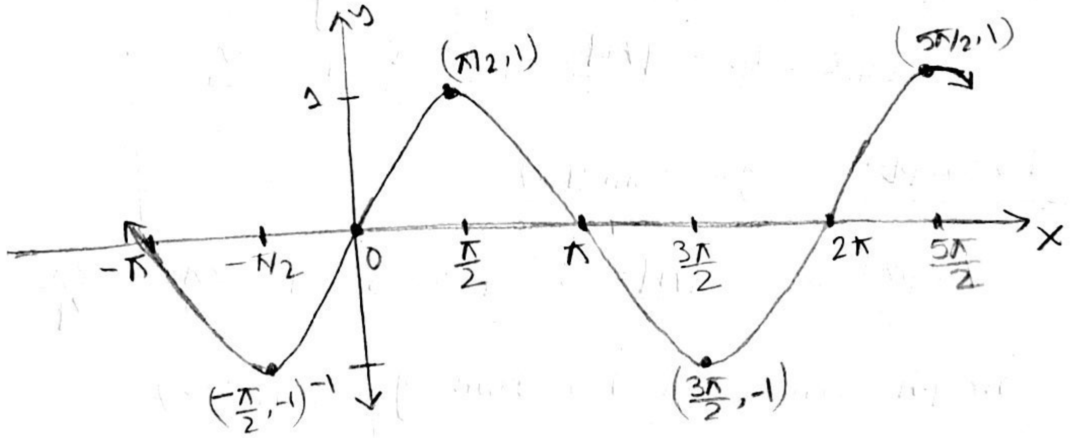
Mat - 116

Graph of Sine and Cosine functions

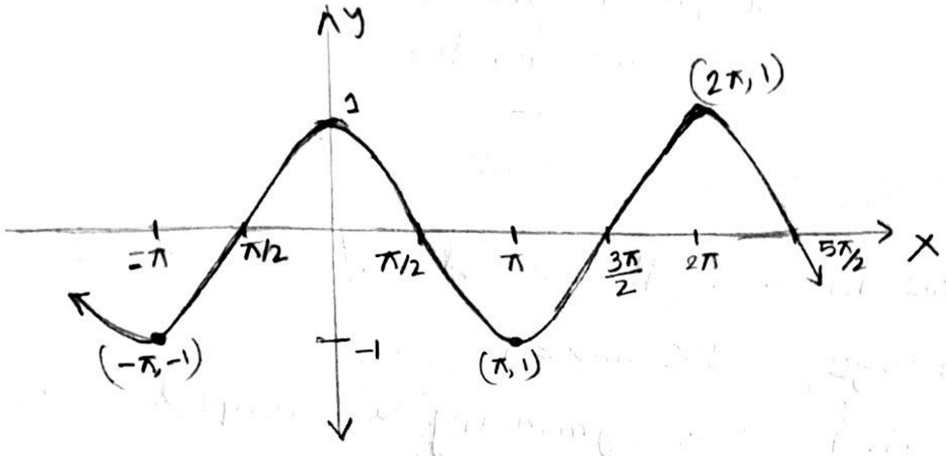
Properties of Sine functions:

1. The domain is the set of all real numbers
2. Range $-1 \le \sin x \le 1$
3. Odd function, symmetry with respect to origin.
4. periodic with period 2π.
5. x-intercept : $\dots -2\pi, -\pi, 0, \pi, 2\pi, 3\pi$; y-intercept $= 0$
6. Absolute max is 1 and occurs at $x = \dots -\frac{3\pi}{2}, \frac{\pi}{2}, \frac{5\pi}{2}, \frac{9\pi}{2} \dots$
7. Absolute min is -1 " occurs at $x = \dots -\frac{\pi}{2}, \frac{3\pi}{2}, \frac{7\pi}{2}, \frac{11\pi}{2} \dots$



Properties of cosine functions:

1. The domain is the set of all real numbers.
2. Range $-1 \le \cos x \le 1$
3. Even function, symmetry with respect to y axis.
4. periodic function with period 2π.
5. x-intercept : $\dots -\frac{3\pi}{2}, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2} \dots$ y intercept is 1.
6. Absolute max is 1 and occurs at $x = \dots -2\pi, 0, 2\pi, 4\pi \dots$
7. Absolute min is -1 and occurs at $x = \dots -\pi, \pi, 3\pi, 5\pi \dots$



$y = \cos x$

Determine Amplitude and period of Sinusoidal functions:

If $\omega > 0$, the amplitude and period of $y = A\sin(\omega x)$ and $y = A\cos(\omega x)$ are given by

$$\text{Amplitude} = |A|, \quad \text{period} = T = \frac{2\pi}{\omega}$$

Example: $y = 3\sin(4x)$

$$\text{Amplitude} = |A| = 3 \quad \text{Period} = T = \frac{2\pi}{\omega} = \frac{2\pi}{4} = \frac{\pi}{2}$$

Graph sinusoidal function: $y = 3\sin(4x)$

Step 1: Find Amplitude and period.

$$A = 3, \quad T = \frac{\pi}{2}$$

Step 2: Divide the interval $\left[0, \frac{2\pi}{\omega}\right]$ into four subintervals of the same length

So divide $\left[0, \frac{\pi}{2}\right]$ into four subintervals, each of length $\frac{\pi}{2} \div 4 = \frac{\pi}{8}$ as follows:

$$\left[0, \frac{\pi}{8}\right], \left[\frac{\pi}{8}, \frac{\pi}{4}\right], \left[\frac{\pi}{4}, \frac{3\pi}{8}\right], \left[\frac{3\pi}{8}, \frac{\pi}{2}\right]$$

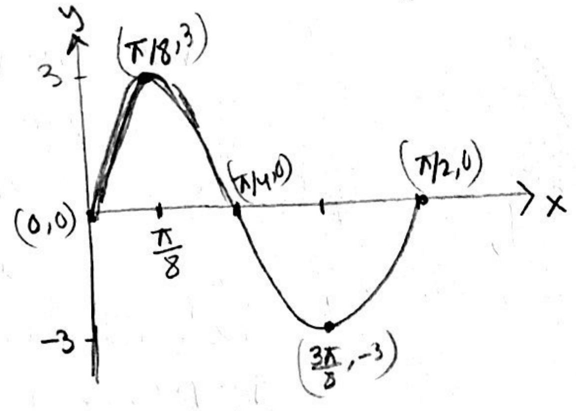
The x-coordinates of the points on the graph are $0, \frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}$

Step:3

To obtain y-coordinates of the points on the graph of $y = 3\sin(4x)$, multiply the y-coordinates of $y = \sin x$ by $A = 3$.

$\therefore$ The points are

$$(0,0), \left(\frac{\pi}{8}, 3\right), \left(\frac{\pi}{4}, 0\right), \left(\frac{3\pi}{8}, -3\right), \left(\frac{\pi}{2}, 0\right)$$



Ex: $y = 2\sin\left(-\frac{\pi}{2}x\right)$

Soln: Since the sine function is odd, then $y = -2\sin\left(\frac{\pi}{2}x\right)$

Amplitude, $|A| = |-2| = 2$ period, $T = \frac{2\pi}{\omega} = \frac{2\pi}{\frac{\pi}{2}} = 4$

Divide $[0,4]$ into four subintervals, each of length $4 \div 4 = 1$.

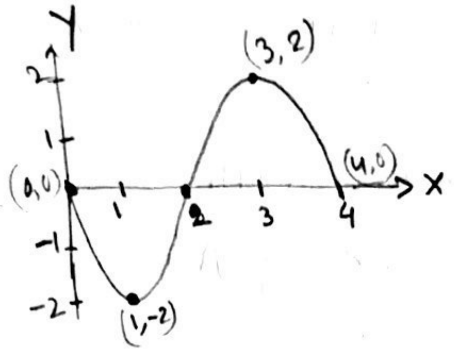
$[0,1], [1,2], [2,3], [3,4]$.

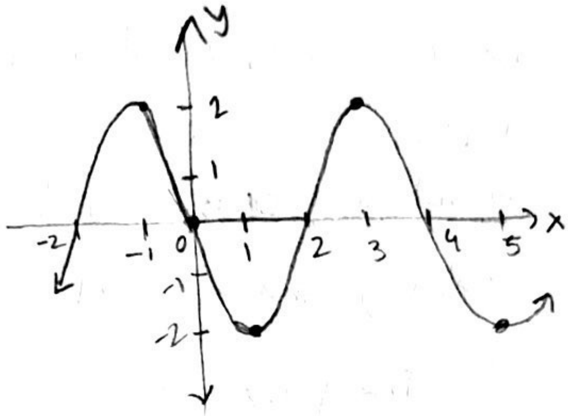
$\therefore$ x-coordinates $0, 1, 2, 3, 4$.

y-coordinates: multiply the y-coordinates of $y = \sin x$ by $A = -2$

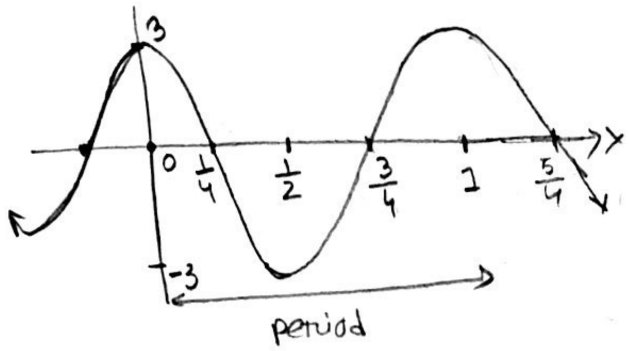
$\therefore$ The points are.

$$(0,0), (1,-2), (2,0), (3,2), (4,0)$$





Ex: Find an eq^n for the graph shown below:



Here, Amplitude = 3

Period = 1

i.e $\frac{2\pi}{\omega} = 1$

$\Rightarrow \omega = 2\pi$

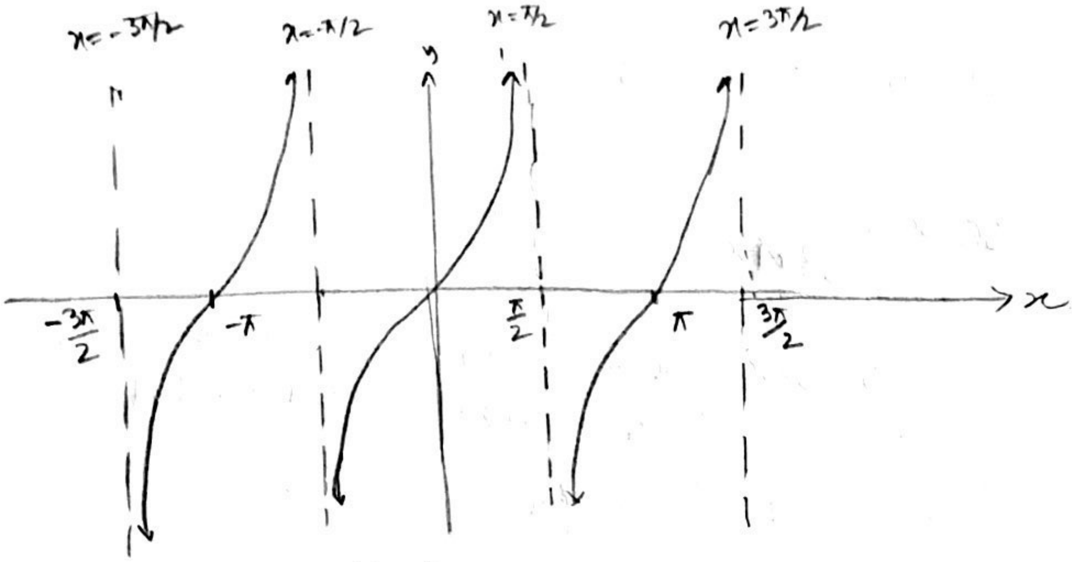
The function is of the form ~~y = Acos~~

$y = A\cos(\omega x)$

$\Rightarrow y = 3\cos(2\pi x)$

Properties of tangent functions:

1. Domain is the set of all real numbers except odd multiples of $\pi/2$
2. Range is $(-\infty, \infty)$
3. tangent function is odd function, so symmetric with respect to origin.
4. tangent function is periodic with period π.
5. x-intercepts are $-2\pi, -\pi, 0, \pi, 2\pi, 3\pi,$
   y-intercept is 0.
6. vertical asymptotes occur at $x = \ldots -\frac{3\pi}{2}, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2} \ldots$



$y = \tan x$

Ex: Draw $y = 2\tan x - 1$

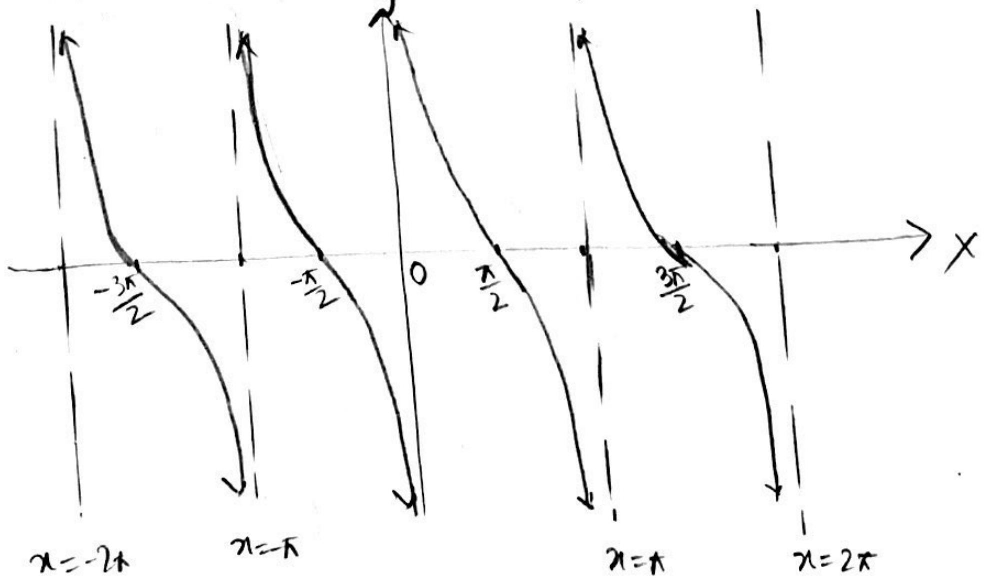
The domain of $y = 2\tan x - 1$ is $\{x \mid x \neq \frac{k\pi}{2}$, k is an odd integer$\}$

Range $= (-\infty, \infty)$

vertical strech by factor of 2

Vertical shift down by 1 unit.

Graph of $\cot\theta$: $\cot\theta = \frac{1}{\tan\theta}$

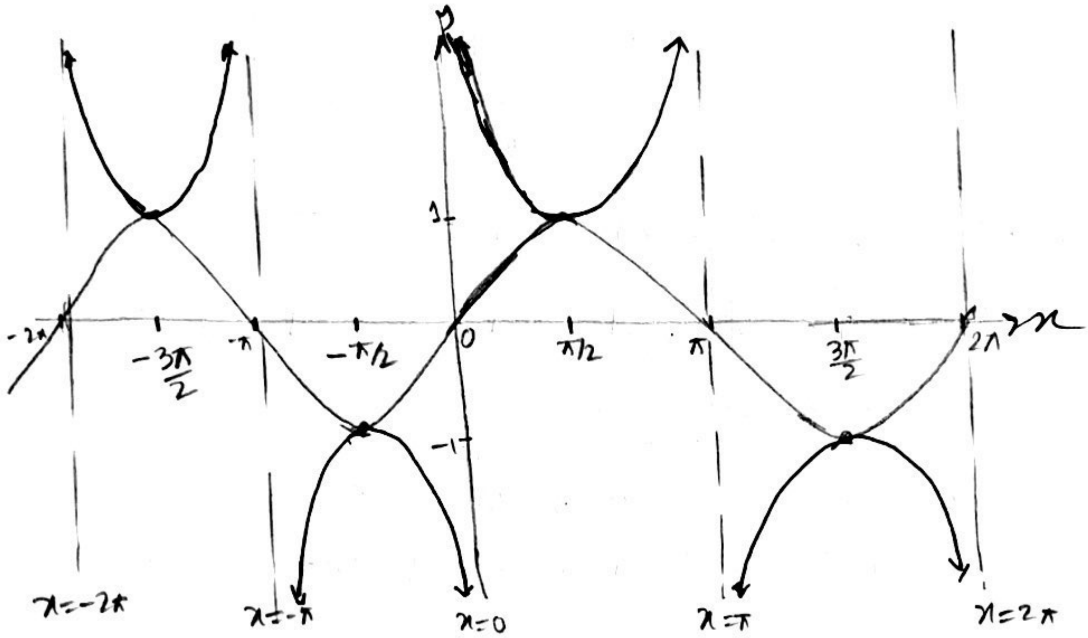


* For both $y = A\tan(\omega x) + B$ and $y = A\cot(\omega x) + B$, the role of A is to provide magnitude of vertical strech and the presence of B indicates that vertical shift is required.

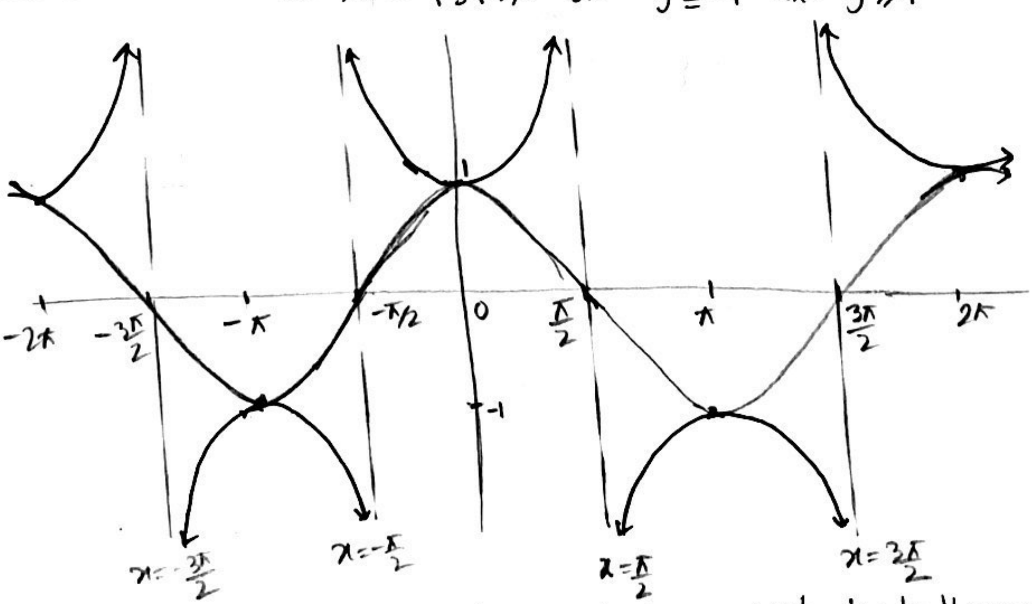
Graph of csc x and sec x:

$$\csc x = \frac{1}{\sin x} \qquad \sec x = \frac{1}{\cos x}$$

$\boxed{\csc x}$ $y = \csc x$, Domain $= \{x \mid x \neq$ integer multiples of $\pi\}$
Range $= |y| \geq 1$ or $y \leq -1$ and $y \geq 1$



$\boxed{\sec x}$ $y = \sec x$, Domain $= \{x \mid x \neq$ odd multiples of $\pi/2$,
Range $= |y| \geq 1$ or $y \leq -1$ and $y \geq 1$



The range of $y = A\csc x$ or $y = A\sec x$ is $\{y \mid |y| \geq |A|\}$, due to the vertical stretch of the graph by factor A. Period $= \frac{2\pi}{\omega}$ due to horizontal compression by factor of $1/\omega$. The presence of B indicates that a vertical shift B required. Ex: Graph $y = 2\csc x - 1$