PHY 107
Oscillation

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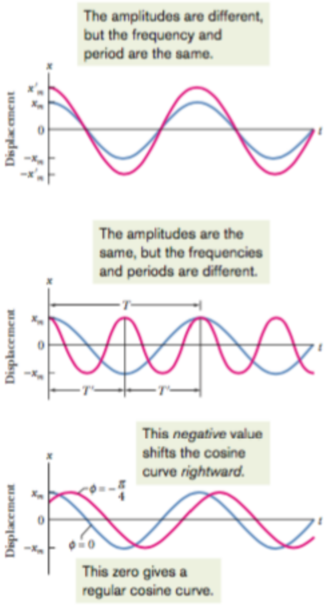
OUTLINE

- Simple Harmonic Motion
- Force Law
- Energy in SHM
- Simple Harmonic Motion and Uniform Circular Motion

Simple Harmonic Motion

Motion that repeats itself at regular intervals is called **periodic motion** or **harmonic motion**.
Displacement x of the particle as a function of time is given as
$x(t) = x_m cos(\omega t + \phi)(displacement)$
x: displacement at time t
x_m: amplitude
ω : angular frequency
t: time
ϕ: phase constant or phase angle


The amplitudes are different, but the frequency and period are the same.
Displacement
x
x'm
xm
0
−xm
−x'm
t
The amplitudes are the same, but the frequencies and periods are different.
Displacement
x
xm
0
−xm
t
T
T'
T'
This negative value shifts the cosine curve rightward.
Displacement
x
xm
0
−xm
t
φ = −π/4
φ = 0
This zero gives a regular cosine curve.

Velocity and Acceleration of SHM

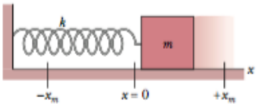
$v(t) = -\omega x_m sin(\omega t + \phi)$
$a(t) = -\omega^2 x_m cos(\omega t + \phi)$
$\rightarrow a(t) = -\omega^2 x(t)$
In SHM, the acceleration is proportional to the displacement but opposite in sign, and the two quantities are related by the square of the angular frequency.

Force Law for SHM

$F = ma = -(m\omega^2)x$
$F = -kx$
$k = m\omega^2$

Simple harmonic motion is the motion executed by a particle subject to a force that is proportional to the displacement of the particle but opposite in sign.

Linear Simple Harmonic Oscillator



$\omega = \sqrt{(\frac{k}{m})}$

Example

Finding SHM phase constant from displacement and velocity

At t $= 0$, the displacement x(0) of the block in a linear oscillator is -8.50 cm. The block's velocity v(0) then is $-0.920 m/s$, and its acceleration a(0) is $47.0 m/s^2$.

a) What is the angular frequency ω of this system?

b) What are the phase constant ϕ and amplitude x_m?

Solution: $\omega = \sqrt{-\frac{a(0)}{x(0)}}$

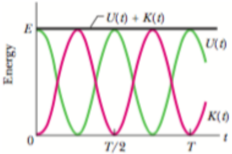
$\frac{v(0)}{x(0)} = -\omega tan(\phi)$

$\phi = -25^\circ$ or $180 + (-25) = 155^\circ$

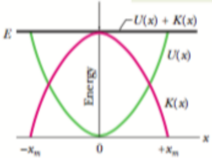
Insert these angles to check the positivity of the amplitude

Energy in SHM

$$U(t) = 0.5kx^2 = 0.5kx_m^2 cos^2(\omega t + \phi)$$
$$K(t) = 0.5mv^2 = 0.5m\omega^2 x_m^2 sin^2(\omega t + \phi)$$
$$E = U + K = 0.5kx_m^2$$



(a) As *time* changes, the energy shifts between the two types, but the total is constant.



(b) As *position* changes, the energy shifts between the two types, but the total is constant.

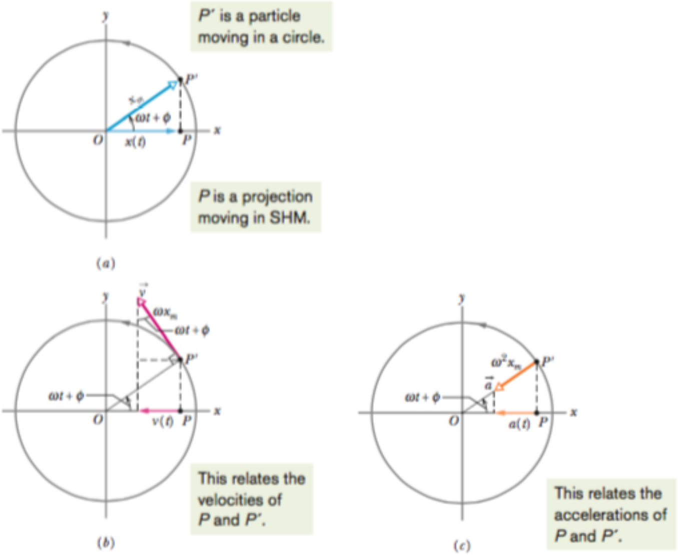
Example on Energy in SHM

EXAMPLE SHM potential energy, kinetic energy, mass dampers
Mass dampers are anti-sway devices to prevent buildings from oscillating in a wind. The device might be a block oscillating at the end of a spring and on a lubricated track. If the building sways, say, eastward, the block also moves eastward but delayed enough so that when it finally moves, the building is then moving back westward. Thus, the motion of the oscillator is out of step with the motion of the building.
Suppose the block has mass $m = 2.72\ x\ 105$ kg and is designed to oscillate at frequency f = 10.0 Hz and with amplitude $x_m = 20.0$ cm.
(a) What is the total mechanical energy E of the spring - block system?
(b) What is the block's speed as it passes through the equilibrium point?

Example

Solution:
a)$\omega = \sqrt{(k/m)} = 2\pi f$
$E = 0.5kx_m^2$
b)$U = 0 \ \rightarrow E = K$
This velocity is the maximum velocity.

SHM and Uniform Circular Motion



SHM is the projection of uniform circular motion on a diameter of the circle in which the circular motion occurs.

Problem of importance

The Force Law for Simple Harmonic Motion: 9 (a,b,c,d)

Reference

Fundamentals of Physics by Halliday and Resnik