Ans to the Ques No 1

Given that,

$$G(i,j) = \sum_{u=0}^{k-1} \sum_{v=0}^{L-1} F(u,v) \cdot \bar{I}(i+u, j+v)$$

$$I = \begin{bmatrix} 7 & 4 & 1 \\ 8 & 5 & 2 \\ 9 & 6 & 3 \end{bmatrix}, \quad \bar{I} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 7 & 4 & 1 & 0 \\ 0 & 8 & 5 & 2 & 0 \\ 0 & 9 & 6 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(a) $$F = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$G = F \times I$ and $G \in \mathbb{R}^{3\times 3}$

according to, ~~F(0,0)~~ formula, F start from, (0,0).

$$G(1,1) = F(0,0) \cdot \bar{I}(1,1) + F(0,1)\bar{I}(1,2) + F(0,2)\bar{I}(1,3)$$
$$+ F(1,0)\bar{I}(2,1) + F(1,1)\bar{I}(2,2) + F(1,2)\bar{I}(2,3)$$
$$+ F(2,0)\bar{I}(3,1) + F(2,1)\bar{I}(3,2) + F(2,2)\bar{I}(3,3)$$
$$= F(1,1)\bar{I}(2,2) = 7$$

as others value of f are 0 without $f(1,1)$ (according to formula) so, we can calculate only $F(1,1)$

$G(1,2) = F(1,1) \cdot \bar{I}(2,3) = 4$

$G(1,3) = F(1,1) \cdot \bar{I}(2,4) = 1$

$G(2,1) = F(1,1) \cdot \bar{I}(3,2) = 8$

$G(2,2) = F(1,1) \cdot \bar{I}(3,3) = 5$

$G(2,3) = F(1,1) \cdot \bar{I}(3,4) = 2$

$G(3,1) = F(1,1) \cdot \bar{I}(4,2) = 9$

$G(3,2) = F(1,1) \cdot \bar{I}(4,3) = 6$

$G(3,3) = F(1,1) \cdot \bar{I}(4,4) = 3$

$$G = \begin{bmatrix} 7 & 4 & 1 \\ 8 & 5 & 2 \\ 9 & 6 & 3 \end{bmatrix}$$

(b) $F = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$F(0,0) = 1$ (according to formula)

$G(1,1) = F(0,0) \cdot \bar{I}(1,1) = 0$

$G(1,2) = F(0,0) \cdot \bar{I}(1,2) = 0$

$G(1,3) = F(0,0) \cdot \bar{I}(1,3) = 0$

$G(2,1) = F(0,0) \cdot \bar{I}(2,1) = 0$

$G(2,2) = F(0,0) \cdot \bar{I}(2,2) = 7$

$G(2,3) = F(0,0) \cdot \bar{I}(2,3) = 4$

$G(3,1) = F(0,0) \cdot \bar{I}(3,1) = 0$

$G(3,2) = F(0,0) \cdot \bar{I}(3,2) = 8$

$G(3,3) = F(0,0) \cdot \bar{I}(3,3) = 5$

$$G = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 7 & 4 \\ 0 & 8 & 5 \end{bmatrix}$$

(e) $F = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ -1 & -1 & -1 \end{bmatrix}$

$$G(1,1) = F(0,0)\bar{I}(1,1) + F(0,1)\bar{I}(1,2) + F(0,2)\bar{I}(1,3)$$
$$+ F(2,0)\bar{I}(3,1) + F(2,1)\bar{I}(3,2) + F(2,2)\bar{I}(3,3)$$
$$= 0 + 0 + 0 + 0 - 8 - 5 = -13$$

$$G(1,2) = F(0,0)\bar{I}(1,2) + F(0,1)\bar{I}(1,3) + F(0,2)\bar{I}(1,4)$$
$$+ F(2,0)\bar{I}(3,2) + F(2,1)\bar{I}(3,3) + F(2,2)\bar{I}(3,4)$$
$$\cancel{= 0 + 0 + 0 - 5 - 2 + 0 = -7}$$
$$= 0 + 0 + 0 - 8 - 5 - 2 = -15$$

$$G(1,3) = F(0,0)\bar{I}(1,3) + F(0,1)\bar{I}(1,4) + F(0,2)\bar{I}(1,5)$$
$$+ F(2,0)\bar{I}(3,3) + F(2,1)\bar{I}(3,4) + F(2,2)\bar{I}(3,5)$$
$$= 0 + 0 + 0 - 5 - 2 = -7$$

$$G(2,1) = F(0,0)\bar{I}(2,1) + F(0,1)\bar{I}(2,2) + F(0,2)\bar{I}(2,3)$$
$$+ F(2,0)\bar{I}(4,1) + \cancel{F(2,0)}F(2,1)\bar{I}(4,2) + F(2,2)\bar{I}(4,3)$$
$$= 0 + 7 + 4 - 0 - 9 - 6 = -4$$

$$G(2,2) = F(0,0)\, I(2,2) + F(0,1)\, I(2,3) + F(0,2)\, I(2,4)$$
$$+ F(2,0)\, I(4,2) + F(2,1)\, I(4,3) + F(2,2)\, I(4,4)$$
$$= 7+4+1-9-6-3 = -6$$

$$G(2,3) = F(0,0)\, I(2,3) + F(0,1)\, I(2,4) + F(0,2)\, I(4,5)$$
$$+ F(2,0)\, I(4,3) + F(2,1)\, I(4,4) + F(2,2)\, I(4,5)$$
$$= 4+1+0 - 6 - 3 - 0 = -4$$

$$G(3,1) = F(0,0)\, I(3,1) + F(0,1)\, I(3,2) + F(0,2)\, I(3,3)$$
$$+ F(2,0)\, I(5,1) + F(2,1)\, I(5,2) + F(2,2)\, I(5,3)$$
$$= 0+8+5+0+0+0 = 13$$

$$G(3,2) = F(0,0)\, I(3,2) + F(0,1)\, I(3,3) + F(0,2)\, I(3,4)$$
$$+ F(2,0)\, I(5,2) + F(2,1)\, I(5,3) + F(2,2)\, I(5,4)$$
$$= 8+5+2 - 0-0-0 = \cancel{13}\ 15$$

$$G(3,3) = F(0,0)\, I(3,3) + F(0,1)\, I(3,4) + F(0,2)\, I(3,5)$$
$$+ F(2,0)\, I(5,3) + F(2,1)\, I(5,4) + F(2,2)\, I(5,5)$$
$$= 5+2+0+0-0 = 7$$

$$G = \begin{bmatrix} -13 & -15 & -7 \\ -4 & -6 & -4 \\ 13 & 15 & 7 \end{bmatrix}$$

(d) ~~The~~ The ~~m~~ Filter is shown masks are mirrored in convolution. Image ~~deriv~~ differentiation filters ~~taking the~~ derivative of an image can be used to identify certain

F is intended to detect horizontal edges in image. It reacts strongly to horizontal egdes when performing shifting operation from dark to light and light to dark. It emphasizes features like horizontal boundaries and edges.

(d) $F' = \begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix}$

Hence, $F(0,1)$, $F(1,1)$ and $F(2,1)$ are 0 [according to formula]

$G(1,1) = F(0,0) I'(1,1) + F(0,2) I'(1,3) + F(1,0) I'(2,1)$
$+ F(1,2) I'(2,3) + F(2,0) I'(3,1) + F(2,2) I'(3,3)$
$= 4 + 5 = 9$

$G(1,2) = F(0,0) I'(1,2) + F(0,2) I'(1,4) + F(1,0) I'(2,2)$
$+ F(1,2) I'(2,4) + F(2,0) I'(3,2) + F(2,2) I'(3,4)$
$= -7 - 8 + 1 + 2 = -12$

$G(1,3) = F(0,0) I'(1,3) + F(0,2) I'(1,5) + F(1,0) I'(2,3)$
$+ F(1,2) I'(2,5) + F(2,0) I'(3,3) + F(2,2) I'(3,5)$
$= -4 - 5 = -9$

$G(2,1) = F(0,0) I'(2,1) + F(0,2) I'(2,3) + F(1,0) I'(3,1)$
$+ F(1,2) I'(3,3) + F(2,0) I'(4,1) + F(2,2) I'(4,3)$
$= 4 + 5 + 6 = 15$

$F(2,2) = F(0,0) I'(2,2) + F(0,2) I'(2,4) + F(1,0) I'(3,2)$
$+ F(1,2) I'(3,4) + F(2,0) I'(4,2) + F(2,2) I'(4,4)$
$= -7 + 1 - 8 + 2 - 9 + 3 = -18$

$$G(2,3) = F(0,0)\,I'(2,3) + F(0,2)\,I'(2,5) + F(1,0)\,I'(3,3)$$
$$+ F(1,2)\,I'(3,5) + F(2,0)\,I'(4,3) + F(2,2)\,I'(4,5)$$
$$= -4 - 5 - 6 = -15$$

$$G(3,1) = F(0,0)\,I'(3,1) + F(0,2)\,I'(3,3) + \cancel{F(0,2)}\,F(1,0)\,I'(4,1)$$
$$+ F(1,2)\,I'(4,3) + F(2,0)\,I'(5,1) + F(2,2)\,I'(5,3)$$
$$= \cancel{8}\; 5 + 6 = 11$$

$$G(3,2) = F(0,0)\,I'(3,2) + F(0,2)\,I'(3,4) + F(1,0)\,I'(4,2)$$
$$+ F(1,2)\,I'(4,4) + F(2,0)\,I'(5,2) + F(2,2)\,I'(5,4)$$
$$= -8 - 9 + 3 + 2 = -12$$

$$G(3,3) = F(0,0)\,I'(3,3) + F(0,2)\,I'(3,5) + F(1,0)\,I'(4,3)$$
$$+ F(1,2)\,I'(4,5) + F(2,0)\,I'(5,3) + F(2,2)\,I'(5,5)$$
$$= -5 - 6 = -11$$

$$G = \begin{bmatrix} 9 & -12 & -9 \\ 15 & -18 & 15 \\ 11 & -12 & -11 \end{bmatrix}$$

Now in filter $F = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ -1 & -1 & -1 \end{bmatrix}$ and

$F' = \begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix}$ have difference in edge

detection.

F is intend'd to detect horizontal

It reacts to horizontal where

edges. dark to light. F'

a shift from

where a transition

detect vertical changes

from drak to light or vice versa. It

Looks the area in vertical direction.

F and F' are designed for edge

detection for defined operation. those are

detect edges in their direction and contributing

to different aspect of image analysis.

(e) $F = \frac{1}{16}\begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}$

$G(1,1) = F(0,0)\,I'(1,1) + F(0,1)\,I'(0,2) + F(0,2)\,I'(0,3)$
$+ F(1,0)\,I'(2,1) + F(1,1)\,I'(2,2) + F(1,2)\,I'(2,3)$
$+ F(2,0)\,I'(3,1) + F(2,1)\,I'(3,2) + F(2,2)\,I'(3,3)$
$= \frac{1}{16}(\cancel{8}\ 4.7 + 2.4 + 2.\cancel{8}2 + 5)$
$= \frac{1}{16} \times 57 = 3.5$

$G(1,2) = F(0,0)\,I'(1,2) + F(0,1)\,I'(1,3) + F(0,2)\,I'(1,4)$
$+ F(1,0)\,I'(2,2) + F(1,1)\,I'(2,3) + F(1,2)\,I'(2,4)$
$+ F(2,0)\,I'(3,2) + F(2,1)\,I'(3,3) + F(2,2)\,I'(3,4)$
$= \frac{1}{16}(2.7 + 4.4 + 2.1 + 1.8 + 2.5 + 1.2)$
$= \frac{1}{16} \times 52 = 3.25$

$G(1,3) = F(0,0)\,I'(1,3) + F(0,1)\,I'(1,4) + F(0,2)\,I'(1,5)$
$+ F(1,0)\,I'(2,3) + F(1,1)\,I'(2,4) + F(1,2)\,I'(2,5)$
$+ F(2,0)\,I'(3,3) + F(2,1)\,I'(3,4) + F(2,2)\,I'(3,5)$
$= \frac{1}{16}(2.4 + 4.1 + 1.5 + 2.2)$
$= \frac{1}{16}.21 = 1.31$

$$G(2,1) = F(0,0)\,I'(2,1) + F(0,1)\,I'(2,2) + F(0,2)\,I'(2,3) + F(1,0)\,I'(3,1) + F(1,1)\,I'(3,2) + F(1,2)\,I'(3,3) + F(2,0)\,I'(4,1) + F(2,1)\,I'(4,2) + F(2,2)\,I'(4,4)$$

$$= \frac{1}{16}(2.7 + 1.4 + 4.8 + 2.5 + 2.9 + 1.6)$$

$$= \frac{1}{16} \cdot 84 = 5.25$$

$$G(2,2) = F(0,0)\,I'(2,2) + F(0,1)\,I'(2,3) + F(0,2)\,I'(2,4) + F(1,0)\,I'(3,2) + F(1,1)\,I'(3,3) + F(1,2)\,I(3,4) + F(2,0)\,I(4,2) + F(2,1)\,I(4,3) + F(2,2)\,I'(4,4)$$

$$= \frac{1}{16}(7 + 2.4 + 1 + 2.8 + 4.5 + 2.2 + 1.9 + 2.6 + 1.3) = \frac{1}{16} \cdot 80 = 5$$

$$G(2,3) = F(0,0)\,I'(2,3) + F(0,1)\,I'(2,4) + F(0,2)\,I'(2,5) + F(1,0)\,I'(3,3) + F(1,1)\,I'(3,4) + F(1,2)\,I'(3,5) + F(2,0)\,I'(4,3) + F(2,1)\,I'(4,4) + F(2,2)\,I(4,5)$$

$$= \frac{1}{16}(4 + 2 + 10 + 8 + 6 + 6) = \frac{1}{16} \cdot 36$$

$$= 2.25$$

$$G(3,1) = F(0,0)\,I'(3,1) + F(0,1)\,I'(3,2) + F(0,2)\,I'(3,3) + F(1,0)\,I'(4,1) + F(1,1)\,I(4,2) + F(1,2)\,I'(4,3) + F(2,0)\,I'(5,1) + F(2,1)\,I(5,2) + F(2,2)\,I'(5,3)$$

$$= \frac{1}{16}(2.8 + 1.5 + 4.9 + 2.[illegible]$$

$$= \frac{1}{16} \cdot 69 = 4.31$$

$$G(3,2) = F(0,0)\,I'(3,2) + F(0,1)\,I'(3,3) + F(0,2)\,I'(3,4)$$
$$+ F(1,0)\,I'(4,2) + F(1,1)\,I'(4,3) + F(1,2)\,I'(4,4)$$
$$+ F(2,0)\,I'(5,2) + F(2,1)\,I'(5,3) + F(2,2)\,I'(5,4)$$
$$= \frac{1}{16}(8 + 10 + 2 + 2.9 + 4.6 + 2.3)$$
$$= \frac{1}{16} \cdot 68 = 4.25$$

$$G(3,3) = F(0,0)\,I'(3,3) + F(0,1)\,I'(3,4) + F(0,2)\,I'(3,5)$$
$$+ F(1,0)\,I'(4,3) + F(1,1)\,I(4,4) + F(1,2)\,I'(4,5)$$
$$+ F(2,0)\,I'(5,3) + F(2,1)\,I'(5,4) + F(2,2)\,I'(5,5)$$
$$= \frac{1}{16} \cdot (5 + 4 + 2.6 + 4.3)$$
$$= \frac{1}{16} \cdot 33 = 2.06$$

$$G = \begin{bmatrix} 3.5 & 3.25 & 1.31 \\ & & 2.25 \\ 5.25 & 5 & \\ & & 2.06 \\ 4.31 & 4.25 & \end{bmatrix}$$

The filter $F = \frac{1}{16}\begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}$ is

Gaussian smoothing filter.

It is working as smoothing and blurring tool. It is designed to smooth image by averaging each pixel. It reduce high frequency noise, and create a less detailed appearance. The normalization factor $\frac{1}{16}$ ensure the smoothing is done in a way which preserve the overall brightness of input image.

(f) $F = \frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

$$G(1,1) = F(0,0)\bar{I}(1,1) + F(0,1)\bar{I}(1,2) + F(0,2)\bar{I}(1,3)$$
$$+ F(1,0)\bar{I}(2,1) + F(1,1)\bar{I}(2,2) + F(1,2)\bar{I}(2,3)$$
$$+ F(2,0)\bar{I}(3,1) + F(2,1)\bar{I}(3,2) + F(2,2)\bar{I}(3,3)$$
$$= \frac{1}{9}(0+0+0+0+7+4+8+5)$$
$$= \frac{1}{9} \cdot 24 = 2.67$$

$$G(1,2) = F(0,0)\bar{I}(1,2) + F(0,1)\bar{I}(1,3) + F(0,2)\bar{I}(1,4)$$
$$+ F(1,0)\bar{I}(2,2) + F(1,1)\bar{I}(2,3) + F(1,2)\bar{I}(2,4)$$
$$+ F(2,0)\bar{I}(3,2) + F(2,1)\bar{I}(3,3) + F(2,2)\bar{I}(3,4)$$
$$= \frac{1}{9}(7+4+1+8+5+2) = \frac{1}{9} \times 27 = 3$$

$$G(1,3) = F(0,0)\bar{I}(1,3) + F(0,1)\bar{I}(1,4) + F(0,2)\bar{I}(1,5)$$
$$+ F(1,0)\bar{I}(2,3) + F(1,1)\bar{I}(2,4) + F(1,2)\bar{I}(2,5)$$
$$+ F(2,0)\bar{I}(3,3) + F(2,1)\bar{I}(3,4) + F(2,2)\bar{I}(3,5)$$

$$= \frac{1}{9}(4+1+5+2) = \frac{1}{9} \times 12$$

$$= 1.33$$

$$G(2,1) = f(0,0)\,I'(2,1) + f(0,1)\,I'(2,2) + f(0,2)\,I'(2,3)$$
$$+ f(1,0)\,\bar{I}(3,1) + f(1,1)\,\bar{I}(3,2) + f(1,2)\,\bar{I}(3,3)$$
$$+ f(2,0)\,\bar{I}(4,1) + f(2,1)\,\bar{I}(4,2) + f(2,2)\,\bar{I}(4,3)$$
$$= \frac{1}{9}(7+4+8+5+9+6) = \frac{1}{9} \times 39$$
$$= 4.33$$

$$G(2,2) = f(0,0)\,\bar{I}(2,2) + f(0,1)\,\bar{I}(2,3) + f(0,2)\,\bar{I}(2,4)$$
$$+ f(1,0)\,\bar{I}(3,2) + f(1,1)\,\bar{I}(3,3) + f(1,2)\,\bar{I}(3,4)$$
$$+ f(2,0)\,\bar{I}(4,2) + f(2,1)\,\bar{I}(4,3) + f(2,2)\,\bar{I}(4,4)$$
$$= \frac{1}{9}(7+4+1+8+5+2+9+6+3)$$
$$= \frac{1}{9} \cdot 45 = 5$$

$$G(2,3) = f(0,0)\,\bar{I}(2,3) + f(0,1)\,\bar{I}(2,4) + f(0,2)\,\bar{I}(2,5)$$
$$+ f(1,0)\,\bar{I}(3,3) + f(1,1)\,\bar{I}(3,4) + f(1,2)\,\bar{I}(3,5)$$
$$+ f(2,0)\,\bar{I}(4,3) + f(2,1)\,\bar{I}(4,4) + f(2,2)\,\bar{I}(4,5)$$
$$= \frac{1}{9}(4+1+5+2+6+3) = \frac{1}{9} \times 21$$
$$= 2.33$$

$$G(3,1) = f(0,0)\,\bar{I}(3,1) + f(0,1)\,\bar{I}(3,2) + f(0,2)\,\bar{I}(3,3)$$
$$+ f(1,0)\,\bar{I}(4,1) + f(1,1)\,\bar{I}(4,2) + f(1,2)\,\bar{I}(4,3)$$
$$+ f(2,0)\,\bar{I}(5,1) + f(2,1)\,\bar{I}(5,2) + f(2,2)\,\bar{I}(5,3)$$
$$= \frac{1}{9}(0+8+5+0+9+6+0+0+0)$$
$$= \frac{1}{9} \times 28 = 3.11$$

$$G(3,2) = F(0,0)I(3,2) + F(0,1)I(3,3) + f(0,2)I(3,4)$$
$$+ f(1,0)I(4,2) + F(1,1)I(4,3) + f(1,2)I(4,4)$$
$$+ F(2,0)I(5,2) + f(2,1)I(5,3) + f(2,2)I(5,4)$$
$$= \frac{1}{9}(8+5+2+9+6+3+0+0+0)$$
$$= \frac{1}{9} \times 33 = 3.67$$

$$G(3,3) = f(0,0)I(3,3) + F(0,1)I(3,4) + f(0,2)I(3,5)$$
$$+ f(1,0)I(4,3) + f(1,1)I(4,4) + F(1,2)I(4,5)$$
$$+ f(2,0)I(5,3) + F(2,1)I(5,4) + F(2,2)I(5,5)$$
$$= \frac{1}{9}(5+2+0+6+3+8+0+0+0)$$
$$= \frac{1}{9} \times 16 = 1.77$$

$$G = \begin{bmatrix} 2.67 & 3 & 1.33 \\ 4.33 & 5 & 2.33 \\ 3.11 & 3.67 & 1.77 \end{bmatrix}$$

~~Here~~ the filters of (e) is gaussian smoothing filter and (f) is ~~[illegible]~~ Average moving filter.

Gaussian smoothing filter used for smoothing or blurring image. And Average moving filter are used for basic smoothing through ~~basic~~ pixel averaging. Gaussian smoothing emphasizes the central pixel, gradually diminishing weights toward the edges in a gaussian distribution. that filter ~~effective~~ for reduce noise and maintain image details. Basic smoothing simplifies the smoothing and less focus on preserving details. It applies uniform weights to all pixels and results a straightforward average of neighboring values without a selective emphasis.

Ans to the Ques No 2

(a)

According to formula, the correlation defined as,

$$G(i,j) = \sum_{u=0}^{k-1} \sum_{v=0}^{l-1} F(u,v) . \bar{I}(i+u, j+v)$$

where, F is the filter, $\bar{I} \in R^{(m+k-1) \times (n+l-1)}$ is the original image, I, padded with zeros along it's edges.

Now, we can define filter F as a vector representation of f,

$$f = vector(F)$$

Also, we can represt $\bar{I}(i,j)$ as $t(i,j)$ which is the vector representation of neighborhood patch of images.

$$t(i,j) = vector(\bar{I}(i-1 : i+2, j-1 : j+2))$$

We have to show that. We can write corrdation as a vector dot product,

$$G(i,j) = f^T t_{ij}$$

As we apply vectorization which turns a matrix into a single column, we have to apply dot product. For performing that dot product we need to use the neighborhood ~~path~~ patch as the dot product of a row vector and a column vector can be expressed as a multiplication. ~~of~~ The corrdation operation as involving corresponding element in neighborhood and summing the result.

$G(i,j) = f^T t_{ij}$ reshaping f and neighborhood path $I(i,j)$. finally we can say that, corrlation operation can perform as a vector dot product.

$$f = [f(0,0) \ldots f(u,v)] \qquad t_{ij} = \begin{bmatrix} I(i,j) \\ I(i, j+1) \\ \vdots \\ I(i+u, j+v) \end{bmatrix}$$