PHY 107
Potential Energy and Conservation of Energy

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OUTLINE

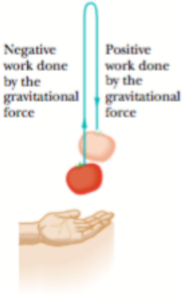
- Work and Potential Energy
- Path Independence of Conservative Forces
- Determining Potential Energy values
- Conservation of Mechanical Energy
- Reading a Potential Energy curve
- Work Done on a system by an external force

Intro

The force between the objects is the gravitational force. The configuration of the system changes (the separation between the jumper and Earth decreases – that is, of course, the thrill of the jump). We can account for the jumper's motion and increase in kinetic energy by defining a gravitational potential energy U.

This is the energy associated with the state of separation between two objects that attract each other by the gravitational force, The configuration of the system changes (the cord stretches). We can account for the jumper's decrease in kinetic energy and the cord's increase in length by defining an elastic potential energy U.

Work and Potential Energy



A tomato is thrown upward. As it rises, the gravitational force does negative work on it, decreasing its kinetic energy. As the tomato descends, the gravitational force does positive work on it, increasing its kinetic energy.
The change ΔU in gravitational potential energy is defined as being equal to the negative of the work done on the tomato by the gravitational force.
$\Delta U = -W$

Work and Potential Energy

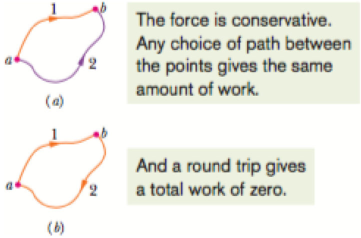
Conservative and Non-conservative forces

1. The system consists of two or more objects
2. A force acts between a particle like object in the system and the rest of the system

Conservative force : Gravitational force and the spring force
Non-conservative force : Frictional force and drag force
We know from experiment that this energy transfer cannot be reversed (thermal energy cannot be transferred back to kinetic energy of the block by the kinetic frictional force).

Path Independence of Conservative Forces

The net work done by a conservative force on a particle moving around any closed path is zero.
Think about the tomato....
The work done by a conservative force on a particle moving between two points does not depend on the path taken by the particle.



This result is powerful because it allows us to simplify difficult problems when only a conservative force is involved.

Determining Potential energy values

The value of the two types of potential energy discussed in this chapter: gravitational potential energy and elastic potential energy.
Let's find a general relation between a conservative force and the associated potential energy.
$\Delta U = -\int_{x_i}^{x_f} F(x)dx$
Gravitational Potential Energy : $U - U_i = mg(y - y_i)$
Elastic Potential Energy: $U(x) = 0.5kx^2$

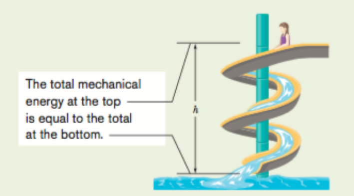
Conservation of Mechanical Energy

The mechanical energy E_{mec} is the sum of its potential energy U and kinetic energy K of the object within it.
We examine what happens to this mechanical energy when only conservative forces cause energy transfers within the system
The system is assumed to be isolated
Under such assumptions: $K_2 + U_2 = K_1 + U_1$
Principle of conservation of mechanical energy: $\Delta K + \Delta U = 0$

Conservation of Mechanical Energy

Example Water slide

A child of mass m is released from rest at the top of a water slide, at height h = 8.5 m above the bottom of the slide. Assuming that the slide is frictionless because of the water on it, find the child's speed at the bottom of the slide.



$E_{mec,b} = E_{mec,t}$

$K_b + U_b = K_t + U_t$

$0.5mv_b^2 + mgy_b = 0.5mv_t^2 + mgy_t$

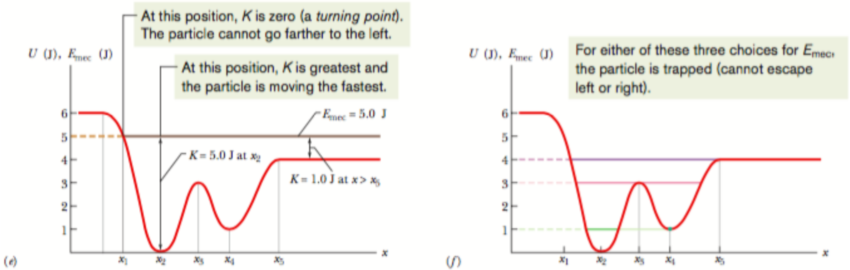
$\underline{v_b = 13m/s}$ This is the same speed that the child would reach if she fell 8.5 m vertically. On an actual slide, some frictional forces would act and the child would not be moving quite so fast.

Reading a potential energy curve

Assume a conservative force acting on a particle moving along an x axis

What can we learn about the motion of the particle from a plot of system's mechanical energy?

Finding the force analytically: $F(x) = -\frac{dU(x)}{dx}$



Note the equilibrium points!

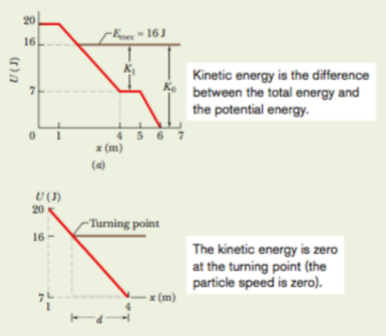
Reading a potential energy curve

A 2.00 kg particle moves along an x axis in one-dimensional motion while a conservative force along that axis acts on it. That is, if the particle were placed at any position between x=0 and x= 7.00 m, it would have the plotted value of U. At x = 6.5 m, the particle has velocity $v_0 = (-4.00 m/s)\hat{i}$

a) Determine the particle's speed at $x_1 = 4.5$m

b) Where is the particle's turning point located?

c) Evaluate the force acting on the particle when it is in the region $1.9m < x < 4m$.



Reading a potential energy curve

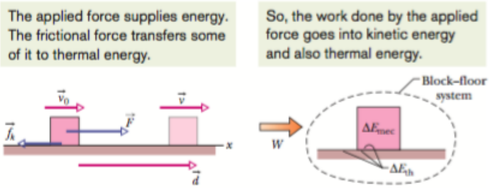
a)$K_1 = E_{mec} - U_1$
$K_1 = 0.5mv_1^2 \rightarrow v_1 = 3m/s$
b)The turning point is where the force momentarily stops and then reverses the particle's motion. That is, it is where the particle momentarily has $v = 0$ and thus $K = 0$.
$\frac{20-7}{1-4} = \frac{20-16}{1-x_t}$
c) $F(x) = -\frac{dU(x)}{dx}$
$F = -\frac{20-7}{1-4} = 4.3N$

Work Done on a system by an external force

Work is energy transferred to or from a system by means of an external force acting on that system.

No friction involved: $W = \Delta K + \Delta U = \Delta E_{mec}$ (work done on system, no friction involved)

Friction involved



$F - f_k = ma$; $v^2 = v_0^2 + 2ad$

$Fd = 0.5mv^2 - 0.5mv_0^2 + f_k d = \Delta K + f_k d$

Here, the thermal energy of the block and floor increases because (1) there is friction between them and (2) there is sliding: $\Delta E_{th} = f_k d$

$$W = \Delta E_{mec} + \Delta E_{th}$$

Work Done on a system by an external force

Example:

A food shipper pushes a wood crate of cabbage heads (total mass m = 14 kg) across a concrete floor with a constant horizontal force $\overrightarrow{F}$ of magnitude 40 N. In a straight-line displacement of magnitude d=0.50 m, the speed of the crate decreases from $v_0 = 0.60 m/s$ to $v = 0.20 m/s$.

a) How much work is done by force $\overrightarrow{F}$, and on what system does it do the work?

b) What is the increase ΔE_{th} in the thermal energy of the crate and floor?

Solution: a) $W = Fdcos(\phi)$=40(0.5) cos(0)=20J

The crate is slowing, so there must be friction and a change ΔE_{th} in thermal energy of the crate and the floor.

$E_{th} = W - (0.5mv^2 - 0.5mv_0^2) \approx 22J$

Reference

Fundamentals of Physics by Halliday and Resnik