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Robust Control Design for Inverted Pendulum System with Uncertain Disturbances

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***Abstract*—This paper inheres presents the designing of robust control of inverted pendulum system which is a class of nonlinear system consist of a wagon, a pole and a dc motor. The mathematical modeling of inverted pendulum system is transformed into an approximate normal form using lie derivatives. The proposed controllers are designed using conventional sliding, integral sliding and super twisting sliding mode control (SMC) approach which is a class of robust control technique. Simulation results are demonstrated to depict the effectiveness of proposed controllers. Finally, a comparative analysis has been made between both the controllers.**

Keywords*—*Inverted Pendulum; Lie Derivative; Nonlinearity; Robustness; Sliding Mode Control; Super Twisting

I. Introduction

The problem of balancing an inverted pendulum presents a unique research opportunity due to its varied applications. It is considered as an ideal benchmark for testing of extensive range of modern and classical control techniques [1]. The most prevalent application is by human beings, as the upright posture of a person need to make constant adjustments to maintain balance while walking or standing. Additional examples includes metronome, seismometers, control of airplanes, launching of satellites, balancing of mobile manipulator and robots [2], [3]. Robust controllers are the key elements to balance systems such as that of inverted pendulum. They are used to make the system robust to any disturbances and stabilize it. Numerous researches have been done in designing the robust controller for inverted pendulum system. Methods such as linear quadratic regulator (LQR), proportional-integral-derivative (PID) control, SMC, fuzzy logic, neural network and various other algorithms have been used to design controllers [3]-[8].

Many researchers have been designed controllers on the basis of linearized models of inverted pendulum. Sliding mode control for the transformation to an inverted pendulum system mode of a mobile robot with wheel-arms is reported in [3]. Linear controllers are designed on the basis of linearized model while the sliding mode controller is designed on the basis of nonlinear model of inverted pendulum. The problem with linear controllers is that they are not very effective in the situations where the system state is far away from equilibrium point [2]-[5]. A dynamics based nonlinear acceleration control of the mobile inverted pendulum (MIP) system with a slider mechanism is designed in [6]. The basic idea is to control the translational acceleration and deceleration of the MIP in a dynamical manner. In [7], a nonlinear disturbance observer (NDO) based dynamic surface controller (DSC) is implemented to control the mobile wheeled inverted pendulum (MWIP) system. An automatic motion control for an under-actuated wheeled inverted pendulum (WIP) model is presented for modeling of a large range of two wheeled modern vehicles [8].

The sliding mode control approach consists two phases: (i) reaching phase and (ii) sliding phases. For the implementation of the SMC based controllers, there are two steps: (i) selection of sliding surface such that the desired close loop performance can be achieved and (ii) designing of robust control law that forces the state trajectories towards the sliding surface and stay thereafter [9], [10]. In recent years, many papers have presented motivating solutions to overcome issues such as uneven uncertainties, systems with time delay and nonlinear jump systems. But these methods are too complex and have huge computation volume. In overcoming the existing challenges, the major one is the reduction and elimination of undesirable chattering in SMC. The chattering can also be eliminated by using saturation function [11], [12]. The higher order system is converted into lower order system in sliding mode approach. It permits control algorithm which is simple to implement and robust in action. Sliding mode controllers demonstrate great control performance with no correlation to the system parameters and disturbances. It is also well acknowledged for its sturdiness to uncertainties [13], [14]. Super twisting and higher order sliding mode observer based controller for the position control of industrial emulator is illustrated in [15], [16].

In this paper, the preferred path has been traced using nonlinear model of inverted pendulum. The robust controllers are designed on the premises of conventional sliding mode, integral sliding mode and super twisting sliding mode control (STSMC) approaches in presence of the disturbances. Comparative analysis has been done to highlight the robustness issues of the proposed control scheme. The reminders of the manuscript are as follows.

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Section II consists of the mathematical modeling of the inverted pendulum system, brief introduction of the Lie derivative and SMC technique. Section III expressing the plant model in the approximate normal form. In Section IV, problem statement is explored. The robust control design using SMC, ISMC and STSMC techniques are presented in Section V. Section VI deals with the vivid simulation of the control schemes. Section VII contains the comparative analysis among all the robust controllers. Finally, the conclusion is reported in the last section.

II. PRELIMINARIES

A. Mathematical Modeling of Inverted Pendulum System

The inverted pendulum taken into consideration in this paper is a fourth order, multi variable unstable system [5]. It consists of a wagon, a pole and a motor which is steered by a controller. The main objective is to balance the bob in upright position and prevent it from falling. The inverted pendulum is shown in Fig. 1.

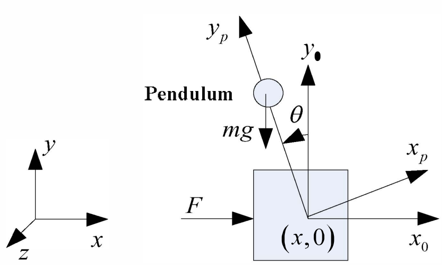

Fig. 1: Inverted Pendulum System

The inverted pendulum system consists of a wagon, a pole and a motor. Under normal conditions the pole of the pendulum would be in downward position. If the pole is held upright it would have the tendency to swing downwards, hence the supply voltage to the rotor works as a controller and moves the wagon to maintain the upright position of the pole and thus forming a robust inverted pendulum system. The system parameters are given in Table I.

TABLE 1: PARAMETERS OF THE INVERTED PENDULUM SYSTEM

Parameters	Values
M = Mass of the wagon	2 kg
m = Pendulum mass	0.026 kg
I_p = Moment of inertia of pendulum	0.000362 kg-m2
l = Half-length of pendulum	0.1 kg
g = Acceleration due to gravity	9.8 m/sec2

The dynamical modeling of the inverted pendulum system is derived using Lagrangian formulation is given by,

$$\left(m+M\right)\ddot{x}-ml\left(\ddot{\theta}\cos\left(\theta\right)-\dot{\theta}^{2}\sin\left(\theta\right)\right)=F \tag{1}$$

$$\left(ml^{2}+I_{m}\right)\ddot{\theta}-ml\cos\left(\theta\right)\ddot{x}-mgl\sin\left(\theta\right)=0 \tag{2}$$

where x =distance travelled by the cart, θ = angle of pendulum, and F = Force exerted to the wagon.

The complete dynamical behavior of the inverted pendulum system in the form of differential equations is as follow,

$$\ddot{x}=\frac{F.a-\left(ac\sin\left(\theta\right)\right)\dot{\theta}^{2}+c^{2}g\cos\left(\theta\right)\sin\left(\theta\right)}{\left(ab-c^{2}\cos^{2}\left(\theta\right)\right)} \tag{3}$$

$$\ddot{\theta}=\frac{c\cos\left(\theta\right)\ddot{x}+cg\sin\left(\theta\right)}{a} \tag{4}$$

where, $a=(ml^{2}+\mathrm{I}_{m})$, $b=(m+M)$, and $c=ml$.

The inverted pendulum system (3), (4) can be expressed in form of state space model given by,

$$\dot{x}_{1}=x_{2} \tag{5a}$$

$$\dot{x}_{2}=\frac{c\sin\left(x_{3}\right)\left(cg\cos\left(x_{3}\right)-ax_{4}^{2}\right)}{ab-c^{2}\cos^{2}\left(x_{3}\right)}+\frac{a}{ab-c^{2}\cos^{2}\left(x_{3}\right)}u \tag{5b}$$

$$\dot{x}_{3}=x_{4} \tag{5c}$$

$$\dot{x}_{4}=\frac{c^{2}\sin\left(x_{3}\right)\cos\left(x_{3}\right)\left(cg\cos\left(x_{3}\right)-ax_{4}^{2}\right)}{a\left(ab-c^{2}\cos^{2}\left(x_{3}\right)\right)}+\frac{cg\sin\left(x_{3}\right)}{a}+\frac{c\cos\left(x_{3}\right)}{ab-c^{2}\cos^{2}\left(x_{3}\right)}u \tag{5d}$$

$$y=x_{3} \tag{5e}$$

where, $X=\left[x\quad\dot{x}\quad\theta\quad\dot{\theta}\right]=\left[x_{1}\quad x_{2}\quad x_{3}\quad x_{4}\right]$=state vector, u =input and y is the output.

B. Lie Derivatives

Consider the nonlinear SISO system described by the state equation,

$$\dot{\bar{x}}=\bar{f}\left(\bar{x}\right)+\bar{g}\left(\bar{x}\right)\bar{u}$$
$$\bar{y}=\bar{h}\left(\bar{x}\right)$$

where $\bar{f}$, $\bar{g}$, and $\bar{h}$ are sufficiently smooth in a domain $\bar{D}\in\Re^{n}$. The mappings $\bar{f}:\bar{D}\rightarrow\Re^{n}$ and $\bar{g}:\bar{D}\rightarrow\Re^{n}$ are called vector fields on $\bar{D}$ and $\bar{x}$ is an $n-$dimensional state vector that is assumed to be measurable, $\bar{u}$ is a scalar input, and $\bar{y}$ is a scalar output. The derivative $\dot{\bar{y}}$ is given by [16],

$$\dot{\bar{y}}=\frac{\partial\bar{h}}{\partial\bar{x}}\left[\bar{f}\left(\bar{x}\right)+\bar{g}\left(\bar{x}\right)\bar{u}\right]=L_{\bar{f}}\bar{h}\left(\bar{x}\right)+L_{\bar{g}}\bar{h}\left(\bar{x}\right)\bar{u}$$

where the Lie derivative of a scalar function $\bar{h}\left(\bar{x}\right)$ with respect to vector function $\bar{f}\left(\bar{x}\right)$ is given by [47] ,

$$L_{\bar{f}}\bar{h}\left(\bar{x}\right)=\frac{\partial\bar{h}}{\partial\bar{x}}\bar{f}\left(\bar{x}\right)$$

This is the familiar notion of the derivative of $\bar{h}$ along the trajectories of the system $\dot{\bar{x}}=\bar{f}\left(\bar{x}\right)$.

C. Sliding Mode Control Technique

Sliding mode control is a robust control method for nonlinear systems. It is a special form of variable structure system. By the application of a discontinuous control signal SMC changes the output of the nonlinear system and forces it to slide along the sliding surface. The sliding surface is based upon the system's normal behavior [2].

The purpose of the control law is to force the plant's state path onto a predefined surface in the state space. It also forces it to maintain that path for a given period of time. This surface is called switching surface. The feedback path's gain vary according to a rule which be governed by the plant's state trajectory whether it is above or below the switching surface [15]. Control function is given as

$$\bar{u} = \begin{cases} \bar{u}^{+}(\bar{x}) & S(\bar{x}) > 0 \\ \bar{u}^{-}(\bar{x}) & S(\bar{x}) < 0 \end{cases}$$

Once, the trajectory is followed the controller maintains the plant's state on the surface for further time and the plant's path slides on this surface.

Let a nonlinear system be defined as

$$\bar{x}^{n} = \bar{f}(\bar{x},t) + \bar{g}(\bar{x},t)\bar{u}(t)$$

Here $\bar{x}(t)$ is the state vector and $\bar{u}(t)$ is the input. $\bar{f}(\bar{x},t)$ and $\bar{g}(\bar{x},t)$ are nonlinear functions of time and states. They are bounded by the upper limits of the continuous function $\bar{x}(t)$. To track the specified trajectory $\bar{x}_d(t)$, a time varying surface $S(\bar{x},t)$ is defined and equated to zero.

$$S(\bar{x},t) = \left(\frac{d}{dt} + \mu\right)^{n-1} \tilde{x}(t) = 0$$

where μ is a strict positive constant and is defined as the bandwidth of the system, n is order of the system. $\tilde{x}(t) = \bar{x}(t) - \bar{x}_d(t)$ is the reference tracking error.

III. Normal form Modeling of the Inverted Pendulum System

Differentiating output y in (5e), the relative degree is obtained by 2 at $x = 0$ [16].

Now, considering,

$$z = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} h(x) \\ L_f h(x) \end{bmatrix} \equiv \begin{bmatrix} x_3 \\ x_4 \end{bmatrix}$$

where, $L_f h(x) = \dfrac{\partial h(x)}{\partial x} f(x)$ is the lie derivative of $h(x)$ w.r.t. $f(x)$.

The dynamics of inverted pendulum system (5) can be rewritten in the normal form given as,

$$\dot{z} = \begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \end{bmatrix} = \begin{bmatrix} L_f h(x) \\ \bar{b}(x) + \bar{a}(x)u \end{bmatrix} \equiv \begin{bmatrix} x_4 \\ f_4(x) + g_4(x)u \end{bmatrix} \quad (6)$$

$$y = z_1 \equiv x_3$$

where, $\bar{a}(x) = g_4(x) = \dfrac{c\cos(x_3)}{ab - c^2\cos^2(x_3)}$,

$\bar{b}(x) = f_4(x)$

$$\bar{b}(x) = \frac{c^2 \sin(x_3)\cos(x_3)\left(cg\cos(x_3) - ax_4^2\right)}{a\left(ab - c^2\cos^2(x_3)\right)} + \frac{cg\sin(x_3)}{a}.$$

IV. Problem Formulations

The dynamics of inverted pendulum system (5) can be rewritten in the normal form given as,

$$\dot{z} = f(z) + g(z)u + d(t)$$
$$y = z_1 \quad (7)$$

where, $f(z) = \begin{bmatrix} x_4 \\ f_4(x) \end{bmatrix}$, $g(z) = \begin{bmatrix} 0 \\ g_4(x) \end{bmatrix}$, and $d(t)$ is the external disturbances which are bounded.

The control objective of this paper is to design a nonlinear robust controller for inverted pendulum system, where the plant output y tracks the desired trajectory y_d, i.e. $\lim_{t\to\infty}(y - y_d) = 0$. In the next section the robust controllers are designed.

V. Nonlinear Robust Controller Design

A. Conventional Sliding Mode Controller Design

The error dynamics of inverted pendulum system (7) is defined as,

$$e = y - y_d \quad (8)$$

where, y_d=desired trajectory. Now, the sliding surface for the system (7) is selected as [17],

$$S_1 = \dot{e} + \lambda_1 e = 0 \quad (9)$$

Differentiating (9), gives

$$\dot{S}_1 = \ddot{e} + \lambda_1 \dot{e} = 0$$

$$\dot{S}_1 = f_4(x) + g_4(x)u - \ddot{y}_d + \lambda_1(x_4 - \dot{y}_d) = 0 \quad (10)$$

Stabilize the dynamical system (10), the sliding mode controller u is chosen as,

$$u = [g_4(x)]^{-1}\left[-\hat{f}_4(x) + \ddot{y}_d - \lambda_1(x_4 - \dot{y}_d) - K_1|S_1|^{\alpha_1}\operatorname{sgn}(S_1)\right] \quad (11)$$

where, K_1 is designed to ensure sliding with $0 < \alpha_1 < 1$.

Using (11), (10) becomes,

$$\dot{S}_1 = \tilde{f}_4(x) - K_1|S_1|^{\alpha_1}\operatorname{sgn}(S_1) = 0 \quad (12)$$

where, $\tilde{f}_4(x) = f_4(x) - \hat{f}_4(x)$.

B. Integral Sliding Mode Controller Design

The integral sliding surface for the system (7) is selected as [17],

$$S_2 = \dot{e} + 2\lambda_2 e + \lambda_2^2 \int_0^t e(t)\,d\tau \tag{13}$$

Differentiating (13), gives

$$\dot{S}_2 = \ddot{e} + 2\lambda_2 e + \lambda_2{}^2 e \tag{14}$$

$$\dot{S}_2 = f_4(x) + g_4(x)u - \ddot{y}_d + 2\lambda_2(x_4 - \dot{y}_d) + \lambda_2{}^2(x_3 - y_d) \tag{15}$$

To stabilize the dynamical system (15), the integral sliding mode controller u is chosen as,

$$u = [g_4(x)]^{-1}\Big[\ddot{y}_d - 2\lambda_2(x_4 - \dot{y}_d) - \lambda_2{}^2(x_3 - y_d) - \hat{f}_4(x) - K_2|S_2|^{\alpha_2}\operatorname{sgn}(S)\Big] \tag{16}$$

where, K_2 ensures sliding with $0 < \alpha_2 < 1$.

Using (16), (15) becomes,

$$\dot{S}_2 = \tilde{f}_4(x) - K_2|S_2|^{\alpha_2}\operatorname{sgn}(S_2) = 0 \tag{17}$$

where, $\tilde{f}_4(x) = f_4(x) - \hat{f}_4(x)$.

C. Super Twisting Sliding Mode Controller Design

The super twisting sliding surface for the system (7) is selected as [16],

$$S_3 = \dot{e} + \lambda_3 e = 0 \tag{18}$$

Differentiating (9), gives

$$\dot{S}_3 = \ddot{e} + \lambda_3 \dot{e} = 0$$

$$\dot{S}_3 = f_4(x) + g_4(x)u - \ddot{y}_d + \lambda_3(x_4 - \dot{y}_d) = 0 \tag{19}$$

Stabilize the dynamical system (10), the sliding mode controller u is chosen as,

$$u = [g_4(x)]^{-1}\Big[-\hat{f}_4(x) + \ddot{y}_d - \lambda_3(x_4 - \dot{y}_d) - K_3|S_3|^{\alpha_3}\operatorname{sgn}(S_3) - \int_0^t K_4 \operatorname{sgn}(S_3)\,d\tau\Big] \tag{20}$$

where, K_3 and K_4 are designed to ensure sliding with $\alpha_3 = 0.5$.

Using (20), (19) becomes,

$$\dot{S}_3 = \tilde{f}_4(x) - K_3|S_3|^{\alpha_3}\operatorname{sgn}(S_3) - \int_0^t K_4 \operatorname{sgn}(S_3)\,d\tau = 0 \tag{21}$$

where, $\tilde{f}_4(x) = f_4(x) - \hat{f}_4(x)$.

The sliding conditions for the above robust controllers are given by, $S_i^T \dot{S}_i < 0$, $S_i \dot{S}_i \le -\eta_i |S_i|$, $\eta_i > 0$, with assumption $|\tilde{f}_4|(x) \le F$, $F_2 > 0$, the choice of $K_i \ge (\eta_i + F_i)$, $\forall\, i = 1,\ldots,4$ ensure sliding. In the next section simulation results are presented for both the robust controllers.

VI. SIMULATION RESULTS

The inverted pendulum system is operated in the closed loop control system using the following parameters,

$K_1 = K_2 = K_3 = K_4 = 5$, $\lambda_1 = \lambda_2 = \lambda_3 = 2$, $\alpha_1 = \alpha_2 = 0.5$.

The initial conditions of the inverted pendulum system are $[0 \quad 0 \quad 0.1 \quad 0]$.

The simulation studies of nonlinear robust controllers are done by adding a disturbance term in the plant given by, $d(t) = [0.02\sin(t) \quad 0.02\sin(t)]^T$.

The desired trajectory for the tracking of angle of pendulum is chosen as, $y_d = 0.3\sin(0.5t) + 0.2\cos(0.5t)$.

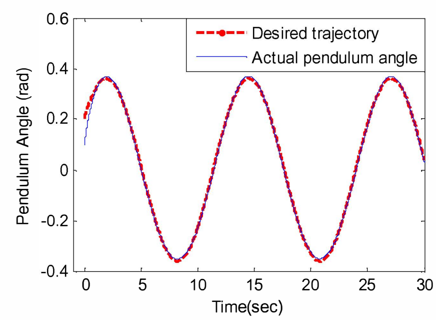

Fig. 2: Tracking of Angle of Pendulum with Disturbances

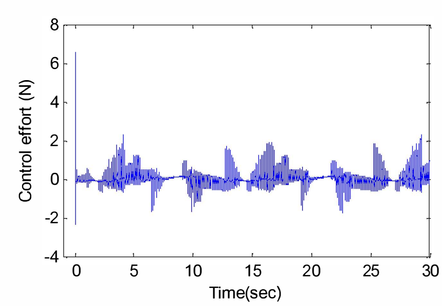

Fig. 3: Control Effort Applied to the Plant with Disturbances

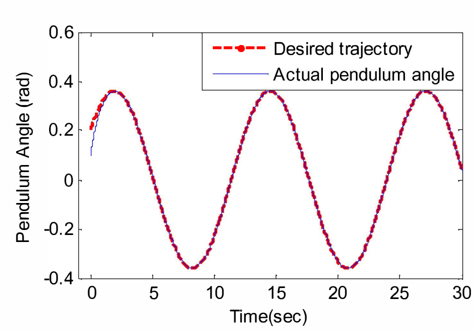

Fig. 4: Tracking of Angle of Pendulum without Disturbances

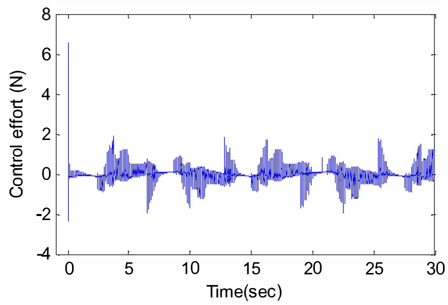

Fig. 5: Control Effort Applied to the Plant without Disturbances

The tracking of angle of pendulum with and without disturbances is given in Fig. 2 and Fig. 4 respectively. It can be seen that the angle of pendulum converge to the desired trajectory very quickly. The control effort which applied to the wagon of the inverted pendulum system with and without disturbances is plotted in Fig. 3 and Fig. 5 respectively.

Case 1: Conventional sliding mode control:

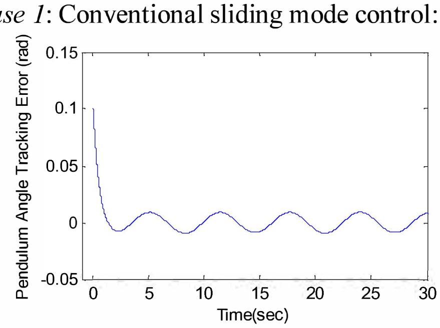

Fig. 6: Tracking Error of Pendulum Angle with Disturbances

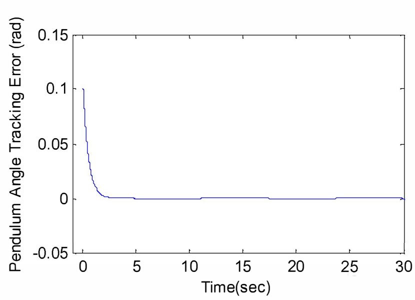

Fig. 7: Tracking Error of Pendulum Angle without Disturbances

Case 2: Integral sliding mode control:

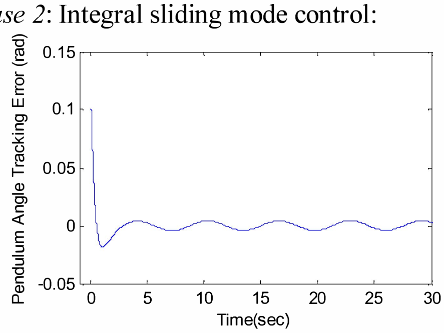

Fig. 8: Tracking Error of Pendulum Angle with Disturbances

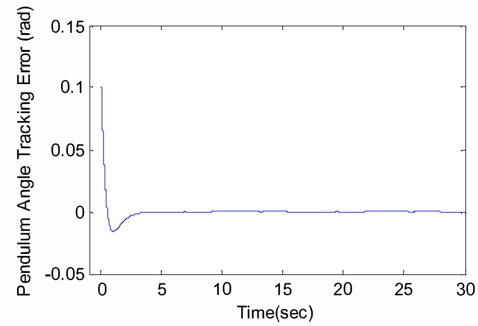

Fig. 9: Tracking Error of Pendulum Angle without Disturbances

Case 3: Super twisting sliding mode control:

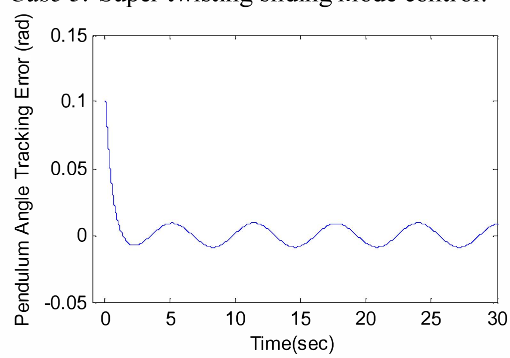

Fig.10. Tracking Error of Pendulum Angle with Disturbances

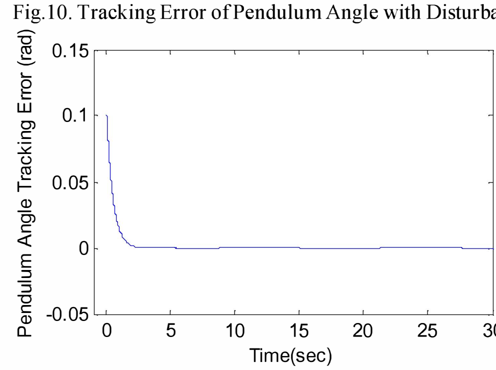

Fig. 11: Tracking Error of Pendulum Angle without Disturbances

Figure 6 to Fig. 11 represents the tracking error of the angle of pendulum using CSMC, ISMC and STSMC with and without disturbances respectively. It can be observed that the tracking error is approximately equals to zero. These simulation results demonstrate the robustness of the proposed control schemes. A comparative analysis has been made among all three robust controller using error analysis (see Table II).

TABLE 2: ROOT MEAN SQUARE VALUES OF PENDULUM ANGLE TRACKING ERROR

Controller Applied to the Inverted Pendulum System	**Pendulum Angle Tracking Error**	
	without Disturbances	**with Disturbances**
CSMC	0.00568	0.008322
ISMC	0.004319	0.005504
STSMC	0.005559	0.00842

VII. Conclusion

The simulation studies of the nonlinear robust controller for inverted pendulum system are presented in this paper. The nonlinearities of the plant are considered as unknown but the bounds of the nonlinearities are known. The controller for inverted pendulum system is designed using SMC, ISMC, and STSMC approaches. A comparative analysis has been worked out between all the three controllers in the presence of external disturbances. The simulation results highlight the effectiveness of the proposed nonlinear robust controller. Implementation of the higher order SMC has been worked out, it may be reported in near future.

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