σ^2

Ch 5: Normal or Gaussian distribution

The **normal** or **Gaussian distribution** has a probability density function

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

for $-\infty \le x \le \infty$, depending upon two parameters, the mean and the variance

$E(X) = \mu$ and $V(X) = \sigma^2$ of the distribution. The probability density function is a bell-shaped curve that is symmetric about μ. The notation

$$X \sim N(\mu, \sigma^2)$$

denotes that the random variable X has a normal distribution with mean μ and

variance σ^2

Standard normal distribution

A normal distribution with mean μ = 0 and variance σ2 = 1 is known as the **standard normal distribution**. Its pdf has the notation *φ(x)* and is given by

$$\varphi(\mathrm{x}) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}, \text{ for } -\infty \leq x \leq \infty$$

The notation φ(x) is used for the cumulative distribution function of a standard normal

distribution, which is calculated from the expression

$$\Phi(\mathrm{x}) = \int_{-\infty}^{x} \varphi(y)dy$$

Standard normal Distribution (cont.)

- The symmetry of the standard normal distribution about 0 implies that if the random variable *Z* has a standard normal distribution, then
- $1 - \phi(x) = P((X \geq x) = P(X \leq -x) = \phi(-x)$, as illustrated in Figure 5.6. This equation can be rearranged to provide the easily remembered relationship

$$\phi(x) + \phi(-x) = 1$$

- The cumulative distribution function of the standard normal distribution $\phi(x)$ is tabulated in Table I at the end of the book. This table provides values of $\phi(x)$ to four decimal places for values of *x* between −3.49 and 3.49. For values of *x* less than −3.49, *(x)* is very close to 0, and for values of *x* greater than 3.49, *(x)* is very close to 1.

Probability Calculations for General Normal Distributions

- A very important general result is that if
- $X \sim N(\mu, \sigma^2)$ then $Z = \frac{X-\mu}{\sigma}$
- has a standard normal distribution. That is, $Z \sim N(0,1)$
- The probability values of any normal distribution can be related to the probability values of a standard normal distribution and, in particular, to the cumulative distribution function $\Phi(x)$.
- Example: $P(a \le X \le b) = P(\frac{a-\mu}{\sigma} \le \frac{X-\mu}{\sigma} \le \frac{b-\mu}{\sigma}) = P(\frac{a-\mu}{\sigma} \le Z \le \frac{b-\mu}{\sigma})$
- . $= \Phi\left(\frac{b-\mu}{\sigma}\right) - \Phi\left(\frac{a-\mu}{\sigma}\right)$