HOMEWORK-2

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Course : CSE495A

Section : 1

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Ans to the Q.No-1

(a)

Given,

$$\dot{x}_1 = u_1$$
$$\dot{x}_2 = u_2$$
$$\dot{x}_3 = x_2 u_1$$

Now,

$$z = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_3 \end{bmatrix}$$

So,

$$x_1 = z_1$$
$$x_3 = z_2$$
$$\therefore \dot{z}_1 = \dot{x}_1 = u_1 \qquad \therefore u_1 = \dot{z}_1$$
$$\therefore \dot{z}_2 = \dot{x}_3 = x_2 u_1$$
$$\Rightarrow x_2 = \frac{\dot{x}_3}{u_1}$$
$$\Rightarrow x_2 = \frac{\dot{x}_3}{\dot{z}_1} \qquad \text{So, } x_2 = \frac{\dot{z}_2}{\dot{z}_1}$$

$$\therefore u_2 = \dot{x}_2$$
$$= \frac{d}{dt}\left(\frac{\dot{z}_2}{\dot{z}_1}\right) = \frac{1}{\dot{z}_1} \cdot \ddot{z}_2 + \dot{z}_2\left(-\frac{1}{\dot{z}_1^2}\right)\ddot{z}_1$$
$$= \frac{\ddot{z}_2}{\dot{z}_1} + \dot{z}_2\left(-\frac{\ddot{z}_1}{\dot{z}_1^2}\right)$$
$$= \frac{\ddot{z}_2}{\dot{z}_1} - \frac{\dot{z}_2 \ddot{z}_1}{\dot{z}_1^2}$$

Therefore, $u_1 = \dot{z}_1$ and $u_2 = \frac{\ddot{z}_2}{\dot{z}_1} - \frac{\dot{z}_2 \ddot{z}_1}{\dot{z}_1^2}$

So, this system is differentially flat for output $z = (x_1, x_3)$.

(showed)

(b)

Given,

$t_i = 0$ and $t_f = T$

Following conditions:

At $t_i = 0$, $x_1(0), x_2(0), x_3(0), \dot{x}_1(0) = 1$

At $t_f = T$, $x_1(T), x_2(T), x_3(T), \dot{x}_1(T) = 1$

We know,

$$z(t) = \sum_{i=1}^{N=4} \alpha_i \psi_i(t)$$

Basis functions $\psi_1 = 1, \psi_2 = t, \psi_3 = t^2, \psi_4 = t^3$

$$z_1(t) = \alpha_{11}\psi_1(t) + \alpha_{12}\psi_2(t) + \alpha_{13}\psi_3(t) + \alpha_{14}\psi_4(t)$$
$$= \alpha_{11} + \alpha_{12}t + \alpha_{13}t^2 + \alpha_{14}t^3$$

$$\dot{z}_1(t) = 0 + \alpha_{12}.1 + 2\alpha_{13}t + 3\alpha_{14}t^2$$
$$= \alpha_{12} + 2\alpha_{13}t + 3\alpha_{14}t^2$$

$$z_2(t) = \alpha_{21}\psi_1(t) + \alpha_{22}\psi_2(t) + \alpha_{23}\psi_3(t) + \alpha_{24}\psi_4(t)$$

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$$= \alpha_{21} + \alpha_{22} t + \alpha_{23} t^2 + \alpha_{24} t^3$$

$$\dot{z}_2(t) = 0 + \alpha_{22}.1 + 2\alpha_{23} t + 3\alpha_{24} t^2$$
$$= \alpha_{22} + 2\alpha_{23} t + 3\alpha_{24} t^2$$

For $t = 0$,

$$z_1(0) = \alpha_{11}$$
$$\dot{z}_1(0) = \alpha_{12}$$
$$z_2(0) = \alpha_{21}$$
$$\dot{z}_2(0) = \alpha_{22}$$

for $t = T$,

$$z_1(T) = \alpha_{11} + \alpha_{12} T + \alpha_{13} T^2 + \alpha_{14} T^3$$
$$\dot{z}_1(T) = \alpha_{12} + 2\alpha_{13} T + 3\alpha_{14} T^2$$
$$z_2(T) = \alpha_{21} + \alpha_{22} T + \alpha_{23} T^2 + \alpha_{24} T^3$$
$$\dot{z}_2(T) = \alpha_{22} + 2\alpha_{23} T + 3\alpha_{24} T^2$$

Therefore, Matrix vector equation: $Ax = b$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & T & T^2 & T^3 & 0 & 0 & 0 & 0 \\ 0 & 1 & 2T & 3T^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & T & T^2 & T^3 \\ 0 & 0 & 0 & 0 & 0 & 1 & 2T & 3T^2 \end{bmatrix}_{8\times8} \begin{bmatrix} \alpha_{11} \\ \alpha_{12} \\ \alpha_{13} \\ \alpha_{14} \\ \alpha_{21} \\ \alpha_{22} \\ \alpha_{23} \\ \alpha_{24} \end{bmatrix}_{8\times1} = \begin{bmatrix} z_1(0) \\ \dot{z}_1(0) \\ z_2(0) \\ \dot{z}_2(0) \\ z_1(T) \\ \dot{z}_1(T) \\ z_2(T) \\ \dot{z}_2(T) \end{bmatrix}_{8\times1}$$ (Ans).

(c)

We know,

$$z(t) = \sum_{i=1}^{N=6} \alpha_i \psi_i(t)$$

Basis functions, $\psi_1 = 1$, $\psi_2 = t$, $\psi_3 = t^2$, $\psi_4 = t^3$, $\psi_5 = t^4$, $\psi_6 = t^5$.

$$\therefore z_1(t) = \alpha_{11}\psi_1(t) + \alpha_{12}\psi_2(t) + \psi_{13}\psi_3(t) + \alpha_{14}\psi_4(t) + \alpha_{15}\psi_5(t) + \alpha_{16}\psi_6(t)$$

$$= \alpha_{11} + \alpha_{12} t + \alpha_{13} t^2 + \alpha_{14} t^3 + \alpha_{15} t^4 + \alpha_{16} t^5$$

$$\therefore \dot{z}_1(t) = 0 + \alpha_{12} . 1 + 2\alpha_{13} t + 3\alpha_{14} t^2 + 4\alpha_{15} t^3 + 5\alpha_{16} t^4$$

$$= \alpha_{12} + 2\alpha_{13} t + 3\alpha_{14} t^2 + 4\alpha_{15} t^3 + 5\alpha_{16} t^4$$

$$\therefore z_2(t) = \alpha_{21}\psi_1(t) + \alpha_{22}\psi_2(t) + \alpha_{23}\psi_3(t) + \alpha_{24}\psi_4(t) + \alpha_{25}\psi_5(t) + \alpha_{26}\psi_6(t).$$

$$= \alpha_{21} + \alpha_{22} t + \alpha_{23} t^2 + \alpha_{24} t^3 + \alpha_{25} t^4 + \alpha_{26} t^5$$

$$\therefore \dot{z}_2(t) = 0 + \alpha_{22} . 1 + 2\alpha_{23} t + 3\alpha_{24} t^2 + 4\alpha_{25} t^3 + 5\alpha_{26} t^4$$

$$= \alpha_{22} + 2\alpha_{23} t + 3\alpha_{24} t^2 + 4\alpha_{25} t^3 + 5\alpha_{26} t^4$$

For $t = 0$,

$z_1(0) = \alpha_{11}$ $\qquad$ $z_2(0) = \alpha_{21}$

$\dot{z}_1(0) = \alpha_{12}$ $\qquad$ $\dot{z}_2(0) = \alpha_{22}$

a

For $t = T$,

$$z_1(T) = \alpha_{11} + \alpha_{12}T + \alpha_{13}T^2 + \alpha_{14}T^3 + \alpha_{15}T^4 + \alpha_{16}T^5$$

$$\dot{z}_1(T) = \alpha_{12} + 2\alpha_{13}T + 3\alpha_{14}T^2 + 4\alpha_{15}T^3 + 5\alpha_{16}T^4$$

$$z_2(T) = \alpha_{21} + \alpha_{22}T + \alpha_{23}T^2 + \alpha_{24}T^3 + \alpha_{25}T^4 + \alpha_{26}T^5$$

$$\dot{z}_2(T) = \alpha_{22} + 2\alpha_{23}T + 3\alpha_{24}T^2 + 4\alpha_{25}T^3 + 5\alpha_{26}T^4$$

Therefore, Matrix vector equation: $Ax = b$.

$$\underset{8\times 12}{\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & T & T^2 & T^3 & T^4 & T^5 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 2T & 3T^2 & 4T^3 & 5T^4 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & T & T^2 & T^3 & T^4 & T^5 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 2T & 3T^2 & 4T^3 & 5T^4 \end{bmatrix}} \underset{12\times 1}{\begin{bmatrix} \alpha_{11} \\ \alpha_{12} \\ \alpha_{13} \\ \alpha_{14} \\ \alpha_{15} \\ \alpha_{16} \\ \alpha_{21} \\ \alpha_{22} \\ \alpha_{23} \\ \alpha_{24} \\ \alpha_{25} \\ \alpha_{26} \end{bmatrix}} = \underset{8\times 1}{\begin{bmatrix} z_1(0) \\ \dot{z}_1(0) \\ z_2(0) \\ \dot{z}_2(0) \\ z_1(T) \\ \dot{z}_1(T) \\ z_2(T) \\ \dot{z}_2(T) \end{bmatrix}}$$

(Ans)

Ans to the Q NO - 2

(a)

Given,

$$\dot{x}(t) = V(t)\cos\theta(t)$$

$$\dot{y}(t) = V(t)\sin\theta(t)$$

$$\dot{V}(t) = \alpha(t)$$

$$\dot{\theta}(t) = \omega(t).$$

Now,

$$Z = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$$

So,

$$x = z_1 \quad \text{and} \quad y = z_2$$

$$\therefore \dot{z}_1 = \dot{x}(t) = V(t)\cos\theta(t)$$

$$\therefore \dot{z}_2 = \dot{y}(t) = V(t)\sin\theta(t)$$

Using pythagorean theorem,

$$V(t) = \sqrt{V^2(t)\cos^2\theta(t) + V^2(t)\sin^2\theta(t)}$$

$$= \sqrt{(\dot{x})^2(t) + (\dot{y})^2(t)}.$$

$$= \sqrt{(\dot{z}_1)^2 + (\dot{z}_2)^2}$$

Again,

$$\frac{\dot{y}(t)}{\dot{x}(t)} = \frac{V(t)\sin\theta(t)}{V(t)\cos\theta(t)}.$$

$$\Rightarrow \frac{\dot{y}(t)}{\dot{x}(t)} = \tan\theta(t)$$

$$\Rightarrow \theta(t) = \tan^{-1}\left(\frac{\dot{y}(t)}{\dot{x}(t)}\right)$$

$$\therefore \theta(t) = \tan^{-1}\left(\frac{\dot{z}_2}{\dot{z}_1}\right)$$

So,

$$x(t) = z_1$$

$$y(t) = z_2$$

$$v(t) = \sqrt{(\dot{z}_1)^2 + (\dot{z}_2)^2}$$

$$\theta(t) = \tan^{-1}\left(\frac{\dot{z}_2}{\dot{z}_1}\right)$$

Therefore, the system is differentially flat with flat output $z = (x, y)$.

(showed)