Chapter 7.8

7. $\int_e^{+\infty} \frac{1}{x\ln^3 x}\,dx$

$= \lim_{k\to+\infty} \int_e^k \frac{1}{x\ln^3 x}\,dx$

Put, $u = \ln x \Rightarrow du = \frac{1}{x}dx$

$\int u^{-3}\,du = -\frac{u^{-2}}{2} = -\frac{1}{2u}$

$= \lim_{k\to+\infty} \left[-\frac{1}{2\ln^2 x}\right]_e^k$

$= \lim_{k\to+\infty} \left[-\frac{1}{2\ln^2 k} + \frac{1}{2}\right] = \frac{1}{2}$

8. $\int_2^{+\infty} \frac{1}{x\sqrt{\ln x}}\,dx$

$= \lim_{k\to+\infty} \int_2^k \frac{1}{x\sqrt{\ln x}}\,dx$

$= \lim_{k\to+\infty} \left[2\sqrt{\ln x}\right]_2^k$

$= \lim_{k\to+\infty} \left(2\sqrt{\ln k} - 2\sqrt{\ln 2}\right) = +\infty$, divergent

9. $\int_{-\infty}^{0} \frac{dx}{(2x-1)^3}$

$= \lim_{k \to -\infty} \left[-\frac{1}{4(2x-1)^2} \right]_k^0 = \lim_{k \to -\infty} \frac{1}{4}\left[-1 + 1/(2k-1)^2\right] = -\frac{1}{4}$

10. $\int_{-\infty}^{3} \frac{dx}{x^2+9}$

$= \lim_{k \to -\infty} \left[\frac{1}{3} \tan^{-1} \frac{x}{3} \right]_{-\infty k}^{3} = \frac{\pi}{4}$

11. $\int_{-\infty}^{0} e^{3x}\, dx$

$= \lim_{k \to -\infty} \left[\frac{e^{3x}}{3} \right]_k^0 = \frac{1}{3}$

12. $\int_{-\infty}^{0} \frac{e^x dx}{3-2e^x}$

$= \lim_{k \to -\infty} \left[-\frac{1}{2} \ln(3-2e^x) \right]_k^0 = \frac{1}{2} \ln 3$

13. $\int_{-\infty}^{+\infty} x\, dx$

$= \int_{0}^{+\infty} x\, dx + \int_{-\infty}^{0} x\, dx$

$\int_0^{+\infty} x\, dx = \lim_{k \to +\infty} \left[\frac{3}{2} x^2 \right]_0^k = \lim_{k \to +\infty} \frac{1}{2} k^2 = +\infty$

So, $\int_{-\infty}^{+\infty} x dx$ is divergent

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14. $\displaystyle\int_{-\infty}^{+\infty} \frac{x}{\sqrt{x^2+2}}\,dx$

$$= \int_{0}^{+\infty} \frac{x}{\sqrt{x^2+2}}\,dx + \int_{-\infty}^{0} \frac{x}{\sqrt{x^2+2}}\,dx$$

$$\int_{0}^{+\infty} \frac{x}{\sqrt{x^2+2}}\,dx = \lim_{k\to+\infty} \left[\sqrt{x^2+2}\right]_0^k = +\infty$$

So, $\displaystyle\int_{-\infty}^{+\infty} \frac{x}{\sqrt{x^2+2}}\,dx$ is divergent

15. $\displaystyle\int_{-\infty}^{+\infty} \frac{x}{(x^2+3)^2}\,dx$

$$= \int_{0}^{+\infty} \frac{x}{(x^2+3)^2}\,dx + \int_{-\infty}^{0} \frac{x}{(x^2+3)^2}\,dx$$

$$\int_{0}^{+\infty} \frac{x}{(x^2+3)^2}\,dx = \lim_{k\to+\infty} \left[-\frac{1}{2(x^2+3)}\right]_0^k = \frac{1}{6}$$

$$\int_{-\infty}^{0} \frac{x}{(x^2+3)^2}\,dx = \lim_{k\to-\infty} \left[\frac{-1}{2(x^2+3)}\right]_k^0 = -\frac{1}{6}$$

So, $\displaystyle\int_{-\infty}^{+\infty} \frac{2x}{(x^2+3)^2} = \frac{1}{6} + \left(-\frac{1}{6}\right) = 0$

16. $\int_{-\infty}^{+\infty} \frac{e^{-t}}{1+e^{-2t}} dt$

$= \int_{0}^{+\infty} \frac{e^{-t}}{1+e^{-2t}} dt + \int_{-\infty}^{0} \frac{e^{-t}}{1+e^{-2t}} dt$

$\int_{0}^{+\infty} \frac{e^{-t}}{1+e^{-2t}} dt = \lim_{k \to +\infty} \left[-\tan^{-1}(e^{-t})\right]_0^k = \frac{\pi}{4}$

$\int_{-\infty}^{0} \frac{e^{-t}}{1+e^{-2t}} dt = \lim_{k \to -\infty} \left[-\tan(e^{-t})\right]_k^0 = \lim_{k \to -\infty} \left[-\frac{\pi}{4} + \tan^{-1}(e^{-k})\right] = \frac{\pi}{4}$

So, $\int_{-\infty}^{+\infty} \frac{e^{-t}}{1+e^{-2t}} dt = \frac{\pi}{4} + \frac{\pi}{4} = \frac{\pi}{2}$

17. $\int_0^4 \frac{dx}{(x-4)^2}$

$= \lim_{k \to 4^-} \left[-\frac{1}{x-4}\right]_0^k = +\infty$, divergent

18. $\int_0^8 \frac{dx}{\sqrt[3]{x}}$

$= \lim_{k \to 0^+} \left[\frac{3}{2} x^{2/3}\right]_k^8 = 6$

19. $\int_0^{\pi/2} \tan x \, dx$

$= \lim_{k \to \frac{\pi}{2}^-} \left[-\ln(\cos x)\right]_0^k = +\infty$, divergent

20. $\displaystyle\int_0^4 \frac{dx}{\sqrt{4-x}}$

$$= \lim_{k \to 4^-} \left[-2\sqrt{4-x} \right]_0^k = 4$$

21. $\displaystyle\int_0^1 \frac{dx}{\sqrt{1-x^2}}$

$$= \lim_{k \to 1^-} \left[\sin^{-1} x \right]_0^k = \frac{\pi}{2}$$

22. $\displaystyle\int_{-3}^1 \frac{x\,dx}{\sqrt{9-x^2}}$

$$= \lim_{k \to -3^+} \left[-\sqrt{9-x^2} \right]_k^1 = \sqrt{8}$$

23. $\displaystyle\int_{\pi/3}^{\pi/2} \frac{\sin x}{\sqrt{1-2\cos x}}\,dx$

$$= \lim_{k \to \frac{\pi}{3}^+} \left[\sqrt{1-2\cos x} \right]_k^{\pi/2} = 1$$

24. $\displaystyle\int_0^{\pi/4} \frac{\sec^2 x}{1-\tan x}\,dx$

$$= \lim_{k \to \frac{\pi}{4}^-} \left[-\ln(1-\tan x) \right]_0^k = +\infty$$, divergent

25. $\int_0^3 \frac{dx}{x-2}$

$\int_0^2 \frac{dx}{x-2} = \lim_{x\to 2^-} \Big[\ln|x-2|\Big]_0^k = -\infty$

So, $\int_0^3 \frac{dx}{x-2}$ is divergent

26. $\int_{-2}^2 \frac{dx}{x^2}$

$= \int_0^2 \frac{dx}{x^2} + \int_{-2}^0 \frac{dx}{x^2}$

$\int_0^2 \frac{dx}{x^2} = \lim_{k\to 0^+} \Big[-\frac{1}{x}\Big]_k^2 = +\infty$, So $\int_{-2}^2 \frac{dx}{x^2}$ is divergent

27. $\int_{-1}^8 x^{-1/3}\,dx$

$= \int_0^8 x^{-1/3}\,dx + \int_{-1}^0 x^{-1/3}\,dx$

$\int_0^8 x^{-1/3}\,dx = \lim_{k\to 0^+} \Big[\frac{3}{2} x^{2/3}\Big]_k^8 = 6$

$\int_{-1}^0 x^{-1/3}\,dx = \lim_{k\to 0^-} \Big[\frac{3}{2} x^{2/3}\Big]_{-1}^k = -\frac{3}{2}$

So, $\int_{-1}^8 x^{-1/3} = 6 + \left(-\frac{3}{2}\right) = \frac{9}{2}$

28. $\int_0^1 \frac{dx}{(x-1)^{2/3}}$

$= \lim_{k\to 1^-} \left[3(x-1)^{1/3}\right]_0^k = 3$

29. $\int_0^{+\infty} \frac{1}{x^2}\,dx$

$= \int_0^a \frac{1}{x^2}\,dx + \int_a^{+\infty} \frac{1}{x^2}\,dx$, where $a>0$, take $a=1$ for convenience

$\int_0^1 \frac{1}{x^2}\,dx = \lim_{l\to 0^+} \left[-\frac{1}{x}\right]_l^1 = +\infty$

So, $\int_0^{+\infty} \frac{1}{x^2}\,dx$ is divergent

30. $\int_1^{+\infty} \frac{dx}{x\sqrt{x^2-1}}$

$= \int_2^{+\infty} \frac{dx}{x\sqrt{x^2-1}} + \int_1^2 \frac{dx}{x\sqrt{x^2-1}}$

$\int_1^2 \frac{dx}{x\sqrt{x^2-1}} = \lim_{k\to 1^+} \left[\sec^{-1} x\right]_k^2 = \frac{\pi}{3}$

$\int_2^{+\infty} \frac{dx}{x\sqrt{x^2-1}} = \lim_{k\to +\infty} \left[\sec^{-1} x\right]_2^k = \frac{\pi}{2} - \frac{\pi}{3}$

So, $\int_1^{+\infty} \frac{dx}{x\sqrt{x^2-1}} = \frac{\pi}{2} - \frac{\pi}{3} + \frac{\pi}{3} = \frac{\pi}{2}$