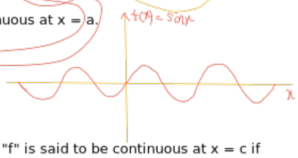
$y = f(x)$

$f(a) = \lim_{x \to a} f(x)$

f is continuous at x = a.




A function "f" is said to be continuous at x = c if

1. f(c) is defined
2. $\lim_{x \to c} f(x)$ exists
3. $f(c) = \lim_{x \to c} f(x)$

$(x+2)$ $\qquad f(x) = \dfrac{x^2 - 4}{x - 2}$

Is f continuous at x = 2?

$f(2) = \dfrac{4-4}{2-2} = \dfrac{0}{0} \to$ undefined

$f(2) \overset{?}{=} \lim_{x \to 2} f(x)$

f is not continuous at $x = 2$.

$g(2)$
$\lim_{x \to 2} g(x)$

$$g(x) = \begin{cases} \dfrac{x^2-4}{x-2} & ; \ x \neq 2 \\ 4 & ; \ x = 2 \end{cases}$$

Is "g" continuous at x = 2?

$g(2) = 4$

$\lim_{x \to 2} g(x) = \lim_{x \to 2} \dfrac{x^2 - 4}{x - 2} = \lim_{x \to 2} \dfrac{(x+2)(x-2)}{x-2}$

$= \lim_{x \to 2} x + 2 = 2 + 2 = 4$

So, "g" is continuous at x = 2.

[2,3] $\qquad$ (2,3) $\qquad$ (2,5)

(5, 8) $\qquad$ 5.27

Continuity on an interval.

$\boxed{c} \in (5,8)$ $\qquad$ 6 $\qquad$ 6.7

if "f" is continuous at every point of the interval (a,b), then it is continuous on the interval (a,b).

Let "c" be an arbitrary point in the interval (a,b), that is, c∈ (a,b).

If "f" is continuous at the arbitrary point "x=c", then "f" is continuous on the interval (a,b).

Investigate whether the function $f(x) = \sqrt{9 - x^2}$ is continuous on the interval (-3,3).

Let "c" be an arbitrary point in the interval (-3,3). Then

$f(c) = \sqrt{9 - c^2}$

$\lim_{x \to c} f(x) = \lim_{x \to c} \sqrt{9 - x^2} = \sqrt{9 - c^2}$

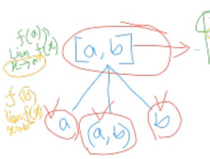
So, "f" is continuous at x = c.

Hence, the function "f" is continuous on the interval (-3,3).

Continuity on a closed interval:

$[a,b] \to a \leq x \leq b$

$\boxed{(a,b) \to a < x < b}$





Investigate whether the function $f(x) = x^3 + x^2 - 2x - 1$ is continuous on the interval [-1,1]

We first check the continuity on the open interval (-1,1).

$x = -1$ $\qquad \boxed{(-1,1)}$ $\qquad x = 1$

Let c be an arbitrary point in the interval (-1,1).

$f(c) = c^3 + c^2 - 2c - 1$

$\lim_{x \to c} f(x) = \lim_{x \to c} x^3 + x^2 - 2x - 1 = c^3 + c^2 - 2c - 1$

So, f is continuous on (-1,1).

For $x = -1$, $\qquad f(-1) = (-1)^3 + (-1)^2 - 2(-1) - 1$
$= 1$

$\lim_{x \to -1^+} f(x) = \lim_{x \to -1^+} x^3 + x^2 - 2x - 1$
$= (-1)^3 + (-1)^2 - 2(-1)$
$= 1$

$x \to a^+$
$x \to b^-$

So, $f(-1) = \lim_{x \to -1^+} f(x)$

Hence, f is continuous from the right at x = -1.

For $x = 1$: $\qquad f(1) = 1^3 + 1^2 - 2(1) - 1 = -1$

$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} (x^3 + x^2 - 2x - 1)$

$= 1^3 + 1^2 - 2 - 1 = -1$

So, $\quad f(1) = \lim_{x \to 1^-} f(x)$

Hence, f is continuous from the left at x = 1.
Therefore, f is continuous on the interval [-1, 1].