Sec-10 quiz-1

1. Find intercepts and Symmetry of $x^4+y^2-xy=81$

Soln: To find x-intercept, let $y=0$

① $x^4=81$

$\therefore x=\pm 3$

$\therefore$ The x-intercepts are +3 and -3.

To find y-intercept, let $x=0$

① $\therefore y^2=81$

$\Rightarrow y=\pm 9$

Therefore, y-intercepts are +9 and -9.

Thus (-3,0), (3,0), (0,-9) and (0, 9) are the intercepts.

symmetry check:

① x-axis → Replace ~~x~~y by -y

$$x^4+(-y)^2-x(-y)-81=0$$

$\Rightarrow x^4+y^2+xy-81=0$ not same as the given eqn, so not symmetric w.r.t x-axis.

① y-axis → Replace x by -x

$$(-x)^4+y^2-(-x)y-81=0$$

$\Rightarrow x^4+y^2+xy-81=0$, not same as given eqn, so not symmetric w.r.t y-axis.

① Origin: Replace x by $-x$, y by $-y$

$$(-x)^4 + (-y)^2 - (-x)(-y) - 81 = 0$$

$\Rightarrow x^4 + y^2 - xy - 81 = 0$, same as the given eq^n, so symmetric w.r.t origin.

b. The point $(5, b)$ is on the graph of $3x+2y=1$.

$\therefore \quad 15 + 2b = 1$

$\Rightarrow \quad 2b = 1 - 15 = -14$

$\Rightarrow \quad b = \frac{-14}{2} = -7$ ②

$\therefore \quad b = -7.$

2. Any point on y-axis is $(0, b)$ 0.5, which is in the same distance from $(6, -6)$ and $(2, 2)$.

$\therefore \sqrt{(6-0)^2 + (-6-b)^2} = \sqrt{(2-0)^2 + (2-b)^2}$ — ①

$\Rightarrow \quad 36 + (6+b)^2 = 4 + (2-b)^2$

$\Rightarrow \quad 36 + 36 + 12b + \cancel{b^2} = 4 + 4 - 4b + \cancel{b^2}$

$\Rightarrow \quad 12b + 72 = -4b + 8$

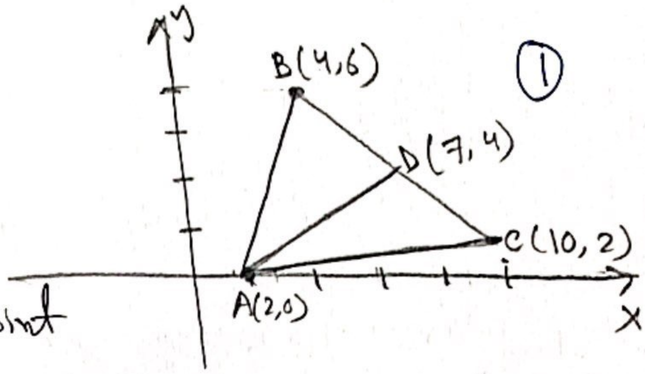
$\Rightarrow \quad 12b + 4b = 8 - 72$

$\Rightarrow \quad 16b = -64$

$\therefore \quad b = -4$ — 0.5

Therefore the point on y-axis is $(0, -4)$.

3. The vertices of the triangle are A(2,0), B(4,6) and C(10,2)



①

we have to find the length of AD.

The point D is the midpoint of the line BC.

$\therefore D(x,y) = \left(\frac{4+10}{2}, \frac{6+2}{2}\right) = \left(\frac{14}{2}, \frac{8}{2}\right)$

$= (7, 4)$ ②

$\therefore$ The length of AD $= \sqrt{(7-2)^2 + (4-0)^2}$

$= \sqrt{(5)^2 + 4^2} = \sqrt{25+16}$

$= \sqrt{41}$ ②.

$\therefore$ The length of AD is $\sqrt{41}$ units.