A series-parallel configuration is one that is formed by a combination of series and parallel elements.

A complex configuration is one in which none of the elements are in series or parallel.

In this chapter, we examine the series-parallel combination using the basic laws introduced for series and parallel circuits. There are no new laws or rules to learn—simply an approach that permits the analysis of such structures. In the next chapter, we consider complex networks using methods of analysis that allow us to analyze any type of network.

The possibilities for series-parallel configurations are infinite. Therefore, you need to examine each network as a separate entity and define the approach that provides the best path to determining the unknown quantities. In time, you will find similarities between configurations

7.2 SERIES-PARALLEL NETWORKS

The network in Fig. 7.1 is a series-parallel network. At first, you must be very careful to determine which elements are in series and which are in parallel. For instance, resistors R_1 and R_2 are *not* in series due to resistor R_3 connected to the common point b between R_1 and R_2. Re-

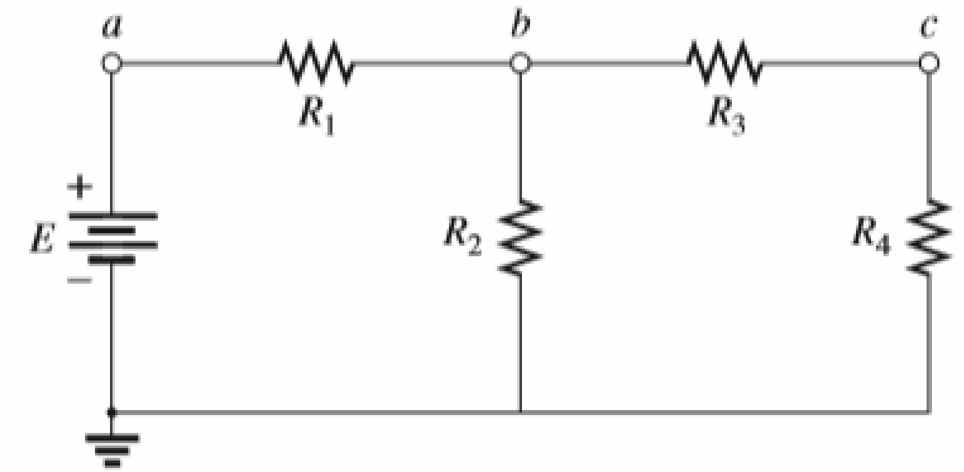


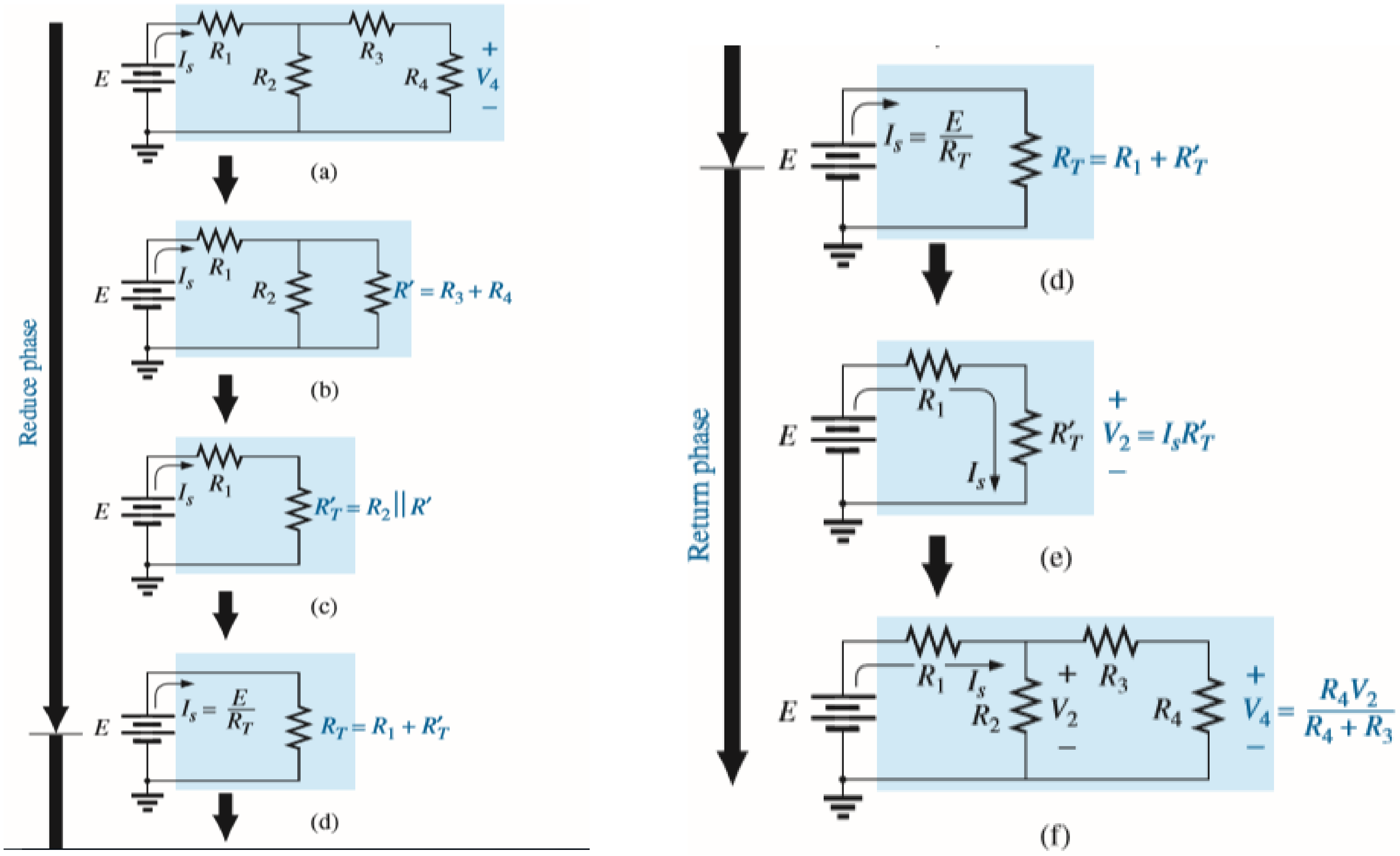
FIG. 7.1
Series-parallel dc network.

Take a moment to study the problem “in total” and make a brief mental sketch of the overall approach you plan to use. The result may be time- and energy-saving shortcuts.

Examine each region of the network independently before tying them together in series-parallel combinations. This usually simpli-

Redraw the network as often as possible with the reduced

7.3 REDUCE AND RETURN APPROACH



We can now proceed with the **return phase** whereby we work our way back to the desired voltage V_4. Due to the resulting series configuration, the source current is also the current through R_1 and R'_T. The voltage across R'_T (and therefore across R_2) can be determined using Ohm's law as shown in Fig. 7.2(e). Finally, the desired voltage V_4 can be determined by an application of the voltage divider rule as shown in Fig. 7.2(f).

The *reduce and return approach* has now been introduced. This process enables you to reduce the network to its simplest form across the source and then determine the source current. In the return phase, you use the resulting source current to work back to the desired unknown. For

EXAMPLE 7.1 Find current I_3 for the series-parallel network in Fig. 7.3.

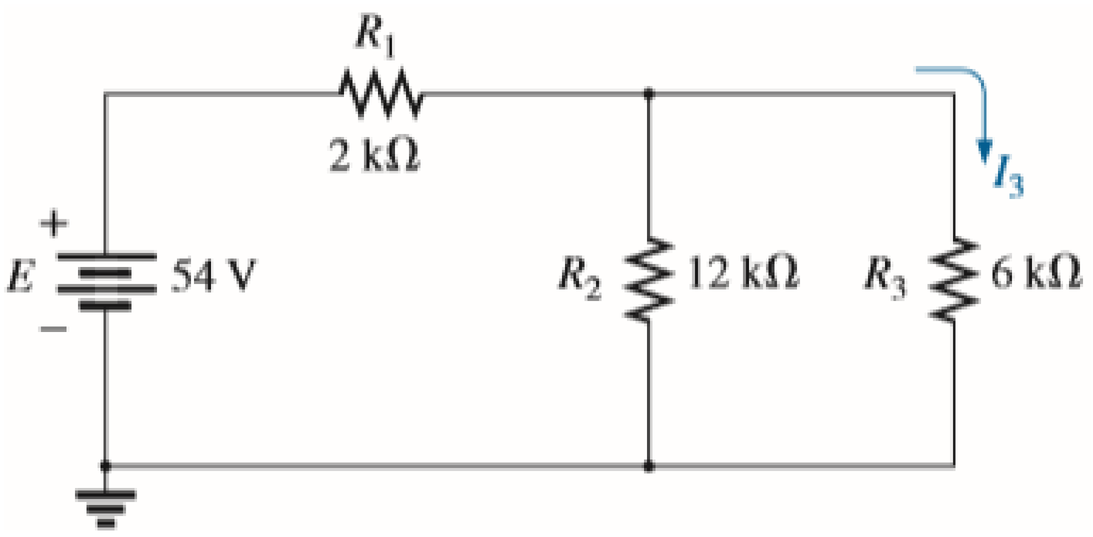


FIG. 7.3

EXAMPLE 7.2 For the network in Fig. 7.5:

a. Determine currents I_4 and I_s and voltage V_2.
b. Insert the meters to measure current I_4 and voltage V_2.

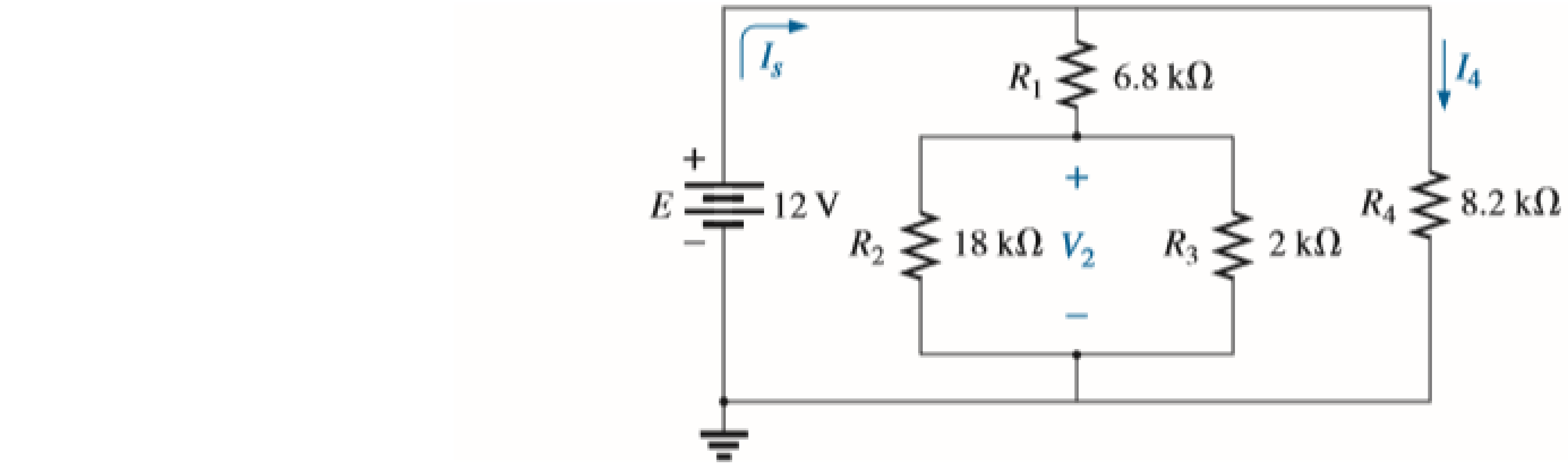


FIG. 7.5

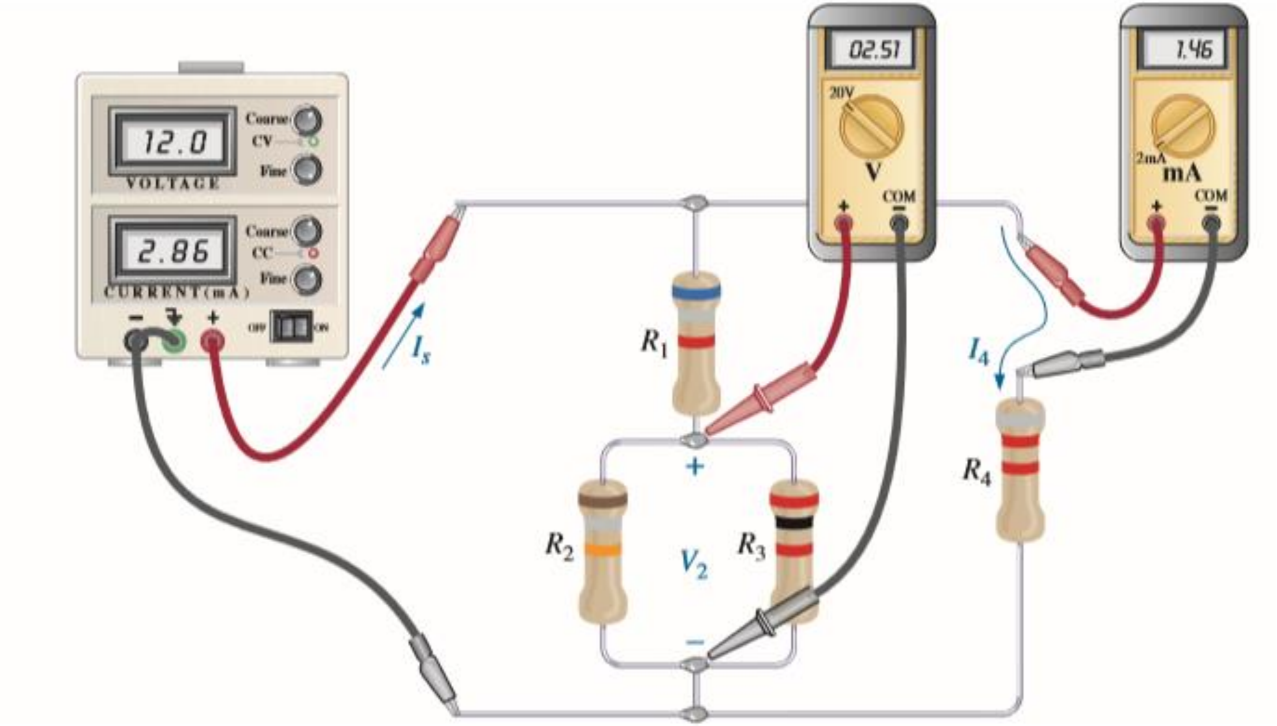


FIG. 7.7
Inserting an ammeter and a voltmeter to measure I_4 and V_2, respectively.

b. The meters have been properly inserted in Fig. 7.7. Note that the voltmeter is across both resistors since the voltage across parallel elements is the same. In addition, note that the ammeter is in series with resistor R_4, forcing the current through the meter to be the same as that through the series resistor. The power supply is displaying the source current.

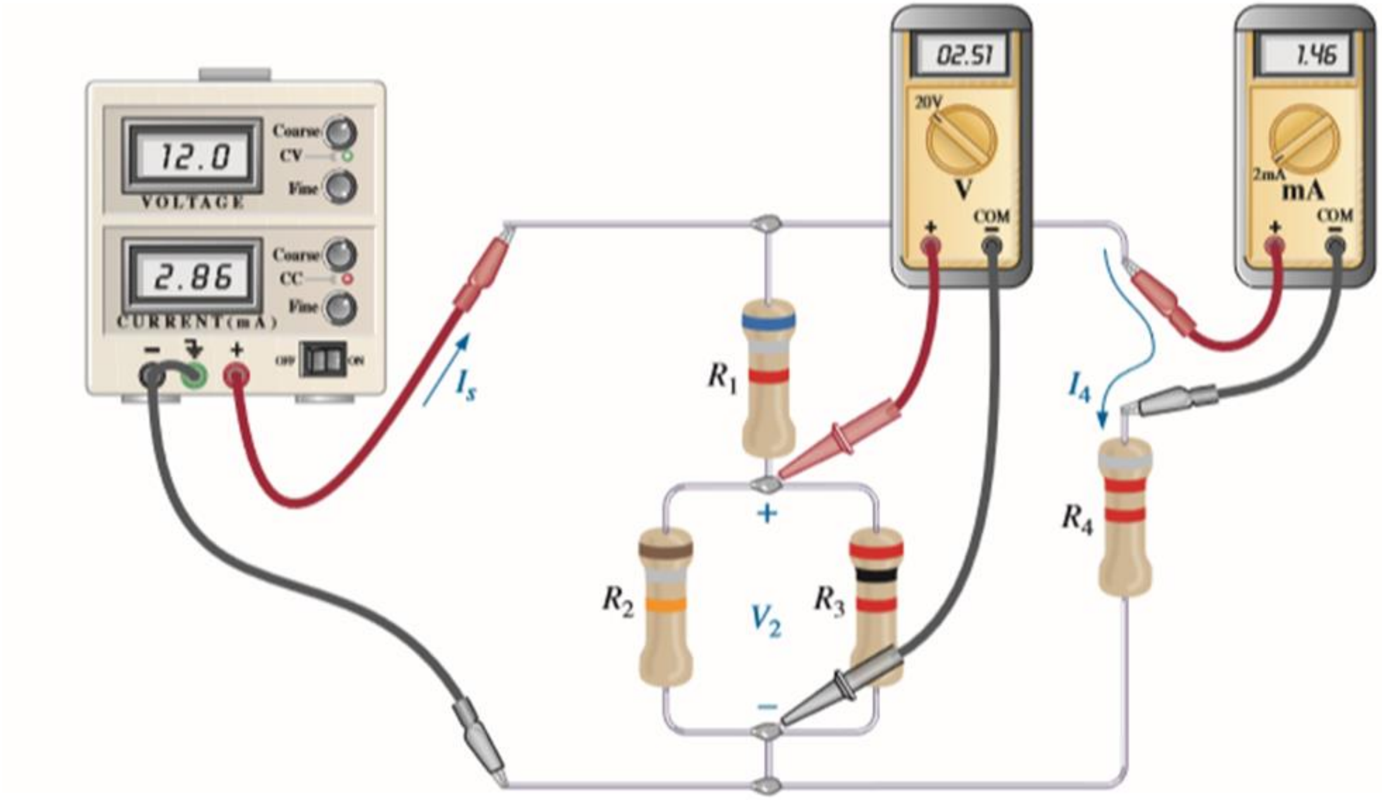


FIG. 7.7

Inserting an ammeter and a voltmeter to measure I_4 and V_2, respectively.

7.4 BLOCK DIAGRAM APPROACH

In the previous example, we used the reduce and return approach to find the desired unknowns. The direction seemed fairly obvious and the solution relatively easy to understand. However, occasionally the approach is not as obvious, and you may need to look at groups of elements rather than the individual components. Once the grouping of elements reveals the most direct approach, you can examine the impact of the individual components in each group. This grouping of elements is called the *block diagram approach* and is used in the following examples.

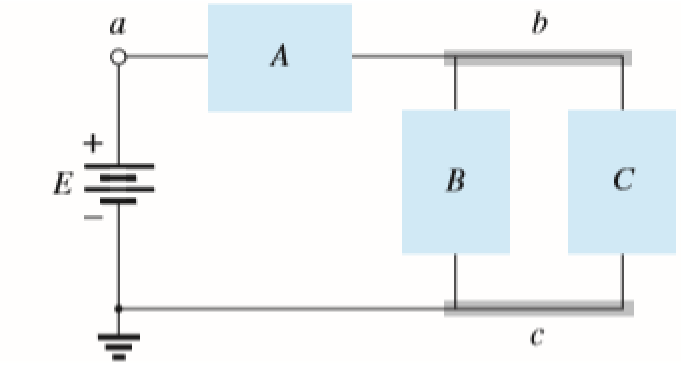


FIG. 7.8
Introducing the block diagram approach.

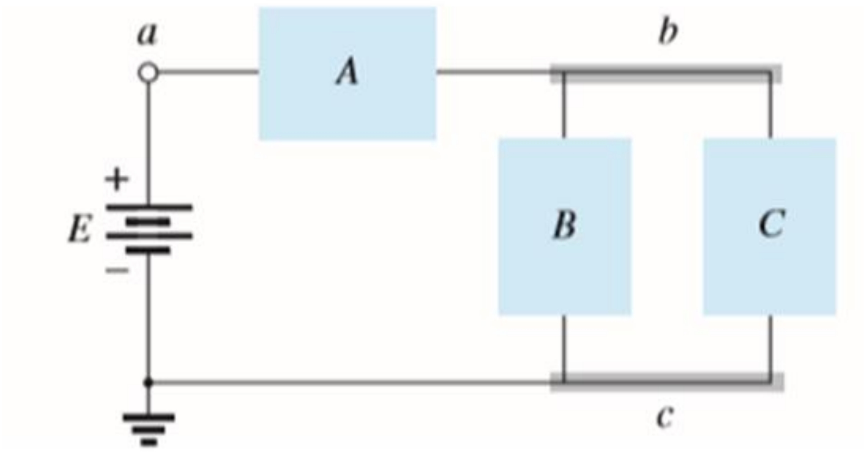


FIG. 7.8
Introducing the block diagram approach.

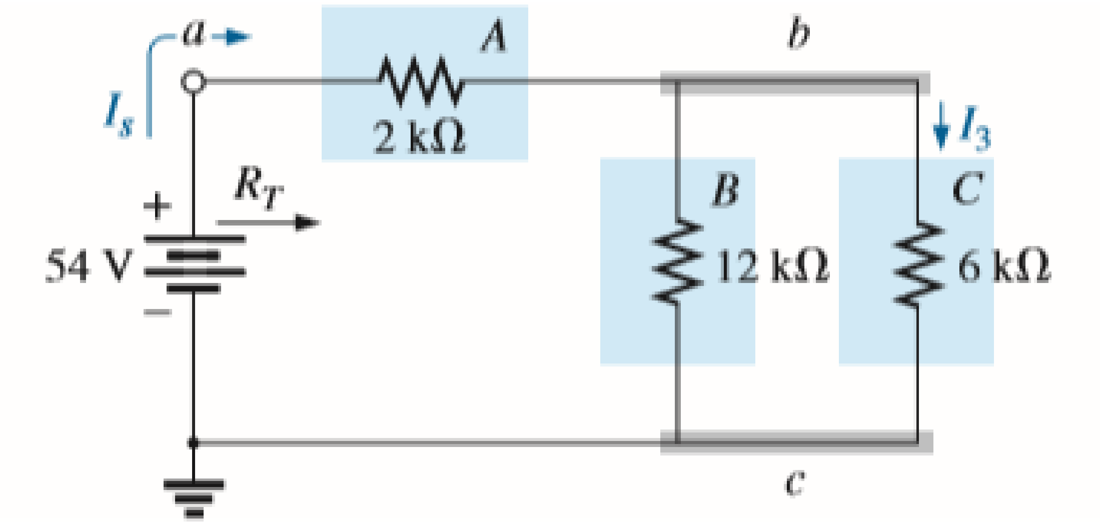


FIG. 7.9
Block diagram format of Fig. 7.3.

EXAMPLE 7.3 Determine all the currents and voltages of the network in Fig. 7.10.

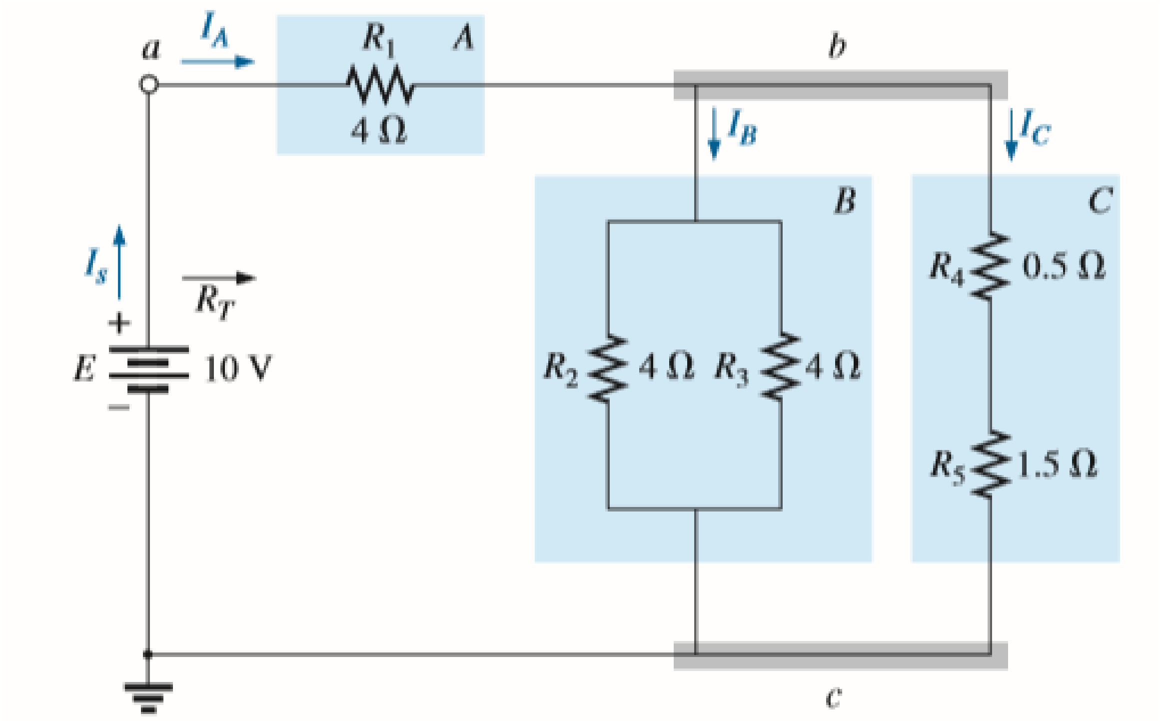


FIG. 7.10

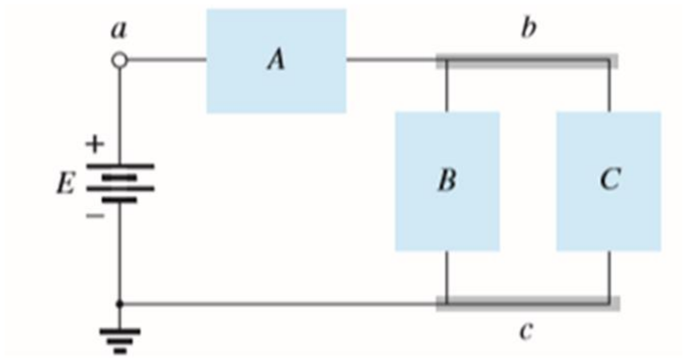


FIG. 7.8
Introducing the block diagram approach.

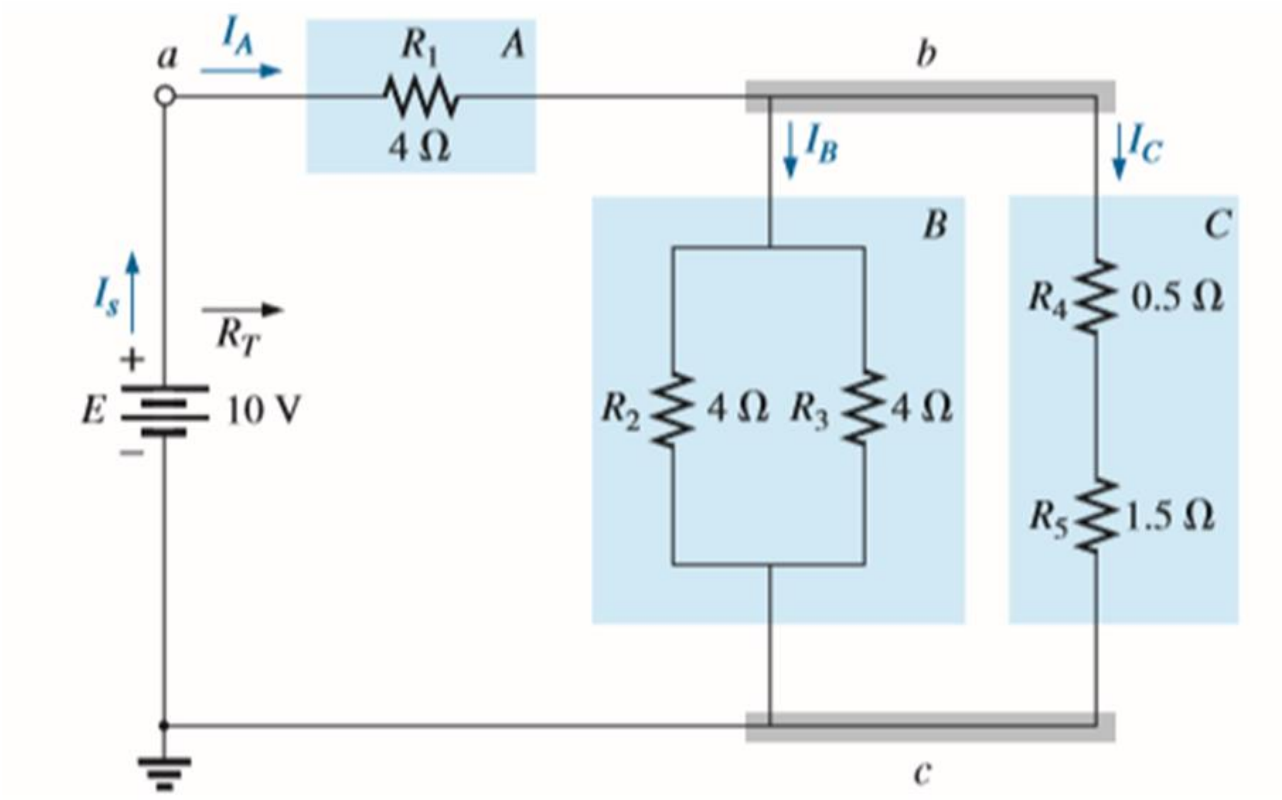


FIG. 7.10

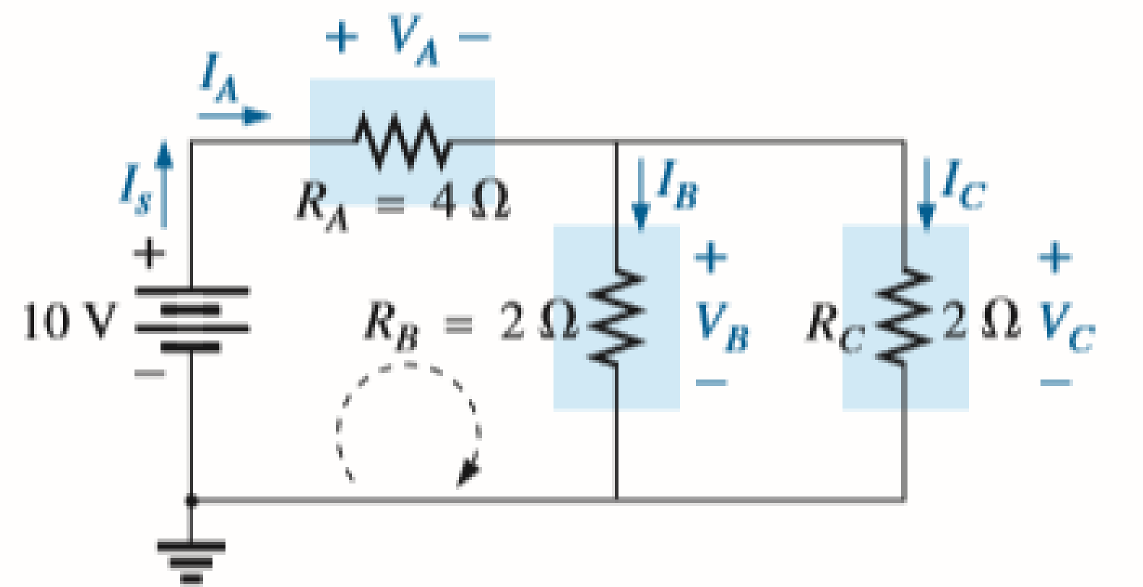


FIG. 7.11

Applying Kirchhoff's voltage law for the loop indicated in Fig. 7.11, we obtain

$$\Sigma_{\circlearrowright} V = E - V_A - V_B = 0$$

$$E = V_A + V_B = 8\ \text{V} + 2\ \text{V}$$

or

$$10\ \text{V} = 10\ \text{V} \quad \text{(checks)}$$

$$I_A = I_s = \mathbf{2\ A}$$

and

$$I_B = I_C = \frac{I_A}{2} = \frac{I_s}{2} = \frac{2\ \text{A}}{2} = \mathbf{1\ A}$$

Returning to the network in Fig. 7.10, we have

$$I_{R_2} = I_{R_3} = \frac{I_B}{2} = \mathbf{0.5\ A}$$

The voltages V_A, V_B, and V_C from either figure are

$$V_A = I_A R_A = (2\ \text{A})(4\ \Omega) = \mathbf{8\ V}$$
$$V_B = I_B R_B = (1\ \text{A})(2\ \Omega) = \mathbf{2\ V}$$
$$V_C = V_B = \mathbf{2\ V}$$

EXAMPLE 7.4 Another possible variation of Fig. 7.8 appears in Fig. 7.12. Determine all the currents and voltages.

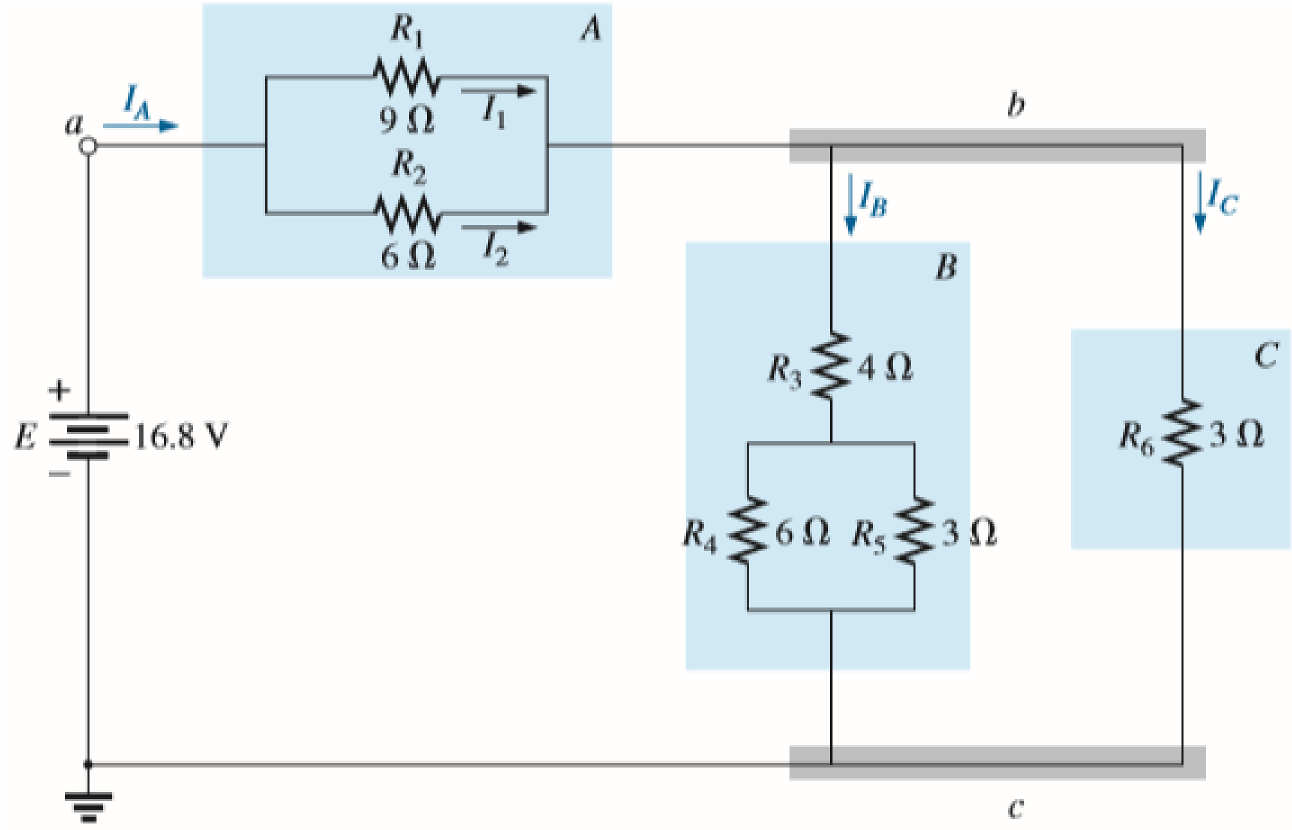


FIG. 7.12

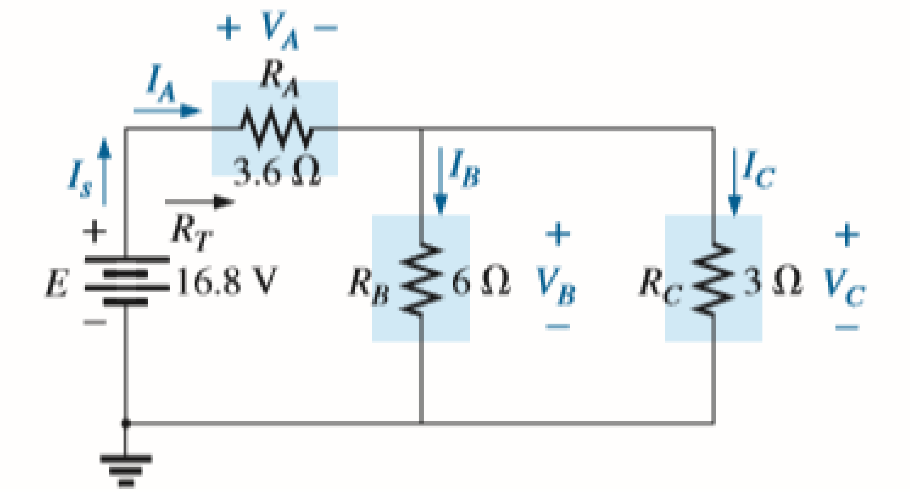


FIG. 7.13

Reduced equivalent of Fig. 7.12.

$$R_T = R_A + R_{B\|C} = 3.6\ \Omega + \frac{(6\ \Omega)(3\ \Omega)}{6\ \Omega + 3\ \Omega}$$

$$= 3.6\ \Omega + 2\ \Omega = \mathbf{5.6\ \Omega}$$

$$I_s = \frac{E}{R_T} = \frac{16.8\ \text{V}}{5.6\ \Omega} = \mathbf{3\ A}$$

$$I_A = I_s = \mathbf{3\ A}$$

Applying the current divider rule yields

$$I_B = \frac{R_C I_A}{R_C + R_B} = \frac{(3\ \Omega)(3\ \text{A})}{3\ \Omega + 6\ \Omega} = \frac{9\ \text{A}}{9} = \mathbf{1\ A}$$

By Kirchhoff's current law,

$$I_C = I_A - I_B = 3\ \text{A} - 1\ \text{A} = \mathbf{2\ A}$$

By Ohm's law,

$$V_A = I_A R_A = (3 \text{ A})(3.6 \ \Omega) = \mathbf{10.8 \ V}$$
$$V_B = I_B R_B = V_C = I_C R_C = (2 \text{ A})(3 \ \Omega) = \mathbf{6 \ V}$$

Returning to the original network (Fig. 7.12) and applying the current divider rule,

$$I_1 = \frac{R_2 I_A}{R_2 + R_1} = \frac{(6 \ \Omega)(3 \text{ A})}{6 \ \Omega + 9 \ \Omega} = \frac{18 \text{ A}}{15} = \mathbf{1.2 \ A}$$

By Kirchhoff's current law,

$$I_2 = I_A - I_1 = 3 \text{ A} - 1.2 \text{ A} = \mathbf{1.8 \ A}$$

The blocks in Fig. 7.8 can be arranged in a variety of ways. In fact, there is no limit on the number of series-parallel configurations that can appear within a given network. In reverse, the block diagram approach can be used effectively to reduce the apparent complexity of a system by identifying the major series and parallel components of the network. This approach is demonstrated in the next few examples.

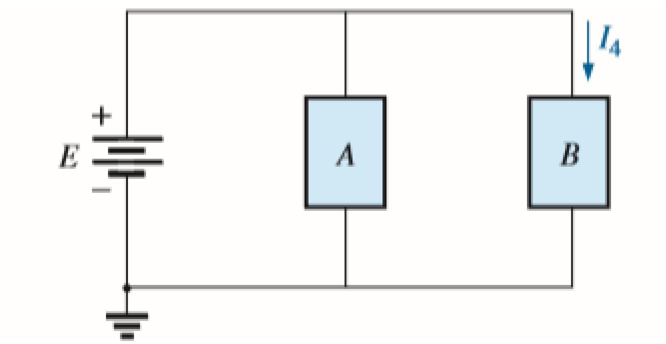


FIG. 7.15
Block diagram of Fig. 7.14.

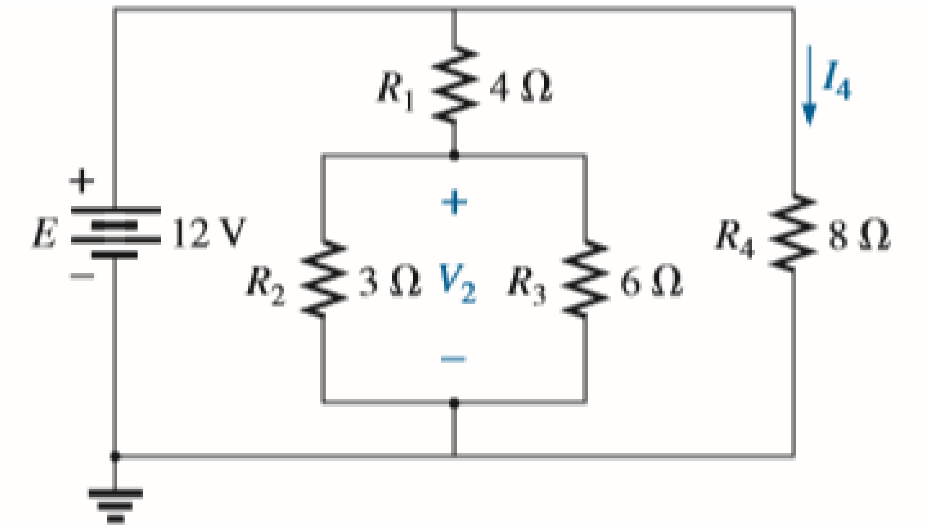


FIG. 7.14
Example 7.5.

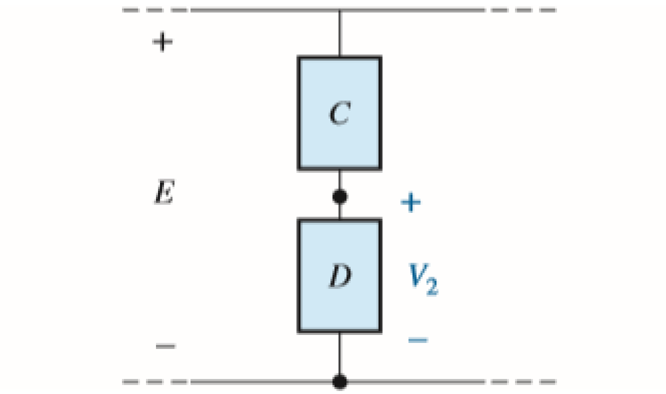


FIG. 7.16

7.5 DESCRIPTIVE EXAMPLES

EXAMPLE 7.5 Find the current I_4 and the voltage V_2 for the network in Fig. 7.14 using the block diagram approach.

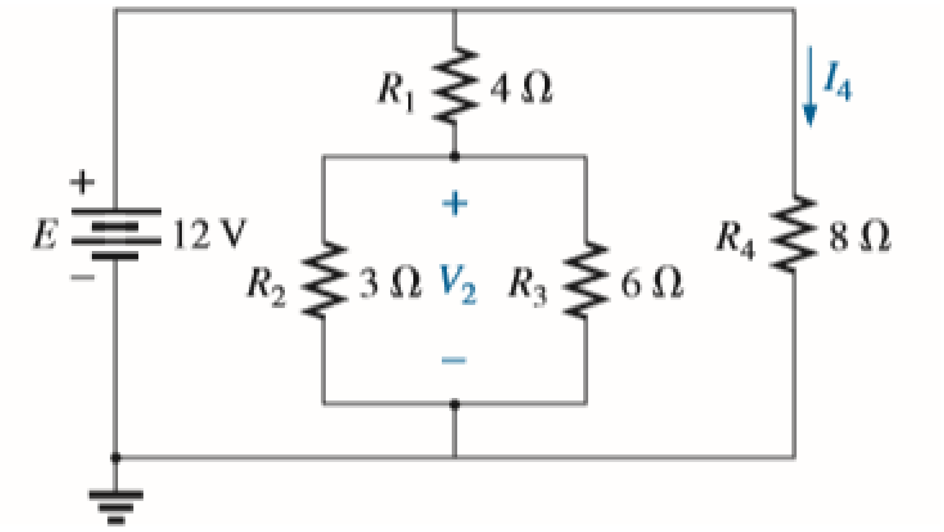


FIG. 7.14

EXAMPLE 7.6 Find the indicated currents and voltages for the network in Fig. 7.17.

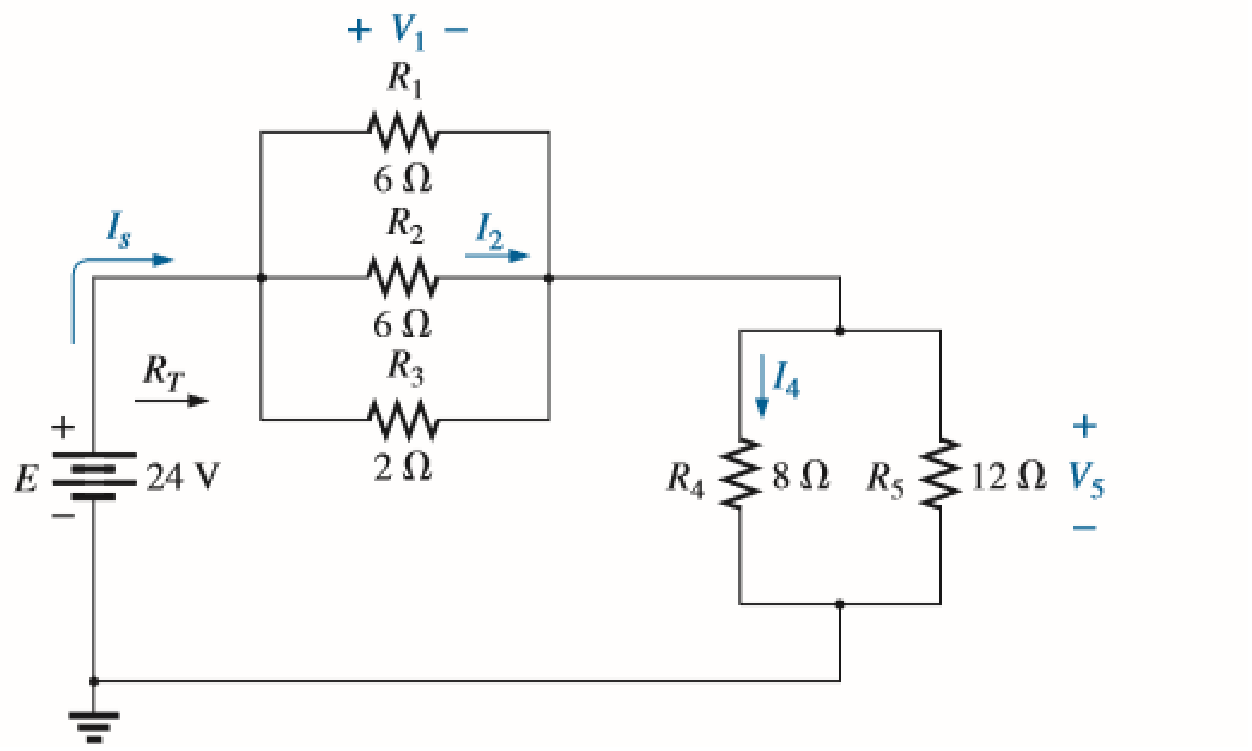


FIG. 7.17
Example 7.6.

EXAMPLE 7.7

a. Find the voltages V_1, V_3, and V_{ab} for the network in Fig. 7.20.
b. Calculate the source current I_s.

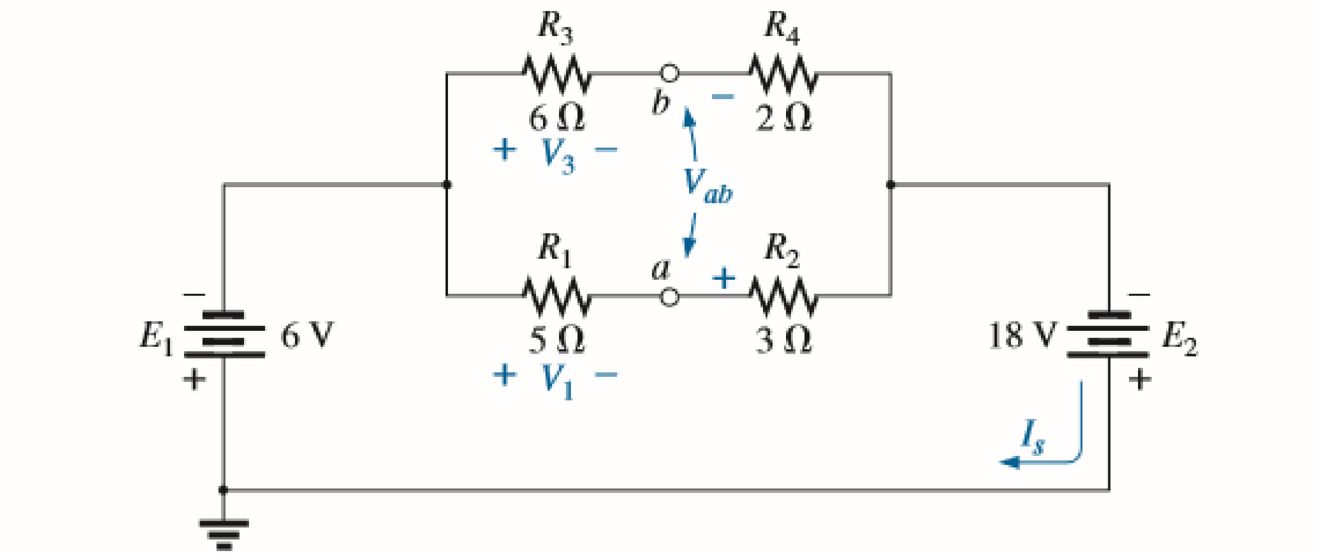


FIG. 7.20
Example 7.7.

EXAMPLE 7.7

a. Find the voltages V_1, V_3, and V_{ab} for the network in Fig. 7.20.
b. Calculate the source current I_s.

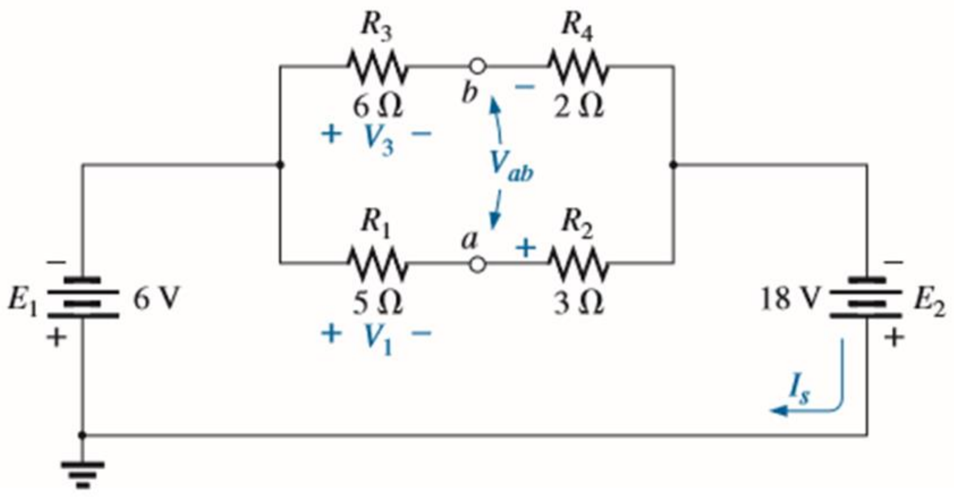


FIG. 7.20
Example 7.7.

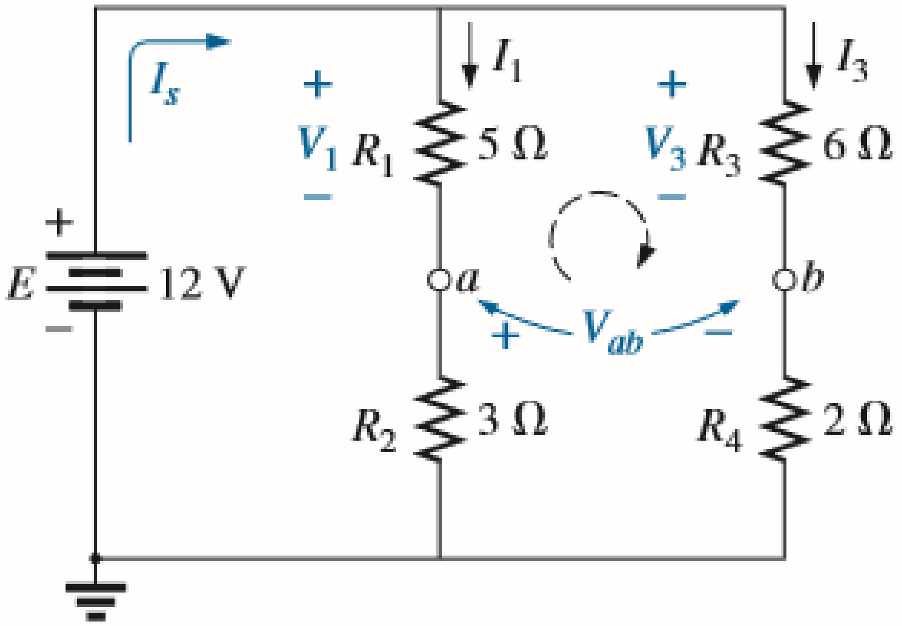


FIG. 7.21
Network in Fig. 7.20 redrawn.

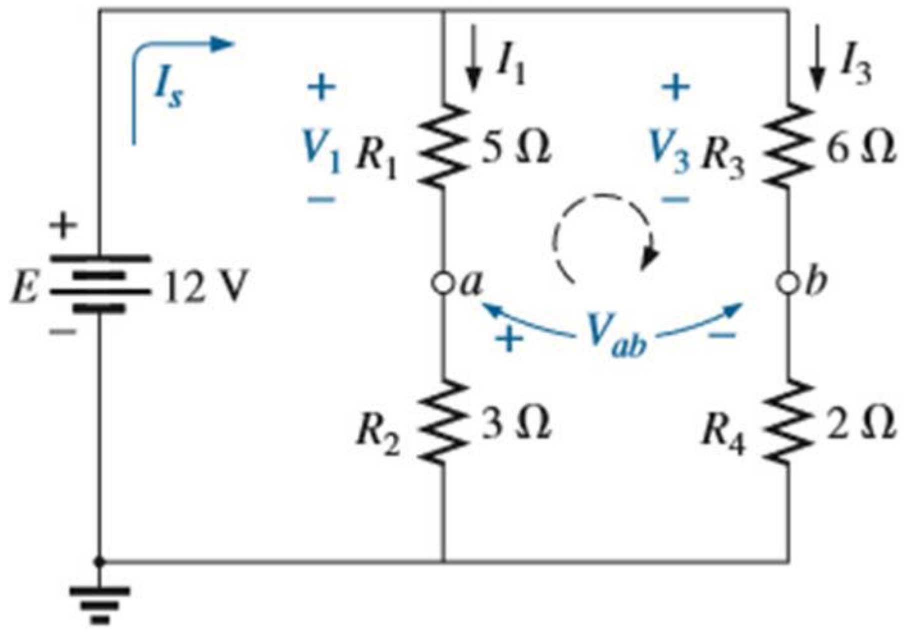


FIG. 7.21
Network in Fig. 7.20 redrawn.

a. Note the similarities with Fig. 7.16, permitting the use of the voltage divider rule to determine V_1 and V_3:

$$V_1 = \frac{R_1 E}{R_1 + R_2} = \frac{(5\,\Omega)(12\text{ V})}{5\,\Omega + 3\,\Omega} = \frac{60\text{ V}}{8} = \mathbf{7.5\text{ V}}$$

$$V_3 = \frac{R_3 E}{R_3 + R_4} = \frac{(6\,\Omega)(12\text{ V})}{6\,\Omega + 2\,\Omega} = \frac{72\text{ V}}{8} = \mathbf{9\text{ V}}$$

The open-circuit voltage V_{ab} is determined by applying Kirchhoff's voltage law around the indicated loop in Fig. 7.21 in the clockwise direction starting at terminal *a*.

$$+V_1 - V_3 + V_{ab} = 0$$

and $$V_{ab} = V_3 - V_1 = 9\text{ V} - 7.5\text{ V} = \mathbf{1.5\text{ V}}$$

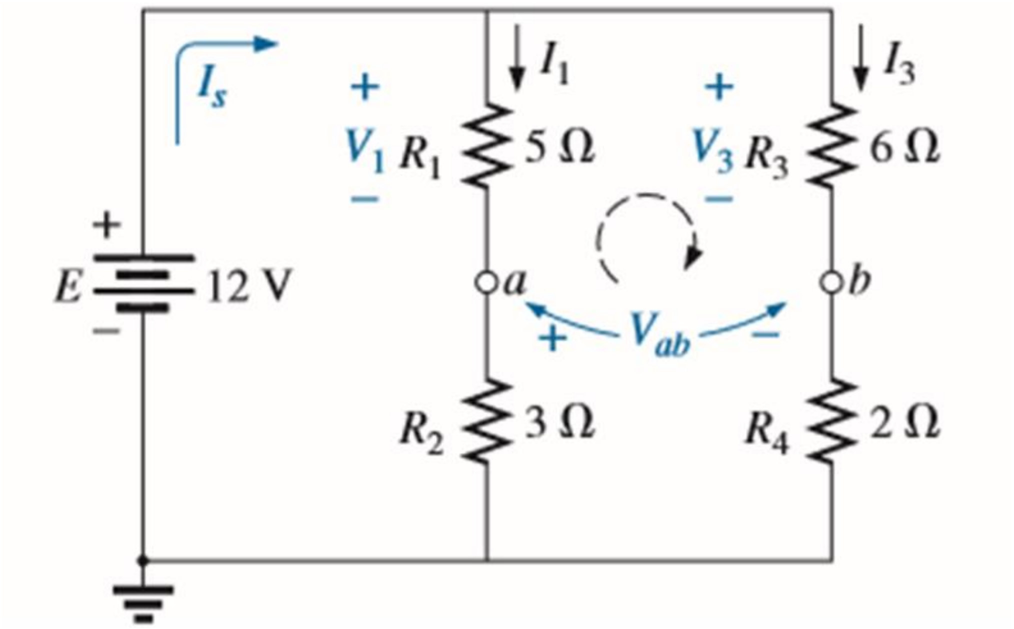


FIG. 7.21
Network in Fig. 7.20 redrawn.

b. By Ohm's law,

$$I_1 = \frac{V_1}{R_1} = \frac{7.5\text{ V}}{5\ \Omega} = 1.5\text{ A}$$

$$I_3 = \frac{V_3}{R_3} = \frac{9\text{ V}}{6\ \Omega} = 1.5\text{ A}$$

Applying Kirchhoff's current law,

$$I_s = I_1 + I_3 = 1.5\text{ A} + 1.5\text{ A} = \mathbf{3\text{ A}}$$

EXAMPLE 7.8 For the network in Fig. 7.22, determine the voltages V_1 and V_2 and the current I.

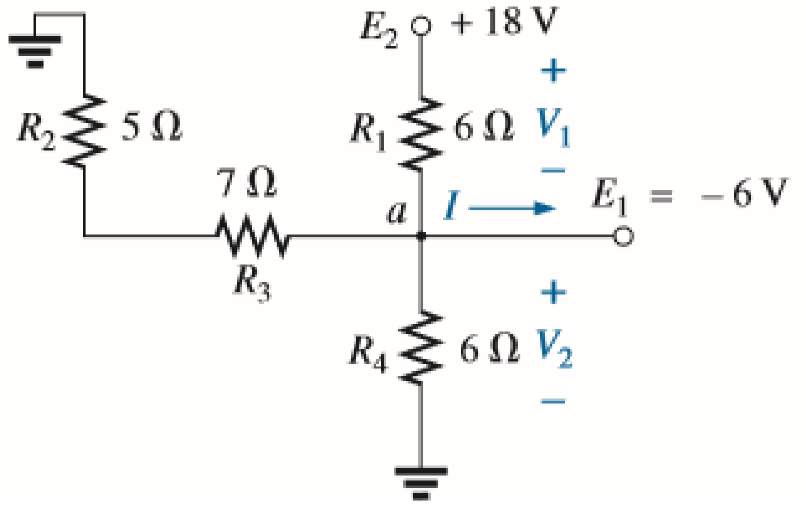


FIG. 7.22
Example 7.8.

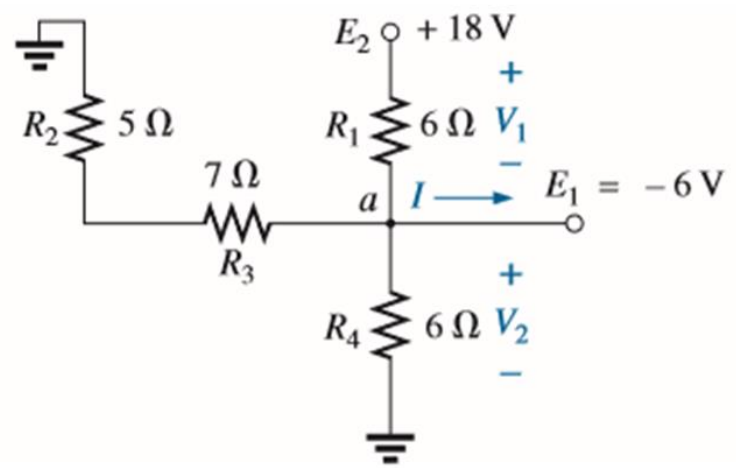


FIG. 7.22
Example 7.8.

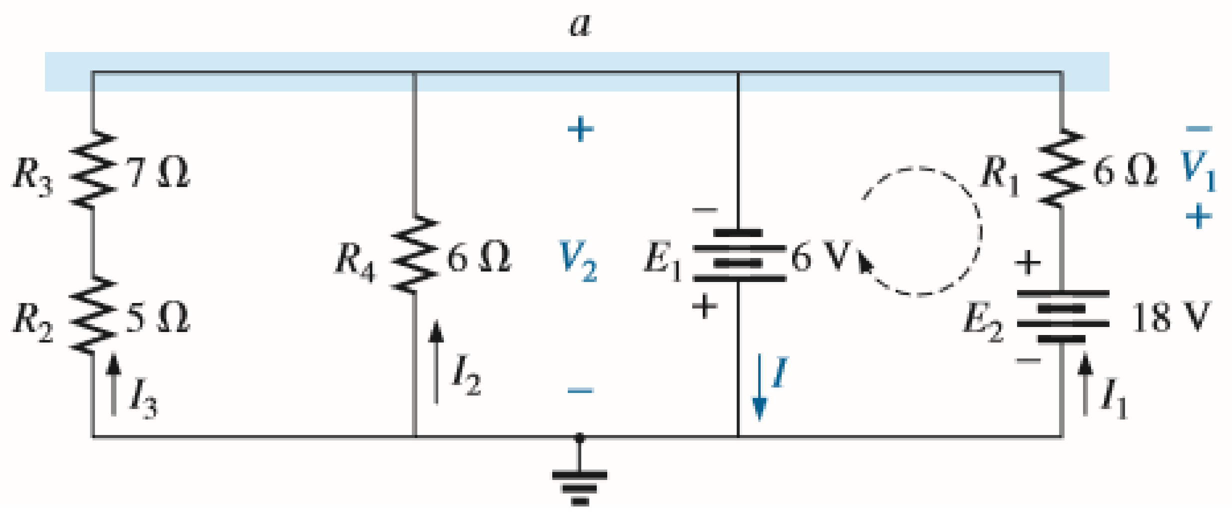


FIG. 7.23
Network in Fig. 7.22 redrawn.

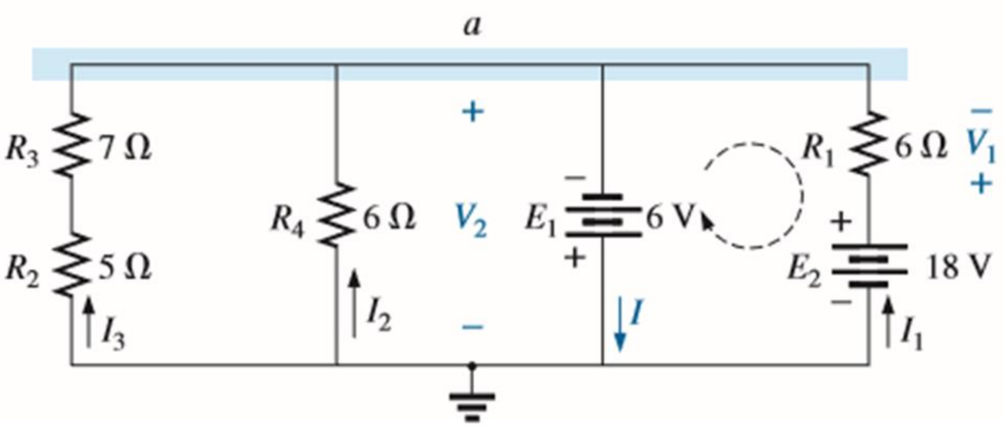


FIG. 7.23
Network in Fig. 7.22 redrawn.

It is now obvious that

$$V_2 = -E_1 = \mathbf{-6\ V}$$

The minus sign simply indicates that the chosen polarity for V_2 in Fig. 7.18 is opposite to that of the actual voltage. Applying Kirchhoff's voltage law to the loop indicated, we obtain

$$-E_1 + V_1 - E_2 = 0$$

and $$V_1 = E_2 + E_1 = 18\ \text{V} + 6\ \text{V} = \mathbf{24\ V}$$

Applying Kirchhoff's current law to note *a* yields

$$\begin{aligned} I &= I_1 + I_2 + I_3 \\ &= \frac{V_1}{R_1} + \frac{E_1}{R_4} + \frac{E_1}{R_2 + R_3} \\ &= \frac{24\ \text{V}}{6\ \Omega} + \frac{6\ \text{V}}{6\ \Omega} + \frac{6\ \text{V}}{12\ \Omega} \\ &= 4\ \text{A} + 1\ \text{A} + 0.5\ \text{A} \\ I &= \mathbf{5.5\ A} \end{aligned}$$

EXAMPLE 7.10 Calculate the indicated currents and voltage in Fig. 7.26.

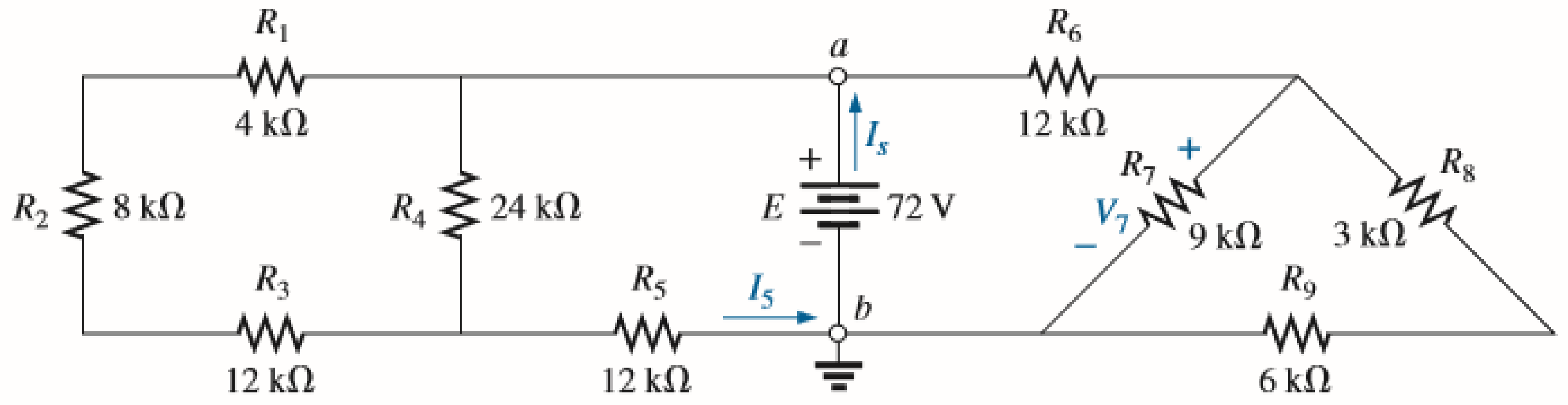


FIG. 7.26
Example 7.10.

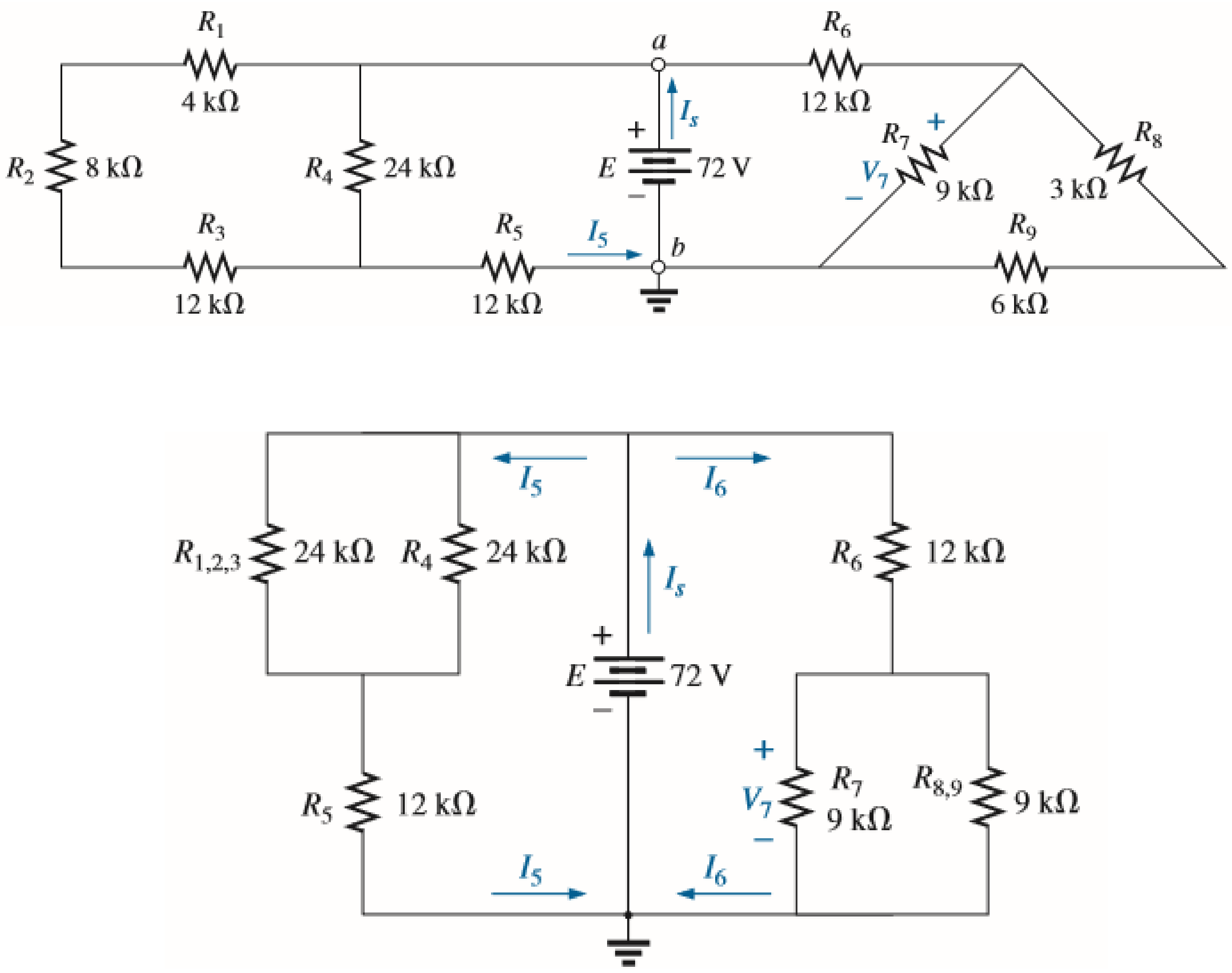


FIG. 7.27

Network in Fig. 7.26 redrawn.

Solution: Redrawing the network after combining series elements yields Fig. 7.27, and

$$I_5 = \frac{E}{R_{(1,2,3)\|4} + R_5} = \frac{72\text{ V}}{12\text{ k}\Omega + 12\text{ k}\Omega} = \frac{72\text{ V}}{24\text{ k}\Omega} = \mathbf{3\text{ mA}}$$

with

$$V_7 = \frac{R_{7\|(8,9)}E}{R_{7\|(8,9)} + R_6} = \frac{(4.5\text{ k}\Omega)(72\text{ V})}{4.5\text{ k}\Omega + 12\text{ k}\Omega} = \frac{324\text{ V}}{16.5} = \mathbf{19.6\text{ V}}$$

$$I_6 = \frac{V_7}{R_{7\|(8,9)}} = \frac{19.6\text{ V}}{4.5\text{ k}\Omega} = \mathbf{4.35\text{ mA}}$$

and $$I_s = I_5 + I_6 = 3\text{ mA} + 4.35\text{ mA} = \mathbf{7.35\text{ mA}}$$

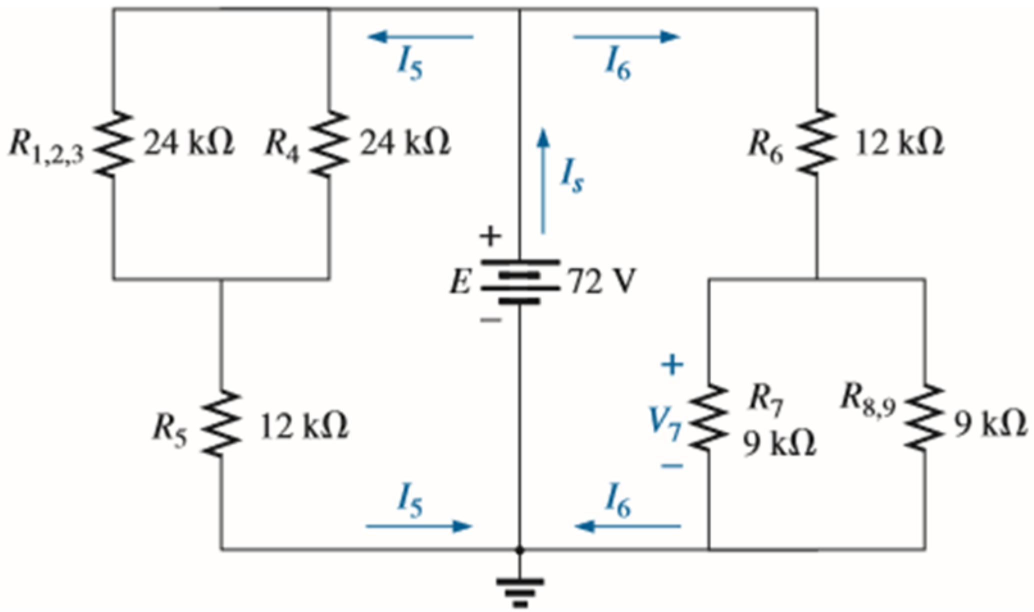
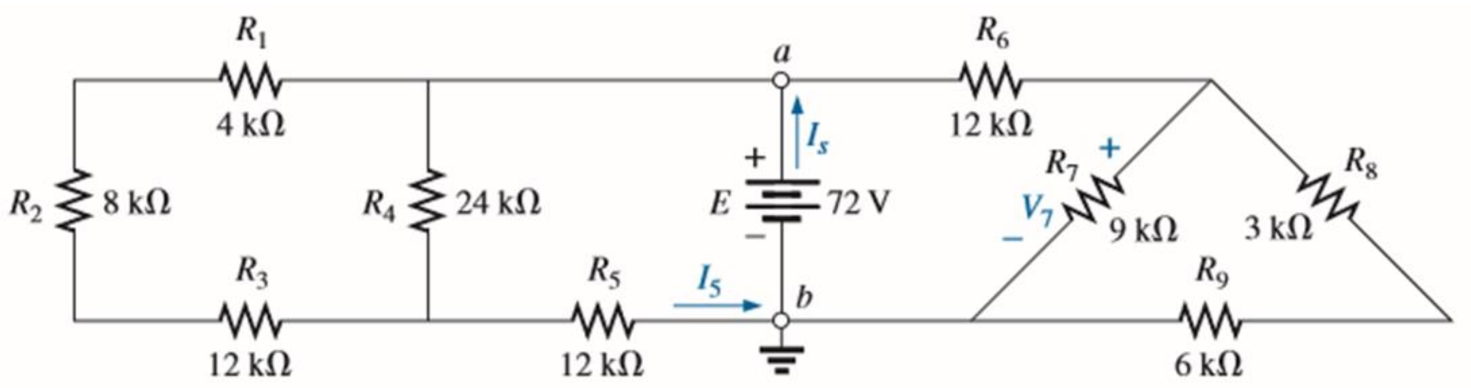


FIG. 7.27
Network in Fig. 7.26 redrawn.



Since the potential difference between points a and b in Fig. 7.26 is fixed at E volts, the circuit to the right or left is unaffected if the network is reconstructed as shown in Fig. 7.28.

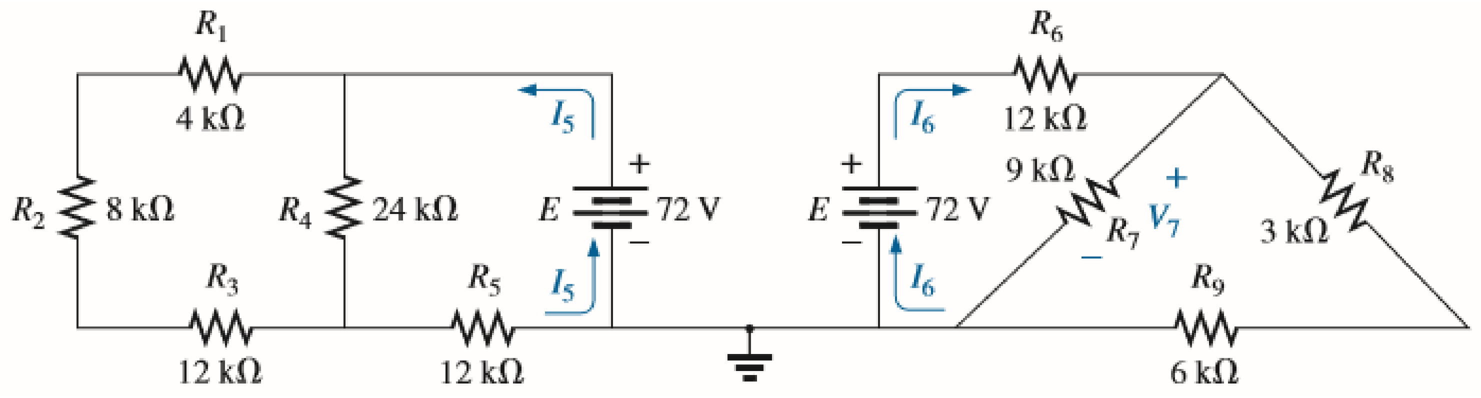


FIG. 7.28
An alternative approach to Example 7.10.

EXAMPLE 7.11 For the network in Fig. 7.29:

a. Determine voltages V_a, V_b, and V_c.
b. Find voltages V_{ac} and V_{bc}.
c. Find current I_2.
d. Find the source current I_{s_3}.
e. Insert voltmeters to measure voltages V_a and V_{bc} and current I_{s_3}.

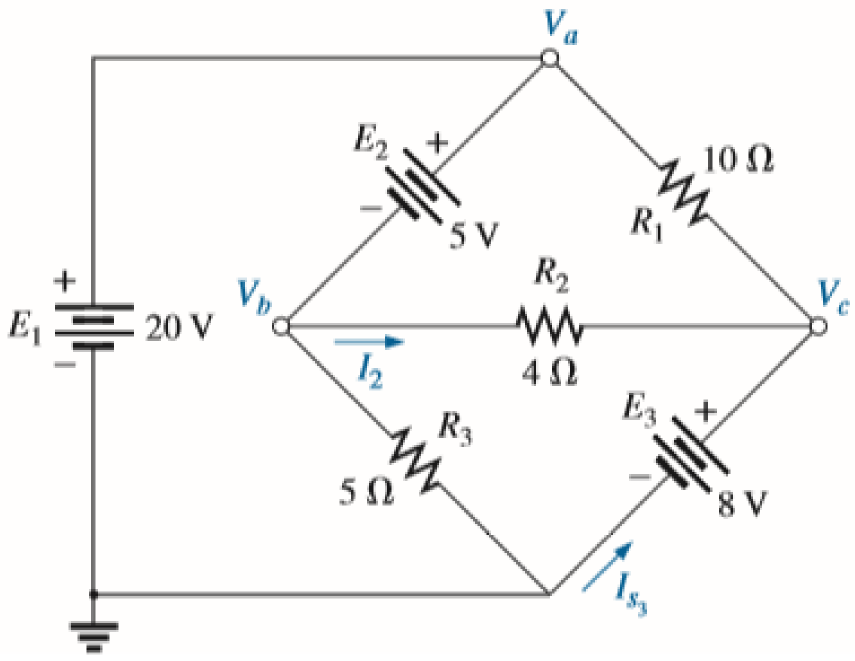


FIG. 7.29

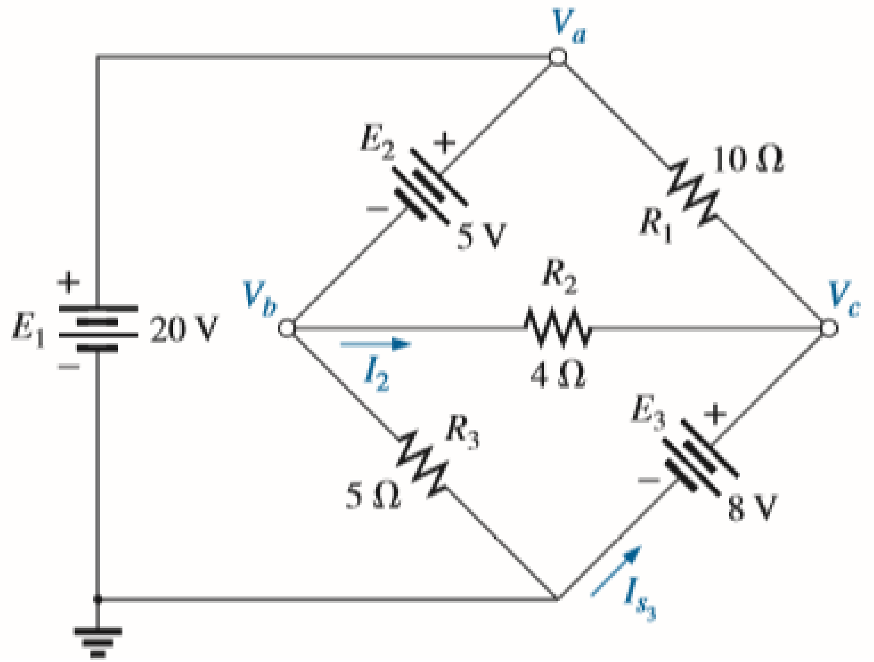


FIG. 7.29

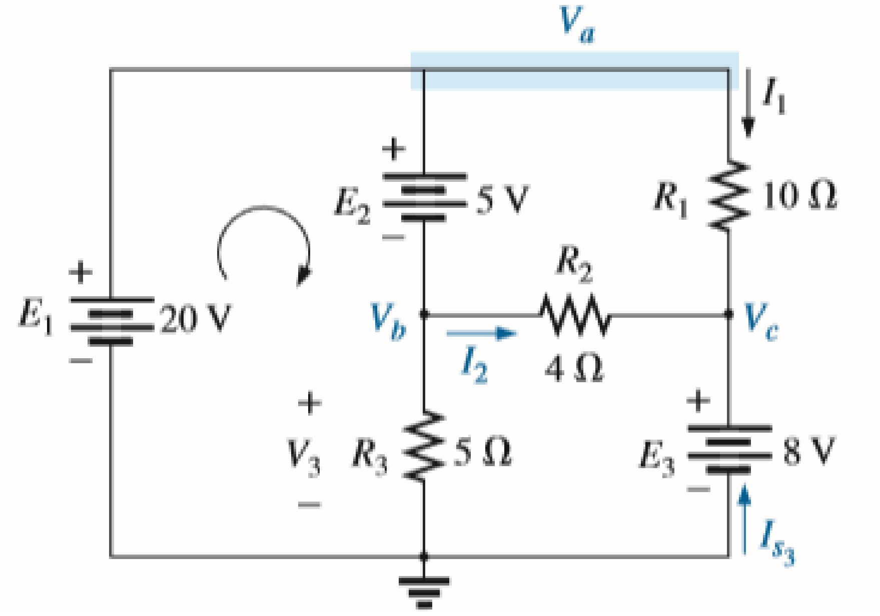


FIG. 7.30

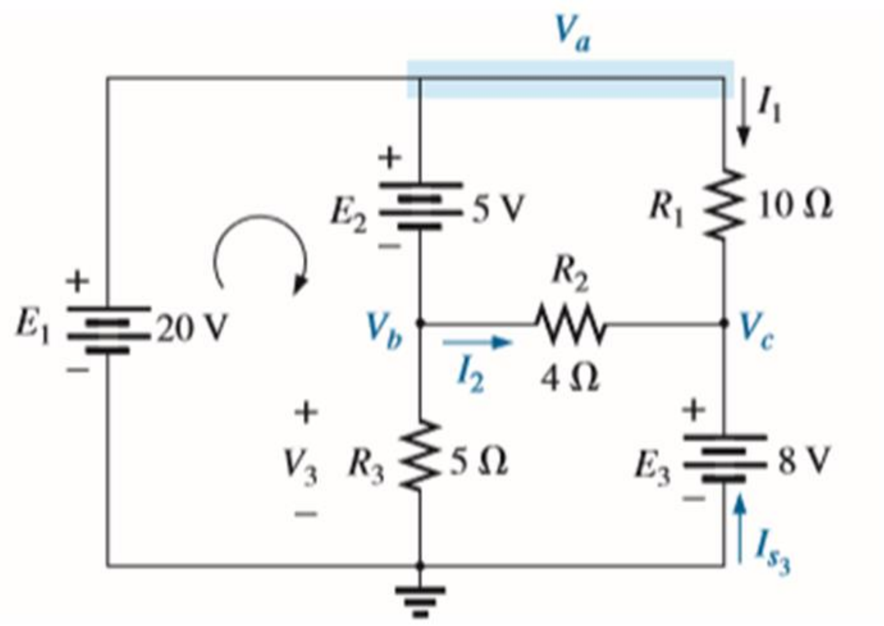


FIG. 7.30

First, note that voltage V_a is directly across voltage source E_1. Therefore,

$$V_a = E_1 = \mathbf{20\ V}$$

The same is true for voltage V_c, which is directly across the voltage source E_3. Therefore,

$$V_c = E_3 = \mathbf{8\ V}$$

To find voltage V_b, which is actually the voltage across R_3, we must apply Kirchhoff's voltage law around loop 1 as follows:

$$+E_1 - E_2 - V_3 = 0$$

and $$V_3 = E_1 - E_2 = 20\ \text{V} - 5\ \text{V} = 15\ \text{V}$$

and $$V_b = V_3 = \mathbf{15\ V}$$

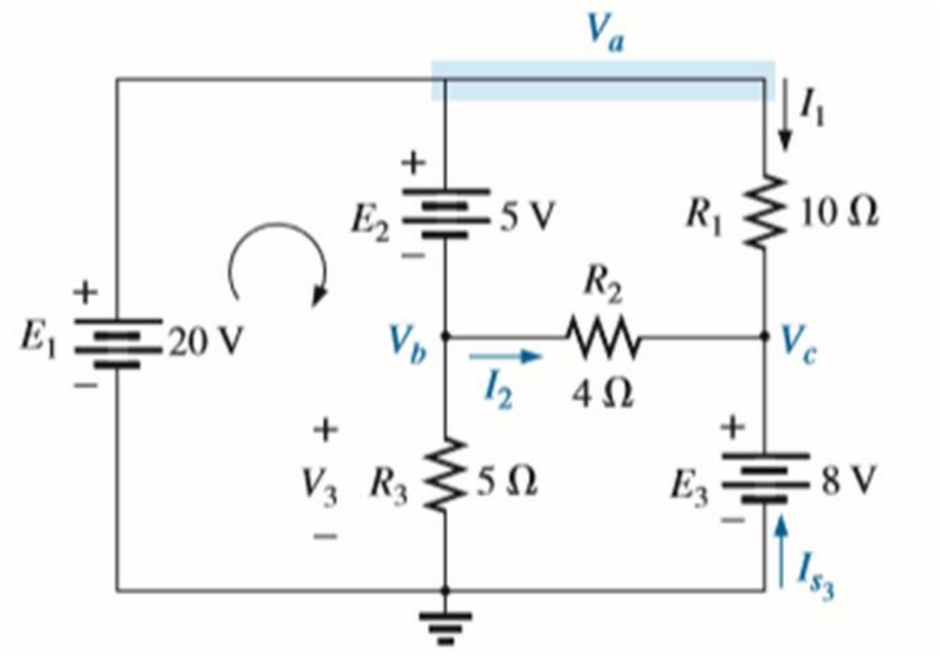


FIG. 7.30

b. Voltage V_{ac}, which is actually the voltage across resistor R_1, can then be determined as follows:

$$V_{ac} = V_a - V_c = 20\text{ V} - 8\text{ V} = \mathbf{12\ V}$$

Similarly, voltage V_{bc}, which is actually the voltage across resistor R_2, can then be determined as follows:

$$V_{bc} = V_b - V_c = 15\text{ V} - 8\text{ V} = \mathbf{7\ V}$$

c. Current I_2 can be determined using Ohm's law:

$$I_2 = \frac{V_2}{R_2} = \frac{V_{bc}}{R_2} = \frac{7\text{ V}}{4\ \Omega} = \mathbf{1.75\ A}$$

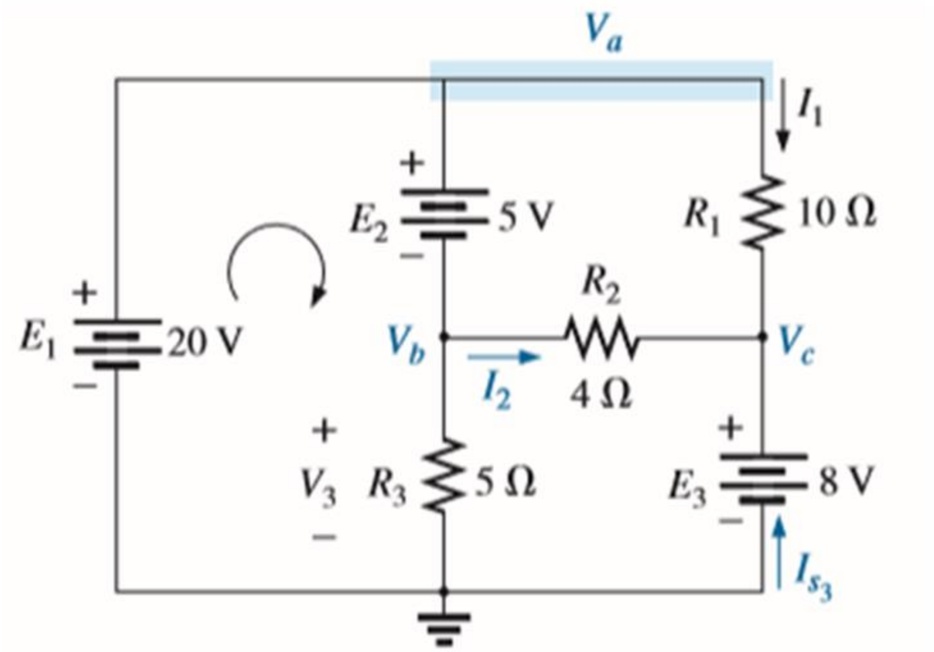


FIG. 7.30

d. The source current I_{s_3} can be determined using Kirchhoff's current law at note c:

$$\Sigma I_i = \Sigma I_o$$
$$I_1 + I_2 + I_{s_3} = 0$$

and
$$I_{s_3} = -I_1 - I_2 = -\frac{V_1}{R_1} - I_2$$

with
$$V_1 = V_{ac} = V_a - V_c = 20\text{ V} - 8\text{ V} = 12\text{ V}$$

so that

$$I_{s_3} = -\frac{12\text{ V}}{10\ \Omega} - 1.75\text{ A} = -1.2\text{ A} - 1.75\text{ A} = \mathbf{-2.95\ A}$$

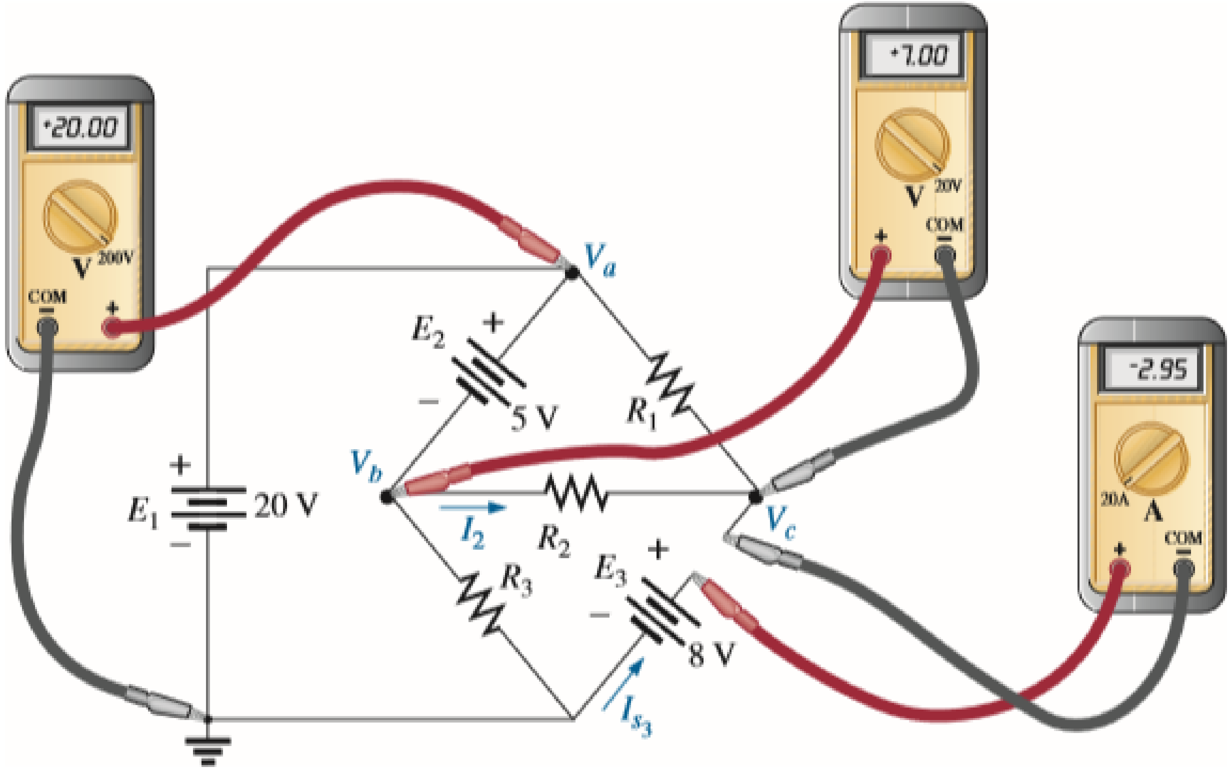


FIG. 7.31
Complex network for Example 7.11.

revealing that current is actually being forced through source E_3 in a direction opposite to that shown in Fig. 7.29.

e. Both voltmeters have a positive reading as shown in Fig. 7.31 while the ammeter has a negative reading.