Gravitation Near Earth's Surface

The magnitude of the gravitational force from Earth on a particle of mass m, located outside Earth E a distance r from Earth's center, is then given by

$$F = \frac{GMm}{r^2}$$

If the particle is released, it will fall toward the center of Earth as a result of gravitational force $\vec{F}$ with an acceleration called gravitational acceleration. $\vec{a}_g$ $\quad F = ma_g$

$$a_g = \frac{GM}{r^2}$$

a_g is significant even at 400 Km.

Check the table 360 page.

Everest — 9.8 m/s^2

We assumed that g has the constant value 9.8 m/s^2 any place on Earth surface. However, any g value measured at a given location will differ from it value 9.8 for three reasons.

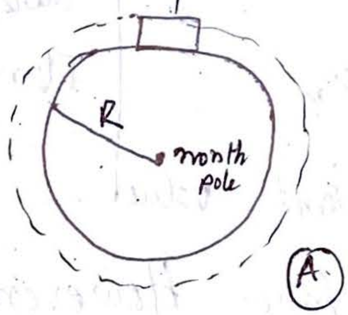
1. Earth's mass is not distributed equally.
2. Earth is not a perfect sphere
3. Earth rotates.

Altitude	
8.8	9.8 m/s^2
36.6	9.71
400	8.70
35,700	0.22

Earth's equatorial radius is greater than its polar radius. $g = 9.83$ at north pole. $.5\%$ more at the poles than the equator.

* The earth is rotating through the north & South poles of earth. An object located on Earth's surface anywhere except at those poles must rotate in a circle about the rotation axis & thus must have a centripetal acceleration directed toward the the center of the circle.

A crate of mass m is on a scale at the equator.



$\vec{F}_N$ normal force upward

$\vec{a}_{centripetal}$ acceleration due to Earth motion

$m\vec{g}$ Gravitational force downward.

As the Earth turns, the crate has a centripetal acceleration $\vec{a}$ directed toward the Earth's center.

$a_n = \omega^2 r$ From Second law, $F_{net,r} = ma_r$

$$\left[a_n = \frac{v^2}{r} = \frac{\omega^2 r^2}{r} = \omega^2 r \right]$$

$$F_N - ma_g = m(-\omega^2 R)$$

$$mg = ma_g - m\omega^2 R$$

measured weight = magnitude of gravitational force $-$ mass times centripetal acceleration

Thus the measured weight is less than the magnitude of the gravitational force on the crate, because of Earth's rotation.

$$g = a_g - \omega^2 R$$

$\boxed{\omega^2 R \text{ is } 0.034 \ m/s^2}$

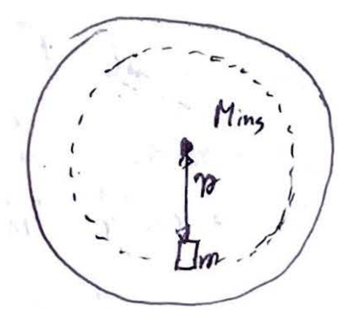
this is 300 time's smaller than $9.8 \ m/s^2$.

problem An astronaut whose height h is 1.70 m floats "feet down" in an orbiting space shuttle at distance $r = 6.77 \times 10^6$ m away from the earth. What is the difference between the gravitational acceleration at her feet and at her head? pag 362

Gravitation Inside Earth:

Newton's shell of matter exerts no net gravitational force on a particle located inside it.

Lets consider the inside Mass M_{ins} is concentrated as a particle at Earth's center.



$F = \frac{G m M_{ins}}{r^2}$, Here we assume a uniform density ρ, we can write this inside mass in terms of Earth's total mass M and its radius R;

$$\text{density} = \frac{\text{inside mass}}{\text{inside vollum}} = \frac{\text{total mass}}{\text{total volume}}$$

$$\rho = \frac{M_{ins}}{\frac{4}{3}\pi r^3} = \frac{M}{\frac{4}{3}\pi R^3}$$

$$M_{ins} = \frac{M r^3}{R^3}$$

The magnitude of gravitational force on the capsule as a function of the capsule's distance r from Earth's center

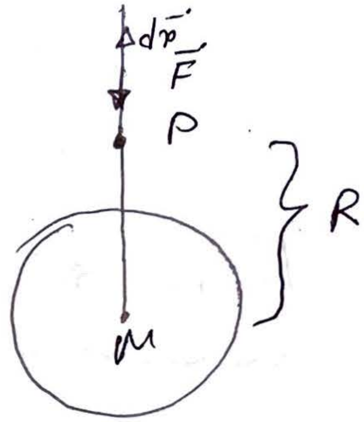
$$F = \frac{G m M}{R^3} r$$

The force decreases linearly as the capsule approaches the center, until it is zero at the center.

* <u>Talk about problem</u>

For the real Earth, which certainly has a nonuniform distribution of mass, the force on the capsule would initially increase as the capsule descends. The force would then reach a maximum at a certain depth and only then would it begin to decrease as the capsule further descends.

Gravitational potential Energy



Let us shoot a baseball directly away from Earth along the path.
What would be the gravitational potential energy U of the ball at point P.
To do so, we first find work done W on the ball by gravitational force as the ball travel from point P to a great distance from Earth.

$$W = \int_R^{\infty} \vec{F}(r)\cdot d\vec{r}$$

→ Gravitational force

$$\vec{F}(r)\cdot d\vec{r} = F(r)\, dr \cos\theta$$

$$= -\frac{GMm}{r^2}\, dr \quad [\ dr \;\&\; F(r) \text{ is in opposite}\ ]$$

$$W = -GMm \int_R^{\infty} r^{-2}\, dr = \left[\frac{GMm}{r}\right]_R^{\infty}$$

$$= -\frac{GMm}{R}$$

where W is the work required to move the ball from point P to infinity.

We know

$$\Delta U = -W$$

$$U_\infty - U = -W$$

The potential energy U_∞ at infinity is zero,

$$U = W = -\frac{GMm}{R}$$ [problem]

Escape speed

If you fire a projectile upward, usually it will slow, stop momentarily & return to Earth. However, there is a certain minimum initial speed that will cause it to move upward forever, theoretically coming to rest only at infinity. The minimum initial speed is called the (Earth) escape speed.

Consider a projectile of mass, m, leaving the surface of a planet with escape speed v. The projective has a kinetic energy $K = \frac{1}{2}mv^2$,

potential energy $U = -\frac{GMm}{R}$

M is the mass of the planet & R its radius.

U=mgh

When the projectile reaches infinity, it stops and thus has no kinetic energy. It has also no potential energy because of infinite seperation between two body. Its total energy at infinity is therefore zero. From the principle of conservation of energy, it's total energy at planet's surface must be also zero

$$K + U = \frac{1}{2}mv^2 + \left(-\frac{GMm}{R}\right) = 0$$

$$V = \sqrt{\frac{2GM}{R}}$$

Body	Escape speed Km/s
moon	2.38
Earth	11.2
sun	61.8
Neutron star	2×10^5
Black hole	?

Check table 13.2 page 367

Check point 3:

problem 13.63 page 368