0.4 Inverse Function:

$5 \times \frac{1}{5} = 1$ $f(x)$, $f^{-1}(x)$

$g(x)$, $g^{-1}(x)$

$f(x)$ $g(x)$

$$\begin{cases} \text{(i)} & (f \circ g)(x) = f(g(x)) = x \\ \text{(ii)} & (g \circ f)(x) = g(f(x)) = x \end{cases} \quad \begin{cases} g = f^{-1}(x) \\ f = g^{-1}(x) \end{cases}$$

Ex 2: (i) $f(x) = 2x$ $\quad f^{-1}(x) = \frac{1}{2}x$ $\quad$ (ii) $g(x) = x^3$, $g^{-1}(x) = x^{1/3}$

Sol: (i)

$$(f \circ f^{-1})(x) = f(f^{-1}(x)) = f\left(\frac{1}{2}x\right) = 2 \cdot \frac{1}{2}x = x$$

$$(f^{-1} \circ f)(x) = f^{-1}(f(x)) = f^{-1}(2x) = \frac{1}{2} \cdot 2x = x$$

$\therefore$ f^{-1} is the inverse of $f(x)$ verified

(ii) $g(x) = x^3$, $g^{-1}(x) = x^{1/3}$

Soln $(g \circ g^{-1})(x) = g(g^{-1}(x)) = g(x^{1/3}) = (x^{1/3})^3 = x$

$(g^{-1} \circ g)(x) = g^{-1}(g(x)) = g^{-1}(x^3) = (x^3)^{1/3} = x$

$\therefore g^{-1}$ is the inverse of $g(x)$. verified.

Domain and Range of Inverse Function :

Domain of f = Range of f^{-1}

Range of f = Domain of f^{-1}

e.g. $\boxed{f(x) = 2x}$, $f^{-1}(x) = \frac{1}{2}x$

D: $(-\infty, +\infty)$ R: $(-\infty, +\infty)$ D: $(-\infty, +\infty)$

R: $(-\infty, +\infty)$

A Method for Finding Inverse Function :

$y = f(x) = 2x$

$y = 2x$

$\boxed{x = \frac{1}{2}y}$

Step 1 : Let $y = f(x)$

Step 2 : Solve for $\boxed{x}$

Step 3 : $\boxed{x = f^{-1}(y)}$

Step 4 : Change x and y

$\boxed{y = f^{-1}(x)}$

Ex 4 : $f(x) = \sqrt{3x-2}$, $f^{-1}(x) = ?$, Domain of $f^{-1} = ?$

10/02/2022

QZ # 01

0.1, 02, 0.3

Solⁿ : Steps :- Let $y = f(x)$

$\Rightarrow y = \sqrt{3x-2}$

Step 2 : $y^2 = 3x - 2 \Rightarrow 3x = y^2 + 2$

$\therefore x = \frac{1}{3}(y^2+2)$

Step 3 : ~~$f^{-1}(y) = y^2 + 2$~~ x

$x = \frac{1}{3}(y^2+2)$

change x and y

$y = \frac{1}{3}(x^2+2)$ $\therefore f^{-1}(x) = \frac{1}{3}(x^2+2)$ Ans

$3x - 2 \geq 0$

$\Rightarrow x \geq 2/3$

D : $[\frac{2}{3}, \infty)$

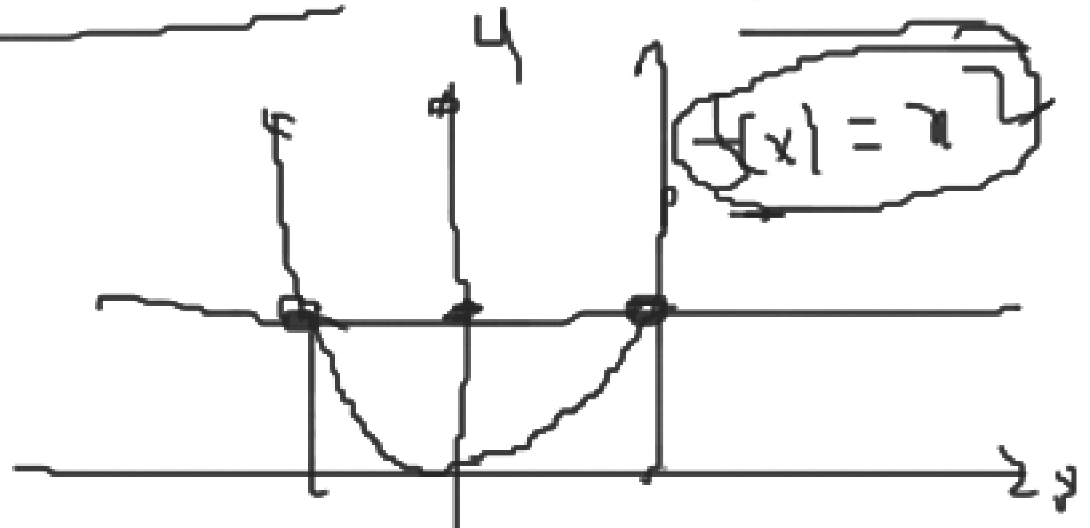
R : $[0, \infty)$

$\therefore$ Domain of f^{-1} is : $[0, \infty)$

Range " f^{-1} " : $[\frac{2}{3}, \infty)$ Ans

Theorem: A function has an inverse iff it is one-to-one.

One-to-one: $f(x_1) = f(x_2) \Rightarrow \boxed{x_1 = x_2}$



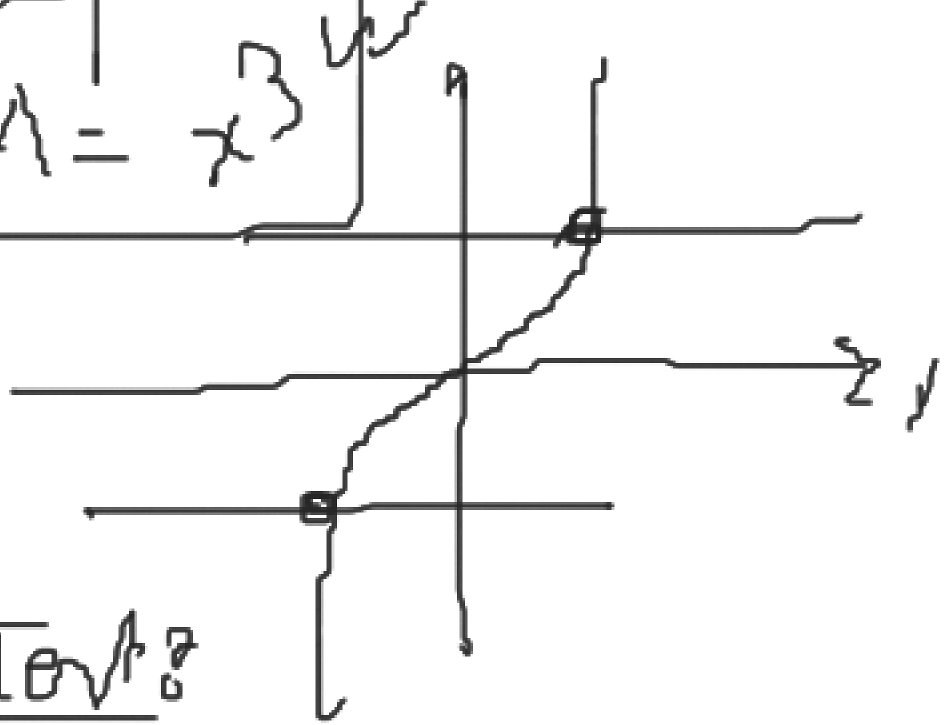
$f(x) = x^2$ X

$y = f(x) = x^2$ X

Let $y = 4$

$4 = x^2 \Rightarrow \boxed{x = \pm 2}$

Ex 2: $\boxed{g(x) = x^3}$



$\therefore y = g(x) = x^3$

$y = 8$

$8 = x^3$

$\boxed{\therefore x = 2}$

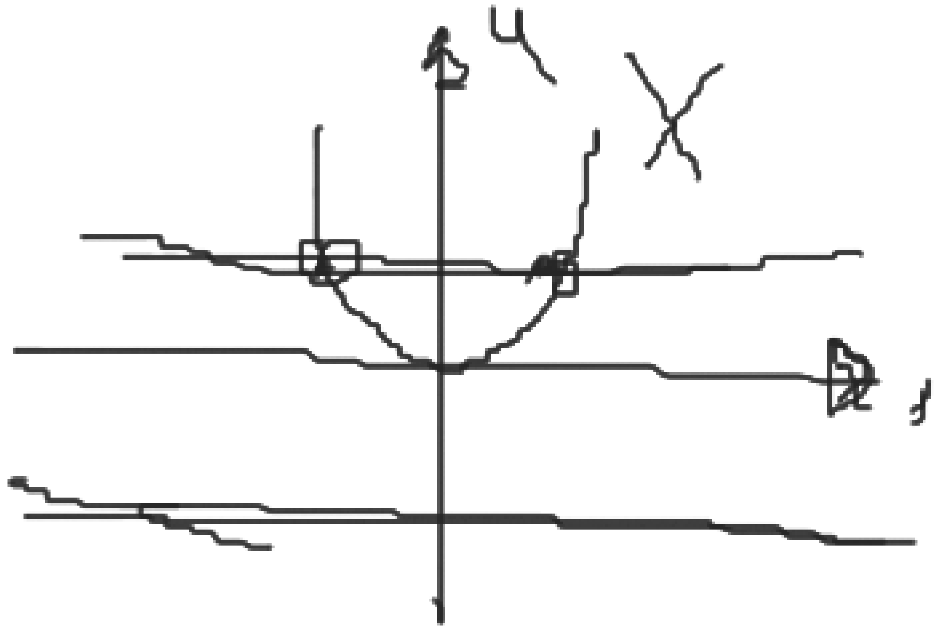
Horizontal Line Test:

\# $f(x)=x^2$ $y=f(x)$. $y=x^2$ X

Let $\boxed{y=4}$; Let $\boxed{y=a}$

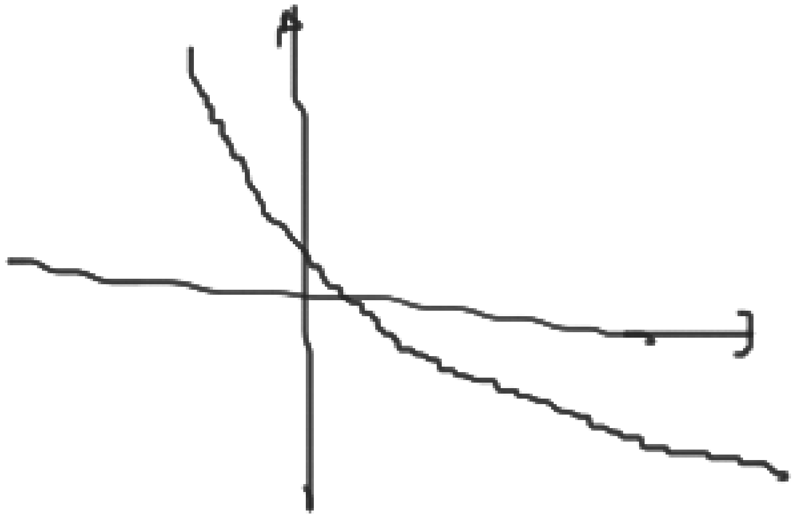
0.1 X 2.3 X

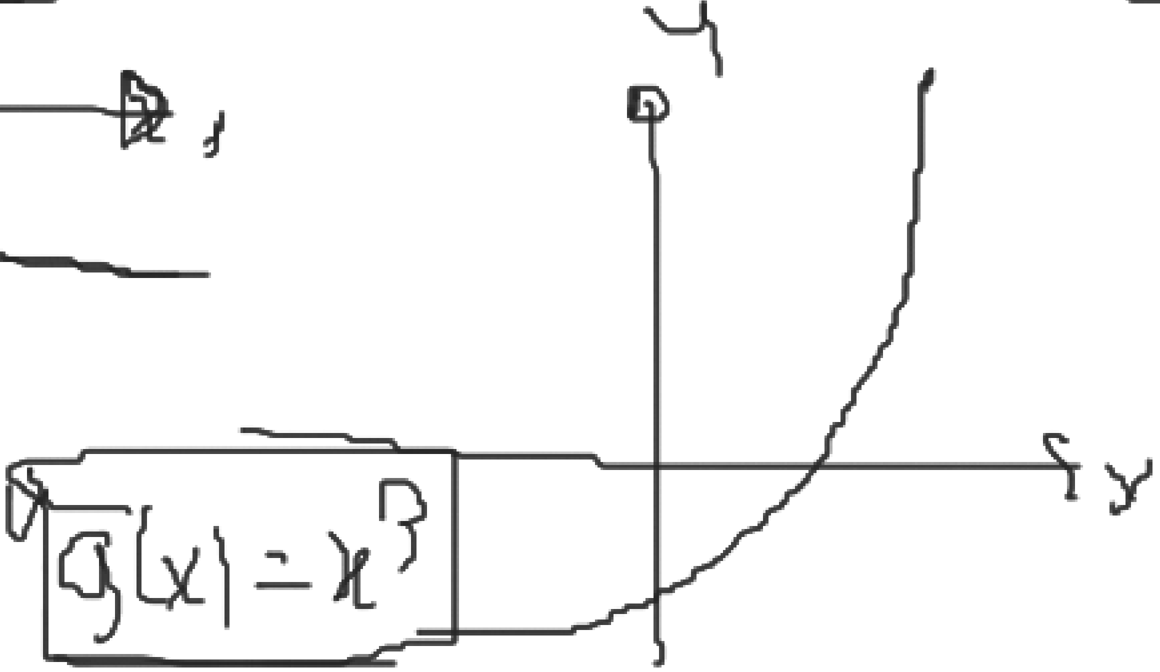
$a=0$ X 1.5 X



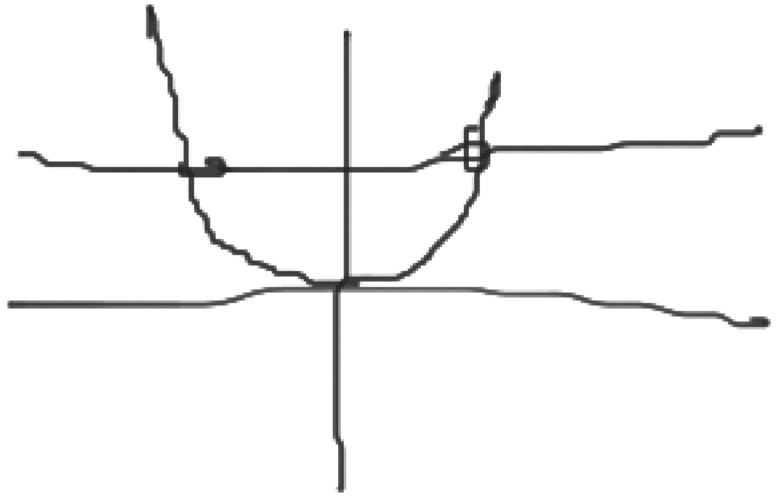
$\therefore a=x^2$

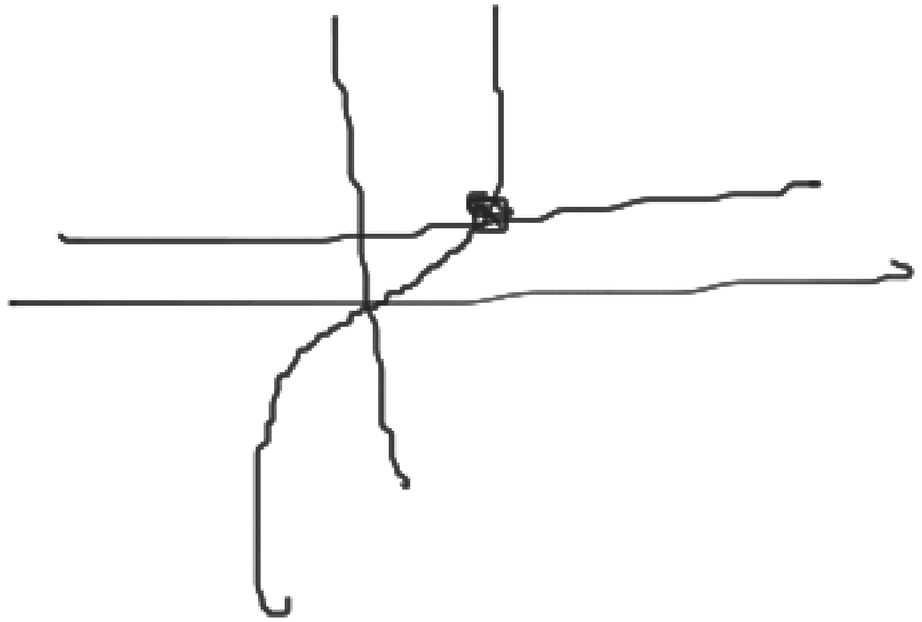
$\Rightarrow \boxed{x=\pm\sqrt{a}}$



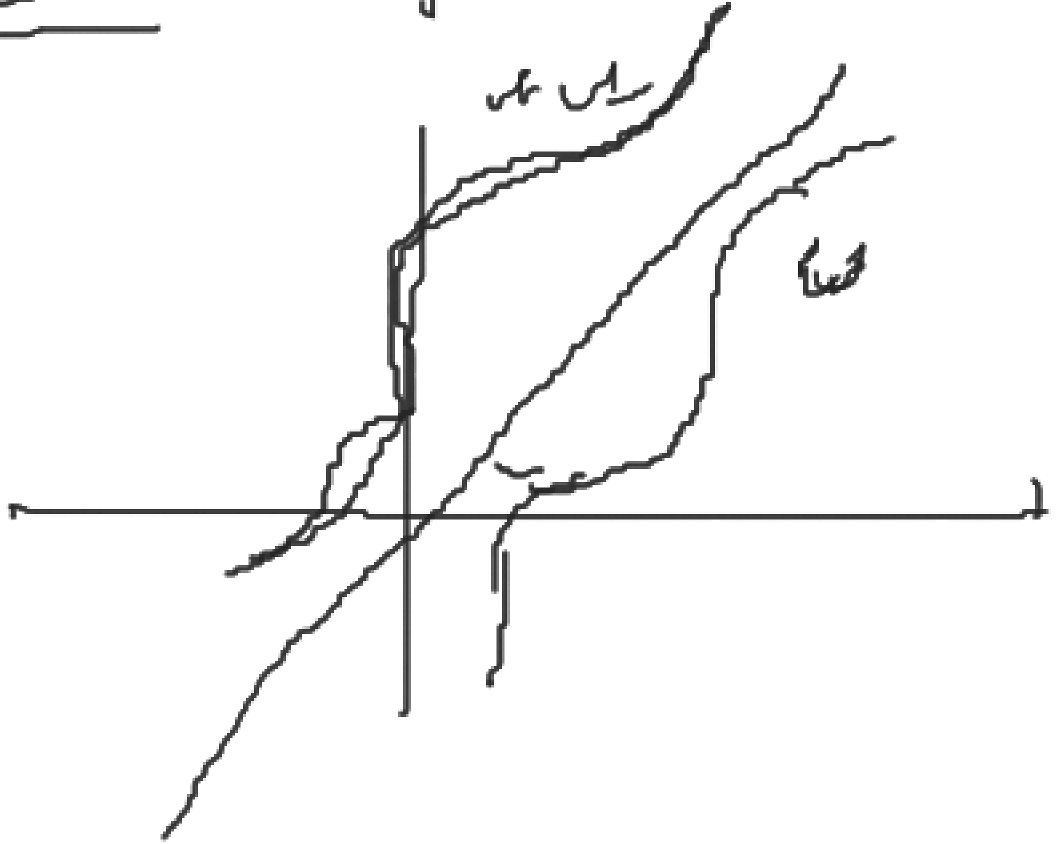


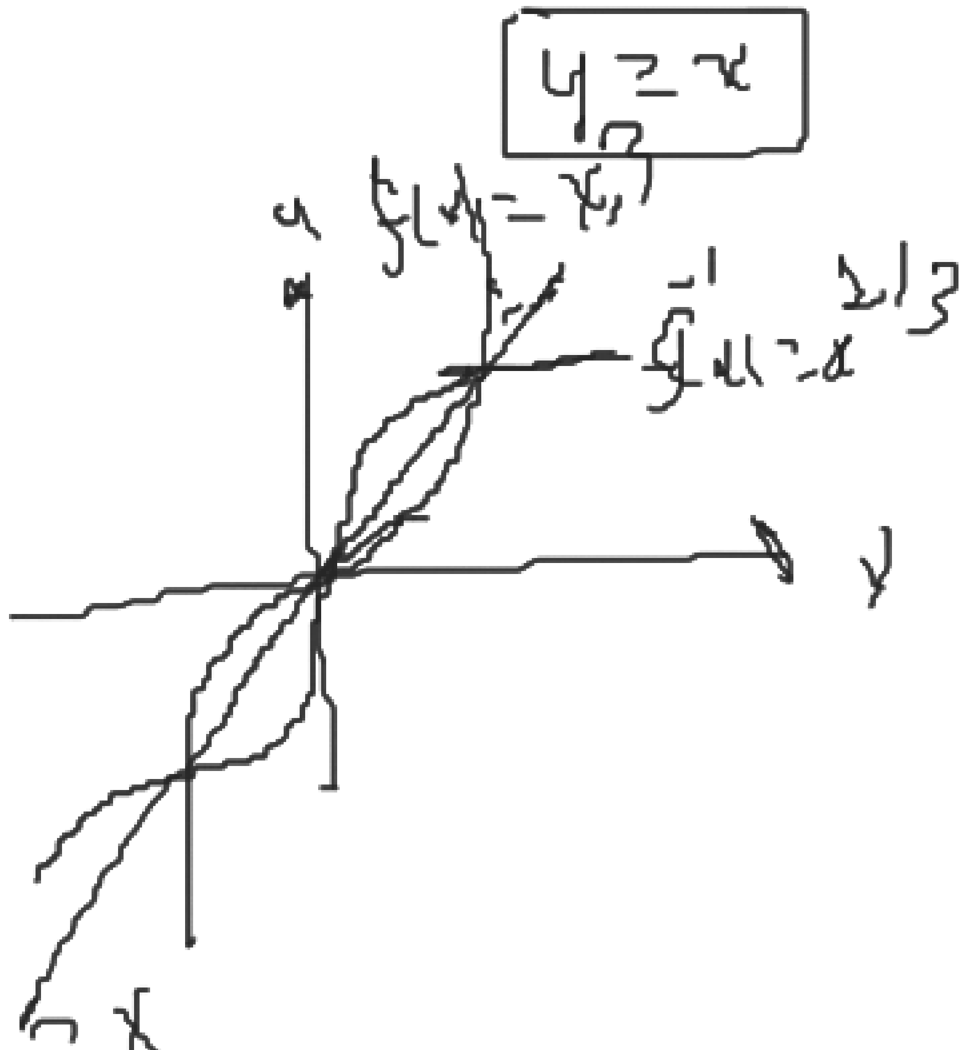
Ex.5 X $\boxed{f(x)=x^4}$, $\boxed{g(x)=x^3}$





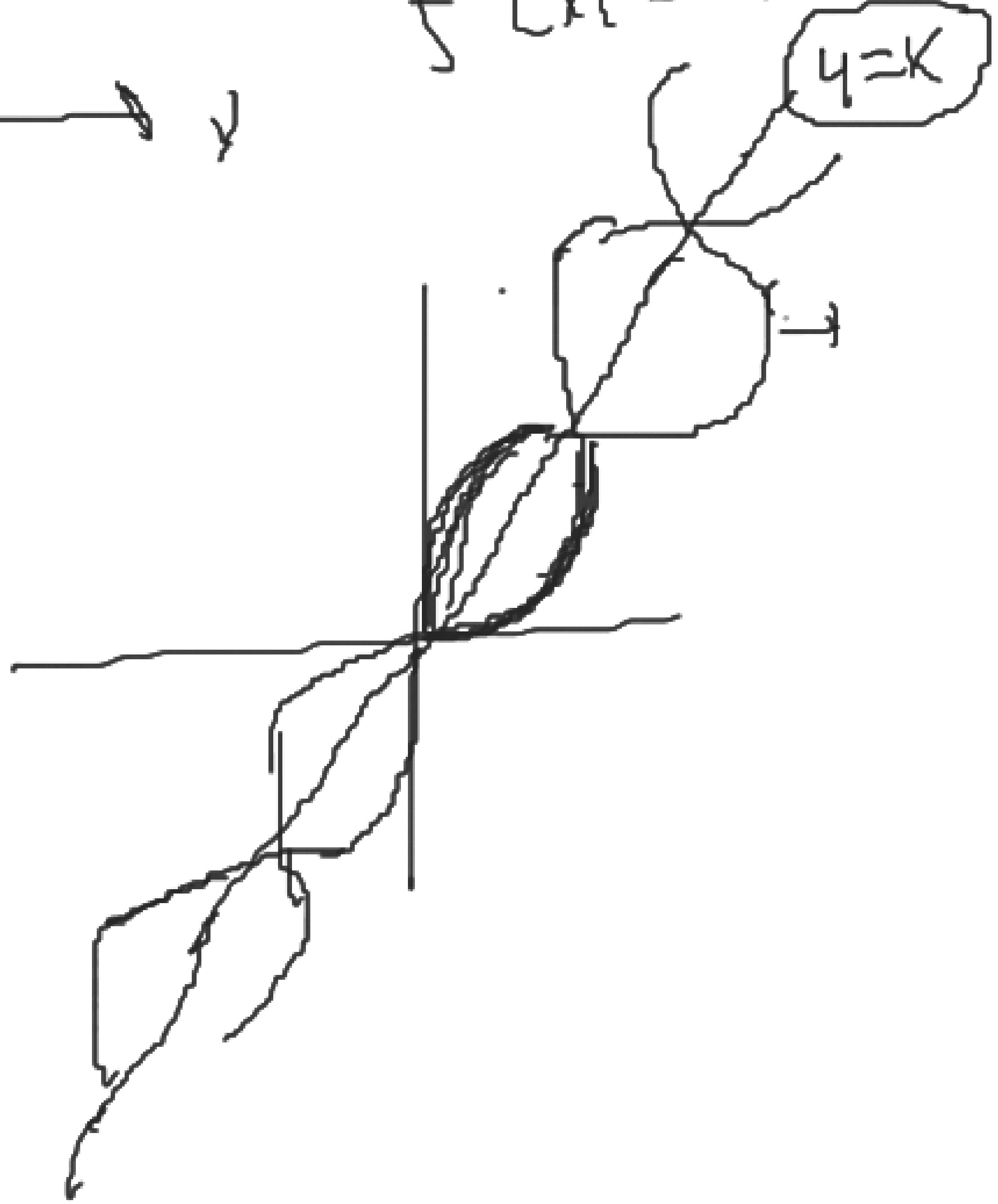
Theorem: $f \to$ has an inverse. $\boxed{y = f(x).}$ $\boxed{y = f^{-1}(x)}$



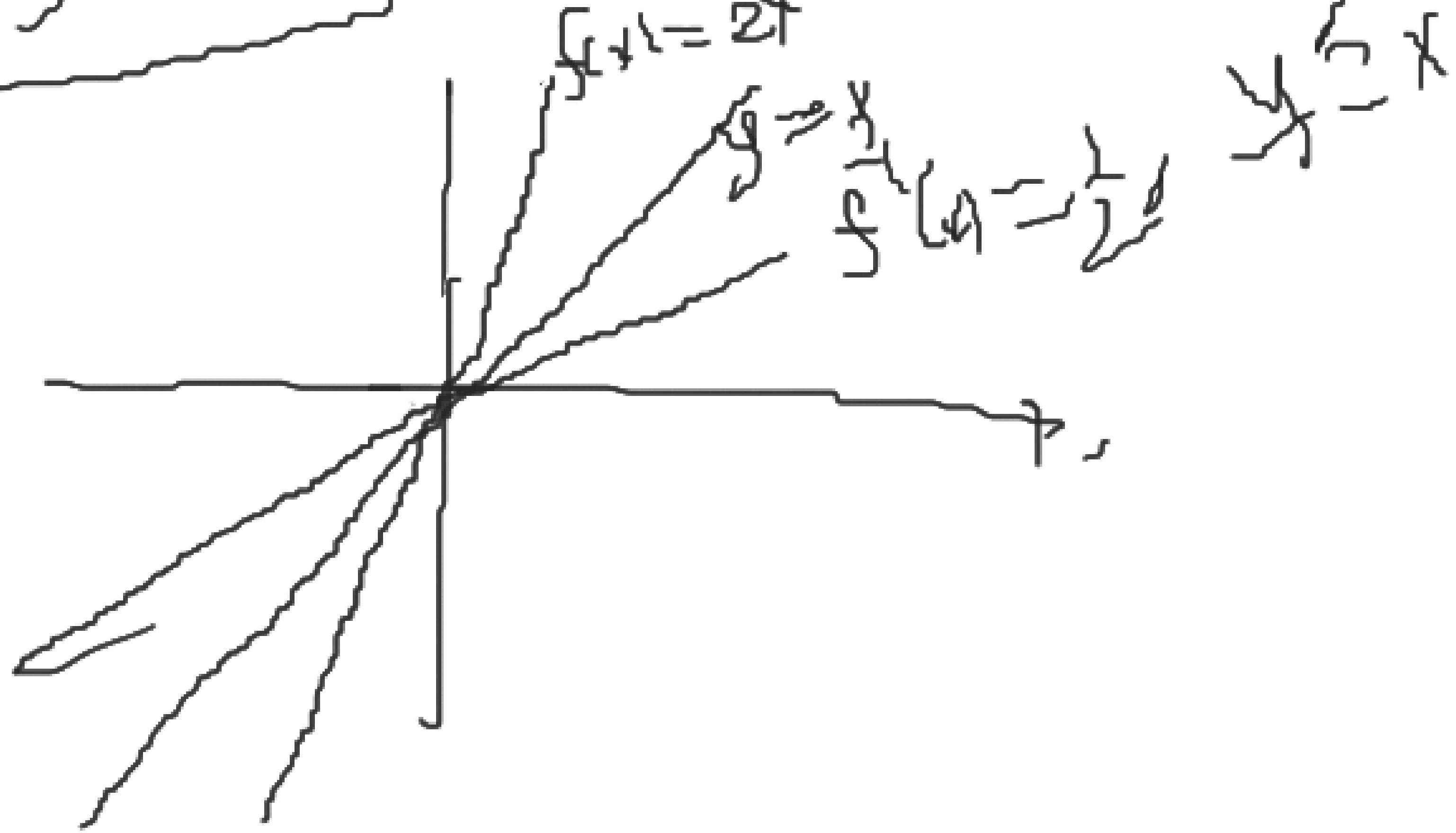


$f(x) = x^3$

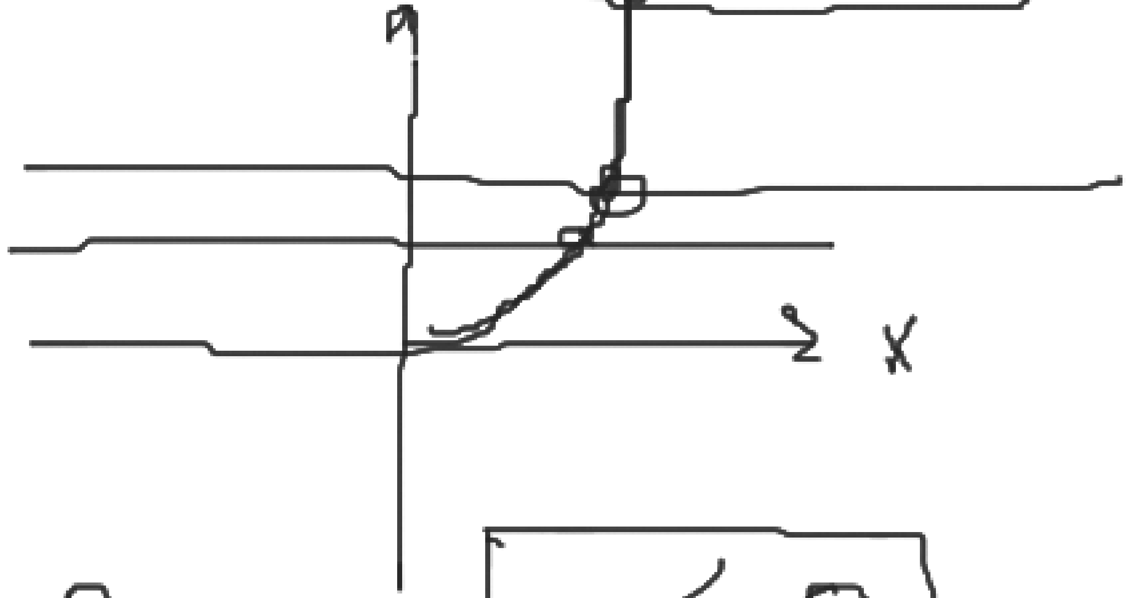
$f^{-1}(x) = x^{1/3}$



$\boxed{f(x) = 2x.}$ $f^{-1}(y) = \frac{1}{2} y$



Restricted Domain: $f(x) = x^2$; $\boxed{x \geq 0}$



Let $y = f(x)$

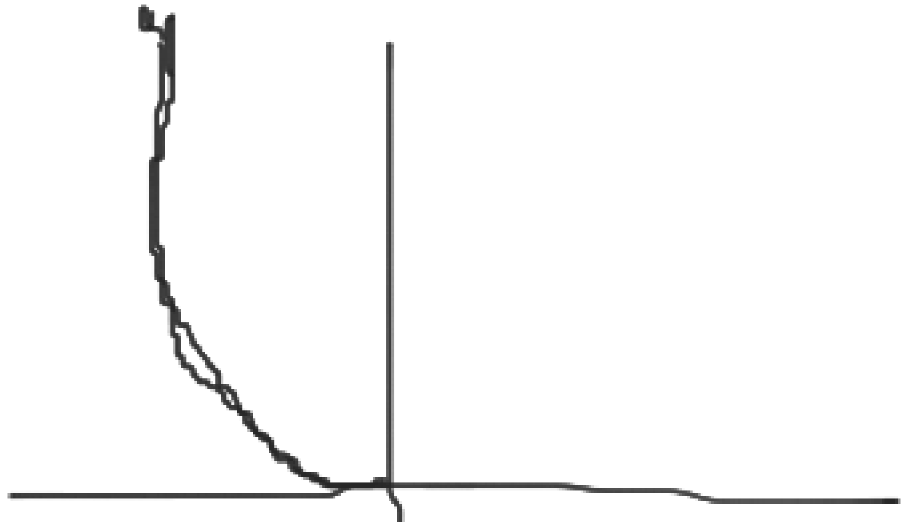
$y = x^2$

$\Rightarrow x = +\sqrt{y}$

$y = +\sqrt{x}$

$\boxed{f^{-1}(x) = +\sqrt{x}}$

$f(x) = x^2$; $\boxed{x \leq 0}$



$y = x^2 \Rightarrow x = -\sqrt{y}$

$y = -\sqrt{x}$

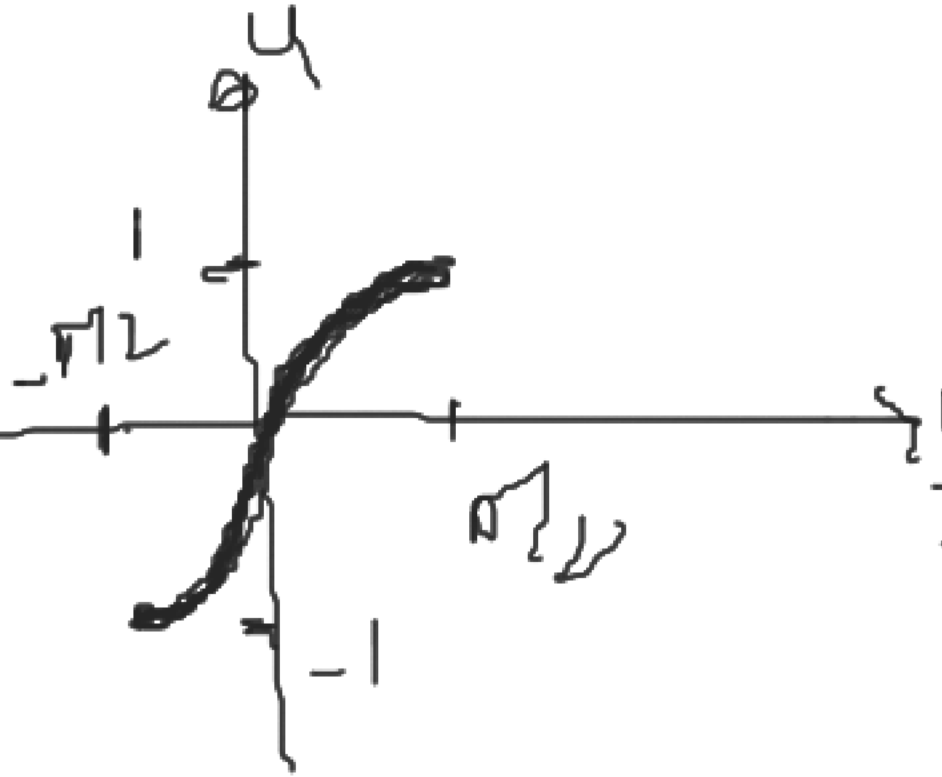
$\therefore f^{-1}(x) = -\sqrt{x}$

Inverse Trigonometric Function: $\boxed{f(x) = \sin x}$

$f(x) = \sin x$; $\boxed{-\pi/2 \le x \le \pi/2}$

D: $[-\pi/2, \pi/2]$

R: $[-1, 1]$



$\boxed{f^{-1}(x)}$, $\boxed{\sin^{-1} x}$

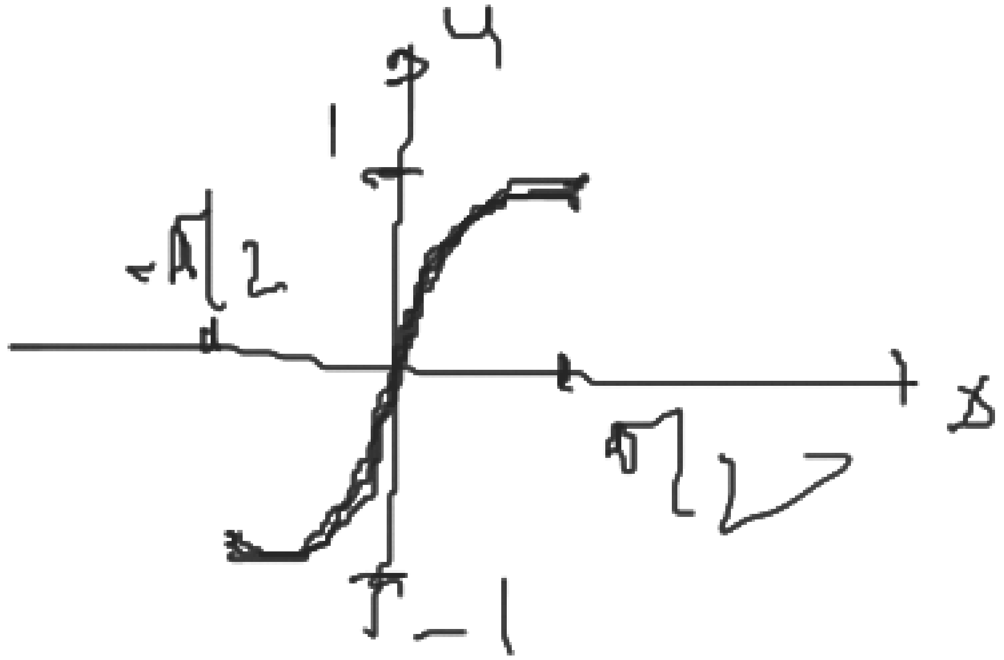
arcsin x

$f(x) = \sin^{-1} x$

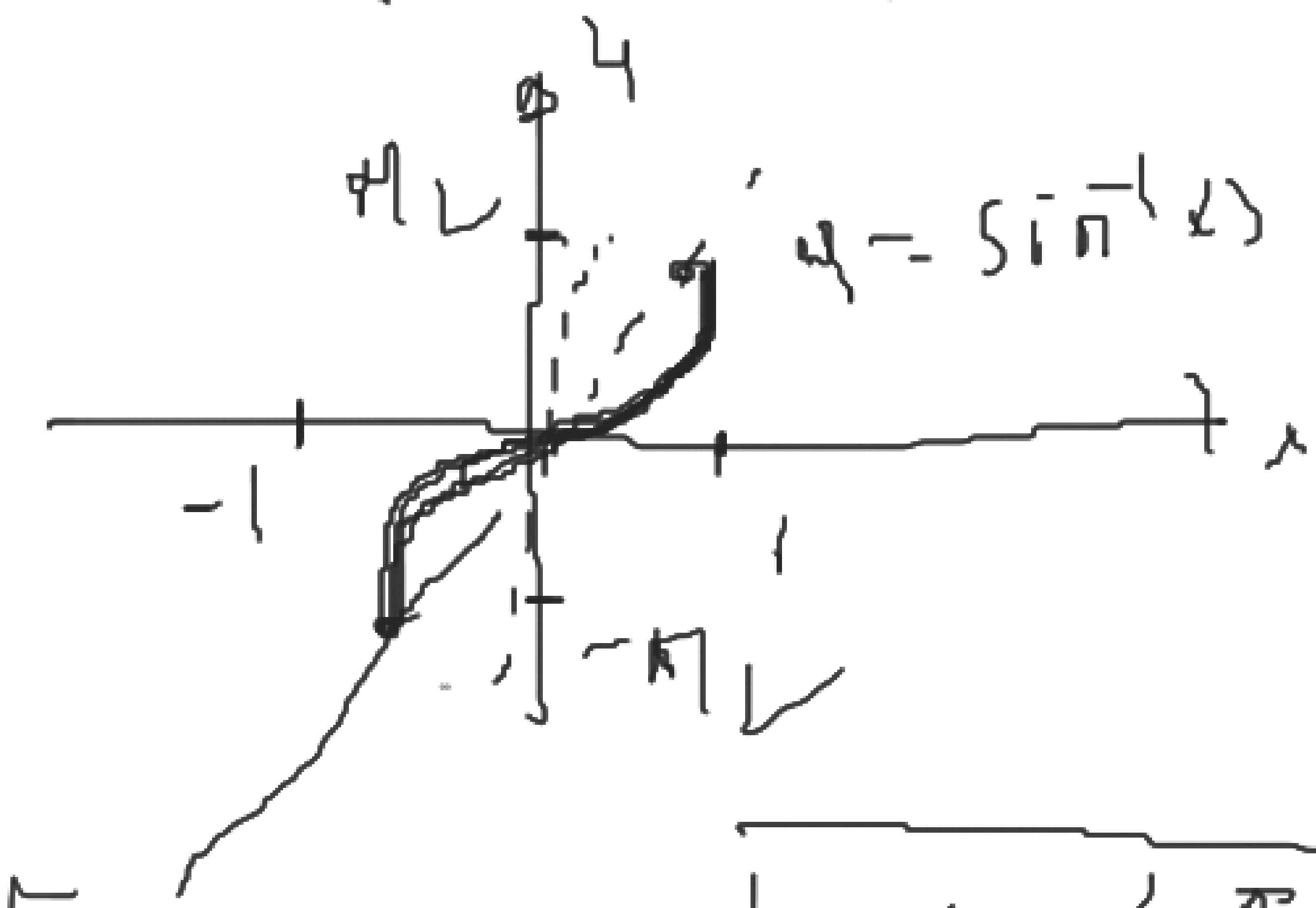
D: $[-1, 1]$

R: $[-\pi/2, \pi/2]$

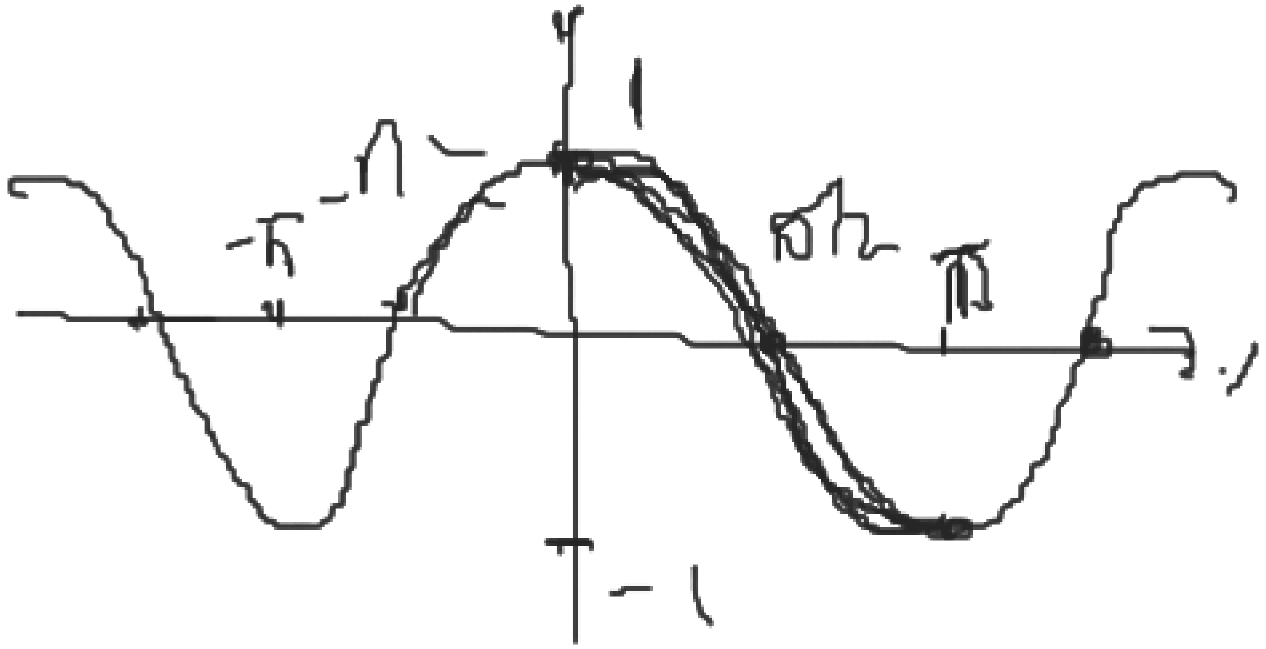
$f(x) = \sin x \qquad -\pi/2 \le x \le \pi/2$



$f^{-1}(x) = \sin^{-1} x$



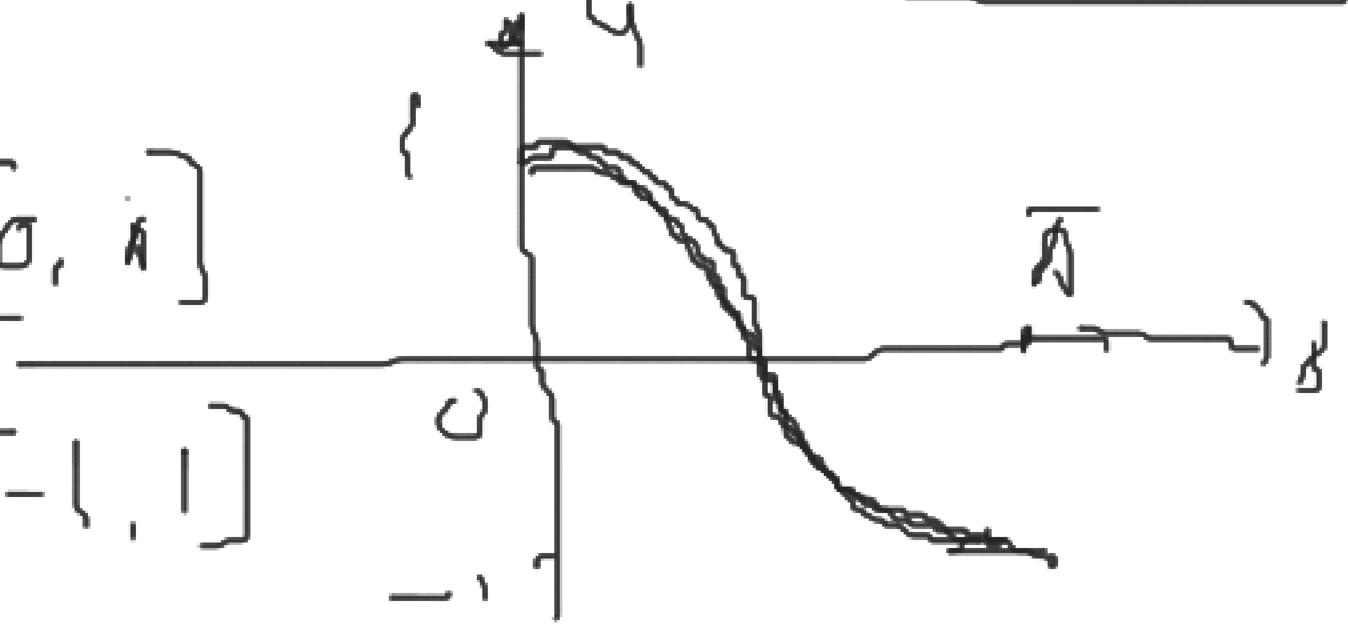
Ex $\quad f(x) = \cos x$



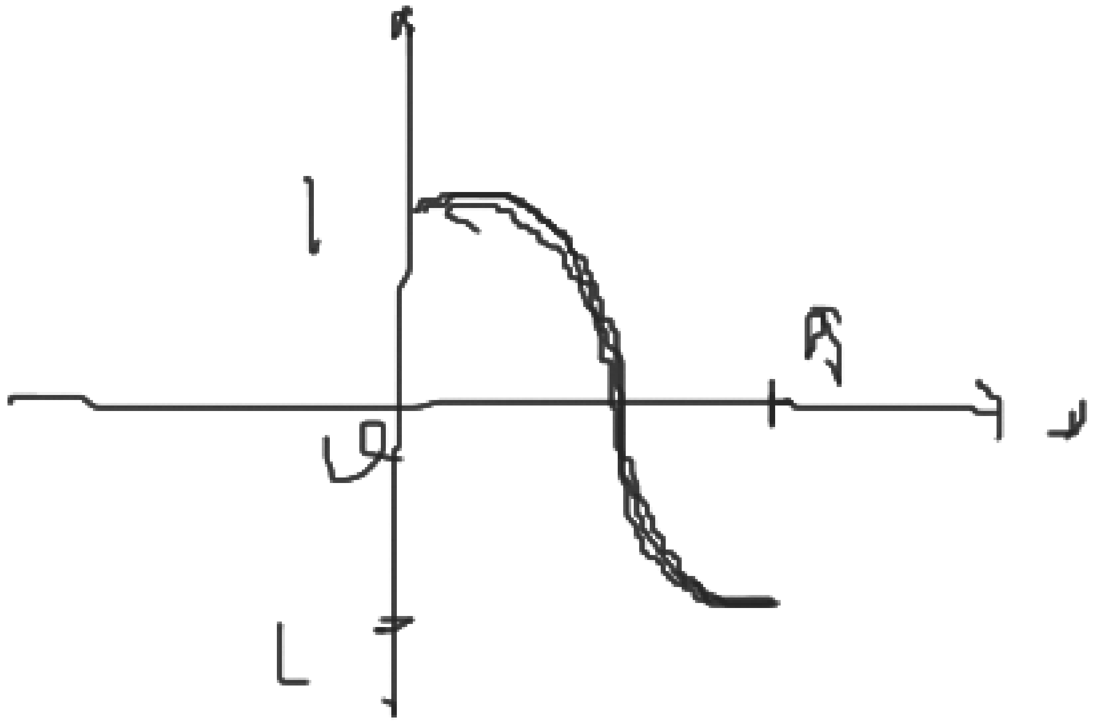
$f(x) = \cos x \;;\; \boxed{0 \le x \le \pi}$

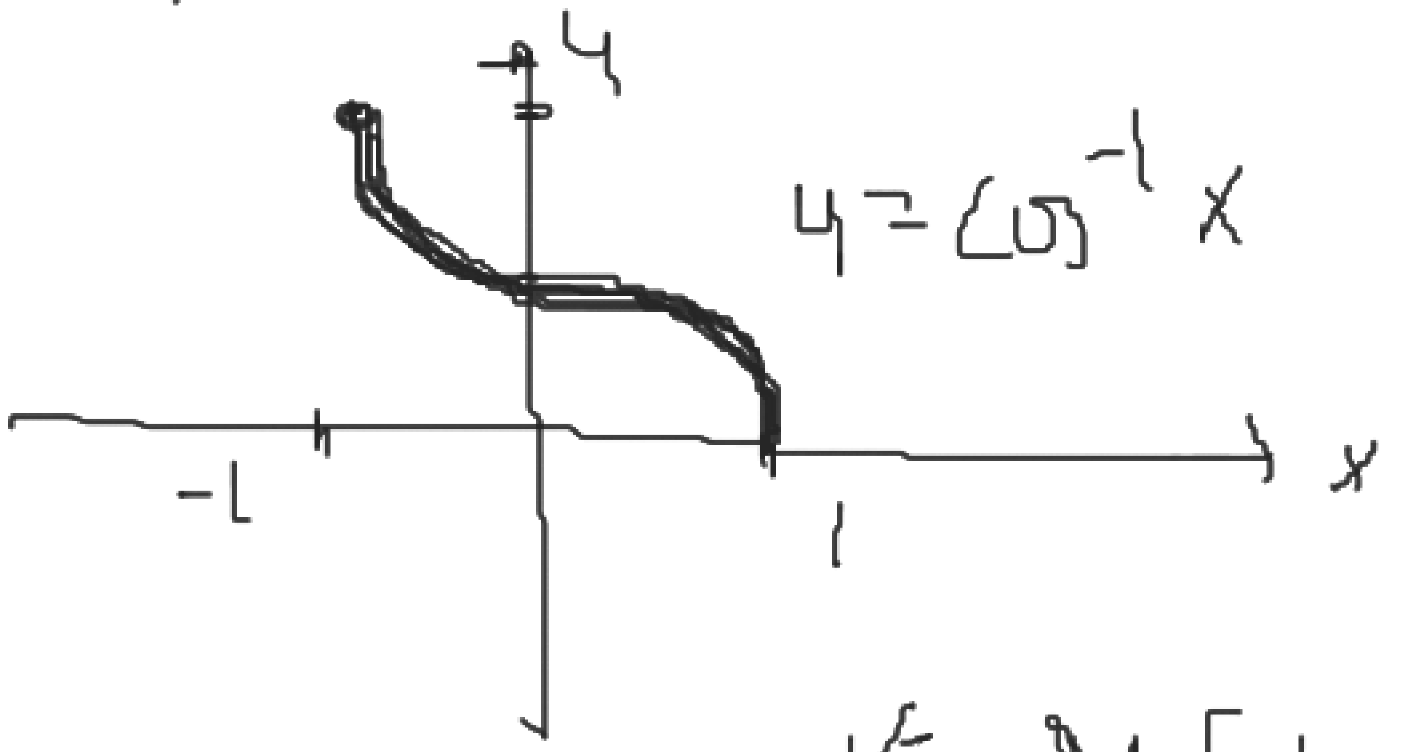
D: $[0, \pi]$

R: $[-1, 1]$



$\# \; f(x) = \cos x \;; \quad 0 \le x \le \pi$ $\qquad$ $g(x) = \cos^{-1} x$





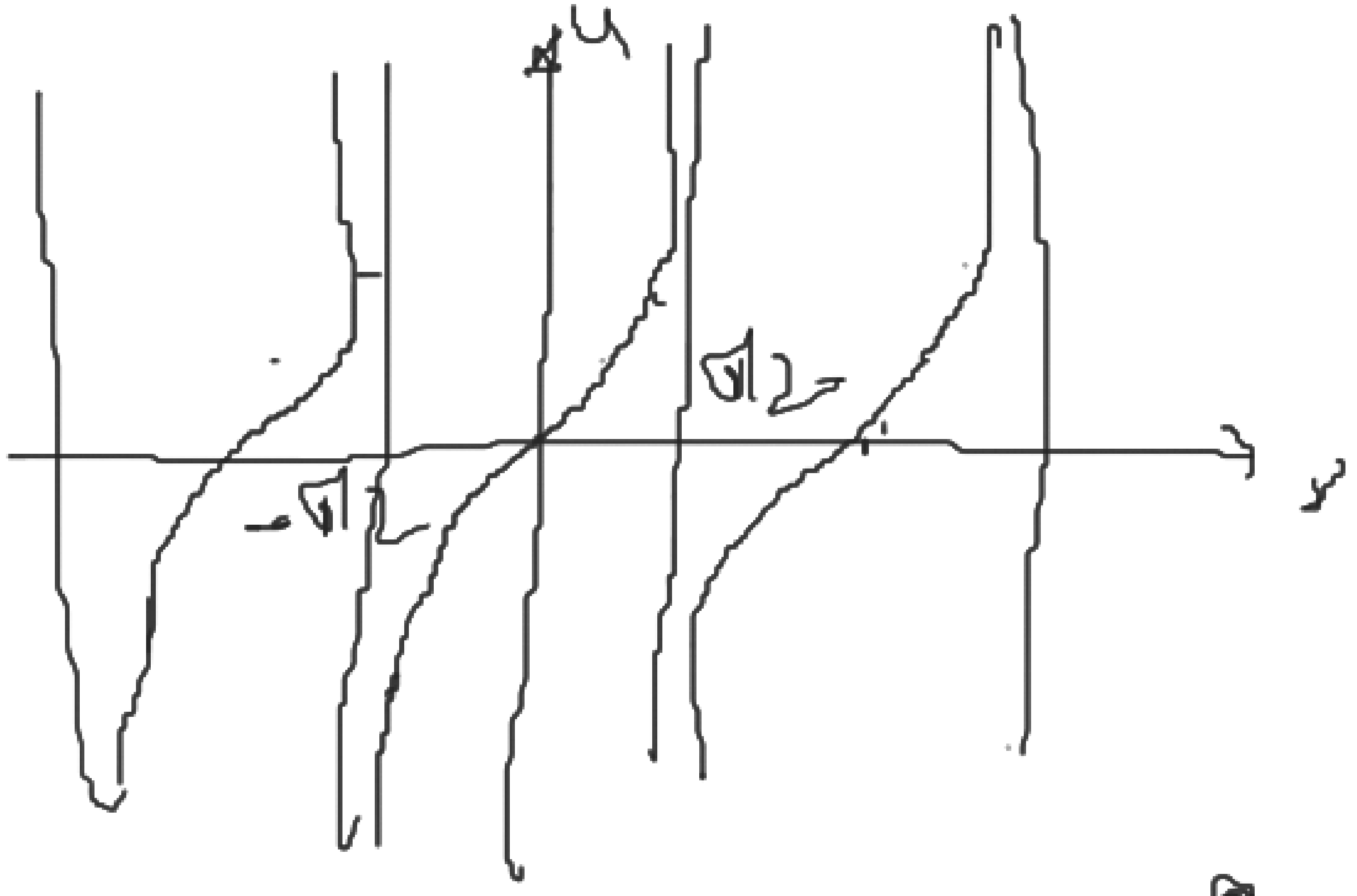
D: $[0, \pi]$

R: $[-1, 1]$

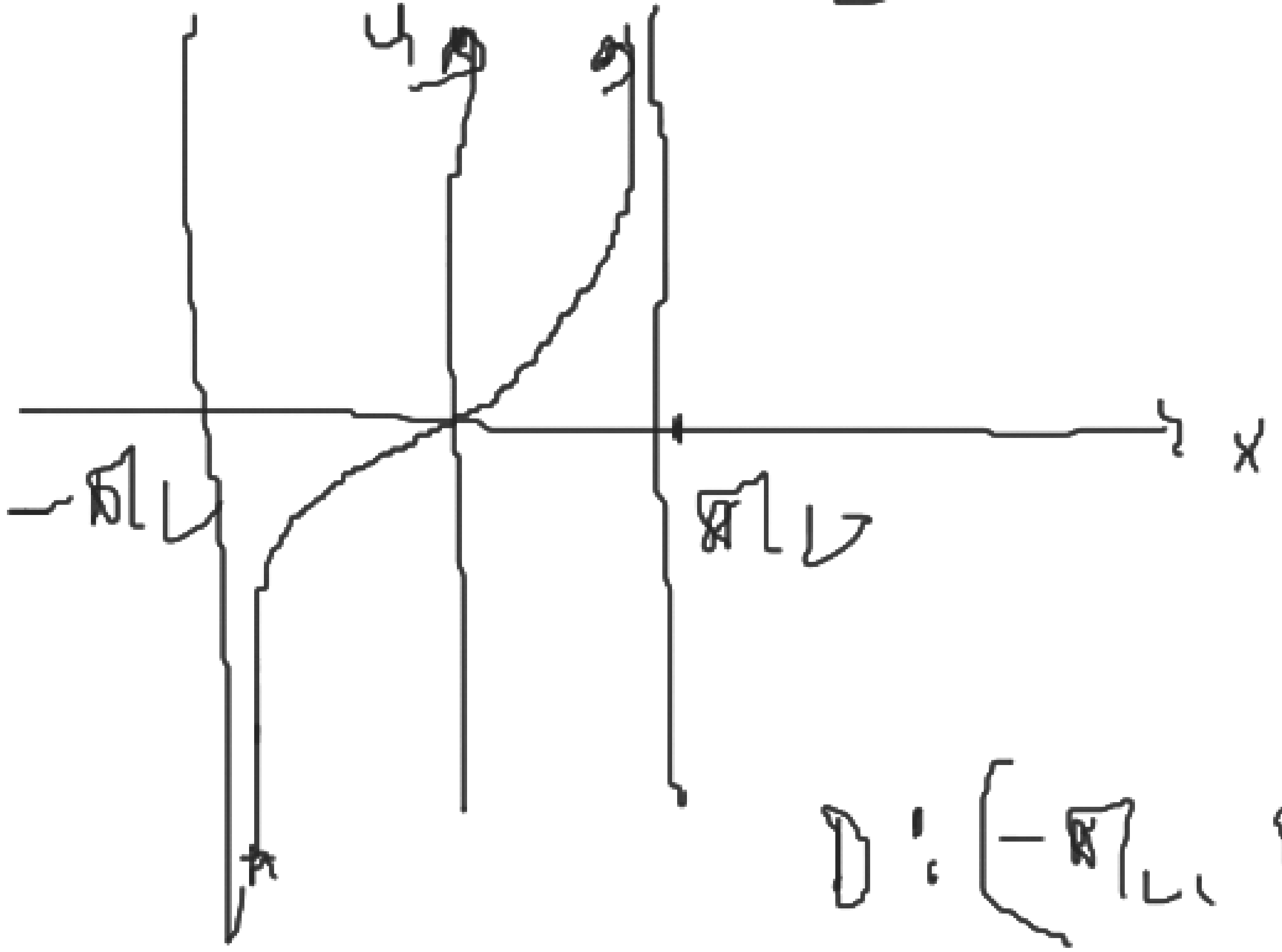
D: $[-1, 1]$

R: $[0, \pi]$

$y = \tan x$



✓ $f(x) = \tan x \;;\; -\pi/2 \leq x \leq \pi/2$



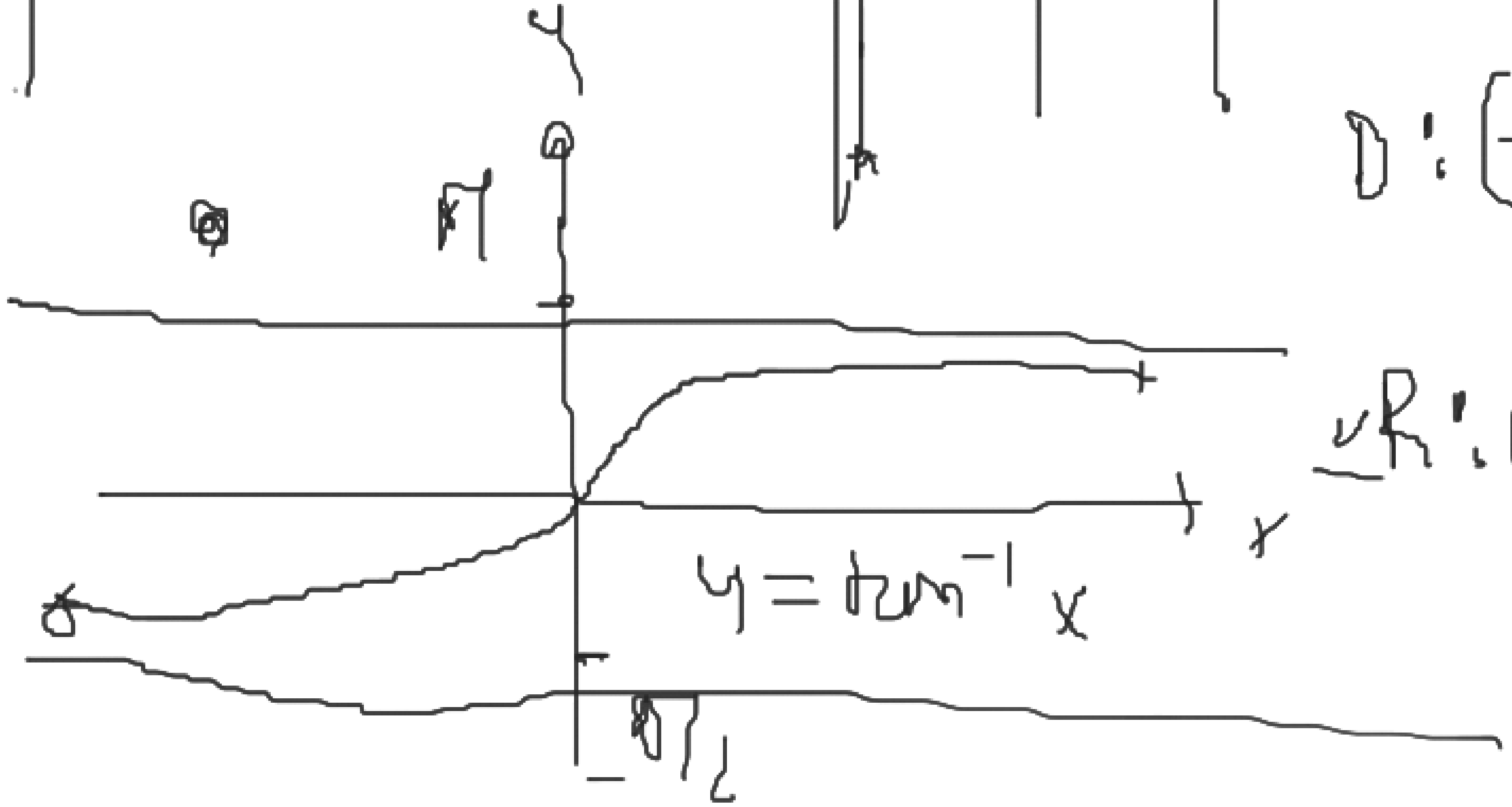
$D: (-\pi/2, \pi/2)$

✓ $R: (-\infty, +\infty)$

$g(x) = \tan^{-1} x$

✓ $D: (-\infty, +\infty)$

✓ $R: (-\pi/2, \pi/2)$



	Domain	Range
$\sin x$	$[-\pi/2, \pi/2]$	$[-1, 1]$
$\sin^{-1} x$	$[-1, 1]$	$[-\pi/2, \pi/2]$
$\cos x$	$[0, \infty]$	$[-1, 1]$
$\cos^{-1}$	$[-1, 1]$	$[0, \pi]$
$\tan x$	$(-\pi/2, \pi/2)$	$(-\infty, +\infty)$
$\tan^{-1} x$	$(-\infty, \infty)$	$(-\pi/2, \pi/2)$

Identities :

$$\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$$

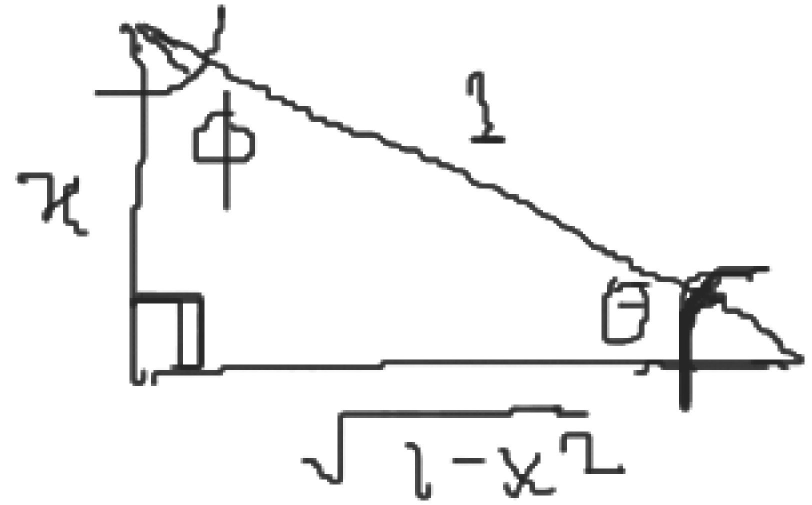
$$\cos(\sin^{-1}x) = \sqrt{1-x^2}$$

$$\sin(\cos^{-1}x) = \sqrt{1-x^2}$$

$$\tan(\sin^{-1}x) = \frac{x}{\sqrt{1-x^2}}$$

H.W. Ex 0.4 : 1 (c, d), 3 (c, e), 9 - 20, 40 & 41

$\sin^{-1}x + \cos^{-1}x = \pi/2$



$\therefore \sin\theta = \frac{x}{1}$

$\Rightarrow \sin\theta = x \quad \therefore \boxed{\theta = \sin^{-1}x}$

$\cos\phi = \frac{x}{1} \Rightarrow \cos\phi = x \quad \therefore \boxed{\phi = \cos^{-1}x}$

Now. $\theta + \phi = \pi/2$

$\Rightarrow \sin^{-1}x + \cos^{-1}x = \pi/2$

$\cos\theta = \frac{\sqrt{1-x^2}}{1}$

$\Rightarrow \cos\theta = \sqrt{1-x^2}$

$\Rightarrow \cos(\sin^{-1}x) = \sqrt{1-x^2}$

proved