Hypothesis Testing I

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What is a Hypothesis?

- A hypothesis is a claim (assumption) about a population parameter:
  - population mean

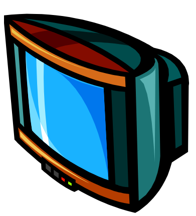
Example: The mean monthly cell phone bill of Dhaka city is μ = Tk. 250

The Null Hypothesis, H_0

- States the assumption (numerical) to be tested

> Example: The average number of TV sets in BD homes is equal to one $(H_0 : \mu = 1)$

- Is always about a population parameter, not about a sample statistic



$H_0 : \mu = 1$

$\cancel{H_0 : \bar{X} = 1}$

The Null Hypothesis, H_0

(continued)

- Begin with the assumption that the null hypothesis is true
  - Similar to the notion of innocent until proven guilty
- May or may not be rejected

The Alternative Hypothesis, H_1

- Is the opposite of the null hypothesis
  - e.g., The average number of TV sets in BD homes is not equal to 1 ($H_1: \mu \neq 1$)

- Is generally the hypothesis that the researcher is trying to support

Level of Significance, α

- **Defines the unlikely values of the sample statistic if the null hypothesis is true**
  - Defines rejection region of the sampling distribution
- Is designated by α , (level of significance)
  - Typical values are .01, .05, or .10
- Is selected by the researcher at the beginning
- Provides the critical value(s) of the test

Level of Significance and the Rejection Region

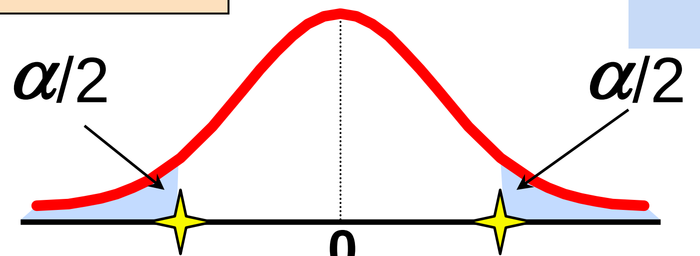
Level of significance = α

Represents critical value

Rejection region is shaded

H_0: $\mu = 3$

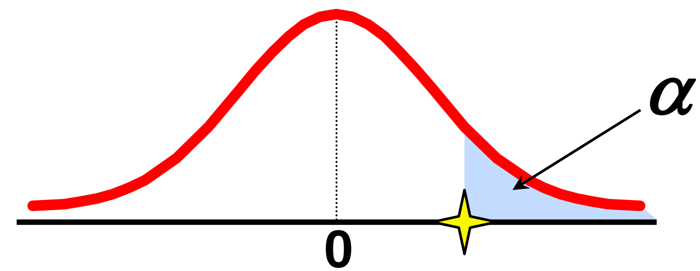
H_1: $\mu \neq 3$



Two-tail test

H_0: $\mu \leq 3$

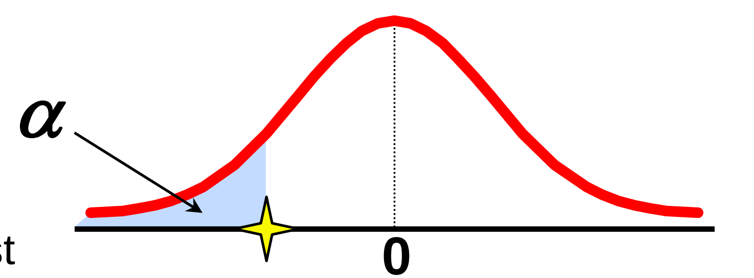
H_1: $\mu > 3$



Upper-tail test

H_0: $\mu \geq 3$

H_1: $\mu < 3$



Lower-tail test

Errors in Making Decisions

- **Type I Error**
  - Reject a true null hypothesis
  - Considered a serious type of error

The probability of Type I Error is α

  - Called level of significance of the test
  - Set by researcher in advance

Errors in Making Decisions

(continued)

- **Type II Error**
  - Fail to reject a false null hypothesis

The probability of Type II Error is β

Outcomes and Probabilities

Possible Hypothesis Test Outcomes

	Actual Situation	
Decision	H_0 True	H_0 False
Do Not Reject **H_0**	**No error** **($1 - \alpha$)**	**Type II Error** **(β)**
Reject **H_0**	**Type I Error** **(α)**	**No Error** **($1 - \beta$)**

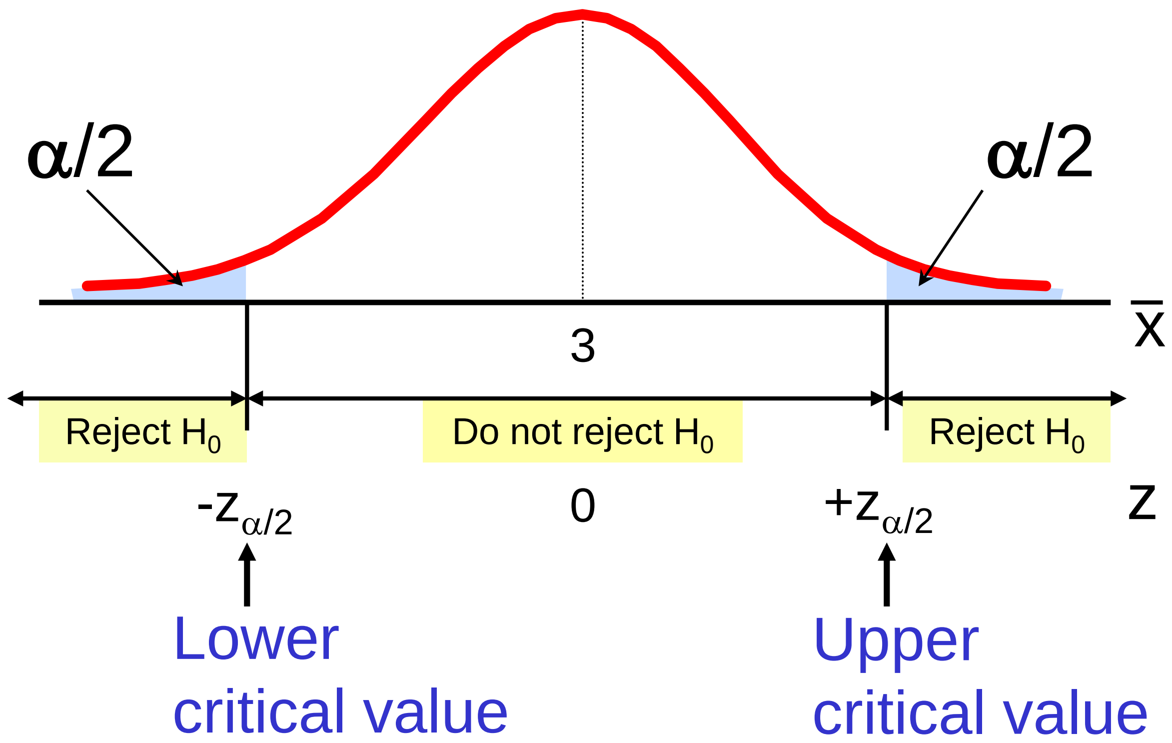
Key:
Outcome
(Probability)

Two-Tail Tests

- In some settings, the alternative hypothesis does not specify a unique direction

H_0: $\mu = 3$
H_1: $\mu \neq 3$

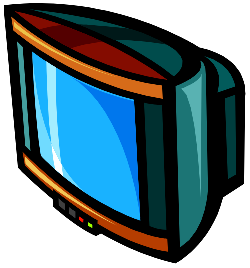
- There are two critical values, defining the two regions of rejection



Hypothesis Testing Example

Test the claim that the true mean # of TV sets in US homes is equal to 3. (Assume σ = 0.8)

- State the appropriate null and alternative hypotheses
  - H_0: $\mu = 3$, H_1: $\mu \neq 3$ (This is a two tailed test)
- Specify the desired level of significance
  - Suppose that $\alpha = .05$ is chosen for this test
- Choose a sample size
  - Suppose a sample of size $n = 100$ is selected



Hypothesis Testing Example

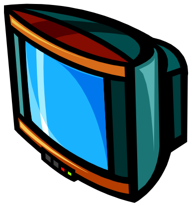
(continued)

- Determine the appropriate technique
  - σ is known so this is a z test
- Set up the critical values
  - For $\alpha = .05$ the critical z values are ±1.96
- Collect the data and compute the test statistic
  - Suppose the sample results are

    $n = 100$, $\bar{x} = 2.84$ ($\sigma = 0.8$ is assumed known)

So the test statistic is:

$$z = \frac{\bar{X} - \mu_0}{\frac{\sigma}{\sqrt{n}}} = \frac{2.84 - 3}{\frac{0.8}{\sqrt{100}}} = \frac{-.16}{.08} = -2.0$$

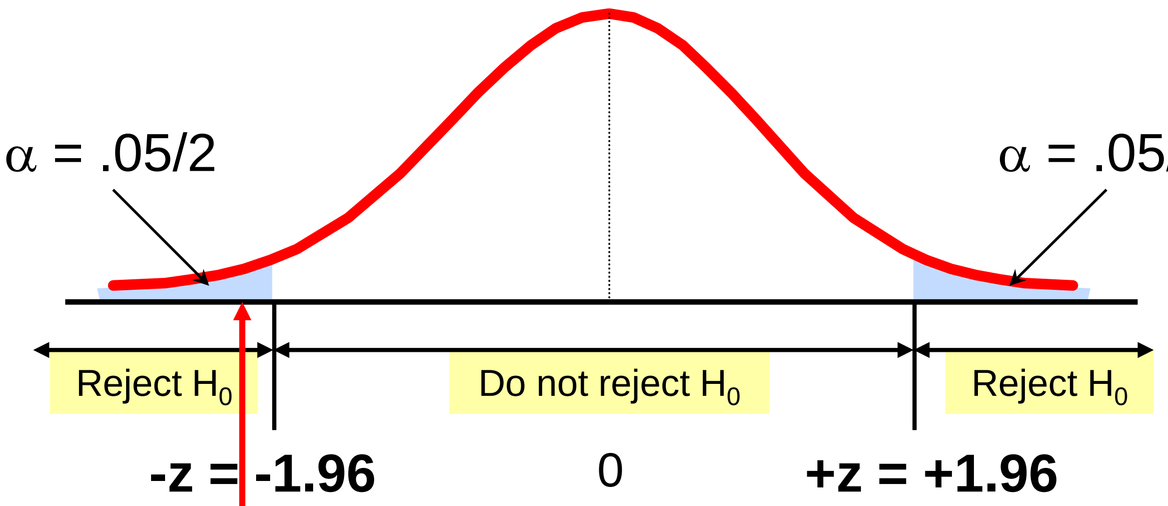


Hypothesis Testing Example

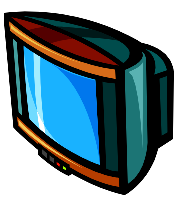
(continued)

- Is the test statistic in the rejection region?

Reject H_0 if $z < -1.96$ or $z > 1.96$; otherwise do not reject H_0



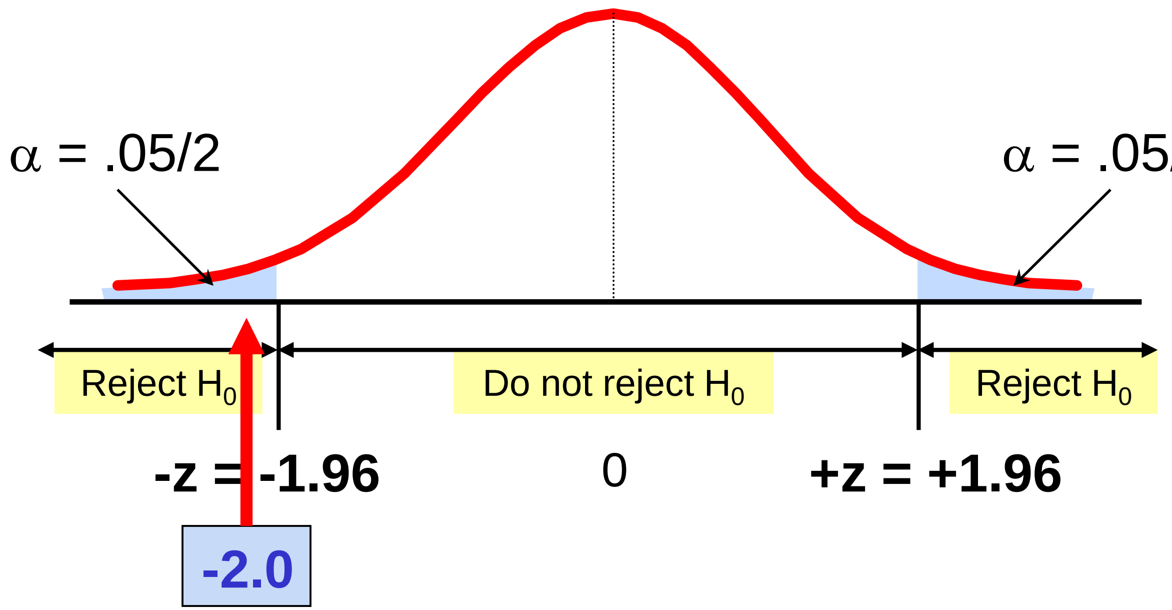
Here, $z = -2.0 < -1.96$, so the test statistic is in the rejection region



Hypothesis Testing Example

(continued)

- Reach a decision and interpret the result



Since $z = -2.0 < -1.96$, we <u>reject the null hypothesis</u> and conclude that there is sufficient evidence that the mean number of TVs in US homes is not equal to 3