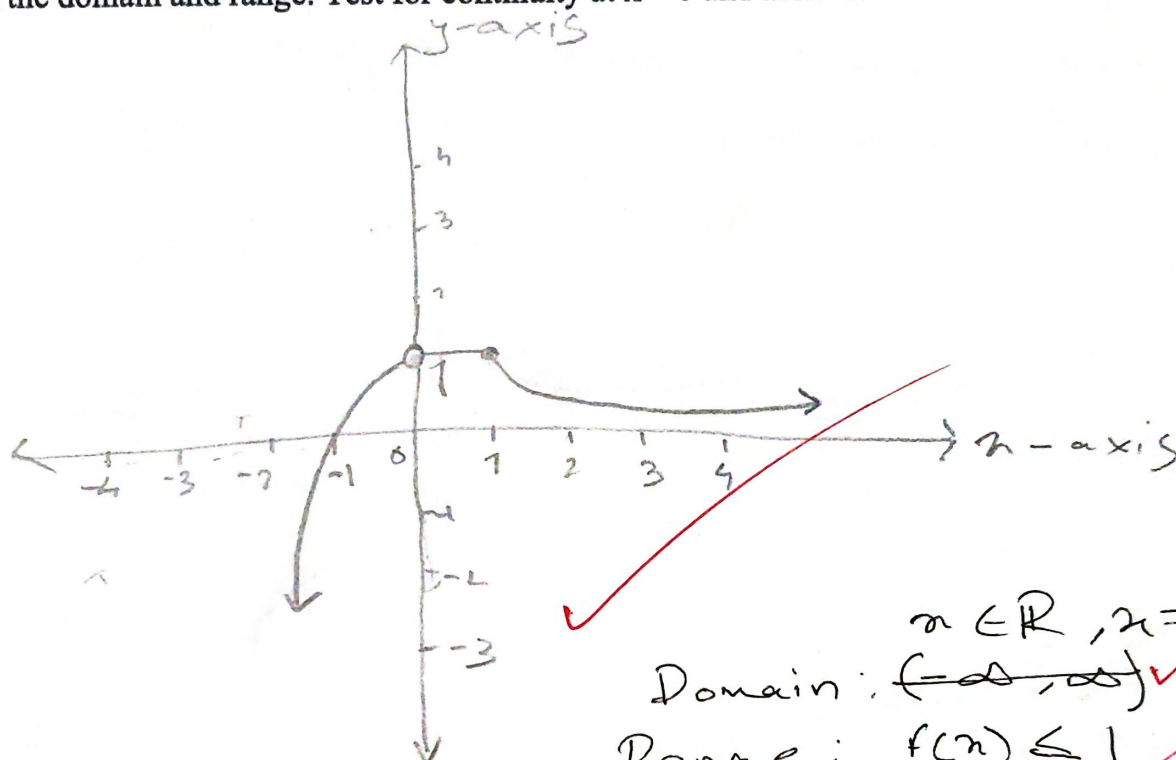10

1. Sketch the graphs of the function $f(x) = \begin{cases} 1 - x^2 & \text{when } x < 0 \\ 1 & \text{when } 0 < x < 1 \\ \frac{1}{x} & \text{when } x \geq 1 \end{cases}$

Write down the domain and range. Test for continuity at x = 0 and at x = 1.

$x \in \mathbb{R}, x \neq 0$

Domain: $(-\infty, \infty)$

Range: $f(x) \leq 1$

Continuity at $x = 0$

$\lim\limits_{x \to 0^+} f(x)$ $\lim\limits_{x \to 0^-} f(x)$ $f(0)$ = Not defined

$= 1$ $= 1$

∴ The function is not continous at $x \neq 0$ as both right hand limit and left hand limit are equal but $f(0)$ is not defined

~~Continuity at $x = 1$~~

Continuity at $x = 1$ $f(1)$

limit $f(x)$ limit $f(x)$ $= \frac{1}{1}$

$x \longrightarrow 1^{+}$ $x \rightarrow 1^{-}$ $= 1$

$= 1$ ✓ $= 1$ ✓

∴ $f(x)$ is continuous at $x = 1$ as both right hand limit and, left hand limit are equal and $f(1)$ are equal ✓