Assignment : 01

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Section : 04

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Classwork of Joint Probability Mass Function

X = the score assigned by the first inspector

Y = the score assigned by the second inspector

Y \ X	1	2	3	4
1	0.09	0.03	0.01	0.01
2	0.02	0.15	0.03	0.01
3	0.01	0.01	0.24	0.04
4	0.00	0.01	0.02	0.32

(i) We know,

$$\sum_{j=1}^{m} \sum_{i=1}^{n} P_{ij} = 1 \quad \text{where } 0 \le P_{ij} \le 1$$

So, $\sum_{j=1}^{4} \sum_{i=1}^{4} P_{ij} = P_{11} + P_{12} + P_{13} + P_{14} + P_{21} + P_{22} + P_{23} + P_{24} + P_{31} + P_{32} + P_{33} + P_{34} + P_{41} + P_{42} + P_{43} + P_{44}$

$= 0.09 + 0.02 + 0.01 + 0.00 + 0.03 + 0.15 + 0.01 + 0.01 + 0.01 + 0.03 + 0.24 + 0.02 + 0.01 + 0.01 + 0.04 + 0.32 = 1$

Therefore, this is a valid joint Probability mass function. (Showed).

(ii) $P(X<2, Y<3) = P_{11} + P_{12} = 0.09 + 0.02$
$= 0.11$
(Ans).

(iii) $P(X=1) = 0.09 + 0.02 + 0.01 + 0.00 = 0.12$

$P(X=2) = 0.03 + 0.15 + 0.01 + 0.01 = 0.2$

$P(X=3) = 0.01 + 0.03 + 0.24 + 0.02 = 0.3$

$P(X=4) = 0.01 + 0.01 + 0.04 + 0.32 = 0.38$

Again,

$P(Y=1) = 0.09 + 0.03 + 0.01 + 0.01 = 0.14$

$P(Y=2) = 0.02 + 0.15 + 0.03 + 0.01 = 0.21$

$P(Y=3) = 0.01 + 0.01 + 0.24 + 0.04 = 0.3$

$P(Y=4) = 0.00 + 0.01 + 0.02 + 0.32 = 0.35$

(iv) $E(x) = \sum_{i=1}^{4} i\, P(X=i)$

$= 1.P(X=1) + 2.P(X=2) + 3.P(X=3) + 4.P(X=4)$

$= 1.(0.12) + 2.(0.2) + 3.(0.3) + 4.(0.38)$

$= 0.12 + 0.4 + 0.9 + 1.52 = 2.94$

$$E(x^2) = \sum_{i=1}^{4} i^2 P(X=i)$$

$$= (1)^2 P(X=1) + (2)^2 P(X=2) + (3)^2 P(X=3) + (4)^2 P(X=4)$$

$$= 1.(0.12) + 4.(0.2) + 9.(0.3) + 16.(0.38)$$

$$= 0.12 + 0.8 + 2.7 + 6.08 = 9.7.$$

$$\therefore V(x) = E(x^2) - (E(x))^2 = 9.7 - (2.94)^2$$

$$= 1.0564$$

(Ans).

(v) $$P(X \mid y=2) = \frac{P(x, y=2)}{P(y=2)}.$$

$\therefore P(y=2) = 0.21$ [From (iii)]

So,

$$P(x=1 \mid y=2) = \frac{P(x=1, y=2)}{P(y=2)} = \frac{0.02}{0.21} = 0.095$$

$$P(x=2 \mid y=2) = \frac{P(x=2, y=2)}{P(y=2)} = \frac{0.15}{0.21} = 0.714$$

$$P(x=3 \mid y=2) = \frac{P(x=3, y=2)}{P(y=2)} = \frac{0.03}{0.21} = 0.143$$

$$P(x=4 \mid y=2) = \frac{P(x=4, y=2)}{P(y=2).} = \frac{0.01}{0.21} = 0.048$$

$E(X|y=2) = \sum_{i=1}^{4} i P(X|y=2)$

$= 1. P(x=1|y=2) + 2. P(x=2|y=2) + 3. P(x=3|y=2)$
$+ 4. P(x=4|y=2)$.

$= 1.(0.095) + 2.(0.714) + 3(0.143) + 4.(0.048)$

$= 0.095 + 1.428 + 0.429 + 0.192 = 2.144$

(Ans).

(vi) $E(y) = \sum_{j=1}^{4} j P(Y=j)$

$= 1. P(Y=1) + 2. P(Y=2) + 3. P(Y=3) + 4. (Y=4)$.

$= 1. (0.14) + 2. (0.21) + 3. (0.3) + 4. (0.35)$

$= 0.14 + 0.42 + 0.9 + 1.4 = 2.86$.

$E(y^2) = \sum_{j=1}^{4} j^2 P(Y=j)$

$= (1)^2. P(Y=1) + (2)^2. P(Y=2) + (3)^2 P(Y=3) + (4)^2 P(Y=4)$

$= 1.(0.14) + 4.(0.21) + 9.(0.3) + 16.(0.35)$.

$= 0.14 + 0.84 + 2.7 + 5.6 = 9.28$

$\therefore V(y) = E(y^2) - (E(y))^2 = 9.28 - (2.86)^2$
$= 1.1004$

$$E(x,y) = \sum_{j=1}^{4}\sum_{i=1}^{4} ij\,P_{ij}$$

$= (1\times1\times P_{11}) + (1\times2\times P_{12}) + (1\times3\times P_{13}) + (1\times4\times P_{14}) +$
$(2\times1\times P_{21}) + (2\times2\times P_{22}) + (2\times3\times P_{23}) + (2\times4\times P_{24}) +$
$(3\times1\times P_{31}) + (3\times2\times P_{32}) + (3\times3\times P_{33}) + (3\times4\times P_{34}) +$
$(4\times1\times P_{41}) + (4\times2\times P_{42}) + (4\times3\times P_{43}) + (4\times4\times P_{44})$

$= (1\times0.09) + (2\times0.02) + (3\times0.01) + (4\times0.00) + (2\times$
$0.03) + (4\times0.15) + (6\times0.01) + (8\times0.01) + (3\times$
$0.01) + (6\times0.03) + (9\times0.24) + (12\times0.02) +$
$(4\times0.01) + (8\times0.01) + (12\times0.04) + (16\times0.32).$

$= 0.09 + 0.04 + 0.03 + 0.00 + 0.06 + 0.6 +$
$0.06 + 0.08 + 0.03 + 0.18 + 2.16 + 0.24 +$
$0.04 + 0.08 + 0.48 + 5.12 = 9.29$

$\therefore Cov(x,y) = E(x,y) - E(x)E(y).$

$= 9.29 - (2.94.\ 2.86) = 0.8816$

$$\therefore Corr(x,y) = \frac{Cov(x,y)}{\sqrt{V(x)V(y)}} = \frac{0.8816}{\sqrt{(1.0564.1.1004)}}$$

$= 0.818$, which is positive.

So, there is a positive relationship between two random variables. If one random variable increases than another random variable will also increase.

(Ans).

Classwork of Bernoulli, Bionomial & Poisson Distribution

~~(a) $P(X=2) = \binom{9}{2} p^2 (1-$~~

(a) Bionomial distribution:

$$P(X=x) = \binom{9}{x} p^x (1-p)^{9-x}, \text{ where } x = 0, 1, 2, 3, \ldots, 9$$

$$\therefore P(X=2) = \binom{9}{2} (0.09)^2 (1-0.09)^{9-2}$$

$$= 36 . 0.0081 . 0.517$$

$$= 0.1508$$

(Ans).

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(b) $P(X \geq 2) = P(X=2) + P(X=3) + P(X=4) + \cdots\cdots + P(X=9)$.

we Know, total probability $= 1$

So,

$P(X=0) + P(X=1) + P(X=2) + P(X=3) + \cdots\cdots + P(X=9) = 1$

$\Rightarrow P(X=2) + P(X=3) + \cdots\cdots + P(X=9) = 1 - [P(X=0) + P(X=1)]$

$= 1 - [\binom{9}{0}(0.09)^0(1-0.09)^{9-0} + \binom{9}{1}(0.09)^1 (1-0.09)^{9-1}]$

$= 1 - [0.428 + 0.381] = 0.191$

$\therefore P(X=2) + P(X=3) + \cdots\cdots + P(X=9) = 0.191$

(Ans).

(c) Expected number of bull's eye scored,

$E(x) = np = 9.(0.09) = 0.81$

(Ans).

(d) Variance, $V(x) = np(1-p)$

$= 9.(0.09).(1-0.09)$

$= 9.(0.09).(0.91)$

$= 0.7371$

$\therefore$ Standard deviation, $SD(x) = \sqrt{V(x)}$

$= \sqrt{0.7371}$

$= 0.859$

(Ans).

Classwork of Geometric & Negative bionomial distribution

(1) Negative bionomial distribution:

$P(X=x) = \binom{x-1}{3-1}(1-0.6)^{x-3}(0.6)^3$ where

$x = 3, 4, 5, 6, \ldots\ldots$

$= \binom{x-1}{2}(0.4)^{x-3}.0.216$

(Ans).

(2) $P(X=6) = \binom{6-1}{2}(0.4)^{6-3}.0.216$

$= 10.(0.064).(0.216)$

$= 0.13824$ (Ans).

(3) $P(X \leq 6) = P(X=3) + P(X=4) + P(X=5) + P(X=6)$

$= \binom{3-1}{2}(0.4)^{3-3} \cdot 0.216 + \binom{4-1}{2}(0.4)^{4-3} \cdot 0.216 +$

$\binom{5-1}{2}(0.4)^{5-3} \cdot 0.216 + \binom{6-1}{2}(0.4)^{6-3} \cdot 0.216$

$= 0.216 + 0.2592 + 0.20736 + 0.13824$

$= 0.8208$

(Ans).

(4) $P(X \geq 6) = P(X=6) + P(X=7) + \cdots\cdots$

We know, total probability $= 1$

So,

$P(X=3) + P(X=4) + P(X=5) + P(X=6) + P(X=7) + \cdots\cdots = 1$

$\Rightarrow P(X=6) + P(X=7) = 1 - [P(X=3) + P(X=4) + P(X=5)]$

$= 1 - [\binom{3-1}{2}(0.4)^{3-3} \cdot 0.216 + \binom{4-1}{2}(0.4)^{4-3} \cdot 0.216 + \binom{5-1}{2}(0.4)^{5-3} \cdot 0.216]$

$= 1 - [0.216 + 0.2592 + 0.20736]$

$= 0.31744$ (Ans).

(5) Expected number of interviews:

$$E(x) = \frac{r}{p} = \frac{3}{0.6} = 5$$

(Ans).

Classwork of Normal & Standard normal distribution

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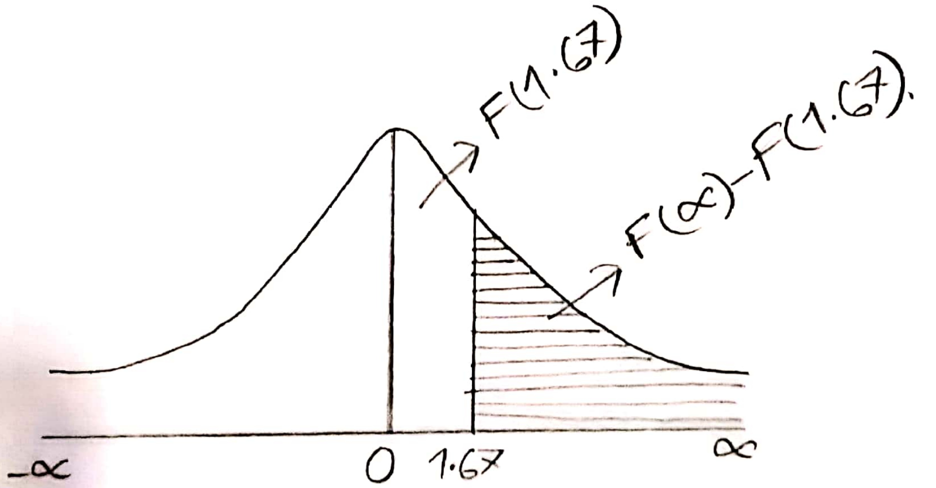
(1) $P(x > 3.2)$.

$= P(3.2 < x < \infty)$.

$= P\left(\frac{3.2-3}{0.12} < \frac{x-3}{0.12} < \frac{\infty-3}{0.12}\right)$.

$= P(1.67 < z < \infty)$.

$= F(\infty) - F(1.67) = 1 - 0.9525 = 0.0475$



(Ans).

(b) $P(x<2.7)$

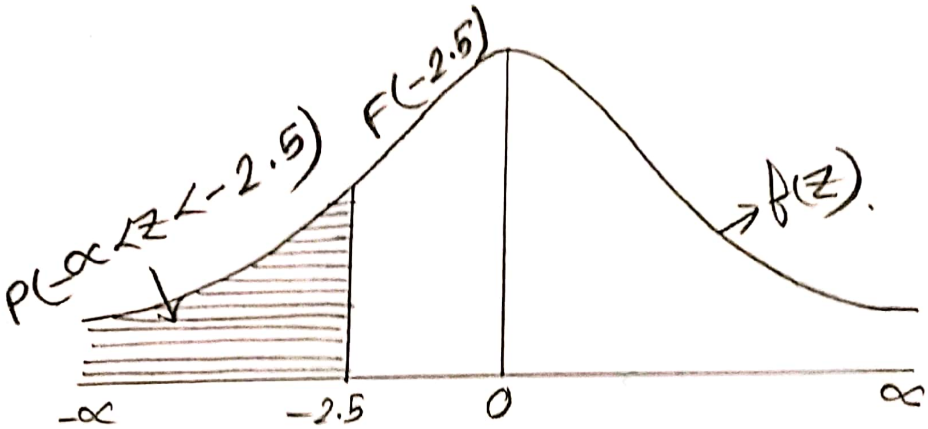
$= P(-\infty < x < 2.7)$

$= P\left(\frac{-\infty-3}{0.12} < \frac{x-3}{0.12} < \frac{2.7-3}{0.12}\right)$

$= P(-\infty < z < -2.5)$

$= F(-2.5)$

$= 0.0062$



(Ans)