Expt-5: Compound Pendulum and simple harmonic motion

OBJECTIVE:

1. To determine the g, the acceleration due to gravity.
2. To determine k, the radius of the gyration of the pendulum.

APPARATUS:

1. Compound pendulum
2. Meter rule
3. Stop watch
4. Digital weigh machine

THEORY:

A physical pendulum or compound pendulum is a rigid object, which is free to rotate about a fixed horizontal axis. In this experiment, we use a special type of compound pendulum which is symmetric about its center of mass. This compound pendulum is nothing but a metal bar, containing a number of holes with equal intervals. The pendulum can be suspended by the help of knife edge passing through different holes. The point of suspension is known as pivot point. If we swing the bar from different holes then the moment of inertia of the pendulum and the time period will change.

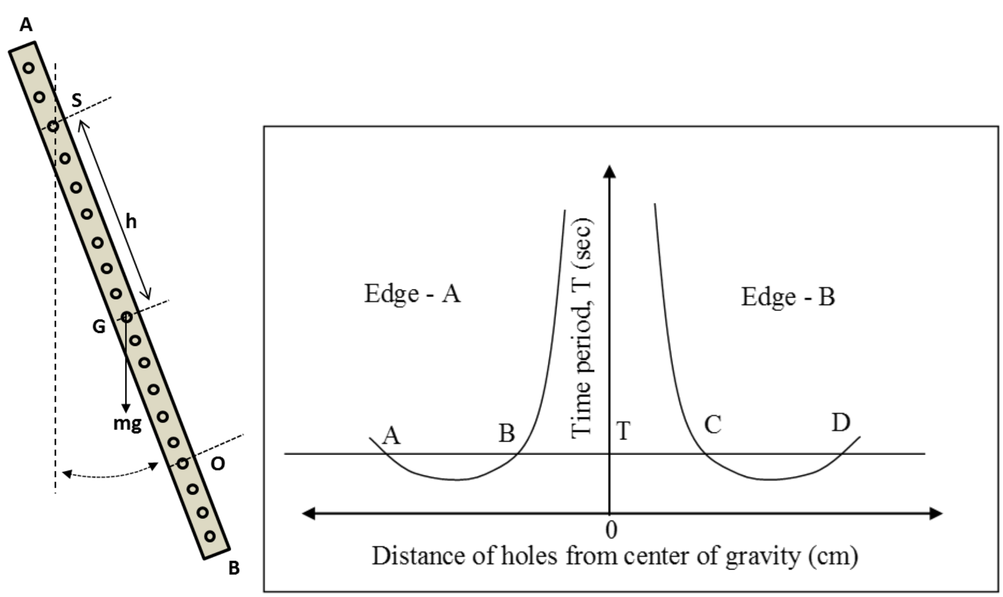

Figure 1: Demonstration of compound pendulum oscillations and corresponding time period vs distance graph.

Allowing the bar to swing it will approximately follow a simple harmonic motion. According to Newton's 2nd law of motion for rotation:

$$\tau = I\,\alpha \qquad (1)$$

where, I is the moment of inertia of the pendulum about the axis of rotation, and α the angular acceleration.

Torque is given by $\tau = -mgl\ sin\phi$**,** here l is the distance of the pivot from the center of the pendulum.

For very small angle of rotation ($\phi < 4^0$) $sin\phi$ can be approximated by $\emptyset$,

$$I\frac{d^2\emptyset}{dt^2} = -mgl \qquad (2)$$

By rearranging the equation,

$$\frac{d^2\emptyset}{dt^2} + \frac{mgl}{I}\emptyset = 0 \qquad (3)$$

This 2nd order differential equation describes the simple harmonic motion analogous to,

$\frac{d^2\emptyset}{dt^2} + \omega^2\emptyset = 0$. Here the angular frequency, $\omega = \frac{2\pi}{T}$

Therefore, we can deduce the time period from (3):

$$T = 2\pi\sqrt{\frac{I}{mgl}} \qquad \ldots(4)$$

Using "Parallel Axis Theorem: moment of inertia, I, of an object about an axis parallel to the axis that passes through the center of mass is $I = I_G + m\,l^2$, where I_G the moment of inertia of the object about the axis through the center of mass, m is the total mass of the objet, and l is the distance between the axes, the Moment of Inertia can be determined as:

$$I = I_G + ml^2 \quad \ldots(5)$$

where I_G is the moment of inertia of the bar about its center of mass.

$$I_G = mK^2 \qquad \ldots(6)$$

Here K is the radius of gyration about the axis passing through the G.

Substituting Eq. 5 and Eq.6 in Eq.4 we get,

$$T = 2\pi\sqrt{\frac{\frac{K^2}{l}+l}{g}} \qquad \ldots(7)$$

Comparing the time period relation for simple pendulum of length L, $T = 2\pi\sqrt{\frac{L}{g}}$, we can deduce,

$$L = l + \frac{k^2}{l} \qquad \ldots(8)$$

From the above equation we can obtain a quadratic equation of l, which has 2 roots l_1 and l_2 such that,

$$L = l_1 + l_2 \qquad \ldots(9)$$

$$K^2 = l_1 l_2 \qquad \ldots(10)$$

The value of K and g can be determined from

$$g = 4\pi^2 \frac{L}{T^2} \qquad \ldots(11)$$

$$K = \sqrt{l_1 l_2} \quad \ldots \ (12)$$

Since the "effective length L is composed of two roots l_1 and l_2, so there are infinite ways to combine $l_1$and l_2 to make the same L.

In this experiment, we will determine the length L and corresponding time period T graphically [see Figure 1].

If we plot a graph using table (1), two curves symmetric about the position of COM should appear. Horizontal lines in the lower portion will intersect the curves in four points. l_1 and l_2 can be determined by measuring the distances from the COM position. Using eqns. (11), acceleration due to gravity g and K can be calculated.

PROCEDURE:

1. With the help of the knife edge suspend the metal bar by passing through the hook to the hole closer to the Edge A.
2. Measure the distance d from the center of gravity (middle hole) to the edge of the hole.
3. Oscillate the metal bar with an angle for a small angle.
4. Record the time for 10 oscillations using a stopwatch. Repeat it for two times and obtain the average time period T for that distance.
5. Repeat the procedure 1-4 for more holes of the bar, except the center of mass.
6. After procedure 5 again repeat the procedure 1-4 by inverting the metal bar (Edge B) for all the holes.
7. Draw the graph $T \ vs \ d$.
8. Draw a suitable horizontal line parallel to x axis. Mark A, B, C and D to the four points of intersection with the graph. Measure the length AC and BD, then find the $L = \frac{AC+BD}{2}$. Find the corresponding T for the line and then find the value of g.
9. Repeat the procedure 8 for by drawing another horizontal lines to different points and find the values of g. Calculate the mean of g.
10. To calculate the value K, determine the length A, B or C, D of the line ABCD. The graph is symmetrical about this vertical line. By using the formula $K = \sqrt{A \times B}$ or $\sqrt{C \times D}$. Repeat the procedure for all the lines and then find the average value of K.

- Minimum 5 holes should be used to get curves like the figure.

Table 1

<table>
<tr><th colspan="2"><u>Hole Number</u></th><th>Distance d (cm)</th><th colspan="2">Time for 10 oscillations (s)</th><th>Mean time t (s)</th><th>Time Period T=$\frac{t}{10}$ (s)</th></tr>
<tr><td rowspan="8"><u>Edge A</u></td><td><u>1</u></td><td></td><td></td><td></td><td></td><td></td></tr>
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TABLE 2 (From the graph)

Observations from the horizontal lines	L (m)	T (sec)	$g = 4\pi^2 \frac{L}{T^2}$ (m/s^2)	Mean g (m/s^2)	K (m)	Mean K (m)
1. ABCD	$L = \frac{AC + BD}{2}$					
2. $A'B'C'D'$	$L' = \frac{A'C' + B'D'}{2}$					

Questions:

1. Why the time period increasing while swinging closer to the middle hole?

2. Do you think compound pendulum in comparison to simple pendulum would show better oscillatory motion in air for measurement of g? Why?

Pre lab 5

1. Distinguish between a simple pendulum and a compound pendulum.

2. What do you understand by moment of inertia and torque?