Instructor Solution Manual

Probability and Statistics for Engineers and Scientists
(4th Edition)

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This instructor solution manual to accompany the fourth edition of

"Probability and Statistics for Engineers and Scientists" by Anthony Hayter

provides worked solutions and answers to almost all of the problems given in the textbook. The student solution manual provides worked solutions and answers to only the odd-numbered problems given at the end of the chapter sections. In addition to the material contained in the student solution manual, this instructor manual therefore provides worked solutions and answers to the even-numbered problems given at the end of the chapter sections together with almost all of the supplementary problems at the end of each chapter.

Contents

Chapter 1

Probability Theory

1.1 Probabilities

1.1.1 $\mathcal{S}$ = {(head, head, head), (head, head, tail), (head, tail, head), (head, tail, tail), (tail, head, head), (tail, head, tail), (tail, tail, head), (tail, tail, tail)}

1.1.2 $\mathcal{S}$ = {0 females, 1 female, 2 females, 3 females, . . . , n females}

1.1.3 $\mathcal{S}$ = {0,1,2,3,4}

1.1.4 $\mathcal{S}$ = {January 1, January 2, ... , February 29, ... , December 31}

1.1.5 $\mathcal{S}$ = {(on time, satisfactory), (on time, unsatisfactory), (late, satisfactory), (late, unsatisfactory)}

1.1.6 $\mathcal{S}$ = {(red, shiny), (red, dull), (blue, shiny), (blue, dull)}

1.1.7 (a) $\frac{p}{1-p} = 1 \Rightarrow p = 0.5$

(b) $\frac{p}{1-p} = 2 \Rightarrow p = \frac{2}{3}$

(c) $p = 0.25 \Rightarrow \frac{p}{1-p} = \frac{1}{3}$

1.1.8 $0.13 + 0.24 + 0.07 + 0.38 + P(V) = 1 \Rightarrow P(V) = 0.18$

1.1.9 $0.08 + 0.20 + 0.33 + P(IV) + P(V) = 1 \;\Rightarrow\; P(IV) + P(V) = 1 - 0.61 = 0.39$
Therefore, $0 \leq P(V) \leq 0.39$.
If $P(IV) = P(V)$ then $P(V) = 0.195$.

1.1.10 $P(I) = 2 \times P(II)$ and $P(II) = 3 \times P(III) \;\Rightarrow\; P(I) = 6 \times P(III)$
Therefore,
$P(I) + P(II) + P(III) = 1$
so that
$(6 \times P(III)) + (3 \times P(III)) + P(III) = 1$.
Consequently,
$P(III) = \frac{1}{10}$, $P(II) = 3 \times P(III) = \frac{3}{10}$
and
$P(I) = 6 \times P(III) = \frac{6}{10}$.

1.1.11 $p = 1 - 0.28 - 0.55 = 0.17$.

1.2 Events

1.2.1 (a) $0.13 + P(b) + 0.48 + 0.02 + 0.22 = 1 \;\Rightarrow\; P(b) = 0.15$

(b) $A = \{c, d\}$ so that $P(A) = P(c) + P(d) = 0.48 + 0.02 = 0.50$

(c) $P(A') = 1 - P(A) = 1 - 0.5 = 0.50$

1.2.2 (a) $P(A) = P(b) + P(c) + P(e) = 0.27$ so $P(b) + 0.11 + 0.06 = 0.27$
and hence $P(b) = 0.10$

(b) $P(A') = 1 - P(A) = 1 - 0.27 = 0.73$

(c) $P(A') = P(a) + P(d) + P(f) = 0.73$ so $0.09 + P(d) + 0.29 = 0.73$
and hence $P(d) = 0.35$

1.2.3 Over a four year period including one leap year, the number of days is
$(3 \times 365) + 366 = 1461$.
The number of January days is $4 \times 31 = 124$
and the number of February days is $(3 \times 28) + 29 = 113$.
The answers are therefore $\frac{124}{1461}$ and $\frac{113}{1461}$.

1.2.4 $1 - 0.03 - 0.18 = 0.79$
$1 - 0.03 = 0.97$

1.2.5 $1 - 0.38 - 0.11 - 0.16 = 0.35$
$0.38 + 0.16 + 0.35 = 0.89$

1.2.6 In Figure 1.10 let (x, y) represent the outcome that the score on the red die is x and the score on the blue die is y. The event that the score on the red die is *strictly greater* than the score on the blue die consists of the following 15 outcomes:
{(2,1), (3,1), (3,2), (4,1), (4,2), (4,3), (5,1), (5,2), (5,3), (5,4), (6,1), (6,2), (6,3), (6,4), (6,5)}
The probability of each outcome is $\frac{1}{36}$ so the required probability is $15 \times \frac{1}{36} = \frac{5}{12}$.
This probability is less than 0.5 because of the possibility that both scores are equal.

The complement of this event is the event that the red die has a score *less than or equal* to the score on the blue die which has a probability of $1 - \frac{5}{12} = \frac{7}{12}$.

1.2.7 $P(\spadesuit \text{ or } \clubsuit) = P(A\spadesuit) + P(K\spadesuit) + \ldots + P(2\spadesuit) + P(A\clubsuit) + P(K\clubsuit) + \ldots + P(2\clubsuit)$
$= \frac{1}{52} + \ldots + \frac{1}{52} = \frac{26}{52} = \frac{1}{2}$

1.2.8 $P(\text{draw an ace}) = P(A\spadesuit) + P(A\clubsuit) + P(A\diamondsuit) + P(A\heartsuit)$
$= \frac{1}{52} + \frac{1}{52} + \frac{1}{52} + \frac{1}{52} = \frac{4}{52} = \frac{1}{13}$

1.2.9 (a) Let the four players be named A, B, C, and T for Terica, and let the notation (X, Y) indicate that player X is the winner and player Y is the runner up.
The sample space consists of the 12 outcomes:
$\mathcal{S}$ = {(A,B), (A,C), (A,T), (B,A), (B,C), (B,T), (C,A), (C,B), (C,T), (T,A), (T,B), (T,C)}
The event *'Terica is winner'* consists of the 3 outcomes {(T,A), (T,B), (T,C)}.
Since each outcome in $\mathcal{S}$ is equally likely to occur with a probability of $\frac{1}{12}$ it follows that
$P(\text{Terica is winner}) = \frac{3}{12} = \frac{1}{4}$.

(b) The event *'Terica is winner or runner up'* consists of 6 out of the 12 outcomes so that
$P(\text{Terica is winner or runner up}) = \frac{6}{12} = \frac{1}{2}$.

1.2.10 (a) See Figure 1.24.
$P(\text{Type I battery lasts longest})$
$= P((II, III, I)) + P((III, II, I))$
$= 0.39 + 0.03 = 0.42$

(b) $P(\text{Type I battery lasts shortest})$
$= P((I, II, III)) + P((I, III, II))$
$= 0.11 + 0.07 = 0.18$

(c) $P(\text{Type I battery does not last longest})$
$= 1 - P(\text{Type I battery lasts longest})$
$= 1 - 0.42 = 0.58$

(d) $P(\text{Type I battery last longer than Type II})$
$= P((II, I, III)) + P((II, III, I)) + P((III, II, I))$
$= 0.24 + 0.39 + 0.03 = 0.66$

1.2.11 (a) See Figure 1.25.
The event *'both assembly lines are shut down'* consists of the single outcome {(S,S)}.
Therefore,

$P(\text{both assembly lines are shut down}) = 0.02.$

(b) The event *'neither assembly line is shut down'* consists of the outcomes
{(P,P), (P,F), (F,P), (F,F)}.
Therefore,
$P(\text{neither assembly line is shut down})$
$= P((P,P)) + P((P,F)) + P((F,P)) + P((F,F))$
$= 0.14 + 0.2 + 0.21 + 0.19 = 0.74.$

(c) The event *'at least one assembly line is at full capacity'* consists of the outcomes
{(S,F), (P,F), (F,F), (F,S), (F,P)}.
Therefore,
$P(\text{at least one assembly line is at full capacity})$
$= P((S,F)) + P((P,F)) + P((F,F)) + P((F,S)) + P((F,P))$
$= 0.05 + 0.2 + 0.19 + 0.06 + 0.21 = 0.71.$

(d) The event *'exactly one assembly line at full capacity'* consists of the outcomes
{(S,F), (P,F), (F,S), (F,P)}.
Therefore,
$P(\text{exactly one assembly line at full capacity})$
$= P((S,F)) + P((P,F)) + P((F,S)) + P((F,P))$
$= 0.05 + 0.20 + 0.06 + 0.21 = 0.52.$

The complement of *'neither assembly line is shut down'* is the event *'at least one assembly line is shut down'* which consists of the outcomes
{(S,S), (S,P), (S,F), (P,S), (F,S)}.

The complement of *'at least one assembly line is at full capacity'* is the event *'neither assembly line is at full capacity'* which consists of the outcomes
{(S,S), (S,P), (P,S), (P,P)}.

1.2.12 The sample space is
$\mathcal{S}$ = {(H,H,H), (H,T,H), (H,T,T), (H,H,T), (T,H,H), (T,H,T), (T,T,H), (T,T,T)}
with each outcome being equally likely with a probability of $\frac{1}{8}$.
The event *'two heads obtained in succession'* consists of the three outcomes
{(H,H,H), (H,H,T), (T,H,H)}
so that $P(\text{two heads in succession}) = \frac{3}{8}$.

1.2.13 $0.26 + 0.36 + 0.11 = 0.73$

1.2.14 $0.18 + 0.43 + 0.29 = 0.90$

1.3 Combinations of Events

1.3.1 The event A contains the outcome 0 while the empty set does not contain any outcomes.

1.3.2 (a) See Figure 1.55.
$P(B) = 0.01 + 0.02 + 0.05 + 0.11 + 0.08 + 0.06 + 0.13 = 0.46$

(b) $P(B \cap C) = 0.02 + 0.05 + 0.11 = 0.18$

(c) $P(A \cup C) = 0.07+0.05+0.01+0.02+0.05+0.08+0.04+0.11+0.07+0.11 = 0.61$

(d) $P(A \cap B \cap C) = 0.02 + 0.05 = 0.07$

(e) $P(A \cup B \cup C) = 1 - 0.03 - 0.04 - 0.05 = 0.88$

(f) $P(A' \cap B) = 0.08 + 0.06 + 0.11 + 0.13 = 0.38$

(g) $P(B' \cup C) = 0.04 + 0.03 + 0.05 + 0.11 + 0.05 + 0.02 + 0.08 + 0.04 + 0.11 + 0.07$
$+ 0.07 + 0.05 = 0.72$

(h) $P(A \cup (B \cap C)) = 0.07 + 0.05 + 0.01 + 0.02 + 0.05 + 0.08 + 0.04 + 0.11 = 0.43$

(i) $P((A \cup B) \cap C) = 0.11 + 0.05 + 0.02 + 0.08 + 0.04 = 0.30$

(j) $P(A' \cup C) = 0.04 + 0.03 + 0.05 + 0.08 + 0.06 + 0.13 + 0.11 + 0.11 + 0.07 + 0.02$
$+ 0.05 + 0.08 + 0.04 = 0.87$
$P(A' \cup C)' = 1 - P(A' \cup C) = 1 - 0.87 = 0.13$

1.3.4 (a) $A \cap B = \{$females with black hair$\}$

(b) $A \cup C' = \{$all females and any man who does not have brown eyes$\}$

(c) $A' \cap B \cap C = \{$males with black hair and brown eyes$\}$

(d) $A \cap (B \cup C) = \{$females with either black hair or brown eyes or both$\}$

1.3.5 Yes, because a card must be drawn from either a red suit or a black suit but it cannot be from both at the same time.

No, because the ace of hearts could be drawn.

1.3.6 $P(A \cup B) = P(A) + P(B) - P(A \cap B) \le 1$

so that

$P(B) \le 1 - 0.4 + 0.3 = 0.9.$

Also, $P(B) \ge P(A \cap B) = 0.3$

so that

$0.3 \le P(B) \le 0.9.$

1.3.7 Since $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

it follows that

$P(B) = P(A \cup B) - P(A) + P(A \cap B)$

$= 0.8 - 0.5 + 0.1 = 0.4.$

1.3.8 C

1.3.9 Yes, the three events are mutually exclusive because the selected card can only be from one suit.

Therefore,

$P(A \cup B \cup C) = P(A) + P(B) + P(C) = \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{3}{4}.$

A' is the event *'a heart is not obtained'* (or similarly the event *'a club, spade, or diamond is obtained'*) so that B is a subset of A'.

1.3.10 (a) $A \cap B = \{A\heartsuit,\ A\diamondsuit\}$

(b) $A \cup C = \{A\heartsuit,\ A\diamondsuit,\ A\clubsuit,\ A\spadesuit,\ K\heartsuit,\ K\diamondsuit,\ K\clubsuit,\ K\spadesuit,\ Q\heartsuit,\ Q\diamondsuit,\ Q\clubsuit,\ Q\spadesuit,\ J\heartsuit,\ J\diamondsuit,\ J\clubsuit,\ J\spadesuit\}$

(c) $B \cap C' = \{A\heartsuit, 2\heartsuit,\ \ldots\ ,\ 10\heartsuit,\ A\diamondsuit,\ 2\diamondsuit,\ \ldots\ ,\ 10\diamondsuit\}$

(d) $B' \cap C = \{K\clubsuit,\ K\spadesuit,\ Q\clubsuit,\ Q\spadesuit,\ J\clubsuit,\ J\spadesuit\}$

$A \cup (B' \cap C) = \{A\heartsuit,\ A\diamondsuit,\ A\clubsuit,\ A\spadesuit,\ K\clubsuit,\ K\spadesuit,\ Q\clubsuit,\ Q\spadesuit,\ J\clubsuit,\ J\spadesuit\}$

1.3.11 Let the event O be an on time repair and let the event S be a satisfactory repair.

It is known that $P(O \cap S) = 0.26$, $P(O) = 0.74$ and $P(S) = 0.41$.

We want to find $P(O' \cap S')$.

Since the event $O' \cap S'$ can be written $(O \cup S)'$ it follows that

$P(O' \cap S') = 1 - P(O \cup S)$

$= 1 - (P(O) + P(S) - P(O \cap S))$

$= 1 - (0.74 + 0.41 - 0.26) = 0.11.$

1.3.12 Let R be the event that a red ball is chosen and let S be the event that a shiny ball is chosen.

It is known that $P(R \cap S) = \frac{55}{200}$, $P(S) = \frac{91}{200}$ and $P(R) = \frac{79}{200}$.

Therefore, the probability that the chosen ball is either shiny or red is

$P(R \cup S) = P(R) + P(S) - P(R \cap S)$

$= \frac{79}{200} + \frac{91}{200} - \frac{55}{200}$

$= \frac{115}{200} = 0.575.$

The probability of a dull blue ball is

$P(R' \cap S') = 1 - P(R \cup S)$

$= 1 - 0.575 = 0.425.$

1.3.13 Let A be the event that the patient is male, let B be the event that the patient is younger than thirty years of age, and let C be the event that the patient is admitted to the hospital.

It is given that $P(A) = 0.45$, $P(B) = 0.30$, $P(A' \cap B' \cap C) = 0.15$,

and $P(A' \cap B) = 0.21$.

The question asks for $P(A' \cap B' \cap C')$.

Notice that

$P(A' \cap B') = P(A') - P(A' \cap B) = (1 - 0.45) - 0.21 = 0.34$

so that

$P(A' \cap B' \cap C') = P(A' \cap B') - P(A' \cap B' \cap C) = 0.34 - 0.15 = 0.19.$

1.3.14 $P(A \cap B) = 0.26$

$P(A \cup B) = 1$

1.3.15 $P(A \cap B) = P(B) = 0.43 + 0.29 = 0.72$

$P(A \cup B) = P(A) = 0.18 + 0.43 + 0.29 = 0.90$

1.4 Conditional Probability

1.4.1 See Figure 1.55.

(a) $P(A \mid B) = \frac{P(A\cap B)}{P(B)} = \frac{0.02+0.05+0.01}{0.02+0.05+0.01+0.11+0.08+0.06+0.13} = 0.1739$

(b) $P(C \mid A) = \frac{P(A\cap C)}{P(A)} = \frac{0.02+0.05+0.08+0.04}{0.02+0.05+0.08+0.04+0.018+0.07+0.05} = 0.59375$

(c) $P(B \mid A\cap B) = \frac{P(B\cap(A\cap B))}{P(A\cap B)} = \frac{P(A\cap B)}{P(A\cap B)} = 1$

(d) $P(B \mid A\cup B) = \frac{P(B\cap(A\cup B))}{P(A\cup B)} = \frac{P(B)}{P(A\cup B)} = \frac{0.46}{0.46+0.32-0.08} = 0.657$

(e) $P(A \mid A\cup B\cup C) = \frac{P(A\cap(A\cup B\cup C))}{P(A\cup B\cup C)} = \frac{P(A)}{P(A\cup B\cup C)} = \frac{0.32}{1-0.04-0.05-0.03} = 0.3636$

(f) $P(A\cap B \mid A\cup B) = \frac{P((A\cap B)\cap(A\cup B))}{P(A\cup B)} = \frac{P(A\cap B)}{P(A\cup B)} = \frac{0.08}{0.7} = 0.1143$

1.4.2 (a) $0.42 \times 0.42 \times 0.42 = 0.0741$

(b) $0.42 \times (1 - 0.63) = 0.1554$

1.4.3 (a) $P(A\heartsuit \mid \text{red suit}) = \frac{P(A\heartsuit\cap\text{red suit})}{P(\text{red suit})} = \frac{P(A\heartsuit)}{P(\text{red suit})} = \frac{\left(\frac{1}{52}\right)}{\left(\frac{26}{52}\right)} = \frac{1}{26}$

(b) $P(\text{heart} \mid \text{red suit}) = \frac{P(\text{heart}\cap\text{red suit})}{P(\text{red suit})} = \frac{P(\text{heart})}{P(\text{red suit})} = \frac{\left(\frac{13}{52}\right)}{\left(\frac{26}{52}\right)} = \frac{13}{26} = \frac{1}{2}$

(c) $P(\text{red suit} \mid \text{heart}) = \frac{P(\text{red suit}\cap\text{heart})}{P(\text{heart})} = \frac{P(\text{heart})}{P(\text{heart})} = 1$

(d) $P(\text{heart} \mid \text{black suit}) = \frac{P(\text{heart}\cap\text{black suit})}{P(\text{black suit})} = \frac{P(\emptyset)}{P(\text{black suit})} = 0$

(e) $P(\text{King} \mid \text{red suit}) = \frac{P(\text{King}\cap\text{red suit})}{P(\text{red suit})} = \frac{P(K\heartsuit,\ K\diamondsuit)}{P(\text{red suit})} = \frac{\left(\frac{2}{52}\right)}{\left(\frac{26}{52}\right)} = \frac{2}{26} = \frac{1}{13}$

(f) $P(\text{King} \mid \text{red picture card}) = \frac{P(\text{King}\cap\text{red picture card})}{P(\text{red picture card})}$

$= \frac{P(K\heartsuit,\ K\diamondsuit)}{P(\text{red picture card})} = \frac{\left(\frac{2}{52}\right)}{\left(\frac{6}{52}\right)} = \frac{2}{6} = \frac{1}{3}$

1.4.4 $P(A)$ is smaller than $P(A \mid B)$.

Event B is a necessary condition for event A and so conditioning on event B increases the probability of event A.

1.4.5 There are 54 blue balls and so there are $150 - 54 = 96$ red balls.

Also, there are 36 shiny, red balls and so there are $96 - 36 = 60$ dull, red balls.

$P(\text{shiny} \mid \text{red}) = \frac{P(\text{shiny} \cap \text{red})}{P(\text{red})} = \frac{\left(\frac{36}{150}\right)}{\left(\frac{96}{150}\right)} = \frac{36}{96} = \frac{3}{8}$

$P(\text{dull} \mid \text{red}) = \frac{P(\text{dull} \cap \text{red})}{P(\text{red})} = \frac{\left(\frac{60}{150}\right)}{\left(\frac{96}{150}\right)} = \frac{60}{96} = \frac{5}{8}$

1.4.6 Let the event O be an on time repair and let the event S be a satisfactory repair.

It is known that $P(S \mid O) = 0.85$ and $P(O) = 0.77$.

The question asks for $P(O \cap S)$ which is

$P(O \cap S) = P(S \mid O) \times P(O) = 0.85 \times 0.77 = 0.6545$.

1.4.7 (a) It depends on the weather patterns in the particular location that is being considered.

(b) It increases since there are proportionally more black haired people among brown eyed people than there are in the general population.

(c) It remains unchanged.

(d) It increases.

1.4.8 Over a four year period including one leap year the total number of days is

$(3 \times 365) + 366 = 1461$.

Of these, $4 \times 12 = 48$ days occur on the first day of a month and so the probability that a birthday falls on the first day of a month is

$\frac{48}{1461} = 0.0329$.

Also, $4 \times 31 = 124$ days occur in March of which 4 days are March 1st.

Consequently, the probability that a birthday falls on March 1st. conditional that it is in March is

$\frac{4}{124} = \frac{1}{31} = 0.0323$.

Finally, $(3 \times 28) + 29 = 113$ days occur in February of which 4 days are February 1st.

Consequently, the probability that a birthday falls on February 1st. conditional that it is in February is

$\frac{4}{113} = 0.0354$.

1.4.9 (a) Let A be the event that *'Type I battery lasts longest'* consisting of the outcomes $\{$(III, II, I), (II, III, I)$\}$.
Let B be the event that *'Type I battery does not fail first'* consisting of the outcomes $\{$(III,II,I), (II,III,I), (II,I,III), (III,I,II)$\}$.
The event $A \cap B = \{$(III,II,I), (II,III,I)$\}$ is the same as event A.
Therefore,
$P(A \mid B) = \frac{P(A\cap B)}{P(B)} = \frac{0.39+0.03}{0.39+0.03+0.24+0.16} = 0.512.$

(b) Let C be the event that *'Type II battery fails first'* consisting of the outcomes $\{$(II,I,III), (II,III,I)$\}$.
Thus, $A \cap C = \{(II, III, I)\}$ and therefore
$P(A \mid C) = \frac{P(A\cap C)}{P(C)} = \frac{0.39}{0.39+0.24} = 0.619.$

(c) Let D be the event that *'Type II battery lasts longest'* consisting of the outcomes $\{$(I,III,II), (III,I,II)$\}$.
Thus, $A \cap D = \emptyset$ and therefore
$P(A \mid D) = \frac{P(A\cap D)}{P(D)} = 0.$

(d) Let E be the event that *'Type II battery does not fail first'* consisting of the outcomes $\{$(I,III,II), (I,II,III), (III,II,I), (III,I,II)$\}$.
Thus, $A \cap E = \{(III, II, I)\}$ and therefore
$P(A \mid E) = \frac{P(A\cap E)}{P(E)} = \frac{0.03}{0.07+0.11+0.03+0.16} = 0.081.$

1.4.10 See Figure 1.25.

(a) Let A be the event *'both lines at full capacity'* consisting of the outcome $\{$(F,F)$\}$.
Let B be the event *'neither line is shut down'* consisting of the outcomes $\{$(P,P), (P,F), (F,P), (F,F)$\}$.
Thus, $A \cap B = \{(F, F)\}$ and therefore
$P(A \mid B) = \frac{P(A\cap B)}{P(B)} = \frac{0.19}{(0.14+0.2+0.21+0.19)} = 0.257.$

(b) Let C be the event *'at least one line at full capacity'* consisting of the outcomes $\{$(F,P), (F,S), (F,F), (S,F), (P,F)$\}$.
Thus, $C \cap B = \{(F, P), (F, F), (P, F)\}$ and therefore
$P(C \mid B) = \frac{P(C\cap B)}{P(B)} = \frac{0.21+0.19+0.2}{0.74} = 0.811.$

(c) Let D be the event that *'one line is at full capacity'* consisting of the outcomes $\{$(F,P), (F,S), (P,F), (S,F)$\}$.
Let E be the event *'one line is shut down'* consisting of the outcomes $\{$(S,P), (S,F), (P,S), (F,S)$\}$.
Thus, $D \cap E = \{(F, S), (S, F)\}$ and therefore
$P(D \mid E) = \frac{P(D\cap E)}{P(E)} = \frac{0.06+0.05}{0.06+0.05+0.07+0.06} = 0.458.$

(d) Let G be the event that *'neither line is at full capacity'* consisting of the outcomes {(S,S), (S,P), (P,S), (P,P)}.
Let H be the event that *'at least one line is at partial capacity'* consisting of the outcomes {(S,P), (P,S), (P,P), (P,F), (F,P)}.
Thus, $F \cap G = \{(S,P), (P,S), (P,P)\}$ and therefore
$P(F \mid G) = \frac{P(F \cap G)}{P(G)} = \frac{0.06+0.07+0.14}{0.06+0.07+0.14+0.2+0.21} = 0.397.$

1.4.11 Let L, W and H be the events that the length, width and height respectively are within the specified tolerance limits.
It is given that $P(W) = 0.86$, $P(L \cap W \cap H) = 0.80$, $P(L \cap W \cap H') = 0.02$, $P(L' \cap W \cap H) = 0.03$ and $P(W \cup H) = 0.92$.

(a) $P(W \cap H) = P(L \cap W \cap H) + P(L' \cap W \cap H) = 0.80 + 0.03 = 0.83$
$P(H) = P(W \cup H) - P(W) + P(W \cap H) = 0.92 - 0.86 + 0.83 = 0.89$
$P(W \cap H \mid H) = \frac{P(W \cap H)}{P(H)} = \frac{0.83}{0.89} = 0.9326$

(b) $P(L \cap W) = P(L \cap W \cap H) + P(L \cap W \cap H') = 0.80 + 0.02 = 0.82$
$P(L \cap W \cap H \mid L \cap W) = \frac{P(L \cap W \cap H)}{P(L \cap W)} = \frac{0.80}{0.82} = 0.9756$

1.4.12 Let A be the event that the gene is of *'type A'*, and let D be the event that the gene is *'dominant'*.
$P(D \mid A') = 0.31$
$P(A' \cap D) = 0.22$
Therefore,
$P(A) = 1 - P(A')$
$= 1 - \frac{P(A' \cap D)}{P(D \mid A')}$
$= 1 - \frac{0.22}{0.31} = 0.290$

1.4.13 (a) Let E be the event that the *'component passes on performance'*, let A be the event that the *'component passes on appearance'*, and let C be the event that the *'component passes on cost'*.
$P(A \cap C) = 0.40$
$P(E \cap A \cap C) = 0.31$
$P(E) = 0.64$
$P(E' \cap A' \cap C') = 0.19$
$P(E' \cap A \cap C') = 0.06$
Therefore,
$P(E' \cap A' \cap C) = P(E' \cap A') - P(E' \cap A' \cap C')$
$= P(E') - P(E' \cap A) - 0.19$

$= 1 - P(E) - P(E' \cap A \cap C) - P(E' \cap A \cap C') - 0.19$
$= 1 - 0.64 - P(A \cap C) + P(E \cap A \cap C) - 0.06 - 0.19$
$= 1 - 0.64 - 0.40 + 0.31 - 0.06 - 0.19 = 0.02$

(b) $P(E \cap A \cap C \mid A \cap C) = \frac{P(E \cap A \cap C)}{P(A \cap C)}$
$= \frac{0.31}{0.40} = 0.775$

1.4.14 (a) Let T be the event *'good taste'*, let S be the event *'good size'*, and let A be the event *'good appearance'*.
$P(T) = 0.78$
$P(T \cap S) = 0.69$
$P(T \cap S' \cap A) = 0.05$
$P(S \cup A) = 0.84$
Therefore,
$P(S \mid T) = \frac{P(T \cap S)}{P(T)} = \frac{0.69}{0.78} = 0.885.$

(b) Notice that
$P(S' \cap A') = 1 - P(S \cup A) = 1 - 0.84 = 0.16.$
Also,
$P(T \cap S') = P(T) - P(T \cap S) = 0.78 - 0.69 = 0.09$
so that
$P(T \cap S' \cap A') = P(T \cap S') - P(T \cap S' \cap A) = 0.09 - 0.05 = 0.04.$
Therefore,
$P(T \mid S' \cap A') = \frac{P(T \cap S' \cap A')}{P(S' \cap A')} = \frac{0.04}{0.16} = 0.25.$

1.4.15 $P(\text{delay}) = (P(\text{delay} \mid \text{technical problems}) \times P(\text{technical problems}))$
$+ (P(\text{delay} \mid \text{no technical problems}) \times P(\text{no technical problems}))$
$= (1 \times 0.04) + (0.33 \times 0.96) = 0.3568$

1.4.16 Let S be the event that a chip *'survives 500 temperature cycles'* and let A be the event that the chip was *'made by company A'*.
$P(S) = 0.42$
$P(A \mid S') = 0.73$
Therefore,
$P(A' \cap S') = P(S') \times P(A' \mid S') = (1 - 0.42) \times (1 - 0.73) = 0.1566.$

1.4.17 $\frac{0.26}{0.26+0.36+0.11} = 0.3562$

1.4.18 $\frac{0.43+0.29}{0.18+0.43+0.29} = 0.8$

1.5 Probabilities of Event Intersections

1.5.1 (a) $P(\text{both cards are picture cards}) = \frac{12}{52} \times \frac{11}{51} = \frac{132}{2652}$

(b) $P(\text{both cards are from red suits}) = \frac{26}{52} \times \frac{25}{51} = \frac{650}{2652}$

(c) $P(\text{one card is from a red suit and one is from black suit})$
$= (P(\text{first card is red}) \times P(\text{2nd card is black} \mid \text{1st card is red}))$
$+ (P(\text{first card is black}) \times P(\text{2nd card is red} \mid \text{1st card is black}))$
$= \left(\frac{26}{52} \times \frac{26}{51}\right) + \left(\frac{26}{52} \times \frac{26}{51}\right) = \frac{676}{2652} \times 2 = \frac{26}{51}$

1.5.2 (a) $P(\text{both cards are picture cards}) = \frac{12}{52} \times \frac{12}{52} = \frac{9}{169}$
The probability increases with replacement.

(b) $P(\text{both cards are from red suits}) = \frac{26}{52} \times \frac{26}{52} = \frac{1}{4}$
The probability increases with replacement.

(c) $P(\text{one card is from a red suit and one is from black suit})$
$= (P(\text{first card is red}) \times P(\text{2nd card is black} \mid \text{1st card is red}))$
$+ (P(\text{first card is black}) \times P(\text{2nd card is red} \mid \text{1st card is black}))$
$= \left(\frac{26}{52} \times \frac{26}{52}\right) + \left(\frac{26}{52} \times \frac{26}{52}\right) = \frac{1}{2}$
The probability decreases with replacement.

1.5.3 (a) No, they are not independent.
Notice that
$P((ii)) = \frac{3}{13} \neq P((ii) \mid (i)) = \frac{11}{51}$.

(b) Yes, they are independent.
Notice that
$P((i) \cap (ii)) = P((i)) \times P((ii))$
since
$P((i)) = \frac{1}{4}$
$P((ii)) = \frac{3}{13}$
and
$P((i) \cap (ii)) = P(\text{first card a heart picture} \cap (ii))$
$+ P(\text{first card a heart but not a picture} \cap (ii))$
$= \left(\frac{3}{52} \times \frac{11}{51}\right) + \left(\frac{10}{52} \times \frac{12}{51}\right) = \frac{153}{2652} = \frac{3}{52}$.

(c) No, they are not independent.
Notice that
$P((ii)) = \frac{1}{2} \neq P((ii) \mid (i)) = \frac{25}{51}$.

(d) Yes, they are independent.
Similar to part (b).

(e) No, they are not independent.

1.5.4 $P(\text{all four cards are hearts}) = P(\text{lst card is a heart})$
$\times P(\text{2nd card is a heart} \mid \text{lst card is a heart})$
$\times P(\text{3rd card is a heart} \mid \text{1st and 2nd cards are hearts})$
$\times P(\text{4th card is a heart} \mid \text{1st, 2nd and 3rd cards are hearts})$
$= \frac{13}{52} \times \frac{12}{51} \times \frac{11}{50} \times \frac{10}{49} = 0.00264$

$P(\text{all 4 cards from red suits}) = P(\text{1st card from red suit})$
$\times P(\text{2nd card is from red suit} \mid \text{lst card is from red suit})$
$\times P(\text{3rd card is from red suit} \mid \text{1st and 2nd cards are from red suits})$
$\times P(\text{4th card is from red suit} \mid \text{1st, 2nd and 3rd cards are from red suits})$
$= \frac{26}{52} \times \frac{25}{51} \times \frac{24}{50} \times \frac{23}{49} = 0.055$

$P(\text{all 4 cards from different suits}) = P(\text{1st card from any suit})$
$\times P(\text{2nd card not from suit of 1st card})$
$\times P(\text{3rd card not from suit of 1st or 2nd cards})$
$\times P(\text{4th card not from suit of 1st, 2nd, or 3rd cards})$
$= 1 \times \frac{39}{51} \times \frac{26}{50} \times \frac{13}{49} = 0.105$

1.5.5 $P(\text{all 4 cards are hearts}) = (\frac{13}{52})^4 = \frac{1}{256}$
The probability increases with replacement.

$P(\text{all 4 cards are from red suits}) = (\frac{26}{52})^4 = \frac{1}{16}$
The probability increases with replacement.

$P(\text{all 4 cards from different suits}) = 1 \times \frac{39}{52} \times \frac{26}{52} \times \frac{13}{52} = \frac{3}{32}$
The probability decreases with replacement.

1.5.6 The events A and B are independent so that $P(A \mid B) = P(A)$, $P(B \mid A) = P(B)$, and $P(A \cap B) = P(A)P(B)$.

To show that two events are independent it needs to be shown that one of the above three conditions holds.

(a) Recall that
$P(A \cap B) + P(A \cap B') = P(A)$
and

$P(B) + P(B') = 1$.
Therefore,
$P(A \mid B') = \frac{P(A \cap B')}{P(B')}$
$= \frac{P(A) - P(A \cap B)}{1 - P(B)}$
$= \frac{P(A) - P(A)P(B)}{1 - P(B)}$
$= \frac{P(A)(1 - P(B))}{1 - P(B)}$
$= \ P(A)$.

(b) Similar to part (a).

(c) $P(A' \cap B') + P(A' \cap B) = P(A')$
so that
$P(A' \cap B') = P(A) - P(A' \cap B) = P(A) - P(A')P(B)$
since the events A' and B are independent.
Therefore,
$P(A' \cap B') = P(A)(1 - P(B)) = P(A')P(B')$.

1.5.7 The only way that a message will not get through the network is if both branches are closed at the same time. The branches are independent since the switches operate independently of each other.

Therefore,

P(message gets through the network)

$= 1 - P$(message cannot get through the top branch or the bottom branch)

$= 1 - (P$(message cannot get through the top branch)

$\times\ P$(message cannot get through the bottom branch)).

Also,

P(message gets through the top branch) $= P$(switch 1 is open $\cap$ switch 2 is open)

$= P$(switch 1 is open) $\times$ P(switch 2 is open)

$= 0.88 \times 0.92 = 0.8096$

since the switches operate independently of each other.

Therefore,

P(message cannot get through the top branch)

$= 1 - P$(message gets through the top branch)

$= 1 - 0.8096 = 0.1904$.

Furthermore,

P(message cannot get through the bottom branch)

$= P(\text{switch 3 is closed}) = 1 - 0.9 = 0.1.$

Therefore,

$P(\text{message gets through the network}) = 1 - (0.1 \times 0.1904) = 0.98096.$

1.5.8 Given the birthday of the first person, the second person has a different birthday with a probability $\frac{364}{365}$.

The third person has a different birthday from the first two people with a probability $\frac{363}{365}$, and so the probability that all three people have different birthdays is

$1 \times \frac{364}{365} \times \frac{363}{365}$.

Continuing in this manner the probability that n people all have different birthdays is therefore

$\frac{364}{365} \times \frac{363}{365} \times \frac{362}{365} \times \ldots \times \frac{366-n}{365}$

and

$P(\text{at least 2 people out of } n \text{ share the same birthday})$

$= 1 - P(n \text{ people all have different birthdays})$

$= 1 - \left(\frac{364}{365} \times \frac{363}{365} \times \ldots \frac{366-n}{365}\right).$

This probability is equal to 0.117 for $n = 10$,

is equal to 0.253 for $n = 15$,

is equal to 0.411 for $n = 20$,

is equal to 0.569 for $n = 25$,

is equal to 0.706 for $n = 30$,

and is equal to 0.814 for $n = 35$.

The smallest values of n for which the probability is greater than 0.5 is $n = 23$.

Note that in these calculations it has been assumed that birthdays are equally likely to occur on any day of the year, although in practice seasonal variations may be observed in the number of births.

1.5.9 $P(\text{no broken bulbs}) = \frac{83}{100} \times \frac{82}{99} \times \frac{81}{98} = 0.5682$

$P(\text{one broken bulb}) = P(\text{broken, not broken, not broken})$

$+ P(\text{not broken, broken, not broken}) + P(\text{not broken, not broken, broken})$

$= \left(\frac{17}{100} \times \frac{83}{99} \times \frac{82}{98}\right) + \left(\frac{83}{100} \times \frac{17}{99} \times \frac{82}{98}\right) + \left(\frac{83}{100} \times \frac{82}{99} \times \frac{17}{98}\right) = 0.3578$

$P(\text{no more than one broken bulb in the sample})$

$= P(\text{no broken bulbs}) + P(\text{one broken bulb})$

$= 0.5682 + 0.3578 = 0.9260$

1.5.10 $P(\text{no broken bulbs}) = \frac{83}{100} \times \frac{83}{100} \times \frac{83}{100} = 0.5718$

$P(\text{one broken bulb}) = P(\text{broken, not broken, not broken})$

$+ P(\text{not broken, broken, not broken}) + P(\text{not broken, not broken, broken})$

$= \left(\frac{17}{100} \times \frac{83}{100} \times \frac{83}{100}\right) + \left(\frac{83}{100} \times \frac{17}{100} \times \frac{83}{100}\right) + \left(\frac{83}{100} \times \frac{83}{100} \times \frac{17}{100}\right) = 0.3513$

$P(\text{no more than one broken bulb in the sample})$

$= P(\text{no broken bulbs}) + P(\text{one broken bulb})$

$= 0.5718 + 0.3513 = 0.9231$

The probability of finding no broken bulbs increases with replacement, but the probability of finding no more than one broken bulb decreases with replacement.

1.5.11 $P(\text{drawing 2 green balls})$

$= P(\text{1st ball is green}) \times P(\text{2nd ball is green} \mid \text{1st ball is green})$

$= \frac{72}{169} \times \frac{71}{168} = 0.180$

$P(\text{two balls same color})$

$= P(\text{two red balls}) + P(\text{two blue balls}) + P(\text{two green balls})$

$= \left(\frac{43}{169} \times \frac{42}{168}\right) + \left(\frac{54}{169} \times \frac{53}{168}\right) + \left(\frac{72}{169} \times \frac{71}{168}\right) = 0.344$

$P(\text{two balls different colors}) = 1 - P(\text{two balls same color})$

$= 1 - 0.344 = 0.656$

1.5.12 $P(\text{drawing 2 green balls}) = \frac{72}{169} \times \frac{72}{169} = 0.182$

$P(\text{two balls same color})$

$= P(\text{two red balls}) + P(\text{two blue balls}) + P(\text{two green balls})$

$= \left(\frac{43}{169} \times \frac{43}{169}\right) + \left(\frac{54}{169} \times \frac{54}{169}\right) + \left(\frac{72}{169} \times \frac{72}{169}\right) = 0.348$

$P(\text{two balls different colors}) = 1 - P(\text{two balls same color})$

$= 1 - 0.348 = 0.652$

The probability that the two balls are green increases with replacement while the probability of drawing two balls of different colors decreases with replacement.

1.5.13 $P(\text{same result on both throws}) = P(\text{both heads}) + P(\text{both tails})$

$= p^2 + (1-p)^2 = 2p^2 - 2p + 1 = 2(p - 0.5)^2 + 0.5$

which is minimized when $p = 0.5$ (a fair coin).

1.5.14 $P(\text{each score is obtained exactly once})$
$= 1 \times \frac{5}{6} \times \frac{4}{6} \times \frac{3}{6} \times \frac{2}{6} \times \frac{1}{6} = \frac{5}{324}$
$P(\text{no sixes in seven rolls}) = \left(\frac{5}{6}\right)^7 = 0.279$

1.5.15 (a) $\left(\frac{1}{2}\right)^5 = \frac{1}{32}$

(b) $1 \times \frac{5}{6} \times \frac{4}{6} = \frac{5}{9}$

(c) $P(BBR) + P(BRB) + P(RBB)$
$= \left(\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}\right) + \left(\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}\right) + \left(\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}\right)$
$= \frac{3}{8}$

(d) $P(BBR) + P(BRB) + P(RBB)$
$= \left(\frac{26}{52} \times \frac{25}{51} \times \frac{26}{50}\right) + \left(\frac{26}{52} \times \frac{26}{51} \times \frac{25}{50}\right) + \left(\frac{26}{52} \times \frac{26}{51} \times \frac{25}{50}\right)$
$= \frac{13}{34}$

1.5.16 $1 - (1 - 0.90)^n \geq 0.995$
is satisfied for $n \geq 3$.

1.5.17 Claims from clients in the same geographical area would not be independent of each other since they would all be affected by the same flooding events.

1.5.18 (a) $P(\text{system works}) = 0.88 \times 0.78 \times 0.92 \times 0.85 = 0.537$

(b) $P(\text{system works}) = 1 - P(\text{no computers working})$
$= 1 - ((1 - 0.88) \times (1 - 0.78) \times (1 - 0.92) \times (1 - 0.85)) = 0.9997$

(c) $P(\text{system works}) = P(\text{all computers working})$
$+ P(\text{computers 1,2,3 working, computer 4 not working})$
$+ P(\text{computers 1,2,4 working, computer 3 not working})$
$+ P(\text{computers 1,3,4 working, computer 2 not working})$
$+ P(\text{computers 2,3,4 working, computer 1 not working})$
$= 0.537 + (0.88 \times 0.78 \times 0.92 \times (1 - 0.85)) + (0.88 \times 0.78 \times (1 - 0.92) \times 0.85)$
$+ (0.88 \times (1 - 0.78) \times 0.92 \times 0.85) + ((1 - 0.88) \times 0.78 \times 0.92 \times 0.85)$
$= 0.903$

1.5.19 $(0.26 + 0.36 + 0.11) \times (0.26 + 0.36 + 0.11) = 0.5329$

1.5.20 $(0.43 + 0.29) \times (0.43 + 0.29) \times (0.43 + 0.29) \times (0.43 + 0.29) = 0.2687$

1.6 Posterior Probabilities

1.6.1 (a) The following information is given:
$P(\text{disease}) = 0.01$
$P(\text{no disease}) = 0.99$
$P(\text{positive blood test} \mid \text{disease}) = 0.97$
$P(\text{positive blood test} \mid \text{no disease}) = 0.06$
Therefore,
$P(\text{positive blood test}) = (P(\text{positive blood test} \mid \text{disease}) \times P(\text{disease}))$
$+ \ (P(\text{positive blood test} \mid \text{no disease}) \times P(\text{no disease}))$
$= (0.97 \times 0.01) + (0.06 \times 0.99) = 0.0691.$

(b) $P(\text{disease} \mid \text{positive blood test})$
$= \frac{P(\text{positive blood test} \cap \text{disease})}{P(\text{positive blood test})}$
$= \frac{P(\text{positive blood test} \mid \text{disease}) \times P(\text{disease})}{P(\text{positive blood test})}$
$= \frac{0.97 \times 0.01}{0.0691} = 0.1404$

(c) $P(\text{no disease} \mid \text{negative blood test})$
$= \frac{P(\text{no disease} \cap \text{negative blood test})}{P(\text{negative blood test})}$
$= \frac{P(\text{negative blood test} \mid \text{no disease}) \times P(\text{no disease})}{1 - P(\text{positive blood test})}$
$= \frac{(1-0.06) \times 0.99}{(1-0.0691)} = 0.9997$

1.6.2 (a) $P(\text{red}) = (P(\text{red} \mid \text{bag 1}) \times P(\text{bag 1})) + (P(\text{red} \mid \text{bag 2}) \times P(\text{bag 2}))$
$+ \ (P(\text{red} \mid \text{bag 3}) \times P(\text{bag 3}))$
$= \left(\frac{1}{3} \times \frac{3}{10}\right) + \left(\frac{1}{3} \times \frac{8}{12}\right) + \left(\frac{1}{3} \times \frac{5}{16}\right) = 0.426$

(b) $P(\text{blue}) = 1 - P(\text{red}) = 1 - 0.426 = 0.574$

(c) $P(\text{red ball from bag 2}) = P(\text{bag 2}) \times P(\text{red ball} \mid \text{bag 2})$
$= \frac{1}{3} \times \frac{8}{12} = \frac{2}{9}$

$P(\text{bag 1} \mid \text{red ball}) = \frac{P(\text{bag 1} \cap \text{red ball})}{P(\text{red ball})}$

$= \frac{P(\text{bag 1}) \times P(\text{red ball} \mid \text{bag 1})}{P(\text{red ball})}$

$= \frac{\frac{1}{3} \times \frac{3}{10}}{0.426} = 0.235$

$P(\text{bag 2} \mid \text{blue ball}) = \frac{P(\text{bag 2} \cap \text{blue ball})}{P(\text{blue ball})}$

$= \frac{P(\text{bag 2}) \times P(\text{blue ball} \mid \text{bag 1})}{P(\text{blue ball})}$

$= \frac{\frac{1}{3}\times\frac{4}{12}}{0.574} = 0.194$

1.6.3 (a) $P(\text{Section I}) = \frac{55}{100}$

(b) $P(\text{grade is A})$
$= (P(\text{A} \mid \text{Section I}) \times P(\text{Section I})) + (P(\text{A} \mid \text{Section II}) \times P(\text{Section II}))$
$= \left(\frac{10}{55} \times \frac{55}{100}\right) + \left(\frac{11}{45} \times \frac{45}{100}\right) = \frac{21}{100}$

(c) $P(\text{A} \mid \text{Section I}) = \frac{10}{55}$

(d) $P(\text{Section I} \mid \text{A}) = \frac{P(\text{A} \cap \text{Section I})}{P(\text{A})}$
$= \frac{P(\text{Section I})\times P(\text{A} \mid \text{Section I})}{P(\text{A})}$
$= \frac{\frac{55}{100}\times\frac{10}{55}}{\frac{21}{100}} = \frac{10}{21}$

1.6.4 The following information is given:

$P(\text{Species 1}) = 0.45$

$P(\text{Species 2}) = 0.38$

$P(\text{Species 3}) = 0.17$

$P(\text{Tagged} \mid \text{Species 1}) = 0.10$

$P(\text{Tagged} \mid \text{Species 2}) = 0.15$

$P(\text{Tagged} \mid \text{Species 3}) = 0.50$

Therefore,

$P(\text{Tagged}) = (P(\text{Tagged} \mid \text{Species 1}) \times P(\text{Species 1}))$

$+ (P(\text{Tagged} \mid \text{Species 2}) \times P(\text{Species 2})) + (P(\text{Tagged} \mid \text{Species 3}) \times P(\text{Species 3}))$

$= (0.10 \times 0.45) + (0.15 \times 0.38) + (0.50 \times 0.17) = 0.187.$

$P(\text{Species 1} \mid \text{Tagged}) = \frac{P(\text{Tagged} \cap \text{Species 1})}{P(\text{Tagged})}$

$= \frac{P(\text{Species 1})\times P(\text{Tagged} \mid \text{Species 1})}{P(\text{Tagged})}$

$= \frac{0.45\times 0.10}{0.187} = 0.2406$

$P(\text{Species 2} \mid \text{Tagged}) = \frac{P(\text{Tagged} \cap \text{Species 2})}{P(\text{Tagged})}$

$= \frac{P(\text{Species 2})\times P(\text{Tagged} \mid \text{Species 2})}{P(\text{Tagged})}$

$= \frac{0.38\times 0.15}{0.187} = 0.3048$

$P(\text{Species 3} \mid \text{Tagged}) = \frac{P(\text{Tagged} \cap \text{Species 3})}{P(\text{Tagged})}$

$= \frac{P(\text{Species 3}) \times P(\text{Tagged} \mid \text{Species 3})}{P(\text{Tagged})}$

$= \frac{0.17 \times 0.50}{0.187} = 0.4545$

1.6.5 (a) $P(\text{fail}) = (0.02 \times 0.77) + (0.10 \times 0.11) + (0.14 \times 0.07) + (0.25 \times 0.05)$
$= 0.0487$

$P(C \mid \text{fail}) = \frac{0.14 \times 0.07}{0.0487} = 0.2012$

$P(D \mid \text{fail}) = \frac{0.25 \times 0.05}{0.0487} = 0.2567$

The answer is $0.2012 + 0.2567 = 0.4579$.

(b) $P(A \mid \text{did not fail}) = \frac{P(A) \times P(\text{did not fail} \mid A)}{P(\text{did not fail})}$
$= \frac{0.77 \times (1-0.02)}{1-0.0487} = 0.7932$

1.6.6 $P(C) = 0.15$
$P(W) = 0.25$
$P(H) = 0.60$
$P(R \mid C) = 0.30$
$P(R \mid W) = 0.40$
$P(R \mid H) = 0.50$
Therefore,
$P(C \mid R') = \frac{P(R'|C)P(C)}{P(R'|C)P(C)+P(R'|W)P(W)+P(R'|H)P(H)}$
$= \frac{(1-0.30) \times 0.15}{((1-0.30) \times 0.15)+((1-0.40) \times 0.25)+((1-0.50) \times 0.60)}$
$= 0.189$

1.6.7 (a) $P(C) = 0.12$
$P(M) = 0.55$
$P(W) = 0.20$
$P(H) = 0.13$
$P(L \mid C) = 0.003$
$P(L \mid M) = 0.009$
$P(L \mid W) = 0.014$
$P(L \mid H) = 0.018$
Therefore,
$P(H \mid L) = \frac{P(L|H)P(H)}{P(L|C)P(C)+P(L|M)P(M)+P(L|W)P(W)+P(L|H)P(H)}$
$= \frac{0.018 \times 0.13}{(0.003 \times 0.12)+(0.009 \times 0.55)+(0.014 \times 0.20)+(0.018 \times 0.13)}$
$= 0.224$

(b) $P(M \mid L') = \frac{P(L'|M)P(M)}{P(L'|C)P(C)+P(L'|M)P(M)+P(L'|W)P(W)+P(L'|H)P(H)}$

$= \frac{0.991\times0.55}{(0.997\times0.12)+(0.991\times0.55)+(0.986\times0.20)+(0.982\times0.13)}$

$= 0.551$

1.6.8 (a) $P(A) = 0.12$

$P(B) = 0.34$

$P(C) = 0.07$

$P(D) = 0.25$

$P(E) = 0.22$

$P(M \mid A) = 0.19$

$P(M \mid B) = 0.50$

$P(M \mid C) = 0.04$

$P(M \mid D) = 0.32$

$P(M \mid E) = 0.76$

Therefore,

$P(C \mid M) = \frac{P(M|C)P(C)}{P(M|A)P(A)+P(M|B)P(B)+P(M|C)P(C)+P(M|D)P(D)+P(M|E)P(E)}$

$= \frac{0.04\times0.07}{(0.19\times0.12)+(0.50\times0.34)+(0.04\times0.07)+(0.32\times0.25)+(0.76\times0.22)}$

$= 0.0063$

(b) $P(D \mid M') = \frac{P(M'|D)P(D)}{P(M'|A)P(A)+P(M'|B)P(B)+P(M'|C)P(C)+P(M'|D)P(D)+P(M'|E)P(E)}$

$= \frac{0.68\times0.25}{(0.81\times0.12)+(0.50\times0.34)+(0.96\times0.07)+(0.68\times0.25)+(0.24\times0.22)}$

$= 0.305$

1.6.9 $(0.08 \times 0.03) + (0.19 \times 0.14) + (0.26 \times 0.60) + (0.36 \times 0.77) + (0.11 \times 0.99) = 0.5711$

$\frac{(0.08\times0.03)+(0.19\times0.14)}{(0.08\times0.03)+(0.19\times0.14)+(0.26\times0.60)+(0.36\times0.77)+(0.11\times0.99)} = 0.0508$

1.6.10 $(0.10 \times 0.74) + (0.18 \times 0.43) + (0.43 \times 0.16) + (0.29 \times 0.01) = 0.2231$

$\frac{(0.18\times0.43)+(0.43\times0.16)+(0.29\times0.01)}{(0.10\times0.74)+(0.18\times0.43)+(0.43\times0.16)+(0.29\times0.01)} = 0.6683$

1.7 Counting Techniques

1.7.1 (a) $7! = 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 5040$

(b) $8! = 8 \times 7! = 40320$

(c) $4! = 4 \times 3 \times 2 \times 1 = 24$

(d) $13! = 13 \times 12 \times 11 \times \ldots \times 1 = 6{,}227{,}020{,}800$

1.7.2 (a) $P_2^7 = \frac{7!}{(7-2)!} = 7 \times 6 = 42$

(b) $P_5^9 = \frac{9!}{(9-5)!} = 9 \times 8 \times 7 \times 6 \times 5 = 15120$

(c) $P_2^5 = \frac{5!}{(5-2)!} = 5 \times 4 = 20$

(d) $P_4^{17} = \frac{17!}{(17-4)!} = 17 \times 16 \times 15 \times 14 = 57120$

1.7.3 (a) $C_2^6 = \frac{6!}{(6-2)! \times 2!} = \frac{6 \times 5}{2} = 15$

(b) $C_4^8 = \frac{8!}{(8-4)! \times 4!} = \frac{8 \times 7 \times 6 \times 5}{24} = 70$

(c) $C_2^5 = \frac{5!}{(5-2)! \times 2!} = \frac{5 \times 4}{2} = 10$

(d) $C_6^{14} = \frac{14!}{(14-6)! \times 6!} = 3003$

1.7.4 The number of full meals is $5 \times 3 \times 7 \times 6 \times 8 = 5040$.

The number of meals with just soup or appetizer is $(5 + 3) \times 7 \times 6 \times 8 = 2688$.

1.7.5 The number of experimental configurations is $3 \times 4 \times 2 = 24$.

1.7.6 (a) Let the notation (2,3,1,4) represent the result that the player who finished 1st in tournament 1 finished 2nd in tournament 2, the player who finished 2nd in tournament 1 finished 3rd in tournament 2, the player who finished 3rd in tournament 1 finished 1st in tournament 2, and the player who finished 4th in tournament 1 finished 4th in tournament 2.

Then the result (1,2,3,4) indicates that each competitor received the same ranking in both tournaments.

Altogether there are $4! = 24$ different results, each equally likely, and so this single result has a probability of $\frac{1}{24}$.

(b) The results where no player receives the same ranking in the two tournaments are:
(2,1,4,3), (2,3,4,1), (2,4,1,3), (3,1,4,2), (3,4,1,2) (3,4,2,1), (4,1,2,3), (4,3,1,2), (4,3,2,1)
There are nine of these results and so the required probability is $\frac{9}{24} = \frac{3}{8}$.

1.7.7 The number of rankings that can be assigned to the top 5 competitors is
$P_5^{20} = \frac{20!}{15!} = 20 \times 19 \times 18 \times 17 \times 16 = 1{,}860{,}480$.

The number of ways in which the best 5 competitors can be chosen is
$C_5^{20} = \frac{20!}{15! \times 5!} = 15504$.

1.7.8 (a) $C_3^{100} = \frac{100!}{97! \times 3!} = \frac{100 \times 99 \times 98}{6} = 161700$

(b) $C_3^{83} = \frac{83!}{80! \times 3!} = \frac{83 \times 82 \times 81}{6} = 91881$

(c) $P(\text{no broken lightbulbs}) = \frac{91881}{161700} = 0.568$

(d) $17 \times C_2^{83} = 17 \times \frac{83 \times 82}{2} = 57851$

(e) The number of samples with 0 or 1 broken bulbs is
$91881 + 57851 = 149732$.
$P(\text{sample contains no more than 1 broken bulb}) = \frac{149732}{161700} = 0.926$

1.7.9 $C_k^{n-1} + C_{k-1}^{n-1} = \frac{(n-1)!}{k!(n-1-k)!} + \frac{(n-1)!}{(k-1)!(n-k)!} = \frac{n!}{k!(n-k)!}\left(\frac{n-k}{n} + \frac{k}{n}\right) = \frac{n!}{k!(n-k)!} = C_k^n$

This relationship can be interpreted in the following manner.

C_k^n is the number of ways that k balls can be selected from n balls. Suppose that one ball is red while the remaining $n-1$ balls are blue. Either all k balls selected are blue or one of the selected balls is red. C_k^{n-1} is the number of ways k blue balls can be selected while C_{k-1}^{n-1} is the number of ways of selecting the one red ball and $k-1$ blue balls.

1.7.10 (a) The number of possible 5 card hands is $C_5^{52} = \frac{52!}{47! \times 5!}$ =2,598,960.

(b) The number of ways to get a hand of 5 hearts is $C_5^{13} = \frac{13!}{8! \times 5!} = 1287$.

(c) The number of ways to get a flush is $4 \times C_5^{13} = 4 \times 1,287 = 5148$.

(d) $P(\text{flush}) = \frac{5148}{2{,}598{,}960} = 0.00198.$

(e) There are 48 choices for the fifth card in the hand and so the number of hands containing all four aces is 48.

(f) $13 \times 48 = 624$

(g) $P(\text{hand has four cards of the same number or picture}) = \frac{624}{2{,}598{,}960} = 0.00024.$

1.7.11 There are $n!$ ways in which n objects can be arranged in a line. If the line is made into a circle and rotations of the circle are considered to be indistinguishable, then there are n arrangements of the line corresponding to each arrangement of the circle. Consequently, there are $\frac{n!}{n} = (n-1)!$ ways to order the objects in a circle.

1.7.12 The number of ways that six people can sit in a line at a cinema is $6! = 720$.

See the previous problem.

The number of ways that six people can sit around a dinner table is $5! = 120$.

1.7.13 Consider 5 blocks, one block being Andrea and Scott and the other four blocks being the other four people. At the cinema these 5 blocks can be arranged in 5! ways, and then Andrea and Scott can be arranged in two different ways within their block, so that the total number of seating arrangements is $2 \times 5! = 240$.

Similarly, the total number of seating arrangements at the dinner table is $2 \times 4! = 48$.

If Andrea refuses to sit next to Scott then the number of seating arrangements can be obtained by subtraction. The total number of seating arrangements at the cinema is $720 - 240 = 480$ and the total number of seating arrangements at the dinner table is $120 - 48 = 72$.

1.7.14 The total number of arrangements of n balls is $n!$ which needs to be divided by $n_1!$ because the rearrangements of the n_1 balls in box 1 are indistinguishable, and similarly it needs to be divided by $n_2! \ldots n_k!$ due to the indistinguishable rearrangements possible in boxes 2 to k.

When $k = 2$ the problem is equivalent to the number of ways of selecting n_1 balls (or n_2 balls) from $n = n_1 + n_2$ balls.

1.7.15 (a) Using the result provided in the previous problem the answer is $\frac{12!}{3! \times 4! \times 5!} = 27720.$

(b) Suppose that the balls in part (a) are labelled from 1 to 12. Then the positions of the three red balls in the line (where the places in the line are labelled 1 to

12) can denote which balls in part (a) are placed in the first box, the positions of the four blue balls in the line can denote which balls in part (a) are placed in the second box, and the positions of the five green balls in the line can denote which balls in part (a) are placed in the third box. Thus, there is a one-to-one correspondence between the positioning of the colored balls in part (b) and the arrangements of the balls in part (a) so that the problems are identical.

1.7.16 $\frac{14!}{3!\times 4!\times 7!} = 120120$

1.7.17 $\frac{15!}{3!\times 3!\times 3!\times 3!\times 3!} = 168{,}168{,}000$

1.7.18 The total number of possible samples is C_{12}^{60}.

(a) The number of samples containing only items which have either excellent or good quality is C_{12}^{43}.
Therefore, the answer is
$\frac{C_{12}^{43}}{C_{12}^{60}} = \frac{43}{60} \times \frac{42}{59} \ldots \times \frac{32}{49} = 0.0110.$

(b) The number of samples that contain three items of excellent quality, three items of good quality, three items of poor quality and three defective items is
$C_3^{18} \times C_3^{25} \times C_3^{12} \times C_3^{5}$ =4,128,960,000.
Therefore, the answer is
$\frac{4{,}128{,}960{,}000}{C_{12}^{60}} = 0.00295.$

1.7.19 The ordering of the visits can be made in $10! = 3{,}628{,}800$ different ways.

The number of different ways the ten cities be split into two groups of five cities is $C_5^{10} = 252$.

1.7.20 $\binom{26}{2} \times \binom{26}{3} = 845000$

1.7.21 (a) $\frac{\binom{39}{8}}{\binom{52}{8}} = \frac{39}{52} \times \frac{38}{51} \times \frac{37}{50} \times \frac{36}{49} \times \frac{35}{48} \times \frac{34}{47} \times \frac{33}{46} \times \frac{32}{45} = 0.082$

(b) $\frac{\binom{13}{2} \times \binom{13}{2} \times \binom{13}{2} \times \binom{13}{2}}{\binom{52}{8}} = 0.049$

1.7.22 $$\frac{\binom{5}{2} \times \binom{30}{4} \times \binom{5}{2}}{\binom{40}{8}} = 0.0356$$

1.7.23 $5 \times 5 \times 5 = 125$

1.7.24 $4 \times 4 \times 4 \times 4 = 256$

1.10 Supplementary Problems

1.10.1 $\mathcal{S} = \{\ 1,\ 1.5,\ 2,\ 2.5,\ 3,\ 3.5,\ 4,\ 4.5,\ 5,\ 5.5,\ 6\}$

1.10.2 If the four contestants are labelled A, B, C, D and the notation (X, Y) is used to indicate that contestant X is the winner and contestant Y is the runner up, then the sample space is:

$\mathcal{S} = \{(A, B), (A, C), (A, D), (B, A), (B, C), (B, D),$

$(C, A), (C, B), (C, D), (D, A), (D, B), (D, C)\}$

1.10.3 One way is to have the two team captains each toss the coin once. If one obtains a head and the other a tail, then the one with the head wins (this could just as well be done the other way around so that the one with the tail wins, as long as it is decided beforehand). If both captains obtain the same result, that is if there are two heads or two tails, then the procedure could be repeated until different results are obtained.

1.10.4 See Figure 1.10.

There are 36 equally likely outcomes, 16 of which have scores differing by no more than one.

Therefore,

$P(\text{the scores on two dice differ by no more than one}) = \frac{16}{36} = \frac{4}{9}$.

1.10.5 The number of ways to pick a card is 52.

The number of ways to pick a diamond picture card is 3.

Therefore,

$P(\text{picking a diamond picture card}) = \frac{3}{52}$.

1.10.6 With replacement:

$P(\text{drawing two hearts}) = \frac{13}{52} \times \frac{13}{52} = \frac{1}{16} = 0.0625$

Without replacement:

$P(\text{drawing two hearts}) = \frac{13}{52} \times \frac{12}{51} = \frac{3}{51} = 0.0588$

The probability decreases without replacement.

1.10.7 $A = \{(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (3, 1)\}$

$B = \{(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)\}$

(a) $A \cap B = \{(1,1),(2,2)\}$
$P(A \cap B) = \frac{2}{36} = \frac{1}{18}$

(b) $A \cup B = \{(1,1),(1,2),(1,3),(2,1),(2,2),(3,1),(3,3),(4,4),(5,5),(6,6)\}$
$P(A \cup B) = \frac{10}{36} = \frac{5}{18}$

(c) $A' \cup B = \{(1,1),(1,4),(1,5),(1,6),(2,2),(2,3),(2,4),(2,5),(2,6),$
$(3,2),(3,3),(3,4),(3,5),(3,6),(4,1),(4,2),(4,3),(4,4),(4,5),(4,6),$
$(5,1),(5,2),(5,3),(5,4),(5,5),(5,6),(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)\}$
$P(A' \cup B) = \frac{32}{36} = \frac{8}{9}$

1.10.8 See Figure 1.10.

Let the notation (x, y) indicate that the score on the red die is x and that the score on the blue die is y.

(a) The event *'the sum of the scores on the two dice is eight'*
consists of the outcomes:
$\{(2,6),(3,5),(4,4),(5,3),(6,2)\}$

Therefore,
$P(\text{red die is 5} \mid \text{sum of scores is 8})$
$= \frac{P(\text{red die is 5} \cap \text{sum of scores is 8})}{P(\text{sum of scores is 8})}$
$= \frac{\left(\frac{1}{36}\right)}{\left(\frac{5}{36}\right)} = \frac{1}{5}.$

(b) $P(\text{either score is 5} \mid \text{sum of scores is 8}) = 2 \times \frac{1}{5} = \frac{2}{5}$

(c) The event *'the score on either die is 5'*
consists of the 11 outcomes:
$\{(1,5),(2,5),(3,5),(4,5),(5,5),(6,5),(5,6),(5,4),(5,3),(5,2),(5,1)\}$

Therefore,
$P(\text{sum of scores is 8} \mid \text{either score is 5})$
$= \frac{P(\text{sum of scores is 8} \cap \text{either score is 5})}{P(\text{either score is 5})}$
$= \frac{\left(\frac{2}{36}\right)}{\left(\frac{11}{36}\right)} = \frac{2}{11}.$

1.10.9 $P(A) = P(\text{either switch 1 or 4 is open or both})$
$= 1 - P(\text{both switches 1 and 4 are closed})$
$= 1 - 0.15^2 = 0.9775$

$P(B) = P(\text{either switch 2 or 5 is open or both})$

$= 1 - P(\text{both switches 2 and 5 are closed})$
$= 1 - 0.15^2 = 0.9775$

$P(C) = P(\text{switches 1 and 2 are both open}) = 0.85^2 = 0.7225$
$P(D) = P(\text{switches 4 and 5 are both open}) = 0.85^2 = 0.7225$

If $E = C \cup D$ then
$P(E) = 1 - (P(C') \times P(D'))$
$= 1 - (1 - 0.85^2)^2 = 0.923.$

Therefore,
$P(\text{message gets through the network})$
$= (P(\text{switch 3 is open}) \times P(A) \times P(B)) + (P(\text{switch 3 closed}) \times P(E))$
$= (0.85 \times (1 - 0.15^2)^2) + (0.15 \times (1 - (1 - 0.85^2)^2)) = 0.9506.$

1.10.10 The sample space for the experiment of two coin tosses consists of the four equally likely outcomes:
$\{(H, H), (H, T), (T, H), (T, T)\}$
Three out of these four outcomes contain at least one head, so that
$P(\text{at least one head in two coin tosses}) = \frac{3}{4}.$

The sample space for four tosses of a coin consists of $2^4 = 16$ equally likely outcomes of which the following 11 outcomes contain at least two heads:
$\{(HHTT), (HTHT), (HTTH), (THHT), (THTH), (TTHH),$
$(HHHT), (HHTH), (HTHH), (THHH), (HHHH)\}$
Therefore,
$P(\text{at least two heads in four coin tosses}) = \frac{11}{16}$
which is smaller than the previous probability.

1.10.11 (a) $P(\text{blue ball}) = (P(\text{bag 1}) \times P(\text{blue ball} \mid \text{bag 1}))$
$+ (P(\text{bag 2}) \times P(\text{blue ball} \mid \text{bag 2}))$
$+ (P(\text{bag 3}) \times P(\text{blue ball} \mid \text{bag 3}))$
$+ (P(\text{bag 4}) \times P(\text{blue ball} \mid \text{bag 4}))$
$= \left(0.15 \times \frac{7}{16}\right) + \left(0.2 \times \frac{8}{18}\right) + \left(0.35 \times \frac{9}{19}\right) + \left(0.3 \times \frac{7}{11}\right) = 0.5112$

(b) $P(\text{bag 4} \mid \text{green ball}) = \frac{P(\text{green ball} \cap \text{bag 4})}{P(\text{green ball})}$
$= \frac{P(\text{bag 4}) \times P(\text{green ball} \mid \text{bag 4})}{P(\text{greenball})}$
$= \frac{0.3 \times 0}{P(\text{green ball})} = 0$

(c) $P(\text{bag 1} \mid \text{blue ball}) = \frac{P(\text{bag 1}) \times P(\text{blue ball} \mid \text{bag 1})}{P(\text{blue ball})}$
$= \frac{0.15 \times \frac{7}{16}}{0.5112} = \frac{0.0656}{0.5112} = 0.128$

1.10.12 (a) $\mathcal{S} = \{1, 2, 3, 4, 5, 6, 10\}$

(b) $P(10) = P(\text{score on die is 5}) \times P(\text{tails})$
$= \frac{1}{6} \times \frac{1}{2} = \frac{1}{12}$

(c) $P(3) = P(\text{score on die is 3}) \times P(\text{heads})$
$= \frac{1}{6} \times \frac{1}{2} = \frac{1}{12}$

(d) $P(6) = P(\text{score on die is 6}) + (P(\text{score on die is 3}) \times P(\text{tails}))$
$= \frac{1}{6} + (\frac{1}{6} \times \frac{1}{2})$
$= \frac{1}{4}$

(e) 0

(f) $P(\text{score on die is odd} \mid \text{6 is recorded})$
$= \frac{P(\text{score on die is odd} \cap \text{6 is recorded})}{P(\text{6 is recorded})}$
$= \frac{P(\text{score on die is 3}) \times P(\text{tails})}{P(\text{6 is recorded})}$
$= \frac{\left(\frac{1}{12}\right)}{\left(\frac{1}{4}\right)} = \frac{1}{3}$

1.10.13 $5^4 = 625$

$4^5 = 1024$

In this case $5^4 < 4^5$, and in general $n_2^{n_1} < n_1^{n_2}$ when $3 \leq n_1 < n_2$.

1.10.14 $\frac{20!}{5! \times 5! \times 5! \times 5!} = 1.17 \times 10^{10}$

$\frac{20!}{4! \times 4! \times 4! \times 4 \times 4!} = 3.06 \times 10^{11}$

1.10.15 $P(X = 0) = \frac{1}{4}$

$P(X = 1) = \frac{1}{2}$

$P(X = 2) = \frac{1}{4}$

$P(X = 0 \mid \text{white}) = \frac{1}{8}$

$P(X = 1 \mid \text{white}) = \frac{1}{2}$

$P(X = 2 \mid \text{white}) = \frac{3}{8}$

$P(X = 0 \mid \text{black}) = \frac{1}{2}$
$P(X = 1 \mid \text{black}) = \frac{1}{2}$
$P(X = 2 \mid \text{black}) = 0$

1.10.16 Let A be the event that *'the order is from a first time customer'* and let B be the event that *'the order is dispatched within one day'*.
It is given that $P(A) = 0.28$, $P(B \mid A) = 0.75$, and $P(A' \cap B') = 0.30$.

Therefore,
$P(A' \cap B) = P(A') - P(A' \cap B')$
$= (1 - 0.28) - 0.30 = 0.42$

$P(A \cap B) = P(A) \times P(B \mid A)$
$= 0.28 \times 0.75 = 0.21$

$P(B) = P(A' \cap B) + P(A \cap B)$
$= 0.42 + 0.21 = 0.63$

and
$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$
$= \frac{0.21}{0.63} = \frac{1}{3}$.

1.10.17 It is given that
$P(\text{Puccini}) = 0.26$
$P(\text{Verdi}) = 0.22$
$P(\text{other composer}) = 0.52$
$P(\text{female} \mid \text{Puccini}) = 0.59$
$P(\text{female} \mid \text{Verdi}) = 0.45$
and
$P(\text{female}) = 0.62$.

(a) Since
$P(\text{female}) = (P(\text{Puccini}) \times P(\text{female} \mid \text{Puccini}))$
$+ (P(\text{Verdi}) \times P(\text{female} \mid \text{Verdi}))$
$+ (P(\text{other composer}) \times P(\text{female} \mid \text{other composer}))$
it follows that
$0.62 = (0.26 \times 0.59) + (0.22 \times 0.45) + (0.52 \times P(\text{female} \mid \text{other composer}))$
so that
$P(\text{female} \mid \text{other composer}) = 0.7069$.

(b) $P(\text{Puccini} \mid \text{male}) = \frac{P(\text{Puccini}) \times P(\text{male} \mid \text{Puccini})}{P(\text{male})}$
$= \frac{0.26 \times (1-0.59)}{1-0.62} = 0.281$

1.10.18 The total number of possible samples is C_{10}^{92}.

(a) The number of samples that do not contain any fibers of polymer B is C_{10}^{75}.
Therefore, the answer is
$\frac{C_{10}^{75}}{C_{10}^{92}} = \frac{75}{92} \times \frac{74}{91} \ldots \times \frac{66}{83} = 0.115.$

(b) The number of samples that contain exactly one fiber of polymer B is $17 \times C_9^{75}$.
Therefore, the answer is
$\frac{17 \times C_9^{75}}{C_{10}^{92}} = 0.296.$

(c) The number of samples that contain three fibers of polymer A, three fibers of polymer B, and four fibers of polymer C is
$C_3^{43} \times C_3^{17} \times C_4^{32}$.
Therefore, the answer is
$\frac{C_3^{43} \times C_3^{17} \times C_4^{32}}{C_{10}^{92}} = 0.042.$

1.10.19 The total number of possible sequences of heads and tails is $2^5 = 32$, with each sequence being equally likely. Of these, sixteen don't include a sequence of three outcomes of the same kind.

Therefore, the required probability is
$\frac{16}{32} = 0.5.$

1.10.20 (a) Calls answered by an experienced operator that last over five minutes.

(b) Successfully handled calls that were answered either within ten seconds or by an inexperienced operator (or both).

(c) Calls answered after ten seconds that lasted more than five minutes and that were not handled successfully.

(d) Calls that were either answered within ten seconds and lasted less than five minutes, or that were answered by an experienced operator and were handled successfully.

1.10.21 (a) $\frac{20!}{7! \times 7! \times 6!} = 133,024,320$

(b) If the first and the second job are assigned to production line I, the number of assignments is
$\frac{18!}{5!\times7!\times6!} = 14,702,688.$

If the first and the second job are assigned to production line II, the number of assignments is
$\frac{18!}{7!\times5!\times6!} = 14,702,688.$

If the first and the second job are assigned to production line III, the number of assignments is

$\frac{18!}{7!\times7!\times4!} = 10,501,920.$

Therefore, the answer is
$14,702,688 + 14,702,688 + 10,501,920 = 39,907,296.$

(c) The answer is $133,024,320 - 39,907,296 = 93,117,024.$

1.10.22 (a) $\dfrac{\binom{13}{3}}{\binom{52}{3}} = \frac{13}{52} \times \frac{12}{51} \times \frac{11}{50} = 0.0129$

(b) $\dfrac{\binom{4}{1}\times\binom{4}{1}\times\binom{4}{1}}{\binom{52}{3}} = \frac{12}{52} \times \frac{8}{51} \times \frac{4}{50} = 0.0029$

1.10.23 (a) $\dfrac{\binom{48}{4}}{\binom{52}{4}} = \frac{48}{52} \times \frac{47}{51} \times \frac{46}{50} \times \frac{45}{49} = 0.719$

(b) $\dfrac{\binom{4}{1}\times\binom{48}{3}}{\binom{52}{4}} = \frac{4\times4\times48\times47\times46}{52\times51\times50\times49} = 0.256$

(c) $\left(\frac{1}{52}\right)^3 = \frac{1}{140608}$

1.10.24 (a) True

(b) False

(c) False

(d) True

(e) True

(f) False

(g) False

1.10.25 Let W be the event that *'the team wins the game'*
and let S be the event that *'the team has a player sent off'*.

$P(W) = 0.55$
$P(S') = 0.85$
$P(W \mid S') = 0.60$

Since
$P(W) = P(W \cap S) + P(W \cap S')$
$= P(W \cap S) + (P(W \mid S') \times P(S'))$
it follows that
$0.55 = P(W \cap S) + (0.60 \times 0.85)$.

Therefore,
$P(W \cap S) = 0.04$.

1.10.26 (a) Let N be the event that the machine is *'new'*
and let G be the event that the machine has *'good quality'*.

$P(N \cap G') = \frac{120}{500}$
$P(N') = \frac{230}{500}$

Therefore,
$P(N \cap G) = P(N) - P(N \cap G')$
$= 1 - \frac{230}{500} - \frac{120}{500} = \frac{150}{500} = 0.3$.

(b) $P(G \mid N) = \frac{P(N \cap G)}{P(N)}$
$= \frac{0.3}{1 - \frac{230}{500}} = \frac{5}{9}$

1.10.27 (a) Let M be the event *'male'*,
let E be the event *'mechanical engineer'*,
and let S be the event *'senior'*.

$P(M) = \frac{113}{250}$
$P(E) = \frac{167}{250}$
$P(M' \cap E') = \frac{52}{250}$
$P(M' \cap E \cap S) = \frac{19}{250}$

Therefore,
$P(M \mid E') = 1 - P(M' \mid E')$
$= 1 - \frac{P(M' \cap E')}{P(E')}$
$= 1 - \frac{52}{250-167} = 0.373.$

(b) $P(S \mid M' \cap E) = \frac{P(M' \cap E \cap S)}{P(M' \cap E)}$
$= \frac{P(M' \cap E \cap S)}{P(M') - P(M' \cap E')}$
$= \frac{19}{250-113-52} = 0.224$

1.10.28 (a) Let T be the event that *'the tax form is filed on time'*,
let S be the event that *'the tax form is from a small business'*,
and let A be the event that *'the tax form is accurate'*.

$P(T \cap S \cap A) = 0.11$
$P(T' \cap S \cap A) = 0.13$
$P(T \cap S) = 0.15$
$P(T' \cap S \cap A') = 0.21$

Therefore,
$P(T \mid S \cap A) = \frac{P(T \cap S \cap A)}{P(S \cap A)}$
$= \frac{P(T \cap S \cap A)}{P(T \cap S \cap A) + P(T' \cap S \cap A)}$
$= \frac{0.11}{0.11+0.13} = \frac{11}{24}.$

(b) $P(S') = 1 - P(S)$
$= 1 - P(T \cap S) - P(T' \cap S)$
$= 1 - P(T \cap S) - P(T' \cap S \cap A) - P(T' \cap S \cap A')$
$= 1 - 0.15 - 0.13 - 0.21 = 0.51$

1.10.29 (a) $P(\text{having exactly two heart cards}) = \frac{C_2^{13} \times C_2^{39}}{C_4^{52}} = 0.213$

(b) $P(\text{having exactly two heart cards and exactly two club cards})$
$= \frac{C_2^{13} \times C_2^{13}}{C_4^{52}} = 0.022$

(c) $P(\text{having 3 heart cards} \mid \text{no club cards})$
$= P(\text{having 3 heart cards from a reduced pack of 39 cards})$
$= \frac{C_3^{13} \times C_1^{26}}{C_4^{39}} = 0.09$

1.10.30 (a) $P(\text{passing the first time}) = 0.26$
$P(\text{passing the second time}) = 0.43$

$P(\text{failing the first time and passing the second time})$
$= P(\text{failing the first time}) \times P(\text{passing the second time})$
$= (1 - 0.26) \times 0.43 = 0.3182$

(b) $1 - P(\text{failing both times}) = 1 - (1 - 0.26) \times (1 - 0.43) = 0.5782$

(c) $P(\text{passing the first time} \mid \text{moving to the next stage})$
$= \frac{P(\text{passing the first time and moving to the next stage})}{P(\text{moving to the next stage})}$
$= \frac{0.26}{0.5782} = 0.45$

1.10.31 The possible outcomes are $(6, 5, 4, 3, 2)$, $(6, 5, 4, 3, 1)$, $(6, 5, 4, 2, 1)$, $(6, 5, 3, 2, 1)$, $(6, 4, 3, 2, 1)$, and $(5, 4, 3, 2, 1)$.
Each outcome has a probability of $\frac{1}{6^5}$ so that the required probability is $\frac{6}{6^5} = \frac{1}{6^4} = \frac{1}{1296}$.

1.10.32 $P(\text{at least one uncorrupted file}) = 1 - P(\text{both files corrupted})$
$= 1 - (0.005 \times 0.01) = 0.99995$

1.10.33 Let C be the event that *'the pump is operating correctly'* and let L be the event that *'the light is on'*.

$P(L \mid C') = 0.992$

$P(L \mid C) = 0.003$

$P(C) = 0.996$

Therefore, using Bayes theorem
$P(C' \mid L) = \frac{P(L \mid C')P(C')}{P(L \mid C')P(C') + P(L \mid C)P(C)}$
$= \frac{0.992 \times 0.004}{(0.992 \times 0.004) + (0.003 \times 0.996)} = 0.57.$

1.10.34 $\dfrac{\binom{4}{2}\times\binom{4}{2}\times\binom{4}{3}\times\binom{4}{3}}{\binom{52}{10}} = \frac{1}{27{,}465{,}320}$

1.10.35 (a) $\dfrac{\binom{7}{3}}{\binom{11}{3}} = \frac{7}{11} \times \frac{6}{10} \times \frac{5}{9} = \frac{7}{33}$

(b) $\dfrac{\binom{7}{1}\times\binom{4}{2}}{\binom{11}{3}} = \frac{14}{55}$

1.10.36 (a) The probability of an infected person having strain A is $P(A) = 0.32$.
The probability of an infected person having strain B is $P(B) = 0.59$.
The probability of an infected person having strain C is $P(C) = 0.09$.

$P(S \mid A) = 0.21$
$P(S \mid B) = 0.16$
$P(S \mid C) = 0.63$

Therefore, the probability of an infected person exhibiting symptoms is
$P(S) = (P(S \mid A) \times P(A)) + (P(S \mid B) \times P(B)) + (P(S \mid C) \times P(C))$
$= 0.2183$
and
$P(C \mid S) = \frac{P(S \mid C)\times P(C)}{P(S)}$
$= \frac{0.63\times 0.09}{0.2183} = 0.26.$

(b) $P(S') = 1 - P(S) = 1 - 0.2183 = 0.7817$
$P(S' \mid A) = 1 - P(S \mid A) = 1 - 0.21 = 0.79$

Therefore,
$P(A \mid S') = \frac{P(S' \mid A)\times P(A)}{P(S')}$
$= \frac{0.79\times 0.32}{0.7817} = 0.323.$

(c) $P(S') = 1 - P(S) = 1 - 0.2183 = 0.7817$

1.10.37 $\frac{0.16\times 0.81}{(0.16\times 0.81)+(0.84\times 0.09)} = 0.6316$