3.4 : The poisson.

$X =$ A poisson r.v $\sim$ Pois (λ).

$$P(X) = \frac{e^{-\lambda} \cdot \lambda^x}{x!} \; ; \; x = 0, 1, 2, \ldots$$

where λ is the parameter.

number of rare ~~of~~ event. (like mis page, death in hors, num of Accident, num of highest paid job holder)

$\lambda =$ rate of event.

x. pmf?

$$\sum_{x=0}^{\infty} \frac{e^{-\lambda} \lambda^{x}}{x!} = e^{-\lambda} + \frac{\lambda}{1!} e^{-\lambda} + \frac{\lambda^2}{2!} e^{-\lambda} + \frac{\lambda^3}{3!} e^{-\lambda} + \ldots$$

$$= e^{-\lambda} \left[1 + \frac{\lambda}{1!} + \frac{\lambda^2}{2!} + \frac{\lambda^3}{3!} + \ldots \right]$$

$$= e^{-\lambda} \times e^{\lambda}$$

$$= 1.$$

~~$E(X) = \sum x \, e^{-\lambda} \lambda x$~~

$$E(X) = \sum_{x=0}^{\infty} \frac{x e^{-\lambda} \lambda^x}{x!}$$

$$= \sum_{x=1}^{\infty} \frac{e^{-\lambda} \lambda^x}{(x-1)!}$$

$$= \lambda e^{-\lambda} + \frac{\lambda^2}{1!} e^{-\lambda} + \frac{\lambda^3}{2!} e^{-\lambda} + \ldots$$

$$= \lambda e^{-\lambda} \left[1 + \frac{\lambda}{1!} + \frac{\lambda^2}{2!} + \cdots\right]$$

$$= \lambda \cancel{e^{-\lambda}} \times \cancel{e^{\lambda}}$$

$$= \lambda.$$

* $E(x^2) = \lambda^2 + \lambda.$

$$v(x) = E(x^2) - [E(x)]^2 = \cancel{\lambda^2} + \lambda - \cancel{\lambda^2}$$
$$= \lambda.$$

poissoin এ mean and variance -এ same.

<u>Example of poissoin :-</u>

* $\lambda = 2$ ঘণ্টায় ২ টা At আসে। 4 টা আসার chance কত?

$$P(x=4) = ?$$

$$= \frac{e^{-2} 2^4}{4!}$$

$$=$$ কোনো At আসবে না।

$$P(x=0) = \frac{e^{-2} 2^0}{0!}$$

কমপক্ষে ১ টা আসার

$$P(x \geq 1) = 1 - P(x < 1) = 1 - P(x=0).$$

$$=$$

ch-4 :– Continuous R. Dist.

$F(x^2) - [E(x)]^2$

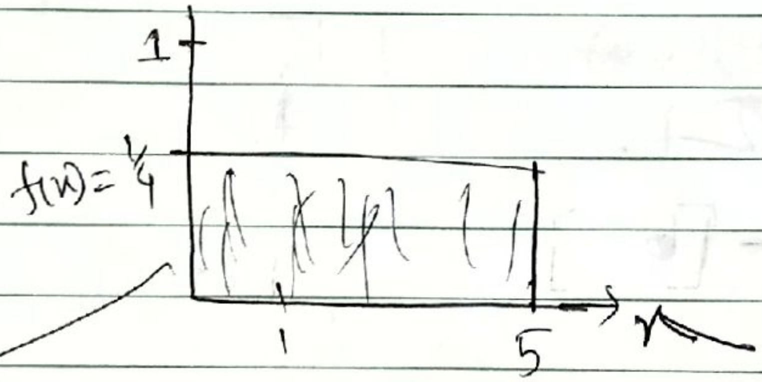
$\boxed{p(1-p)}$

4.1 → The uniform Dist :–

X = A uniform r.v $\sim$ Uni (a, b).

$f(x) = \frac{1}{b-a}$; $a \leq x \leq b$.

$1 \leq x \leq 5$. $\quad f(x) = \frac{1}{4}$



uniform distribution :– x এর প্রত্যেকটার value-এর সমান chance থাকবে।

is pdf?

$$\int f(x)\,dx = \int_a^b \frac{1}{b-a}\,dx$$

$$= \frac{1}{b-a}\int_a^b dx$$

$$= \frac{1}{\cancel{b-a}} \times \cancel{[b-a]}$$

$$= 1.$$

$$E(X) = \int x\, f(x)\,dx$$

$$= \int x \frac{1}{b-a}$$

$$= \frac{1}{b-a} \int_a^b x \, dx$$

$$= \frac{1}{b-a} \left[\frac{x^2}{2}\right]_a^b$$

$$= \frac{1}{2(b-a)} \left[b^2 - a^2\right]$$

$$= \frac{(b+a)\cancel{(b-a)}}{2\cancel{(b-a)}}$$

$$= \frac{1}{2}\left[b+a\right]$$

$E(x^2) =$

$$V(x) = \frac{(b-a)^2}{12}$$

CdF

$$F(x) = P(X \le x)$$

$$= \int_a^x \frac{1}{b-a} \, dx$$

$$= \frac{x-a}{b-a}.$$

Example :-

[container এর milk থাকার chance]

$$f(x) = \frac{1}{10} ; \quad 1 \le x \le 8$$

$$P(X \le 5) = \frac{5-1}{8-1}$$

$$= \frac{4}{7}$$

$$= \frac{4}{7}.$$

4.2 :- Exponential distribution :-

X = expon. r. variable $\sim Exp(\lambda)$.

$f(x) = \lambda e^{-\lambda x}$; $x > 0$.

x = waiting time. | doctor appointment

$$E(x) = \int_0^{\infty} x \lambda e^{-\lambda x} dx$$

$$= \frac{1}{\lambda}$$

$E(x^2) = ?$

$V(x) = \frac{1}{\lambda^2}$.

$$F(x) = P(X \leq x) = \int_0^{x} \lambda e^{-\lambda x} dx$$

$$= \lambda \int_0^{x} e^{-\lambda x} dx$$

$$= \cancel{\lambda} \left[\frac{e^{-\lambda x}}{\cancel{-\lambda}} \right]_0^{x}$$

$$= \left[-e^{-\lambda x} \right]_0^{x}$$

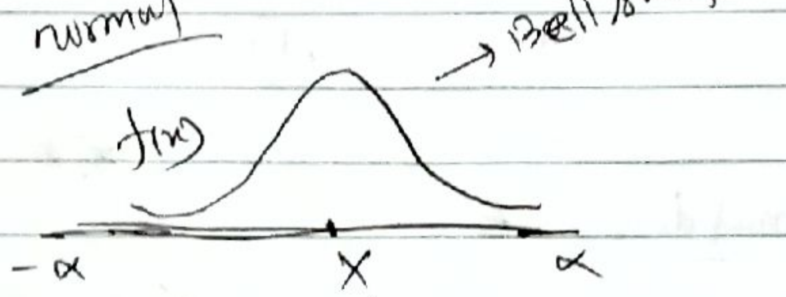
$$= 1 - e^{-\lambda x}$$

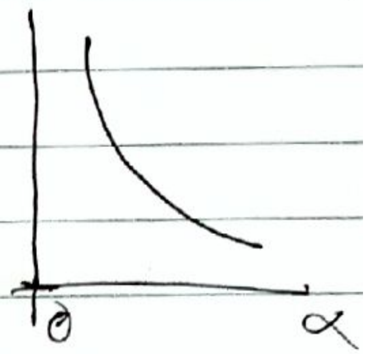
5.1 :- The Normal Dis :- [continious r.v]

X = a normal r.v.

exponential

normal





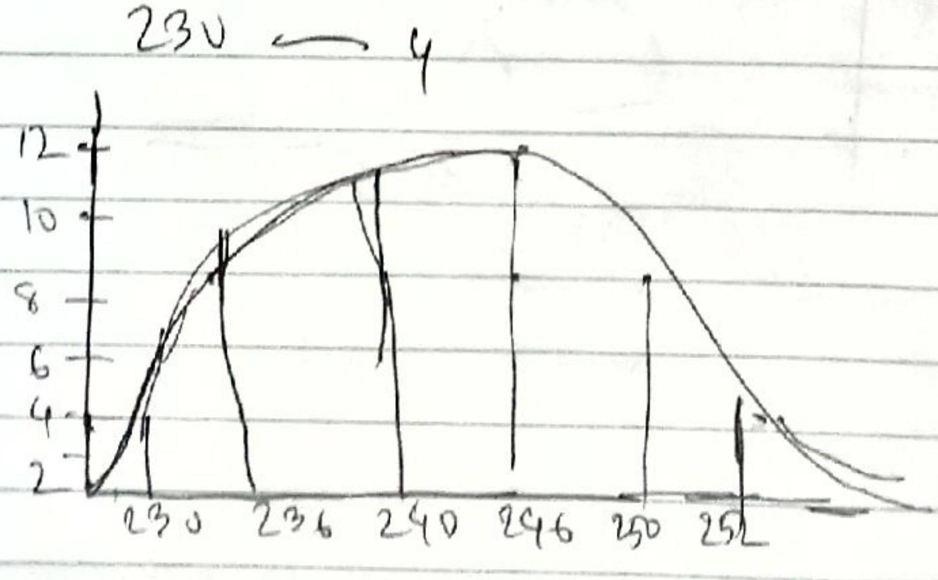
μ

$\mu = E(X)$

$P(X \leq \mu) = .50$

$P(X \geq \mu) = .50$

252 — 4
250 — 8
246 — 12
240 — 10
236 — 10
230 — 4



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not memorize just understand the parameter

$$\left[f(x) = P(X=x) = \frac{1}{\sqrt{2\pi\sigma^2}} \right] e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \quad [-\alpha \le x \le \alpha]$$

mean

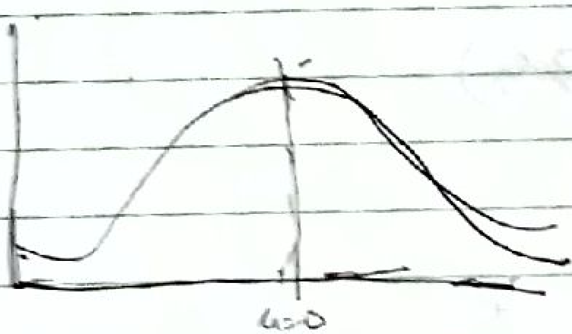
variance

$$Z \sim N(0,1)$$

<u>Standard normal dis:-</u> $f(z) = \frac{1}{\sqrt{2}} e^{-\frac{1}{2}z^2}$ $\left[Z = \frac{x-\mu}{\sigma}\right]$

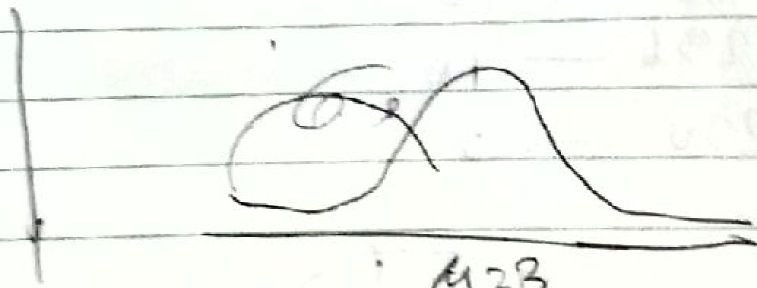
$$= \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}}$$

$$= \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}}$$



* [Standard normal Variable = Z]

$N=(3,2)$



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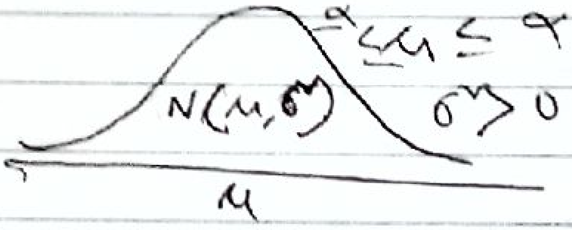
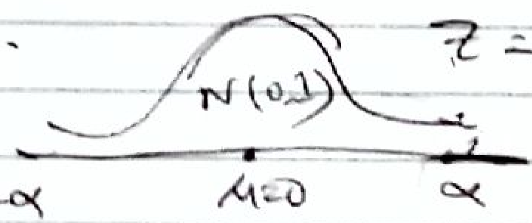
5.1 :- The normal distribution.

$X \sim$



$Z \sim$

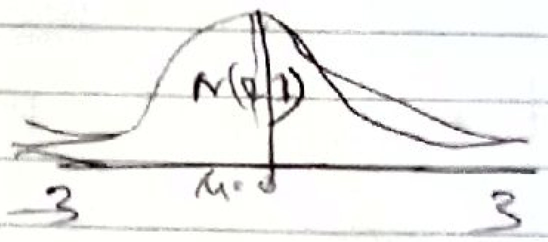


$Z = \frac{X-\mu}{\sigma}$

-3 (Practical) → 3

X - (money) - $N(500, 60)$

$Z = \frac{X-500}{\sqrt{60}}$



$P(Z \le 0) = .50$

$P(Z \geq 0) = .50$

$P(Z > 3) = 0$

$P(Z \le 3) = 0$

$P(Z > -3) = 1$

$$P(a \le x \le b) = ?\quad P\left(\frac{a-\mu}{\sigma} \le \frac{x-\mu}{\sigma} \le \frac{b-\mu}{\sigma}\right)$$

$$P(21 \le x < 22) = ? = P\left(\frac{a-\mu}{\sigma} \le z \le \frac{b-\mu}{\sigma}\right)$$

$$x \sim N(\mu, \sigma^2) \qquad = \phi\left(\frac{b-\mu}{\sigma}\right) - \phi\left(\frac{a-\mu}{\sigma}\right)$$

$$P(Z \le z) = \phi(z)$$

$$\phi(0) = \tfrac{1}{2}$$

$$\phi(3) = 1$$

$$\phi(-3) = 0$$

$$\cancel{\phi}\ P(z > 3) = \cancel{\phi}\ 1 - P(z \le 3)$$

$$= 1 - \phi(3)$$

$$= 1 - 1$$

$$= 0$$

$$* \quad P(x = 20) = P\left(\frac{x-\mu}{\sigma} = \frac{20-\mu}{\sigma}\right)$$

$$= P\left(z = \frac{20-23}{\sqrt{3}}\right)$$

$$= P(z = -\sqrt{3})$$

$$= P(z = -1.73)$$

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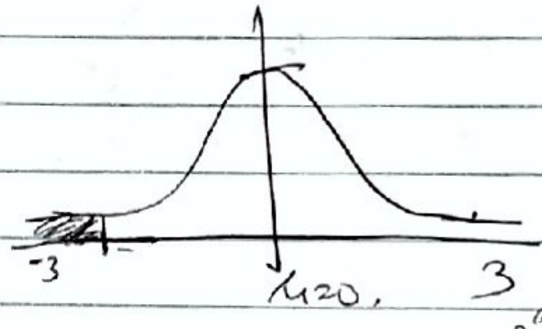
$$* \; P(X \le 20)$$

$$= P\left(Z \le \frac{20-23}{\sqrt{3}}\right)$$

$$= \phi(-1.73)$$

$$=$$

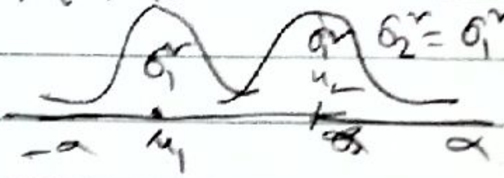


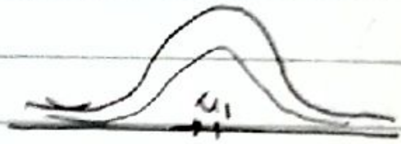
$$\phi\left(\frac{x_T - 23}{3}\right) = .5$$

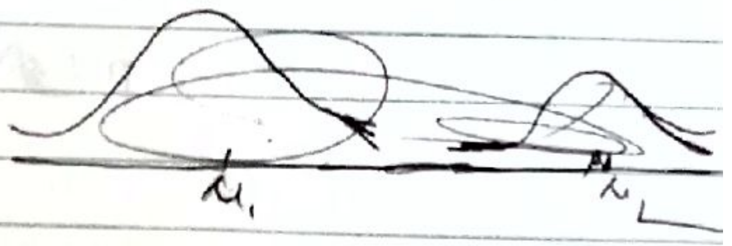
$$\Rightarrow \frac{x_T - 23}{3} = \phi^{-1}(.5)$$

$N(\mu_1, \sigma_1^2)$ $N(\mu_2\, \sigma_1^2)$

$\sigma_2^2 = \sigma_1^2$

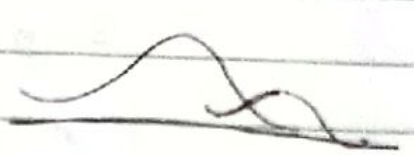


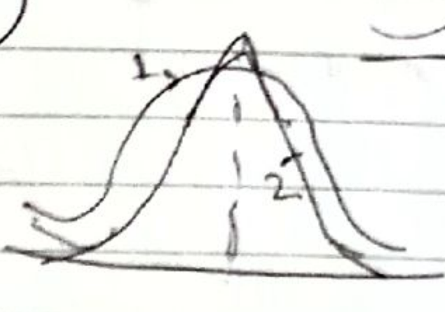




$N(\mu, \sigma_1^2)$

$N(\mu_1, \sigma_2^2)$





variance= mean [illegible]

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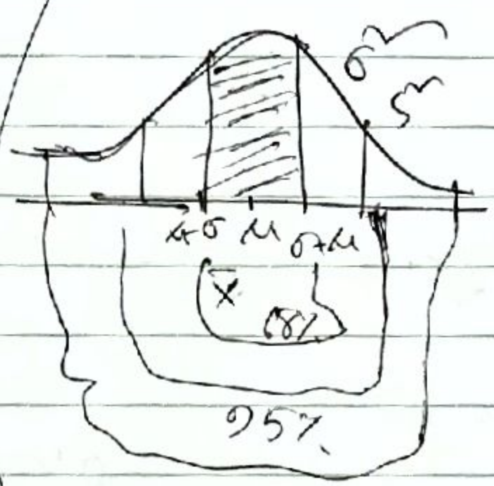
Ex' I $X \text{ (marks)} \sim N(12, 4)$



$\mu \pm 1\sigma$ $(\mu-\sigma, \mu+\sigma) = 68\%$

$\mu \pm 2\sigma$ $= 95\%$

$\mu \pm 3\sigma$ $= 99\%$

$\boxed{N(0,1)}$

$(-1, 1) = 68\%$

$(-2, 2) = 95\%$

$(3, 3) = 99\%$

$P(X \le 9) = ?$

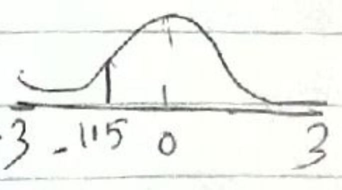
$P(Z) = \frac{9-12}{}$

$= P\left(Z < \frac{9-12}{\sqrt{4}}\right)$

$= P\left(Z < \frac{-3}{2}\right)$

$= P(Z \le -1.5)$

$= \varphi(-1.5)$



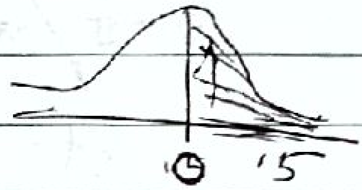
=

(*) $P(X \geq 12) = 1 - P(X \leq 12)$

$= 1 - P\left(Z \leq \frac{12-12}{\sqrt{4}}\right)$

$= 1 - P(Z \leq 0)$

$= 1 - \phi(0)$



$= 1 - .5$

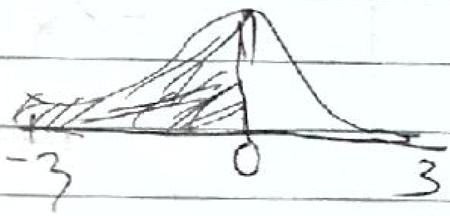
$= .5$.

(d) $P(6 \leq X \leq 12) = ?$

$P\left(\frac{6-12}{\sqrt{4}} \leq Z \leq \frac{12-12}{\sqrt{4}}\right)$

$= P(-3 \leq Z \leq 0)$

$= \phi(0) - \phi(-3)$



$= .5 - 0$

$= .5$

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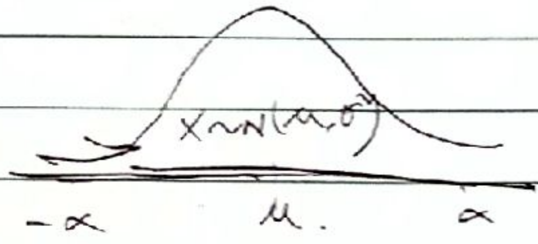
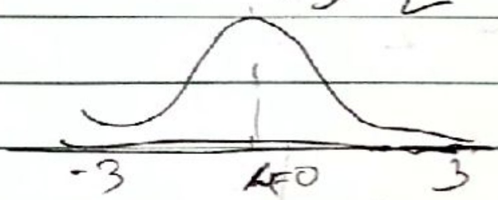


$X \sim N(\mu, \sigma^2)$

$Z = \frac{X-\mu}{\sigma}$

$Z \sim N(0, 1)$

$\phi(0) = 1/2$ $\phi(z) = P(Z \le z)$ = cdf of Z.



$\phi(3) = 1$

$\phi(-3) = 0$.

$X \sim (12, 4)$

$P(X \le 8)$

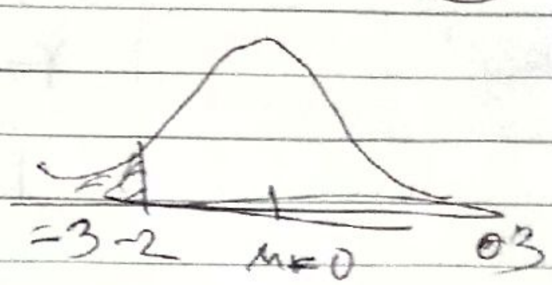
$= P\left(\frac{X-12}{2} \le \frac{8-12}{2}\right)$

$= P(Z \le -2)$ $\frac{.025}{5} = (\cdot) x (.5)$

$= P(Z \le -2)$

$= \phi(-2)$

$=$



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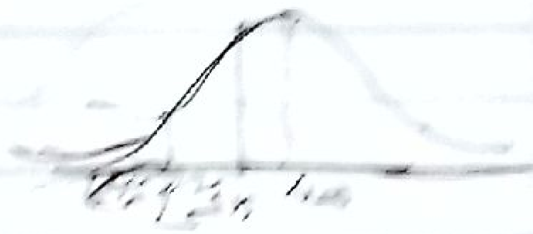
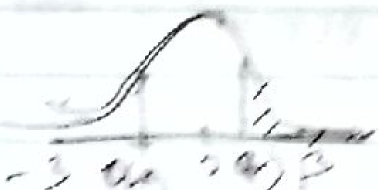
$P(8 \le X \le 15)$

$\left(\frac{8-12}{2} \le z \le \frac{15-12}{2}\right)$



$= \Phi(1.5) - \Phi(-2)$

$= (\ )$



$\to 1-\Phi(x)$

$1-\Phi(x) = \Phi(-x)$

$\Rightarrow \Phi(-x) = 1-\Phi(x)$

$\Phi(x) + \Phi(-x) = 1$

52

$X_1 \sim N_1(\mu_1, \sigma_1^2)$

$X_2 \sim N_2(\mu_2, \sigma_2^2)$

$Y = X_1 + X_2$, $Y \sim N(\mu_1+\mu_2, \sigma_1^2+\sigma_2^2)$

$E(Y) = E(X_1) + E(X_2)$
$= \mu_1 + \mu_2$

$V(Y) = V(X_1) + V(X_2)$
$= \sigma_1^2 + \sigma_2^2$

$Y = X_1 - X_2$

$Y \sim N(\mu_1 - \mu_2, \sigma_1^2 + \sigma_2^2)$

10.5

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$X_1 \sim N(12, 4)$

$X_2 \sim N(12, 9)$

$V(cX) = c^2 V(X)$

$V(-X) = (-1)^2 V(X)$

$= V(X)$

$Y = X_1 - X_2 \sim N(0, 13)$

$E(Y) = 0$

$V(X) = 13.$

$$P(Y \leq 3) = P\left(\frac{Y-0}{\sqrt{13}} \leq \frac{3-0}{\sqrt{13}}\right)$$

$$= P\left(Z \leq \frac{3}{\sqrt{13}}\right)$$

$$= \phi(.83)$$

$=$

5.3 CLT (Central Limit theoram)

$X_1 \ldots X_n \sim ?(\mu, \sigma^2)$

$\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$.

5.3:- Approximating Dist with the normal Distn.

~~5.3~~:- $x_1, x_2 \cdots x_n \sim f(\mu, \sigma^2)$

$$\bar{x} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$$

$$X \sim B(n, p)$$

$$n \to \infty / \text{large.}$$

$$p \to 1 \text{ or } 0$$

$$X \sim N(np, np(1-p))$$

$$P(X \le x_0) = P(X \le x_0 + .5)$$

Ex:- $n = 100$

$p = .85$ || $\mu = np = 85$ $\sigma^2 = np(1-p) = 12.75$

$P(X \geq 50) = ?$

Since $n \to \infty$, $p \to 1$, with use normal approximation

$$P(X \geq 50) = 1 - P(X < 50)$$

$$= 1 - P(X < 50 - .5)$$

$$= 1 - P(X < 49.5)$$

$$= 1 - P\left(\frac{X - 85}{\sqrt{12.75}} < \frac{49.5 - 85}{\sqrt{12.75}}\right)$$

$$= 1 - P(Z < \quad)$$

$$= 1 - \phi(-8)$$

$$= 1 - 0$$

$$= 1.$$