EEE363

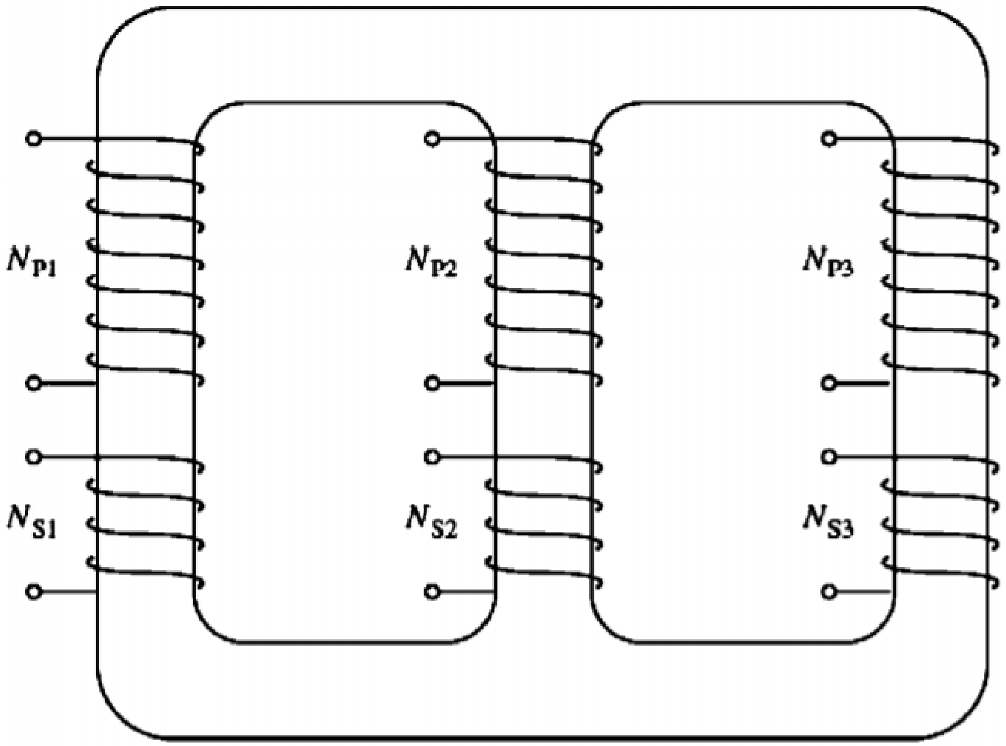
Electrical Machines

Lecture # 14

Dr Atiqur Rahman

3-Phase Transformer

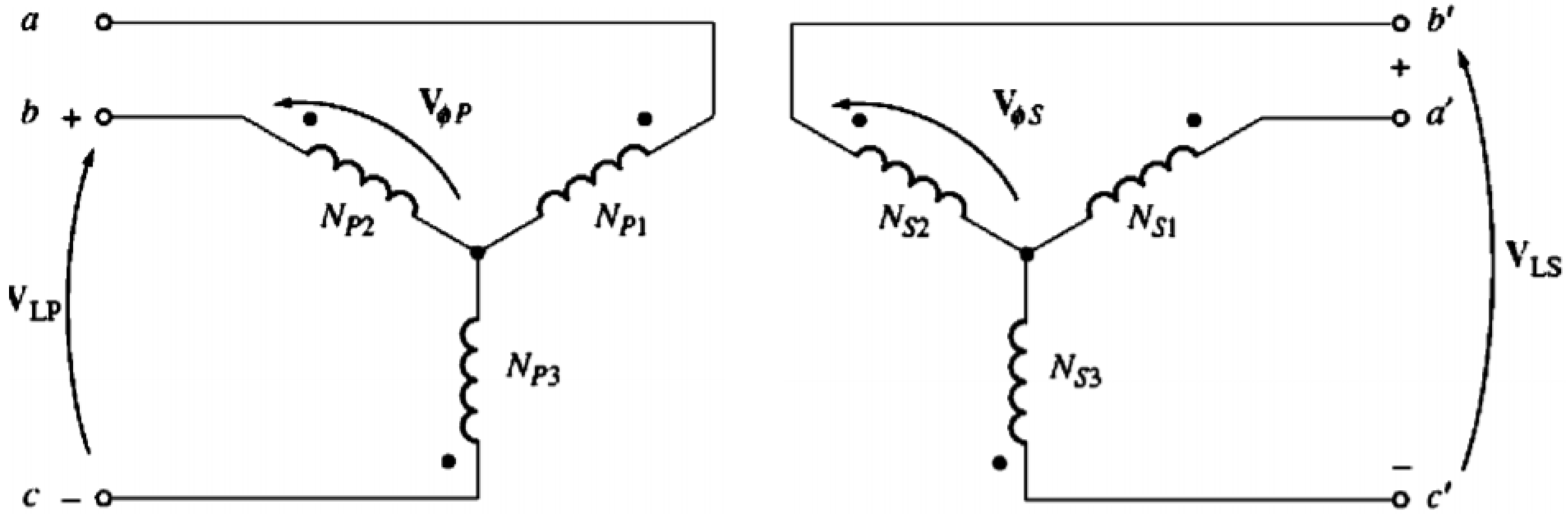
Almost all the major power generation and distribution systems in the world today are three-phase ac systems.



3-phase Transformer connections

1. Wye–wye (Y–Y)
2. Wye–delta (Y–Δ)
3. Delta–wye (Δ–Y)
4. Delta–delta (Δ–Δ)

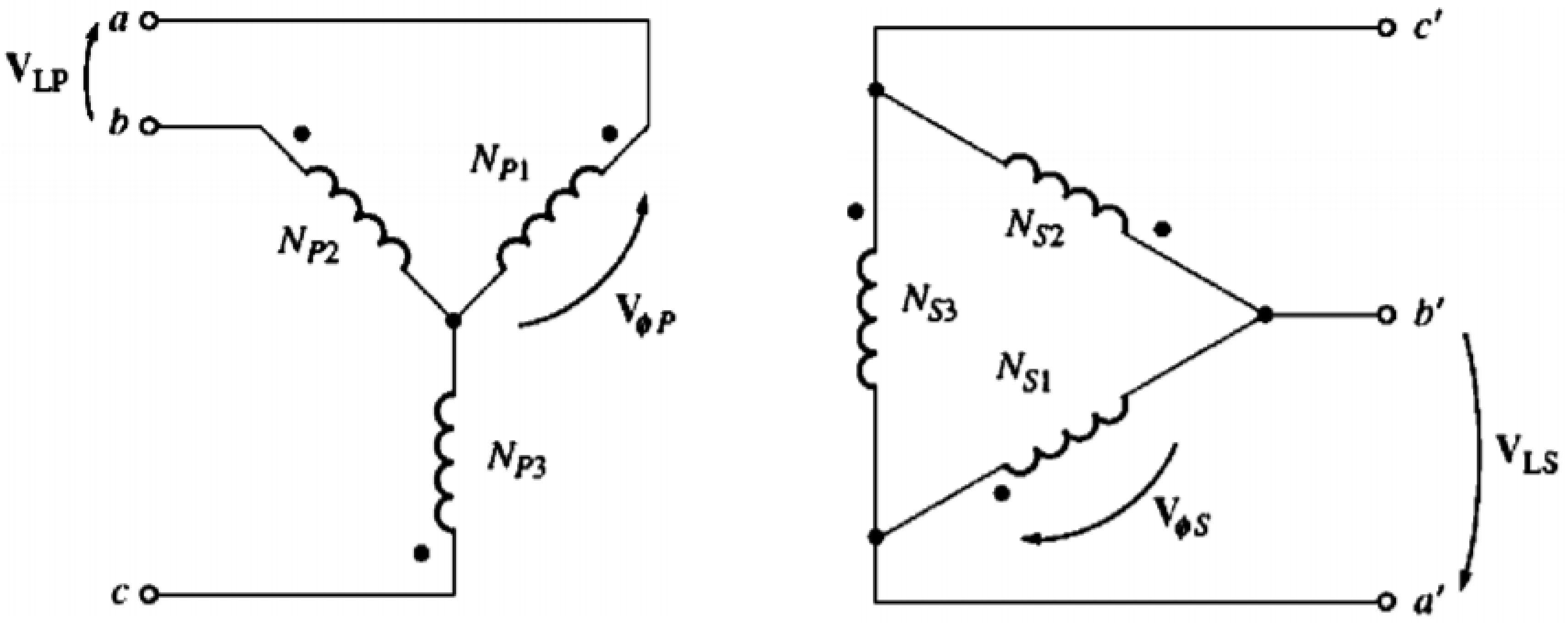
Y – Y Connection



$$\frac{V_{\phi P}}{V_{\phi S}} = a$$

$$\boxed{\frac{V_{LP}}{V_{LS}} = \frac{\sqrt{3}V_{\phi P}}{\sqrt{3}V_{\phi S}} = a \qquad \text{Y−Y}}$$

Y – Δ Connection

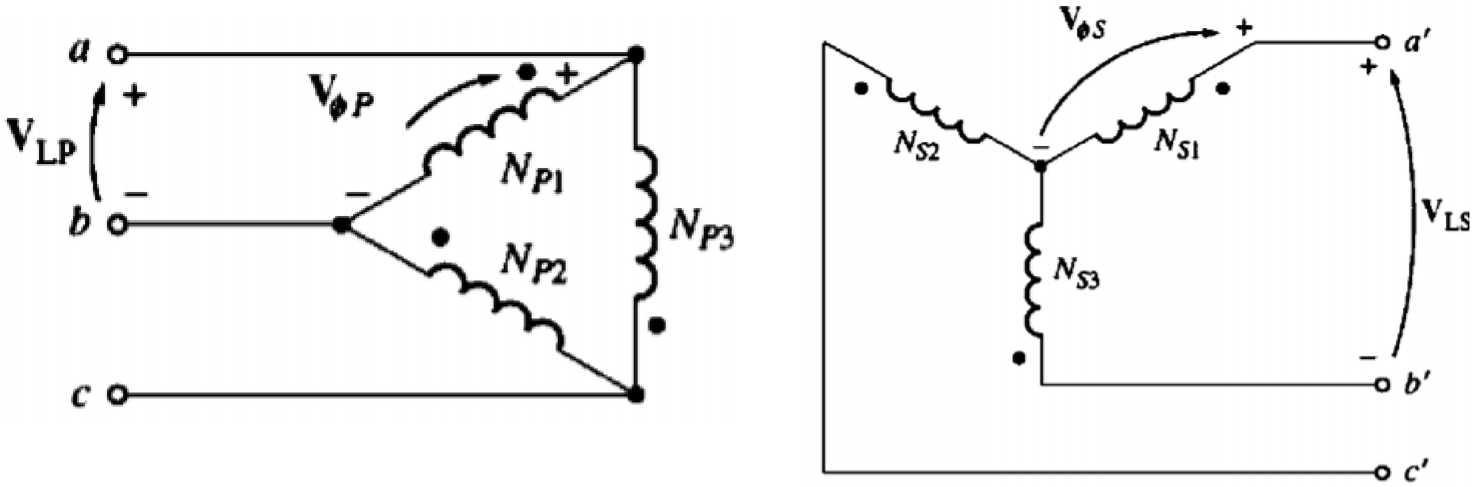


$$\frac{V_{\phi P}}{V_{\phi S}} = a \qquad \frac{V_{LP}}{V_{LS}} = \frac{\sqrt{3}V_{\phi P}}{V_{\phi S}}$$

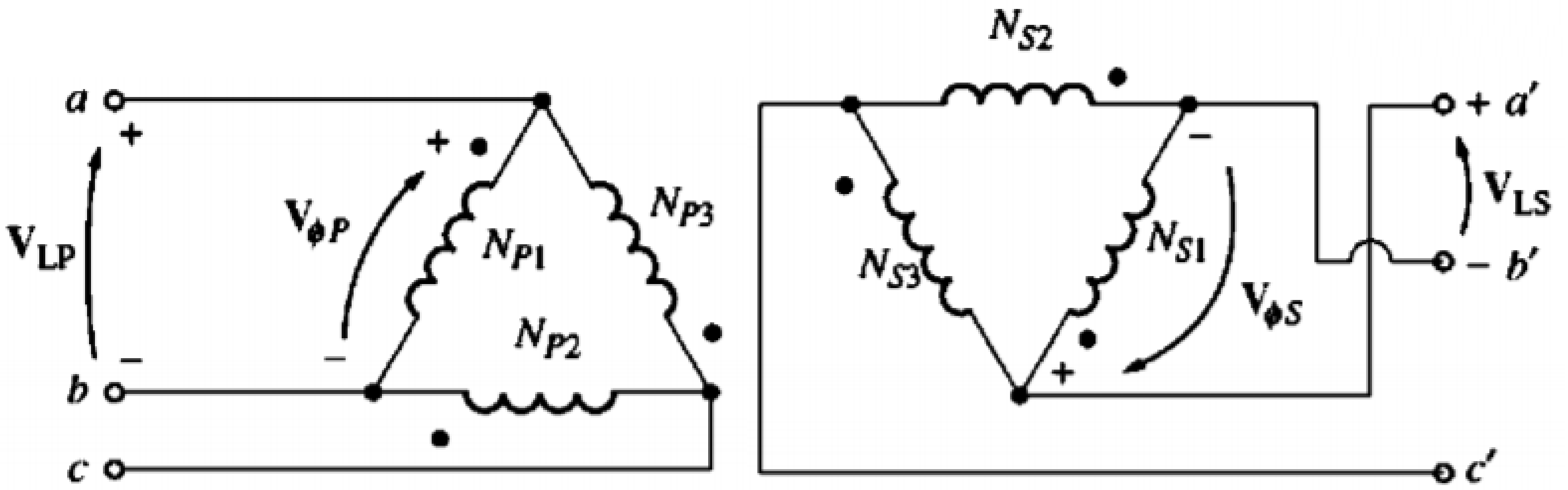
$$\boxed{\frac{V_{LP}}{V_{LS}} = \sqrt{3}a \qquad \text{Y}-\Delta}$$

Other connections

$$\frac{V_{LP}}{V_{LS}} = \frac{\sqrt{3}}{a} \qquad \Delta - Y$$



$$\frac{V_{LP}}{V_{LS}} = \frac{V_{\phi P}}{V_{\phi S}} = a \qquad \Delta - \Delta$$



PU system for 3-phase transformer

$$S_{1\phi,\text{base}} = \frac{S_{\text{base}}}{3}$$

$$Z_{\text{base}} = \frac{(V_{\phi,\text{base}})^2}{S_{1\phi,\text{base}}}$$

$$I_{\phi,\text{base}} = \frac{S_{1\phi,\text{base}}}{V_{\phi,\text{base}}}$$

$$\boxed{Z_{\text{base}} = \frac{3(V_{\phi,\text{base}})^2}{S_{\text{base}}}}$$

$$\boxed{I_{\phi,\text{base}} = \frac{S_{\text{base}}}{3\,V_{\phi,\text{base}}}}$$

Problem # 1

Example 2–9. A 50-kVA 13,800/208-V Δ-Y distribution transformer has a resistance of 1 percent and a reactance of 7 percent per unit.

(a) What is the transformer's phase impedance referred to the high-voltage side?

(b) Calculate this transformer's voltage regulation at full load and 0.8 PF lagging, using the calculated high-side impedance.

(c) Calculate this transformer's voltage regulation under the same conditions, using the per-unit system.

$$Z_{\text{base}} = \frac{3(V_{\phi,\text{base}})^2}{S_{\text{base}}} = \frac{3(13{,}800\text{ V})^2}{50{,}000\text{ VA}} = 11{,}426\ \Omega$$

PU impedance of the transformer is $Z_{\text{eq}} = 0.01 + j0.07\text{ pu}$

So the high side impedance in ohms is $Z_{\text{eq}} = Z_{\text{eq,pu}} Z_{\text{base}} = (0.01 + j0.07\text{ pu})(11{,}426\ \Omega)$

$$= 114.2 + j800\ \Omega$$

Regulation calculation

$$\mathrm{VR} = \frac{V_{\phi P} - aV_{\phi S}}{aV_{\phi S}} \times 100\%$$

$$I_\phi = \frac{S}{3V_\phi}$$

$$\mathrm{I}_\phi = \frac{50{,}000 \text{ VA}}{3(13{,}800 \text{ V})} = 1.208 \text{ A}$$

$$\begin{aligned}
\mathbf{V}_{\phi P} &= a\mathbf{V}_{\phi S} + R_{\text{eq}}\mathbf{I}_\phi + jX_{\text{eq}}\mathbf{I}_\phi \\
&= 13{,}800\angle 0^\circ \text{ V} + (114.2\ \Omega)(1.208\angle -36.87^\circ \text{ A}) + (j800\ \Omega)(1.208\angle -36.87^\circ \text{ A}) \\
&= 13{,}800 + 138\angle -36.87^\circ + 966.4\angle 53.13^\circ \\
&= 13{,}800 + 110.4 - j82.8 + 579.8 + j773.1 \\
&= 14{,}490 + j690.3 = 14{,}506\angle 2.73^\circ \text{ V}
\end{aligned}$$

$$\mathrm{VR} = \frac{V_{\phi P} - aV_{\phi S}}{aV_{\phi S}} \times 100\% = \frac{14{,}506 - 13{,}800}{13{,}800} \times 100\% = 5.1\%$$

Regulation calculation using PU system

$$V_P = 1\angle 0° + (0.01)(1\angle -36.87°) + (j0.07)(1\angle -36.87°)$$

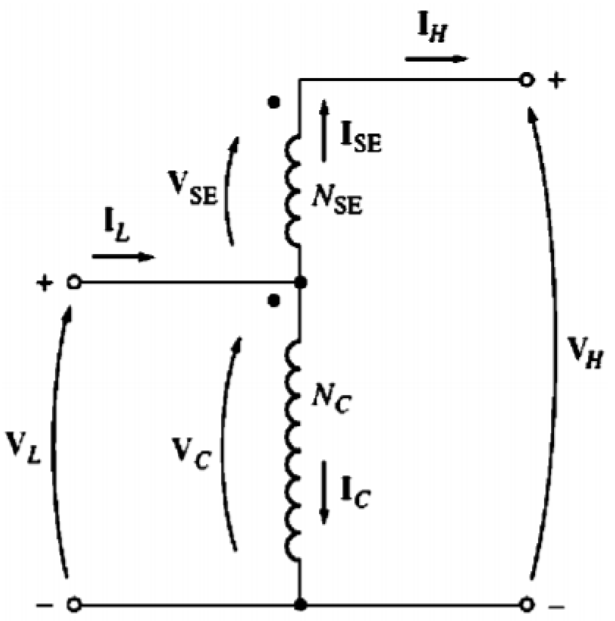
$$= 1 + 0.008 - j0.006 + 0.042 + j0.056$$

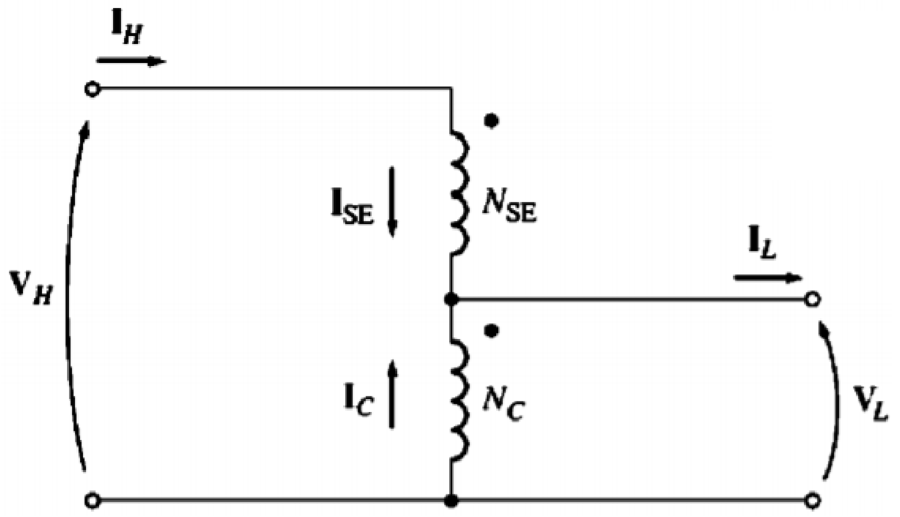
$$= 1.05 + j0.05 = 1.051\angle 2.73°$$

$$\mathrm{VR} = \frac{1.051 - 1.0}{1.0} \times 100\% \quad = 5.1\%$$

Autotransformer

- Has one winding only
- To cater for the need to change the voltage slightly.
- To give a small boost to a distribution cable to correct the voltage drop.
- A two-winding transformer is expensive in this case.





Voltage and current relationship

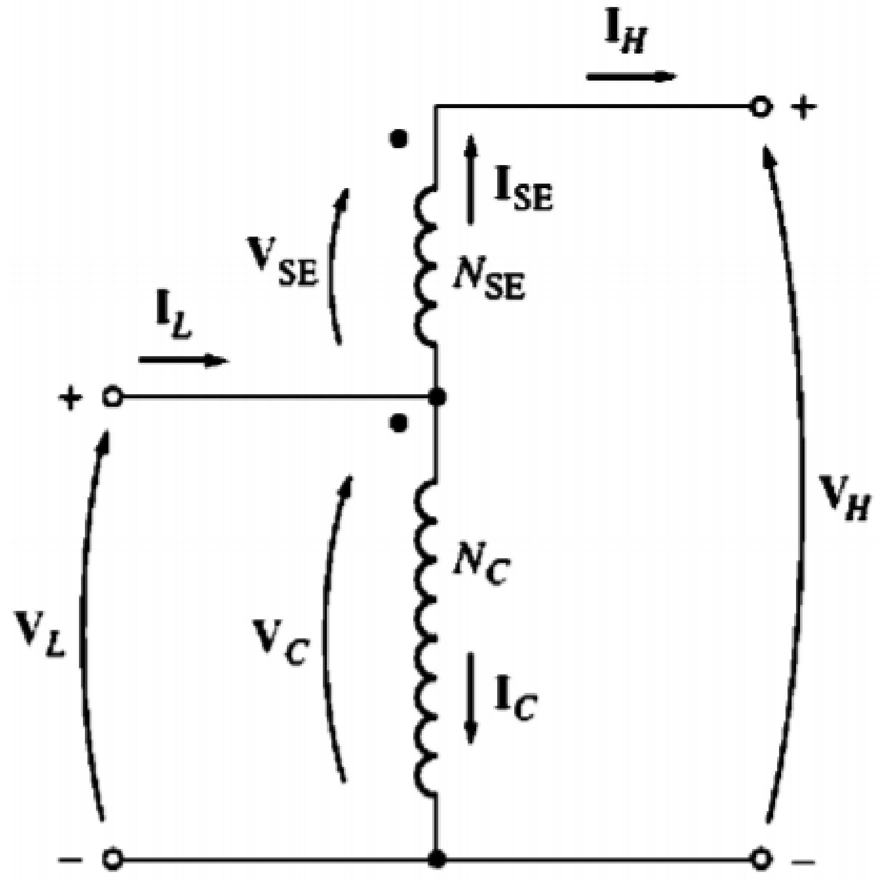
$$\frac{\mathbf{V}_C}{\mathbf{V}_{SE}} = \frac{N_C}{N_{SE}} \qquad N_C \mathbf{I}_C = N_{SE} \mathbf{I}_{SE}$$

$$\mathbf{V}_L = \mathbf{V}_C \qquad \mathbf{V}_H = \mathbf{V}_C + \mathbf{V}_{SE}$$

$$\mathbf{V}_H = \mathbf{V}_C + \mathbf{V}_{SE}$$

$$\mathbf{V}_H = \mathbf{V}_C + \frac{N_{SE}}{N_C}\mathbf{V}_C = \mathbf{V}_L + \frac{N_{SE}}{N_C}\mathbf{V}_L = \frac{N_{SE} + N_C}{N_C}\mathbf{V}_L$$

$$\boxed{\frac{\mathbf{V}_L}{\mathbf{V}_H} = \frac{N_C}{N_{SE} + N_C}}$$



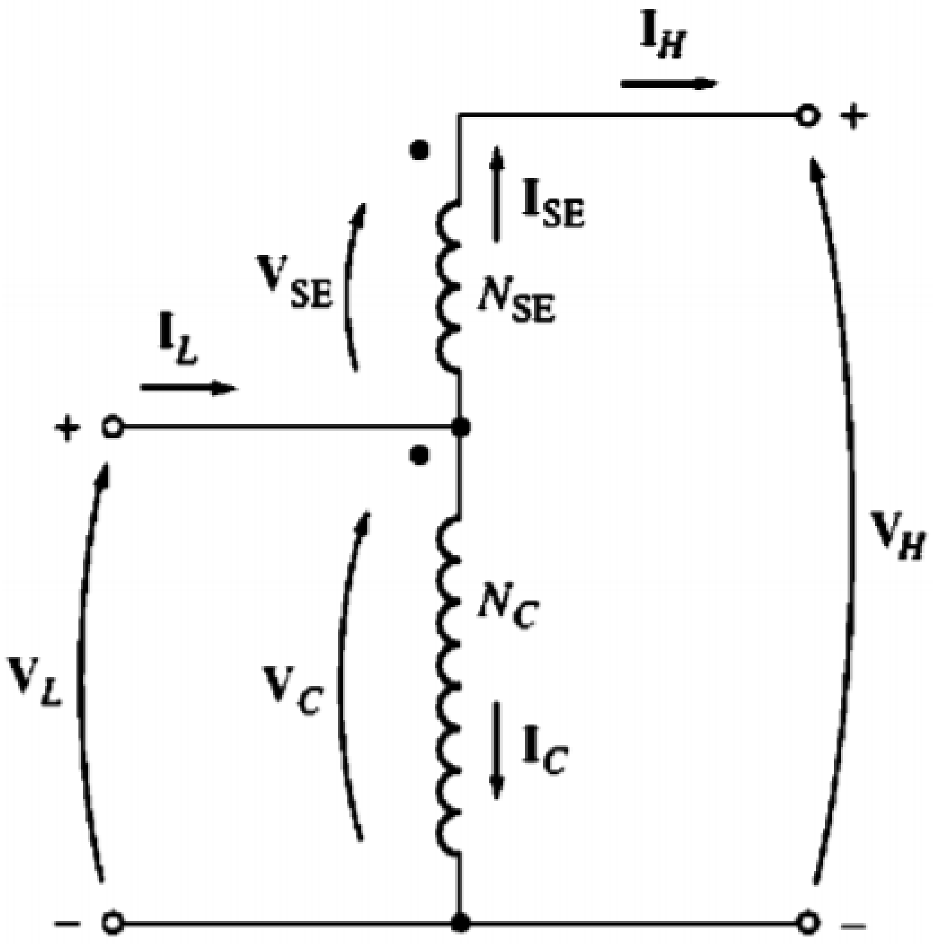
Voltage and current relationship

$$\mathbf{I}_L = \mathbf{I}_C + \mathbf{I}_{SE} \quad = \frac{N_{SE}}{N_C}\mathbf{I}_{SE} + \mathbf{I}_{SE}$$

$$= \frac{N_{SE}}{N_C}\mathbf{I}_H + \mathbf{I}_H$$

$$= \frac{N_{SE} + N_C}{N_C}\mathbf{I}_H$$

$$\boxed{\frac{\mathbf{I}_L}{\mathbf{I}_H} = \frac{N_{SE} + N_C}{N_C}}$$



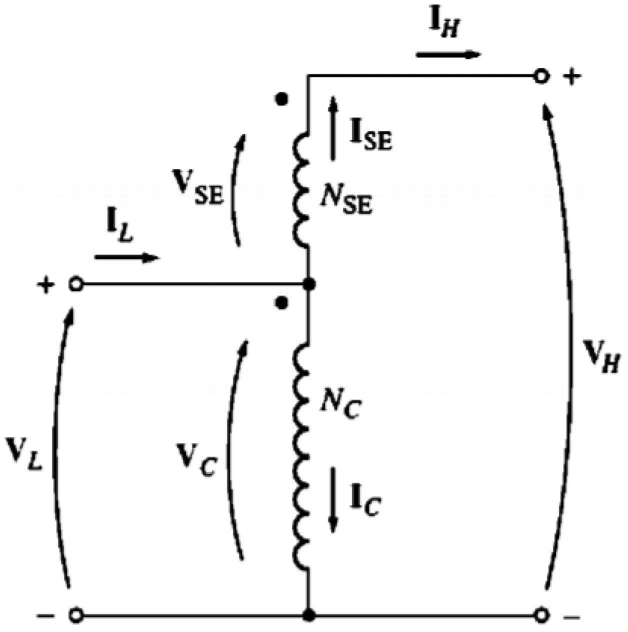
Apparent power rating advantage of Autotransformer

Input apparent power of the transformer $S_{in} = V_L I_L$

Output apparent power of the transformer $S_{out} = V_H I_H$

$$S_{in} = S_{out} = V_L I_L = V_H I_H = S_{IO}$$

The apparent power in the transformer windings is $S_W = V_C I_C = V_{SE} I_{SE}$



$$S_W = V_C I_C = V_L(I_L - I_H)$$

$$= V_L I_L - V_L I_L \frac{N_C}{N_{SE} + N_C} = V_L I_L \frac{(N_{SE} + N_C) - N_C}{N_{SE} + N_C} = S_{IO} \frac{N_{SE}}{N_{SE} + N_C}$$

$$\boxed{\frac{S_{IO}}{S_W} = \frac{N_{SE} + N_C}{N_{SE}}}$$

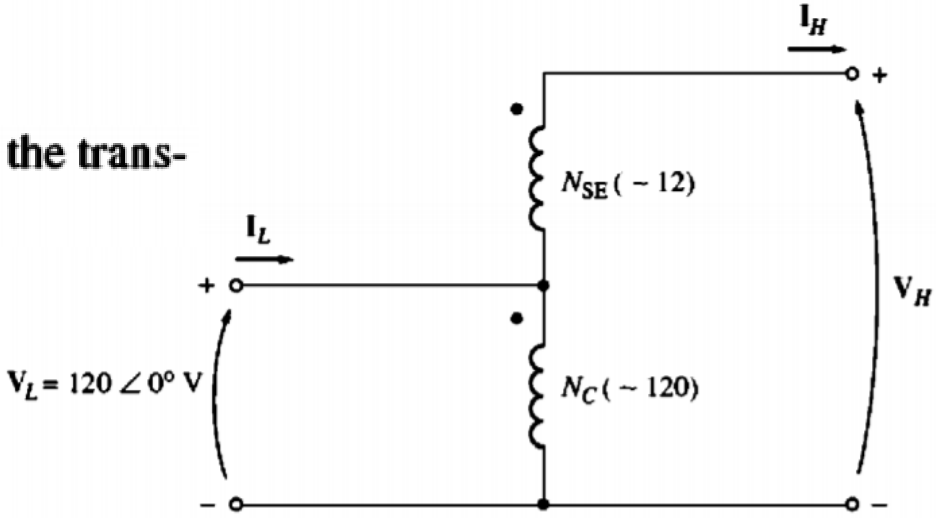
Problem # 2

Example 2–7. A 100-VA 120/12–V transformer is to be connected so as to form a step-up autotransformer (see Figure 2–34). A primary voltage of 120 V is applied to the transformer.

(*a*) What is the secondary voltage of the transformer?

(*b*) What is its maximum voltampere rating in this mode of operation?

(*c*) Calculate the rating advantage of this autotransformer connection over the transformer's rating in conventional 120/12–V operation.



$$\mathbf{V}_H = \frac{N_{SE} + N_C}{N_C}\mathbf{V}_L = \frac{12 + 120}{120}120 \text{ V} = 132 \text{ V}$$

$$\frac{S_{IO}}{S_W} = \frac{N_{SE} + N_C}{N_{SE}} = \frac{12 + 120}{12} = \frac{132}{12} = 11$$ *Rating advantage*

Rating of the autotransformer = 11 x 100 = 1100 VA

Problem # 2

The transformer's impedance in a per-unit system when connected in the conventional manner is

$$Z_{eq} = 0.01 + j0.08 \text{ pu}$$

The apparent power advantage of this autotransformer is 11, so the per-unit impedance of the autotransformer connected as described is

$$Z_{eq} = \frac{0.01 + j0.08}{11}$$

$$= 0.00091 + j0.00727 \text{ pu}$$

Problem of initial current inrush

Suppose that voltage $v(t) = V_M \sin(\omega t + \theta)$ is applied at the moment the transformer is first connected to the power line

If the initial voltage is $v(t) = V_M \sin(\omega t + 90°) = V_M \cos \omega t$ and the initial flux in the core is zero, then the maximum flux during the first half cycle will just equal the maximum flux at steady state:

$$\phi_{\max} = \frac{V_{\max}}{\omega N_P}$$

This flux level is just the steady-state flux, so it causes no special problems. But if the applied voltage happens to be

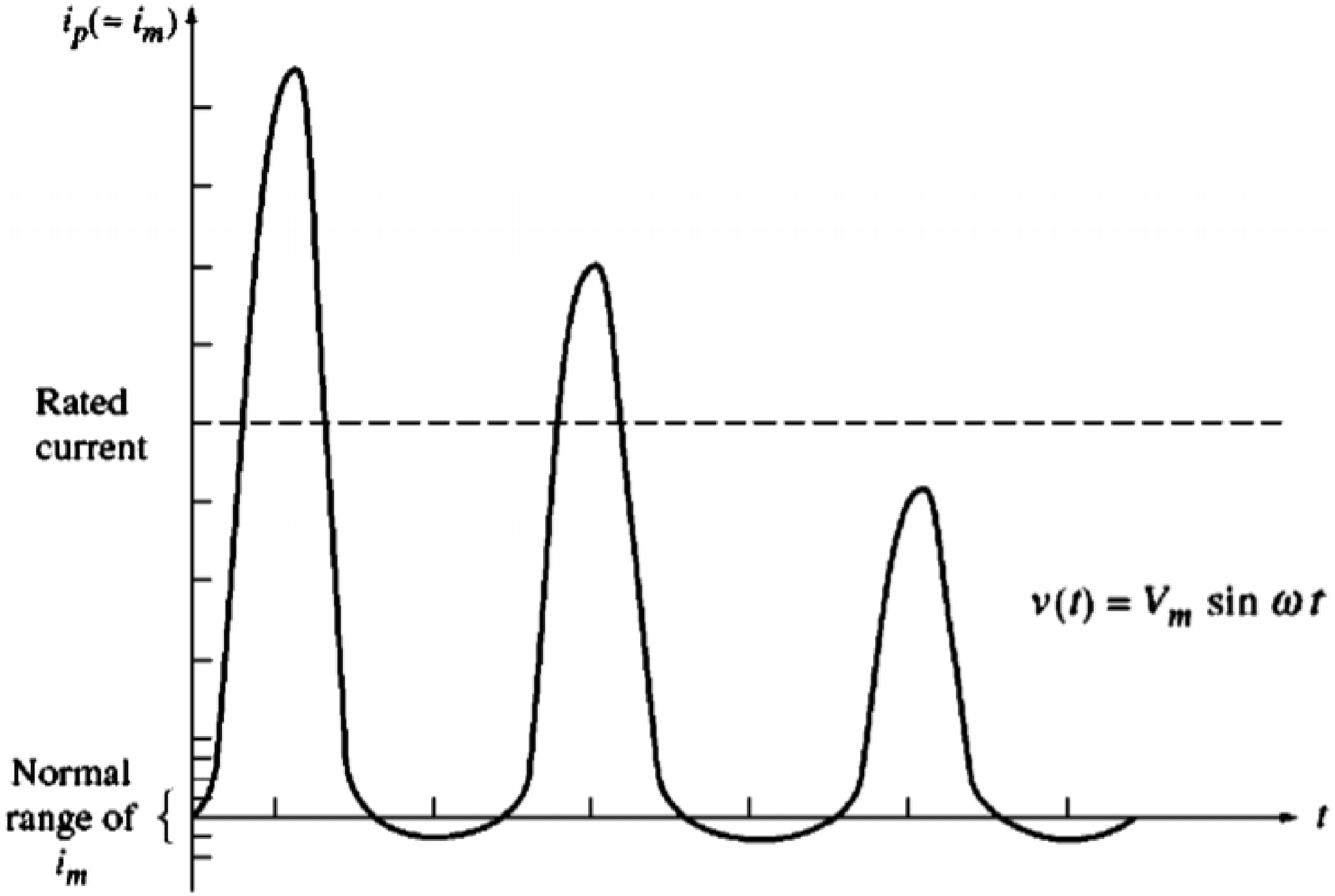
$$v(t) = V_M \sin \omega t$$

$$\phi(t) = \frac{1}{N_P}\int_0^{\pi/\omega} V_M \sin \omega t \, dt = -\frac{V_M}{\omega N_P} \cos \omega t \Big|_0^{\pi/\omega} = -\frac{V_M}{\omega N_P}[(-1) - (1)] \rightarrow \boxed{\phi_{\max} = \frac{2V_{\max}}{\omega N_P}}$$

Problem of initial current inrush

$$\phi_{max} = \frac{2V_{max}}{\omega N_P}$$

- ✓ *This maximum flux is twice as high as the normal steady-state flux.*
- ✓ *This doubling of the flux in the core results in an enormous magnetization current.*
- ✓ *In fact, for part of the cycle, the transformer looks like a short circuit, and a very large current flows*



Maintenance

- Transformers are generally housed in tightly-filled sheet steel metal cage.
- Tanks are filled with special insulating oil. This keeps coil cool.
- Oil should be free from alkalis, sulphur and moisture.
- Sides of the tank is corrugated or are fitted with radiators in order to allow for faster cooling.
- For large transformers, breathers are provided in order to allow for the expansion/contraction of the oil in the tank as the temperature increases or decreases.
- Moisture trapper is provided to block moisture