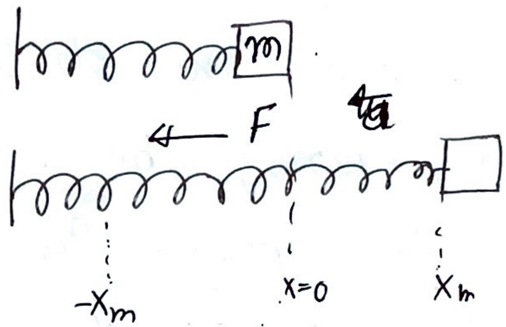
Hooke' law:

If we have spring at relaxation position $x=0$.



If ~~we~~ the spring gets an extension(x_m) in $+x$ direction, there will be a force F ~~which~~ working in opposite direction. According to Hooke's law

$$F \propto -x$$

$F=-kx$, K is called the spring constant.

If the mass of the load is m, and acceleration, a due to force F, we can write

$$F = ma = -kx$$

$$\text{or. } ma + kx = 0$$

$$\text{or. } a + \frac{k}{m}x = 0$$

$$\text{or } \frac{d^2x}{dt^2} + \frac{k}{m}x = 0$$

$$\boxed{\text{or } \ddot{x} + \omega^2 x = 0} \quad \text{--- (1)} \qquad \text{where } \omega = \sqrt{\frac{k}{m}}$$

This is equation of simple harmonic motion.

If the acceleration is a function of time

$$a(t) = -\omega^2 x(t)$$

This is the hall mark of S.H.M.

① particle's acceleration is always opposite of its displacement.

② The quantities are related by a constant ω^2.

ω is called the angular frequency.

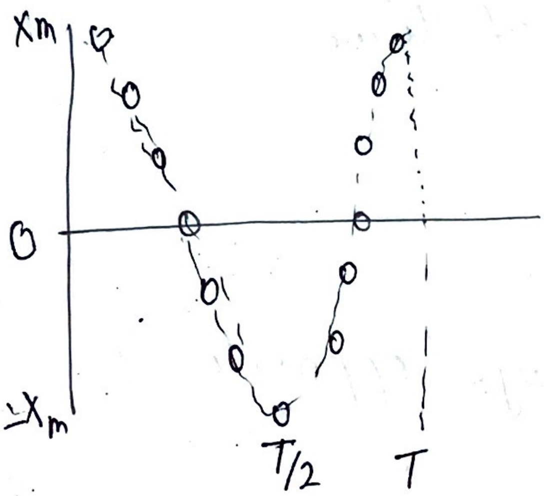
The solution of equation ① is

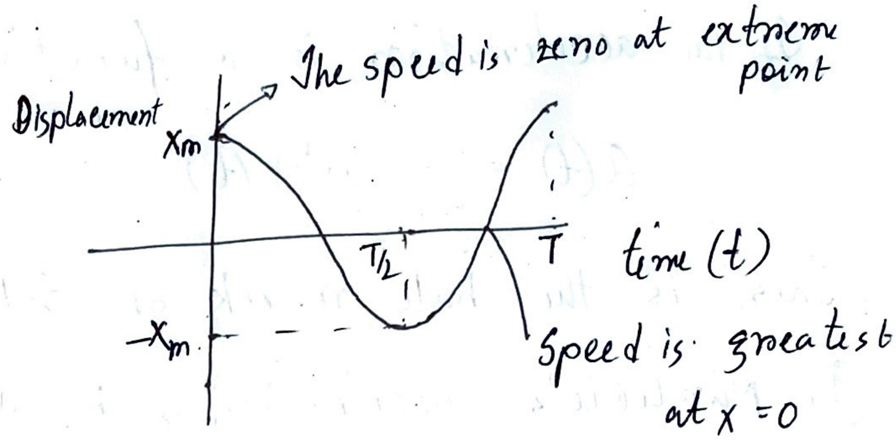
$$x(t) = x_m \cos(\omega t + \phi)$$

frequency: The frequency f of the oscillation is the number of times per second that it completes a full oscillation.

$$f = \frac{1}{T}$$

T is the time period for one full cycle of oscillation.





$$x(t) = x_m \cos(\omega t + \phi)$$

$x(t)$: Displacement at time t.

x_m: amplitude

ω: angular frequency

ϕ: phase angle

- Amplitude defines how far the particle moves in its oscillations.

- ϕ, phase angle define where the particle is in its oscillation when the clock time $t = 0$.

- Angular frequency: If $x(t)$ is the position at some chosen time t, & the particle returns ~~at~~ to same position at $t + T$. & if $\phi = 0$

$$x_m \cos \omega t = x_m \cos \omega (t + T)$$

$$\omega(t+T) = \omega t + 2\pi$$

$$\omega T = 2\pi$$

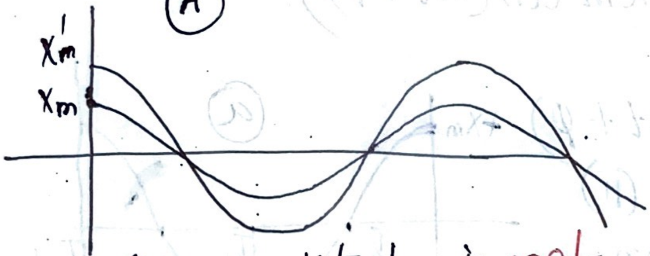
$$\boxed{\text{or, } \omega = \frac{2\pi}{T} = 2\pi f}$$

For spring

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{K}}$$

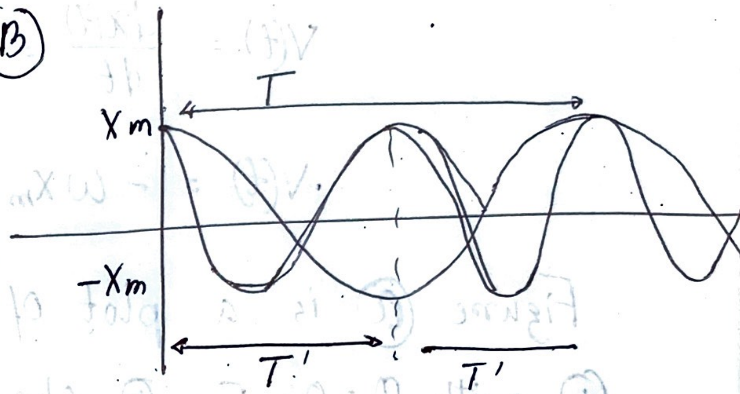
The SI unit of angular frequency is radian per second.

Ⓐ



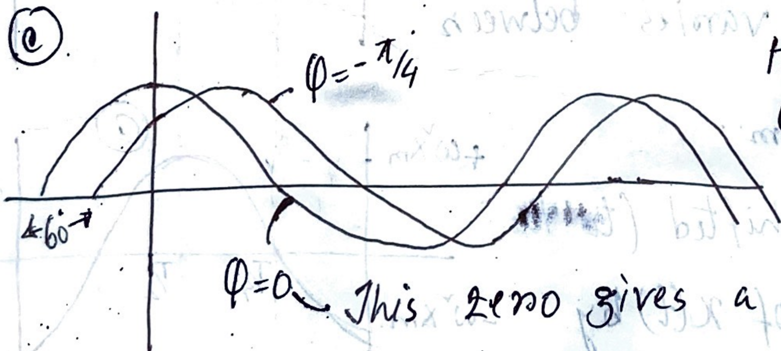
The amplitude is not same, the frequency & period are the same

Ⓑ



The amplitude are the same but the frequencys and the periods are different

Ⓒ



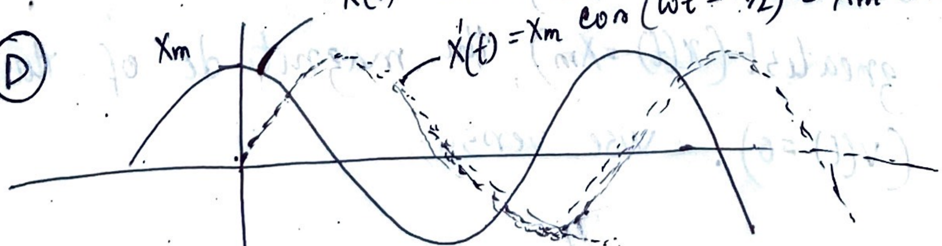
The negative value shifts the cosine curve rightward

$\phi = 0$ This zero gives a regular Cosine curve

$x(t) = X_m \cos(\omega t)$

$x(t) = X_m \cos(\omega t - \pi/2) = X_m \sin \omega t$

Ⓓ



Velocity of SHM

$-X_m,\ X_m,\ 0$

We know that

$$x(t) = x_m \cos(\omega t + \phi) \quad --- \quad (1)$$

We can find an expression for the velocity of a particle moving with simple harmonic motion, that is

$$V(t) = \frac{dx(t)}{dt} = \frac{d}{dt}\left(x_m \cos(\omega t + \phi)\right)$$

$$\boxed{V(t) = -\omega X_m \sin(\omega t + \phi)} \quad --- \quad (11)$$

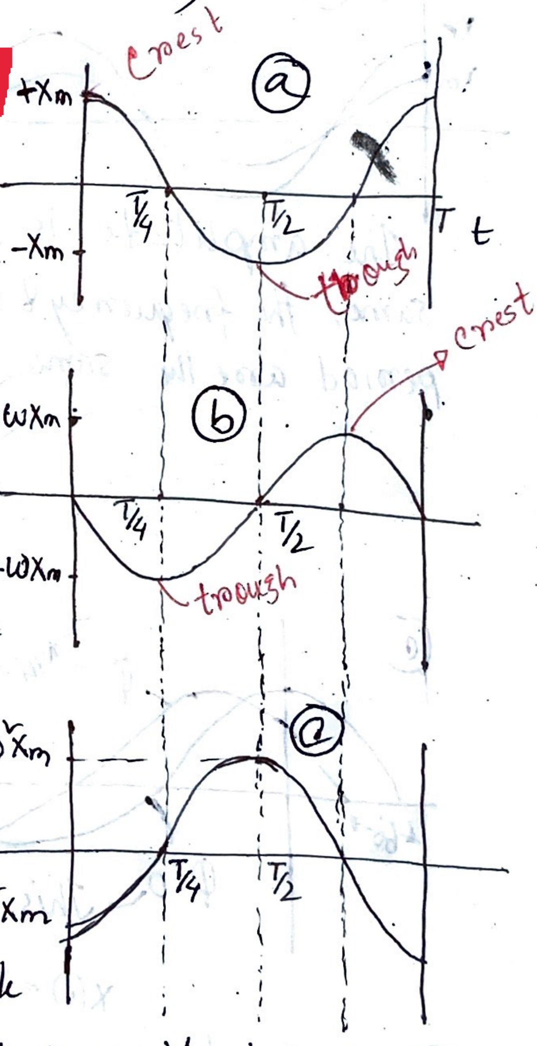


Figure (a) is a plot of equation (1) with $\phi = 0$. Fig (b) shows equation (11) with $\phi = 0$. Analogous to the amplitude X_m in equation (1), ωX_m is called the velocity amplitude. The velocity of oscillating particle varies between the limits $\pm V_m = \pm \omega X_m$.

The curve of $V(t)$ is shifted (to the left) from the curve of $x(t)$ by one-quarter period. When the magnitude of displacement is greatest ($x(t) = X_m$), the magnitude of the velocity is least ($V(t) = 0$). - vise versa.

~~The~~ acceleration of S.H.M

$$a(t) = \frac{dv(t)}{dt} = \frac{d}{dt}\left(-\omega x_m \sin(\omega t + \phi)\right)$$

$$\boxed{a(t) = -\omega^2 x_m \cos(\omega t + \phi) \quad \text{---} \; (III)}$$

Equation (III) shows is plotted in Fig (c). for the case $\phi = 0$.
The quantity $\omega^2 x_m$ is called acceleration amplitude a_m.
The acceleration curve $a(t)$ is shifted to the left by $T/4$ relative to the velocity curve. $v(t)$.

$$a(t) = -\omega^2 x(t)$$

This is a hallmark of Simple Harmonic motion.

Check point 1, page 389 A particle undergoing simple harmonic oscillation of period T is at $-X_m$ at time $t = 0$. Is it at $-X_m$, at $+X_m$, at 0, between $-X_m$ and 0 or between 0 and $+X_m$ when (a) $t = 2.00T$, (b) $t = 3.50T$ (c) $t = 5.25T$

$-X_m$ $\quad$ $+X_m$ $\quad$ 0

Check point 2: Which of the following relationships between the force F on a particle and the particle's position x implies simple harmonic oscillation (a) $F = -5x$ (b) $F = -400x^2$ (c) $F = 10x$ (d) $F = 3x^2$. $\boxed{a}$

Assignment: A block whose mass is ~~680~~ 100 g is fastened to spring whose spring constant K is 10 N/m. The block is pulled a distance $x = 10$ cm from its equilibrium position at $x = 0$ on a frictionless surface and released from rest at $x = 0$.

(A) What are the angular frequency, the frequency and the period of the resulting motion

(B) What is the amplitude of the oscillation?

(C) What is the maximum speed V_m of the oscillating block, and where is the block when it has this speed

(d) What is the magnitude a_m of the maximum acceleration of the block?