1) several equality means
(n equal, means)

$H_0: \mu_1 = \mu_2 = \mu_3 = \dots = \mu_n$

$H_1:$ at least two of means are not equal.

S.V	S.S	df	M.S	Fcomp
treatment	SSA	$K-1$	$S_1^2 = \frac{SSA}{K-1}$	$F = \frac{S_1^2}{S^2}$
Error	SSE	$K(n-1)$	$S^2 = \frac{SSE}{K(n-1)}$	
total	SST	$Kn-1$		

Ftable $(\alpha, K-1, K(n-1))$ n → rows, k → column

if $F_{comp} > F_{table}$ → reject H_0

2) several non equality means
(n not equal, means)

$H_0: \mu_1 = \mu_2 = \dots = \mu_n$

$H_1:$ at least two of means are not equal

S.V	S.S	df	M.S	Fcomp
treatment	SSA	$K-1$	$S_1^2 = \frac{SSA}{K-1}$	$F = \frac{S_1^2}{S^2}$
Error	SSE	$N-K$	$S^2 = \frac{SSE}{N-K}$	
total	SST			

Ftable $(\alpha, K-1, N-K)$, $N = n_1 + n_2 + \dots$

if $F_{comp} > F_{table}$ → reject H_0

3) several equality variances
Cochran's test (equal, S)

$H_0: \delta_1^2 = \delta_2^2 = \dots = \delta_k^2$

$H_1:$ all variances are equal

$$g = \frac{\text{large}(S_i^2)}{\sum_{i=1}^{k} S_i^2}, \quad g_\alpha$$

if $g > g_\alpha$ → reject H_0

4) bartlett's test (not equal, S)

$H_0: \delta_1^2 = \delta_2^2 = \dots = \delta_k^2$

$H_1:$ all variances are not equal

$N = n_1 + n_2 + n_3 + \dots + n_k$

$$S_p^2 = \frac{1}{N-K}\sum_{i=1}^{k}(n_i - 1)\, S_i^2$$

$$b = \frac{\left[(S_1^2)^{n_1-1} \cdot (S_2^2)^{n_2-1} \cdots (S_k^2)^{n_k-1}\right]^{\frac{1}{N-k}}}{S_p^2}$$

$b_k(\alpha, n_1, n_2, \dots, n_k)$

$$= \frac{n_1 b_k(\alpha, n_1) + n_2 b_k(\alpha, n_2) + \dots}{N}$$

if $b_k > b$ → reject H_0

Tests for each pair information

1) tukey test

$\bar{y}_1 \quad \bar{y}_2 \quad \bar{y}_3$

$H_0: \mu_i = \mu_j$

$H_1: \mu_i \neq \mu_j$

→ sort means of sample (asc)

→ get the diff of each pair of μ (after sorting)

→ Compare each diff with the value of $q_{(\alpha, K, \nu)} \cdot \sqrt{\frac{S^2}{n}}$

if diff $> q_{(\alpha, K, \nu)} \cdot \sqrt{\frac{S^2}{n}}$ → reject H_0

ν → $K(n-1)$

2) Dunnett's test (control test)

$\bar{y}_0 \quad \bar{y}_1 \quad \bar{y}_2$

$H_0: \mu_0 = \mu_i$

$H_1: \mu_0 \neq \mu_i$

control

$\bar{y}_0 = -, \bar{y}_1 = -, \bar{y}_2 = -, \bar{y}_3 = -$

$$d_i = \frac{\bar{y}_i - \bar{y}_0}{\sqrt{2\frac{S^2}{n}}}$$

S^2 → means square error

$d_{\frac{\alpha}{2}}(K-1, K(n-1))$

if $|d_i| > d_{\frac{\alpha}{2}}(K-1, K(n-1))$ → reject H_0

3) Duncan test

$\bar{y}_1 \quad \bar{y}_2 \quad \bar{y}_3$

$H_0: \mu_i = \mu_j$

$H_1: \mu_i \neq \mu_j$

→ sort $\bar{y}_{\square}$ (asc)

	$\bar{y}_6$	$\bar{y}_5$	$\bar{y}_4$	$\bar{y}_3$	$\bar{y}_2$	$\bar{y}_1$
$\bar{y}_7$	$\bar{y}_7 - \bar{y}_6$ R_6	$\bar{y}_7 - \bar{y}_5$ R_5	R_4	R_3	$\bar{y}_7 - \bar{y}_2$ R_2	
$\bar{y}_6$	R_5	R_4	R_3	R_2		
$\bar{y}_5$	R_4	R_3	R_2			
$\bar{y}_4$	R_3	R_2				
$\bar{y}_3$	R_2			Symmetry		
$\bar{y}_2$	$\bar{y}_2 - \bar{y}_1$					
$\bar{y}_1$						

Rs	2	3	4	5	6
rp	—	—	—	—	—

From table

$R_p = r_p \sqrt{\frac{S^2}{n}}$

Rs	2	3	4	5	6
Rp	$\square$	$\square$	$\square$	$\square$	$\square$

Compare $R_S = \bar{y}_{\square} - \bar{y}_{\square}$ with R_S in table

if $R_S = \bar{y}_{\square} - \bar{y}_{\square} > R_S$ in table → reject H_0

so $\mu_i \neq \mu_j$

Randomized complete block design

$b_1 \quad b_2 \quad b_3$ ... k

b

$H_0: \mu_1 = \mu_2 = \dots = \mu_n$

$H_1:$ the means are not equal

$$S_1^2 = \frac{SSA}{K-1}, \quad S_2^2 = \frac{SSB}{b-1}$$

$$S^2 = \frac{SSE}{(K-1)(b-1)}, \quad F = \frac{S_1^2}{S^2}$$

Ftable $(\alpha, K-1, (K-1)(b-1))$

if $F_{comp} > F_{table}$ → reject H_0

model of hypothesis for the two factors

$\mu_{ij} = \mu + \alpha_i + \beta_j + (\alpha\beta)_{ij}$

α_i → the effect of factor A

β_j → the effect of factor B

$\alpha\beta_j$ → the " " " AB

$\sum_{i=1}^{a} \alpha_i = 0, \quad \sum_{j=1}^{b} \beta_j = 0$

$\sum_{i=1}^{a} (\alpha\beta)_{ij} = \sum_{j=1}^{b} (\alpha\beta)_{ij} = 0$

	b_1	b_2	b_3	b_4
a_1	≡	≡	≡	≡
a_2	≡	≡	≡	≡
a_3	≡	≡	≡	≡

→ $a = 3, b = 4, n = 3$

sol

$H_0: \alpha_1 = \alpha_2 = \dots = \alpha_a$

$H_1:$ at least one of α_i is not equal

$H_0': B_1 = B_2 = \dots = B_b$

$H_1':$ at least one of B_i is not equal

$H_0'': (\alpha B)_{11} = (\alpha B)_{12} = \dots = (\alpha B)_{ab}$

$H_1'':$ at least one of $(\alpha B)_{ij}$ is not equal

$$S_1^2 = \frac{SSA}{a-1}, \quad S_2^2 = \frac{SSB}{b-1}$$

$$S_3^2 = \frac{SSAB}{(a-1)(b-1)}, \quad S^2 = \frac{SSE}{ab(n-1)}$$

$$F_1 = \frac{S_1^2}{S^2}, \quad F_2 = \frac{S_2^2}{S^2}, \quad F_3 = \frac{S_3^2}{S^2}$$

F_1table $(\alpha, a-1, ab(n-1))$

F_2table $(\alpha, b-1, ab(n-1))$

F_3table $(\alpha, (a-1)(b-1), ab(n-1))$

if F_1comp $> F_1$table → reject H_0

F_2comp $> F_2$table → reject H_0'

F_3comp $> F_3$table → reject H_0''

the three factor experiments

the method of three factor

$$y_{ijkl} = \mu + \alpha_i + \beta_j + \gamma_k + (\alpha\beta)_{ij} + (\alpha\gamma)_{ik} + (\beta\gamma)_{jk} + (\alpha\beta\gamma)_{ijk} + \epsilon_{ijkl}$$

$i = 1, 2, 3, \dots, a$
$j = 1, 2, 3, \dots, b$
$k = 1, 2, 3, \dots, c$

$\alpha_i, \beta_j, \gamma_k$ are main effects of Factors A, B, C

$(\alpha\beta)_{ij}, (\alpha\gamma)_{ik}, (\beta\gamma)_{jk}$ are two factor interaction.

$(\alpha\beta\gamma)_{ijk}$ is three factor interaction effect.

	c_1			c_2		
	b_1	b_2	b_3	b_1	b_2	b_3
a_1	≡	≡	≡	≡	≡	≡
a_2	≡	≡	≡	≡	≡	≡
a_3	≡	≡	≡	≡	≡	≡

$a = 3,\ b = 3,\ c = 2,\ n = 4$

Sol

S.S	df	M.S	F
SSA	$a-1$	$S_1^2 = \frac{SS}{df}$	$F_1 = \frac{S_1^2}{S^2}$
SSB	$b-1$	⋮	$F_2 = \frac{S_2^2}{S^2}$
SSC	$c-1$	⋮	⋮
SSAB	$(a-1)(b-1)$	⋮	$F_i = \frac{S_i^2}{S^2}$
SSAC	$(a-1)(c-1)$	⋮	
SSBC	$(b-1)(c-1)$	⋮	
SSABC	$(a-1)(b-1)(c-1)$		
SSE	$abc(n-1)$	$S^2 = \frac{SSE}{abc(n-1)}$	

$F_{table}(\alpha, df, abc(n-1))$

if $F_{i\,comp} > F_{table} \Rightarrow$ reject

the unbaised estimates of variances
(block case randomized)

$\hat{\delta}^2 = S^2$, $\quad \hat{\delta}_\alpha^2 = \dfrac{S_1^2 - S^2}{b} \rightarrow \square\square\square$ -

$\hat{\delta}_\beta^2 = \dfrac{S_2^2 - S^2}{k} \rightarrow$ ≡

Estimated variance in case of two factor

$\hat{\delta}^2 = S^2$, $\quad \hat{\delta}_\alpha^2 = \dfrac{S_1^2 - S_2^2}{b \times n}$

$\hat{\delta}_B^2 = \dfrac{S_2^2 - S_3^2}{a \times n}$, $\quad \hat{\delta}_{\alpha B}^2 = \dfrac{S_3^2 - S^2}{n}$

construct comparison

$w_1 = q_1\mu_1 + q_2\mu_2 + q_3\mu_3 + \dots$
$w_2 = k_1\mu_1 + k_2\mu_2 + k_3\mu_3 + \dots$

$\sum c_i = \sum q_i \times k_i = 0 \Rightarrow$ لو مساوي صفر reject ومتعامدة

$$SSw_1 = \frac{(\sum q_i \bar{y}_i)^2}{n \sum q_i^2} \quad ≡$$

$$= \frac{(q_1\mu_1 + q_2\mu_2 + q_3\mu_3 + \dots)^2}{n(q_1^2 + q_2^2 + q_3^2 + \dots)} = X$$

$$SSw_2 = \frac{(\sum k_i \bar{y}_i)^2}{n \sum k_i^2}$$

$$= \frac{(k_1\mu_1 - k_2\mu_2 + k_3\mu_3 + \dots)^2}{n(k_1^2 + (-k_2)^2 + (k_3)^2 + \dots)} = y$$

source of variation	S.S	df	M.S	F
aggregates	SS_{agg}	$k-1$	$S_1^2 = \frac{SS_{agg}}{k-1}$	$F_{agg} = \frac{S_1^2}{S^2}$
$(+)$ and $(--)$	X	1	$S_2^2 = \frac{SS(\ )vs(\ )}{1}$	$F_{()vs()} = \frac{S_2^2}{S^2}$
$(--)$ and $(-+)$	y	1	$S_3^2 = \frac{SS(\ )vs(\ )}{1}$	$F_{()vs()} = \frac{S_3^2}{S^2}$
Error	SSE	$k(n-1)$	$S^2 = \frac{SSE}{k(n-1)}$	

if $F_{i\,()vs()} > F_{agg} \Rightarrow$ reject w_j

○ ⇒ given