

Decision Making Under Uncertainty

CARS TUTORIAL

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WHAT IS THIS TUTORIAL?

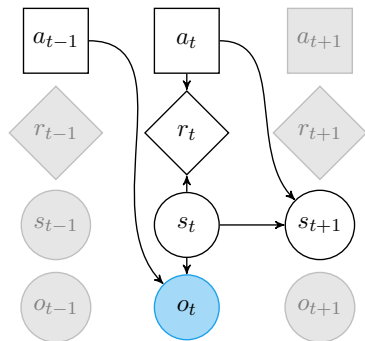



Figure: POMDP Sequence.

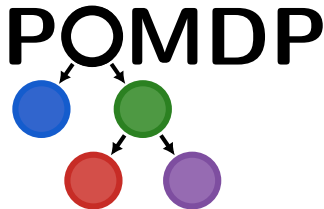
- A peek into the POMDPs.jl ecosystem of  packages
- “But what *are* POMDPs?”
 - POMDPs are a *problem formulation* that enable optimal¹ sequential decisions to be made in uncertain environments.
- Teaching *by example* using interactive Pluto.jl notebooks
 - No prior knowledge of MDPs/POMDPs necessary—all are welcome!
 - Can also be used as a refresher on *decision making under uncertainty*.
 - Target audience is wide, but familiarity with Julia is helpful.
- <https://github.com/mossr/CARS-tutorial>

¹or *approximately* optimal.

TOPICS COVERED IN THIS COURSE

All topics highlight packages that adhere to the `POMDPs.jl` interface.

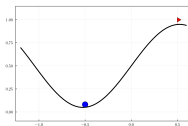
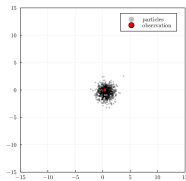
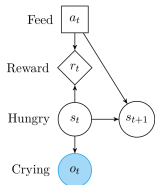
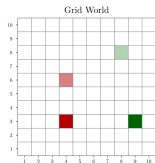
- **Sequential Decision Making**
 - *Markov decision processes* (MDPs)
 - *Partially observable Markov decision processes* (POMDPs)
- **Solution Methods:** Algorithms to solve MDPs/POMDPs
 - *Online* and *offline* solvers
 - *Value function approximation*
- **Simulations**
- **State Estimation using Particle Filters**
- **Reinforcement Learning**
- **Deep Reinforcement Learning**
- **Imitation Learning**



EXAMPLE PROBLEMS COVERED IN THIS COURSE

Common problems in the literature are used as running examples.

- (MDP) **Grid World**: Agent moving around a grid world, looking for rewards.
- (POMDP) **Crying Baby**: When to feed a baby, based on crying observations.
- (MDP) **1D Random Walk**: Agent moves around the number line.
- (POMDP) **2D Random Walk**: Estimating state of a moving agent based on observations.
- (MDP) **Mountain Car**: Reach a goal up a hill, starting in a valley.
- (MDP) **Swinging Pendulum**: Balance a swinging pendulum upright.



POMDPs.jl PACKAGE ECOSYSTEM

The POMDPs.jl package itself contains the interface to define problem definitions.

Other packages provide supporting tools that contain most of the functionality:¹

- QuickPOMDPs.jl
- POMDPModelTools.jl
- POMDPPolicies.jl
- POMDPSimulators.jl
- POMDPModels.jl
- POMDPGallery.jl
- BeliefUpdaters.jl
- ParticleFilters.jl
- POMDPModelChecking.jl
- POMDPStressTesting.jl
- DiscreteValueIteration.jl
- LocalApproximationValueIteration.jl
- GlobalApproximationValueIteration.jl
- MCTS.jl
- TabularTDLearning.jl
- DeepQLearning.jl
- Crux.jl
- QMDP.jl
- FIB.jl
- BeliefGridValueIteration.jl
- Sarsop.jl
- BasicPOMCP.jl
- ARDESPOT.jl
- MCVI.jl
- POMDPSolve.jl
- IncrementalPruning.jl
- POMCPOW.jl
- AEMS.jl
- PointBasedValueIteration.jl

¹ Key: Tools, Extensions, MDP solvers, POMDP solvers.

OTHER RESOURCES

There are many *excellent* resources on MDPs/POMDPs and reinforcement learning:

- ***Algorithms for Decision Making*, Kochenderfer, Wheeler, & Wray**
(<https://algorithmsbook.com/>)
- ***Reinforcement Learning: An Introduction*, Sutton & Barto**
(<http://incompleteideas.net/book/the-book.html>)
- ***POMDPs.jl: A Framework for Sequential Decision Making under Uncertainty*, Egorov, Sunberg, et al., Journal of Machine Learning Research, 2017**
(<https://www.jmlr.org/papers/volume18/16-300/16-300.pdf>)
- **Introduction to Reinforcement Learning with David Silver**
(<https://deepmind.com/learning-resources/-introduction-reinforcement-learning-david-silver>)

LECTURE BREAKDOWN

Each lecture has an associated **Pluto** notebook detailing the material.

1. MDPs: Markov Decision Processes

– Includes: *planning, reinforcement learning, online/offline solvers, simulations*

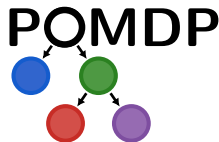
2. POMDPs: Partially Observable Markov Decision Processes

3. State Estimation using Particle Filtering

4. Approximate Methods for Continuous Spaces

5. Deep Reinforcement Learning

6. Imitation Learning: Learn from Demonstrations



MDPs: MARKOV DECISION PROCESSES

CARS TUTORIAL: POMDPs.JL

DECISION MAKING UNDER UNCERTAINTY

WHAT IS AN MDP?

Definition: MDP. A *Markov decision process* (MDP) is a *problem formulation* that defines how an agent takes sequential *actions* from *states* in its environment, guided by *rewards*—using uncertainty in how it *transitions* from state to state.

- Formally, an MDP is defined by the following:

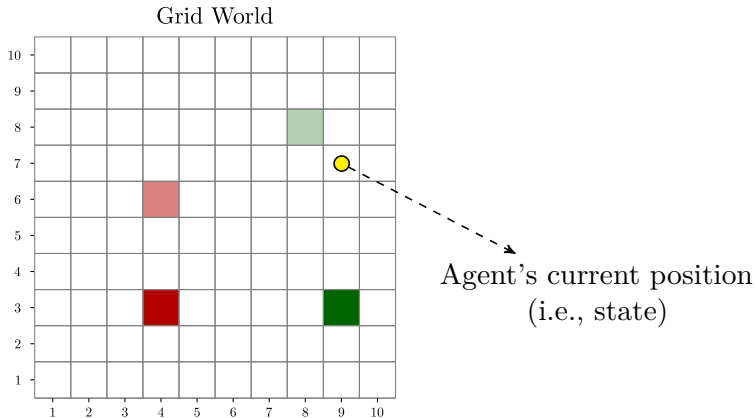
Table: MDP Problem Formulation: $\langle \mathcal{S}, \mathcal{A}, T, R, \gamma \rangle$

Variable	Description	POMDPs Interface
\mathcal{S}	State space	POMDPs.states
\mathcal{A}	Action space	POMDPs.actions
$T(s' s, a)$	Transition function	POMDPs.transition
$R(s, a)$	Reward function	POMDPs.reward
$\gamma \in [0, 1]$	Discount factor	POMDPs.discount

Remember, an MDP is a *problem formulation* and *not an algorithm*.
An MDP formulation enables the use of solution methods, i.e. algorithms.

MDP EXAMPLE: GRID WORLD

In the **Grid World** problem, an *agent* moves around a grid attempting to collect as much reward (**green cells**) as possible, avoiding negative rewards (**red cells**).



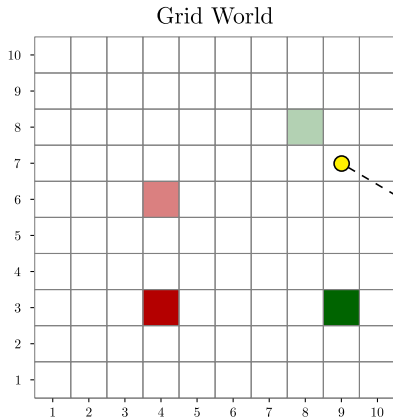
MDP: STATE SPACE

Definition: State space \mathcal{S} .

A set of all possible *states* an agent can be in (discrete or continuous).

Grid World example:

All possible (x, y)
cells in a 10×10 grid
(i.e., 100 discrete states)



State
 (x, y) of $(9, 7)$

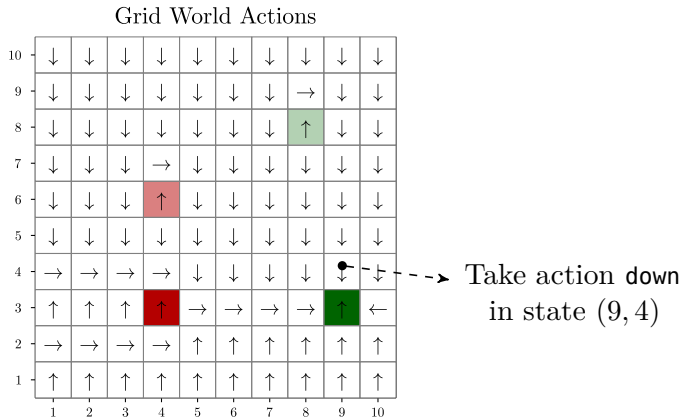
MDP: ACTION SPACE

Definition: Action space \mathcal{A} .

A set of all possible *actions* an agent can take (discrete or continuous).

Grid World example:

The four (discrete)
cardinal directions:
[up, down, left, right]



MDP: TRANSITION FUNCTION

Definition: Transition function¹ $T(s' | s, a)$.

Defines how the agent *transitions* from the current state s to the next state s' when taking action a .
Returns a *probability distribution* over all possible next states s' given (s, a) .

Grid World example:

Stochastic transitions (incorporates randomness/uncertainty).

Action a = up from state s .

70% chance of transitioning correctly.

30% chance ($10\% \times 3$) of transitioning incorrectly.²

	0.7	
0.1	s \uparrow a	0.1
	0.1	

¹Sometimes called the *transition model*.

²i.e., a different action is taken.

MDP: REWARD FUNCTION

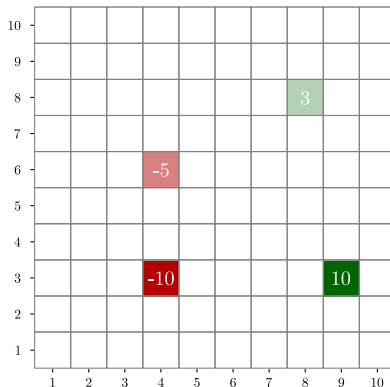
Definition: Reward function¹ $R(s, a)$.

Defines the *reward* an agent receives when taking action a from state s .

Grid World example:

Two cells contain **positive rewards**
and two cells contain **negative rewards**,
all others are **zero**.

Grid World Rewards

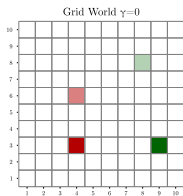


¹Sometimes called the *reward model*.

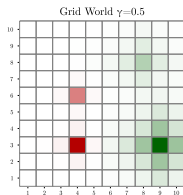
MDP: DISCOUNT FACTOR

Definition: Discount factor $\gamma \in [0, 1]$.

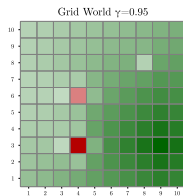
The *discount factor* controls how myopic (short-sighted) the agent is in its decision making (e.g., when $\gamma = 0$, the agent only cares about immediate rewards (myopic) and as $\gamma \rightarrow 1$, the agent takes in potential future information in its decision making process).



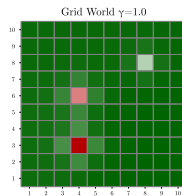
(a) Short-sighted
(no reward spread)



(b) Some future
reward¹ is spread



(c) Future reward
is nicely spread



(d) Dominated by
the future reward

¹The sum of the *discounted future rewards* is called the *utility* $U(s)$ or the *value* $V(s)$ of a state.

QuickPOMDPs: GRID WORLD

```
using POMDPs, POMDPModelTools, QuickPOMDPs

struct State; x::Int; y::Int end # State definition
@enum Action UP DOWN LEFT RIGHT # Action definition

s = [(State(x,y) for x=1:10, y=1:10)..., State(-1,-1)] # State-space
A = [UP, DOWN, LEFT, RIGHT] # Action-space

const MOVEMENTS = Dict{UP⇒State(0,1), DOWN⇒State(0,-1), LEFT⇒State(-1,0), RIGHT⇒State(1,0)}
Base.:*(s1::State, s2::State) = State(s1.x + s2.x, s1.y + s2.y) # Helper for applying actions

function T(s, a) # Transition function
    R(s) != 0 && return Deterministic(State(-1,-1))
    Ns = length(A)
    next_states = Vector{State}{}(undef, Ns + 1)
    probabilities = zeros{Ns + 1}
    for (i, a') in enumerate(A)
        prob = (a' == a) ? 0.7 : (1 - 0.7) / (Ns - 1)
        destination = s + MOVEMENTS[a']
        next_states[i+1] = destination
        if 1 ≤ destination.x ≤ 10 && 1 ≤ destination.y ≤ 10
            probabilities[i+1] += prob
        end
    end
    (next_states[1], probabilities[1]) = (s, 1 - sum(probabilities))
    return SparseCat(next_states, probabilities)
end

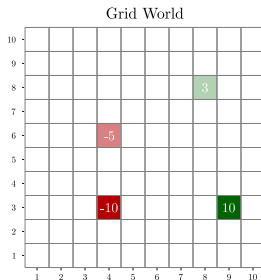
function R(s, a=missing) # Reward function
    if s == State(4,3)
        return -10
    elseif s == State(4,6)
        return -5
    elseif s == State(9,3)
        return 10
    elseif s == State(8,8)
        return 3
    end
    return 0
end

abstract type GridWorld <: MDP{State, Action} end

mdp = QuickMDP(GridWorld,
    states = s,
    actions = A,
    transition = T,
    reward = R,
    discount = 0.95,
    isterminal = s→s==State(-1,-1));
```

- This code^a defines the entire *Grid World* problem using QuickPOMDPs.jl

- Just a sneak-peek: we'll walk through this in detail in the Pluto notebooks



^aYes, this is self-contained—copy and paste it into a notebook or REPL!

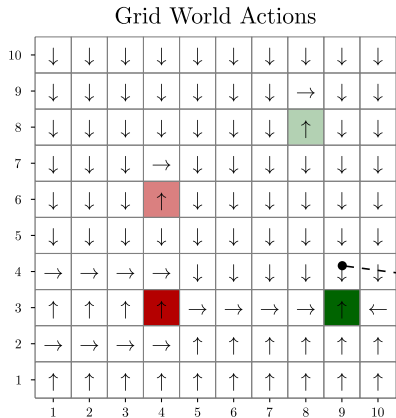
MDP: POLICY

Definition: Policy $\pi : \mathcal{S} \rightarrow \mathcal{A}$.

A mapping from states $s \in \mathcal{S}$ to actions $a \in \mathcal{A}$. As a function: $a = \pi(s)$

Grid World example:

The four (discrete)
cardinal directions:
[up, down, left, right]



Take action down
in state (9,4). So
 $\pi((9,4)) = \text{down}$

MDP SOLVERS

A number of ways to solve MDPs are implemented in the following packages.

Table: MDP Solution Methods

Package	Online/Offline	State Spaces	Actions Spaces
DiscreteValueIteration.jl	Offline	Discrete	Discrete
LocalApproximationValueIteration.jl	Offline	Continuous	Discrete
GlobalApproximationValueIteration.jl	Offline	Continuous	Discrete
MCTS.jl*	Online	Continuous	Continuous

* Monte Carlo Tree Search.

When defining your problem, the *type* of state and action space is very important!

REINFORCEMENT LEARNING SOLVERS

Certain problems are better suited in the *reinforcement learning* (RL) domain. Several RL solvers that adhere to the `POMDPs.jl` interface are implemented in the following packages.

Table: Reinforcement Learning Solution Methods

Package	State Spaces	Actions Spaces	Algorithms Implemented
TabularTDLearning.jl	Discrete	Discrete	Q-learning, SARSA, SARSA- λ
DeepQLearning.jl	Continuous	Discrete	DQN, Double DQN, Dueling DQN, Recurrent Q-learning
Crux.jl	Discrete/Continuous	Discrete/Continuous	DQN, REINFORCE, PPO, A2C, DDPG, TD3, SAC, Behavior Cloning, GAIL, AdVIL, AdRIL, SQIL, ASAF

When defining your problem, the *type* of state, action, and observation space is very important!

POMDPs: PARTIALLY OBSERVABLE MARKOV DECISION PROCESSES

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DECISION MAKING UNDER UNCERTAINTY

WHAT IS A POMDP?

Definition: POMDP. A *Partially observable Markov decision process* (POMDP) is an MDP with *state uncertainty*—meaning we cannot know the *true* state, only a *belief* about the true state using *observations*.

- Formally, a POMDP is defined by the following:

Table: MDP Problem Formulation: $\langle \mathcal{S}, \mathcal{A}, \mathcal{O}, T, R, \mathcal{O}, \gamma \rangle$

Variable	Description	POMDPs Interface
\mathcal{S}	State space	POMDPs.states
\mathcal{A}	Action space	POMDPs.actions
\mathcal{O}	Observation space	POMDPs.observations
$T(s' s, a)$	Transition function	POMDPs.transition
$R(s, a)$	Reward function	POMDPs.reward
$O(o s')$	Observation function	POMDPs.observation
$\gamma \in [0, 1]$	Discount factor	POMDPs.discount

Remember, a POMDP is a *problem formulation* and *not an algorithm*.

HOW ARE POMDPs DIFFERENT THAN MDPs?

- A POMDP² is an MDP with *state uncertainty*

MDP: $\langle \mathcal{S}, \mathcal{A}, T, R, \gamma \rangle$

POMDP: $\langle \mathcal{S}, \mathcal{A}, \mathcal{O}, T, R, \mathcal{O}, \gamma \rangle$

- The agent receives an *observation* of the current state rather than the true state (potentially imperfect observations)
- Using past observations, the agent builds a *belief* of their underlying state
 - Which can be represented by a probability distribution over true states

²“Partially observable” is key in understanding beliefs.

EXAMPLE POMDP: CRYING BABY PROBLEM

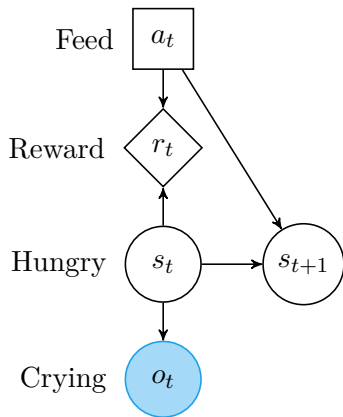


Figure: The crying baby POMDP.

- A simple POMDP with 2 states, 2 actions, and 2 observations:

$$\mathcal{S} = \{\text{hungry}, \text{full}\}$$

$$\mathcal{A} = \{\text{feed}, \text{ignore}\}$$

$$\mathcal{O} = \{\text{crying}, \text{quiet}\}$$

- We cannot directly tell if the baby is truly hungry, but we can observe that it's crying and update our *belief* about the true state using this information.

QuickPOMDPs: CRYING BABY

```
using POMDPs, POMDPModelTools, QuickPOMDPs

@enum State hungry full
@enum Action feed ignore
@enum Observation crying quiet

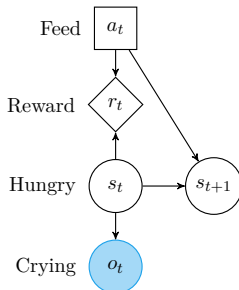
pomdp = QuickPOMDP(
    states      = [hungry, full], # S
    actions     = [feed, ignore], # A
    observations = [crying, quiet], # O
    initialstate = [full], # Deterministic
    discount    = 0.9, # γ

    transition = function T(s, a)
        if a == feed
            return SparseCat([hungry, full], [0, 1])
        elseif s == hungry && a == ignore
            return SparseCat([hungry, full], [1, 0])
        elseif s == full && a == ignore
            return SparseCat([hungry, full], [0.1, 0.9])
        end
    end,

    observation = function O(s, a, s')
        if s' == hungry
            return SparseCat([crying, quiet], [0.8, 0.2])
        elseif s' == full
            return SparseCat([crying, quiet], [0.1, 0.9])
        end
    end,

    reward = (s,a)->(s == hungry ? -10 : 0) + (a == feed ? -5 : 0)
)
```

- This code^a defines the entire *Crying Baby* POMDP using QuickPOMDPs.jl
 - Just a sneak-peek: we'll walk through this in detail in the Pluto notebooks



^aYes, this is self-contained—copy and paste it into a notebook or REPL!

POMDP SOLVERS

A number of ways to solve POMDPs are implemented in the following packages.

Table: POMDP Solution Methods

Package	Online/Offline	State Spaces	Actions Spaces	Observation Spaces
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FIB.jl	Offline	Discrete	Discrete	Discrete
BeliefGridValueIteration.jl	Offline	Discrete	Discrete	Discrete
SARSOP.jl	Offline	Discrete	Discrete	Discrete
BasicPOMCP.jl	Online	Continuous	Discrete	Discrete
ARDESPOT.jl	Online	Continuous	Discrete	Discrete
MCVI.jl	Offline	Continuous	Discrete	Continuous
POMDPSolve.jl	Offline	Discrete	Discrete	Discrete
IncrementalPruning.jl	Offline	Discrete	Discrete	Discrete
POMCPOW.jl	Online	Continuous	Continuous	Continuous
AEMS.jl	Online	Discrete	Discrete	Discrete
PointBasedValueIteration.jl	Offline	Discrete	Discrete	Discrete

When defining your problem, the *type* of state, action, and observation space is very important!