Sampling theory

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$\boxed{1}$ Gamma distn

$$f(x,\alpha,\beta) = \frac{1}{\beta^{\alpha}\Gamma\alpha} e^{-x/\beta} x^{\alpha-1}$$

Moment $M_x(t) = E(e^{tx})$ $= (1-\beta t)^{-\alpha}$

cont / dis: $E(x) = \int x f(x)\,dx = \sum x f(x)$

look You led!

$\eta = \frac{\partial M_x(t)}{\partial t}\Big|_{t=0} = E(x)$ $E(x^2) = \frac{\partial^2 M_x(t)}{\partial t^2}\Big|_{t=0}$

$\boxed{2}$ The random variable x has uniform distribution with pdf $f(x) = 1$ for $0 < x < 1$ and zero elsewhere find p.d.f of $Z = -2\ln x$

Uniform dist $f(x) = \frac{1}{b-a}$ $a<x<b \Rightarrow f(x) = \frac{1}{1-0} = 1$, $Z = \ln x^{-2}$

$$M_Z(t) = E(e^{tZ}) = E\left(e^{t\ln x^{-2}}\right) = \int_0^1 e^{t\ln x^{-2}} * 1\,dx = \int_0^1 e^{\ln(x^{-2})^t}\,dx$$

$$= \int_0^1 x^{-2t}\,dx = \frac{x^{-2t+1}}{-2t+1}\Big|_0^1 = \frac{1}{1-2t} = (1-2t)^{-1}$$

Then compare $(1-2t)^{-1}$ by $(1-\beta t)^{-\alpha}$ $\alpha = 1$, $\beta = 2$

Then $f(x,\alpha,\beta) = \frac{1}{2^1\Gamma 1} e^{-x/2} x^0 = \frac{1}{2} e^{-x/2}$

$\boxed{3}$ The random variable x has gamma distribution with $\alpha = r/2$ where $r > 0$ and $\beta > 0$ as its parameters find the pdf of $y = 2x/\beta$

r is integer

The gamma pdf is $f(x,\alpha,\beta) = \int_0^\infty \frac{1}{\beta^\alpha\Gamma\alpha} e^{-x/\beta} x^{\alpha-1}\,dx \rightarrow \alpha = r/2$

$$f(x, r/2, \beta) = \int_0^\infty \frac{1}{\beta^{r/2}\Gamma r/2} e^{-x/\beta} x^{r/2-1}\,dx$$

$$M_y(t) = E(e^{yt}) = \int_0^\infty e^{yt} f(x)\,dx = \int_0^\infty e^{yt} \frac{1}{\beta^{r/2}\Gamma r/2} \cdot e^{-x/\beta} \cdot x^{r/2-1}\,dx$$

$$= \frac{1}{\beta^{r/2}\Gamma r/2} \int_0^\infty e^{(yt - \frac{x}{\beta})} x^{r/2-1}\,dx \qquad y = \frac{2x}{\beta}$$

$$= \frac{1}{\beta^{r/2}\Gamma r/2} \int_0^\infty e^{(\frac{2x}{\beta}t - \frac{x}{\beta})} x^{r/2-1}\,dx$$

$$= \frac{1}{\beta^{r/2}\Gamma r/2} \int_0^\infty e^{-\frac{x}{\beta}(1-2t)} x^{r/2-1}\,dx$$

let $\frac{x}{\beta}(1-2t) = u \Rightarrow x = \frac{\beta}{1-2t} u \Rightarrow dx = \frac{\beta}{1-2t}\,du$

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$$M_y(t) = \frac{1}{B^{r/2}\,\Gamma_{r/2}} \int_0^\infty e^{-u}\left(\frac{B}{1-2t}\,u\right)^{\frac{r}{2}-1}\left(\frac{B}{1-2t}\right)du$$

$$= \frac{1}{B^{r/2}\,\Gamma_{r/2}}\left(\frac{B}{1-2t}\right)^{r/2} \underbrace{\int_0^\infty e^{-u}\,u^{\frac{r}{2}-1}\,du}_{\Gamma_{r/2}}$$

$$= (1-2t)^{-r/2}$$

Compare by $(1-Bt)^{-\alpha}$

$\alpha = \frac{r}{2}$ $\quad B = 2$

pdf $\Rightarrow f(x, \frac{r}{2}, 2) = \frac{1}{2^{r/2}\,\Gamma_{r/2}} \cdot e^{-x/2} \cdot x^{\frac{r}{2}-1}$

$\boxed{4}$ Prove that if $Z \sim N(0,1)$ and $y \sim \chi_{(r)}$ are two independent variables then $X = \frac{Z}{\sqrt{y/r}} \sim t_r$

$Z \sim N(0,1) \Rightarrow f_1(z) = \frac{1}{\sqrt{2\pi}}\, e^{-\frac{1}{2}z^2}$

$y \sim \chi^2_{(r)} \Rightarrow f_2(y) = \frac{1}{2^{r/2}\,\Gamma_{r/2}}\, e^{-y/2}\, y^{r/2 - 1}$

$g(z,y) = f_1(z) * f_2(y) = \frac{1}{\sqrt{2\pi}\; 2^{r/2}\,\Gamma_{r/2}}\, e^{-\frac{1}{2}(z^2+y)}\, y^{\frac{r}{2}-1}$

$g(x,y) = g(z,y)\big|_{z=x}\, |J|$

$X = Z/\sqrt{y/r} \Rightarrow Z = X\sqrt{\frac{y}{r}} \Rightarrow \frac{dz}{dx} = \sqrt{\frac{y}{r}} = |J|$

$-\frac{1}{2}(z^2+y)$

$-\frac{1}{2}(x^2\frac{y}{r}+y)$

$g(x,y) = \frac{1}{\sqrt{2\pi r}\; 2^{r/2}\,\Gamma_{r/2}}\, e^{-\frac{y}{2}\left(\frac{x^2}{r}+1\right)} \cdot y^{\frac{r-1}{2}}$

$h(x) = \int_0^\infty g(x,y)\,dy = \frac{1}{\sqrt{2\pi r}\; 2^{r/2}\,\Gamma_{r/2}} \int_0^\infty e^{-\frac{y}{2}\left(\frac{x^2}{r}+1\right)} \cdot y^{\frac{r-1}{2}}\,dy$

let $u = \frac{y}{2}\left(\frac{x^2}{r}+1\right) \Rightarrow y = 2u/\left(\frac{x^2}{r}+1\right) \Rightarrow dy = 2/\left(\frac{x^2}{r}+1\right)du$

$$h(x) = \frac{2^{\frac{r+1}{2}}\left(\frac{x^2}{r}+1\right)^{-\left(\frac{r+1}{2}\right)}}{\sqrt{2\pi r}\; 2^{r/2}\;\Gamma_{r/2}} \int_0^\infty e^{-u}\, u^{\frac{r-1}{2}}\, du$$

distn of $t_{(r)}$

$$h(x) = \frac{\left(\frac{x^2}{r}+1\right)^{-\left(\frac{r+1}{2}\right)} \cdot \Gamma_{\frac{r+1}{2}}}{\sqrt{\pi r}\;\;\Gamma_{r/2}}$$

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5 ~~If $u \sim \chi^2_{(r_1)}$ and $v \sim \chi^2_{(r_2)}$ are two independent variables then prove~~

$$X = \frac{u/r_1}{v/r_2} \sim F(r_1, r_2)$$

$$u \sim \chi^2_{r_1} \Rightarrow f_1(u) = \frac{1}{2^{r_1/2}\Gamma\frac{r_1}{2}} u^{\frac{r_1}{2}-1} e^{-u/2}$$

$$v \sim \chi^2_{r_2} \Rightarrow f_2(v) = \frac{1}{2^{r_2/2}\Gamma\frac{r_2}{2}} v^{\frac{r_2}{2}-1} e^{-v/2}$$

$$g(u,v) = f_1(u) \cdot f_2(v) = \frac{1}{2^{\frac{r_1+r_2}{2}}\Gamma\frac{r_1}{2}\Gamma\frac{r_2}{2}} u^{\frac{r_1}{2}-1} v^{\frac{r_2}{2}-1} e^{-\frac{1}{2}(u+v)}$$

$$h(x,v) = g(u,v)\big|_{u=x} |J|$$

$$X = \frac{u/r_1}{v/r_2} \Rightarrow u = \frac{r_1}{r_2}vx \Rightarrow \frac{du}{dx} = \frac{r_1}{r_2}v = |J|$$

$$h(x,v) = \frac{1}{2^{\frac{r_1+r_2}{2}}\Gamma\frac{r_1}{2}\Gamma\frac{r_2}{2}} \cdot \left(\frac{r_1}{r_2}vx\right)^{\frac{r_1}{2}-1} \cdot v^{\frac{r_2}{2}-1} \cdot e^{-\frac{1}{2}\left(\frac{r_1}{r_2}xv+v\right)} \cdot \frac{r_1 v}{r_2}$$

$$F(x) = \int_0^\infty h(x,v)\,dv$$

$$F(x) = \frac{\left(\frac{r_1}{r_2}\right)^{r_1/2} x^{\frac{r_1}{2}-1}}{2^{\frac{r_1+r_2}{2}}\Gamma\frac{r_1}{2}\Gamma\frac{r_2}{2}} \int_0^\infty v^{\frac{r_1+r_2}{2}-1} e^{-\frac{v}{2}\left(\frac{r_1}{r_2}x+1\right)} dv$$

~~let~~ $w = \frac{v}{2}\left(\frac{r_1}{r_2}x+1\right) \Rightarrow v = \frac{2w}{\frac{r_1}{r_2}x+1} \Rightarrow dv = \frac{2}{\frac{r_1}{r_2}x+1}dw$

$$F(x) = \frac{\left(\frac{r_1}{r_2}\right)^{r_1/2} x^{\frac{r_1}{2}-1}}{2^{\frac{r_1+r_2}{2}}\Gamma\frac{r_1}{2}\Gamma\frac{r_2}{2}} \cdot 2^{\frac{r_1+r_2}{2}} \cdot \left(\frac{r_1}{r_2}x+1\right)^{-\left(\frac{r_1+r_2}{2}\right)} \int_0^\infty w^{\frac{r_1+r_2}{2}-1} e^{-w} dw$$

$F(r_1, r_2) \sim$
$$F(x) = \frac{\left(\frac{r_1}{r_2}x+1\right)^{-\left(\frac{r_1+r_2}{2}\right)} \cdot \left(\frac{r_1}{r_2}\right)^{r_1/2} \cdot x^{\frac{r_1}{2}-1} \cdot \Gamma\frac{r_1+r_2}{2}}{\Gamma\frac{r_1}{2}\Gamma\frac{r_2}{2}}$$

$\boxed{6}$ Show that if $X \sim F(v_1, v_2)$ then $\frac{1}{X} \sim F(v_2, v_1)$ [illegible]

يبدل r_1, r_2 بـ v_1, v_2 وبعد ذلك التالي

$$f(x) = \frac{\Gamma\left(\frac{r_1+r_2}{2}\right)}{\Gamma\left(\frac{r_1}{2}\right)\Gamma\left(\frac{r_2}{2}\right)} \cdot \left(\frac{r_1}{r_2}\right)^{\frac{r_1}{2}} \cdot x^{\frac{r_1}{2}-1} \cdot \left(1 + \frac{r_1}{r_2} x\right)^{-\left(\frac{r_1+r_2}{2}\right)}$$

let $y = \frac{1}{x} \Rightarrow x = \frac{1}{y} \Rightarrow \frac{dx}{dy} = \frac{1}{y^2} = |J|$

$$h(y) = f(x)\big|_{x=y} \cdot |J|$$

$$h(y) = \frac{\Gamma\left(\frac{r_1+r_2}{2}\right)}{\Gamma\left(\frac{r_1}{2}\right)\Gamma\left(\frac{r_2}{2}\right)} \cdot \left(\frac{r_1}{r_2}\right)^{\frac{r_1}{2}} \cdot y^{-\left(\frac{r_1}{2}-1\right)} \cdot \left(1 + \frac{r_1}{r_2}\frac{1}{y}\right)^{-\left(\frac{r_1+r_2}{2}\right)}$$

Multiply with $\dfrac{\left(\frac{r_2}{r_1} y\right)^{-\left(\frac{r_1+r_2}{2}\right)}}{\left(\frac{r_2}{r_1} y\right)^{-\left(\frac{r_1+r_2}{2}\right)}}$

$$h(y) = \frac{\Gamma\left(\frac{r_1+r_2}{2}\right)}{\Gamma\left(\frac{r_1}{2}\right)\Gamma\left(\frac{r_2}{2}\right)} \cdot \left(\frac{r_2}{r_1}\right)^{\frac{r_2}{2}} \cdot y^{\frac{r_2}{2}-1} \cdot \left(1 + \frac{r_2}{r_1} y\right)^{-\left(\frac{r_1+r_2}{2}\right)}$$

(*) look pdf الشرح $\bar{x}, s^2$ is unbiased estimator for μ, σ^2

$\boxed{7}$ R.S size n Note $P(->-) = 1 - P(-<-)$ R.S size n_1, n_2

$P(a<z<b) = f(b) - f(a)$

$$\frac{\bar{X} - \mu}{\sqrt{\frac{\sigma^2}{n}}} \sim N(\mu, \sigma^2) \qquad \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} \sim N(\mu, \sigma^2)$$

look Ex proportion in pdf

$$\frac{x}{n} \sim N\left(\overset{\mu}{p}, \overset{\sigma^2}{\frac{p(1-p)}{n}}\right) \qquad \frac{r - p}{\frac{p(1-p)}{n}} \sim N(0,1) \text{ where } r = \frac{x}{n}$$

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8 test of hypotheses mean

1- Mean

① $H_0: \mu = \mu_0$

$H_a: \mu \neq \mu_0$ or $\mu > \mu_0$ or $\mu < \mu_0$

②

δ^2 Known → $Z_{comp} = \dfrac{\bar{X} - \mu_0}{\delta / \sqrt{n}}$ (var pop)

S^2 unknown:

- $n \geq 30$: $Z = \dfrac{\bar{X} - \mu_0}{S/\sqrt{n}}$ (var sample)
- $n < 30$: $t = \dfrac{\bar{X} - \mu_0}{S/\sqrt{n}}$
  - $\neq$: $\pm t(\frac{\alpha}{2}, n-1)$
  - $>$: $+t(\alpha, n-1)$
  - $<$: $-t(\alpha, n-1)$

③ Z table: rej | acc | rej ($-Z_{\frac{\alpha}{2}}$, $Z_{\frac{\alpha}{2}}$); acc | rej (Z_α); rej | acc ($-Z_\alpha$)

④ Compare $Z_{compt} > Z_{table}$ ⑤ reject H_0

2- Two main

① $H_0: \mu_1 = \mu_2$

$H_a: \mu_1 \neq \mu_2$ or $\mu_1 > \mu_2$ or $\mu_1 < \mu_2$

②

δ_1^2, δ_2^2 Known: $Z = \dfrac{\bar{X}_1 - \bar{X}_2}{\sqrt{\dfrac{\delta_1^2}{n_1} + \dfrac{\delta_2^2}{n_2}}}$

S_1^2, S_2^2 unknown:

- $n_1, n_2 \geq 30$: $Z = \dfrac{\bar{X}_1 - \bar{X}_2}{\sqrt{\dfrac{S_1^2}{n_1} + \dfrac{S_2^2}{n_2}}}$
- $n_1, n_2 < 30$: $t = \dfrac{\bar{X}_1 - \bar{X}_2}{S_p \sqrt{\dfrac{1}{n_1} + \dfrac{1}{n_2}}}$

$\pm t(\frac{\alpha}{2}, n_1 + n_2 - 2)$ $+t(\alpha, n_1 + n_2 - 2)$ or $-$ " " "

$S_p^2 = \dfrac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{n_1 + n_2 - 2}$

③ Z table: rej | acc | rej ($-Z_{\frac{\alpha}{2}}$, $+Z_{\frac{\alpha}{2}}$); acc | rej (Z_α); rej | acc ($-Z_\alpha$)

④ Compare $Z_{compt} > Z_{table}$ ⑤ reject H_0

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9. Test the variance of population

1- Variance

① $H_0: \delta^2 = \delta_0^2$

$H_a: \delta^2 \neq \delta_0^2$ or $\delta^2 > \delta_0^2$ or $\delta^2 < \delta_0^2$

② $X^2_{comp} = \dfrac{(n-1)s^2}{\delta_0^2}$

Desh

③ X^2_{table}

$\neq$: rej | acc | rej — $X^2_{(1-\frac{\alpha}{2}, n-1)}$, $X^2_{(\frac{\alpha}{2}, n-1)}$

$>$: acc | rej — $X^2_{(\alpha, n-1)}$

$<$: rej | acc — $X^2_{(1-\alpha, n-1)}$

④ $X^2_{comp} > X^2_{table}$ ⑤ reject H_0

2- Two variance

① $H_0: \delta_1^2 = \delta_2^2$

$H_a: \delta_1^2 \neq \delta_2^2$ or $\delta_1^2 > \delta_2^2$ or $\delta_1^2 < \delta_2^2$

Desh

② $f_{comp} = \dfrac{s_1^2}{s_2^2}$ البسط دائما الأكبر سواء كانت s_1 or s_2

Desh

③ f_{table}

$\neq$: $f_{(1-\frac{\alpha}{2}, n_1-1, n_2-1)} \rightarrow f_{(\frac{\alpha}{2}, n_1-1, n_2-1)}$

$>$: $f(\alpha, n_1-1, n_2-1)$

$<$: $f_{(1-\alpha, n_1-1, n_2-1)}$

④ $f_{comp} > f_{table}$ ⑤ reject H_0

Desh

10. Test of sample proportion

1- Proportion ① H_0: ~~...~~ $P = P_0$

$H_a: P \neq P_0$ or $P > P_0$ or $P < P_0$

② $Z_{comp} = \dfrac{P - P_0}{\sqrt{\dfrac{P_0(1-P_0)}{n}}}$

$P = \dfrac{x}{n}$

③ Z_{table}

$\neq$: rej | acc | rej — $-Z_{\frac{\alpha}{2}}$, $+Z_{\frac{\alpha}{2}}$

$>$: acc | rej — $+Z_{\alpha}$

$<$: rej | acc — $-Z_{\alpha}$

④ $Z_{comp} > Z_{table}$ ⑤ reject H_0

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* 2- Two Proportion ① $H_0: P_1 = P_2$
$H_a: P_1 \neq P_2$ or $P_1 > P_2$ or $P_1 < P_2$

② $$Z_{comp} = \frac{P_1 - P_2}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}}$$ $$\hat{p} = \frac{x_1 + x_2}{n_1 + n_2}$$

③ Z_{table} $\neq$ rej | acc | rej $-Z_{\frac{\alpha}{2}}$ $+Z_{\frac{\alpha}{2}}$; $>$ acc | rej $+Z_\alpha$; $<$ rej | acc $-Z_\alpha$

④ $Z_{comp} > Z_{table}$ ⑤ reject H_0

[11] χ^2 test ① H_0: The die is fair
H_a: The die isnot fair

Expect دة لو معطى

② $$\chi^2 = \sum \frac{(O-E)^2}{E}$$

③ $\chi^2_{table} = \chi^2_{(\alpha, n-1)}$ ④ $\chi^2_{comp} > \chi^2_{table}$ ⑤ reject H_0

دة لو Expect عند سابه ماخلاني

① H_0: two factors are independent
H_a: two factors are not independent

② $$\chi^2 = \sum \frac{(O-E)^2}{E}$$ ③ $\chi^2_{table}(\alpha, M-1, n-1)$

no. of M, no. of n

	A		Σ
B	1,1	1,2	S_1
	2,1	2,2	S_2
Σ	T_1	T_2	N

$E(1,1) = \frac{T_1 * S_1}{N}$, $E(2,1) = \frac{T_1 * S_1}{N}$

$E(1,2) = \frac{T_1 * S_2}{N}$, $E(2,2) = \frac{T_2 * S_2}{N}$

$\chi^2_{comp} > \chi^2_{table} \Rightarrow$ accept H_0

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2022
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Experimental design

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$\boxed{1}$ f-test Mean

Source of variation	Sum of squares	degree of freedom $n =$	$\neq$	Mean square $\left(\frac{\text{sum}}{df}\right)$ $n =$	$\neq$	f_{computed}
treatment	SSA	$K-1$	$K-1$	$s_1^2 = \frac{SSA}{K-1}$	$s_1^2 = \frac{SSA}{K-1}$	$f = \frac{s_1^2}{s^2}$
Error	SSE	$K(n-1)$	$N-K$	$s^2 = \frac{SSE}{K(n-1)}$	$s^2 = \frac{SSE}{N-K}$	
total	SST	$Kn-1$	$Kn-1$			

f_{table} $n =$ $f_\alpha(K-1, K(n-1))$ from table A.6 في عندك الـ K و n

(Ex 13.1) $n \neq$ $f_\alpha(K-1, N-K)$ $f_{comp} >$ reject H_0 no. of n ↓ no. of K →

$H_0: \mu_1 = \mu_2 = \mu_3 = \mu_4 = \mu_5 \dots$

H_a: at least two of the means are not equal

$\boxed{2}$ Bartlett Test

Several equality variance تباع الـ (Ex 13.3)

$H_0: \delta_1^2 = \delta_2^2 = \delta_3^2 \dots \delta_K^2$ $b_{comp} < b_K(\alpha; n_i)$

H_1: all variances are not equal reject H_0

b_{table} $b_K(\alpha; n_1 \dots n_K) = \frac{1}{N}\left[n_1 b_K(\alpha; n_1) \dots n_K b_K(\alpha; n_K)\right]$

$$Sp^2 = \frac{1}{N-K}\sum_{i=1}^{K}(n_i - 1)s_i^2$$

$$b_{comp} = \frac{1}{Sp^2}\left[(s_1^2)^{n_1-1} \times \dots (s_K^2)^{n_K-1}\right]^{1/N-K}$$

from table A.10

The sample sizes equal Cochran's test نفس فرض الـ b-test وكلة واشتغل بالـ G

$G = \frac{\text{largest } s_i^2}{\sum_{i=1}^{K} s_i^2} = \square$ $g_\alpha = \square$ from table A.11 (Ex 13.1)

$G > g_\alpha$ reject H_0

$\boxed{3}$ Test $H_0: \mu_i = \mu_j$ $H_a: \mu_i \neq \mu_j$

Tukey test | Duncan test | Dunnett test

Dunnett test (Ex 13.5)

$H_0: \mu_0 = \mu_i$ $H_a: \mu_0 \neq \mu_i$

$$|d_i| = \frac{\bar{y}_i - \bar{y}_0}{\sqrt{\frac{2s^2}{n}}}$$

$s^2 \rightarrow$ Mean square error

table A.14 reject H_0 نكتب | | للناتج لو بالسالب

$d_i \rightarrow |d_i| > d(\frac{\alpha}{2}, K, v)$ $v = K(n-1)$ $K = K-1$ علشان مقيدة واحدة

Tukey test كرواحد منفرد

ذكر y_K فرقة من الكل ثم المعروف

$q_{comp} = q_{\text{table}} \times \sqrt{\frac{s^2}{n}}$ ← نحط في جدول الفروق ونحسب →

table A.12

الفروق $> q_{comp}$ reject H_0 الفروق $> R_p$

Duncan test رتب تنازلي وانزل واقف

رتب تصاعدي صفًا وتنازلي عموديًا

$R_p = r_p \sqrt{\frac{s^2}{n}}$

table A.13

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ولا تنسى القرمن

~~Tukey test~~ Page 524 اشرح على المثال Tukey test جدول الفروق على نفس المثال

$\bar{y}_2$	كبرها	$\bar{y}_5$	$\bar{y}_1$	$\bar{y}_3$	$\bar{y}_6$	$\bar{y}_4$
14.50	صغرها	16.75	19.84	21.12	22.90	23.20

هكذا التطبيق

$\bar{y}_4-\bar{y}_2$ ثم $\bar{y}_6-\bar{y}_2$ ثم $\bar{y}_3-\bar{y}_2$ ثم $\bar{y}_1-\bar{y}_2$ وهكذا $\bar{y}_5-\bar{y}_2$

كبرها $\bar{y}_4-\bar{y}_5$ الاصغر $\bar{y}_6-\bar{y}_5$ الاصغر $\bar{y}_3-\bar{y}_5$ الاصغر وطبق $\bar{y}_1-\bar{y}_5$ شش نشهى هنا القاهرة

صغرها $\bar{y}_4-\bar{y}_1$ كبرها $\bar{y}_6-\bar{y}_1$ وطبق $\bar{y}_3-\bar{y}_1$

$\bar{y}_4-\bar{y}_3$ صغرها $\bar{y}_6-\bar{y}_3$ كلها ركب ١١ = عدد موجب

$\bar{y}_4-\bar{y}_6$ عدد العينات $n = K-1$

$$q_{comp} = q_{table} * \sqrt{\frac{s^2}{n}} \qquad q_{table} \Rightarrow q(\alpha, K, \nu) \quad \text{A.12} \qquad \nu = K(n-1)$$

$H_0: m_i = m_j \quad H_a: m_i \neq m_j$ لما نعكس كل القيم بال q_{comp} والفرق أكبر نرفض H_0

~~Duncan test~~ $H_0: m_i = m_j \quad H_a: m_i \neq m_j$ الاول الطرح تمت

جدول الفروق على نفس المثال رتب تنازلي وافقى تصاعدي نازل تنازلي وافقى ال R الانزل بالترتيب

	$\bar{y}_2$	$\bar{y}_5$	$\bar{y}_1$	$\bar{y}_3$	$\bar{y}_6$
$\bar{y}_4$	$\pm \to R_6$	$\pm \to R_5$	$\pm \to R_4$	$\pm \to R_3$	$\pm \to R_2$
$\bar{y}_6$	$\pm \to R_5$	$\pm \to R_4$	$\pm \to R_3$	$\pm \to R_2$	
$\bar{y}_3$	$\pm \to R_4$	$\pm \to R_3$	$\pm \to R_2$		
$\bar{y}_1$	$\pm \to R_3$	$\pm \to R_2$			
$\bar{y}_5$	$\pm \to R_2$				

نبدأ بـ 2 وننتهي لحد K

Note $P \to 2\ 3\ 4\ 5\ 6 \dots K$

جدول نقيم $r_p \to$ A.13 عند (α, ν)

~~reject H_0~~ الفرق $>$ $R_p \to r_p \sqrt{\frac{s^2}{n}} = R_p$

4 Estimation of variance components $s_\alpha^2 = \frac{s_1^2 - s^2}{n}$ look Ex 13.7

Estimation block $s_\alpha^2 = \frac{s_1^2 - s^2}{b} \qquad s_\beta^2 = \frac{s_2^2 - s^2}{K}$ اعمل Estimation Ex 13.6

Estimated Two factor

$s_\alpha^2 = \frac{s_1^2 - s_3^2}{bn} \qquad s_\beta^2 = \frac{s_2^2 - s_3^2}{an} \qquad s_{\alpha\beta}^2 = \frac{s_3^2 - s^2}{n}$ look Ex 14.6

5 Randomized compelet block design Ex 13.6

$H_0: m_1 = m_2 = \dots m_K$

H_a: The m_i are not equal

no. of b, no of K

Source of variation	Sum of square	df	MS $= \frac{\text{Sum}}{\text{df}}$	f_{comp}
treatment	SSA	K-1	$s_1^2 = \frac{SSA}{K-1}$	
blocks	SSB	b-1	$s_2^2 = \frac{SSB}{b-1}$	$f = \frac{s_1^2}{s^2}$
Error	SSE	(K-1)(b-1)	$s^2 = \frac{SSE}{(K-1)(b-1)}$	
Total	SST			

$f_{comp} > f_{table}$ reject H_0 A.6 $f_\alpha(K-1, (K-1)(b-1))$

6 Two factor test

$H_0: \alpha_1 = \alpha_2 \cdots \alpha_a = 0$

H_a: at least one of the α_i is not equal

$f_{comp} > f_{table}$ reject H_0

$H_0': \beta_1 = \beta_2 \cdots \beta_b = 0$

H_a': at least one of the β_j is not equal

table A-6

Ex 14.1

$H_0'': (\alpha\beta)_{11} = (\alpha\beta)_{12} \cdots (\alpha\beta)_{ab} = 0$

H_a'': at least one of the $(\alpha\beta)_{ij}$ is not equal

no. of → b, no. of a ↓, n → no. of all, a×b

S.O.V	SS	df	MS $\frac{SS}{df}$	fcomp	f table
A	SSA	$a-1$	$s_1^2 = SSA/a-1$	$f_1 = s_1^2/s^2$	$\alpha, a-1, ab(n-1)$
B	SSB	$b-1$	$s_2^2 = SSB/b-1$	$f_2 = s_2^2/s^2$	$\alpha, b-1, ab(n-1)$
AB	SSAB	$(a-1)(b-1)$	$s_3^2 = SSAB/(a-1)(b-1)$	$f_3 = s_3^2/s^2$	$\alpha, (a-1)(b-1), ab(n-1)$
Error	SSE	$ab(n-1)$	$s^2 = SEE/ab(n-1)$		$\alpha, df, ab(n-1)$ (A B AB)
Total	SST				

7 Three factor test

$f_{comp} > f_{table}$ reject H_0 no. of a ↓ → no. of c → no. of b

Ex 14.4

S.O.V	SS	df	MS $\frac{SS}{df}$	fcomp	f table
A	SSA	$a-1$	$s_1^2 = SSA/a-1$	$f_1 = s_1^2/s^2$	α
B	SSB	$b-1$	$s_2^2 = SSB/b-1$	$f_2 = s_2^2/s^2$	df
C	SSC	$c-1$	$s_3^2 = SSC/c-1$	$f_3 = s_3^2/s^2$	$abc(n-1)$
AB	SSAB	$(a-1)(b-1)$	$s_4^2 = SSAB/(a-1)(b-1)$	$f_4 = s_4^2/s^2$	df
AC	SSAC	$(a-1)(c-1)$	$s_5^2 = SSAC/(a-1)(c-1)$	$f_5 = s_5^2/s^2$	عند
BC	SSBC	$(b-1)(c-1)$	$s_6^2 = SSBC/(b-1)(c-1)$	$f_6 = s_6^2/s^2$	A AB ABC B AC C BC
ABC	SSABC	$(a-1)(b-1)(c-1)$	$s_7^2 = SSABC/(a-1)(b-1)(c-1)$	$f_7 = s_7^2/s^2$	
Error	SSE	$abc(n-1)$	$s^2 = SSE/abc(n-1)$		
Total	SST				

8

$w_1 = \mu_1 + \mu_2 - \mu_3 - \mu_5$ $\quad w_2 = \mu_1 + \mu_2 + \mu_3 - 4\mu_4 + \mu_5$

$\sum c_i = 1*1 + 1*1 + (-1)*1 + 0*(-4) + (-1)*1 - 1 = 0$

$SS_w = \frac{(\sum c_i y_i)^2}{n \sum c_i^2}$ $\quad SSw_1 =$ Continue look book Ex 13.1

$SSw_2 =$

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