NORTH SOUTH UNIVERSITY

DEPARTMENT OF MATHEMATICS & PHYSICS

SUMMER 2019

ASSIGNMENT # 2

INTRODUCTION TO LINEAR ALGEBRA

MAT 125 SECTION 07, 08

DUE DATE: DECEMBER 10, 2019

Submitted by:

Name:
ID#:
Serial #:
Section:
E-Mail:
Department:
Date of Submission:

Submitted to:

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Number of problems given in the assignment: 15

Number of solved problems:

N.B.:

1. Please use **A4** size papers and add this sheet as a cover page
2. Assignment will not be **accepted** after the due date
3. Your score will be **zero** for any copy or plagiarism

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SHOW ALL THE WORK.

1. Find the scalar product, norms, and distance between u and v where
   (i) $u = (-1,1,0,4,-3)$ and $v = (-2,-2,0,2,-1)$
   (ii) $u = (2,1,-3,0,4)$ and $v = (5,-3,-1,2,7)$
   (iii) $u = (-4,6,-5,1)$ and $u = (2,1,-2,8)$
   Also verify Cauchy-Schwartz, Minkowski's (triangle) inequality and Pythagorean Theorem.
2. Let W is a subset of the vector space V. Determine whether or not W is a subspace of V.
   (i) $W = \{(a,b,c)|a \geq b\}$
   (ii) $W = \{(a,b,c,d)|2a - 3b + 5c - d = 0\}$
   (iii) $W = \{(a,b,c)|a + b = 0\}$
3. Determine with proof, whether $W = \left\{\begin{bmatrix} a & b \\ 0 & c \end{bmatrix} \middle| a - c = 1 \text{ } and \text{ } a,b,c \in \boldsymbol{R}\right\}$ is a subspace of M_{22}.
4. Determine whether $W = \{a_0 + a_1x + a_2x^2 + a_3x^3 | a_0 + a_2 = a_3 \, a_1\}$ is a subspace of P_3.
5. Find conditions on a, b, c so that $v = (a,b,c)$ in$\boldsymbol{R^3}$ belongs to $W = span(u_1, u_2, u_3)$, where $u_1 = (1,2,-3)$, $u_2 = (2,6,-11)$, $u_3 = (2,-4,14)$.
6. Write the vector $(1,2,6)$ as a linear combination of $(2,1,0), (1,-1,2)$ and $(0,3,-4)$. Verify your answer.
7. Write the vector $(3,9,-4,-2)$ as a linear combination of $(1,-2,0,3), (2,3,0,-1)$, and $(2,-1,2,1)$.Verify your answer.
8. Write the polynomial $p(t) = 2t^2 - 3t + 1$ in P_2 as a linear combination of the polynomials $p_1(t) = (t-1)^2$, $p_2(t) = (t-1)$and $p_3(t) = 3$ if possible.
9. Express the matrix $A = \begin{pmatrix} 5 & 1 \\ -2 & 3 \end{pmatrix}$ as a linear combination of the matrices $B = \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix}$, $C = \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}$ and $D = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$. Verify your answer.
10. Determine whether or not the following vectors span $\boldsymbol{R^3}$.
    (i) $u_1 = (1,1,2), u_2 = (1,-1,2), u_3 = (1,0,1)$
    (ii) $u_1 = (-1,1,0), u_2 = (-1,0,1), u_3 = (1,1,1)$
11. Show that the polynomials $(1-t)^3, (1-t)^2, (1-t), and$ 1 generates the space of polynomials of degree $\leq$ 3.
12. Find the conditions on $a, b, and\ c$ so that $(a,b,c) \in \boldsymbol{R^3}$ belongs to the space generated by $u = (2,1,0), v = (1,-1,2)$, and $w = (0,3,-4)$.
13. Show that the yz plane $W = \{(0,b,c)|b,c \in \boldsymbol{R}\}$ in $\boldsymbol{R^3}$ is generated by $u = (0,1,2), v = (0,2,3)$, and $w = (0,3,1)$
14. Determine whether the vectors $u = (1,-3,7), v = (3,-1,-1)$, and $w = (2,4,-5)$ in R^3 are linearly independent or dependent.
15. Let V be the vector space of 2×2 matrices over R. Determine whether the matrices $A, B, C \in V$ are dependent or independent where:
    $A = \begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix}, B = \begin{bmatrix} 3 & -1 \\ 2 & 2 \end{bmatrix}, C = \begin{bmatrix} 1 & -5 \\ -4 & 0 \end{bmatrix}$
16. Let V be the vector space of polynomials of degree $\leq$3 over R. Determine whether $u, v, w \in V$ are independent or dependent where:
    $u = t^3 + 4t^2 - 2t + 3, v = t^3 + 6t^2 - t + 4, w = 3t^3 + 8t^2 - 8t + 7$
17. Let V be the vector space of functions from **R** into **R.** Show that $f, g, h \in V$ are independent where:
    $f(x) = e^{2x}$, $g(x) = x^2$ and $h(x) = x$
18. Use Wronskian to show that the following functions are linearly independent $f(t) = e^t$, $g(t) = cost$ and $h(t) = \sin t$.