Hooke's Law and the Simple Harmonic Motion

OBJECTIVE:

1. To determine the spring constant by studying the simple harmonic motion of an oscillating mass.
2. To determine the mass of the spring.

APPARATUS:

Stand with a clamp, a set of slotted masses, meter stick, geometry box, stop watch, digital balance.

THEORY:

Hooke's Law

A spring with a varying amounts of force applied is proportional to its displacement. This proportionality constant is called the spring constant, k, and the entire relation is referred to as Hooke's Law:

$$F = -kX. \tag{1}$$

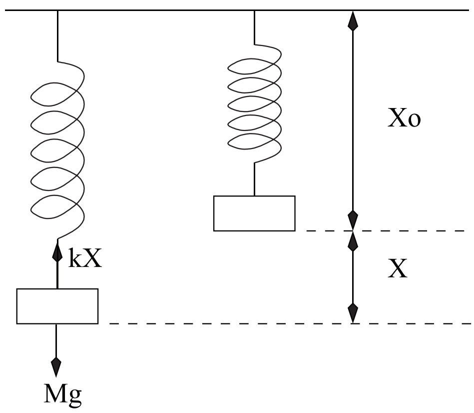

Figure 1: Hooke's Law Setup.

The minus sign is only used to describe the "restoring" force. This force opposes the applied force on the spring and tries to return the spring to equilibrium. In figure 1, X_o is the length of the spring with the mass holder hanging at rest at the equilibrium point. The displacement, X, is measured relative to the equilibrium point. Hooke's Law becomes invalid only when the elastic limit of the spring is surpassed.

When a mass of an ideal or friction free spring is displaced its oscillation is known as simple harmonic motion. If this motion is graphed it would look sine-like with its amplitude remaining constant. Taking Hooke's Law into account with Newton's Second Law we get $F = -kX = Ma$ and then rewrite as

$$a + (k/M)X = 0. \tag{2}$$

This is the equation for simple harmonic motion. The corresponding position *(i.e.* the solution) as a function of time is

$$X = A\cos(\omega t + \phi), \tag{3}$$

where $\omega = \sqrt{k/M}$. Since ω is the angular frequency, we have $\omega = 2\pi f = 2\pi/T$. Therefore the period, T, can be written as follows

$$T = 2\pi/\omega = 2\pi\sqrt{\frac{M}{k}}. \tag{4}$$

The mass M in equation-(4) is equal to the mass added including the holder M_a plus a contribution from the mass of the spring (m), that is, $M = M_a + m$). In this experiment, the dependence of period upon the mass is investigated. It can be shown that m equals 1/3 of the spring mass M_s.

PROCEDURE:

1. The apparatus should look similar to Figure-2.

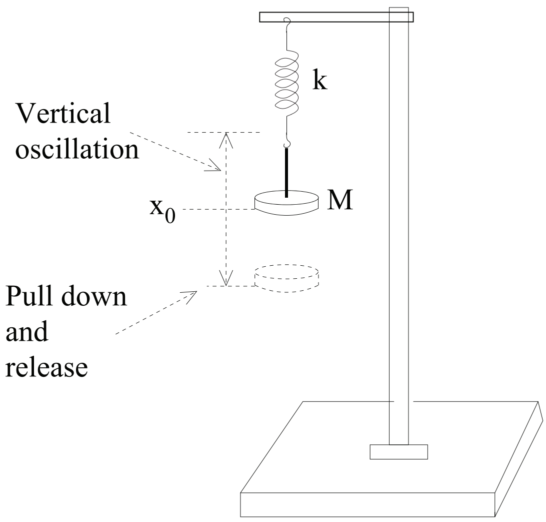

Figure 2: The Experimental Setup

2. Start with enough mass on the holder to separate the coils of the spring. The initial mass should be about 0.200 kg.

3. Pull down the mass holder by a relatively small amount and then release it - do not apply any external forces. The mass should be oscillating straight up and down. If not, stop it and try again until you get vertical oscillation.

4. With the digital stop watch count the time for 20 cycles for each mass added three times and then take the average. Note: All of your time values should have to the same first two digits - otherwise your experimental setup might have some flaws. Record your results in Table 1.

5. To gather the best results one has to be very careful and consistent. The base of the apparatus should be firmly held and not allowed to move. Any motion of the apparatus outside the spring and the holder will increase the error in your results.

6. Construct a graph of Period^2 vs. Mass Added. The scale on the x axis should start at zero.

Calculations:

1. Measure the actual mass of the spring (M_s) by using the digital balance and record it under the table-2.

2. Rewriting from Eq.(4) by using $M = M_a + m$, we obtain,

$$T^2 = \left(\frac{4\pi^2}{k}\right) M_a + \frac{4\pi^2 m}{k} \ .$$

   So T^2 vs. M_a graph is straight line with

$$\text{Slope} = \frac{4\pi^2}{k} \qquad \text{and} \qquad \text{Intercept} = \frac{4\pi^2 m}{k} \ .$$

3. From the graph, find the slope of the best fit line. Use the slope to calculate the spring constant k and record Table-3.

4. From the graph, find the y-intercept, and use this and the spring constant just computed in the previous step to calculate m which is the contribution of the mass of the spring and record Table-2.

5. Compute the spring mass from the data in the previous steps and then compute the percent error.

Lab Report

Name of the Experiment	:	Determination of **Spring Constant** and **Effective Mass** of a spring from simple harmonic motion
Your Name	:	
Your ID #	:	
Section	:	
Namer of the lab Partner	:	
Date	:	

Table 1. Period Dependence on Mass

<table>
<tr><th>Mass Added to Holder (kg)</th><th>Time for 15 Oscillations (sec)</th><th>Time Period (T) (sec)</th><th>Average Time Period (T_{av}) (sec)</th><th>Time Period^2 (T^2) (sec^2)</th></tr>
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Detail Calculation

Calculation for Spring Constant:

Calculation for Effective Mass:

Table 2. Calculation spring constant and mass

Mass of spring by digital balance, M_s	kg
Slope of the best fit line	
y-intercept of the graph	
Spring constant, k	N/m
Mass contribution from the spring, m	kg
Experimental value of Mass of the spring, $M_{\mathrm{s,exp}}$	kg
Percent error in the mass of spring	

Result:

QUESTIONS:

1. What would be value of the y-intercept if the spring were a ideal massless spring? Explain.

2. How does your calculated and measured values of the spring mass compare? What might be source of the differences (if any)

3. How the value of the spring constant compare with value found in a previous experiment?

Guideline for plotting graph

0. Use pencil to plot graph.
1. Name it at the botton of the graph. (e.g.: *Y vs X graph*)
2. Label the dimension with proper unit of each axis. (e.g.: Mass(kg), Time Period (sec)).
3. Use full graph paper with maximum area to plot the graph
4. Choose origin properly.
5. Scale the axes before plotting the points.
6. Do not use data points to calculate slope. Use any arbitrary two points.
7. Mark/highlight the data point and slope point with different symbol.
8. If necessary, indicate the Y intersection point clearly.
9. Show the calculation of slope at one isolated corner of your graph.
10. Use ruler to draw straight line
11. Learn to draw “Best-fit line”
12. Do not forget to write your name+id no. in the graph.

Prelab:

1. An object oscillates from a massless vertical spring with a frequency of 4.00 Hz. Hanging at rest from this spring, how much would the object stretch the spring?

2. A 100 grams hanging from a spring oscillates vertically with time period $T = 1.5\,\text{sec}$ on the surface of earth. On the moon surface what would be the time period (same spring and same mass). The value of g on moon surface is 1/6-th times on earth.