Expt-1: Introduction to Measurement and Statistical Error

Objective:

1. To familiarize the student with random error and bias in laboratory measurements (ruler, vernier caliper).

2. To introduce the concepts of arithmetic mean, standard deviation and probable error.

Apparatus:

Vernier caliper, ruler (meter stick) and a mass block (wooden, metal, etc.).

Theory:

Introduction to Measurement:

The vernier caliper is designed to facilitate the estimation of a fractional part of a scale or ruler. Ten units of the vernier scale have the same length as nine units of the main scale. Each unit on the vernier scale is therefore 1/10 mm smaller than the smallest unit of the main scale. Thus, the line on the vernier which is aligned with a line on the main scale indicates the number of tenths of a millimeter that the index is past the last whole millimeter of the main scale. The index in Figure 1 is located to the right of 3.2 centimeters. Hence, the reading is a little more than 3.2 centimeters, but not as large as 3.3 centimeters. The line indicating the third unit of the vernier scale is directly beneath a line on the main scale. It is the fifth line on the vernier scale that lines up. This tells us that the reading is 3.25 centimeters.

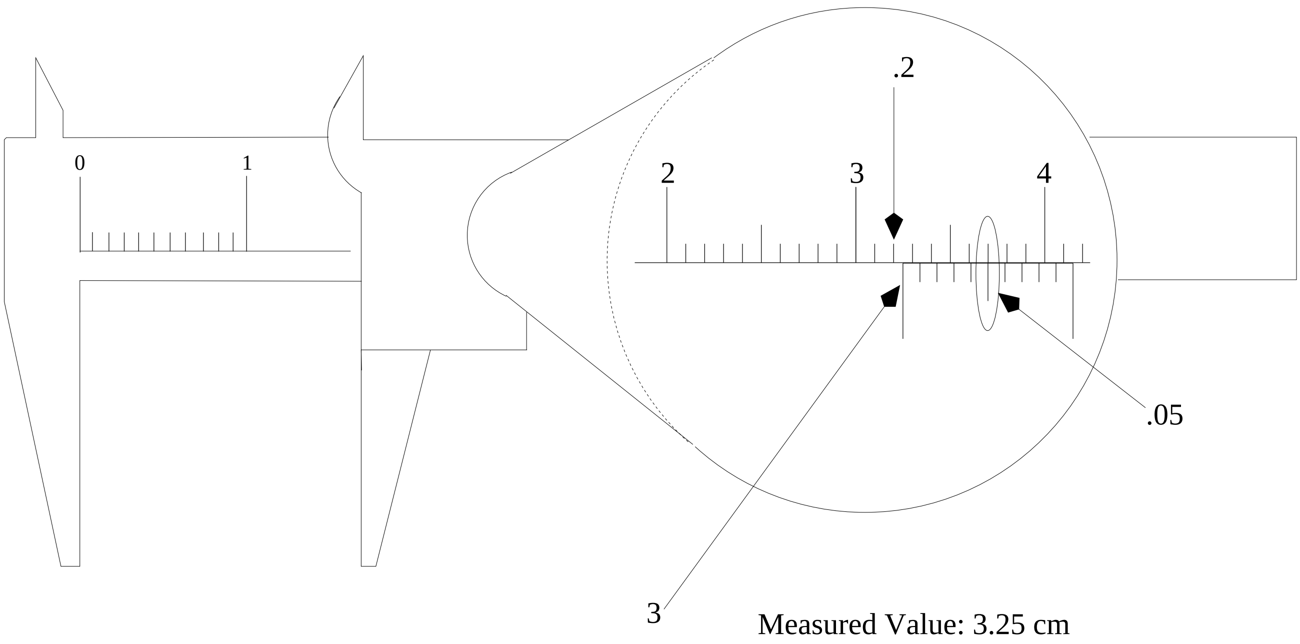


Figure 1: How to Read a Vernier Caliper.

Statistical Error:

We will assume that the errors in the following experiment is due to random error. The distribution of data may be biased or skewed, but we will assume that the data approximates a normal distribution; e.g. one for which the probability of a measurement being greater than the mean value is the same as the probability of a measurement being smaller than the mean. Most of the measurements will be grouped close to the mean value, the so called “normal” distribution, or bell-shaped curve. The distribution of values should be symmetric with respect to the mean.

Suppose X is a physical variable, like mass, length, etc. The measurement of X may involve some error as discussed in the previous paragraph, called the standard deviation σ_X. The standard deviation can be calculated using the method of statistical analysis. Suppose, N number of trials has been taken to measure the value of X. Then the average value (or the mean value) of X and its standard deviation σ_X can be calculated by using the following formula,

$$X_{\text{av}} = \frac{\sum_{i=1}^{N} X_i}{N} , \tag{1}$$

$$\text{and} \quad \sigma_X = \sqrt{\frac{\sum_{i=1}^{N}(X_i - X_{\text{av}})^2}{N-1}} , \tag{2}$$

where $i = 1, 2, \cdots, N$ is the number of trials for the measurement. Therefore, the measured value of X can be written as

$$X = X_{\text{av}} \pm \sigma_X . \tag{3}$$

Propagation of Error

Suppose A and B are two physical quantities with standard deviations σ_A and σ_B respectively. Let F defines a new physical variable that is determined by $F = f(A, B)$. Using statistical analysis, the average value and the standard deviation of F can be calculated as follows:

If $F = f(A, B) = A \pm B$, the average value and the standard deviation are given by

$$F = \overline{A} \pm \overline{B} , \tag{4}$$

$$\text{and} \quad \sigma_F = \sqrt{\sigma_A^2 + \sigma_B^2} . \tag{5}$$

If $F = f(A, B) = AB$, the average value and the standard deviation are given by

$$F = \overline{A}\,\overline{B} , \tag{6}$$

$$\text{and} \quad \sigma_F = |F| \times \sqrt{\left(\frac{\sigma_A}{\overline{A}}\right)^2 + \left(\frac{\sigma_B}{\overline{B}}\right)^2} . \tag{7}$$

Significant Figures

According to the discussion in the previous sections, it is clear that the accuracy of the measurement depends on the number of trials in addition to other factors. The question is how many digits we need to keep in a calculation or measurement. There is no fixed answer, however, more digits means better accuracy. As for example, the numbers 2, 2.0, 2.00, looks same. But 2.00 has better accuracy than 2. This feature is expressed by the notion of significant figures. The rule to find the significant figures in a number is the following: express the number in the scientific form

$$abcd... = a.bcd... \times 10^d ,$$

where a, b, c, d etc are digits (i.e., 0,1,2,3, ...). The number of nonzero digits before the exponential factor is called the significant figure. That is, 2 is a one significant number, 2.0 a two significant number, 2.00 a three significant number, and so on. Similarly 2.05, 0.00375, 9.11×10^{-11} are all three significant numbers.

Procedure:

1. The length and width of any of the six faces of the rectangular block will be measured both by ordinary ruler and vernier ruler. The area of the same face will also be calculated including the error for vernier measurements. To do that follow the instructions below.

2. There are six faces and **choose** any one face (say front face) and the opposite face (say the back face) for your measurement. There will be twelve data values, six for each partner. Three data for the front face and three data for the back face.

3. Using a ruler, measure the length and width of the selected face and record the values in third and fourth columns in Table-1. Use eyeball guessing to approximate the value when necessary.

4. Now using the vernier ruler the same process will be repeated and recorded in Table-2. Note that each data will be calculated using the formula:

$$\text{Data value} = \text{Main scale reading} + \text{Vernier Reading} \times \text{Vernier constant}$$

   Your instruction will show you how to use the Vernier ruler and calculate the data value.

5. Now put the block in between the jaws of the vernier scale to measure the same length you used for ordinary ruler. Read the main scale and vernier scale readings, and record these in the third and fourth columns in Table-2. Compute the total reading using the formula given in the previous step and write these in the fifth column in the same table.

6. Repeat the previous procedure for length measurement two more times and record them in Table-2. Now do the same for three more data value for length of the back face. You partner will repeat the procedure and collect six more data value for length. In total there are twelve data values for the length.

7. Compute the average length $\overline{L}$ and record it in the sixth column in Table-2. Fill-up the seventh column and using Eq.(2), compute the standard deviation of the length measurement (or simply called error in measurement), and write it in the last column in Table-2.

8. Each of you repeat the previous three steps to measure the average width and its error for the same faces (front and back), and fill-up Table-3 accordingly.

Calculations:

1. Now the area of the selected face will be computed. Show the detail calculations in the designated area in page- 7 (below Table-3).

2. First compute the area of the face using the results from Table-1.

3. Using Eq.(6), compute the area of same face using the results from Tables-(2,3).

4. Using Eq.(7), compute the error in the area.

5. Answer the questions in the Questions section.

Lab Report

Name of the Experiment	:
Your Name	:
Your ID #	:
Name of the Lab Partner	:
Date	:

Instructor's comments:

Table 1: Ruler measurements

Face	Data No.	Length, L (cm)	Width, W (cm)	$\overline{L}$ (cm)	$\overline{W}$ (cm)
Front	1				
	2				
	3				
	4				
	5				
	6				
Back	7				
	8				
	9				
	10				
	11				
	12				

Table 2: Vernier ruler measurements for Length

Vernier constant: ________________ cm

Faces	Data No.	Main Scale reading (cm)	Vernier reading	Length (cm)	$\overline{L}$ (cm)	$(\overline{L} - L_i)^2$ (cm^2)	σ_{L} (cm)
Front	1						
	2						
	3						
	4						
	5						
	6						
Back	7						
	8						
	9						
	10						
	11						
	12						

Table 3: Vernier ruler measurements for Width

Vernier constant: ________________cm

Faces	Data No.	Main Scale reading (cm)	Vernier reading	Width (cm)	$\overline{W}$ (cm)	$(\overline{W} - W_i)^2$ (cm^2)	σ_W (cm)
Front	1						
	2						
	3						
	4						
	5						
	6						
Back	7						
	8						
	9						
	10						
	11						
	12						

Calculation for area and its error:

1. Ordinary ruler: Area of the face , $A =$

2. Vernier ruler: Area of the face, $A =$

3. Error in area for vernier ruler measurement, $\sigma_A =$

4. Final result, $A \pm \sigma_A =$

Questions:

1. How many of the length readings lie in the interval $L_{\text{av}} \pm \sigma_L$?

2. What fraction of the 12 readings is this?

3. How does the percentage compare with 68.3 % ?

4. Does the distribution of your data agree with that predicted for a normal distribution?

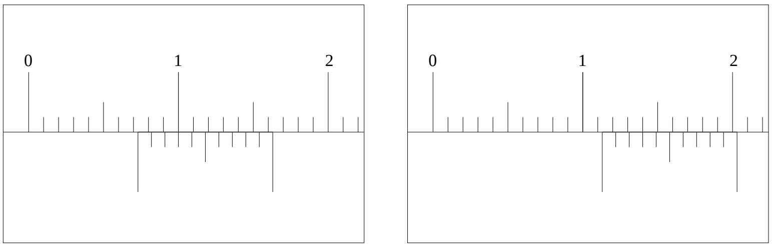
5. Which is a more precise measuring tool: ruler or vernier caliper? Why?

Prelab: Expt-1

Your name: ID# :

The following questions must be answered and turned in at the beginning of the lab.

1. Look at the figure below and correctly read each vernier caliper reading.



Caliper 1 = ____________ *cm* Caliper 2 = ____________ *cm*

2. Suppose two students measure the length of a piece of wire three times each. Student A obtains results of 8.0, 8.1, and 8.2 centimeters. Student B obtains determines a length of 8.1 centimeters for all of his three measurements. Although the average length found for each student's measurements was 8.1 cm, what <u>qualitative</u> statements can be made concerning the measurements made by students A and B?