CIT-644 Scientific Computing - Coursework 1 - Copy

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Please click the binder badge above to run this notebook live . Also please check our github repo for this project. All the code in this notebook xan be compiled and ran from the `main.cpp` file in the repo. However, for the purpose of this report we have chosen to use a jupyter notebook.

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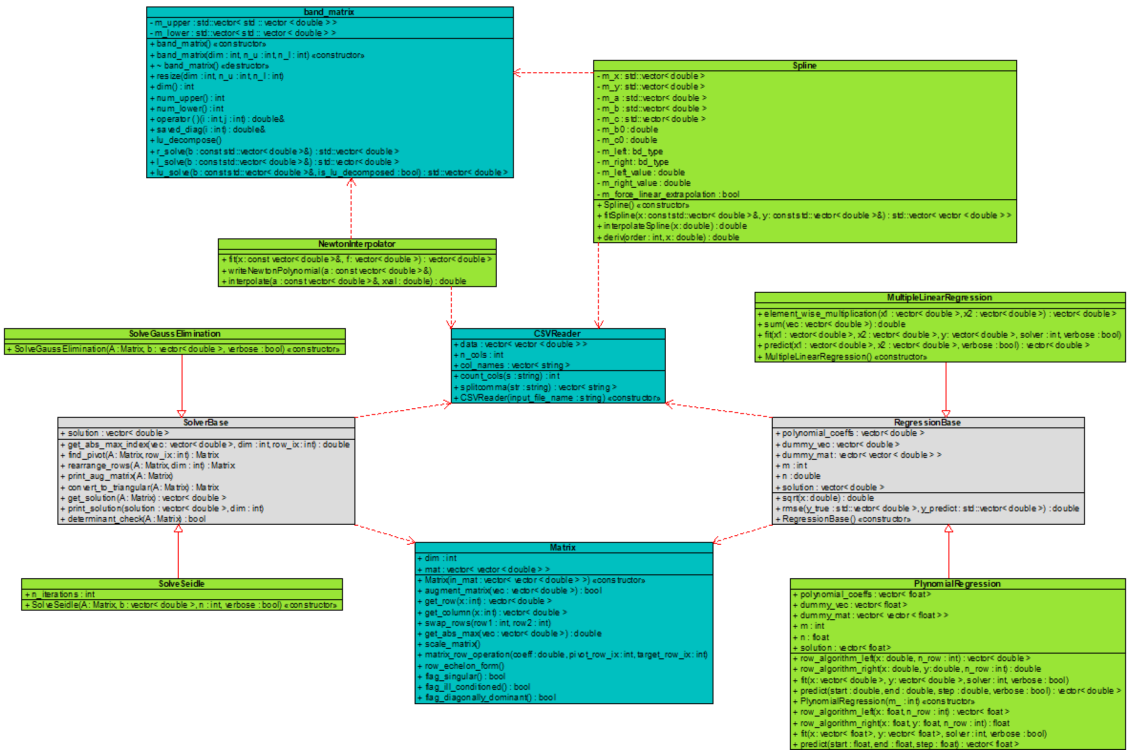
1 Documentation

We used OOP paradigm to make a library which is imported below using `#include"Coursework1/Coursework1lib.h"`.

1.1 Class Diagram



1.2 Library Dependencies:

1.2.1 - CSVReader.hpp

This File contains the `CSVReader` class whith with its helper methods and attributes that to read given datasets

`CSVReader` Constructor Arguments:

- `input_file_name` takes the path to the csv file as a string

`CSVReader` Attributes:

- `n_cols` represent number of columns in the dataset
- `col_names` represent the names of columns

`CSVReader` Methods:

- `splitcomma()` converts a string to a vector of strings by splitting at commas.
- `count_cols()` used to get the number of columns in the file.

1.2.2 - Matrix.hpp

This File contains the `Matrix` class whith with its helper methods and attributes that are used mainly to solve both part 1 and part 2

`Matrix` Constructor Arguments:

- `in_mat` a vector of vectors of doubles

`Matrix` Attributes:

- `dim` an integer denoting the matrix dimensions

`Matrix` Methods:

- `augment_matrix()` is used to add the coefficient of f(x)/y among the equation.
- `get_row ()` / `get_column()` are used to catch specific rows/columns respectively and make any calculations on them, their inputs are the row and column number respectively.
- `swap_rows()` used to swap rows according to the greatest coefficient for pivoting, its inputs are the rows numbers that needed to be swapped.
- `get_abs_max()` used to get the maximum coefficient in the row, its input is coefficient integer.
- `scale_matrix()` used for scaling as dividing all coefficients in each row by the greatest coefficient in it.
- `matrix_row_operation()` takes a coefficient multiplies it by a given row and adds the result to a second row.
- `row_echelon_form()` helps to establish a lower triangular matrix.
- `flag_singular()` / `flag_ill_conditioned()` are used to test the condition of the system.
- `flag_diagonally_dominant()` used specifically for Gauss Seidel as it requires that the coefficients in the absolute values on the diagonal of the matrix are larger than the sum of other elements in their rows.

1.2.3 - SolverBase.hpp

This file contains a base class called `SolverBase` that contains all steps that are shared between `SolveGaussElimination` and `SolveSeidle` . This class should be inherited in both solvers as it contains common steps in both solvers.

`SolverBase` Attributes:

- `solution` a vector that contains the solution for the system of equations.

`SolverBase` Methods:

- `get_abs_max_index()` returns the index of the absolute maximum element in a vector.
- `find_pivot()` used to choose the pivot row.
- `rearrange_rows()` called recursively to rearrange rows according to find_pivot.
- `print_aug_matrix()` prints the augemented matrix in a formatted manner.
- `convert_to_triangular()` puts the matrix in the upper triangular form.
- `print_solution()` prints the solution to consule in formatted manner.
- `determinant_check()` calculate the determinant value by getting the product of element on the diagonal, then checks if the value != 0.

1.2.4 - SolveSeidle.hpp

This file contains the SolveSeidle class that implements the Gauss-Seidle method for solving equations.

This class will print a warning when the given matrix is not diagonally dominant, indicating that convergence is not guaranteed. However it will proceed and attempt to find a solution.

This class will only attempt a solution if `flag_singular()` and `flag_ill_conditioned()` are both true.

`SolveSeidle` Attributes:

- all of `SolverBase` attributes.
- `n_iterations` an Integer. Number of iterations.

`SolveSeidle` Methods:

- all of `SolverBase` methods.

`SolveSeidle` Constructor Arguments:

- `A` a `Matrix` object. This is the matrix of coefficients.
- `b` a vector of doubles. This the y-values.
- `n` an Integer. Number of iterations.
- `verbose` a boolean. if true prints the solution after every iteration. Otherwise prints the solution every fifth iteration.

1.2.5 - SolveGaussElimination.hpp

This file contains the SolveSeidle class that implements the Gauss-Seidle method for solving equations.

This class will only attempt a solution if `flag_singular()` and `flag_ill_conditioned()` are both true.

`SolveGaussElimination` Constructor Arguments:

- `A` a `Matrix` object. This is the matrix of coefficients.
- `b` a vector of doubles. This the y-values.
- `verbose` a boolean. if true prints the steps of Gauss Elimination methods for the problem being solbed

`SolveGaussElimination` Attributes:

- all of `SolverBase` attributes.

`SolveGaussElimination` Methods:

- all of `SolverBase` methods.

1.2.6 - RegressionBase.hpp

This File contains the `RegressionBase` base class, containing common steps in `PolynomialRegression` and `MultipleLinearRegression` like calculation of root mean square error (rmse) so this class should be inherited in both regressors.

`RegressionBase` Attributes:

- `polynomial_coeffs` is a vector containing the coefficients obtained after solving the system of equations.
- `m` is the degree of the polynomial in case of using polynomial regression.
- `n` if n is equal to 1 the system of equations will be solved using the Gauss Elimination Method. if n is equal to 2 Gauss Seidle will be used instead. 1 is default.
- `solution` a vector that stores the solution to the equation.

`RegressionBase` Methods:

- `sqrt()` calculates the square root of a given double.
- `rmse()` claculates the root main square error given true values of y and predicted ones.

1.2.7 - PolynomialRegression.hpp

This File contains the `PolynomialRegression` Class. This class fits a polynomial of order `m` to a given dataset and can predict pints based on the calculated polynomial.

`PolynomialRegression` Constructor Arguments:

- `m` the order of the polynomial required.

`PolynomialRegression` Attributes:

- all of `RegressionBase` attributes.

`PolynomialRegression` Methods:

- all of `RegressionBase` methods.
- `row_algorithm_left()` generates each row of $\mathbf{A}$ where $\mathbf{A} \times \mathbf{x} = \mathbf{b}$ the size of the row is dictated by `m`
- `row_algorithm_right()` generates each row of $\mathbf{b}$ where $\mathbf{A} \times \mathbf{x} = \mathbf{b}$ the size of the row is dictated by `m`
- `fit()` takes the values of $\mathbf{x}$ and $\mathbf{y}$ of a given dataset, fits a polynomial using the solver specified by the argument `solver`. the argument `verbose` is a bool that controls how much information this function prints to the console.
- `predict()` substitutes values from `start` to `end` for a given step size `step` and returns values of predicted $\mathbf{y}$.

1.2.8 - MultipleLinearRegression.hpp

This File contains the `MultipleLinearRegression` Class. This class fits a plane to two given vectors of the same size $\mathbf{x_1}$ and $\mathbf{x_2}$

`MultipleLinearRegression` Constructor Arguments:

- `m` the order of the polynomial required.

`MultipleLinearRegression` Attributes:

- all of `RegressionBase` attributes.

`MultipleLinearRegression` Methods:

- all of `RegressionBase` methods.
- `element_wise_multiplication()` takes two vectors and performs element wise multiplication to produce the returned vector,
- `sum()` returns the sum of a given vector.
- `fit()` takes the values of $\mathbf{x_1}$, $\mathbf{x_2}$ and $\mathbf{y}$ of a given dataset, fits a plane using the solver specified by the argument `solver`. the argument `verbose` is a bool that controls how much information this function prints to the console.
- `predict()` substitutes values from $\mathbf{x_1}$ and $\mathbf{x_2}$ and returns values of predicted $\mathbf{y}$.

1.2.9 - CubicSpline.hpp

This File contains the `band_matrix` and `Spline` classes.

`band_matrix` Constructor Arguments:

- `dim` matrix dimension
- `n_u` number of upper band layers
- `n_l` number of lower band layers

`band_matrix` Attributes:

- `m_upper` upper band matrix elements
- `m_lower` lower band matrix elements

`band_matrix` Methods:

- `resize()` takes matrix dimension and number of upper and lower diagonals and resizes them to fill matrix shape.
- `dim()` return matrix dimension (longest diagonal)
- `operator()` method defined to access matrix elements by A(i,j)
- `num_upper()` given the elements of the upper band as vector of vectors, returns num upper diagonals
- `num_lower()` given the elements of the lower band as vector of vectors, returns num lower diagonals
- `lu_decompose()` method takes no arguments and performs the decomposition of the matrix instance.
- `l_solve()` solves system Ly=b to get y
- `r_solve()` solves system Rx=y to get x
- `lu_solve()` solves the system of equations using LU decomposition

`Spline` Constructor Arguments:

- `x` the original x points observed (independent variable)
- `y` the original y points observed (dependent variable)

`Spline` Attributes:

- `m_x` set of x points (independent variable)
- `m_y` set of y points (dependent variable)
- `m_a` 3rd order spline coefficients
- `m_b` 2nd order spline coefficients
- `m_c` 1st order spline coefficients
- `m_b0` used for left extrapolation (values of x smaller than the smallest value given)
- `m_c0` used for left extrapolation (values of x smaller than the smallest value given)
- `m_left` this attribute is used for left boundary handling, it is enumerated type of (bd_type) or boundary type. It can hold first derivate or second derivative and with the value the will be defined in m_left_value.
- `m_right` similar to the above for the rightmost edge. It is assumed that second deriv is 0 for both ends in our case to extrapolate on zero curvature.
- `m_left_value` this is the value of the boundary type specified in m_left.
- `m_right_value` similar to the above for the rightmost edge.

- `m_force_linear_extrapolation` a boolean attribute whether the fitted curve will continue linearly outside given range (extrapolation area)

`Spline` Methods:

- `fitSpline()` this method takes the observed original points and calculates the intervals and returns the cubic spline coefficients for all intervals (fitted curve).
- `interpolateSpline()` this method is used to substitute with new values of X to get new values of Y based on the fitted curve.

1.2.10 - NewtonInterpolator.hpp

This File contains the `NewtonInterpolator` class.

`NewtonInterpolator` Constructor Arguments:

- `x` set of x points (independent variable)
- `y` set of y points (dependent variable)

`NewtonInterpolator` Methods:

- `fit()` this method takes the observed original points and calculates the coefficients of the newton polynomial that fits in all given points.
- `writeNewtonPolynomial()` this method takes the calculated coefficients then writes the newton polynomial to the console.
- `interpolate()` takes the calculated newton polynomial coefficients and a given value x, it substitutes and returns the interpolated value "y".

1.3 Plots in this Report

All plots in this report are created using matplotlib-cpp. We created a header file which can be found at `Coursework1/PlottingFunctions.h` in the github repository to pgenerate the plots we need

2 Reportng our work

2.1 Importing the Library

[1]:
```
#include "Coursework1/Coursework1lib.h"
using namespace std;
```

2.2 Part 1 - Solving a System of Linear Equations:

In this section we will test the `SolveGaussElimination` and `SolveSeidle` outlined above on for different examples. Note that Example 1 is the example included in the assignment description.

We included examples that demonstrate the ability of our solvers to solve equaations of variable order. We also included examples that clearly show Guass Seidle limitatins.

Guass Seidle's main criteria for convergence is that the given **A** matrix should be diagonally dominant, i.e. the absolute elements on the diagonal of **A** must be larger than the sum of all other elements. Otherwise convergence is not guaranteed.

Therefore, although we are aware that we can perform row operations on possibly any given matrix and swap rows/columns to eventually put it in diagonally dominant form, we have chosen not do so in our `SolveSeidle` implementation as this is going to be very computationally expensive. Another reason is that it doesn't make much sense to perform these kind of operations just to use an iterative solver and get an approximated solution when we can use `SolveGaussElimination` to get an exact solution.

2.2.1 Example 1

$$\begin{bmatrix} 2 & 1 & -1 \\ 1 & 4 & 2 \\ -1 & 2 & 7 \end{bmatrix} \times \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 14 \\ 30 \end{bmatrix}$$

[2]:
```
vector<vector<double>> v ={{2, 1, -1}, {1, 4, 3}, {-1 ,2 ,7}};
Matrix A = Matrix(v);
vector<double> b = {0, 14, 30};
```

Solution using Gauss Elimination Method:

[3]:
```
SolveGaussElimination x = SolveGaussElimination(A, b, true);
```

```
Matrix Rearranged
                col 0      col 1      col 2      col 3


      Row 0:        2          1         -1          0


      Row 1:        1          4          3         14
```

```
      Row 2:         -1         2         7        30



Matrix Scaled
                col 0      col 1      col 2      col 3

      Row 0:          1       0.5      -0.5         0

      Row 1:       0.25         1      0.75       3.5

      Row 2:  -0.142857  0.285714         1   4.28571



Matrix in Row Echelon Form
                col 0      col 1      col 2      col 3

      Row 0:          1       0.5      -0.5         0

      Row 1:          0         1         1         4

      Row 2:          0         0         1         5



The Solution is:
(3, -1, 5)
```

Solution using Gauss-Seidle Method:

[4]:
```
SolveSeidle y = SolveSeidle(A, b, 30, false);
```

```
iteration 1:
(0, 3.5, 3.28571)


iteration 6:
(2.74616, -0.813117, 4.91034)


iteration 11:
(2.98782, -0.991033, 4.9957)


iteration 16:
(2.99942, -0.99957, 4.99979)
```

```
iteration 21:
(2.99997, -0.999979, 4.99999)


iteration 26:
(3, -0.999999, 5)


(3, -1, 5)
```

2.2.2 Example 2

$$\begin{bmatrix} 2 & -1 & 0 \\ 1 & -3 & 1 \\ -1 & 1 & -3 \end{bmatrix} \times \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \\ 6 \end{bmatrix}$$

[5]:
```
vector<vector<double>> v ={{2, -1, 0}, {1, -3, 1}, {-1, 1 ,-3}};
Matrix A = Matrix(v);
vector<double> b = {2, -2, 6};
```

Solution using Gauss Elimination Method:

[6]:
```
SolveGaussElimination x = SolveGaussElimination(A, b, true);
```

```
Matrix Rearranged
                col 0      col 1      col 2      col 3

      Row 0:        2         -1          0          2

      Row 1:        1         -3          1         -2

      Row 2:       -1          1         -3          6



Matrix Scaled
                col 0      col 1      col 2      col 3

      Row 0:        1       -0.5          0          1

      Row 1:  0.333333        -1   0.333333  -0.666667

      Row 2: -0.333333  0.333333         -1          2
```

```
Matrix in Row Echelon Form
                col 0      col 1      col 2      col 3

      Row 0:        1       -0.5          0          1

      Row 1:        0          1       -0.4        1.2

      Row 2:        0          0          1   -2.28571



The Solution is:
(1.14286, 0.285714, -2.28571)
```

Solution using Gauss-Seidle Method:

[7]:
```
SolveSeidle y = SolveSeidle(A, b, 31, false);
```

```
iteration 2:
(1.5, 0.5, -2.33333)


iteration 7:
(1.14263, 0.285751, -2.28563)


iteration 12:
(1.14286, 0.285714, -2.28571)


iteration 17:
(1.14286, 0.285714, -2.28571)


iteration 22:
(1.14286, 0.285714, -2.28571)


iteration 27:
(1.14286, 0.285714, -2.28571)


(1.14286, 0.285714, -2.28571)
```

2.2.3 Example 3

$$\begin{bmatrix} 5 & 6 & 7 \\ 6 & 3 & 9 \\ 7 & 9 & 10 \end{bmatrix} \times \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 18 \\ 18 \\ 26 \end{bmatrix}$$

[8]:
```
vector<vector<double>> v ={{5, 6, 7}, {6, 3, 9}, {7, 9, 10}};
Matrix A = Matrix(v);
vector<double> b = {18, 18, 26};
```

Solution using Gauss Elimination Method:

[9]:
```
SolveGaussElimination x = SolveGaussElimination(A, b, true);
```

```
Matrix Rearranged
                col 0      col 1      col 2      col 3


      Row 0:        7          9         10         26


      Row 1:        5          6          7         18


      Row 2:        6          3          9         18




Matrix Scaled
                col 0      col 1      col 2      col 3


      Row 0:      0.7        0.9          1        2.6


      Row 1: 0.714286   0.857143          1    2.57143


      Row 2: 0.666667   0.333333          1          2




Matrix in Row Echelon Form
                col 0      col 1      col 2      col 3


      Row 0:        1    1.28571    1.42857    3.71429


      Row 1:        0          1   0.333333    1.33333


      Row 2:        0          0          1          1
```

```
The Solution is:
(1, 1, 1)
```

Solution using Gauss-Seidle Method:

[10]:
```
SolveSeidle y = SolveSeidle(A, b, 30, false);
```

```
This matrix is not in diagonally dominant form. Convergence is not guaranteed!
iteration 1:
(2.57143, 0.857143, 0.888889)


iteration 6:
(-16.2997, 6.70171, 11.5214)


iteration 11:
(-58.6386, 17.0658, 36.2927)


iteration 16:
(-141.786, 36.1943, 85.3481)


iteration 21:
(-299.796, 71.9214, 178.779)


iteration 26:
(-597.383, 138.88, 354.851)


(-1014.84, 232.685, 601.884)
```

2.3 Example 4

$$\begin{bmatrix} 10 & 2 & -1 & 2 \\ 1 & 5 & 1 & 0 \\ 1 & -2 & -5 & 1 \\ 3 & 0 & 0 & 9 \end{bmatrix} \times \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} -4 \\ 1 \\ 2 \\ 10 \end{bmatrix}$$

[11]:
```
vector<vector<double>> v = {{10, 2, -1, 2}, {1, 5, 1, 0}, {1, -2, -5, 1}, {3,
 ↪0, 0, 9}};
Matrix A = Matrix(v);
vector<double> b = {-4, 1, 2, 10};
```

Solution using Gauss Elimination Method:

[12]:
```
SolveGaussElimination x = SolveGaussElimination(A, b, true);
```

```
Matrix Rearranged
                col 0      col 1      col 2      col 3      col 4

      Row 0:        10          2         -1          2         -4

      Row 1:         1          5          1          0          1

      Row 2:         1         -2         -5          1          2

      Row 3:         3          0          0          9         10



Matrix Scaled
                col 0      col 1      col 2      col 3      col 4

      Row 0:         1        0.2       -0.1        0.2       -0.4

      Row 1:       0.2          1        0.2          0        0.2

      Row 2:       0.2       -0.4         -1        0.2        0.4

      Row 3:  0.333333          0          0          1    1.11111



Matrix in Row Echelon Form
                col 0      col 1      col 2      col 3      col 4

      Row 0:         1        0.2       -0.1        0.2       -0.4

      Row 1:         0          1   0.229167-0.0416667   0.291667

      Row 2:         0          0          1 -0.161137 -0.691943

      Row 3:         0          0          0          1    1.38272



The Solution is:
(-0.814815, 0.45679, -0.469136, 1.38272)
```

Solution using Gauss-Seidle Method:

[13]:
```
SolveSeidle y = SolveSeidle(A, b, 30, false);
```

```
iteration 1:
(-0.4, 0.28, -0.592, 1.24444)


iteration 6:
(-0.814909, 0.456827, -0.469136, 1.38275)


iteration 11:
(-0.814815, 0.45679, -0.469136, 1.38272)


iteration 16:
(-0.814815, 0.45679, -0.469136, 1.38272)


iteration 21:
(-0.814815, 0.45679, -0.469136, 1.38272)


iteration 26:
(-0.814815, 0.45679, -0.469136, 1.38272)


(-0.814815, 0.45679, -0.469136, 1.38272)
```

2.4 Example 5

$$\begin{bmatrix} -1 & 1 & -1 & 1 \\ 1 & 1 & 1 & 1 \\ 8 & 4 & 2 & 1 \\ 27 & 9 & 3 & 1 \end{bmatrix} \times \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ -2 \\ 1 \end{bmatrix}$$

[14]:
```
vector<vector<double>> v ={{-1,1,-1,1}, {1, 1, 1, 1}, {8, 4 ,2 ,1}, {27, 9, 3,␣
 ↪1}};
Matrix A = Matrix(v);
vector<double> b = {1, 1, -2, 1};
```

Solution using Gauss Elimination Method:

[15]:
```
SolveGaussElimination x = SolveGaussElimination(A, b, true);
```

```
Matrix Rearranged
                col 0      col 1      col 2      col 3      col 4
```

```
      Row 0:        27         9         3         1         1

      Row 1:         8         4         2         1        -2

      Row 2:         1         1         1         1         1

      Row 3:        -1         1        -1         1         1
```

```
Matrix Scaled
                col 0      col 1      col 2      col 3      col 4

      Row 0:         1  0.333333  0.111111  0.037037  0.037037

      Row 1:         1       0.5      0.25     0.125     -0.25

      Row 2:         1         1         1         1         1

      Row 3:        -1         1        -1         1         1
```

```
Matrix in Row Echelon Form
                col 0      col 1      col 2      col 3      col 4

      Row 0:         1  0.333333  0.111111  0.037037  0.037037

      Row 1:         0         1  0.833333  0.527778  -1.72222

      Row 2:         0         0         1   1.83333   6.33333

      Row 3:         0         0         0         1         4
```

```
The Solution is:
(1, -3, -1, 4)
```

Solution using Gauss-Seidle Method:

[16]:
```
SolveSeidle y = SolveSeidle(A, b, 30, false);
```

```
This matrix is not in diagonally dominant form. Convergence is not guaranteed!
iteration 1:
(0.037037, 0.175926, -2.21296, -1.35185)
```

```
iteration 6:
(-0.61628, 2.89132, -0.919571, -3.42717)


iteration 11:
(-0.752839, 3.00523, -0.747436, -3.5055)


iteration 16:
(-0.750172, 3.00018, -0.749638, -3.49999)


iteration 21:
(-0.749997, 2.99999, -0.750013, -3.5)


iteration 26:
(-0.75, 3, -0.75, -3.5)


(-0.75, 3, -0.75, -3.5)
```

2.5 Part 2 - Regression:

2.5.1 Reading CSV Files

in this part we will need to read CSV files so we will demonstrate the functionality of the `CSVReader` classs outlined above

[17]:
```
CSVReader csv = CSVReader("datasets/part_2/a/2_a_dataset_1.csv");
```

[18]:
```
csv.n_cols
```

[18]: 2

[19]:
```
csv.data
```

[19]:
```
{ { 0.0000000, 1.0000000, 2.0000000, 3.0000000, 4.0000000, 5.0000000, 6.0000000,
7.0000000, 8.0000000, 9.0000000 }, { 2.7539933, 1.8948100, 5.6677209, 8.5087095,
10.926519, 13.530086, 11.835347, 17.623997, 21.183804, 23.939619 } }
```

[20]:
```
csv.col_names
```

[20]:
```
{ "x", "y" }
```

2.5.2 A. Curve Fitting

below we will demonstrate how our `PlynomialRegression` class fits `2_a_dataset_1.csv` and `2_a_dataset_2.csv` using each solver

We will also need to import our plotting dependencies

[21]:
```
!python3-config --includes
```

```
-I/srv/conda/envs/notebook/include/python3.7m
-I/srv/conda/envs/notebook/include/python3.7m
```

[22]:
```
!python3-config --libs
```

```
-lpython3.7m -lcrypt -lpthread -ldl  -lutil -lrt -lm
```

[23]:
```
#pragma cling add_include_path("/srv/conda/envs/notebook/include/python3.7m")
#pragma cling add_library_path("/srv/conda/envs/notebook/lib")
#pragma cling load("python3.7m")
```

[24]:
```
#define WITHOUT_NUMPY 1
#include "Coursework1/PlottingFunctions.h"
```

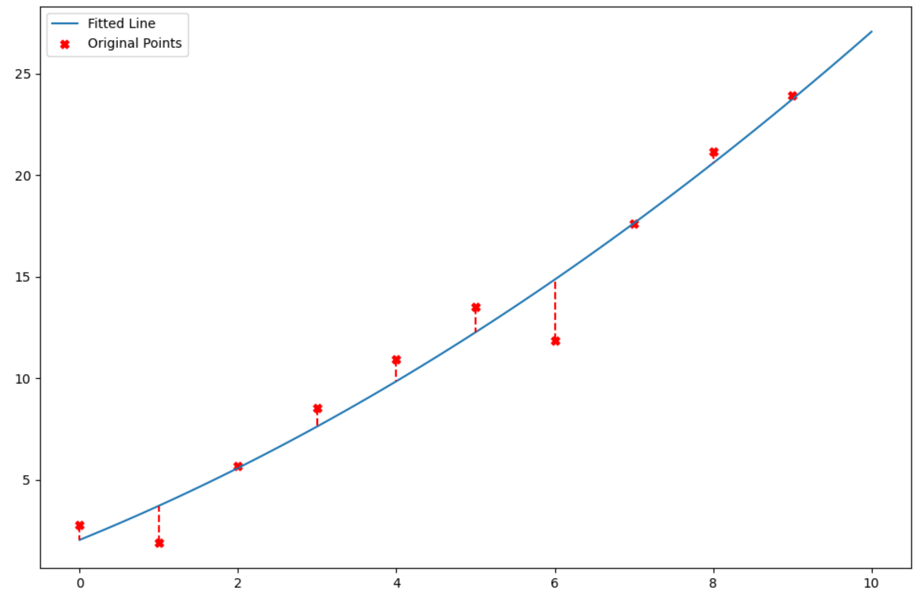
`2_a_dataset_1.csv` Using `SolveGaussElimination`

[25]:
```
PlynomialRegression P(2);
CSVReader csv = CSVReader("./datasets/part_2/a/2_a_dataset_1.csv");
P.fit(csv.data[0], csv.data[1], 1, true);
vector<double> y = P.predict(0, 10, 0.1);
```

```
Matrix Rearranged
                col 0      col 1      col 2      col 3

      Row 0:        285       2025      15333    5198.74

      Row 1:         45        285       2025     729.42

      Row 2:         10         45        285    117.865



Matrix Scaled
                col 0      col 1      col 2      col 3

      Row 0:  0.0185874   0.132068          1   0.339056

      Row 1:  0.0222222   0.140741          1   0.360207

      Row 2:  0.0350877   0.157895          1    0.41356



Matrix in Row Echelon Form
                col 0      col 1      col 2      col 3

      Row 0:          1    7.10526       53.8    18.2412

      Row 1:          0          1       11.4    2.63219

      Row 2:          0          0          1  0.0915477



The Solution is:
(2.02889, 1.58855, 0.0915477)
```

[26]:
```
vector<double> x;
for(double i=0; i<=10; i += 0.1) {
    x.push_back(i);
}

PlotLineScatter Plot(x, y, csv.data[0], csv.data[1], "charts/chart1-1.png");
```

```
Plot.gen_plot()
```



[27]:
```
P.rmse(csv.data[1], y)
```

[27]: 3.5188506

2_a_dataset_1.csv Using SolveSeidle

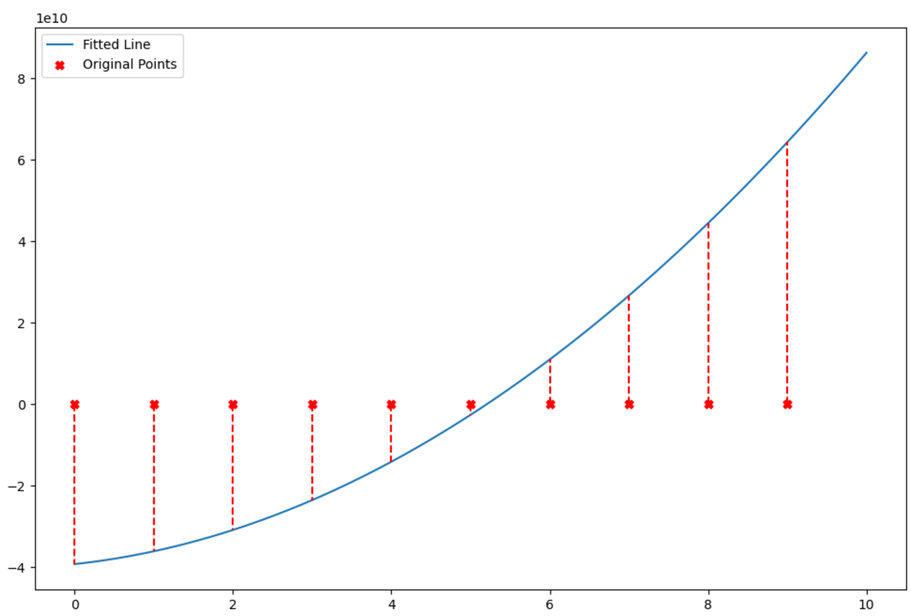
[28]:
```
P.fit(csv.data[0], csv.data[1], 2, false);
vector<double> y = P.predict(0, 10, 0.1);
```

```
Please Specify the number of iterations
30
This matrix is not in diagonally dominant form. Convergence is not guaranteed!
iteration 1:
(0.41356, 2.49407, 17.8329)


iteration 6:
(-23096.6, 1120.73, 651.692)


iteration 11:
(-479624, 25451.3, 12828.5)
```

```
iteration 16:
(-9.4339e+06, 504635, 251353)


iteration 21:
(-1.85001e+08, 9.90178e+06, 4.92785e+06)


iteration 26:
(-3.62731e+09, 1.94151e+08, 9.66185e+07)


(-3.92201e+10, 2.09925e+09, 1.04468e+09)
```

[29]:

```
vector<double> x;
for(double i=0; i<=10; i += 0.1) {
    x.push_back(i);
}

PlotLineScatter Plot(x, y, csv.data[0], csv.data[1], "charts/chart1-2.png");
Plot.gen_plot()
```



[30]:

```
P.rmse(csv.data[1], y)
```

[30]: 8.7206189e+15

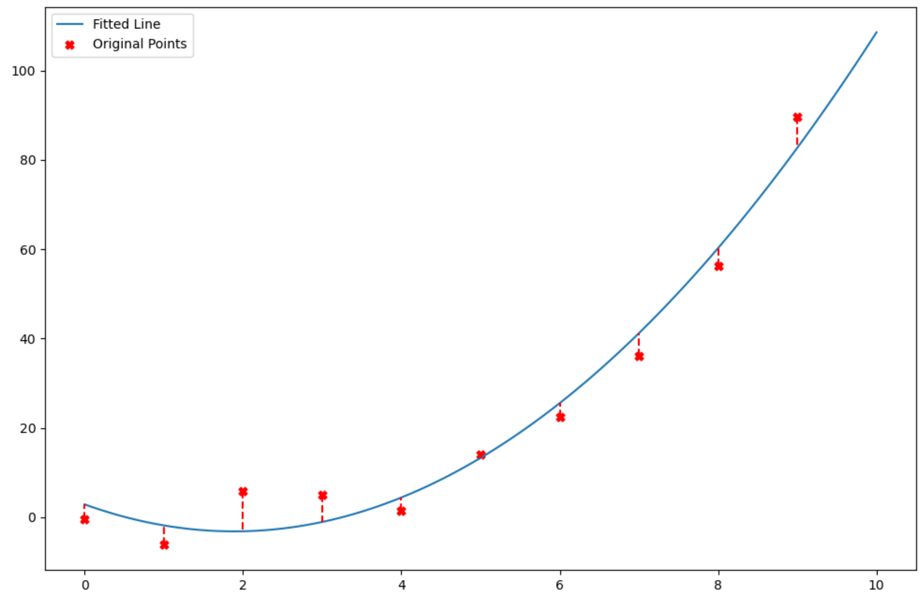
2_a_dataset_2.csv Using SolveGaussElimination

[31]:
```
PlynomialRegression P_2(2);
CSVReader csv_2 = CSVReader("./datasets/part_2/a/2_a_dataset_2.csv");
P_2.fit(csv_2.data[0], csv_2.data[1], 1, true);
vector<double> y = P_2.predict(0, 10, 0.1);
```

```
Matrix Rearranged
                col 0      col 1      col 2      col 3

      Row 0:      285       2025      15333    13874.5

      Row 1:       45        285       2025    1741.22

      Row 2:       10         45        285    224.363



Matrix Scaled
                col 0      col 1      col 2      col 3

      Row 0:  0.0185874  0.132068         1   0.904878

      Row 1:  0.0222222  0.140741         1   0.859861

      Row 2:  0.0350877  0.157895         1   0.787238



Matrix in Row Echelon Form
                col 0      col 1      col 2      col 3

      Row 0:        1    7.10526       53.8    48.6824

      Row 1:        0          1       11.4    12.9399

      Row 2:        0          0          1    1.69675



The Solution is:
(2.89269, -6.40308, 1.69675)
```

[32]:
```
vector<double> x;
for(double i=0; i<=10; i += 0.1) {
    x.push_back(i);
}
```

```
PlotLineScatter Plot(x, y, csv_2.data[0], csv_2.data[1], "charts/chart2-1.png");
Plot.gen_plot()
```



[33]:
```
P_2.rmse(csv_2.data[1], y)
```

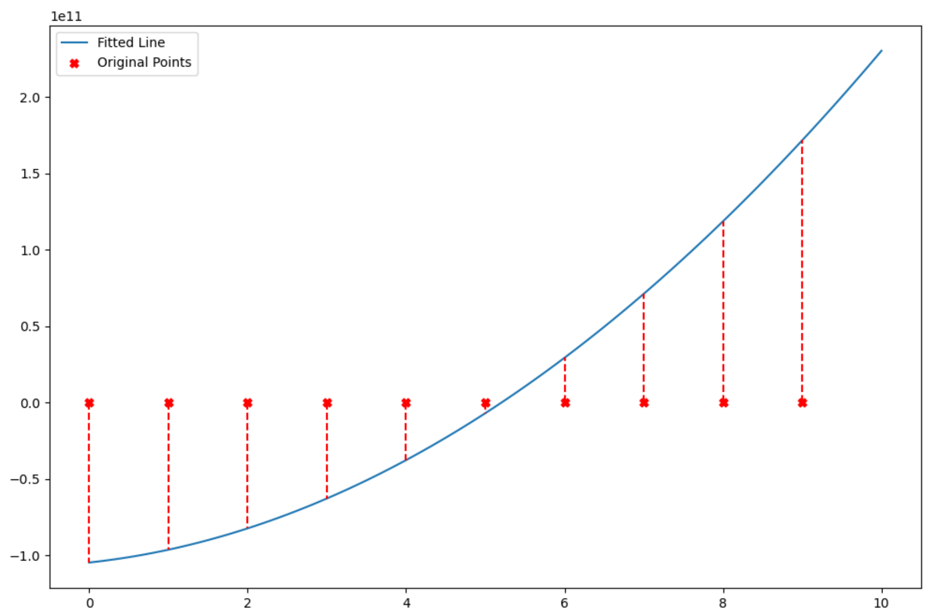
[33]: 11.680771

2_a_dataset_2.csv Using SolveSeidle

[34]:
```
PlynomialRegression P_2(2);
CSVReader csv_2 = CSVReader("./datasets/part_2/a/2_a_dataset_2.csv");
P_2.fit(csv_2.data[0], csv_2.data[1], 2, false);
vector<double> y = P_2.predict(0, 10, 0.1);

vector<double> x;
for(double i=0; i<=10; i += 0.1) {
    x.push_back(i);
}

PlotLineScatter Plot(x, y, csv_2.data[0], csv_2.data[1], "charts/chart2-2.png");
Plot.gen_plot()
```

```
Please Specify the number of iterations
30
This matrix is not in diagonally dominant form. Convergence is not guaranteed!
```

```
iteration 1:
(0.787238, 5.98524, 47.7098)


iteration 6:
(-61696.2, 2990.1, 1741.34)


iteration 11:
(-1.28127e+06, 67984.5, 34271)


iteration 16:
(-2.52019e+07, 1.34808e+06, 671470)


iteration 21:
(-4.94215e+08, 2.64518e+07, 1.31643e+07)


iteration 26:
(-9.69004e+09, 5.18658e+08, 2.58108e+08)


(-1.04773e+11, 5.60798e+09, 2.79078e+09)
```



[35]:
```
P_2.rmse(csv_2.data[1], y)
```

[35]: 6.2234378e+16

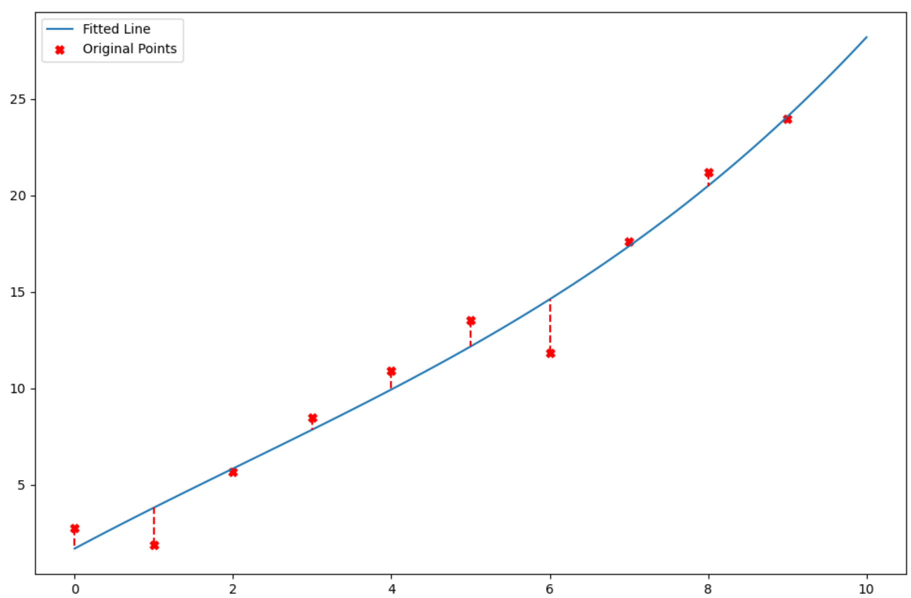
Both the datasets given in this assignment failed to converge using Guass Seidle method during 30 iterations. and that's becuase the system of equations produced from these datasets contained matrices that are non-diagonally dominant.

Demonstration of higher order polynomials

[36]:
```
PlynomialRegression P_3(3);
P_3.fit(csv.data[0], csv.data[1], 1, true);
vector<double> y_3_1 = P_3.predict(0, 10, 0.1);

vector<double> x;
for(double i=0; i<=10; i += 0.1) {
    x.push_back(i);
}

PlotLineScatter Plot(x, y_3_1, csv.data[0], csv.data[1], "charts/chart2-3-1.
↪png");
Plot.gen_plot();
```

```
Matrix Rearranged
                col 0      col 1      col 2      col 3      col 4

      Row 0:       2025      15333     120825     978405    39567.1

      Row 1:        285       2025      15333     120825    5198.74

      Row 2:         45        285       2025      15333     729.42

      Row 3:         10         45        285       2025    117.865



Matrix Scaled
                col 0      col 1      col 2      col 3      col 4

      Row 0:   0.0020697  0.0156714   0.123492          1  0.0404404

      Row 1:  0.00235878  0.0167598   0.126903          1   0.043027

      Row 2:  0.00293485  0.0185874   0.132068          1  0.0475719

      Row 3:  0.00493827  0.0222222   0.140741          1  0.0582047
```

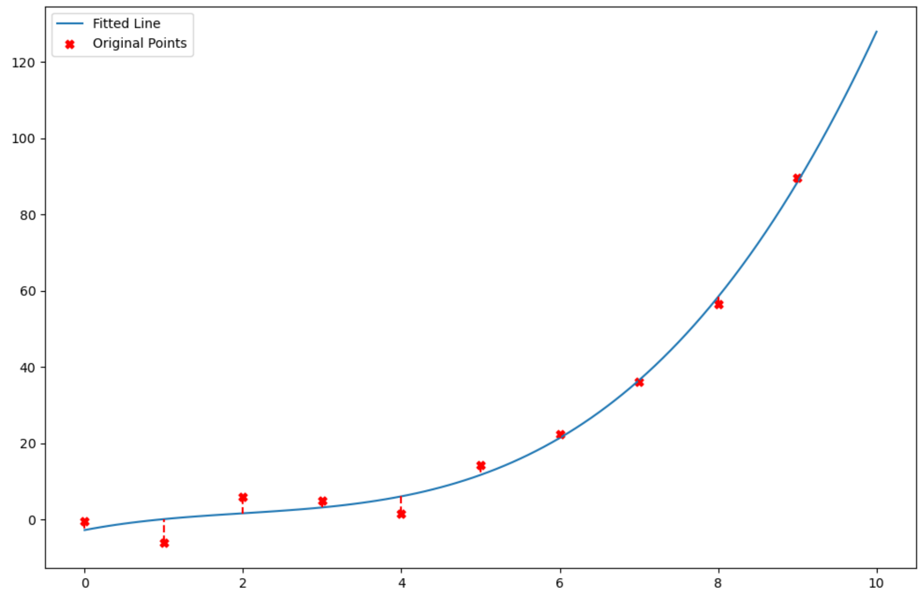
```
Matrix in Row Echelon Form
                col 0      col 1      col 2      col 3      col 4

      Row 0:          1    7.57185    59.6667    483.163    19.5393

      Row 1:          0          1    12.5735    126.912    2.78211

      Row 2:          0-1.63122e-16          1    16.2857   0.127756

      Row 3:          0-9.9245e-17          0          1 0.0129977


The Solution is:
(1.70135, 2.18774, -0.0839212, 0.0129977)
```



Fitting 2_a_dataset_1.csv with a 3rd degree polynomial

[37]:
```
P_3.fit(csv_2.data[0], csv_2.data[1], 1, false);
vector<double> y_3_2 = P_3.predict(0, 10, 0.1);

vector<double> x;
for(double i=0; i<=10; i += 0.1) {
    x.push_back(i);
```

```
}

PlotLineScatter Plot(x, y_3_2, csv_2.data[0], csv_2.data[1], "charts/chart2-3-2.
↪png");
Plot.gen_plot();
```

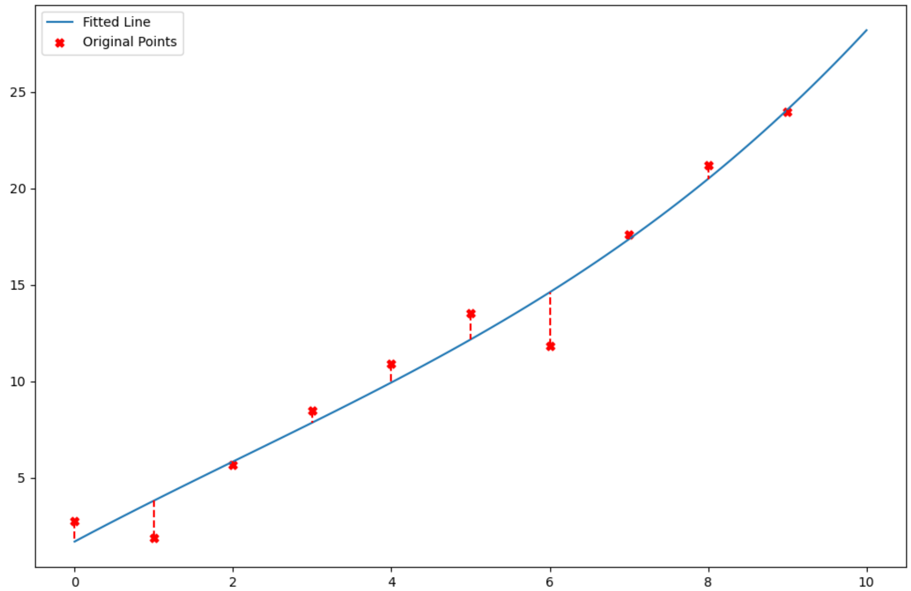


Fitting 2_a_dataset_2.csv with a 3rd degree polynomial

[38]:
```
PlynomialRegression P_4(4);
P_4.fit(csv.data[0], csv.data[1], 1, true);
vector<double> y_4_1 = P_4.predict(0, 10, 0.1);

vector<double> x;
for(double i=0; i<=10; i += 0.1) {
    x.push_back(i);
}

PlotLineScatter Plot(x, y_3_1, csv.data[0], csv.data[1], "charts/chart2-4-1.
↪png");
Plot.gen_plot();
```

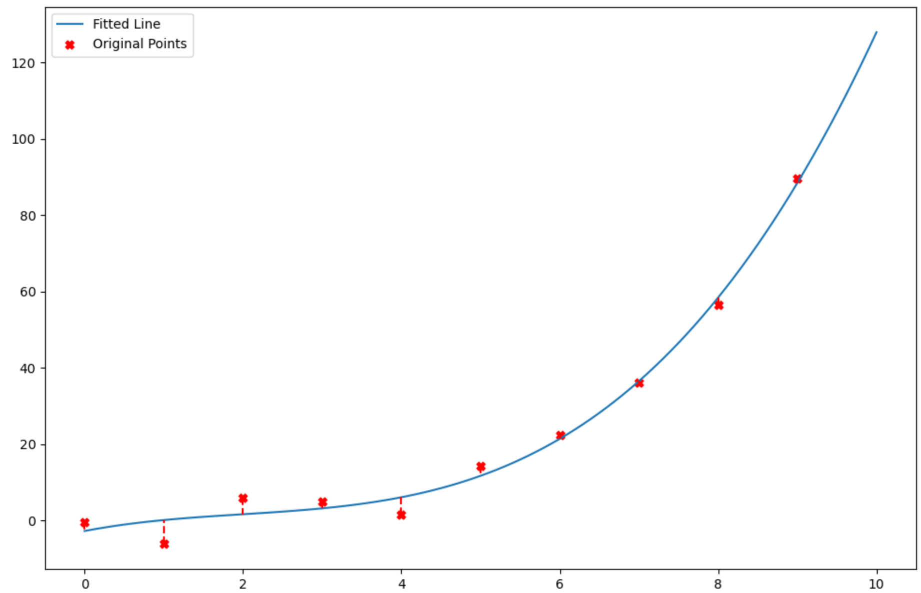
```
Matrix Rearranged
                col 0      col 1      col 2      col 3      col 4      col 5

      Row 0:       15333      120825     9784058.08042e+066.77313e+07     313526
```

```
     Row 1:      2025     15333    120825    9784058.08042e+06   39567.1

     Row 2:       285      2025     15333    120825    978405   5198.74

     Row 3:        45       285      2025     15333    120825    729.42

     Row 4:        10        45       285      2025     15333   117.865


Matrix Scaled
                col 0      col 1      col 2      col 3      col 4      col 5

     Row 0: 0.000226380.00178389 0.0144454  0.119301         10.00462896

     Row 1: 0.0002506060.00189755 0.0149528  0.121083         10.00489666

     Row 2: 0.00029129 0.0020697 0.0156714  0.123492         10.00531349

     Row 3: 0.0003724390.00235878 0.0167598  0.126903          1  0.006037

     Row 4: 0.0006521880.00293485 0.0185874  0.132068         10.00768699


Matrix in Row Echelon Form
                col 0      col 1      col 2      col 3      col 4      col 5

     Row 0:          1   7.88006   63.8104   526.996   4417.36   20.4478

     Row 1:          0         1   13.4445   142.217   1385.49   2.94759

     Row 2:          0-2.28972e-16         1   17.5722   219.301  0.189984

     Row 3:          0-3.89937e-16         0         1   21.1111 0.0483727

     Row 4:          0-1.62593e-16         0         0         1 0.0113705


The Solution is:
(2.19256, 0.141043, 1.0645, -0.191672, 0.0113705)
```



Fitting 2_a_dataset_1.csv with a 4th degree polynomial

[39]:
```
P_4.fit(csv_2.data[0], csv_2.data[1], 1, false);
vector<double> y_4_2 = P_4.predict(0, 10, 0.1);

vector<double> x;
for(double i=0; i<=10; i += 0.1) {
    x.push_back(i);
}

PlotLineScatter Plot(x, y_3_2, csv_2.data[0], csv_2.data[1], "charts/chart2-4-2.
↪png");
Plot.gen_plot();
```


Fitted Line
Original Points
120
100
80
60
40
20
0
0
2
4
6
8
10

Fitting `2_a_dataset_2.csv` with a 4th degree polynomial

2.5.3 B. Multivariable Linear Regression

We will use our `MultipleLinearRegression` to fit the house pricing dataset.

Reading the dataset

[40]:
```
CSVReader csv_3 = CSVReader("./datasets/kaggle_Housing.csv");
```

[41]:
```
csv_3.n_cols
```

[41]: 6

[42]:
```
csv_3.col_names = {"price", "lotsize", "bedrooms", "bathrms", "stories",␣
 ↪"garagepl"}
```

[42]: { "price", "lotsize", "bedrooms", "bathrms", "stories", "garagepl" }

Chosing the best two features based on the RMSE value

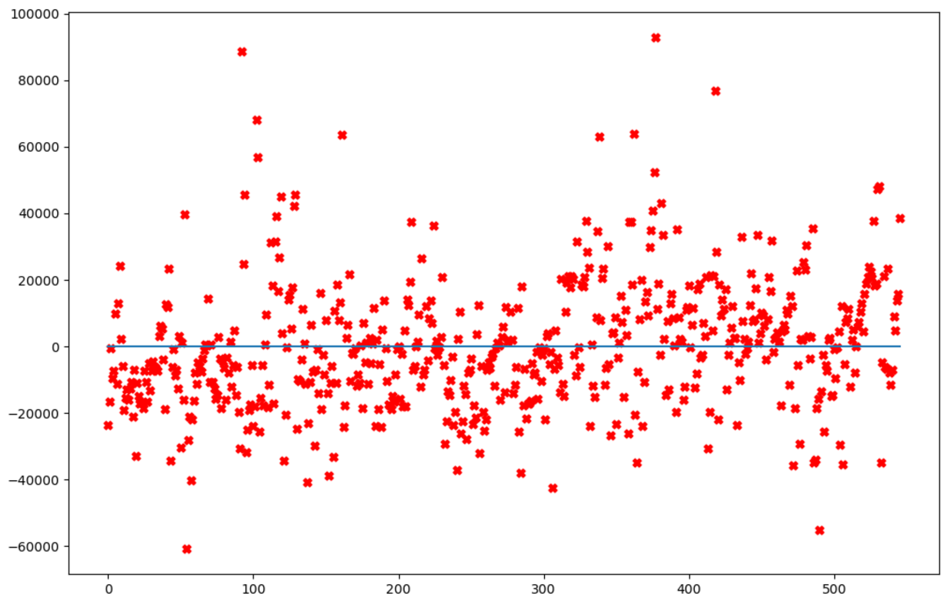
[43]:
```
MultipleLinearRegression M;
for (int i=1; i < 6; ++i)
{
    for (int j=i+1; j<6; ++j)
    {
        M.fit(csv_3.data[i], csv_3.data[j],  csv_3.data[0], 1, false);
        vector<double> y = M.predict(csv_3.data[i], csv_3.data[j]);
        double rmse_value = M.rmse(csv_3.data[0], y);
        cout << "using " << csv_3.col_names[i] << " & " << csv_3.col_names[j]␣
 ↪<< ": the RMSE value is " << rmse_value << endl;
    }
}
```

```
using lotsize & bedrooms: the RMSE value is 32610.7
using lotsize & bathrms: the RMSE value is 28497
using lotsize & stories: the RMSE value is 30007.9
using lotsize & garagepl: the RMSE value is 34364.4
using bedrooms & bathrms: the RMSE value is 35599.3
using bedrooms & stories: the RMSE value is 39052.6
using bedrooms & garagepl: the RMSE value is 38002.1
using bathrms & stories: the RMSE value is 33981.1
using bathrms & garagepl: the RMSE value is 33296.3
using stories & garagepl: the RMSE value is 35207.1
```

Based on the values above we choose *lotsize* & *bathrms* as our two features since they give us the lowest RMSE value.

We will fit a plane using *lotsize* & *bathrms* below and plot the residual values

[44]:
```
M.fit(csv_3.data[1], csv_3.data[3],  csv_3.data[0], 1, false);
vector<double> y = M.predict(csv_3.data[1], csv_3.data[3]);
double rmse_value = M.rmse(csv_3.data[0], y);

PlotResiduals Plot(csv_3.data[0], y, "charts/residuals.png");
Plot.gen_plot()
```



2.6 Part 3 - Interpolation

2.6.1 We will start by reading both given datasets

[45]:
```
std::vector<std::pair<std::string, std::vector<double>>> dataset_1 = read_csv(".
↪/datasets/part_3/3_dataset_1.csv");
std::vector<std::pair<std::string, std::vector<double>>> dataset_2 = read_csv(".
↪/datasets/part_3/3_dataset_2.csv");

vector<double> x1 = dataset_1.at(0).second;
vector<double> y1 = dataset_1.at(1).second;
vector<double> x2 = dataset_2.at(0).second;
vector<double> y2 = dataset_2.at(1).second;
```

2.6.2 We will fit `NewtonInterpolator` to both datasets below and print the polynomial

[46]:
```
vector<double> x1 = dataset_1.at(0).second;
vector<double> y1 = dataset_1.at(1).second;

NewtonInterpolator Newton;

vector<double> a1 = Newton.fit(x1,y1);
cout << "Newton Interpolation Polynomial: "; Newton.writeNewtonPolynomial(a1);
```

```
Newton Interpolation Polynomial: 10+20x^1+1x^2-0.2x^3+1.02969e-13x^4-1.72975e-14
x^5-4.80697e-15x^6-5.08368e-17x^7+4.59488e-17x^8+2.53217e-18x^9
```

[47]:
```
vector<double> a2 = Newton.fit(x2,y2);
cout << "Newton Interpolation Polynomial: "; Newton.writeNewtonPolynomial(a2);
```

```
Newton Interpolation Polynomial: 1.10059+0.555967x^1-0.248617x^2-0.243468x^3-0.0
0947879x^4+0.0356979x^5+0.00708407x^6-0.00222214x^7-0.000773607x^8+4.02274e-05x^
9+3.83602e-05x^10+2.01025e-06x^11-9.20882e-07x^12-1.17225e-07x^13+7.73236e-09x^1
4+2.16861e-09x^15+6.52763e-11x^16-1.25152e-11x^17-1.14218e-12x^18-2.91053e-14x^1
9
```

[48]:
```
std::vector<string> colnames(2);
colnames[0] = "Newton Coefficients Dataset 1";
colnames[1] = "Newton Coefficients Dataset 2";
std::vector<vector<double>> coeff(2);
coeff[0] = a1;
coeff[1] = a2;
write_csv("output_csvs/Newton Polynomial Coefficients.csv",colnames,coeff);
```

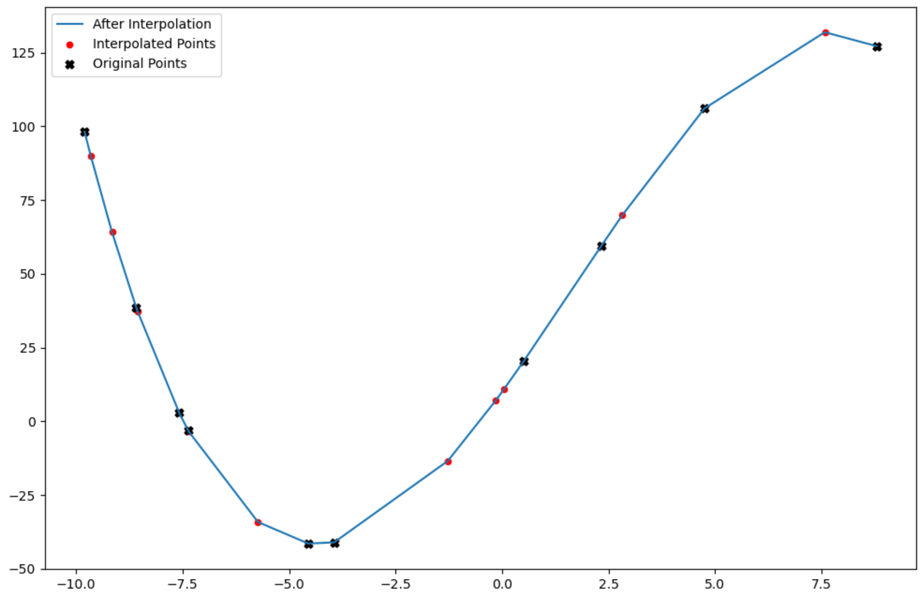
2.6.3 We will use `NewtonInterpolator` to double and quadruple the given points in the given data set and plot the results

The resulting pints are given in the `output_csvs` folder

Doubling `3_dataset_1.csv` using `NewtonInterpolator`

[49]:
```
vector<double> x1 = dataset_1.at(0).second;
vector<double> y1 = dataset_1.at(1).second;

std::vector<string> colnamesnewp(2);
colnamesnewp[0] = "X";
colnamesnewp[1] = "Y";

vector<vector<double>> newPointsNewton2D1 = getPointsNewton(x1,y1,x1.
↪size(),Newton,a1);
write_csv("output_csvs/Newton 2 Times Dataset1.
↪csv",colnamesnewp,newPointsNewton2D1);

PlotInterpolation Plot(x1, y1, newPointsNewton2D1[0], newPointsNewton2D1[1],␣
↪"charts/chart2.png");
Plot.gen_plot()
```

```
x = -7.35324
Newton Interpolation: -3.4766
x = -1.27357
Newton Interpolation: -13.4363
x = -5.72844
Newton Interpolation: -34.158
x = 2.8193
Newton Interpolation: 69.8527
x = 7.57409
Newton Interpolation: 131.948
x = -0.144181
Newton Interpolation: 7.13778
x = -9.15543
Newton Interpolation: 64.1983
x = 0.0469531
Newton Interpolation: 10.9412
x = -9.6549
Newton Interpolation: 90.1196
x = -8.55566
Newton Interpolation: 37.3398
```

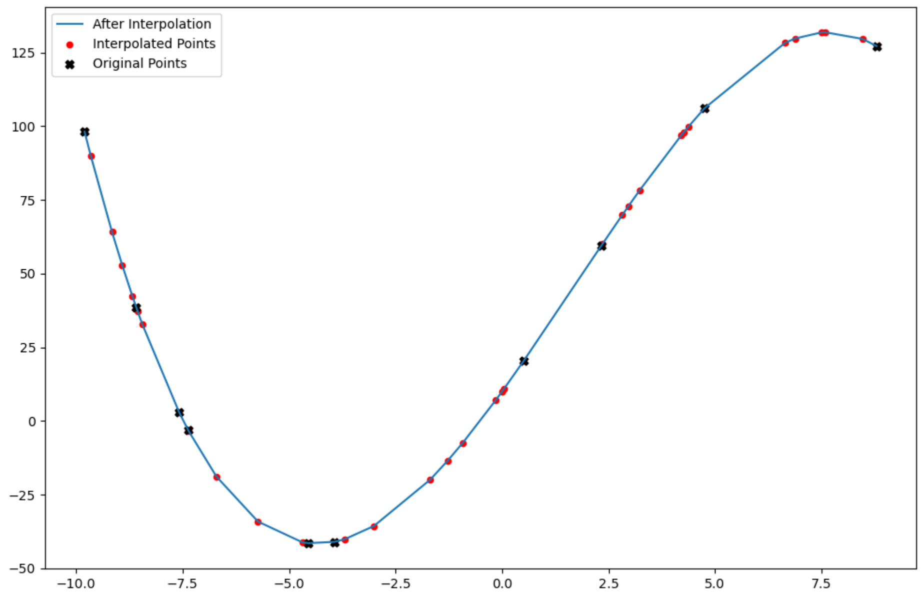


Quadrupling 3_dataset_1.csv using NewtonInterpolator

[50]:
```
vector<double> x1 = dataset_1.at(0).second;
vector<double> y1 = dataset_1.at(1).second;
vector<vector<double>> newPointsNewton4D1 = getPointsNewton(x1,y1,x1.
↪size()*3,Newton,a1);
write_csv("output_csvs/Newton 4 Times Dataset1.
↪csv",colnamesnewp,newPointsNewton4D1);

PlotInterpolation Plot(x1, y1, newPointsNewton4D1[0], newPointsNewton4D1[1],␣
↪"charts/chart3.png");
Plot.gen_plot()
```

```
x = -7.35324
Newton Interpolation: -3.4766
x = -1.27357
Newton Interpolation: -13.4363
x = -5.72844
Newton Interpolation: -34.158
x = 2.8193
Newton Interpolation: 69.8527
x = 7.57409
Newton Interpolation: 131.948
x = -0.144181
Newton Interpolation: 7.13778
x = -9.15543
```

```
Newton Interpolation: 64.1983
x = 0.0469531
Newton Interpolation: 10.9412
x = -9.6549
Newton Interpolation: 90.1196
x = -8.55566
Newton Interpolation: 37.3398
x = 2.96628
Newton Interpolation: 72.9045
x = 7.49498
Newton Interpolation: 131.869
x = -0.00455605
Newton Interpolation: 9.9089
x = 2.35567
Newton Interpolation: 60.0481
x = 3.23425
Newton Interpolation: 78.3791
x = 4.36813
Newton Interpolation: 99.7738
x = -8.91581
Newton Interpolation: 52.922
x = -3.69746
Newton Interpolation: -40.1683
x = 4.26056
Newton Interpolation: 97.8957
x = -3.00785
Newton Interpolation: -35.6673
x = 8.46356
Newton Interpolation: 129.651
x = 4.20379
Newton Interpolation: 96.8898
x = -8.44705
Newton Interpolation: 32.9556
x = 6.64506
Newton Interpolation: 128.373
x = -1.6869
Newton Interpolation: -19.9323
x = -0.918925
Newton Interpolation: -7.37888
x = -4.6886
Newton Interpolation: -41.1752
x = -6.7033
Newton Interpolation: -18.8902
x = 6.88573
Newton Interpolation: 129.833
x = -8.67234
Newton Interpolation: 42.2111
```

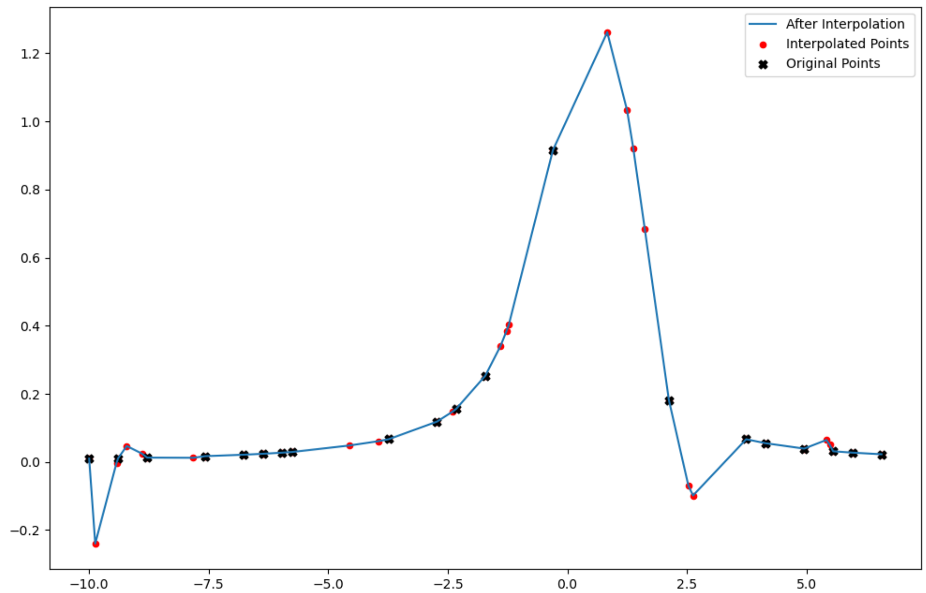


Doubling 3_dataset_2.csv using NewtonInterpolator

[51]:
```
vector<double> x2 = dataset_2.at(0).second;
vector<double> y2 = dataset_2.at(1).second;
vector<vector<double>> newPointsNewton2D2 = getPointsNewton(x2,y2,x2.
 ↪size(),Newton,a2);
write_csv("output_csvs/Newton 2 Times Dataset2.
 ↪csv",colnamesnewp,newPointsNewton2D2);

PlotInterpolation Plot(x2, y2, newPointsNewton2D2[0], newPointsNewton2D2[1],␣
 ↪"charts/chart4.png");
Plot.gen_plot()
```

```
x = -7.82099
Newton Interpolation: 0.01227
x = -2.40216
Newton Interpolation: 0.147797
x = -6.3728
Newton Interpolation: 0.0240288
x = 1.24584
Newton Interpolation: 1.03316
x = 5.4838
Newton Interpolation: 0.050436
x = -1.39553
Newton Interpolation: 0.341593
x = -9.42729
```

```
Newton Interpolation: -0.00242728
x = -1.22517
Newton Interpolation: 0.403353
x = -9.87247
Newton Interpolation: -0.239619
x = -8.89271
Newton Interpolation: 0.0243353
x = 1.37684
Newton Interpolation: 0.920963
x = 5.41329
Newton Interpolation: 0.064648
x = -1.27108
Newton Interpolation: 0.38559
x = 0.832597
Newton Interpolation: 1.26192
x = 1.61568
Newton Interpolation: 0.685382
x = 2.62631
Newton Interpolation: -0.0983455
x = -9.21372
Newton Interpolation: 0.0468638
x = -4.56258
Newton Interpolation: 0.0482861
x = 2.53044
Newton Interpolation: -0.0704881
x = -3.94793
Newton Interpolation: 0.061169
```



Quadrupling 3_dataset_2.csv using NewtonInterpolator

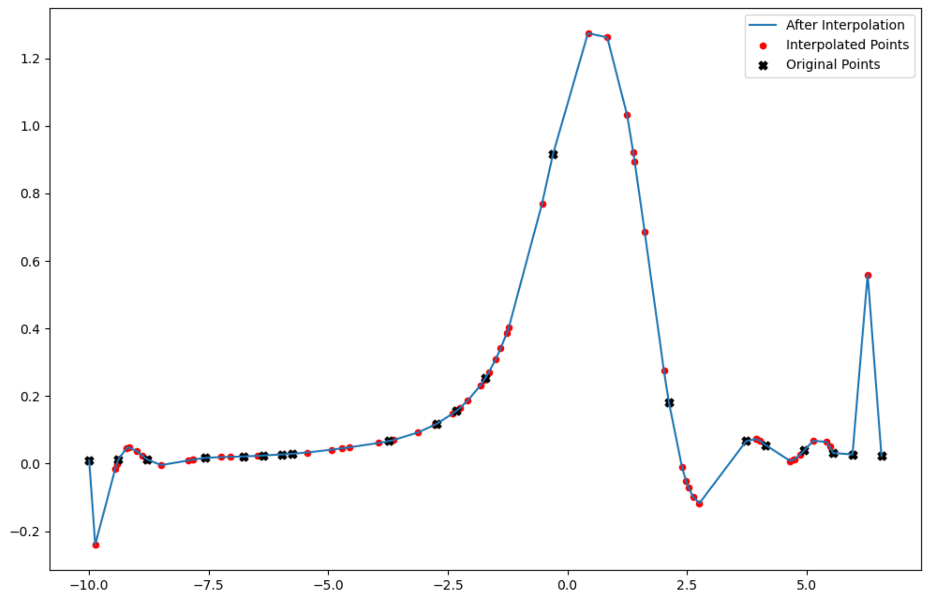
[52]:
```
vector<double> x2 = dataset_2.at(0).second;
vector<double> y2 = dataset_2.at(1).second;
vector<vector<double>> newPointsNewton4D2 = getPointsNewton(x2,y2,x2.
↪size()*3,Newton,a2);
write_csv("output_csvs/Newton 4 Times Dataset2.
↪csv",colnamesnewp,newPointsNewton4D2);

PlotInterpolation Plot(x2, y2, newPointsNewton4D2[0], newPointsNewton4D2[1],␣
↪"charts/chart5.png");
Plot.gen_plot()
```

```
x = -7.82099
Newton Interpolation: 0.01227
x = -2.40216
Newton Interpolation: 0.147797
x = -6.3728
Newton Interpolation: 0.0240288
x = 1.24584
Newton Interpolation: 1.03316
x = 5.4838
Newton Interpolation: 0.050436
x = -1.39553
Newton Interpolation: 0.341593
x = -9.42729
```

```
Newton Interpolation: -0.00242728
x = -1.22517
Newton Interpolation: 0.403353
x = -9.87247
Newton Interpolation: -0.239619
x = -8.89271
Newton Interpolation: 0.0243353
x = 1.37684
Newton Interpolation: 0.920963
x = 5.41329
Newton Interpolation: 0.064648
x = -1.27108
Newton Interpolation: 0.38559
x = 0.832597
Newton Interpolation: 1.26192
x = 1.61568
Newton Interpolation: 0.685382
x = 2.62631
Newton Interpolation: -0.0983455
x = -9.21372
Newton Interpolation: 0.0468638
x = -4.56258
Newton Interpolation: 0.0482861
x = 2.53044
Newton Interpolation: -0.0704881
x = -3.94793
Newton Interpolation: 0.061169
x = 6.27659
Newton Interpolation: 0.560015
x = 2.47983
Newton Interpolation: -0.0508226
x = -8.79591
Newton Interpolation: 0.0135965
x = 4.65575
Newton Interpolation: 0.00818451
x = -2.77056
Newton Interpolation: 0.115192
x = -2.08606
Newton Interpolation: 0.186477
x = -5.44599
Newton Interpolation: 0.0329312
x = -7.2417
Newton Interpolation: 0.0195496
x = 4.87027
Newton Interpolation: 0.0270976
x = -8.99671
Newton Interpolation: 0.0363538
x = -1.64225
```

```
Newton Interpolation: 0.270883
x = -4.71501
Newton Interpolation: 0.0453435
x = -1.81695
Newton Interpolation: 0.232151
x = -8.49695
Newton Interpolation: -0.00424079
x = -8.7783
Newton Interpolation: 0.0118335
x = -3.63643
Newton Interpolation: 0.0699557
x = 5.13799
Newton Interpolation: 0.0675253
x = -2.30615
Newton Interpolation: 0.158245
x = -9.17033
Newton Interpolation: 0.0479083
x = 2.75894
Newton Interpolation: -0.11726
x = -7.92324
Newton Interpolation: 0.00923891
x = 1.40471
Newton Interpolation: 0.895221
x = 0.4288
Newton Interpolation: 1.27431
x = 2.01693
Newton Interpolation: 0.275353
x = 4.71978
Newton Interpolation: 0.0110592
x = -4.92558
Newton Interpolation: 0.0413765
x = -1.49728
Newton Interpolation: 0.309915
x = 4.01424
Newton Interpolation: 0.0682795
x = 3.94018
Newton Interpolation: 0.0725632
x = -3.11872
Newton Interpolation: 0.0925036
x = -2.24864
Newton Interpolation: 0.164998
x = -7.04588
Newton Interpolation: 0.0201793
x = -0.530163
Newton Interpolation: 0.770173
x = -9.45244
Newton Interpolation: -0.0143489
x = -1.74235
```

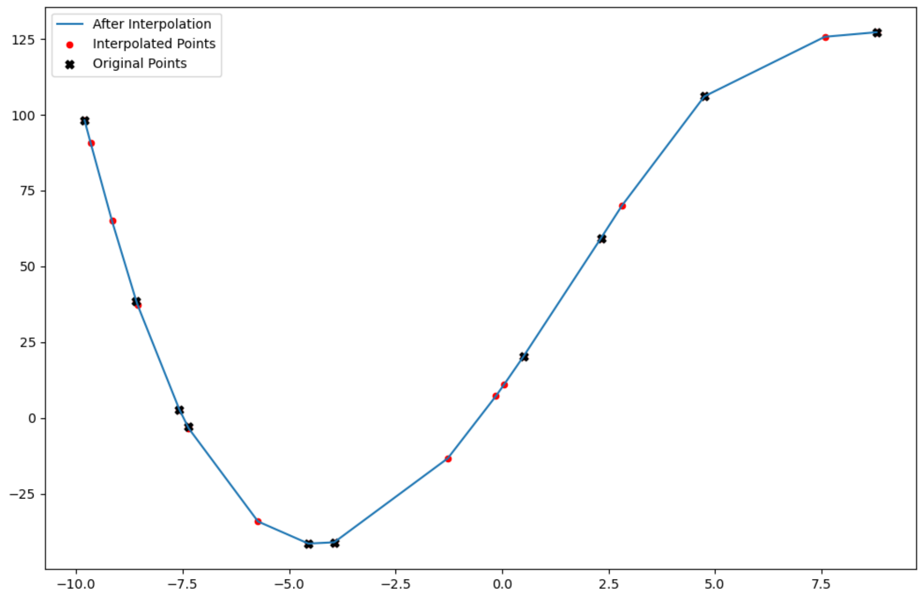
```
Newton Interpolation: 0.247679
x = 2.39596
Newton Interpolation: -0.0106893
x = 4.75565
Newton Interpolation: 0.0137791
x = 3.94894
Newton Interpolation: 0.0722156
x = -6.47563
Newton Interpolation: 0.0232603
x = -7.83939
Newton Interpolation: 0.0117659
```



Doubling 3_dataset_1.csv using Spline

[53]:
```
vector<double> x1 = dataset_1.at(0).second;
vector<double> y1 = dataset_1.at(1).second;
Spline s;
vector<vector<double>> cSpline1 = s.fitSpline(x1,y1);

vector<vector<double>> newPointsSpline2D1 = getPointsSpline(x1,y1,x1.size(),s);
write_csv("output_csvs/Spline 2 Times Dataset1.
↪csv",colnamesnewp,newPointsSpline2D1);

PlotInterpolation Plot(x1, y1, newPointsSpline2D1[0], newPointsSpline2D1[1],␣
↪"charts/chart6.png");
Plot.gen_plot()
```

```
x = -7.35324
Cubic Spline Interpolation: -3.47768
x = -1.27357
Cubic Spline Interpolation: -13.3258
x = -5.72844
Cubic Spline Interpolation: -34.1737
x = 2.8193
Cubic Spline Interpolation: 70.1556
x = 7.57409
Cubic Spline Interpolation: 125.746
x = -0.144181
Cubic Spline Interpolation: 7.21725
x = -9.15543
Cubic Spline Interpolation: 65.0448
x = 0.0469531
Cubic Spline Interpolation: 11.0029
x = -9.6549
Cubic Spline Interpolation: 90.6614
x = -8.55566
Cubic Spline Interpolation: 37.3083
```

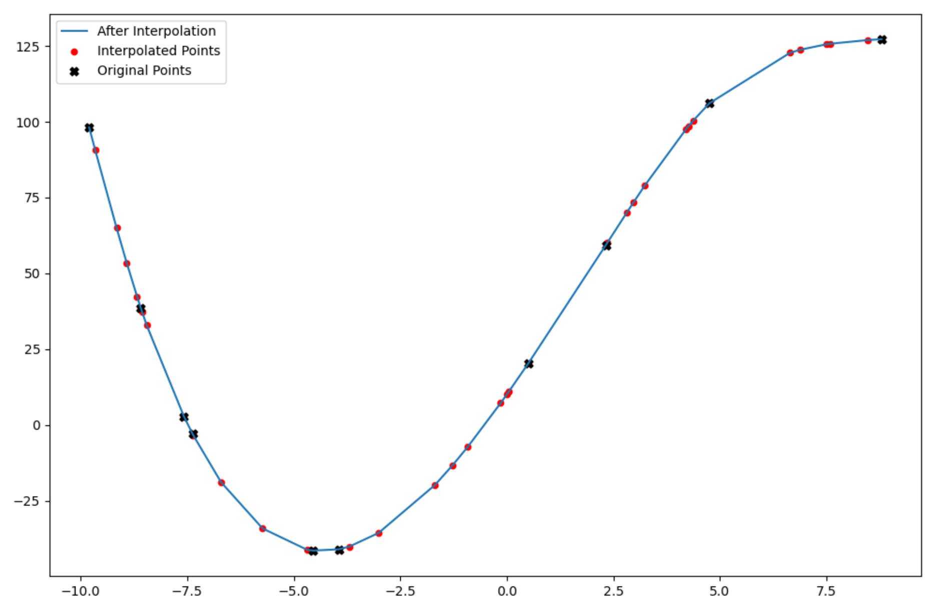

After Interpolation
Interpolated Points
Original Points
125
100
75
50
25
0
−25
−10.0
−7.5
−5.0
−2.5
0.0
2.5
5.0
7.5

Quadrupling 3_dataset_1.csv using Spline

[54]:
```
vector<double> x1 = dataset_1.at(0).second;
vector<double> y1 = dataset_1.at(1).second;
vector<vector<double>> newPointsSpline4D1 = getPointsSpline(x1,y1,x1.
↪size()*3,s);
write_csv("output_csvs/Spline 4 Times Dataset1.
↪csv",colnamesnewp,newPointsSpline4D1);

PlotInterpolation Plot(x1, y1, newPointsSpline4D1[0], newPointsSpline4D1[1],␣
↪"charts/chart7.png");
Plot.gen_plot()
```

```
x = -7.35324
Cubic Spline Interpolation: -3.47768
x = -1.27357
Cubic Spline Interpolation: -13.3258
x = -5.72844
Cubic Spline Interpolation: -34.1737
x = 2.8193
Cubic Spline Interpolation: 70.1556
x = 7.57409
Cubic Spline Interpolation: 125.746
x = -0.144181
Cubic Spline Interpolation: 7.21725
x = -9.15543
Cubic Spline Interpolation: 65.0448
x = 0.0469531
Cubic Spline Interpolation: 11.0029
x = -9.6549
Cubic Spline Interpolation: 90.6614
x = -8.55566
Cubic Spline Interpolation: 37.3083
x = 2.96628
Cubic Spline Interpolation: 73.3096
x = 7.49498
Cubic Spline Interpolation: 125.579
x = -0.00455605
Cubic Spline Interpolation: 9.9758
x = 2.35567
Cubic Spline Interpolation: 60.0636
x = 3.23425
Cubic Spline Interpolation: 78.9598
x = 4.36813
Cubic Spline Interpolation: 100.322
x = -8.91581
Cubic Spline Interpolation: 53.4236
x = -3.69746
```

```
Cubic Spline Interpolation: -40.1641
x = 4.26056
Cubic Spline Interpolation: 98.5371
x = -3.00785
Cubic Spline Interpolation: -35.636
x = 8.46356
Cubic Spline Interpolation: 126.969
x = 4.20379
Cubic Spline Interpolation: 97.5704
x = -8.44705
Cubic Spline Interpolation: 32.8396
x = 6.64506
Cubic Spline Interpolation: 122.795
x = -1.6869
Cubic Spline Interpolation: -19.8334
x = -0.918925
Cubic Spline Interpolation: -7.26656
x = -4.6886
Cubic Spline Interpolation: -41.1756
x = -6.7033
Cubic Spline Interpolation: -18.9111
x = 6.88573
Cubic Spline Interpolation: 123.794
x = -8.67234
Cubic Spline Interpolation: 42.3205
```

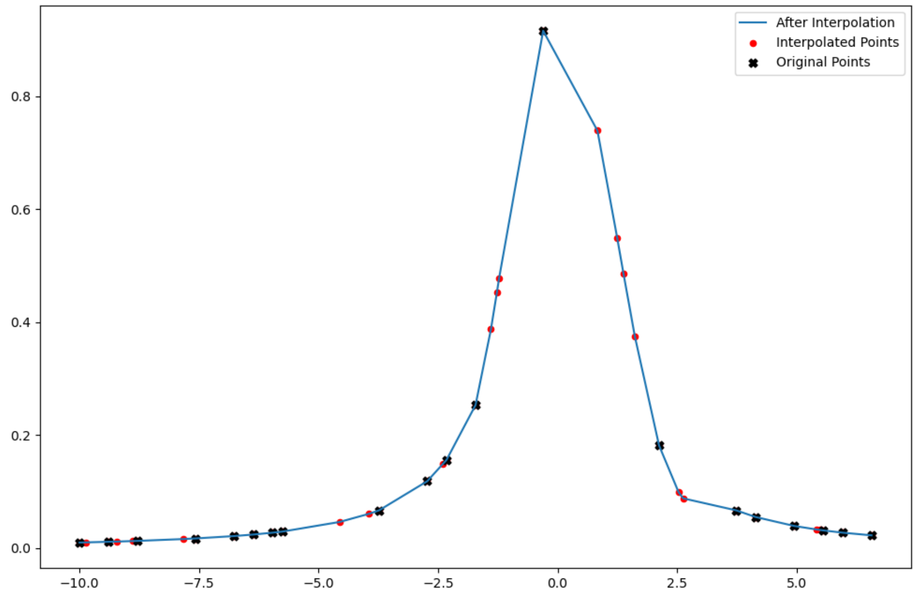


Doubling 3_dataset_2.csv using Spline

[55]:
```
vector<double> x2 = dataset_2.at(0).second;
vector<double> y2 = dataset_2.at(1).second;
Spline s2;
vector<vector<double>> cSpline2 = s2.fitSpline(x2,y2);

vector<vector<double>> newPointsSpline2D2 = getPointsSpline(x2,y2,x2.size(),s2);
write_csv("output_csvs/Spline 2 Times Dataset2.
↪csv",colnamesnewp,newPointsSpline2D2);

PlotInterpolation Plot(x2, y2, newPointsSpline2D2[0], newPointsSpline2D2[1],␣
↪"charts/chart8.png");
Plot.gen_plot()
```

```
x = -7.82099
Cubic Spline Interpolation: 0.0160849
x = -2.40216
Cubic Spline Interpolation: 0.148882
x = -6.3728
Cubic Spline Interpolation: 0.0240312
x = 1.24584
Cubic Spline Interpolation: 0.549258
x = 5.4838
Cubic Spline Interpolation: 0.0322147
x = -1.39553
Cubic Spline Interpolation: 0.387296
x = -9.42729
Cubic Spline Interpolation: 0.0111277
x = -1.22517
Cubic Spline Interpolation: 0.477508
x = -9.87247
Cubic Spline Interpolation: 0.0101644
x = -8.89271
Cubic Spline Interpolation: 0.0124866
x = 1.37684
Cubic Spline Interpolation: 0.486327
x = 5.41329
Cubic Spline Interpolation: 0.0330678
x = -1.27108
Cubic Spline Interpolation: 0.45239
x = 0.832597
Cubic Spline Interpolation: 0.739015
x = 1.61568
Cubic Spline Interpolation: 0.374616
x = 2.62631
Cubic Spline Interpolation: 0.0881143
x = -9.21372
```

```
Cubic Spline Interpolation: 0.0116398
x = -4.56258
Cubic Spline Interpolation: 0.0462939
x = 2.53044
Cubic Spline Interpolation: 0.0989651
x = -3.94793
Cubic Spline Interpolation: 0.0607052
```



Quadrupling 3_dataset_2.csv using Spline

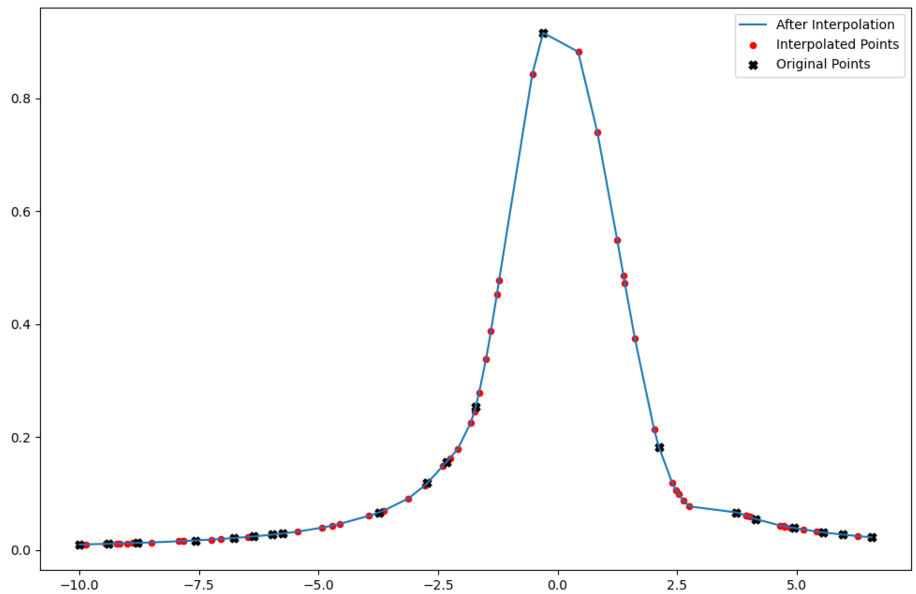
[56]:
```
vector<double> x2 = dataset_2.at(0).second;
vector<double> y2 = dataset_2.at(1).second;
vector<vector<double>> newPointsSpline4D2 = getPointsSpline(x2,y2,x2.
 ↪size()*3,s2);
write_csv("output_csvs/Spline 4 Times Dataset2.
 ↪csv",colnamesnewp,newPointsSpline4D2);

PlotInterpolation Plot(x2, y2, newPointsSpline4D2[0], newPointsSpline4D2[1],␣
 ↪"charts/chart9.png");
Plot.gen_plot()
```

```
x = -7.82099
Cubic Spline Interpolation: 0.0160849
x = -2.40216
Cubic Spline Interpolation: 0.148882
x = -6.3728
```

```
Cubic Spline Interpolation: 0.0240312
x = 1.24584
Cubic Spline Interpolation: 0.549258
x = 5.4838
Cubic Spline Interpolation: 0.0322147
x = -1.39553
Cubic Spline Interpolation: 0.387296
x = -9.42729
Cubic Spline Interpolation: 0.0111277
x = -1.22517
Cubic Spline Interpolation: 0.477508
x = -9.87247
Cubic Spline Interpolation: 0.0101644
x = -8.89271
Cubic Spline Interpolation: 0.0124866
x = 1.37684
Cubic Spline Interpolation: 0.486327
x = 5.41329
Cubic Spline Interpolation: 0.0330678
x = -1.27108
Cubic Spline Interpolation: 0.45239
x = 0.832597
Cubic Spline Interpolation: 0.739015
x = 1.61568
Cubic Spline Interpolation: 0.374616
x = 2.62631
Cubic Spline Interpolation: 0.0881143
x = -9.21372
Cubic Spline Interpolation: 0.0116398
x = -4.56258
Cubic Spline Interpolation: 0.0462939
x = 2.53044
Cubic Spline Interpolation: 0.0989651
x = -3.94793
Cubic Spline Interpolation: 0.0607052
x = 6.27659
Cubic Spline Interpolation: 0.0248143
x = 2.47983
Cubic Spline Interpolation: 0.10584
x = -8.79591
Cubic Spline Interpolation: 0.0127603
x = 4.65575
Cubic Spline Interpolation: 0.0434686
x = -2.77056
Cubic Spline Interpolation: 0.114925
x = -2.08606
Cubic Spline Interpolation: 0.179729
x = -5.44599
```

```
Cubic Spline Interpolation: 0.0326501
x = -7.2417
Cubic Spline Interpolation: 0.0187108
x = 4.87027
Cubic Spline Interpolation: 0.0403091
x = -8.99671
Cubic Spline Interpolation: 0.0122021
x = -1.64225
Cubic Spline Interpolation: 0.278717
x = -4.71501
Cubic Spline Interpolation: 0.0434136
x = -1.81695
Cubic Spline Interpolation: 0.225788
x = -8.49695
Cubic Spline Interpolation: 0.0136614
x = -8.7783
Cubic Spline Interpolation: 0.012811
x = -3.63643
Cubic Spline Interpolation: 0.0699828
x = 5.13799
Cubic Spline Interpolation: 0.0366476
x = -2.30615
Cubic Spline Interpolation: 0.157884
x = -9.17033
Cubic Spline Interpolation: 0.0117489
x = 2.75894
Cubic Spline Interpolation: 0.0772795
x = -7.92324
Cubic Spline Interpolation: 0.0156785
x = 1.40471
Cubic Spline Interpolation: 0.473001
x = 0.4288
Cubic Spline Interpolation: 0.882363
x = 2.01693
Cubic Spline Interpolation: 0.214244
x = 4.71978
Cubic Spline Interpolation: 0.042473
x = -4.92558
Cubic Spline Interpolation: 0.0398277
x = -1.49728
Cubic Spline Interpolation: 0.338558
x = 4.01424
Cubic Spline Interpolation: 0.0593278
x = 3.94018
Cubic Spline Interpolation: 0.061778
x = -3.11872
Cubic Spline Interpolation: 0.0916173
x = -2.24864
```

```
Cubic Spline Interpolation: 0.163144
x = -7.04588
Cubic Spline Interpolation: 0.0197446
x = -0.530163
Cubic Spline Interpolation: 0.842672
x = -9.45244
Cubic Spline Interpolation: 0.01107
x = -1.74235
Cubic Spline Interpolation: 0.245642
x = 2.39596
Cubic Spline Interpolation: 0.119142
x = 4.75565
Cubic Spline Interpolation: 0.0419386
x = 3.94894
Cubic Spline Interpolation: 0.0614965
x = -6.47563
Cubic Spline Interpolation: 0.0232916
x = -7.83939
Cubic Spline Interpolation: 0.0160106
```



Saving results from our implementation of the cubic spline method in CSV format

[57]:
```
vector<double> x1 = dataset_1.at(0).second;
vector<double> y1 = dataset_1.at(1).second;
vector<double> x2 = dataset_2.at(0).second;
vector<double> y2 = dataset_2.at(1).second;
```

```
std::vector<string> colnamesSpline1(5);
colnamesSpline1[0] = "Spline x Dataset 1";
colnamesSpline1[1] = "Spline a Dataset 1";
colnamesSpline1[2] = "Spline b Dataset 1";
colnamesSpline1[3] = "Spline c Dataset 1";
colnamesSpline1[4] = "Spline y Dataset 1";

write_csv("output_csvs/Cubic Spline Interpolator Dataset 1.
↪csv",colnamesSpline1,cSpline1);
```

[58]:
```
vector<double> x1 = dataset_1.at(0).second;
vector<double> y1 = dataset_1.at(1).second;
vector<double> x2 = dataset_2.at(0).second;
vector<double> y2 = dataset_2.at(1).second;

vector<vector<double>> cSpline2 = s.fitSpline(x2,y2);
std::vector<string> colnamesSpline2(5);
colnamesSpline2[0] = "Spline x Dataset 2";
colnamesSpline2[1] = "Spline a Dataset 2";
colnamesSpline2[2] = "Spline b Dataset 2";
colnamesSpline2[3] = "Spline c Dataset 2";
colnamesSpline2[4] = "Spline y Dataset 2";

write_csv("output_csvs/Cubic Spline Interpolator Dataset 2.
↪csv",colnamesSpline2,cSpline2);
```

2.6.4 Our comment on the difference between Newton's and cubic spline performance on each dataset.

From visually inspecting the plots above we can tell the did fit both datasets better. This is due to the fact that cubic spline is more complex, it gets a more stable solution (less variations in the function after interpolation)