

- 1- A measurable space (X, A) is a set X and a σ -algebra A on X . True Definition
- 2- if f is bounded on $[a, b]$ & if the points of discontinuity of f on $[a, b]$ form a null set then $f \in L^1[a, b]$. True Theorem
- 3- A series of real numbers is absolutely convergent if and only if it can be expressed as the difference of two convergent series of positive real numbers. True Theorem Proposition
 $|a_n| = a_n^+ + a_n^-$ $\sum a_n^+$ & $\sum a_n^-$ are convergent
- 4- Any countable set on the real line is null and so the rational numbers form a null set. True Theorem
- 5- Any step function may be expressed as a finite linear combination of characteristic function of disjoint intervals. True Theorem
- 6- if S is a null set on \mathbb{R} then there is an increasing sequence $\{\varphi_n\}$ of step function for which the sequence $\{\varphi_n\}$ converges and such that $\{\varphi_n(x)\}$ diverges for every x in S . False
 Almost everywhere $\sum \varphi_n$ converge and the point fail to converge $\{\varphi_n\}$ in $x \in S$
- 7- Any increasing sequence $\{s_n\}$ of real numbers is convergent if and only if, it is bounded above. True
 Axiom of Completeness theorem
- 8- A compact subset of \mathbb{R}^k has finite measure. True (Proposition) (Theorem)
- 9- Any countable set on the real line is null. True proposition Theorem
- 10- A ring is closed under taking the set operations \cup, \cap, \setminus . True Definition \mathbb{R} let S, T be measurable sets, then so are $S \cup T, S \cap T$

Mostafa Abdelkarem

De Sher
2022

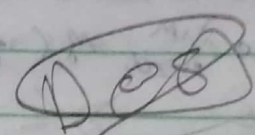
May 2017

- 1- The Lebesgue integral is a linear, Positive absolute operator on the linear space L^1 . True Definition
- 2- if f is monotone on $[a, b]$ then f is integrable on $[a, b]$. True Theorem Corollary
- 3- All Continuous & all integrable functions are measurable. True Theorem
- 4- $M(S) = 0$ if and only if S is null. False $M(S) = 0$ iff S is a null set
- 6- Fatou's Lemma states that let f_n be a sequence of +ve measurable functions on E . if $f_n \rightarrow f$ in Measure on E , then $\int_E f \leq \liminf_{n \rightarrow \infty} \int_E f_n$. True Theorem
- 7- $\int_{-\infty}^{\infty} e^{-|x|} dx = 2 = \int_0^1 x^{-1/2} dx$
- 8- By means of the Monotone Convergence theorem we can evaluate the integrals such as $\int_{-\infty}^{\infty} e^{-|x|} dx = \lim \int f_n = 2 = \int_0^1 x^{-1/2} dx$. True & Example in book Page 99
- 10- Assume A is a Lebesgue measurable subset of \mathbb{R} of finite measure and $\phi(x) = |A \cap (-\infty, x]|$. Then ϕ is Continuous at each $x \in \mathbb{R}$. True

Mostafa Abdelkareem

Deha
2022

May 2019

1. A set function is a function whose range is a class of set
Domain ~~could be false~~
2. Extended Real Valued Set function is a set function of
range of Complex numbers and also could be $\pm \infty$
~~could be false~~
3. If μ is a measure on a ring R a set E in R is said to have
finite measure if $\mu(E) \leq \infty$
~~false~~ $\mu(E) < \infty$ 
4. The measure of E is σ -finite if there exists a sequence
 $\{E_n\}$ of sets in R such that $E \subset \bigcup_{n=1}^{\infty} E_n$ & $\mu(E_n) < \infty$,
 $n = 1, 2, \dots$ True Definition
5. An extended real valued set function μ on a class G is not
Continuous from below at a set E (in G) if for every increasing
sequence $\{E_n\}$ of sets in G for which $\lim_{n \rightarrow \infty} E_n = E$, we have
 $\lim_{n \rightarrow \infty} \mu(E_n) \neq \mu(E)$ false Continuous
6. A hereditary class is σ -ring iff it's not closed under the
formation of countable unions false Closed
7. An outer measure μ^* is a set function from a hereditary
 σ -ring H to the positive extended real values such that (non-negative)
(monotonic) and (Countable subadditive) false non-negative
8. Let \mathcal{E} be a σ -algebra over a set X and μ be a measure on \mathcal{E} .
Then the inner measure μ_* induced by μ is defined by $\mu_*(T) = \inf_{S \in \mathcal{E} \text{ and } S \subset T} \mu(S)$
 \sup false
9. If μ is σ -finite measure on a ring R , then there is a unique
measure $\bar{\mu}$ on the σ -ring $\mathcal{S}(R)$, such that for E in R ,
 $\bar{\mu}(E) \neq \mu(E)$; the measure $\bar{\mu}$ is σ -finite $\bar{\mu}(E) = \mu(E)$ false
10. A set X is non-measurable set iff $\mu^*(X) > \mu_*(X)$; which means
inner and outer measure of X are equal false
 $\mu^*(X) > \mu_*(X)$ or $\mu^*(X) = \mu_*(X) = \infty$

Mastab Abdulkarim

DeSha 2022

May 2019

- 11 - A Measurable function is a function between two unmeasurable spaces such that the preimage of any measurable set is measurable. false measurable
- 12 - A sequence $\{f_n\}$ of a finite valued measurable functions converges in measure to the measurable function f , if for every any $\epsilon > 0$ $\lim_{n \rightarrow \infty} \mu(\{x: |f_n(x) - f(x)| \leq \epsilon\}) = 0$. false $\geq \epsilon$
- 13 - So long as there are non measurable sets in a measure space, there aren't non-measurable functions from that space. false are
- 14 - If μ is a measure on a ring R , then μ is monotone or subtractive. True Theorem monotone & ring
- 15 - Every set E in $H(S)$ has a measurable kernel. True Theorem
- 16 - Every countable set is a Borel set of measure zero. True Theorem
- 17 - The measure of a singleton is zero. True Theorem
- 18 - Let a closed set F be a subset of a bounded closed set G . Then $\mu(F) \leq \mu(G)$. True Theorem
- 19 - ~~ACCA~~ Repeated
- 20 - If f is measurable and $|f|$ is integrable, then f is integrable. True Proposition

Mustafa Abdelkareem



May 2018

- 1- if (X, A, μ) is a measure space and $1 \leq p \leq \infty$ then $L^p(X)$ is a Banach space True Theorem
- 2-
- 3-
- 4- if a +ve function has Lebesgue integral equal to zero then the function is zero a.e. ω True Theorem
- 5- The function $f(x) = \frac{1}{1+x^2}$ has supremum = max^m and infimum = min^m False max = sup = 1, min \neq inf
- 6-9
- 10- The function $f: \mathbb{R}^k \rightarrow \mathbb{R}$ is measurable if the truncated function mid $[-g, f, g]$ is integrable for every positive integrable function g in $L^2(\mathbb{R}^k)$ True Definition Pg 120
- 11-12.
- 13- The smallest linear space containing L^{loc} is denoted by L^1 which is the set of all function of the form $f = g - h$ where $g, h \in L^{loc}$ and so L^1 is called the space of Lebesgue integrable functions True Definition
- 14-15.
- 16- The union and intersection of a countable number of measurable sets are measurable sets. True Theorem
- 17-
- 18- Any ring is a semi-ring True Theorem
- 19- The Lebesgue measure of $S(MCS) = \infty$ if X_S is not integrable True Definition Measure Pg 125
- 20- The rational numbers are countable set True Theorem

Mosab
Alhakeem
Deshi

May 2018

Math

- 1- let S, T be measurable sets then $M(S \cup T) + M(S \cap T) = M(S) + M(T)$ True Theorem
- 2- Holder inequality let $p, q > 1$ satisfy the relation $\frac{1}{p} + \frac{1}{q} = 1$ if $f \in L^p$ and $g \in L^q$ then $fg \in L^1$ and $|fg| \leq \|f\|_p \|g\|_q$ True Theorem
- 3- \mathbb{R} has the property of completeness of the real numbers True Theorem
- 4- let $\{Q_n\}$ be a decreasing sequence of positive step functions converging a.e. to zero then $\int Q_n \rightarrow 0$ True Theorem
- 5- The inner measure $M_*(E)$ of E is the least upper bound of measure of closed sets F contained in the set E :
 $M_*(E) = \sup \{ M(F) : F \subseteq E, F \text{ is closed} \}$ false
 The set $F \subseteq E$ must be closed and bdd \equiv compact
- 6- If M is a measure on X De
- 7- 8- De
- 9- A measurable space is a set X and a σ -ring S of subsets of X with the property that $U S = X$ True Theorem
- 10- A measure space is a measurable space (X, S) and a measure M on S True Definition
- L^1 is the space of Lebesgue integrable functions.

De

Mostafa Abdelkareem

2022

Q1

~~May~~ May 2017

- 1-d 2-d 3-a 4-c 5-d
6-d 7-d 8-a 9-d 10-

Q3 Repeated

May 2016

1- L^{inc} denotes the set of all function f where f is the limit almost every where of an increasing sequence of step function whose integrals are bounded.

True Definition

2- The Monotone function f is continuous at the point P if and only if, $f(P-0) = f(P+0)$. True Theorem

3- $M^*(E_1) - M^*(E_2) \leq 2M^*(E_1 \Delta E_2) + 2M^*(E_1 \cap E_2)$, where M^* is the Lebesgue outer measure on \mathbb{R} & $E_1, E_2 \subset \mathbb{R}$

True

4- ✓

5- ✓

6- The symmetric difference of 2 sets is $A \Delta B = (A/B) \cup (B/A)$

True

7-8-9-10- ✓

Skip must prove 2 lipas
جواب كل أسئلة يجب أن يثبت

Mustafa Abdelkarem
Deshay

4/6/2022

May 2015

- 1- let $A \subset \mathbb{R}$ be a Lebesgue measurable set if $0 \leq b \leq m(A)$ then there is a Lebesgue measurable set $B \subset A$ with $m(B) = b$
True
- 2- كـ
- 3- if $|A \cap I| \leq b|I|$ for all open intervals $I \subset \mathbb{R}$ & $b < 1$, then $|A| = 0$
من حاله صح، تصدق طمس T
- 4- if $a|I| \leq |A \cap I|$ for all open intervals $I \subset \mathbb{R}$ & $a > 0$ then $|A| = \infty$
من حاله صح، زي افتمها
- 5- كـ
- 6- A measurable map $T: X \rightarrow X$ on a measure space (X, \mathcal{A}, μ) is said to be measure preserving if $\mu(T^{-1}(A)) = \mu(A)$.
True
- 7- كـ
- 8- All closed sets in \mathbb{R}^k are measurable True Theorem
- 9- كـ
- 10- كـ

Mostafa Abdelkayem

Desha
Mansy

الى اللقاء يا بهرجين

انشاء الله خير تجميع For F

قولوا لى حيا منى، يا بهرجين يا بهرجين
فتمت لوز يا كازيها

May 2021

1. The Axiom of Completeness assures that $\left\{\left(1 + \frac{1}{n}\right)^n\right\}$ converges to a real number between 2 and 3

True Theorem

13. Let $\{f_n\}$ be an increasing sequence of function in L^{∞} where integrals are bounded. Then $\{f_n\}$ converges almost everywhere to a function f , where f lies in L^{∞} and $\int f = \lim \int f_n$.

True Theorem

15. Let $f: \mathbb{R}^k \rightarrow \mathbb{R}$ be the limit almost everywhere of a sequence of integrable function and suppose that $|f| \leq g$ for positive integrable function g . Then f is integrable.

True Theorem

16. The product of two integrable function may not be integrable while the product of any two measurable function is measurable

True Theorem

19. If $f_n \rightarrow f$ almost everywhere and f_n is measurable for $n = 1, 2, \dots$ then f is measurable

True Theorem

17. All continuous and all integrable function are measurable

True Theorem

Desha

Desha Mansy

Mesbaba Abdelkarim

Senior 2022

Desha Mansy

May 2019

1- \mathbb{R}^3 the open sphere reduces to open interval in \mathbb{R}^1 .
True Theorem

2- The space defined by the Cartesian product \mathbb{R}^n is called n dimensional Euclidean space. True Theorem

3- if a bounded set S in \mathbb{R}^n is infinite and bounded; then there is at least one point in \mathbb{R}^n which is an accumulation point of S . True Theorem

6- Fatou's Lemma implies the monotone convergence theorem. True Theorem

12- it is necessary and sufficient for the sequence $\{s_n\}$ of real number to converge to a real number if there exists an integer N such that $|s_m - s_n| < \epsilon$ for $m, n \geq N, \epsilon > 0$. True Theorem

Refered math 2021

Refered math

Deena 2022
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2022 Senior