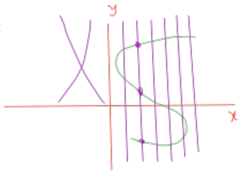
Vertical Line Test



$(-\infty, 0) \cup (0, \infty)$

Find the domain and range of the function $f(x) = \frac{1}{x-2}$

∞

$R_f = \mathbb{R} - \{0\}$

$\mathbb{R}$

$\frac{1}{x-2} \longrightarrow \infty$

$\propto$ ∞

$(-\infty, \infty)$

$\mathbb{R}$

$x-2=0$

$\Rightarrow x=2$

$D_f = \mathbb{R} - \{2\}$

$\frac{1}{x-2}$ $\neq 0$??

$x^2 - 7$

$f(x) = x^2 - 7$

$x^2 - 7 \rightarrow -ve$ if $x^2 < 7$

$D_f = \mathbb{R}$

$R_f = [-7, \infty)$

$f(x)$

$R_f = [-7, \infty)$

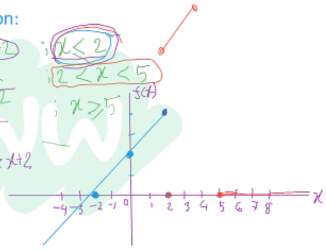
$0 < x^2 < 7$

Piecewise defined function:

$$f(x) = \begin{cases} x+2 & ; x < 2 \\ 5x & ; 2 < x < 5 \\ x^2 & ; x \geq 5 \end{cases}$$

$x \leq 2 \rightarrow f(x) = x+2$

$2 < x < 5 \rightarrow f(x) = 5x$



Operations on functions:

1. $f(x) + g(x) = (f+g)(x)$
2. $f(x) - g(x) = (f-g)(x)$
3. $f(x)\,g(x) = fg\,(x)$
4. $\frac{f(x)}{g(x)} = \frac{f}{g}(x)$ $[g(x) \neq 0]$

$2 + 3 = 5$

$9 \times 2 = 18$

$f(x) = 1 + \sqrt{x-4}$ $\quad g(x) = x+2$

$(f+g)(x) = 1 + \sqrt{x-4} + x + 2 = 3 + x + \sqrt{x-4}$

$(fg)(x) = (1+\sqrt{x-4})(x+2)$

$= x + x\sqrt{x-4} + 2 + 2\sqrt{x-4}$

Composition of functions:

$g(x)$

g

$$\begin{cases} f \circ g(x) = f(g(x)) \\ g \circ f(x) = g(f(x)) \end{cases}$$

$g(x)$ $\quad f(g(x))$

$f(x) = x+2$ $\quad g(x) = \sqrt{x}$

$f \circ g(x) = f(g(x)) = f(\sqrt{x})$ $\quad D_f = \mathbb{R}$

$= \sqrt{x} + 2$ $\quad R_f = \mathbb{R}$

$\mathbb{R}$ $\quad [0, \infty)$

$g \circ f(x) = g(f(x)) = g(x+2) = \sqrt{x+2}$

$D_{g \circ f} = [-2, \infty)$

$x + 2$

$x + 2 \geq 0$

$\Rightarrow x \geq -2$