What's a Minterm?

- Otherwise known as a *standard product*
- Possible AND combinations of *n* variables
- Example? For two variables *a* and *b:*

a	b	minterm	
0	0	a'b'	m_0
0	1	a'b	m_1
1	0	ab'	m_2
1	1	ab	m_3

What's a Maxterm?

- Otherwise known as a *standard sum*
- Possible OR combinations of *n* variables
- Example? For two variables *a* and *b:*

a	b	Maxterm	
1	1	a' + b'	M_3
1	0	a' + b	M_2
0	1	a + b'	M_1
0	0	a + b	M_0

Minterms & Functions

- Given a truth table, a function can be put into algebraic form by OR-ing all of the **minterms** that have a 1 in the result column
- Example? F = (ab') + (a'b)

a	b	minterm		F
0	0	a'b'	m_0	0
0	1	a'b	m_1	1
1	0	ab'	m_2	1
1	1	ab	m_3	0

- ANY Boolean function can expressed this way

Maxterms & Functions

- Given a truth table, a function can be put into algebraic form by AND-ing all of the **maxterms** that have a 0 in the result column
- Example? F =(a+b)(a'+b')

a	b	minterm		maxterm		F
0	0	a'b'	m_0	a+b	M_0	0
0	1	a'b	m_1	a+b'	M_1	1
1	0	ab'	m_2	a'+b	M_2	1
1	1	ab	m_3	a'+b'	M_3	0

- ANY Boolean function can expressed this way

Canonical Form

- Boolean functions expressed algebraically as either…
  - A sum of minterms
  - A product of maxterms

Canonical Form → Complements

- The Complement for Sum of Minterms is…
  - …the sum of minterms missing from the original function
  - Two-variable example (4 minterms)?

$$F = \sum(0,2) \quad \text{so} \quad F' = \sum(1,3)$$

- Converting to a Product of Maxterms?
  - Remember that any minterm is the complement of its corresponding maxterm (e.g. $m_0' = M_0$)
  - So, using the example above:

$$F = \sum(0,2) \quad \text{so} \quad F' = \sum(1,3) \quad \text{so} \quad (F')' = \prod(1,3)$$

Canonical Form - Examples

a	b	c	minterm		F
0	0	0	a'b'c'	m_0	0
0	0	1	a'b'c	m_1	1
0	1	0	a'bc'	m_2	0
0	1	1	a'bc	m_3	0
1	0	0	ab'c'	m_4	1
1	0	1	ab'c	m_5	1
1	1	0	abc'	m_6	0
1	1	1	abc	m_7	0

Sum of Minterms? $F = m_1 + m_4 + m_5 = \sum(1,4,5)$

Product of Maxterms? $F = M_0 \cdot M_2 \cdot M_3 \cdot M_6 \cdot M_7 = \prod(0,2,3,6,7)$

Table 2.3
Minterms and Maxterms for Three Binary Variables

			Minterms		Maxterms	
x	y	z	**Term**	**Designation**	**Term**	**Designation**
0	0	0	$x'y'z'$	m_0	$x + y + z$	M_0
0	0	1	$x'y'z$	m_1	$x + y + z'$	M_1
0	1	0	$x'yz'$	m_2	$x + y' + z$	M_2
0	1	1	$x'yz$	m_3	$x + y' + z'$	M_3
1	0	0	$xy'z'$	m_4	$x' + y + z$	M_4
1	0	1	$xy'z$	m_5	$x' + y + z'$	M_5
1	1	0	xyz'	m_6	$x' + y' + z$	M_6
1	1	1	xyz	m_7	$x' + y' + z'$	M_7

Table 2.4
Functions of Three Variables

x	y	z	Function f_1	Function f_2
0	0	0	0	0
0	0	1	1	0
0	1	0	0	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

$$f_1 = x'y'z + xy'z' + xyz = m_1 + m_4 + m_7$$

$$f_1' = x'y'z' + x'yz' + x'yz + xy'z + xyz'$$

$$\begin{aligned} f_1 &= (x + y + z)(x + y' + z)(x' + y + z')(x' + y' + z) \\ &= M_0 \cdot M_2 \cdot M_3 \cdot M_5 \cdot M_6 \end{aligned}$$

$$f_2 = x'yz + xy'z + xyz' + xyz = m_3 + m_5 + m_6 + m_7$$

$$\begin{aligned} f_2 &= (x + y + z)(x + y + z')(x + y' + z)(x' + y + z) \\ &= M_0M_1M_2M_4 \end{aligned}$$