Show that $f(x)=|x|$ is continuous everywhere.

We write.

$$f(x)=|x| = \begin{cases} -x & ; x<0 \to (-\infty,0) \\ 0 & ; x=0 \\ x & ; x>0 \to (0,\infty) \end{cases}$$

$x=-7$

$|x| = -x$

$= -(-7) = 7$

$y = f(x) = |x|$

$(-\infty, \infty)$

$$|x| = \begin{cases} -x & \text{if } x<0 \\ 0 & \text{if } x=0 \\ x & \text{if } x>0 \end{cases}$$

$f(x) =$ $x=-3$

$|x| = |-3| = 3$ $\quad -(-3) = 3$

$\lim_{x\to 0^-}$

$\lim_{x\to 0^+}$

For $x<0$, i.e. $x \in (-\infty, 0)$

Let "c" an arbitrary point in the interval $(-\infty,0)$.

$f(c) = -c$ $\qquad \lim_{x\to c} f(x) = \lim_{x\to c} (-x) = -c$

So, f is continuous on the interval $(-\infty,0)$

For $x=0$, $\quad f(0) = 0$

$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} x = 0$

$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} (-x) = 0$

f is continuous at x = 0.

For $x>0$, i.e. $x \in (0,\infty)$

Let "a" be an arbitrary point in the interval $(0, \infty)$

$f(a) = a$ $\qquad \lim_{x\to a} f(x) = \lim_{x\to a} x = a$

So, f is continuous on the interval $(0,\infty)$

Therefore, f is continuous everywhere.

$[a,b]$

$(-\infty,\infty)$

$$f(x) = \begin{cases} 2x & ; x<2 \to (-\infty,2) \\ x^2+2 & ; 2\le x\le 5 \to [2,5] \\ 5 & ; x>5 \to (5,\infty) \end{cases}$$

$x=2 \quad (2,5) \quad x=5$

$$f(x) = \frac{x^2-9}{x^2-5x+6}$$

$x^2-5x+6=0$

$\Rightarrow x^2-3x-2x+6=0$

$\Rightarrow (x-3)(x-2)=0$

$\Rightarrow x=3,2$

f is discontinuous at x = 3 and x = 2.

$$f(x) = \begin{cases} 2x+3 & ; x\le 4 \\ 7+\frac{16}{x} & ; x>4 \end{cases}$$

$x=4$

$f(4) = 2\cdot 4+3 = 11$

$\lim_{x\to 4^+} f(x) = \lim_{x\to 4^+} \left(7+\frac{16}{x}\right) = 7+\frac{16}{4}$

$= 7+4 = 11$

$\lim_{x\to 4^-} f(x) = \lim_{x\to 4^-} 2x+3 = 2\cdot 4+3 = 11$

$f(x) = \sqrt{x} \to [0,\infty)$

$[-2,-1]$

$$f(x) = \frac{x^2+6x+9}{|x|+3} = \begin{cases} \frac{x^2+6x+9}{-x+3} & ; x<0 \\ \frac{x^2+6x+9}{3} & ; x=0 \\ \frac{x^2+6x+9}{x+3} & ; x>0 \end{cases}$$

$|x|+3=0 \to |x|=-3!$

$y=f(x)$ is continuous $[a,b]$

$f(x)=k$

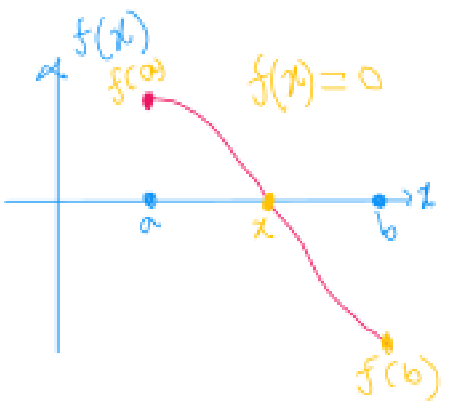
$a<x<b$

$y=f(x) \to [a,b]$

$f(a), f(b) \ne 0$

opposite signs

$a<x<b$ s.t. $f(x)=0$



$x^3+x^2-2x-1=0 \to [-1,1]$

$f(x) = x^3+x^2-2x-1 \to [-1,1]$

$f(-1) = (-1)^3+(-1)^2-2(-1)-1 = 1$

$f(1) = 1+1-2-1 = -1$

By intermediate value theorem, we find that there is at least one root of f(x) = 0 in the interval [-1,1].