Experiment (2)
Signal Transformations and Properties

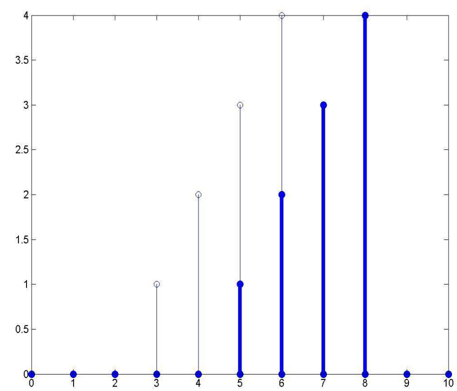
Introduction:

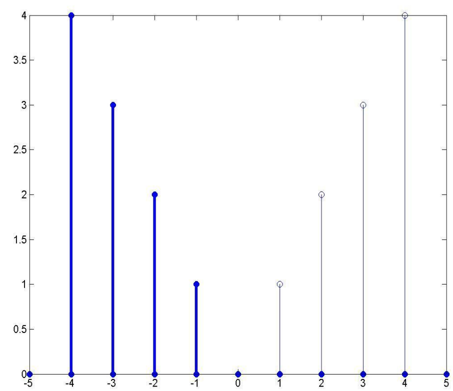
The objectives of this experiment are:

1) To understand the topic of "transformation of the independent variables".
2) To comprehend the meaning of periodicity of the discrete time signals.
3) To have the ability to calculate the Energy and Power for discrete time signals.

1) Transformation of independent variables:

Three types of transformations are common in the signal processing algorithms: inversion, shifting, and compression. These transformations are for the independent variable. Assuming that the time is the independent variable in most of signals related to our studies; we invert the signal in time, shift it and scale it in time.





Signal shift in the time domain Signal inversion in the time domain

"Shifting with respect to the independent variable" "Inversion with respect to the independent variable"

In the above two figures, a signal and a shifted version of this signal are shown in the first figure. Note that the original signal starts at time 3, and the shifted version starts at time 5. This means the signal now starts *late*; which implies a *delay* in the signal. The second figure shows a signal and an inverted version of the signal. An example of the inverted signal is a video signal seen on the screen when the *backward* button of the video set is pressed. The events that really happened late in time come first, then those events that happened first. You watch the end of the film and then the beginning of it.

The video signal can also be used to demonstrate the compression that can happen to a signal. The video can be shown in a rate much faster than the original rate of the video when the forward button is pressed while the video is running. This shows the compression of the time axis. What you should see in time t=10 you see now at time t=1. Things appear faster than they really are. Now, what do you think about slow motion? You see the video slow, this means you made expansion for the time axis, you see things that really happened at time t=1 at time t=10. Everything became slower.

So, we can categorize the operations that can be done on the independent variable of the signal to:

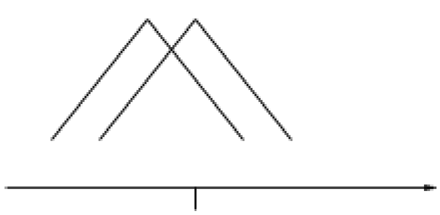
1- Shift:
   a. Delay   b. Advance
2- Inversion
3- Scaling
   a. Compression   b. Expansion

We can also make any combination of the above operations; we can make inversion then delay, or compression then inversion,... etc.

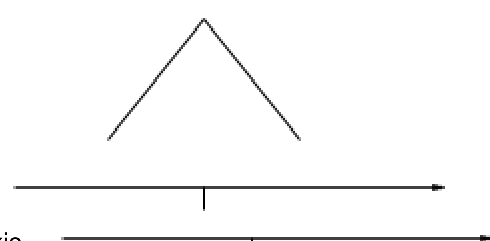
- But how these transformations really occur??

The shift, inversion or scaling can be done either by the signal itself or by changing the axis. Try to think of the following: you are in a closed room and you have a stop watch. A friend of yours has a cassette and stands behind the door of the room. It is expected that when you shout “Start”; your friend is expected to start playing the audio and you start counting the time using the stop watch. Now, you shout “Start” and your friend starts playing the audio. In this case, the signal starts at time zero. In another trial, you shouted “Start” and he became late in playing the audio, and he starts playing it after two seconds. This means the signal was *delayed* by two seconds. In the last trial, your friend started the audio before you start your stop watch by three seconds; this means the signal was *advanced* in time by three seconds.

Now, consider the following case. You have a stored audio, you don’t want to listen to it from the beginning; you want to listen to it from the middle of the file. You will have to forward the tape till you reach the middle of the file, this means simply you moved the time axis. The origin of the new axis now is in the middle of the file, this is completely equivalent to advancing the signal.



A signal (solid line) and an advanced version (dashed)



A signal (solid line) and an advanced version (dashed axis)

Consider an arbitrary signal x[n], the signal y[n] = x[n-N] is considered as a signal shifted to the right of the current axis, or we can interpret it also as fixing the signal and shifting the time axis to the left. The latter interpretation is more accurate for the transformation of independent variables. So what happens in fact is that we want to know (where is "the old" x[n] now?) so x[0] appears at y[N], x[1] appears at y[N+1],.... etc, so we will fix the signal and have new INDEPENDENT VARIABLE (m) where m=n+N, and this is the relation between the new time axis and the old time axis.

To interpret the time inversion, consider the signal y[n] = x[-n], we can think of it as inversion in the time axis itself, not in the signal, so the old sample that was at n= -1 will be now at n=1, this simply means we have inverted the time axis.

How to draw discrete signals?

1) If you know the equation of the signal:

If you are required to draw the signal x[n] = 3n+1 from n=-3 □ n=7, then you start by defining the time axis you will use:

```
>> nx=[-3:7]
```

Then you initialize the signal to be all zeros (here it is not important but in general it is a good practice to do so)

```
>> x=zeros(length(nx),1);
>> x=3 * nx +1
>> stem (nx,x)
```

For continuous time signals, you can only deal with samples in MATLAB, so you just need to construct the time axis with smaller steps; i.e.:

```
>>tx=[-3:.001:7]
```

And then follow the same procedure as before.

2) If you don't know the equation:

The values must be generated.

For example if you want to draw x = [1 3 5 3 2] for n= -1 □ 3

```
>> nx=[-1:3];
>> x=zeros(length(nx),1);
>> x=[1 3 5 3 2]
>> stem(nx,x);
```

Students experiment 1:

a) Define and stem the signal x[n] where:

x[n]=2 at n=0, 1 at n=2, -1 at n=3, 3 at n=4, 0 otherwise

Draw it from n= -3 □ 7

b) Draw the following signals by defining the new axes:

y1[n]=x[n-2];%delayed by 2 samples
y2[n]=x[n+1]; %advanced by on sample
y3[n]=x[-n];%flipped version
y4[n]=x[-n+1];%flipped then advanced

2) Periodicity of discrete time signals:

For discrete time signals, the signals are periodic if x[n] = x[n+N]. For discrete time sinusoids, x[n] = cos(Omega*n) will be periodic if and only if

Omega=2*pi*m/N

Where **m** is an integer representing the number of periods in the continuous domain included in one period in the discrete domain, and **N** is the period in samples in the discrete domain.

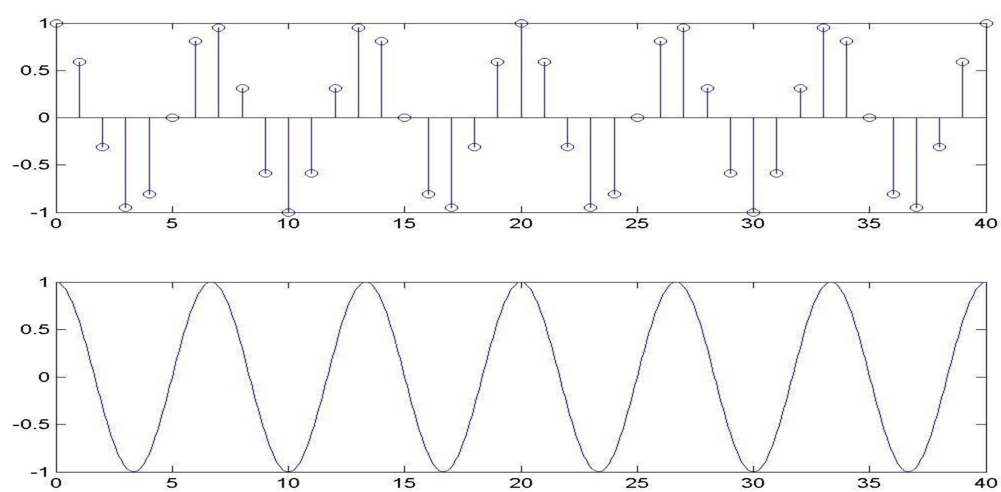
Now, from the following examples, the following should be clear:

1) What's the meaning of (**m**)

2) If **Omega** is in any form other than rational number multiplied by 2*pi, the signal *will not* be periodic

ex1:

```
>>n=[0:40];
>>x=cos(2*pi*3/20*n);
%this is periodic signal with period 20 and the discrete %period
includes 3 continuous periods
>>subplot(2,1,1);
>>stem(n,x);%discrete plotting
>>t=[0:.1:40]; %Now let's draw a continuous version of the
%function, we will take more samples between 0 and 40
%"10 samples in every second"
>>xt=cos(2*pi*3/20*t);
>>subplot(2,1,2);
>>plot(t,xt);%continuous plotting
```



You should note that every one discrete period includes 3 continuous periods

<u>Students experiment 2 :</u>

a) Consider the discrete time signal

$X_M[n]$=sin(2*pi*M*n/N)

And assume N=12. For M=4, 5, 7 and 10, plot $X_M[n]$ on the interval n=0□2N-1. What is the fundamental period of each signal? In general, how can the fundamental period be determined from arbitrary integer values of M and N? Be sure to consider the case in which M>N.

b) Consider the signal

Xk[n]=sin(wk*n)

Where w=2*pi /5. Stem Xk for k=1,2,4,6 from n=0□9 . How many *unique* signals have you plotted? Explain.

3) Calculation of Energy and Power of discrete time signals:

For discrete time signal, we define the energy of the signal as E = sum(x.^2). This is the energy of the whole signal, so we should square the samples, and then add these squares for all time indices.

We define also the energy of a part of the signal from N1 to N2 as E=sum(x(N1:N2).^2). The power of discrete signal between N1 and N2 is defined as the energy divided by (N2-N1+1) which represents the No. of samples in the interval from N1 to N2.

The power of the whole signal equals to 0 if the signal is limited, and equals to the Power of one period if the signal is periodic.

ex:

```
>>x=randn([1,51]);
 % x is a random signal normally distributed with mean 0 and variance 1
>>subplot(2,1,1);
>>stem (x);
>>Etot=sum(x.^2); %this is the energy of the whole signal
>>Ptot=Etot/length(x); % this is the power

>>j=0;
>>for N1=1:5:46
>>j=j+1;
>>E(j)=sum(x(N1:N1+4).^2);% energy for every 5 samples
>>P(j)=sum(x(N1:N1+4).^2/5;% average power
>>end;
>>subplot(2,1,2);
>>stem(E);
```

Students experiment 3:

In this experiment, you are required to calculate the power of a periodic signal in two ways:

The power is formally defined as the average energy with respect to time. But for *periodic signals*, the power of the whole signal equals to the power of one period.

a) Define a sinusoidal signal (for one period only) with period = 10, stem it and calculate the power of this signal.
b) Define the above signal from n= 0 □12, calculate the energy of the signal, calculate its power. Compare it with the result in (a)
c) Define the signal from n=0 □1002, calculate the energy of the signal, calculate its power. Compare it with the result in (a).

Do you expect for b,c to be periodic or not, so what is the expected energy and power for it ?

Comment on your observations

Name	Sec	B. N.

Experiment 2 Results sheet:

1- a) Code and plot for x[n]

b) Write the definition of the new axis and plot the signal in the table below:

2- a)

M	Plotting	Fundamental period

2- b)

K	Stem of x

3)

a	
B	
C	

:Comments

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