Day-1 , 15th February, 2021

Assignment:1

A list of formulas for differentiation and integration from Chapters 2, 3 and 5.

→Due: 22nd February, 2021

MAT-130: Calculus II → Chapters 7, 6 and 10

$$\frac{d}{dx}\left(\int f(x)\ dx\right) = \int \frac{d}{dx}(f(x))\ dx = f(x) + C$$

Note: $\int \frac{1}{\sqrt{1-x^2}}\ dx = \sin^{-1}x + C$

Section 7.1: Overview

7.1 → 5.3 : Integration by Substitution

Method: Integration by Substitution [u-substitution]

Recall Chain Rule: $\frac{d}{dx}\left(f\left(g(x)\right)\right) = f'(g(x)) \cdot g'(x)$; $[\frac{d}{dx}(f(u) = f'(u) \cdot \frac{du}{dx}]$

For example, $\frac{d}{dx}[\sin^2\left(e^{\sqrt{2x}}\right)] = \frac{d}{dx}[\sin\left(e^{\sqrt{2x}}\right)]^2 = 2\ \sin\left(e^{\sqrt{2x}}\right) \cdot \cos\left(e^{\sqrt{2x}}\right) \cdot e^{\sqrt{2x}} \cdot \frac{1}{2\sqrt{2x}} \cdot 2$

Backward Chain Rule: $\int f'\left(g(x)\right) \cdot g'(x)\ dx.$

- Integral function must be a product of two functions, where one of them is a composite function
- The other one must be the derivative or a constant multiple of the derivative of the input function in the composition.

Understanding: Identify the composite function. Set the input function as u, **whose derivatives is given** (somehow, scalar multiple is given).

Now,

$$\int f'\left(g(x)\right) \cdot g'(x)\ dx\ ;$$

Set $u = g(x)$. Then $\frac{du}{dx} = g'(x)$, than is, $du = g'(x)dx.$

Then,

$$\int f'\big(g(x)\big)\cdot\ g'(x)\ \ dx$$

$$= \int f'(u)\,du\ = f(u) + C\ = f\big(g(x)\big) + C,\ \ \text{where}\ \ C\ \text{is a constant}$$

Examples

1) $(a)\ \displaystyle\int \frac{\mathbf{1}}{\sqrt{\boldsymbol{x}}}\ \mathbf{sec^2}(\sqrt{\boldsymbol{x}}\,)\ \ \boldsymbol{dx} \qquad = 2\int \frac{1}{2\sqrt{x}}\ \sec^2(\sqrt{x}\,)\ dx$

$= 2\displaystyle\int \sec^2(\sqrt{x}\,)\ \frac{1}{2\sqrt{x}}\,dx$

$Set\ \ u = \sqrt{x}$ implies $\frac{du}{dx} = \frac{1}{2\sqrt{x}}$.

That is, $du\ = \frac{1}{2\sqrt{x}}\ dx$

Now,

$$\int \frac{1}{\sqrt{x}}\ \sec^2(\sqrt{x}\,)\ dx = 2\int \frac{1}{2\sqrt{x}}\ \sec^2(\sqrt{x}\,)\ dx = \ 2\int \sec^2(\sqrt{x}\,)\ \frac{1}{2\sqrt{x}}\,dx$$

$= 2\ \int \sec^2(u\,)\,du\ \ = 2\tan u\ + C\ \ = 2\ \tan(\sqrt{x})\ + C$; [Formula : $\int \sec^2 x\ dx = \ \tan x\ + C$]

$(b)\ \displaystyle\int \frac{\mathbf{1}}{\sqrt{\boldsymbol{x}}}\ \ \mathbf{csc^2}(\sqrt{\boldsymbol{x}})\ \ \boldsymbol{dx}$

Solution:

$$\int \frac{1}{\sqrt{x}}\ \csc^2(\sqrt{x})\ dx\ \ = 2\int \frac{1}{2\sqrt{x}}\ \csc^2(\sqrt{x})\ dx$$

$= 2\displaystyle\int \csc^2(\sqrt{x})\ \frac{1}{2\sqrt{x}}\ \ dx$; Set $u = \sqrt{x}$, then $\frac{du}{dx} = \frac{1}{2\sqrt{x}}$, that is, $du = \frac{1}{2\sqrt{x}}\ dx$

$$= 2\int \csc^2(\,u)\ du\ \ = -2\cot u + C\ \ \ = -2\cot\big(\sqrt{x}\big)\ + C$$

OR

$\displaystyle\int \frac{1}{\sqrt{x}}\ \csc^2(\sqrt{x})\ dx$; Set $u = \sqrt{x}$, then $\frac{du}{dx} = \frac{1}{2\sqrt{x}}$, that is, $2\,du = \frac{1}{\sqrt{x}}\ dx$

$$\int \csc^2(\sqrt{x})\ \frac{1}{\sqrt{x}}\,dx = \int \csc^2(\,u)\ 2\ du = 2\int \csc^2(\,u)\ du\ \ = -2\cot u + C\ \ \ = -2\cot\big(\sqrt{x}\big)\ + C$$

2) $(a) \int \frac{x}{\sqrt{1-x^4}}\, dx \quad = \int \frac{x}{\sqrt{1-(x^2)^2}}\, dx$

Set $u = x^2, \quad du = 2x\, dx$. So, $x\, dx \; = \frac{1}{2} du$

Now, $\int \frac{x}{\sqrt{1-x^4}}\, dx \quad = \int \frac{x}{\sqrt{1-(x^2)^2}}\, dx$

$= \int \frac{1}{\sqrt{1-(x^2)^2}} \quad x\, dx \quad = \int \frac{1}{\sqrt{1-u^2}} \quad \frac{1}{2} \; du$

$$= \frac{1}{2} \int \frac{1}{\sqrt{1-u^2}} \quad du$$

$$= \frac{1}{2} \sin^{-1} u \; + \quad C$$

$= \frac{1}{2} \; \sin^{-1}(x^2) \; + \quad C$

$$2) \; (b) \int \frac{x^2}{\sqrt{1-x^3}}\, dx \; ; \quad Set \quad u = 1 - x^3 \, , then \; -\frac{1}{3}\, du = x^2 dx$$

$$\int \frac{x^2}{\sqrt{1-x^3}}\, dx \; = \int \frac{1}{\sqrt{u}} \left(-\frac{1}{3}\right) du \quad = -\frac{1}{3} \int u^{-\frac{1}{2}}\, du$$

$$= -\frac{1}{3} \frac{u^{\frac{1}{2}}}{\frac{1}{2}} + C$$

$$= -\frac{2}{3} \; \sqrt{1-x^3} \; + C$$

Please practice few exercises from section 7.1

3) $(a) \int \frac{1}{x(\ln x\,)^3} \; dx$

Set $u = \ln x, \quad$ then $\frac{du}{dx} = \frac{1}{x}$, that is, $\quad du = \frac{1}{x}\, dx$

$$\int \frac{1}{x(\ln x\,)^3} \; dx \; = \quad \int \frac{1}{(\ln x\,)^3} \cdot \frac{1}{x}\, dx$$

$$= \int \frac{1}{u^3}\ du \ = \int u^{-3}\, du = \frac{u^{-3+1}}{-3+1} + C$$

$$= -\frac{1}{2}\, u^{-2} + C \ = -\frac{1}{2u^2} + C$$

$$= -\frac{1}{2(\ln x)^2} + C$$

3) $(b) \int x\sqrt{1-x^2}\ dx$; $Set\ \ u = 1 - x^2$

3) $(c) \int \sqrt{1-x^2}\ dx$

3) $(d) \int \frac{1}{\sqrt{1+x^2}}\ dx$

3) $(e) \int \frac{x}{\sqrt{1+x^2}}\ dx$; $Set\ u = 1 + x^2$

Note: parts (c) and (d) need the concept of trigonometric substitution.

4) $\int \frac{x^2}{\sqrt{1-x^6}}\ dx \ = \int \frac{x^2}{\sqrt{1-(x^3)^2}}\ dx \ = \ \int \frac{1}{\sqrt{1-(x^3)^2}} \cdot x^2\, dx$

Set $u = x^3$ Then $x^2 dx = \frac{1}{3}\ du$

$$\int \frac{x^2}{\sqrt{1-x^6}}\ dx \ = \ \int \frac{1}{\sqrt{1-(x^3)^2}} \cdot x^2\, dx$$

$$= \int \frac{1}{\sqrt{1-u^2}} \cdot \frac{1}{3}\ du$$

$= \frac{1}{3}\sin^{-1} u\ + C = \ \frac{1}{3}\sin^{-1}(x^3)\ + C$; Formula: $\int \frac{1}{\sqrt{1-u^2}}\, du = \sin^{-1} u\ + C$