Exercise 7.3

30. $\int \tan^5 x \sec^4 x \, dx$

$= \int \tan^5 x \cdot \sec^2 x \cdot \sec^2 x \, dx$

$= \int \tan^5 x \cdot (1 + \tan^2 x) \cdot \sec^2 x \, dx$

$= \int (\tan^5 x + \tan^7 x) \sec^2 x \, dx$

$= \dfrac{\tan^6 x}{6} + \dfrac{\tan^8 x}{8} + C$

31. $\int \tan 4x \sec^4 4x \, dx$

$= \int \tan 4x \cdot \sec^2 4x \cdot \sec^2 4x \, dx$

$= \int \tan 4x \, (1 + \tan^2 4x) \sec^2 4x \, dx$

$= \int (\tan 4x + \tan^3 4x) \sec^2 4x \, dx$

$= \dfrac{\tan^2 4x}{8} + \dfrac{\tan^4 4x}{16} + C$

32. $\int \tan^4 \theta \sec^4 \theta \, d\theta$

$= \int \tan^4 \theta \cdot \sec^2 \theta \cdot \sec^2 \theta \, d\theta$

$= \int \tan^4 \theta \, (1 + \tan^2 \theta) \sec^2 \theta \, d\theta$

$= \int (\tan^4 \theta + \tan^6 \theta) \sec^2 \theta \, d\theta$

$= \dfrac{\tan^5 \theta}{5} + \dfrac{\tan^7 \theta}{7} + C$

33. $\int \sec^5 x \tan^3 x \, dx$

$= \int \sec^4 x \cdot \tan^2 x \cdot \sec x \tan x \, dx$

$= \int \sec^4 x \cdot (\sec^2 x - 1) \sec x \tan x \, dx$

$= \int (\sec^6 x - \sec^4 x) \sec x \tan x \, dx$

$= \dfrac{\sec^7 x}{7} - \dfrac{\sec^5 x}{5} + c$

34. $\int \tan^5 \theta \sec \theta \, d\theta$

$= \int \tan^4 \theta \cdot \sec\theta \tan\theta \, d\theta$

$= \int (\sec^2\theta - 1)^2 \sec\theta \tan\theta \, d\theta$

$= \int (\sec^4\theta - 2\sec^2\theta + 1) \sec\theta \tan\theta \, d\theta$

$= \dfrac{\sec^5\theta}{5} - \dfrac{2\sec^3\theta}{3} + \sec\theta + C$

35. $\int \tan^4 x \sec x \, dx$

$= \int (\sec^2 x - 1)^2 \sec x \, dx$

$= \int (\sec^5 x - 2\sec^3 x + \sec x) \, dx$

$= \int \sec^5 x \, dx - 2\int \sec^3 x \, dx + \int \sec x \, dx$

$= \dfrac{\sec^{5-2} x \tan x}{5-1} + \dfrac{5-2}{5-1} \int \sec^{5-2} x \, dx - 2\int \sec^3 x \, dx + \int \sec x \, dx$

$= \dfrac{\sec^3 x \tan x}{4} + \dfrac{3}{4} \int \sec^3 x \, dx - 2\int \sec^3 x \, dx + \int \sec x \, dx$

$= \dfrac{\sec^3 x \tan x}{4} - \dfrac{5}{4} \times \dfrac{\sec^{3-2} x \tan x}{3-1} - \dfrac{5}{4} \times \dfrac{3-2}{3-1} \int \sec^{3-2} x \, dx + \int \sec x \, dx$

$= \dfrac{\sec^3 x \tan x}{4} - \dfrac{5 \sec x \tan x}{8} - \dfrac{5}{8} \int \sec x \, dx + \int \sec x \, dx$

$= \dfrac{\sec^3 x \tan x}{4} - \dfrac{5 \sec x \tan x}{8} + \dfrac{3}{8} \ln|\sec x + \tan x| + C$

36. $\int \tan^2 x \sec^3 x\,dx$

$= \int (\sec^2 x - 1)\sec^3 x\,dx$

$= \int \sec^5 x\,dx - \int \sec^3 x\,dx$

$= \dfrac{\sec^{5-2}x \tan x}{5-1} + \dfrac{5-2}{5-1}\int \sec^{5-2}x\,dx - \int \sec^3 x\,dx$

$= \dfrac{\sec^3 x \tan x}{4} - \dfrac{1}{4}\int \sec^3 x\,dx$

$= \dfrac{\sec^3 x \tan x}{4} - \dfrac{1}{4} \times \dfrac{\sec^{3-2}x \tan x}{3-1} - \dfrac{1}{4} \times \dfrac{3-2}{3-1}\int \sec^{3-2}x\,dx$

$= \dfrac{\sec^3 x \tan x}{4} - \dfrac{\sec x \tan x}{8} - \dfrac{1}{8}\ln|\sec x + \tan x| + C$

37. $\int \tan t \sec^3 t\,dt$

$= \int \sec^2 t \cdot \sec t \cdot \tan t\,dt \quad = \dfrac{1}{3}\sec^3 t + C$

38. $\int \tan x \sec^5 x\,dx$

$= \int \sec^4 x \cdot \sec x \tan x\,dx \quad = \dfrac{\sec^5 x}{5} + C$

39. $\int \sec^4 x\,dx \quad = \dfrac{\sec^{4-2}x \tan x}{4-1} + \dfrac{4-2}{4-1}\int \sec^{4-2}x\,dx$

$= \int \sec^2 x \cdot \sec^2 x\,dx$

$= \int (1+\tan^2 x)\sec^2 x\,dx \quad = \int \sec^2 x\,dx + \int \sec^2 x \tan^2 x\,dx$

$= \tan x + \dfrac{\tan^3 x}{3} + C$

40. $\int \sec^5 x \, dx$

$$= \frac{\sec^{5-2} x \tan x}{5-1} + \frac{5-2}{5-1} \int \sec^{5-2} x \, dx$$

$$= \frac{\sec^3 x \tan x}{4} + \frac{3}{4} \int \sec^3 x \, dx$$

$$= \frac{\sec^3 x \tan x}{4} + \frac{3}{4} \times \frac{\sec^{3-2} x \tan x}{3-1} + \frac{3}{4} \times \frac{3-2}{3-1} \int \sec^{3-2} x \, dx$$

$$= \frac{\sec^3 x \tan x}{4} + \frac{3}{8} \sec x \tan x + \frac{3}{8} \ln|\sec x + \tan x| + C$$

41. $\int \tan^3 4x \, dx = \frac{1}{4} \int \tan^3 u \, du$ | let, $u = 4x \Rightarrow \frac{du}{4} = dx$

$$= \left[\frac{\tan^{3-1} 4x}{3-1} - \int \tan^{3-2} 4x \, dx \right] \frac{1}{4}$$

$$= \frac{\tan^2 4x}{8} - \frac{1}{8} \ln|\sec 4x| + C$$

42. $\int \tan^4 x \, dx$

$$= \frac{\tan^{4-1} x}{4-1} - \int \tan^{4-2} x \, dx$$

$$= \frac{\tan^3 x}{3} + \int dx - \int \sec^2 x \, dx \quad [\tan^2 x = \sec^2 x - 1]$$

$$= \frac{\tan^3 x}{3} + x - \tan x + C$$

43. $\int \sqrt{\tan x}\, \sec^4 x \, dx$

$= \int \sqrt{\tan x} \cdot \sec^2 x \cdot \sec^2 x \, dx$

$= \int \sqrt{\tan x}\,(1 + \tan^2 x) \sec^2 x \, dx$

$= \frac{2}{3} \tan^{3/2} x + \frac{2}{7} \tan^{7/2} x + C$

44. $\int \tan x \sec^{3/2} x \, dx$

$= \int \sec^{1/2} x \cdot \sec x \tan x \, dx$

$= \frac{2}{3} \sec^{3/2} x + C$

46. $\int_0^{\pi/6} \sec^3 2\theta \tan 2\theta \, d\theta$

$= \int_0^{\pi/6} \sec^2 2\theta \cdot \sec 2\theta \tan 2\theta \, d\theta$

$= \left[\frac{1}{3} \times \frac{1}{2} \sec^3 2\theta\right]_0^{\pi/6} = \frac{7}{6}$

48. $\int_0^{1/4} \sec \pi x \tan \pi x \, dx$

let, $u = \pi x \Rightarrow \frac{du}{\pi} = dx$

$x = \frac{1}{4}, u = \frac{\pi}{4}$ $\quad x = 0, u = 0$

$\frac{1}{\pi} \int_0^{\pi/4} \sec u \tan u \, du$

$= \frac{1}{\pi} [\sec u]_0^{\pi/4} = (\sqrt{2} - 1)/\pi$

45. $\int_0^{\pi/8} \tan^2 2x \, dx$

$= \int_0^{\pi/8} (\sec^2 2x - 1) \, dx$

$= \left[\frac{1}{2} \tan 2x - x\right]_0^{\pi/8} = \frac{1}{2} - \frac{\pi}{8}$

47. $\int_0^{\pi/2} \tan^5 \frac{x}{2} \, dx$

let, $u = \frac{x}{2} \Rightarrow 2du = dx$

$x = \frac{\pi}{2}, u = \frac{\pi}{4} \cdot x = 0, u = 0$

$2 \int_0^{\pi/4} \tan^5 u \, du$

$= 2\left[\frac{\tan^{5-1} u}{5-1} - \int \tan^{5-2} du\right]_0^{\pi/4}$

$= 2\left[\frac{\tan^4 u}{4} - \frac{\tan^{3-1} u}{3-1} + \int \tan^{3-2} u \, du\right]_0^{\pi/4}$

$= 2\left[\frac{\tan^4 u}{4} - \frac{\tan^2 u}{2} + \ln|\sec u|\right]_0^{\pi/4}$

$= -\frac{1}{2} - 2\ln(1/\sqrt{2})$

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49. $\int \cot^3 x \operatorname{cosec}^3 x \, dx$

$= \int (\operatorname{cosec}^2 x - 1) \operatorname{cosec}^2 x (\operatorname{cosec} x \cot x) \, dx$

$= \int (\operatorname{cosec}^4 x - \operatorname{cosec}^2 x) \operatorname{cosec} x \cot x \, dx$

$= -\frac{1}{5} \operatorname{cosec}^5 x + \frac{1}{3} \operatorname{cosec}^3 x + C$

Put, $u = \operatorname{cosec} x$

$\Rightarrow -du = \operatorname{cosec} x \cdot \cot x \, dx$

50. $\int \cot^2 3t \sec 3t \, dt$

$= \int \frac{\cos^2 3t}{\sin^2 3t} \times \frac{1}{\cos 3t} \, dt$

$= \int \operatorname{cosec} 3t \cot 3t \, dt$

$= -\frac{1}{3} \operatorname{cosec} 3t + C$

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