ASSIGNMENT-04

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Course : CSE231

Section : 10

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Date : 14-03-2021.

Minterm

* Otherwise known as a standard product.
* Possible AND combinations of n variables.
* Examples? For two variables a and b:

a	b	minterm	
0	0	$a'b'$	m_0
0	1	$a'b$	m_1
1	0	ab'	m_2
1	1	ab	m_3

Maxterm

* Otherwise known as a standard sum.
* Possible OR combinations of n variables.
* Examples? For two variables a and b:

a	b	Maxterm	
1	1	$a'+b'$	M_3
1	0	$a'+b$	M_2
0	1	$a+b'$	M_1
0	0	$a+b$	M_0

Minterms & Functions

* Given a truth table, a function can be put into algebraic form by OR-ing all of the minterms that have a 1 in the result column.

* Example ? $F = (ab') + (a'b)$

a	b	minterm		F
0	0	$a'b'$	m_0	0
0	1	$a'b$	m_1	1
1	0	ab'	m_2	1
1	1	ab	m_3	0

* ANY Boolean function can expressed this way.

Max Term & Functions

* Given a truth table, a function can be put into algebraic form by AND-ing all of the maxterms that have a 0 in the result column.

* Example ? $F = (a+b)(a'+b')$

a	b	minterm		maxterm		F
0	0	$a'b'$	m_0	$a+b$	M_0	0
0	1	$a'b$	m_1	$a+b'$	M_1	1
1	0	ab'	m_2	$a'+b$	M_2	1
1	1	ab	m_3	$a'+b'$	M_3	0

* ANY Boolean function can expressed this way.

Canonical form

+ Boolean functions expressed algebraically as either
- A sum of minterms
- A product of maxterms

Canonical Form → Complements

+ The Complement for Sum of Minterms is
-- the sum of minterms missing from the original function.
- Two-variable example (4 minterm)?

$$F = \sum(0,2) \quad \text{so} \quad F' = \sum(1,3)$$

+ Converting to a Product of Maxterms?
- Remember that any minterm is the complement of its corresponding maxterm (e.g. $m_0' = M_0$)
- So, using the example above:

$$F = \sum(0,2) \quad \text{so} \quad F' = \sum(1,3) \quad \text{so} \quad (F')' = \Pi(1,3)$$

Canonical Form - Examples

a	b	c	minterm		F
0	0	0	$a'b'c'$	m_0	0
0	0	1	$a'b'c$	m_1	1
0	1	0	$a'bc'$	m_2	0
0	1	1	$a'bc$	m_3	0
1	0	0	$ab'c'$	m_4	1
1	0	1	$ab'c$	m_5	1
1	1	0	abc'	m_6	0
1	1	1	abc	m_7	0

Sum of Minterms? $F = m_1 + m_4 + m_5 = \Sigma(1,4,5)$

Product of Maxterms? $F = M_0 \cdot M_2 \cdot M_3 \cdot M_6 \cdot M_7 = \Pi(0,2,3,6,7)$

Table 2.3

Minterms and Maxterms for Three Binary Variables.

			Minterms		Maxterms	
x	y	z	Term	Designation	Term	Designation
0	0	0	$x'y'z'$	m_0	$x+y+z$	M_0
0	0	1	$x'y'z$	m_1	$x+y+z'$	M_1
0	1	0	$x'yz'$	m_2	$x+y'+z$	M_2
0	1	1	$x'yz$	m_3	$x+y'+z'$	M_3
1	0	0	$xy'z'$	m_4	$x'+y+z$	M_4
1	0	1	$xy'z$	m_5	$x'+y+z'$	M_5
1	1	0	xyz'	m_6	$x'+y'+z$	M_6
1	1	1	xyz	m_7	$x'+y'+z'$	M_7

Table 2.4

Functions of Three Variables

x	y	z	Function f_1	Function f_2
0	0	0	0	0
0	0	1	1	0
0	1	0	0	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

$$f_1 = x'y'z + xy'z' + xyz. = m_1 + m_4 + m_7.$$

$$f'_1 = x'y'z' + x'yz' + x'yz + xy'z + xyz'$$

$$f_1 = (x+y+z)(x+y'+z)(x+y'+z')(x'+y+z')(x'+y'+z)$$

$$= M_0 . M_2 . M_3 . M_5 . M_6 .$$

$$f_2 = x'yz + xy'z + xyz' + xyz. = m_3 + m_5 + m_6 + m_7$$

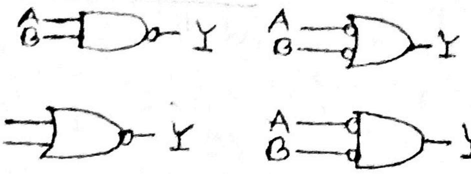
$$f_2 = (x+y+z)(x+y+z')(x+y'+z)(x'+y+z)$$

$$= M_0 M_1 M_2 M_4$$

De Morgan's Theorem

$Y = \overline{AB} = \bar{A} + \bar{B}$

$Y = \overline{A+B} = \bar{A} \cdot \bar{B}$



Complementing Functions

* Use DeMorgan's Theorem:
  (1) Interchange AND and OR operators
  (2) Complement each constant and literal

Example: Complement $F = \bar{x}y\bar{z} + x\bar{y}\bar{z}$

$\bar{F} = (x+\bar{y}+z)(\bar{x}+y+z)$

Example: Complement $G = (\bar{a}+bc)\bar{d} + e$

$\bar{G} = (a(\bar{b}+\bar{c})+d)\bar{e}$

Expression Simplification

* An application of Boolean algebra
* Simplify to contain the smallest number of literals (variables that may or may not be complemented).

$AB + \bar{A}CD + \bar{A}BD + \bar{A}C\bar{D} + ABCD$

$= AB + ABCD + \bar{A}CD + \bar{A}C\bar{D} + \bar{A}BD$

$= AB + AB(CD) + \bar{A}C(D+\bar{D}) + \bar{A}BD$

$= AB + \bar{A}C + \bar{A}BD = B(A+\bar{A}D) + \bar{A}C = B(A+D) + \bar{A}C$ (has only 5 literals)

Canonical Forms

- Minterms and Maxterms
- Sum of Minterm (SOM) Canonical form.
- Product of Maxterm (POM) Canonical form.
- Representation of Complements of Functions.
- Conversions between Representations.

Minterms

- Minterms are AND terms with every variable present in either true or complemented form.
- Given that each binary variable may appear normal (e.g., ...x) or complemented (e.g. ...$\bar{x}$), there are 2^n minterms for n variables.
- Example: Two variables (X and Y) produce $2 \times 2 = 4$ combinations:

XY (both normal)
$X\bar{Y}$ (X normal, Y complemented)
$\bar{X}Y$ (X complemented, Y normal)
$\bar{X}\bar{Y}$ (both complemented)

- Thus there are four minterms of two variables.

Maxterms

* Maxterms are OR terms with every variable in true or complemented form
* Given that each binary variable may appear normal (e.g., ... x) or complemented (e.g., ... $\bar{x}$), there are 2^n maxterms for n variables.

<u>Example</u>: Two variables (X and Y) produce $2 \times 2 = 4$ combinations:

$X+Y$ (both normal)

$X+\bar{Y}$ (x normal, y complemented)

$\bar{X}+Y$ (x complemented, y normal)

$\bar{X}+\bar{Y}$ (both complemented)

Minterms & Maxterms for 2 variables

* Two variable minterms and maxterms.

x	y	Index	Minterm	Maxterm
0	0	0	$m_0 = \bar{x}\bar{y}$	$M_0 = x+y$
0	1	1	$m_1 = \bar{x}y$	$M_1 = x+\bar{y}$
1	0	2	$m_2 = x\bar{y}$	$M_2 = \bar{x}+y$
1	1	3	$m_3 = xy$	$M_3 = \bar{x}+\bar{y}$

* The minterm m_1, should evaluate to 1 for each combination of x and y.
* The maxterm is the complement of the minterm.

Minterms & Maxterms for variables

x	y	z	Index	Mi term	Maxterm
0	0	0	0	$m0 = \bar{x}\bar{y}\bar{z}$	$M0 = x+y+z$
0	0	1	1	$m1 = \bar{x}\bar{y}z$	$M1 = x+y+\bar{z}$
0	1	0	2	$m2 = \bar{x}y\bar{z}$	$M2 = x+\bar{y}+z$
0	1	1	3	$m3 = \bar{x}yz$	$M3 = x+\bar{y}+\bar{z}$
1	0	0	4	$m4 = x\bar{y}\bar{z}$	$M4 = \bar{x}+y+z$
1	0	1	5	$m_5 = x\bar{y}z$	$M5 = \bar{x}+y+\bar{z}$
1	1	0	6	$m_6 = xy\bar{z}$	$M6 = \bar{x}+\bar{y}+z$
1	1	1	7	$m_7 = xyz$	$M7 = \bar{x}+\bar{y}+\bar{z}$

Maxterm M_i is the complement of minterm m_i.

$M_i = \overline{m_i}$ and $m_i = \overline{M_i}$.

Purpose of the Index

- Minterms and Maxterms are designated with an index.
- The index number corresponds to a binary pattern.
- The index for the minterm or maxterm, expressed as a binary number, is used to determine

whether the variable is shown in the true or complemented form.

* For Minterms:
- '1' means the variable is "Not Complemented" and
- '0' means the variable is "Complemented".

* For Maxterms:
- '0' means the variable is "NOT Complemented" and
- '1' means the variable is "Complemented".

Standard Order

* All variables should be present in minterm or maxterm and should be listed in the same order (usually alphabetically)
* Example: For variables a, b, c:
- Maxterms $(a+b+\bar{c})$, $(\bar{a}+b+\bar{c})$ are in standard order.
- However, $(b+\bar{a}+c)$ is NOT in standard order.
- $(\bar{a}+c)$ does NOT contain all variables.
- Minterms $(ab\bar{c})$ and $(\bar{a}b\bar{c})$ are in standard order.
- However, $(ba\bar{c})$ is not all standard order.
- $(\bar{a}c)$ doesnot contain all variable.

Sum of Minterm (SOM)

+ Sum of Minterm (SOM) canonical form:

Sum of minterm of entries that evaluate to '1'

x	y	z	F	Minterm
0	0	0	0	
0	0	1	1	$m_1 = \bar{x}\bar{y}z$
0	1	0	0	
0	1	1	0	
1	0	0	0	
1	0	1	0	
1	1	0	1	$m_6 = xy\bar{z}$
1	1	1	1	$m_7 = xyz$

Focus on the '1' entries.

$$F = m_1 + m_6 + m_7 = \sum(1,6,7) = \bar{x}\bar{y}z + xy\bar{z} + xyz.$$

Sum of Minterm Examples

+ $F(a,b,c,d) = \sum(2,3,6,10,11)$

+ $F(a,b,c,d) = m_2 + m_3 + m_6 + m_{10} + m_{11}$

$\bar{a}\bar{b}c\bar{d} + \bar{a}\bar{b}cd + \bar{a}bc\bar{d} + a\bar{b}c\bar{d} + a\bar{b}cd$

+ $G(a,b,c,d) = \sum(0,1,12,15)$

+ $G(a,b,c,d) = m_0 + m_1 + m_{12} + m_{15}$

$\bar{a}\bar{b}\bar{c}\bar{d} + \bar{a}\bar{b}\bar{c}d + ab\bar{c}\bar{d} + abcd.$

Product of Maxterm (POM)

+ Product-of-Maxterm (POM) canonical form:

Product of maxterms of entries that evaluate to '0'

x	y	z	F	Max-term
0	0	0	1	
0	0	1	1	
0	1	0	0	$M_2 = (x+\bar{y}+z)$
0	1	1	1	
1	0	0	0	$M_4 = (\bar{x}+y+z)$
1	0	1	1	
1	1	0	0	$M_6 = (\bar{x}+\bar{y}+z)$
1	1	1	1	

Focus on the '0' entries

$$F = M_2 . M_4 . M_6 = \Pi(2,4,6) = (x+\bar{y}+z)(\bar{x}+y+z)(\bar{x}+\bar{y}+z)$$

Product-Of-Maxterm Examples

+ $F(a,b,c,d) = \Pi(1,3,6,11)$

+ $F(a,b,c,d) = M_1 \cdot M_3 \cdot M_6 \cdot M_{11}$

$(a+b+c+\bar{d})(a+b+\bar{c}+\bar{d})(a+\bar{b}+\bar{c}+d)(\bar{a}+b+\bar{c}+\bar{d})$

+ $G(a,b,c,d) = \Pi(0,4,12,15)$

+ $G(a,b,c,d) = M_0 \cdot M_4 \cdot M_{12} \cdot M_{15}$

$(a+b+c+d)(a+\bar{b}+c+d)(\bar{a}+\bar{b}+c+d)(\bar{a}+\bar{b}+\bar{c}+\bar{d})$

Observations

- We can implement any function by "ORing" the minterm corresponding to the '1' entries in the function table. A minterm evaluates to '1' for its corresponding entry.
- We can implement any function by "ANDing" the maxterms corresponding to the '0' entries in the function table. A maxterm evaluates to '0' for its corresponding entry.
- The same Boolean function can be expressed in two canonical ways: Sum-of-Minterms (SOM) and Product of Maxterms (POM).
- If a Boolean function has fewer '1' entries then the SOM canonical form will contain fewer literals than POM. However if it has fewer '0' entries then the POM form will have fewer literals than SOM.

Converting to Sum-of-Minterms Form

* A function that is not in the Sum of Minterms form can be converted to that form by means of a truth table.
* Consider $F = \bar{y} + \bar{x}\bar{z}$

x	y	z	F	Min term
0	0	0	1	$m_0 = \bar{x}\bar{y}\bar{z}$
0	0	1	1	$m_1 = \bar{x}\bar{y}z$
0	1	0	1	$m_2 = \bar{x}y\bar{z}$
0	1	1	0	
1	0	0	1	$m_4 = x\bar{y}\bar{z}$
1	0	1	1	$m_5 = x\bar{y}z$
1	1	0	0	
1	1	1	0	

$F = \Sigma(0,1,2,4,5) = m_0 + m_1 + m_2 + m_4 + m_5 = \bar{x}\bar{y}\bar{z} + \bar{x}\bar{y}z + \bar{x}y\bar{z} + x\bar{y}\bar{z} + x\bar{y}z.$

Converting to Product of Maxterms Form

* A function that is not in the Product of Maxterms form can be converted to that form by means of a truth table.
* Consider again: $F = \bar{y} + \bar{x}\bar{z}$

x	y	z	F	Maxterm
0	0	0	1	
0	0	1	1	
0	1	0	1	
0	1	1	0	$M_3 = x+\bar{y}+\bar{z}$
1	0	0	1	
1	0	1	1	
1	1	0	0	$M_6 = \bar{x}+\bar{y}+z$
1	1	1	0	$M_7 = \bar{x}+\bar{y}+\bar{z}$

$F = \Pi(3,6,7) = M_3 \cdot M_6 \cdot M_7 = (x+\bar{y}+\bar{z})(\bar{x}+\bar{y}+z)(\bar{x}+\bar{y}+\bar{z})$

Conversion Between Canonical Forms

x	y	z	F	Minterm	Maxterm
0	0	0	0		$M_0 = x+y+z$
0	0	1	1	$m_1 = \bar{x}\bar{y}z$	
0	1	0	1	$m_2\ \bar{x}y\bar{z}$	
0	1	1	1	$m_3 = \bar{x}yz$	
1	0	0	0		$M_4 = \bar{x}+y+z$
1	0	1	1	$m_5 = x\bar{y}z$	
1	1	0	0		$M_6 = \bar{x}+\bar{y}+z$
1	1	1	1	$m_7 = xyz$	

$$F = m_1 + m_2 + m_3 + m_5 + m_7 = \Sigma(1,2,3,5,7) =$$

$$\bar{x}\bar{y}z + \bar{x}y\bar{z} + \bar{x}yz + x\bar{y}z + xyz$$

$$F = M_0 \cdot M \cdot M_6 = \Pi(0,4,6) = (x+y+z)(\bar{x}+y+z)(\bar{x}+\bar{y}+z)$$

Algebraic Conversion to Sum-of-Minterms

- Expand all terms first to explicitly list all minterms
- AND any term missing a variable v with $(v+\bar{v})$
- Example 1: $f = x + \bar{x}\bar{y}$ (2 variables).

$$f = x(y+\bar{y}) + \bar{x}\bar{y}$$

$$f = xy + x\bar{y} + \bar{x}\bar{y}$$

$$f = m_3 + m_2 + m_0 = \Sigma(0,2,3).$$

→ Example 2 : $g = a + \bar{b}c$ (3 variables)

$g = a(b+\bar{b})(c+\bar{c}) + (a+\bar{a})\bar{b}c$

$g = abc + ab\bar{c} + a\bar{b}c + a\bar{b}\bar{c} + a\bar{b}c + \bar{a}\bar{b}c$

$g = \bar{a}\bar{b}c + a\bar{b}\bar{c} + a\bar{b}c + ab\bar{c} + abc.$

$g = m_1 + m_4 + m_5 + m_6 + m_7 = \Sigma(1,4,5,6,7)$

Algebraic Conversion to Product of Maxterms

→ Expand all terms first to explicitly list all maxterms

→ OR any term missing a variable v with $v.\bar{v}$.

→ Example 1 : $f = x + \bar{x}\bar{y}$ (2 variables).

Apply 2nd distributive law :

$f = (x+\bar{x})(x+\bar{y}) = 1.(x+\bar{y}) = (x+\bar{y}) = M_1$

→ Example 2 : $g = a\bar{c} + bc + \bar{a}\bar{b}$ (3 variables).

$g = (a\bar{c} + bc + \bar{a})(a\bar{c} + bc + \bar{b})$ (distributive)

$g = (\bar{c} + bc + \bar{a})(a\bar{c} + c + \bar{b})$ $(x + \bar{x}y = x + y)$

$g = (\bar{c} + b + \bar{a})(a + c + \bar{b})$ $(x + \bar{x}y = x + y)$.

$g = (\bar{a} + b + \bar{c})(a + \bar{b} + c) = M_5 \cdot M_2 = \Pi(2,5)$.

Function Complements

- The complement of a function expressed as a sum of minterms is constructed by selecting the minterms missing in the sum-of-minterms canonical form.
- Alternatively, the complement of a function expressed by a Sum of Minterms form is simply the Product of Maxterms with the same indices.
- Example: Given $F(x,y,z) = \Sigma(1,3,5,7)$

$$\bar{F}(x,y,z) = \Sigma(0,2,4,6)$$

$$\bar{F}(x,y,z) = \Pi(1,3,5,7)$$

Summary of Minterms and Maxterms

- There are 2^n minterms and maxterms for Boolean functions with n variables.
- Minterms and maxterms are indexed from 0 to $2^n - 1$.

- Any Boolean function can be expressed as a logical sum of minterms and as a logical product of maxterms.
- The complement of a function contains those minterms not included in the original function.
- The complement of a sum-of-minterms is a product of maxterms with the same indices.

Standard Forms

- Standard Sum-of-Products (SOP) form: equations are written as an OR of AND terms.
- Standard Product of Sum (POS) form: equations are written as an AND of OR terms.
- Examples:

- SOP : $ABC + \bar{A}\bar{B}C + B$

- POS : $(A+B) \cdot (A+\bar{B}+\bar{C}) \cdot C$

- These "mixed" form are neither SOP nor POS

- $(AB + C)(A+C)$

- $AB\bar{C} + AC(A+B)$

Standard Sum of Products (SOP)

+ A sum of minterms from for n variables can be written down directly from a truth table.

- Implementation of this from is a two level network of gates such that:

- The first level consists of n-input AND gates.
- The second level is a single OR gate.

+ This form often can be simplified so that the corresponding circuit is simpler.

+ A Simplification Example:

$$F(A,B,C) = \sum(1,4,5,6,7)$$

+ Writing the minterm expression:

$$F = \bar{A}\bar{B}C + A\bar{B}\bar{C} + A\bar{B}C + AB\bar{C} + ABC$$

+ Simplifying:

$$F = \bar{A}\bar{B}C + A(\bar{B}\bar{C} + \bar{B}C + B\bar{C} + BC)$$

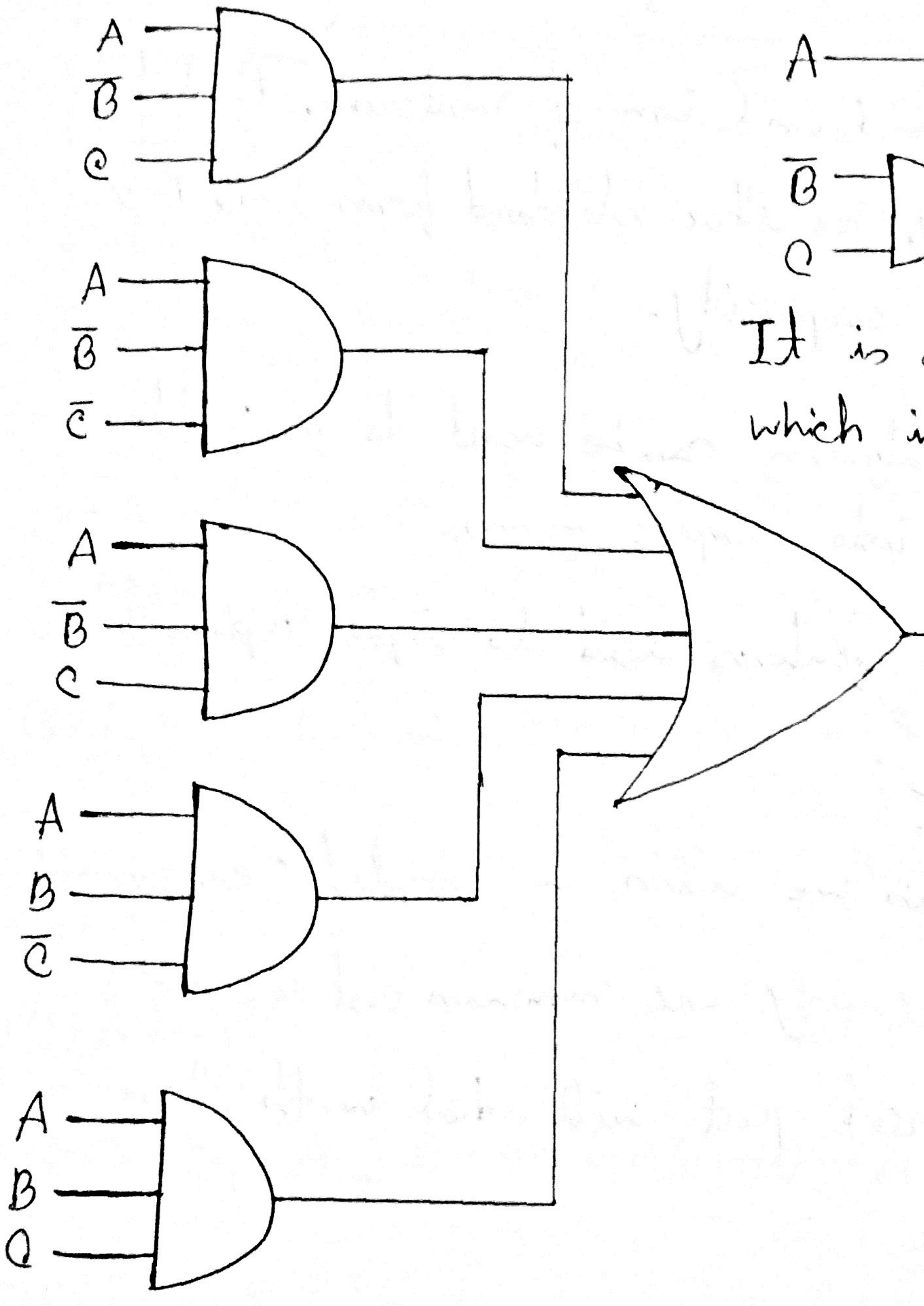
$$F = \bar{A}\bar{B}C + A(\bar{B}(\bar{C}+C) + B(\bar{C}+C))$$

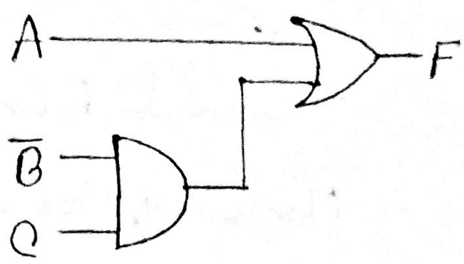
$$F = \bar{A}\bar{B}C + A(\bar{B}+B) = \bar{A}\bar{B}C + A = \bar{B}C + A$$

+ Simplified F contains 3 literals compared to 15.

AND/OR Two-Level Implementation

* The two implementations for F are shown below





It is quite apparent which is simpler!

SOP and POS observations

+ The previous examples show that:

- Canonical From (Sum-of-minterms, Product-of-Maxterms), or other standard forms (SOP, POS) differ in complexity.

- Boolean algebra can be used to manipulate equations into simpler forms.

- Simpler equations lead to simpler implementations.

+ Questions:

- How can we attain a "simplest" expression?

- Is there only one minimum cost circuit?

- The next part will deal with these issues.