Second Mid-Term Examination, Spring 2018

Department of Mathematics and Physics, North South University
Course MAT 361: Probability and Statistics, Section: 6
Time: 1 Hour 15 Minutes, Full Marks: 20

(Answer ALL questions)

1. Suppose that two continuous random variables X and Y have a joint pdf

$$f(x,y) = A(e^{x+y} + e^{2x-y})$$

for $1 \le x \le 2$ and $0 \le y \le 3$ and $f(x,y) = 0$ elsewhere.

(a) Find the value of A.

(b) Construct the marginal probability functions $f_X(x)$ and $f_Y(y)$.

2. Suppose that X_1, X_2 and X_3 are independent random variables each with a mean μ and a variance σ^2. Compare the mean and variances of $Y = 3X_1$ and $Z = X_1 + X_2 + X_3$.

3. Suppose $X_1, X_2, \cdots, X_n$ are distributed as Bernouli(p). Find

(a) $E(X)$ and $V(X)$ if $\mathbf{X} = X_1 + X_2 + \cdots + X_n$.

(b) What is the name of the distribution of $\mathbf{X}$? Give two examples of such distribution of $\mathbf{X}$.

(c) If X denotes the number of students who got $A+$ grade and if each student has chance 0.08 to get $A+$. Find the probability that no student will get $A+$ grade.

4. Consider a fair six -sided dice. The dice is rolled until 6 is obtained for the 3rd time. What is the probability that it will take 20 throwing to get 6 the 3rd times. If Y denotes the number of throwing to get the first 6, then what is the name of the distribution of Y. Derive the mean and variance of Y.

5. (a) A jar contains 20 red balls and 10 green balls. If 7 balls are drawn then what is the probability that of them 6 are red balls and 1 is green ball.

(b) Show that mean of poisson distribution is λ if $X \sim Pois(\lambda)$. If average misprint of a book is 5 in a library then what is the probability that a book will contain 0 misprint.

6. (a) Define the pdf of Uniform distribution. Find $F(X)$.

(b) If $X \sim Exponential(\lambda)$, then show that $E(X) = 1/\lambda$.

(c) If $X \sim N(0, 1)$, find $\phi(0)$.

3 (a) The Binomial Distribution

$E(x) = np$

$\text{var}(x) = np(1-p)$

(b) The distribution of x is Binomial

Example 1

$$P(X=x) = \binom{n}{x} p^x (1-p)^{n-x}$$

x = the number of success = 15

n = " " " trails = 18

P = " probability of success = 0.85

$$P(x=15) = \binom{18}{15} (0.85)^{15} (1 - 0.85)^{18-15}$$

$$P(15) = 0.239$$

Example

A Batsman ball faced $n = 30$

Run score $x = 15$

$p = 0.85$

$$P(x=x) = \binom{n}{x} p^x (1-p)^{n-x}$$

$$= \binom{30}{15} p^{15} (1-p)^{30-15}$$

$$= \binom{30}{15} (0.85)^{15} (1-0.85)^{30-15}$$

$$= (155117520)(0.85)^{15} (1-0.85)^{15}$$

$$= 5.93 \times 10^{-6} \text{ (Ans)}$$

(e) $P(x=0) = p(1-p)^{x-1}$

$$P(x=0) = 0.08(1-0.08)^{0-1}$$

$$= 0.08 (1-0.08)^{-1}$$

$$= 0.08 \left(\frac{25}{23}\right)$$

$$= \frac{2}{23} \text{ (Ans)}$$

4 Solution

Given at

$p = \frac{1}{6}$ $r = 3$ $x = 20$

$$P(X = x) = \binom{x-1}{r-1} (1-p)^{x-r} p^r$$

$$= \binom{20-1}{3-1} \left(1-\frac{1}{6}\right)^{20-3} (1/6)^3$$

$$= \binom{19}{2} \left(1-\frac{1}{6}\right)^{17} (1/6)^3$$

$$= (171)(0.045)\left(\frac{1}{216}\right)$$

$$= \frac{57}{1600} \text{ (Ans)}$$

5(a)

Grivanfat

$N = 20 + 10 = 30$

$r = 20$

$n = 7$

$$P(x=6) = \frac{\binom{r}{x}\binom{N-r}{n-x}}{\binom{N}{n}}$$

$$= \frac{\binom{20}{6}\binom{30-20}{7-6}}{\binom{30}{7}}$$

$$= \frac{\binom{20}{6}\binom{10}{1}}{\binom{30}{7}}$$

$$= \frac{(210)(10)}{2035800}$$

$$= \frac{2100}{2035800}$$

$$= \frac{7}{6766} \quad (\text{Ans})$$

$$P(x=1) = \frac{\binom{20}{1}\binom{30-20}{7-1}}{\binom{30}{7}}$$

$$= \frac{\binom{20}{1}\binom{30-20}{7-1}}{\binom{30}{7}}$$

$$= \frac{(20)\ (210)}{2035800}$$

$$= \frac{4200}{2035800}$$

$$= \frac{7}{33593} \text{ (Ans)}$$

5(b)

(b) Poisson distribution is λ if $x \sim P(\lambda)$

$$P(x=x) = \frac{e^{\lambda} \lambda^x}{x!}$$

$$x \sim P(\lambda)$$

$$\sum_{x=0}^{\infty} P(x=x) = \sum_{x=0}^{\infty} \frac{e^{-\lambda} \lambda^x}{x!}$$

$$= \left(\frac{1}{1} + \frac{\lambda}{1} + \frac{\lambda^2}{2!} + \frac{\lambda^3}{3!} + \frac{\lambda^4}{4!} \cdots \right)$$

$$\Rightarrow e^{-\lambda} \times \lambda^x = 1$$

Therefore,

$$E(x) = \text{Var}(x) = \lambda \quad \text{(Shown)}$$

Given that $\lambda = 5$

$x = 0$

$$P(x=x) = \frac{e^{-\lambda} \lambda^x}{x!}$$

$$P(x=0) = \frac{e^{-5} (5)^0}{0!}$$

$$= e^{-5}$$

$$= 6.73 \times 10^{-3} \text{ (Ans)}$$

G(a) $x \sim U(a,b)$

In order Area under the pdf is equal to 1, so it heig

is $\frac{1}{b-a}$, so that $f(x) = \frac{1}{b-a}$

for $a \leq x \leq b$ and $f(x)$

where else cdf is

$$F(x) = \int_{y=a}^{x} \frac{1}{b-a}\, dy = \frac{x-a}{b-a}$$

for $a \leq x \leq b$

6(b) $E(x) = \frac{1}{\lambda}$

$f(x) = \lambda e^{-\lambda x}$

$E(x) = \int_0^{\infty} x f(x)\, dx$

$= \int_0^{\infty} x \cdot \lambda e^{-\lambda x} - \int_0^{\infty} \frac{dx}{dx} \int e^{-\lambda x}\, dx$

$= \lambda x \int_0^{\infty} e^{-\lambda x} - \int_0^{\infty} \frac{dx}{dx} \int e^{-\lambda x}\, dx$

$= \frac{\lambda}{\lambda} \left| -x e^{-\lambda x} \right|_0^{\infty} + \frac{\lambda}{\lambda} \int e^{-\lambda x}\, dx$

$= \frac{\lambda}{\lambda} \left[-\infty e^{-\infty} + 0 \times e^{0} \right] + \frac{\lambda}{\lambda^2} \left[-e^{\lambda x} \right]_0^{\infty}$

$= \frac{\lambda}{\lambda} \left[-\infty e^{-\infty} + 0 \times e^{0} \right] + \frac{1}{\lambda} \left[-e^{-\infty} + e^{0} \right]$

$= [0 + 0] + \frac{1}{\lambda} [0 + 1]$

$= \frac{1}{\lambda}$

$\therefore E(x) = \frac{1}{\lambda}$ [proved]

Final Examination, Spring 2018

Department of Mathematics and Physics, North South University
Course MAT 361: Probability and Statistics, Section: 10
Time: 1 Hour 15 Minutes, Full Marks: 40

(Answer ALL questions)

1. (a) From the experiment of throwing two dices together find
   (i) Sample space.
   (ii) Event A that consists of both times only odd numbers.
   (iii) event B that consists of both times only even numbers.

   (b) Suppose in a city of UK, 60% of the people are British (B), 70% of the people are White (W), 50% of the people are both British and White. What is the probability that a person is White (W), given that the person is British (B)?

2. (a) Define continuous random variable.

   (b) Define continuous Uniform distribution? Find its cumulative distribution function.

   (c) Show how does the binomial distribution can be approximated with the normal distribution.

3. (a) If your expected marks in Fial exam of MAT361.10 has $N(30, 6)$. How many students will expect to get between 21 and 30 marks. Find the median mark.

   (b) Suppose scores on a mathematics test have a mean 60 and standard deviation 20 while scores on an English test have a mean 60 and standard deviation 10. (a) If Bob gets 80 on both tests, on which test did he do better relative to his class mates? (b) If the scores on the tests each follow a normal distribution how many students did better than Bob in each case?

4. (a) Briefly discuss the properties of point estimates. Find the sampling distribution of $\bar{X}$. Is $\bar{X}$ an unbiased estimator of μ?

   (b) Find the MLE for λ if $X_1, X_2, \cdots, X_{10} \sim \text{Pois}(\lambda)$. Is this bias?

5. (a) What is 95% t-confidence interval for μ? Briefly discuss the effect of sample size on the confidence interval.

   (b) Suppose the sample mean age of 36 workers in the Samsung electronics industries in South Korea is 25 years and a sample standard deviation is $S = 4$. Construct a 95% two-sided t-confidence interval for population mean μ. [if $t_{0.05,35} = 2.1$)].

6. (a) What is hypothesis and p-value in test of hypothesis? Briefly mention the steps of the test of hypothesis procedure for single sample mean test.

   (b) Test the claim that true mean age of the workers in the Samsung electronics industries in South Korea is 25 years. Suppose, the level of significance is $\alpha = 5\%$ and a randomly 64 workers are chosen for this test with sample mean age $\bar{X} = 26$ and $S = 4$. [if $P(X > |6|) = 0$].

Final

Chapter-1

Question-1

1.1 Probability

Sample space: It is a set of all possible outcome that can occur

SP18 a

Sample space throwing a two dice:

a(i)

	1	2	3	4	5	6
1	(1,1),	(1,2),	(1,3),	(1,4),	(1,5),	(1,6)
2	(2,1),	(2,2),	(2,3),	(2,4),	(2,5),	(2,6)
3	(3,1),	(3,2),	(3,3),	(3,4),	(3,5),	(3,6)
4	(4,1),	(4,2),	(4,3),	(4,4),	(4,5),	(4,6)
5	(5,1)	(5,2),	(5,3),	(5,4),	(5,5),	(5,6)
6	(6,1),	(6,2),	(6,3),	(6,4),	(6,5),	(6,6)

Sample space : {1,2,3,4,5,6}

ii) Event A that consists of both time only odd number

(ii) $S = \{(1,1), (1,3), (1,5), (3,1), (3,3), (3,5), (5,1), (5,3), (5,5)\}$

$= 9$

$P(A) = \frac{9}{36}$

$S = \{(2,2), (2,4), (2,6), (4,2), (4,4), (4,6), (6,2), (6,4), (6,6)\}$

$= 9$

$P(B) = \frac{9}{36}$

ii) Event B that consists of both times any even number

Conditional probability class Example : 2

SP18 1 (D)

Q. Suppose in a city of UK, 60% of the people are British (B), 70% of the people are white (W), 50% of the people are Both British and white.

What is the probability that the person in white (W), given that the person is British (B)

Ans Given that

$$P(B) = \frac{60}{100} = 3/5$$

$$P(W) = \frac{70}{100} = \frac{7}{10}$$

$$P(B \cap W) = \frac{50}{100} = \frac{1}{2}$$

$$P(W \mid B) = \frac{P(W \cap B)}{P(B)}$$

$$= \frac{1/2}{3/5}$$

$$= \frac{5}{6} \text{ (Ans)}$$

Final

Chap: 4

Continuous Probability Distribution

4.1 Uniform Distribution

SP18 2(a),(b)

Define continous random variable

Ans:- A continous random variable is a variable where the data can take infinitely many values

Defiul continuous uniform Distribution

Ans:- The continuous uniform is the probability distribution of random number selection from the continous interval between a and b.

It's pdf

$$f(x) = \frac{1}{b-a} \quad \text{where} \quad a \leq x \leq b$$

[illegible]

for $a \leq x \leq b$ and $f(x) = 0$

$$F(x) = \int_{y=a}^{x} \frac{1}{b-a}\, dy = \frac{x-a}{b-a}$$

Question. 2

Final

Chapter: 3 and 4

Chapter: 3

SP 18 2 (c)

: Show how does the binomial distribution can be pproximated with the normal distribution

Ans:- The normal distribution can be used as an pproximation to the binomial distribution, under ircumstances, namely if $x \sim B(n,p)$ and if n is larger r p is closer to $1/2$, then x is approximately

$\sim N(np, npq)$

[SP18]

3(a) $x \sim N(30, 6)$

$\mu = 30$

$\sigma^2 = 6$

$\sigma = \sqrt{6}$

$$P(21 \le x \le 30) = \Phi\left(\frac{b-\mu}{\sigma}\right) - \Phi\left(\frac{a-\mu}{\sigma}\right)$$

$$= \Phi\left(\frac{30-30}{\sqrt{6}}\right) - \Phi\left(\frac{21-30}{\sqrt{6}}\right)$$

$$= \Phi\left(\frac{9}{\sqrt{6}}\right)$$

$$= \Phi(3.7)$$

$$= 0.4999 \text{ (Ans)}$$

3(b) Math Test $(60, 20^2)$

$\mu_1 = 80 \times 60 = 1600$

$G_1^2 = 80 \times 20^2 = 3200$

English Test $(60, 10^2)$

$\mu_2 = 80 \times 60 = 1600$

$G_2^2 = 80 \times 10^2 = 8000$

Mean for Math and English Test (x)

$\mu = 1600 - 1600 = 0$

Standard deviation for math and Test (y)

$G^2 = G_1^2 + G_2^2$

$G^2 = 3200 + 8000$

$\therefore G = 0.105$

$x \sim N(0, 0.105)$

$P(x - y \geq 0) = 1 - P(x - y \leq 0)$

$= \Phi\left(\frac{0-0}{0.105}\right) - \Phi\left(\frac{x-0}{0.105}\right)$

$= 1 - [0.07]$

$= \approx 1$

Both Math and English are Same

7.2 Properties of Point Estimate.

$E(\hat{\theta}) = \theta$

$\hat{p} = \frac{x}{n}$

$x \sim B(n, p)$

$E(x) = np$

$E(\hat{p}) = E\left(\frac{x}{n}\right) = \frac{1}{n} E(x) = \frac{1}{n} np = p$

Hence $\hat{p}$ is the unbiased point estimate of the success probability p

Example: 3

Find MLE for λ if $x_1, x_2 \ldots x_{10} \sim P(\lambda)$

$$E(x) = \lambda$$

$$\hat{\lambda} = \bar{x}$$

$$f(x,\lambda) = \frac{e^{-\lambda}\lambda^x}{x!}$$

$$L(x_1, \ldots, x_n, \lambda) = \prod_{i=1}^{n} f(x,\lambda) = \frac{e^{-n\lambda}\lambda^{(x_1 + \ldots + x_n)}}{(x_1! \times \ldots \times x_n!)}$$

Therefore

$$\ln(L) = n\lambda + (x_1 + \ldots + x_n)\ln(\lambda) - \ln(x_1! \times \ldots x_n!)$$

$$\frac{d\ln(L)}{d\lambda} = -n + \frac{(x_1 + \ldots + x_n)}{\lambda}$$

$$\hat{\lambda} = \bar{x}$$

$$\hat{\lambda} = 0.517$$

$$\text{var}(\bar{x}) = \frac{\lambda}{n}$$

SP 6a

(6a)

The word hypothesis indicates that this state will be tested with an appropriate data set.

It is useful to associate a null hypothesis with an alternative hypothesis

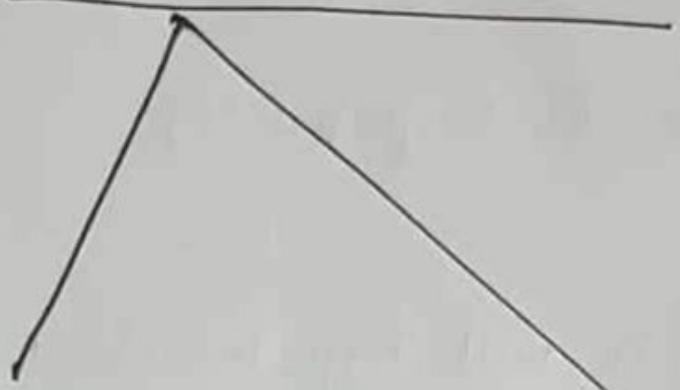
The plausibility of a null hypothesis is measured with a p-value, which is a probability that takes a value between 0 and 1.

Step of Hypothysis testing

Step 1: constesting Hypothysis

Step 2: fix $\alpha = 5\%$

Step 3: Test Statistic



$t = \frac{\bar{x} - M}{s\sqrt{n}}$ (t-test)

(z-test)

$z = \frac{\bar{x} - M}{\sigma\sqrt{n}}$

Step 4: P-value calculation

Step 5: $P(t \geq t_0) + P(t \leq -t_0)$

Step 6: Reject H_0 iff (if and only if)

6 (B)

Given that

$\alpha = 5\%$

$n = 25$

$S = 4$

$\bar{x} = 28$

$\mu_0 = 64$

$$t = \frac{\bar{x} - \mu_0}{s\sqrt{n}}$$

$$= \frac{28 - \mu_0}{4\sqrt{25}}$$

$$= \frac{28 - 64}{4\sqrt{25}}$$

$$= \frac{-36}{20}$$

$$= -1.8$$

$$P(t < -1.8) + P(t > 1.8)$$