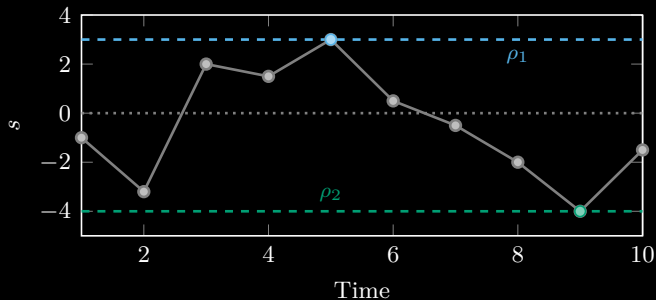


SignalTemporalLogic.jl

INTRODUCTION

$$\psi_1 = \Diamond(s_t > 0) \quad \psi_2 = \Box(s_t > 0)$$



ROBERT MOSS

STANFORD AA228V/CS238V

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You can install the `SignalTemporalLogic.jl` package via:

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using Pkg  
Pkg.add("SignalTemporalLogic")
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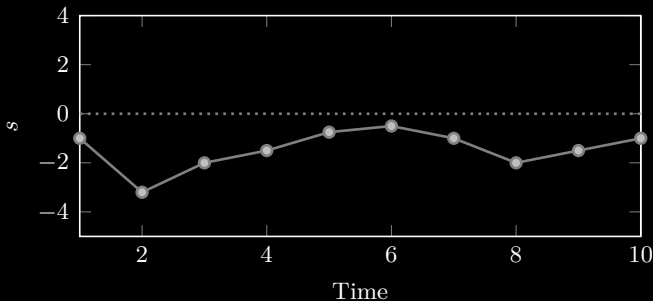
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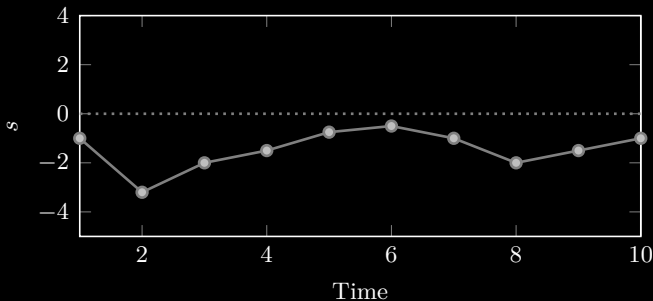
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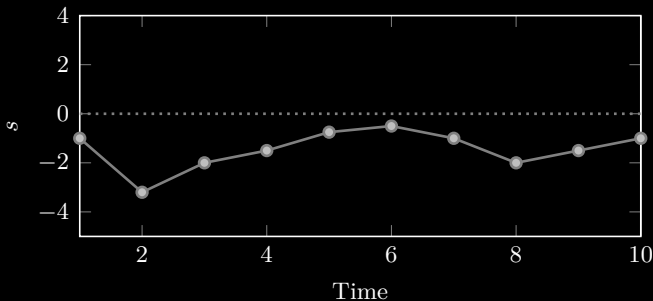
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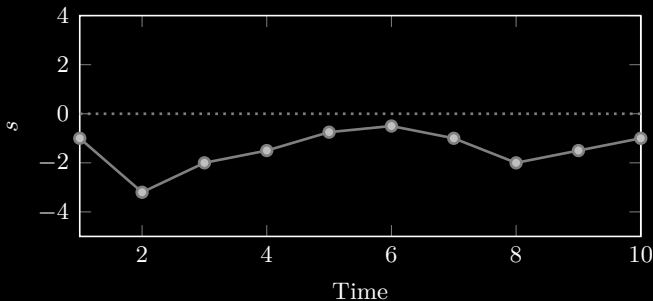
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```
 $\tau$  # \tau<TAB>
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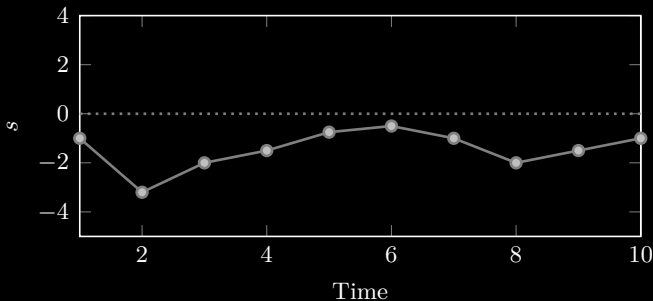
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julia>  $\psi(\tau)$ 
false
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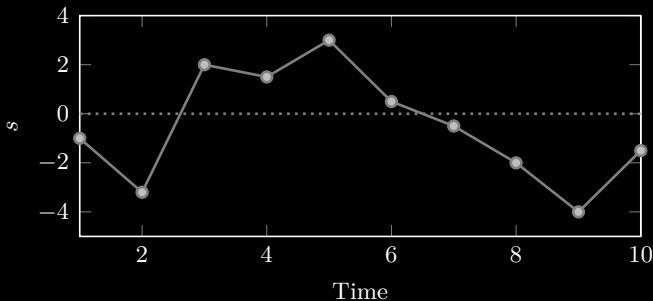
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ROBUSTNESS

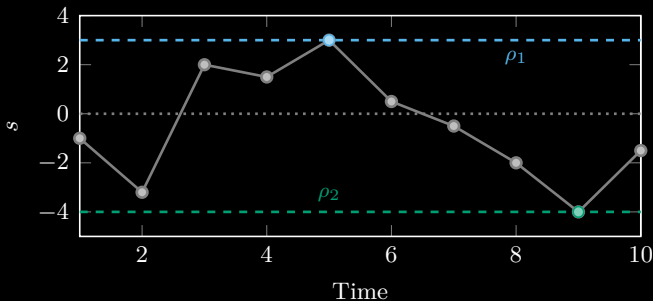
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mutable struct Eventually <: Formula
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$$\Diamond_{[a,b]}\psi = \top \mathcal{U}_{[a,b]}\psi$$

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```
( $\Diamond$ ::Eventually)(x) = any( $\Diamond$ . $\psi$ (x[t]) for t in interval( $\Diamond$ ,x)) # Evaluate formula  $\psi$ (x)
```


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 $\rho(x, \Diamond::\text{Eventually}) = \text{maximum}(\rho(x[t], \Diamond.\psi) \text{ for } t \in \text{interval}(\Diamond, x)) \text{ \# Robustness}$ 
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ρ(x, ◇::Eventually) = maximum(ρ(x[t], ◇.ψ) for t ∈ interval(◇, x)) # Robustness
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 $\text{smoothmax}(x; w=1) = (w == 0) ? \text{maximum}(x) : \text{sum}(x_i * \exp(x_i/w) \text{ for } x_i \text{ in } x) / \text{sum}(\exp(x_j/w) \text{ for } x_j \text{ in } x)$ 
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-0.03149700775134967
-0.012828520312110245
-0.002782850376909859
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```
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`robustness(s, ψ) = ρ (s, ψ)`

`smooth_robustness(s, ψ ; w=1) = $\tilde{\rho}$ (s, ψ ; w=1)`

USE IN PROJECTS

Wrappers are provided in the textbook/projects:

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Linear temporal logic (LTL)

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struct LTLSpecification <: Specification
    formula # formula specified using SignalTemporalLogic.jl
end
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USE IN PROJECTS

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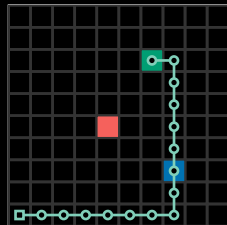
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Signal temporal logic (STL, includes time interval)

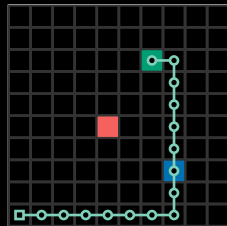
```
struct STLSpecification <: Specification
    formula # formula specified using SignalTemporalLogic.jl
    I       # time interval (e.g. 3:10)
end
evaluate( $\psi$ ::STLSpecification,  $\tau$ ) =  $\psi$ .formula([step.s for step in  $\tau$ [ $\psi$ .I]])
```

GRID WORLD



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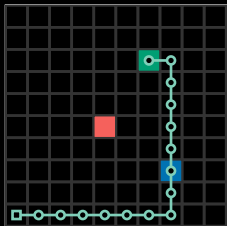
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$G(s_t)$: the state s at time t is the goal

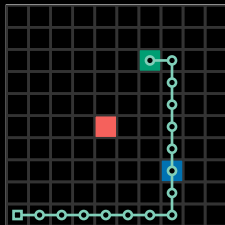


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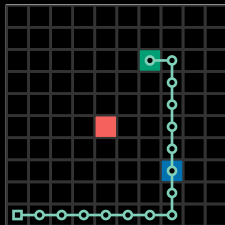
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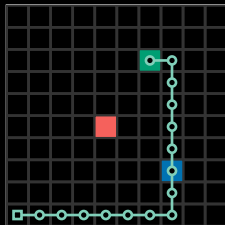
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$C(s_t)$: the state s at time t is the checkpoint

$$\psi = \underbrace{\Diamond G(s_t)}_{\text{reaches goal}} \wedge \underbrace{\neg C(s_t) \mathcal{U} G(s_t)}_{\text{reach checkpoint before goal}}$$



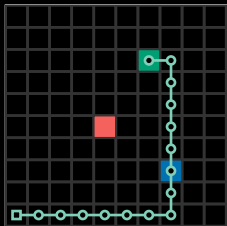
GRID WORLD

$F(s_t)$: the state s at time t contains an obstacle

$G(s_t)$: the state s at time t is the goal

$C(s_t)$: the state s at time t is the checkpoint

$$\psi = \underbrace{\Diamond G(s_t)}_{\text{reaches goal}} \wedge \underbrace{\neg C(s_t) \mathcal{U} G(s_t)}_{\text{reach checkpoint before goal}} \wedge \underbrace{\Box \neg F(s_t)}_{\text{always avoid obstacles}}$$



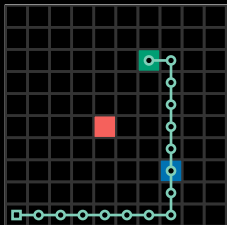
GRID WORLD

$F(s_t)$: the state s at time t contains an obstacle

$G(s_t)$: the state s at time t is the goal

$C(s_t)$: the state s at time t is the checkpoint

$$\psi = \underbrace{\Diamond G(s_t)}_{\text{reaches goal}} \wedge \underbrace{\neg C(s_t) \mathcal{U} G(s_t)}_{\text{reach checkpoint before goal}} \wedge \underbrace{\Box \neg F(s_t)}_{\text{always avoid obstacles}}$$




```
F = @formula s_t -> s_t == [5, 5]
G = @formula s_t -> s_t == [7, 8]
C = @formula s_t -> s_t == [8, 3]
ψ = LTLSpecification(@formula ◇(G) ∧ ℳ(¬G, C) ∧ □(¬F))
```


CONTINUUM WORLD

System	Property	Implementation


CONTINUUM WORLD

System	Property	Implementation
Continuum World		
		

CONTINUUM WORLD

System	Property	Implementation
Continuum World 	<i>“Reach the goal without hitting the obstacle”</i> $G(s_t)$: s_t is in the goal region $F(s_t)$: s_t is in the obstacle region $\psi = \Diamond G(s_t) \wedge \Box \neg F(s_t)$	

CONTINUUM WORLD

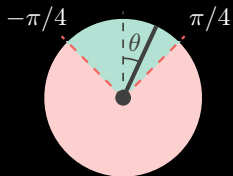
System	Property	Implementation
Continuum World 	<i>“Reach the goal without hitting the obstacle”</i> $G(s_t)$: s_t is in the goal region $F(s_t)$: s_t is in the obstacle region $\psi = \Diamond G(s_t) \wedge \Box \neg F(s_t)$	$G = \text{@formula } s \rightarrow \text{norm}(s - [6.5, 7.5]) \leq 0.5$ $F = \text{@formula } s \rightarrow \text{norm}(s - [4.5, 4.5]) \leq 0.5$ $\psi = \text{@formula } \Diamond(G) \wedge \Box(\neg F)$

INVERTED PENDULUM

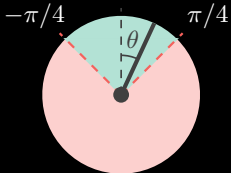
System	Property	Implementation

INVERTED PENDULUM

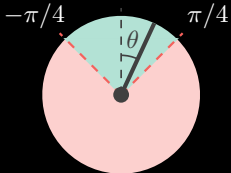
System	Property	Implementation
Inverted Pendulum		



INVERTED PENDULUM

System	Property	Implementation
Inverted Pendulum 	<i>“Keep the pendulum balanced”</i> $B(s_t): \theta_t \leq \pi/4$ $\psi = \Box B(s_t)$	

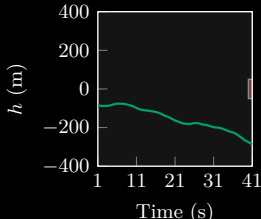
INVERTED PENDULUM

System	Property	Implementation
<p>Inverted Pendulum</p> 	<p><i>“Keep the pendulum balanced”</i></p> $B(s_t): \theta_t \leq \pi/4$ $\psi = \Box B(s_t)$	$B = \text{@formula } s \rightarrow \text{abs}(s[1]) \leq \pi/4$ $\psi = \text{@formula } \Box(B)$

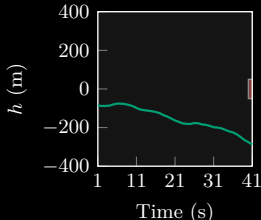
AIRCRAFT COLLISION AVOIDANCE

System	Property	Implementation

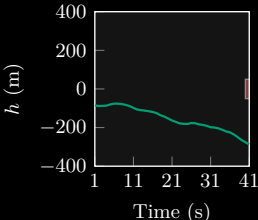
AIRCRAFT COLLISION AVOIDANCE

System	Property	Implementation
Aircraft Collision Avoidance		
		

AIRCRAFT COLLISION AVOIDANCE

System	Property	Implementation
<p>Aircraft Collision Avoidance</p>  <p>Time (s)</p>	<p><i>“Ensure at least 50 meters relative altitude between 40 and 41 seconds”</i></p> <p>$S(s_t): h_t \geq 50$</p> <p>$\psi = \Box_{[40,41]} S(s_t)$</p>	

AIRCRAFT COLLISION AVOIDANCE

System	Property	Implementation
<p>Aircraft Collision Avoidance</p>  <p>Time (s)</p>	<p><i>“Ensure at least 50 meters relative altitude between 40 and 41 seconds”</i></p> $S(s_t): h_t \geq 50$ $\psi = \Box_{[40,41]} S(s_t)$	$S = \text{@formula } s \rightarrow \text{abs}(s[1]) \geq 50$ $\psi = \text{@formula } \Box(40:41, S)$

RESOURCES

RESOURCES



`github.com/sisl/SignalTemporalLogic.jl`

RESOURCES



github.com/sisl/SignalTemporalLogic.jl



github.com/mossr/STL-mini-lecture