Assignment 1

MAT 361

Probability and Statistics

Section 4

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Assignment-1

1] red denoted by r, blue by b and dull denoted by d and shiny by s.

$S = \{(r,s),(r,d),(b,s),(b,d)\}$ Answer

2] $A = \{\heartsuit A, \diamondsuit A, \clubsuit A, \spadesuit A\}$

$n = 4, \quad N = 52.$

$\therefore P(A) = \frac{4}{52}$ Answer

3] $P(A) = 0.5$

$P(A \cap B) = 0.1$

$P(A \cup B) = 0.8$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow P(B) = P(A \cup B) + P(A \cap B) - P(A)$

$= 0.8 + 0.1 - 0.5$

$\therefore P(B) = 0.4.$ Answer

4] $P(I) = 2 \times P(II) \quad \ldots\ldots (i)$

$P(II) = 3 \times P(III) \quad \ldots\ldots (ii)$

$P(I) = 2 \times 3 \times P(III)$

$= 6 \times P(III)$

$\Sigma P = 1.$

$\Rightarrow P(I) + P(II) + P(III) = 1$

$\Rightarrow 6 \times P(III) + 3 P(III) + P(III) = 1$

$\Rightarrow P(III) = \frac{1}{10}$

$P(I) = 6 \times \frac{1}{10} = \frac{6}{10}$

$P(II) = 3 \times \left(\frac{1}{10}\right) = \frac{3}{10}$ Answer

5] $\Sigma P = 1$

$\therefore P = 1 - 0.28 - 0.55$

$= 0.17$ Answer

6] $A = \{c, d\}$, $P(A) = 0.48 + 0.2 = 0.50$

$A' = \{a, b, e\}$, $P(A') = 0.13 + 0.22 + P(b)$.

@ $P(A) + P(A') = 1$

$\Rightarrow 0.50 + 0.35 + P(b) = 1$

$\therefore P(b) = 1 - 0.85 = 0.15$ Answer

(b) $P(A) = 0.50$ Answer

(c) $P(A') = 1 - 0.50$

$= 0.50$ Answer

7] $P(A) = 0.27. = P(b) + P(c) + P(e)$

@ $P(b) = 0.27 - 0.11 - 0.06$

$= 0.10$ Answer

(b) $P(A') = 1 - P(A) = 1 - 0.27$

$= 0.73$ Answer

(c) $P(A') = P(a) + P(f) + P(d) = 0.73$

$\Rightarrow P(d) = 0.73 - 0.09 - 0.29$

$= 0.35$ Answer

8|

(a) $A = \{(\text{III}, \text{II}, \text{I}), (\text{II}, \text{III}, \text{I})\}$

$P(A) = 0.03 + 0.39$

$= 0.42$ Answer

(b)

Last shortest means type I battery fails first.

$A = \{(\text{I}, \text{II}, \text{III}), (\text{I}, \text{III}, \text{II})\}$

$P(A) = 0.11 + 0.07$

$= 0.18$ Answer

(c)

type I does not last long.

$\therefore A = \{(\text{III}, \text{I}, \text{II}), (\text{I}, \text{III}, \text{II}), (\text{II}, \text{I}, \text{III}), (\text{I}, \text{II}, \text{III})\}$

$P(A) = 0.16 + 0.07 + 0.24 + 0.11$

$= 0.58$ Answer

(d)

type I battery lasts longer than the type II battery. $\Rightarrow$ II, I.

let, $A = \{(\text{III}, \text{II}, \text{I}), (\text{II}, \text{I}, \text{III}), (\text{II}, \text{III}, \text{I})\}$

$P(A) = 0.03 + 0.24 + 0.39$

$= 0.66$ Answer

9] 1

(a) both assembly lines are shut down.

$\therefore A = \{(S,S)\}$

$P(A) = 0.02$ Answer

(b) neither assembly line is shut down.

$\therefore A = \{(P,P),(P,F),(F,P),(F,F)\}$

$P(A) = 0.14 + 0.20 + 0.21 + 0.19$

$= 0.74$ Answer

(c) at least one assembly line is at full capacity?.

$A = \{(F,S),(F,P),(F,F),(P,F),(S,F)\}$

$P(A) = 0.06 + 0.21 + 0.19 + 0.20 + 0.05$

$= 0.71$ Answer

(d) Exactly one assembly line is at full capacity?.

$A = \{(F,S),(F,P),(P,F),(S,F)\}$

$P(A) = 0.06 + 0.21 + 0.20 + 0.05$

$= 0.52$ Answer

If A is the set of events neither assembly line is shut down, so

$A = \{(P,P),(P,F),(F,P),(F,F)\}$

So the complement of A, $A^c = \{(S,S),(S,P),(S,F),(P,S),(F,S)\}$

If B is the events set of at least one assembly line is at full capacity Answer

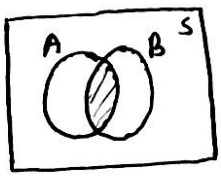
$B = \{(F,S),(F,P),(F,F),(P,F),(S,F)\}$

So the complement of B, $B^c = \{(S,S),(S,P),(P,S),(P,P)\}$

Answer

10] $P(A) = 0.4$

$P(A \cap B) = 0.3$



We know, $P(A \cup B) \leq 1$

$\Rightarrow P(A) + P(B) - P(A \cap B) \leq 1$

$\Rightarrow 0.4 + P(B) - 0.3 \leq 1$

$\Rightarrow P(B) + 0.1 \leq 1$

$\Rightarrow 0 \leq P(B) \leq 0.9$ ..... ①

But $P(B) \geq P(A \cap B)$

$\therefore P(B) \geq 0.3$ ..... ⑪

From ① and ⑪, we get $0.3 \leq P(B) \leq 0.9$ Answer

11] $A = \{13\heartsuit\}$, $P(A) = \frac{13}{52} = \frac{1}{4}$

$B = \{13\clubsuit\}$, $P(B) = 13/52$

$C = \{13\diamondsuit\}$, $P(C) = 13/52$

Yes, these three events mutually exclusive. Because they have no common cards. Answer

$P(A \cup B \cup C) = \frac{13}{52} + \frac{1}{4} + \frac{1}{4}$

$= \frac{3}{4}$ Answer

If $A = \{13\heartsuit\}$, then $A' = \{13\diamondsuit, 13\clubsuit, 13\spadesuit\}$.

and $B = \{13\clubsuit\}$

So, B is a subset of A' or $B \subset A'$ Answer

12)

shiny red ball, $(r, s) \Rightarrow (r \cap s) = 55$

shiny ball, $s = 91$

red ball, $r = 79$

$\therefore P(r \cap s) = \frac{55}{200}$, $P(s) = \frac{91}{200}$, $P(r) = \frac{79}{200}$

probablity that it is either a shiny ball or a red ball,

$$P(s \cup r) = P(s) + P(r) - P(r \cap s)$$
$$= \frac{91}{200} + \frac{79}{200} - \frac{55}{200}$$
$$= \frac{115}{200}$$ Answer

13) There are 150 balls.

$r \cap s = 36 \quad \therefore P(r \cap s) = \frac{36}{150}$

$b = 54. \quad \therefore P(b) = \frac{54}{150}$

$\therefore r = 150 - 54 = 96 \quad \therefore P(r) = \frac{96}{150}$

a

Probablity of the chosen ball being shiny conditional on it being red,

$$P(A|B) = \frac{P(A \cap B)}{P(B}$$

$$P(s|r) = \frac{P(r \cap s)}{P(r)} = \frac{36/150}{96/150} = \frac{36}{96} = \frac{6}{16} = \frac{3}{8}$$ Answer