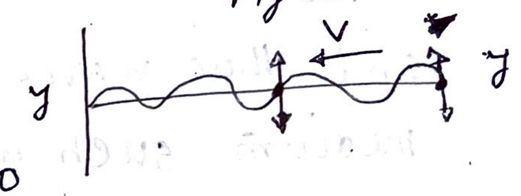
Transverse Wave: If The motion of the element is perpen-dicular to the direction of travel of the wave, this motion is said to be transverse and wave is said to be a transverse wave. Example: wave in water.

Fig A



Here we consider only an ideal string in Fig A, in which no friction-like forces within the string cause the wave to die out as it travels along the string. In addition, we assume that the string is so long that we need not consider a wave rebounding from the far end.

longitudinal wave: If the motion of the element is paral-lel to the direction of the wave's travel, the motion is said to be longitudinal and the wave is said to be a longitudinal wave.

Both transverse wave and longitudinal wave are said to be traveling waves because they both travel from one point to another. Note that, it is the wave that moves from end to end, not the material through which the wave moves.

Wave Equation: To completely describe a wave on a string, we need a function that gives the shape of the wave. Therefore, we need a relation in the form

$$y = h(x,t)$$

y is the transverse displacement of any string as a function h of the time t & the position x of the element along the string.

Imagine a sinusoidal wave traveling in the positive direction of an x axis. At time t, the displacement y of the element

$$y(x,t) = \underbrace{y_m}_{\text{Amplitude}} \sin(\underbrace{kx - \omega t}_{\text{phase}}) \quad \text{--- (1)}$$

wave number

angular frequency.

Amplitude The amplitude y_m of a wave is the magnitude of the maximum displacement of the elements from their equilibrium position as the wave passes through them. Because y_m is a magnitude, it is always a positive quantity, even if it is measured downward instead of upward.

Phase: The phase of the wave is the argument $kx - \omega t$ of sine. As the wave sweeps through a string element at a particular postion x, the phase changes linearly with time t. This means the sine also changes, oscillating between +1 and -1.

<u>Wave length:</u> The wave length λ of a wave is a distance between repetitions of the shape of the wave.

at $t=0$

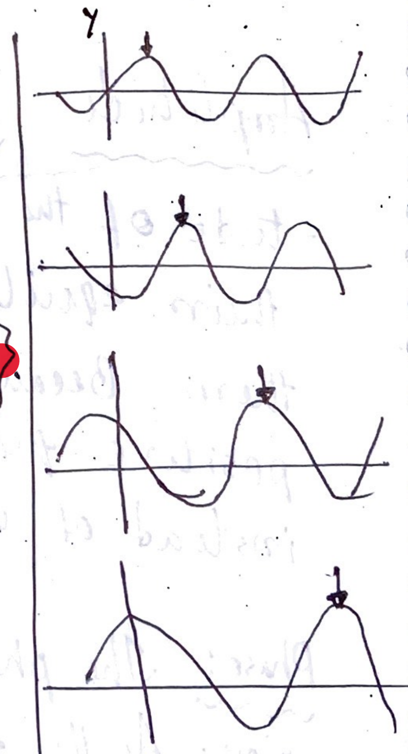
$$y(x,0) = y_m \sin Kx$$

By definition, the displacement y is same at both ends of this wave length, that $x = x_1$, $x = x_1 + \lambda$.

$$y_m \sin Kx_1 = y_m \sin K(x_1 + \lambda)$$

$$= y_m \sin (Kx_1 + K\lambda)$$

A sine function repeat itself when its angle is increased by 2π rad, hence



$$K\lambda = 2\pi$$

$$\text{or } K = \frac{2\pi}{\lambda}$$

K is called angular wave number.

unit, rad/m.

Period, Angular frequency, frequency:

If, ~~$x=0$~~ You were to monitor the string, you would see that the single element of the string at that position moves up and down in simple harmonic motion. with $x=0$

$$y(0,t) = y_m \sin(-\omega t)$$
$$= -y_m \sin \omega t \quad (x=0)$$

We define the period of oscillation, T, of a wave to be the time any string element takes to move through one full oscillation. Therefore

$$-y_m \sin \omega t_1 = -y_m \sin \omega(t_1 + T)$$
$$= -y_m \sin(\omega t_1 + \omega T)$$

$$\omega T = 2\pi$$

$$\omega = \frac{2\pi}{T} \qquad \omega \text{ is angular frequency.}$$

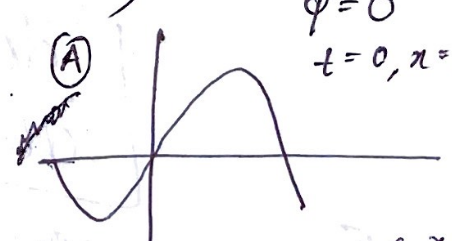
& $$f = \frac{1}{T} = \frac{\omega}{2\pi}$$

phase constant $\qquad y = y_m \sin(kx - \omega t + \phi)$

at $x=0, t=0$ $\quad y = y_m \sin(\phi)$

$$y = y_m \sin(\pi/5)$$

(A)



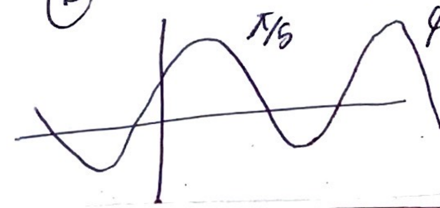
$\phi = 0$

$t=0, x=0$

(B)

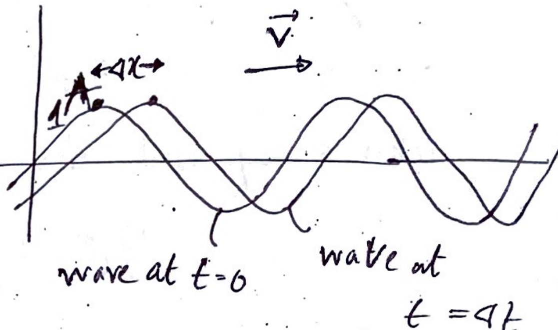
$t=0, x=0$

$\phi = \pi/5$



The speed of a travelling wave:

A wave (1) is to moving in $+x$ direction. If point A retains its displacement as it moves, the phase must remain a constant.



$$Kx - \omega t = \text{a constant} \quad \text{— — (1)}$$

Although this argument is constant, both x and t are changing. In fact as t increases, x must also to the argument constant. To find the wave speed, v, we take the derivative of equation (1)

$$K\frac{dx}{dt} - \omega \cancel{t} = 0, \quad \frac{dx}{dt} = \frac{\omega}{K}\cancel{t} \quad E$$

$$\frac{dx}{dt} = \boxed{V = \frac{\omega}{K} = \frac{\lambda}{T}} = \lambda f \left[\omega = \frac{2\pi}{T}, \; K = \frac{2\pi}{\lambda} \right.$$

$$\boxed{V = \lambda f}$$

If the wave traveling in $-X$ direction

$$\boxed{\frac{dx}{dt} = -\frac{\omega}{K}}$$

For a arbitrary shape wave

$$y(x,t) = h(Kx \pm \omega t)$$

SAC - 207
19Ap - 9.40 - 12.30
10 - 3 - 6

where h represents any function, the sine function being one possibility.

Check point 2   p-450

Here are the equations of three waves:

(1) $y(x,t) = 2\sin(4x-2t)$,   (2) $y(x,t) = \sin(3x-4t)$

(3) $y(x,t) = 2\sin(3x-3t)$, Rank the waves according to their (a) wave speed. (b) Maximum speed perpendi-

~~We know~~; $v = \frac{\omega}{k}$

-cular to the wave's direction of travel, greatest first.

(a) $V = \frac{w}{K}$ for 1: $V = \frac{w}{K} = \frac{2}{4} = \frac{1}{2}\ ms^{-1}$

2: $V = \frac{w}{K} = \frac{4}{3} = 1.33\ ms^{-1}$

2>3>1

3: $V = \frac{w}{K} = \frac{3}{3} = 1\ ms^{-1}$

(b) for 1: $\frac{dy}{dt} = \frac{d(2\sin(4x-2t))}{dt} = -4\cos(4x-2t)$

for 2: $\frac{dy}{dt} = \frac{d(\sin(3x-4t))}{dt} = -4\cos(3x-4t)$

for 3: $\frac{dy}{dt} = \frac{d(2\sin(3x-3t))}{dt} = -6\cos(3x-3t)$

We took $\frac{dy}{dt}$, v_y as y is perpendicular to the wave's direction of travel,

$3 > 2 = 1$ Ans.

Check the Sample problem 16.01 page 450

Draw the wave with values, & make problems from the waves.

Interference of waves:

When several wave effects occur simultaneously, their net effect is the sum of the individual effects. That means, Overlapping waves algebraically add to produce a resultant wave (net wave)

Suppose two waves travel simultaneously along the same stretched string. Let $y_1(x,t)$ & $y_2(x,t)$ be the displacements that the string would experience if each wave traveled alone.

The resultant wave

$$y'(x,t) = y_1(x,t) + y_2(x,t)$$