OneNote

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CSE495A

Wednesday, October 9, 2024 10:42 AM

$Ax = b$

$\Rightarrow x = A^{\dagger} b$

$\dagger$ = dagger

$A^{\dagger}$ is called pseudo inverse

$A^{\dagger}$ exist for all matrices

A^{-1} exists only for square full rank matrices

$A_{m \times n}$

$m > n$	$m = n$	$m < n$
$\boxed{A}$	$\boxed{A}$	$\boxed{A}$

SS

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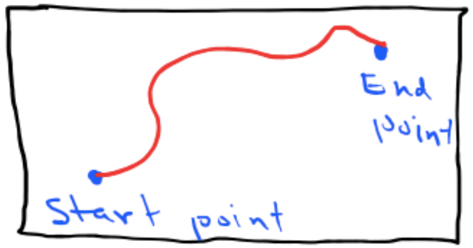
Example:

$$\dot{x}_1 = u_1$$
$$\dot{x}_2 = u_2$$
$$\dot{x}_3 = x_2 u_1$$

$$0 \le t \le T$$

Given

$x_1(0)$	$x_1(T)$
$x_2(0)$	$x_2(T)$
$x_3(0)$	$x_3(T)$
Start position at $t=0$	End position at $t=T$



— trajectory you want to calculate using differential flatness method

Solution: In last class we showed the system is differentially flat for

$$z = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_3 \end{bmatrix}$$

$$j = 1, 2$$

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$$z_1 = x_1 = \sum_{i=1}^{n} a_{1i}\, \psi_i(t) \qquad z_2 = x_3 = \sum_{i=1}^{n} a_{2i}\, \psi_i(t)$$

The more basis functions you use the more accurate your trajectory approximation.

Let's choose $\boxed{n=4}$ & polynomial basis functions

$$\psi_1(t) = 1, \quad \psi_2(t) = t, \quad \psi_3(t) = t^2, \quad \psi_4(t) = t^3$$

Choosing n, also depends on the degree of derivative.

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$$z_1 = x_1 = a_{11}\psi_1(t) + a_{12}\psi_2(t) + a_{13}\psi_3(t) + a_{14}\psi_4(t)$$

$$\Rightarrow \boxed{\begin{aligned} z_1 &= x_1 = a_{11} + a_{12}t + a_{13}t^2 + a_{14}t^3 \\ z_2 &= x_3 = a_{21} + a_{22}t + a_{23}t^2 + a_{24}t^3 \end{aligned}}$$

for $n = 4$

$$x_1 = z_1 \qquad x_2 = \frac{\dot{z}_2}{\dot{z}_1} \qquad x_3 = z_2$$

$$\boxed{\begin{aligned} \dot{z}_1 &= a_{12} + 2a_{13}t + 3a_{14}t^2 \\ \dot{z}_2 &= a_{22} + 2a_{23}t + 3a_{24}t^2 \end{aligned}}$$

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$x_1 = z_1 \qquad x_2 = \dfrac{\dot{z}_2}{\dot{z}_1}, \qquad x_3 = z_2 \qquad j = 1, 2$

$x_1(t) = z_1(t)$

$z_1(0) = x_1(0)$

$\dot{z}_1(0) = 1$

$z_2(0) = x_3(0)$

$\dot{z}_2(0) = x_2(0)$

$z_1(T) = x_1(T)$

$\dot{z}_1(T) = 1$

$z_2(T) = x_3(T)$

$\dot{z}_2(T) = x_2(T)$

Set

$\dot{z}_1(0) = 1$

$\Rightarrow \dot{z}_2(0) = x_2(0)$

$\dot{z}_1(T) = 1$

$\Rightarrow \dot{z}_2(T) = x_2(T)$

$$\begin{bmatrix} z_1(0) \\ \dot{z}_1(0) \\ z_2(0) \\ \dot{z}_2(0) \\ z_1(T) \\ \dot{z}_1(T) \\ z_2(T) \\ \dot{z}_2(T) \end{bmatrix}_{8\times 1}$$

$x_2(t) = \dfrac{\dot{z}_2(t)}{\dot{z}_1(t)}$

$t = 0$

$x_2(0) = \dfrac{\dot{z}_2(0)}{\dot{z}_1(0)}$

$$z_1 = a_{11} + a_{12}t + a_{13}t^2 + a_{14}t^3 \qquad z_2 = a_{21} + a_{22}t + a_{23}t^2 + a_{24}t^3$$

$$\dot{z}_1 = a_{12} + 2a_{13}t + 3a_{14}t^2 \qquad \dot{z}_2 = a_{22} + 2a_{23}t + 3a_{24}t^2$$

$$\underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & T & T^2 & T^3 & 0 & 0 & 0 & 0 \\ 0 & 1 & 2T & 3T^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & T & T^2 & T^3 \\ 0 & 0 & 0 & 0 & 0 & 1 & 2T & 3T^2 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} a_{11} \\ a_{12} \\ a_{13} \\ a_{14} \\ a_{21} \\ a_{22} \\ a_{23} \\ a_{24} \end{bmatrix}}_{X} = \underbrace{\begin{bmatrix} z_1(0) \\ \dot{z}_1(0) \\ z_2(0) \\ \dot{z}_2(0) \\ z_1(T) \\ \dot{z}_1(T) \\ z_2(T) \\ \dot{z}_2(T) \end{bmatrix}}_{b}$$

$$X = A^{+} b$$

x is the vector of unknown parameters