Digital Logic Design :

Lecture 5

The Standard POS form :

A Standard POS expression is one in which all the variables in the domain appear in each sum term in the expression.

example : $(A + \bar{B} + C + \bar{D})(\bar{A} + B + C + \bar{D})(\bar{A} + B + \bar{C} + D)$

Convert the following Boolean expression into standard POS form.

$(A + \bar{B})(B + \bar{C} + D)$

now,

$$A + \bar{B} = A + \bar{B} + C \cdot \bar{C}$$

$$= (A + \bar{B} + C)(A + B + \bar{C}) \quad \left[\begin{array}{l} \text{Rule 12} \\ A + BC = (A + B)(A + C) \end{array} \right]$$

$$= (A + \bar{B} + C + D \cdot \bar{D})(A + B + \bar{C} + D \cdot \bar{D})$$

$$= (A + \bar{B} + C + D)(A + \bar{B} + C + \bar{D})(A + B + \bar{C} + D)(A + B + \bar{C} + \bar{D})$$

$$B + \bar{C} + D = B + \bar{C} + D + A \cdot \bar{A}$$

$$= (A + B + \bar{C} + D)(\bar{A} + B + \bar{C} + D)$$

$$\therefore (A + \bar{B})(B + \bar{C} + D) = (A + \bar{B} + C + D)(A + \bar{B} + C + \bar{D})(A + B + \bar{C} + D)$$
$$(A + \bar{B} + \bar{C} + \bar{D})(A + B + \bar{C} + D)(\bar{A} + B + \bar{C} + D)$$

Developing truth table for standard SOP expression:

let $X = \bar{A}\bar{B}\bar{C} + A\bar{B}\bar{C} + ABC$

Inputs			Output	Output	Output
A	B	C	X	Y	Z
0	0	0	0	0	1
0	0	1	1	1	0
0	1	0	0	0	1
0	1	1	0	0	1
1	0	0	1	1	0
1	0	1	0	0	0
1	1	0	0	0	1
1	1	1	1	1	1

Converting POS Expression to Truth Table Format:

let $Y = (A+B+C)(A+\bar{B}+C)(A+\bar{B}+\bar{C})(\bar{A}+B+\bar{C})(\bar{A}+\bar{B}+C)$

Determining Standard Expression from a Truth Table standard SOP expression:

$Z = \bar{A}\bar{B}\bar{C} + \bar{A}B\bar{C} + \bar{A}BC + AB\bar{C} + ABC$

Standard POS expression:

$Z = (A+B+\bar{C})(\bar{A}+B+C)(\bar{A}+B+\bar{C})$

3 variable karnaugh map:

AB \ C	0	1
00	0	1
01	0	0
11	0	1
10	1	0

Map the following SOP expression in the karnaugh map.

$$\bar{A}\bar{B}C + A\bar{B}\bar{C} + ABC$$

Map the following SOP expression on a K map:

$$\bar{A}\bar{B}C \quad + \quad \bar{A}B\bar{C} \quad + \quad AB\bar{C} \quad + \quad ABC$$
$$\quad 001 \qquad\qquad 010 \qquad\quad 110 \qquad 111$$

AB \ C	0	1
00	0	1
01	1	0
11	1	1
10	0	0

Map the following SOP expression on a K map:

$$\bar{A} + A\bar{B} + AB\bar{C}$$

000	100	110
001	101	
010		
011		

AB \ C	0	1
00	1	1
01	1	1
11	1	0
10	1	1

K map simplification of SOP expression :

Use a K map to minimize the following expression :

$$A\bar{B}C + \bar{A}BC + \bar{A}\bar{B}C + \bar{A}\bar{B}\bar{C} + A\bar{B}\bar{C}$$

$\quad\;\; 1\;0\;1 \qquad\quad 0\;1\;1 \qquad\quad 0\;0\;1 \qquad\quad 0\;0\;0 \qquad\quad 1\;0\;0$

Simplified SOP expression

$$\bar{B} + \bar{A}C$$
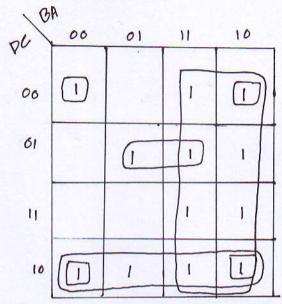
⊞ 1's in adjacent cells can be grouped. Each group can

have 2, 4, 8, 16 . . . . . ones.

Use K map to minimize the following expression :

$$\bar{A}\bar{B}\bar{C}\bar{D} + \bar{A}\bar{B}\bar{C}D + \bar{A}B\bar{C}\bar{D} + \bar{A}B\bar{C}D + \bar{A}BCD + \bar{A}BC\bar{D}$$

$\;\; 0\;0\;0\;0 \qquad 0\;0\;0\;1 \qquad 0\;1\;0\;0 \qquad 0\;1\;0\;1 \qquad 0\;1\;1\;1 \qquad 0\;1\;1\;0$

Simplified SOP expression :

$$\bar{A}\bar{C} + \bar{A}B + A\bar{B}D$$

Simplify the following SOP expression using K map:

$$\bar{D}\bar{C}\bar{B}\bar{A} + \bar{D}\bar{C}B\bar{A} + \bar{D}\bar{C}BA + \bar{D}C\bar{B}A + \bar{D}CB\bar{A} + \bar{D}CBA + D\bar{C}\bar{B}\bar{A}$$

0 0 0 0   0 0 1 0   0 0 1 1   0 1 0 1   0 1 1 0   0 1 1 1   1 0 0 0

$$+ D\bar{C}B\bar{A} + DCBA + DCB\bar{A} + D\bar{C}BA + D\bar{C}\bar{B}A$$

1 0 0 1

BA

DC

DC \ BA	00	01	11	10
00	1		1	1
01		1	1	1
11			1	1
10	1	1	1	1

Simplified expression :

$$B + D\bar{C} + \bar{C}\bar{A} + \bar{D}CA$$

Simplify the function expressed in sum of minterms form

$$F(W, X, Y, Z) = \sum (0, 1, 2, 4, 5, 6, 8, 9, 12, 13, 14)$$

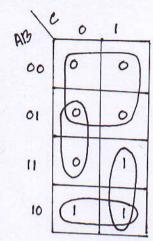no. of minterms = 11 (minterms means standard
product, ref. : morris mano
Pg 48)

YZ

WX

WX \ YZ	00	01	11	10
00	0 1	1 1	3	2 1
01	4 1	5 1	7	6 1
11	12 1	13 1	15	14 1
10	8 1	9 1	11	10

$$\therefore F = \bar{Y} + \bar{W}\bar{Z} + X\bar{Z}$$

Use a K map to minimize the following :

POS expression :

$(A+B+c)$ $(A+B+\bar{z})$ $(A+\bar{B}+c)$ $(A+\bar{B}+\bar{z})$ $(\bar{A}+\bar{B}+c)$

000   001   010   011   110

Simplified POS expression :

$$\bar{F} = \bar{A} + B\bar{C}$$

$$F = \overline{\bar{A} + B\bar{C}}$$

$$= \bar{\bar{A}}\,(\overline{B\bar{C}})$$

$$= A\,(\bar{B}+c)$$

$F = A\bar{B} + AC$

$= A(\bar{B}+c)$

simplify the function expressed in product of maxterm form.

$F(x,y,z) = \pi\,(0,2,5,7)$

no. of maxterm or sum = 4

$$\bar{F} = \bar{x}\bar{z} + xz$$

$$F = \overline{\bar{x}\bar{z} + xz}$$

$$= \overline{\bar{x}\bar{z}} + \overline{xz}$$

$$= (\bar{\bar{x}} + \bar{\bar{z}})(\bar{x} + \bar{z})$$

$$= (x+z)(\bar{x} + \bar{z})$$

The '1' functions represents the minterms and
The '0' functions represents the maxterms.