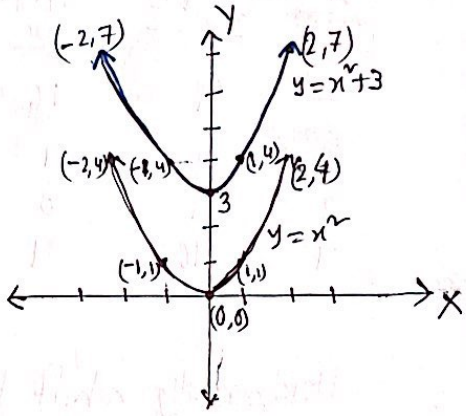
Mat-116 Lecture: 9

Graphing techniques

Vertical shift up:

Use the graph $f(x)=x^2$ to obtain the graph $g(x)=x^2+3$.

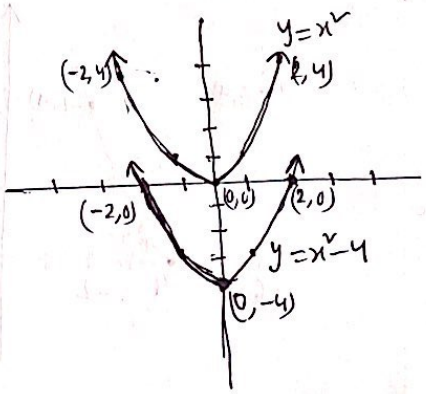
x	$f(x)=x^2$	$g(x)=x^2+3$
-2	4	7
-1	1	4
0	0	3
+1	1	4
2	4	7



Vertical shift down:

Use the graph $f(x)=x^2$ to obtain the graph of $g(x)=x^2-4$

x	$f(x)=x^2$	$g(x)=x^2-4$
-2	4	0
-1	1	-3
0	0	-4
1	1	-3
2	4	0

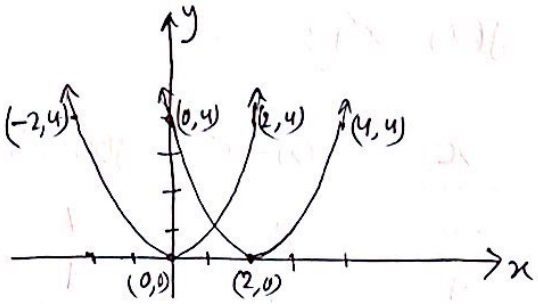


* $y=f(x)+k \Rightarrow f$ is shifted vertically up by k units.
* $y=f(x)-k \Rightarrow f$ is shifted vertically down k units.

Horizontally shift to right:

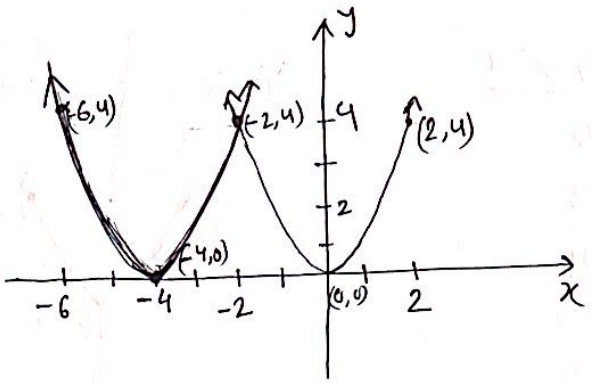
Use the graph $f(x) = x^2$ to obtain the graph of $g(x) = (x-2)^2$

x	$f(x) = x^2$	$g(x) = (x-2)^2$
-2	4	16
0	0	4
2	4	0
4	16	4



Horizontally shift to the left:

Use the graph of $f(x) = x^2$ to obtain the graph of $g(x) = (x+4)^2$.



x	x^2	$(x+4)^2$
-6	36	4
-4	16	0
-2	4	4
0	0	16

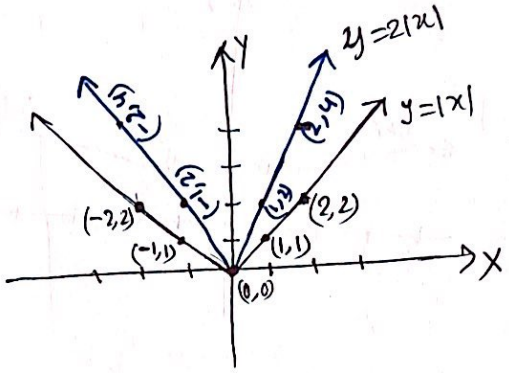
Example.

Graph the function $f(x) = (x+3)^2 - 5$

Vertical stretched:

Use the graph $f(x) = |x|$ to obtain the graph $g(x) = 2|x|$

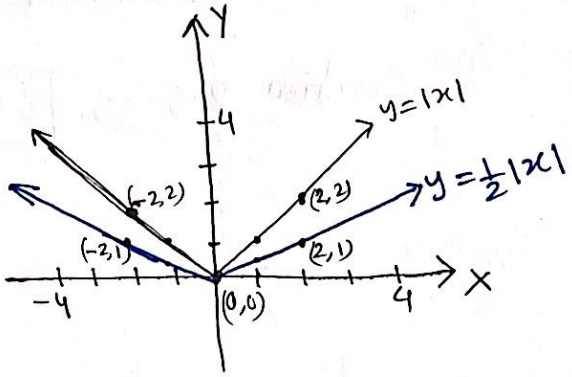
x	$f(x) = \lvert x \rvert$	$g(x) = 2\lvert x \rvert$
-2	2	4
-1	1	2
0	0	0
1	1	2
2	2	4



Vertical compression:

Use the graph $f(x) = |x|$ to obtain the graph of $g(x) = \frac{1}{2}|x|$

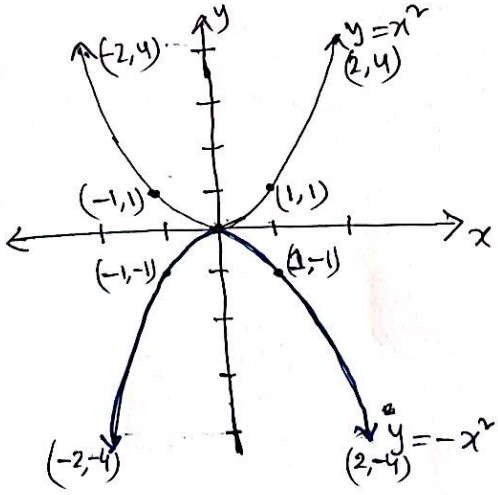
x	$f(x) = \lvert x \rvert$	$g(x) = \frac{1}{2}\lvert x \rvert$
-2	2	1
-1	1	$1/2$
0	0	0
1	1	$1/2$
2	2	1



Reflection about x-axis:

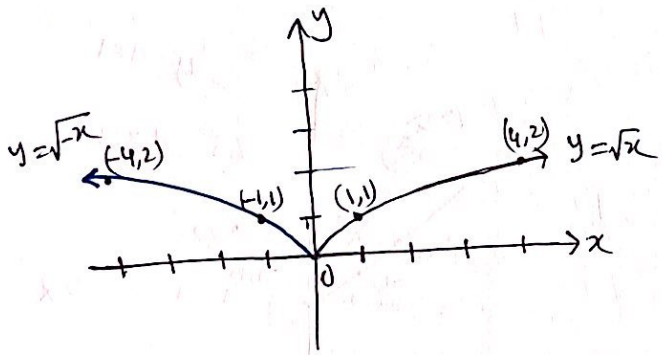
Graph the function $f(x) = -x^2$

x	x^2	$-x^2$
-2	4	-4
-1	1	-1
0	0	0
1	1	-1
2	4	-4



Reflection about y-axis:

Graph the function $f(x) = \sqrt{-x}$



Follow the summary of graphing techniques from book.

Exercise: From book:

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Example:

Graph the function $f(x) = \frac{3}{x-2} + 1$

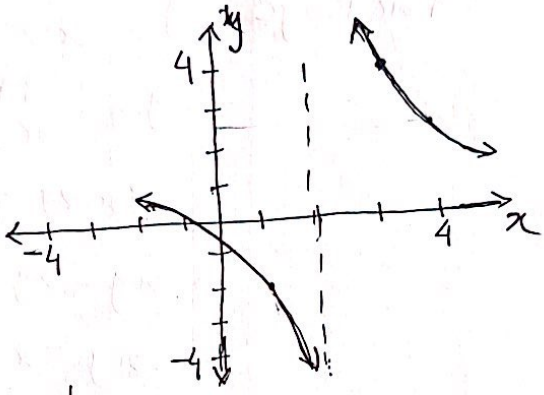
Soln:

Step 1: $y = \frac{1}{x}$

Step 2: $y = \frac{3}{x}$

Step 3: $y = \frac{3}{x-2}$

Step 4: $y = \frac{3}{x-2} + 1$



Domain $= \{x \mid x \neq 2\}$ Range $= \{y \mid y \neq 1\}$

Example:

Graph the function $f(x) = \sqrt{1-x} + 2$

Soln:

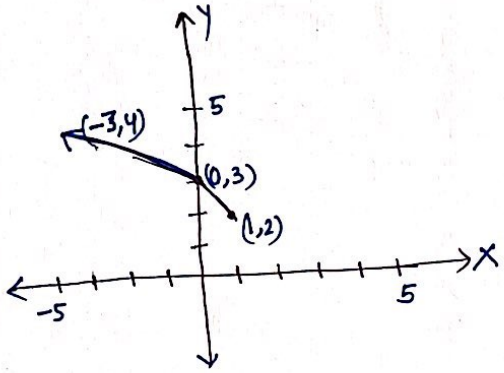
Step 1: $y = \sqrt{x}$

Step 2: $y = \sqrt{-x}$

Step 3: $y = \sqrt{-(x-1)}$

$= \sqrt{1-x}$

Step 4: $y = \sqrt{1-x} + 2$



Domain $= (-\infty, 1]$ and range $= [2, \infty)$

#1 $f(x) = \frac{3}{x-2} + 1$

For domain $x-2 \neq 0$ i.e $x \neq 2$

For range $y = \frac{3}{x-2} + 1$

$\Rightarrow y = \frac{3+x-2}{x-2}$

$\Rightarrow y(x-2) = x+1$

$\Rightarrow xy - 2y = x+1$

$\Rightarrow xy - x = 2y+1$

$\Rightarrow x(y-1) = 2y+1$

$\Rightarrow x = \frac{2y+1}{y-1} = f^{-1}(y)$

$\therefore$ range $= \{y \mid y \neq 1\}$.