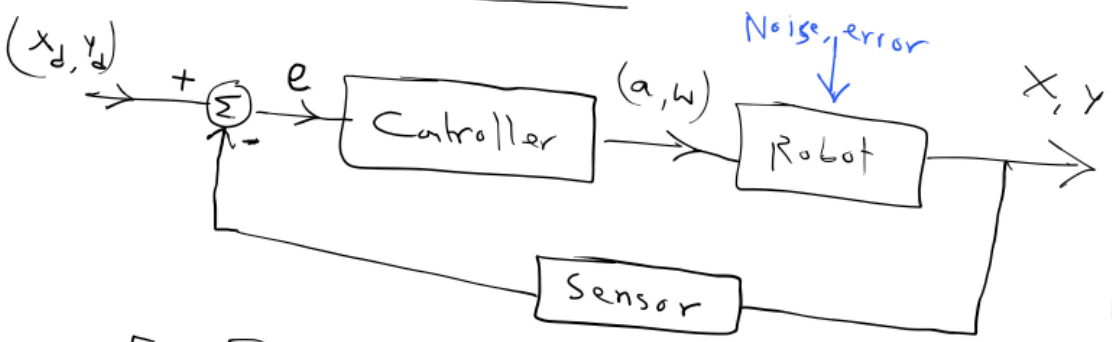
Sunday, April 21, 2024 10:37 AM

Midterm: Sunday, April 28

Closed loop tracking controller

(x, y) sensor measurement

(x_d, y_d) desired trajectory



e = error

You want $e = 0$

then $x_d = x$
$y_d = y$

$$e = \begin{bmatrix} e_x \\ e_y \end{bmatrix} = \begin{bmatrix} x_d - x \\ y_d - y \end{bmatrix} \Rightarrow \dot{e} = \begin{bmatrix} \dot{x}_d - \dot{x} \\ \dot{y}_d - \dot{y} \end{bmatrix}$$

$$\ddot{e} = \begin{bmatrix} \ddot{x}_d - \ddot{x} \\ \ddot{y}_d - \ddot{y} \end{bmatrix}$$

OneNote

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To Do

Important

Dictate

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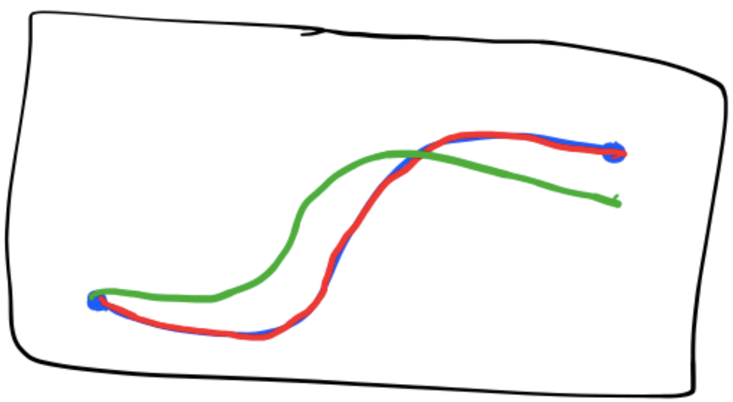

— desired (x_d, y_d) trajectory

— Robot (x, y)

$(x_d, y_d) = (x, y)$

— Robot with error (x, y)

$(x, y) \neq (x_d, y_d)$

SS

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Recall 2^{nd} ODE

$$\ddot{q}(t) + 3\dot{q}(t) + 2q(t) = 0$$

I.C

$q(0) = $ something

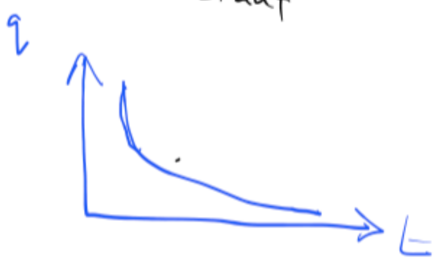
$\dot{q}(0) = $ something

Assume $q(t) = e^{rt}$

r is some constant

$\dot{q}(\cancel{t}) =$

$\ddot{q}(t) =$



$$\boxed{r = -2, -1}$$

If $q(t) \to 0$ as $t \to \infty$
$\Rightarrow$ Stable system

If $q(t) \to \infty$ as $t \to \infty$
$\Rightarrow$ unstable system

$q_1(t) = e^{-2t}$ $\qquad$ $q_2(t) = e^{-t}$

As $t \to \infty \Rightarrow q_1(t) \to 0,\ q_2(t) \to 0 \Rightarrow$ Stable system

OneNote

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Closed-loop controller

$$\ddot{e}(t) + k_d \dot{e}(t) + k_p e(t) = 0$$

Pick k_d & k_p so that $\boxed{e(t) \to 0}$ as $t \to \infty$

$\Rightarrow$ stable system

$$\ddot{x}_d(t) - \ddot{x}(t) + k_d(\dot{x}_d - \dot{x}) + k_p(x_d - x) = 0$$

$$\Rightarrow \boxed{\begin{aligned} \ddot{x}(t) &= \ddot{x}_d(t) + k_d(\dot{x}_d - \dot{x}) + k_p(x_d - x) \\ \ddot{y}(t) &= \ddot{y}_d(t) + k_d(\dot{y}_d - \dot{y}) + k_p(y_d - y) \end{aligned}}$$

SS

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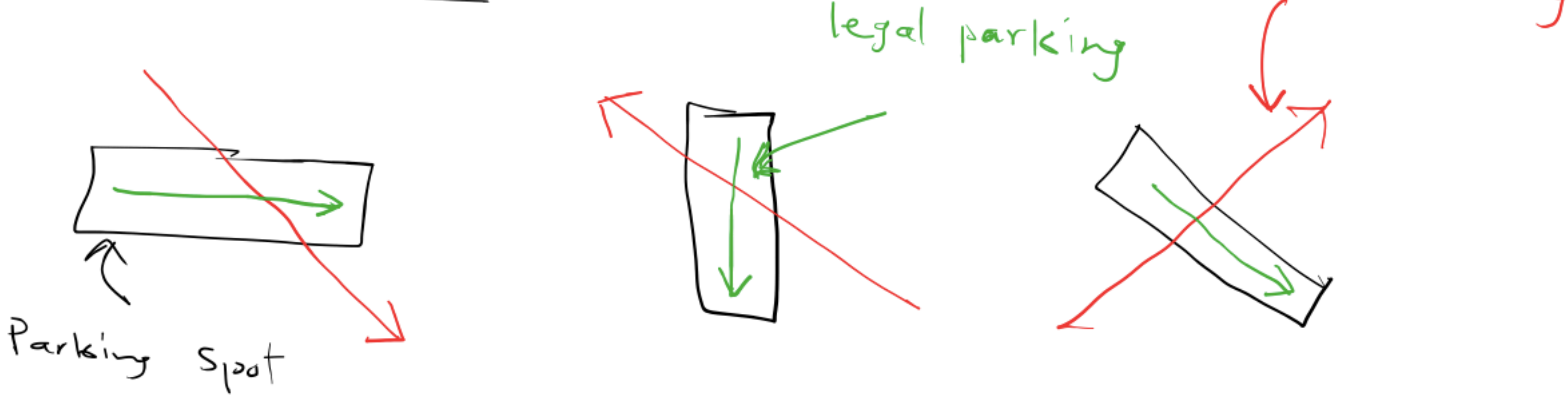
Pose stabilization
Pose regulation

$\Rightarrow$ Solve using Lyapunov Stability Method

↳ orientation (θ variable)

Example: Parking Problem



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Lyapunov stability method

Given a function $\dot{x} = f(x, u)$, the system is stable if there exists a function $V(x, u)$ such that

Lyapunov function

1. $V(0, 0) = 0$ for $x = 0$, $u = 0$
2. $V(x, u) > 0$ for $x \neq 0$, $u \neq 0$
3. $\dot{V}(x, u) < 0$ for $x = 0$, $u = 0$

Stable: $x(t) \to 0$ as $t \to \infty$

Unstable: $x(t) \to \infty$ as $t \to \infty$