

## \* Gamma distn

$$F(x, \alpha, \beta) = \frac{1}{\beta^\alpha \Gamma(\alpha)} e^{-\frac{x}{\beta}} x^{\alpha-1} \quad \text{Cont} \quad \text{Dis}$$

$$\text{Moment } M_x(t) = E(e^{tx}) = (1 - \beta t)^{-\alpha} \quad E(x) = \int x F(x) dx = \sum x F(x)$$

$$\eta = \frac{\partial M_x(t)}{\partial t} \bigg|_{t=0} = E(x)$$

$$E(x^2) = \frac{\partial^2 M_x(t)}{\partial t^2} \bigg|_{t=0}$$

2] The random variable  $x$  has uniform distribution with pdf  $F(x) = 1$  for  $0 < x < 1$  and zero elsewhere find P.d.f of  $Z = -2 \ln x$

Uniform dist  $F(x) = \frac{1}{b-a} \quad a < x < b \Rightarrow F(x) = \frac{1}{1-0} = 1, Z = \ln x^{-2}$

$$M_z(t) = E(e^{tz}) = E(e^{t \ln x^{-2}}) = \int_0^1 e^{t \ln x^{-2}} \times 1 dx = \int_0^1 e^{\ln(x^{-2})^t} dx$$

$$= \int_0^1 x^{-2t} dx = \frac{x^{-2t+1}}{-2t+1} \bigg|_0^1 = \frac{1}{1-2t} = (1-2t)^{-1}$$

Then compare  $(1-2t)^{-1}$  by  $(1-\beta t)^{-\alpha} \quad \alpha=1, \beta=2$

Then  $F(x, \alpha, \beta) = \frac{1}{\beta^\alpha \Gamma(\alpha)} e^{-\frac{x}{\beta}} x^{\alpha-1} = \frac{1}{2} e^{-\frac{x}{2}}$

3] The random variable  $x$  has gamma distribution with  $\alpha = r/2$  where  $r > 0$  and  $\beta > 0$  as its parameters find the pdf of  $y = ex$ .  
The gamma pdf is  $F(x, \alpha, \beta) = \int_0^\infty \frac{1}{\beta^\alpha \Gamma(\alpha)} e^{-\frac{x}{\beta}} x^{\alpha-1} dx \rightarrow \alpha = r/2$

$$F(x, \frac{r}{2}, \beta) = \int_0^\infty \frac{1}{\beta^{r/2} \Gamma(\frac{r}{2})} e^{-\frac{x}{\beta}} x^{\frac{r}{2}-1} dx$$

$$M_y(t) = E(e^{yt}) = \int_0^\infty e^{yt} F(x) dx = \int_0^\infty e^{yt} \frac{1}{\beta^{r/2} \Gamma(\frac{r}{2})} e^{-\frac{x}{\beta}} x^{\frac{r}{2}-1} dx$$

$$= \frac{1}{\beta^{r/2} \Gamma(\frac{r}{2})} \int_0^\infty e^{(yt - \frac{x}{\beta})} x^{\frac{r}{2}-1} dx \quad y = \frac{2x}{\beta}$$

$$= \frac{1}{\beta^{r/2} \Gamma(\frac{r}{2})} \int_0^\infty e^{(\frac{2x}{\beta} t - \frac{x}{\beta})} x^{\frac{r}{2}-1} dx$$

$$= \frac{1}{\beta^{r/2} \Gamma(\frac{r}{2})} \int_0^\infty e^{-\frac{x}{\beta} (1-2t)} x^{\frac{r}{2}-1} dx$$

let  $\frac{x}{\beta} (1-2t) = u \Rightarrow x = \frac{\beta}{1-2t} u \Rightarrow dx = \frac{\beta}{1-2t} du$



$$\begin{aligned}
 *M_y(t) &= \frac{1}{B^{r/2} \Gamma_{r/2}} \int_0^\infty e^{-u} \left( \frac{B}{1-2t} u \right)^{\frac{r}{2}-1} \left( \frac{B}{1-2t} \right) du \\
 &= \frac{1}{B^{r/2} \Gamma_{r/2}} \left( \frac{B}{1-2t} \right)^{\frac{r}{2}} \int_0^\infty e^{-u} u^{\frac{r}{2}-1} du \\
 &= (1-2t)^{-\frac{r}{2}}
 \end{aligned}$$

Compare by  $(1-Bt)^{-\alpha}$ 

$$\alpha = \frac{r}{2}$$

$$B=2$$

$$PDF \rightarrow F(x, \frac{r}{2}, 2) = \frac{1}{2^{r/2} \Gamma_{r/2}} \cdot e^{-x/2} \cdot x^{\frac{r}{2}-1}$$

4] Prove that if  $z \sim N(0,1)$  and  $y \sim \chi_r^2$  are two independent variables then  $X = \frac{z}{\sqrt{y/r}} \sim t_r$

$$z \sim N(0,1) \Rightarrow f_1(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2}$$

$$y \sim \chi_r^2 \Rightarrow f_2(y) = \frac{1}{2^{r/2} \Gamma_{r/2}} e^{-\frac{y}{2}} y^{\frac{r}{2}-1}$$

$$g(z,y) = f_1(z) * f_2(y) = \frac{1}{\sqrt{2\pi} 2^{r/2} \Gamma_{r/2}} e^{-\frac{1}{2}(z^2+y)} y^{\frac{r}{2}-1}$$

$$g(x,y) = g(z,y)|_{z=x} |J|$$

$$X = \frac{z}{\sqrt{y/r}} \Rightarrow z = x \sqrt{\frac{y}{r}} \Rightarrow \frac{dz}{dx} = \sqrt{\frac{y}{r}} = |J|$$

$$g(x,y) = \frac{1}{\sqrt{2\pi r} 2^{r/2} \Gamma_{r/2}} e^{-\frac{y}{2} \left( \frac{x^2}{r} + 1 \right)} \cdot y^{\frac{r-1}{2}}$$

$$h(x) = \int_0^\infty g(x,y) dy = \frac{1}{\sqrt{2\pi r} 2^{r/2} \Gamma_{r/2}} \int_0^\infty e^{-\frac{y}{2} \left( \frac{x^2}{r} + 1 \right)} \cdot y^{\frac{r-1}{2}} dy$$

$$\text{let } u = \frac{y}{2} \left( \frac{x^2}{r} + 1 \right) \Rightarrow y = 2u / \left( \frac{x^2}{r} + 1 \right) \Rightarrow dy = 2 / \left( \frac{x^2}{r} + 1 \right) du$$

$$\begin{aligned}
 h(x) &= \frac{2^{\frac{r+1}{2}}}{\sqrt{2\pi r} 2^{r/2} \Gamma_{r/2}} \left( \frac{x^2}{r} + 1 \right)^{-\left(\frac{r+1}{2}\right)} \int_0^\infty e^{-u} u^{\frac{r-1}{2}} du \\
 &= \frac{2^{\frac{r+1}{2}}}{\sqrt{2\pi r} 2^{r/2} \Gamma_{r/2}} \left( \frac{x^2}{r} + 1 \right)^{-\left(\frac{r+1}{2}\right)} \cdot \Gamma_{\frac{r+1}{2}}
 \end{aligned}$$

distn of  $t_r$



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5. If  $U \sim X_{r_1}^2$  and  $V \sim X_{r_2}^2$  are two independent variables then  
Prove

$$X = \frac{U/r_1}{V/r_2} \sim F(r_1, r_2)$$

$$U \sim X_{r_1}^2 \Rightarrow f_1(u) = \frac{1}{2^{r_1/2} \Gamma(r_1/2)} u^{r_1/2 - 1} e^{-u/2}$$

$$V \sim X_{r_2}^2 \Rightarrow f_2(v) = \frac{1}{2^{r_2/2} \Gamma(r_2/2)} v^{r_2/2 - 1} e^{-v/2}$$

$$g(u, v) = f_1(u) \cdot f_2(v) = \frac{1}{2^{(r_1+r_2)/2} \Gamma(r_1/2) \Gamma(r_2/2)} u^{r_1/2 - 1} v^{r_2/2 - 1} e^{-\frac{1}{2}(u+v)}$$

$$h(x, v) = g(u, v) \Big|_{u=x} |J|$$

$$X = \frac{U/r_1}{V/r_2} \Rightarrow U = \frac{r_1}{r_2} v x \Rightarrow \frac{du}{dx} = \frac{r_1}{r_2} v = |J|$$

$$h(x, v) = \frac{1}{2^{(r_1+r_2)/2} \Gamma(r_1/2) \Gamma(r_2/2)} \cdot \left(\frac{r_1}{r_2} v x\right)^{r_1/2 - 1} \cdot v^{r_2/2 - 1} \cdot e^{-\frac{1}{2}\left(\frac{r_1}{r_2} x v + v\right)} \cdot \frac{r_1}{r_2} v$$

$$f(x) = \int_0^\infty h(x, v) dv$$

$$f(x) = \frac{\left(\frac{r_1}{r_2}\right)^{r_1/2} x^{r_1/2 - 1}}{2^{(r_1+r_2)/2} \Gamma(r_1/2) \Gamma(r_2/2)} \int_0^\infty v^{(r_1+r_2)/2 - 1} e^{-\frac{v}{2}\left(\frac{r_1}{r_2} x + 1\right)} dv$$

$$w = \frac{v}{2} \left(\frac{r_1}{r_2} x + 1\right) \Rightarrow v = \frac{2w}{\frac{r_1}{r_2} x + 1} \Rightarrow dv = \frac{2}{\frac{r_1}{r_2} x + 1} dw$$

$$f(x) = \frac{\left(\frac{r_1}{r_2}\right)^{r_1/2} x^{r_1/2 - 1}}{2^{(r_1+r_2)/2} \Gamma(r_1/2) \Gamma(r_2/2)} \cdot 2 \cdot \frac{(r_1+r_2)/2 - 1}{\left(\frac{r_1}{r_2} x + 1\right)} \int_0^\infty w^{(r_1+r_2)/2 - 1} e^{-w} dw$$

$$f(x) \sim F(x) = \frac{\left(\frac{r_1}{r_2} x + 1\right)^{-\left(\frac{r_1+r_2}{2}\right)} \cdot \left(\frac{r_1}{r_2}\right)^{r_1/2} \cdot x^{r_1/2 - 1} \cdot \sqrt{\frac{r_1+r_2}{2}}}{\Gamma(r_1/2) \Gamma(r_2/2)}$$



\* Show that if  $X \sim P(r_1, r_2)$  then  $\frac{1}{X} \sim P(r_2, r_1)$

$$f(x) = \frac{\sqrt{\frac{r_1+r_2}{2}}}{\sqrt{\frac{r_1}{2}} \sqrt{\frac{r_2}{2}}} \cdot \left(\frac{r_1}{r_2}\right)^{\frac{r_1}{2}} \cdot x^{\frac{r_1}{2}-1} \cdot \left(1 + \frac{r_1}{r_2} x\right)^{-\left(\frac{r_1+r_2}{2}\right)}$$

let  $y = \frac{1}{x} \Rightarrow x = \frac{1}{y} \Rightarrow \frac{dx}{dy} = \frac{1}{y^2} = |J|$

$$h(y) = f(x)|_{x=y} \cdot |J|$$

$$h(y) = \frac{\sqrt{\frac{r_1+r_2}{2}}}{\sqrt{\frac{r_1}{2}} \sqrt{\frac{r_2}{2}}} \cdot \left(\frac{r_1}{r_2}\right)^{\frac{r_1}{2}} \cdot y^{-\left(\frac{r_1}{2}-1\right)} \cdot \left(1 + \frac{r_1}{r_2} \frac{1}{y}\right)^{-\left(\frac{r_1+r_2}{2}\right)}$$

Multiply with

$$\left(\frac{r_2}{r_1} y\right)^{-\left(\frac{r_1+r_2}{2}\right)}$$

$$h(y) = \frac{\sqrt{\frac{r_1+r_2}{2}}}{\sqrt{\frac{r_1}{2}} \sqrt{\frac{r_2}{2}}} \cdot \left(\frac{r_2}{r_1}\right)^{\frac{r_2}{2}} \cdot y^{\frac{r_2}{2}-1} \cdot \left(1 + \frac{r_2}{r_1} y\right)^{-\left(\frac{r_1+r_2}{2}\right)}$$

\* look pdf  $\bar{X}, S^2$  is unbiased estimator for  $\mu, \sigma^2$

[7] R.S size n

Note  $P(-\rightarrow-) = 1 - P(-\leftarrow-)$   
 $P(a < z < b) = F(b) - F(a)$

R.S size  $n_1, n_2$

$$\frac{\bar{X} - \mu}{\sqrt{\frac{S^2}{n}}} \sim N(\mu, \sigma^2)$$

$$\frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}} \sim N(\mu, \sigma^2)$$

look Ex Proportion in pdf

$$\frac{X}{n} \sim N\left(p, \frac{p(1-p)}{n}\right)$$

$$\frac{r-p}{\frac{p(1-p)}{n}} \sim N(0,1) \text{ where } r = \frac{X}{n}$$



# 8] Test of hypotheses mean

## 1-Mean

$$① H_0: \mu = \mu_0$$

$$H_a: \mu \neq \mu_0 \quad \text{or} \quad \mu > \mu_0 \quad \text{or} \quad \mu < \mu_0$$

②

$S^2$  Known

$$Z_{\text{comp}} = \frac{\bar{X} - \mu_0}{\text{Var Pop } \sigma / \sqrt{n}}$$

$$n \geq 30$$

$$Z = \frac{\bar{X} - \mu_0}{\text{Var Sample } S / \sqrt{n}}$$

$S^2$  unknown

$$n < 30$$

$$t = \frac{\bar{X} - \mu_0}{S / \sqrt{n}}$$

$\neq$

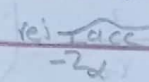
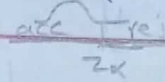
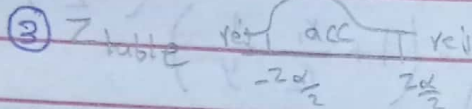
$$\pm t(\frac{\alpha}{2}, n-1)$$

$>$

$$+ t(\alpha, n-1)$$

$<$

$$- t(\alpha, n-1)$$



④ Compare  $Z_{\text{comp}} > Z_{\text{table}}$

⑤ reject  $H_0$

## 2 Two main

$$① H_0: \mu_1 = \mu_2$$

$$H_a: \mu_1 \neq \mu_2 \quad \text{or} \quad \mu_1 > \mu_2 \quad \text{or} \quad \mu_1 < \mu_2$$

②

$S_1^2, S_2^2$  Known

$$Z = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}}$$

$S_1^2, S_2^2$  unknown

$$n_1, n_2 \geq 30$$

$$Z = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}}$$

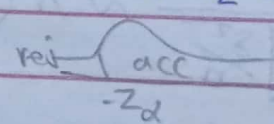
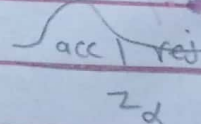
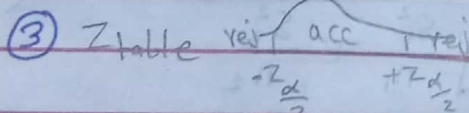
$$n_1, n_2 < 30$$

$$t = \frac{\bar{X}_1 - \bar{X}_2}{SP \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

$$\pm t(\frac{\alpha}{2}, n_1 + n_2 - 2)$$

$$+ t(\alpha, n_1 + n_2 - 2)$$

$$SP^2 = \frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{n_1 + n_2 - 2}$$



④ Compare  $Z_{\text{comp}} > Z_{\text{table}}$

⑤ reject  $H_0$



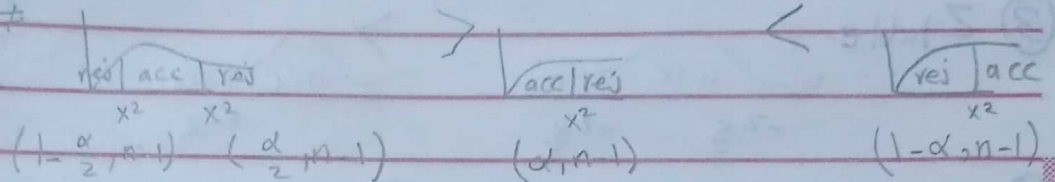
## Test the Variance of Population

1. Variance ①  $H_0: \sigma^2 = \sigma_0^2$

$$H_a: \sigma^2 \neq \sigma_0^2 \quad \text{or} \quad \sigma^2 > \sigma_0^2 \quad \text{or} \quad \sigma^2 < \sigma_0^2$$

$$\textcircled{2} \chi^2_{\text{comp}} = \frac{(n-1)s^2}{\sigma_0^2}$$

$$\textcircled{3} \chi^2_{\text{table}} \neq$$



$$\textcircled{4} \chi^2_{\text{comp}} > \chi^2_{\text{table}}$$

⑤ reject  $H_0$

2. Two variance ①  $H_0: \sigma_1^2 = \sigma_2^2$

$$H_a: \sigma_1^2 \neq \sigma_2^2 \quad \text{or} \quad \sigma_1^2 > \sigma_2^2 \quad \text{or} \quad \sigma_1^2 < \sigma_2^2$$

$$\textcircled{2} F_{\text{comp}} = \frac{s_1^2}{s_2^2}$$

or  $\frac{s_2^2}{s_1^2}$

$$\textcircled{3} F_{\text{table}} \neq$$



$$\textcircled{4} F_{\text{comp}} > F_{\text{table}}$$

⑤ reject  $H_0$

## Test of Sample Proportion

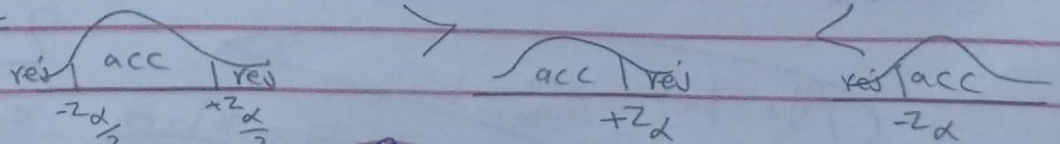
1. Proportion ①  $H_0: P = P_0$

$$H_a: P \neq P_0 \quad \text{or} \quad P > P_0 \quad \text{or} \quad P < P_0$$

$$\textcircled{2} Z_{\text{comp}} = \frac{P - P_0}{\sqrt{\frac{P_0(1-P_0)}{n}}}$$

$$P = \frac{x}{n}$$

$$\textcircled{3} Z_{\text{table}} \neq$$



$$\textcircled{4} Z_{\text{comp}} > Z_{\text{table}}$$

⑤ reject  $H_0$



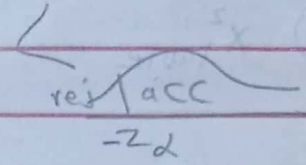
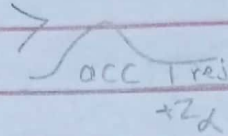
\* 2- Two Proportion ①  $H_0: P_1 = P_2$ 

$$H_a: P_1 \neq P_2 \quad \text{or} \quad P_1 > P_2 \quad \text{or} \quad P_1 < P_2$$

$$② Z_{\text{comp}} = \frac{P_1 - P_2}{\sqrt{\hat{P}(1-\hat{P})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

$$\hat{P} = \frac{x_1 + x_2}{n_1 + n_2}$$

$$③ Z_{\text{table}} \neq$$



$$④ Z_{\text{comp}} > Z_{\text{table}}$$

⑤ reject  $H_0$ II  $\chi^2$  test ①  $H_0$ : The die is fair $H_a$ : The die is not fairExpect  $\alpha$  و  $\beta$ 

$$② \chi^2 = \sum \frac{(O-E)^2}{E}$$

$$③ \chi^2_{\text{table}} = \chi^2_{(\alpha, n-1)}$$

$$④ \chi^2_{\text{comp}} > \chi^2_{\text{table}}$$

⑤ reject  $H_0$ توقع  $\alpha$  و  $\beta$  في Expect①  $H_0$ : two factors are independent $H_a$ : two factors are not independent

$$② \chi^2 = \sum \frac{(O-E)^2}{E}$$

$$③ \chi^2_{\text{table}} (\alpha, M-1, n-1)$$

	A		$\Sigma$
B	1,1	1,2	$S_1$
	2,1	2,2	$S_2$
$\Sigma$	$T_1$	$T_2$	$N$

$$E(1,1) = \frac{T_1 * S_1}{N}, \quad E(2,1) = \frac{T_1 * S_1}{N}$$

$$E(1,2) = \frac{T_1 * S_2}{N}, \quad E(2,2) = \frac{T_2 * S_2}{N}$$

$$\chi^2_{\text{comp}} > \chi^2_{\text{table}} \Rightarrow \text{accept } H_0$$

Mostafa Abdel Kareem

Deshg



# Experimental Design

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## \* F-test Mean

Source of variation	Sum of squares	degree of freedom	Mean square	F <sub>comp</sub>
treatment	SSA	K-1	$S_1^2 = \frac{SSA}{K-1}$	$F = \frac{S_1^2}{S^2}$
Error	SSE	K(n-1)	$S^2 = \frac{SSE}{K(n-1)}$	
total	SST	Kn-1		

**F<sub>table</sub>**  $r = F_{\alpha}(K-1, K(n-1))$  From table A.6  
 $r \neq F_{\alpha}(K-1, N-K)$   $F_{comp} > r$  reject  $H_0$   
 $H_0: \mu_1 = \mu_2 = \mu_3 = \mu_4 = \mu_5 \dots$   
 $H_a: \text{at least two of the means are not equal}$

## [2] Bartlett Test Several equality variance

$$H_0: S_1^2 = S_2^2 = S_3^2 = \dots = S_K^2$$

$H_1: \text{all variances are not equal}$

$$b_{comp} < b_K(\alpha; n_1, \dots, n_K)$$

reject  $H_0$

**b<sub>table</sub>**  $b_K(\alpha; n_1, \dots, n_K) = \frac{1}{N} [n_1 b_K(\alpha; n_1) + \dots + n_K b_K(\alpha; n_K)]$

$$SP^2 = \frac{1}{N-K} \sum_{i=1}^K (n_i - 1) S_i^2$$

$$b_{comp} = \frac{1}{SP^2} \left[ (S_1^2)^{n_1-1} + \dots + (S_K^2)^{n_K-1} \right]^{\frac{1}{N-K}}$$

From table A.10

The sample sizes equal

Cochran's test

$$G = \frac{\text{largest } S_i^2}{\sum_{i=1}^K S_i^2} = \square$$

$\alpha = \square$  From table A.11

$G > \alpha$  reject  $H_0$

## [3] Test $H_0: \mu_i = \mu_j$

$H_a: \mu_i \neq \mu_j$

Tukey test

Duncan test

Dunnnett test

Dunnnett test

$$H_0: \mu_0 = \mu_i$$

$$H_a: \mu_0 \neq \mu_i$$

$S^2$  Mean square error

$$|d_i| = \bar{y}_i - \bar{y}_0$$

$$\sqrt{\frac{2S^2}{n}}$$

الناتج لوجوالية

Table A.14

reject  $H_0$

$$|d_i| > d(\frac{\alpha}{2}, K, \nu)$$

$$\nu = K(n-1)$$

على أنه مثبتة واحدة  $K = K-1$

Tukey test

كروا - كروا

Duncan test

أكبر 4 فرقة من الكل لا فرق

$$q_{comp} = q_{table}$$

$$\sqrt{\frac{S^2}{n}}$$

بمعدل جدول الفروق  $q_{table}$

Table A.12

الفرق  $> q_{comp}$

reject  $H_0$

$$R_p = r_p \sqrt{\frac{S^2}{n}}$$

Table A.13

الفرق  $> R_p$



# **Tukey test**

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أشرح على مثال

# **Tukey test**

الفرق بين المتوسطات

$\bar{y}_2$	كبيرة	$\bar{y}_5$	$\bar{y}_1$	$\bar{y}_3$	$\bar{y}_6$	$\bar{y}_4$
14.50	متوسطة	16.75	19.84	21.12	22.90	23.20
$\bar{y}_4 - \bar{y}_2$	كبيرة	$\bar{y}_5 - \bar{y}_2$	كبيرة	$\bar{y}_3 - \bar{y}_2$	$\bar{y}_6 - \bar{y}_2$	$\bar{y}_4 - \bar{y}_2$
$\bar{y}_4 - \bar{y}_5$	كبيرة	$\bar{y}_5 - \bar{y}_3$	كبيرة	$\bar{y}_3 - \bar{y}_5$	$\bar{y}_6 - \bar{y}_5$	$\bar{y}_4 - \bar{y}_5$
$\bar{y}_4 - \bar{y}_1$	كبيرة	$\bar{y}_5 - \bar{y}_1$	كبيرة	$\bar{y}_3 - \bar{y}_1$	$\bar{y}_6 - \bar{y}_1$	$\bar{y}_4 - \bar{y}_1$
$\bar{y}_4 - \bar{y}_3$	كبيرة	$\bar{y}_5 - \bar{y}_3$	كبيرة	$\bar{y}_3 - \bar{y}_3$	$\bar{y}_6 - \bar{y}_3$	$\bar{y}_4 - \bar{y}_3$
$\bar{y}_4 - \bar{y}_6$	كبيرة	$\bar{y}_5 - \bar{y}_6$	كبيرة	$\bar{y}_3 - \bar{y}_6$	$\bar{y}_6 - \bar{y}_6$	$\bar{y}_4 - \bar{y}_6$

$$q_{comp} = q_{table} * \sqrt{\frac{s^2}{n}}$$

$$q_{table} \rightarrow q(\alpha, K, v) \quad \alpha = K(n-1)$$

$H_0: \mu_i = \mu_j$   $H_a: \mu_i \neq \mu_j$   $H_0$  لا توجد فروق بين المتوسطات  $q_{comp}$  أكبر من  $q_{table}$  نرفض  $H_0$

## **Duncan test**

$H_0: \mu_i = \mu_j$   $H_a: \mu_i \neq \mu_j$

جدول الفرق على نفس الترتيب نأخذ أقل واحد ونصاعدها ثم نقارننا واقف الى R انزل بالتوازي

$\bar{y}_2$	$\bar{y}_3$	$\bar{y}_1$	$\bar{y}_3$	$\bar{y}_6$
$\bar{y}_4$	$\downarrow R_6$	$\downarrow R_5$	$\downarrow R_4$	$\downarrow R_3$
$\bar{y}_6$	$\downarrow R_5$	$\downarrow R_4$	$\downarrow R_3$	$\downarrow R_2$
$\bar{y}_3$	$\downarrow R_4$	$\downarrow R_3$	$\downarrow R_2$	$\downarrow R_1$
$\bar{y}_1$	$\downarrow R_3$	$\downarrow R_2$	$\downarrow R_1$	$\downarrow R_0$
$\bar{y}_5$	$\downarrow R_2$	$\downarrow R_1$	$\downarrow R_0$	$\downarrow R_{-1}$

note  $P \rightarrow 2 \ 3 \ 4 \ 5 \ 6 \dots K$

$(\alpha, v)$  عند  $r_p \rightarrow A.13$

## **Estimation of Variance Components**

$$S^2_{\alpha} = \frac{s^2 - s^2}{b}$$

$$S^2_{\alpha} = \frac{s^2 - s^2}{b}$$

$$S^2_{\beta} = \frac{s^2 - s^2}{K}$$

Estimated Two Factor

$$S^2_{\alpha} = \frac{s^2 - s^2}{bn}$$

$$S^2_{\beta} = \frac{s^2 - s^2}{an}$$

$$S^2_{\alpha\beta} = \frac{s^2 - s^2}{n}$$

## **Randomized Complete block design**

Ex 13.6

$H_0: \mu_1 = \mu_2 = \dots = \mu_K$

$H_a: \text{The } \mu_i \text{ are not equal}$

Source of variation	Sum of square	df	MS = $\frac{Sum}{df}$	F <sub>comp</sub>
Treatment	SSA	K-1	$S^2_1 = \frac{SSA}{K-1}$	$F = \frac{S^2_1}{S^2}$
blocks	SSB	b-1	$S^2_2 = \frac{SSB}{b-1}$	
Error	SSE	(K-1)(b-1)	$S^2 = \frac{SSE}{(K-1)(b-1)}$	
Total	SST			

$F_{comp} > F_{table}$  Reject  $H_0$   $A.6$   $F_{\alpha}(K-1, (K-1)(b-1))$



## Two Factor test

$$H_0: \alpha_1 = \alpha_2 = \dots = \alpha_a = 0$$

$H_a$ : at least one of the  $\alpha_i$  is not equal

$$F_{\text{comp}} > F_{\text{table}} \rightarrow \text{reject } H_0$$

$$H_0: \beta_1 = \beta_2 = \dots = \beta_b = 0$$

$H_a$ : at least one of the  $\beta_j$  is not equal

Table A-6

Ex 14.1

$$H_0: (\alpha\beta)_{11} = (\alpha\beta)_{12} = \dots = (\alpha\beta)_{ab} = 0$$

$H_a$ : at least one of the  $(\alpha\beta)_{ij}$  is not equal

no. of  $\rightarrow b$   
no. of  $\downarrow$  a  $\rightarrow$  no. of fall  
a x b

S.o.V	SS	df	MS $\frac{SS}{df}$	$F_{\text{comp}}$	$F_{\text{table}}$
A	SSA	a-1	$S_1^2 = SSA/a-1$	$F_1 = S_1^2/S^2$	$\alpha, a-1, ab(a-1)$
B	SSB	b-1	$S_2^2 = SSB/b-1$	$F_2 = S_2^2/S^2$	$\alpha, b-1, ab(b-1)$
AB	SSAB	(a-1)(b-1)	$S_3^2 = SSAB/(a-1)(b-1)$	$F_3 = S_3^2/S^2$	$\alpha, (a-1)(b-1), ab(a-1)(b-1)$
Error	SSE	ab(n-1)	$S^2 = SSE/ab(n-1)$		$\alpha, df, ab(n-1)$
Total	SST				

## Three factor test

$$F_{\text{comp}} > F_{\text{table}} \rightarrow \text{reject } H_0$$

no. of a  $\rightarrow$  no. of b

S.o.V	SS	df	MS $\frac{SS}{df}$	$F_{\text{comp}}$	$F_{\text{table}}$
A	SSA	a-1	$S_1^2 = SSA/a-1$	$F_1 = S_1^2/S^2$	$\alpha$
B	SSB	b-1	$S_2^2 = SSB/b-1$	$F_2 = S_2^2/S^2$	df
C	SSC	c-1	$S_3^2 = SSC/c-1$	$F_3 = S_3^2/S^2$	$abc(n-1)$
AB	SSAB	(a-1)(b-1)	$S_4^2 = SSAB/(a-1)(b-1)$	$F_4 = S_4^2/S^2$	df
AC	SSAC	(a-1)(c-1)	$S_5^2 = SSAC/(a-1)(c-1)$	$F_5 = S_5^2/S^2$	df
BC	SSBC	(b-1)(c-1)	$S_6^2 = SSBC/(b-1)(c-1)$	$F_6 = S_6^2/S^2$	df
ABC	SSABC	(a-1)(b-1)(c-1)	$S_7^2 = SSABC/(a-1)(b-1)(c-1)$	$F_7 = S_7^2/S^2$	df
Error	SSE	abc(n-1)	$S^2 = SSE/abc(n-1)$		
Total	SST				

$$W_1 = M_1 + M_2 - M_3 - M_5$$

$$W_2 = M_1 + M_2 + M_3 - 4M_4 + M_5$$

$$\sum c_i = 1 \times 1 + 1 \times 1 + (-1) \times 1 + 0 \times (-4) + (-1) \times 1 = 0$$

$$SSW = \frac{(\sum c_i y_i)^2}{n \sum c_i^2}$$

Continue look book Ex 13.1

$$SSW_2 =$$

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