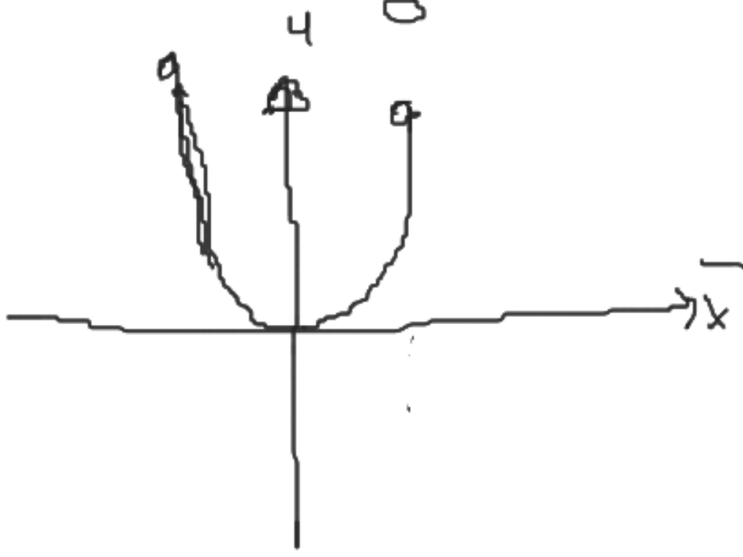
0.2 New function for old?

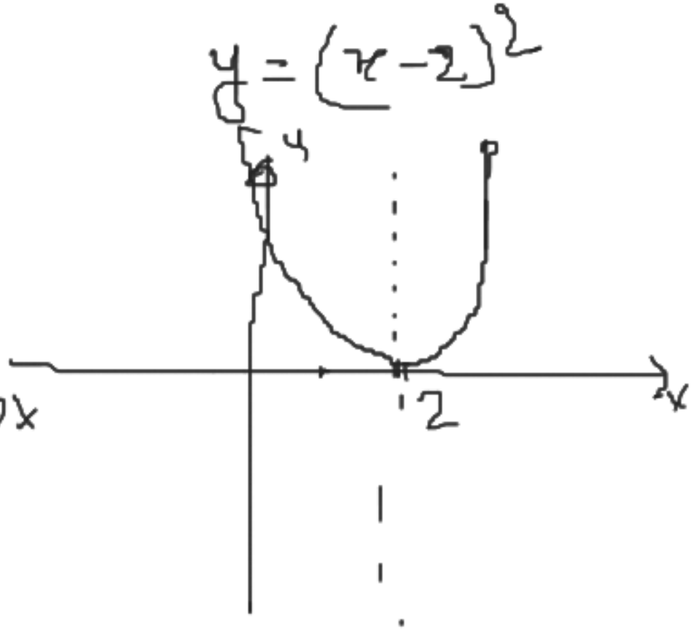
Ex 8: $y = x^2 - 4x + 5$

Sol^n: $y = x^2 - 4x + 5 = x^2 - 4x + 4 + 1 = (x-2)^2 + 1$

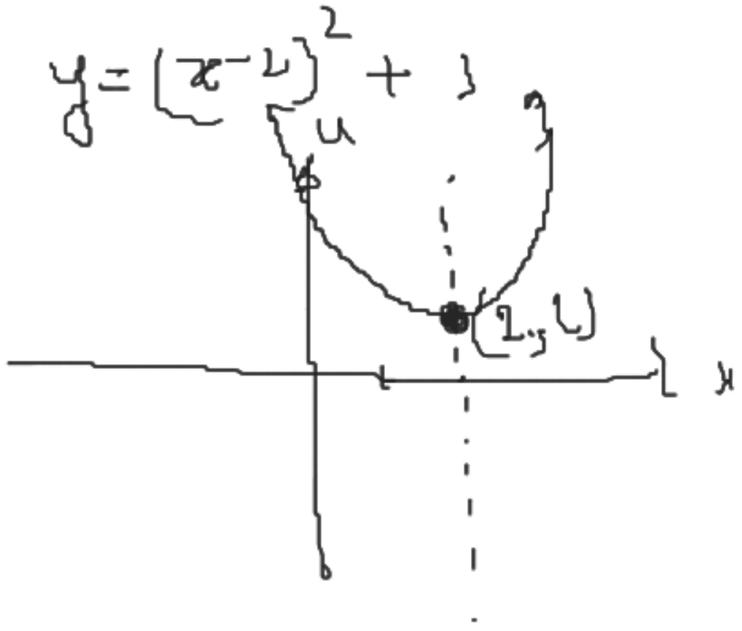
$y = x^2$



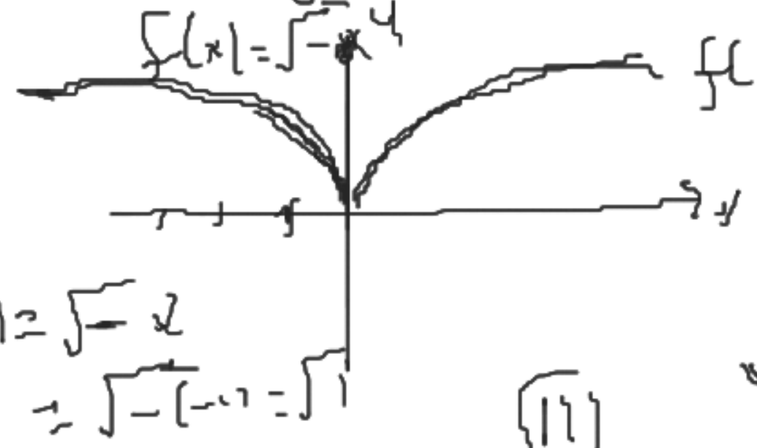
$y = (x-2)^2$



$y = (x-2)^2 + 1$



Reflections: (I) $y = f(-x)$ of $y = f(x)$ about y-axis

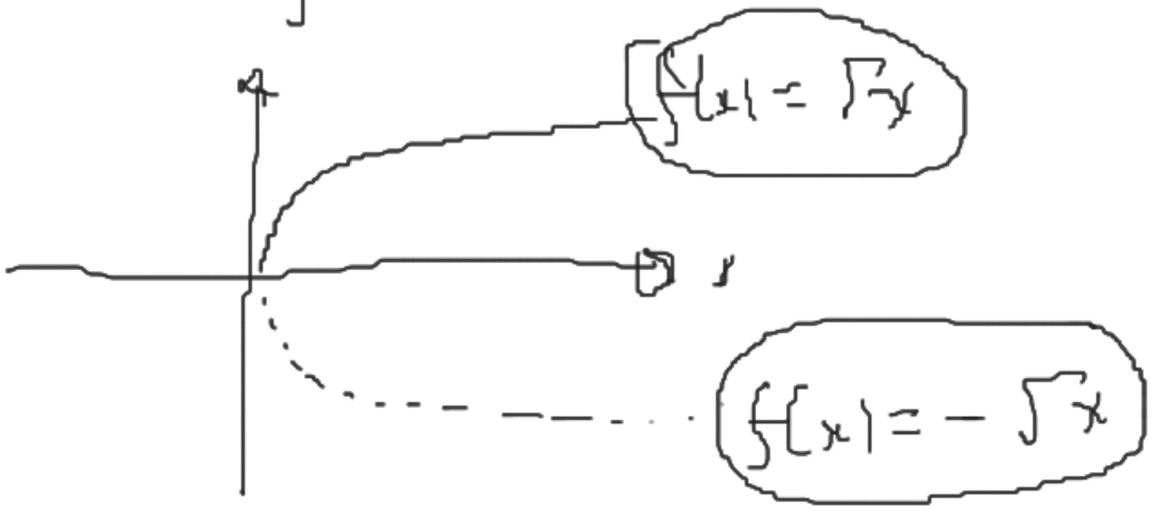


$f(x) = \sqrt{-x}$

$= \sqrt{-(-1)} = \sqrt{1}$

(II) $y = -f(x)$ of $y = f(x)$ about x-axis

$f(x) = \sqrt{x}$, $f(x) = -\sqrt{x}$



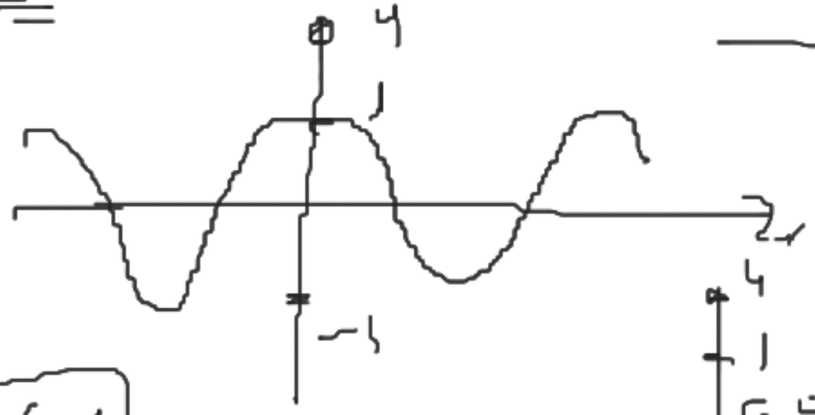
Stretches and Compressions

$y = f(x)$

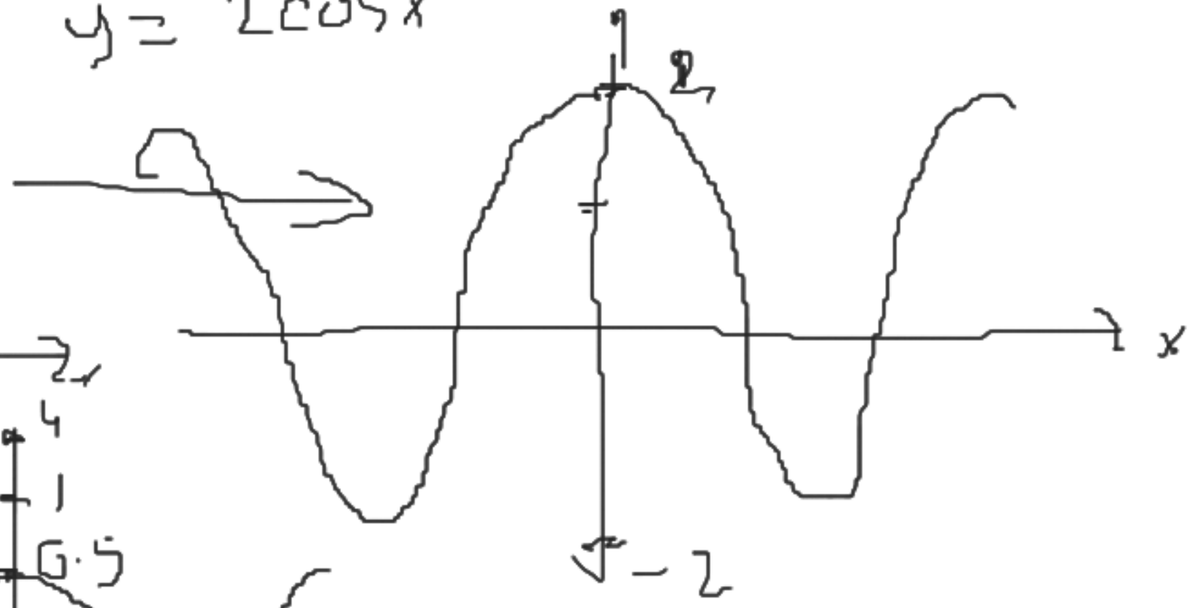
$y = c\,f(x)$; $c > 0$

① If $c > 1$: $y = cf(x)$ ← stretches on y-axis

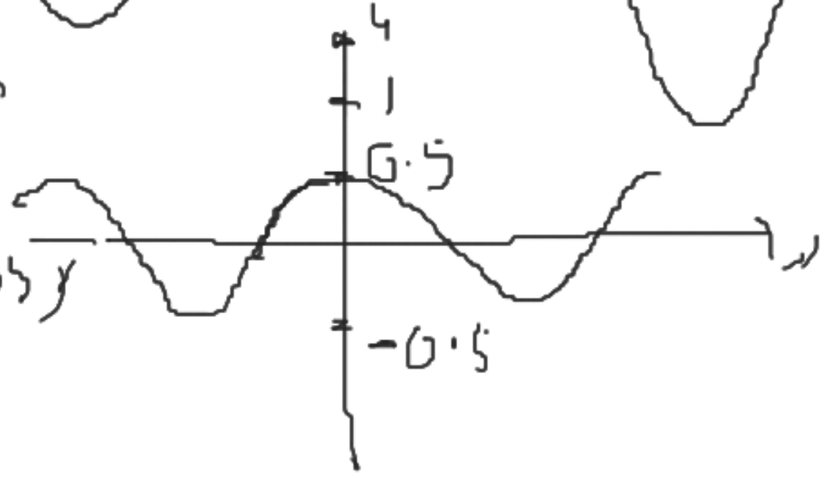
Ex $y = \cos x$



$y = 2\cos x$



If $0 < c < 1$

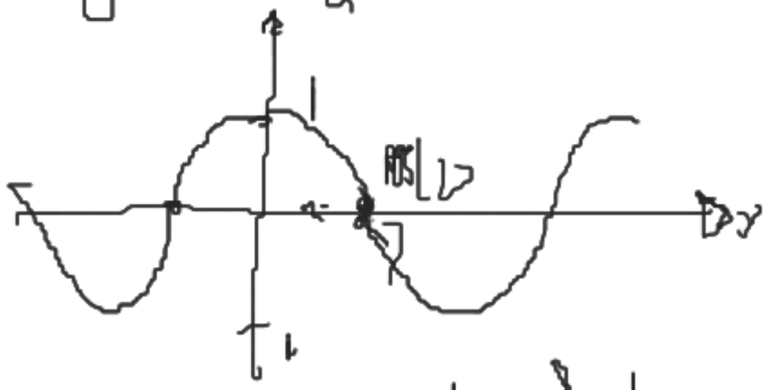


$y = 0.5\cos x$

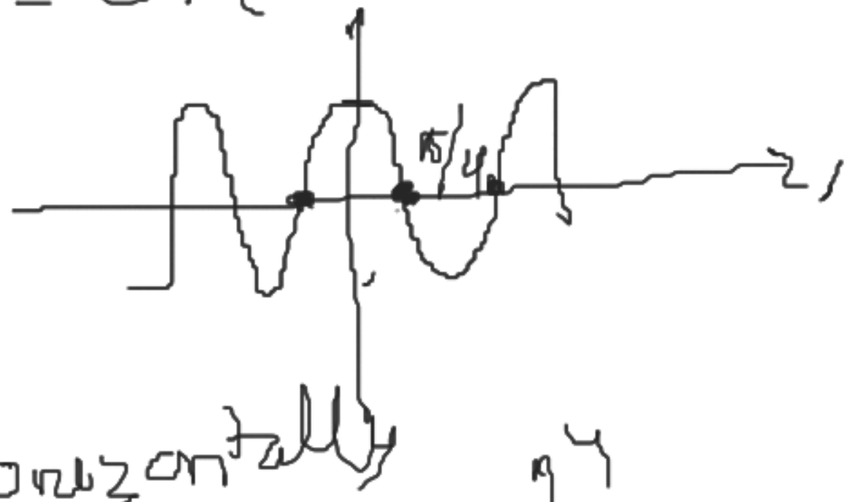
(11) $y = f(x)$ $y = f(cx)$ i $c > 0$

If $c > 1$: Compressed the graph horizontally

Ex: $y = \cos x$

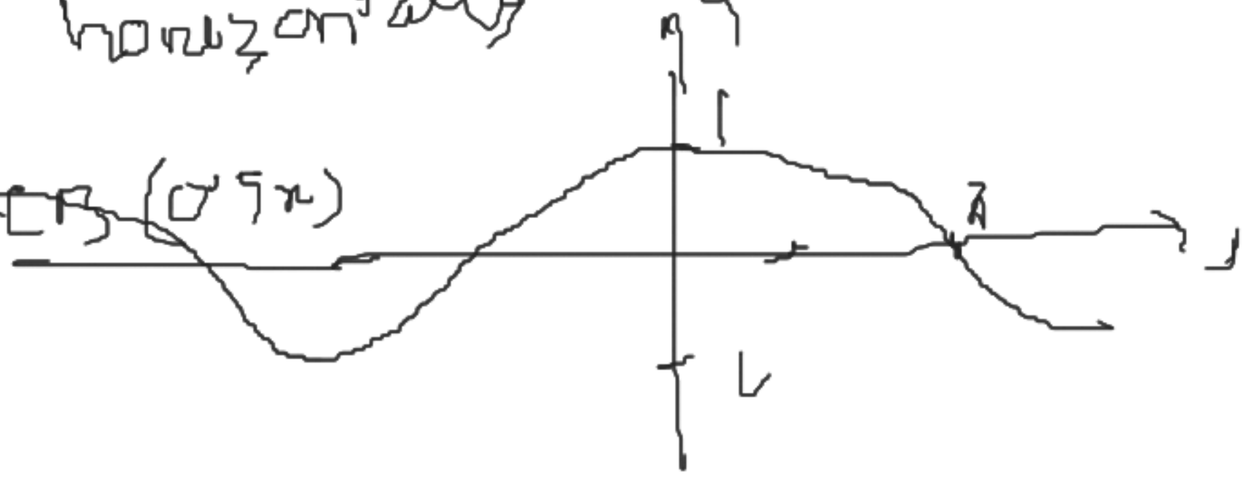


$y = \cos(2x)$

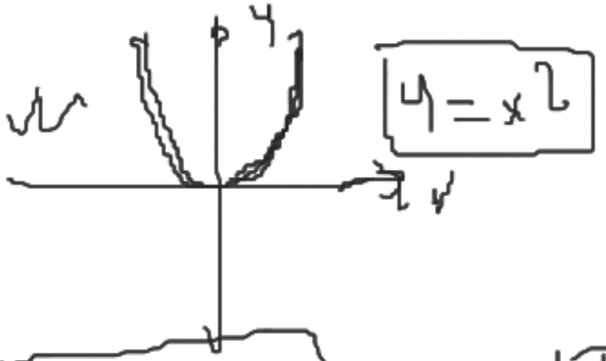


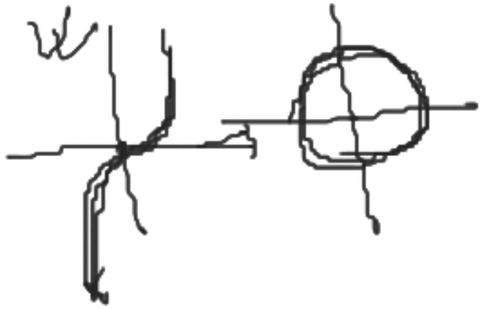
If $0 < c < 1$ Stretches horizontally

$y = \cos x$, $y = \cos(0.5x)$



Symmetry :





Test : (i) $x \to -x$ Ex: $y(x) = x^2$

$y(-x) = (-x)^2 = x^2$

(ii) $y \to -y$ Ex: $y^2 = x$

$(-y)^2 = x$: $y^2 = x$

(iii) If $x \to -x$ & $y \to -y$ symmetric about origin

Ex: $y = x^3$

$-y = (-x)^3 \Rightarrow -y = -x^3$

$\therefore y = x^3$

Even & Odd : $f(-x) = f(x)$ — Even

$f(-x) = -f(x)$ — Odd

Ex. $f(x) = x^2$ — Even

$\therefore f(-x) = (-x)^2 = x^2 = f(x)$

Ex. $f(x) = x^3$

$f(-x) = (-x)^3 = -x^3 = -f(x)$

— Odd

03 Families of Functions:

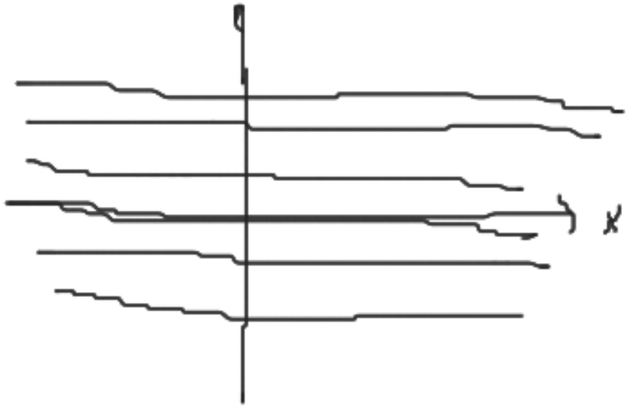
$f(x) = c$

$f(x) = 1$

$f(x) = 2$

$f(x) = 3$

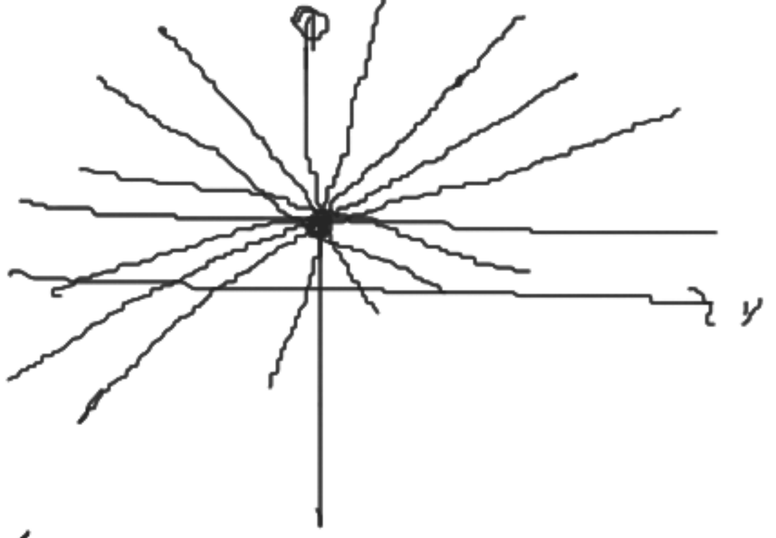
$f(x) = 0$



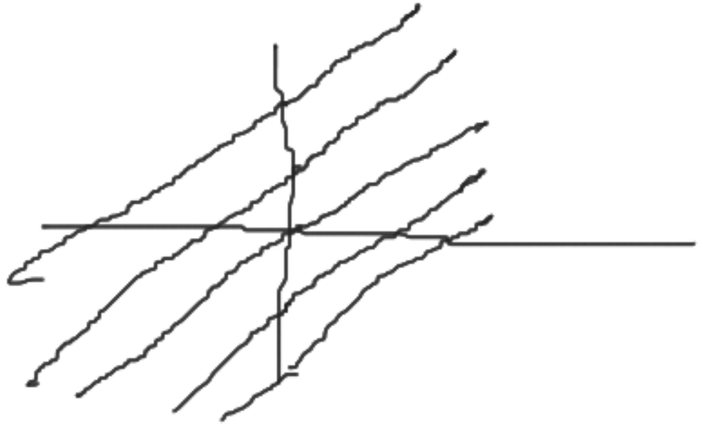
$y = mx + b$ ve

b - is ~~constant~~ fixed

Let $b = 1$, m - vary



Let $m = \pi/4$, b - vary.



Power Functions: $y = x^n$

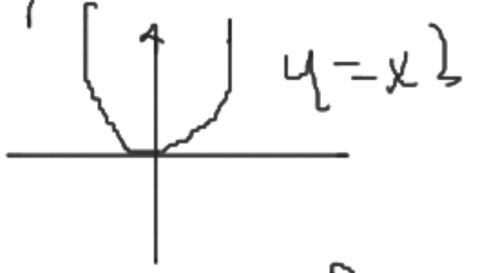
For n is positive integer

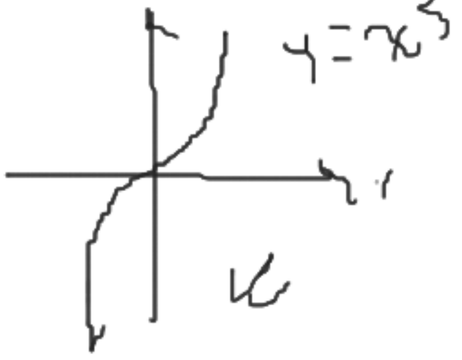
If $n=1$: $y = x$

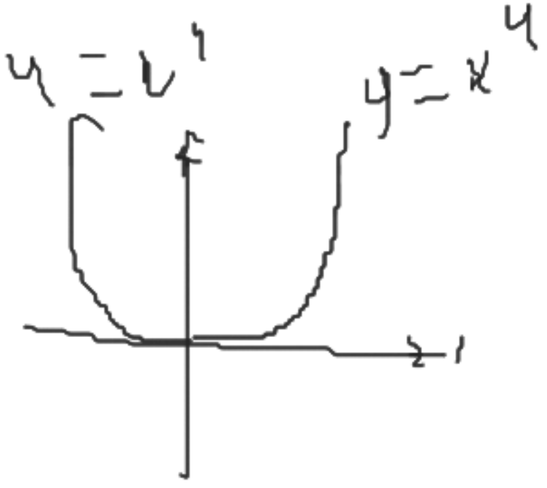
" $n=2$: $y = x^2$

" $n=3$: $y = x^3$

" $n=4$: $y = x^4$







If $n \geq 2$

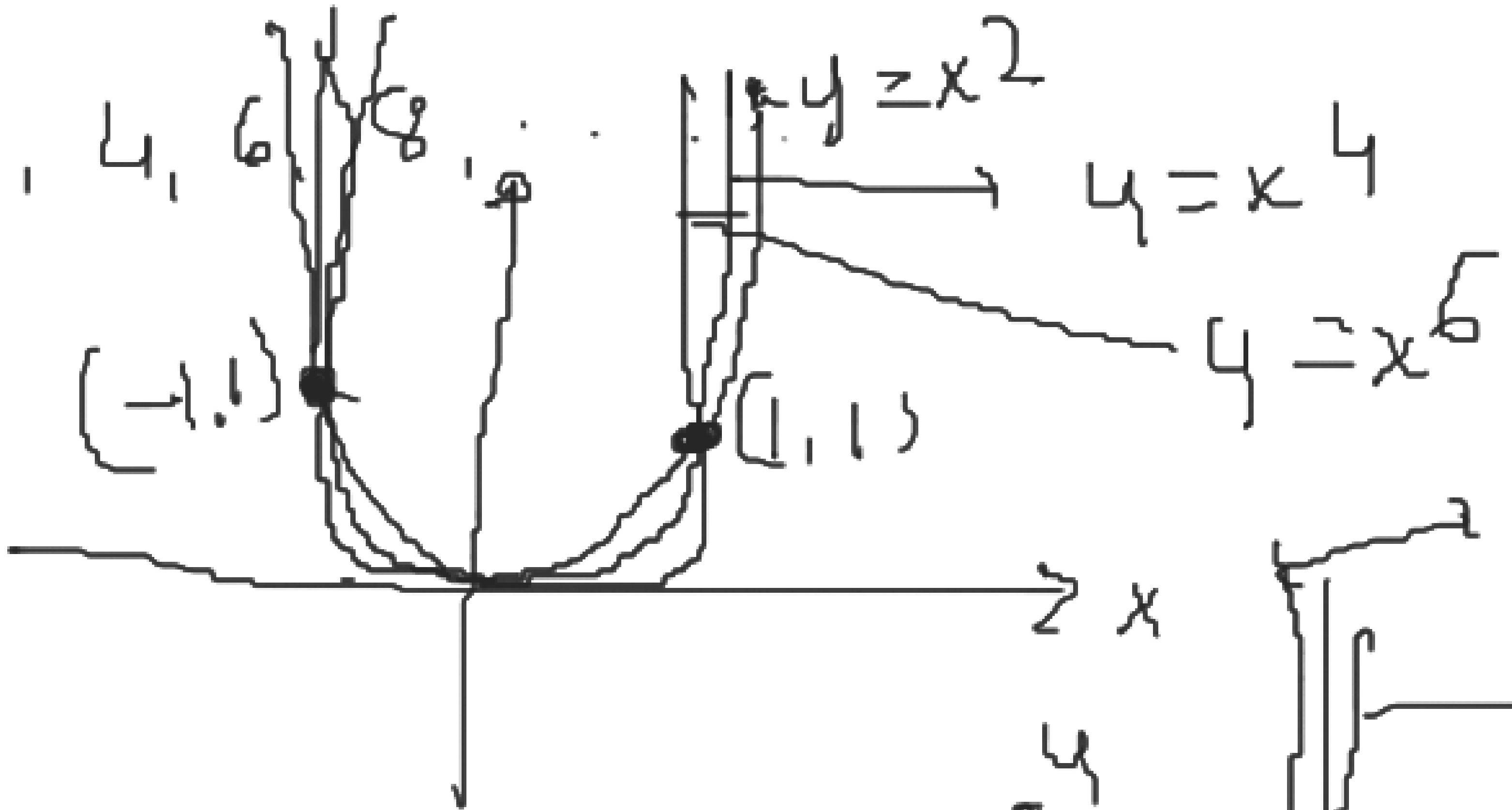
(i) If n is even: $n = 2, 4, 6, 8, \ldots$

$y = x^2$

$y = x^4$

$y = x^n$

$y = x^6$ ✓



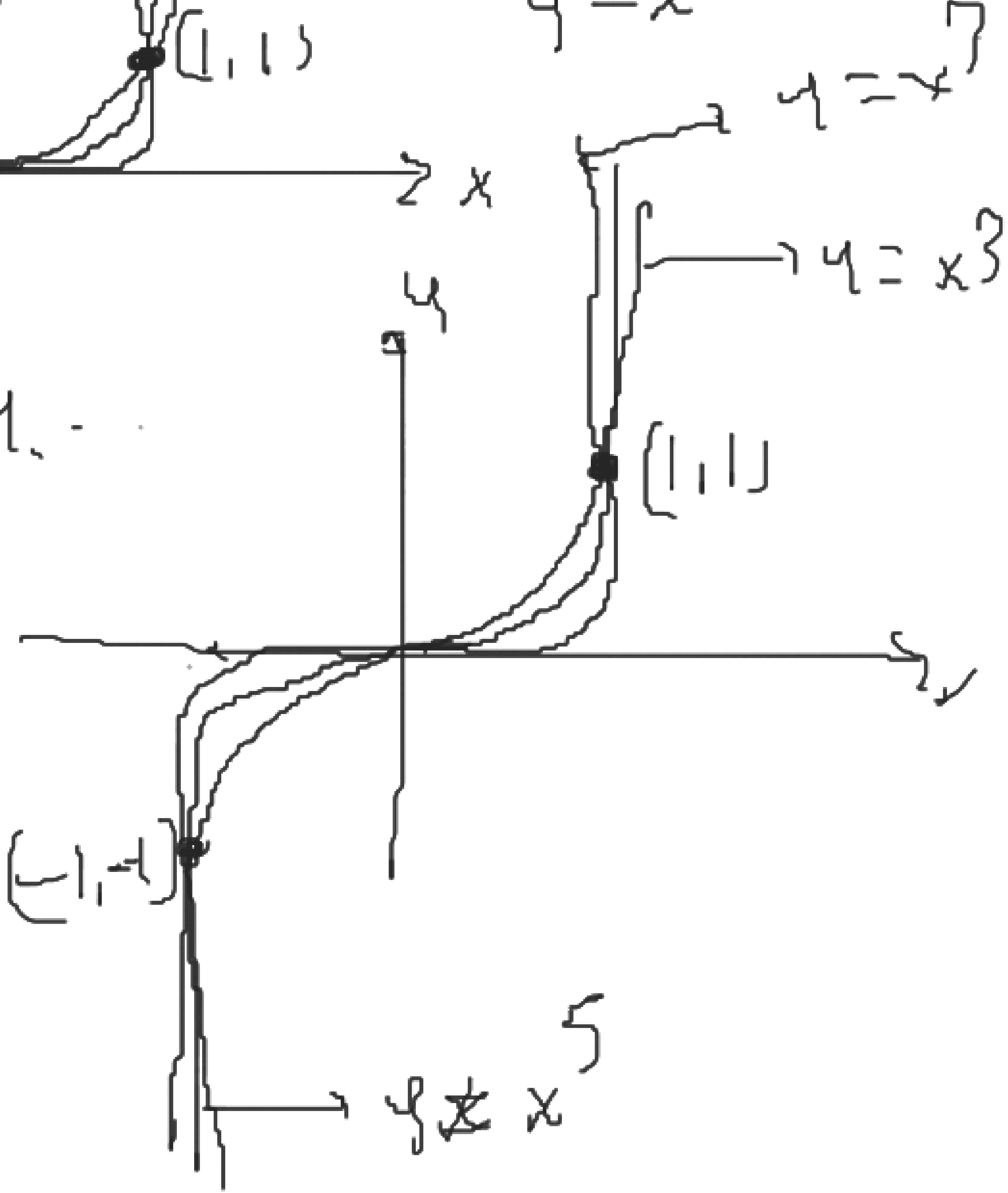
(ii) If n is odd: $n = 3, 5, 7, 9, \ldots$

$y = x^3$

$y = x^5$

$y = x^n$

$y = x^7$ ✓

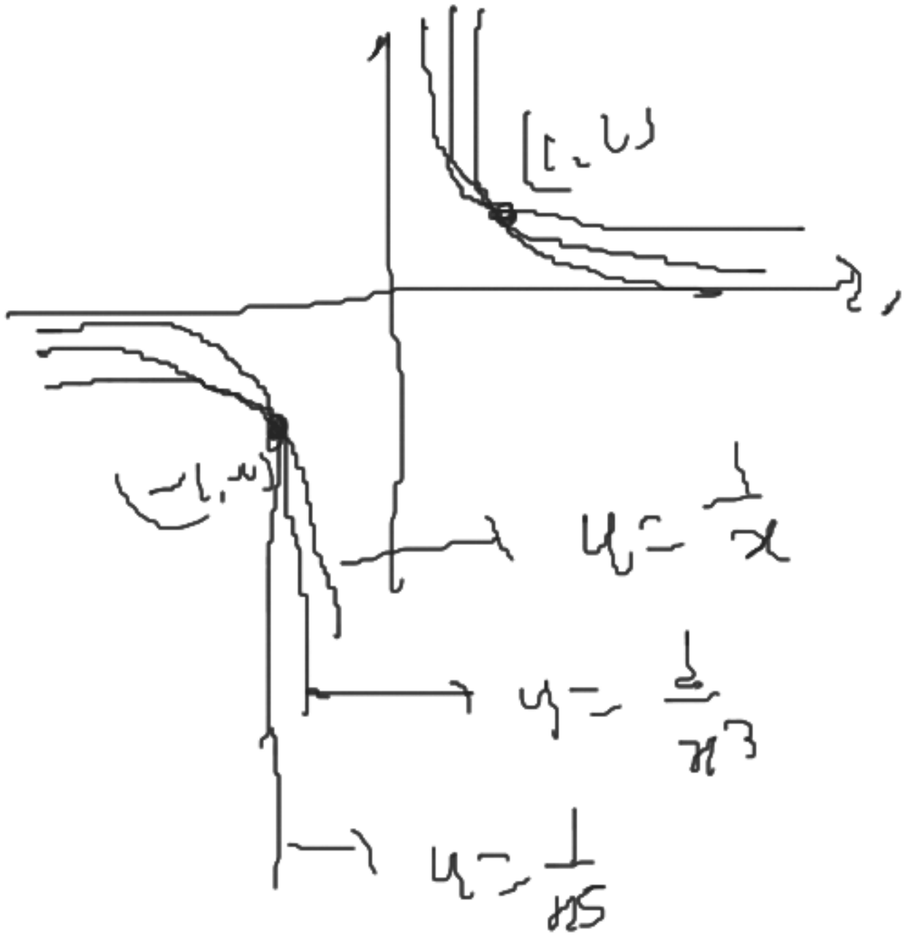


$y = x^{-n}$

① 当 $n=1$, $y = x^{-1} \Rightarrow y = \frac{1}{x}$

② $n=3$. $y = \frac{1}{x^3}$

③ $n=5$. $y = \frac{1}{x^5}$

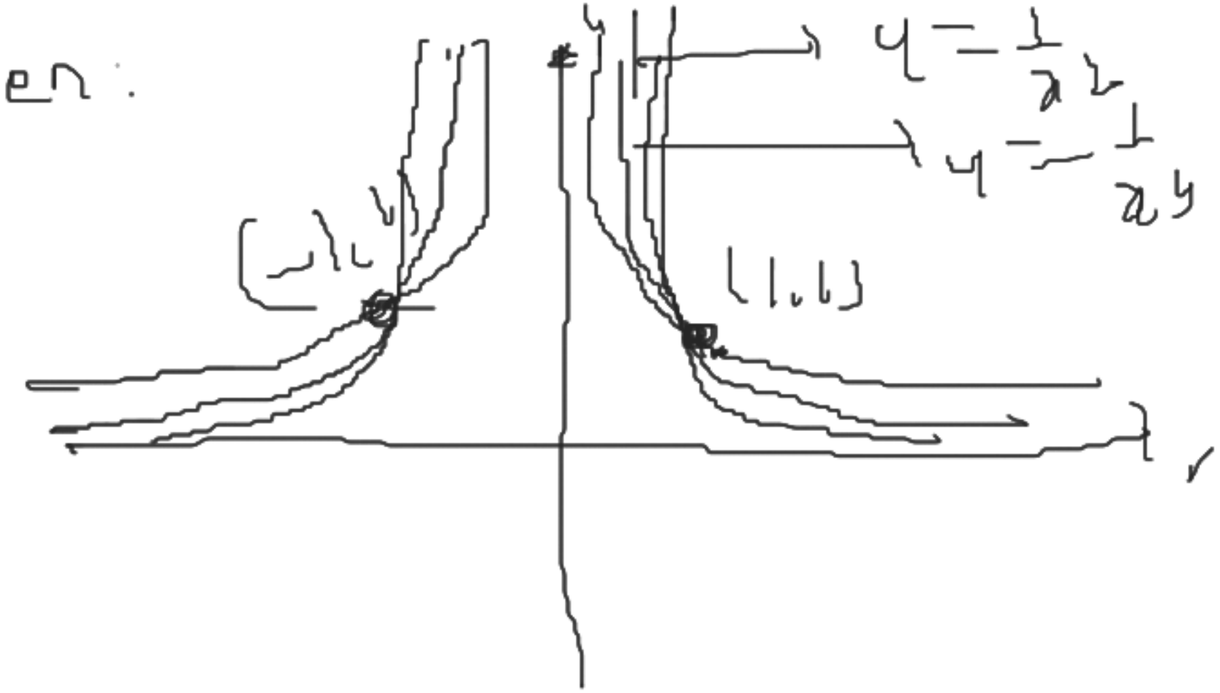


(11) $y = x^{-n}$, if n is even.

$y = \frac{1}{x^2}$

$y = \frac{1}{x^4}$

$y = \frac{1}{x^6}$



Polynomial : cx^n

$c_0 + c_1 x + c_2 x^2 + c_3 x^3 + \cdots + c_n x^n$

Ex: 2 , $2+x$, $2+x+3x^{\textcircled{2}}$, x^3+3

$x^7 - 3x + \sqrt{2}$, $\sqrt{x}+3$ ✗

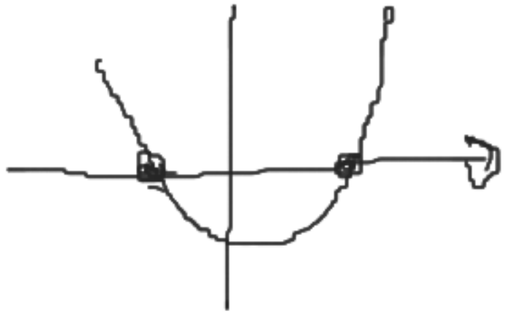
$x^{-1} + x^3$ ✗

$2 = 2x^{\textcircled{0}}$

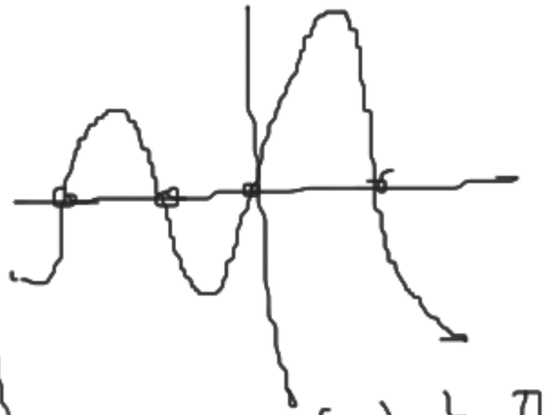
$2+x = 2+x^{\textcircled{1}}$

$0 = 0x$ $\quad 0 \cdot x^2 = 0 \cdot x^7$

x^1 ; $D: [0, \infty)$



$f(x) = 5 = 5 \cdot x^0$



Rational Functions:

$$f(x) = \frac{P(x)}{Q(x)}$$

$\therefore Q(x) \neq 0$

Ex. $f(x) = \frac{2}{x+2}$

$x \neq -2$

If $f(x) = \frac{x^3}{x^2+2x-1}$

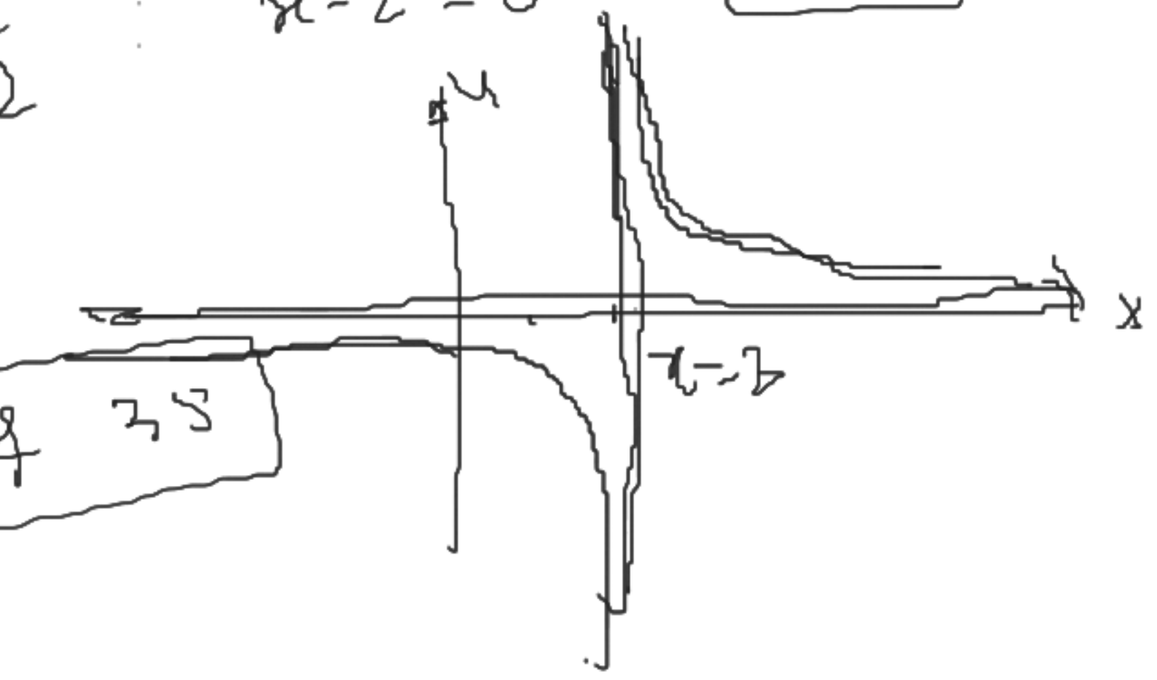
$f(x) = \frac{\sqrt{x}}{x-1}$ $x \geq 0$

(i) Continuum, the $f(x) = \frac{P(x)}{Q(x)}$ is discontinuum at $Q(x) = 0$

(ii) Rational function not defined at $Q(x) = 0$

— Vertical asymptote

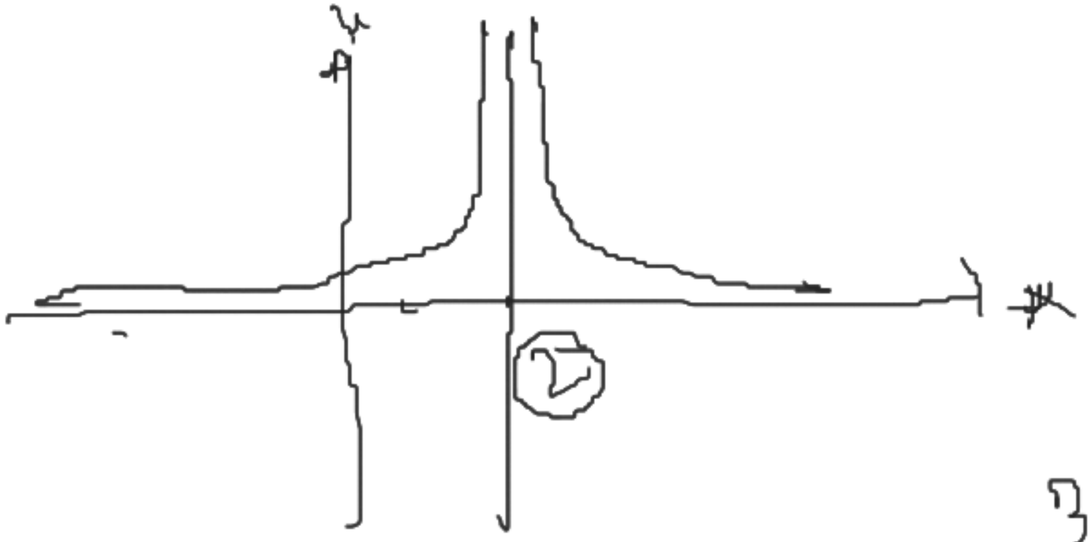
$f(x) = \frac{1}{x-2}$ $\quad x - 2 = 0 \Rightarrow x = 2$



(iii)

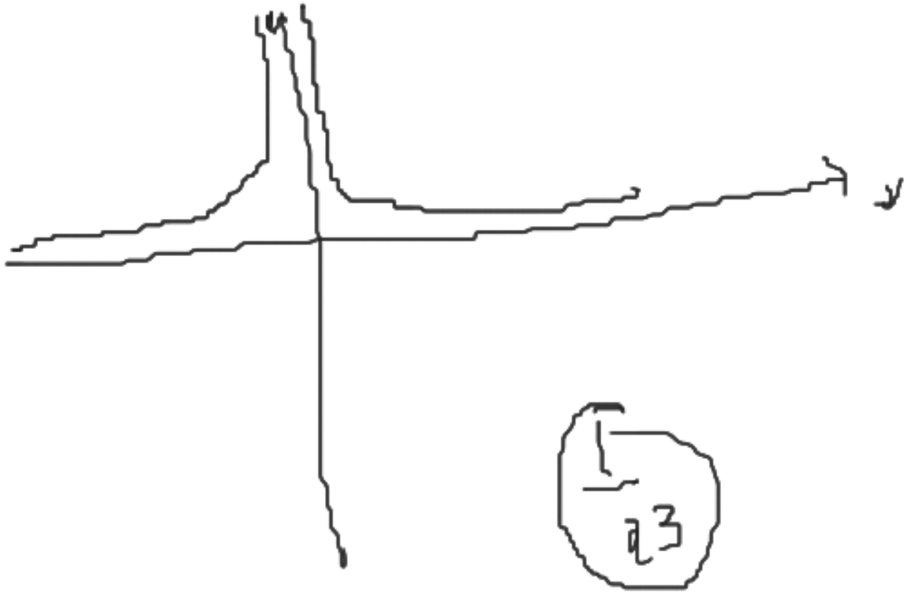
H.W. Ex 0.3: 13-17 & 35

4 $y = \frac{1}{(x-2)^2}$: $x = 2$



$y = \frac{3}{(x-1)^3}$

$y = \frac{1}{x^2}$



$\frac{1}{a^3}$