Periodic function A function $f(x)$ is said to be a periodic function if

$$f(x+T) = f(x)$$

where, T = be the period of the given function.

Example: $f(x) = \sin x$

$\therefore f(x+2\pi) = \sin(x+2\pi) = \sin x$
$= f(x)$

So 2π be the period of $\sin x$

The general form of $\sin x / \cos x$

$$f(x) = a \sin(bx - c) + d$$

and $f(x) = a \cos(bx - c) + d$

where a = amplitude

$bx - c$ = angular momentum

d = shifting phase

* The general form of Sinosoidal/cos (?)

$f(x) = a \sin(\omega x)$ where $\omega = bx - c$ is angular momentum

or $f(x) = a \cos(\omega x)$

where a = amplitude

$(bx - c) = \omega$ = angular momentum

T = period $= 2\pi/b = \frac{2\pi}{\omega}$.

starting point $\Rightarrow bx - c = \omega = 0$

$\therefore \boxed{bx - c = 0}$

x - scale point $= \frac{\text{Period}}{4}$

Ending point $\Rightarrow bx - c = 2\pi \Rightarrow \omega = 2\pi$

$\therefore \boxed{bx - c = 2\pi}$

The above properties is used for graphing Sinusoidal Function of the form

$$\boxed{y = A \sin(\omega x) = a \sin(bx - c)}$$

$$\boxed{y = A \cos(\omega x) = a \cos(bx - c)}$$

Problem: Graphing a sinusodial function

$$y = 2\sin(-\pi/2\, x)$$

Solution:

$$y = 2\sin(-\pi/2\, x)$$
$$y = -2\sin(\pi/2\, x)$$

Amplitude $= |a| = |-2| = 2$

period $= 2\pi/b = \frac{2\pi}{\pi/2} = 2\pi/\pi/2 = 4$

starting point $bx - c = 0$

$\Rightarrow \pi/2\, x = 0 \Rightarrow \boxed{x = 0}$

Scale point $= \frac{\text{Period}}{4} = 4/4 = 1$

Ending point $\Rightarrow bx - c = 2\pi$

$\Rightarrow \pi/2\, x = 2\pi$

$\Rightarrow \boxed{x = 4}$

Now starting point $= 0$; $f(0) = 0$

next point $0+1 = 1$; $f(1) = -2$

next point $1+1 = 2$; $f(2) = 0$

next point $2+1 = 3$; $f(3) = 2$

next point $3+1 = 4$; $f(4) = 0$

The graph lies between -2 to 2.

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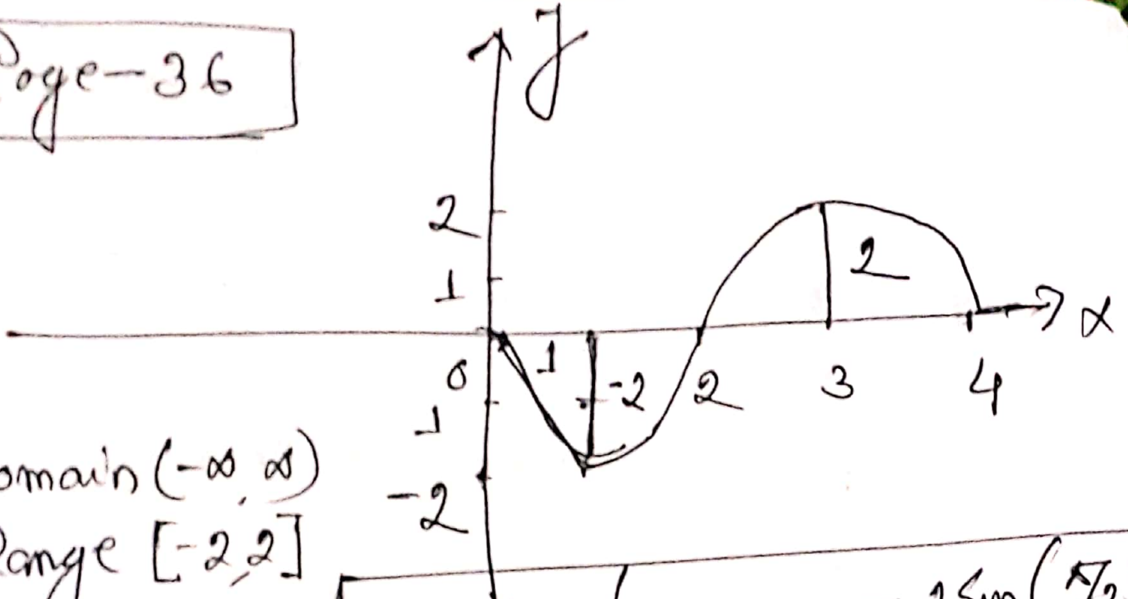


Domain $(-\infty, \infty)$
Range $[-2, 2]$

$\boxed{\text{graph of } y = -2\sin\left(\frac{\pi}{2}x\right)}$

Problem: Draw the graph of the function and determine domain and range.

$$y = -6\sin\left(\frac{\pi}{3}x\right) + 4$$

Solution: Amplitude $= |a| = |-6| = 6$

Period $= \frac{2\pi}{b} = \frac{2\pi}{\pi/3} = 6$

Starting point $bx - c = 0$

$\Rightarrow \frac{\pi}{3}x = 0 \quad \therefore \boxed{x = 0}$

Scale point $= \frac{\text{Period}}{4} = \frac{6}{4} = \frac{3}{2}$

Ending point $\Rightarrow bx - c = 2\pi$

$\Rightarrow \frac{\pi}{3}x = 2\pi$

$\therefore \boxed{x = 6}$

Starting point $x=0$, $f(0)=4$

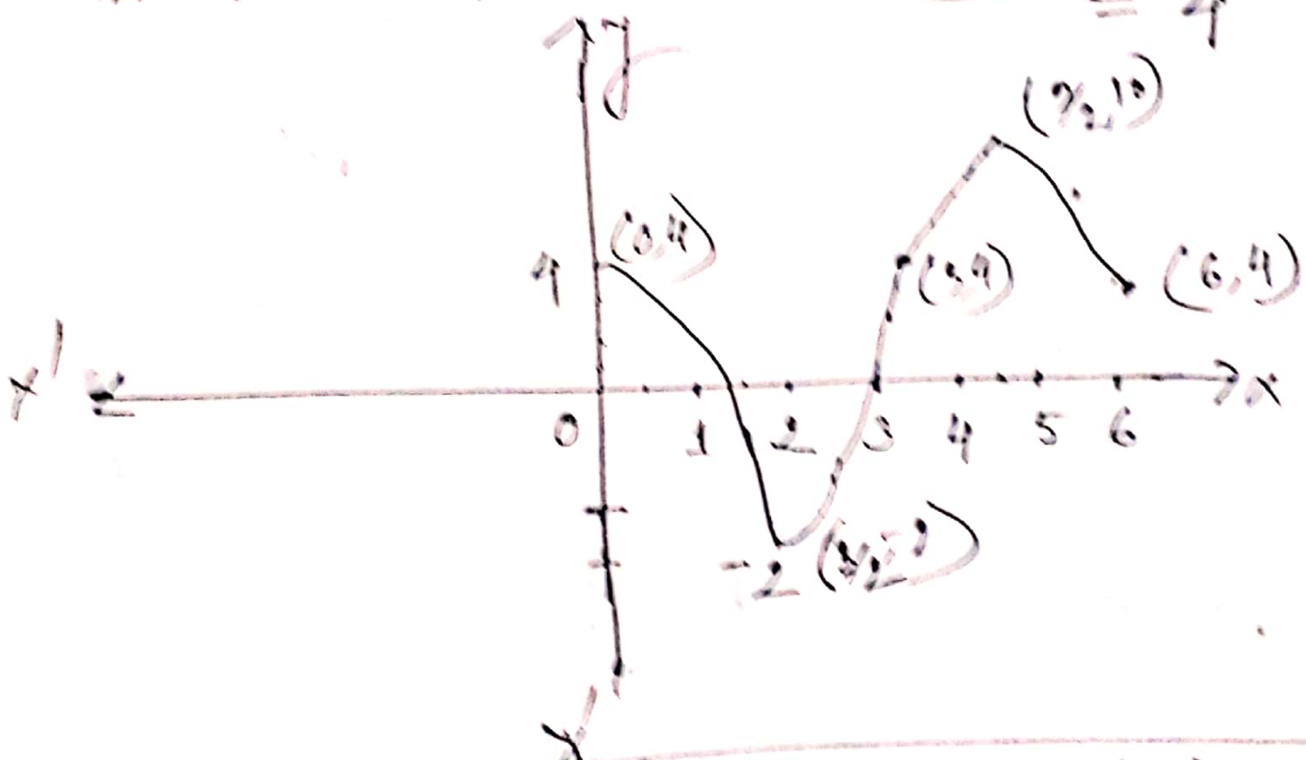
next point $0+\frac{3}{2}=\frac{3}{2}$ ∴ $f(\frac{3}{2})=-6+4=-2$

next point $\frac{3}{2}+\frac{3}{2}=3$ ∴ $f(3)=-6\cdot 0+4=4$

next point $3+\frac{3}{2}=\frac{9}{2}$ ∴ $f(\frac{9}{2})=6+4=10$

next point $\frac{9}{2}+\frac{3}{2}=6$ $f(6)=-6\cdot 0+4$

$=4$



$$\boxed{\text{graph of } y=-6\sin\left(\frac{\pi x}{3}\right)+4}$$

Domain $=(-\infty,\infty)$

Range $=[-2,10]$

Problem: Draw the graph of the given function and find the domain of range.

$$y = 2\cos\left(\frac{x}{2}\right)$$

Soln

Amplitude $= |a| = |2| = 2$

Period $= \frac{2\pi}{b} = \frac{2\pi}{\omega} = \frac{2\pi}{1/2} = 4\pi$

Starting point $= bx - c = 0$

$\Rightarrow \frac{x}{2} = 0 \Rightarrow \boxed{x = 0}$

x-scale point $= \frac{\text{Period}}{4}$

$\therefore$ $\boxed{\text{x-scale point} = \frac{4\pi}{4} = \pi}$

Ending point $\Rightarrow bx - c = 2\pi$

$\Rightarrow \frac{x}{2} = 2\pi \Rightarrow \boxed{x = 4\pi}$

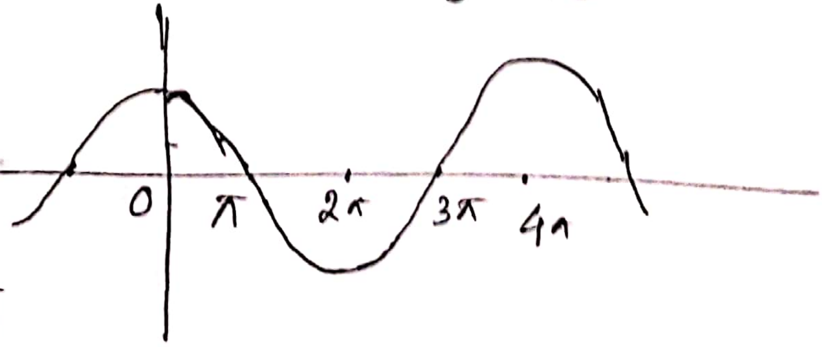
Starting point $x = 0$, $f(0) = 2$

next point $x = 0 + \pi$, $f(\pi) = 0$

next point $\pi + \pi = 2\pi$ $\therefore f(2\pi) = -2$

next point $2\pi + \pi = 3\pi$ $\therefore f(3\pi) = 0$

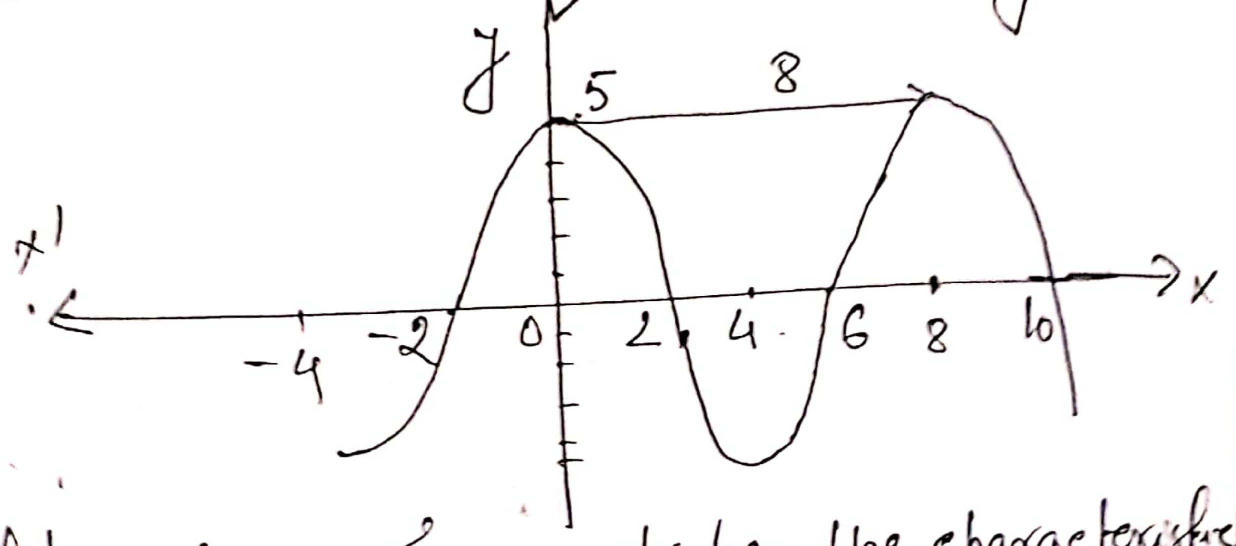
next point $3\pi + \pi = 4\pi$ $\therefore f(4\pi) = 2$



Domain $(-\infty, \infty)$

Range $[-2, 2]$

Problem Find the equation for the graph.



Solution: The graph has the characteristic of a cosine function. The maximum value 5 occur at $x=0$

So the equation is a cosine function

$$y = A\cos(\omega x) = a\cos(bx-c)$$

amplitude $= 5$, period $= 8$

$$\Rightarrow \frac{2\pi}{\omega} = 8$$

$$\Rightarrow \omega = 2\pi/8 = \pi/4$$

So, the cosine function whoes graph is given is $\boxed{y = 5\cos\left(\frac{\pi}{4}x\right)}$.

Home work: $\boxed{11-76}$ Exercise 6.4

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Periodic function A function $f(x)$ is said to be a periodic function if

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$= f(x)$

So 2π be the period of $\sin x$

The general form of $\sin x / \cos x$

$$\boxed{\begin{aligned} f(x) &= a\sin(bx-c)+d \\ \text{and } f(x) &= a\cos(bx-c)+d \end{aligned}}$$

where a = amplitude
$bx-c$ = angular momentum
d = shifting phase

Example: Graphing cosine with period change and phase shifting of the function $f(x) = -3\cos(2x+\pi)$

Solution: Given that, $f(x) = -3\cos(2x+\pi)$

Now, $\boxed{f(x) = -3\cos(2x+\pi)}$

amplitude $= |a| = |-3| = 3$

Period $= \frac{2\pi}{b} = \frac{2\pi}{2} = \pi$

starting point $= bx - c = 0$

$\Rightarrow 2x + \pi = 0$

$\Rightarrow \boxed{x = -\pi/2}$

x-scale point $= \frac{\text{Period}}{4} = \pi/4$

Ending point: $\Rightarrow bx - c = 2\pi$

$\Rightarrow 2x + \pi = 2\pi$

$\Rightarrow 2x = \pi$

$\Rightarrow \boxed{x = \pi/2}$

starting point $= -\pi/2$; $f(-\pi/2) = -3\cos(\pi+\pi)$
$= -3$

2nd point : starting point + x-scale point

$= \boxed{-\pi/2 + \pi/4 = -\pi/4}$

$\therefore f(-\pi/4) = -3\cos(2(-\pi/4)+\pi) = 0$

Next point : $\boxed{-\pi/4 + \pi/4 = 0}$

$\therefore f(0) = -3(\cos 0 + \pi) = 3$

Next point : $\boxed{0 + \pi/4 = \pi/4}$

$\therefore f(\pi/4) = -3\cos(2\cdot\pi/4 + \pi)$
$= -3\cos(3\pi/2) = 0$

Next point : $\boxed{\pi/4 + \pi/4 = 2\pi/4 = \pi/2}$

$\therefore f(\pi/2) = -3\cos(2\cdot\pi/2 + \pi)$
$= -3$.

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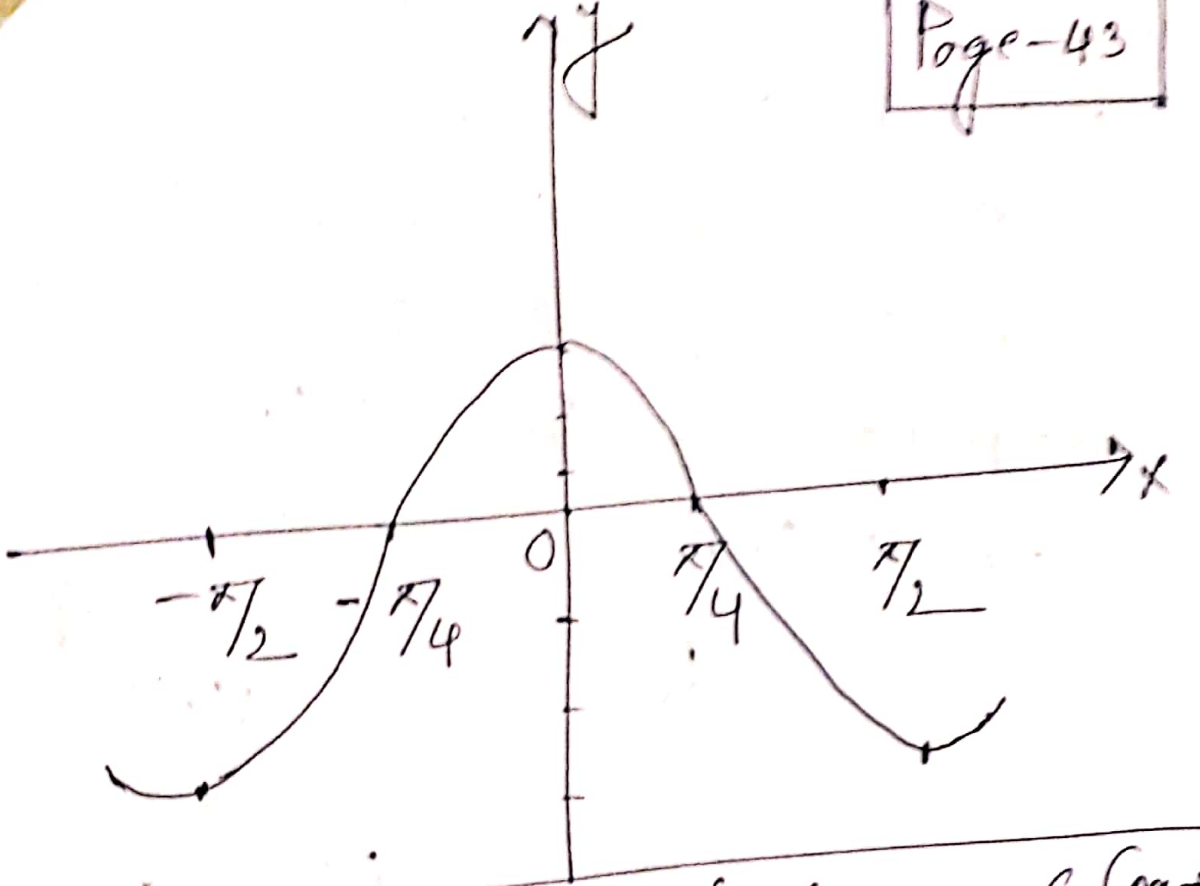

The graph of $f(x) = -3\cos(2x+\pi)$

Home-work Exercise 35-58 (6.4)

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Problem : Draw the graph
$f(x) = -\cos(x - \pi/4)$

Solution : we know the general form
$f(x) = a\cos(bx - c) + d$

Given that, $f(x) = -\cos(x - \pi/4)$

amplitude $= |a| = |-1| = 1$

Period $= 2\pi/b = 2\pi/1 = 2\pi$

x-scale $= 2\pi/4 = \frac{\text{Period}}{4} = 2\pi/4 = \pi/2$

starting point : $bx - c = 0$

$\Rightarrow x - \pi/4 = 0$

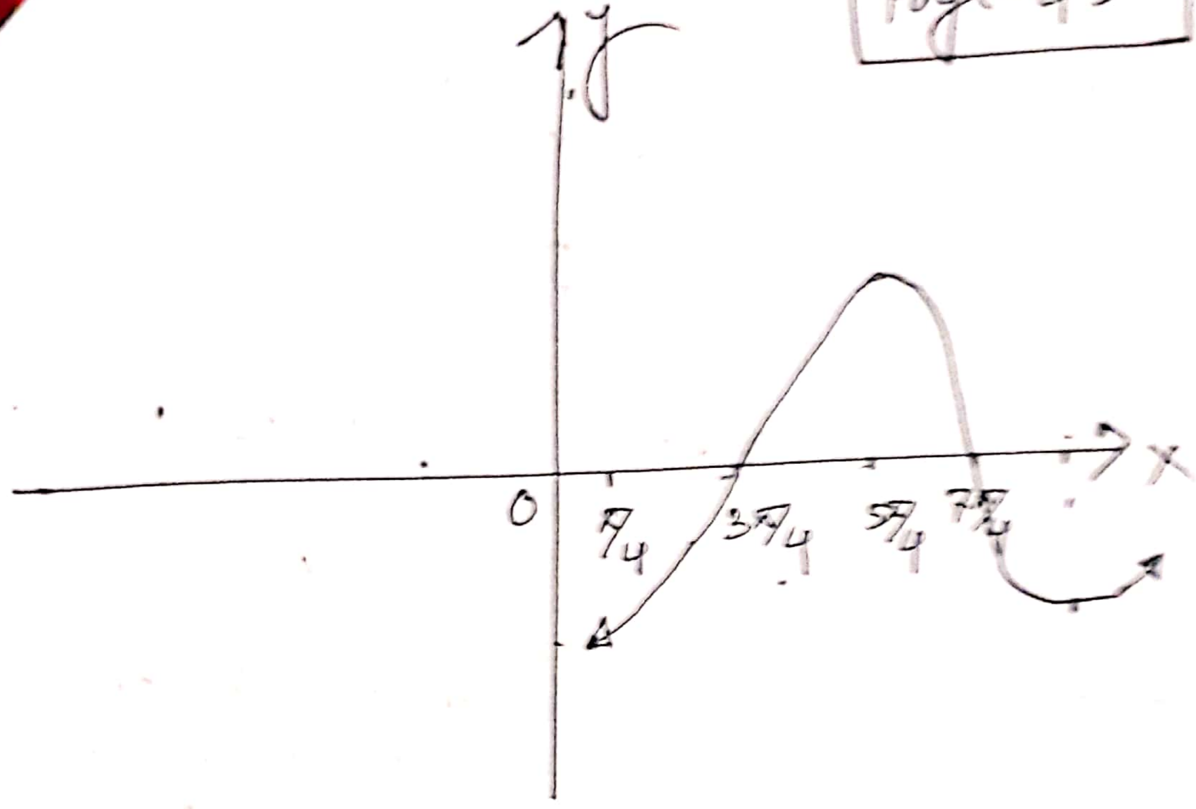
$\therefore \boxed{x = \pi/4}$

Ending point : $bx - c = 2\pi$

$\Rightarrow x - \pi/4 = 2\pi$

$\Rightarrow x = 2\pi + \pi/4$

$\boxed{x = 9\pi/4}$



Starting point $= \frac{\pi}{4}$: $f\left(\frac{\pi}{4}\right) = -\cos\left(\frac{\pi}{4} - \frac{\pi}{2}\right)$
$= -1$

next point $= \frac{\pi}{4} + \frac{\pi}{2} = \frac{3\pi}{4}$: $f\left(\frac{3\pi}{4}\right) = 0$

next point $= \frac{3\pi}{4} + \frac{\pi}{2} = \frac{5\pi}{4}$: $f\left(\frac{5\pi}{4}\right) = 1$

next point $= \frac{5\pi}{4} + \frac{\pi}{2} = \frac{7\pi}{4}$: $f\left(\frac{7\pi}{4}\right) = 0$

next point $= \frac{7\pi}{4} + \frac{\pi}{2} = \frac{9\pi}{4}$: $f\left(\frac{9\pi}{4}\right) = -1$