$$y = \frac{1}{\sqrt{x-1}}$$

$$f(x) = \frac{1}{\sqrt{x-1}} \qquad f(x+h) = \frac{1}{\sqrt{x+h-1}}$$

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{1}{\sqrt{x-1}}\right) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0}\frac{\frac{1}{\sqrt{x+h-1}} - \frac{1}{\sqrt{x-1}}}{h}$$

$$= \lim_{h\to 0}\frac{\sqrt{x-1}-\sqrt{x+h-1}}{h\left(\sqrt{(x-1)(x+h-1)}\right)}$$

$$= \lim_{h\to 0}\frac{\left(\sqrt{x-1}-\sqrt{x+h-1}\right)\left(\sqrt{x-1}+\sqrt{x+h-1}\right)}{h\left(\sqrt{x^2-x+hx-h-x+1}\right)\left(\sqrt{x-1}+\sqrt{x+h-1}\right)}$$

$$\Rightarrow \lim_{h\to 0}\frac{x-1-(x+h-1)}{h\left(\sqrt{x^2-2x+hx-h+1}\right)\left(\sqrt{x-1}+\sqrt{x+h-1}\right)}$$

$$\Rightarrow \lim_{h\to 0}\frac{-h}{h\left(\sqrt{}\right)\left(\sqrt{x-1}+\sqrt{x+h-1}\right)}$$

$$\Rightarrow \lim_{h\to 0}\frac{-1}{\sqrt{x^2-2x+hx+1-h}\left(\sqrt{x-1}+\sqrt{x+h-1}\right)}$$

$$= \frac{1}{\sqrt{x^2-2x+1}\quad\left(\sqrt{x-1}+\sqrt{x-1}\right)}$$

$$= \frac{1}{\sqrt{(x-1)^2}\quad 2\sqrt{x-1}}$$

$$= \frac{1}{2(x-1)\sqrt{x-1}} = \frac{1}{2(x-1)^{3/2}}$$

$$y = \frac{1+x+x^2+x^3+x^4+x^5+x^6}{x^3}$$

$$y = \frac{1}{x^3}+\frac{x}{x^3}+\frac{x^2}{x^3}+\frac{x^4}{x^3}+\frac{x^5}{x^3}+\frac{x^6}{x^3}$$

$$y = x^{-3}+x^{-2}+x^{-1}+x+x^2+x^3$$

$$\frac{dy}{dx} = \frac{d}{dx}\left(x^{-3}+x^{-2}+x^{-1}+x+x^2+x^3\right)$$

$$= \frac{d}{dx}(x^{-3})+\frac{d}{dx}(x^{-2})+\frac{d}{dx}(x^{-1})+\frac{d}{dx}(x) + \frac{d}{dx}(x^2)+\frac{d}{dx}(x^3)$$

$$= (-3)x^{-3-1}+(-2)x^{-2-1}+(-1)x^{-1-1}+x^{1-1}+2x^{2-1}+3x^{3-1}$$

$$= -3x^{-4}-2x^{-3}-x^{-2}+1+2x+3x^2.$$

$$y = f(x)$$

$$\frac{dy}{dx} = \frac{d}{dx}(f(x)) = f'(x)$$

Inst Instantaneous rate of change of "y" w.r.t. "x"

slope

1st Derivative $\Rightarrow$ $\boxed{\frac{dy}{dx} = f'(x)}$

$$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}\left(f'(x)\right)$$

2nd order Derivative $\Rightarrow$ $\frac{d^2y}{dx^2} = f''(x)$

Instantaneous rate of change of "$\frac{dy}{dx}$" w.r.t. "x"

Second Derivative of "y" w.r.t. "x".

Second derivative of "f" w.r.t. "x"

3rd order Derivative

$$\frac{d}{dx}\left(\frac{d^2y}{dx^2}\right) = \frac{d}{dx}\left(f''(x)\right)$$

$$\Rightarrow \frac{d^3y}{dx^3} = f'''(x)$$

Higher Derivatives

Find $\frac{d^2y}{dx^2}$ if $y=(5x^2-3)(7x^3+x)$

$$y = (5x^2-3)(7x^3+x)$$

$$\Rightarrow y = 35x^5 \underbrace{-21x^3+5x^3}-3x$$

$$\frac{dy}{dx} = \frac{d}{dx}\left(35x^5-16x^3-3x\right)$$

$$\Rightarrow \frac{dy}{dx} = \frac{d}{dx}(35x^5)-\frac{d}{dx}(16x^3)-\frac{d}{dx}(3x)$$

$$= 35\cdot 5x^{5-1}-16\cdot 3x^{3-1}-3x^{1-1}$$

$$\Rightarrow \frac{dy}{dx} = 175x^4-48x^2-3$$

$$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}\left(175x^4-48x^2-3\right)$$

$$\Rightarrow \frac{d^2y}{dx^2} = 175\cdot 4x^{4-1}-48\cdot 2x^{2-1}-0$$

$$\Rightarrow \frac{d^2y}{dx^2} = 700x^3-96x$$

Find $f'''(2)$ if $f(x)=2x^3-3$

$$f'(x) = \frac{d}{dx}(2x^3-3)$$

$$\Rightarrow f'(x) = 6x^2$$

$$f''(x) = 12x$$

$$f'''(x) = 12$$

So, $f'''(2) = 12$

$$\frac{d}{dx}\left(f(x)\pm g(x)\right) = \frac{d}{dx}(f(x)) \pm \frac{d}{dx}(g(x))$$

$$\frac{d}{dx}\left(f(x)g(x)\right) = ?$$

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = ?$$

Product Rule of Differentiation $\rightarrow \frac{d}{dx}\left(f(x)\,g(x)\right) = f(x)\frac{d}{dx}(g(x)) + g(x)\frac{d}{dx}(f(x))$

Quotient Rule of Differentiation

Find the derivative of

$$y = 2x\ln x$$

$$\frac{dy}{dx} = \frac{d}{dx}(2x\ln x)$$

$$= 2x\frac{d}{dx}(\ln x) + \ln x\frac{d}{dx}(x)$$

$$= 2x\frac{1}{x} + \ln x\,(1)$$

$$= 2+\ln x$$