PHY 107
Rolling

Mohammad Murshed
Department of Math and Physics

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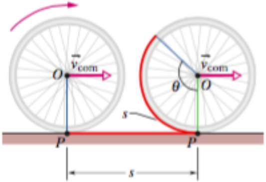
OUTLINE

- Rolling
- Torque
- Angular momentum

Rolling

Rolling: a combination of translation of the center of mass and rotation of the rest of the object around that center.

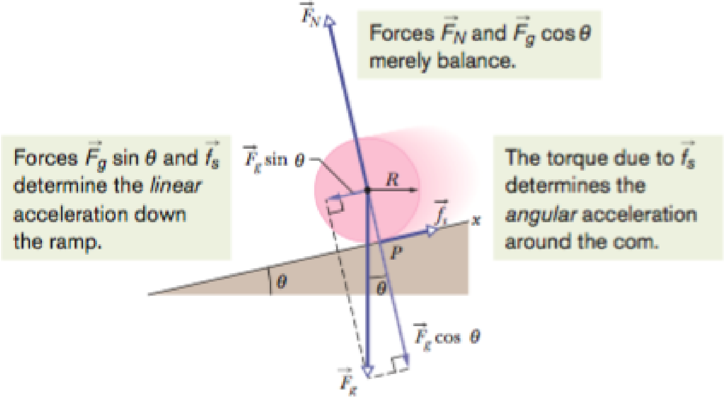


Kinetic Energy in rolling: A rolling object has two types of kinetic energy: a rotational kinetic energy $0.5 I_{com}\omega^2$ due to its rotation about its center of mass and a translational kinetic energy $0.5 M v_{com}^2$ due to translation of its center of mass.

Friction and Rolling: If the wheel does not slide, the force is a static frictional force f_s and the motion is smooth rolling.

Rolling down a ramp

We want to find an expression for the body's acceleration $a_{com,x}$ down the ramp.



$$a_{com,x} = -\frac{g\sin(\theta)}{1+I_{com}/MR^2}$$

Ball rolling down a ramp

EXAMPLE

A uniform ball, of mass M $= 6.00$ kg and radius R, rolls smoothly from rest down a ramp at angle $\theta = 30^\circ$

(a) The ball descends a vertical height h $= 1.20$ m to reach the bottom of the ramp.What is its speed at the bottom?

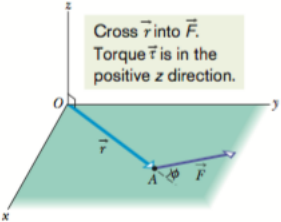
(b) What are the magnitude and direction of the frictional force on the ball as it rolls down the ramp?

Ball rolling down a ramp

Solution:
a)The frictional force on the ball from the ramp does not transfer any energy to thermal energy because the ball does not slide (it rolls smoothly).
$K_f + U_f = K_i + U_i$
$0.5 I_{com}\omega^2 + 0.5 M v_{com}^2 + 0 = 0 + Mgh$
$\omega = v_{com}/R$
$v_{com} = \sqrt{(\frac{10}{7}gh)}$
b) $a_{com,x} = -\frac{g sin(\theta)}{1+I_{com}/MR^2}$
$f_s = -I_{com}\frac{a_{com,x}}{R^2}$

Torque

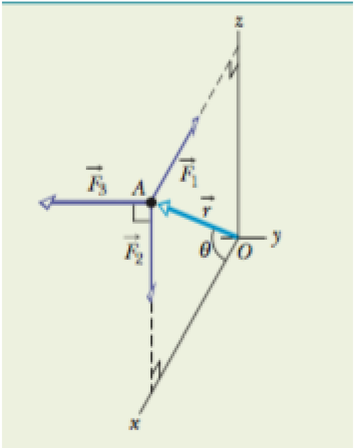
$$\vec{\tau} = \vec{r} X \vec{F}$$



$\tau = rFsin(\phi)$

EX: Three forces, each of magnitude 2.0 N, act on a particle. The particle is in the xz plane at point A given by position vector $\vec{r}$, where r $= 3.0$ m and $\theta = 30^\circ$. Force $\vec{F_1}$ is parallel to the x axis, force $\vec{F_2}$ is parallel to the z axis, and force $\vec{F_3}$ is parallel to the y axis. What is the torque, about the origin O, due to each force?

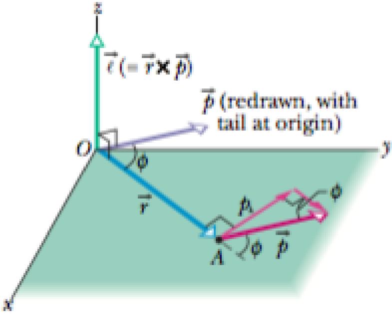
Torque



$$\tau_1 = rF_1 sin(\phi_1)$$
$$\tau_2 = rF_2 sin(\phi_2)$$
$$\tau_3 = rF_3 sin(\phi_3)$$

Angular momentum

A particle of mass m with linear momentum $\overrightarrow{p} = m\overrightarrow{v}$ as it passes through point A in an xy plane.



The angular momentum $\overrightarrow{l}$ of this particle with respect to the origin O is a vector quantity
$\overrightarrow{l} = \overrightarrow{r} X \overrightarrow{p} = m(\overrightarrow{r} X \overrightarrow{v})$

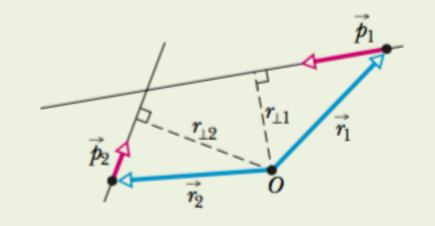
Angular momentum

$l = rmvsin(\phi)$
$l = rp_{\perp} = rmv_{\perp}$
$l = r_{\perp}p = r_{\perp}mv$
Two features:
(1) Angular momentum has meaning only with respect to a specified origin
(2) Its direction is always perpendicular to the plane formed by the position and linear momentum vectors $\overrightarrow{r}$ and $\overrightarrow{p}$

Angular Momentum

Angular momentum of a two particle system

Figure shows an overhead view of two particles moving at constant momentum along horizontal paths. Particle 1, with momentum magnitude $p_1 = 5.0kg\ m/s$, has position vector $\overrightarrow{r_1}$ and will pass 2.0 m from point O. Particle 2, with momentum magnitude $p_2 = 2.0kgm/s$, has position vector $\overrightarrow{r_2}$ and will pass 4.0 m from point O. What are the magnitude and direction of the net angular momentum $\overrightarrow{L}$ about point O of the two - particle system?



Solution

$l_1 = r_{\perp 1} p_1 = 2(5) = 10 kg\ m^2/s$

$l_2 = r_{\perp 2} p_2 = 4(2) = 8 kg\ m^2/s$

Taking direction into account...., the net angular momentum:

$L = l_1 + l_2 = 10 - 8 = 2 kg\ m^2/s$

Problems of importance

Reference (Extended 9th edition)...
The forces of rolling: 3
Torque Revisited: 19
Angular Momentum: 26

Reference

Fundamentals of Physics by Halliday and Resnik