PHY 107
Motion in two and three dimensions

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OUTLINE

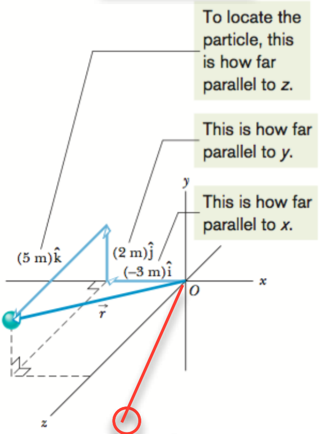
- Motion
- Position and Displacement
- Average Velocity and Instantaneous Velocity
- Average Acceleration and Instantaneous Acceleration
- Projectile Motion
- Uniform Circular Motion

Motion

We live in a space of 3 dimensions. So, it is important to get familiar with how things operate in 3D. Such understanding would help us lead life more efficiently.
e.g. Soccer players intend to hit the ball taking into account the fact that the soccer ball can be both translated and rotated as it is displaced.

Position and Displacement

The position of a particle is denoted by a position vector
$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$
$\overrightarrow{\Delta r} = (\underline{x_2\hat{i} + y_2\hat{j} + z_2\hat{k}}) - (\underline{x_1\hat{i} + y_1\hat{j} + z_1\hat{k}}) = \underline{\Delta x\hat{i} + \Delta y\hat{j} + \Delta z\hat{k}}$



$\Delta \vec{r} = \vec{r}_2 - \vec{r}_1$

r1=-3i+2j+5k

r2= _i+_j+_k

Average Velocity and Instantaneous Velocity

A particle moves through a displacement $\Delta \overrightarrow{r}$ in a time interval Δt

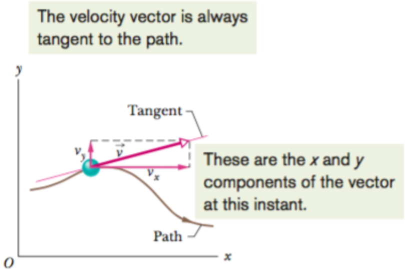
$$v_{avg} = \frac{\Delta \overrightarrow{r}}{\Delta t} \tag{1}$$

Instantaneous velocity:

$\overrightarrow{v} = \frac{d\overrightarrow{r}}{dt}$

The direction of the instantaneous velocity $\overrightarrow{v}$ of a particle is always tangent to the particle's path at the particle's position.

$\overrightarrow{v} = \frac{d}{dt}(x\hat{i} + y\hat{j} + z\hat{k}) = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} + \frac{dz}{dt}\hat{k} = v_x\hat{i} + v_y\hat{j} + v_z\hat{k}$



Average Acceleration and Instantaneous Acceleration

A particle goes through a change in velocity $\Delta\vec{v}$ in a time interval Δt

$$a_{avg} = \frac{\Delta\vec{v}}{\Delta t}$$

Instantaneous acceleration:

$$\vec{a} = \frac{d\vec{v}}{dt}$$

$$\vec{a} = \frac{d}{dt}(v_x\hat{i} + v_y\hat{j} + v_z\hat{k}) = \frac{dv_x}{dt}\hat{i} + \frac{dv_y}{dt}\hat{j} + \frac{dv_z}{dt}\hat{k}$$

$$\vec{v} = 2t\hat{i} + t^3\hat{j}$$

$$\vec{a} = 2\hat{i} + 3t^2\hat{j}$$

Projectile

A particle is in motion in a vertical plane with some initial velocity $\vec{v}_0$

-acceleration is the free fall acceleration (downward)

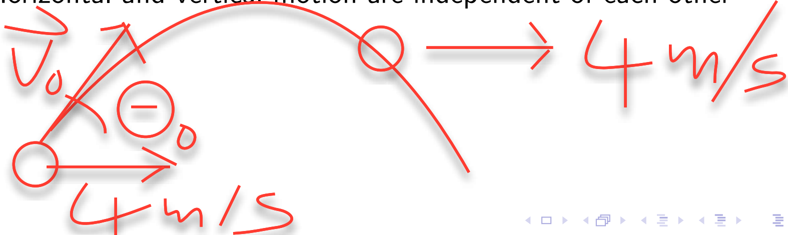
-**AIR has NO effect on the projectile**

The projectile is launched with an initial velocity $\vec{v}_0 = v_{0x}\hat{i} + v_{0y}\hat{j}$

$v_{0x} = v_0 cos(\theta_0)$, $v_{0y} = v_0 sin(\theta_0)$

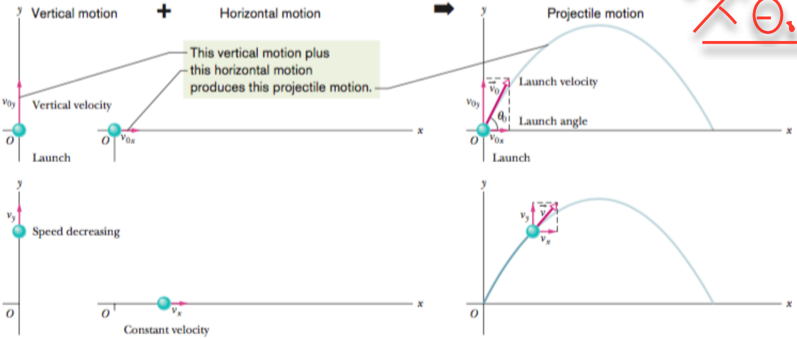
-No horizontal acceleration

-Horizontal and vertical motion are independent of each other

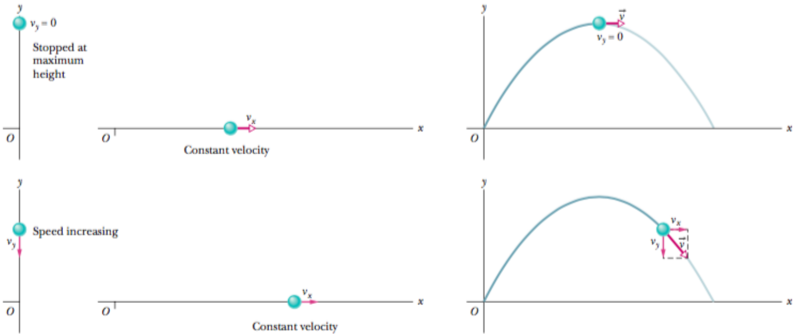


Projectile

v_0 θ_0



Projectile



$v_y = 0$
Stopped at maximum height
v_x
Constant velocity
$\vec{v}$
$v_y = 0$
Speed increasing
v_y
v_x
Constant velocity
v_x
v_y
$\vec{v}$

Analysis of the projectile motion

Horizontal Motion: $x - x_0 = (v_0 cos(\theta_0))t$

Vertical Motion: 1. $y - y_0 = v_{0y} t - 0.5gt^2$

$y - y_0 = v_0 sin(\theta_0)t - 0.5gt^2$

2. $v_y = v_0 sin(\theta_0) - gt$

3. $v_y^2 = (v_0 sin(\theta_0))^2 - 2g(y - y_0)$

The Equation of the path:

$y = tan(\theta_0)x - \frac{gx^2}{2(v_0 cos(\theta_0))^2} \rightarrow PARABOLIC$

Analysis of the projectile motion

The Horizontal Range: horizontal distance the projectile has traveled when it returns to its initial height

$x - x_0 = R$

$R = v_0 cos(\theta_0) t$

$0 = v_0 sin(\theta_0) t - 0.5 g t^2 \rightarrow R = \frac{v_0^2}{g} sin(2\theta_0)$

R is max when $sin(2\theta_0) = 1$

Examples on projectile motion

Example 1 A projectile's launch speed is five times its speed at maximum height. Find launch angle θ_0.

Example 2 A small ball rolls horizontally off the edge of a tabletop that is 1.20 m high. It strikes the floor at a point 1.52 m horizontally from the table edge.

(a) How long is the ball in the air?

(b) What is its speed at the instant it leaves the table?

Uniform Circular Motion

The particle travels around a circle at constant speed
-the speed does not vary
-the particle accelerates since the velocity changes its direction
Direction of velocity and acceleration:
-velocity is directed tangent to the circle in the direction of motion
-acceleration is always directed radially inward (centripetal acceleration)
The magnitude of this acceleration $\vec{a}$:
$a = \frac{v^2}{r}$
Time taken by the particle to travel the whole circumference:
$T = \frac{2\pi r}{v}$: Period of oscillation
Proof can be found in the book

Reference

Fundamentals of Physics by Halliday and Resnik