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(d) $\int_0^{\infty} \frac{x}{x^2-1} dx$.

The function $f(x) = \frac{x}{x^2-1}$ is discounting only at $x=1$ over infinite interval $(0,\infty)$.

$$\int_0^{\infty} \frac{x}{x^2-1} dx = \int_0^{1} \frac{x}{x^2-1} dx + \int_1^{\infty} \frac{x}{x^2-1} dx.$$

$$= \int_0^{1} \frac{x}{x^2-1} dx + \int_1^{2} \frac{x}{x^2-1} dx + \int_2^{\infty} \frac{x}{x^2-1} dx.$$

$$= \lim_{b \to 1^-} \int_a^{b} \frac{x}{x^2-1} dx + \lim_{a \to 1} \int_0^{2} \frac{x}{x^2-1} dx +$$

$$\lim_{a \to \infty} \int_2^{\infty} \frac{x}{x^2-1} dx$$

Now,

$$\int \frac{x}{x^2-1}\,dx.$$

$\therefore$ let, $u = x^2-1$

$$\Rightarrow \frac{du}{dx} = 2x.$$

$$\therefore dx = \frac{du}{2x}.$$

$$= \frac{1}{2}\int \frac{1}{u}\,du$$

$$= \frac{\ln(u)}{2}$$

$$= \frac{\ln(x^2-1)}{2}$$

So,

$$\lim_{b\to 1^-}\left[\frac{1}{2}(\ln(x^2-1))\right]_0^b + \lim_{a\to 1^+}\left[\frac{1}{2}\ln(x^2-1)\right]_a^2 + \lim_{a\to\infty}\left[\frac{1}{2}\ln(x^2-1)\right]_2^\infty$$

$$= \lim_{b\to 1^-}\left[\frac{1}{2}\ln(b^2-1) - \frac{1}{2}\ln(0^2-1)\right] + \lim_{a\to 1^+}\left[\frac{1}{2}\ln(3) - \frac{1}{2}\ln(a^2-1)\right] + \lim_{a\to\infty}\left[\frac{1}{2}\ln\left(\frac{1}{0}-\right)^2 - \frac{1}{2}(\ln(2-1))\right]$$

$$= \lim_{b\to 1^-}\left[\frac{1}{2}\ln(b^2-1) + \frac{1}{2}\ln 1\right] + \lim_{a\to 1^+}\left[\frac{1}{2}\ln 3 - \frac{1}{2}(a^2-1)\right] + \lim_{a\to\infty}\left[\frac{1}{2}\ln\left(\left(\frac{1}{0}\right)^2-1\right) - \frac{1}{2}\ln 3\right]$$

$= \infty$, hence the integral is divergent.

(Ans)