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① 3rd December → Monday, 11 November 7.30 pm

② Today's class makeup for previous cancelled class

Final Exam : HW 3, 4 (To be determined)

6pm Dec 17 (To be confirmed)

Mobile Robot → computionally constrained

↳ Power supply

↳ CPU

↳ memory

⇒ optimized, effecient algorithms

cloud computing ⇒ Theoritically infinite resources

SS

Trajectory tracking (PID control)

$$e = \begin{bmatrix} x_d - x \\ y_d - y \end{bmatrix}$$

x_d, y_d : desired trajectory

x, y : measured trajectory

Linear system assumption

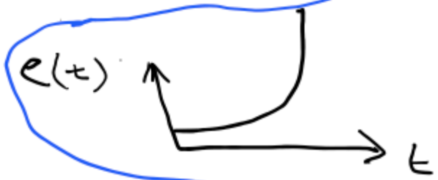
$$\ddot{e} + k_d \dot{e} + k_p e = 0$$

k_p : proportional gain

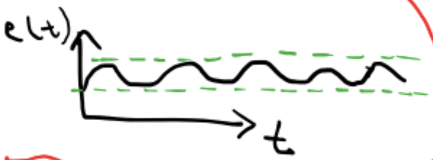
k_d : derivative gain

Choose k_p, k_d so that $e(t) \to 0$ as $t \to \infty$

$\Rightarrow$ stable system



$e(t) \to \infty$ as $t \to \infty$ unstable system



$\to$ Neutrally Stable

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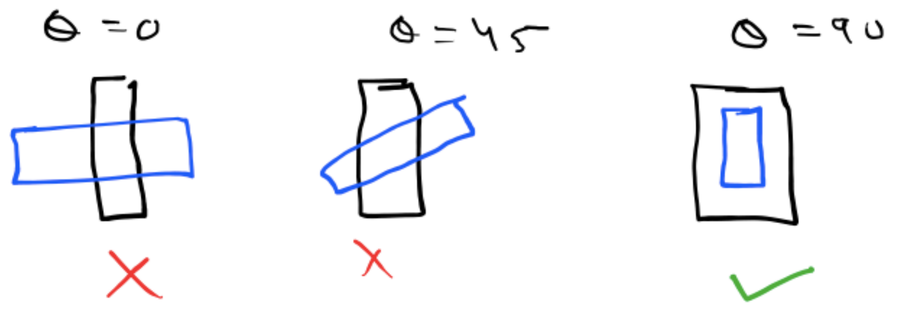
Lyapunov stability Method



illegal parking

Pose stabilization (Regulation) $\Rightarrow$ Calculate ω such that we acheive the desired x, y, θ

use $\Rightarrow$ Lyapunov Method

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Lyapunov stability Theory

Given a system $\dot{x} = f(x, u)$, the system is stable if there exists a function $V(x, u)$ [called the Lyapunov function] such that :

(1) $V(0, 0) = 0$ for $x = 0$, $u = 0$

(2) $V(x, u) > 0$ for $x \neq 0$, $u \neq 0$

(3) $\dot{V}(x, u) < 0$ for $x \neq 0$, $u \neq 0$

} x, u can be vectors or scalars

Proof : In text books (you can check)

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Example:
$$\dot{x}_1 = -x_1$$
$$\dot{x}_2 = -x_2$$

Show that this system is stable for the Lyapunov function $V = \frac{1}{2}x_1^2 + \frac{1}{2}x_2^2$

$x_1 = -1$
$x_2 = -2$
$\Rightarrow V = \frac{1}{2} + \frac{4}{2}$
$= 2\frac{1}{2} > 0$

Solution: Apply Lyapunov's theorem:

(1) $x_1 = 0$ & $x_2 = 0 \Rightarrow V = 0$ ✓

(2) $x_1 \neq 0$ & $x_2 \neq 0 \Rightarrow V > 0$ ✓

(3) $\dot{V} = \frac{dV}{dt} = x_1\dot{x}_1 + x_2\dot{x}_2 = x_1(-x_1) + x_2(-x_2)$

$= -x_1^2 - x_2^2 < 0$ ✓

for $x_1 \neq 0$ & $x_2 \neq 0$

So, this system is stable.

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Example: $\dot{x}_1 = u_1$ States $\rightarrow x_1, x_2$

$\dot{x}_2 = u_2$ controls $\rightarrow u_1, u_2$

Find the controls u_1 & u_2 such that the system is stable using the Lyapunov function $V = \frac{1}{2}x_1^2 + \frac{1}{2}x_2^2$

Solution:

1. For $x_1 = 0$ & $x_2 = 0$, $V = 0$ ✓
2. For $x_1 \neq 0$ & $x_2 \neq 0 \Rightarrow V > 0$ ✓
3. $\dot{V} = \frac{dV}{dt} = x_1\dot{x}_1 + x_2\dot{x}_2 = x_1u_1 + x_2u_2$

We want $\dot{V} < 0$, so pick $\boxed{u_1 = -x_1}$ & $\boxed{u_2 = -x_2}$

then $\dot{V} = -x_1^2 - x_2^2 < 0$ ✓

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Trajector & motion planning

State space based method

(vehicles dynamics are taken into account)

(differential flatness, optimal control, Lyapunov, Trajectory tracking)

Configuration space based methods

(c-space) [Geometry based]

(1) Search based motion planning (Chapter 5) Grid based search methods

(2) Sampling based motion planning (chapter 6)

(a) PRM (Probabilistic Road Map)

(b) RRT (Rapidly Exploring Random Trees)