - \frac{e^{-2.4} \times 2.4^0}{0!}$

$= 1 - 0.209 - 0.2613 - 0.2177 - 0.0907$

$= 0.2213.$ Answer

5] Given, $\mu = 3.00$ mm

standard deviation of $\sigma = 0.12$ mm

$\therefore$ variance $= \sigma^2$

(i) $P(x > 3.2)$

$= P(3.2 < x < \infty)$

$= P\left(\frac{3.2-\mu}{\sigma} < \frac{x-\mu}{\sigma} < \frac{\infty-\mu}{\sigma}\right)$

$= P\left(\frac{3.2-3}{0.12} < \frac{x-3}{0.12} < \infty\right)$

$= F(\infty) - F(1.67)$

$= 1 - F(1.67)$

$= 1 - 0.9525$

$= 0.0475$ Answer

(ii) ~~$P(x < b)$~~ $P(x < 2.7)$

$= P(-\infty < x < 2.7) = P\left(\frac{-\infty-3}{0.12} < \frac{x-3}{0.12} < \frac{2.7-3}{0.12}\right)$

$= P\left(-\infty < \frac{x-3}{0.12} < -2.5\right)$

$= F(-2.5)$

$= 0.0062$ Answer