Exercise 6.1

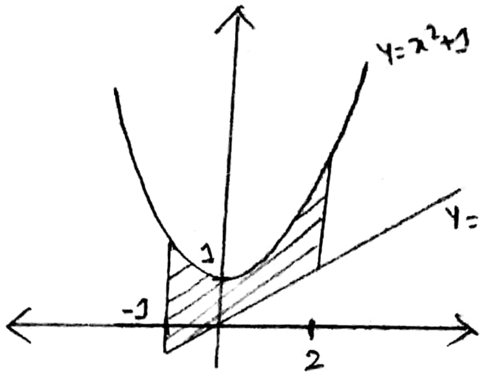
1.

$$A = \int_{-1}^{2} [f(x) - g(x)]\,dx$$

$$= \int_{-1}^{2} (x^2+1-x)\,dx$$

$$= \left[\frac{x^3}{3} + x - \frac{x^2}{2}\right]_{-1}^{2}$$

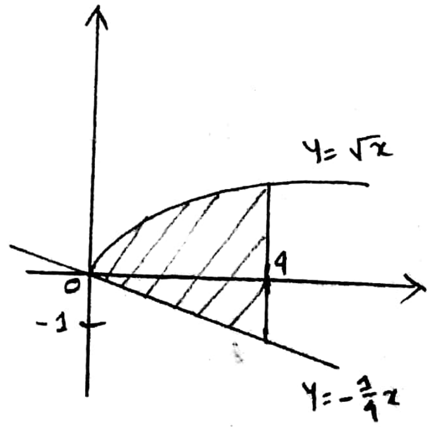
$$= \frac{9}{2}$$



2. $A = \int_0^4 \left[\sqrt{x} - \left(-\frac{x}{4}\right)\right] dx$

$$= \left[\frac{2x^{3/2}}{3} + \frac{x^2}{8}\right]_0^4$$

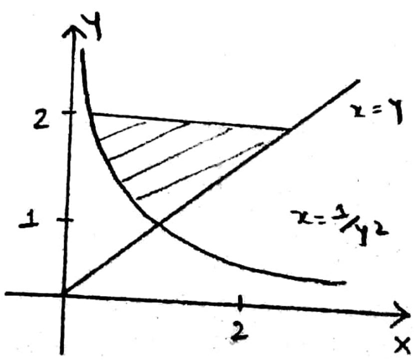
$$= \frac{22}{3}$$



3. $A = \int_1^2 \left(y - \frac{1}{y^2}\right) dy$

$$= \left[\frac{y^2}{2} + \frac{1}{y}\right]_1^2$$

$$= 1$$



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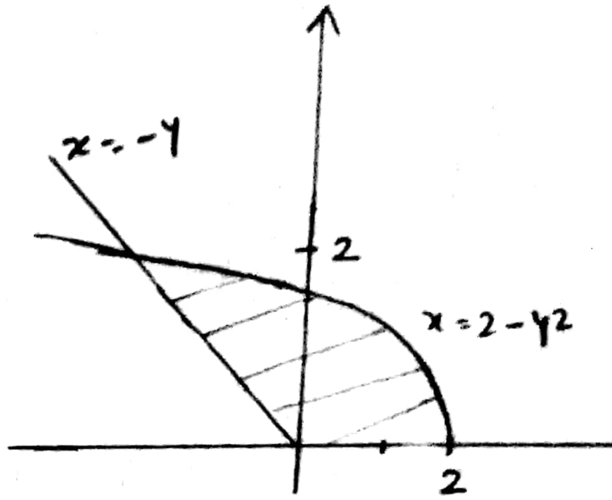
4. $A = \int_0^2 \left[(2-y^2)-(-y)\right] dy$

$= \left[2y - \frac{y^3}{3} + \frac{y^2}{2}\right]_0^2$

$= \frac{10}{3}$

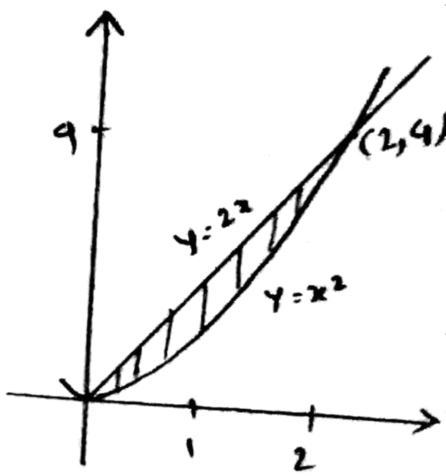


5.

(a) integrating aith respecting to x,

$A = \int_0^2 (2x - x^2)\, dx$

$= \left[x^2 - \frac{x^3}{3}\right]_0^2 = \frac{4}{3}$



(b) Integrating with respect to y,

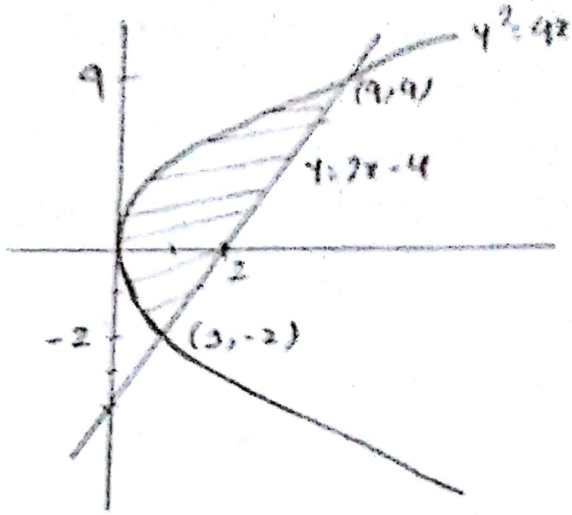
$A = \int_0^4 \left(\sqrt{y} - \frac{y}{2}\right) dy$

$= \left[\frac{2y^{3/2}}{3} - \frac{y^2}{4}\right]_0^4$

$= \frac{4}{3}$

6.

(a) Integrating with respect to x



$$A = \int_0^1 \left[(2\sqrt{x}) - (-2\sqrt{x})\right] dx \; + \int_1^4 \left[2\sqrt{x} - (2x-4)\right] dx$$

$$= \int_0^1 4\sqrt{x}\, dx + \int_1^4 (2\sqrt{x} - 2x + 4)\, dx = \frac{8}{3} + \frac{19}{3} = 9$$

(b) Integrating with respect to y

$$A = \int_{-2}^{4} \left[\left(\frac{y}{2} + 2\right) - y^2/4\right] dy$$

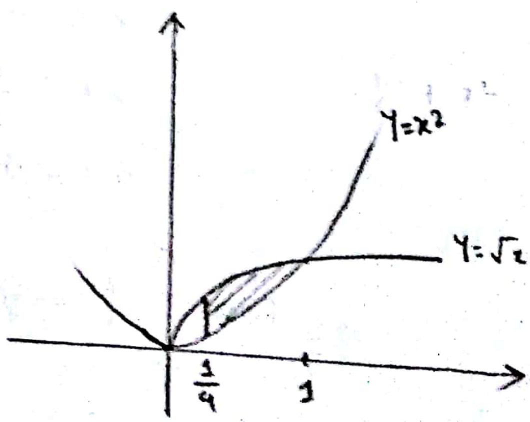
$$= \left[\frac{y^2}{4} + 2y - \frac{y^3}{12}\right]_{-2}^{4}$$

$$= 9$$

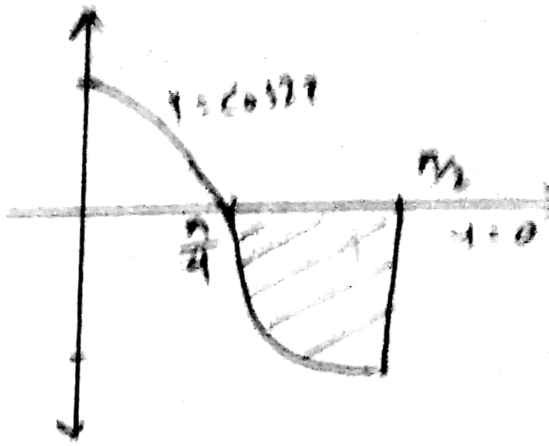
7. $A = \int_{\frac{1}{4}}^{1} (\sqrt{x} - x^2)\, dx$

$$= \left[\frac{2x^{3/2}}{3} - \frac{x^3}{3}\right]_{\frac{1}{4}}^{1}$$

$$= \frac{49}{192}$$



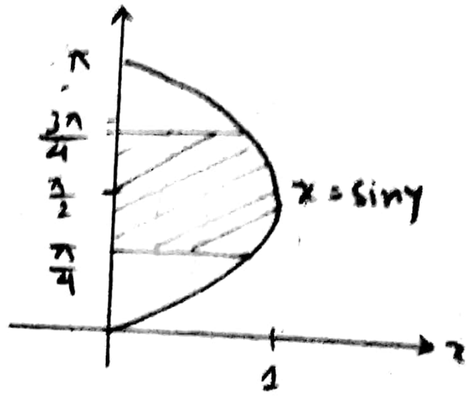
9. $y = \cos 2x$, $y = 0$, $x = \frac{\pi}{4}$, $x = \frac{\pi}{2}$



$$A = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (0 - \cos 2x)\, dx$$

$$= \left[-\frac{\sin 2x}{2} \right]_{\pi/4}^{\pi/2} = \frac{1}{2}$$

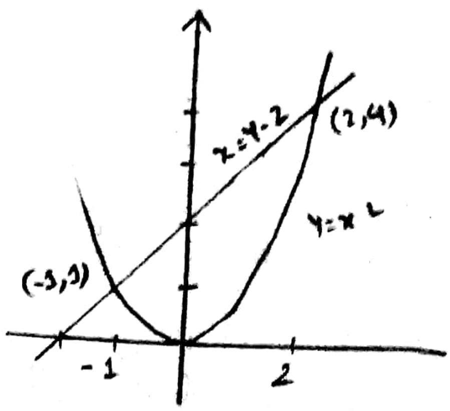
11. $x = \sin y$, $x = 0$, $y = \frac{\pi}{4}$, $y = \frac{3\pi}{4}$



$$A = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} [\sin y - 0]\, dy$$

$$= \left[-\cos y \right]_{\frac{\pi}{4}}^{\frac{3\pi}{4}} = \sqrt{2}$$

12. $x^2 = y$, $x = y - 2$



$$x^2 = x + 2 \Rightarrow x^2 - x - 2 = 0$$

$$\Rightarrow x^2 - 2x + x - 2 = 0$$

$$\Rightarrow (x-2)(x+1) = 0$$

$$\Rightarrow x = -1, 2$$

So, $A = \int_{-1}^{2} [(x+2) - x^2]\, dx = \left[\frac{x^2}{2} + 2x - \frac{x^3}{3} \right]_{-1}^{2}$

$$= \frac{9}{2}$$

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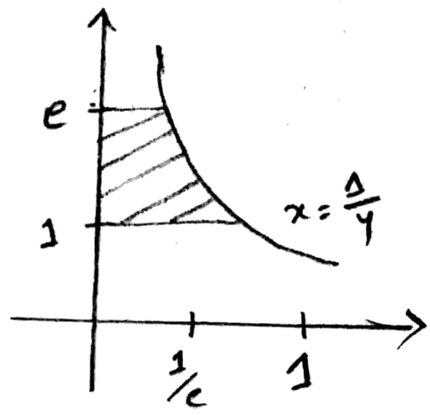
14. $x = \frac{1}{y}$, $x = 0$, $y = 1$, $y = e$

$$A = \int_1^e \left[\frac{1}{y} - 0 \right] dy$$

$$= \left[\ln y \right]_1^e = 1$$

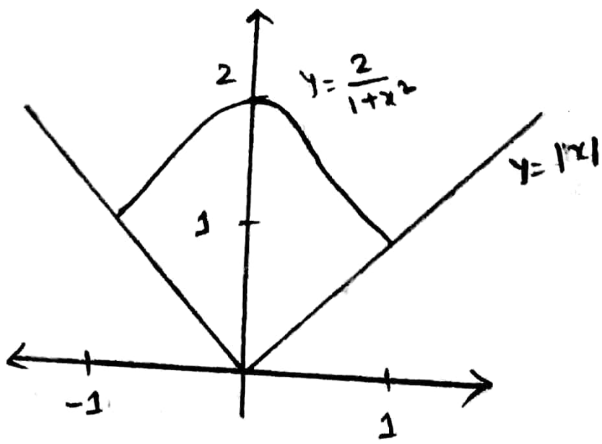


15. $y = \frac{2}{1+x^2}$, $y = |x|$

$$|x| = \frac{2}{1+x^2}$$

$$x = \frac{2}{1+x^2} .$$

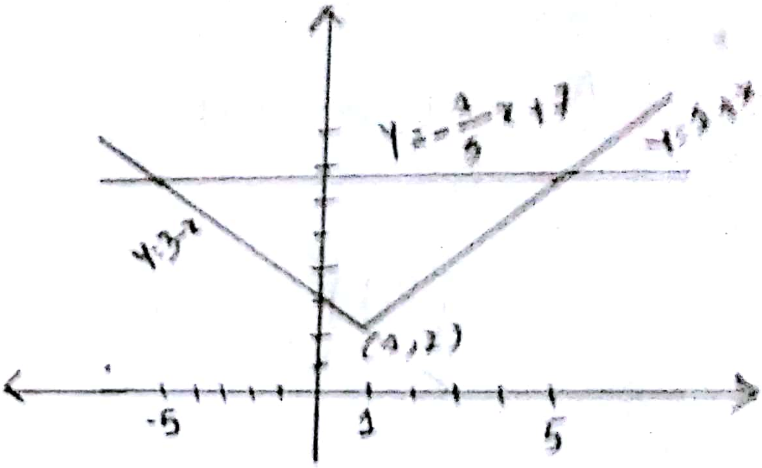
$$\Rightarrow x + x^3 - 2 = 0$$



17. $y = 2+|x-1|$, $y = -\frac{1}{5}x+7$

$y = 2+|x-1|$

$= \begin{cases} 1+x, & x \ge 1 \\ 3-x, & x < 1 \end{cases}$



$y = -\frac{1}{5}x+7$

$\Rightarrow \frac{y}{7} + \frac{x}{35} = 1$

$1+x = 3-x \Rightarrow x+x = 3-1 \Rightarrow x = 1$

$1+x = -\frac{1}{5}x+7$

$\Rightarrow 5+5x = -x+35$

$\Rightarrow 5x+x = 35-5$

$\Rightarrow 6x = 30$

$\Rightarrow x = 5$

$3-x = -\frac{1}{5}x+7$

$\Rightarrow 15-5x = -x+35$

$\Rightarrow -5x+x = 35-15$

$\Rightarrow -4x = 20$

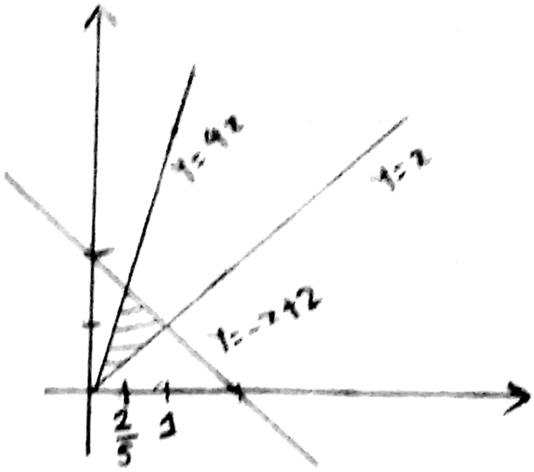
$\Rightarrow x = -5$

$A = \int_{-5}^{1}\left[\left(-\frac{1}{5}x+7\right)-(3-x)\right]dx + \int_{1}^{5}\left[\left(-\frac{1}{5}x+7\right)-(1+x)\right]dx$

$= \int_{-5}^{1}\left(\frac{4}{5}x+4\right)dx + \int_{1}^{5}\left(6-\frac{6}{5}x\right)dx$

$= \left[\frac{4x^2}{10}+4x\right]_{-5}^{1} + \left[6x-\frac{6x^2}{10}\right]_{1}^{5} = \frac{72}{5} + \frac{48}{5} = 24$

18. $y = x$, $y = 9x$, $y = -x+2$



For bounded area between

$y = 4x$ and $y = x$ is A_1.

and for bounded area between

$y = -x+2$ and $y = x$ is A_2.

For A_1, $4x = -x+2 \Rightarrow 5x = 2 \Rightarrow x = \frac{2}{5}$

So, $A_1 = \int_0^{\frac{2}{5}} [4x - x]\,dx = \int_0^{\frac{2}{5}} 3x\,dx = \left[\frac{3x^2}{2}\right]_0^{2/5}$

for A_2, $-x+2 = x \Rightarrow 2x = 2 \Rightarrow x = 1$

So, $A_2 = \int_{\frac{2}{5}}^{1} [(-x+2) - x]\,dx = \int_{\frac{2}{5}}^{1} (-2x+2)\,dx = \left[-\frac{2x^2}{2} + 2x\right]_{2/5}^{1}$

So, $A = A_1 + A_2 = \frac{3}{5}$