Probability density function:

Probability density function is a function of a continuous random variable.

The probability that the random variable lies between two values a and b is obtained by integrating the probability density function between these two values. A valid probability density function *f (x)* must integrate to one over the whole sample space, so that the total probability is equal to 1.

Question:

Suppose that the diameter of a metal cylinder has a probability density function $f(x) = 1.5 - 6(x - 50.0)^2$ for $49.5 \le x \le 50.5$

i) Show that total area under the probability density function = 1 or Prove that this is a valid PDF.

ii) Calculate The probability that a metal cylinder has a diameter between 49.8 and 50.1 mm.

iii) Find cumulative distribution function.

Solution: i) $\int_{49.5}^{50.5} 1.5 - 6(x - 50.0)^2 \, dx$

$= \left[1.5x - \frac{6(x-50)^3}{3}\right]_{49.5}^{50.5}$ [As, $\int c\, dx = cx, \int (x-a)^n dx = \frac{(x-a)^{n+1}}{n+1}$]

= [1.5*50.5 - $2(50.5 - 50)^3$] - [1.5*49.5 - $2(49.5 - 50)^3$]

= 75.5 – 74.5

= 1

That is 100% of the cylinders will have diameters within these limits.

Since over the whole sample space, the integration of f(x) = 1, so this is a valid PDF.

ii) $\int_{49.8}^{50.1} 1.5 - 6(x - 50.0)^2 \, dx$

$= \left[1.5x - \frac{6(x-50)^3}{3}\right]_{49.8}^{50.1}$ [As, $\int c\,dx = cx$, $\int (x-a)^n dx = \frac{(x-a)^{n+1}}{n+1}$]

= [1.5*50.1 - $2(50.1 - 50)^3$] - [1.5*49.8 - $2(49.8 - 50)^3$]

= 75.148 – 74.716

= 0.43

That is, 43% of the cylinders will have diameters within these limits.

iii) F(x) = $\int_{49.5}^{x} 1.5 - 6(x - 50.0)^2 \, dx$

$= \left[1.5x - \frac{6(x-50)^3}{3}\right]_{49.5}^{x}$ [As, $\int c\,dx = cx$, $\int (x-a)^n dx = \frac{(x-a)^{n+1}}{n+1}$]

= [1.5*x - $2(x - 50)^3$] - [1.5*49.5 - $2(49.5 - 50)^3$]

= 1.5*x - $2(x - 50)^3$ – 74.5

Question: f (x) = A(0.5 − (x − 0.25)^2) for $0.125 \le x \le 0.5$
(a) Find the value of *A*
(b) Construct the cumulative distribution function.
(c) What is the probability that the paint thickness at a particular point is less than 0.2 mm?

Solution: a) $\int_{0.125}^{0.5} A(0.5\ -\ (x-0.25)^2)\ dx = 1$

$\Rightarrow A\left[0.5X - \frac{(x-0.25)^3}{3}\right]_{0.125}^{0.5} = 1$

$\Rightarrow$ A[[0.5*0.5 - $\frac{(0.5-0.25)^3}{3}$] - $[0.5 * 0.125 - \frac{(0.125-0.25)^3}{3}$]] = 1

$\Rightarrow$ A [$\frac{47}{192} - \frac{97}{1536}$] = 1

$\Rightarrow$A$\frac{93}{512}$ = 1

i.e A = $\frac{512}{93}$

b) F(x) = $\int_{0.125}^{x} \frac{512}{93}(0.5\ -\ (x-0.25)^2)\ dx$

$= \frac{512}{93}\left[0.5x - \frac{(x-0.25)^3}{3}\right]_{0.125}^{x}$

$= \frac{512}{93}\left[\left[0.5x - \frac{(x-0.25)^3}{3}\right] - [0.5 * 0.125 - \frac{(0.125-0.25)^3}{3}]\right]$

$= \frac{512}{93}\left[\left[0.5x - \frac{(x-0.25)^3}{3}\right] - \frac{97}{1536}\right]$

c) $\int_{0.125}^{0.2} \frac{512}{93}(0.5\ -\ (x-0.25)^2)\ dx$

$= \frac{512}{93}\left[\left[0.5 * 0.2 - \frac{(0.2-0.25)^3}{3}\right] - \frac{97}{1536}\right]$

$=0.203$