# EEE363

## Electrical Machines
(DC Motor)

Lecture # 9

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# Armature torque

Armature developed power $= T_a \times \omega = E_b\, I_a$

$$T_a = \frac{E_b\, I_a}{\omega} = \frac{1}{\omega} \times \frac{\varphi ZN}{60} \times \frac{P}{A} \times I_a \quad = \frac{1}{2\pi}\, \varphi Z I_a \cdot \frac{P}{A}$$

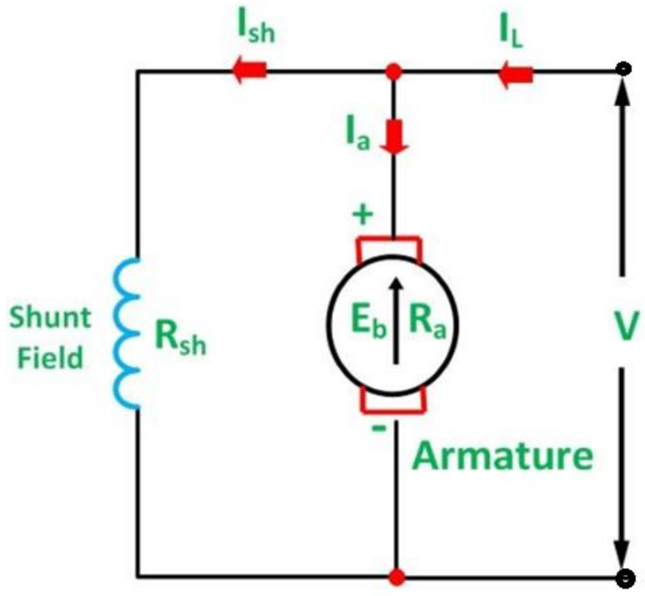
Thus $T_a \propto \varphi I_a$

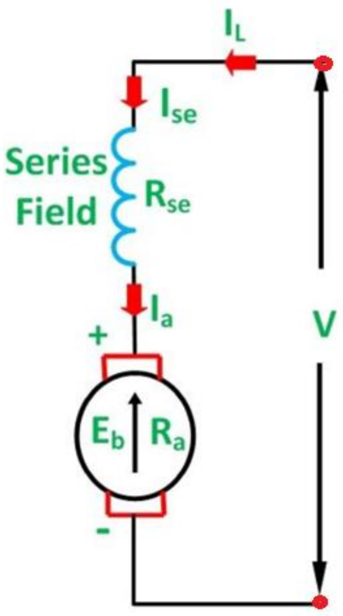
**Shunt motor**

$T_a \propto I_a$

**Series motor**

$T_a \propto I_a^2$



# Problem#1

Determine developed torque and shaft torque of 220-V, 4-pole series motor with 800 conductors wave-connected supplying a load of 8.2 kW by taking 45 A from the mains. The flux per pole is 25 mWb and its armature circuit resistance is 0.6 Ω.

$$T_a = \frac{1}{2\pi}\,\varphi Z I_a \cdot \frac{P}{A} = 0.159 \times 25 \times 10^{-3} \times 800 \times 45\,(4/2) = 286.2 \text{ N-m}$$

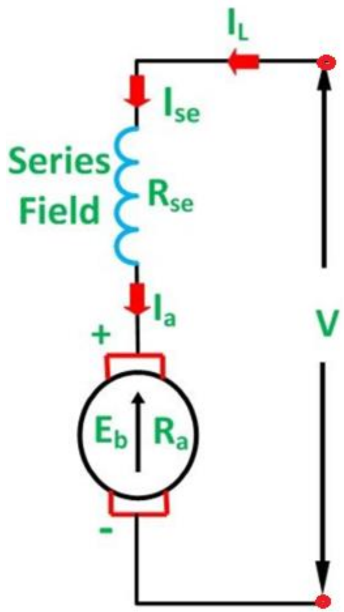
$$E_b = V - I_a R_a = 220 - 45 \times 0.6 = 193 \text{ V}$$

$$E_b = \Phi ZN\,(P/A) \text{ or } 193 = 25 \times 10^{-3} \times 800 \times N\,\pi \times (4/2)$$

$$N = 4.825 \text{ r.p.s.}$$

$$P = T_{sh} \times \omega$$

$$T_{sh} = 270.5 \text{ N-m}$$



# Speed of a DC Motor

$$E_b = V - I_a R_a \rightarrow \frac{\Phi Z N}{60}\left(\frac{P}{A}\right) = V - I_a R_a \rightarrow N = \frac{V - I_a R_a}{\Phi} \times \left(\frac{60A}{ZP}\right)$$

$$N = \frac{E_b}{\Phi} \times \left(\frac{60A}{ZP}\right) \rightarrow N = K\frac{E_b}{\Phi}$$

Series motor $\rightarrow$ $\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{I_{a1}}{I_{a2}}$ $\quad \Phi \propto I_a$

Shunt motor $\rightarrow$ $\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\Phi_1}{\Phi_2} \rightarrow \frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}}$

# Speed Regulation

- Defines change in speed with load

$$\%\ \text{speed regulation} = \frac{\text{N.L. speed} - \text{F.L. speed}}{\text{F.L. speed}} \times 100 = \frac{dN}{N} \times 100$$

# Torque & Speed of a DC motor

$$N = K\frac{V - I_a R_a}{\phi} = \frac{KE_b}{\Phi}$$

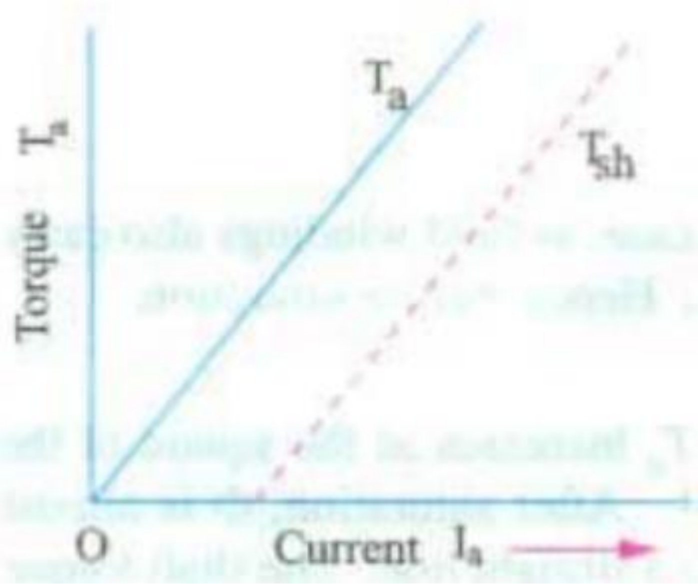
$$T_a \propto \Phi I_a$$

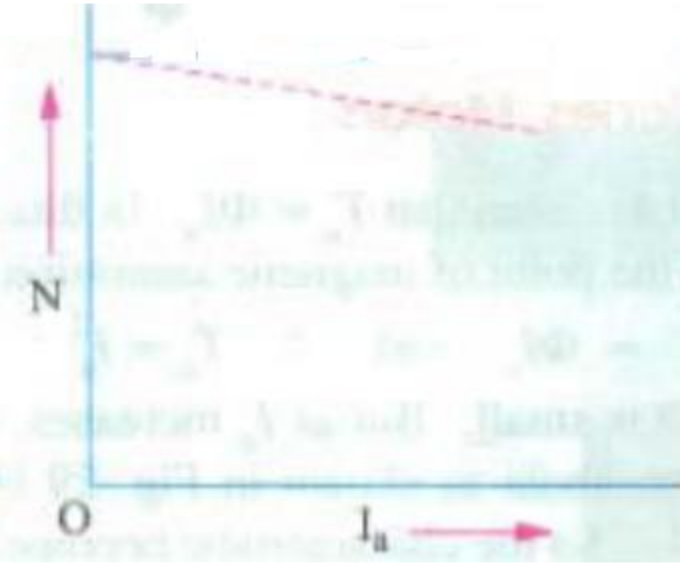
*There appears to be a conflict between these two expressions*

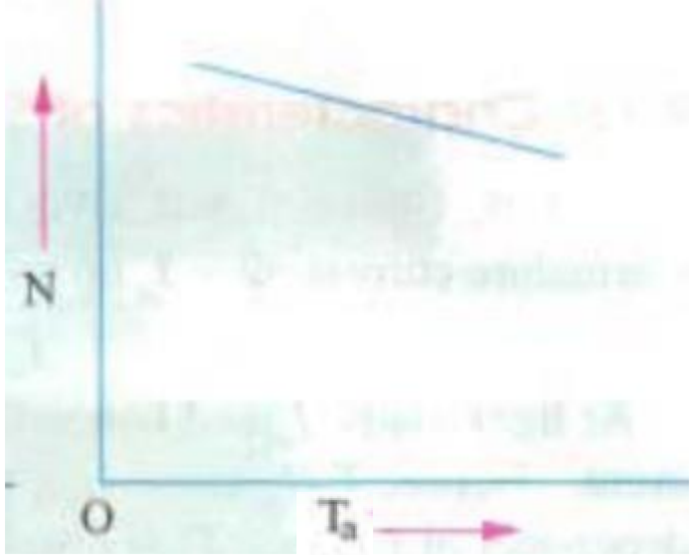
- Paradox between these two expressions can be explained in the following way:

1. Back e.m.f. $E_b$ ($= N\Phi/K$) drops instantly (the speed remains constant because of inertia of the heavy armature).
2. Due to decrease in $E_b$, $I_a$ is increased because $I_a = (V - E_b)/R_a$. Moreover, a small reduction in flux produces a proportionately large increase in armature current.
3. Hence, the equation $T_a \propto \Phi I_a$, a small decrease in $\phi$ is more than counterbalanced by a *large* increase in $I_a$ with the result that there is a net *increase* in $T_a$.
4. This increase in $T_a$ produces an increase in motor speed.

# Shunt motor characteristics



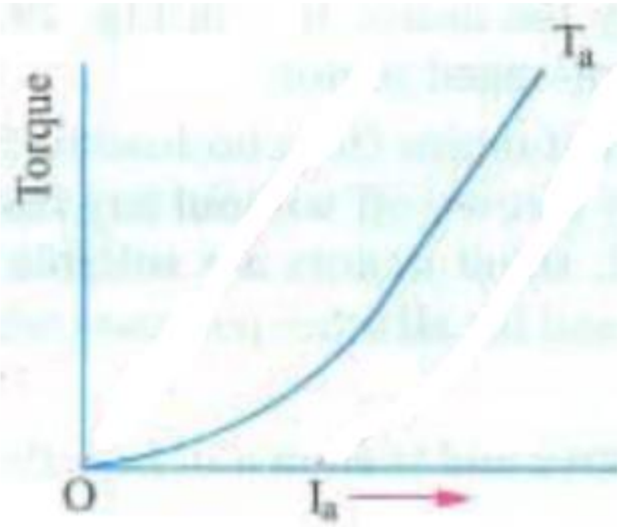



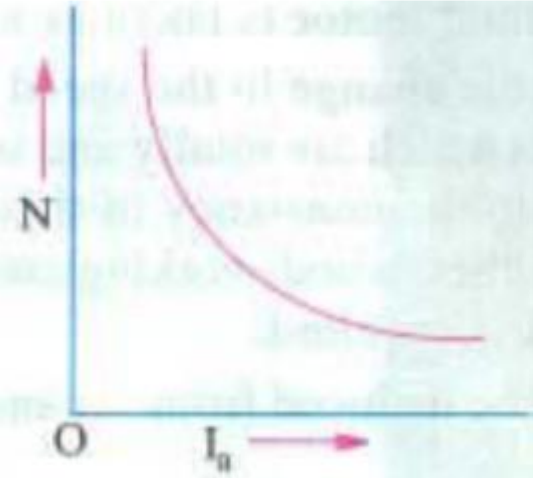



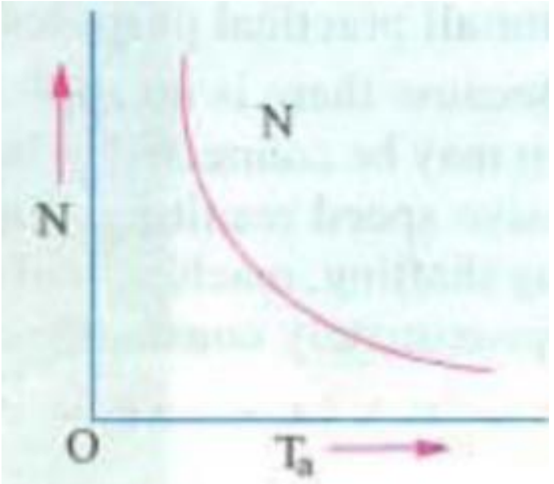

$$N = \frac{KE_b}{\Phi}$$

$$T_a \propto \Phi I_a$$

# Series motor characteristics




$$N = \frac{KE_b}{\Phi}$$

$$T_a \propto \Phi I_a$$

# Comparison of shunt & series motor

## Shunt motor

- The speed of a shunt motor is sufficiently constant.
- For the same current input its starting torque is not as high as that of a series motor.

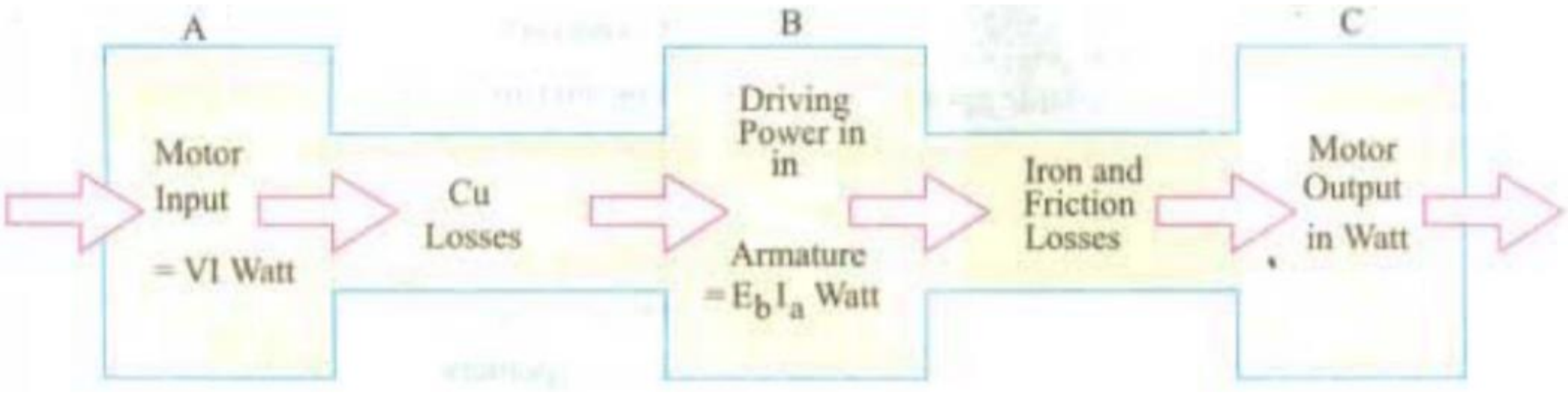
## Series motor

- Has relatively large starting torque.
- Has low speed at high loads and very high speed at light load.

# Comparison of shunt & series motor

| *Type of motor* | *Characteristics* | *Applications* |
|---|---|---|
| Shunt | Approximately constant speed Adjustable speed<br>Medium starting torque (Up to 1.5 F.L. torque) | For driving constant speed line shafting<br>Lathes<br>Centrifugal pumps<br>Machine tools<br>Blowers and fans<br>Reciprocating pumps |
| Series | Variable speed<br>Adjustable variying speed<br>High Starting torque | For traction work *i.e.*<br>Electric locomotives<br>Rapid transit systems<br>Trolley, cars etc.<br>Cranes and hoists<br>Conveyors |

# Power stages



# Problem # 2

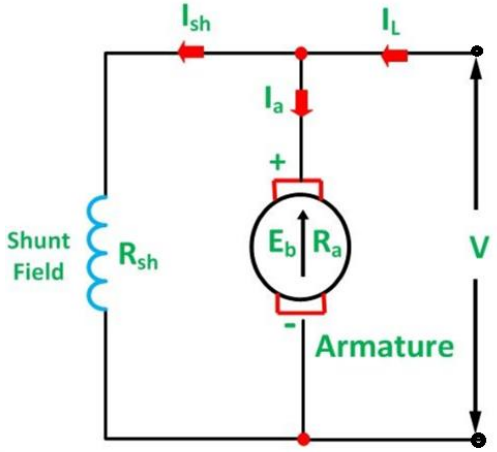
*A d.c. shunt machine while running as generator develops a voltage of 250 V at 1000 r.p.m. on no-load. It has armature resistance of 0.5 Ω and field resistance of 250 Ω. When the machine runs as motor, input to it at no-load is 4 A at 250 V. Calculate the speed and efficiency of the machine when it runs as a motor taking 40 A at 250 V. Armature reaction weakens the field by 4 %.*

$$\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\Phi_1}{\Phi_2}$$

Now, when running as a generator, the machine gives 250 V at 1000 r.p.m. If this machine was running as motor at 1000 r.p.m., it will, obviously, have a back e.m.f. of 250 V produced in its armature. Hence $N_1 = 1000$ r.p.m. and $E_{b1} = 250$ V.

When it runs as a motor, drawing 40 A, the back e.m.f. induced in its armature is $E_{b2} = 250 - (40 - 1) \times 0.5 = 230.5$ V;

$$\frac{N_2}{1000} = \frac{230.5}{250} \times \frac{\Phi_1}{0.96\,\Phi_1}; \; N_2 = \textbf{960 r.p.m.}$$




**Efficiency**

No-load input represents motor losses which consists of

(*a*) armature Cu loss $= I_a^2 R_a$ which is variable.

(*b*) constant losses $W_c$ which consists of (*i*) shunt Cu loss (*ii*) magnetic losses and (*iii*) mechanical losses.

# Problem # 2 contd...

No-load input or total losses = $250 \times 4 = 1000$ W

Arm. Cu loss = $I_a^2 R_a = 3^2 \times 0.5 = 4.5$ W, $\therefore$ $W_c = 1000 - 4.5 = 995.5$ W

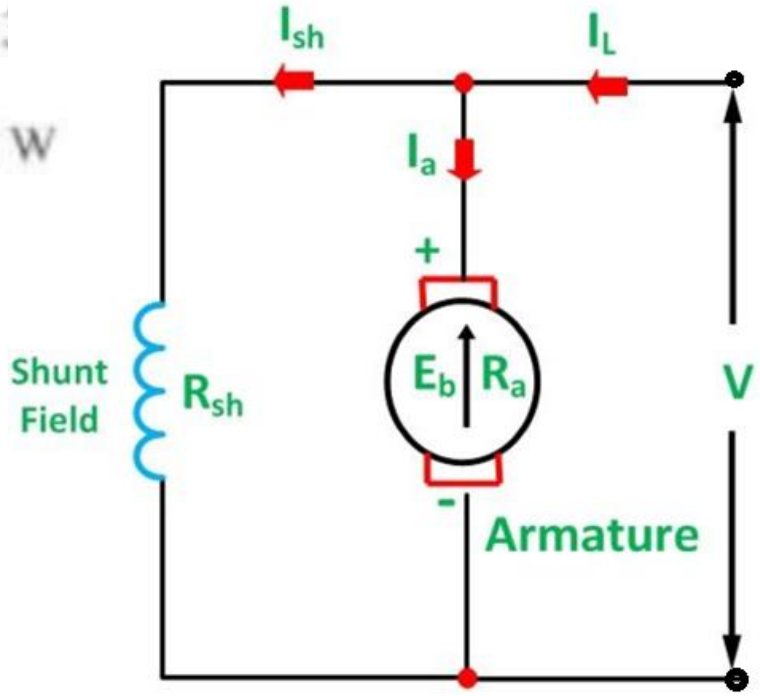
When motor draws a line current of 40 A, its armature current is $(40 - 1) =$

Arm. Cu loss = $39^2 \times 0.5 = 760.5$ W; Total losses = $760.5 + 955.5 = 1756$ W

Input = $250 \times 40 = 10{,}000$ W;

output = $10{,}000 - 1756 = 8{,}244$ W

$\eta = 8{,}244 \times 100/10{,}000 = 82.44\%$



# Speed control of DC Motor

Factors controlling motor speed

$$N = 60\,\frac{V - I_a R_a}{Z\Phi} \cdot \left(\frac{A}{P}\right)$$

## I. Flux control method

- ✓ This method is very efficient.
- ✓ In non-interpolar m/c speed can be varied at a ratio 2:1.
- ✓ For machines with interpoles the ratio of maximum to minimum speed of 6:1 is common.