

$$\lim_{x \rightarrow a} (-x) = -a \quad \lim_{x \rightarrow a} f(x) = \lim_{x \rightarrow a} x = a$$



$$\lim_{x \rightarrow 0} f(x) = ??$$

$$\begin{aligned} \lim_{x \rightarrow 1^+} \frac{x^4 - 1}{x - 1} &= \lim_{x \rightarrow 1^+} \frac{(x^2 + 1)(x^2 - 1)}{x - 1} \\ &= \lim_{x \rightarrow 1^+} \frac{(x^2 + 1)(x + 1)(x - 1)}{x - 1} = \lim_{x \rightarrow 1^+} (x^2 + 1)(x + 1) \\ &= \lim_{x \rightarrow 1^+} (x^2 + 1) \lim_{x \rightarrow 1^+} (x + 1) \\ &= (1 + 1)(1 + 1) = 4 \end{aligned}$$

$$\lim_{x \rightarrow 1^+} \frac{x^4 - 1}{x - 1} = 4$$

Find $\lim_{x \rightarrow 0} f(x)$ and $\lim_{x \rightarrow 3} f(x)$

$$f(x) = \begin{cases} x - 1; & x \leq 3 \\ 3x - 7; & x > 3 \end{cases}$$

$$\begin{aligned} \textcircled{1} \lim_{x \rightarrow 0^+} f(x) &= \lim_{x \rightarrow 0^+} (x - 1) = -1 \\ \lim_{x \rightarrow 0^-} f(x) &= \lim_{x \rightarrow 0^-} (x - 1) = -1 \end{aligned}$$

$$\text{so, } \lim_{x \rightarrow 0} f(x) = -1$$

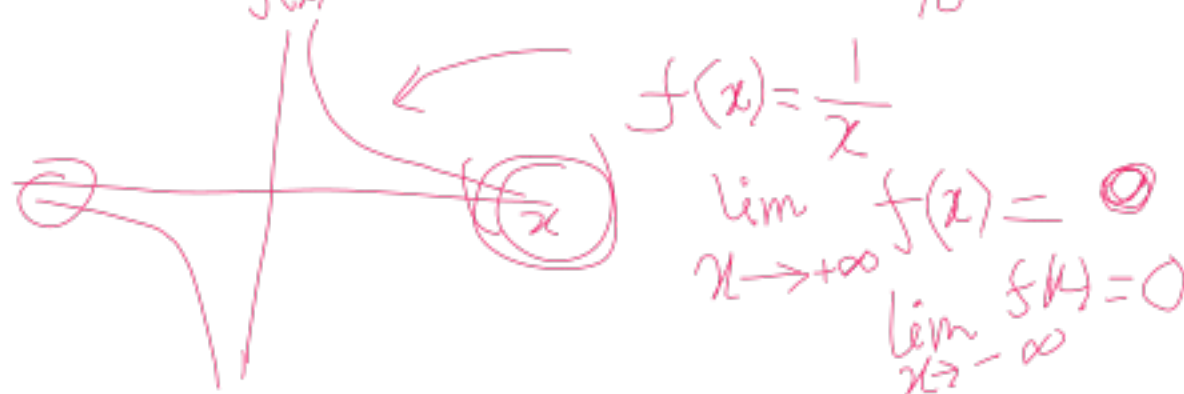
$$\begin{aligned} \lim_{x \rightarrow 3^+} f(x) &= \lim_{x \rightarrow 3^+} (3x - 7) = 3 \cdot 3 - 7 = 2 \\ \lim_{x \rightarrow 3^-} f(x) &= \lim_{x \rightarrow 3^-} (x - 1) = 3 - 1 = 2 \end{aligned}$$

$$\text{so, } \lim_{x \rightarrow 3} f(x) = 2$$

$$\begin{aligned} \frac{1}{x} \lim_{x \rightarrow 0} \frac{\sqrt{x+4} - 2}{x} &= \frac{1}{4} \\ &= \lim_{x \rightarrow 0} \frac{(\sqrt{x+4} - 2)(\sqrt{x+4} + 2)}{x(\sqrt{x+4} + 2)} \\ &= \lim_{x \rightarrow 0} \frac{(\sqrt{x+4})^2 - 2^2}{x(\sqrt{x+4} + 2)} \\ &= \lim_{x \rightarrow 0} \frac{x + 4 - 4}{x(\sqrt{x+4} + 2)} = \lim_{x \rightarrow 0} \frac{1}{\sqrt{x+4} + 2} \\ &= \frac{1}{\sqrt{0+4} + 2} = \frac{1}{4} \end{aligned}$$

$$\begin{aligned} \lim_{t \rightarrow 2} \frac{t^3 + 3t^2 - 12t + 4}{t^3 - 4t} &= \lim_{t \rightarrow 2} \frac{t^3 + 2t^2 + t + 2t - 14t - 28 + 4}{t(t^2 - 4)} \\ &= \lim_{t \rightarrow 2} \frac{t^3 - 2t^2 + 5t - 2}{t(t^2 - 4)} \\ &= \lim_{t \rightarrow 2} \frac{t^2(t-2) + 5t(t-2) - 2(t-2)}{t(t+2)(t-2)} \\ &= \lim_{t \rightarrow 2} \frac{(t-2)(t^2 + 5t - 2)}{t(t+2)(t-2)} \\ &= \lim_{t \rightarrow 2} \frac{t^2 + 5t - 2}{t(t+2)} = \frac{4 + 10 - 2}{2(2+2)} = \frac{12}{8} = \frac{3}{2} \end{aligned}$$

$$\lim_{x \rightarrow \infty} f(x) = ?? \quad \lim_{x \rightarrow -\infty} f(x) = ?$$



$$\begin{aligned} f(x) &= \frac{1}{x} \\ \lim_{x \rightarrow \infty} f(x) &= 0 \\ \lim_{x \rightarrow -\infty} f(x) &= 0 \end{aligned}$$