Assignment 3

MAT 361

Probability and Statistics

Section 4

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Assignment 03

$$f(x,y) = A(x-3)y \quad -2 \le x \le 3 \ ; \ 4 \le y \le 6$$

(a)
$$\int_4^6 \int_{-2}^3 f(x,y)\,dx\,dy = 1 \quad [\because \text{pdf} = 1, \text{for total area}]$$

$$\Rightarrow A \int_4^6 \int_{-2}^3 (x-3)y \, dx\, dy = 1$$

$$\Rightarrow A \int_4^6 \left[\frac{x^2}{2} - 3x\right]_{-2}^3 y\,dy = 1$$

$$\Rightarrow A \int_4^6 \left[\frac{9}{2} - 9 - \frac{4}{2} - 6\right] y\,dy = 1.$$

$$\Rightarrow \left(-\frac{25A}{2}\right)\left[\frac{y^2}{2}\right]_4^6 = 1$$

$$\Rightarrow -\frac{25}{2} \cdot A \cdot \frac{20}{2} = 1$$

$$\therefore A = -1/125 \quad \text{Answer}$$

(b) $P(0 \le x \le 1, 4 \le y \le 5)$

$$= \int_4^5 \int_0^1 \frac{-1}{125}(x-3)y \, dx\, dy$$

$$= -\frac{1}{125} \int_4^5 \left[\frac{x^2}{2} - 3x\right]_0^1 y\,dy$$

$$= -1/125 \cdot \left[\frac{1}{2} - 3\right]\left[\frac{y^2}{2}\right]_4^5$$

$$= \left(-\frac{1}{125}\right) \cdot \left(-\frac{5}{2}\right) \cdot \left(\frac{9}{2}\right)$$

$$= 0.09. \quad \text{Answer}$$

(c) Marginal probablity density functions,

$$g(x) = \int_4^6 -\frac{1}{125}(x-3)y\,dy$$

$$= -\frac{1}{125}(x-3)\left[\frac{y^2}{2}\right]_4^6$$

$$= -\frac{1}{125} \cdot 10\,(x-3)$$

$$\therefore g(x) = -\frac{2}{25}(x-3) \text{ . Answer}$$

$$h(y) = \int_{-2}^{3} -\frac{1}{125}(x-3)y\,dx$$

$$= -\frac{1}{125}y\left[\frac{x^2}{2} - 3x\right]_{-2}^{3}$$

$$= -\frac{y}{125}\left[\frac{9}{2} - 9 - 2 - 6\right]$$

$$\therefore h(y) = y/10 \text{ Answer}$$

(d) Two random variable are said to be independent if

$$g(x) \cdot h(y) = f(x,y)$$

$$\therefore g(x) \cdot h(y) = -\frac{2}{25}(x-3) \cdot \frac{y}{10}$$

$$= -\frac{1}{125}(x-3)y$$

$$= f(x,y).$$

So, x and y are independent. Answer

(e) $f(y=5) = \dfrac{f(x, y=5)}{h(y=5)} = \dfrac{-\frac{1}{125}(x-3) \cdot 5}{5/10}$

$$= -\frac{2}{25}(x-3) \text{ Answer}$$

(f) $E(x) = \int_{-2}^{3} x\, g(x)\, dx$

$= \int_{-2}^{3} -\frac{2x}{25}(x-3)\, dx$

$= -\frac{2}{25}\int_{-2}^{3}(x^2-3x)\, dx$

$= -\frac{2}{25}\left[\frac{x^3}{3}-\frac{3x^2}{2}\right]_{-2}^{3}$

$= -\frac{2}{25}\left[9-\frac{27}{2}+\frac{8}{3}+6\right]$

$= -\frac{2}{25}\cdot\frac{25}{6}$

$\therefore\ E(x) = -1/3$. Answer

$E(y) = \int_{4}^{6} y\, h(y)\, dy = \int_{4}^{6} y \cdot y/10\, dy$

$= \frac{1}{10}\left[\frac{y^3}{3}\right]_4^6$

$= \frac{1}{10}\left[\frac{6^3}{3}-\frac{4^3}{3}\right] = \frac{1}{10}\ \ \frac{152}{3}$

$E(y) = 76/15$ Answer

$E(x^2) = \int_{-2}^{3} x^2 g(x)\, dx = -\frac{2}{25}\int_{-2}^{3} x^2 (x-3)\, dx$

$= -\frac{2}{25}\left[\frac{x^4}{4}-\frac{3x^3}{3}\right]_{-2}^{3}$

$= -\frac{2}{25}\left[\frac{3^4}{4}-3^3-\frac{(-2)^4}{4}+(-2)^3\right]$

$= \left(-\frac{2}{25}\right)\cdot\left(-\frac{75}{4}\right)$

$\therefore E(x^2) = 3/2$

$$E(y^2) = \int_4^6 y^2 h(y)\,dy$$

$$= \int_4^6 y^2 \cdot \frac{y}{10}\,dy = \int_4^6 \frac{1}{10} y^3\,dy$$

$$= \frac{1}{10}\left[\frac{y^4}{4}\right]_4^6$$

$$= \frac{1}{10}\cdot\left[6^4/4 - 4^3\right] = \frac{1}{10}\cdot 260$$

$$\therefore E(y^2) = 26.$$

$$\therefore \text{varience},\ V(x) = E(x^2) - (E(x))^2$$

$$= \frac{3}{2} - \left(-1/3\right)^2 = \frac{3}{2} - \frac{1}{9}$$

$$= \frac{25}{18} \quad \text{Answer}$$

$$V(y) = E(y^2) - (E(y))^2$$

$$= 26 - \left(\frac{76}{15}\right)^2$$

$$= \frac{5774}{225} \quad \text{Answer}$$

(g) Covariance of X and Y, $\text{cov}(x,y) = E(xy) - E(x)\cdot E(y)$

Here, $E(xy) = \int_4^6 \int_{-2}^3 (xy)\left(-\frac{1}{125}(x-3)y\right)dx\,dy.$

$$= -\frac{1}{125}\int_4^6\int_{-2}^3 (x^2-3x)y^2\,dx\,dy$$

$$= -\frac{1}{125}\int_4^6 y^2\left[\frac{x^3}{3} - \frac{3x^2}{2}\right]_{-2}^3 dy$$

$$= -\frac{1}{125}\int_4^6 y^2\,dy\cdot\left[9 - \frac{27}{2} + \frac{8}{3} + 6\right]$$

$$= -\frac{1}{125}\cdot\frac{25}{6}\cdot\left[y^3/3\right]_4^6$$

$$= -\frac{1}{125}\cdot\frac{25}{6}\cdot\frac{152}{3}.$$

$$= -\frac{76}{45} \quad \text{Answer}$$

$$\text{cov}(x,y) = E(xy) - E(x)E(y) = -\frac{76}{45} - \left[\left(-1/3\right)\cdot\frac{76}{15}\right] = -\frac{76}{45} + \frac{76}{45} = 0 \quad \text{Ans}$$

(h) correlation between X and Y,

$$\text{corr}(x,y) = \frac{\text{cov}(x,y)}{\sqrt{v(x)\cdot v(y)}}$$

$$= \frac{0}{\sqrt{v(x)\cdot v(y)}}$$

$$= 0$$

Answer

Classwork

Given, $f(x) = \frac{1}{x \ln(1.5)}$; $4 \le x \le 6$ and $f(x) = 0$ elsewhere

(a)

$$\text{CDF} = F(x) = \int_4^x \frac{1}{x \cdot \ln(1.5)}\, dx$$

$$= \frac{1}{\ln(1.5)} \left[\ln x\right]_4^x$$

$$= \frac{1}{\ln(1.5)} \left[\ln x - \ln 4\right]$$

$$= \frac{1}{\ln(1.5)} \times \left[\ln \frac{x}{4}\right]$$

2^{nd} quartile = median : $F(x) = 0.50$

$$\frac{1}{\ln(1.5)} \times \ln\frac{x}{4} = 0.5$$

$$\Rightarrow \ln\frac{x}{4} = 0.5 \times \ln 1.5$$

$$\Rightarrow \ln\frac{x}{4} = 0.203 \Rightarrow \frac{x}{4} = e^{0.203}$$

$$\Rightarrow x = 4e^{0.203}$$

$\therefore$ median $= 4.899$ Answer

(b)

lower quartiles,

$F(x) = 0.25$

$\Rightarrow \frac{1}{\ln(1.5)} \cdot \ln\left(\frac{x}{4}\right) = 0.25$

$\Rightarrow \ln(x/4) = 0.25 \times \ln(1.5)$

$\Rightarrow \ln\left(\frac{x}{4}\right) = 0.1014$

$\Rightarrow x = 4e^{0.1014}$

$\therefore x = 4.43$

$\therefore$ lower quartile $= 4.43$ Answer

~~(c)~~ For upper quartiles,

$F(x) = 0.75$

$\Rightarrow \ln(x/4) = 0.75 \times \ln(1.5)$

$\Rightarrow x = 4e^{(0.75 \times \ln(1.5))}$

$= 5.42$

$\therefore$ upper quartiles $= 5.422$ Answer

(c) Interquartile range,

IQR = upper quartile − lower quartile

$= Q_3 - Q_1$

$= 5.422 - 4.43$

$= \frac{124}{125}$

$= 0.992$ Answer