Exercise 6.5

1. $y = 7x \quad , \quad 0 \leq x \leq 1$ revolve about x axis

$f(x) = 7x$

$\Rightarrow f'(x) = 7 \quad \Rightarrow 1 + [f'(x)^2] = 49 + 1 = 50$

So, $S = \int_0^1 2\pi f(x) \sqrt{1 + [f'(x)]^2}\, dx$

$= \int_0^1 2\pi \times 7x \times \sqrt{50}\, dx \quad = 2\pi\sqrt{50} \times 7 \times \frac{1}{2} [x^2]_0^1 = 155.5$

2. $y = \sqrt{x} \quad , \quad 1 \leq x \leq 4$ revolve about x axis

$f(x) = x^{\frac{1}{2}}$

$\Rightarrow f'(x) = \frac{1}{2} x^{-\frac{1}{2}}$

$\Rightarrow 1 + [f'(x)]^2 = 1 + \frac{1}{4x} = \frac{4x+1}{4x}$

$S = \int_1^4 2\pi f(x) \sqrt{1 + [f'(x)]^2}\, dx$

$= \int_1^4 2\pi \times x^{1/2} \sqrt{\frac{4x+1}{4x}}\, dx$

$= \int_1^4 2\sqrt{x} \times \pi \times \frac{\sqrt{4x+1}}{2\sqrt{x}}\, dx$

$$= \int_1^4 \pi \sqrt{4x+1}\, dx$$

Put, $4x+1 = u \Rightarrow 4dx = du \Rightarrow dx = \frac{du}{4}$

$x = 4,\ u = 17 \qquad x = 1,\ u = 5$

So, $S = \int_5^{17} \frac{1}{4}\pi u^{1/2}\, du$

$$= \pi \times \frac{2}{3}\left[u^{3/2}\right]_5^{17} \times \frac{1}{4}$$

$$= 30.85$$

3. $y = \sqrt{4-x^2}$, $-1 \le x \le 1$ Revolve about x axis

$f(x) = \sqrt{4-x^2}$

$\Rightarrow f'(x) = \frac{1}{2} \times (4-x^2)^{-\frac{1}{2}} \times -2x = \frac{-x}{\sqrt{4-x^2}}$

$\Rightarrow 1 + [f'(x)]^2 = 1 + \frac{x^2}{4-x^2} = \frac{4-x^2+x^2}{4-x^2} = \frac{4}{4-x^2}$

So, $L = \int_1^{-1} 2\pi f(x) \sqrt{1+[f'(x)]^2}\, dx$

$$= \int_1^{-1} 2\pi \times \sqrt{4-x^2} \times \sqrt{\frac{4}{4-x^2}}\, dx$$

$$= \int_1^{-1} 4\pi\, dx = 4\pi [x]_1^{-1} = 8\pi$$

4. $x = \sqrt[3]{y}$, $1 \le y \le 8$ revolve about x axis

$x^3 = \sqrt[3]{y}$ so, $y = f(x) = x^3$

$y = 1$, $x^3 = 1$ $\Rightarrow x = 1$

$y = 8$, $x = \sqrt[3]{8} = 2$

$f(x) = x^3$ $\Rightarrow f'(x) = 3x^2$

$\Rightarrow 1 + [f'(x)]^2 = 1 + 9x^4$

So, $S = \int_1^2 2\pi f(x) \sqrt{1 + [f'(x)]^2} \, dx$

$= \int_1^2 2\pi \times x^3 \times \sqrt{1 + 9x^4} \, dx$

Put, $1 + 9x^4 = u$ $\Rightarrow 36x^3 \, dx = du$ $\Rightarrow 2x^3 \, dx = \frac{du}{18}$

$x = 1$, $u = 1 + 9 = 10$ $x = 2$, $u = 1 + 2^4 \times 9 = 145$

So, $S = \int_2^{145} \pi \frac{1}{18} u^{1/2} \, du = \frac{\pi}{18} \times \frac{2}{3} \left[u^{3/2} \right]^{145}$

$= 199.48$

5. $x = 9y+1, \quad 0 \le y \le 2$ revolve about y axis

$g(y) = 9y+1$

$\Rightarrow g'(y) = 9$

$\Rightarrow 1+[g'(y)]^2 = 1+81 = 82$

$S = \int_0^2 2\pi \, g(y) \sqrt{1+[g'(y)]^2} \, dy$

$= \int_0^2 2\pi \times (9y+1) \times \sqrt{82} \, dy$

$= 2\pi\sqrt{82} \left[9 \times \frac{y^2}{2} + y \right]_0^2 = 1137.93$

6. $x = y^3, \quad 0 \le y \le 1$ revolve about y axis

$g(y) = y^3$

$\Rightarrow g'(y) = 3y^2$

$\Rightarrow 1+[g'(y)]^2 = 1+9y^4$

$S = \int_0^1 2\pi \, g(y) \sqrt{1+[g'(y)]^2} \, dy$

$= \int_0^1 2\pi \times y^3 \times \sqrt{1+9y^4} \, dy$

Put, $1+9y^4 = u \Rightarrow 36y^3\,dy = du \Rightarrow 2y^3\,dy = \frac{du}{18}$

$y=0,\ u = \cancel{0}1 \qquad y=1,\ u = 1+9 = 10$

So, $S = \int_1^{10} \pi \times \frac{1}{18} \times u^{1/2}\,du$

$= \frac{\pi}{18} \times \frac{2}{3} \times \left[u^{3/2}\right]_1^{10}$

$= 3.56$

7. $x = \sqrt{9-y^2}$, $-2 \le y \le 2$ revolve about y axis

$g(y) = \sqrt{9-y^2}$

$\Rightarrow g'(y) = \frac{1}{2}(9-y^2)^{-1/2} \times 2y = \frac{y}{\sqrt{9-y^2}}$

$\Rightarrow 1+[g'(y)]^2 = 1 + \frac{y^2}{9-y^2} = \frac{9}{9-y^2}$

So, $S = \int_{-2}^{2} 2\pi\, g(y) \sqrt{1+[g'(y)]^2}\,dy$

$= \int_{-2}^{2} 2\pi \times \sqrt{9-y^2} \times \frac{3}{\sqrt{9-y^2}}\,dy$

$= 6\pi\,[y]_{-2}^{2} = 24\pi$

8. $x = 2\sqrt{1-y}$, $-1 \le y \le 0$ revolve about Y axis

$$g(y) = 2\sqrt{1-y}$$

$$\Rightarrow g'(y) = 2 \times \frac{1}{2} \times (1-y)^{-\frac{1}{2}} \times (-1) = -(1-y)^{-1/2}$$

$$\Rightarrow 1 + [g'(y)]^2 = 1 + \frac{1}{1-y} = \frac{2-y}{1-y}$$

So, $$S = \int_{-1}^{0} 2\pi g(y) \sqrt{1+[g'(y)]^2}\, dy$$

$$= \int_{-1}^{0} 2\pi \times 2\sqrt{1-y} \times \frac{\sqrt{2-y}}{\sqrt{1-y}}\, dy$$

$$= 4\pi \int_{-1}^{0} \sqrt{2-y}\, dy$$

Put, $2-y = u \Rightarrow -dy = du \Rightarrow -du = dy$

$y=0, u=2 \qquad y=-1, u=3$

So, $$S = 4\pi \int_{3}^{2} -u^{1/2}\, du$$

$$= 4\pi \times \frac{2}{3}\left[-u^{3/2}\right]_3^2 = 19.84$$

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