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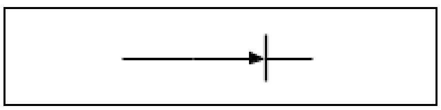
Experiment (2)
Systems Properties and Convolution

1) Linearity and time invariance:

Linearity and Time invariance are characteristics of the *system*. To claim that a certain system is linear; it should be linear for *any* input signal, not just one or two inputs.
Linearity is a double faceted property. Firstly if *any* input to the system is scaled by *any* value, the output of the system will be scaled by the same value, and secondly, if *any* two inputs to the system were added, then the new output is the result of the addition of the two inputs. Again, we persist on that this should be valid for any input. A system may be nonlinear but for a certain input, or set of inputs, it looks to be linear. For example, consider the following system:



It's an ideal diode. The diode looks to be linear if the input was greater than zero. If the input signal to the diode is a DC=5 v, the output is also a DC=5 v, and if the input was scaled by 2 to be 10 v, the output will also be scaled by 2 to be 10v. But what happens if we scaled the input by (-1) (the new input = -5 v), the output will be zero. The system then is not linear although it looks to be linear for positive inputs and positive scales.

Time invariance also should be proved for any input and any time, not just certain input at certain time. For example, consider an On-OFF valve. This valve has a control signal that opens and closes it in a certain schedule. When the valve is open, its output signal equals to its input signal. But when it is closed, it's output signal always equals to zero. So, the system response to the input *changes with time*, so it is a time varying system. Note that if someone looks to the valve only when the valve is open, he can claim that it is a time invariant system, but in fact, one should prove the time invariance for any time.

Note that in practical life, we are interested to keep the system linear and time invariant only in the period of the experiment and for the inputs that are used in the system. There is no system in our life that is ALWAYS linear and ALWAYS time invariant for all inputs. The simple amplifier works well as long as the input of the amplifier is less than the source voltage, but it saturates if we applied a higher voltage. Also the output will change if the components changed as a result of heating or any other reasons. So the system output also changes with time. So practically, all systems are time variant and nonlinear.

In the signal analysis, to prove linearity and time invariance, we must prove this for all signals and all times. This can't be done except analytically. To prove that the property is not applied (*time variance* or *nonlinearity)*, it is sufficient to prove this *even for one input to the system*.

Students experiment 1:

a) Discover, analytically, whether the following systems are linear and/or time invariant:

   a) y[n]=x[n]-x[n-1]-x[n-2]
   b) y[n]=cos(x[n])
   c) y[n]=nx[n]

b) Stem the response of the three systems to the following inputs:

   1) delta(n)
   2) delta(n-1)
   3) delta(n)+2* delta(n-1)

Using the previous inputs, verify the results you got using the analytical solution

delta(n) can be defined as follows:

```
nx1=[-10:10], x1=[zeros(1,10) 1 zeros(1,10)]
```

Note also that the second input is a delayed version of the first one. The third input equals to the first input+ scaled version of the second input.

Name	Sec	B. N.

Experiment 2 –Part 1 Results sheet:

1- Analytical solution to discover time invariance and linearity of the systems:

a)

b)

c)

Write the code used to input the first signal to the three systems

System	Code
a	
b	
c	

Plotting for the responses of the systems to the three inputs:

	System (1)	System (2)	System (3)
Input(1)			
Input(2)			

Input(3)			
Comment			

Part 2:

1) Discrete time convolution:

The convolution sum expresses the relation between the input signal of a linear time invariant system and its output. The expression of the discrete time convolution is:

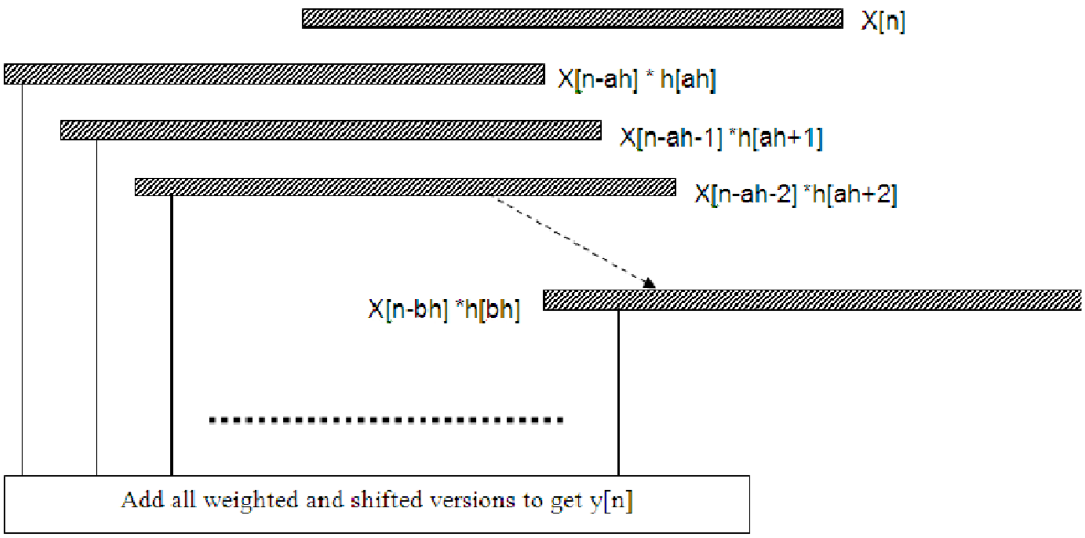
$$y[n]=\sum_{i=-\infty}^{\infty}h[i]x[n-i]$$

We can think about the convolution sum as a *weighted sum* of *shifted versions* of input vector. The weights of the different versions are determined by the impulse response of the system

y[n]= ……………+h[-1] x[n+1] +h[0] x[n]+h[1] x[n-1] +…………………

So, to get the result of convolution, you have to add different versions of the signal multiplied by a number that depends on the system.

Assume x[n] starts from **ax**, and ends at **bx**
Assume h[n] starts from **ah**, and ends at **bh**
Then y[n] starts at **ax+ah** and ends at **bx+bh**



In MATLAB, we treat the signal and its time axis independently. The complete algorithm to do the convolution is:

1- Define nx, x , nh, h
2- ny (the time index of the result) as a vector starting from nx(1)+nh(1) and ends at nx(end)+nh(end)
3- Initialize the output vector y to all zeros, its length should be equal to length(x)+length(h)-1
4- Loop over all elements of the vector h, at each iteration add a shifted version of the signal to the old y
5- End

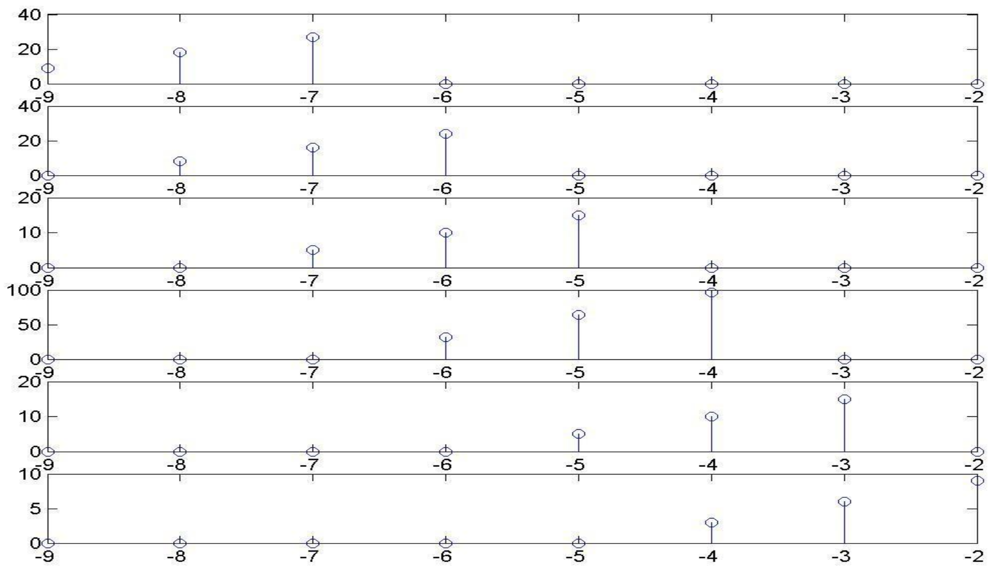
Students experiment 1:

a) An outline of the code required to do the previous algorithm follows. You are required to complete the code, and stem the result of convolution between the two following vectors:

```
nx=[-3 -2 -1];
x =[1 2 3];
nh=[-6 -5 -4 -3 -2 -1];
h =[9 8 5 32 5 3];
```

The outline is:

```
M=length(x);
N=length(h);
ny= ……
y=zeros(1,…..);
for u=1:……
    x1=h(u)*[zeros(1,u-1) x zeros(1,………)];
    y=………+x1;
End
```



b) Use the MATLAB help and try to use the **conv** command to verify your results in the previous parts.

Name	**Sec**	**B. N.**

Experiment 2- Part 2 Results sheet:

a) Convolution complete code (1)

Using stem function to plot the final output y.

c) The conv command:

Using stem function to plot its final output y.

2) Fourier Series:

Fourier series representation for the periodic signals is concerned by expressing any periodic signal as a summation of weighted complex exponentials that have frequencies equal to integer multiples of the frequency of the periodic signal (harmonics).

It is very important to notice that for LTI systems, these complex exponentials are the Eigen functions. If the input is represented as a summation of weighted complex exponentials, the output can be easily written as the summation of the input weighted complex exponentials where each is multiplied by a complex number.

The following program gets the Fourier Series coefficients of a periodic signal **x**, where **x** represents one period of the periodic signal.

```
function a=f_series(x)
N=length(x)
n=0:N-1
for k=0:N-1
     a(k+1)=1/N*sum(x.*exp(-2*pi*i*k*n/N));
end
```

The Fourier series coefficients are in general complex numbers.

Students experiment 1:

a) Write a MATLAB function that implements the inverse Fourier series given certain coefficients ak
b) Use `f_series` code and your code to test your answer for the following inputs:
   a. x=[1 2 3 4]
   b. x=[1 2 2 1]
   c. x=[0 1 2 -2 -1]
Find ak for them using `f_series` code then find x signal using your code.

c) Apply the Fourier series algorithm on the following signals:

a. x[n]=cos(2*pi*n*3/7)
b. x[n]=sin(2*pi*n*3/7)
c. x[n]= exp(j*2*pi*n*3/7)

Check your result with your analytical solution.

Experiment 2 Results sheet:

a)Inverse Fourier Series Code	
b)Fourier series of the three signals:	
X signal for the previous Fourier series coefficients	

c) analytical solution of the three signals:	
Simulation output of the three signals:	