Page: Date:

Numerical Analysis

Newton

h Step size difference ▷ يعمل الجدول ونقسم على الـ x بشكل مائل

$$y = y_0 + (x-x_0)\Delta y_0 + (x-x_0)(x-x_1)\Delta^2 y_0 + (x-x_0)(x-x_1)(x-x_2)\Delta^3 y_0 + \dots + (x-x_0)\dots(x-x_n)\Delta^{n+1} y_0$$

h Step size equal 2 type forward & backward

forward المطلوب قيمة في أول الجدول

$$y = y_0 + \frac{p}{1!}\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0 + \dots$$

where $p = \frac{x-x_0}{h}$ يعمل نفس الجدول بشكل مثلث مائل ▷ من غير قسمة

backword المطلوب قيمة في آخر الجدول

$$y = y_n + \frac{p}{1!}\Delta y_n + \frac{p(p+1)}{2!}\Delta^2 y_n + \frac{p(p+1)(p+2)}{3!}\Delta^3 y_n + \dots$$

where $p = \frac{x-x_n}{h}$

Differentiation

1- forward

$$\frac{dy}{dx} = \frac{1}{h}\left[\Delta y_0 + \frac{(2p-1)}{2!}\Delta^2 y_0 + \frac{3p^2-6p+2}{3!}\Delta^3 y_0 + \dots\right]$$

$$\frac{d^2y}{dx^2} = \frac{1}{h^2}\left[\Delta^2 y_0 + (p-1)\Delta^3 y_0 + \dots\right]$$

2- backward

$$\frac{dy}{dx} = \frac{1}{h}\left[\Delta y_n + \frac{(2p+1)}{2!}\Delta^2 y_n + \frac{3p^2+6p+2}{3!}\Delta^3 y_n + \dots\right]$$

$$\frac{d^2y}{dx^2} = \frac{1}{h^2}\left[\Delta^2 y_n + \frac{6p+6}{3!}\Delta^3 y_n + \dots\right]$$

⟹ 3- Stirling formula

$$y = y_0 + \frac{p}{1!}\left[\frac{\Delta y_0 + \Delta y_{-1}}{2}\right] + \frac{p^2}{2!}\Delta^2 y_{-1} + \frac{p(p^2-1^2)}{3!}\left[\frac{\Delta^3 y_{-1} + \Delta^3 y_{-2}}{2}\right] + \frac{p^2(p^2-1^2)}{4!}\Delta^4 y_{-2} + \dots$$

where $p = \frac{x-x_0}{h}$

$$\frac{dy}{dx} = \frac{1}{h}\left[\frac{\Delta y_0 + \Delta y_{-1}}{2} + p\,\Delta^2 y_{-1} + \frac{3p^2-1}{6}\,\frac{\Delta^3 y_{-1} + \Delta^3 y_{-2}}{2} + \dots\right]$$

$$\frac{d^2y}{dx^2} = \frac{1}{h^2}\left[\Delta^2 y_{-1} + p\,\frac{\Delta^3 y_{-1} + \Delta^3 y_{-2}}{2} + \frac{6p^2-1}{12}\Delta^4 y_{-2} + \dots\right]$$

Page: .... Date: ....

Integration

1- Trapezoidal Rule

2 باقي + آخر + أول

$$\int_{x_0}^{x_0+nh} f(x)\,dx = \frac{h}{2}\left[(y_0+y_n)+2(y_1+y_2+y_3+\cdots+y_{n-1})\right]$$

2- Simpson's 1/3 Rule

Note: Area $= x^2$, Volume $= \pi x^3$

أول + آخر + 4 odd + 2 even

$$\int_{x_0}^{x_0+nh} f(x)\,dx = \frac{h}{3}\left[(y_0+y_n)+4(y_1+y_3+\cdots+y_{n-1})+2(y_2+y_4+\cdots+y_{n-2})\right]$$

3- Simpson's 3/8 Rule

أول + آخر + 2 index 3 + 3 باقي

$$\int_{x_0}^{x_0+nh} f(x)\,dx = \frac{3h}{8}\left[(y_0+y_n)+2(y_3+y_6+y_9+\cdots y_{n-3})+3(y_1\cdots y_{n-1})\right]$$

Maclaurin if $x_0=0$: $y(x) = y(x_0) + \frac{x-x_0}{1!}y'(x_0) + \frac{(x-x_0)^2}{2!}y''(x_0) + \cdots$

Taylor's Series $y_n = y(x_n) = y_{n-1} + \frac{h}{1!}y'_{n-1} + \frac{h^2}{2!}y''_{n-1} + \cdots$

Euler Method $y_{n+1} = y_n + h\,f(x_n, y_n)$

Improved $y_{n+1} = y_n + \frac{h}{2}\left[f(x_n,y_n) + f(x_n+h,\ y_n + h f(x_n,y_n))\right]$

Modified $y_{n+1} = y_n + h\left[f\left(x_n+\frac{h}{2},\ y_n+\frac{h}{2}f(x_n,y_n)\right)\right]$

Runge Kutta method

Second order	Third order	Forth order
$K_1 = hf(x_0,y_0)$	$K_1 = hf(x_0,y_0)$	$K_1 = hf(x_0,y_0)$
$K_2 = hf(x_0+h,\ y_0+K_1)$	$K_2 = hf(x_0+\frac{h}{2},\ y_0+\frac{K_1}{2})$	$K_2 = hf(x_0+\frac{h}{2},\ y_0+\frac{K_1}{2})$
	$K_3 = hf(x_0+h,\ y_0+K_2)$	$K_3 = hf(x_0+\frac{h}{2},\ y_0+\frac{K_2}{2})$
$K = \frac{1}{2}(K_1+K_2)$		$K_4 = hf(x_0+h,\ y_0+K_3)$
$y_1 = y_0 + K$	$K = \frac{1}{6}(K_1+4K_2+K_3)$	$K = \frac{1}{6}(K_1+2K_2+2K_3+K_4)$
	$y_1 = y_0+K$	$y_1 = y_0+K$

Page: .... Date: ....

~~*Runge Kutta method system~~

$\frac{dy}{dx} = f_1(x,y,z)$ $\qquad \frac{dz}{dx} = f_2(x,y,z)$ $\qquad y(x_0) = y_0 ,\ z(x_0) = z_0$

$K_1 = h f_1(x_0, y_0, z_0)$ $\qquad L_1 = h f_2(x_0, y_0, z_0)$

$K_2 = h f_1(x_0 + \frac{h}{2}, y_0 + \frac{K_1}{2}, z_0 + \frac{L_1}{2})$ $\qquad L_2 = h f_2(x_0 + h/2, y_0 + \frac{K_1}{2}, z_0 + \frac{L_1}{2})$

$K_3 = h f_1(x_0 + \frac{h}{2}, y_0 + \frac{K_2}{2}, z_0 + \frac{L_2}{2})$ $\qquad L_3 = h f_2(x_0 + \frac{h}{2}, y_0 + \frac{K_2}{2}, z_0 + \frac{L_2}{2})$

$K_4 = h f_1(x_0 + h, y_0 + K_3, z_0 + L_3)$ $\qquad L_4 = h f_2(x_0 + h, y_0 + K_3, z_0 + L_3)$

$K = \frac{1}{6}[K_1 + 2K_2 + 2K_3 + K_4]$ $\qquad L = \frac{1}{6}[L_1 + 2L_2 + 2L_3 + L_4]$

$y_1 = y_0 + K$ $\qquad z_1 = z_0 + L$

Desha

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