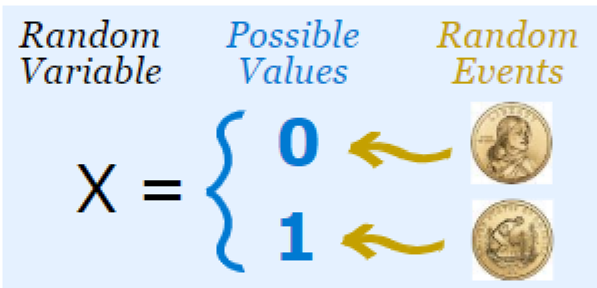
RANDOM VARIABLES

Random variable:

A random variable is obtained by assigning a numerical value to each outcome of a particular experiment.



So:

- We have an **experiment** (such as tossing a coin)
- We give **values** to each event
- The **set of values** is a **Random Variable**

Types of a Random Variables:

Random Variables can be either Discrete or Continuous:
Discrete random variable can only take **certain values** (such as 0,1)
Continuous random variable can take any value **within a range** (ex: A company manufactures metal cylinders that are used in the construction of a particular type of engine. The company discovers that the cylinders it manufactures can have a diameter anywhere between 49.5 and 50.5 mm. Suppose that the random variable X is the diameter of a randomly chosen cylinder manufactured by the company. Since this random variable can take any value between 49.5 and 50.5, it is a continuous random variable.)

X: 0,1
$49.5 \leq X \leq 50.5$.

Probability mass function (PMF):

PMF is a function of a **discrete random variable** that gives the probability that a discrete random variable is exactly equal to a value. Note that by definition the PMF is a probability measure, so it satisfies all properties of a probability. i.e The properties of PMF:

i) $0 \leq P_i \leq 1$

ii) $\sum P_i = 1$

Ex: I toss a coin twice. Let X be the number of heads. Find the PMF of X.

Solution: Here, the random variable X be the number of heads.

If I toss a coin twice, the possible outcomes are HH, HT, TH, TT.

S = {HH, HT, TH, TT}

So the value of the X = {0, 1, 2}.

Here P(HH) = P(HT) = P(TH) = P(TT) = 1/4.

PMF is given by P(0) = P(TT) = 1/4

P(1) = P(HT) + P(TH) = (1/4) + (1/4) = 1/2

P(2) = P(HH) = ¼.

Note: **I toss a coin. Let X be the number of heads. Find the PMF of X.**

Solution: X: {0,1}

S= {H,T}

P(H) = ½, P(T) = 1/2

P(0) = P(T) = 1/2

P(1) = P(H) = 1/ 2.

X: {1,2,3,4,5,6} (**the score shown on top face**)

S={1,2,3,4,5,6}

P(1) = P(2) = P(3) = P(4) = P(5) = P(6) = 1/6

PMF:

P(1) = P(2) = P(3) = P(4) = P(5) = P(6) = 1/6

*****If you throw a dice twice. Let X be the number of 6 comes on the two dice. Find the probability mass function of X.**
X: {0, 1, 2}

PMF:
P(0) = 25/36 (the probability that no 6 comes on the two dices)
P(1) = 10/36 (the probability that one 6 comes on the two dices)
P(2) = 1/36 (the probability that two 6 comes on the two dices)

P(0) + P(1) + P(2) =1

Cumulative distribution function (cdf):

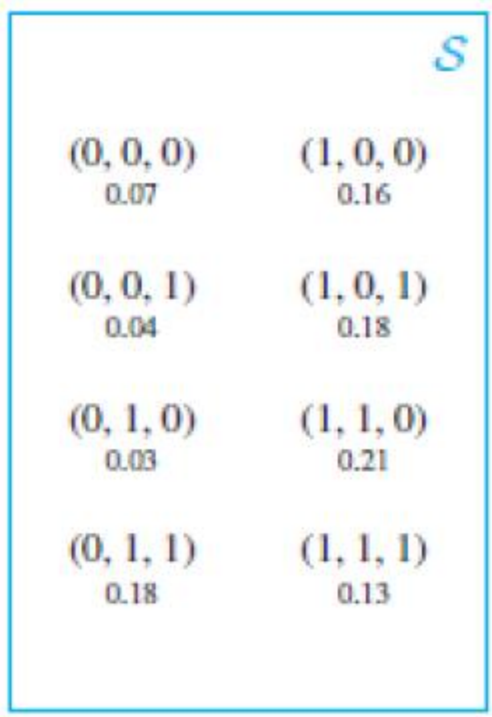
CDF is a function that starts at a small values of *x* (numerical values of the random variable X) and increases to a large values of *x*.

Ex: I toss a coin twice. Let X be the number of heads. Find the CDF of X.

Solution: X: {0,1,2**} (from previous example)**
PMF is given by P(0) = P(TT) = 1/4
P(1) = P(HT) + P(TH) = (1/4) + (1/4) = 1/2
P(2) = P(HH) = ¼.

F(0) = P(0) = 1/4 (from previous solution) (probability that no head occur)
F(1) = P(0) + P(1) = 1/4 + 1/**2 =** (probability that not more 1 head occur)
F(2) = P(0) + P(1) + P(2) = 1 (probability that no more than two head occur).

Question: A manager supervises the operation of three power plants, plant X, plant Y, and plant Z. At any given time, each of the three plants can be classified as either generating electricity (1) or being idle (0). With the notation (0, 1, 0) used to represent the situation where plant Y is generating electricity but plants X and Z are not generating electricity. Here, random variable X be the number of plants generating electricity.



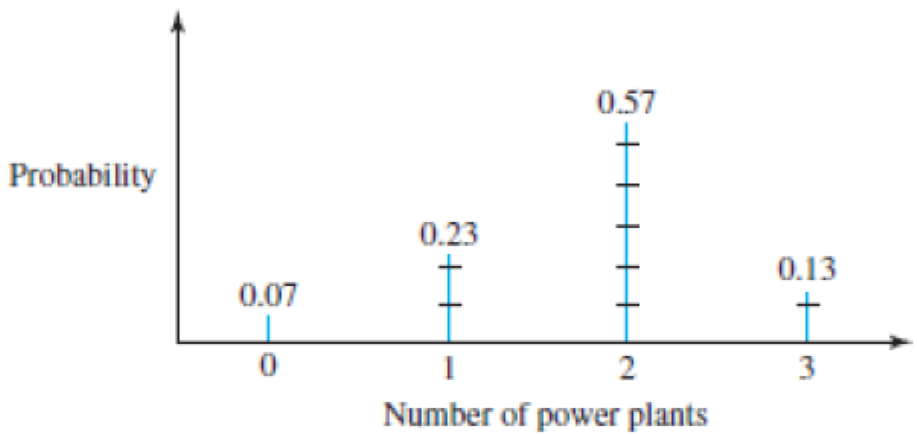
1) Calculate the probability mass function
2) Draw PMF
3) Calculate CDF
4) Draw CDF

Solution: Here, random variable X be the number of plants generating electricity. X = {0,1, 2, 3}.

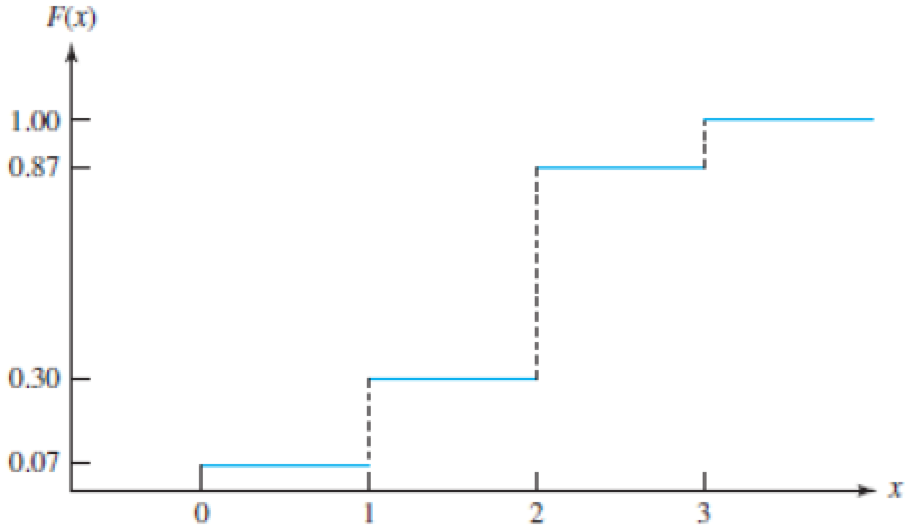
1) P(0) = 0.07 (probability that no plants are generating electricity)
   P(1) = 0.23 (probability that one plant is generating electricity)
   P(2) = 0.57 (probability that two plants are generating electricity)
   P(3) = 0.13 (probability that three plants are generating electricity)

x_i	0	1	2	3
p_i	0.07	0.23	0.57	0.13

2)



3) F(0) = P(0) = 0.07 (probability that no plants are generating electricity)
F(1) = P(0) + P(1) = 0.3 (probability that no more than one plant is generating electricity)
F(2) = P(0) + P(1) + P(2) = 0.87 (probability that no more than two plants are generating electricity)
F(3) = P(0) + P(1) + P(2) + P(3) = 1 (probability that no more than three plants are generating electricity).



F(x)
1.00
0.87
0.30
0.07
0
1
2
3
x