

PHY 107

1D motion

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OUTLINE

- ▶ Motion
- ▶ Position and Displacement
- ▶ Velocity
- ▶ Acceleration
- ▶ Concept of instantaneous and average quantities
- ▶ Constant Acceleration Equations
- ▶ Free Fall
- ▶ Graphical Integration

Motion

Kinematics: Classification and comparison of motions

Assumptions:

1. Motion is along a straight line (horizontal, vertical, slanted)
2. No interest in force
3. The object is treated as a particle

Displacement

It means change in position.

Displacement is a vector quantity (magnitude and direction)

1. A particle moves from $x=5\text{m}$ to $x=7\text{m} \rightarrow \text{Displacement} = 2\text{m}$
2. A particle is at position $x=2\text{m}$. It then moves to $x=100\text{m}$ and finally comes to position $x=2\text{m}$

Displacement = $x_2 - x_1 = 0\text{ m}$

What about distance?

Average Velocity

It is the ratio of the displacement Δx that occurs in a time interval Δt to the time interval Δt

$$v_{avg} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1}$$

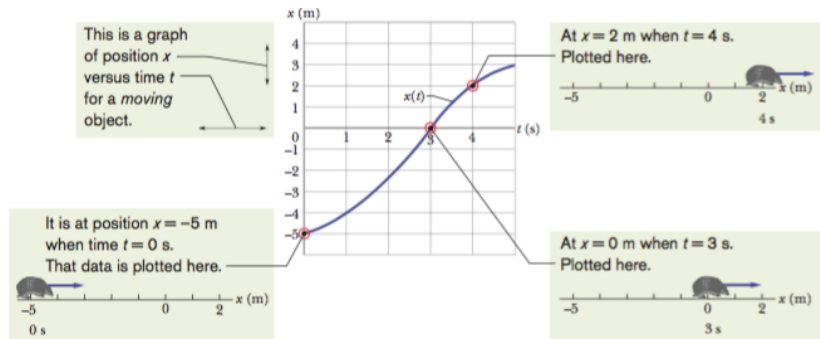
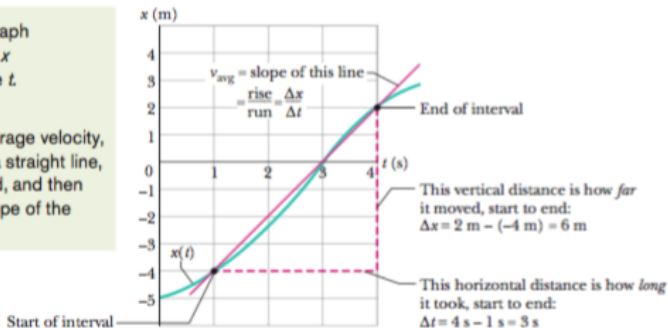


Fig. 2-3 The graph of $x(t)$ for a moving armadillo. The path associated with the graph is also shown, at three times.

Average Velocity

This is a graph of position x versus time t .

To find average velocity, first draw a straight line, start to end, and then find the slope of the line.



$$v_{avg} = \frac{\Delta x}{\Delta t} = \frac{6 \text{ m}}{3 \text{ s}} = 2 \text{ m/s}$$

Instantaneous Velocity

The velocity at any instant is obtained from average velocity by shrinking the time interval Δt closer to zero (but NOT equal to zero)

$$v_{avg} = \lim_{\Delta t \rightarrow 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

Calculus comes into play!

Acceleration

Acceleration is the change in velocity in a given amount of time

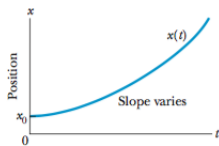
$$a_{avg} = \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{t_2 - t_1}$$

Instantaneous acceleration: $a = \frac{dv}{dt} = \frac{d}{dt} \frac{dx}{dt} = \frac{d^2x}{dt^2}$

Example A particle's position on the x-axis is given by $x = 4 - 27t + t^3$ with x in meters and t in seconds

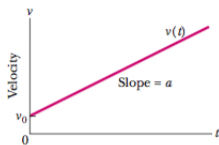
1. Find $v(t)$ and $a(t)$
2. Is there ever a time when $v=0$?
3. Describe the particle's motion for $t \geq 0$

Constant acceleration



(a)

Slopes of the position graph are plotted on the velocity graph.



(b)

Slope of the velocity graph is plotted on the acceleration graph.



(c)

Table 2-1

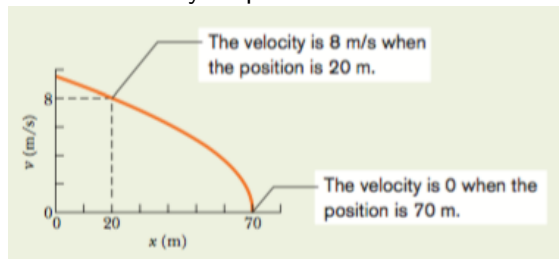
Equations for Motion with Constant Acceleration^a

Equation Number	Equation	Missing Quantity
2-11	$v = v_0 + at$	$x - x_0$
2-15	$x - x_0 = v_0t + \frac{1}{2}at^2$	v
2-16	$v^2 = v_0^2 + 2a(x - x_0)$	t
2-17	$x - x_0 = \frac{1}{2}(v_0 + v)t$	a
2-18	$x - x_0 = vt - \frac{1}{2}at^2$	v_0

^aMake sure that the acceleration is indeed constant before using the equations in this table.

Constant acceleration

EXAMPLE The plot below shows a particle's velocity versus its position as it moves along an x axis with constant acceleration. Find its velocity at position $x=0$.



1. $v^2 = v_0^2 + 2a(x - x_0)$
2. Use two points on the curve to find acceleration.
3. Use the computed a to find $v(x = 0)$

Free Fall Acceleration

Toss an object up or down

Eliminate effects of air on the flight

Free fall acceleration: constant downward acceleration of the object

Mass, density or shape have no impact on this acceleration.



Fig. 2-10 A feather and an apple free fall in vacuum at the same magnitude of acceleration g . The acceleration increases the distance between successive images. In the absence of air, the feather and apple fall together. (Jim Sugar/Corbis Images)

The directions of motion are along the y axis with the positive direction of y upward.

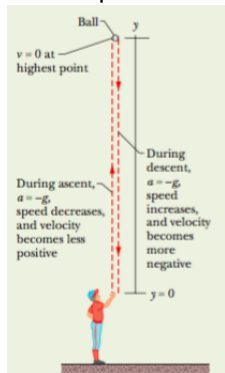
The free-fall acceleration near Earth's surface:

$$a = -g = -9.8\text{m/s}^2$$

Free fall acceleration

EXAMPLE A pitcher tosses a baseball up along a y axis, with an initial speed of 12 m/s .

- How long does the ball take to reach its maximum height?
- What is the ball's max height above its release point?
- How long does the ball take to reach a point 5.0 m above its release point?



Free fall acceleration

$$(a) t = \frac{v-v_0}{a} \text{ Ans: } 1.2 \text{ s}$$

$$(b) y = \frac{v^2-v_0^2}{2a} \text{ Ans: } 7.3 \text{ m}$$

$$(c) y = v_0 t - 0.5gt^2$$

$$5 = 12t - 0.5(9.8)t^2$$

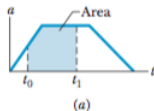
$$4.9t^2 - 12t + 5 = 0$$

$$t = 0.53 \text{ s and } t = 1.9 \text{ s}$$

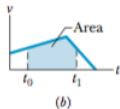
Graphical Integration in Motion Analysis

$$a = \frac{dv}{dt}: \text{Fundamental theorem of Calculus: } v_1 - v_0 = \int_{t_0}^{t_1} a \, dt$$

$$v = \frac{dx}{dt}: \text{Fundamental theorem of Calculus: } x_1 - x_0 = \int_{t_0}^{t_1} v \, dt$$



This area gives the change in velocity.



This area gives the change in position.

Reference

[1] Fundamentals of Physics by Halliday and Resnik