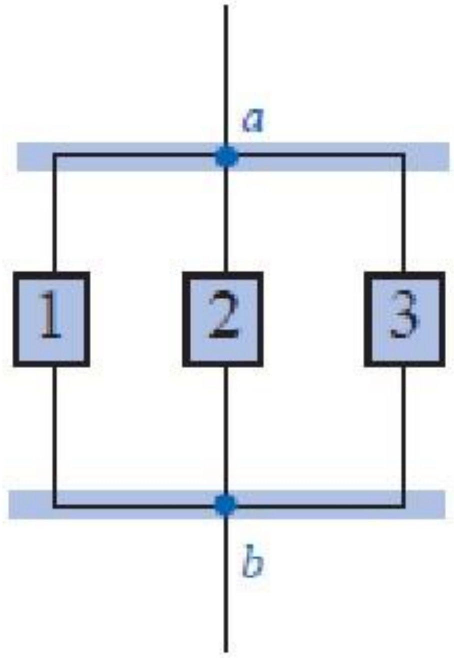
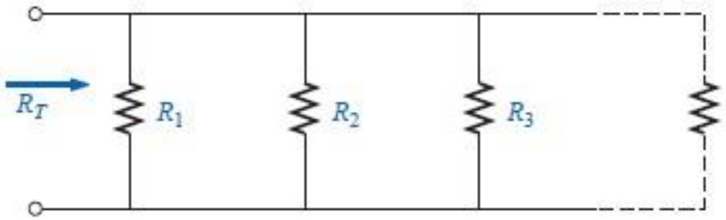
Parallel Circuit

Two elements, branches, or networks are in* parallel *if they have two points in common.



Parallel Circuit

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots + \frac{1}{R_N}$$



The total resistance of parallel resistors is always less than the value of the smallest resistor.

parallel elements can be interchanged without changing the total resistance or input current.

For parallel resistors, the total resistance will always decrease as additional elements are added in parallel.

The voltage across parallel elements is the same.

Current seeks the path of least resistance.

PARALLEL RESISTORS AND CURRENT DIVISION

When two resistors are connected in parallel, they have the same voltage across them.

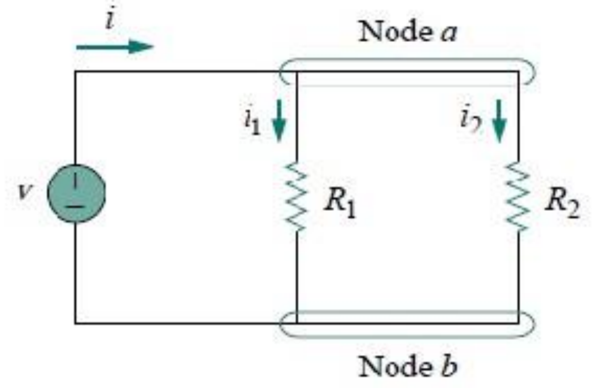
$$v = i_1 R_1 = i_2 R_2$$

$$i_1 = \frac{v}{R_1}, \qquad i_2 = \frac{v}{R_2}$$

$$i = i_1 + i_2$$

$$i = \frac{v}{R_1} + \frac{v}{R_2} = v\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = \frac{v}{R_{eq}}$$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

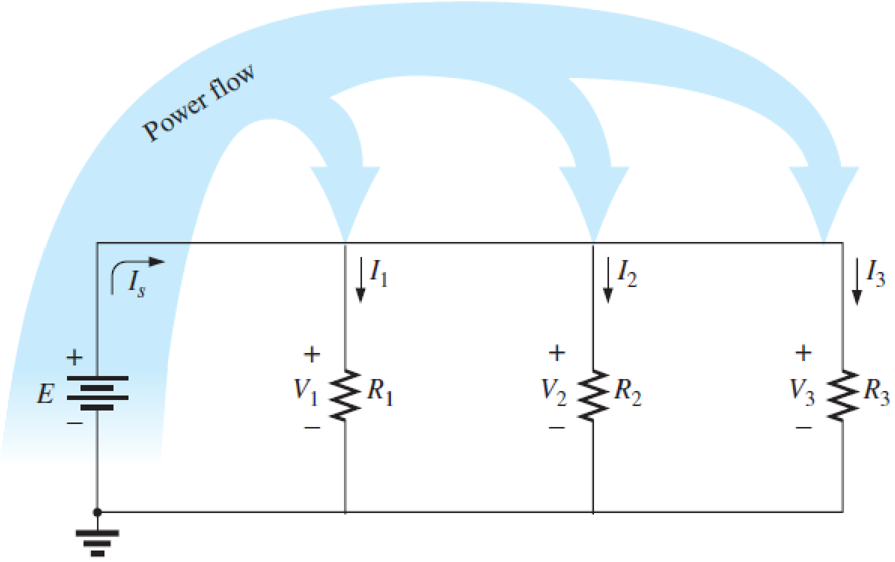


$$\boxed{R_{eq} = \frac{R_1 R_2}{R_1 + R_2}}$$

The equivalent resistance of two parallel resistors is equal to the product of their resistances divided by their sum.

Power distribution in a Parallel Circuit

- For any network composed of resistive elements, the power applied by the battery will equal that dissipated by the resistive elements.



$$\boxed{P_E = P_{R_1} + P_{R_2} + P_{R_3}}$$

$$\boxed{P_1 = V_1 I_1 = I_1^2 R_1 = \frac{V_1^2}{R_1}}$$

- In a parallel resistive network, the larger the resistor, the less is the power absorbed.

KIRCHHOFF'S CURRENT LAW

Kirchhoff's current law (KCL) states that the algebraic sum of currents entering a node (or a closed boundary) is zero.

The sum of the currents entering a node is equal to the sum of the currents leaving the node.

$$i_1 + i_3 + i_4 = i_2 + i_5$$

$$\sum_{n=1}^{N} i_n = 0$$

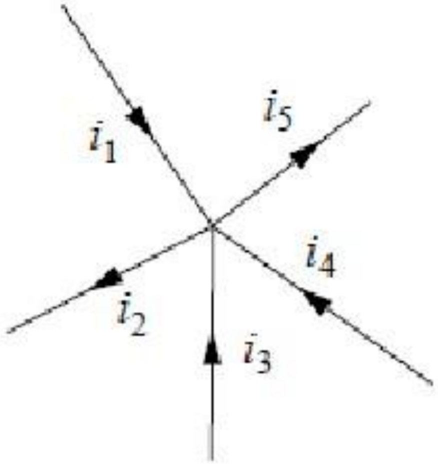
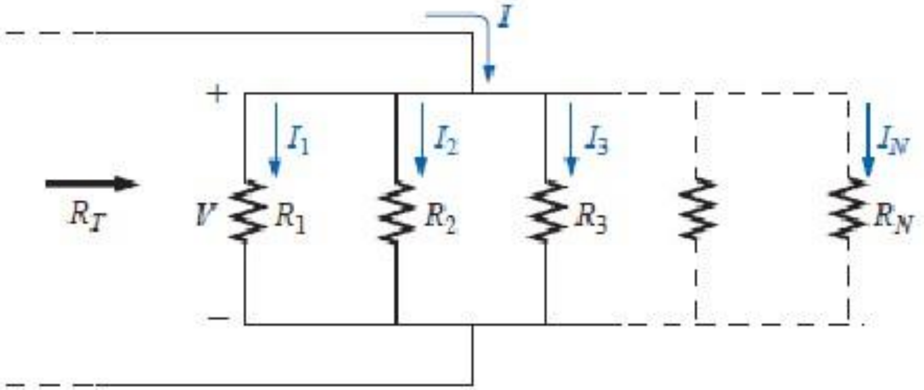
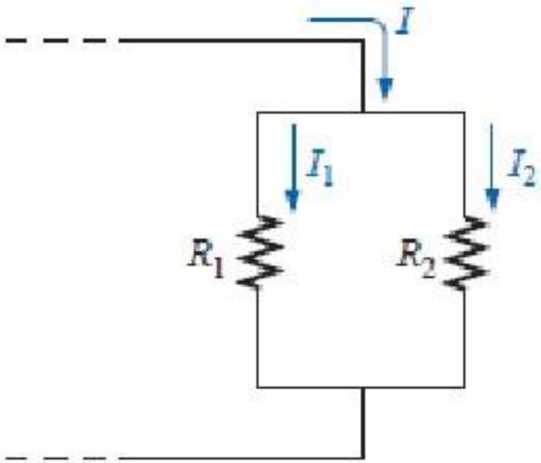


Figure 2.16 Currents at a node illustrating KCL.

CURRENT DIVIDER RULE





$$I = \frac{V}{R_T} = \frac{I_x R_x}{R_T}$$

$$\boxed{I_x = \frac{R_T}{R_x} I}$$

$$I_1 = \frac{R_T}{R_1} I$$

$$I_2 = \frac{R_T}{R_2} I$$

$$R_T = \frac{R_1 R_2}{R_1 + R_2}$$

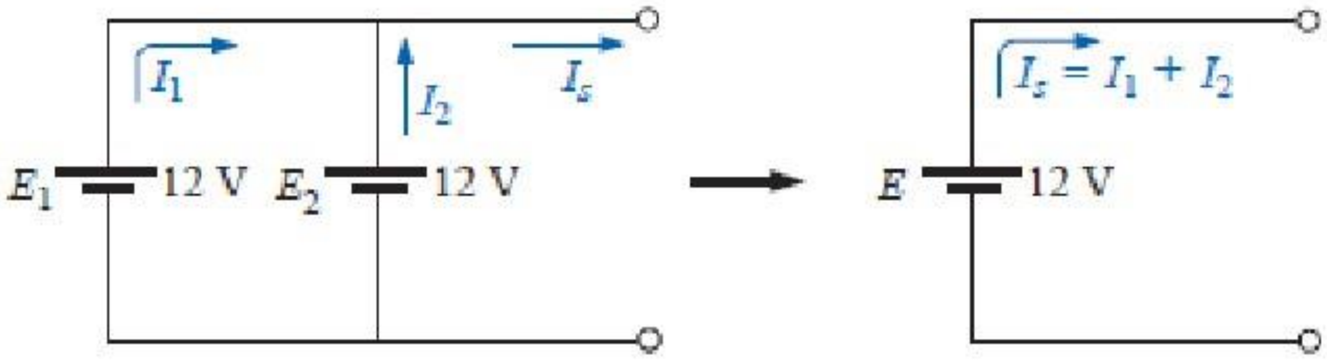
$$I_1 = \frac{R_T}{R_1} I = \frac{\dfrac{R_1 R_2}{R_1 + R_2}}{R_1} I$$

Note difference in subscripts.

$$\boxed{I_1 = \frac{R_2 I}{R_1 + R_2}}$$

$$\boxed{I_2 = \frac{R_1 I}{R_1 + R_2}}$$

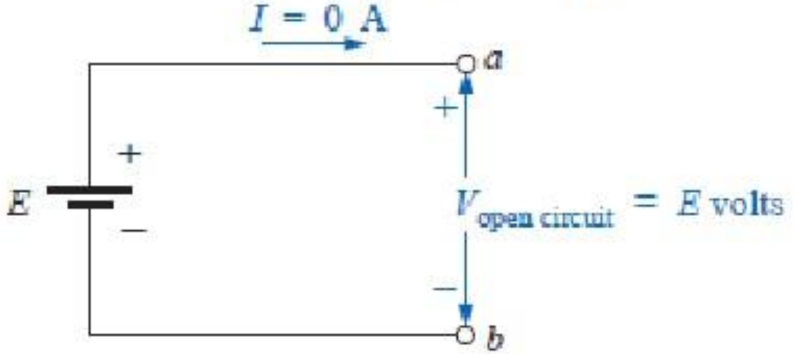
Voltage Sources In Parallel



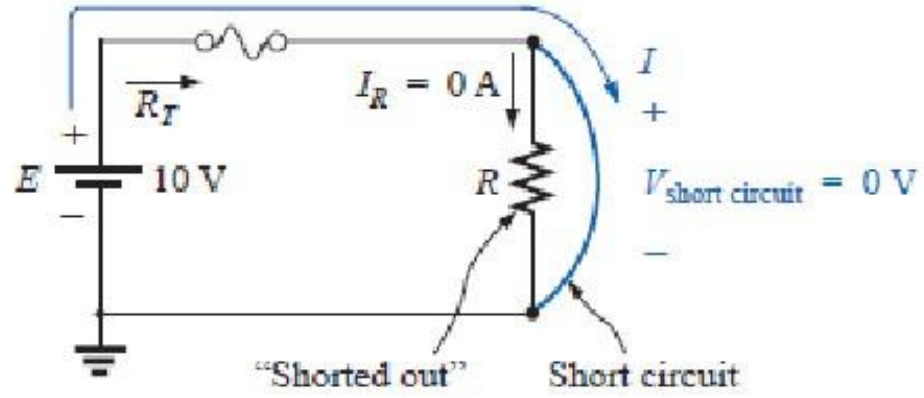
- Voltage sources can be connected in parallel if they have the same voltage rating.
- Having two or more batteries with same terminal voltage will increase the current rating.
- If two batteries with different terminal voltages are connected in parallel, both would be left damaged.
- The larger battery would try to drop rapidly to that of the lower supply.

OPEN AND SHORT CIRCUITS

an open circuit can have a potential difference (voltage) across its terminals, but the current is always zero amperes.



a short circuit can carry a current of a level determined by the external circuit, but the potential difference (voltage) across its terminals is always zero volts.



Voltmeter loading effects

- The internal resistance of a voltmeter appears as parallel to the circuit.

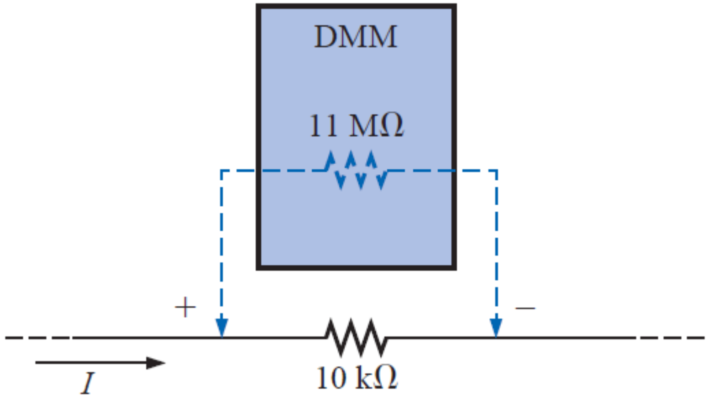


FIG. 6.54
Voltmeter loading.

$$R_T = 10\ \text{k}\Omega \parallel 11\ \text{M}\Omega = \frac{(10^4\ \Omega)(11 \times 10^6\ \Omega)}{10^4\ \Omega + (11 \times 10^6\ \Omega)} = 9.99\ \text{k}\Omega$$

- The ideal level for the internal resistance of a voltmeter would be infinite ohms, just as zero ohms would be ideal for an ammeter.