Probability theory

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- **Outline**: 1) Probability
  2) Sample space with example
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  4) Event
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  7) Mutually exclusive events
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- **Statistics:** Statistics is the branch of knowledge that deals with the collection, summarization, analysis and interpretation of data.

- **Introduction**: Probability theory is a branch of statistics that has been developed to deal with uncertainty. Probability tells us how often some event will happen after many repeated trials.

- **Sample space:** The sample space S of an experiment is a set consisting of all of the possible experimental outcomes.
  Example: 1) A usual six-sides die has a sample space S = {1,2,3,4,5,6}

  2) Sample space for rolling two dice

					S
(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

3) The toss of a single coin has a sample space S = {H,T}
4) The toss of two coins has a sample space S = {HH,HT,TH,TT}
5) The toss of three coins has a sample space S = {HHH,HHT,HTH,HTT,THH,THT,TTH,TTT}

Classwork

1) What is the sample space when a coin is tossed four times?
2) What is the sample space for choosing a prime number less than 15 at random?

(Note: A prime number is number greater than one that is only divisble by one and itself.)

- **Probability Values:** Each outcome in the sample space is assigned a probability value that is a number between zero and one. The probabilities are chosen so that the sum of the probability values over all of the elements in the sample space is one. An intuitive interpretation of a set of probability values is that the larger the probability value of a particular outcome, the more likely it is to happen. Mathematically, Sample space, S = $\{O_1, O_2, ..., O_n\}$ consists of some probabilities $p_1, p_2, ..., p_n$ that satisfy $0 \leq p_1 \leq 1, 0 \leq p_2 \leq 1, ..., 0 \leq p_n \leq 1$ and $p_1 + p_2 + ... + p_n = 1$
The probability of outcome O_i occurring is said to be p_i, and this is written $P(O_i) = p_i$.

Note: In practice these probabilities would have to be estimated from prior experiences. If there are n outcomes in the sample space that are equally likely (If all the outcomes of a sample space have the same chance of occurrence, then it is known as equally likely outcomes), then each probability value will be 1/n.
if a sample space S consists of N equally likely outcomes, of which n are contained within the event A, then the probability of the event A is P(A) = n/N

Example: A fair die have six (n) outcomes and each of the six outcomes must have a probability of 1/6, i.e P(1) = P(2) = P(3) = P(4) = P(5) = P(6) = 1/6
An example of a biased die would be one for which P(1) = 0.10, P(2) = 0.15, P(3) = 0.15, P(4) = 0.15, P(5) = 0.15, P(6) = 0.30. In this case the die is most likely to score a 6. Scores of 2, 3, 4, and 5 are equally likely, and a score of 1 is the least likely event.

Note : On a fair die, every number has an equal chance of being rolled. On a biased die, some numbers are more likely to be rolled than others. This may be due to the die's shape. (If a red die and a blue die are thrown, with each of the 36 outcomes. let B be the event that at least one 6 is obtained on the two dice with a probability of P(B) = 11/ 36)

Homework

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- **Events**: An event A is a subset of the sample space S.
- **Complements of Events:** The complement of an event A, is the event consisting of everything in the sample space S that is not contained within the event A.

  Example: A usual six-sides die has a sample space S = {1,2,3,4,5,6}, If Event A = {1,3,6} then complementary of event A is $A' = \{2, 4, 5\}$.
  If P(1) = 0.10, P(2) = 0.15, P(3) = 0.15, P(4) = 0.15, P(5) = 0.15, P(6) = 0.30.
  The probability of the event A is calculated as the sum of the probabilities of these three events, so that P(A) = P(1) + P(3) + P(6) = 0.10 + 0.15 + 0.30 = 0.55.
  The probability of the complement of A is obtained by summing the probabilities of the three outcomes not contained within A, so that P(A′) = 0.15 + 0.15 + 0.15 = 0.45
  Notice that P(A) + P(A′) = 1.

Homework

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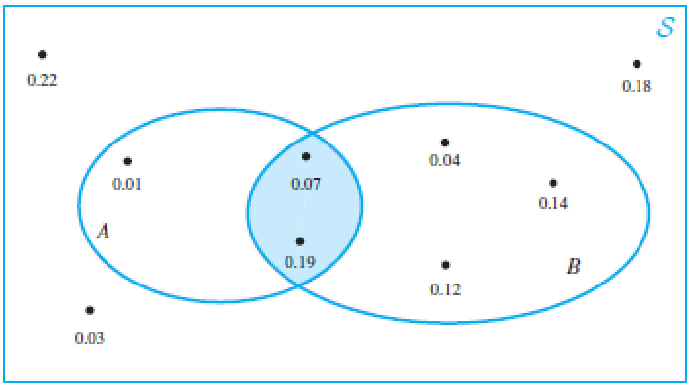
- **Intersections of Events:** The event A ∩ B is the intersection of the events A and B and consists of the outcomes that are contained within both events A and B. The probability of this event, P(A ∩ B), is the probability that both events A and B occur simultaneously.

FIGURE 1.27

The event $A \cap B$



- **Mutually Exclusive Events:** Two events A and B are said to be mutually exclusive if A ∩ B = ∅ so that they have no outcomes in common.

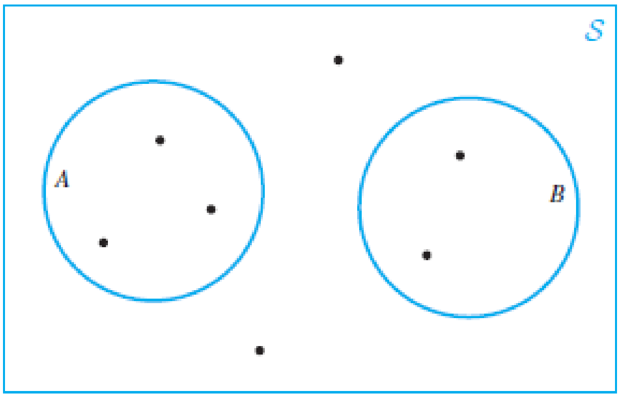


FIGURE 1.30

A and *B* are mutually exclusive events

Unions of Events: The event $A \cup B$ is the union of events A and B and consists of the outcomes that are contained within at least one of the events A and B. The probability of this event, $P(A \cup B)$, is the probability that at least one of the events A and B occurs.

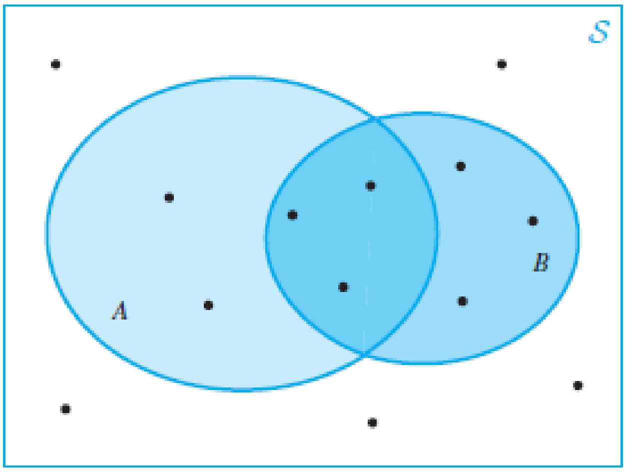


FIGURE 1.33

Decomposition of the event $A \cup B$

Note: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Classwork

If the events A and B are mutually exclusive, then $P(A \cup B) = \ldots$

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page no. 31: 1.3.1, 1.3.2 (a,b,c,d,e), 1.3.5, 1.3.6, 1.3.7

Conditional Probability: For experiments with two or more events of interest, attention is often directed not only at the probabilities of individual events, but also at the probability of an event occurring conditional on the knowledge that another event has occurred. Probabilities such as these are important and very useful since they provide appropriate revisions of a set of probabilities once a particular event is known to have occurred. The probability that event A occurs conditional on event B having occurred is written P(A|B) where P(A|B) $= P(A \cap$B)/P(B)

Example: A manager supervises the operation of three power plants, plant X, plant Y and plant Z. At any given time, each of the three plants can be classified as either generating electricity 91) or not generating electricity (0). With the notation (0,1,0) used to represent the situation where plant Y is generating electricity but plants X and Z are both not generating electricity.

Example 4
Power Plant Operation

The probability that plant X is idle is $P(A) = 0.32$. However, suppose it is known that at least two out of the three plants are generating electricity (event B). How does this change the probability of plant X being idle?

The probability that plant X is idle (event A) conditional on at least two out of the three plants generating electricity (event B) is

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0.18}{0.70} = 0.257$$

as shown in Figure 1.58. Therefore, whereas plant X is idle about 32% of the time, it is idle only about 25.7% of the time when at least two of the plants are generating electricity.

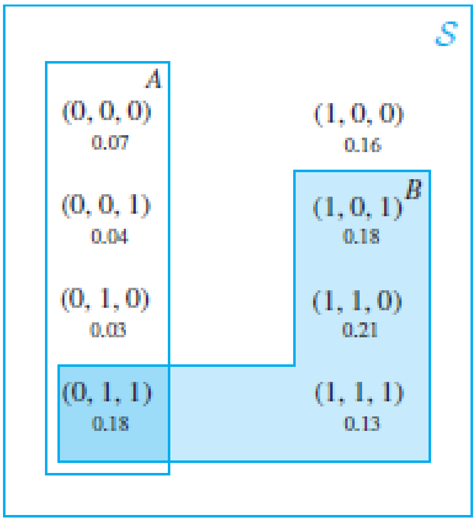

FIGURE 1.58

$P(A|B) = P(A \cap B)/P(B)$

Homework

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