Why are data collected?

What kind of data are collected?

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- Medical & pharmaceutical sciences...

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How to do it in a proper way?

Syllabus

□ Introduction & Descriptive Statistics	*chapter 6*
□ Introduction to Probability	*chapter 1*
□ Random Variables	*chapter 2*
□ Discrete and Continuous Probability Distributions	*chapters 3,4*
□ The Normal Probability Distribution	*chapter 5*
□ Sampling Distributions, Random Sample	*chapter 7*
□ Inferences on a Population Mean	*chapter 8*
□ Comparing Two Population Means	*chapter 9*
□ Simple Linear Regression and Correlation	*chapter 12*
□ Inferences on a Population Proportion	*chapter 10*
□ The Analysis of Variance	*chapter 11*

How to do it in a proper way?

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Probability $\overset{}{\underset{\text{vs.}}{\longleftrightarrow}}$ Statistics

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How to do it in a proper way?

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- □ Random mechanism $\rightarrow$ produces random outcomes;
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- □ **Probability** - theoretical background for a random mechanism;
  (describes how the mechanism works - but the principle is unknown)
- □ **Statistics** - uses random outcomes to draw conclusions;
  (using statistics we can very precisely describe the mechanism behind)

Statistics in Real Life...

- ❑ In probability, we could have a precise description of the random mechanism behind which would be accurate, but it is unknown!
- ❑ In statistics, we have data (measurements) that are quite often not precise, but the final conclusions drawn from such data are very accurate!

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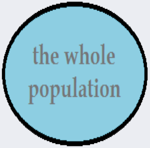
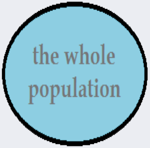
- "**I only believe in statistics that I doctored myself.**"
  Winston S. Churchill (1874 – 1965)

Population vs. Sample



Population vs. Sample

Probability Theory
Random Distribution



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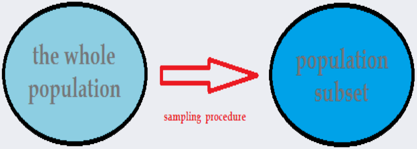

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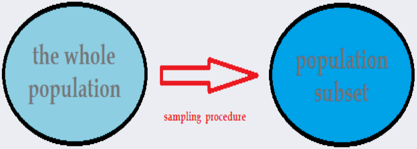
Empirical Distribution



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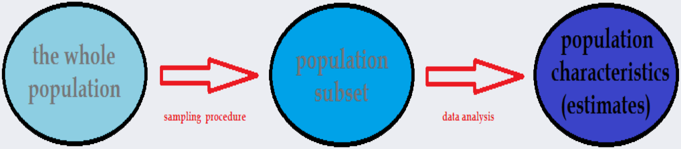

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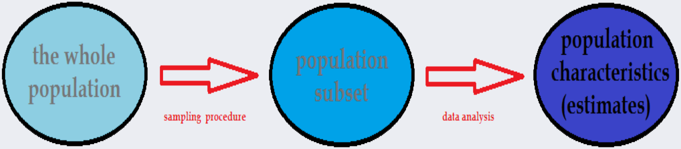
Sample Statistics



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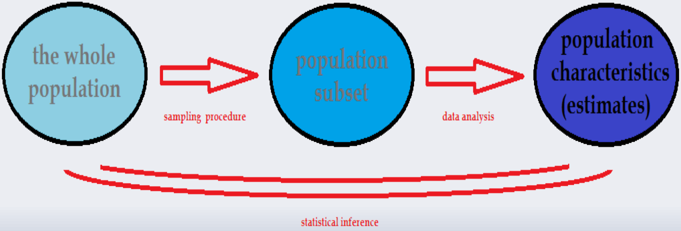
Statistical Inference



Different Data types

- **Categorical variables**

Different Data types

- ❑ **Categorical variables**

- ❑ **Numerical variables**

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  - Nominal categories
    - categories with no ordering;
      What kind of transportation do you use to get to work?
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  - Real values
    - any real value is possible...
      length, weight, height, temperature, etc.

Data exploration

- ❑ What is the nature of data?
- ❑ What are the limitations of the experiment behind?

- ❑ What is the main question of interest?
- ❑ Is it possible to use data available to answer it?

- ❑ Is it important to visualize the data?
- ❑ How can one get an insight in the data?

Binary Data

- the simplest data type...
- different coding possible...
  - logical: TRUE | FALSE; 1|0; YES|NO;
  - categorical: A|B; 1|2; HOME|ABROAD;
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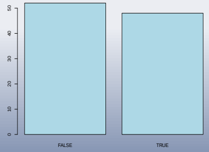

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- **Nominal categories...**
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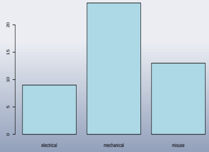

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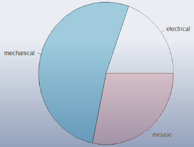

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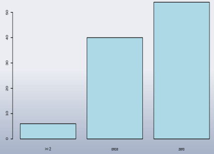

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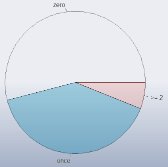

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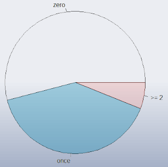

- What is the main difference between nominal and ordinal categories?

Numerical Data

- **Integer values - counts...**
  - How many hours can a machine work until it breaks down?

Numerical Data

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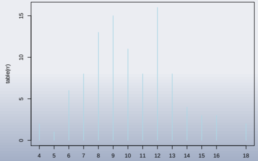

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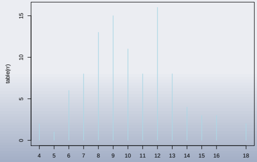

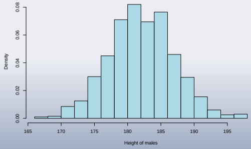

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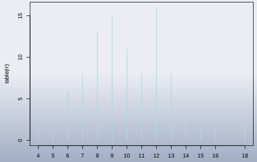

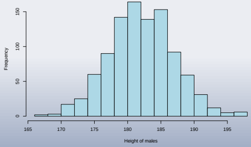

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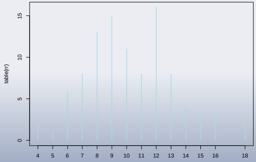

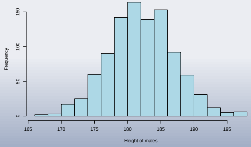

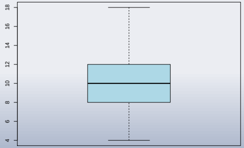

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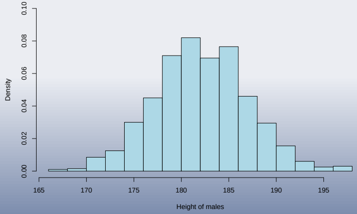

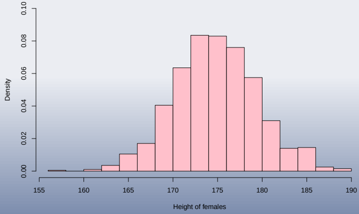

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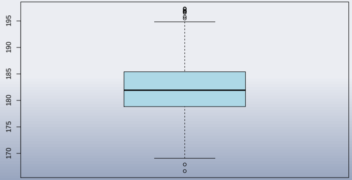

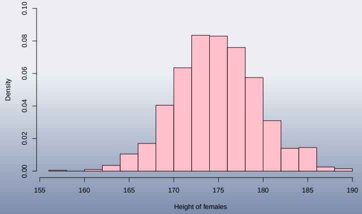

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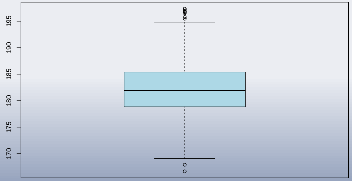

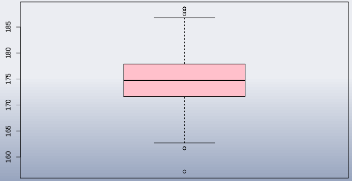

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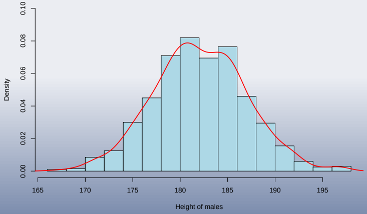

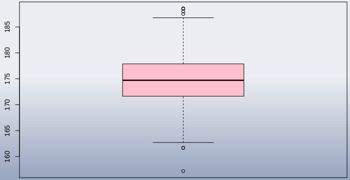

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- **Real values...**
  - We could improve a precision on the previous example... *data → positive real values...*
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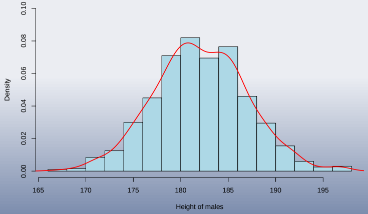

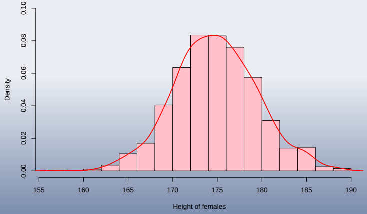

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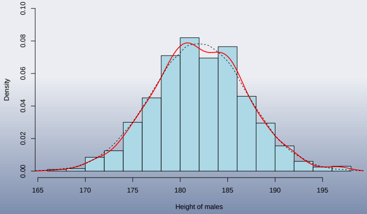

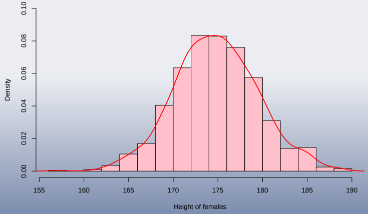

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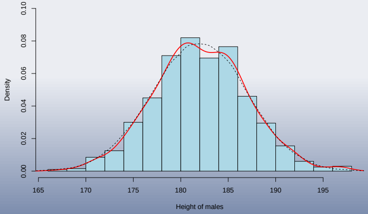

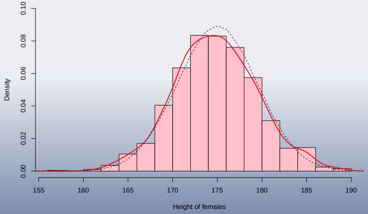

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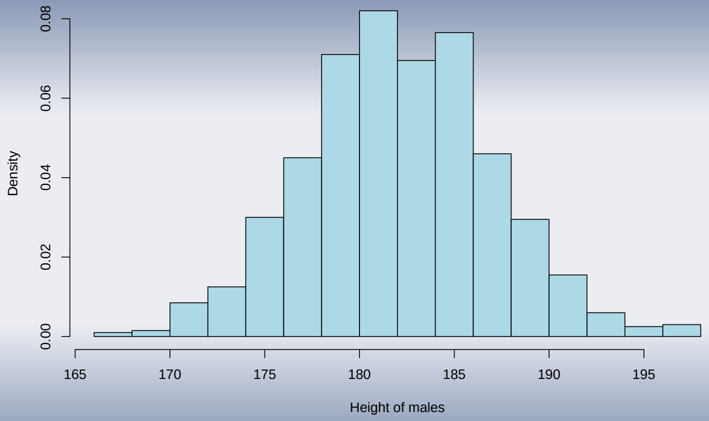
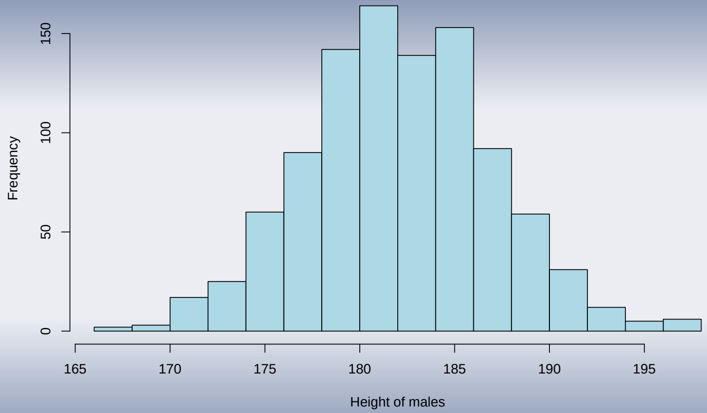

Density
0.00
0.02
0.04
0.06
0.08
165
170
175
180
185
190
195
Height of males

Histogram...


Frequency
0
50
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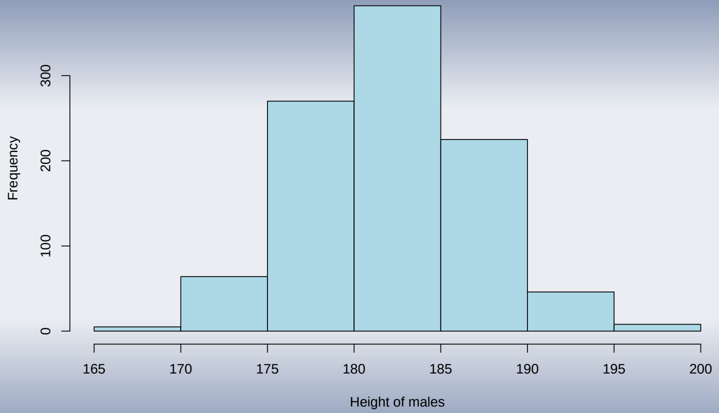
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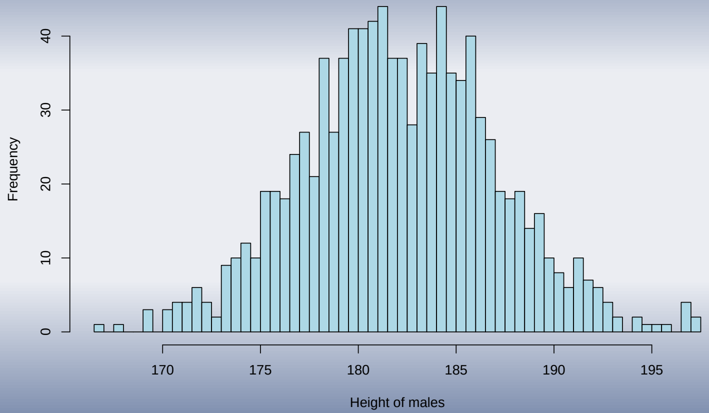

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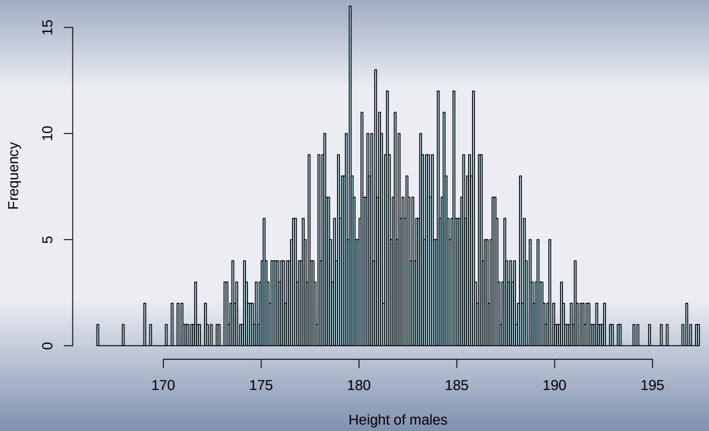
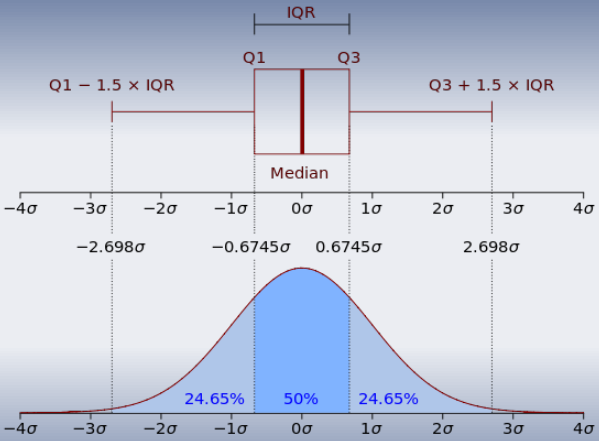


Boxplot figure...



Some nice properties

What can be directly observed from histogram (boxplot)?

❏ symmetric/non-symmetric data distribution;

❏ unimodal or multimodal data distribution;

❏ skewness of data distribution;

❏ indications of outlying observations;

❏ ...

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- ❑ ...

It is not so much important now when we only focus on data exploration part however, it will become really crucial when considering more sophisticated statistical modeling approaches.

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❏ Could we expect more reliability and confidence if more data is available?

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- ❏ The more data we obtain the more we have to pay for...
- ❏ How to find a reasonable balance?

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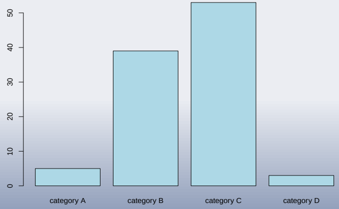

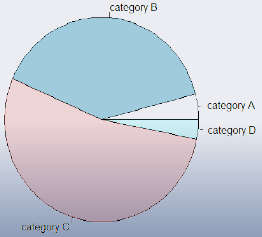

Higher dimensional data

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- more dimensional data $\Rightarrow$ it gets even difficult to plot;
- multivariate statistical methods were proposed to be used instead;

Titanic survivals

```
      Sex                                     Sex
Class  Male Female                  Class  Male Female
  1st     0      0                    1st     5      1
  2nd     0      0                    2nd    11     13
  3rd    35     17                    3rd    13     14
  Crew    0      0                    Crew    0      0

,, Age = Child, Survived = No       ,, Age = Child, Survived = Yes



      Sex                                     Sex
Class  Male Female                  Class  Male Female
  1st   118      4                    1st    57    140
  2nd   154     13                    2nd    14     80
  3rd   387     89                    3rd    75     76
  Crew  670      3                    Crew  192     20

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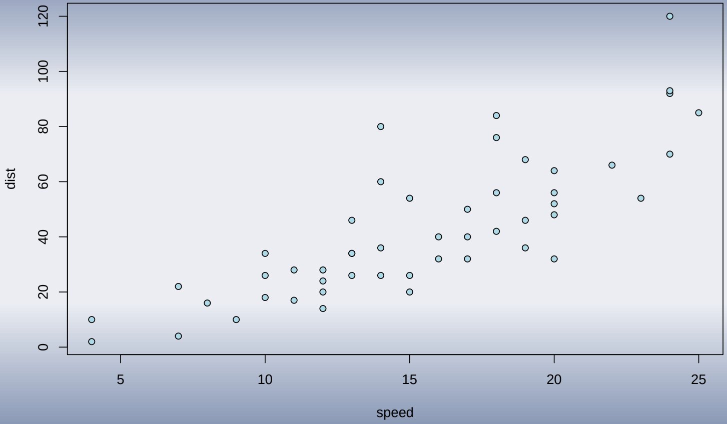
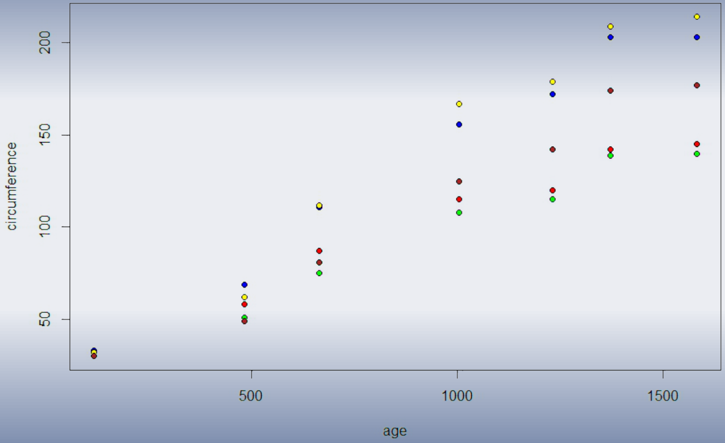
Car Breaking Distance



Orange Trees



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population $\Longrightarrow$ population sample $\Longrightarrow$ statistical inference

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- ❑ random sample → independent and identically distributed observations;

Sample Statistics

Two different worlds

...

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- ❑ Some other proposals: sample trimmed mean, weighted mean, etc.;

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- middle observation from all ordered observations;
- $\tilde{x}_n = x_{(\frac{n+1}{2})}$ for n even and $\tilde{x}_n = [x_{(n/2)} + x_{(\frac{n+1}{2})}]/2$ for n odd;

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- ❑ Some other proposals as well ...;

Some other sample characteristics

- **sample mode (for categorical variables mostly);**
  What is the relation between sample mean, median and mode?
- **sample variance & sample standard error;**
  mostly it is difficult to imagine → *sample range used "instead"*
- **sample quantiles (quartiles, percentiles, etc.);**
  some of them are more important than others...
- **coefficient of variation;**
  spread of data relative to its middle value

To be continued...

- Probability, probability concepts;
- Random events, combination of events;
- Conditional probability;
- Independence principle;
- Law of Total Probability;
- Bayes Theorem in Probability;