Course Name : Physics – I
Course # PHY 107

Examples on Traveling Waves

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Example # 16.10: The equation of a transverse wave traveling along a very long string is $y = 6.0 \sin(0.020\pi x + 4.0\pi t)$, where x and y are expressed in centimeters and t is in seconds. Determine (a) the amplitude, (b) the wavelength, (c) the frequency, (d) the speed, (e) the direction of propagation of the wave, and (f) the maximum transverse speed of a particle in the string. (g) What is the transverse displacement at $x = 3.5\,\mathrm{cm}$ when $t = 0.26\,\mathrm{s}$?

Solution: Comparing to the standard equation: $y(x,t) = A\sin(kx + \omega t)$, we easily get:

(a) Amplitude, $A = |y_{\mathrm{max}}| = 6.0\,\mathrm{cm}$.

(b) Wave number, $k = 0.020\pi\,\mathrm{r/cm}$. $\therefore\ \lambda = 2\pi/k = 100\,\mathrm{cm}$.

(c) Angular frequency, $\omega = 4.0\pi\,\mathrm{r/s}$. $\therefore\ f = \omega/2\pi = 2\,\mathrm{Hz}$.

(d) Wave speed, $v = f\lambda = 200\,\mathrm{cm/s}$.

(e) The direction of propagation is negative.

(f) The Max. Transverse Speed $= \left|\frac{\partial y}{\partial t}\right| = (6.0)(4.0\pi)\,\mathrm{cm/s} = 24\pi\,\mathrm{cm/s}$.

(g) $$y(3.5\mathrm{cm}, 0.26\mathrm{sec}) = (6.0\,\mathrm{cm})\sin(0.020\pi \times 3.5 + 4.0\pi \times 0.06)$$
$$= (6.0\,\mathrm{cm})\sin(1.11\pi) = -2.03\,\mathrm{cm}.$$

Example # 16.17: The linear mass density of a string is $1.6 \times 10^{-4}\,\mathrm{kg/m}$. A transverse wave is setup that obeys

$$y(x,t) \;=\; \left(0.021\,\mathrm{m}\right) \sin\left[(2.0\,\mathrm{m}^{-1})x + (30\,\mathrm{s}^{-1})t\right] \; .$$

Find the wave speed and the tension in the string.
Solution: Given: $\mu = 1.6 \times 10^{-4}\,\mathrm{kg/m}$. Now compare to the standard equation: $y = A\sin(kx + \omega t)$, we easily find that amplitude $A = 0.021\,\mathrm{m}$, wave number $k = 2.0\,\mathrm{r/m}$ and angular frequency $\omega = 30\,\mathrm{r/s}$.
Therefore, the wave speed is: $v = (\omega/k) = (30/2)\mathrm{m/s} = 15\,\mathrm{m/s}$. And the tension in the string is: $T = \mu v^2 = 1.6 \times 10^{-4} \times (15)^2\,\mathrm{N} = 0.036\,\mathrm{N}$.

Example # 16.53: A string is oscillating according to

$$y(x,t) \;=\; (0.50\,\mathrm{cm})\,\sin\left[\left(\frac{\pi}{3}\,\mathrm{cm}^{-1}\right)x\right]\,\cos\left[\left(40\pi\,\mathrm{s}^{-1}\right)t\right]\;,$$

which is a standing wave. What are: amplitude, speed of the waves and the distance between the nearest nodes.

Solution: Comparing to the standard equation: $y = 2A\sin(kx)\,\cos(\omega t)$, we easily find that amplitude $A = (0.50\,\mathrm{cm})/2 = 0.25\,\mathrm{cm}$, wave number $k = (\pi/3)\,\mathrm{r/cm}$ and angular frequency $\omega = 40\pi\,\mathrm{r/s}$. So, the wavelength is $\lambda = 2\pi/k = 2\pi/(\pi/3)\,\mathrm{cm} = 6\,\mathrm{cm}$. Therefore, the wave speed is: $v = (\omega/k) = (40\pi)/(\pi/3)\mathrm{cm/s} = 120\,\mathrm{cm/s}$.

Now, the nodes are located at $x_n = n(\lambda/2)$, where n is positive integers. Therefore, the distance between any two nearest nodes is

$$\Delta x = x_{\mathrm{n+1}} - x_{\mathrm{n}} = (n+1)\frac{\lambda}{2} - n\frac{\lambda}{2} = \frac{\lambda}{2} = \frac{6}{2}\mathrm{cm} = 3.0\,\mathrm{cm}\;.$$