Calculus 3

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Section 3.1: Vector Valued Functions and Space Curves

Definition

A **vector-valued function** is a function of the form

$$r(t) = f(t)i + g(t)j \quad or \quad r(t) = f(t)i + g(t)j + h(t)k$$

where the **component functions** f. g. and h, are real-valued functions of the parameter t. Vector-valued functions are also written in the form

$$r(t) = \langle f(t), g(t) \rangle \quad or \quad r(t) = \langle f(t), g(t), h(t) \rangle$$

In both cases, the first form of the function defines a two-dimensional vector valued function; the second form describes a three-dimensional vector valued function.

- The parameter t can lie between two real numbers: $a \leq t \leq b$.
- Another possibility is that the value of t might take on all real numbers.
- Last, the component functions themselves may have domain restrictions that enforce restrictions on the value of t.
- We often use t as a parameter because t can represent time.

Example: Evaluate the functions at $r(0)$ and $r\left(\frac{\pi}{2}\right)$

$r(t) = \langle t, 1 \rangle$

At $r(0)$

$$r(0) = \langle 0, 1 \rangle$$

At $r\left(\frac{\pi}{2}\right)$

$$r(\frac{\pi}{2}) = \left\langle \frac{\pi}{2}, 1 \right\rangle$$

$r(t) = \langle t^2, t \rangle$

At $r(0)$

$$r(0) = \langle 0^2, 0 \rangle$$
$$r(0) = \langle 0, 0 \rangle$$

At $r\left(\frac{\pi}{2}\right)$

$$r(\frac{\pi}{2}) = \left\langle \frac{\pi}{2}^2, \frac{\pi}{2} \right\rangle$$
$$r(\frac{\pi}{2}) = \left\langle \frac{\pi^2}{4}, \frac{\pi}{2} \right\rangle$$