Exercise 7.5

15. $\int \frac{x^2-8}{x+3}\,dx$

$$\frac{x^2-8}{x+3} = \frac{x^2-9+1}{x+3} = \frac{x^2-9}{x+3} + \frac{1}{x+3} = x-3+\frac{1}{x+3}$$

So, $\int \left(x-3+\frac{1}{x+3}\right)dx = \frac{1}{2}x^2 - 3x + \ln|x+3| + C$

16. $\int \frac{x^2+1}{x-1}\,dx$

$$\frac{x^2+1}{x-1} = \frac{x^2-1+2}{x-1} = \frac{x^2-1}{x-1} + \frac{2}{x-1} = x+1+\frac{2}{x-1}$$

So, $\int \left(x+1+\frac{2}{x-1}\right)dx = \frac{1}{2}x^2 + x + 2\ln|x-1| + C$

17. $\int \frac{3x^2-10}{x^2-4x+4}\,dx$

$$\frac{3x^2-10}{x^2-4x+4} = 3 + \frac{12x-22}{(x-2)^2}$$

$$\frac{12x-22}{(x-2)^2} = \frac{A}{x-2} + \frac{B}{(x-2)^2}$$

$\Rightarrow 12x-22 = A(x-2)+B = Ax-2A+B$

$A = 12 \qquad -2\times 12 + B = -22 \Rightarrow B = 2$

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So, $\int \frac{3x^2-10}{x^2-4x+4}\,dx$

$= 3\int dx + 12\int \frac{dx}{x-2} + 2\int \frac{dx}{(x-2)^2}$

$= 3x + 12\ln|x-2| - \frac{2}{x-2} + C$

18. $\int \frac{x^2}{x^2-3x+2}\,dx$

$\frac{x^2}{x^2-3x+2} = 1 + \frac{3x-2}{x^2-3x+2} = 1 + \frac{3x-2}{(x-1)(x-2)}$

$\frac{3x-2}{(x-1)(x-2)} = \frac{A}{x-1} + \frac{B}{x-2}$

$\Rightarrow 3x-2 = A(x-2) + B(x-1)$

Put $x=2$, $B=4$ Put, $x=1$, $A=-1$

So, $\int \frac{x^2}{x^2-3x+2}\,dx$

$= \int dx - \int \frac{dx}{x-1} + 4\int \frac{dx}{x-2}$

$= x - \ln|x-1| + 4\ln|x-2| + C$

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19. $\int \frac{2x-3}{x^2-3x-10}\,dx$

Put, $u = x^2-3x-10 \Rightarrow du = (2x-3)\,dx$

So, $\int \frac{du}{u} = \ln u + c = \ln|2x-3| + c$

20. $\int \frac{3x+1}{3x^2+2x-1}\,dx$

Put, $u = 3x^2+2x-1 \Rightarrow du = 6x+2 = 2(3x+1)$

$\frac{1}{2}\int \frac{du}{u} = \frac{1}{2}\ln u + c = \frac{1}{2}\ln|3x^2+2x-1| + c$

21. $\int \frac{x^5+x^2+2}{x^3-x}\,dx$

$\frac{x^5+x^2+2}{x^3-x} = x^2+1 + \frac{x^2+x+2}{x^3-x} = x^2+1+\frac{x^2+x+2}{x(x+1)(x-1)}$

$\frac{x^2+x+2}{x(x+1)(x-1)} = \frac{A}{x} + \frac{B}{x+1} + \frac{C}{x-1}$

$\Rightarrow x^2+x+2 = A(x+1)(x-1) + Bx(x-1) + C(x+1)x$

Put $x = 0$, $A = -2$

Put $x = 1$, $C = 2$

Put $x = -1$, $B = 1$

So, $\int \frac{x^5+x^2+2}{x^3-x}\,dx$

$= \int (x^2+1)\,dx - 2\int \frac{1}{x}\,dx + \int \frac{dx}{x+1} + 2\int \frac{dx}{x-1}$

$= \frac{x^3}{3} + x - 2\ln x + \ln|x+1| + 2\ln|x-1| + C_1$

22. $\int \frac{x^5-4x^3+1}{x^3-4x}\,dx$

$\frac{x^5-4x^3+1}{x^3-4x} = x^2 + \frac{1}{x^3-4x} = x^2 + \frac{1}{x(x+2)(x-2)}$

$\frac{1}{x(x+2)(x-2)} = \frac{A}{x} + \frac{B}{x+2} + \frac{C}{x-2}$

$\Rightarrow 1 = A(x+2)(x-2) + B(x-2)x + Cx(x+2)$

Put $x=0$, $A = -\frac{1}{4}$

Put $x=2$, $C = \frac{1}{8}$

Put $x=-2$, $B = \frac{1}{8}$

So, $\int \frac{x^5-4x^3+1}{x^3-4x}\,dx$

$= \int x^2\,dx + \frac{1}{8}\int \frac{1}{x+2}\,dx + \frac{1}{8}\int \frac{dx}{x-2} - \frac{1}{4}\int \frac{1}{x}\,dx$

$= \frac{x^3}{3} + \frac{1}{8}\ln|x+2| + \frac{1}{8}\ln|x-2| - \frac{1}{4}\ln x + C_1$

23. $\int \frac{2x^2+3}{x(x-1)^2}\,dx$

$$\frac{2x^2+3}{x(x-1)^2} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{(x-1)^2}$$

$$\Rightarrow 2x^2+3 = A(x-1)^2 + Bx(x-1) + Cx \quad \text{— (1)}$$

Put $x=0$, $A=3$

Put $x=1$, $C=5$

$$(1) \Rightarrow 2x^2+3 = Ax^2 - 2Ax + A + Bx^2 - Bx + Cx$$

$$= x^2(A+B) + (C-2A-B)x + A$$

$A+B=2 \Rightarrow B=-1$

So, $\int \frac{2x^2+3}{x(x-1)^2}\,dx$

$$= 3\int\frac{dx}{x} - \int\frac{dx}{x-1} + 5\int\frac{dx}{(x-1)^2}$$

$$= 3\ln|x| - \ln|x-1| - \frac{5}{x-1} + C_1$$

21. $\displaystyle\int \frac{3x^2-x+1}{x^3-x^2}\,dx$

$$\frac{3x^2-x+1}{x^3-x^2} = \frac{3x^2-x+1}{x^2(x-1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-1}$$

$$\Rightarrow 3x^2-x+1 = Ax(x-1) + B(x-1) + Cx^2$$

$$= Ax^2 - Ax + Bx - B + Cx^2$$

$$= (A+C)x^2 + (B-A)x - B$$

$B = -1$

$B - A = -1 \Rightarrow A = 0$

$A + C = 3 \Rightarrow C = 3$

So, $\displaystyle\int \frac{3x^2-x+1}{x^3-x^2}\,dx$

$$= -\int \frac{dx}{x^2} + 3\int \frac{dx}{x-1} = \frac{1}{x} + 3\ln|x-1| + C$$

25. $\int \frac{2x^2-10x+4}{(x+1)(x-3)^2}\,dx$

$$\frac{2x^2-10x+4}{(x+1)(x-3)^2} = \frac{A}{x+1} + \frac{B}{x-3} + \frac{C}{(x-3)^2}$$

$$\Rightarrow 2x^2-10x+4 = A(x-3)^2 + B(x+1)(x-3) + C(x+1) \quad \text{— (1)}$$

$$= Ax^2 - 6Ax + 9A + Bx^2 - 2Bx - 3B + Cx + C$$

$$= (A+B)x^2 + -(6A+2B-C)x + (9A-3B+C)$$

From (1),

Put $x=3$, $C=-2$ Put $x=-1$, $A=1$

$A+B=2 \Rightarrow B=1$

So, $\int \frac{2x^2-10x+4}{(x+1)(x-3)^2}\,dx$

$$= \int \frac{dx}{x+1} + \int \frac{dx}{x-3} - 2\int \frac{dx}{(x-3)^2}$$

$$= \ln|x+1| + \ln|x-3| + \frac{2}{x-3} + C_1$$

26. $\int \frac{2x^2-2x-1}{x^2(x-1)}\,dx$

$$\frac{2x^2-2x-1}{x^2(x-1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-1}$$

$$\Rightarrow 2x^2-2x-1 = Ax(x-1) + B(x-1) + Cx^2$$

$$= Ax^2 - Ax + Bx - B + Cx^2$$

$$= (A+C)x^2 - (A-B)x - B$$

$B = 1$

$A - B = 2 \Rightarrow A = 3$

$A + C = 2 \Rightarrow C = -1$

So, $\int \frac{2x^2-2x-1}{x^2(x-1)}\,dx$

$$= 3\int \frac{dx}{x} - \int \frac{dx}{x-1} + \int \frac{dx}{x^2}$$

$$= 3\ln x - \frac{1}{x} - \ln|x-1| + C_1$$

27. $\int \frac{x^2}{(x+1)^3} dx$

$$\frac{x^2}{(x+1)^3} = \frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{C}{(x+1)^3}$$

$$\Rightarrow x^2 = A(x+1)^2 + B(x+1) + C \quad \text{---} \ (1)$$

$$= Ax^2 + 2Ax + A + Bx + B + C$$

$$= Ax^2 + (2A+B)x + (A+B+C)$$

from (1), put $x = -1$, $C = 1$

$A = 1 \qquad 2A+B = 0 \Rightarrow B = -2$

So, $\int \frac{x^2}{(x+1)^3} dx$

$$= \int \frac{dx}{x+1} - 2\int \frac{dx}{(x+1)^2} + \int \frac{dx}{(x+1)^3}$$

$$= \ln|x+1| + \frac{2}{x+1} - \frac{1}{2(x+1)^2} + C_1$$

28. $\int \frac{2x^2+3x+3}{(x+1)^3}\,dx$

$$\frac{2x^2+3x+3}{(x+1)^3} = \frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{C}{(x+1)^3}$$

$$\Rightarrow 2x^2+3x+3 = A(x+1)^2 + B(x+1) + C$$

$$= Ax^2 + (2A+B)x + (C+A+B)$$

$A = 2$

$2A + B = 3 \Rightarrow B = -1$

$A + B + C = 3 \Rightarrow C = 2$

So, $\int \frac{2x^2+3x+3}{(x+1)^3}\,dx$

$$= 2\int \frac{dx}{x+1} - \int \frac{dx}{(x+1)^2} + 2\int \frac{dx}{(x+1)^3}$$

$$= 2\ln|x+1| + \frac{1}{x+1} - \frac{1}{(x+1)^2} + c_1$$

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29. $\int \frac{2x^2-1}{(4x-1)(x^2+1)}\,dx$

$$\frac{2x^2-1}{(4x-1)(x^2+1)} = \frac{A}{4x-1} + \frac{Bx+C}{x^2+1}$$

$$\Rightarrow 2x^2-1 = A(x^2+1) + (Bx+C)(4x-1) \quad \text{--- (1)}$$

$$= Ax^2 + A + 4Bx^2 + 4Cx - Bx - C$$

$$= (A+4B)x^2 + (4C-B)x + (A-C)$$

From (1), Put $x = \frac{1}{4}$, $A = -\frac{14}{17}$

$A+4B = 2 \quad \Rightarrow B = \frac{12}{17}$

$A-C = -1 \quad \Rightarrow C = \frac{3}{17}$

So, $\int \frac{2x^2-1}{(4x-1)(x^2+1)}\,dx$

$$= -\frac{14}{17}\int \frac{dx}{4x-1} + \int \frac{\frac{12x}{17}+\frac{3}{17}}{x^2+1}\,dx$$

$$= -\frac{7}{34}\ln|4x-1| + \frac{1}{17}\int \frac{12x+3}{x^2+1}\,dx$$

$$= -\frac{7}{34}\ln|4x-1| + \frac{12}{17}\int \frac{x\,dx}{x^2+1} + \frac{3}{17}\int \frac{dx}{x^2+1}$$

$$= -\frac{7}{34}\ln|4x-1| + \frac{6}{17}\ln|x^2+1| + \frac{3}{17}\tan^{-1}x + C_1$$

30. $\displaystyle\int \frac{dx}{x^3+2x}$

$$\frac{1}{x^3+2x} = \frac{1}{x(x^2+2)} = \frac{A}{x} + \frac{Bx+C}{x^2+2}$$

$$\Rightarrow 1 = A(x^2+2) + (Bx+C)x$$

$$= Ax^2 + 2A + Bx^2 + Cx$$

$$= (A+B)x^2 + Cx + 2A$$

$2A = 1 \Rightarrow A = \frac{1}{2}$ $\quad C = 0 \quad$ $A + B = 0 \Rightarrow B = -\frac{1}{2}$

So, $\displaystyle\int \frac{dx}{x^3+2x} = \frac{1}{2}\int \frac{dx}{x} + \int \frac{-\frac{x}{2}+0}{x^2+2}\,dx$

$$= \frac{1}{2}\ln x - \frac{1}{2}\int \frac{x\,dx}{x^2+2} = \frac{1}{2}\ln x - \frac{1}{4}\ln|x^2+2| + C_1$$

31. $\displaystyle\int \frac{x^3+3x^2+x+9}{(x^2+1)(x^2+3)}\,dx$

$$\frac{x^3+3x^2+x+9}{(x^2+1)(x^2+3)} = \frac{Ax+B}{x^2+1} + \frac{Cx+D}{x^2+3}$$

$$\Rightarrow x^3+3x^2+x+9 = (Ax+B)(x^2+3) + (Cx+D)(x^2+1)$$

$$= Ax^3 + 3Ax + Bx^2 + 3B + Cx^3 + Cx + Dx^2 + D$$

$$= (A+C)x^3 + (B+D)x^2 + (3A+C)x + (3B+D)$$

$A + C = 1$ — (1) $\quad$ From $3\times(1) - (2) \Rightarrow C = 1$

$3A + C = 1$ — (2) $\quad$ So, From (1) $\Rightarrow A = 0$

$B+D=3 \quad -(3)$ From $3\times(3)-(4) \Rightarrow D=0$

$3B+D=9 \quad -(4)$ From $(3) \Rightarrow B=3$

So, $\displaystyle\int \frac{x^3+3x^2+x+9}{(x^2+1)(x^2+3)}dx$

$\displaystyle = 3\int \frac{dx}{x^2+1} + \int \frac{x}{x^2+3}dx$

$= 3\tan^{-1}x + \frac{1}{2}\ln|x^2+3| + C_1$

32. $\displaystyle\int \frac{x^3+x^2+x+2}{(x^2+1)(x^2+2)}dx$

$$\frac{x^3+x^2+x+2}{(x^2+1)(x^2+2)} = \frac{Ax+B}{x^2+1} + \frac{Cx+D}{x^2+2}$$

$\Rightarrow x^3+x^2+x+2 = (Ax+B)(x^2+2) + (Cx+D)(x^2+1)$

$= Ax^3+2Ax+Bx^2+2B+Cx^3+Cx+Dx^2+D$

$= (A+C)x^3+(B+D)x^2+(2A+C)x+(2B+D)$

$A+C=1 \quad -(1)$ From $2\times(1)-(2) \Rightarrow C=1$

$2A+C=1 \quad -(2)$ From $(1) \Rightarrow A=0$

$B+D=1 \quad -(3)$ From $2\times(3)-(4) \Rightarrow D=0$

$2B+D=2 \quad -(4)$ From $(3) \Rightarrow B=1$

So, $\int \frac{x^3+x^2+x+2}{(x^2+1)(x^2+2)}\,dx$

$= \int \frac{dx}{x^2+1} + \int \frac{x}{x^2+2}\,dx$

$= \tan^{-1}x + \frac{1}{2}\ln|x^2+2| + C_1$

33. $\int \frac{x^3-2x^2+2x-2}{x^2+1}\,dx$

$\frac{x^3-2x^2+2x-2}{x^2+1} = \frac{(x^2+1)(x-2)}{x^2+1} + \frac{x}{x^2+1} = x-2+\frac{x}{x^2+1}$

$\int \frac{x^3-2x^2+2x-2}{x^2+1}\,dx = \int (x-2)\,dx + \int \frac{x}{x^2+1}\,dx$

$= \frac{1}{2}x^2 - 2x + \frac{1}{2}\ln|x^2+1| + C$

34. $\int \frac{x^4+6x^3+10x^2+x}{x^2+6x+10}\,dx$

$\frac{x^4+6x^3+10x^2+x}{x^2+6x+10} = x^2 + \frac{x}{x^2+6x+10} = x^2 + \frac{x}{(x+3)^2+1}$

$\int \frac{x}{(x+3)^2+1}\,dx$

Put $u = x+3 \Rightarrow du = dx$

So, $\int \frac{u-3}{u^2+1}\,du = \int \frac{u}{u^2+1}\,du - 3\int \frac{du}{u^2+1}$

$$= \frac{1}{2}\ln|u^2+1| - 3\tan^{-1}u$$

$$= \frac{1}{2}\ln|x^2+6x+10| - 3\tan^{-1}(x+3) + C_1$$

So, $\displaystyle\int \frac{x^4+6x^3+10x^2+x}{x^2+6x+10}\,dx$

$$= \int x^2\,dx + \int \frac{x\,dx}{(x+3)^2+1}$$

$$= \frac{x^3}{3} + \frac{1}{2}\ln|x^2+6x+10| - 3\tan^{-1}(x+3) + C$$