Course Name : Physics – I
Course # PHY 107

Examples on Simple Harmonic Motion (SHM)

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Examples : Simple Harmonic Motion

Example # 1: A mass of $1.0 \times 10^{-20}\,\mathrm{kg}$ is oscillating with time period $1.0 \times 10^{-5}\,\mathrm{sec}$ and maximum speed of $1.0 \times 10^{3}\,\mathrm{m/s}$. Calculate: the angular frequency, amplitude and the force constant.

Solution: Given that: $m = 1.0 \times 10^{-20}\,\mathrm{kg}$, $T = 1.0 \times 10^{-5}\,\mathrm{sec}$ and $v_{\mathrm{m}} = 1.0 \times 10^{3}\,\mathrm{m/s}$.

Now, using the definition the angular frequency is

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{1.0 \times 10^{-5}}\,\mathrm{r/s} = 6.28 \times 10^{5}\,\mathrm{r/s}\ .$$

Since $v_{\mathrm{m}} = \omega x_{\mathrm{m}}$, we find:

$$x_{\mathrm{m}} = \frac{v_{\mathrm{m}}}{\omega} = \frac{1.0 \times 10^{3}}{6.28 \times 10^{5}}\,\mathrm{m} = 1.59 \times 10^{-3}\,\mathrm{m}\ .$$

The force constant is:

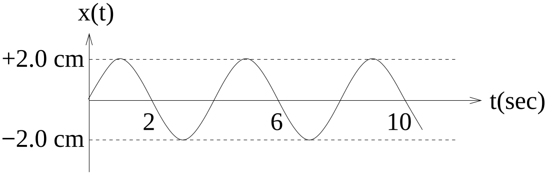
$$k = m\omega^2 = (1.0 \times 10^{-20}) \times (6.28 \times 10^{5})^2\mathrm{N/m} = 3.94 \times 10^{-9}\,\mathrm{N/m}\ .$$

Example # 2: The function $x(t) = (6.0\,\mathrm{m})\cos\left[(3\pi)t + \pi/3\right]$, is a SHM. At $t = 0$, what are: x, v, a, Phase, f and T.

Solution: Comparing to the standard equation, $x = A\cos(\omega t + \phi)$, we identify that the amplitude $A = (6.0\,\mathrm{m})$, the angular frequency is $\omega = 3\pi\,\mathrm{r/s}$ and the initial phase is $\phi = \pi/3\,\mathrm{rad}$. Now, using the definitions, we find:

$$
\begin{aligned}
x(t=0) &= (6.0\,\mathrm{m})\cos\left[3\pi \times 0 + \pi/3\right] = 3.0\,\mathrm{m}\ . \\
v(t=0) &= \left.\frac{dx}{dt}\right|_{t=0} = -\,3\pi(6.0\,\mathrm{m})\sin\left[3\pi \times 0 + \pi/3\right]_{t=0} \\
&= -18\pi\sin(\pi/3)\,\mathrm{m/s} = -\,49\,\mathrm{m/s}\ . \\
a(t=0) &= -\omega^2 x(t=0) = (3\pi)^2 \times 3.0\,\mathrm{m/s^2} = -\,266\,\mathrm{m/s^2}\ . \\
\mathrm{Phase}(t=0) &= \phi = \pi/3\,\mathrm{rad}\ . \\
f &= \frac{\omega}{2\pi} = \frac{3\pi}{2\pi}\,\mathrm{Hz} = 1.5\,\mathrm{Hz}\ . \\
T &= \frac{1}{f} = \frac{1}{1.5\,\mathrm{Hz}} = 0.67\,\mathrm{Hz}\ .
\end{aligned}
$$

Example # 3: A mass m attached with an ideal spring of constant $25\,\mathrm{N/m}$ is undergoing SHM. It's position as a function of time is given by the adjacent graph. Find: the angular frequency, mass and the maximum kinetic energy.



Solution: From the given graph, we can readily identify the amplitude $A = 2\,\mathrm{cm} = 0.02\mathrm{m}$ and the time period $T = (6-2)\mathrm{sec} = 4\,\mathrm{sec}$. The spring constant is $k = 25\,\mathrm{N/m}$. Using the definitions and properties, we easily find:

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{4}\,\mathrm{r/s} = 1.57\,\mathrm{r/s}\ .$$

$$m = \frac{k}{\omega^2} = \frac{25}{(1.57)^2}\,\mathrm{kg} = 10.1\,\mathrm{kg}\ .$$

$$K_{\mathrm{max}} = U_{\mathrm{max}} = \frac{1}{2}kA^2 = (0.5)(25)(0.02)^2\,\mathrm{J} = 5.0\times 10^{-3}\,\mathrm{J} = 5.0\,\mathrm{mJ}\ .$$

Example # 4:

A SHM consists of a block of mass 2.0 kg that is attached with a spring of constant 100 N/m and it is oscillating. At $t = 1\,\mathrm{sec}$, it's position is $x_1 = 0.129\,\mathrm{m}$ and speed is $v_1 = 3.415\,\mathrm{m/s}$. What are: the amplitude; position and speed at $t = 0$?
Solution: We know that $m = 2.0\,\mathrm{kg}$, $k = 100\,\mathrm{N/m}$, $t = 1.0\,\mathrm{sec}$, $x_1 = 0.129\,\mathrm{m}$ and speed is $v_1 = 3.415\,\mathrm{m/s}$. Using these, we easily find that $\omega = \sqrt{k/m} = 7.07\,\mathrm{r/s}$ and $T = 2\pi/\omega = 0.89\,\mathrm{sec}$. Let's choose the standard equation of the SHM motion is $x = A\cos(\omega t + \phi)$. Using this equation and its first derivative for speed, and then substituting $t = 1$, we find:

$$
\begin{aligned}
x_1 &= A\cos(\omega + \phi)\ , \\
0.129 &= A\cos(7.07 + \phi)\ , \qquad (1) \\
v_1 &= -\omega A\sin(\omega + \phi)\ , \\
3.415 &= -7.07A\sin(7.07 + \phi)\ . \qquad (2)
\end{aligned}
$$

From Eqs.(1, 2), we easily find:

$$\cos(\omega + \phi) = \frac{0.129}{A} \quad \text{and} \quad \sin(\omega + \phi) = \frac{3.415}{-7.07A} \ .$$

Squaring and adding, we find that:

$$\left(\frac{0.29}{A}\right)^2 + \left(\frac{3.415}{7.07A}\right)^2 = 1 \quad \Longrightarrow \therefore \ A = 4.83\,\mathrm{m} \ .$$

Putting $A = 4.83\,\mathrm{m}$ back into Eq.(1) and solving for ϕ gives that

$$\phi = \cos^{-1}\left(\frac{x_1}{A}\right) - \omega = \cos^{-1}\left(\frac{0.129}{4.83}\right) - 7.07 = -5.52\,\mathrm{rad} \approx \frac{7\pi}{4} \ .$$

Therefore, at $t = 0$, we find:

$$\begin{aligned} x_0 &= (4.83\,\mathrm{m})\cos(0 - 5.52) = 3.49\,\mathrm{m} \ . \\ v_0 &= -(7.07)(4.83)\sin(0 - 5.52)\,\mathrm{m/s} = -23.6\,\mathrm{m/s} \ . \end{aligned}$$