We know,

$$\binom{n}{r} = \frac{n!}{r! \times (n-r)!}, \quad 1 < r < n \text{.....} (i)$$

We have to prove,

$$\binom{n}{r} = \binom{n-1}{r-1} + \binom{n-1}{r}, \quad 1 < r < n$$

Let,

$$\binom{n}{r} = \binom{n-1}{r-1} + \binom{n-1}{r} \text{........} (ii)$$

From (i),

$$\binom{n}{r} = \frac{n!}{r! \times (n-r)!}$$

$$\Rightarrow \binom{n+1}{r} = \frac{(n+1)!}{r! \times (n+1-r)!}$$

$$= \frac{(n+1)\, n!}{r! \times (n-r+1)(n-r)!}$$

$$= \frac{(n+1)}{(n-r+1)} \times \frac{n!}{r!\,(n-r)!}$$

$$= \frac{(n+1)}{(n-r+1)} \times \binom{n}{r}$$

$$= \frac{(n+1)}{(n-r+1)} \times \left[\binom{n-1}{r-1} + \binom{n-1}{r} \right] \text{ [From (ii)]}$$

$$= \frac{(n+1)}{(n-r+1)} \times \left[\frac{(n-1)!}{(r-1)!\,(n-r-2)!} + \frac{(n-1)!}{r!\,(n-r-1)!} \right]$$

$$= \frac{(n+1).n.(n-1)!}{(r-1)! \times (n+1-r)(n-r)(n-1-r)(n-2-r)!} +$$

$$\frac{(n+1).n.(n-1)!}{r! \times (n+1-r)(n-r)(n-1-r)!}$$

$$= \binom{n-1}{r-1} + \binom{n-1}{r} ; \quad 1 < r < n$$

$$\therefore \binom{n}{r} = \binom{n-1}{r-1} + \binom{n-1}{r} ; \quad 1 < r < n$$

(proved).