Exam Introduction Robotics (4L160)

4/11/2009, Wednesday, 14.00-17.00

General

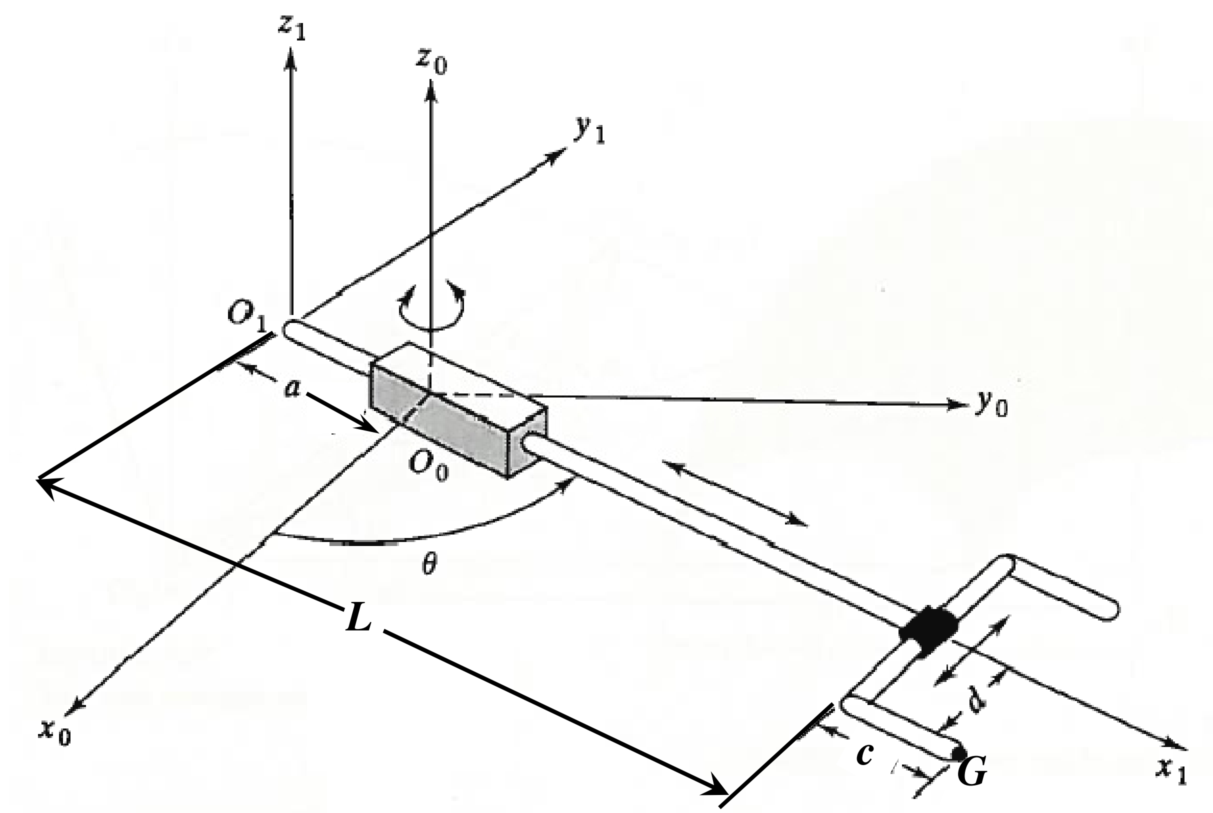
- You are allowed to use the book, the slides of the lectures, your notes, and your laptop.
- There are some typographical errors in the book. It is completely your responsibility to make sure that what you quote from the book is accurate.
- Grades:

problem	points
1	20
2	25
3	25
4	30
5	20
total	120

Consider the manipulator with two degrees of freedom shown in the figure below. The base frame is $O_0x_0y_0z_0$ and the frame $O_1x_1y_1z_1$ is attached to the manipulator arm. The arrows shown in the figure indicate the possible motions for the manipulator and the end-effector. The length of the manipulator arm is $L = 0.3$ [m], while the length of the side of the end-effector equals $c = 0.05$ [m]. At some time instant t, the shortest distance between the origins O_0 and O_1 equals $a(t)=0.05$ [m] and the corresponding linear velocity is $\mathrm{d}a(t)/\mathrm{d}t=0.4$ [m/s]. At the same time, the end-effector is opened at $d(t)=0.04$ [m] and has linear velocity of $\mathrm{d}d(t)/\mathrm{d}t=0.6$ [m/s]. At this t, the angular displacement of the arm relative to the base equals $\theta(t)=\pi/3$ [rad] and the corresponding angular velocity is $\mathrm{d}\theta(t)/\mathrm{d}t=3$ [rad/s]. For this t, calculate the following quantities:

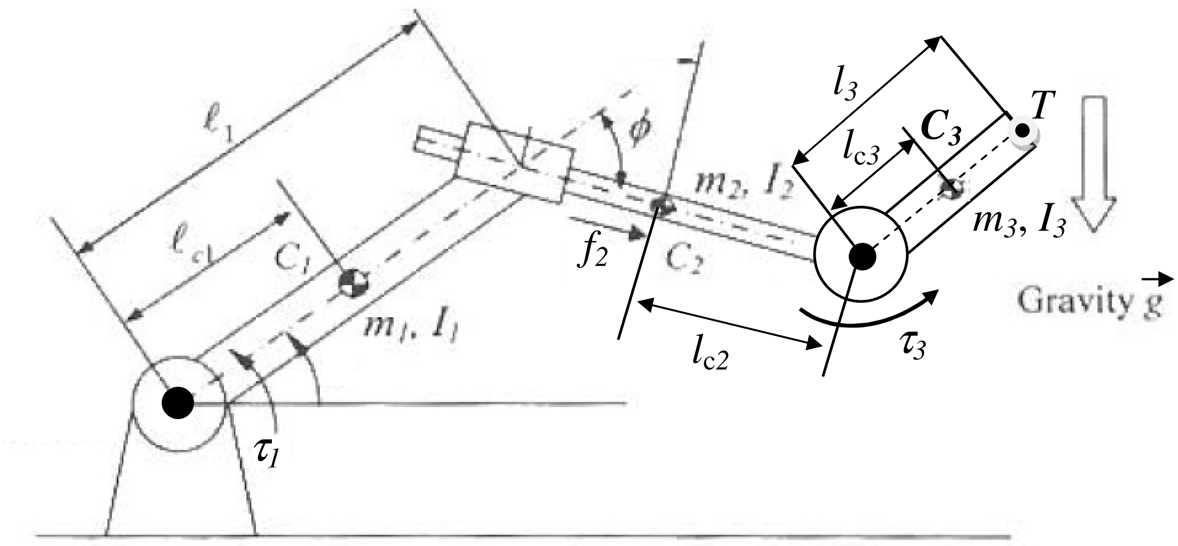
1. the homogenous transformation matrix $\mathbf{H}_1^0(t)$ which describes the position and orientation of the frame $O_1x_1y_1z_1$ relative to $O_0x_0y_0z_0$,

2. linear velocity of the end-effector point G relative to $O_0x_0y_0z_0$.



2

Assign coordinate frames according to the Denavits-Hartenberg convention to the RPR robot manipulator shown in the figure below. Derive the forward kinematics equations (homogenous transformation matrix $\mathbf{H}_3^0$) for this manipulator. It is known that $l_1 = 0.3$ [m], $l_3 = 0.2$ [m] and $\phi = \pi/6$ [rad].



3

For the manipulator considered in the problem 2, determine the generalized coordinates q_1, q_2, and q_3 that correspond to the location in the task-space specified in terms of the task-space variables o_x, o_y, o_z and $r_{i,j}$ $(i,j \in \{1,2,3\})$:

$$\begin{bmatrix} r_{1,1} & r_{1,2} & r_{1,3} & o_x \\ r_{2,1} & r_{2,2} & r_{2,3} & o_y \\ r_{3,1} & r_{3,2} & r_{3,3} & o_z \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & \frac{11}{200}+\frac{3\sqrt{3}}{20} \\ -1 & 0 & 0 & -\frac{1}{20}-\frac{11\sqrt{3}}{200} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

Hint: The Matlab function 'solve.m' can be used to solve the resulting system of nonlinear equations.

For the manipulator considered in the problem 2, masses of links 1, 2, and 3 are equal to 5 [kg], 2 [kg], and 0.5 [kg], respectively. The inertia tensor $\mathbf{I}_i$ of each link i (i=1,2,3), expressed relative to the coordinate frame attached to the link center of the mass, is equal to

$$\mathbf{I}_i = \begin{bmatrix} 0.2 & 0 & 0 \\ 0 & 0.2 & 0 \\ 0 & 0 & 0.2 \end{bmatrix}.$$

The link lengths l_1 and l_3 are already specified in the problem 2. The locations of the link centers of masses C_i, i=1,2,3, are given by $l_{c1}=0.5\, l_1$, $l_{c2}=0.12$ [m], and $l_{c3}=0.5\, l_3$. Direction of the gravity vector $\vec{g}$ ($g = 9.81$ [m/s^2]) is also indicated in the figure shown in the problem 2.

For this manipulator:

1. Write down the total kinetic energy.
2. Write down the total potential energy.
3. Derive inertia matrix.
4. Derive elements of the vector of the centripetal/Coriolis effects.
5. Derive elements of the gravity vector.

5

For the manipulator considered in the problem 2, determine values of the generalized forces τ_1, f_2 and τ_3 that are needed to counteract the static forces $f_x = 10\ [N]$ and $f_y = 15\ [N]$, together with the static moment $n_z = 0.3\ [Nm]$, acting at the point T located at the tip of the manipulator. These static forces and the moment are given relative to the coordinate frame at the manipulator base. Dimensions of the manipulator are specified in the problems 2 and 4. Consider the following configuration in the manipulator joints (according to the Denavits-Hartenberg convention): $q_1 = q_3 = \frac{\pi}{4}$ [rad] and $q_2 = 0.2\ [m]$.