Sample problems

QUIZ 1

June 28, 2018

This document contains two sample problems (with solution). You can expect similar type question on the quiz. Note that there are several ways a problem can be solved, if you are consistent with the signs and units, you should get the right answer.

Projectile dropped from airplane

In Fig. 4-14, a rescue plane flies at 198 km/h (= 55.0 m/s) and constant height $h = 500$ m toward a point directly over a victim, where a rescue capsule is to land.

(a) What should be the angle ϕ of the pilot's line of sight to the victim when the capsule release is made?

KEY IDEAS

Once released, the capsule is a projectile, so its horizontal and vertical motions can be considered separately (we need not consider the actual curved path of the capsule).

Calculations: In Fig. 4-14, we see that ϕ is given by

$$\phi = \tan^{-1}\frac{x}{h}, \qquad (4\text{-}27)$$

where x is the horizontal coordinate of the victim (and of the capsule when it hits the water) and $h = 500$ m. We should be able to find x with Eq. 4-21:

$$x - x_0 = (v_0 \cos\theta_0)t. \qquad (4\text{-}28)$$

Here we know that $x_0 = 0$ because the origin is placed at the point of release. Because the capsule is *released* and not shot from the plane, its initial velocity $\vec{v}_0$ is equal to the plane's velocity. Thus, we know also that the initial velocity has magnitude $v_0 = 55.0$ m/s and angle $\theta_0 = 0°$ (measured relative to the positive direction of the x axis). However, we do not know the time t the capsule takes to move from the plane to the victim.

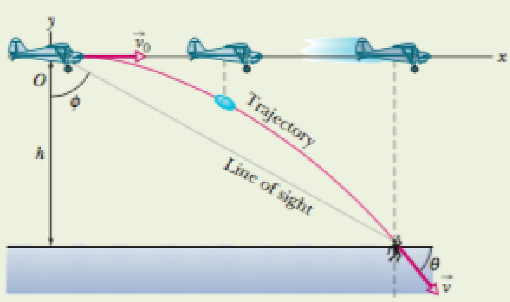


Fig. 4-14 A plane drops a rescue capsule while moving at constant velocity in level flight. While falling, the capsule remains under the plane.

To find t, we next consider the *vertical* motion and specifically Eq. 4-22:

$$y - y_0 = (v_0 \sin\theta_0)t - \tfrac{1}{2}gt^2. \qquad (4\text{-}29)$$

Here the vertical displacement $y - y_0$ of the capsule is -500 m (the negative value indicates that the capsule moves *downward*). So,

$$-500\text{ m} = (55.0\text{ m/s})(\sin 0°)t - \tfrac{1}{2}(9.8\text{ m/s}^2)t^2. \quad (4\text{-}30)$$

Solving for t, we find $t = 10.1$ s. Using that value in Eq. 4-28 yields

$$x - 0 = (55.0\text{ m/s})(\cos 0°)(10.1\text{ s}), \qquad (4\text{-}31)$$

or

$$x = 555.5\text{ m}.$$

Then Eq. 4-27 gives us

$$\phi = \tan^{-1}\frac{555.5\text{ m}}{500\text{ m}} = 48.0°. \qquad \text{(Answer)}$$

(b) As the capsule reaches the water, what is its velocity $\vec{v}$ in unit-vector notation and in magnitude-angle notation?

KEY IDEAS

(1) The horizontal and vertical components of the capsule's velocity are independent. (2) Component v_x does not change from its initial value $v_{0x} = v_0 \cos\theta_0$ because there is no horizontal acceleration. (3) Component v_y changes from its initial value $v_{0y} = v_0 \sin\theta_0$ because there is a vertical acceleration.

Calculations: When the capsule reaches the water,

$$v_x = v_0 \cos\theta_0 = (55.0\text{ m/s})(\cos 0°) = 55.0\text{ m/s}.$$

Using Eq. 4-23 and the capsule's time of fall $t = 10.1$ s, we also find that when the capsule reaches the water,

$$\begin{aligned} v_y &= v_0 \sin\theta_0 - gt \qquad (4\text{-}32)\\ &= (55.0\text{ m/s})(\sin 0°) - (9.8\text{ m/s}^2)(10.1\text{ s})\\ &= -99.0\text{ m/s}. \end{aligned}$$

Thus, at the water

$$\vec{v} = (55.0\text{ m/s})\hat{\imath} - (99.0\text{ m/s})\hat{\jmath}. \qquad \text{(Answer)}$$

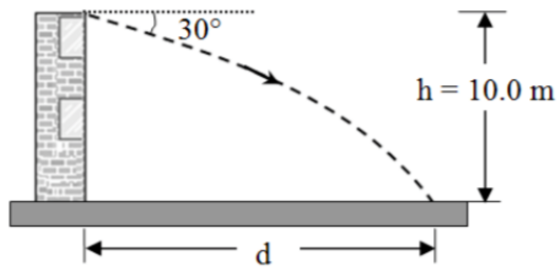
Using Eq. 3-6 as a guide, we find that the magnitude and the angle of $\vec{v}$ are

$$v = 113\text{ m/s} \quad\text{and}\quad \theta = -60.9°. \qquad \text{(Answer)}$$

You throw a ball from a window at a height h =10.0 m above the ground, with an initial speed of 20 m/s at an angle 30° below the horizontal, see **Figure 6**. At what horizontal distance d will the ball hit the ground? Ignore air resistance.

A) 12.7 m
B) 34.6 m
C) 22.4 m
D) 10.0 m
E) 20.0 m

Figure 6



$$y = V_0 \sin\theta_0 t - \frac{1}{2} g t^2$$

$$-10 = -20 \cdot \frac{1}{2}\, t - 4.9\, t^2$$

Find t

$$x = v_{0x} t = 20\,\cos(30)\;(0.73)$$