Course Name: Physics – I
Course # PHY 107

Topic-1 : Introduction and
Review on Vector Algebra

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Introduction to Physics –I
(PHY 107)

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Lecture-1: Topics to be covered

1. Physical quantities
2. Base units and dimensions
3. Significant figures
4. Reviews on vector algebra
   a. What is vector
   b. How to represent a vector
   c. Relation to the Pythagorean theorem

Physical Quantities:

Name	Symbol	Units (SI)	Dimension
Mass	m	Kg	M
Length	x	m	L
Time	t	s (or sec)	T

1. These are called base quantities because any other physical quantities can be expressed in terms of these.
2. As for example, the force is expressed symbolically by the letter F and its unit is N (for Newton). This is not base unit because

Force = Mass × acceleration

Unit of F = Units of (Mass × acceleration)

$$1\text{ N} = 1\,\frac{kg.m}{s^2}$$

$$\text{Dimension of F} = \text{Dimensions of (Mass × acceleration)}$$

$$[\,\text{F}\,] = [\,MLT^{-2}\,]$$

Significant figures:

- This feature is related to the accuracy level of a measurement.
- As for example: the measurement of masses 2kg and 2.0 kg are not same. The measuring method used for 2.0 kg measurement can measure one-tenth of a kilogram accurately, whereas the instrument used to measure 2kg can not.
- The fraction of a kilogram has been rounded off, whereas for the other method the fraction of one-tenth of a kilogram is rounded off. So the method or instrument used to measure 2.0 kg is more accurate.
- This way of writing a number indicating its accuracy level is known as the significant figure.
- To find out the significant figure, each number must be expressed in the exponent form:

$$\text{Any Number} = (a.bcd\,.......) \times 10^{\pm \#}$$

The number of digits inside the parentheses without the exponential factor is the significant figure of the number.

- As for example: 1.60×10^{-3} is a three significant number, whereas 1.6×10^{-3} is a two significant number.
- Another example: 0.002357 is a four significant number because it can be expressed as 2.357×10^{-3}.
- The sig. fig. rules for addition, subtraction, product and division of numbers:
  - For addition, subtraction and product, the result will be expressed in the smallest significant figures, where as for division, the result will be expresses in the largest significant figures.
  - As for example:

$$\text{a)}\ 2.35\times3.6792 = 8.64612 \approx 8.65 \ \text{(rounded off to 3 sig. fig)}$$

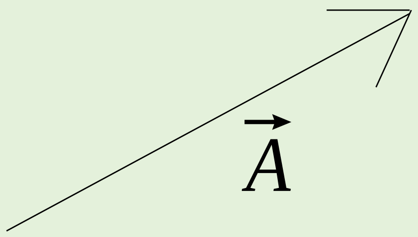
$$\text{b)}\ 2.35\pm3.6792 \ \ 6.0292\,\text{or}-1.3292 = 6.03\,\text{or}-1.33$$

$$\text{c)}\ 2.35\div3.6792 = 0.63872581 \approx 0.63873 \ \text{(rounded off to 5 sig. fig)}.$$

This is the scientific method of writing and expressing any numbers. This needs to be followed for the rest of the course and also during your lab classes (if any).

Vectors: Definitions and Properties:

- Definition: A vector is an abstract quantity which has \`Length' and a \`Sense of direction'. This is denoted by $\vec{A}$ and geometrically represented by a directed arrow, like,

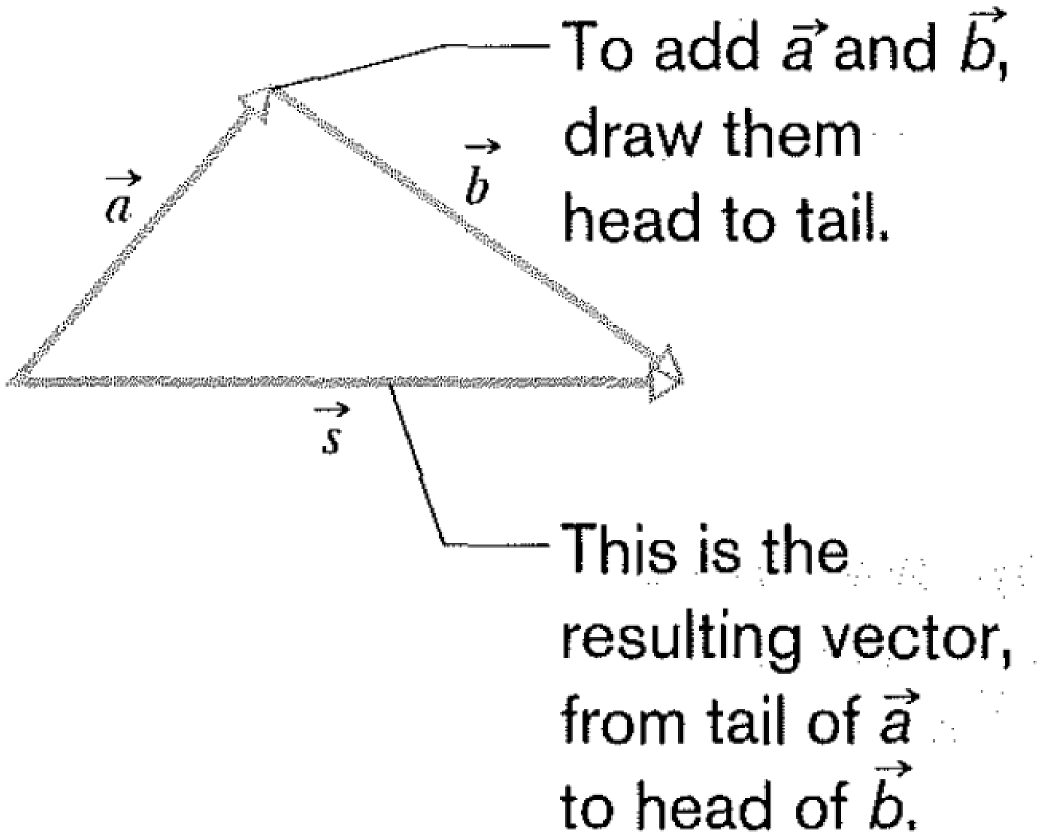


The number of parameters required to express a vector algebraically depends on the dimensions of space. In simple terms, it is the number of coordinates, called the dimensions of space. In one dimension, there is only one coordinate, like, the x-axis only. In three dimensions, there are three axes: x, y and z-axes.

- How many parameters do we need to represent the length and direction?

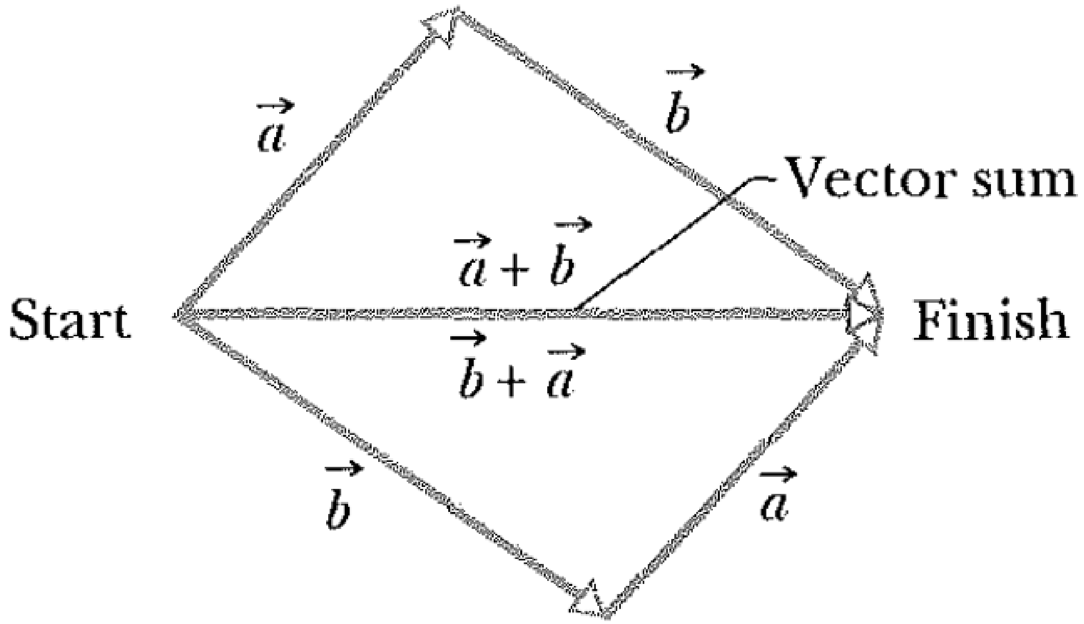
Vector addition:

- Triangular or Tip-to-Toe rule:



Source: The diagram is taken from the Textbook

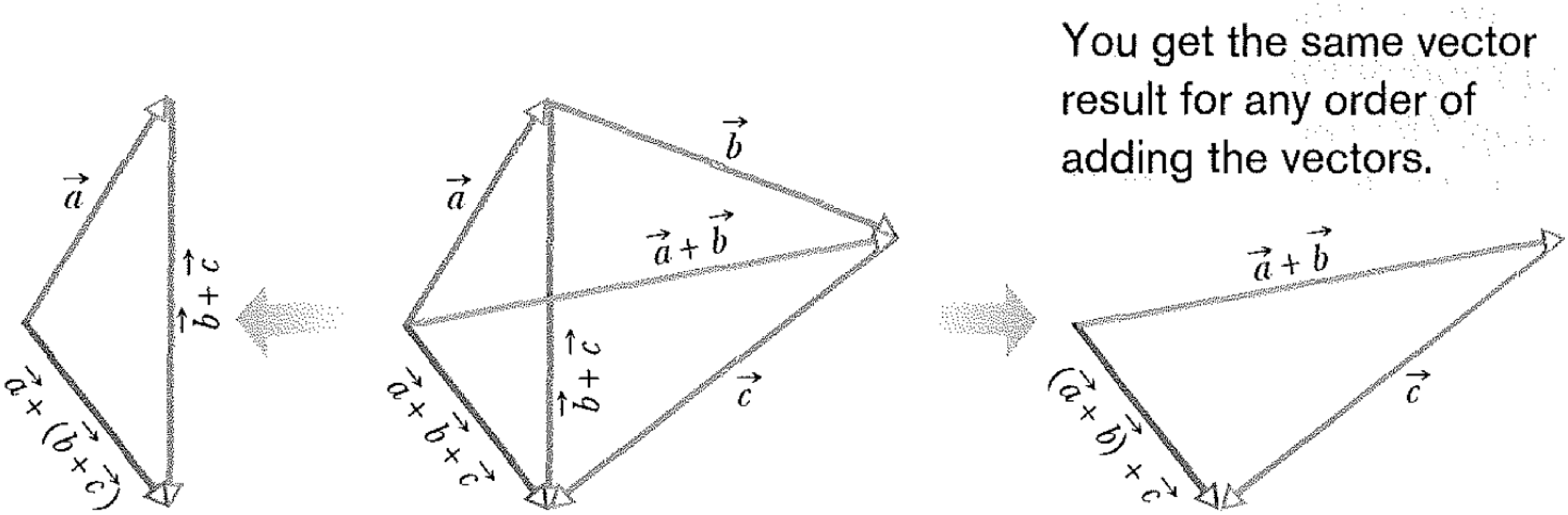
- Parallelogram rule:



You get the same vector result for either order of adding vectors.

Source: The diagram is taken from the Textbook

- Example: addition of three vectors

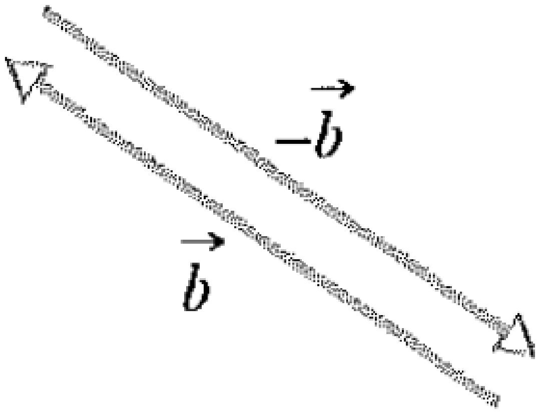


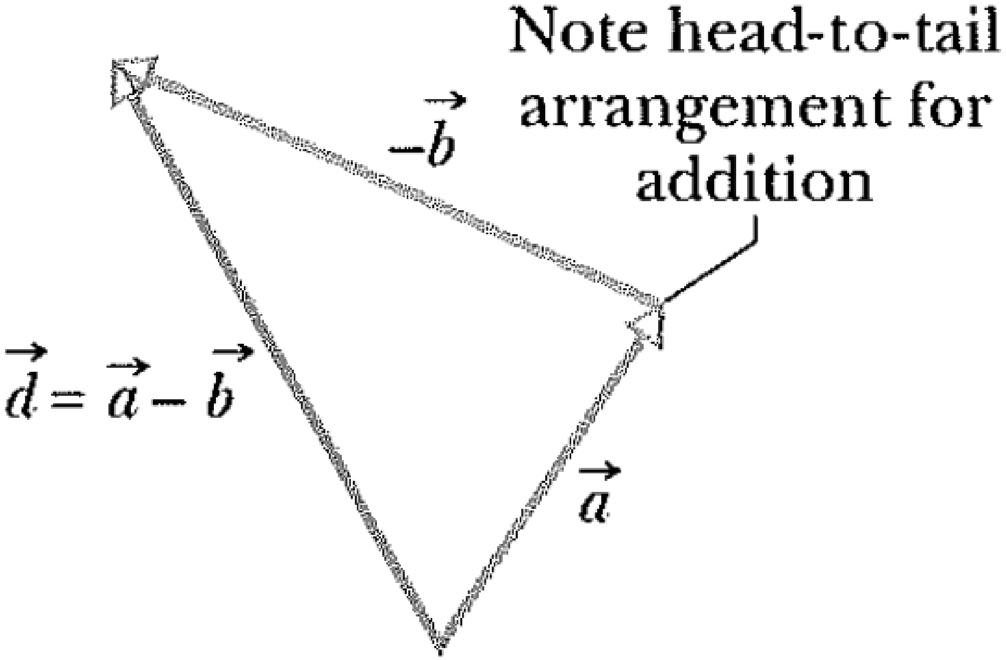
Very clearly from the above diagrams that $\vec{a}+(\vec{b}+\vec{c})=(\vec{a}+\vec{b})+\vec{c}$

Source: The diagram is taken from the Textbook

- Subtraction:

  A negative is a vector whose direction has been reversed, and the length or magnitude remains same (Left diagram).





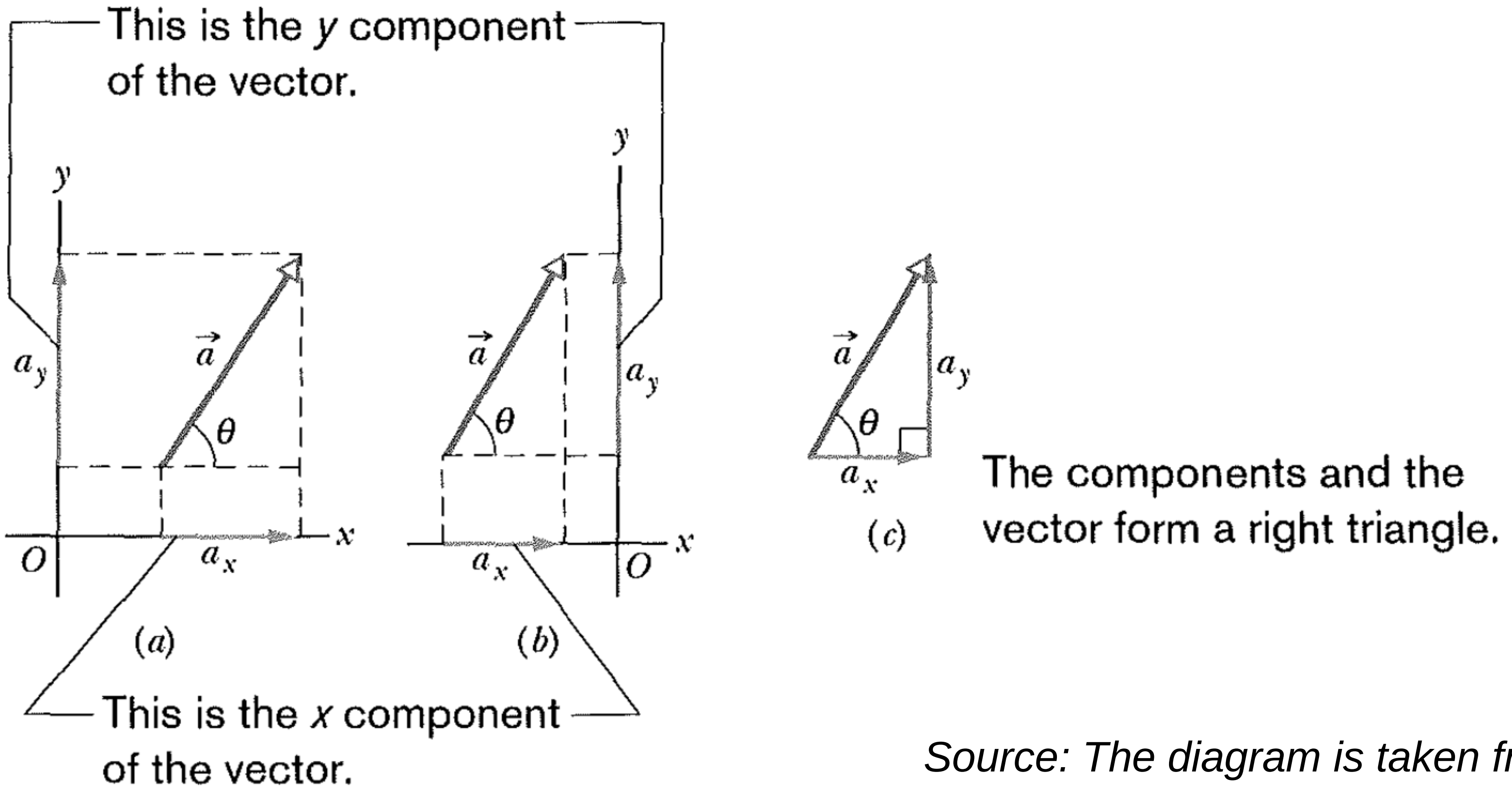
Subtracting a vector is really an a

(Right diagram).

Source: The diagram is taken from the Textbook

Algebraic form of a vector in two dimensions: We will represent a vector algebraically in two ways: Polar form and Cartesian (or Component) forms.

$$\vec{a} = (\text{Magnitude, angle}) = (\text{x-component, y-component})$$
$$= (a, \theta_a) = (a_x, a_y)$$

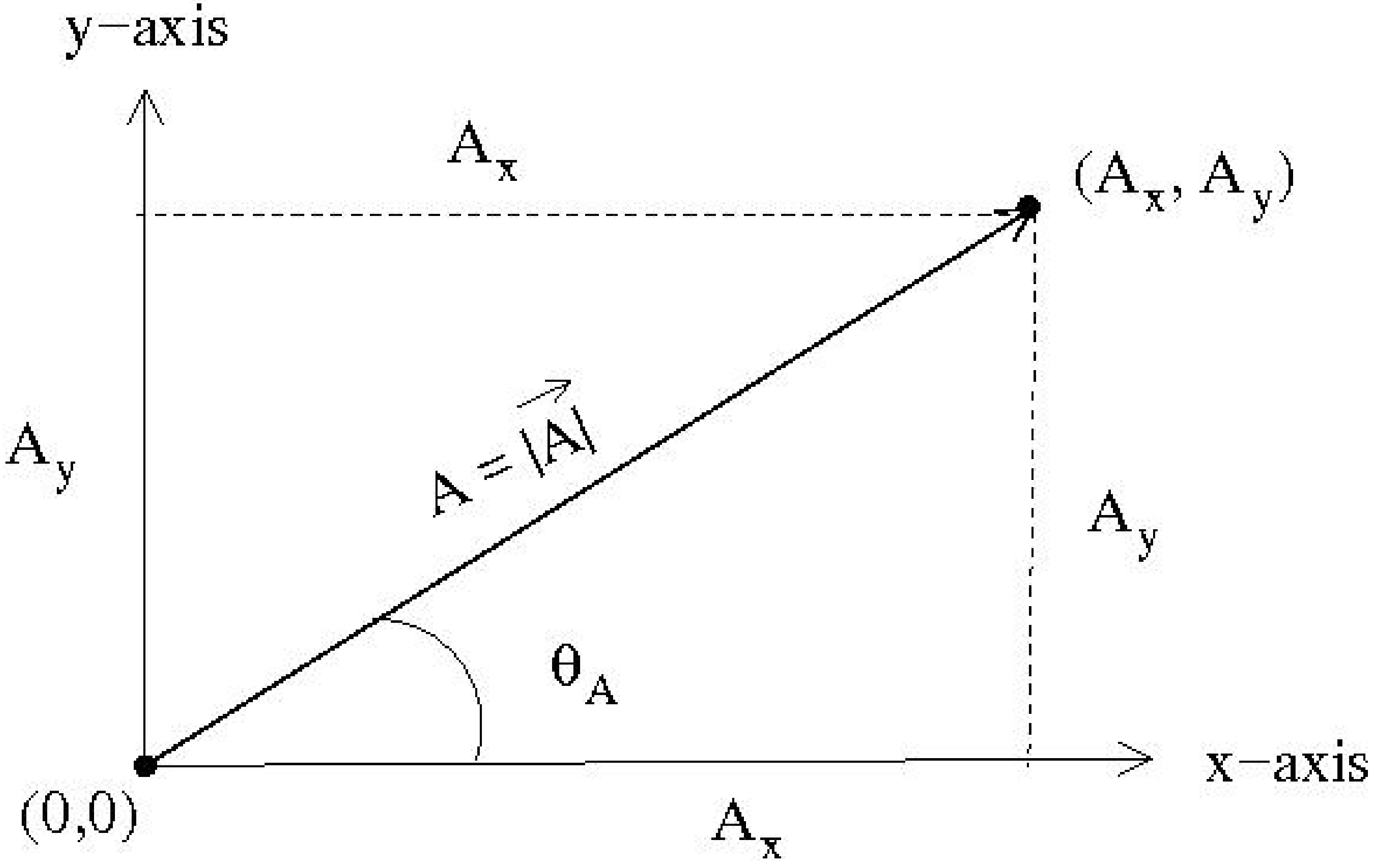


Source: The diagram is taken from the Textbook

Pythagorean Theorem and the Representation of a vector:

Let’s choose two dimensions (i.e. two coordinate axes) for simplicity.

A general vector can be expressed as: $\vec{A}=(A_x, A_y)=(A, \theta_A)$



- From the above diagrams, it is clear that the length of the vector (hypotenuse), the x-component (base) and the y-component (height) form a right-angle triangle

- The angle, θ_A, gives the direction of the vector, and is measured from the base to the hypotenuse counter-clockwise (positive angle).

- By Pythagorean Theorem, we can easily write:

$$\left.\begin{aligned} A^2 &= A_x^2 + A_y^2 \\ \tan\theta_A &= \left(\frac{A_y}{A_x}\right) \end{aligned}\right\}$$

This gives the conversion from the Cartesian to the Polar form of a vector

$$\left.\begin{aligned} A_x &= A\cos\theta_A \\ A_y &= A\sin\theta_A \end{aligned}\right\}$$

This gives the conversion from the polar to the Cartesian form of a vector.

Note:

- The Pythagorean theorem is defined only for the 1^{st} quadrant.
- Need to extend to all four quadrants.

How ?

- Consider the following two vectors given in Cartesian coordinates as

$$\vec{A}=(1,1) \rightarrow \theta_A=\tan^{-1}\left(\frac{1}{1}\right)=45^0$$

$$\vec{B}=(-1,-1) \rightarrow \theta_B=\tan^{-1}\left(\frac{-1}{-1}\right)=45^0!!!!!$$

- Certainly these two vectors have the same directions (as the algebra shows)
- But this is impossible since one vector is in the 1st quadrant, and the other is in the 3rd quadrant.
- Thus these two vectors have different directions. Note that the length these two vectors are same by Pythagorean theorem which is $\sqrt{2}$.
- The solution is:

$$\theta_B=\tan^{-1}\left(\frac{-1}{-1}\right)=180^0+\tan^{-1}\left(\frac{1}{1}\right)=180^0+45^0=225^0.$$

- Similar problem also occur between the vectors in the 2nd and 4th quadrant.
- The solutions are:

$$\theta = 180^0 - \tan^{-1}\left(\frac{|A_y|}{|A_x|}\right) \quad \text{(For a vector in the 2nd quadrant)}$$

$$= 360^0 - \tan^{-1}\left(\frac{|A_y|}{|A_x|}\right) \quad \text{(For a vector in the 4th quadrant)}$$

- Note that the angle is positive by convention when measured counter-clockwise.
- In three dimension, we need two angles to fix the direction which is known as the spherical polar coordinates (MAT250), and hence in more than two dimensions, we will mainly use Cartesian coordinates (or the component form of a vector)