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$$\int e^x \sqrt{1-e^{2x}}\,dx.$$

Set,

$a = 1$ and $u = e^x$.

So,

$u = a\sin\theta$

$\Rightarrow e^x = \sin x$

$\Rightarrow e^x dx = \cos\theta\, d\theta.$

Therefore,

$$\int \sqrt{1-(e^x)^2}\,.\,e^x dx = \int \sqrt{1-\sin^2\theta}\,\cos\theta\, d\theta$$

$$= \int \sqrt{\cos^2\theta}\,\cos\theta\, d\theta \quad \left[\because \sin^2\theta + \cos^2\theta = 1 \Rightarrow \cos^2\theta = 1 - \sin^2\theta\right]$$

$$= \int \cos^2\theta \, d\theta$$

$$= \int \left(\frac{1+\cos 2\theta}{2}\right) d\theta.$$

$$= \frac{1}{2}\int (1+\cos 2\theta)\, d\theta.$$

$$= \frac{1}{2}\left(\theta + \frac{1}{2}\sin 2\theta\right) + C.$$

$$= \frac{\theta}{2} + \frac{1}{4}\sin 2\theta + C.$$

$$= \frac{\theta}{2} + \frac{1}{4}(2\sin\theta\cos\theta) + C.$$

$$= \frac{\theta}{2} + \frac{1}{2}\sin\theta\cos\theta + C.$$

$$= \frac{\sin^{-1}e^x}{2} + \frac{1}{2}(e^x)\left(\sqrt{1-e^{2x}}\right) + C.$$

$$\left[\because \sin\theta = e^x \Rightarrow \theta = \sin^{-1}(e^x) \quad \cos\theta = \sqrt{1-e^{2x}}\right]$$

$$= \frac{\sin^{-1}(e^{x}) + e^{x}\sqrt{1-e^{2x}}}{2} + C.$$

(Ans).