PHY 107
Center of mass and Linear momentum

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OUTLINE

- Center of mass
- Newton's second law for a system of particles
- Linear Momentum
- Linear Momentum of a system of particles
- Collision and Impulse
- Conservation of linear momentum
- Momentum and Kinetic Energy in collisions
- Elastic and Inelastic collision in 1D

Center of mass

The center of mass of a system of particles is the point that moves as though (1) all of the system's mass were concentrated there and (2) all external forces were applied there.

System of particles

n particles are strung out along the x-axis:

$x_{com} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3 + ...m_n x_n}{m_1 + m_2 + ...m_n}$

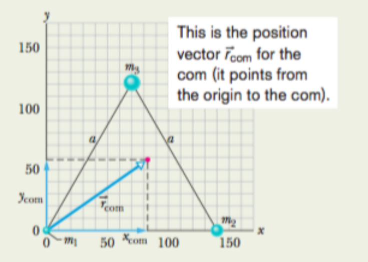
Solid bodies

$x_{com} = \frac{1}{M} \int x \ dm$, $y_{com} = \frac{1}{M} \int y \ dm$, $z_{com} = \frac{1}{M} \int z \ dm$ We look into uniform objects

1. Point, line or plane of symmetry
2. The center of mass of an object need not lie within the object.

Center of mass

EXAMPLE Three particles of masses $m_1 = 1.2kg$, $m_2 = 2.5kg$, and $m_3 = 3.4kg$ form an equilateral triangle of edge length a 140 cm. Where is the center of mass of this system?



Particle	Mass (kg)	x (cm)	y (cm)
1	1.2	0	0
2	2.5	140	0
3	3.4	70	120

$x_{com} = \frac{1}{M}\sum_{i=1}^{3} m_i x_i = 83$ cm

$y_{com} = \frac{1}{M}\sum_{i=1}^{3} m_i y_i = 58$ cm

Newton's Second law for a system of particles

The governing equation of the motion of the center of mass of a system of particles is

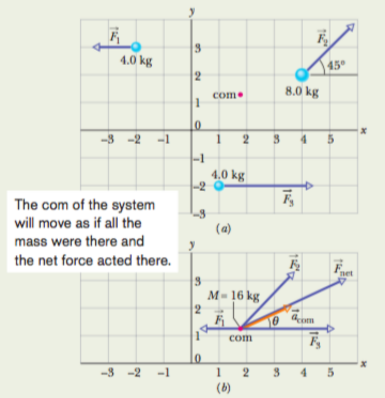
$\overrightarrow{F_{net}} = M\overrightarrow{a_{com}}$ (system of particles)

What exactly are these three quantities?

Newton's second law for a system of particles

EXAMPLE Motion of the com of three particles

The three particles are initially at rest. Each experiences an external force due to bodies outside the three-particle system. The directions are indicated, and the magnitudes are $F_1 = 6.0N$, $F_2 = 12N$, and $F_3 = 14N$.What is the acceleration of the center of mass of the system, and in what direction does it move?



Newton's second law for a system of particles

Solution

$a_{com,x} = \frac{F_{1x}+F_{2x}+F_{3x}}{M}$

$a_{com,y} = \frac{F_{1y}+F_{2y}+F_{3y}}{M}$

If you have a in unit vector notation, then you can easily find the magnitude and the angle

Linear Momentum

The linear momentum of a particle is a vector quantity $\overrightarrow{p}$ that is defined as: $\overrightarrow{p} = m\overrightarrow{v}$

Newtons' second law: $\overrightarrow{F_{net}} = \frac{d\overrightarrow{p}}{dt}$

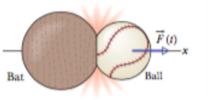
The system as a whole has a total linear momentum $\overrightarrow{P}$, which is defined to be the vector sum of the individual particles' linear momenta.

$\overrightarrow{P} = \overrightarrow{p_1} + \overrightarrow{p_2} + ... + \overrightarrow{p_n} = m_1\overrightarrow{v_1} + m_2\overrightarrow{v_2} + ...m_n\overrightarrow{v_n}$

The linear momentum of a system of particles is equal to the product of the total mass M of the system and the velocity of the center of mass

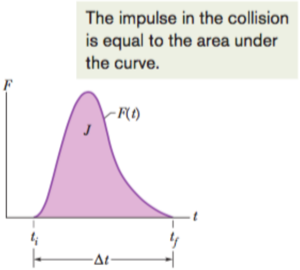
Collision and Impulse

Collisions are common in the world and we aim to study collisions where a moving particle like body collides with some other body (a target)



$$\int_{t_i}^{t_f} d\vec{p} = \int_{t_i}^{t_f} \vec{F}(t)dt$$

$\Delta\vec{p} = \vec{J}$ (linear momentum-impulse theorem)



Conservation of linear momentum

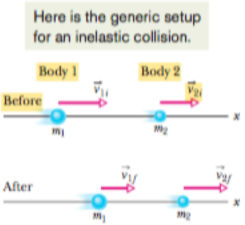
The net external force $\overrightarrow{F_{net}}$ (and thus the net impulse $\overrightarrow{J}$) acting on a system of particles is zero (the system is isolated) and that no particles leave or enter the system (the system is closed)
$\overrightarrow{P} = constant$
$\overrightarrow{P_i} = \overrightarrow{P_f}$ (closed, isolated system)
If the component of the net external force on a closed system is zero along an axis, then the component of the linear momentum of the system along that axis cannot change.

Momentum and Kinetic Energy in collisions

Elastic Collision Total kinetic energy of a system of two colliding bodies remains the same.
Inelastic collision Total kinetic energy of the system is not conserved.
What happens when we drop a ball on a surface?

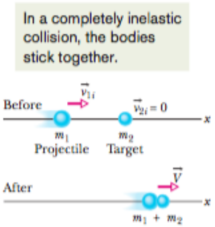
Inelastic collision in 1D

1D inelastic collision



$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

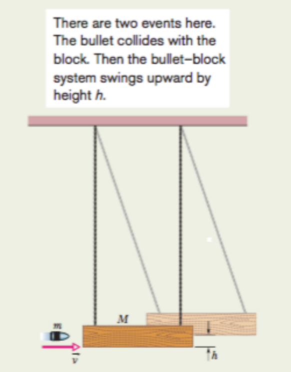
1D completely inelastic collision



$$m_1 v_{1i} = (m_1 + m_2)V$$

EXAMPLE Ballistic Pendulum

The ballistic pendulum was used to measure the speeds of bullets before electronic timing devices were developed. The version below consists of a large block of wood of mass M = 5.4 kg, hanging from two long cords. A bullet of mass m = 9.5 g is fired into the block, coming quickly to rest. The block bullet then swing upward, their center of mass rising a vertical distance h=6.3 cm before the pendulum comes momentarily to rest at the end of its arc. What is the speed of the bullet just prior to the collision?



EXAMPLE Ballistic Pendulum

Solution:

Total momentum before collision =Total momentum after collision
The mechanical energy of the bullet-block-Earth system is conserved: Mechanical energy at bottom= mechanical energy at top:

$$0.5(m + M)V^2 = (m + M)gh$$

The ballistic pendulum is a kind of 'transformer' exchanging the high speed of a light object (the bullet) for the low and thus more easily measurable speed of a massive object (the block).

Elastic Collisions in 1D

In an elastic collision, the kinetic energy of each colliding body may change, but the total kinetic energy of the system does not change.

Two cases:

1. Stationary target
2. Moving target

Stationary target:

A projectile body of mass m_1 and initial velocity v_{1i} moves toward a target body of mass m_2 that is initially at rest $v_{2i} = 0$.

$m_1 v_{1i} = m_1 v_{1f} + m_2 v_{2f}$(linear momentum).

$0.5 m_1 v_{1i}^2 = 0.5 m_1 v_{1f}^2 + 0.5 m_2 v_{2f}^2$

$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i}$

$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i}$

Stationary target continued....

A few special situations:

1. Equal masses: $v_{1f} = 0, v_{2f} = v_{1i}$
2. A massive target: $v_{1f} = -v_{1i}, v_{2f} \approx (\frac{2m_1}{m_2})v_{1i}$
3. A massive projectile: $v_{1f} \approx v_{1i}, v_{2f} \approx 2v_{1i}$

Moving target

$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$

$0.5 m_1 v_{1i}^2 + 0.5 m_2 v_{2i}^2 = 0.5 m_1 v_{1f}^2 + 0.5 m_2 v_{2f}^2$

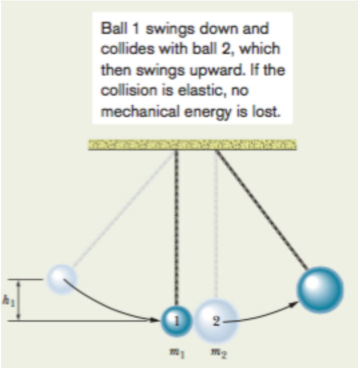
$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} + \frac{2m_2}{m_1 + m_2} v_{2i}$

$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i} + \frac{m_2 - m_1}{m_1 + m_2} v_{2i}$

Elastic collision

EXAMPLE Elastic Collision, Two pendulums

Two metal spheres, suspended by vertical cords, initially just touch. Sphere 1, with mass $m_1 = 30g$, is pulled to the left to height $h_1 = 8.0cm$, and then released from rest. After swinging down, it undergoes an elastic collision with sphere 2, whose mass $m_2 = 75g$. What is the velocity v_{1f} of sphere 1 just after the collision?



Elastic collision

Solution
Apply COE for the descent of sphere 1
Think about conservation of momentum for the two sphere collision
$0.5 m_1 v_{1i}^2 = m_1 g h_1$
$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} \approx -0.54$ m/s

Problems of importance:
Check the book (Edition: Extended 9th)
Center of mass: 1
Newton's Second Law for a system of particles:11
The linear momentum of a system of particles: 19
Collision and impulse: 25
Conservation of Linear Momentum: 39
Inelastic collisions in 1D: 49, 51
Elastic collisions in 1D: 61, 63, 65

Reference

Fundamentals of Physics by Halliday and Resnik