NORTH SOUTH UNIVERSITY

Department of Mathematics and Physics
Final Assignment, Spring 2020
Subject: MAT 116 (Pre-calculus)
Section: 23 Date: 18 May, 2020

Full Marks: 30 **Submission Date: 4 June, 2020**

Important notes

a) you have to answer all the questions.
b) you must prepare it by hand writing, for submission please use pdf only.
c) please write your answers neatly in clear white paper and scan the answers using your phone.
d) Write page number and ID at the top of each page of your assignment.
e) you have to solve the assignment with honesty and integrity.
f) submit the assignment soon once you complete it.
g) you should not share your solutions with others.
h) it may go through 'plagiarism test' on your assignment, significant similarity (copying from others) would severely reduce marks from both.
i) this submission will carry maximum 30% marks for grading.

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1. (a) Suppose that the quantity supplied S and quantity demanded D of hot dogs at a baseball game are given by the following functions:

   $$S = -2000 + 3000p \quad \text{and} \quad D = 10{,}000 - 1000p$$

   where p is the price of a hot dog. Graph $S = S(p)$, $D = D(p)$ and label the equilibrium price for hot dogs.

   (b) Graph $f(x) = -2x^2 + 2x - 3$ by determining whether its graph opens up or down and by finding its vertex, axis of symmetry, y-intercept, and x-intercepts, if any. Then determine the domain and the range of f. Finally, determine where f is increasing and where it is decreasing.

   (c) Analyze the graph of the polynomial function $f(x) = -\frac{1}{2}(x+4)(x-1)^3$ by following all necessary steps for graphing polynomial functions.

   (d) Consider the polynomial function $f(x) = \left(x - \frac{1}{3}\right)^2 (x-1)^3$.

   (i) List each real zero, its multiplicity and determine whether the graph crosses or touches the x-axis at each x-intercept,
   (ii) Determine the behavior of the graph near each x-intercept,
   (iii) Determine the maximum number of turning points on the graph,
   (iv) Determine the end behavior of the graph.

2. (a) Using transformations analyze the graph of $R(x)=\dfrac{1}{(x-3)^2}+2.$

(b) Find the horizontal or oblique asymptote, if one exists, of the graph of

$R(x)=\dfrac{8x^2+26x-7}{4x-1}.$

(c) Analyze the graph of the rational function $R(x)=\dfrac{x^2+x-12}{x^2-4}$ by following all necessary steps for graphing rational functions.

(d) Solve $R(x)=\dfrac{2x+4}{x-1}\geq 0$ using the graph of the rational function.

3. (a) Use the Rational Zeros Theorem to find all the real zeros of the polynomial function $f(x)=2x^4-x^3-5x^2+2x+2.$ Use the zeros to factor *f* over the real numbers.

(b) For the given functions *f* and *g*, find $f\circ g,\ f\circ f$ and $g\circ f,$ where

(i) $f(x)=\sqrt{x-2}$ and $g(x)=1-2x,$ (ii) $f(x)=\dfrac{2x-1}{x-2},$ and $g(x)=\dfrac{x+4}{2x-5}.$

Also, state the domain of each composite function.

(c) Verify that the functions $f(x)=\dfrac{x-5}{2x+3}$ and $g(x)=\dfrac{3x+5}{1-2x}$ are inverses of each other by showing that $(f\circ g)(x)=x$ and $(g\circ f)(x)=x.$ Give any values of *x* that need to be excluded from the domain of *f* and the domain of *g*. Also, compare the vertical and horizontal asymptotes of *f* and *g*.

4. (a) Show that the function $f(x)=\dfrac{2x+3}{x+2}$ is one-to-one. Find its inverse and check your answer.

(b) The function $f(x)=x^2+9,\ x\geq 0$ is one-to-one. Find its inverse and check your answer. Graph $f,\ f^{-1}$ and $y=x$ on the same coordinate plane.

(c) Given $f(x)=\dfrac{ax+b}{cx+d},$ find $f^{-1}(x).$ If $c\neq 0,$ under what conditions on *a*, *b*, *c* and *d*, is $f=f^{-1}$?

5. (a) Use transformations to graph the function $f(x)=1-2^{x-3}.$ Determine the domain, range and horizontal asymptote of each function.

(b) Begin with the graph of $y=e^x,$ use transformations to graph the function $f(x)=7-3e^{2x}.$ Determine the domain, range and horizontal asymptote of *f*.

(c) Find the domain of the logarithmic function $f(x)=\log_3\left(\dfrac{x}{x-1}\right).$

Given $f(x)=2-\log_3(x+1).$ From the graph, determine the domain, range and any asymptotes of *f*. Then find f^{-1} and graph f^{-1} in the same coordinate plane.

6. (a) Solve the following equations:

(i) $\log_5(x^2+x+4)=2$, (ii) $9^{2x}.\ 27^{x^2}=3^{-1}$.

(b) Solve the following equations:

(i) $\log_3(3x^2)^{1/4}=3$,

(ii) $a^{4/3}=81$,

(iii) $\ln x^2-\ln(x+2)=\ln 6$.

(c) Graph the function $y=-\frac{1}{2}\sin\left(\frac{\pi}{8}x\right)+\frac{3}{2}$. Label key points and show at least two cycles. Use the graph to determine the domain and the range of the function.
