Department of Electrical & Computer Engineering

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Bonus Assignment

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Course: Theory of Electromagnetics (EEE361)

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Bonus Assignment :

Why the electric field in one frame (with no magnetic field) of reference can be observed as an altered electric field & magnetic field in another frame of reference?

Answer:

Electric & magnetic field, both follows Maxwell's equation. From Einstein's Theory of Special Relativity, we get to know how space & time changes on basis of inertial reference frame.

It is seen that, special relativity is contained in Maxwell's equation. Indeed, Einstein discovered special relativity by observing & understanding Maxwell's equation. Thus, using maxwell's equation in relativistic form, how electric & magnetic fields transform from one reference frame to another.

For example, if I measure an electric field in a room as a motionless observer, then, through the realivistic frame transformation, I can figure out what the fields will look to another observer who is ~~fixed~~ running around me in a circle.

At first, special relativity says that, all inertial reference frames are ~~equa~~ valid & fundamental. For example, let there is a moving train with ~~two~~ ~~ob~~ a passenger & there are two observer; one is inside the train & other is outside. Now, for the observer who is insidethe train ~~can~~ states that, passenger in the train is not moving, he's standstill. But for the observer who is outside the train will state passenger is moving ~~as~~ as distance between them is changing. The fact is two observer, see the situation differently. It means, the situation is measured from two different reference frames. Thus, ~~there are~~ none of them are less fundamental. Therefore, as there exists an inertial reference frame in which a electric field exists without ~~an~~ magnetic field. ~~for this~~ & since every inertial frame is real & fundamental, this means that, in other reference frame it can be observed as an altered electric & magnetic field.

Secondly, as per electromagnetic relativistic frame transformation equation, we can obtain that, ~~there~~ is ~~no way to start with a~~ depending on what reference frame the observer is in, a particular electromagnetic field will look more electric & less magnetic, or more magnetic & less electric. But, they are both fundamental & both part of same unified entity.

Till now, ~~we have~~ ~~in~~ quantum effects has been ignored. ~~But~~, However, the accurate description of electromagnetic fields is currently not the original Maxwell's equation but, the quantum form of maxwell's equation. Since, these new ~~qua~~ form simply extends the old equation rather than replacing, thus, ~~we~~ it can be stated that the concepts are valid.

Now, ~~lets I will describe~~ here is a description of the fields transformation betwee inertial frames.

First, we would consider two frames. One frame is moving primarily relative to the frame at velocity V. Field that defines the prime frame are indicated as prime sign (') in equation & other ~~of~~ one would be in plain form. Field, component parallel to velocity will be denote as ~~(B)~~ (PL) & components perpendicular to V denote as (PP). Both in of these frame, E & B field are related through,

$$E_{PL}' = E_{PL} \quad , \quad B_{PL}' = B_{PL}$$

$$\& \quad E_{PP}' = \gamma \left(E_{PP} + V \times B \right).$$

$$B_{PP}' = \gamma \left(B_{PP} - \frac{1}{c^2} V \times E \right).$$

[As per, electromagnetic theory].

Here, $\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$; it known as Lorentz factor.

Alternatively,

$$E' = \gamma (E + v \times B) - (\gamma - 1)(E \cdot \hat{v})\hat{v}$$

$$\& \quad B' = \gamma\left(B - \frac{v \times E}{c^2}\right) - (\gamma - 1)(B \cdot \hat{v})\hat{v}$$

Here, $\hat{v}$ is the unit vector of velocity.

Here, $(E \cdot \hat{v}) \cdot \hat{v} = E_{PL}$ & $(B \cdot \hat{v})\hat{v} = B_{PL}$.

If, one of the field i zero in one reference frame, that does not mean it is will be zero in all other reference frame. Making the unprimed electric field zero in the transformation to the prime electric field, we can see, depending on the orientation of the magnetic field the prime observer might see an electric field even there is none in the other one. It is not like two completely different events are occuring in two frames rather, same sequence is described in different ways. To clear this paradox, we can here is a example:

This example is based on intermixing of electric & magnetic phenomena in different frames known as moving magnet & conductor problem.

If a conductor moves with a constant velocity throug a field of stationary magnet, eddy current is produced due to a magnetic force on the electrons in the conductor. But, in the rest frame of the conductor, the magnet will ~~be~~ move & conductor stationary. Classical electromagnetic theory says that, precisely the same microscopic eddy currents is produced, but they are due to an electric force

If a particle charge, q moves with velocity u in a frame S, then, Lorentz force in this frame is:

$$F = qE + qu \times B.$$

for S' frame, $F' = qE' + qu' \times B'$

If, both frame in aligned axes:

$$u'_x = \frac{u_x + V}{1 + (Vu_x)/c^2}$$

$$u'_y = \frac{\cancel{u_y/\gamma}}{1} \quad \frac{u_y/\gamma}{1 + (Vu_x)/c^2}$$

$$u'_z = \frac{u_z/\gamma}{1 + \frac{Vu_x}{c^2}}$$

for a special case $u = 0$:

the relative motion along x axis,

$$E'_x = E_x \qquad B'_x = B_x$$

$$E'_y = \gamma(E_y = vB_z) \qquad B'_y = \gamma\left(B_y + \frac{v}{c^2}E_z\right).$$

$$E'_z = \gamma(E_z + vB_y) \qquad B'_z = \gamma\left(B_z - \frac{v}{c^2}E_y\right)$$

This way of transformation can be made more compact through electromagnetic tensor, whic is a way to describe the electromagnetic field in spacetime.