Thursday, September 5, 2024 7:11 PM

- Create mathematical model of robot using differential equations.

- Represent the differential equation is state space form.

$$\dot{x} = Ax + Bu \qquad y = Cx + Du$$

x is called the state vector

u is called the control vector

A is called dynamics matrix

B is called the control matrix

Last class, I showed how to obtain state space representation of simple car.

Thursday, September 5, 2024 7:51 PM

Aside: To solve differential equations you must either,

1. initial conditions
2. forcing function (input, control)
3. both 1 & 2

Example 1: $\ddot{x} + \dot{x} + 6x = 0$, $\underbrace{x(0) = 1, \ \dot{x}(0) = 2}_{\text{initial condition}}$

Example 2: $\ddot{x} + 3\dot{x} + 4x = u$, $x(0) = $

Example 1: $\ddot{x} + \dot{x} + 6x = 0$, $\underbrace{x(0) = 1, \quad \dot{x}(0) = 2}_{\text{initial condition}}$

Example 2: $\ddot{x} + 3\dot{x} + 4x = u$, $x(0) = 2$, $\dot{x}(0) = 3$

u is called the input or control

To solve example 2, I have to give you IC & u.

Thursday, September 5, 2024 7:45 PM

Example: You are given the differential equations of a system: $\dddot{x} + 6\ddot{x} + 11\dot{x} + 6x = u$

Express this in state space form.

Solution: State vector x is 3×1 because 3^{rd} order equation

Define three variables,

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix},$$ u is the control/input (single variable in this case)

$$\dddot{x} = -6\ddot{x} - 11\dot{x} - 6x + u$$

$$= -6x_3 - 11x_2 - 6x_1 + u$$

Thursday, September 5, 2024 8:07 PM

Set $x_1 = x$, $x_2 = \dot{x}$, $x_3 = \ddot{x}$

$\dot{x}_1 = \dot{x} = x_2$

$\dot{x}_2 = \ddot{x} = x_3$

$\dot{x} = Ax + Bu$

$$\dot{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix}}_{A} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}}_{B} u$$

$x_1 = x$

$x_2 = \dot{x}$

$x_3 = \ddot{x}$

$\dot{x}_1 = \dot{x} = x_2$

$\dot{x}_3 = \dddot{x}$

$$\dddot{x} + 6\ddot{x} + 11\dot{x} + 6x = u$$

$$\Rightarrow \dddot{x} = -6\ddot{x} - 11\dot{x} - 6x + u$$

$$\Rightarrow \dot{x}_3 = -6x_3 - 11x_2 - 6x_1 + u$$

$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ state vector

Say you want $y = x_1$

$$\dddot{x} + 6\ddot{x} + 11\dot{x} + 6x = u \qquad x_3 = \ddot{x}$$

$$\Rightarrow \dddot{x} = -6\ddot{x} - 11\dot{x} - 6x + u$$

$$\Rightarrow \dot{x}_3 = -6x_3 - 11x_2 - 6x_1 + u$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \text{ state vector}$$

Say you want $y = x_2$ [y is called the output]

$$y = \underbrace{[0 \;\; 1 \;\; 0]}_{C} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \underbrace{[0]}_{D} u$$

$$\dot{x} = Ax + Bu$$

$$y = Cx + Du$$

Say want

$$y = x_2 + x_3$$

$$y = \underbrace{[0 \;\; 1 \;\; 1]}_{C} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + [0]\, u$$

S...

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$$\underbrace{[\,0 \;\; 1 \;\; 0\,]}_{C} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \underbrace{[\,0\,]}_{D} u$$

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$

Say want

$$y = x_2 + x_3$$

$$y = \underbrace{[\,0 \;\; 1 \;\; 1\,]}_{C} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + [\,0\,] u$$

Say you want, $y = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}_{2\times 1}$

$$y = \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}_{2\times 3}}_{C} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}_{3\times 1} + \begin{bmatrix} 0 \\ 0 \end{bmatrix}_{2\times 1} u$$

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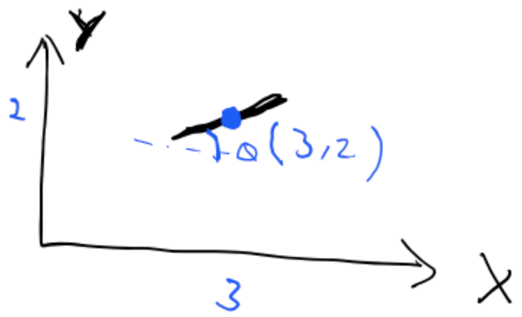
Thursday, September 5, 2024 8:36 PM



θ is measured with respect to x-axis

Unicycle robot :

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 \\ \sin\theta & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v \\ w \end{bmatrix}$$