## 1] several equality means
(n equal, means)

$H_0: \mu_1 = \mu_2 = \mu_3 = \dots = \mu_n$

$H_1$: at least two of means are not equal.

| S.V | S.S | df | M.S | Fcomp |
|---|---|---|---|---|
| treatment | SSA | $K-1$ | $S_1^2 = \frac{SSA}{K-1}$ | $F = \frac{S_1^2}{S^2}$ |
| Error | SSE | $K(n-1)$ | $S^2 = \frac{SSE}{K(n-1)}$ | |
| total | SST | $Kn-1$ | | |

$F_{table}(\alpha, K-1, K(n-1))$ n ⇒ rows, K ⇒ column

if $F_{comp} > F_{table}$ ⇒ reject $H_0$

## 2] several non equality means
(n not equal, means)

$H_0: \mu_1 = \mu_2 = \dots = \mu_k$

$H_1$: at least two of means are not equal

| S.V | S.S | df | M.S | Fcomp |
|---|---|---|---|---|
| treatment | SSA | $K-1$ | $S_1^2 = \frac{SSA}{K-1}$ | $F = \frac{S_1^2}{S^2}$ |
| Error | SSE | $N-K$ | $S^2 = \frac{SSE}{N-K}$ | |
| total | SST | | | |

$F_{table}(\alpha, k-1, N-k)$, $N = n_1 + n_2 + \dots + n_k$

if $F_{comp} > F_{table}$ ⇒ reject $H_0$

## 3] several equality variances
Cochran's test ([illegible]equal, S)

$H_0: \delta_1^2 = \delta_2^2 = \dots = \delta_k^2$

$H_1$: all variances are equal

$$g = \frac{\text{large}(S_i^2)}{\sum_{i=1}^{k} S_i^2}, \quad g_\alpha$$

if $g > g_\alpha$ ⇒ reject $H_0$

## 4] Bartlett's test ([illegible]nequal, S)

$H_0: \delta_1^2 = \delta_2^2 = \dots = \delta_k^2$

$H_1$: all variances are not equal

$N = n_1 + n_2 + n_3 + \dots + n_k$

$$S_p^2 = \frac{1}{N-K} \sum_{i=1}^{k} (n_i - 1) S_i^2$$

$$b = \frac{\left[ (S_1^2)^{n_1-1} \cdot (S_2^2)^{n_2-1} \dots (S_k^2)^{n_k-1} \right]^{\frac{1}{N-K}}}{S_p^2}$$

$b_k(\alpha, n_1, n_2, \dots, n_k)$

$$= \frac{n_1 b_k(\alpha, n_1) + n_2 b_k(\alpha, n_2) + \dots}{N}$$

if $b_k > b$ ⇒ reject $H_0$

# Tests for each pair information

## 1] Tukey test

$\bar{y}_1 \quad \bar{y}_2 \quad \bar{y}_3 \ \dots$

$H_0: \mu_i = \mu_j$

$H_1: \mu_i \neq \mu_j$

⇒ sort means of sample (asc)

⇒ get the diff of each pair of $\mu$ (after sorting)

⇒ compare each diff with the value of $q_{(\alpha, K, \nu)} \cdot \sqrt{\frac{S^2}{n}}$

if diff $> q_{(\alpha, K, \nu)} \cdot \sqrt{\frac{S^2}{n}}$ ⇒ reject $H_0$

$\nu$ ⇒ $K(n-1)$

## 2] Dunnett's test (control test)

$\bar{y}_0 \quad \bar{y}_1 \quad \bar{y}_2 \ \dots$

↳ control

$H_0: \mu_0 = \mu_i$

$H_1: \mu_0 \neq \mu_i$

$\bar{y}_0 = \dots, \bar{y}_1 = \dots, \bar{y}_2 = \dots, \bar{y}_3 = \dots$

$$d_i = \frac{\bar{y}_i - \bar{y}_0}{\sqrt{2 \frac{S^2}{n}}}$$

$S^2$ ⇒ means square error

$d_{\frac{\alpha}{2}}(K-1, K(n-1))$

if $|d_i| > d_{\frac{\alpha}{2}}(k-1, K(n-1))$ ⇒ reject $H_0$

## 3] Duncan test

$\bar{y}_1 \quad \bar{y}_2 \quad \bar{y}_3 \ \dots$

$H_0: \mu_i = \mu_j$

$H_1: \mu_i \neq \mu_j$

⇒ sort $\bar{y}_{\square}$ (asc)

| | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | $\bar{y}_1$ |
|---|---|---|---|---|---|---|
| $\bar{y}_F$ | R6 | R5 | R4 | R3 | R2 | |
| $\bar{y}_e$ | R5 | R4 | R3 | R2 | | |
| $\bar{y}_d$ | R4 | R3 | R2 | | | |
| $\bar{y}_c$ | R3 | R2 | | | | |
| $\bar{y}_b$ | R2 | | | | | |
| $\bar{y}_a$ | | | | | | |

Symmetry

| Rs | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| rp | — | — | — | — | — |

From table

$R_p = r_p \sqrt{\frac{S^2}{n}}$

| Rs | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| Rp | ☐ | ☐ | ☐ | ☐ | ☐ |

Compare $R_s = |\bar{y}_{\square} - \bar{y}_{\square}|$ with $R_s$ in table

if $R_s = \bar{y}_{\square} - \bar{y}_{\square} > R_s$ in table

reject $H_0$

so $\mu_i \neq \mu_j$

# Randomized complete block design

$b_1 \quad b_2 \quad b_3 \ \dots$ } $k$

$\underbrace{\qquad\qquad}_{b}$

$H_0: \mu_1 = \mu_2 = \dots = \mu_n$

$H_1$: the means are not equal

$$S_1^2 = \frac{SSA}{K-1}, \quad S_2^2 = \frac{SSB}{b-1}$$

$$S^2 = \frac{SSE}{(K-1)(b-1)}, \quad F = \frac{S_1^2}{S^2}$$

$F_{table}(\alpha, K-1, (K-1)(b-1))$

if $F_{comp} > F_{table}$ ⇒ reject $H_0$

## model of hypothesis for the two factors

$\mu_{ij} = \mu + \alpha_i + \beta_j + (\alpha\beta)_{ij}$

$\alpha_i$ ⇒ the effect of Factor A

$\beta_j$ ⇒ the effect of Factor B

$\alpha\beta_{ij}$ ⇒ the " " " AB

$\sum_{i=1}^{a} \alpha_i = 0, \quad \sum_{j=1}^{b} \beta_j = 0$

$\sum_{i=1}^{a} (\alpha\beta)_{ij} = \sum_{j=1}^{b} (\alpha\beta)_{ij} = 0$

| | $b_1$ | $b_2$ | $b_3$ | $b_4$ |
|---|---|---|---|---|
| $a_1$ | ≡ | ≡ | ≡ | ≡ |
| $a_2$ | ≡ | ≡ | ≡ | ≡ |
| $a_3$ | ≡ | ≡ | ≡ | ≡ |

⇒ $a = 3, b = 4, n = 3$

sol

$H_0: \alpha_1 = \alpha_2 = \dots = \alpha_a$

$H_1$: at least one of $\alpha_i$ is not equal

$H_0': \beta_1 = \beta_2 = \dots = \beta_b$

$H_1'$: at least one of $\beta_j$ is not equal

$H_0'': (\alpha\beta)_{11} = (\alpha\beta)_{12} = \dots = (\alpha\beta)_{ab}$

$H_1''$: at least one of $(\alpha\beta)_{ij}$ is not equal

$$S_1^2 = \frac{SSA}{a-1}, \quad S_2^2 = \frac{SSB}{b-1}$$

$$S_3^2 = \frac{SSAB}{(a-1)(b-1)}, \quad S^2 = \frac{SSE}{ab(n-1)}$$

$$F_1 = \frac{S_1^2}{S^2}, \quad F_2 = \frac{S_2^2}{S^2}, \quad F_3 = \frac{S_3^2}{S^2}$$

$F_1$ table $(\alpha, a-1, ab(n-1))$

$F_2$ table $(\alpha, b-1, ab(n-1))$

$F_3$ table $(\alpha, (a-1)(b-1), ab(n-1))$

if $F_1$ comp $> F_1$ table ⇒ reject $H_0$

$F_2$ comp $> F_2$ table ⇒ reject $H_0'$

$F_3$ comp $> F_3$ table ⇒ reject $H_0''$

## the three Factor experiments

the method of three Factor

$$y_{ijkl} = \mu + \alpha_i + \beta_j + \gamma_k + (\alpha\beta)_{ij} + (\alpha\gamma)_{ik} + (\beta\gamma)_{jk} + (\alpha\beta\gamma)_{ijk} + \epsilon_{ijkl}$$

$i = 1, 2, 3, \dots, a$

$j = 1, 2, 3, \dots, b$

$k = 1, 2, 3, \dots, c$

$\alpha_i, \beta_j, \gamma_k$ are main effects of Factors A, B, C

$(\alpha\beta)_{ij}, (\alpha\gamma)_{ik}, (\beta\gamma)_{jk}$ are two Factor interaction.

$(\alpha\beta\gamma)_{ijk}$ is three factor interaction effect.

| | $c_1$ | | | $c_2$ | | |
|---|---|---|---|---|---|---|
| | $b_1$ | $b_2$ | $b_3$ | $b_1$ | $b_2$ | $b_3$ |
| $a_1$ | ≡ | ≡ | ≡ | ≡ | ≡ | ≡ |
| $a_2$ | ≡ | ≡ | ≡ | ≡ | ≡ | ≡ |
| $a_3$ | ≡ | ≡ | ≡ | ≡ | ≡ | ≡ |

$a = 3$, $b = 3$, $c = 2$, $n = 4$

sol

| S.S | df | M.S | F |
|---|---|---|---|
| SSA | $a-1$ | $s_1^2 = \frac{SS}{df}$ | $F_1 = \frac{s_1^2}{s^2}$ |
| SSB | $b-1$ | ⋮ | $F_2 = \frac{s_2^2}{s^2}$ |
| SSC | $c-1$ | ⋮ | ⋮ |
| SSAB | $(a-1)(b-1)$ | ⋮ | |
| SSAC | $(a-1)(c-1)$ | ⋮ | $F_i = \frac{s_i^2}{s^2}$ |
| SSBC | $(b-1)(c-1)$ | ⋮ | |
| SSABC | $(a-1)(b-1)(c-1)$ | | |
| SSE | $abc(n-1)$ | $s^2 = \frac{SSE}{abc(n-1)}$ | |

$F_{table}(\alpha, df, abc(n-1))$

if $F_{i\,comp} > F_{table} \Rightarrow$ reject

## the unbaised estimates of variances (block case randomized)

$\hat{\delta}^2 = s^2$, $\hat{\delta}_\alpha^2 = \frac{s_1^2 - s^2}{b} \to$ □ □ □ —

$\hat{\delta}_\beta^2 = \frac{s_2^2 - s^2}{k} \to$ ≡

## Estimated variance in case of two Factor

$\hat{\delta}^2 = s^2$, $\hat{\delta}_\alpha^2 = \frac{s_1^2 - s_2^2}{b * n}$

$\hat{\delta}_\beta^2 = \frac{s_2^2 - s_3^2}{a * n}$, $\hat{\delta}_{\alpha\beta}^2 = \frac{s_3^2 - s^2}{n}$

## construct comparison

$w_1 = q_1\mu_1 + q_2\mu_2 + q_3\mu_3 + \dots$

$w_2 = k_1\mu_1 - k_2\mu_2 + k_3\mu_3 + \dots$

$\sum c_i = \sum q_i * k_i = 0 \Rightarrow$ لو مش مساوي reject ⇒ ومتكملش

$$SSw_1 = \frac{(\sum q_i \bar{y}_i)^2}{n \sum q_i^2}$$

$$= \frac{(q_1\mu_1 + q_2\mu_2 + q_3\mu_3 + \dots)^2}{n(q_1^2 + q_2^2 + q_3^2 + \dots)} = X$$

$$SSw_2 = \frac{(\sum k_i \bar{y}_i)^2}{n \sum k_i^2}$$

$$= \frac{(k_1\mu_1 - k_2\mu_2 + k_3\mu_3 + \dots)^2}{n(k_1^2 + (-k_2)^2 + (k_3)^2 + \dots)} = y$$

| source of variation | S.S | df | M.S | F |
|---|---|---|---|---|
| aggregates | $SS_{agg}$ | $k-1$ | $s_1^2 = \frac{SS_{agg}}{k-1}$ | $F_{agg} = \frac{s_1^2}{s^2}$ |
| (-) and (---) | X | 1 | $s_2^2 = \frac{SS_{()vs()}}{1}$ | $F_{()vs()} = \frac{s_2^2}{s^2}$ |
| (----) and (--) | y | 1 | $s_3^2 = \frac{SS_{()vs()}}{1}$ | $F_{()vs()} = \frac{s_3^2}{s^2}$ |
| Error | SSE | $k(n-1)$ | $s^2 = \frac{SSE}{k(n-1)}$ | |

if $F_{i\,()vs()} > F_{agg} \Rightarrow$ reject $w_j$

○ ⇒ given