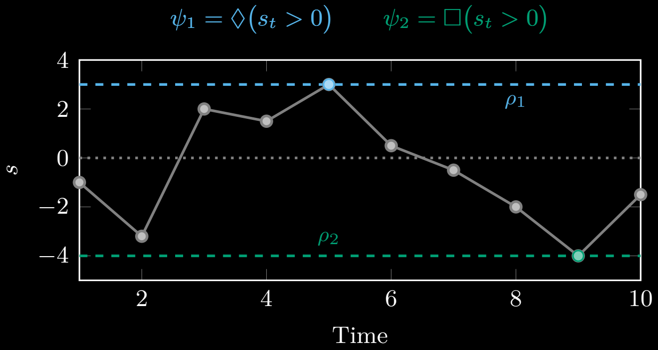
SignalTemporalLogic.jl

Introduction



Robert Moss
Stanford AA228V/CS238V
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Installation

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You can install the `SignalTemporalLogic.jl` package via:

```julia
using Pkg
Pkg.add("SignalTemporalLogic")
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Then you can run this to use the package:

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Specifications

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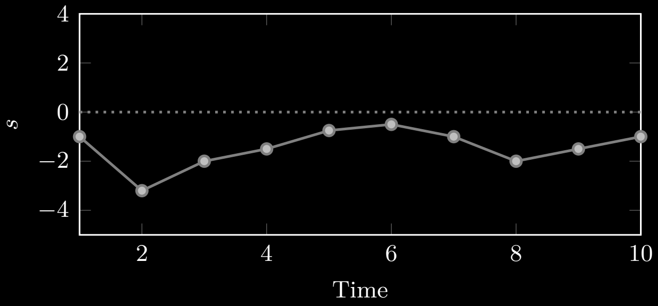
julia> using SignalTemporalLogic
julia> τ = [-1.0, -3.2, -2.0, -1.5, -0.75, -0.5, -1.0, -2.0, -1.5, -1.0];
```



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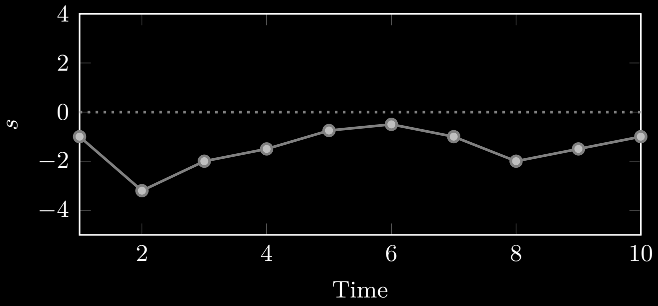

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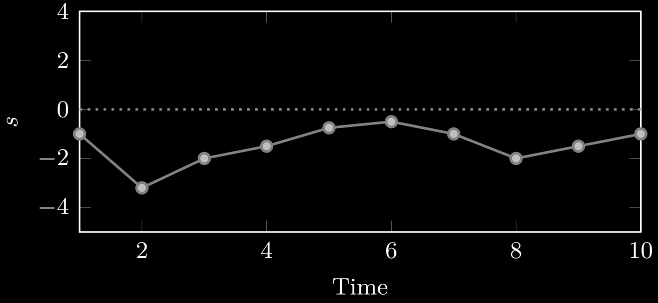

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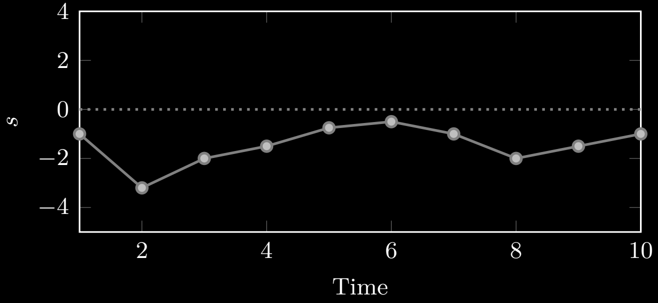

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julia> ψ(τ)
false
```

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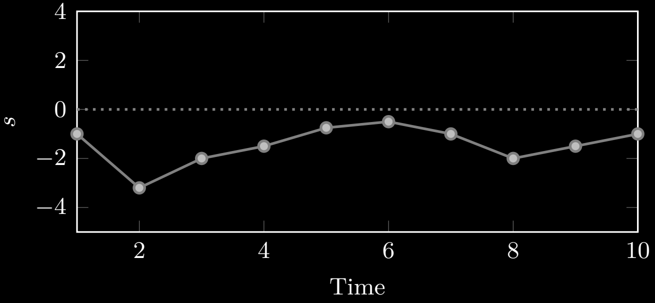

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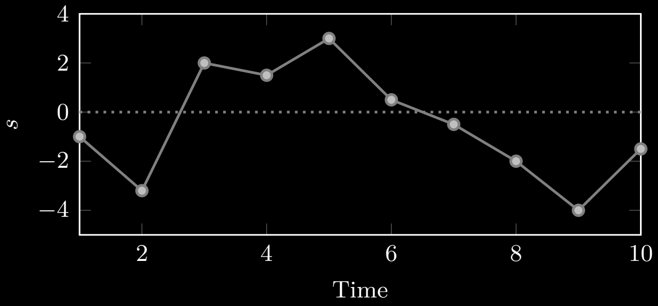

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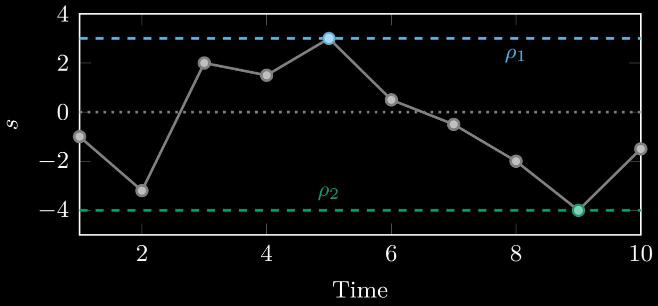
$\psi_1 = \Diamond(s_t > 0)$ $\psi_2 = \Box(s_t > 0)$



Robustness and Smooth Robustness

Example: Eventually $\Diamond$

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\end{aligned}
$$

```julia
(◊::Eventually)(x) = any(◊.ψ(x[t]) for t ∈ interval(◊,x)) # Evaluate formula ψ(x)
```

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$$\rho(x_t, \Diamond_{[a,b]}\psi) = \max_{t' \in [t+a, t+b]} \rho(x_{t'}, \psi)$$

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ρ(x, ◊::Eventually) = maximum(ρ(x[t], ◊.ψ) for t ∈ interval(◊,x)) # Robustness
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ρ̃(x, ◊::Eventually; w=1) = smoothmax(ρ̃(x[t], ◊.ψ; w) for t ∈ interval(◊,x); w) # Smooth robustness
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```
smoothmax(x; w=1) = (w == 0) ? maximum(x) : sum(xᵢ*exp(xᵢ/w) for xᵢ in x) / sum(exp(xⱼ/w) for xⱼ in x)
```

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Example: Eventually $\Diamond$

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Robustness and Smooth Robustness

Example: Eventually ◊

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julia> import ForwardDiff: gradient
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Robustness and Smooth Robustness

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julia> gradient(τ→ρ(τ, ψ), τ)
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 -0.03149700775134967
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Robustness and Smooth Robustness

Example: Eventually $\Diamond$

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```
robustness(s, ψ) = ρ(s, ψ)
smooth_robustness(s, ψ; w=1) = ρ̃(s, ψ; w=1)
```

Use in Projects

Wrappers are provided in the textbook/projects:

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Linear temporal logic (LTL)

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struct LTLSpecification <: Specification
    formula # formula specified using SignalTemporalLogic.jl
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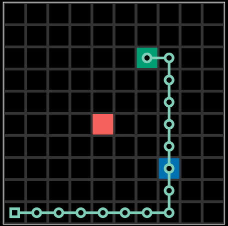
Signal temporal logic (STL, includes time interval)

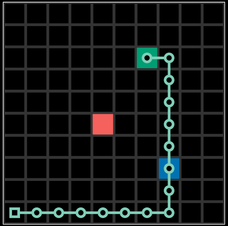
```
struct STLSpecification <: Specification
    formula # formula specified using SignalTemporalLogic.jl
    I       # time interval (e.g. 3:10)
end
evaluate(ψ::STLSpecification, τ) = ψ.formula([step.s for step in τ[ψ.I]])
```

Grid World



Grid World

$F(s_t)$: the state s at time t contains an obstacle



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$F(s_t)$: the state s at time t contains an obstacle

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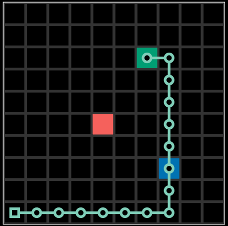

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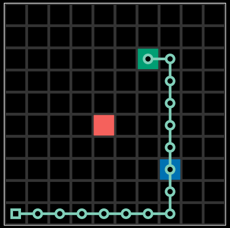

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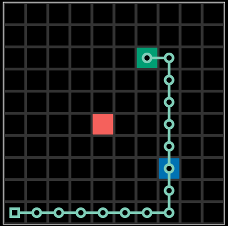

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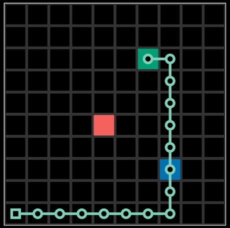

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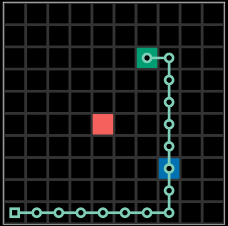
$C(s_t)$: the state s at time t is the checkpoint

$$\psi = \underbrace{\Diamond G(s_t)}_{\text{reaches goal}} \wedge \underbrace{\neg C(s_t)\, \mathcal{U}\, G(s_t)}_{\text{reach checkpoint before goal}} \wedge \underbrace{\Box \neg F(s_t)}_{\text{always avoid obstacles}}$$



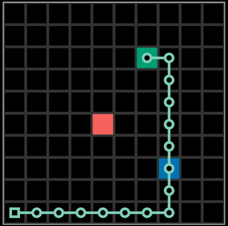
Grid World

$F(s_t)$: the state s at time t contains an obstacle

$G(s_t)$: the state s at time t is the goal

$C(s_t)$: the state s at time t is the checkpoint

$$\psi = \underbrace{\Diamond G(s_t)}_{\text{reaches goal}} \wedge \underbrace{\neg C(s_t) \,\mathcal{U}\, G(s_t)}_{\text{reach checkpoint before goal}} \wedge \underbrace{\square \neg F(s_t)}_{\text{always avoid obstacles}}$$

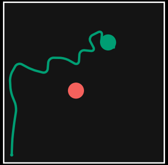


```
F = @formula sₜ -> sₜ == [5, 5]
G = @formula sₜ -> sₜ == [7, 8]
C = @formula sₜ -> sₜ == [8, 3]
ψ = LTLSpecification(@formula ◊(G) ∧ 𝒰(¬G, C) ∧ □(¬F))
```

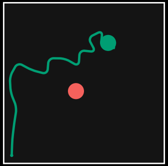
Continuum World

System	Property	Implementation

Continuum World

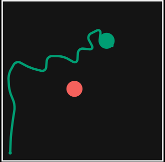
System	Property	Implementation
Continuum World 		

Continuum World

System	Property	Implementation
Continuum World 	*"Reach the goal without hitting the obstacle"* $G(s_t)$: s_t is in the goal region $F(s_t)$: s_t is in the obstacle region $\psi = \Diamond G(s_t) \wedge \Box \neg F(s_t)$	

Continuum World

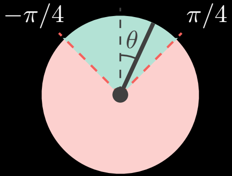
System	Property	Implementation

Continuum World 	*“Reach the goal without hitting the obstacle”*
	$G(s_t)$: s_t is in the goal region
	$F(s_t)$: s_t is in the obstacle region
	$\psi = \Diamond G(s_t) \wedge \Box \neg F(s_t)$
		`G = @formula s→norm(s.-[6.5,7.5])≤0.5`
		`F = @formula s→norm(s.-[4.5,4.5])≤0.5`
		`ψ = @formula ◊(G) ∧ □(¬F)`

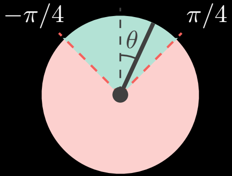
Inverted Pendulum

System	Property	Implementation

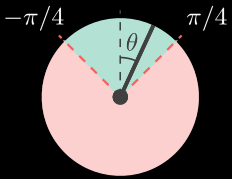
Inverted Pendulum

System	Property	Implementation
Inverted Pendulum 		

Inverted Pendulum

System	Property	Implementation
Inverted Pendulum 	*"Keep the pendulum balanced"* $B(s_t)$: $\lvert\theta_t\rvert \leq \pi/4$ $\psi = \Box B(s_t)$	

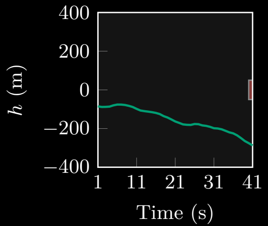
Inverted Pendulum

System	Property	Implementation
Inverted Pendulum 	*"Keep the pendulum balanced"* $B(s_t)$: $\lvert\theta_t\rvert \leq \pi/4$ $\psi = \Box B(s_t)$	`B = @formula s→abs(s[1])≤π/4` `ψ = @formula □(B)`

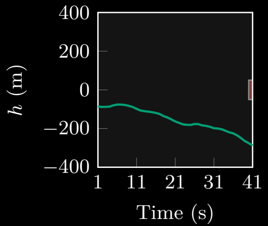
Aircraft Collision Avoidance

System	Property	Implementation

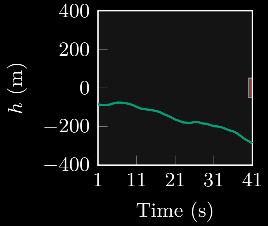
Aircraft Collision Avoidance

System	Property	Implementation
Aircraft Collision Avoidance 		

Aircraft Collision Avoidance

System	Property	Implementation
Aircraft Collision Avoidance 	*"Ensure at least 50 meters relative altitude between 40 and 41 seconds"* $S(s_t)$: $\lvert h_t \rvert \geq 50$ $\psi = \Box_{[40,41]} S(s_t)$	

Aircraft Collision Avoidance

System	Property	Implementation
Aircraft Collision Avoidance 	*"Ensure at least 50 meters relative altitude between 40 and 41 seconds"* $S(s_t)$: $\lvert h_t \rvert \geq 50$ $\psi = \square_{[40,41]} S(s_t)$	`S = @formula s->abs(s[1])≥50` `ψ = @formula □(40:41, S)`

Resources

Resources

github.com/sisl/SignalTemporalLogic.jl

Resources

github.com/sisl/SignalTemporalLogic.jl

github.com/mossr/STL-mini-lecture