A Markov Decision Process-based approach for trajectory planning with clothoid tentacles

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I. Background

History (pt. 1)

- Bernoulli (1694) - the ‘elastica’
- Euler (1744)
  - Tackled inverse problem of ‘straightening out’ a curved lamina
  - Original (still viable) series expansion for integral defining the curve
- Euler (1781) - the ‘Euler spiral’
  - Determined limits to integral expression

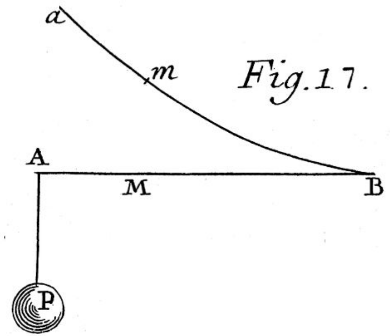


Figure 4: Euler’s drawing of his spiral, from Tabula V of the Additamentum.

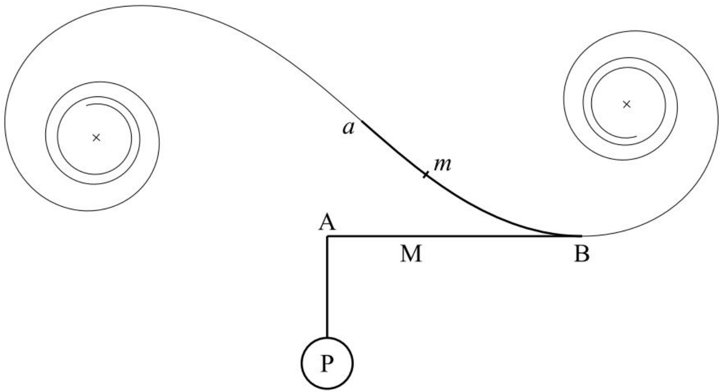
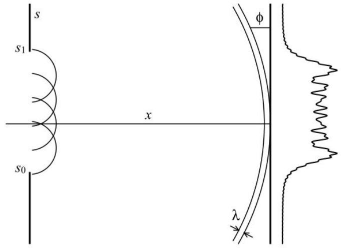


Figure 5: Reconstruction of Euler’s Fig. 17, with complete spiral superimposed.

History (pt. 2)

- Fresnel (1818) - the ‘Fresnel Integral’
  - Diffraction of a monochromatic light source through a slit
  - Independent derivation to describe the resulting intensity
- Cornu (1874) - the ‘Cornu Spiral’
  - Found that intensity is inversely proportional to arclength
- Cesaro (1886) - the ‘Clothoid’



$$S(z) = \int_0^z \sin\left(\frac{\pi t^2}{2}\right) dt$$

$$C(z) = \int_0^z \cos\left(\frac{\pi t^2}{2}\right) dt$$

Figure 8: Diffraction through a slit.

$$I = \left(S(s_1) - S(s_0)\right)^2 + \left(C(s_1) - C(s_0)\right)^2$$

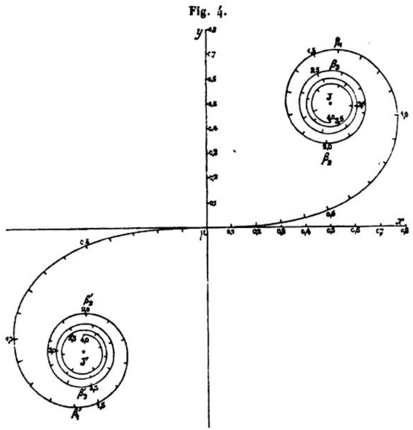


Figure 9: Cornu’s plot of the Fresnel integrals.

History (pt. 3)

- Talbot (1890) - the ‘Railway Transition Spiral’
  - Independent derivation of a curve with the following properties:
    - Curvature increases proportionately to the distance along that curve from the point of a spiral
    - Minimizes variation of curvature across total length
    - Keeps passengers comfy!

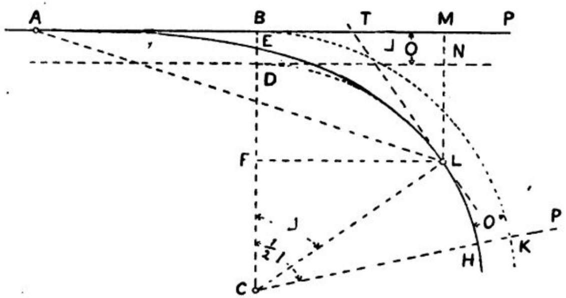


Figure 10: Talbot’s Railway Transition Spiral.

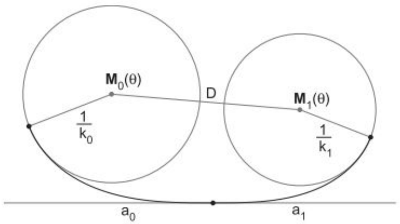


Fig. 4. A pair of clothoids from circle to circle forming a *C*-curve.

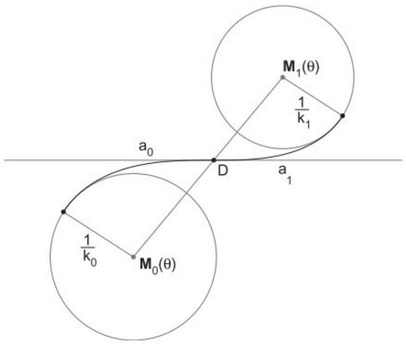


Fig. 3. A pair of clothoids from circle to circle forming an *S*-curve.

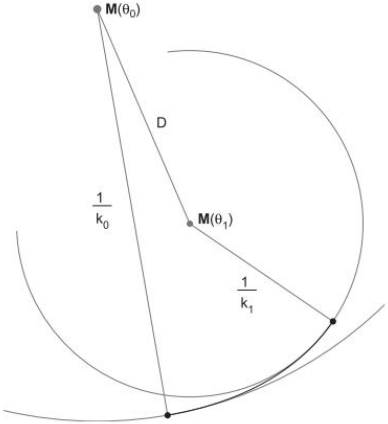
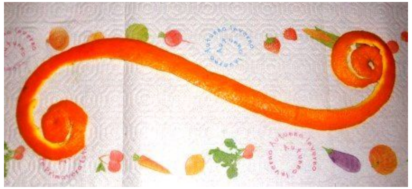


Fig. 2. A single clothoid from one circle to another circle.

Approximation of the clothoid curve



- costly to compute Fresnel integrals
- Taylor series used historically
- Hermitian splines used more recently

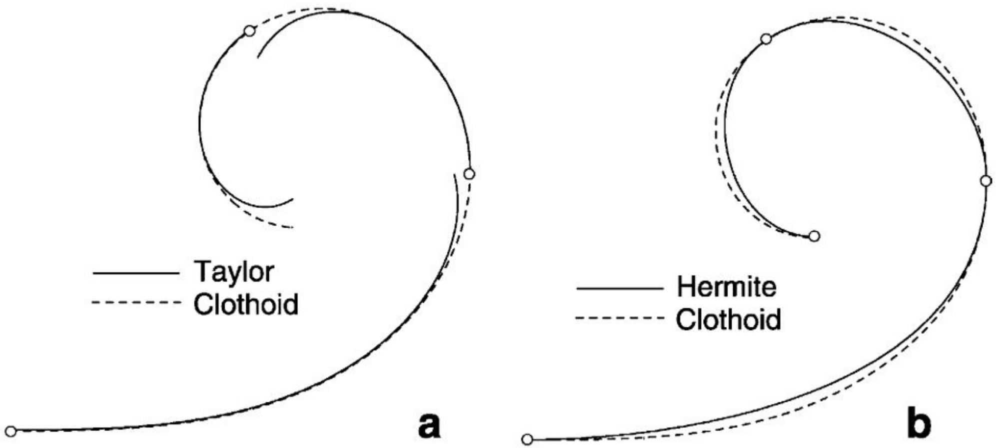


Fig. 2. Piecewise polynomial approximations of a clothoid: (a) Hermite, (b) Taylor.

II. A Markov Decision Process-based approach for trajectory planning with clothoid tentacles

(Mouhagir et al., 2016)

The goal

reactive local trajectory planning

contextualized by global reference path

contextual choice of best candidate trajectory (MDP)

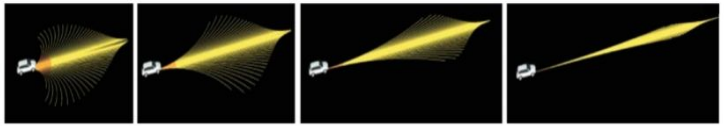
designed to respond to obstacles in real-time

trajectories modeled as clothoid curves

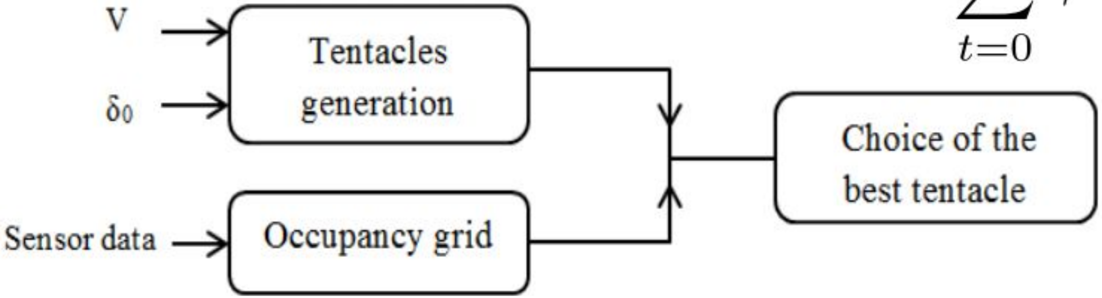
Prior work

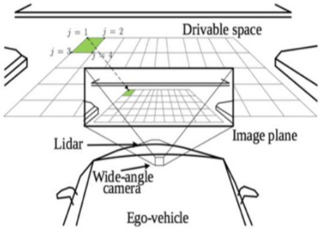
- RRT
  - good for path planning involving obstacles + non-holonomic constraints
  - under presence of heavy traffic, RRTs check every possible collision for every expanded node
- Lattice planners
  - guaranteed optimality, smoothness, resolution complete
  - curvature discontinuity
- Lateral curve shifting w/ cost function
  - not very robust (prone to discontinuities)

Methods (pt. 1) - Overview



$$V = \sum_{t=0}^{h} \gamma^t r_t$$





Methods (pt. 2) - Occupancy Grid

- 800x800 Bayesian occupancy grid
- binary representation (free/occupied)

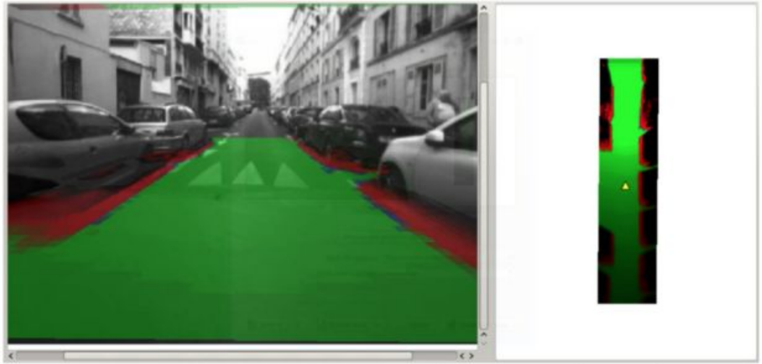
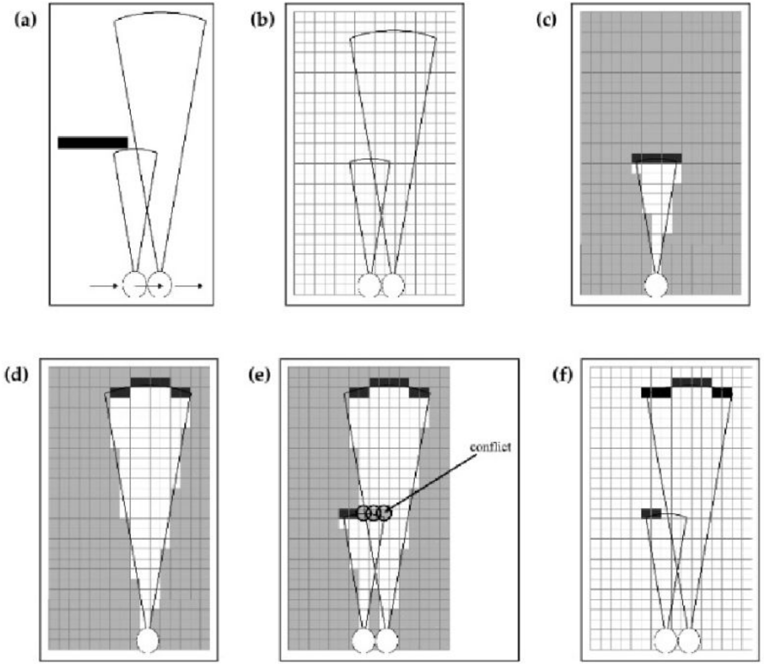


Figure 3: Occupancy grid example of a road



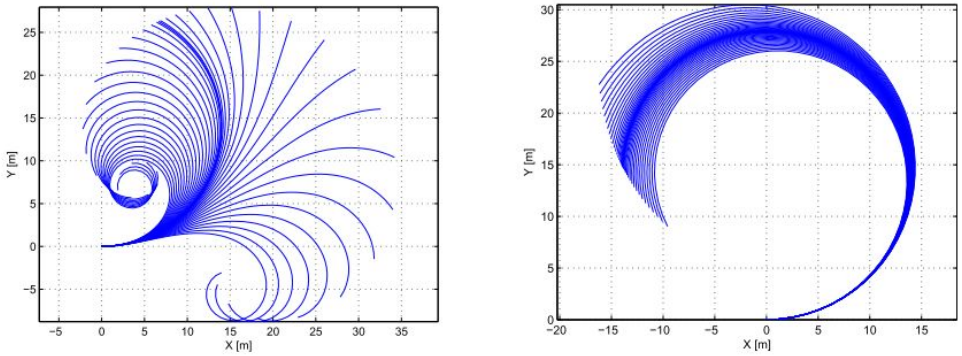
Methods (pt. 3) - Generation of clothoid tentacles

- Fixed number of tentacles generated (n_t)
- approximated at n_s timesteps

$t_0 = 7s \qquad L_0 = 5m.$

$$L_{tentacle}(m) = \begin{cases} t_0 \, V_x - L_0 & V_x > 1(m/s) \\ 2(m) & V_x \leq 1(m/s) \end{cases} \qquad (2)$$

$$\rho_0 = \frac{\tan \delta_0}{L} \qquad \Bigg| \qquad \rho = \frac{2}{k^2} s$$



(a) $V_x = 6 \, m/s \, , \delta_0 = 0.3 \, rad$ (b) $V_x = 10 \, m/s \, , \delta_0 = 0.3 \, rad$

Figure 5: Examples of set of clothoid tentacles

Methods (pt. 4) - Choosing the best tentacle

- Typical methods are computationally costly
  - e.g., geometric ripple minimization
- MDP formulation + series approximation of clothoids

$$V = \sum_{t=0}^{h} \gamma^t r_t$$

estimate distance to first obstacle

estimate smoothness of steering angle variation

reflects similarity to reference trajectory

$$V_{combined} = a_0 V_{clearance} + a_1 V_{curvature} + a_2 V_{trajectory}$$

Methods (pt. 6) - MDP formulation; S, A, T

- **State space** :
  - circles at each step in trajectories
  - width of vehicle + security margin
  - each tentacle composed of n_s states
  - deterministic state space reduction

- **Action space** :
  - # of tentacles (n_t)

- **Transition probabilities** :
  - no possible transition between tentacles
  - p(s’ | s, a) = 1 for a unique s’ in S
  - p(s’’ | s, a) = 0 for all s’’ =/= s’

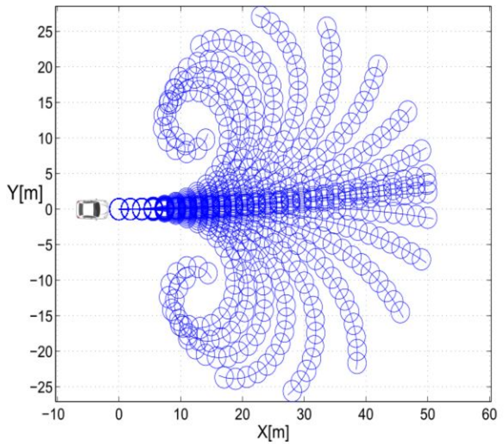


Figure 7: States of MDP model with clothoid tentacles

Methods (pt. 7a) - MDP formulation; R

reward term representing closeness to reference trajectory

negative reward due to occupied cells

$$R(s_f) = \sum_{k=0}^{n_s} \gamma_t^k R(s_{i_k|trajectory}) + \sum_{k=0}^{n_s} \gamma_o^k R(s_{i_k|occupied}) + \sum_{k=0}^{n_s} \gamma_f^k R(s_{i_k|free}) + R(s_f \mid left) \tag{5}$$

positive reward due to free cells

positive reward for overtaking on the left

Methods (pt. 7b) - MDP formulation; R

$$l_c = \frac{V_x^2}{2a_m} + l_s$$

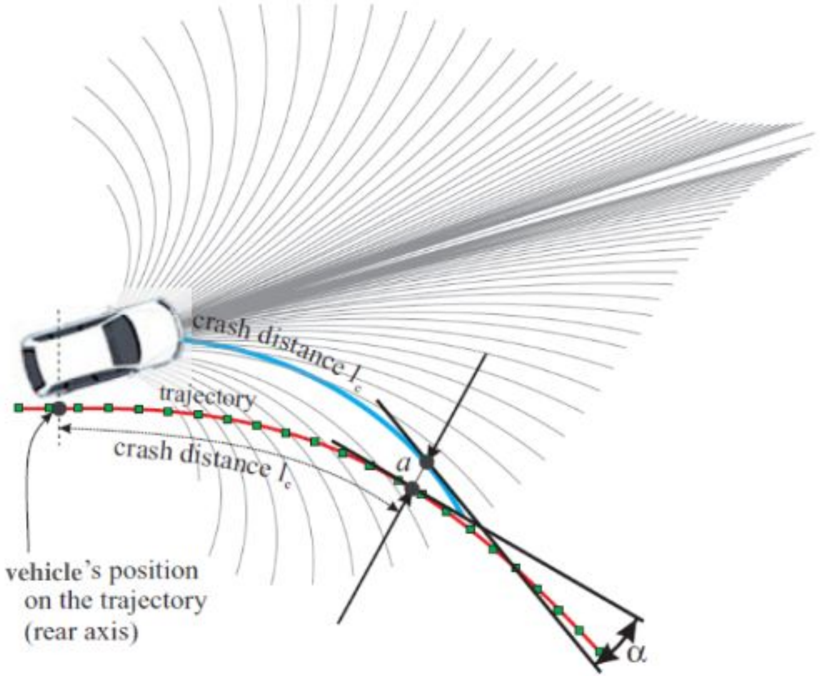
(crash distance)

$$d_i = a_i + c_\alpha \alpha_i$$

(combined distance metric)

$$d = \sum_{i=1}^{n} \lambda_i d_i$$

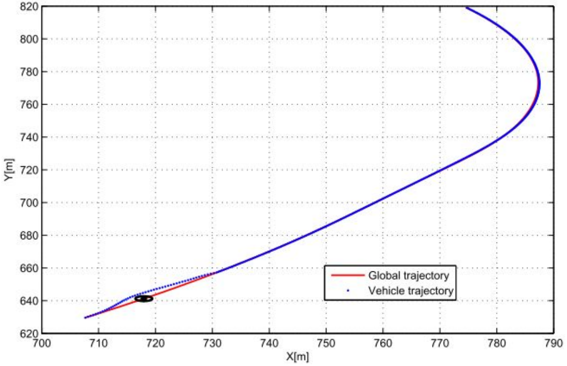
$$R(s_{|trajectory}) = R_{trajectory} - d$$



Results

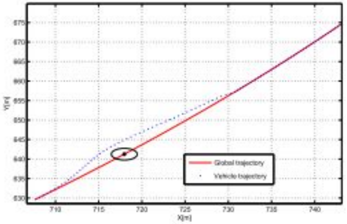
c_α	γ_t	γ_o	γ_f	κ_1	κ_2	κ_3	λ_1	λ_2	λ_3
0.7	0.99	0.95	0.99	$\frac{1}{10}$	$\frac{1}{2}$	1	10	2	$\frac{1}{3}$

Table I: The values of various parameters

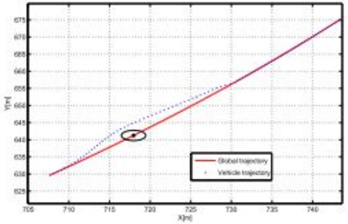


mean err. :$0.08\,m$
min err. :$0\,m$
max err. :$0.2\,m$
std deviation: $0.09\,m$

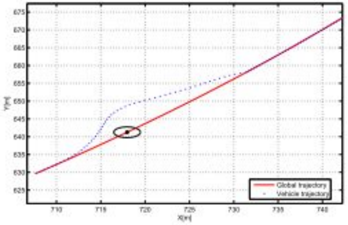
Figure 10: Overtaking manoeuver and following the reference trajectory and different performance measures



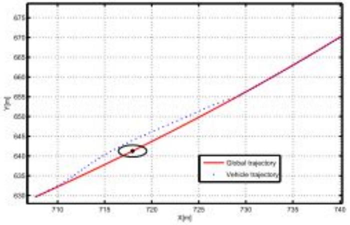
(a) $n_s = 16\ \gamma_o = 0.95$



(b) $n_s = 16\ \gamma_o = 0.83$



(c) $n_s = 10\ \ \gamma_o = 0.95$



(d) $n_s = 25\ \ \gamma_o = 0.95$

Figure 11: Influence of parameters γ_o and n_s with $V = 6\,m/s$

Strengths

- memoryless
- tractable curve evaluation
- tunable behavior of approach

Weaknesses

- question of approximation method
- results are for low velocities
    - (5 m/s ~ 11 mph)

Future work

- incorporation of stochastic transitions (between clothoids)
- evidential occupancy grid (better track external dynamics)
- implementation of algorithm on hardware

Citations

1. H. Mouhagir, R. Talj, V. Cherfaoui, F. Guillemard, and F. Aioun, "**A Markov Decision Process-based approach for trajectory planning with clothoid tentacles**," *IEEE Intell. Veh. Symp. Proc.*, vol. 2016-Augus, pp. 1254–1259, 2016, doi: 10.1109/IVS.2016.7535551.
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