Mid-Term Assessment (Spring 2021)
Department of Mathematics & Physics
School of Engineering & Physical Sciences
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Course Instructor:
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COURSE CODE: MAT 250 **TITLE: CALCULUS & ANALYTICAL GEOMETRY III** **SET: αLPHA**

Instructions:

- *You must answer all the questions.*
- *You may prepare the assignment by typing or by handwriting. For handwritten, please write your answers neatly in a clear white paper and compile your work into a single PDF.*
- *Write your ID at the top of each page of your assignment.*

Important Notes:

- ***You have to solve the assignment with honesty and integrity.***
- ***Submit the assignment as soon as you complete it.***
- ***You should not share your solutions with others. Each submission will be carefully examined, and it may go through 'plagiarism test' on your assignment***
- ***Significant similarity (copying from others) would severely reduce marks from both.***
- ***This submission will carry 20% marks for grading***
- ***Please note that a viva for 5 marks will be taken later on the topics/problems of assignment***

Problem 01:

(a) Find the domain of the following functions and then graph the domain in the xy-plane. Use a solid curve to indicate that the domain includes the boundary and a dashed curve to indicate that the domain excludes the boundary.

$z = \sqrt{x+5} + \ln(y^2 - 4x)$	$z = \sqrt{\dfrac{x^2+y^2}{x^2-y^2}}$	$z = \dfrac{y}{\sqrt{9-x^2-y^2}}$

(b) Sketch the graph of the following surface:

$z = 4x^2 + y^2$	$z = \sqrt{\dfrac{x^2}{4^2} + \dfrac{y^2}{4^2}}$

(c) Graph the level curves corresponding to the given values of c

$z = x^2 - y^2$ at $c = 0,1,4,9$	$z = y - \ln x$ at $c = 1,2,4$

Problem 02:

(a) Show that $\lim\limits_{(x,y)\to(0,0)} \frac{x^2y}{x^4+y^2}$ does not exist.

(b) Find the gradient of the surface $\ln(2x^2 + y - z^3 + 1) = x$ at the point $(0,-8,-2)$ along x-direction and y-direction.

(c) The temperature T (in degree Celsius) on a metal plate, located in the xy-plane, at any point (x, y) is given by $T(x, y) = x^2ye^{xy}$. Find the rate at which temperature changes if you start at the point $(1,2)$ and move vertically upward.

Problem 03:

(a) What is the meaning of directional derivative of $f(x, y)$?

(b) The temperature T (in degree Celsius) on a metal plate, located in the xy-plane, at any point (x, y) is given by $T(x, y) = e^x(\sin x + \sin y)$.

(i) What is the rate of change of T at $(0,0)$ in the direction of $3\mathbf{i} - 4\mathbf{j}$?

(ii) Find the directions in which the temperature increases and decreases most rapidly at $(-3, 4)$.

(iii) Find the directions in which the rate of change of T at $(-3,4)$ is equal to 0?

Problem 04:

(a) Find an equation of the tangent plane to the following surface at the given point.

(b) Find symmetric equations of the normal line to the following surface at the given point.

$$x^{2/3} + y^{2/3} + z^{2/3} = 9 \text{ at } (1, 8, -8)$$

(c) Two surfaces are said to be **tangent at a common point** P_0 if each has the same tangent plane atP_0. Show that the surfaces $x^2 + 4y + z^2 = 0$ and $x^2 + y^2 + z^2 - 6z + 7 = 0$ are tangent at the point $(0, -1, 2)$.

Problem 05:

A manufacturer wants to make an open rectangular box of volume $V = 500$ cm^3 using the least possible amount of material. Find the dimensions of the box.

Problem 06:

(a) State Fubini's theorem for double integral

(b) Use Fubini's Theorem to find the following integrals.

$\iint\limits_R x^3 \cos(x^2 y)\ dA, \quad 0 \le x \le \frac{\pi}{2}, 0 \le y \le 1$	$\iint\limits_R x \sec^2 y\ dA, \quad 0 \le x \le 3, 0 \le y \le \frac{\pi}{4}$