Binary Search Trees: Basic Operations

[Data Structures]

[Data Structures and Algorithms]

Learning Objectives

- Implement basic operations on Binary Search Trees.
- Understand some of the difficulties with making updates.

Outline

Find

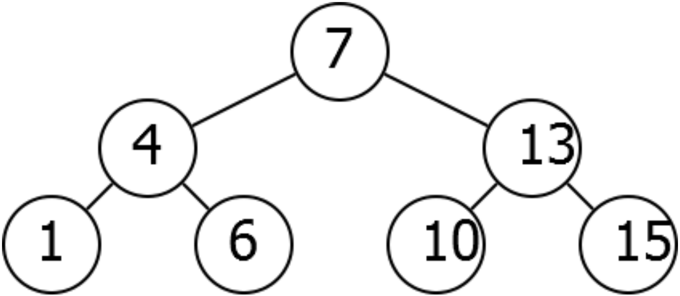
Find

Input: Key k, Root R

Output: The node in the tree of R with key k

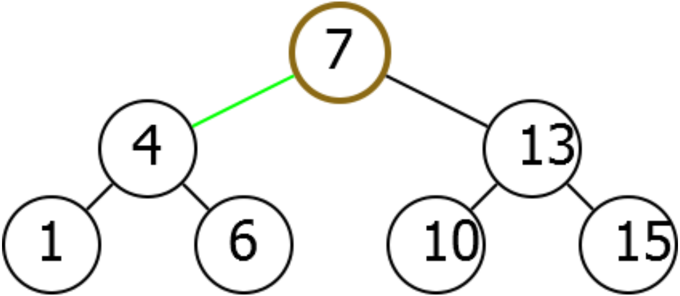
Idea

`Find`(6)



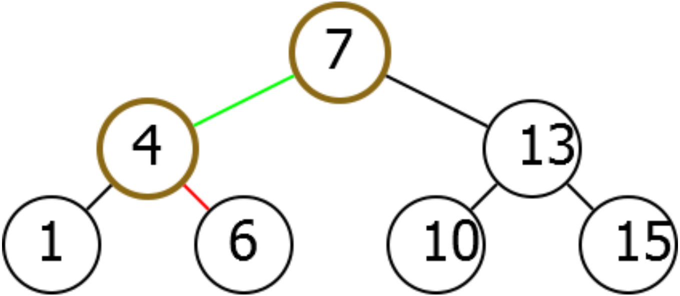
Idea

`Find`(6)



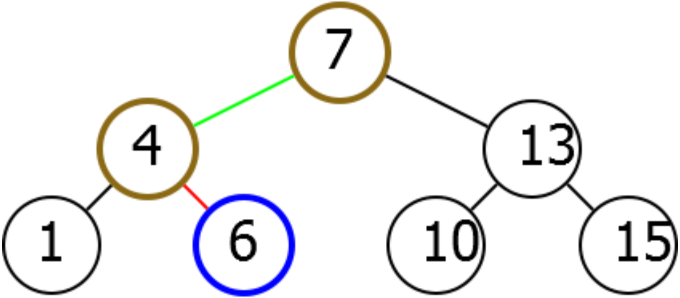
Idea

`Find`(6)



Idea

`Find`(6)



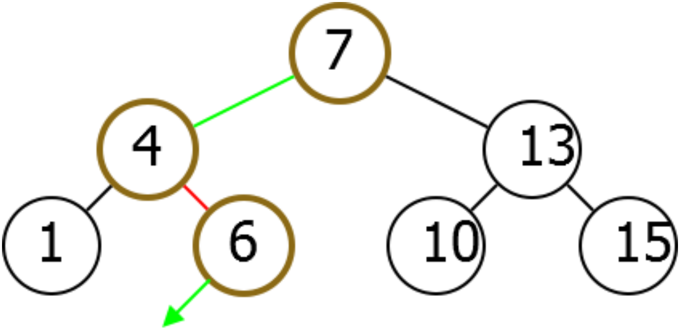
Algorithm

$\texttt{Find}(k, R)$

```
if R.Key = k:
  return R
else if R.Key > k:
  return Find(k, R.Left)
else if R.Key < k:
  return Find(k, R.Right)
```

Missing Key

Run `Find`(5).



Key not in tree. Did find point where it should be.

Missing Key

If you stop before reaching a null pointer, you find the place in the tree where *k* would fit.

Modification

```
Find (modified)
```

else if $R.\text{Key} > k$:

  if $R.\text{Left} \neq \text{null}$:

    return $\texttt{Find}(k, R.\text{Left})$

  return R

Outline

1. Find
2. Next Element
3. Search
4. Insert
5. Delete

Adjacent Elements

Given a node *N* in a Binary Search Tree, would like to find adjacent elements.

Next

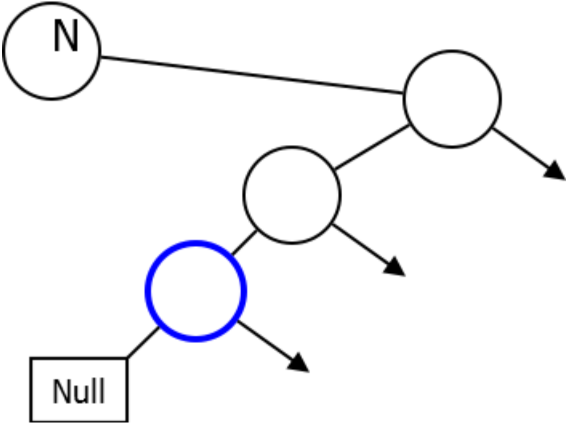
Next

Input: Node N

Output: The node in the tree with the next largest key.

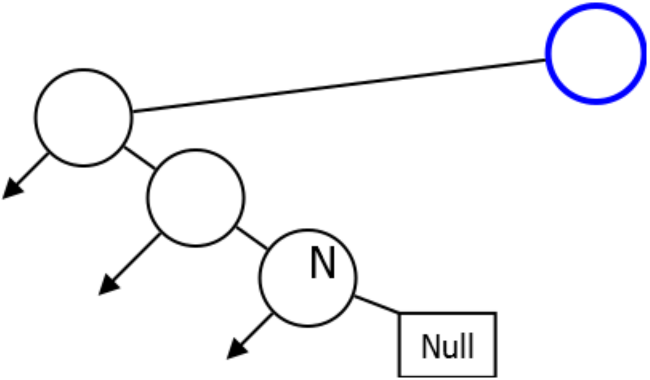
Case I

If you have right child.



Case II

No right child.

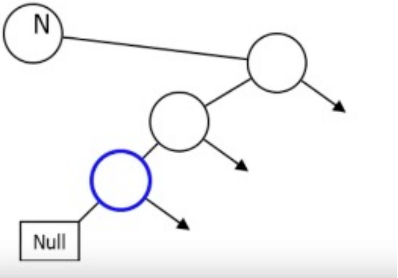


Next

```
Next(N)
```

if N.Right $\neq$ null:
  return LeftDescendant(N.Right)
else:
  return RightAncestor(N)

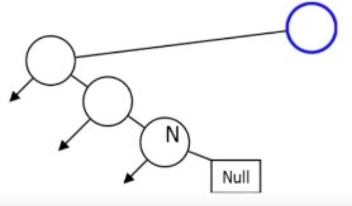
Left Descendant



LeftDescendant(N)

```
if N.Left = null
  return N
else:
  return LeftDescendant(N.Left)
```

Right Ancestor



RightAncestor(N)

```
if N.Key < N.Parent.Key

return N.Parent
else:
   return RightAncestor(N.Parent)
```

Range Search

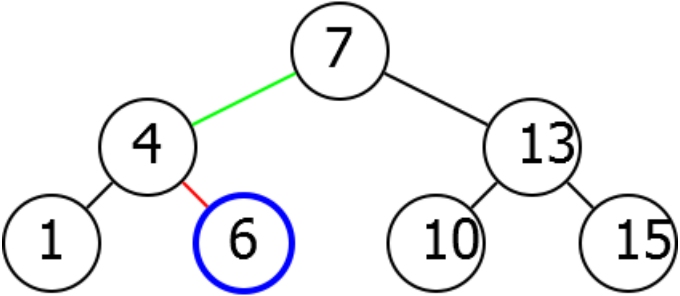
Range Search

Input: Numbers x, y, root R

Output: A list of nodes with key between x and y

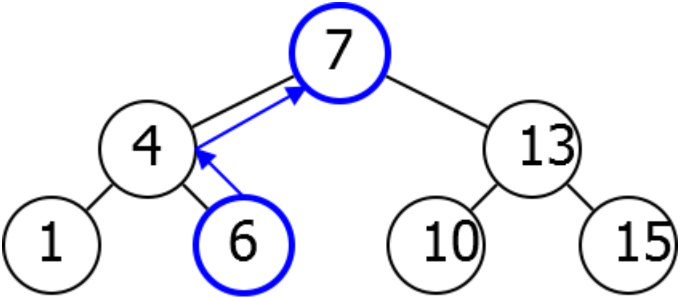
Idea

`RangeSearch`(5, 12).



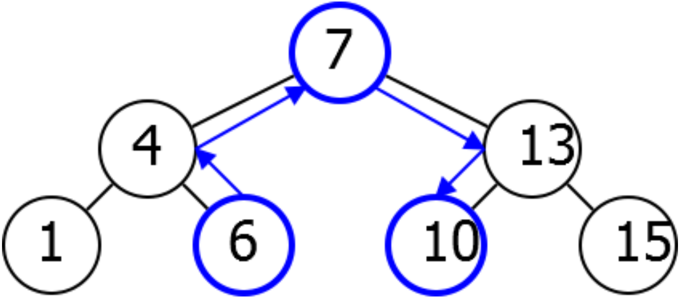
Idea

`RangeSearch`(5, 12).



Idea

`RangeSearch`(5, 12).



Implementation

RangeSearch(x, y, R)

```
L ← ∅
N ← Find(x, R)
while N.Key ≤ y
   if N.Key ≥ x:
      L ← L.Append(N)
      N ← Next(N)
return L
```

Insert

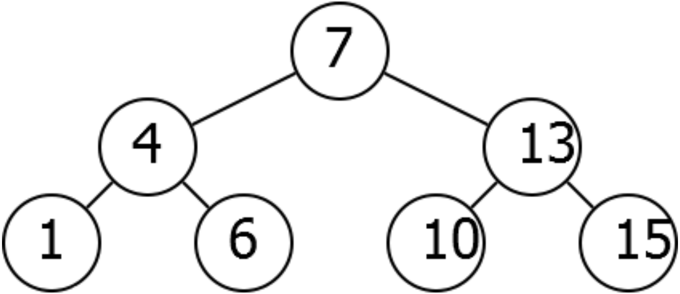
Insert

Input: Key k and root R

Output: Adds node with key k to the tree

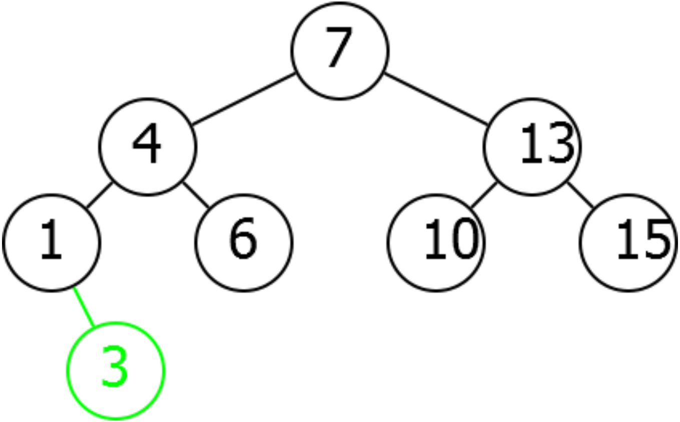
Insert Idea

`Insert`(3)



Insert Idea

`Insert`(3)



Implementation

Insert(k, R)

$P \leftarrow$ `Find`(k, R)

`Add new node with key` k `as child of` P

Delete

Delete

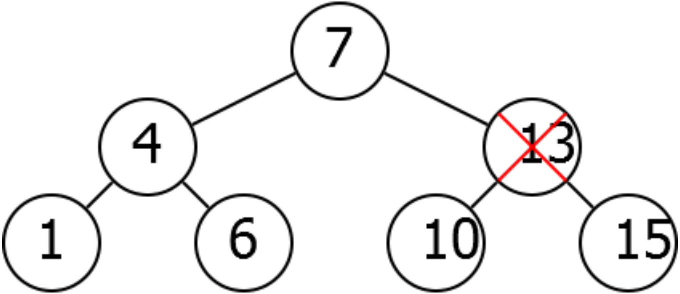
Input: Node N

Output: Removes node N from the tree

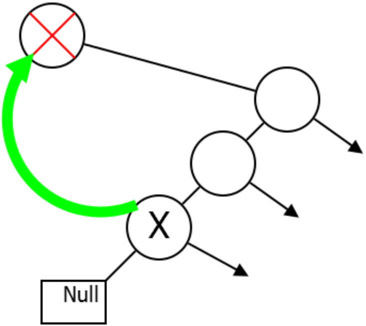
Difficulty

Cannot simply remove.

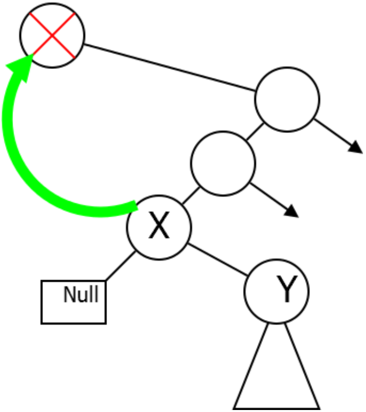
`Delete`(13)



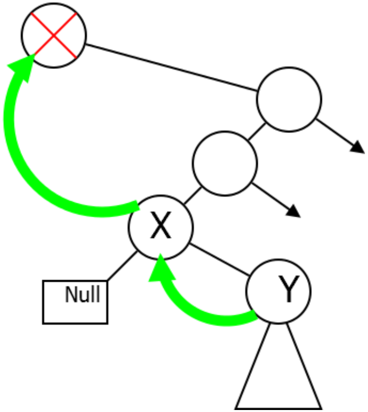
Idea



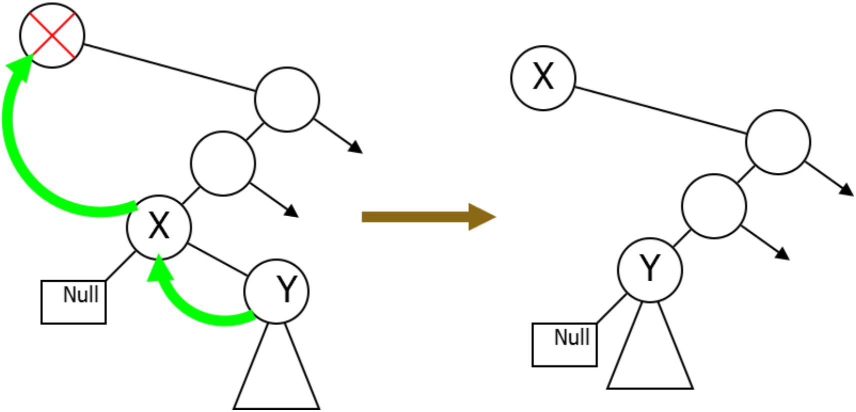
Idea



Idea



Idea



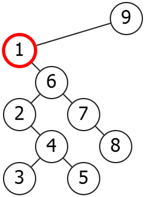
Implementation

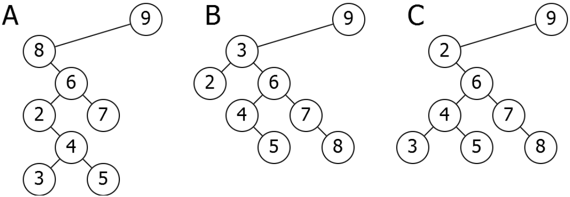
```
Delete(N)

if N.Right = null:
  Remove N, promote
N.Left else:
  X ← Next(N)
  \\ X.Left =
       null
  Replace N by X, promote
  X.Right
```

Problem

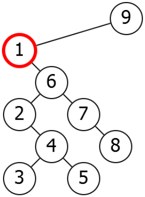
Which of the following trees is obtained when the selected node is deleted?



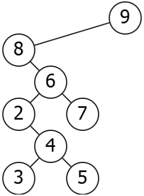


Problem

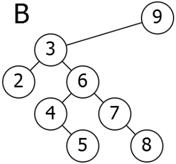
Which of the following trees is obtained when the selected node is deleted?



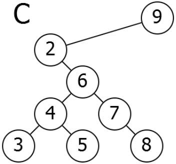
A



B



C

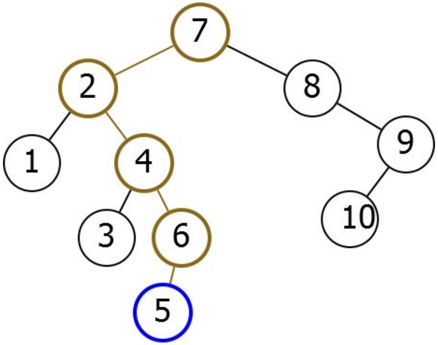


Runtime

How long do Binary Search Tree operations take?

Find

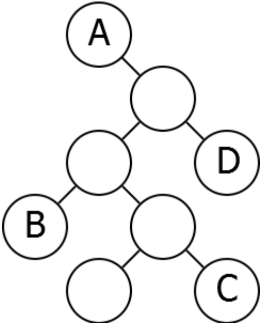
`Find`(5)



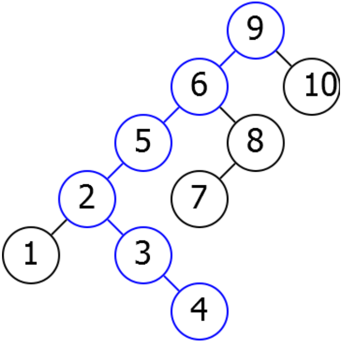
Number of operations = O(Depth)

Problem

Which nodes will be faster to search for in the following tree?



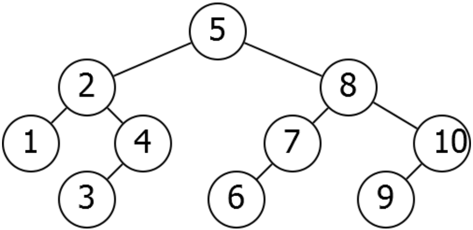
Example I



Depth can be as bad as *n*.

Outline

Example II



Depth can be much smaller.

Balance

- Want left and right subtrees to have approximately the same size.

Balance

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- Suppose perfectly balanced:

Balance

- Want left and right subtrees to have approximately the same size.
- Suppose perfectly balanced:
  - Each subtree half the size of its parent.
  - After $\log_2(n)$ levels, subtree of size
  - 1.

    Operations run in $O(\log(n))$ time.