1. $\int_0^1 \cot^{-1} x \, dx$

Let, $u = \cot^{-1} x$ $\qquad$ $dv = dx$

$\Rightarrow du = -\frac{1}{1+x^2} dx$ $\qquad$ $\therefore v = x$

Now,

$$\int_0^1 \cot^{-1} x \, dx = \int_0^1 u \, dv = uv\Big]_0^1 - \int_0^1 v \, du$$

$$= x \cot^{-1} x\Big]_0^1 - \int_0^1 x \frac{-1}{1+x^2} dx$$

But, $\int_0^1 x \cdot \frac{-1}{1+x^2} dx = -\frac{1}{2}\int_0^1 \frac{2x}{1+x^2} dx = -\frac{1}{2} \ln(1+x^2)\Big]_0^1$

$$= -\frac{1}{2} \ln 2$$

So, $\int_0^1 \cot^{-1} x \, dx = x \cot^{-1} x\Big]_0^1 - \left(-\frac{1}{2} \ln 2\right)$

$$= x \cot^{-1} x\Big]_0^1 + \frac{1}{2} \ln 2$$

$$= \left(\frac{\pi}{4} - 0\right) + \frac{1}{2} \ln 2$$

2. $\int \sin^3 x \cos^9 x$

$= \int \sin^2 x \cos^9 x \sin x \, dx$

$= \int (1 - \cos^2 x) \cos^9 x \sin x \, dx$

Let, $\cos x = u$

$-dx \sin x = du$

$dx \sin x = -du$

$= -\int (1-u^2) u^9 \, dx$

$= -\int u^9 - u^{11} \, dx$

$= -\frac{u^{10}}{10} + \frac{u^{12}}{12}$

$= -\frac{\cos^{10} x}{10} + \frac{\cos^{12} x}{12} + c$

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3. $\int e^x \cos(2x)\,dx$

$= \cos(2x) \cdot e^x - \int e^x (-2\sin 2x)\,dx$

Let,
$u = \cos 2x$
$du = -\sin 2x . 2dx$
$dv = e^x$
$v = e^x$

$= e^x \cos 2x + 2\int e^x \sin 2x\,dx$

$= e^x \cos 2x + 2\left[\sin 2x \cdot e^x - \int e^x\, 2 . \cos 2x\,dx\right]$

Let
$u = \sin 2x$
$du = 2\cos 2x\,dx$
$dv = e^x$
$v = e^x$

$= e^x \cos 2x + 2\sin 2x\, e^x - 4\int e^x . \cos 2x\,dx$

$\therefore I = e^x \cos 2x + 2e^x \sin 2x - 4I$

Let,
$\int e^x \cos 2x = I$

$\Rightarrow 5I = e^x \cos 2x + 2e^x \sin 2x$

$I = \frac{1}{5}\left(e^x \cos 2x + 2e^x \sin 2x\right)$

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