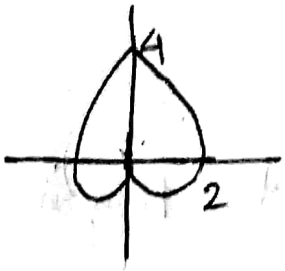
Chapter 10.3

29. Area of the region that is enclosed by the cardoid $r = 2 + 2\sin\theta$



$$A = \int_0^{2\pi} \frac{1}{2} r^2 d\theta$$

$$= \int_0^{2\pi} \frac{1}{2} (2 + 2\sin\theta)^2 d\theta$$

$$= \int_0^{2\pi} (2 + 2\sin\theta + 2\sin^2\theta) d\theta$$

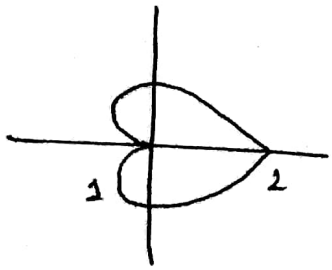
$$= \int_0^{2\pi} (2 + 2\sin\theta + 1 - \cos 2\theta) d\theta$$

$$= \left[2\theta - 2\cos\theta + \theta - \frac{1}{2}\sin 2\theta\right]_0^{2\pi}$$

$$= 6\pi$$

30. Area of the region in the first quadrant within the cardoid $r = 1 + \cos\theta$

$$A = \int_0^{\pi/2} \frac{1}{2} r^2 d\theta = \int_0^{\pi/2} \frac{1}{2} (1 + \cos\theta)^2 d\theta$$



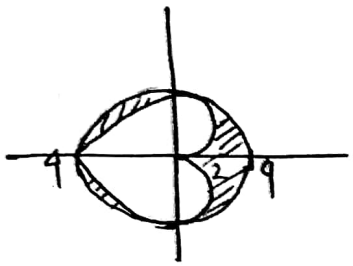
$$= \int_0^{\pi/2} \frac{1}{2} (1 + 2\cos\theta + \cos^2\theta) d\theta$$

$$= \int_0^{\pi/2} \left(\frac{1}{2} + \cos\theta + \frac{1}{4}\cos 2\theta + \frac{1}{4}\right) d\theta$$

$$= \left[\frac{1}{2}\theta + \frac{1}{4}\theta + \sin\theta + \frac{1}{8}\sin 2\theta\right]_0^{\pi/2}$$

$$= \frac{3\pi}{8} + 1$$

40. Area of the region outside the cardoid $r = 2 - 2\cos\theta$ and inside the circle $r = 4$



$$A = 2\int_0^{\pi} \frac{1}{2}\left[4^2 - (2-2\cos\theta)^2\right] d\theta$$

$$= 2\int_0^{\pi} \frac{1}{2}\left(16 - 4 + 8\cos\theta - 4\cos^2\theta\right) d\theta$$

$$= \int_0^{\pi} \left(12 + 8\cos\theta - 2\cos 2\theta - 2\right) d\theta$$

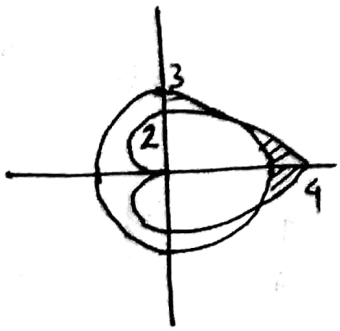
$$= \left[10\theta + 8\sin\theta - \sin 2\theta\right]_0^{\pi}$$

$4 = 2 - 2\cos\theta$

$\Rightarrow \cos\theta = -1$

$\Rightarrow \theta = \pi$

11. Area of the region inside the cardoid $r = 2+2\cos\theta$ and outside $r = 3$



$$3 = 2 + 2\cos\theta$$

$$\Rightarrow 2\cos\theta = 1$$

$$\Rightarrow \cos\theta = \frac{1}{2}$$

$$\Rightarrow \theta = \frac{\pi}{3}$$

$$A = 2\int_0^{\pi/3} \frac{1}{2}\left[(2+2\cos\theta)^2 - 3^2\right] d\theta$$

$$= \int_0^{\pi/3} \left(4 + 8\cos\theta + 4\cos^2\theta - 9\right) d\theta$$

$$= \int_0^{\pi/3} \left(-5 + 8\cos\theta + 2\cos 2\theta + 2\right) d\theta$$

$$= \left[-3\theta + 8\sin\theta + \sin 2\theta\right]_0^{\pi/3}$$

$$= \frac{9\sqrt{3}}{2} - \pi$$