Page:

Ch1 Queuing theory

1- The probability that the system may be busy = $\rho = \frac{\lambda}{\eta}$

2- Average number of units expected in the system $L_s = \frac{\rho}{1-\rho}$

3- Average number of units expected in the queue $= L_q = \frac{\rho^2}{1-\rho}$

4- Average expected units waiting time in the system $W_s = \frac{1}{\eta(1-\rho)}$

5- Average expected units waiting time in the queue $W_q = \frac{\rho}{\eta(1-\rho)}$

6- The possibility that no one is in the system $P_0 = 1-\rho$

7- The probability that there are n units in the system

$$P_n = \rho^n P_0 = \rho^n (1-\rho)$$

Ex1 If the number of individuals who come to the post office every day 7 people depending on the Poisson distribution and 10 hours working day, and competent employee serves 10 individuals per hour depending on the exponential distribution. Required:

1- Average number of individuals in front of officer?

2- The possibility of there being more than two in the system

(M|M|1) with $\lambda = 7$, $\eta = 10$. $\rho = \frac{\lambda}{\eta} = 0.7$

1- $L_s = \frac{\rho}{1-\rho} = \frac{7}{3}$

2- $P(n>2) = P(n \geq 3) = \rho^3 = 0.7^3 = 0.343$

Ex2 The university Hospital can receive patients in emergencies and clinics at a rate of 600 patients per hour and the rate of arrival of patients to the hospital is 450 patients per hour Required:

1- The probability that the hospital clinics may be busy?

2- Average number of patients expected in the system?

3- Average number of patients expected in the queue?

4- Average expected patient waiting time in the system?

5- Average expected patient waiting time in the queue?

(M|M|1) With $\lambda = 450$, $\eta = 600$

1- $\rho = \lambda/\eta = 450/600 = 0.75$

2- $L_s = \frac{\rho}{1-\rho} = \frac{0.75}{1-0.75} = 3$

3- $L_q = \frac{\rho^2}{1-\rho} = \frac{0.75^2}{1-075} = 2.25$

4- $W_s = \frac{1}{\eta(1-\rho)} = \frac{1}{600(1-0.75)} = \frac{1}{150} = 0.006667$

5- $W_q = \frac{\rho}{\eta(1-\rho)} = \frac{0.75}{600(1-0.75)} = \frac{0.75}{150} = 0.005$

Ex3 The restaurant can receive customers at the restaurant at a rate of 150 Customer per hour and the rate of arrival of customer to the restaurant is 140 customers per hour Required:

1- The probability that the restaurant busy?
2- Percentage of lost idle time?
3- Average number of customers expected in the system?
4- Average number of customers expected in the queue?
5- Average expected customer waiting time in the system?
6- Average expected customer waiting time in the queue

(M/M/1) With $\lambda = 140$, $\eta = 150$

1- $\rho = \lambda/\eta = 140/150 = 0.933$

2- $P_0 = 1 - \rho = 1 - 0.933 = 0.0666 = 6.6\%$

3- $L_s = \frac{\rho}{1-\rho} = \frac{0.933}{1-0.933} = 14$

4- $L_q = \frac{\rho^2}{1-\rho} = \frac{0.933^2}{1-0.933} = 13$

5- $W_s = \frac{1}{\eta(1-\rho)} = \frac{1}{150(1-0.933)} = 0.1$ 6 Minuts

6- $W_q = \frac{\rho}{\eta(1-\rho)} = \frac{0.933}{150(1-0.933)} = 7/75 = 0.09333$ 5.95 Min

Mostafa Abdelkarem

Lec 2 Linear Programming

Ex1 The area of a parking lot is 500 square meters. A car requires 5 square meters and a bus requires 20 square meters of space. The lot can handle a maximum of 50 vehicles. If a car costs 3$ and a bus cost 8$ to park in the lot, determine the number of each vehicle to maximize the amount collected?

Define your unknowns let X = the number of car
y = the number of buses

Express the objective and the constraints

Profit Max $Z = 3X + 8y$

$X + y \leq 50$ $\quad 5X + 20y \leq 500$

$X, y \geq 0$

Ex2 A diet is to include at least 140 milligrams of vitamin A and at least 145 milligrams of vitamin B. These requirements can be obtained from two type of food. Type X contains 10 milligrams of vitamin A and 20 milligrams of vitamin B per pound. Type y contains 30 milligrams of vitamin A and 15 milligrams of vitamin B per pound. If type X food costs 12$ per pound and type y food costs 8$ per pound how many pounds of each type of food should be purchased to satisfy the requirements at the minimum cost?

Define your unknowns let X = the number of units for food X
y = the number of units for food y

Express the objective and the constraints

Cost Min $P = 12X + 8y$

$10X + 30y \geq 140$ $\quad 20X + 15y \geq 145$

$X, y \geq 0$

Mostafa Abdelhaleem

Desha

Page: Date:

Ch3 The Assignment Model

Hungarian method

امشي باقل صفوص وبعدين اقل عمود

step1 Subtract row minima ركب خراطيش على الصفر والاقل اطرح باقي

step2 Subtract column minima واجمع مشبك على ثنان جمعان

step3 Cover all zeros with a minimum number of lines

step4 Create additional zeros الخرطوش على الصفر مساوي

والف حنا وشفا عليك

Example1 The funny Toys Company has four men available for work on four separate jobs. only one man can work on any one job. The cost of assigning each man to each job is given in the following table. The objective is to assign men to jobs in such a way that the total cost of assignment is minimum.

Person \ Job	1	2	3	4
A	20	25	22	28
B	15	18	23	17
C	19	17	21	24
D	25	23	24	24

A1+B4+C2+D3

20+17+17+24

78 $

1

	1	2	3	4
A	0	5	2	8
B	0	3	8	2
C	2	0	4	7
D	2	0	1	1

rows

Solution

2

	1	2	3	4
A	0	5	1	7
B	0	3	7	1
C	2	0	3	6
D	2	0	0	0

Column

3

	1	2	3	4
A	0	5	1	7
B	0	3	7	1
C	2	0	3	6
D	2	0	0	0

4

	1	2	3	4
A	0	4	0	6
B	0	2	6	0
C	3	0	3	6
D	3	0	0	0

Page: Date:

Example2

Four persons A, B, C and D are to be assigned four jobs I II, III and IV. The cost matrix is given as under, find the proper assignment.

Jobs \ Man	A	B	C	D
I	8	10	17	9
II	3	8	5	6
III	10	12	11	9
IV	6	13	9	7

Solution

(1)

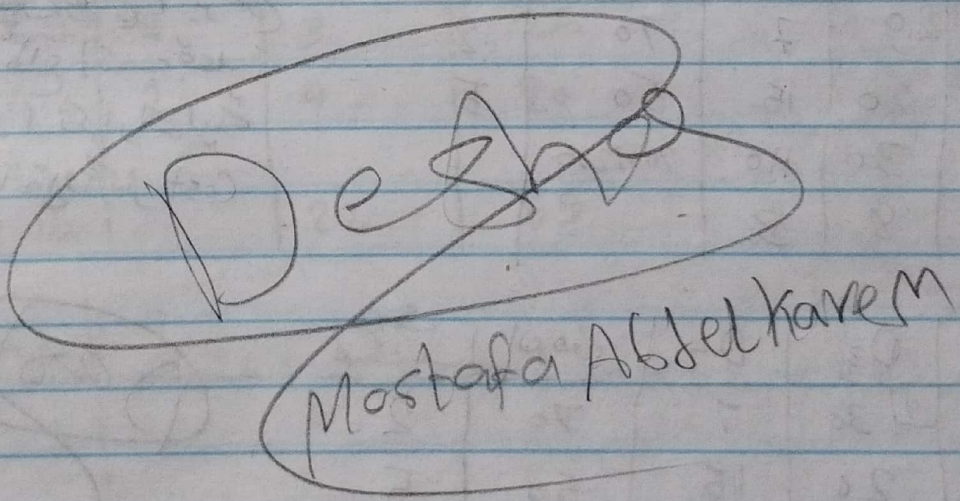
	A	B	C	D
I	0	2	9	1
II	0	5	2	3
III	1	3	2	0
IV	0	7	3	1

(2)

	A	B	C	D
I	0	0	7	1
II	0	3	0	3
III	1	1	0	0
IV	0	5	1	1

The Total Cost Assignment: $AIV + BI + CII + DIII$

$6 + 10 + 5 + 9 = 30$



Transportation Problems

Vogel's Approximation method

العرض / الطلب	D_1	D_2	D_3	D_4	العرض
S_1	20	22	17	4	120
S_2	24	37	9	7	70
S_3	32	37	20	15	50
الطلب	60	40	30	120	240/240

Solve

	D_1	D_2	D_3	D_4	العرض	فروق الصفوف
S_1	20	22 [40]	17	4	120	13
S_2	24	37	9	7	70	2
S_3	32	37	20	15	50	5
طلب	60	40	30	110	240/240	
فروق الأعمدة	4	15	8	3		

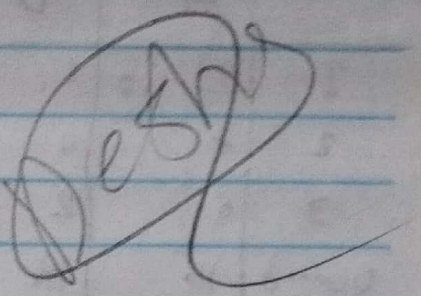
	D_1	D_3	D_4	العرض	فروق الصفوف
S_1	20	17	[4] 80	80	13
S_2	24	9	7	70	2
S_3	32	20	15	50	5
الطلب	60	30	110	200/200	
الفروق الأعمدة	4	8	3		

امشي بأقل 2 صوص في الصوصات والخراطيش واطرح أكبر طرح دخل وطلع اتعب قعد وهكذا تكرار العملية لحد أقل وأكثر Cost

	D_1	D_3	D_4	العرض	فروق صفوف
S_2	24	[9] 30	7	70	2
S_3	32	20	15	50	5
الطلب	60	30	30	120/120	
فروق الأعمدة	8	11	8		

Done

	D_1	D_4	العرض	فروق الصفوف
S_2	[24] 10	[7] 30	40	17
S_3	[32] 50	15	50	17
الطلب	60	30	90/90	
الفروق العمودية	8	8		

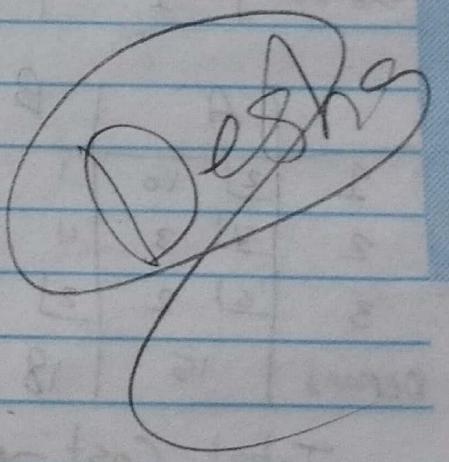


السوق / المصنع	D_1	D_2	D_3	D_4	العرض
S_1	20	[22] 40	17	[4] 80	120
S_2	[24] 10	37	[9] 30	[7] 30	70
S_3	[32] 50	37	20	15	50
الطلب	60	40	30	110	240/240

$$Z = \sum_{j=1}^{4} \sum_{i=1}^{3} C_{ij} X_{ij} = 10*24 + 50*32 + 40*22 + 30*9 + 80*4 + 30*7 = 3520$$

Example Solve the following transportation problem using Vogel's Approximation Method

From \ To	A	B	C	Supply
1	2	1	8	10
2	7	4	3	25
3	6	2	4	20
Demand	15		22	55



	A
1	2
2	7
3	6
Demand	

Page: Date:

*	A	B	C	Supply	عروض مفقود
1	[2] 10	1	8	10	1
2	7	4	3	25	1
3	6	2	4	20	2
Demand	15	18	22	55	
عروض مفقود	4	1	1		

	A	B	C	Supply	عروض مفقود
2	7	4	3	25	1
3	6	[2] 18	4	20	2
Demand	5	18	22	55	
عروض مفقود	1	2	1		

	A		C		Supply	عروض مفقود
2	7	3	[3]	22	25	4
3	6	2	4		2	2
Demand	5		22		55	
عروض مفقود	1		1			

	A		B		C	Supply
1	[2]	10	1		8	10
2	[7]	3	4		[3] 22	25
3	[6]	2	[2]	18	4	20
Demand	15		18		22	55

Total Cost →

$10*2 + 7*3 + 6*2 + 2*18 + 3*22$

155

كل عام وانتم بخير

Mostafa Abdelkarke

2022