Course Name : Physics – I
Course # PHY 107

Notes-5 : Newton's laws - Part One

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Topics to be studied

- What is dynamics?
- Cause of motion – Concept of Force
- Newton's 1^{st} law and its equivalent statements
- Newton's 2^{nd} law
- Newton's 3^{rd} law
- Examples

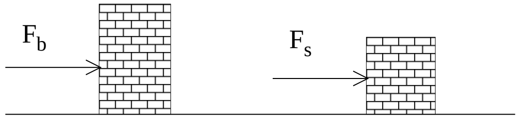
Dynamics:

- The main objective of dynamics is to explain the observed motion. Mainly how and why an object is accelerated or not?
- This is done in two steps:
  - Firstly, how to understand that there is an external cause acting on the object. The answer to this question is known as the Newton's 1^{st} law.
  - Secondly, if there is an external cause, how to quantitatively express the effect. The answer to this question is known as the Newton's 2^{nd} law.
- The 1^{st} law defines what is known as the 'Inertia' of an object.
- The 2^{nd} law provides a way to measure the amount of inertia.
- The 3^{rd} law explains how two objects interact with each other.

Cause of Motion: Force

- Let's consider some observation. An object is at rest implies that it's position does not change with respect to time (*i.e.* zero velocity). If we apply a 'Push', it's position changes. So it acquired the velocity. Here 'Push' is the cause of motion, and the change in velocity (from zero to some nonzero value) is the effect.
- The 'Push' is called the 'Force' and is denoted by $\vec{F}$ because it is a vector quantity (has both magnitude and direction).
- To understand the factor that determine the amount of change in velocity under a force, let's consider some phenomena.
- We apply the same Force on two objects that are of same material, but different size, to produce same change in velocity on both.
- We also apply Force on two different objects that are of equal size, but different material to produce the same change in velocity.

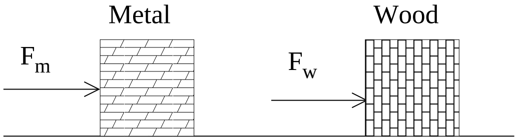
- Here we assume that the surface is 'perfectly smooth' (*i.e.* ideal slippery) so that it does not apply any 'influence' on the objects.
- Case-I: Force required to change velocity



The blocks are of same material, but different size

- Here it is clear from the observation that $\vec{F}_b > \vec{F}_s$.
- Bigger size object has more 'inertness', or simply it has more 'inertia'.

- Case-II: Force required to change velocity



The blocks are of same size, but different material.

- Here it is clear from the observation that $\vec{F}_m > \vec{F}_w$.
- Metallic (or heavier) object has more 'inertness', or simply it has more 'inertia'.

Newton's 1^{st} law:

- The previous two observations imply that 'inertia' is an inherent property of all objects. It does not depend on the size.
- Higher inertia requires more force to change the velocity.
- Because of the inertia, the object resist to change and hence requires the external 'Cause", called the Force, to change it's motion.
- 1^{st} law: If no force acts on an object, the velocity of the object cannot change; that is, the body cannot accelerate.
- In other words, if the object is at rest, it stays at rest. If it is moving, it continues to move with the same velocity (same magnitude and same direction).
- But what about if more than one force acts on an object? In this case, the net force needs to be considered, because a given object CAN NOT have different changes in velocities simultaneously. The object must respond to the net force.
- If no net force acts on an object ($\vec{F}_{\text{net}} = \sum \vec{F} = 0$), the object's velocity cannot change; that is, the body cannot accelerate.

- Finally, the Newton's 1^{st} law can be expressed as

$$\begin{array}{ll} 1) & \text{The object is in equilibrium.} \\ 2) & \vec{v} = \text{constant.} \\ 3) & \vec{a} = 0. \\ 4) & \sum \vec{F} \equiv \vec{F}_{\text{tot}} = 0. \end{array}$$

- It is very important to understand that these four statements are equivalent.
- This means that if any of these four is correct, then automatically all four statements are correct.
- In any frame where Newton's laws hold is called an 'Inertial Reference Frame'.
- A rotating frame is not an Inertial Reference Frame, because of rotation there is a sideway force due to rotation of the frame.
- This is why the Earth is not an Inertial Reference Frame in the strict sense.

Newton's 2nd law:

- Let's now understand how to measure the 'Inertia'.
- Suppose an object with fixed 'Inertia' is moving with a constant velocity $\vec{v}_i$. When a force $\vec{F}$ acts on it for a time interval of Δt. it's velocity changes to $\vec{v}_f$.
- So, the acceleration under this force is $\vec{a} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t} = \frac{\Delta \vec{v}}{\Delta t}$.
- It is found from observation that, if the applied force is increased, the observed acceleration also increases proportionally.
- That is, $\vec{a} \propto \vec{F}$.
- If more than one force is applied, then the acceleration to proportional to the net force, *i.e.*, $\vec{a} \propto \sum \vec{F}$.
- Therefore, mathematically, we can write, $\vec{a} = \frac{1}{m} \sum \vec{F}$. More rigorously this is written as,

$$\boxed{\sum \vec{F} = m\vec{a}}\,.$$

- The proportionality constant m is called the mass of the object, and it is the measure of the 'Inertia' of the object.
- The mass is a scalar quantity, and in SI unit it is measured in 'Kilogram'.
- The previous analysis also clearly indicates that any object must follow either the 1^{st} law or the 2^{nd} law.
- These two laws explain the phenomena what happens when a force acts on a single object. These does not explain the interaction phenomena.

Newton's 3^{rd} law

- This law explains the interaction between the objects.
- If two object's apply force on each other, the 3^{rd} law states that $\vec{F}_{12} = -\vec{F}_{21}$. That is, these two forces are equal in magnitude, but opposite in directions. Hence, $|\vec{F}_{12}| = |\vec{F}_{21}|$. These are called 'Action' and 'Reaction' forces.
- For every 'Action' (*i.e.* applied force), there exists a 'Reaction' (*i.e.* opposite force) such that their vector sum is always zero.
- Note that because of the the 3^{rd} law, we are able walk on the surface. When we try to walk, we apply a force backward (the 'Action'), and as a result a force by the floor is applied on us forward (the 'reaction'), and we move forward.

Problem # 5.11:

A 2.0 kg particle moving along the x-axis is acted on by a force. Because the force, it's position as a function of time changes as

$$x(t) = 3.0\,\mathrm{m} + (4.0\,\mathrm{m/s})t + ct^2 - (2.0\,\mathrm{m/s^3})t^3 ,$$

where c is a constant, and x is in meters and t is in seconds. At $t = 3.0\,\mathrm{sec}$, the force has a magnitude of $36.0\,\mathrm{N}$, and is in the negative direction of the axis. Find the constant c.

Solution: At $t = 3.0\,\mathrm{sec}$, the force is $\vec{F} = -36\,\mathrm{N}\,\hat{i}$. Now, the acceleration at time $t = 3.0\,\mathrm{sec}$ is

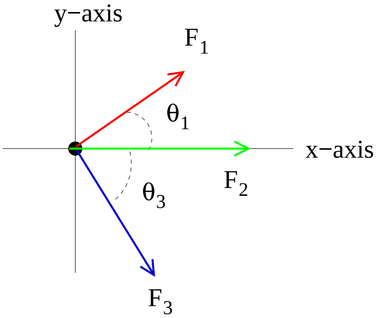
$$a = \left.\frac{d^2x}{dt^2}\right|_{t=3} = \Big(2c - 12t\Big)_{t=3} = 2c - 36\,\mathrm{m/s^2} .$$

Therefore, by the 2^{nd} law, we find at time $t = 3.0\,\mathrm{sec}$ that $\vec{F} = m\vec{a}$, and we get,

$$-36 = 2.0\big(2c - 36\big) \qquad \therefore \quad \boxed{c = 9.0\,\mathrm{m/s^2}}.$$

Problem # 5.5:

An asteroid of mass $m = 120\,\mathrm{kg}$ is guided by three forces: $\vec{F}_1 = (32\,\mathrm{N}\,,\,30^\circ)$, $\vec{F}_2 = (55\,\mathrm{N}\,,\,0^\circ)$ and $\vec{F}_3 = (41\,\mathrm{N}\,,\,-60^\circ)$. Find the acceleration in (i) unit vector notation and in (ii) polar form.



Solution: By Newton's 2$^{\text{nd}}$ law, we can write:
$\sum \vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 = m\vec{a}$. In Unit vector notation, we write,

$$
\begin{aligned}
\vec{a} &= \frac{1}{m}\left[\left(F_{1x} + F_{2x} + F_{3x}\right)\hat{i} + \left(F_{1y} + F_{2y} + F_{3y}\right)\hat{j}\right] \ , \\
&= \frac{1}{m}\left[\left(F_1 \cos\theta_1 + F_2 \cos\theta_2 + F_3 \cos\theta_3\right)\hat{i} + \left(F_1 \sin\theta_1 + F_2 \sin\theta_2 + F_3 \sin\theta_3\right)\hat{j}\right] \ , \\
&= \frac{1}{120}\Big[\left(32\cos 30^\circ + 55\cos 0^\circ + 41\cos 300^\circ\right)\hat{i} \\
&\qquad + \left(32\sin 30^\circ + 55\sin 0^\circ + 41\sin 300^\circ\right)\hat{j}\Big]\,\mathrm{m/s^2} \ , \\
\therefore\ \vec{a} &= \left(0.86\hat{i} - 0.16\hat{j}\right)\mathrm{m/s^2} = \left(0.87\,\mathrm{m/s^2}\,,\,349^\circ\right) .
\end{aligned}
$$