1.

$$x_1 + 2x_2 - 3x_3 + 4x_4 = 2$$

$$2x_1 + 5x_2 - 2x_3 + x_4 = 1$$

$$5x_1 + 12x_2 - 7x_3 + 6x_4 = 3$$

Augmented matrix :-

$$\begin{bmatrix} 1 & 2 & -3 & 4 & 2 \\ 2 & 5 & -2 & 1 & 1 \\ 5 & 12 & -7 & 6 & 3 \end{bmatrix}$$

$$\xrightarrow{} \begin{bmatrix} 1 & 2 & -3 & 4 & 2 \\ 0 & 1 & 4 & -7 & -3 \\ 0 & 2 & 8 & -14 & -7 \end{bmatrix}$$

$$\approx \begin{bmatrix} 1 & 0 & -11 & 18 & -4 \\ 0 & 1 & 4 & -7 & -3 \\ 0 & 0 & 0 & 0 & -1 \end{bmatrix}$$

$$\approx \begin{bmatrix} 1 & 0 & -11 & 18 & 0 \\ 0 & 1 & 4 & -7 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\therefore \quad x_1 - 11x_3 + 18x_4 = 0$$

$$x_2 + 4x_3 - 7x_4 = 0$$

$$x_3 = 0$$

$$\left(\quad \right) |$$

$$\therefore x_1 + 18x_4 = 0$$

$$x_2 - 7x_4 = 0$$

$$x_3 = 0$$

$$\therefore \text{sol}^n = (0, 0, 0$$

$$x_4 = S$$

$$S \in \mathbb{R}$$

$$\therefore (-18, 7, 0, 1)$$

2

$$A = \begin{bmatrix} 5 & -7 & 1 \\ -7 & 8 & 2 \\ 1 & 2 & -4 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 5 & -7 & 1 \\ -7 & 8 & 2 \\ 1 & 2 & -4 \end{bmatrix} \times \begin{bmatrix} 5 & -7 & 1 \\ -7 & 8 & 2 \\ 1 & 2 & -4 \end{bmatrix}$$

$$= \begin{bmatrix} 75 & -89 & -13 \\ -89 & 117 & 1 \\ -13 & 1 & 13 \end{bmatrix}$$

$$2A = 2 \begin{bmatrix} 5 & -7 & 1 \\ -7 & 8 & 2 \\ 1 & 2 & -4 \end{bmatrix}$$

$$= \begin{bmatrix} 10 & -14 & 2 \\ -14 & 16 & 4 \\ 2 & 4 & -8 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 5 & -7 & 1 \\ -7 & 8 & 2 \\ 1 & 2 & -4 \end{bmatrix}$$

$$tr(A^T) = 9$$

49
64
4
117

$$\therefore \ A^2 + 2A + tr(A^t)$$

$$= \begin{bmatrix} 75 & -89 & -13 \\ -89 & 117 & 1 \\ -13 & 1 & 1 \end{bmatrix} + \begin{bmatrix} 10 & -14 & 2 \\ -14 & 16 & 4 \\ 2 & 4 & -8 \end{bmatrix} + 9I$$

$$= \begin{bmatrix} 85 & -113 & -11 \\ -113 & 133 & 5 \\ -11 & 5 & 5 \end{bmatrix} + \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

$$= \begin{bmatrix} 94 & -113 & -11 \\ -113 & 142 & 5 \\ -11 & 5 & 14 \end{bmatrix} \quad Ans:-$$