Cairo University

Faculty of Engineering

Computer Engineering Department

Third year

DIGITAL COMMUNICATION

Assignment (2)

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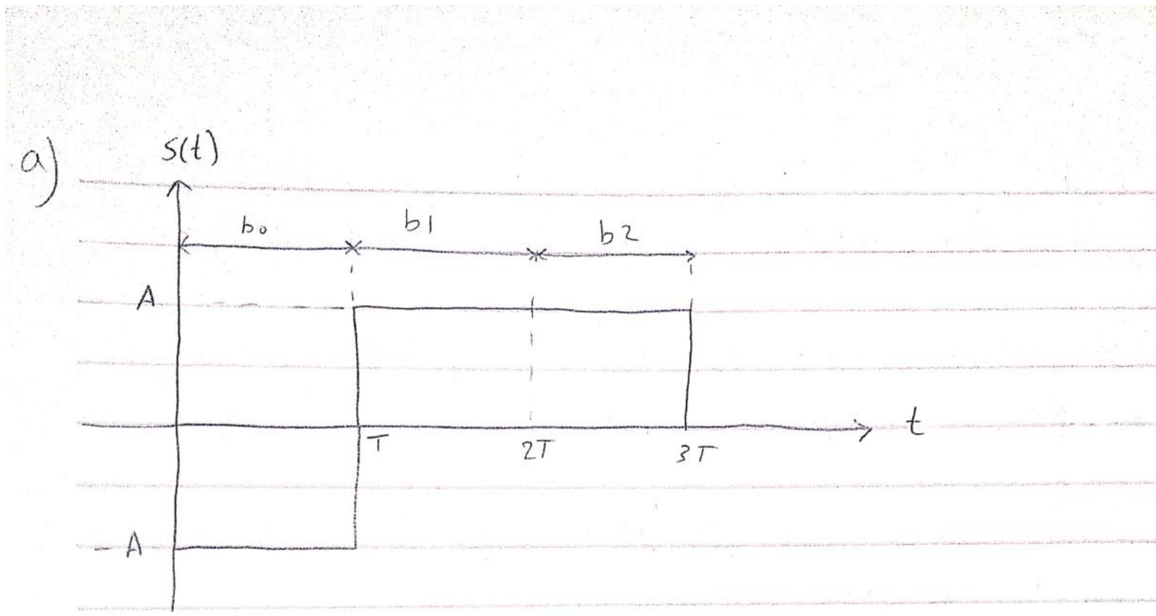
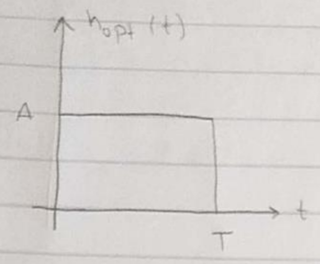
Part I

a)

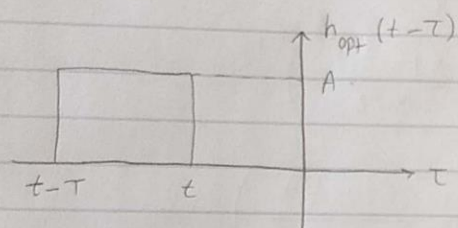


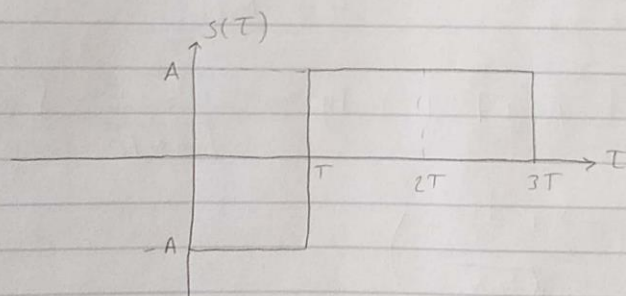
b)

b) $h_{opt}(t) = g(T-t) = g(t)$



$$y(t) = s(t) * h_{opt}(t) = \int_{-\infty}^{\infty} s(\tau)\, h_{opt}(t-\tau)\, d\tau$$





- $t < 0$: $y(t) = 0$
- $t \geqslant 0$ & $t < T \longrightarrow 0 \leq t < T$:

$$y(t) = \int_0^t -A^2\, d\tau = -A^2 t$$

- $t \geqslant T$ & $t - T < T \longrightarrow T \leqslant t < 2T$:

$$y(t) = \int_{t-T}^{T} -A^2\, d\tau + \int_{T}^{t} A^2\, d\tau = -A^2(2T-t) + A^2(t-T)$$

$$y(t) = A^2(t - T - 2T + t) = A^2(2t - 3T)$$

- $t-T \geq T$ & $t < 3T \longrightarrow 2T \leq t < 3T$:

$$y(t) = \int_{t-T}^{t} A^2 \, d\tau = A^2 T$$

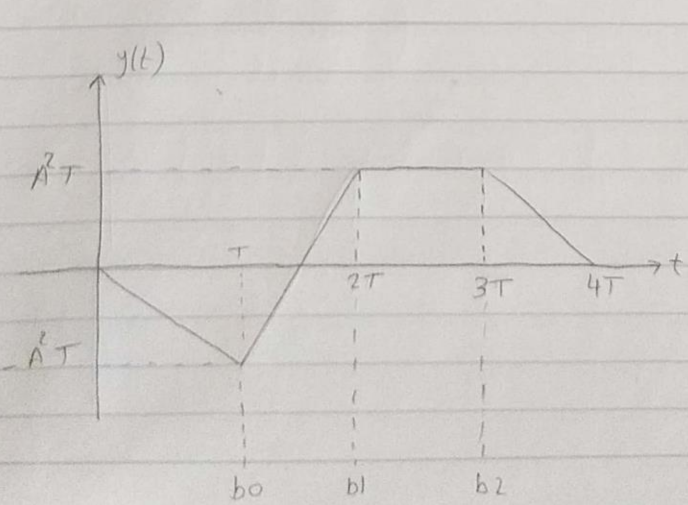
- $t \geq 3T$ & $t - T < 3T \longrightarrow 3T \leq t < 4T$:

$$y(t) = \int_{t-T}^{3T} A^2 \, d\tau = A^2 (4T - t)$$

- $t - T \geq 3T \longrightarrow t \geq 4T$:

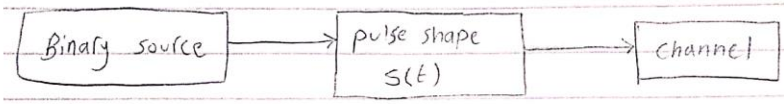
$$y(t) = 0$$

$$y(t) = \begin{cases} 0 & , \quad t < 0 \\ -A^2 t & , \quad 0 \leq t < T \\ A^2(2t - 3T) & , \quad T \leq t < 2T \\ A^2 T & , \quad 2T \leq t < 3T \\ A^2(4T - t) & , \quad 3T \leq t < 4T \\ 0 & , \quad 4T \leq t \end{cases}$$

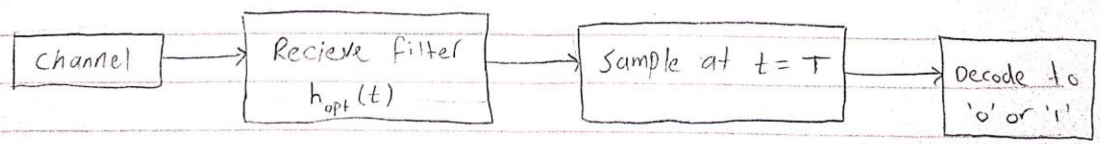


d, e)

d) Block diagram of the transmitter:

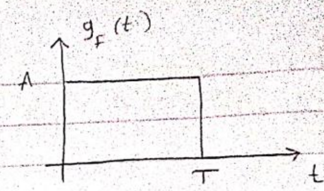


e) Block diagram of the reciever:



Part II

1) a)



a — $h(t) = g_F(T-t) = g_F(t)$

$$g_0(t) = g(t) * h(t) = \int_{-\infty}^{\infty} g(\tau)\, h(t-\tau)\, d\tau$$

$$g_0(t) = \int_{-\infty}^{\infty} g(\tau)\, g_F(T-t+\tau)\, d\tau$$

$$g_0(T) = \int_{-\infty}^{\infty} g(\tau)\, g_F(\tau)\, d\tau$$

$$g(t) = \begin{cases} A, & 0 \leq t \leq T \quad (\text{if '1' is transmitted}) \\ -A, & 0 \leq t \leq T \quad (\text{if '0' is transmitted}) \end{cases}$$

$$g_0(T) = \begin{cases} \int_0^T A^2\, d\tau = A^2 T, & 0 \leq t \leq T \\ \int_0^T -A^2\, d\tau = -A^2 T, & 0 \leq t \leq T \end{cases}$$

$$y(t) = (g(t) + w(t)) * h(t)$$

$$y(T) = \left[g(t) * h(t)\right]_{t=T} + \left[w(t) * h(t)\right]_{t=T}$$

$$y(T) = g_0(T) + n(T)$$

$$y(T) = \pm A^2 T + n(T)$$

$$y(T) = \pm 1 + n(T)$$

$$n(T) = \Big[w(t) * h(t)\Big]_{t=T} = \int_{-\infty}^{\infty} w(\tau)\, h(T-\tau)\, d\tau$$

$$E[n(T)] = E\Big[\int_{-\infty}^{\infty} w(\tau)\, h(T-\tau)\, d\tau\Big]$$

$$E[n(T)] = \int_{-\infty}^{\infty} E[w(\tau)\, h(T-\tau)]\, d\tau$$

$$E[n(T)] = \int_{-\infty}^{\infty} h(T-\tau)\, E[w(\tau)]\, d\tau = 0$$

$$\mathrm{var}[n(T)] = E[n^2(T)] - E^2[n(T)]$$

$$= \int_{-\infty}^{\infty} S_n(f)\, df = \int_{-\infty}^{\infty} S_w(f)\, |H(f)|^2\, df$$

$$= \frac{N_0}{2} \int_{-\infty}^{\infty} |H(f)|^2\, df = \frac{N_0}{2} \int_0^1 |h(t)|^2\, dt$$

$$= \frac{N_0}{2} \int_0^1 (1)^2\, dt = \frac{N_0}{2}$$

$$n(T) \sim N\left(\mu = 0,\ \sigma^2 = \frac{N_0}{2}\right)$$

$$y(T) \sim N\left(\mu = \pm 1,\ \sigma^2 = \frac{N_0}{2}\right)$$

$$p(y|\text{"1"}) = \frac{1}{\sqrt{\pi N_0}} \exp\left(-\frac{(y-1)^2}{N_0}\right)$$

$$p(y|\text{"0"}) = \frac{1}{\sqrt{\pi N_0}} \exp\left(-\frac{(y+1)^2}{N_0}\right)$$

$$p(e|\text{"1"}) = p(y < \lambda|\text{"1"}) = \int_{-\infty}^{\lambda} p(y|\text{"1"})\, dy$$

$$p(e|\text{"1"}) = \frac{1}{\sqrt{2\pi\left(\frac{N_0}{2}\right)}} \int_{-\infty}^{\lambda} \exp\left(-\frac{1}{2}\frac{(y-1)^2}{0.5N_0}\right) dy$$

Let: $Z = -\dfrac{(y-1)}{\sqrt{0.5N_0}}$

$$dZ = -\frac{1}{\sqrt{\frac{N_0}{2}}}\, dy$$

$$p(e|\text{"1"}) = \frac{1}{\sqrt{2\pi}} \int_{\frac{-\lambda+1}{\sqrt{0.5N_0}}}^{\infty} \exp\left(-\frac{1}{2}Z^2\right) dZ = Q\left(\frac{1-\lambda}{\sqrt{0.5N_0}}\right)$$

$$p(e|\text{"0"}) = p(y>\lambda\,|\,\text{"0"}) = \int_{\lambda}^{\infty} p(y|\text{"0"})\, dy$$

$$p(e|\text{"0"}) = \frac{1}{\sqrt{2\pi\frac{N_0}{2}}} \int_{\lambda}^{\infty} \exp\left(-\frac{1}{2}\frac{(y+1)^2}{0.5N_0}\right) = Q\left(\frac{\lambda+1}{\sqrt{0.5N_0}}\right)$$

$$p(\text{error}) = p(e|\text{"0"})\,p(\text{"0"}) + p(e|\text{"1"})\,p(\text{"1"})$$

b)

b- $h(t) = \delta(t)$

$$g_o(t) = g(t) * h(t) = g(t)$$

$$y(t) = g(t) + w(t)$$

$$y(T) = \pm A + w(T)$$

$$y(T) = \pm 1 + w(T)$$

$$w(T) \sim N\left(\mu = 0, \ \sigma^2 = \frac{N_0}{2}\right)$$

$$y(T) \sim N\left(\mu = \pm 1, \ \sigma^2 = \frac{N_0}{2}\right)$$

$$p(e \mid \text{"1"}) = Q\left(\frac{1 - \lambda}{\sqrt{0.5N_0}}\right)$$

$$p(e \mid \text{"0"}) = Q\left(\frac{\lambda + 1}{\sqrt{0.5N_0}}\right)$$

$$p(\text{error}) = p(e \mid \text{"0"})\,p(\text{"0"}) + p(e \mid \text{"1"})\,p(\text{"1"})$$

c)

$$c-\ n(T) = \left[h(t) * w(t)\right]_{t=T}$$

$$n(T) = \int_{-\infty}^{\infty} w(\tau)\ h(T-\tau)\ d\tau$$

$$E[n(T)] = E\left[\int_{-\infty}^{\infty} w(\tau)h(T-\tau)\,d\tau\right]$$

$$E[n(T)] = \int_{-\infty}^{\infty} E\left[w(\tau)h(T-\tau)\right] d\tau$$

$$E[n(T)] = \int_{-\infty}^{\infty} h(T-\tau)\ E[w(\tau)]\ d\tau \quad = \quad 0$$

$$var[n(T)] = E[n^2(T)] - E^2[n(T)]$$

$$var[n(T)] = \int_{-\infty}^{\infty} S_n(f)\,df = \int_{-\infty}^{\infty} S_w(f)\ |H(f)|^2\,df$$

$$var[n(T)] = \frac{N_0}{2}\int_0^1 |h(t)|^2\,dt = \frac{N_0}{2}\int_0^1 3t^2\,dt$$

$$var[n(T)] = \frac{N_0}{2}$$

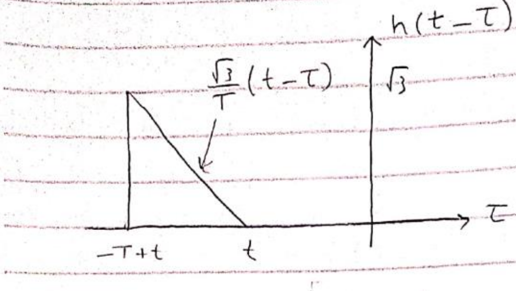
$$y(t) = (g(t) + w(t)) * h(t)$$

$$y(T) = \left[g(t) * h(t)\right]_{t=T} + \left[w(t) * h(t)\right]_{t=T}$$

$$y(T) = g_0(T) + n(T)$$

Evaluating $g_o(T)$:

$$g_o(t) = \int_{-\infty}^{\infty} g(\tau)\, h(t-\tau)\, d\tau$$



- $t < 0$: $g_o(t) = 0$

- $t \geq 0$ & $-T+t < 0 \longrightarrow 0 \leq t < T$:

$$g_o(t) = \int_0^t \frac{\sqrt{3}}{T}(t-\tau)\, d\tau = \frac{\sqrt{3}}{T}\left[t\tau - \frac{\tau^2}{2}\right]_0^t$$

$$g_o(t) = \frac{\sqrt{3}}{T}\left(t^2 - \frac{t^2}{2}\right) = \frac{\sqrt{3}}{2T}t^2 = \frac{\sqrt{3}}{2}t^2$$

- $-T+t \geq 0$ & $-T+t < T \longrightarrow T \leq t < 2T$:

$$g_o(t) = \int_{-T+t}^{T} \frac{\sqrt{3}}{T}(t-\tau)\, d\tau = \frac{\sqrt{3}}{T}\left[t\tau - \frac{\tau^2}{2}\right]_{-T+t}^{T}$$

$$g_o(t) = \sqrt{3}\left[\left(tT - \frac{T^2}{2}\right) - \left(t(t-T) - \frac{(t-T)^2}{2}\right)\right]$$

$$g_o(t) = \sqrt{3}\left[tT - \frac{T^2}{2} - t^2 + tT + \frac{t^2}{2} - tT + \frac{T^2}{2}\right]$$

$$g_o(t) = \sqrt{3}\left[tT - \frac{t^2}{2}\right]$$

- $-T+t \geq T \longrightarrow t \geq 2T$:

$$g_o(t) = 0$$

$$g_0(T) = \begin{cases} \frac{\sqrt{3}}{2} T^2 = \frac{\sqrt{3}}{2}, & 0 \le t \le T \quad (\text{if '1' is transmitted}) \\ -\frac{\sqrt{3}}{2} T^2 = -\frac{\sqrt{3}}{2}, & 0 \le t \le T \quad (\text{if '0' is transmited}) \end{cases}$$

$$y(T) = \pm \frac{\sqrt{3}}{2} + n(T)$$

$$y(T) \sim N\left(\mu = \pm \frac{\sqrt{3}}{2}, \ \sigma^2 = \frac{N_0}{2}\right)$$

$$p(y|"1") = \frac{1}{\sqrt{2\pi N_0/2}} \exp\left(-\frac{1}{2} \frac{(y - \frac{\sqrt{3}}{2})^2}{N_0/2}\right)$$

$$p(y|"0") = \frac{1}{\sqrt{2\pi N_0/2}} \exp\left(-\frac{1}{2} \frac{(y + \frac{\sqrt{3}}{2})^2}{N_0/2}\right)$$

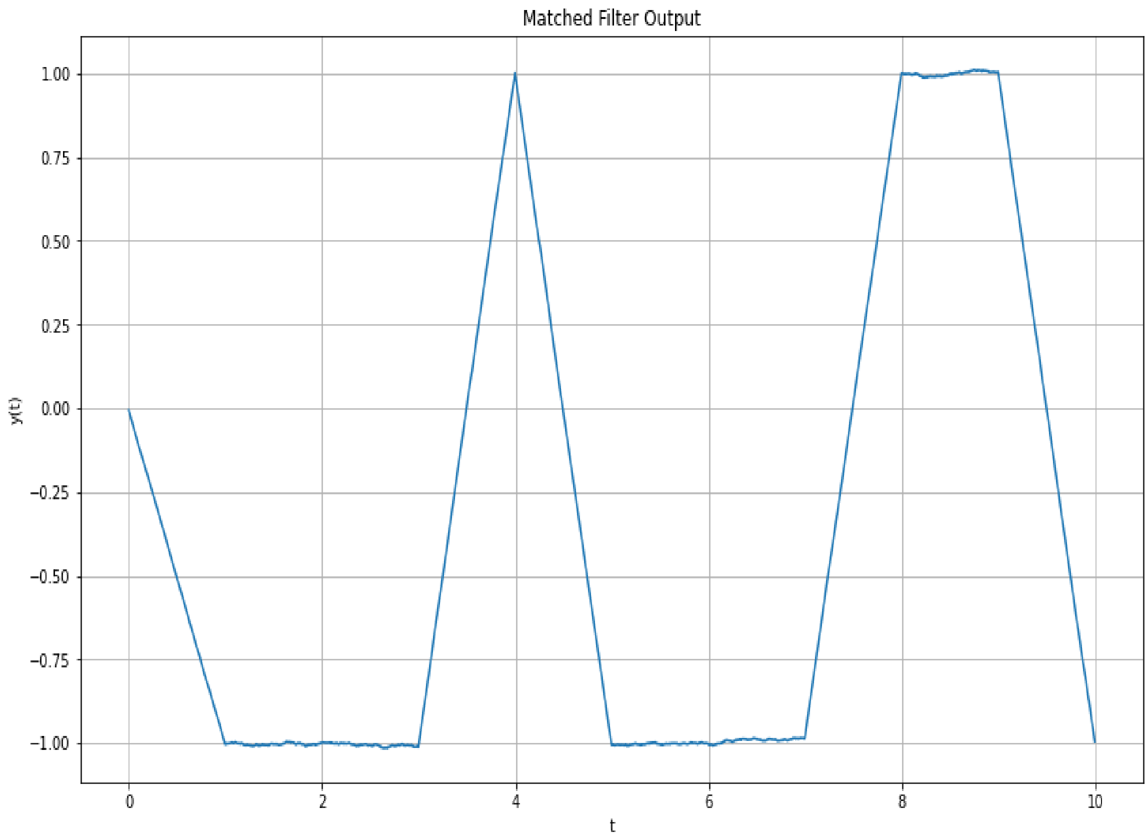
$$p(e|"1") = \int_{-\infty}^{\lambda} p(y|"1")\, dy = \frac{1}{\sqrt{2\pi N_0/2}} \int_{-\infty}^{\lambda} \exp\left(-\frac{1}{2} \frac{(y - \frac{\sqrt{3}}{2})^2}{N_0/2}\right)$$

$$p(e|"1") = Q\left(\frac{-\lambda + \frac{\sqrt{3}}{2}}{\sqrt{0.5 N_0}}\right)$$

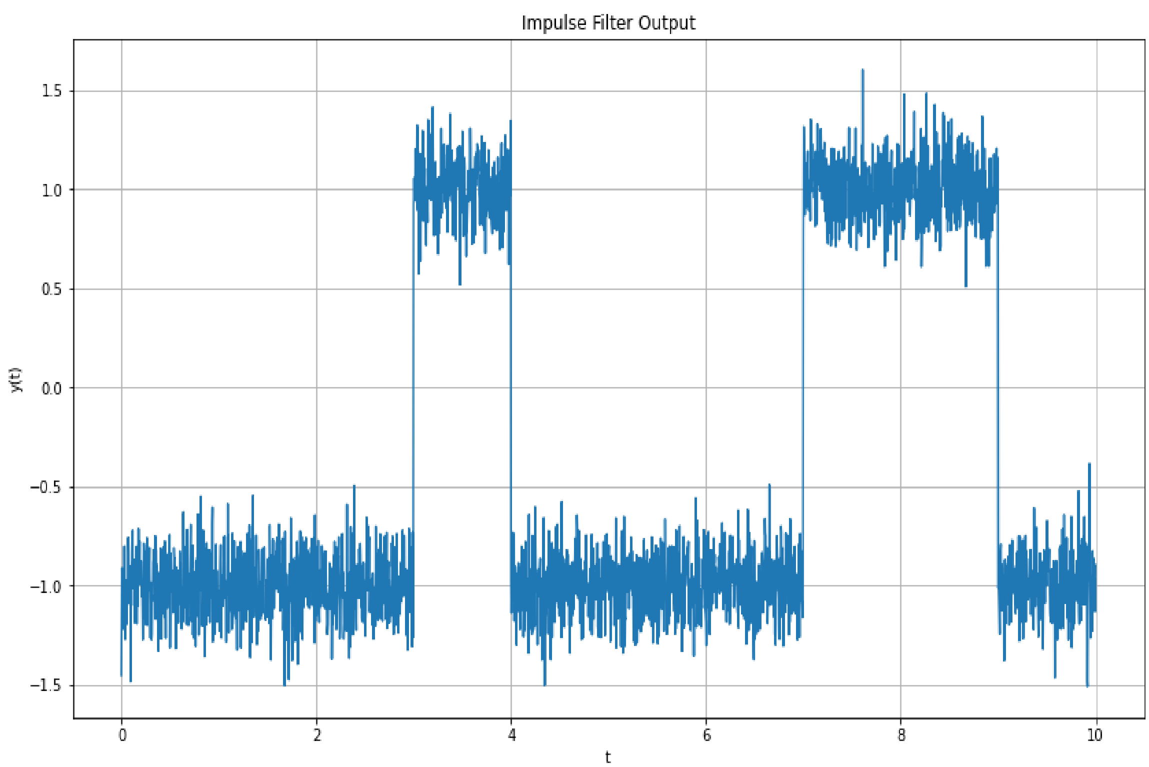
$$p(e|"0") = Q\left(\frac{\lambda + \frac{\sqrt{3}}{2}}{\sqrt{0.5 N_0}}\right)$$

$$P(error) = p(e|"1")\, p("1") + p(e|"0")\, p("0")$$

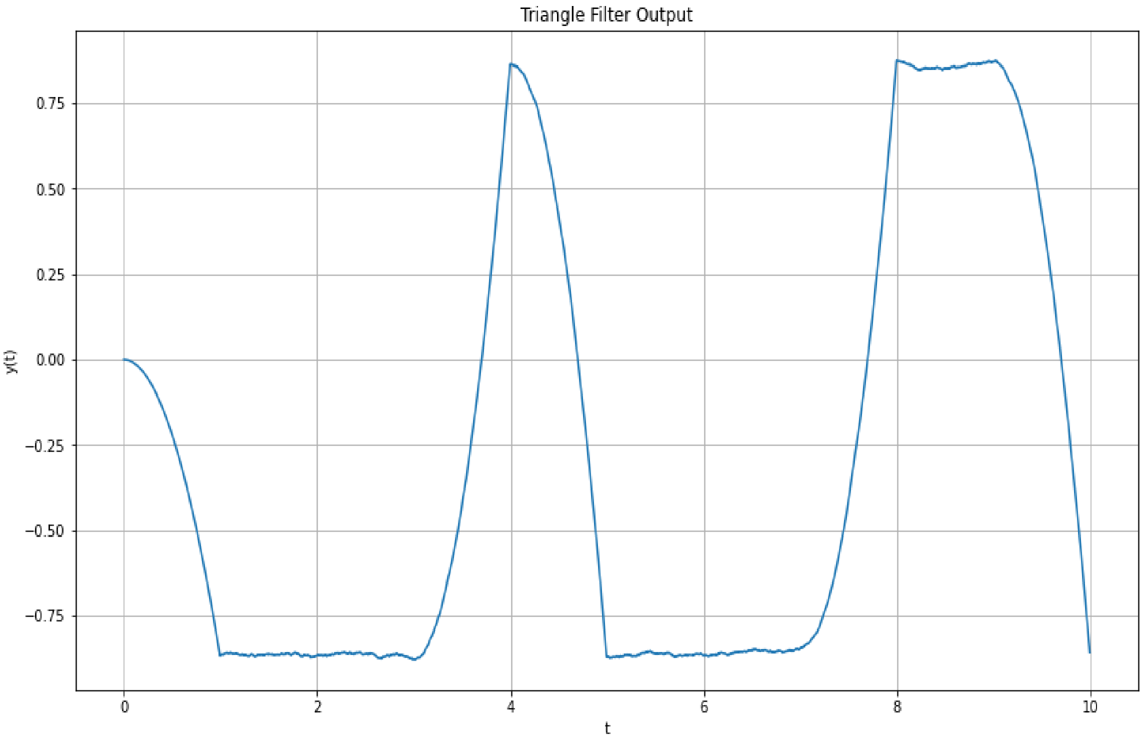
3)



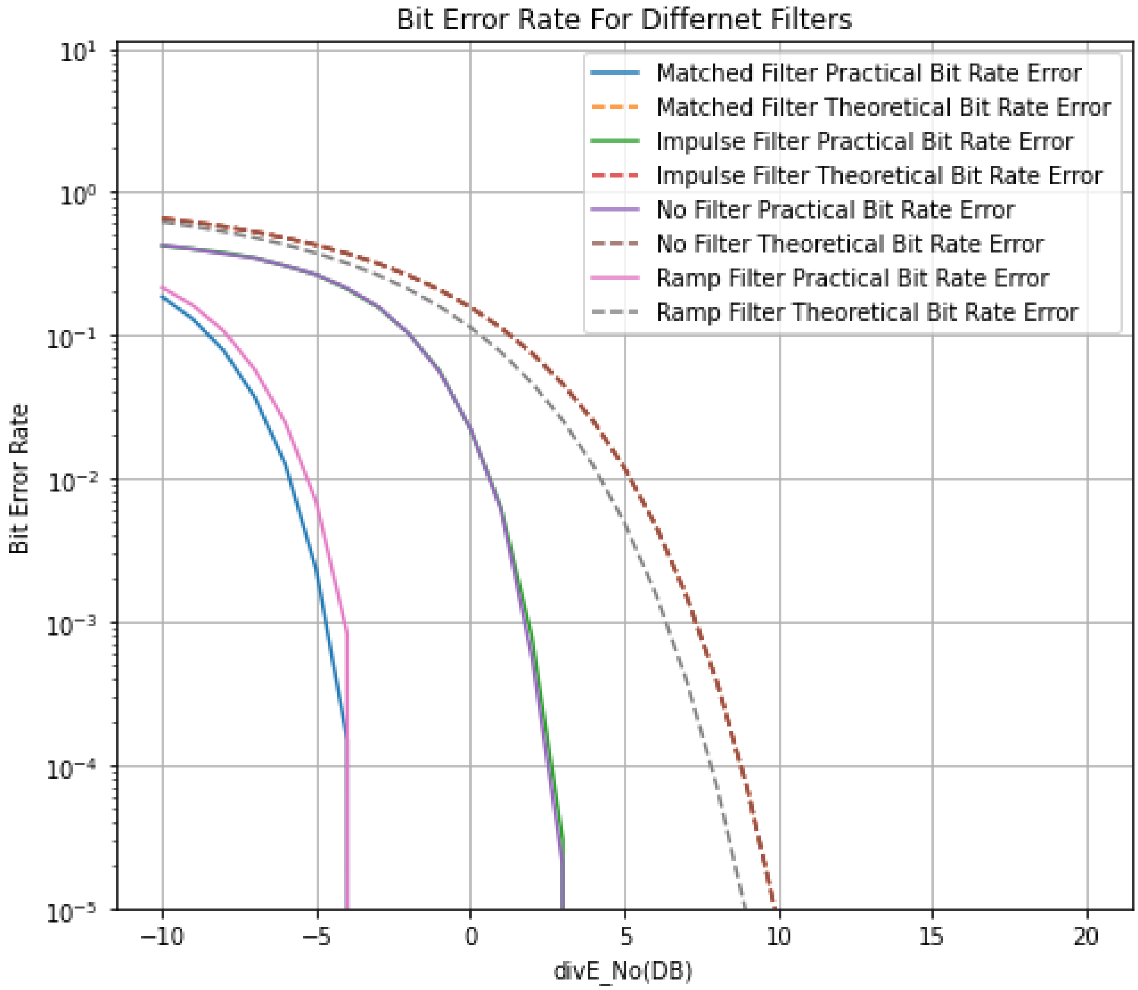
Matched Filter Output
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
y(t)
0
2
4
6
8
10
t



Impulse Filter Output
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
y(t)
0
2
4
6
8
10
t


Triangle Filter Output
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
y(t)
0
2
4
6
8
10
t

4)



5) BER is a decreasing function of E/N0

As E/N0 increases, the noise decreases (because E/N0 represents the signal to noise ratio) and the bits at the channel are less prone to noise and the error is not significant.

6) The lowest case is the Matched Filter case

Because the matched filter is an optimal filter that maximizes peak pulse SNR, therefore the BER is minimum.