Mat-116 lecture-7

Propetrties of functions

Even function:

A function f is even, if for every number x in its domain, the number $-x$ is also in the domain and

$f(-x) = f(x)$ Example: $y = x^2$

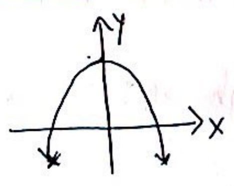
Odd function:

A function f is odd, if for every number x in its domain, the number $-x$ is also in the domain and

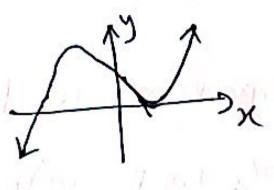
$f(-x) = -f(x)$ Example: $y = x^3$

* A function is even if and only if the graph is symmetric with respect to the y-axis.
* A function is odd if and only if the graph is symmetric with respect to the origin.

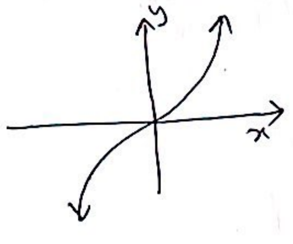
Example:



→ symmetric with respect to y-axis. So the function is even.



→ not symmetric with respect to y-axis nor origin. So the function is neither even nor odd.



→ symmetric with respect to ~~to~~ origin. So the function is odd.

Identifying even and odd function Algebrically:

i) $f(x) = x^2 - 5$

Replace x by $-x$, we have

$f(-x) = (-x)^2 - 5 = x^2 - 5 = f(x)$

Since $f(-x) = f(x)$, so the given function is even.

ii) $g(x) = x^3 - 1$

Replace x by $-x$, we have

$f(-x) = (-x)^3 - 1 = -x^3 - 1$

Since $g(-x) \neq g(x)$ nor $g(-x) \neq -g(x)$. So we can say that the given function is neither even nor odd.

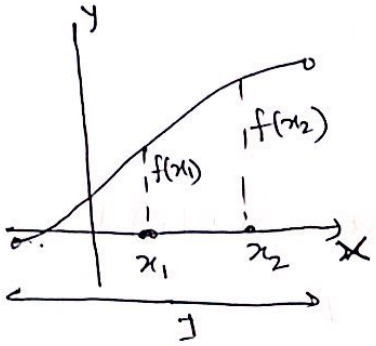
iii) $h(x) = 5x^3 - x$ iv) $f(x) = |x|$

Determine where a function is increasing or decreasing?

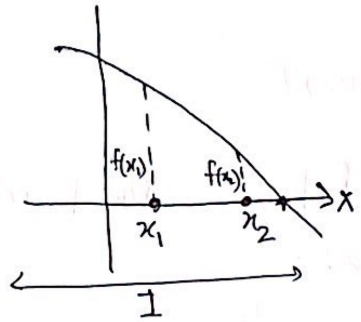
* A function is increasing on an open Interval I, if for any choice of x_1 and x_2 in I, with $x_1 < x_2$ we have $f(x_1) < f(x_2)$.

* A function is decreasing on an open interval I, if for any choice of x_1 and x_2 in I, with $x_1 < x_2$ we have $f(x_1) > f(x_2)$

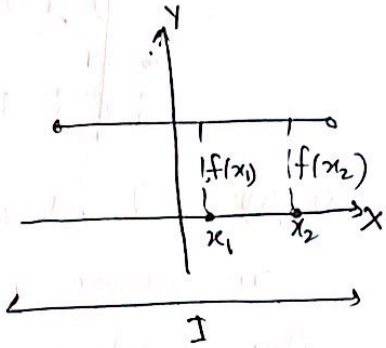
A function is constant on an open interval I, if for all choices of x in I, the values of $f(x)$ are equal.



For $x_1 < x_2$ in I,
$f(x_1) < f(x_2)$
f is increasing on I.



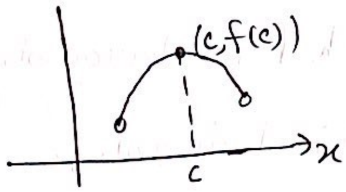
For $x_1 < x_2$ in I,
$f(x_1) > f(x_2)$;
f is decreasing on I.



for all x in I, the values of f are equal, f is constant on I.

Local maximum:

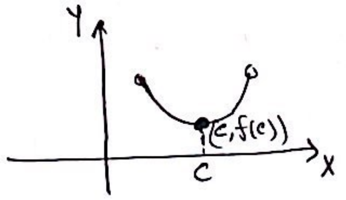
A function f has a local maximum at c if there is an open interval I containing c so that for all x in I, $f(x) \leq f(c)$. We call $f(c)$ a local maximum value of f.



f has a local maximum at c.

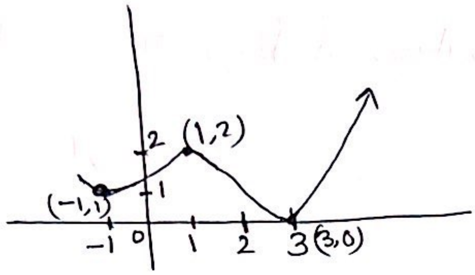
Local minimum:

A function f has a local minimum at c if there is an open interval I containing c so that for all x in I, $f(x) \geq f(c)$. we call $f(c)$ a local minimum value of f.



f has a local minimum at c.

Example:



(a) At what values f has a local maximum? List the maximum values.

Ans: local maximum at $x=1$, maximum values $f(1)=2$.

(b) At what values f has a local minimum? List the minimum values?

Ans. local minimum at $x=-1$ and $x=3$. minimum values $f(-1)=1$ and $f(3)=0$

(c) Find the intervals on which f is increasing.

Ans: On the interval $(-1,1)$ and $(3,\infty)$, f is increasing.

(d) Find the intervals on which f is decreasing.

Ans: on the interval $(1,3)$, f is decreasing.

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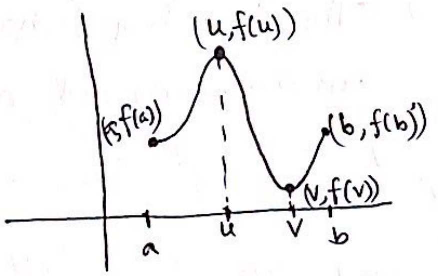
Absolute maximum and absolute minimum:

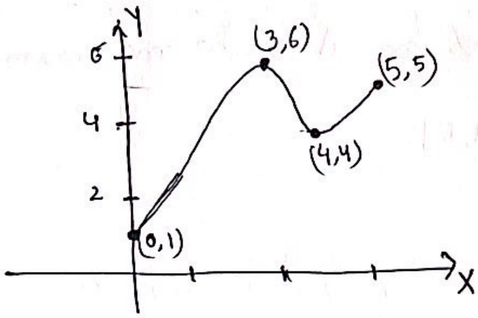
Let f denote a function defined on some interval I. If there is a number u in I for which $f(x) \le f(u)$ for all x in I, then $f(u)$ is the absolute maximum of f on I, we say the absolute maximum of f occurs at u.

If there is a number v for which $f(x) \geq f(v)$ for all x in I, then $f(v)$ is the absolute minimum of f on I and we say the absolute minimum of f occurs at v.

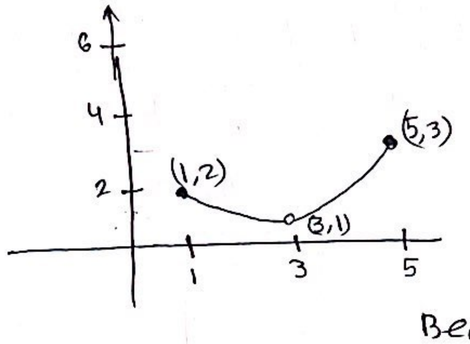


domain $[a, b]$
absolute maximum $f(u)$
" minimum $f(v)$

Example:



Domain $[0, 5]$
Absolute maximum at $x=3$
and the value is $f(3)=6$
Absolute minimum is at $x=0$
and the minimum value is $f(0)=1$



Domain $\{x \mid 1 \le x \le 5, x \neq 3\}$
Absolute maximum at $x=5$
and the maximum value $f(5)=3$
There is no absolute minimum.
Because as we go closer and closer to $(3,1)$
there is no single smallest value.

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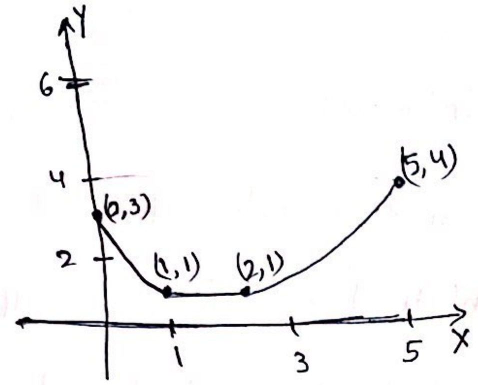
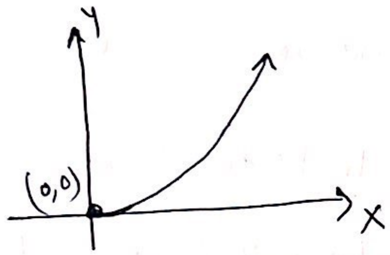


Domain $[0, 5]$

Absolute maximum at $x = 5$ and the absolute maximum value is $f(5) = 4$.

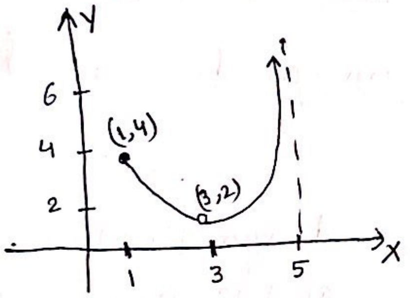
The absolute minimum is 1 and that occurs in the interval $[1, 2]$



Domain $[0, \infty)$

The function has no absolute maximum.

the absolute minimum is $f(0) = 0$



Domain $= \{x \mid 1 \leq x < 5,\ x \neq 3\}$

The function f has no absolute maximum and no absolute minimum.

Extreme value theorem:

If f is a continuous function whose domain is a closed Interval $[a, b]$, then f has an absolute maximum and an absolute minimum on $[a, b]$.

Average rate of change of a function:

If a and b, $a \neq b$, are in the domain of a function $y=f(x)$, the average rate of change of f from a to b is defined as

$$\text{Average rate of change} = \frac{\Delta y}{\Delta x} = \frac{f(b)-f(a)}{b-a}, \quad a \neq b$$

<u>Example:</u>

Find the average rate of change of $f(x)=3x^2$

(a) from 1 to 3 (b) From 1 to 5 (c) From 1 to 7.

<u>Soln.</u>

(a) The average rate of change of $f(x)=3x^2$ from 1 to 3 is

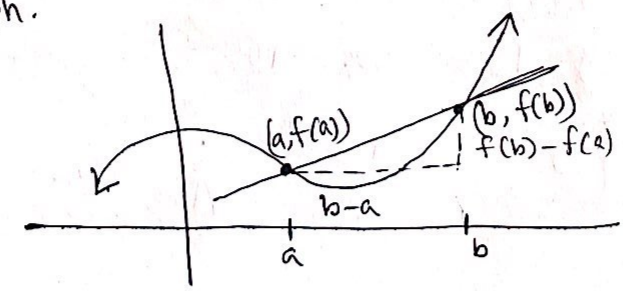
$$\frac{\Delta y}{\Delta x} = \frac{f(3)-f(1)}{3-1} = \frac{3.3^2-3}{2} = \frac{27-3}{2} = \frac{24}{2} = 12.$$

(b)

$$\frac{\Delta y}{\Delta x} = \frac{f(5)-f(1)}{5-1} = \frac{3(25)-3}{4} = \frac{75-3}{4} = \frac{72}{4} = 18$$

Slope of secant line:

The average rate of change of a function from a to b equals the slope of the secant line containing the two points $(a, f(a))$ and $(b, f(b))$ on its graph.



$$m_{sec} = \frac{f(b)-f(a)}{b-a}$$

Example:

Suppose that $g(x) = 3x^2 - 2x + 3$

(a) Find the average rate of change of g from -2 to 1.

(b) Find an equation of the secant line containing $(-2, g(-2))$ and $(1, g(1))$.

Solution:

(a) Average rate of change $= \dfrac{g(1)-g(-2)}{1-(-2)}$

$$= \frac{4-19}{3} = \frac{-15}{3} = -5$$

(b) The slope of the secant line containing $(-2, g(-2)) = (-2, 19)$ and $(1, g(1)) = (1, 4)$ is $m_{sec} = -5$.

Using the point-slope form we can find the eqn of secant line.

$$y - y_1 = m_{sec}(x - x_1)$$

$$\Rightarrow y - 19 = -5(x-(-2))$$

$$\Rightarrow y - 19 = -5(x+2)$$

$$\Rightarrow y - 19 = -5x - 10$$

$$\Rightarrow \boxed{y = -5x + 9}$$

Ans.