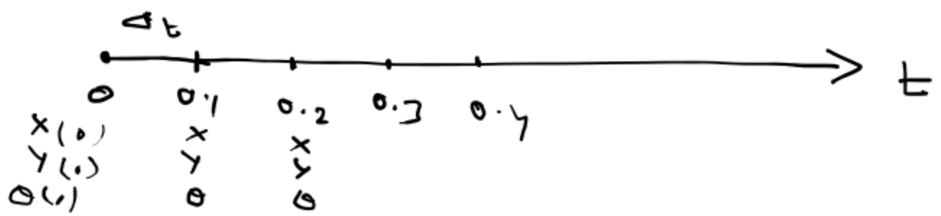
You can use different Δt's

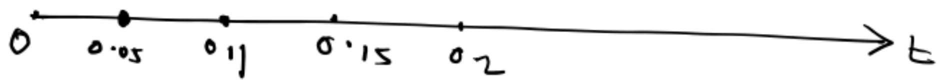
$\Delta t = 0.01$
$\Delta t = 0.2$
$\Delta t = 0.15$
$\Delta t = 0.05$

$0 \leq t \leq 10$

$\Delta t = 0.1$



$\Delta t = 0.05$



The smaller the time step, the larger the size of the vector

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Optimal control provides the best (i.e optimal) solution but these methods are usually computionally intensive. Requiring lots of processing, memory, time.

But many times "feasible" solutions are acceptable. My robot is a real-time system. So we cannot always wait for the optimal solution. "Feasible" solutions can be computed very quickly.

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Differential Flatness :

A method to compute "feasible" solutions for differentially flat systems.

Differentially Flat system

A system $\dot{x} = f(x, u, t)$ is differentially flat if there exists a function z such that the states & controls can be expressed as functions of z & z's derivatives.

SS

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If a system is differentially flat we can solve a matrix-vector system to calculate the trajectory parameters. (This will be home work 2)

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SS

Show that the simple car model is differentially flat

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{pmatrix} = \begin{pmatrix} v\cos\theta \\ v\sin\theta \\ \frac{v}{L}\tan\phi \end{pmatrix}$$

states $\rightarrow$ x, y, θ ✓

controls $\rightarrow$ v, ϕ

Let's pick $z = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$

So, $z_1 = x$ & $z_2 = y$ $\Rightarrow$ $\dot{x} = \dot{z}_1$, $\dot{y} = \dot{z}_2$

$\dot{x} = v\cos\theta$ --- (I)

$\dot{y} = v\sin\theta$ --- (II)

$$\boxed{\begin{aligned} x &= z_1 \\ y &= z_2 \end{aligned}}$$

$\frac{(II)}{(I)} \Rightarrow \frac{v\sin\theta}{v\cos\theta} = \frac{\dot{y}}{\dot{x}}$

$$y = z_2$$

$$\frac{(11)}{(1)} \Rightarrow \frac{V\sin\theta}{V\cos\theta} = \frac{\dot{y}}{\dot{x}}$$

$$\Rightarrow \tan\theta = \frac{\dot{z}_2}{\dot{z}_1} \Rightarrow \boxed{\theta = \tan^{-1}\left(\frac{\dot{z}_2}{\dot{z}_1}\right)}$$

We have shown the state variables are functions of z & it's derivatives.

$$\dot{\theta} = \frac{1}{\left(1+\frac{\dot{z}_2^2}{\dot{z}_1^2}\right)} \frac{d}{dt}\left(\frac{\dot{z}_2}{\dot{z}_1}\right)$$

$$\Rightarrow \boxed{\dot{\theta} = \frac{1}{\left(1+\frac{\dot{z}_2^2}{\dot{z}_1^2}\right)} \left[\frac{1}{\dot{z}_1}\ddot{z}_2 + \dot{z}_2(-1)\frac{1}{\dot{z}_1^2}\right]}$$

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$$\dot{x} = v\cos\theta$$

$$\Rightarrow \dot{z}_1 = v\cos\tan^{-1}\left(\frac{\dot{z}_2}{\dot{z}_1}\right)$$

$$\Rightarrow \boxed{v = \frac{\dot{z}_1}{\cos\tan^{-1}\left(\frac{\dot{z}_2}{\dot{z}_1}\right)}}$$

$$\dot{\theta} = \frac{v}{L}\tan\phi$$

$$\Rightarrow \tan\phi = \frac{\dot{\theta}L}{v}$$

$$\Rightarrow \phi = \tan^{-1}\left(\frac{\dot{\theta}L}{v}\right)$$

$$\dot{x} = v\cos\theta \quad \ldots (i)$$

$$\dot{y} = v\sin\theta \quad \text{--} (ii)$$

$$v^2\cos^2\theta + v^2\sin^2\theta = \dot{x}^2 + \dot{y}^2$$

$$\Rightarrow v^2 = \dot{x}^2 + \dot{y}^2$$

$$\Rightarrow v = \sqrt{\dot{x}^2 + \dot{y}^2}$$

$$\Rightarrow \boxed{v = \sqrt{\dot{z}_1^2 + \dot{z}_2^2}}$$

So

v, ϕ are functions of $z, \dot{z}, \ddot{z}$

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trajectory ⇒ path

If the system is DF

⇒ the trajectory can be written as a linear combination of <u>basis</u> functions

⇒ we can solve a matrix-vector system to calculate the trajectory.