Answers to Exercises

Exercise Set 1.1 (page 6)

1. (a), (c), (f)

3.

(a) $$\begin{aligned} x &= \frac{3}{7} + \frac{5}{7}t \\ y &= t \end{aligned}$$

(b) $$\begin{aligned} x_1 &= \frac{5}{3}s - \frac{4}{3}t + \frac{7}{3} \\ x_2 &= s \\ x_3 &= t \end{aligned} \qquad \begin{aligned} x_1 &= \frac{1}{4}r - \frac{5}{8}s + \frac{3}{4}t - \frac{1}{8} \\ x_2 &= r \\ x_3 &= s \\ x_4 &= t \end{aligned} \qquad \begin{aligned} v &= \frac{8}{3}q - \frac{2}{3}r + \frac{1}{3}s - \frac{4}{3}t \\ w &= q \\ x &= r \\ y &= s \\ z &= t \end{aligned}$$

4.

(a) $$\begin{bmatrix} 3 & -2 & -1 \\ 4 & 5 & 3 \\ 7 & 3 & 2 \end{bmatrix}$$

(b) $$\begin{bmatrix} 2 & 0 & 2 & 1 \\ 3 & -1 & 4 & 7 \\ 6 & 1 & -1 & 0 \end{bmatrix}$$

(c) $$\begin{bmatrix} 1 & 2 & 0 & -1 & 1 & 1 \\ 0 & 3 & 1 & 0 & -1 & 2 \\ 0 & 0 & 1 & 7 & 0 & 1 \end{bmatrix}$$

(d) $$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{bmatrix}$$

5.

(a) $$\begin{aligned} 2x_1 \phantom{{}- 4x_2} &= 0 \\ 3x_1 - 4x_2 &= 0 \\ x_2 &= 1 \end{aligned}$$

(b) $$\begin{aligned} 3x_1 \phantom{{}+ x_2} - 2x_3 &= 5 \\ 7x_1 + x_2 + 4x_3 &= -3 \\ -2x_2 + x_3 &= 7 \end{aligned}$$

(c) $$\begin{aligned} 7x_1 + 2x_2 + x_3 - 3x_4 &= 5 \\ x_1 + 2x_2 + 4x_3 \phantom{{}- 3x_4} &= 1 \end{aligned}$$

(d)
$$\begin{aligned} x_1 \quad\quad\quad &= 7 \\ x_2 \quad\quad &= -2 \\ x_3 \quad &= 3 \\ x_4 &= 4 \end{aligned}$$

6.

(a) $x - 2y = 5$

(b) Let $x = t$; then $t - 2y = 5$. Solving for *y* yields $y = \frac{1}{2}t - \frac{5}{2}$.

12.

(a) The lines have no common point of intersection.

(b) The lines intersect in exactly one point.

(c) The three lines coincide.

Exercise Set 1.2 (page 19)

1. (a), (b), (c), (d), (h), (i), (j)

3.

(a) Both

(b) Neither

(c) Both

(d) Row-echelon

(e) Neither

(f) Both

4.

(a) $x_1 = -3, x_2 = 0, x_3 = 7$

(b) $x_1 = 7t + 8, x_2 = -3t + 2, x_3 = -t - 5, x_4 = t$

(c) $x_1 = 6s - 3t - 2,\ x_2 = s,\ x_3 = -4t + 7,\ x_4 = -5t + 8,\ x_5 = t$

(d) Inconsistent

6.

(a) $x_1 = 3,\ x_2 = 1,\ x_3 = 2$

(b) $x_1 = -\frac{1}{7} - \frac{3}{7}t,\ x_2 = \frac{1}{7} - \frac{4}{7},\ x_3 = t$

(c) $x = t - 1,\ y = 2s,\ z = s,\ w = t$

(d) Inconsistent

8.

(a) Inconsistent

(b) $x_1 = -4,\ x_2 = 2,\ x_3 = 7$

(c) $x_1 = 3 + 2t,\ x_2 = t$

(d) $x = \frac{8}{5} - \frac{3}{5}t - \frac{3}{5}s,\ y = \frac{1}{10} + \frac{2}{5}t - \frac{1}{10}s,\ z = t,\ w = s$

12. (a), (c), (d)

13.

(a) $x_1 = 0,\ x_2 = 0,\ x_3 = 0$

(b) $x_1 = -s,\ x_2 = -t - s,\ x_3 = 4s,\ x_4 = t$

(c) $w = t,\ x = -t,\ y = t,\ z = 0$

14.

(a) Only the trivial solution

(b) $u = 7s - 5t,\ v = -6s + 4t,\ w = 2s,\ x = 2t$

(c) Only the trivial solution

15. **(a)** $I_1 = -1, I_2 = 0, I_3 = 1, I_4 = 2$

(b) $Z_1 = -s - t, Z_2 = s, Z_3 = -t, Z_4 = 0, Z_5 = t$

19. $\begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$ and $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ are possible answers.

20. $\alpha = \pi / 2, \beta = \pi, \gamma = 0$

23. If $\lambda = 1$, then $x_1 = x_2 = -\frac{1}{2}s, x_3 = s$

If $\lambda = 2$, then $x_1 = -\frac{1}{2}s, x_2 = 0, x_3 = s$

24. $x = -13/7, y = 91/54, z = -91/8$

25. $a = 1, b = -6, c = 2, d = 10$

30. **(a)** Three lines, at least two of which are distinct

(b) Three identical lines

32. **(a)** False

(b) False

(c) False

(d) False

Exercise Set 1.3 (page 34)

1. **(a)** Undefined

4×2

(b)

Undefined

(c)

Undefined

(d)

5×5

(e)

5×2

(f)

Undefined

(g)

5×2

(h)

2. $a = 5, b = -3, c = 4, d = 1$

4.

(a) $\begin{bmatrix} 7 & 2 & 4 \\ 3 & 5 & 7 \end{bmatrix}$

(b) $\begin{bmatrix} -5 & 0 & -1 \\ 4 & -1 & 1 \\ -1 & -1 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} -5 & 0 & -1 \\ 4 & -1 & 1 \\ -1 & -1 & 1 \end{bmatrix}$

Undefined

(d)

(e) $\begin{bmatrix} -\frac{1}{4} & \frac{3}{2} \\ \frac{9}{4} & 0 \\ \frac{3}{4} & \frac{9}{4} \end{bmatrix}$

(f) $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

(g) $\begin{bmatrix} 9 & 1 & -1 \\ -13 & 2 & -4 \\ 0 & 1 & -6 \end{bmatrix}$

(h) $\begin{bmatrix} 9 & -13 & 0 \\ 1 & 2 & 1 \\ -1 & -4 & -6 \end{bmatrix}$

5.

(a) $\begin{bmatrix} 12 & -3 \\ -4 & 5 \\ 4 & 1 \end{bmatrix}$

(b) Undefined

(c) $\begin{bmatrix} 42 & 108 & 75 \\ 12 & -3 & 21 \\ 36 & 78 & 63 \end{bmatrix}$

(d) $\begin{bmatrix} 3 & 45 & 9 \\ 11 & -11 & 17 \\ 7 & 17 & 13 \end{bmatrix}$

(e) $\begin{bmatrix} 3 & 45 & 9 \\ 11 & -11 & 17 \\ 7 & 17 & 13 \end{bmatrix}$

(f) $\begin{bmatrix} 21 & 17 \\ 17 & 35 \end{bmatrix}$

(g) $\begin{bmatrix} 0 & -2 & 11 \\ 12 & 1 & 8 \end{bmatrix}$

(h) $\begin{bmatrix} 12 & 6 & 9 \\ 48 & -20 & 14 \\ 24 & 8 & 16 \end{bmatrix}$

(i) 61

(j) 35

(k) (28)

8.

(a)

$$\begin{bmatrix} 67 \\ 64 \\ 63 \end{bmatrix} = 6\begin{bmatrix} 3 \\ 6 \\ 0 \end{bmatrix} + 0\begin{bmatrix} -2 \\ 5 \\ 4 \end{bmatrix} + 7\begin{bmatrix} 7 \\ 4 \\ 9 \end{bmatrix}$$

$$\begin{bmatrix} 41 \\ 21 \\ 67 \end{bmatrix} = -2\begin{bmatrix} 3 \\ 6 \\ 0 \end{bmatrix} + 1\begin{bmatrix} -2 \\ 5 \\ 4 \end{bmatrix} + 7\begin{bmatrix} 7 \\ 4 \\ 9 \end{bmatrix}$$

$$\begin{bmatrix} 41 \\ 59 \\ 57 \end{bmatrix} = 4\begin{bmatrix} 3 \\ 6 \\ 0 \end{bmatrix} + 3\begin{bmatrix} -2 \\ 5 \\ 4 \end{bmatrix} + 5\begin{bmatrix} 7 \\ 4 \\ 9 \end{bmatrix}$$

(b) $$\begin{bmatrix} 6 \\ 6 \\ 63 \end{bmatrix} = 3\begin{bmatrix} 6 \\ 0 \\ 7 \end{bmatrix} + 6\begin{bmatrix} -2 \\ 1 \\ 7 \end{bmatrix} + 0\begin{bmatrix} 4 \\ 3 \\ 5 \end{bmatrix}$$

$$\begin{bmatrix} -6 \\ 17 \\ 41 \end{bmatrix} = -2\begin{bmatrix} 6 \\ 0 \\ 7 \end{bmatrix} + 5\begin{bmatrix} -2 \\ 1 \\ 7 \end{bmatrix} + 4\begin{bmatrix} 4 \\ 3 \\ 5 \end{bmatrix}$$

$$\begin{bmatrix} 70 \\ 31 \\ 122 \end{bmatrix} = 7\begin{bmatrix} 6 \\ 0 \\ 7 \end{bmatrix} + 4\begin{bmatrix} -2 \\ 1 \\ 7 \end{bmatrix} + 9\begin{bmatrix} 4 \\ 3 \\ 5 \end{bmatrix}$$

13.

(a) $$A = \begin{bmatrix} 2 & -3 & 5 \\ 9 & -1 & 1 \\ 1 & 5 & 4 \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 7 \\ -1 \\ 0 \end{bmatrix}$$

(b) $$A = \begin{bmatrix} 4 & 0 & -3 & 1 \\ 5 & 1 & 0 & -8 \\ 2 & -5 & 9 & -1 \\ 0 & 3 & -1 & 7 \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 1 \\ 3 \\ 0 \\ 2 \end{bmatrix}$$

16.

(a) $$\begin{bmatrix} -3 & -15 & -11 \\ 21 & -15 & 44 \end{bmatrix}$$

(b) $$\begin{bmatrix} 4 & -7 & -19 & -43 \\ 2 & 2 & 18 & 17 \\ 0 & 5 & 25 & 35 \\ 2 & 3 & 23 & 24 \end{bmatrix}$$

(c) $$\begin{bmatrix} 3 & 3 \\ -1 & 4 \\ 1 & 5 \\ 4 & -4 \\ 0 & 14 \end{bmatrix}$$

17.

(a) A_{11} is a 2×3 matrix and B_{11} is a 2×2 matrix. $A_{11}B_{11}$ does not exist.

(b) $$\begin{bmatrix} -1 & 23 & -10 \\ 37 & -13 & 8 \\ 29 & 23 & 41 \end{bmatrix}$$

21.

(a) $$\begin{bmatrix} a_{11} & 0 & 0 & 0 & 0 & 0 \\ 0 & a_{22} & 0 & 0 & 0 & 0 \\ 0 & 0 & a_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & a_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & a_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & a_{66} \end{bmatrix}$$

(b) $\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} & a_{16} \\ 0 & a_{22} & a_{23} & a_{24} & a_{25} & a_{26} \\ 0 & 0 & a_{33} & a_{34} & a_{35} & a_{36} \\ 0 & 0 & 0 & a_{44} & a_{45} & a_{46} \\ 0 & 0 & 0 & 0 & a_{55} & a_{56} \\ 0 & 0 & 0 & 0 & 0 & a_{66} \end{bmatrix}$

(c) $\begin{bmatrix} a_{11} & 0 & 0 & 0 & 0 & 0 \\ a_{21} & a_{22} & 0 & 0 & 0 & 0 \\ a_{31} & a_{32} & a_{33} & 0 & 0 & 0 \\ a_{41} & a_{42} & a_{43} & a_{44} & 0 & 0 \\ a_{51} & a_{52} & a_{53} & a_{54} & a_{55} & 0 \\ a_{61} & a_{62} & a_{63} & a_{64} & a_{65} & a_{66} \end{bmatrix}$

(d) $\begin{bmatrix} a_{11} & a_{12} & 0 & 0 & 0 & 0 \\ a_{21} & a_{22} & a_{23} & 0 & 0 & 0 \\ 0 & a_{32} & a_{33} & a_{34} & 0 & 0 \\ 0 & 0 & a_{43} & a_{44} & a_{45} & 0 \\ 0 & 0 & 0 & a_{54} & a_{55} & a_{56} \\ 0 & 0 & 0 & 0 & a_{65} & a_{66} \end{bmatrix}$

27. One; namely, $A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

30.

(a) Yes; for example, $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$

(b) Yes; for example, $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$

32.

(a) True

(b) False; for example, $A = \begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix}$

(c) True

(d) True

Exercise Set 1.4 (page 48)

4. $A^{-1}=\begin{bmatrix} 2 & -1 \\ -5 & 3 \end{bmatrix}$, $B^{-1}=\begin{bmatrix} \frac{1}{5} & \frac{3}{20} \\ -\frac{1}{5} & \frac{1}{10} \end{bmatrix}$, $C^{-1}=\begin{bmatrix} -\frac{1}{2} & -2 \\ 1 & 3 \end{bmatrix}$, $D^{-1}=\begin{bmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{3} \end{bmatrix}$

7.

(a) $A=\begin{bmatrix} \frac{5}{13} & \frac{1}{13} \\ -\frac{3}{13} & \frac{2}{13} \end{bmatrix}$

(b) $A=\begin{bmatrix} \frac{2}{7} & 1 \\ \frac{1}{7} & \frac{3}{7} \end{bmatrix}$

(c) $A=\begin{bmatrix} -\frac{2}{5} & 1 \\ -\frac{1}{5} & \frac{3}{5} \end{bmatrix}$

(d) $A=\begin{bmatrix} -\frac{9}{13} & \frac{1}{13} \\ \frac{2}{13} & -\frac{6}{13} \end{bmatrix}$

9.

(a) $p(A)=\begin{bmatrix} 1 & 1 \\ 2 & -1 \end{bmatrix}$

(b) $p(A)=\begin{bmatrix} 20 & 7 \\ 14 & 6 \end{bmatrix}$

(c) $p(A)=\begin{bmatrix} 39 & 13 \\ 26 & 13 \end{bmatrix}$

11. $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

13.

$$A^{-1}=\begin{bmatrix} \frac{1}{a_{11}} & 0 & \cdots & 0 \\ 0 & \frac{1}{a_{22}} & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & \frac{1}{a_{nn}} \end{bmatrix}$$

18. $C=-A^{-1}BA^{-1}$

19.

(a) $\begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & -\frac{1}{2} \\ -1 & 0 & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$

(b) $\begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

20.

(a) One example is $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \\ 3 & 4 & 5 \end{bmatrix}$.

(b) One example is $\begin{bmatrix} 0 & -1 & -1 \\ 1 & 0 & -1 \\ 1 & 1 & 0 \end{bmatrix}$.

22. Yes

23.

$$A^{-1} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

33. $\begin{bmatrix} \pm 1 & 0 & 0 \\ 0 & \pm 1 & 0 \\ 0 & 0 & \pm 1 \end{bmatrix}$

34.

(a) If A is invertible, then A^T is invertible.

(b) True

Exercise Set 1.5 (page 57)

1. (a), (c), (d), (f)

3.

(a) $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

(b) $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}$

6.

(a) $\begin{bmatrix} -7 & 4 \\ 2 & -1 \end{bmatrix}$

(b) $\begin{bmatrix} -\frac{5}{39} & \frac{2}{13} \\ \frac{4}{39} & \frac{1}{13} \end{bmatrix}$

(c) Not invertible

8.

(a) $\begin{bmatrix} 1 & 3 & 1 \\ 0 & 1 & -1 \\ -2 & 2 & 0 \end{bmatrix}$

(b) $\begin{bmatrix} \frac{\sqrt{2}}{26} & \frac{-3\sqrt{2}}{26} & 0 \\ \frac{4\sqrt{2}}{26} & \frac{\sqrt{2}}{26} & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ -\frac{1}{3} & \frac{1}{3} & 0 & 0 \\ 0 & -\frac{1}{5} & \frac{1}{5} & 0 \\ 0 & 0 & -\frac{1}{7} & \frac{1}{7} \end{bmatrix}$

(d) Not invertible

(e) $\begin{bmatrix} -\frac{4}{5} & \frac{3}{5} & \frac{1}{5} & \frac{1}{5} \\ \frac{3}{2} & 0 & -1 & 0 \\ \frac{1}{2} & 0 & 0 & 0 \\ \frac{4}{5} & \frac{2}{5} & -\frac{1}{5} & -\frac{1}{5} \end{bmatrix}$

10. **(a)** $E_1 = \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix}, \quad E_2 = \begin{bmatrix} 1 & 0 \\ 0 & \frac{1}{2} \end{bmatrix}$

(b) $A^{-1} = E_2 E_1$

(c) $A = E_1^{-1} E_2^{-1}$

11. **(a)** $\begin{bmatrix} 1 & -4 & 7 \\ 4 & 5 & -3 \\ 2 & -1 & 0 \end{bmatrix}$

(b) $\begin{bmatrix} 2 & -1 & 0 \\ \frac{4}{3} & \frac{5}{3} & -1 \\ 1 & -4 & 7 \end{bmatrix}$

(c) $\begin{bmatrix} 10 & 9 & -6 \\ 4 & 5 & -3 \\ 1 & -4 & 7 \end{bmatrix}$

14. $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 3 & 3 & 8 \\ 0 & 1 & 7 & 8 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

19. **(b)** Add −1 times the first row to the second row.

Add −1 times the first row to the third row.

Add −1 times the second row to the first row.

Add the second row to the third row.

24. In general, no. Try $b = 1, a = c = d = 0$.

Exercise Set 1.6 (page 66)

1. $x_1 = 3, x_2 = -1$

4. $x_1 = 1, x_2 = -11, x_3 = 16$

6. $w = -6, x = 1, y = 10, z = -7$

9.

(a) $x_1 = \frac{16}{3}, x_2 = -\frac{4}{3}, x_3 = -\frac{11}{3}$

(b) $x_1 = -\frac{5}{3}, x_2 = \frac{5}{3}, x_3 = \frac{10}{3}$

(c) $x_1 = 3, x_2 = 0, x_3 = -4$

11.

(a) $x_1 = \frac{22}{17}, x_2 = \frac{1}{17}$

(b) $x_1 = \frac{21}{17}, x_2 = \frac{11}{17}$

13.

(a) $x_1 = \frac{7}{15}, x_2 = \frac{4}{15}$

(b) $x_1 = \frac{34}{15}, x_2 = \frac{28}{15}$

(c) $x_1 = \frac{19}{15}, x_2 = \frac{13}{15}$

(d) $x_1 = -\frac{1}{5}, x_2 = \frac{3}{5}$

15.

(a) $x_1 = -12 - 3t, x_2 = -5 - t, x_3 = t$

(b) $x_1 = 7 - 3t, x_2 = 3 - t, x_3 = t$

19. $b_1 = b_3 + b_4, b_2 = 2b_3 + b_4$

21. $X = \begin{bmatrix} 11 & 12 & -3 & 27 & 26 \\ -6 & -8 & 1 & -18 & -17 \\ -15 & -21 & 9 & -38 & -35 \end{bmatrix}$

22.

(a) Only the trivial solution $x_1 = x_2 = x_3 = x_4 = 0$; invertible

(b) Infinitely many solutions; not invertible

28.

(a) $I - A$ is invertible.

(b) $\mathbf{x} = (I - A)^{-1}\mathbf{b}$

30. Yes, for nonsquare matrices

Exercise Set 1.7 (page 73)

1.

(a) $\begin{bmatrix} \frac{1}{2} & 0 \\ 0 & -\frac{1}{5} \end{bmatrix}$

(b) Not invertible

(c) $\begin{bmatrix} -1 & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & 3 \end{bmatrix}$

3.

(a) $A^2 = \begin{bmatrix} 1 & 0 \\ 0 & 4 \end{bmatrix}$, $A^{-2} = \begin{bmatrix} 1 & 0 \\ 0 & \frac{1}{4} \end{bmatrix}$, $A^{-k} = \begin{bmatrix} 1 & 0 \\ 0 & 1/(-2)^k \end{bmatrix}$

(b) $A^2 = \begin{bmatrix} \frac{1}{4} & 0 & 0 \\ 0 & \frac{1}{9} & 0 \\ 0 & 0 & \frac{1}{16} \end{bmatrix}$, $A^{-2} = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 16 \end{bmatrix}$, $A^{-k} = \begin{bmatrix} 2^k & 0 & 0 \\ 0 & 3^k & 0 \\ 0 & 0 & 4^k \end{bmatrix}$

5. (a)

7. $a = 2, b = -1$

10.

(a) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

(b) $\begin{bmatrix} \pm\frac{1}{3} & 0 & 0 \\ 0 & \pm\frac{1}{2} & 0 \\ 0 & 0 & \pm 1 \end{bmatrix}$

11.

(a) $\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}\begin{bmatrix} 3 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 7 \end{bmatrix}$

(b) No

16.

(b) Yes

17. Yes

19. $\begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$, $\begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & -1 \end{bmatrix}$, $\begin{bmatrix} 4 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 4 \end{bmatrix}$, $\begin{bmatrix} -1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$, $\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 4 \end{bmatrix}$, $\begin{bmatrix} -1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & -1 \end{bmatrix}$, $\begin{bmatrix} 4 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$, $\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

20.

(a) Yes

(b) No (unless $n = 1$)

(c) Yes

(d) No (unless $n = 1$)

24.

(a) $x_1 = \frac{7}{4}, x_2 = 1, x_3 = -\frac{1}{2}$

(b) $x_1 = -8, x_2 = -4, x_3 = 3$

25. $A = \begin{bmatrix} 1 & 10 \\ 0 & -2 \end{bmatrix}$

26. $\frac{n}{2}(1 + n)$

Supplementary Exercises (page 76)

1. $x' = \frac{3}{5}x + \frac{4}{5}y,\ y' = -\frac{4}{5}x + \frac{3}{5}y$

3. One possible answer is

$$\begin{aligned} x_1 - 2x_2 - x_3 - x_4 &= 0 \\ x_1 + 5x_2 + 2x_4 &= 0 \end{aligned}$$

5. $x = 4,\ y = 2,\ z = 3$

7.

(a) $a \neq 0,\ b \neq 2$

(b) $a \neq 0,\ b = 2$

(c) $a = 0,\ b = 2$

(d) $a = 0,\ b \neq 2$

9. $K = \begin{bmatrix} 0 & 2 \\ 1 & 1 \end{bmatrix}$

11.

(a) $X = \begin{bmatrix} -1 & 3 & -1 \\ 6 & 0 & 1 \end{bmatrix}$

(b) $X = \begin{bmatrix} 1 & -2 \\ 3 & 1 \end{bmatrix}$

(c) $X = \begin{bmatrix} -\frac{113}{37} & -\frac{160}{37} \\ -\frac{20}{37} & -\frac{46}{37} \end{bmatrix}$

13. *mpn* multiplications and $mp(n-1)$ additions

15. $a = 1,\ b = -2,\ c = 3$

16. $a = 1,\ b = -4,\ c = -5$

26. $A = -\frac{7}{5},\ B = \frac{4}{5},\ C = \frac{3}{5}$

29. **(b)** $\begin{bmatrix} a^n & 0 & 0 \\ 0 & b^n & 0 \\ d & 0 & c^n \end{bmatrix}$, where $d = \begin{cases} \dfrac{a^n - c^n}{a - c} & \text{if } a \neq c \\ na^{n-1} & \text{if } a = c \end{cases}$

Exercise Set 2.1 (page 94)

1. **(a)** $M_{11} = 29, M_{12} = 21, M_{13} = 27, M_{21} = -11, M_{22} = 13, M_{23} = -5, M_{31} = -19, M_{32} = -19, M_{33} = 19$

(b) $C_{11} = 29, C_{12} = -21, C_{13} = 27, C_{21} = 11, C_{22} = 13, C_{23} = 5, C_{31} = -19, C_{32} = 19, C_{33} = 19$

3. 152

4. **(a)** $\operatorname{adj}(A) = \begin{bmatrix} 29 & 11 & -19 \\ -21 & 13 & 19 \\ 27 & 5 & 19 \end{bmatrix}$

(b) $A^{-1} = \begin{bmatrix} \frac{29}{152} & \frac{11}{152} & -\frac{19}{152} \\ -\frac{21}{152} & \frac{13}{152} & \frac{19}{152} \\ \frac{27}{152} & \frac{5}{152} & \frac{19}{152} \end{bmatrix}$

6. −66

8. $k^3 - 8k^2 - 10k + 95$

11. $A^{-1} = \begin{bmatrix} 3 & -5 & -5 \\ -3 & 4 & 5 \\ 2 & -2 & -3 \end{bmatrix}$

13. $A^{-1} = \begin{bmatrix} \frac{1}{2} & \frac{3}{2} & 1 \\ 0 & 1 & \frac{3}{2} \\ 0 & 0 & \frac{1}{2} \end{bmatrix}$

15. $A^{-1} = \begin{bmatrix} -4 & 3 & 0 & -1 \\ 2 & -1 & 0 & 0 \\ -7 & 0 & -1 & 8 \\ 6 & 0 & 1 & -7 \end{bmatrix}$

16. $x_1 = 1, x_2 = 2$

18. $x = -\frac{144}{55}, y = -\frac{61}{55}, z = \frac{46}{11}$

21. Cramer's rule does not apply.

22. $A^{-1} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

24. $x = 1, y = 0, z = 2, w = 0$

31. $\det(A) = 10 \times (-108) = -1080$

34. One

Exercise Set 2.2 (page 101)

2.

(a) −30

(b) −2

(c) 0

(d) 0

4. 30

6. −17

8. 39

11. −2

12.

(a) −6

(b) 72

(c) −6

(d) 18

16.

(a) $\det(A) = -1$

(b) $\det(A) = 1$

18. $x = 0$, −1, $\frac{1}{2}$

Exercise Set 2.3 (page 109)

1.

(a) $\det(2A) = -40 = 2^2 \det(A)$

(b) $\det(-2A) = -448 = (-2)^3 \det(A)$

4.

(a) Invertible

(b) Not invertible

(c) Not invertible

(d) Not invertible

6. If $x = 0$, the first and third rows are proportional.

If $x = 2$, the first and second rows are proportional.

12.

(a) $k = \dfrac{5 \pm \sqrt{17}}{2}$

$k = -1$

(b)

14. **(a)** $\begin{bmatrix} \lambda - 1 & -2 \\ -2 & \lambda - 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

(b) $\begin{bmatrix} \lambda - 2 & -3 \\ -4 & \lambda - 3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

(c) $\begin{bmatrix} \lambda - 3 & -1 \\ 5 & \lambda + 3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

15. **i.** $\lambda^2 - 2\lambda - 3 = 0$

ii. $\lambda = -1, \lambda = 3$

iii. $\begin{bmatrix} -t \\ t \end{bmatrix}, \begin{bmatrix} t \\ t \end{bmatrix}$

i. $\lambda^2 - 5\lambda - 6 = 0$

ii. $\lambda = -1, \lambda = 6$

iii. $\begin{bmatrix} -t \\ t \end{bmatrix}, \begin{bmatrix} \frac{3}{4}t \\ t \end{bmatrix}$

i. $\lambda^2 - 4 = 0$

ii. $\lambda = -2, \lambda = 2$

iii. $\begin{bmatrix} -\frac{t}{5} \\ t \end{bmatrix}, \begin{bmatrix} -t \\ t \end{bmatrix}$

20. No

21. AB is singular.

22. **(a)** Flase

(b) True

(c) Flase

(d) True

23.
(a) True

(b) True

(c) Flase

(d) True

Exercise Set 2.4 (page 117)

1.
(a) 5

(b) 9

(c) 6

(d) 10

(e) 0

(f) 2

3. 22

5. 52

7. $a^2 - 5a + 21$

9. −65

11. −123

13.

(a) $\lambda = 1, \lambda = -3$

(b) $\lambda = -2, \lambda = 3, \lambda = 4$

16. 275

17.

(a) = −120

(b) = −120

18. $x = \frac{3 \pm \sqrt{33}}{4}$

22. Equals 0 if $n > 1$

Supplementary Exercises (page 118)

1. $x' = \frac{3}{5}x + \frac{4}{5}y,\ y' = -\frac{4}{5}x + \frac{3}{5}y$

4. 2

5. $\cos\beta = \frac{c^2 + a^2 - b^2}{2ac},\ \cos\gamma = \frac{a^2 + b^2 - c^2}{2ab}$

12. $\det(B) = (-1)^{n(n-1)/2} \det(A)$

13.

(a) The *i*th and *j*th columns will be interchanged.

(b) The *i*th column will be divided by *c*.

(c) $-c$ times the jth column will be added to the ith column.

15.

(a) $\lambda^3+(-a_{11}-a_{22}-a_{33})\lambda^2$
$+(a_{11}a_{22}+a_{11}a_{33}+a_{22}a_{33}-a_{12}a_{21}-a_{13}a_{31}-a_{23}a_{32})\lambda$
$+(a_{11}a_{23}a_{32}+a_{12}a_{21}a_{33}+a_{13}a_{22}a_{31}-a_{11}a_{22}a_{33}-a_{12}a_{23}a_{31}-a_{13}a_{21}a_{32})$

18.

(a) $\lambda=-5, \lambda=2, \lambda=4; \begin{bmatrix} -2t \\ t \\ t \end{bmatrix}, \begin{bmatrix} 5t \\ t \\ t \end{bmatrix}, \begin{bmatrix} 7t \\ 19t \\ t \end{bmatrix}$

(b) $\lambda=1; \begin{bmatrix} \frac{1}{2}t \\ -\frac{1}{2}t \\ t \end{bmatrix}$

Exercise Set 3.1 (page 130)

3.

(a) $\overrightarrow{P_1P_2}=(-1,-1)$

(b) $\overrightarrow{P_1P_2}=(-7,-2)$

(c) $\overrightarrow{P_1P_2}=(2,1)$

(d) $\overrightarrow{P_1P_2}=(a,b)$

(e) $\overrightarrow{P_1P_2}=(-5,12,-6)$

(f) $\overrightarrow{P_1P_2}=(1,-1,-2)$

(g) $\overrightarrow{P_1P_2}=(-a,-b,-c)$

(h) $\overrightarrow{P_1P_2}=(a,b,c)$

5.

(a) $P(-1,2,-4)$ is one possible answer.

(b) $P(7,-2,-6)$ is one possible answer.

6.

(a) $(-2, 1, -4)$

(b) $(-10, 6, 4)$

(c) $(-7, 1, 10)$

(d) $(80, -20, -80)$

(e) $(132, -24, -72)$

(f) $(-77, 8, 94)$

8. $c_1 = 2, c_2 = -1, c_3 = 2$

10. $c_1 = c_2 = c_3 = 0$

12.

(a) $x' = 5, y' = 8$

(b) $x = -1, y = 3$

15. $\mathbf{u} = \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right), \mathbf{v} = \left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right), \mathbf{u} + \mathbf{v} = \left(\frac{\sqrt{3}-1}{2}, \frac{1-\sqrt{3}}{2}\right), \mathbf{u} - \mathbf{v} = \left(\frac{\sqrt{3}+1}{2}, \frac{\sqrt{3}+1}{2}\right)$

Exercise Set 3.2 (page 134)

1.

(a) 5

(b) $\sqrt{13}$

(c) 5

(d) $2\sqrt{3}$

(e) $3\sqrt{6}$

(f) 6

3.

(a) $\sqrt{83}$

(b) $\sqrt{17} + \sqrt{26}$

(c) $4\sqrt{17}$

(d) $\sqrt{466}$

(e) $\left(\frac{3}{\sqrt{61}}, \frac{6}{\sqrt{61}}, -\frac{4}{\sqrt{61}}\right)$

(f) 1

9.

(b) $\left(\frac{3}{5}, \frac{4}{5}\right)$

(c) $\left(\frac{2}{7}, -\frac{3}{7}, \frac{6}{7}\right)$

10. A sphere of radius 1 centered at (x_0, y_0, z_0)

16.

(a) $a = c = 0$

(b) At least one of *a* or *c* is not zero, that is, $a^2 + c^2 > 0$

17.

(a) The distance from *x* to the origin is less than 1.

(b) $\|x - x_0\| > 1$

Exercise Set 3.3 (page 142)

1.

(a) -11

(b) -24

(c) 0

(d) 0

3.

(a) Orthogonal

(b) Obtuse

(c) Acute

(d) Obtuse

5.

(a) (6, 2)

(b) $\left(-\frac{21}{13}, -\frac{14}{13}\right)$

(c) $\left(\frac{55}{13}, 1, -\frac{11}{13}\right)$

(d) $\left(\frac{73}{89}, -\frac{12}{89}, -\frac{32}{89}\right)$

8.

(b) $(3k, 2k)$ for any scalar k

(c) $\left(\frac{4}{5}, \frac{3}{5}\right), \left(-\frac{4}{5}, -\frac{3}{5}\right)$

11. $\cos\theta_1 = \frac{\sqrt{10}}{10}, \cos\theta_2 = \frac{3\sqrt{10}}{10}, \cos\theta_3 = 0$

13. $\pm\left(1/\sqrt{3}, 1/\sqrt{3}, -1/\sqrt{3}\right)$

16.

(a) $\frac{10}{3}$

(b) $-\frac{6}{5}$

(c) $\frac{-60+34\sqrt{3}}{33}$

(d) $\frac{1}{2}$

20. $\cos^{-1}\left(\frac{2}{\sqrt{6}}\right)$

21.

(b) $\cos\beta = \frac{b}{\|\mathbf{v}\|}, \cos\gamma = \frac{c}{\|\mathbf{v}\|}$

27.

(a) The vector ***u*** is dotted with a scalar.

(b) A scalar is added to the vector ***w***.

(c) Scalars do not have norms.

(d) The scalar *k* is dotted with a vector.

29. No; it merely says that ***u*** is orthogonal to $\mathbf{v}-\mathbf{w}$.

30. $\mathbf{r} = (\mathbf{u}\cdot\mathbf{r})\frac{\mathbf{u}}{\|\mathbf{u}\|^2} + (\mathbf{v}\cdot\mathbf{r})\frac{\mathbf{v}}{\|\mathbf{v}\|^2} + (\mathbf{w}\cdot\mathbf{r})\frac{\mathbf{w}}{\|\mathbf{w}\|^2}$

31. Theorem of Pythagoras

Exercise Set 3.4 (page 153)

1.

(a) (32, −6, −4)

(b) (−14, −20, −82)

(c) (27, 40, −42)

(d) (0, 176, −264)

(e) (−44, 55, −22)

(f) (−8, −3, −8)

3.

(a) $\sqrt{59}$

(b) $\sqrt{101}$

(c) 0

7. For example, $(1, 1, 1) \times (2, -3, 5) = (8, -3, -5)$

9.

(a) −3

(b) 3

(c) 3

(d) −3

(e) −3

(f) 0

11.

(a) No

(b) Yes

(c) No

13. $\left(\frac{6}{\sqrt{61}}, -\frac{3}{\sqrt{61}}, \frac{4}{\sqrt{61}}\right), \left(-\frac{6}{\sqrt{61}}, \frac{3}{\sqrt{61}}, -\frac{4}{\sqrt{61}}\right)$

15. $2(\mathbf{v} \times \mathbf{u})$

17.

(a) $\frac{\sqrt{26}}{2}$

(b) $\frac{\sqrt{26}}{3}$

21.

(a) $\sqrt{122}$

(b) $\theta \approx 40°19''$

23.

(a) $\mathbf{m} = (0, 1, 0)$ and $\mathbf{n} = (1, 0, 0)$

(b) (−1, 0, 0)

(c) (0 ,0, −l)

28. (−8, 0, −8)

31.

(a) $\frac{2}{3}$

(b) $\frac{1}{2}$

35.

(b) $\mathbf{u} \cdot \mathbf{w} \neq 0, \mathbf{v} \cdot \mathbf{w} = 0$

36. No, the equation is equivalent to $\mathbf{u} \times (\mathbf{v} - \mathbf{w}) = 0$ and hence to $\mathbf{v} - \mathbf{w} = k\mathbf{u}$ for some scalar *k*.

38. The are collinear.

Exercise Set 3.5 (page 162)

1.

(a) $-2(x+1)+(y-3)-(z+2)=0$

(b) $(x-1)+9(y-1)+8(z-4)=0$

(c) $2z=0$

(d) $x+2y+3z=0$

3.

(a) (0, 0, 5) is a point in the plane and $\mathbf{n}=(-3, 7, 2)$ is a normal vector so that $-3(x-0)+7(y-0)+2(z-5)=0$ is a point-normal form; other points and normals yield other correct answers.

(b) $(x-0)+0(y-0)-4(z-0)=0$ is a possibility

5.

(a) Not parallel

(b) Parrllel

(c) Parallel

9.

(a) $x=3+2t,\ y=-1+t,\ z=2+3t$

(b) $x=-2+6t,\ y=3-6t,\ z=-3-2t$

(c) $x=2,\ y=2+t,\ z=6$

(d) $x=t,\ y=-2t,\ z=3t$

11.

(a) $x=-12-7t,\ y=-41-23t,\ z=t$

(b) $x=\frac{5}{2}t,\ y=0,\ z=t$

13.

(a) Parallel

(b) Not parallel

17. $2x+3y-5z+36=0$

19.

(a) $z-z_0=0$

(b) $x-x_0=0$

(c) $y-y_0=0$

21. $5x-2y+z-34=0$

23. $y+2z-9=0$

27. $x+5y+3z-18=0$

29. $4x+13y-z-17=0$

31. $3x-y-z-2=0$

37.

(a) $x=\frac{11}{23}+\frac{7}{23}t,\ y=-\frac{41}{23}-\frac{1}{23}t,\ z=t$

(b) $x=-\frac{2}{5}t,\ y=0,\ z=t$

39.

(a) $\frac{5}{3}$

(b) $\frac{1}{\sqrt{29}}$

(c) $\frac{4}{\sqrt{3}}$

43.

(a) $\frac{x-3}{2}=y+1=\frac{z-2}{3}$

(b) $\frac{x+2}{6}=-\frac{y-3}{6}=-\frac{z+3}{2}$

44.

(a) $x - 2y - 17 = 0$ and $x + 4z - 27 = 0$ is one possible answer.

(b) $x - 2y = 0$ and $-7y + 2z = 0$ is one possible answer.

45.

(a) $\theta \approx 35°$

(b) $\theta \approx 79°$

47. They are identical.

Exercise Set 4.1 (page 178)

1.

(a) (−1, 9, −11, 1)

(b) (22, 53, −19, 14)

(c) (−13, 13, −36, −2)

(d) (−90, −114, 60, −36)

(e) (−9, −5, −5, −3)

(f) (27, 29, −27, 9)

3. $c_1 = 1, c_2 = 1, c_3 = -1, c_4 = 1$

5.

(a) $\sqrt{29}$

(b) 3

(c) 13

(d) $\sqrt{31}$

8. $k = \pm\frac{5}{7}$

10. (a) $\left(\frac{1}{\sqrt{10}}, \frac{3}{\sqrt{10}}\right), \left(-\frac{1}{\sqrt{10}}, -\frac{3}{\sqrt{10}}\right)$

14. (a) Yes

(b) No

(c) Yes

(d) No

(e) No

(f) Yes

15. (a) $k = -3$

(b) $k = -2, k = -3$

19. $x_1 = 1, x_2 = -1, x_3 = 2$

22. The component in the ***a*** direction is $\text{proj}_{\mathbf{a}}\mathbf{u} = \frac{4}{15}(-1, 1, 2, 3)$; the orthogonal component is $\frac{1}{15}(34, 11, 52, -27)$.

23. The do not intersect.

33. (a) Euclidean measure of “box” in R^n: $a_1 a_2 .. a_n$

(b) Length of diagonal: $\sqrt{a_1^2 + a_2^2 + \cdots + a_n^2}$

35.

(a) $d(\mathbf{u}, \mathbf{v}) = \sqrt{2}$

37.

(a) True

(b) True

(c) False

(d) True

(e) True, unless $\mathbf{u} = \mathbf{0}$

Exercise Set 4.2 (page 193)

1.

(a) Linear; $R^3 \longrightarrow R^2$

(b) Nonlinear; $R^2 \longrightarrow R^3$

(c) Linear; $R^3 \longrightarrow R^3$

(d) Nonlinear; $R^4 \longrightarrow R^2$

3. $\begin{bmatrix} 3 & 5 & -1 \\ 4 & -1 & 1 \\ 3 & 2 & -1 \end{bmatrix}$; $T(-1, 2, 4) = (3, -2, -3)$

5.

(a) $\begin{bmatrix} 0 & 1 \\ -1 & 0 \\ 1 & 3 \\ 1 & -1 \end{bmatrix}$

(b) $\begin{bmatrix} 7 & 2 & -1 & 1 \\ 0 & 1 & 1 & 0 \\ -1 & 0 & 0 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

(d) $\begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & -1 & 0 \end{bmatrix}$

7.

(a) $T(-1, 4) = (5, 4)$

(b) $T(2, 1, -3) = (0, -2, 0)$

9.

(a) $(2, -5, -3)$

(b) $(2, 5, 3)$

(c) $(-2, -5, 3)$

13.

(a) $\left(-2, \frac{\sqrt{3}-2}{2}, \frac{1+2\sqrt{3}}{2}\right)$

(b) $\left(0, 1, 2\sqrt{2}\right)$

(c) $(-1, -2, 2)$

15.

(a) $\left(-2, \frac{\sqrt{3}+2}{2}, \frac{-1+2\sqrt{3}}{2}\right)$

(b) $\left(-2\sqrt{2}, 1, 0\right)$

(c) $(1, 2, 2)$

17.

(a) $\begin{bmatrix} 0 & 0 \\ 1/2 & -\sqrt{3}/2 \end{bmatrix}$

(b) $\begin{bmatrix} -\sqrt{2} & \sqrt{2} \\ \sqrt{2} & \sqrt{2} \end{bmatrix}$

(c) $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

19.

(a) $\begin{bmatrix} \sqrt{3}/8 & -\sqrt{3}/16 & 1/16 \\ 1/8 & 3/16 & -\sqrt{3}/16 \\ 0 & 1/8 & \sqrt{3}/8 \end{bmatrix}$

(b) $\begin{bmatrix} 0 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

(c) $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & -1 \\ -1 & 0 & 0 \end{bmatrix}$

21.

(a) Yes

(b) No

24. $$\begin{bmatrix} \frac{1}{3}(1-\cos\theta)+\cos\theta & \frac{1}{3}(1-\cos\theta)-\frac{1}{\sqrt{3}}\sin\theta & \frac{1}{3}(1-\cos\theta)-\frac{1}{\sqrt{3}}\sin\theta \\ \frac{1}{3}(1-\cos\theta)-\frac{1}{\sqrt{3}}\sin\theta & \frac{1}{3}(1-\cos\theta)+\cos\theta & \frac{1}{3}(1-\cos\theta)-\frac{1}{\sqrt{3}}\sin\theta \\ \frac{1}{3}(1-\cos\theta)-\frac{1}{\sqrt{3}}\sin\theta & \frac{1}{3}(1-\cos\theta)-\frac{1}{\sqrt{3}}\sin\theta & \frac{1}{3}(1-\cos\theta)+\cos\theta \end{bmatrix}$$

28.

(c) 90°

29.

(a) Twice the orthogonal projection on the x-axis

(b) Twice the reflection about the x-axis

30.

(a) The x-coordinate is stretched by a factor of 2 and the y-coordinate is stretched by a factor of 3.

(b) Rotation through 30°

31. Rotation through the angle 2θ

34. Only if $b = 0$.

Exercise Set 4.3 (page 206)

1.
(a) Not one-to-one

(b) One-to-one

(c) One-to-one

(d) One-to-one

(e) One-to-one

(f) One-to-one

(g) One-to-one

3. For example, the vector (1, 3) is not in the range.

5.
(a) One-to-one; $\begin{bmatrix} \frac{1}{3} & -\frac{2}{3} \\ \frac{1}{3} & \frac{1}{3} \end{bmatrix}$; $T^{-1}(w_1, w_2) = \left(\frac{1}{3}w_1 - \frac{2}{3}w_2, \frac{1}{3}w_1 + \frac{1}{3}w_2\right)$

(b) Not one-to-one

(c) One-to-one; $\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$; $T^{-1}(w_1, w_2) = (-w_2, -w_1)$

(d) Not one-to-one

7.
(a) Reflection about the *x*-axis

(b) Rotation through the angle $-\pi/4$

(c) Contraction by a factor of $\frac{1}{3}$

(d) Reflection about the yz-plane

(e) Dilation by a factor of 5

9.

(a) Linear

(b) Nonlinear

(c) Linear

(d) Nonlinear

12.

(a) For a reflection about the y-axis, $T(\mathbf{e}_1) = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$ and $T(\mathbf{e}_2) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$. Thus, $T = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$.

(b) For a reflection about the xz-plane, $T(\mathbf{e}_1) = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $T(\mathbf{e}_2) = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}$, and $T(\mathbf{e}_3) = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$. Thus, $T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

.

(c) For an orthogonal projection on the x-axis, $T(\mathbf{e}_1) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $T(\mathbf{e}_2) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$. Thus, $T = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$.

(d) For an orthogonal projection on the yz-plane, $T(\mathbf{e}_1) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$, $T(\mathbf{e}_2) = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$, and $T(\mathbf{e}_3) = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$. Thus,

$T = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

(e) For a rotation through a positive angle θ, $T(\mathbf{e}_1) = \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix}$ and $T(\mathbf{e}_2) = \begin{bmatrix} -\sin\theta \\ \cos\theta \end{bmatrix}$. Thus,

$T = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$.

(f) For a dilation by a factor $k \geq 1$, $T(\mathbf{e}_1) = \begin{bmatrix} k \\ 0 \\ 0 \end{bmatrix}$, $T(\mathbf{e}_2) = \begin{bmatrix} 0 \\ k \\ 0 \end{bmatrix}$, and $T(\mathbf{e}_3) = \begin{bmatrix} 0 \\ 0 \\ k \end{bmatrix}$. Thus, $T = \begin{bmatrix} k & 0 & 0 \\ 0 & k & 0 \\ 0 & 0 & k \end{bmatrix}$.

13.

(a) $T(\mathbf{e}_1) = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$ and $T(\mathbf{e}_2) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$. Thus, $T = \begin{bmatrix} -1 & 0 \\ 0 & 0 \end{bmatrix}$.

(b) $T(\mathbf{e}_1)=\begin{bmatrix}0\\-1\end{bmatrix}$ and $T(\mathbf{e}_2)=\begin{bmatrix}1\\0\end{bmatrix}$. Thus, $T=\begin{bmatrix}0&1\\-1&0\end{bmatrix}$.

(c) $T(\mathbf{e}_1)=\begin{bmatrix}0\\3\end{bmatrix}$ and $T(\mathbf{e}_2)=\begin{bmatrix}0\\0\end{bmatrix}$. Thus, $T=\begin{bmatrix}0&0\\3&0\end{bmatrix}$.

16.

(a) Linear transformation from $R^2 \longrightarrow R^3$; one-to-one

(b) Linear transformation from $R^3 \longrightarrow R^2$; not one-to-one

17.

(a) $\left(\frac{1}{2}, \frac{1}{2}\right)$

(b) $\left(\frac{3}{4}, \frac{\sqrt{3}}{4}\right)$

(c) $\left(\frac{1-5\sqrt{3}}{4}, \frac{15-\sqrt{3}}{4}\right)$

19.

(a) $\lambda=1;\ \begin{bmatrix}0\\s\\t\end{bmatrix}\quad \lambda=-1\begin{bmatrix}t\\0\\0\end{bmatrix}$

(b) $\lambda=1;\ \begin{bmatrix}s\\0\\t\end{bmatrix}\quad \lambda=0\begin{bmatrix}0\\t\\0\end{bmatrix}$

(c) $\lambda=2$; all vectors in R^3 are eigenvectors

(d) $\lambda=1;\ \begin{bmatrix}0\\0\\t\end{bmatrix}$

23.

(a) $\begin{bmatrix}\cos 2\theta & \sin 2\theta\\ \sin 2\theta & -\cos 2\theta\end{bmatrix}$

(b) $\left(\frac{1+5\sqrt{3}}{2}, \frac{\sqrt{3}-5}{2}\right)$

27.

(a) The range of T is a proper subset of R^n.

(b) T must map infinitely many vectors to 0.

Exercise Set 4.4 (page 217)

1. **(a)** $x^2 + 2x - 1 - 2\left(3x^2 + 2\right) = -5x^2 + 2x - 5$

4. **(a)** Yes; $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

7. **(a)** $L\colon\ P_1 \longrightarrow P_1$ where L maps $ax + b$ to $(a + b)x + a - b$

9. **(a)** $3e^t + 3e^{-t} = 6\cosh(t)$

(b) Yes

12. $y = 2x^2$

14. **(a)** $y = x^3 - x$

15. **(a)** $y = 2x^3 - 2x + 2$

18. **(a)** No, because of the arbitrary constant of integration

(b) No (except for P_0)

21. **(a)** Each $L_i(x)$ is a polynomial of degree at most *n* and hence so is the sum $y_0 L(x) + \cdots + y_n L(x)$; also, $p(x_i) = 0 + 0 + \cdots + 0 + y_i \cdot L_i(x_i) + 0 + \cdots + 0 + 0 = y_i$, showing that this function is an interpolant of degree at most *n*.

(b) It is $I_{n+1}\mathbf{c} = \mathbf{y}$ where ***c*** is the vector of c_i values and ***y*** is the vector of *y*-values.

Exercise Set 5.1 (page 226)

1. Not a vector space. Axiom 8 fails.

3. Not a vector space. Axioms 9 and 10 fail.

5. The set is a vector space under the given operations.

7. The set is a vector space under the given operations.

9. Not a vector space. Axioms 1, 4, 5, and 6 fail.

11. The set is a vector space under the given operations.

13. The set is a vector space under the given operations.

25. No. A vector space must have a zero element.

26. No. Axioms 1, 4, and 6 will fail.

29.
   1. Axiom 7
   2. Axiom 4
   3. Axiom 5
   4. Follows from statement 2
   5. Axiom 3
   6. Axiom 5
   7. Axiom 4

32. No; $\mathbf{0}_1 = \mathbf{0}_1 + \mathbf{0}_2 = \mathbf{0}_2$

Exercise Set 5.2 (page 238)

1. (a), (c)

3. (a), (b), (d)

5. (a), (b), (d)

6.

(a) Line; $x = -\frac{1}{2}t,\ y = -\frac{3}{2}t,\ z = t$

(b) Line; $x = 2t,\ y = t,\ z = 0$

(c) Origin

(d) Origin

(e) Line; $x = -3t,\ y = -2t,\ z = t$

(f) Plane; $x - 3y + z = 0$

9.

(a) $-9 - 7x - 15x^2 = -2\mathbf{p}_1 + \mathbf{p}_2 - 2\mathbf{p}_3$

(b) $6 + 11x + 6x^2 = 4\mathbf{p}_1 - 5\mathbf{p}_2 + \mathbf{p}_3$

(c) $0 = 0\mathbf{p}_1 + 0\mathbf{p}_2 + 0\mathbf{p}_3$

(d) $7 + 8x + 9x^2 = 0\mathbf{p}_1 - 2\mathbf{p}_2 + 3\mathbf{p}_3$

11.

(a) The vectors span.

(b) The vectors do not span.

(c) The vectors do not span.

(d) The vectors span.

12. (a), (c), (e)

15. $y = z$

24.

(a) They span a line if they are collinear and not both 0. They span a plane if they are not collinear.

(b) If $\mathbf{u} = a\mathbf{v}$ and $\mathbf{v} = b\mathbf{u}$ for some real numbers *a*, *b*

(c) We must have $\mathbf{b} = \mathbf{0}$ since a subspace must contain $\mathbf{x} = \mathbf{0}$ and then $\mathbf{b} = A\mathbf{0} = \mathbf{0}$.

26.

(a) For example, $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$

(b) The set of matrices having one entry equal to 1 and all other entries equal to 0

Exercise Set 5.3 (page 248)

1.

(a) $\mathbf{u}_2$ is a scalar multiple of $\mathbf{u}_1$.

(b) The vectors are linearly dependent by Theorem 5.3.3.

(c) $\mathbf{p}_2$ is a scalar multiple of $\mathbf{p}_1$.

(d) *B* is a scalar multiple of *A*.

3. None

5.

(a) They do not lie in a plane.

(b) They do lie in a plane.

7.

(b) $\mathbf{v}_1 = \frac{2}{7}\mathbf{v}_2 - \frac{3}{7}\mathbf{v}_3,\ \mathbf{v}_2 = \frac{7}{2}\mathbf{v}_1 + \frac{3}{2}\mathbf{v}_3,\ \mathbf{v}_3 = -\frac{7}{3}\mathbf{v}_1 + \frac{2}{3}\mathbf{v}_2$

9. $\lambda = -\frac{1}{2}, \lambda = 1$

18. If and only if the vector is not zero

19.

(a) They are linearly independent since $\mathbf{v}_1$, $\mathbf{v}_2$, and $\mathbf{v}_3$ do not lie in the same plane when they are placed with their initial points at the origin.

(b) The are not linearly independent since $\mathbf{v}_1$, $\mathbf{v}_2$, and $\mathbf{v}_3$ lie in the same plane when they are placed with their initial points at the origin.

20. (a), (d), (e), (f)

24.

(a) False

(b) False

(c) True

(d) False

27.

(a) Yes

Exercise Set 5.4 (page 263)

1.

(a) A basis for R^2 has two linearly independent vectors.

(b) A basis for R^3 has three linearly independent vectors.

(c) A basis for P_2 has three linearly independent vectors.

(d) A basis for M_{22} has four linearly independent vectors.

3. (a), (b)

7.

(a) $(\mathbf{w})_S = (3, -7)$

(b) $(\mathbf{w})_S = \left(\frac{5}{28}, \frac{3}{14}\right)$

(c) $(\mathbf{w})_S = \left(a, \frac{b-a}{2}\right)$

9.

(a) $(\mathbf{v})_S = (3, -2, 1)$

(b) $(\mathbf{v})_S = (-2, 0, 1)$

11. $(A)_S = (-1, 1, -1, 3)$

13. Basis: $\left(-\frac{1}{4}, -\frac{1}{4}, 1, 0\right)$, (0, −1, 0, 1); $\text{dimension} = 2$

15. Basis: (3, 1, 0), (−1, 0, 1); $\text{dimension} = 2$

19.

(a) 3-dimensional

(b) 2-dimensional

(c) 1-dimensional

20. 3-dimensional

21.

(a) $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{e}_1\}$ or $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{e}_2\}$

(b) $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{e}_1\}$ or $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{e}_2\}$ or $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{e}_3\}$

27.

(a) One possible answer is $\left\{-1 + x - 2x^2, 3 + 3x + 6x^2, 9\right\}$.

(b) One possible answer is $\left\{1 + x, x^2, -2 + 2x^2\right\}$.

(c) One possible answer is $\left\{1 + x - 3x^2\right\}$.

29.

(a) (2, 0)

(b) $\left(\frac{2}{\sqrt{3}}, -\frac{1}{\sqrt{3}}\right)$

(c) (0, 1)

(d) $\left(\frac{2}{\sqrt{3}}a, b-\frac{a}{\sqrt{3}}\right)$

31. Yes; for example, $\begin{bmatrix} 1 & 0 \\ 0 & \pm 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ \pm 1 & 0 \end{bmatrix}$

32.

(a) n

(b) $n(n+1)/2$

(c) $n(n+1)/2$

35.

(a) The dimension is $n-1$.

(b) (1, 0, 0, … , 0, −1), (0, 1, 0, … , 0, −1), (0, 0, 1, … , 0, −1), … , (0, 0, 0, … , 1, −1) is a basis of size $n-1$.

Exercise Set 5.5 (page 276)

1.

$\mathbf{r}_1 = (2, -1, 0, 1)$, $\mathbf{r}_2 = (3, 5, 7, -1)$, $\mathbf{r}_3 = (1, 4, 2, 7)$; $\mathbf{c}_1 = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$, $\mathbf{c}_2 = \begin{bmatrix} -1 \\ 5 \\ 4 \end{bmatrix}$, $\mathbf{c}_3 = \begin{bmatrix} 0 \\ 7 \\ 2 \end{bmatrix}$, $\mathbf{c}_4 = \begin{bmatrix} 1 \\ -1 \\ 7 \end{bmatrix}$

3.

(a) $\begin{bmatrix} -2 \\ 10 \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \end{bmatrix} - \begin{bmatrix} 3 \\ -6 \end{bmatrix}$

(b) ***b*** is not in the column space of *A*.

(c) $\begin{bmatrix} 1 \\ 9 \\ 1 \end{bmatrix} - 3\begin{bmatrix} -1 \\ 3 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 1 \\ -1 \end{bmatrix}$

(d) $\begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} + (t-1)\begin{bmatrix} -1 \\ 1 \\ -1 \end{bmatrix} + t\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$

(e) $\begin{bmatrix} 4 \\ 3 \\ 5 \\ 7 \end{bmatrix} = -26\begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} + 13\begin{bmatrix} 2 \\ 1 \\ 2 \\ 1 \end{bmatrix} - 7\begin{bmatrix} 0 \\ 2 \\ 1 \\ 2 \end{bmatrix} + 4\begin{bmatrix} 1 \\ 1 \\ 3 \\ 2 \end{bmatrix}$

5.

(a) $\begin{bmatrix} 1 \\ 0 \end{bmatrix} + t\begin{bmatrix} 3 \\ 1 \end{bmatrix}; t\begin{bmatrix} 3 \\ 1 \end{bmatrix}$

(b) $\begin{bmatrix} -2 \\ 7 \\ 0 \end{bmatrix} + t\begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}; t\begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}$

(c) $\begin{bmatrix} -1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + r\begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \end{bmatrix} + s\begin{bmatrix} -1 \\ 0 \\ 1 \\ 0 \end{bmatrix} + t\begin{bmatrix} -2 \\ 0 \\ 0 \\ 1 \end{bmatrix}; r\begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \end{bmatrix} + s\begin{bmatrix} -1 \\ 0 \\ 1 \\ 0 \end{bmatrix} + t\begin{bmatrix} -2 \\ 0 \\ 0 \\ 1 \end{bmatrix}$

(d) $\begin{bmatrix} \frac{6}{5} \\ \frac{7}{5} \\ 0 \\ 0 \end{bmatrix} + s\begin{bmatrix} \frac{7}{5} \\ \frac{4}{5} \\ 1 \\ 0 \end{bmatrix} + t\begin{bmatrix} \frac{1}{5} \\ -\frac{3}{5} \\ 0 \\ 1 \end{bmatrix}; s\begin{bmatrix} \frac{7}{5} \\ \frac{4}{5} \\ 1 \\ 0 \end{bmatrix} + t\begin{bmatrix} \frac{1}{5} \\ -\frac{3}{5} \\ 0 \\ 1 \end{bmatrix}$

7.

(a) $\mathbf{r}_1 = [1 \quad 0 \quad 2], \mathbf{r}_2 = [0 \quad 0 \quad 1], \mathbf{c} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \mathbf{c}_2 = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$

(b) $\mathbf{r}_1 = [1 \quad -3 \quad 0 \quad 0], \mathbf{r}_2 = [0 \quad 1 \quad 0 \quad 0], \mathbf{c}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \mathbf{c}_2 = \begin{bmatrix} -3 \\ 1 \\ 0 \\ 0 \end{bmatrix}$

(c) $\mathbf{r}_1 = [1 \quad 2 \quad 4 \quad 5], \mathbf{r}_2 = [0 \quad 1 \quad -3 \quad 0], \mathbf{r}_3 = [0 \quad 0 \quad 1 \quad -3], \mathbf{r}_4 = [0 \quad 0 \quad 0 \quad 1], \mathbf{c}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \mathbf{c}_2 = \begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix},$

$\mathbf{c}_3 = \begin{bmatrix} 4 \\ -3 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \mathbf{c}_4 = \begin{bmatrix} 5 \\ 0 \\ -3 \\ 1 \\ 0 \end{bmatrix}$

(d) $\mathbf{r}_1 = [1 \quad 2 \quad -1 \quad 5], \mathbf{r}_2 = [0 \quad 1 \quad 4 \quad 3], \mathbf{r}_3 = [0 \quad 0 \quad 1 \quad -7], \mathbf{r}_4 = [0 \quad 0 \quad 0 \quad 1], \mathbf{c}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \mathbf{c}_2 = \begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \end{bmatrix},$

$$c_3 = \begin{bmatrix} -1 \\ 4 \\ 1 \\ 0 \end{bmatrix}, c_4 = \begin{bmatrix} 5 \\ 3 \\ -7 \\ 1 \end{bmatrix}$$

9.

(a) $\begin{bmatrix} 1 \\ 5 \\ 7 \end{bmatrix}, \begin{bmatrix} -1 \\ -4 \\ -6 \end{bmatrix}$

(b) $\begin{bmatrix} 2 \\ 4 \\ 0 \end{bmatrix}$

(c) $\begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}, \begin{bmatrix} 4 \\ 1 \\ 3 \end{bmatrix}$

(d) $\begin{bmatrix} 1 \\ 3 \\ -1 \\ 2 \end{bmatrix}, \begin{bmatrix} 4 \\ -2 \\ 0 \\ 3 \end{bmatrix}$

(e) $\begin{bmatrix} 1 \\ 0 \\ 2 \\ 3 \\ -2 \end{bmatrix}, \begin{bmatrix} -3 \\ 3 \\ -3 \\ -6 \\ 9 \end{bmatrix}, \begin{bmatrix} 2 \\ 6 \\ -2 \\ 0 \\ 2 \end{bmatrix}$

11.

(a) (1, 1, −4, −3), (0, 1, −5, −2), $\left(0, 0, 1, -\frac{1}{2}\right)$

(b) (1, −1, 2, 0), (0, 1, 0, 0), $\left(0, 0, 1, -\frac{1}{6}\right)$

(c) (1, 1, 0, 0), (0, 1, 1, 1), (0, 0, 1, 1), (0, 0, 0, 1)

14.

(b) $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

17. $\begin{bmatrix} 3a & -5a \\ 3b & -5b \end{bmatrix}$ for all real numbers a, b not both 0.

Exercise Set 5.6 (page 288)

1. $\text{Rank}\,(A) = \text{rank}\left(A^T\right) = 2$

3.

(a) 2; 1

(b) 1; 2

(c) 2; 2

(d) 2; 3

(e) 3: 2

5.

(a) $\text{Rank} = 4$, $\text{nullity} = 0$

(b) $\text{Rank} = 3$, $\text{nullity} = 2$

(c) $\text{Rank} = 3$, $\text{nullity} = 0$

7.

(a) Yes, 0

(b) No

(c) Yes, 2

(d) Yes, 7

(e) No

(f) Yes, 4

(g) Yes, 0

9. $b_1 = r$, $b_2 = s$, $b_3 = 4s - 3r$, $b_4 = 2r - s$, $b_5 = 8s - 7r$

11. No

13. Rank is 2 if $r = 2$ and $s = 1$; the rank is never 1.

16.

(a) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

(b) A line through the origin

(c) A plane through the origin

(d) The nullspace is a line through the origin and the row space is a plane through the origin.

19.

(a) 3

(b) 5

(c) 3

(d) 3

Supplementary Exercises (page 290)

1.

(a) All of R^3

(b) Plane: $2x - 3y + z = 0$

(c) Line: $x = 2t,\ y = t,\ z = 0$

(d) The origin: (0, 0, 0)

3.

(a) $a(4, 1, 1) + b(0, -1, 2)$

(b) $(a + c)(3, -1, 2) + b(1, 4, 1)$

(c) $a(2, 3, 0) + b(-1, 0, 4) + c(4, -1, 1)$

5.

(a) $\mathbf{v} = (-1 + r)\mathbf{v}_1 + \left(\frac{2}{3} - r\right)\mathbf{v}_2 + r\mathbf{v}_3$; r arbitrary

7. No

9.

(a) $\text{Rank} = 2$, $\text{nullity} = 1$

(b) $\text{Rank} = 3$, $\text{nullity} = 2$

(c) $\text{Rank} = n + 1$, $\text{nullity} = n$

11. $\{1, x^2, x^3, x^4, x^5, x^6, \ldots, x^n\}$

13.

(a) 2

(b) 1

(c) 2

(d) 3

Exercise Set 6.1 (page 304)

1.

(a) 2

(b) 11

(c) −13

(d) −8

(e) 0

3.

(a) 3

(b) 56

5.

(b) 29

7.

(a) $\begin{bmatrix} \sqrt{3} & 0 \\ 0 & \sqrt{5} \end{bmatrix}$

(b) $\begin{bmatrix} 2 & 0 \\ 0 & \sqrt{6} \end{bmatrix}$

9.

(a) No. Axiom 4 fails.

(b) No. Axioms 2 and 3 fail.

(c) Yes

(d) No. Axiom 4 fails.

11.

(a) $3\sqrt{2}$

(b) $3\sqrt{5}$

(c) $3\sqrt{13}$

13.

(a) $\sqrt{74}$

(b) 0

15.

(a) $\sqrt{105}$

(b) $\sqrt{47}$

17.

(a) $\sqrt{2}, \frac{1}{3}\sqrt{6}, \frac{1}{5}\sqrt{10}$

(b) $\frac{2}{3}\sqrt{6}$

19. $\langle \mathbf{u}, \mathbf{v} \rangle = \frac{1}{9}\mathbf{u}_1\mathbf{v}_1 + \mathbf{u}_2\mathbf{v}_2$

23. No for P_3, since $\mathbf{p} = x\left(x - \frac{1}{2}\right)(x-1)$ satisfies $\langle \mathbf{p}, \mathbf{p} \rangle = 0$

27.

(a) $-\frac{28}{15}$

(b) 0

34. $a = 1/25, b = 1/16$

Exercise Set 6.2 (page 315)

1.

(a) Yes

(b) No

(c) Yes

(d) No

(e) No

(f) Yes

5.

(a) $-\frac{1}{\sqrt{2}}$

(b) $-\frac{3}{\sqrt{73}}$

(c) 0

(d) $-\frac{20}{9\sqrt{10}}$

(e) $-\frac{1}{\sqrt{2}}$

(f) $\frac{2}{\sqrt{55}}$

9.

(a) Orthogonal

(b) Orthogonal

(c) Orthogonal

(d) Not orthogonal

11. $\pm\frac{1}{57}(-34, 44, -6, 11)$

15.

(a) $x = t, y = -2t, z = -3t$

(b) $2x - 5y + 4z = 0$

(c) $x - z = 0$

17.

(a) $\begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}; \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$

18.

(a) (16, 19, 1)

(b) $(0, 1, 0), \left(\frac{1}{2}, 0, 1\right)$

(c) $(-1, -1, 1, 0), \left(\frac{2}{7}, -\frac{4}{7}, 0, 1\right)$

(d) $(-1, -1, 1, 0, 0), (-2, -1, 0, 1, 0), (-1, -2, 0, 0, 1)$

32. $\langle \mathbf{u}, \mathbf{v} \rangle = \frac{1}{2}\mathbf{u}_1\mathbf{v}_1 + \frac{1}{6}\mathbf{u}_2\mathbf{v}_2$

35.

(a) The line $y = -x$

(b) The xz-plane

(c) The x-axis

37.

(a) False

(b) True

(c) True

(d) False

Exercise Set 6.3 (page 328)

1. (a), (b), (d)

3. (b), (d)

5. (a)

7.

(a) $\left(-\frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}\right), \left(\frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}}\right)$

(b) $\left(\frac{1}{\sqrt{2}}, 0, -\frac{1}{\sqrt{2}}\right), \left(\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right), (0, 1, 0)$

(c) $\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right), \left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right), \left(\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, -\frac{2}{\sqrt{6}}\right)$

9.

(a) $-\frac{7}{5}\mathbf{v}_1 + \frac{1}{5}\mathbf{v}_2 + 2\mathbf{v}_3$

(b) $-\frac{37}{5}\mathbf{v}_1 - \frac{9}{5}\mathbf{v}_2 + 4\mathbf{v}_3$

(c) $-\frac{3}{7}\mathbf{v}_1 - \frac{1}{7}\mathbf{v}_2 + \frac{5}{7}\mathbf{v}_3$

11.

(a) $(\mathbf{w})_S = \left(-2\sqrt{2}, 5\sqrt{2}\right)$

(b) $(\mathbf{w})_S = (0, -2, 1)$

13.

(a) $\mathbf{u} = \left(1, \frac{14}{5}, -\frac{2}{5}\right), \mathbf{v} = \left(0, -\frac{17}{5}, \frac{6}{5}\right), \mathbf{w} = \left(-4, -\frac{11}{5}, \frac{23}{5}\right)$

(b) $\|\mathbf{v}\| = \sqrt{13}, d(\mathbf{u}, \mathbf{v}) = 5\sqrt{3}, \langle \mathbf{w}, \mathbf{v}\rangle = 13$

15.

(b) $\mathbf{u} = -\frac{4}{5}\mathbf{v}_1 - \frac{11}{10}\mathbf{v}_2 + 0\mathbf{v}_3 + \frac{1}{2}\mathbf{v}_4$

17.

(a) $\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right), \left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right), \left(\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, -\frac{2}{\sqrt{6}}\right)$

(b) $(1, 0, 0), \left(0, \frac{7}{\sqrt{53}}, -\frac{2}{\sqrt{53}}\right), \left(0, \frac{2}{\sqrt{53}}, \frac{7}{\sqrt{53}}\right)$

19. $\left(0, \frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}\right), \left(-\frac{\sqrt{5}}{\sqrt{6}}, -\frac{2}{\sqrt{30}}, \frac{1}{\sqrt{30}}\right)$

21. $\mathbf{w}_1 = \left(-\frac{4}{5}, 2, \frac{3}{5}\right), \mathbf{w}_2 = \left(\frac{9}{5}, 0, \frac{12}{5}\right)$

24.

(a) $\begin{bmatrix} \frac{1}{\sqrt{5}} & -\frac{2}{\sqrt{5}} \\ \frac{2}{\sqrt{5}} & \frac{1}{\sqrt{5}} \end{bmatrix} \begin{bmatrix} \sqrt{5} & \sqrt{5} \\ 0 & \sqrt{5} \end{bmatrix}$

(b) $\begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{3}} \\ 0 & \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{3}} \end{bmatrix} \begin{bmatrix} \sqrt{2} & 3\sqrt{2} \\ 0 & \sqrt{3} \end{bmatrix}$

(c) $\begin{bmatrix} \frac{1}{3} & \frac{8}{\sqrt{234}} \\ -\frac{2}{3} & \frac{11}{\sqrt{234}} \\ \frac{2}{3} & \frac{7}{\sqrt{234}} \end{bmatrix} \begin{bmatrix} 3 & \frac{1}{3} \\ 0 & \frac{\sqrt{26}}{3} \end{bmatrix}$

(d) $\begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{6}} \\ 0 & \frac{1}{\sqrt{3}} & \frac{2}{\sqrt{6}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{6}} \end{bmatrix} \begin{bmatrix} \sqrt{2} & \sqrt{2} & \sqrt{2} \\ 0 & \sqrt{3} & -\frac{1}{\sqrt{3}} \\ 0 & 0 & \frac{4}{\sqrt{6}} \end{bmatrix}$

(e) $\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{\sqrt{2}}{2\sqrt{19}} & -\frac{3}{\sqrt{19}} \\ \frac{1}{\sqrt{2}} & -\frac{\sqrt{2}}{2\sqrt{19}} & \frac{3}{\sqrt{19}} \\ 0 & \frac{3\sqrt{2}}{\sqrt{19}} & \frac{1}{\sqrt{19}} \end{bmatrix} \begin{bmatrix} \sqrt{2} & \frac{3}{\sqrt{2}} & \sqrt{2} \\ 0 & \frac{\sqrt{19}}{\sqrt{2}} & \frac{3\sqrt{2}}{\sqrt{19}} \\ 0 & 0 & \frac{1}{\sqrt{19}} \end{bmatrix}$

(f) Columns not linearly independent

29. $\mathbf{v}_1 = \frac{1}{\sqrt{2}}, \mathbf{v}_2 = \sqrt{\frac{3}{2}}x, \mathbf{v}_3 = \frac{\sqrt{5}}{2\sqrt{2}}\left(3x^2 - 1\right)$

31. $\mathbf{v}_1 = 1, \mathbf{v}_2 = \sqrt{3}(2x - 1), \mathbf{v}_3 = \sqrt{5}\left(6x^2 - 6x + 1\right)$

35. $\left(1/\sqrt{5}, 1/\sqrt{5}\right), \left(2/\sqrt{30}, -3/\sqrt{30}\right)$

Exercise Set 6.4 (page 339)

1. **(a)** $\begin{bmatrix} 21 & 25 \\ 25 & 35 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 20 \\ 20 \end{bmatrix}$

(b) $\begin{bmatrix} 15 & -1 & 5 \\ -1 & 22 & 30 \\ 5 & 30 & 45 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -1 \\ 9 \\ 13 \end{bmatrix}$

3.

(a) $x_1 = 5, x_2 = \frac{1}{2}; \begin{bmatrix} \frac{11}{2} \\ -\frac{9}{2} \\ -4 \end{bmatrix}$

(b) $x_1 = \frac{3}{7}, x_2 = -\frac{2}{3}; \begin{bmatrix} \frac{46}{21} \\ -\frac{5}{21} \\ \frac{13}{21} \end{bmatrix}$

(c) $x_1 = 12, x_2 = -3, x_3 = 9; \begin{bmatrix} 3 \\ 3 \\ 9 \\ 0 \end{bmatrix}$

(d) $x_1 = 14, x_2 = 30, x_3 = 26; \begin{bmatrix} 2 \\ 6 \\ -2 \\ 4 \end{bmatrix}$

5.

(a) $(7, 2, 9, 5)$

(b) $\left(-\frac{12}{5}, -\frac{4}{5}, \frac{12}{5}, \frac{16}{5}\right)$

7.

(a) $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$

(b) $\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$

11.

(a) $\mathbf{v}_1 = (2, -1, 4)$

(b) $\begin{bmatrix} \frac{4}{21} & -\frac{2}{21} & \frac{8}{21} \\ -\frac{2}{21} & \frac{1}{21} & -\frac{4}{21} \\ \frac{8}{21} & -\frac{4}{21} & \frac{16}{21} \end{bmatrix}$

(c) $\begin{bmatrix} \frac{4}{21}x_0 - \frac{2}{21}y_0 + \frac{8}{21}z_0 \\ -\frac{2}{21}x_0 + \frac{1}{21}y_0 - \frac{4}{21}z_0 \\ \frac{8}{21}x_0 - \frac{4}{21}y_0 + \frac{16}{21}z_0 \end{bmatrix}$

(d) $\frac{\sqrt{497}}{7}$

17. $[P] = A^T\left(AA^T\right)^{-1}A$

18.

1. Since $A^T\mathbf{0} = \mathbf{0}$
2. Since A^TA is invertible
3. Since the nullspace of *A* is nonzero if and only if the columns of *A* are dependent

Exercise Set 6.5 (page 345)

1.

(a) $[\mathbf{w}]_S = \begin{bmatrix} 3 \\ -7 \end{bmatrix}$

(b) $[\mathbf{w}]_S = \begin{bmatrix} \frac{5}{28} \\ \frac{3}{14} \end{bmatrix}$

(c) $[\mathbf{w}]_S = \begin{bmatrix} a \\ \frac{b-a}{2} \end{bmatrix}$

3.

(a) $(\mathbf{p})_S = (4, -3, 1)$, $[\mathbf{p}]_S = \begin{bmatrix} 4 \\ -3 \\ 1 \end{bmatrix}$

(b) $(\mathbf{p})_S = (0, 2, -1)$, $[\mathbf{p}]_S = \begin{bmatrix} 0 \\ 2 \\ -1 \end{bmatrix}$

5.

(a) $\mathbf{w} = (16, 10, 12)$

(b) $\mathbf{q} = 3 + 4x^2$

(c) $B = \begin{bmatrix} 15 & -1 \\ 6 & 3 \end{bmatrix}$

7.

(a) $\begin{bmatrix} \frac{13}{10} & -\frac{1}{2} \\ -\frac{2}{5} & 0 \end{bmatrix}$

(b) $\begin{bmatrix} 0 & -\frac{5}{2} \\ -2 & -\frac{13}{2} \end{bmatrix}$

(c) $[\mathbf{w}]_B = \begin{bmatrix} -\frac{17}{10} \\ \frac{8}{5} \end{bmatrix}$, $[\mathbf{w}]_{B'} = \begin{bmatrix} -4 \\ -7 \end{bmatrix}$

9.

(a) $\begin{bmatrix} 3 & 2 & \frac{5}{2} \\ -2 & -3 & -\frac{1}{2} \\ 5 & 1 & 6 \end{bmatrix}$

(b) $\begin{bmatrix} -\frac{7}{2} \\ \frac{23}{2} \\ 6 \end{bmatrix}$

11.

(b) $\begin{bmatrix} 2 & 0 \\ 1 & 3 \end{bmatrix}$

(c) $\begin{bmatrix} \frac{1}{2} & 0 \\ -\frac{1}{6} & \frac{1}{3} \end{bmatrix}$

(d) $[\mathbf{h}]_B = \begin{bmatrix} 2 \\ -5 \end{bmatrix}$, $[\mathbf{h}]_{B'} = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$

Exercise Set 6.6 (page 353)

1.

(b) $\begin{bmatrix} \frac{4}{5} & -\frac{9}{25} & \frac{12}{25} \\ 0 & \frac{4}{5} & \frac{3}{5} \\ -\frac{3}{5} & -\frac{12}{25} & \frac{16}{25} \end{bmatrix}$

3.

(a) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(b) $\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$

(d) $\begin{bmatrix} -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{2}{\sqrt{6}} & \frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{bmatrix}$

(e) $\begin{bmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{5}{6} & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{2} & \frac{1}{6} & \frac{1}{6} & -\frac{5}{6} \\ \frac{1}{2} & \frac{1}{6} & -\frac{5}{6} & \frac{1}{6} \end{bmatrix}$

7.

(a) $\left(-1+3\sqrt{3}, 3+\sqrt{3}\right)$

(b) $\left(\frac{5}{2}-\sqrt{3}, \frac{5}{2}\sqrt{3}+1\right)$

9.

(a) $\left(-\frac{1}{2}-\frac{5}{2}\sqrt{3}, 2, \frac{5}{2}-\frac{1}{2}\sqrt{3}\right)$

(b) $\left(\frac{1}{2}-\frac{3}{2}\sqrt{3}, 6, -\frac{3}{2}-\frac{1}{2}\sqrt{3}\right)$

11.

(a) $A=\begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix}$

(b) $A=\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{bmatrix}$

12. $\begin{bmatrix} \frac{\sqrt{2}}{4} & \frac{\sqrt{6}}{4} & -\frac{\sqrt{2}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ \frac{\sqrt{2}}{4} & \frac{\sqrt{6}}{4} & \frac{\sqrt{2}}{2} \end{bmatrix}$

16.
(a) Rotation

(b) Rotation followed by a reflection

20.
(a) Rotation and reflection

(b) Rotation and dilation

(c) Any rigid operator is angle preserving. Any dilation or contraction with $k \neq 0$, 1 is angle preserving but not rigid.

22. $a = 0, b = \sqrt{2/3}, c = -\sqrt{1/3}$ or $a = 0, b = -\sqrt{2/3}, c = \sqrt{1/3}$

Supplementary Exercises (page 356)

1.
(a) $(0, a, a, 0)$ with $a \neq 0$

(b) $\pm\left(0, \frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}}, 0\right)$

6. $\pm\left(\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right)$

7. $w_k = \frac{1}{k}, k = 1, 2, \ldots, n$

11. θ approaches $\frac{\pi}{2}$

12.
(b) The diagonals of a parallelogram are perpendicular if and only if its sides have the same length.

Exercise Set 7.1 (page 367)

1.
(a) $\lambda^2 - 2\lambda - 3 = 0$

(b) $\lambda^2 - 8\lambda + 16 = 0$

(c) $\lambda^2 - 12 = 0$

(d) $\lambda^2 + 3 = 0$

(e) $\lambda^2 = 0$

(f) $\lambda^2 - 2\lambda + 1 = 0$

3.

(a) Basis for eigenspace corresponding to $\lambda = 3$: $\begin{bmatrix} \frac{1}{2} \\ 1 \end{bmatrix}$; basis for eigenspace corresponding to $\lambda = -1$: $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$

(b) Basis for eigenspace corresponding to $\lambda = 4$: $\begin{bmatrix} \frac{3}{2} \\ 1 \end{bmatrix}$

(c) Basis for eigenspace corresponding to $\lambda = \sqrt{12}$: $\begin{bmatrix} \frac{3}{\sqrt{12}} \\ 1 \end{bmatrix}$; basis for eigenspace corresponding to $\lambda = -\sqrt{12}$: $\begin{bmatrix} -\frac{3}{\sqrt{12}} \\ 1 \end{bmatrix}$

(d) There are no eigenspaces.

(e) Basis for eigenspace corresponding to $\lambda = 0$: $\begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

(f) Basis for eigenspace corresponding to $\lambda = 1$: $\begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

6.

(a) $\lambda = 1$: basis $\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$; $\lambda = 2$: basis $\begin{bmatrix} -\frac{1}{2} \\ 1 \\ 1 \end{bmatrix}$; $\lambda = 3$: basis $\begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$

(b) $\lambda = 0$: basis $\begin{bmatrix} \frac{5}{3} \\ \frac{1}{3} \\ 1 \end{bmatrix}$; $\lambda = \sqrt{2}$: basis $\begin{bmatrix} \frac{1}{7}\left(15 + 5\sqrt{2}\right) \\ \frac{1}{7}\left(-1 + 2\sqrt{2}\right) \\ 1 \end{bmatrix}$; $\lambda = -\sqrt{2}$: basis $\begin{bmatrix} \frac{1}{7}\left(15 - 5\sqrt{2}\right) \\ \frac{1}{7}\left(-1 - 2\sqrt{2}\right) \\ 1 \end{bmatrix}$

(c) $\lambda = -8$: basis $\begin{bmatrix} -\frac{1}{6} \\ -\frac{1}{6} \\ 1 \end{bmatrix}$

(d) $\lambda = 2$: basis $\begin{bmatrix} \frac{1}{3} \\ \frac{1}{3} \\ 1 \end{bmatrix}$

(e) $\lambda = 2$: basis $\begin{bmatrix} -\frac{1}{3} \\ -\frac{1}{3} \\ 1 \end{bmatrix}$

(f) $\lambda = -4$: basis $\begin{bmatrix} -2 \\ \frac{8}{3} \\ 1 \end{bmatrix}$; $\lambda = 3$: basis $\begin{bmatrix} 5 \\ -2 \\ 1 \end{bmatrix}$

8.

(a) $\lambda = 1, \lambda = -2, \lambda = -1$

(b) $\lambda = 4$

10.

(a) $\lambda = -1, \lambda = 5$

(b) $\lambda = 3, \lambda = 7, \lambda = 1$

(c) $\lambda = -\frac{1}{3}, \lambda = 1, \lambda = \frac{1}{2}$

13.

(a) $y = x$ and $y = 2x$

(b) No lines

(c) $y = 0$

14.

(a) −5

(b) 7

22.

(a) $\lambda_1 = 1$: $\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$; $\lambda_2 = \frac{1}{2}$: $\begin{bmatrix} \frac{1}{2} \\ 1 \\ 0 \end{bmatrix}$; $\lambda_3 = \frac{1}{3}$: $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

(b) $\lambda_1 = -2: \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}; \lambda_2 = -1: \begin{bmatrix} \frac{1}{2} \\ 1 \\ 0 \end{bmatrix}; \lambda_3 = 0: \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

(c) $\lambda_1 = 3: \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}; \lambda_2 = 4: \begin{bmatrix} \frac{1}{2} \\ 1 \\ 0 \end{bmatrix}; \lambda_3 = 5: \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

25.

(a) A is 6×6

(b) A is invertible.

(c) A has three eigenspaces.

Exercise Set 7.2 (page 378)

1. $\lambda = 0: 1$ or 2; $\lambda = 1: 1$; $\lambda = 2: 1$, 2, or 3

3. Not diagonalizable

5. Not diagonalizable

7. Not diagonalizable

9. $P = \begin{bmatrix} \frac{1}{3} & 0 \\ 1 & 1 \end{bmatrix}; P^{-1}AP = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

11. $P = \begin{bmatrix} -2 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}; P^{-1}AP = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

13. $P = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 3 & 3 \\ 1 & 3 & 4 \end{bmatrix}; P^{-1}AP = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

16. Not diagonalizable

17. $P=\begin{bmatrix}1&1&0&0\\0&1&1&0\\0&0&1&1\\0&0&0&1\end{bmatrix}$; $P^{-1}AP=\begin{bmatrix}-2&0&0&0\\0&-2&0&0\\0&0&3&0\\0&0&0&3\end{bmatrix}$

19. $\begin{bmatrix}-1&10237&-2047\\0&1&0\\0&10245&-2048\end{bmatrix}$

21. $A^n=PD^nP^{-1}=\begin{bmatrix}1&1&1\\2&0&-1\\1&-1&1\end{bmatrix}\begin{bmatrix}1^n&0&0\\0&3^n&0\\0&0&4^n\end{bmatrix}\begin{bmatrix}\frac{1}{6}&\frac{1}{3}&\frac{1}{6}\\\frac{1}{2}&0&-\frac{1}{2}\\\frac{1}{3}&-\frac{1}{3}&\frac{1}{3}\end{bmatrix}$ One possibility is $P=\begin{bmatrix}-b&-b\\a-\lambda_1&a-\lambda_2\end{bmatrix}$ where λ_1 and λ_2 are as in Exercise 18 of Section 7.1.

25.
(a) False

(b) False

(c) True

(d) True

(e) True

27.
(a) Eigenvalues λ must satisfy $-1<\lambda\leq 1$.

(b) If $A=PDP^{-1}$ with *D* diagonal, then $\lim_{k\to+\infty}A^k=PD'P^{-1}$, where D' is obtained from *D* by setting all diagonal entries that are not 1 to 0.

Exercise Set 7.3 (page 383)

1.
(a) $\lambda^2-5\lambda=0$; $\lambda=0$: one-dimensional; $\lambda=5$: one-dimensional

(b) $\lambda^3-27\lambda-54=0$; $\lambda=6$: one-dimensional; $\lambda=-3$: two-dimensional

(c) $\lambda^3-3\lambda^2=0$; $\lambda=3$: one-dimensional; $\lambda=0$: two-dimensional

(d) $\lambda^3-12\lambda^2+36\lambda-32=0$; $\lambda=2$: two-dimensional; $\lambda=8$: one-dimensional

(e) $\lambda^4-8\lambda^3=0$; $\lambda=0$: three-dimensional; $\lambda=8$: one-dimensional

(f) $\lambda^4-8\lambda^3+22\lambda^2-24\lambda+9=0$; $\lambda=1$: two-dimensional; $\lambda=3$: two-dimensional

3.

$$P=\begin{bmatrix} -\frac{2}{\sqrt{7}} & \frac{\sqrt{3}}{\sqrt{7}} \\ \frac{\sqrt{3}}{\sqrt{7}} & \frac{2}{\sqrt{7}} \end{bmatrix};\ P^{-1}AP=\begin{bmatrix} 3 & 0 \\ 0 & 10 \end{bmatrix}$$

5.

$$P=\begin{bmatrix} -\frac{4}{5} & 0 & \frac{3}{5} \\ 0 & 1 & 0 \\ \frac{3}{5} & 0 & \frac{4}{5} \end{bmatrix};\ P^{-1}AP=\begin{bmatrix} 25 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & -50 \end{bmatrix}$$

7.

$$P=\begin{bmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{3}} & -\frac{2}{\sqrt{6}} & 0 \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{2}} \end{bmatrix}\begin{bmatrix} 0 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

9.

$$P=\begin{bmatrix} -\frac{4}{5} & \frac{3}{5} & 0 & 0 \\ \frac{3}{5} & \frac{4}{5} & 0 & 0 \\ 0 & 0 & -\frac{4}{5} & \frac{3}{5} \\ 0 & 0 & \frac{3}{5} & \frac{4}{5} \end{bmatrix};\ P^{-1}AP=\begin{bmatrix} -25 & 0 & 0 & 0 \\ 0 & 25 & 0 & 0 \\ 0 & 0 & -25 & 0 \\ 0 & 0 & 0 & 25 \end{bmatrix}$$

12.

(b)

$$\begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ 0 & 1 & 0 \\ -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{bmatrix}$$

15. Yes; take $A = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 4 \\ 0 & 4 & 3 \end{bmatrix}$.

Supplementary Exercises (page 384)

1. **(b)** The transformation rotates vectors through the angle θ; therefore, if $0 < \theta < \pi$, then no nonzero vector is transformed into a vector in the same or opposite direction.

3. **(c)** $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 3 \end{bmatrix}$

9. $A^2 = \begin{bmatrix} 15 & 30 \\ 5 & 10 \end{bmatrix}$, $A^3 = \begin{bmatrix} 75 & 150 \\ 25 & 50 \end{bmatrix}$, $A^4 = \begin{bmatrix} 375 & 750 \\ 125 & 250 \end{bmatrix}$, $A^5 = \begin{bmatrix} 1875 & 3750 \\ 625 & 1250 \end{bmatrix}$

12. **(b)** $\begin{bmatrix} 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -3 \end{bmatrix}$

13. The are all 0, 1, or −1.

15. $\begin{bmatrix} 1 & 0 & 0 \\ -1 & -\frac{1}{2} & -\frac{1}{2} \\ 1 & -\frac{1}{2} & -\frac{1}{2} \end{bmatrix}$

Exercise Set 8.1 (page 398)

3. Nonlinear

5. Linear

9. **(a)** Linear

(b) Nonlinear

13. $T(x_1, x_2) = \frac{1}{7}(3x_1 - x_2, \ -9x_1 - 4x_2, 5x_1 + 10x_2)$; $T(2, \ -3) = \left(\frac{9}{7}, \ -\frac{6}{7}, \ -\frac{20}{7}\right)$

15. $T(x_1, x_2, x_3) = (-41x_1 + 9x_2 + 24x_3, 14x_1 - 3x_2 - 8x_3)$; $T(7, 13, 7) = (-2, 3)$

17.

(a) Domain: R^2; codomain: R^2; $(T_2 \circ T_1)(x, y) = (2x - 3y, 2x + 3y)$

(b) Domain: R^2; codomain: R^2; $(T_2 \circ T_1)(x, y) = (4x - 12y, 3x - 9y)$

(c) Domain: R^2; codomain: R^2; $(T_2 \circ T_1)(x, y) = (2x + 3y, x - 2y)$

(d) Domain: R^2; codomain: R^2; $(T_2 \circ T_1)(x, y) = (0, 2x)$

19.

(a) $a + d$

(b) $(T_2 \circ T_1)(A)$ does not exist since $T_1(A)$ is not a 2×2 matrix.

22. $(T_2 \circ T_1)\left(a_0 + a_1x + a_2x^2\right) = (a_0 + a_1 + a_2)x + (a_1 + 2a_2)x^2 + a_2x^3$

26.

(b) $(3T)(x_1, x_2) = (6x_1 - 3x_2, 3x_2 + 3x_1)$

28.

(b) No

31.

(a) $x^2 + 3x$

(b) $\sin x$

(c) $e^x - 1$

Exercise Set 8.2 (page 405)

1. (a), (c)

3. (a), (b), (c)

5. (b)

7.

(a) $\left(\frac{1}{2}, 1\right)$

(b) $\left(\frac{3}{2}, -4, 1, 0\right)$

(c) No basis exists.

11.

(a) $\begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$

(b) $\begin{bmatrix} \frac{1}{2} \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

(c) $\text{Rank}(T) = 1$, $\text{nullity}(T) = 2$

(d) $\text{Rank}(A) = 1$, $\text{nullity}(A) = 2$

13.

(a) $\begin{bmatrix} 1 \\ 3 \\ -1 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -\frac{2}{7} \\ \frac{5}{14} \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$

(b) $\begin{bmatrix} -1 \\ -1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ -2 \\ 0 \\ 0 \\ 1 \end{bmatrix}$

(c) $\text{Rank}(T) = 3$, $\text{nullity}(T) = 2$

(d) $\text{Rank}(A) = 3$, $\text{nullity}(A) = 2$

15. $\ker(T) = \{\mathbf{0}\}$; $R(T) = V$

17. $\text{Nullity}(T) = 0$, $\text{rank}(T) = 6$

21.

(a) $x = -t, y = -t, z = t, \ -\infty < t < +\infty$

(b) $14x - 8y - 5z = 0$

25. $\ker(D)$ consists of all constant polynomials.

27. $\ker(D \circ D)$ consists of all functions of the form $ax + b$; $\ker(D \circ D \circ D)$ consists of all functions of the form $ax^2 + bx + c$.

30.

(a) $D \circ D \circ D \circ D$, where D is differentiation

(b) $D \circ D \circ \cdots \circ D(n + 1 \text{ times})$

Exercise Set 8.3 (page 413)

1.

(a) $\ker(T) = \{\mathbf{0}\}$; T is one-to-one.

(b) $\ker(T) = \left\{k\left(-\frac{3}{2}, 1\right)\right\}$; T is not one-to-one

(c) $\ker(T) = \{\mathbf{0}\}$; T is one-to-one

(d) $\ker(T) = \{\mathbf{0}\}$; T is one-to-one

(e) $\ker(T) = \{k(1, 1)\}$; T is not one-to-one

(f) $\ker(T) = \{k(0, 1, -1)\}$; T is not one-to-one

3.

(a) T has no inverse.

(b) $$T^{-1}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \frac{1}{8}x_1 + \frac{1}{8}x_2 - \frac{3}{4}x_3 \\ \frac{1}{8}x_1 + \frac{1}{8}x_2 + \frac{1}{4}x_3 \\ -\frac{3}{8}x_1 + \frac{5}{8}x_2 + \frac{1}{4}x_3 \end{bmatrix}$$

(c) $$T^{-1}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \frac{1}{2}x_1 - \frac{1}{2}x_2 + \frac{1}{2}x_3 \\ -\frac{1}{2}x_1 + \frac{1}{2}x_2 + \frac{1}{2}x_3 \\ \frac{1}{2}x_1 + \frac{1}{2}x_2 - \frac{1}{2}x_3 \end{bmatrix}$$

(d) $$T^{-1}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3x_1 + 3x_2 - x_3 \\ -2x_1 - 2x_2 + x_3 \\ -4x_1 - 5x_2 + 2x_3 \end{bmatrix}$$

5.

(a) $\ker(T) = \{k(-1, 1)\}$

(b) T is not one-to-one since $\ker(T) \neq \{\mathbf{0}\}$.

7.

(a) T is one-to-one.

(b) T is not one-to-one.

(c) T is not one-to-one.

(d) T is one-to-one.

11.

(a) $a_i \neq 0$ for $i = 1, 2, 3, \ldots, n$

(b) $T^{-1}(x_1, x_2, x_3, \ldots, x_n) = \left(\frac{1}{a_1}x_1, \frac{1}{a_2}x_2, \frac{1}{a_3}x_3, \ldots, \frac{1}{a_n}x_n\right)$

13.

(a) $T_1^{-1}(p(x)) = \frac{p(x)}{x}$; $T_2^{-1}(p(x)) = p(x-1)$; $(T_2 \circ T_1)^{-1}(p(x)) = \frac{1}{x}p(x-1)$

15.

(a) $(1, -1)$

(d) $T^{-1}(2, 3) = 2 + x$

17.

(a) T is not one-to-one.

(b) T is one-to-one. $T^{-1}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a & c \\ b & d \end{bmatrix}$

(c) T is one-to-one. $T^{-1}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

21. T is not one-to-one since, for example, $f(x) = x^2(x-1)^2$ is in its kernel.

25. Yes; it is one-to-one.

Exercise Set 8.4 (page 426)

1.

(a) $\begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

3.

(a) $\begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}$

5.

(a) $\begin{bmatrix} 0 & 0 \\ -\frac{1}{2} & 1 \\ \frac{8}{3} & \frac{4}{3} \end{bmatrix}$

7.

(a) $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 4 \\ 0 & 0 & 4 \end{bmatrix}$

(b) $3 + 10x + 16x^2$

9.

(a) $[T(\mathbf{v}_1)]_B = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$, $[T(\mathbf{v}_2)]_B = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$

(b) $T(\mathbf{v}_1) = \begin{bmatrix} 3 \\ -5 \end{bmatrix}$, $T(\mathbf{v}_2) = \begin{bmatrix} -2 \\ 29 \end{bmatrix}$

(c) $T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} \frac{18}{7} & \frac{1}{7} \\ -\frac{107}{7} & \frac{24}{7} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

(d) $\begin{bmatrix} \frac{19}{7} \\ -\frac{83}{7} \end{bmatrix}$

11.

(a) $[T(\mathbf{v}_1)]_B = \begin{bmatrix} 1 \\ 2 \\ 6 \end{bmatrix}, [T(\mathbf{v}_2)]_B = \begin{bmatrix} 3 \\ 0 \\ -2 \end{bmatrix}, [T(\mathbf{v}_3)]_B = \begin{bmatrix} -1 \\ 5 \\ 4 \end{bmatrix}$

(b) $T(\mathbf{v}_1) = 16 + 51x + 19x^2, T(\mathbf{v}_2) = -6 - 5x + 5x^2, T(\mathbf{v}_3) = 7 + 40x + 15x^2$

(c) $T\left(a_0 + a_1x + a_2x^2\right) = \frac{239a_0 - 161a_1 + 289a_2}{24} + \frac{201a_0 - 111a_1 + 247a_2}{8}x + \frac{61a_0 - 31a_1 + 107a_2}{12}x^2$

(d) $T\left(1 + x^2\right) = 22 + 56x + 14x^2$

13.

(a) $[T_2 \circ T_1]_{B',B} = \begin{bmatrix} 0 & 0 \\ 6 & 0 \\ 0 & 0 \\ 0 & -9 \end{bmatrix}, [T_2]_{B',B''} = \begin{bmatrix} 0 & 0 & 0 \\ 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}, [T_1]_{B'',B} = \begin{bmatrix} 2 & 0 \\ 0 & 0 \\ 0 & -3 \end{bmatrix}$

(b) $[T_2 \circ T_1]_{B',B} = [T_2]_{B',B''}[T_1]_{B'',B}$

19.

(a) $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$

(b) $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

(c) $\begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 2 \\ 0 & 0 & 2 \end{bmatrix}$

(d) $14e^{2x} - 8xe^{2x} - 20x^2e^{2x}$ since $\begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 2 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} 4 \\ 6 \\ -10 \end{bmatrix} = \begin{bmatrix} 14 \\ -8 \\ -20 \end{bmatrix}$

21.

(a) B', B''

(b) B', B'''

22. We can easily compute kernels, ranges, and compositions of linear transformations.

Exercise Set 8.5 (page 439)

1.
$$[T]_B = \begin{bmatrix} 1 & -2 \\ 0 & -1 \end{bmatrix},\ [T]_{B'} = \begin{bmatrix} -\frac{3}{11} & -\frac{56}{11} \\ -\frac{2}{11} & \frac{3}{11} \end{bmatrix}$$

3.
$$[T]_B = \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix},\ [T]_{B'} = \begin{bmatrix} \frac{13}{11\sqrt{2}} & -\frac{25}{11\sqrt{2}} \\ \frac{5}{11\sqrt{2}} & \frac{9}{11\sqrt{2}} \end{bmatrix}$$

5.
$$[T]_B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix},\ [T]_{B'} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

8.

(a) $\det(T) = 17$

(b) $\det(T) = 0$

(c) $\det(T) = 1$

10.

(a) $[T]_B = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 2 & 4 & 6 & 8 \\ 0 & 0 & 4 & 12 & 24 \\ 0 & 0 & 0 & 8 & 32 \\ 0 & 0 & 0 & 0 & 16 \end{bmatrix}$, where B is the standard basis for P_4; $\text{rank}\ (T) = 5$ and $\text{nullity}\ (T) = 0$.

(b) T is one-to-one.

12.

(a) $\mathbf{u}_1' = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix},\ \mathbf{u}_2' = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix},\ \mathbf{u}_3' = \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}$

(b) $\mathbf{u}_1' = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix},\ \mathbf{u}_2' = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix},\ \mathbf{u}_3' = \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}$

(c) $\mathbf{u}_1' = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$, $\mathbf{u}_2' = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$, $\mathbf{u}_3' = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$

14.

(a) $\lambda = 1, \lambda = -2, \lambda = -1$

(b) Basis for eigenspace corresponding to $\lambda = 1$: $\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$ and $\begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix}$;

basis for eigenspace corresponding to $\lambda = -2$: $\begin{bmatrix} -1 & 0 \\ 1 & 0 \end{bmatrix}$;

basis for eigenspace corresponding to $\lambda = -1$: $\begin{bmatrix} -2 & 1 \\ 1 & 0 \end{bmatrix}$

21.

1. $B = P^{-1}AP$ is similar to *A*.

2. $I = P^{-1}P$

3. The distributive law for matrices

4. The determinant of a product is the product of the determinants.

5. The commutative law for real multiplication

6. $\det\left(P^{-1}\right) = 1 / \det(P)$

23. The choice of an appropriate basis can yield a better understanding of the linear operator.

Exercise Set 8.6 (page 445)

2. When *A* is noninvertible.

5.

(a) No (not onto)

(b) Yes

(c) No (not one-to-one)

(d) No (not one-to-one)

11. The matrix is $\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -2 \\ 0 & 0 & 0 & 2 & 0 \end{bmatrix}$.

Supplementary Exercises (page 446)

1. No. $T(\mathbf{x}_1+\mathbf{x}_2)=A(\mathbf{x}_1+\mathbf{x}_2)+B\neq(A\mathbf{x}_1+B)+(A\mathbf{x}_2+B)=T(\mathbf{x}_1)+T(\mathbf{x}_2)$, and if $c\neq 1$, then $T(c\mathbf{x})=cA\mathbf{x}+B\neq c(A\mathbf{x}+B)=cT(\mathbf{x})$.

5.

(a) $T(\mathbf{e}_3)$ and any two of $T(\mathbf{e}_1)$, $T(\mathbf{e}_2)$, and $T(\mathbf{e}_4)$ form bases for the range; (−1, 1, 0, 1) is a basis for the kernel.

(b) $\text{Rank}=3$, $\text{nullity}=1$

7.

(a) $\text{Rank}\ (T)=2$ and $\text{nullity}\ (T)=2$

(b) T is not one-to-one.

11. $\text{Rank}=3$, $\text{nullity}=1$

13. $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

14.

(a) $\mathbf{v}_1=2\mathbf{u}_1+\mathbf{u}_2$, $\mathbf{v}_2=-\mathbf{u}_1+\mathbf{u}_2+\mathbf{u}_3$, $\mathbf{v}_3=3\mathbf{u}_1+4\mathbf{u}_2+2\mathbf{u}_3$

(b) $\mathbf{u}_1=-2\mathbf{v}_1-2\mathbf{v}_2+\mathbf{v}_3$, $\mathbf{u}_2=5\mathbf{v}_1+4\mathbf{v}_2-2\mathbf{v}_3$, $\mathbf{u}_3=-7\mathbf{v}_1-5\mathbf{v}_2+3\mathbf{v}_3$

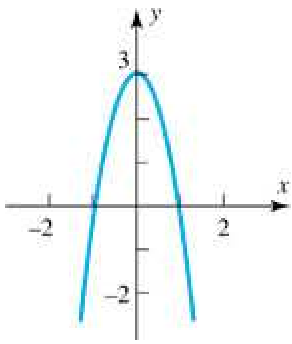
17. $[T]_B=\begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & -1 \end{bmatrix}$

20.

(a) $\begin{bmatrix} 2 \\ 6 \\ 12 \end{bmatrix}$

(d) $-3x^2+3$

(e)



21. The points are on the graph.

24. $\begin{bmatrix} 0 & 1 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & 0 & \cdots & 0 \end{bmatrix}$

Exercise Set 9.1 (page 456)

1.

(a)
$$y_1 = c_1e^{5x} - 2c_2e^{-x}$$
$$y_2 = c_1e^{5x} + c_2e^{-x}$$

(b)
$$y_1 = 0$$
$$y_2 = 0$$

3.

(a)
$$y_1 = -c_2e^{2x} + c_3e^{3x}$$
$$y_2 = c_1e^{x} + 2c_2e^{2x} - c_3e^{3x}$$
$$y_3 = 2c_2e^{2x} - c_3e^{3x}$$

(b)
$$y_1 = e^{2x} - 2e^{3x}$$
$$y_2 = e^{x} - 2e^{2x} + 2e^{3x}$$
$$y_3 = -2e^{2x} + 2e^{3x}$$

7. $y = c_1e^{3x} + c_2e^{-2x}$

9. $y = c_1e^{x} + c_2e^{2x} + c_3e^{3x}$

Exercise Set 9.2 (page 466)

1. **(a)** $\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$

(b) $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$

(d) $\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$

3. **(a)** $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

5. **(a)** $\begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$

7. Rectangle with vertices at (0, 0), (−3, 0), (0, 1), (−3, 1)

9. **(a)** $\begin{bmatrix} 1 & 0 \\ 4 & 1 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix}$

10. **(a)** $\begin{bmatrix} 1 & 0 \\ 0 & \frac{1}{3} \end{bmatrix}$

(b) $\begin{bmatrix} 6 & 0 \\ 0 & 1 \end{bmatrix}$

12.

(a) $\begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix}$; expansion in the y-direction by a factor of 3, then expansion in the x-direction by a factor of 2

(b) $\begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 4 \\ 0 & 1 \end{bmatrix}$; shear in the x-direction by a factor of 4, then shear in the y-direction by a factor of 2

(c) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 4 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & -2 \end{bmatrix}$; expansion in the y-direction by a factor of −2, then expansion in the x-direction by a factor of 4, then reflection about $y = x$

(d) $\begin{bmatrix} 1 & 0 \\ 4 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 1 & 18 \end{bmatrix}\begin{bmatrix} 1 & -3 \\ 0 & 1 \end{bmatrix}$; shear in the x-direction by a factor of −3, then expansion in the y-direction by a factor of 18, then shear in the y-direction by a factor of 4

14.

(a) $\begin{bmatrix} 0 & 1 \\ -5 & 0 \end{bmatrix}$

(b) $\frac{1}{2}\begin{bmatrix} \sqrt{3} & -1 \\ -6\sqrt{3}+3 & 6+3\sqrt{3} \end{bmatrix}$

17.

(a) $y = \frac{2}{7}x$

(b) $y = x$

(c) $y = \frac{1}{2}x$

(d) $y = -2x$

22.

(a) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

(b) $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

24.

(a) $\lambda_1 = 1$: $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$; $\lambda_2 = -1$: $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$

(b) $\lambda_1 = 1$: $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$; $\lambda_2 = -1$: $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$

(c) $\lambda_1 = 1: \begin{bmatrix} 1 \\ 1 \end{bmatrix}; \lambda_2 = -1: \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

(d) $\lambda = 1: \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

(e) $\lambda = 1: \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

(f) (θ an odd integer multiple of π) $\lambda = -1$: (1, 0), (0, 1)

(θ an even integer multiple of π) $\lambda = 1$: (1, 0), (0, 1)

(θ not an integer multiple of π) no real eigenvalues

Exercise Set 9.3 (page 473)

1. $y = -\frac{1}{2} + \frac{7}{2}x$

3. $y = 2 + 5x - 3x^2$

8. $y = 4 - .2x + .2x^2$; if $x = 12$, then $y = 30.4$ ($30.4 thousand)

Exercise Set 9.4 (page 479)

1. **(a)** $(1+\pi) - 2\sin x - \sin 2x$

(b) $(1+\pi) - 2\left[\sin x + \frac{\sin 2x}{2} + \frac{\sin 3x}{3} + \cdots + \frac{\sin nx}{n}\right]$

3. **(a)** 1

(b) 1

5. **(a)** $\frac{3}{\pi}$

(b) 1

8. $\sum_{k=1}^{\infty} \frac{2}{k} \sin$

Exercise Set 9.5 (page 485)

1. (a), (c), (e), (g), (h)

3. **(a)** $A = \begin{bmatrix} 9 & 3 & -4 \\ 3 & -1 & \frac{1}{2} \\ -4 & \frac{1}{2} & 4 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & -\frac{5}{2} & \frac{9}{2} \\ -\frac{5}{2} & 1 & 0 \\ \frac{9}{2} & 0 & -3 \end{bmatrix}$

(c) $A = \begin{bmatrix} 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}$

(d) $A=\begin{bmatrix} \sqrt{2} & \sqrt{2} & -4\sqrt{3} \\ \sqrt{2} & 0 & 0 \\ -4\sqrt{3} & 0 & -\sqrt{3} \end{bmatrix}$

(e) $A=\begin{bmatrix} 1 & 1 & 0 & -5 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 2 \\ -5 & 0 & 2 & -1 \end{bmatrix}$

5.

(a) $\text{max value} = 5$ at $\pm(1, 0)$; $\text{min value} = -1$ at $\pm(0, 1)$

(b) $\text{max value} = \dfrac{11+\sqrt{10}}{2}$ at $\pm\left(\dfrac{1}{\sqrt{20-6\sqrt{10}}}, \dfrac{1}{\sqrt{20+6\sqrt{10}}}\right)$; $\text{min value} = \dfrac{11-\sqrt{10}}{2}$ at $\pm\left(\dfrac{-1}{\sqrt{20+6\sqrt{10}}}, \dfrac{1}{\sqrt{20-6\sqrt{10}}}\right)$

(c) $\text{max value} = \dfrac{7+\sqrt{10}}{2}$ at $\pm\left(\dfrac{1}{\sqrt{20-6\sqrt{10}}}, \dfrac{-1}{\sqrt{20-6\sqrt{10}}}\right)$; $\text{min value} = \dfrac{7-\sqrt{10}}{2}$ at $\pm\left(\dfrac{1}{\sqrt{20+6\sqrt{10}}}, \dfrac{1}{\sqrt{20-6\sqrt{10}}}\right)$

(d) $\text{max value} = \dfrac{3+\sqrt{10}}{2}$ at $\pm\left(\dfrac{3}{\sqrt{20-2\sqrt{10}}}, \dfrac{3}{\sqrt{20+2\sqrt{10}}}\right)$; $\text{min value} = \dfrac{3-\sqrt{10}}{2}$ at $\pm\left(\dfrac{3}{\sqrt{20+2\sqrt{10}}}, \dfrac{-3}{\sqrt{20-2\sqrt{10}}}\right)$

7. (b)

9. (a)

11.

(a) Positive definite

(b) Negative definite

(c) Positive semidefinite

(d) Negative semidefinite

(e) Indefinite

(f) Indefinite

13. **(c)**

16.

(a)
$$A=\begin{bmatrix} \frac{1}{n} & \frac{-1}{n(n-1)} & \frac{-1}{n(n-1)} & \cdots & \frac{-1}{n(n-1)} \\ \frac{-1}{n(n-1)} & \frac{1}{n} & \frac{-1}{n(n-1)} & \cdots & \frac{-1}{n(n-1)} \\ \vdots & \vdots & \vdots & & \vdots \\ \frac{-1}{n(n-1)} & \frac{-1}{n(n-1)} & \frac{-1}{n(n-1)} & \cdots & \frac{1}{n} \end{bmatrix}$$

(b) Positive semidefinite

Exercise Set 9.6 (page 496)

1.

(a)
$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}=\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}\begin{bmatrix} y_1 \\ y_2 \end{bmatrix};\ y_1^2+3y_2^2$$

(b)
$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}=\begin{bmatrix} \frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}} \\ -\frac{2}{\sqrt{5}} & \frac{1}{\sqrt{5}} \end{bmatrix}\begin{bmatrix} y_1 \\ y_2 \end{bmatrix};\ y_1^2+6y_2^2$$

(c)
$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}=\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}\begin{bmatrix} y_1 \\ y_2 \end{bmatrix};\ y_1^2-y_2^2$$

(d)
$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}=\begin{bmatrix} \frac{\sqrt{17}-4}{\sqrt{34-8\sqrt{17}}} & \frac{\sqrt{17}+4}{\sqrt{34+8\sqrt{17}}} \\ \frac{1}{\sqrt{34-8\sqrt{17}}} & \frac{-1}{\sqrt{34+8\sqrt{17}}} \end{bmatrix}\begin{bmatrix} y_1 \\ y_2 \end{bmatrix};\ \left(1+\sqrt{17}\right)y_1^2+\left(1-\sqrt{17}\right)y_2^2$$

3.

(a) $2x^2-3xy+4y^2$

(b) x^2-xy

(c) $5xy$

(d) $4x^2 - 2y^2$

(e) y^2

5.

(a) $[x \quad y]\begin{bmatrix} 2 & -\frac{3}{2} \\ -\frac{3}{2} & 4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + [-7 \quad 2]\begin{bmatrix} x \\ y \end{bmatrix} + 7 = 0$

(b) $[x \quad y]\begin{bmatrix} 1 & -\frac{1}{2} \\ -\frac{1}{2} & 0 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + [5 \quad 8]\begin{bmatrix} x \\ y \end{bmatrix} - 3 = 0$

(c) $[x \quad y]\begin{bmatrix} 0 & \frac{5}{2} \\ \frac{5}{2} & 0 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} - 8 = 0$

(d) $[x \quad y]\begin{bmatrix} 4 & 0 \\ 0 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} - 7 = 0$

(e) $[x \quad y]\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + [7 \quad -8]\begin{bmatrix} x \\ y \end{bmatrix} - 5 = 0$

7.

(a) $9x'^2 + 4y'^2 = 36$, ellipse

(b) $x'^2 - 16y'^2 = 16$, hyperbola

(c) $y'^2 = 8x'$, parabola

(d) $x'^2 + y'^2 = 16$, circle

(e) $18y'^2 - 12x'^2 = 419$, hyperbola

(f) $y' = -\frac{1}{7}x'^2$, parabola

9. $2x''^2 + y''^2 = 6$, ellipse

11. $2x''^2 - 3y''^2 = 24$, hyperbola

15.

(a) Two intersecting lines, $y=x$ and $y=-x$

(b) No graph

(c) The graph is the single point (0, 0).

(d) The graph is the line $y=x$.

(e) The graph consists of two parallel lines $\frac{3}{\sqrt{13}}x+\frac{2}{\sqrt{13}}y=\pm 2$.

(f) The graph is the single point (1, 2).

Exercise Set 9.7 (page 501)

1.

(a) $x^2+2y^2-z^2+4xy-5yz$

(b) $3x^2+7z^2+2xy-3xz+4yz$

(c) $xy+xz+yz$

(d) $x^2+y^2-z^2$

(e) $3z^2+3xz$

(f) $2z^2+2xz+y^2$

3.

(a) $$[x\ \ y\ \ z]\begin{bmatrix}1 & 2 & 0\\ 2 & 2 & -\frac{5}{2}\\ 0 & -\frac{5}{2} & -1\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix}+[7\ \ 0\ \ 2]\begin{bmatrix}x\\ y\\ z\end{bmatrix}-3=0$$

(b) $$[x\ \ y\ \ z]\begin{bmatrix}3 & 1 & -\frac{3}{2}\\ 1 & 0 & 2\\ -\frac{3}{2} & 2 & 7\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix}+[-3\ \ 0\ \ 0]\begin{bmatrix}x\\ y\\ z\end{bmatrix}-4=0$$

(c) $$\begin{bmatrix} x & y & z \end{bmatrix}\begin{bmatrix} 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} - 1 = 0$$

(d) $$\begin{bmatrix} x & y & z \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} - 7 = 0$$

(e) $$\begin{bmatrix} x & y & z \end{bmatrix}\begin{bmatrix} 0 & 0 & \frac{3}{2} \\ 0 & 0 & 0 \\ \frac{3}{2} & 0 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} + \begin{bmatrix} 0 & -14 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} + 9 = 0$$

(f) $$\begin{bmatrix} x & y & z \end{bmatrix}\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} + \begin{bmatrix} 2 & -1 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

7.

(a) $9x'^2 + 36y'^2 + 4z'^2 = 36$, ellipsoid

(b) $6x'^2 + 3y'^2 - 2z'^2 = 18$, hyperboloid of one sheet

(c) $3x'^2 - 3y'^2 - z'^2 = 3$, hyperboloid of two sheets

(d) $4x'^2 + 9y'^2 - z'^2 = 0$, elliptic cone

(e) $x'^2 + 16y'^2 - 16z' = 0$, elliptic paraboloid

(f) $7x'^2 - 3y'^2 + z' = 0$, hyperbolic paraboloid

(g) $x'^2 + y'^2 + z'^2 = 25$, sphere

9. $x''^2 + y''^2 - 2z''^2 = -1$, hyperboloid of two sheets

11. $x''^2 - y''^2 + z'' = 0$, hyperbolic paraboloid

Exercise Set 9.8 (page 509)

1. Multiplications: *mpn*; additions: $mp(n-1)$

3.

	$n=5$	$n=10$	$n=100$	$n=1000$
Solve $A\mathbf{x}=\mathbf{b}$ by Gauss–Jordan elimination	+: 50 ×: 65	+: 375 ×: 430	+: 383,250 ×: 343,300	+: 333,283,500 ×: 334,333,000
Solve $A\mathbf{x}=\mathbf{b}$ by Gaussian elimination	+: 50 ×: 65	+: 375 ×: 430	+: 383,250 ×: 343,300	+: 333,283,500 ×: 334,333,000
Find A^{-1} by reducing $[A \mid I]$ to $[I \mid A^{-1}]$	+: 80 ×: 125	+: 810 ×: 1000	+: 980,100 ×: 1,000,000	+: 998,001,000 ×: 1,000,000,000
Solve $A\mathbf{x}=\mathbf{b}$ as $\mathbf{x}=A^{-1}\mathbf{b}$	+: 100 ×: 150	+: 900 ×: 1100	+: 990,000 ×: 1,010,000	+: 999,000,000 ×: 1,001,000,000
Find $\det(A)$ by row reduction	+: 30 ×: 44	+: 285 ×: 339	+: 328,350 ×: 333,399	+: 332,833,500 ×: 333,333,999
Solve $A\mathbf{x}=\mathbf{b}$ by Cramer's Rule	+: 180 ×: 264	+: 3135 ×: 3729	+: 33,163,350 ×: 33,673,399	+: $33{,}316{,}633 \times 10^4$ ×: $33{,}366{,}733 \times 10^4$

4.

	$n=5$	$n=10$	$n=100$	$n=1000$
	Execution Time (sec)	**Execution Time (sec)**	**Execution Time (sec)**	**Execution Time (sec)**
Solve $A\mathbf{x}=\mathbf{b}$ by Gauss–Jordan elimination	1.55×10^{-4}	1.05×10^{-3}	.878	836
Solve $A\mathbf{x}=\mathbf{b}$ by Gaussian elimination	1.55×10^{-4}	1.05×10^{-3}	.878	836
Find A^{-1} by reducing $[A \mid I]$ to $[I \mid A^{-1}]$	2.84×10^{-4}	2.41×10^{-3}	2.49	2499
Solve $A\mathbf{x}=\mathbf{b}$ as $\mathbf{x}=A^{-1}\mathbf{b}$	3.50×10^{-4}	2.65×10^{-3}	2.52	2502
Find $\det(A)$ by row reduction	1.03×10^{-4}	8.21×10^{-4}	.831	833

	$n=5$	$n=10$	$n=100$	$n=1000$
	Execution Time (sec)	**Execution Time (sec)**	**Execution Time (sec)**	**Execution Time (sec)**
Solve $A\mathbf{x}=\mathbf{b}$ by Cramer's Rule	6.18×10^{-4}	90.3×10^{-4}	83.9	834×10^{3}

Exercise Set 9.9 (page 517)

1. $x_1=2, x_2=1$

3. $x_1=3, x_2=-1$

5. $x_1=-1, x_2=1, x_3=0$

7. $x_1=-1, x_2=1, x_3=0$

9. $x_1=-3, x_2=1, x_3=2, x_4=1$

11.

(a) $A=LU=\begin{bmatrix} 2 & 0 & 0 \\ -2 & 1 & 0 \\ 2 & 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & \frac{1}{2} & -\frac{1}{2} \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

(b) $A=L_1DU=\begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & \frac{1}{2} & -\frac{1}{2} \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

(c) $A=L_2U_2=\begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} 2 & 1 & -1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

13.

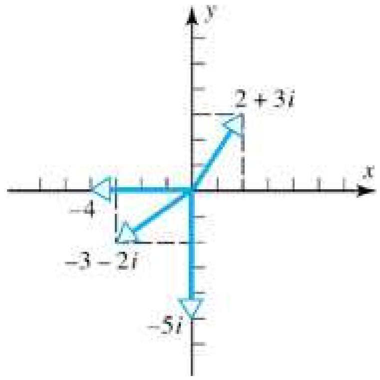
(b) $\begin{bmatrix} a & b \\ c & d \end{bmatrix}=\begin{bmatrix} 1 & 0 \\ \frac{c}{a} & 1 \end{bmatrix}\begin{bmatrix} a & b \\ 0 & \frac{ad-bc}{a} \end{bmatrix}$

18.

$$A=PLU=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 3 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & -\frac{1}{3} & 0 \\ 0 & 1 & \frac{1}{2} \\ 0 & 0 & 1 \end{bmatrix}$$

Exercise Set 10.1 (page 526)

1. (a–d)



3.

(a) $x = -2, y = -3$

(b) $x = 2, y = 1$

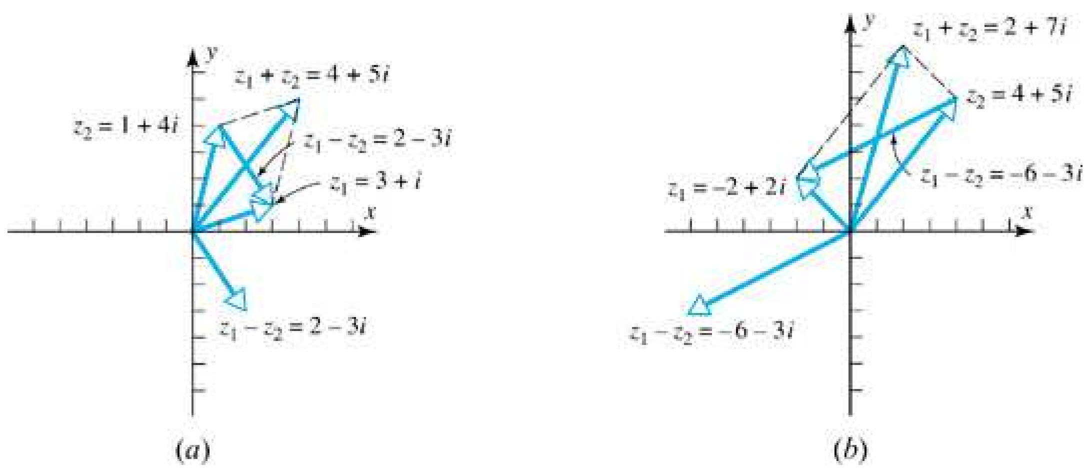
5.

(a) $2 + 3i$

(b) $-1 - 2i$

(c) $-2 + 9i$

6.



9.

(a) $z_1 z_2 = 3 + 3i, z_1^2 = -9, z_2^2 = -2i$

(b) $z_1 z_2 = 26, z_1^2 = -20 + 48i, z_2^2 = -5 - 12i$

(c) $z_1 z_2 = \frac{11}{3} - i$, $z_1^2 = \frac{4}{9}(-3+4i)$, $z_2^2 = -6 - \frac{5}{2}i$

11. $76 - 88i$

12. $26 - 18i$

16. $\left(2+\sqrt{2}\right) + i\left(1-\sqrt{2}\right)$

18. $-24i$

20.

(a) $\begin{bmatrix} 13+13i & -8+12i & -33-22i \\ 1+i & 0 & i \\ 7+9i & -6+6i & -16-16i \end{bmatrix}$

(b) $\begin{bmatrix} 6+2i & -11+19i \\ -1+6i & -9-5i \end{bmatrix}$

(c) $\begin{bmatrix} 6i & 1+i \\ -6-i & 5-9i \end{bmatrix}$

(d) $\begin{bmatrix} 22-7i & 2+10i \\ -5-4i & 6-8i \\ 9-i & -1-i \end{bmatrix}$

22.

(a) $z = -1 \pm i$

(b) $z = \frac{1}{2} \pm \frac{\sqrt{3}}{2}i$

Exercise Set 10.2 (page 531)

1.

(a) $2 - 7i$

(b) $-3 + 5i$

(c) $-5i$

(d) i

(e) -9

(f) 0

5.

(a) $-i$

(b) $\frac{1}{26} + \frac{5}{26}i$

(c) $7i$

7. $\frac{1}{2} + \frac{1}{2}i$

9. $-\frac{7}{625} - \frac{24}{625}i$

11. $\frac{1-\sqrt{3}}{4} + \frac{1+\sqrt{3}}{4}i$

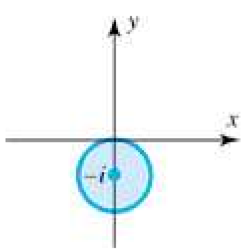
15.

(a) $-1-2i$

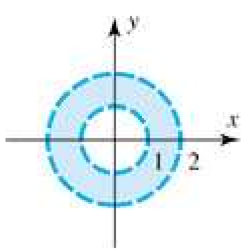
(b) $-\frac{3}{25} - \frac{4}{25}i$

18.

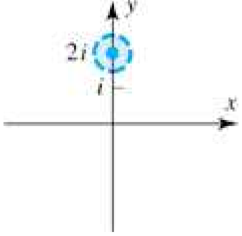
(a)

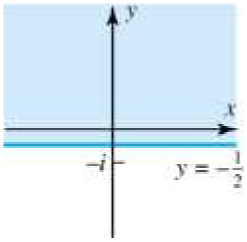


(b)



(c)



(d) 

23.

(a) $\dfrac{x_1x_2+y_1y_2}{x_2^2+y_2^2}$

(b) $\dfrac{x_2y_1-x_1y_2}{x_2^2+y_2^2}$

27.

(c) Yes, if $z \neq 0$.

30. $x_1=\dfrac{1}{2}+i,\ x_2=2,\ x_3=\dfrac{1}{2}-i$

33. $x_1=(1+i)t,\ x_2=2t$

35.

(a) $\begin{bmatrix} i & 2 \\ -1 & i \end{bmatrix}$

(b) $\begin{bmatrix} 0 & 1 \\ -i & 2i \end{bmatrix}$

39.

(a) $\begin{bmatrix} -i & -2-2i & -1+i \\ 1 & 2 & -i \\ i & i & 1 \end{bmatrix}$

(b) $\begin{bmatrix} 1+i & -i & 1 \\ -7+6i & 5-i & 1+4i \\ 1+2i & -i & 1 \end{bmatrix}$

Exercise Set 10.3 (page 539)

1.

(a) 0

(b) $\pi / 2$

(c) $-\pi / 2$

(d) $\pi / 4$

(e) $2\pi / 3$

(f) $-\pi / 4$

3.

(a) $2\left[\cos\left(\frac{\pi}{2}\right) + i \sin\left(\frac{\pi}{2}\right)\right]$

(b) $4\left[\cos \pi + i \sin \pi\right]$

(c) $5\sqrt{2}\left[\cos\left(\frac{\pi}{4}\right) + i \sin\left(\frac{\pi}{4}\right)\right]$

(d) $12\left[\cos\left(\frac{2\pi}{3}\right) + i \sin\left(\frac{2\pi}{3}\right)\right]$

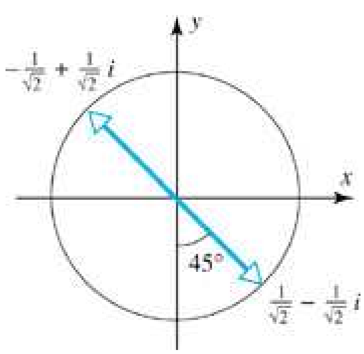
(e) $3\sqrt{2}\left[\cos\left(-\frac{3\pi}{4}\right) + i \sin\left(-\frac{3\pi}{4}\right)\right]$

(f) $4\left[\cos\left(-\frac{\pi}{6}\right) + i \sin\left(-\frac{\pi}{6}\right)\right]$

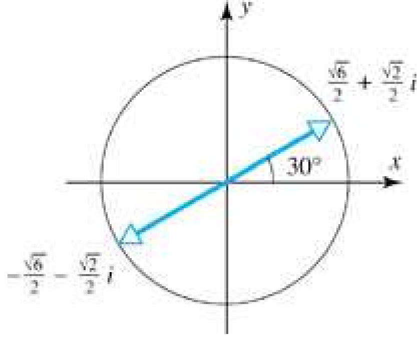
5. 1

7.

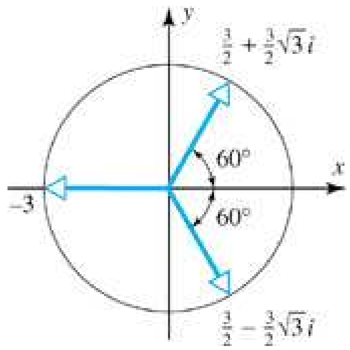
(a)



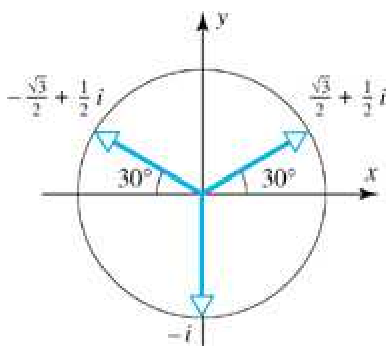
(b)



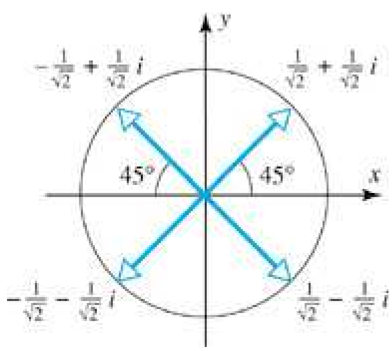
(c)



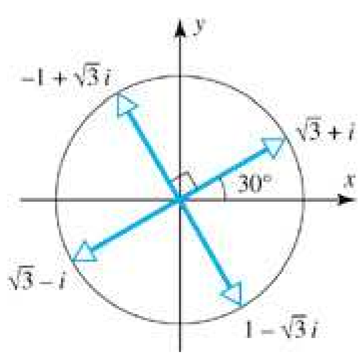
(d)



(e)



(f)



10. $\sqrt[4]{2}\left[\cos\left(\frac{\pi}{8}\right) + i\sin\left(\frac{\pi}{8}\right)\right], \sqrt[4]{2}\left[\cos\left(\frac{9\pi}{8}\right) + i\sin\left(\frac{9\pi}{8}\right)\right]$

12. The roots are $\pm\left(2^{1/4} + 2^{1/4} i\right)$, $\pm\left(2^{1/4} - 2^{1/4} i\right)$ and the factorization is

$$z^4 + 8 = \left(z^2 - 2^{5/4}z + 2^{3/2}\right)\left(z^2 + 2^{5/4}z + 2^{3/2}\right).$$

15.

(a) $\mathrm{Re}(z) = -3, \mathrm{Im}(z) = 0$

(b) $\mathrm{Re}(z) = -3, \mathrm{Im}(z) = 0$

(c) $\mathrm{Re}(z) = 0, \mathrm{Im}(z) = -\sqrt{2}$

(d) $\mathrm{Re}(z) = -3, \mathrm{Im}(z) = 0$

20. $\cos 2\theta = \cos^2\theta - \sin^2\theta, \sin 2\theta = 2\sin\theta\cos\theta,$

$\cos 3\theta = \cos^3\theta - 3\sin^2\theta\cos\theta, \sin 3\theta = 3\sin\theta\cos^2\theta - \sin^3\theta$

Exercise Set 10.4 (page 544)

1.

(a) $(3i, -i, -2-i, 4)$

(b) $(3+2i, -1-2i, -3+5i, -i)$

(c) $(-1-2i, 2i, 2-i, -1)$

(d) $(-3+9i, 3-3i, -3-6i, 12+3i)$

(e) $(-3+2i, 3, -3-3i, i)$

(f) $(-1-5i, 3i, 4, -5)$

5.

(a) $\sqrt{2}$

(b) $2\sqrt{3}$

(c) $\sqrt{10}$

(d) $\sqrt{37}$

9.

(a) 3

(b) $2-27i$

(c) $-5-10i$

11. Not a vector space. Axiom 6 fails; that is, the set is not closed under scalar multiplication. (Multiply by *i*, for example.)

13. $\ker T$ is all multiples of $\begin{bmatrix} 1+3i \\ 1+i \\ -2 \end{bmatrix}$; nullity of $T=1$

17.

(a) $(-3-2i)\mathbf{u}+(3-i)\mathbf{v}+(1+2i)\mathbf{w}$

(b) $(2+i)\mathbf{u}+(-1+i)\mathbf{v}+(-1-i)\mathbf{w}$

(c) $0\mathbf{u}+0\mathbf{v}+0\mathbf{w}$

(d) $(-5-4i)\mathbf{u}+(5+2i)\mathbf{v}+(2+4i)\mathbf{w}$

19. (a), (b), (c)

21. (b), (c)

23. $\mathbf{f}-3\mathbf{g}-3\mathbf{h}=\mathbf{0}$

25. (a), (b)

27. $(-1-i,\ 1)$; dimension $=1$

30. $\left(\frac{5}{2}i,\ -\frac{1}{2},\ 1,\ 0\right),\ \left(-\frac{1}{4},\ \frac{3}{4}i,\ 0,\ 1\right)$; dimension $=2$

Exercise Set 10.5 (page 551)

2.

(a) -12

(b) 0

(c) $2i$

(d) 37

4.

(a) $-4+5i$

(b) 0

(c) $4-4i$

(d) 42

6. $-9-5i$

8. No. Axiom 4 fails.

10.

(a) $\sqrt{10}$

(b) 2

(c) $\sqrt{5}$

(d) 0

12.

(a) $3\sqrt{10}$

(b) $\sqrt{14}$

14.

(a) 2

(b) $2\sqrt{2}$

16.

(a) $7\sqrt{2}$

(b) $2\sqrt{3}$

20. (b)

23. $\left(\frac{i}{\sqrt{2}}, 0, 0, \frac{i}{\sqrt{2}}\right), \left(-\frac{i}{\sqrt{6}}, 0, \frac{2i}{\sqrt{6}}, \frac{i}{\sqrt{6}}\right), \left(\frac{2i}{\sqrt{21}}, \frac{3i}{\sqrt{21}}, \frac{2i}{\sqrt{21}}, \frac{-2i}{\sqrt{21}}\right), \left(-\frac{i}{\sqrt{7}}, \frac{2i}{\sqrt{7}}, -\frac{i}{\sqrt{7}}, \frac{i}{\sqrt{7}}\right)$

25.

(a) $\mathbf{v}_1 = \left(\frac{i}{\sqrt{3}}, \frac{i}{\sqrt{3}}, \frac{i}{\sqrt{3}}\right), \mathbf{v}_2 = \left(-\frac{i}{\sqrt{2}}, \frac{i}{\sqrt{2}}, 0\right), \mathbf{v}_3 = \left(\frac{i}{\sqrt{6}}, \frac{i}{\sqrt{6}}, -\frac{2i}{\sqrt{6}}\right)$

(b) $\mathbf{v}_1 = (i, 0, 0), \mathbf{v}_2 = \left(0, \frac{7i}{\sqrt{53}}, \frac{-2i}{\sqrt{53}}\right), \mathbf{v}_3 = \left(0, \frac{2i}{\sqrt{53}}, \frac{7i}{\sqrt{53}}\right)$

27. $\mathbf{v}_1 = \left(0, \frac{i}{\sqrt{3}}, \frac{1-i}{\sqrt{3}}\right), \mathbf{v}_2 = \left(-\frac{3i}{\sqrt{15}}, \frac{2}{\sqrt{15}}, \frac{1+i}{\sqrt{15}}\right)$

36. $\mathbf{u} = -\sqrt{3}i\mathbf{v}_1 + \frac{3}{\sqrt{6}}\mathbf{v}_2 - \frac{1}{\sqrt{2}}\mathbf{v}_3$

Exercise Set 10.6 (page 561)

1.

(a) $\begin{bmatrix} -2i & 4 & 5-i \\ 1+i & 3-i & 0 \end{bmatrix}$

(b) $\begin{bmatrix} -2i & 4 & -i \\ 1+i & 5+7i & 3 \\ -1-i & i & 1 \end{bmatrix}$

(c) $\begin{bmatrix} -7i \\ 0 \\ 3i \end{bmatrix}$

(d) $\begin{bmatrix} \bar{a}_{11} & \bar{a}_{21} \\ \bar{a}_{12} & \bar{a}_{22} \\ \bar{a}_{13} & \bar{a}_{23} \end{bmatrix}$

3. $k = 3 + 5i,\ l = i,\ m = 2 - 4i$

4. (a), (b)

5.

(a) $A^{-1} = \begin{bmatrix} \frac{3}{5} & -\frac{4}{5} \\ -\frac{4}{5}i & -\frac{3}{5}i \end{bmatrix}$

(b) $A^{-1} = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{-1+i}{2} \\ \frac{1}{\sqrt{2}} & \frac{1-i}{2} \end{bmatrix}$

(c) $A^{-1} = \begin{bmatrix} \frac{1}{2\sqrt{2}}\left(\sqrt{3}-i\right) & \frac{1}{2\sqrt{2}}\left(1-\sqrt{3}i\right) \\ \frac{1}{2\sqrt{2}}\left(1+\sqrt{3}i\right) & \frac{1}{2\sqrt{2}}\left(-\sqrt{3}-i\right) \end{bmatrix}$

(d) $A^{-1} = \begin{bmatrix} \frac{1-i}{2} & -\frac{i}{\sqrt{3}} & \frac{3-i}{2\sqrt{15}} \\ -\frac{1}{2} & \frac{1}{\sqrt{3}} & \frac{4-3i}{2\sqrt{15}} \\ \frac{1}{2} & \frac{i}{\sqrt{3}} & -\frac{5i}{2\sqrt{15}} \end{bmatrix}$

7. $P = \begin{bmatrix} \frac{-1+i}{\sqrt{3}} & \frac{1-i}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & \frac{2}{\sqrt{6}} \end{bmatrix};\ P^{-1}AP = \begin{bmatrix} 3 & 0 \\ 0 & 6 \end{bmatrix}$

9. $P = \begin{bmatrix} -\frac{1+i}{\sqrt{6}} & \frac{1+i}{\sqrt{3}} \\ \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} \end{bmatrix};\ P^{-1}AP = \begin{bmatrix} 2 & 0 \\ 0 & 8 \end{bmatrix}$

11. $P = \begin{bmatrix} 0 & 1 & 0 \\ -\frac{1-i}{\sqrt{6}} & 0 & \frac{1-i}{\sqrt{3}} \\ \frac{2}{\sqrt{6}} & 0 & \frac{1}{\sqrt{3}} \end{bmatrix};\ P^{-1}AP = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & -2 \end{bmatrix}$

14.

(a) $\begin{bmatrix} 0 & i \\ -i & 0 \end{bmatrix}$ is one possibility.

Supplementary Exercises (page 563)

3. $\begin{bmatrix} -i \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ is one possibility.

5. $\lambda = 1$, ω, $\omega^2 (= \bar{\omega})$

Exercise Set 11.1 (page 572)

1.

(a) $y = 3x - 4$

(b) $y = -2x + 1$

2.

(a) $x^2 + y^2 - 4x - 6y + 4 = 0$ or $(x-2)^2 + (y-3)^2 = 9$

(b) $x^2 + y^2 + 2x - 4y - 20 = 0$ or $(x+1)^2 + (y-2)^2 = 25$

3. $x^2 + 2xy + y^2 - 2x + y = 0$ (a parabola)

4.

(a) $x + 2y + z = 0$

(b) $-x + y - 2z + 1 = 0$

5.

(a) $\begin{bmatrix} x & y & z & 0 \\ x_1 & y_1 & z_1 & 1 \\ x_2 & y_2 & z_2 & 1 \\ x_3 & y_3 & z_3 & 1 \end{bmatrix} = 0$

(b) $x + 2y + z = 0; \ -x + y - 2z = 0$

6.

(a) $x^2+y^2+z^2-2x-4y-2z=-2$ or $(x-1)^2+(y-2)^2+(z-1)^2=4$

(b) $x^2+y^2+z^2-2x-2y=3$ or $(x-1)^2+(y-1)^2+z^2=5$

10. $\begin{vmatrix} y & x^2 & x & 1 \\ y_1 & x_1^2 & x_1 & 1 \\ y_2 & x_2^2 & x_2 & 1 \\ y_3 & x_3^2 & x_3 & 1 \end{vmatrix}=0$

11. The equation of the line through the three collinear points

12. $0=0$

13. The equation of the plane through the four coplanar points

Exercise Set 11.2 (page 576)

1. $I_1=\frac{255}{317}, I_2=\frac{97}{317}, I_3=\frac{158}{317}$

2. $I_1=\frac{13}{5}, I_2=-\frac{2}{5}, I_3=\frac{11}{5}$

3. $I_1=-\frac{5}{22}, I_2=\frac{7}{22}, I_3=\frac{6}{11}$

4. $I_1=\frac{1}{2}, I_2=0, I_3=0, I_4=\frac{1}{2}, I_5=\frac{1}{2}, I_6=\frac{1}{2}$

Exercise Set 11.3 (page 588)

1. $x_1=2, x_2=\frac{2}{3}$; maximum value of $z=\frac{22}{3}$

2. No feasible solutions

3. Unbounded solution

4. Invest \$6000 in bond *A* and \$4000 in bond *B*; the annual yield is \$880.

5. $\frac{7}{9}$ cup of milk, $\frac{25}{18}$ ounces of corn flakes; $\text{minimum cost} = \frac{335}{18} = 18.6\,¢$

6.

(a) $x_1 \geq 0$ and $x_2 \geq 0$ are nonbinding; $2x_1 + 3x_2 \leq 24$ is binding

(b) $x_1 - x_2 \leq v$ for $v < -3$ is binding and for $v < -6$ yields the empty set.

(c) $x_2 \leq v$ for $v < 8$ is binding and for $v < 0$ yields the empty set.

7. 550 containers from company *A* and 300 containers from company *B*; $\text{maximum shipping charges} = \$2110$

8. 925 containers from company *A* and no containers from company *B*; $\text{maximum shipping charges} = \$2312.50$

9. 0.4 pound of ingredient *A* and 2.4 pounds of ingredient *B*; $\text{minimum cost} = 24.8\,¢$

Exercise Set 11.4 (page 595)

1. 700

2.

(a) 5

(b) 4

4.

(a) Ox, $\frac{34}{21}$ units; sheep, $\frac{20}{21}$ units

(b) First kind, $\frac{9}{25}$ measures; second kind, $\frac{7}{25}$ measures; third kind, $\frac{4}{25}$ measures

5.

(a) $x_1 = \dfrac{(a_2 + a_3 + \cdots + a_n) - a_1}{n - 2}$, $x_i = a_i - x_1$, $i = 2, 3, \ldots, n$

(b) Exercise 7(b); gold, $30\frac{1}{2}$ minae; brass, $9\frac{1}{2}$ minae; tin, $14\frac{1}{2}$ minae; iron, $5\frac{1}{2}$ minae

6.

(a)
$$5x+y+z-K=0$$
$$x+7y+z-K=0$$
$$x+y+8z-K=0$$

$x=\frac{21t}{131}$, $y=\frac{14t}{131}$, $z=\frac{12t}{131}$, $K=t$ where t is an arbitrary number

(b) Take $t=131$, so that $x=21$, $y=14$, $z=12$, $K=131$.

(c) Take $t=262$, so that $x=42$, $y=28$, $z=24$, $K=262$.

7.

(a) Legitimate son, $577\frac{7}{9}$ staters; illegitimate son, $422\frac{2}{9}$ staters

(b) Gold, $30\frac{1}{2}$ minae; brass, $9\frac{1}{2}$ minae; tin, $14\frac{1}{2}$ minae; iron, $5\frac{1}{2}$ minae

(c) First person, 45; second person, $37\frac{1}{2}$; third person, $22\frac{1}{2}$

Exercise Set 11.5 (page 606)

2.

(a) $S(x)=-.12643(x-.4)^3-.20211(x-.4)^2+.92158(x-.4)+.38942$

(b) $S(.5)=.47943$; error $=0\%$

3.

(a) The cubic runout spline

(b) $S(x)=3x^3-2x^2+5x+1$

4.

$$S(x)=\begin{cases} -.00000042(x+10)^3 & & +.000214(x+10) & +.99815, & -10\le x\le 0 \\ .00000024(x)^3 & -.0000126(x)^2 & +.000088(x) & +.99987, & 0\le x\le 10 \\ -.00000004(x-10)^3 & -.0000054(x-10)^2 & -.000092(x-10) & +.99973, & 10\le x\le 20 \\ .00000022(x-20)^3 & -.0000066(x-20)^2 & -.000212(x-20) & +.99823, & 20\le x\le 30 \end{cases}$$

Maximum at $(x,S(x))=(3.93,\ 1.00004)$

5.

$$S(x)=\begin{cases} .0000009(x+10)^3-.0000121(x+10)^2+.000282(x+10)+.99815, & -10\le x\le 0\\ .0000009(x)^3-.0000093(x)^2+.000070(x)+.99987, & 0\le x\le 10\\ .0000004(x-10)^3-.0000066(x-10)^2-.000087(x-10)+.99973, & 10\le x\le 20\\ .0000004(x-20)^3-.0000053(x-20)^2-.000207(x-20)+.99823, & 20\le x\le 30\end{cases}$$

Maximum at $(x, S(x)) = (4.00, 1.00001)$

6.

(a) $S(x)=\begin{cases}-4x^3+3x & 0\le x\le 0.5\\ 4x^3-12x^2+9x-1 & 0.5\le x\le 1\end{cases}$

(b) $S(x)=\begin{cases}2-2x & 0.5\le x\le 1\\ 2-2x & 1\le x\le 1.5\end{cases}$

(c) The three data points are collinear.

7.

(b) $$\begin{bmatrix} 4 & 1 & 0 & 0 & \cdots & 0 & 0 & 0 & 1\\ 1 & 4 & 1 & 0 & \cdots & 0 & 0 & 0 & 0\\ 0 & 1 & 4 & 1 & \cdots & 0 & 0 & 0 & 0\\ \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots & \vdots\\ 0 & 0 & 0 & 0 & \cdots & 0 & 1 & 4 & 1\\ 1 & 0 & 0 & 0 & \cdots & 0 & 0 & 1 & 4\end{bmatrix}\begin{bmatrix} M_1\\ M_2\\ M_3\\ \vdots\\ M_{n-2}\\ M_{n-1}\end{bmatrix}=\frac{6}{h^2}\begin{bmatrix} y_{n-1}-2y_1+y_2\\ y_1-2y_2+y_3\\ y_2-2y_3+y_4\\ \vdots\\ y_{n-3}-2y_{n-2}+y_{n-1}\\ y_{n-2}-2y_{n-1}+y_1\end{bmatrix}$$

8.

(b) $$\begin{bmatrix} 2 & 1 & 0 & 0 & \cdots & 0 & 0 & 0 & 1\\ 1 & 4 & 1 & 0 & \cdots & 0 & 0 & 0 & 0\\ 0 & 1 & 4 & 1 & \cdots & 0 & 0 & 0 & 0\\ \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots & \vdots\\ 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 4 & 1\\ 0 & 0 & 0 & 0 & \cdots & 0 & 1 & 1 & 2\end{bmatrix}\begin{bmatrix} M_1\\ M_2\\ M_3\\ \vdots\\ M_{n-1}\\ M_n\end{bmatrix}=\frac{6}{h^2}\begin{bmatrix} -hy_1'-y_1+y_2\\ y_1-2y_2+y_3\\ y_2-2y_3+y_4\\ \vdots\\ y_{n-2}-2y_{n-1}+y_n\\ y_{n-1}-y_n+hy_n'\end{bmatrix}$$

Exercise Set 11.6 (page 617)

1.

(a) $\mathbf{x}^{(1)}=\begin{bmatrix}.4\\ .6\end{bmatrix}, \mathbf{x}^{(2)}=\begin{bmatrix}.46\\ .54\end{bmatrix}, \mathbf{x}^{(3)}=\begin{bmatrix}.454\\ .546\end{bmatrix}, \mathbf{x}^{(4)}=\begin{bmatrix}.4546\\ .5454\end{bmatrix}, \mathbf{x}^{(5)}=\begin{bmatrix}.45454\\ .54546\end{bmatrix}$

(b) *P* is regular, since all entries of *P* are positive; $\mathbf{q}=\begin{bmatrix}\frac{5}{11}\\ \frac{6}{11}\end{bmatrix}$

2.

(a) $\mathbf{x}^{(1)}=\begin{bmatrix}.7\\ .2\\ .1\end{bmatrix}, \mathbf{x}^{(2)}=\begin{bmatrix}.23\\ .52\\ .25\end{bmatrix}, \mathbf{x}^{(3)}=\begin{bmatrix}.273\\ .396\\ .331\end{bmatrix}$

(b) P is regular, since all entries of P are positive: $\mathbf{q} = \begin{bmatrix} \frac{22}{72} \\ \frac{29}{72} \\ \frac{21}{72} \end{bmatrix}$

3.

(a) $\begin{bmatrix} \frac{9}{17} \\ \frac{8}{17} \end{bmatrix}$

(b) $\begin{bmatrix} \frac{26}{45} \\ \frac{19}{45} \end{bmatrix}$

(c) $\begin{bmatrix} \frac{3}{19} \\ \frac{4}{19} \\ \frac{12}{19} \end{bmatrix}$

4.

(a) $P^n = \begin{bmatrix} \left(\frac{1}{2}\right)^n & 0 \\ 1 - \left(\frac{1}{2}\right)^n & 1 \end{bmatrix}$, $n = 1, 2, \ldots$. Thus, no integer power of P has all positive entries.

(b) $P^n \to \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}$ as n increases, so $P^n \mathbf{x}^{(0)} \to \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ for any $\mathbf{x}^{(0)}$ as n increases.

(c) The entries of the limiting vector $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ are not all positive.

6. $P^2 = \begin{bmatrix} \frac{1}{2} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{2} \end{bmatrix}$ has all positive entries; $\mathbf{q} = \begin{bmatrix} \frac{1}{3} \\ \frac{1}{3} \\ \frac{1}{3} \end{bmatrix}$

7. $\frac{10}{13}$

8. $54\frac{1}{6}\%$ in region 1, $16\frac{2}{3}\%$ in region 2, and $29\frac{1}{6}\%$ in region 3

Exercise Set 11.7 (page 627)

1.

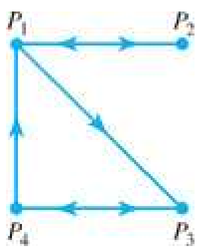
(a) $\begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

(b) $\begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$

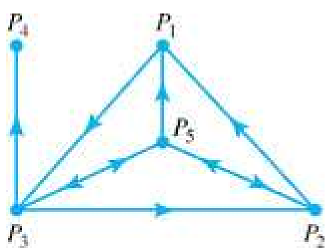
(c) $\begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \end{bmatrix}$

2.

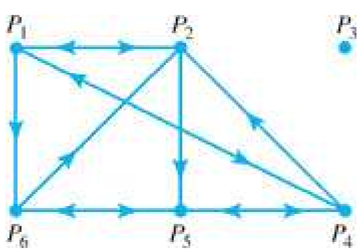
(a)



(b)

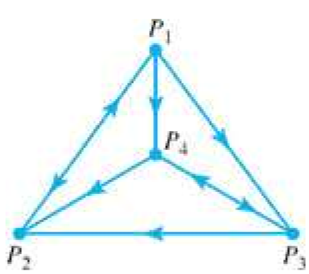


(c)



3.

(a)



(b) 1-step: $P_1 \longrightarrow P_2$

2-step: $P_1 \longrightarrow P_4 \longrightarrow P_2$

$P_1 \longrightarrow P_3 \longrightarrow P_2$

3-step: $P_1 \longrightarrow P_2 \longrightarrow P_1 \longrightarrow P_2$

$P_1 \longrightarrow P_3 \longrightarrow P_4 \longrightarrow P_2$

$P_1 \longrightarrow P_4 \longrightarrow P_3 \longrightarrow P_2$

(c) 1-step: $P_1 \longrightarrow P_4$

2-step: $P_1 \longrightarrow P_3 \longrightarrow P_4$

3-step: $P_1 \longrightarrow P_2 \longrightarrow P_1 \longrightarrow P_4$

$P_1 \longrightarrow P_4 \longrightarrow P_3 \longrightarrow P_4$

4. **(a)** $\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 1 & 2 \end{bmatrix}$

(c) The ijth entry is the number of family members who influence both the ith and jth family members.

5. **(a)** $\{P_1, P_2, P_3\}$

(b) $\{P_3, P_4, P_5\}$

(c) $\{P_2, P_4, P_6, P_8\}$ and $\{P_4, P_5, P_6\}$

6. **(a)** None

(b) $\{P_3, P_4, P_6\}$

7. $\begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}$ Power of $P_1 = 5$, Power of $P_2 = 3$, Power of $P_3 = 4$, Power of $P_4 = 2$

8. First, A; second, B and E (tie); fourth, C; fifth, D

Exercise Set 11.8 (page 637)

1.

(a) $-5/8$

(b) $[0 \quad 1 \quad 0]$

(c) $[1 \quad 0 \quad 0 \quad 0]^T$

2. Let $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$, for example.

3.

(a) $\mathbf{p}^* = [0 \quad 1], \mathbf{q}^* = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, v = 3$

(b) $\mathbf{p}^* = [0 \quad 1 \quad 0], \mathbf{q}^* = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, v = 2$

(c) $\mathbf{p}^* = [0 \quad 0 \quad 1], \mathbf{q}^* = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, v = 2$

(d) $\mathbf{p}^* = [0 \quad 1 \quad 0 \quad 0], \mathbf{q}^* = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, v = -2$

4.

(a) $\mathbf{p}^* = \begin{bmatrix} \frac{5}{8} & \frac{3}{8} \end{bmatrix}, \mathbf{q}^* = \begin{bmatrix} \frac{1}{8} \\ \frac{7}{8} \end{bmatrix}, v = \frac{27}{8}$

(b) $\mathbf{p}^* = \begin{bmatrix} \frac{2}{3} & \frac{1}{3} \end{bmatrix}, \mathbf{q}^* = \begin{bmatrix} \frac{1}{6} \\ \frac{5}{6} \end{bmatrix}, v = \frac{70}{3}$

(c) $\mathbf{p}^* = [1 \quad 0], \mathbf{q}^* = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, v = 3$

(d) $\mathbf{p}^* = \begin{bmatrix} \frac{3}{5} & \frac{2}{5} \end{bmatrix}, \mathbf{q}^* = \begin{bmatrix} \frac{3}{5} \\ \frac{2}{5} \end{bmatrix}, v = \frac{19}{5}$

(e) $\mathbf{p}^* = \begin{bmatrix} \frac{3}{13} & \frac{10}{13} \end{bmatrix}, \mathbf{q}^* = \begin{bmatrix} \frac{1}{13} \\ \frac{12}{13} \end{bmatrix}, v = \frac{-29}{13}$

5.

$\mathbf{p}^* = \begin{bmatrix} \frac{13}{20} & \frac{7}{20} \end{bmatrix}, \mathbf{q}^* = \begin{bmatrix} \frac{11}{20} \\ \frac{9}{20} \end{bmatrix}, v = -\frac{3}{20}$

Exercise Set 11.9 (page 646)

1.

(a) $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$

(b) $\begin{bmatrix} 6 \\ 5 \\ 6 \end{bmatrix}$

(c) $\begin{bmatrix} 78 \\ 54 \\ 79 \end{bmatrix}$

2.

(a) Use Corollary 11.9.4; all row sums are less than one.

(b) Use Corollary 11.9.5; all column sums are less than one.

(c) Use Theorem 11.9.3, with $\mathbf{x} = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} > C\mathbf{x} = \begin{bmatrix} 1.9 \\ .9 \\ .9 \end{bmatrix}$.

3. E^2 has all positive entries.

4. Price of tomatoes, $120.00; price of corn, $100.00; price of lettuce, $106.67

5. $1256 for the CE, $1448 for the EE, $1556 for the ME

6.

(b) $\frac{542}{503}$

Exercise Set 11.10 (page 655)

1. The second class; $15,000

2. $223

3. $1:1.90:3.02:4.24:5.00$

5. $s / \left(g_1^{-1} + g_2^{-1} + \cdots + g_{k-1}^{-1}\right)$

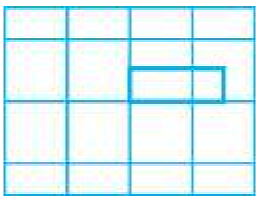
6. $1{:}2{:}3{:}\cdots{:}n-1$

Exercise Set 11.11 (page 662)

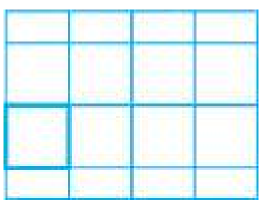
1.

(a) $\begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

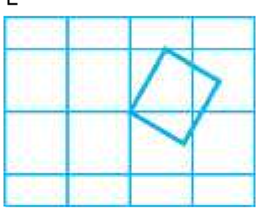
(b) $\begin{bmatrix} 0 & \frac{3}{2} & \frac{3}{2} & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 \end{bmatrix}$



(c) $\begin{bmatrix} -2 & -1 & -1 & -2 \\ -1 & -1 & 0 & 0 \\ 3 & 3 & 3 & 3 \end{bmatrix}$



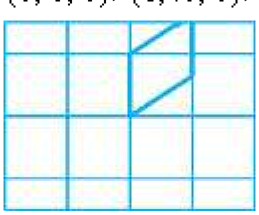
(d) $\begin{bmatrix} 0 & .866 & 1.366 & .500 \\ 0 & -.500 & .366 & .866 \\ 0 & 0 & 0 & 0 \end{bmatrix}$



2.

(b) $(0, 0, 0)$, $(1, 0, 0)$, $\left(1\frac{1}{2}, 1, 0\right)$, and $\left(\frac{1}{2}, 1, 0\right)$

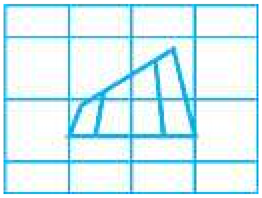
(c) $(0, 0, 0)$, $(1, .6, 0)$, $(1, 1.6, 0)$, $(0, 1, 0)$



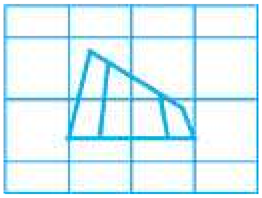
3.

(a) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(b) $\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$



(c) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$



4.

(a)

$$M_1 = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & \frac{1}{3} \end{bmatrix}, M_2 = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & \cdots & \frac{1}{2} \\ 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \end{bmatrix}, M_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos 20° & -\sin 20° \\ 0 & \sin 20° & \cos 20° \end{bmatrix},$$

$$M_4 = \begin{bmatrix} \cos(-45°) & 0 & \sin(-45°) \\ 0 & 1 & 0 \\ -\sin(-45°) & 0 & \cos(-45°) \end{bmatrix}, M_5 = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(b)

$$P' = M_5 M_4 M_3 (M_1 P + M_2)$$

5.

(a)

$$M_1 = \begin{bmatrix} .3 & 0 & 0 \\ 0 & .5 & 0 \\ 0 & 0 & 1 \end{bmatrix}, M_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos 45° & -\sin 45° \\ 0 & \sin 45° & \cos 45° \end{bmatrix}, M_3 = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \end{bmatrix}, M_4 = \begin{bmatrix} \cos 35° & 0 & \sin 35° \\ 0 & 1 & 0 \\ -\sin 35° & 0 & \cos 35° \end{bmatrix},$$

$$M_5 = \begin{bmatrix} \cos(-45°) & -\sin(-45°) & 0 \\ \sin(-45°) & \cos(-45°) & 0 \\ 0 & 0 & 1 \end{bmatrix}, M_6 = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ 1 & 1 & \cdots & 1 \end{bmatrix}, M_7 = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(b)

$$P' = M_7 (M_5 M_4 (M_2 M_1 P + M_3) + M_6)$$

6.

$$R_1 = \begin{bmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{bmatrix}, R_2 = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}, R_3 = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}, R_4 = \begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

$$R_5 = \begin{bmatrix} \cos\beta & 0 & -\sin\beta \\ 0 & 1 & 0 \\ \sin\beta & 0 & \cos\beta \end{bmatrix}$$

7.

(a)

$$M=\begin{bmatrix}1&0&0&x_0\\0&1&0&y_0\\0&0&1&z_0\\0&0&0&1\end{bmatrix}$$

(b)

$$\begin{bmatrix}1&0&0&-5\\0&1&0&9\\0&0&1&-3\\0&0&0&1\end{bmatrix}$$

Exercise Set 11.12 (page 673)

1.

(a)

$$\begin{bmatrix}t_1\\t_2\\t_3\\t_4\end{bmatrix}=\begin{bmatrix}0&\frac{1}{4}&\frac{1}{4}&0\\\frac{1}{4}&0&0&\frac{1}{4}\\\frac{1}{4}&0&0&\frac{1}{4}\\0&\frac{1}{4}&\frac{1}{4}&0\end{bmatrix}\begin{bmatrix}t_1\\t_2\\t_3\\t_4\end{bmatrix}+\begin{bmatrix}0\\\frac{1}{2}\\0\\\frac{1}{2}\end{bmatrix}$$

(b)

$$\mathbf{t}=\begin{bmatrix}\frac{1}{4}\\\frac{3}{4}\\\frac{1}{4}\\\frac{3}{4}\end{bmatrix}$$

(c)

$$\mathbf{t}^{(1)}=\begin{bmatrix}0\\\frac{1}{2}\\0\\\frac{1}{2}\end{bmatrix},\ \mathbf{t}^{(2)}=\begin{bmatrix}\frac{1}{8}\\\frac{5}{8}\\\frac{1}{8}\\\frac{5}{8}\end{bmatrix},\ \mathbf{t}^{(3)}=\begin{bmatrix}\frac{3}{16}\\\frac{11}{16}\\\frac{3}{16}\\\frac{11}{16}\end{bmatrix},\ \mathbf{t}^{(4)}=\begin{bmatrix}\frac{7}{32}\\\frac{23}{32}\\\frac{7}{32}\\\frac{23}{32}\end{bmatrix},\ \mathbf{t}^{(5)}=\begin{bmatrix}\frac{15}{64}\\\frac{47}{64}\\\frac{15}{64}\\\frac{47}{64}\end{bmatrix},\ \mathbf{t}^{(5)}-\mathbf{t}=\begin{bmatrix}-\frac{1}{64}\\-\frac{1}{64}\\-\frac{1}{64}\\-\frac{1}{64}\end{bmatrix}$$

(d) for t_1, 4.5%; for t_2, − 1.8%

2. $\frac{1}{2}$

3.

$$\mathbf{t}^{(1)}=\begin{bmatrix}\frac{3}{4}&\frac{5}{4}&\frac{2}{4}&\frac{5}{4}&\frac{4}{4}&\frac{2}{4}&\frac{5}{4}&\frac{4}{4}&\frac{3}{4}\end{bmatrix}^T$$

$$\mathbf{t}^{(2)}=\begin{bmatrix}\frac{13}{16}&\frac{18}{16}&\frac{9}{16}&\frac{22}{16}&\frac{13}{16}&\frac{7}{16}&\frac{21}{16}&\frac{16}{16}&\frac{10}{16}\end{bmatrix}^T$$

Exercise Set 11.13 (page 685)

1.

(c) $x_3^* = \left(\frac{31}{22}, \frac{27}{22}\right)$

2.

(a) $\mathbf{x}_3^{(1)} = (1.40000,\ 1.20000)$

$\mathbf{x}_3^{(2)} = (1.41000,\ 1.23000)$

$\mathbf{x}_3^{(3)} = (1.40900,\ 1.22700)$

$\mathbf{x}_3^{(4)} = (1.40910,\ 1.22730)$

$\mathbf{x}_3^{(5)} = (1.40909,\ 1.22727)$

$\mathbf{x}_3^{(6)} = (1.40909,\ 1.22727)$

(b) Same as part (a)

(c) $\mathbf{x}_3^{(1)} = (9.55000,\ 25.65000)$

$\mathbf{x}_3^{(2)} = (.59500,\ -1.21500)$

$\mathbf{x}_3^{(3)} = (1.49050,\ 1.47150)$

$\mathbf{x}_3^{(4)} = (1.40095,\ 1.20285)$

$\mathbf{x}_3^{(5)} = (1.40991,\ 1.22972)$

$\mathbf{x}_3^{(6)} = (1.40901,\ 1.22703)$

4. $\mathbf{x}_1^* = (1,\ 1)$, $\mathbf{x}_2^* = (2,\ 0)$, $\mathbf{x}_3^* = (1,\ 1)$

7.

$$\begin{aligned}
x_7 + x_8 + x_9 &= 13.00\\
x_4 + x_5 + x_6 &= 15.00\\
x_1 + x_2 + x_3 &= 8.00\\
.82843(x_6 + x_8) + .58579x_9 &= 14.79\\
1.41421(x_3 + x_5 + x_7) &= 14.31\\
.82843(x_2 + x_4) + .58579x_1 &= 3.81\\
x_3 + x_6 + x_9 &= 18.00\\
x_2 + x_5 + x_8 &= 12.00\\
x_1 + x_4 + x_7 &= 6.00\\
.82843(x_2 + x_6) + .58579x_3 &= 10.51\\
1.41421(x_1 + x_5 + x_9) &= 16.13\\
.82843(x_4 + x_8) + .58579x_7 &= 7.04
\end{aligned}$$

8.

$$
\begin{aligned}
x_7 + x_8 + x_9 &= 13.00\\
x_4 + x_5 + x_6 &= 15.00\\
x_1 + x_2 + x_3 &= 8.00\\
.04289(x_3 + x_5 + x_7) + .75000(x_6 + x_8) + .61396x_9 &= 14.79\\
.91421(x_3 + x_5 + x_7) + .25000(x_2 + x_4 + x_6 + x_8) &= 14.31\\
.04289(x_3 + x_5 + x_7) + .75000(x_2 + x_4) + .61396x_1 &= 3.81\\
x_3 + x_6 + x_9 &= 18.00\\
x_2 + x_5 + x_8 &= 12.00\\
x_1 + x_4 + x_7 &= 6.00\\
.04289(x_1 + x_5 + x_9) + .75000(x_2 + x_6) + .61396x_3 &= 10.51\\
.91421(x_1 + x_5 + x_9) + .25000(x_2 + x_4 + x_6 + x_8) &= 16.13\\
.04289(x_1 + x_5 + x_9) + .75000(x_4 + x_8) + .61396x_7 &= 7.04
\end{aligned}
$$

Exercise Set 11.14 (page 702)

1. $T_i\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \frac{12}{25}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} e_i \\ f_i \end{bmatrix}$, i = 1, 2, 3, 4, where the four values of $\begin{bmatrix} e_i \\ f_i \end{bmatrix}$ are $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$, $\begin{bmatrix} \frac{13}{25} \\ 0 \end{bmatrix}$, $\begin{bmatrix} 0 \\ \frac{13}{25} \end{bmatrix}$, and $\begin{bmatrix} \frac{13}{25} \\ \frac{13}{25} \end{bmatrix}$;

$d_H(S) = \ln(4) / \ln\left(\frac{25}{12}\right) = 1.888\ldots$

2. $s \approx .47$; $d_H(S) \approx \ln(4) / \ln(1 / .47) = 1.8\ldots$ Rotation angles: 0° (upper left); 90° (upper right); 180° (lower left); 180° (lower right)

3.

(a)

i. $s = \frac{1}{3}$;

ii. all rotation angles are 0°;

iii. $d_H(S) = \ln(7) / \ln(3) = 1.771\ldots$

This set is a fractal.

(b)

i. $s = \frac{1}{2}$;

ii. all rotation angles are 180°;

iii. $d_H(S) = \ln(3) / \ln(2) = 1.584\ldots$

This set is a fractal.

(c)

i. $s = \frac{1}{2}$;

ii. rotation angles: 90° (top); 180° (lower left); 180° (lower right);

iii. $d_H(S) = \ln(3) / \ln(2) = 1.584\ldots$

This set is a fractal.

(d)

i. $s = \frac{1}{2}$;

ii. rotation angles: 90° (upper left); 180° (upper right); 180°(lower right);

iii. $d_H(S) = \ln(3) / \ln(2) = 1.584\ldots$

This set is a fractal.

4. $s = .8509\ldots, \theta = -2.69°\ldots$

5. (0.766, 0.996) rounded to three decimal places

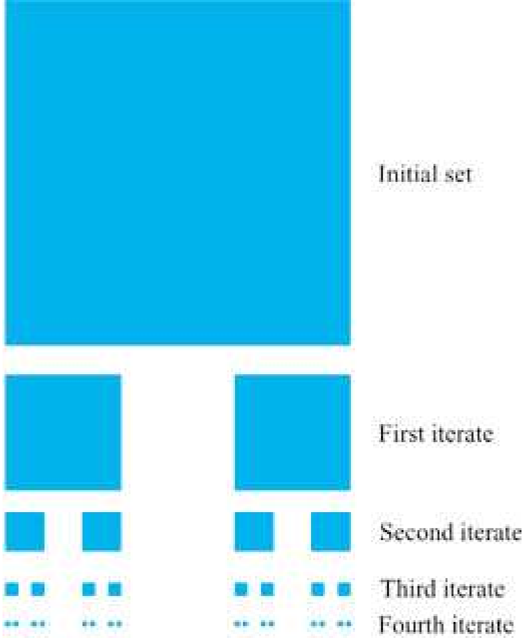
6. $d_H(S) = \ln(16) / \ln(4) = 2$

7. $\ln(4) / \ln\left(\frac{4}{3}\right) = 4.818\ldots$

8. $d_H(S) = \ln(8) / \ln(2) = 3$; the cube is not a fractal.

9. $k = 20$; $s = \frac{1}{3}$; $d_H(S) = \ln(20) / \ln(3) = 2.726\ldots$; the set is a fractal

10.



$d_H(S) = \ln(2)/\ln(3) = 0.6309\ldots$

11. Area of $S_0 = 1$; area of $S_1 = \frac{8}{9} = 0.888\ldots$; area of $S_2 = \left(\frac{8}{9}\right)^2 = 0.790\ldots$; area of $S_3 = \left(\frac{8}{9}\right)^3 = 0.702\ldots$; area of $S_4 = \left(\frac{8}{9}\right)^4 = 0.624\ldots$

Exercise Set 11.15 (page 716)

1. $\Pi(250) = 750,\ \Pi(25) = 50,\ \Pi(125) = 250,\ \Pi(30) = 60,\ \Pi(10) = 30,\ \Pi(50) = 150,\ \Pi(3750) = 7500,$ $\Pi(6) = 12,\ \Pi(5) = 10$

2. One 1-cycle: $\{(0, 0)\}$; one 3-cycle: $\left\{\left(\frac{3}{6}, 0\right), \left(\frac{3}{6}, \frac{3}{6}\right), \left(0, \frac{3}{6}\right)\right\}$;

two 4-cycles: $\left\{\left(\frac{4}{6}, 0\right), \left(\frac{4}{6}, \frac{4}{6}\right), \left(\frac{2}{6}, 0\right), \left(\frac{2}{6}, \frac{2}{6}\right)\right\}$ and $\left\{\left(0, \frac{2}{6}\right), \left(\frac{2}{6}, \frac{4}{6}\right), \left(0, \frac{4}{6}\right), \left(\frac{4}{6}, \frac{2}{6}\right)\right\}$;

two 12-cycles: $\left\{\left(0, \frac{1}{6}\right), \left(\frac{1}{6}, \frac{2}{6}\right), \left(\frac{3}{6}, \frac{5}{6}\right), \left(\frac{2}{6}, \frac{1}{6}\right), \left(\frac{3}{6}, \frac{4}{6}\right), \left(\frac{1}{6}, \frac{5}{6}\right), \left(0, \frac{5}{6}\right), \left(\frac{5}{6}, \frac{4}{6}\right), \left(\frac{3}{6}, \frac{1}{6}\right), \left(\frac{4}{6}, \frac{5}{6}\right), \left(\frac{3}{6}, \frac{2}{6}\right), \left(\frac{5}{6}, \frac{1}{6}\right)\right\}$ and $\left\{\left(\frac{1}{6}, 0\right), \left(\frac{1}{6}, \frac{1}{6}\right), \left(\frac{2}{6}, \frac{3}{6}\right), \left(\frac{5}{6}, \frac{2}{6}\right), \left(\frac{1}{6}, \frac{3}{6}\right), \left(\frac{4}{6}, \frac{1}{6}\right), \left(\frac{5}{6}, 0\right), \left(\frac{5}{6}, \frac{5}{6}\right), \left(\frac{4}{6}, \frac{3}{6}\right), \left(\frac{1}{6}, \frac{4}{6}\right), \left(\frac{5}{6}, \frac{3}{6}\right), \left(\frac{2}{6}, \frac{5}{6}\right)\right\}$,
$\Pi(6) = 12$

3.

(a) 3,7, 10, 2, 12, 14, 11, 10, 6, 1, 7, 8, 0, 8, 8, 1, 9, 10, 4, 14, 3, 2, 5, 7, 12, 4, 1, 5, 6, 11, 2, 13, 0, 13, 13, 11, 9, 5, 14, 4, 3, 7,…

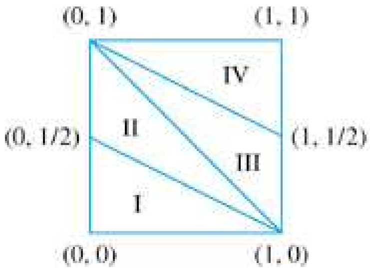
(c) (5,5), (10,15), (4,19), (2,0), (2,2), (4,6), (10,16), (5,0), (5,5),…

4.

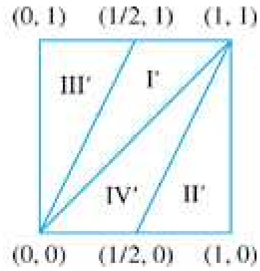
(c) The first five iterates of $\left(\frac{1}{101}, 0\right)$ are $\left(\frac{1}{101}, \frac{1}{101}\right), \left(\frac{2}{101}, \frac{3}{101}\right), \left(\frac{5}{101}, \frac{8}{101}\right), \left(\frac{13}{101}, \frac{21}{101}\right)$, and $\left(\frac{34}{101}, \frac{55}{101}\right)$.

6. **(b)** The matrices of Anosov automorphisms are $\begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$ and $\begin{bmatrix} 5 & 7 \\ 2 & 3 \end{bmatrix}$.

(c) The transformation affects a rotation of S through 90° in the clockwise direction.

9. 

$$\begin{bmatrix} x \\ y \end{bmatrix} \to \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} a \\ b \end{bmatrix}$$



In region I: $\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$; in region II: $\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$; in region III: $\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$; in region IV: $\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} -1 \\ -2 \end{bmatrix}$

12. $\left(\frac{1}{5}, \frac{3}{5}\right)$ and $\left(\frac{4}{5}, \frac{2}{5}\right)$ form one 2-cycle, and $\left(\frac{2}{5}, \frac{1}{5}\right)$ and $\left(\frac{3}{5}, \frac{4}{5}\right)$ form another 2-cycle.

Exercise Set 11.16 (page 729)

1. **(a)** *GIYUOKEVBH*

(b) *SFANEFZWJH*

2. **(a)** $A^{-1} = \begin{bmatrix} 12 & 7 \\ 23 & 15 \end{bmatrix}$

(b) Not invertible

(c) $A^1 = \begin{bmatrix} 1 & 19 \\ 23 & 24 \end{bmatrix}$

(d) Not invertible

(e) Not invertible

(f) $A^{-1} = \begin{bmatrix} 15 & 12 \\ 21 & 5 \end{bmatrix}$

3. *WE LOVE MATH*

4. Deciphering matrix $= \begin{bmatrix} 7 & 15 \\ 6 & 5 \end{bmatrix}$; enciphering matrix $= \begin{bmatrix} 7 & 5 \\ 2 & 15 \end{bmatrix}$

5. *THEY SPLIT THE ATOM*

6. *I HAVE COME TO BURY CAESAR*

7. **(a)** 010100001

(b) $\begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$

8. A is invertible modulo 29 if and only if $\det(A) \neq 0 \pmod{29}$.

Exercise Set 11.17 (page 741)

2.
$$\left.\begin{aligned} a_n &= \frac{1}{4} + \left(\frac{1}{2}\right)^{n+1}(a_0 - c_0) \\ b_n &= \frac{1}{2} \\ c_n &= \frac{1}{4} - \left(\frac{1}{2}\right)^{n+1}(a_0 - c_0) \end{aligned}\right\} n = 1, 2, \ldots \quad \left.\begin{aligned} a_n &\to \frac{1}{4} \\ b_n &\to \frac{1}{2} \\ c_n &\to \frac{1}{4} \end{aligned}\right\} \text{as } n \to \infty$$

3.
$$\left.\begin{aligned} a_{2n+1} &= \frac{2}{3} + \frac{1}{6(4)^n}(2a_0 - b_0 - 4c_0) \\ b_{2n+1} &= \frac{1}{3} - \frac{1}{6(4)^n}(2a_0 - b_0 - 4c_0) \\ c_{2n+1} &= 0 \end{aligned}\right\} n = 0, 1, 2, \ldots$$

$$\left.\begin{aligned} a_{2n} &= \frac{5}{12} + \frac{1}{6(4)^n}(2a_0 - b_0 - 4c_0) \\ b_{2n} &= \frac{1}{2} \\ c_{2n} &= \frac{1}{12} - \frac{1}{6(4)^n}(2a_0 - b_0 - 4c_0) \end{aligned}\right\} n = 1, 2, \ldots$$

4. Eigenvalues: $\lambda_1 = 1$, $\lambda_2 = \frac{1}{2}$; eigenvectors: $\mathbf{e}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $\mathbf{e}_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

5. 12 generations; .006%

6.
$$\mathbf{x}^{(n)} = \begin{bmatrix} \frac{1}{2} + \frac{1}{2^{2n+3}}\left[\left(-3-\sqrt{5}\right)\left(1+\sqrt{5}\right)^{n+1} + \left(-3+\sqrt{5}\right)\left(1-\sqrt{5}\right)^{n+1}\right] \\ \frac{1}{2^{2n+1}}\left[\left(1+\sqrt{5}\right)^{n+1} + \left(1-\sqrt{5}\right)^{n+1}\right] \\ \frac{1}{2^{2n+1}}\left[\left(1+\sqrt{5}\right)^{n} + \left(1-\sqrt{5}\right)^{n}\right] \\ \frac{1}{2^{2n+1}}\left[\left(1+\sqrt{5}\right)^{n} + \left(1-\sqrt{5}\right)^{n}\right] \\ \frac{1}{2^{2n+1}}\left[\left(1+\sqrt{5}\right)^{n+1} + \left(1-\sqrt{5}\right)^{n+1}\right] \\ \frac{1}{2} + \frac{1}{2^{2n+3}}\left[\left(-3-\sqrt{5}\right)\left(1+\sqrt{5}\right)^{n+1} + \left(-3+\sqrt{5}\right)\left(1-\sqrt{5}\right)^{n+1}\right] \end{bmatrix}; \mathbf{x}^{(n)} \to \begin{bmatrix} \frac{1}{2} \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{2} \end{bmatrix} \text{ as } n \to \infty$$

8. $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

Exercise Set 11.18 (page 751)

1.

(a) $\lambda_1 = \frac{3}{2}, \mathbf{x}_1 = \begin{bmatrix} 1 \\ \frac{1}{3} \end{bmatrix}$

(b) $\mathbf{x}^{(1)} = \begin{bmatrix} 100 \\ 50 \end{bmatrix}, \mathbf{x}^{(2)} = \begin{bmatrix} 175 \\ 50 \end{bmatrix}, \mathbf{x}^{(3)} = \begin{bmatrix} 250 \\ 88 \end{bmatrix}, \mathbf{x}^{(4)} = \begin{bmatrix} 382 \\ 125 \end{bmatrix}, \mathbf{x}^{(5)} = \begin{bmatrix} 570 \\ 191 \end{bmatrix}$

(c) $\mathbf{x}^{(6)} = L\mathbf{x}^{(5)} = \begin{bmatrix} 857 \\ 285 \end{bmatrix}, \mathbf{x}^{(6)} \simeq \lambda_1 \mathbf{x}^{(5)} = \begin{bmatrix} 855 \\ 287 \end{bmatrix}$

7. 2.375

8. 1.49611

Exercise Set 11.19 (page 760)

1.

(a) $\text{Yield} = 33\frac{1}{3}\%$ of population; $\mathbf{x}_1 = \begin{bmatrix} 1 \\ \frac{1}{3} \\ \frac{1}{18} \end{bmatrix}$

(b)

Yield $= 45.8\%$ of population; $\mathbf{x}_1 = \begin{bmatrix} 1 \\ \frac{1}{2} \\ \frac{1}{8} \end{bmatrix}$; harvest 57.9% of youngest age class

2. $$\mathbf{x}_1 = \begin{bmatrix} 1.000 \\ .845 \\ .824 \\ .795 \\ .755 \\ .699 \\ .626 \\ .532 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, L\mathbf{x}_1 = \begin{bmatrix} 2.090 \\ .845 \\ .824 \\ .795 \\ .755 \\ .699 \\ .626 \\ .532 \\ .418 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \frac{1.089 + .418}{7.584} = .199$$

4. $h_I = (R - 1) / (a_I b_1 b_2 \cdots b_{I-1} + \cdots + a_n b_1 b_2 \cdots b_{n-1})$

5. $$h_I = \frac{a_1 + a_2 b_1 + \cdots + (a_{J-1} b_1 b_2 \cdots b_{J-2}) - 1}{a_I b_1 b_2 \cdots b_{I-1} + \cdots + a_{J-1} b_1 b_2 \cdots b_{J-2}}$$

Exercise Set 11.20 (page 767)

1. $\frac{\pi^2}{3} + 4\cos t + \cos 2t + \frac{4}{9}\cos 3t$

2. $$\frac{T^2}{3} + \frac{T^2}{\pi^2}\left(\cos\frac{2\pi}{T}t + \frac{1}{2^2}\cos\frac{4\pi}{T}t + \frac{1}{3^2}\cos\frac{6\pi}{T}t + \frac{1}{4^2}\cos\frac{8\pi}{T}t\right)$$
$$-\frac{T^2}{\pi}\left(\sin\frac{2\pi}{T}t + \frac{1}{2}\sin\frac{4\pi}{T}t + \frac{1}{3}\sin\frac{6\pi}{T}t + \frac{1}{4}\sin\frac{8\pi}{T}t\right)$$

3. $\frac{1}{\pi} + \frac{1}{2}\sin t - \frac{2}{3\pi}\cos 2t - \frac{2}{15\pi}\cos 4t$

4. $$\frac{4}{\pi}\left(\frac{1}{2} - \frac{1}{1\cdot 3}\cos t - \frac{1}{3\cdot 5}\cos 2t - \frac{1}{5\cdot 7}\cos 3t - \cdots - \frac{1}{(2n-1)(2n+1)}\cos nt\right)$$

5. $$\frac{T}{4} - \frac{8T}{\pi^2}\left(\frac{1}{2^2}\cos\frac{2\pi t}{T} + \frac{1}{6^2}\cos\frac{6\pi t}{T} + \frac{1}{10^2}\cos\frac{10\pi T}{t} + \cdots + \frac{1}{(2n)^2}\cos\frac{2n\pi t}{T}\right)$$

Exercise Set 11.21 (page 775)

1.

(a) Yes; $\mathbf{v} = \frac{1}{5}\mathbf{v}_1 + \frac{2}{5}\mathbf{v}_2 + \frac{2}{5}\mathbf{v}_3$

(b) No; $\mathbf{v} = \frac{2}{5}\mathbf{v}_1 + \frac{4}{5}\mathbf{v}_2 - \frac{1}{5}\mathbf{v}_3$

(c) Yes; $\mathbf{v} = \frac{2}{5}\mathbf{v}_1 + \frac{3}{5}\mathbf{v}_2 + 0\mathbf{v}_3$

(d) Yes; $\mathbf{v} = \frac{4}{15}\mathbf{v}_1 + \frac{6}{15}\mathbf{v}_2 + \frac{5}{15}\mathbf{v}_3$

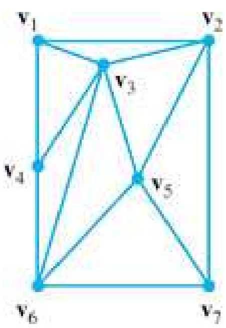
2. $m = \text{number of traingles} = 7$, $n = \text{number of vertex points} = 7$, $k = \text{number of boundary vertex points} = 5$; Equation (7) is $7 = 2(7) - 2 - 5$.

3.

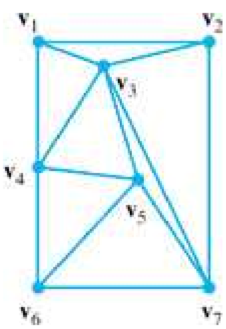
$$\begin{aligned}\mathbf{w} &= M\mathbf{v} + \mathbf{b} = M(c_1\mathbf{v}_1 + c_2\mathbf{v}_2 + c_3\mathbf{v}_3) + (c_1 + c_2 + c_3)\mathbf{b} \\ &= c_1(M\mathbf{v}_1 + \mathbf{b}) + c_2(M\mathbf{v}_2 + \mathbf{b}) + c_3(M\mathbf{v}_3 + \mathbf{b}) \\ &= c_1\mathbf{w}_1 + c_2\mathbf{w}_2 + c_3\mathbf{w}_3\end{aligned}$$

4.

(a)



(b)



5.

(a) $M = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$, $\mathbf{b} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

(b) $M = \begin{bmatrix} 3 & -1 \\ 1 & 1 \end{bmatrix}$, $\mathbf{b} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

(c) $M = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $\mathbf{b} = \begin{bmatrix} 2 \\ -3 \end{bmatrix}$

(d) $M = \begin{bmatrix} \frac{1}{2} & 1 \\ 2 & 0 \end{bmatrix}$, $\mathbf{b} = \begin{bmatrix} \frac{1}{2} \\ -1 \end{bmatrix}$

7.

(a) Two of the coefficients are zero

(b) At least one of the coefficients is zero

(c) None of the coefficients are zero

8.

(a) $\frac{1}{3}\mathbf{v}_1 + \frac{1}{3}\mathbf{v}_2 + \frac{1}{3}\mathbf{v}_3$

(b) $\begin{bmatrix} 8/3 \\ 2 \end{bmatrix}$