

Library Functions:

i) constant function:

$$f(x) = b ; b \text{ is a real number.}$$

Properties:

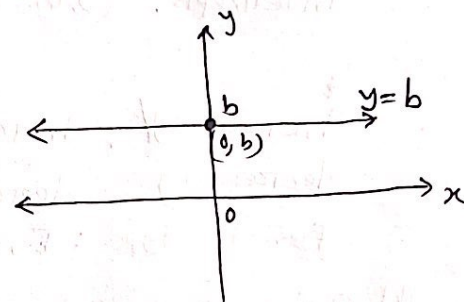
Domain : set of all real numbers

Range : A single number b .

Graph : Horizontal line

Intercepts: x -intercept \rightarrow N/A
 y -intercept $= b$

Function type : Even Function



ii) Identity Function:

$$f(x) = x$$

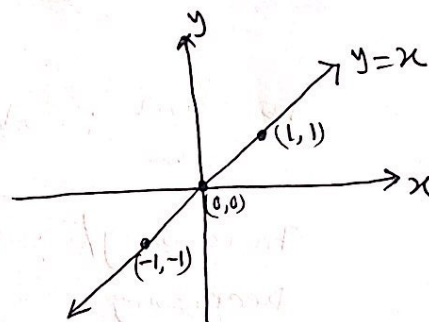
Domain: Set of all real numbers

Range: Set of all real numbers

Intercepts: x -intercept $= 0$
 y -intercept $= 0$

Function type : odd function

Increasing : Increasing over
 or decreasing its domain



III) Square Function:

$$f(x) = x^2$$

Domain: set of all real numbers

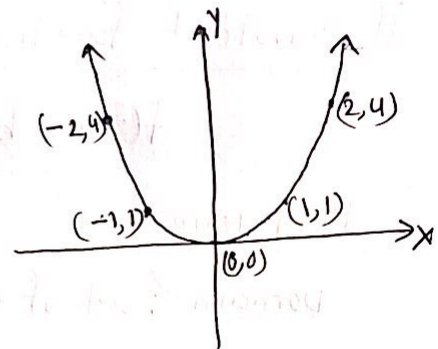
Range: set of non-negative real numbers.

Graph: Parabola.

Intercepts: $(0,0)$; x -intercept = 0
 y -intercept = 0

Increasing/Decreasing: Increasing on the interval $(0, \infty)$
decreasing on the interval $(-\infty, 0)$

Function type: Even function



IV) Cube function:

$$f(x) = x^3$$

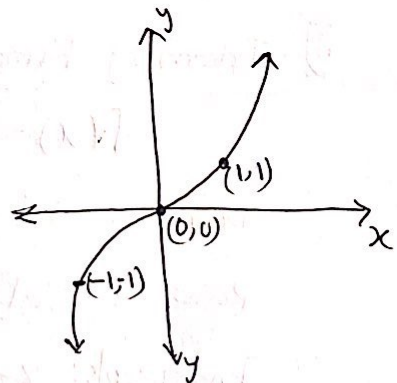
Domain: set of all real numbers.

Range: set of all real numbers.

Intercepts: $(0,0)$, x -intercept = 0
 y -intercept = 0

Function type: odd function

Increasing/Decreasing: Increasing on the interval $(-\infty, \infty)$



VI] Square root Function:

$$f(x) = \sqrt{x}$$

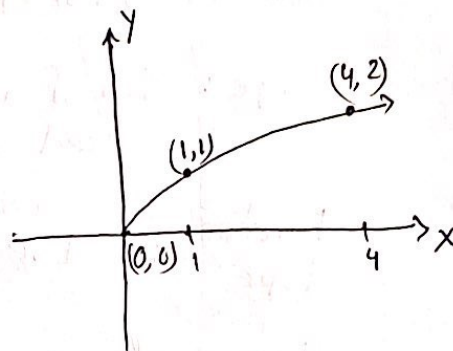
Domain: set of nonnegative real numbers.

Range: set of non-negative real numbers.

Intercepts: $(0,0)$; x -intercept $= 0$
 y " $= 0$

Function type: neither odd nor even.

Increasing/decreasing: Increasing on the interval $(0, \infty)$



VII] Cube root Function:

$$f(x) = \sqrt[3]{x}$$

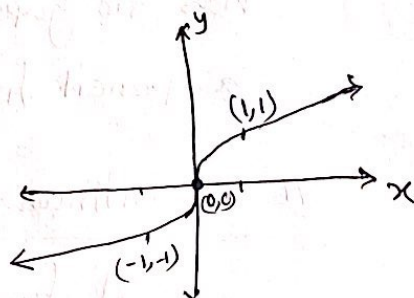
Domain: set of all real numbers

Range: set of all real numbers.

Intercepts: $(0,0)$; x -intercept $= 0$
 y " $= 0$

Function type: odd function

Increasing/decreasing: Increasing on the interval $(-\infty, \infty)$



VIII] Reciprocal function:

$$f(x) = \frac{1}{x}$$

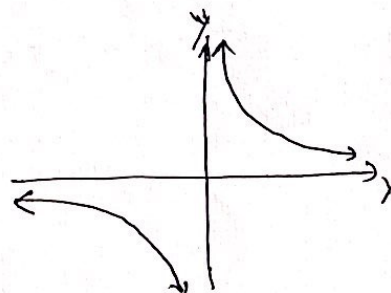
Domain: set of all nonzero real numbers

Range: set of all nonzero real numbers

Intercepts: No

Function type: odd function

Increasing/decreasing: decreasing on the intervals $(-\infty, 0)$ and $(0, \infty)$



VIII) Absolute value function:

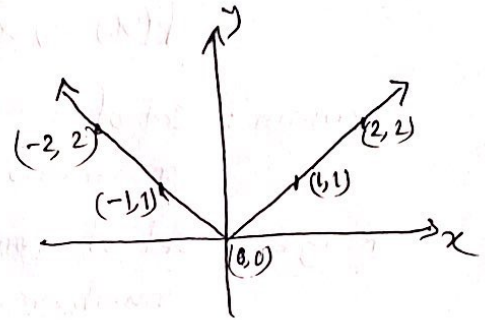
$$f(x) = |x|$$

Domain: Set of all real numbers.

Range: Set of nonnegative real numbers

Function type: Even function.

Increasing/ : increasing on the interval $(0, \infty)$
decreasing : decreasing on the interval $(-\infty, 0)$



continuous function:

A function is said to be continuous if its graph has no gaps or holes and can be drawn without lifting a pencil from the paper.

#1 Discontinuous function:

A function is said to be discontinuous if its graph has gaps or holes so that its graph cannot be drawn without lifting a pencil from the paper.

Piecewise Function:

When a function is defined by different equations on different parts of its domain, then that function is called Piecewise function.

$$f(x) = |x| = \begin{cases} x & \text{if } x > 0 \\ -x & \text{if } x < 0 \end{cases}$$

Example:

The function is defined as

$$f(x) = \begin{cases} -2x+1 & \text{if } -3 \leq x < 1 \\ 2 & \text{if } x = 1 \\ x^2 & \text{if } x > 1 \end{cases}$$

(a) Find $f(-2)$, $f(1)$, $f(2)$.

Solⁿ: $f(-2)$, observe that when $x = -2$, the equation for f is given by $f(x) = -2x + 1$. So

$$f(-2) = -2(-2) + 1 = 5$$

When $x = 1$, the equation for f is $f(x) = 2$

$$\text{i.e. } f(1) = 2$$

When $x = 2$, the equation for f is $f(x) = x^2$

$$\therefore f(2) = 2^2 = 4$$

b) Determine the domain of f .

Solⁿ: f is defined for all x greater than or equal to -3 , so the domain of f is $\{x \mid x \geq -3\}$ or $[-3, \infty)$.

c) Locate any intercepts.

Solⁿ: To find y -intercepts, let's put $x=0$ in the function $y=f(x)=-2x+1$. Because only $f(x)=-2x+1$ is the function where we get y intercepts.

$$f(0) = -2(0) + 1 \Rightarrow f(0) = 1$$

So y -intercepts = 1

To find x -intercepts we have to put $f(x)=0$

$$\therefore f(x) = 0$$

$$-2x + 1 = 0$$

$$\Rightarrow -2x = -1$$

$$\Rightarrow x = \frac{1}{2}$$

$$f(x) = 0$$

$$2 = 0$$

no solution

$$f(x) = 0$$

$$\Rightarrow x = 0$$

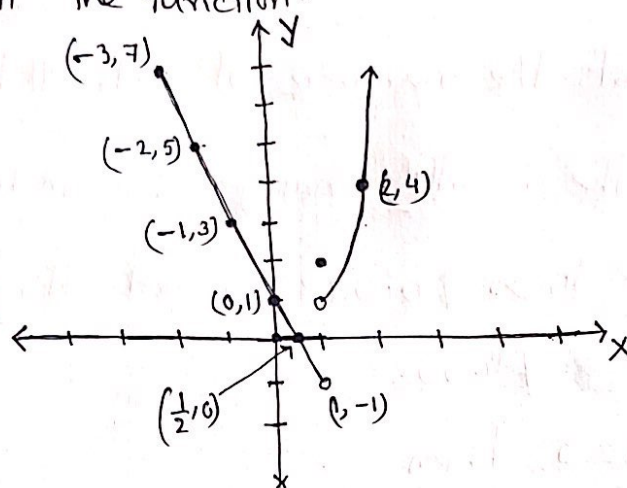
$$\Rightarrow x = 0$$

But $x=0$ does not satisfy the condition $x > -3$

So the only x -intercept is $\frac{1}{2}$

The intercepts are $(0, 1)$ and $(\frac{1}{2}, 0)$.

d) Graph the function.



e) Use the graph to find the range of f .

Soln: From the graph, we conclude that the range of f is $\{y \mid y > -1\}$ or the interval $(-1, \infty)$

f) The function f is not continuous because there is a jump in the graph at $x = 1$.

Application:

In the summer of 2009, Duke energy supplied electricity to residents of Ohio for a monthly customer charge of \$4.50 plus 4.235¢ per kilowatt-hour for the first 1000 kWh supplied in the month and 5.362¢ per kWh for all usage over 1000 kWh in the month.

If C is the monthly charge for x kWh, develop a model relating the monthly charge and kilowatt-hours used. Express C as a function of x .

Solution:

Let x represents the number of kilowatt hours used.

If $0 \leq x \leq 1000$, the monthly charge C can be found by multiplying x times \$0.042345 and adding the monthly customer charge of \$4.50.

So if $0 \leq x \leq 1000$, then

$$C(x) = 0.042345x + 4.50$$

For $x > 1000$, the charge is $0.042345(1000) + 4.50$
 $+ 0.053622(x - 1000)$.

i.e if $x > 1000$ then

$$\begin{aligned} C(x) &= 0.042345(1000) + 4.50 + 0.053622(x - 1000) \\ &= 46.845 + 0.053622(x - 1000) \\ &= 0.053622x - 6.777 \end{aligned}$$

Thus finally,

$$C(x) = \begin{cases} 0.042345x + 4.50 & \text{if } 0 \leq x \leq 1000. \\ 0.053622x - 6.77 & \text{if } x > 1000 \end{cases}$$