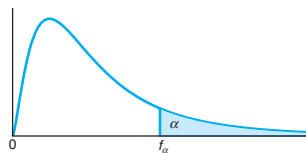


Table A.6 Critical Values of the *F*-Distribution

| $f_{0.05}(v_1, v_2)$ | | | | | | | | | |
|----------------------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| v_2 | v_1 | | | | | | | | |
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 1 | 161.45 | 199.50 | 215.71 | 224.58 | 230.16 | 233.99 | 236.77 | 238.88 | 240.54 |
| 2 | 18.51 | 19.00 | 19.16 | 19.25 | 19.30 | 19.33 | 19.35 | 19.37 | 19.38 |
| 3 | 10.13 | 9.55 | 9.28 | 9.12 | 9.01 | 8.94 | 8.89 | 8.85 | 8.81 |
| 4 | 7.71 | 6.94 | 6.59 | 6.39 | 6.26 | 6.16 | 6.09 | 6.04 | 6.00 |
| 5 | 6.61 | 5.79 | 5.41 | 5.19 | 5.05 | 4.95 | 4.88 | 4.82 | 4.77 |
| 6 | 5.99 | 5.14 | 4.76 | 4.53 | 4.39 | 4.28 | 4.21 | 4.15 | 4.10 |
| 7 | 5.59 | 4.74 | 4.35 | 4.12 | 3.97 | 3.87 | 3.79 | 3.73 | 3.68 |
| 8 | 5.32 | 4.46 | 4.07 | 3.84 | 3.69 | 3.58 | 3.50 | 3.44 | 3.39 |
| 9 | 5.12 | 4.26 | 3.86 | 3.63 | 3.48 | 3.37 | 3.29 | 3.23 | 3.18 |
| 10 | 4.96 | 4.10 | 3.71 | 3.48 | 3.33 | 3.22 | 3.14 | 3.07 | 3.02 |
| 11 | 4.84 | 3.98 | 3.59 | 3.36 | 3.20 | 3.09 | 3.01 | 2.95 | 2.90 |
| 12 | 4.75 | 3.89 | 3.49 | 3.26 | 3.11 | 3.00 | 2.91 | 2.85 | 2.80 |
| 13 | 4.67 | 3.81 | 3.41 | 3.18 | 3.03 | 2.92 | 2.83 | 2.77 | 2.71 |
| 14 | 4.60 | 3.74 | 3.34 | 3.11 | 2.96 | 2.85 | 2.76 | 2.70 | 2.65 |
| 15 | 4.54 | 3.68 | 3.29 | 3.06 | 2.90 | 2.79 | 2.71 | 2.64 | 2.59 |
| 16 | 4.49 | 3.63 | 3.24 | 3.01 | 2.85 | 2.74 | 2.66 | 2.59 | 2.54 |
| 17 | 4.45 | 3.59 | 3.20 | 2.96 | 2.81 | 2.70 | 2.61 | 2.55 | 2.49 |
| 18 | 4.41 | 3.55 | 3.16 | 2.93 | 2.77 | 2.66 | 2.58 | 2.51 | 2.46 |
| 19 | 4.38 | 3.52 | 3.13 | 2.90 | 2.74 | 2.63 | 2.54 | 2.48 | 2.42 |
| 20 | 4.35 | 3.49 | 3.10 | 2.87 | 2.71 | 2.60 | 2.51 | 2.45 | 2.39 |
| 21 | 4.32 | 3.47 | 3.07 | 2.84 | 2.68 | 2.57 | 2.49 | 2.42 | 2.37 |
| 22 | 4.30 | 3.44 | 3.05 | 2.82 | 2.66 | 2.55 | 2.46 | 2.40 | 2.34 |
| 23 | 4.28 | 3.42 | 3.03 | 2.80 | 2.64 | 2.53 | 2.44 | 2.37 | 2.32 |
| 24 | 4.26 | 3.40 | 3.01 | 2.78 | 2.62 | 2.51 | 2.42 | 2.36 | 2.30 |
| 25 | 4.24 | 3.39 | 2.99 | 2.76 | 2.60 | 2.49 | 2.40 | 2.34 | 2.28 |
| 26 | 4.23 | 3.37 | 2.98 | 2.74 | 2.59 | 2.47 | 2.39 | 2.32 | 2.27 |
| 27 | 4.21 | 3.35 | 2.96 | 2.73 | 2.57 | 2.46 | 2.37 | 2.31 | 2.25 |
| 28 | 4.20 | 3.34 | 2.95 | 2.71 | 2.56 | 2.45 | 2.36 | 2.29 | 2.24 |
| 29 | 4.18 | 3.33 | 2.93 | 2.70 | 2.55 | 2.43 | 2.35 | 2.28 | 2.22 |
| 30 | 4.17 | 3.32 | 2.92 | 2.69 | 2.53 | 2.42 | 2.33 | 2.27 | 2.21 |
| 40 | 4.08 | 3.23 | 2.84 | 2.61 | 2.45 | 2.34 | 2.25 | 2.18 | 2.12 |
| 60 | 4.00 | 3.15 | 2.76 | 2.53 | 2.37 | 2.25 | 2.17 | 2.10 | 2.04 |
| 120 | 3.92 | 3.07 | 2.68 | 2.45 | 2.29 | 2.18 | 2.09 | 2.02 | 1.96 |
| ∞ | 3.84 | 3.00 | 2.60 | 2.37 | 2.21 | 2.10 | 2.01 | 1.94 | 1.88 |

Reproduced from Table 18 of *Biometrika Tables for Statisticians*, Vol. I, by permission of E.S. Pearson and the Biometrika Trustees.

Table A.6 (continued) Critical Values of the F -Distribution

| v_2 | $f_{0.05}(v_1, v_2)$ | | | | | | | | | |
|----------|----------------------|--------|--------|--------|--------|--------|--------|--------|--------|----------|
| | v_1 | | | | | | | | | |
| | 10 | 12 | 15 | 20 | 24 | 30 | 40 | 60 | 120 | ∞ |
| 1 | 241.88 | 243.91 | 245.95 | 248.01 | 249.05 | 250.10 | 251.14 | 252.20 | 253.25 | 254.31 |
| 2 | 19.40 | 19.41 | 19.43 | 19.45 | 19.45 | 19.46 | 19.47 | 19.48 | 19.49 | 19.50 |
| 3 | 8.79 | 8.74 | 8.70 | 8.66 | 8.64 | 8.62 | 8.59 | 8.57 | 8.55 | 8.53 |
| 4 | 5.96 | 5.91 | 5.86 | 5.80 | 5.77 | 5.75 | 5.72 | 5.69 | 5.66 | 5.63 |
| 5 | 4.74 | 4.68 | 4.62 | 4.56 | 4.53 | 4.50 | 4.46 | 4.43 | 4.40 | 4.36 |
| 6 | 4.06 | 4.00 | 3.94 | 3.87 | 3.84 | 3.81 | 3.77 | 3.74 | 3.70 | 3.67 |
| 7 | 3.64 | 3.57 | 3.51 | 3.44 | 3.41 | 3.38 | 3.34 | 3.30 | 3.27 | 3.23 |
| 8 | 3.35 | 3.28 | 3.22 | 3.15 | 3.12 | 3.08 | 3.04 | 3.01 | 2.97 | 2.93 |
| 9 | 3.14 | 3.07 | 3.01 | 2.94 | 2.90 | 2.86 | 2.83 | 2.79 | 2.75 | 2.71 |
| 10 | 2.98 | 2.91 | 2.85 | 2.77 | 2.74 | 2.70 | 2.66 | 2.62 | 2.58 | 2.54 |
| 11 | 2.85 | 2.79 | 2.72 | 2.65 | 2.61 | 2.57 | 2.53 | 2.49 | 2.45 | 2.40 |
| 12 | 2.75 | 2.69 | 2.62 | 2.54 | 2.51 | 2.47 | 2.43 | 2.38 | 2.34 | 2.30 |
| 13 | 2.67 | 2.60 | 2.53 | 2.46 | 2.42 | 2.38 | 2.34 | 2.30 | 2.25 | 2.21 |
| 14 | 2.60 | 2.53 | 2.46 | 2.39 | 2.35 | 2.31 | 2.27 | 2.22 | 2.18 | 2.13 |
| 15 | 2.54 | 2.48 | 2.40 | 2.33 | 2.29 | 2.25 | 2.20 | 2.16 | 2.11 | 2.07 |
| 16 | 2.49 | 2.42 | 2.35 | 2.28 | 2.24 | 2.19 | 2.15 | 2.11 | 2.06 | 2.01 |
| 17 | 2.45 | 2.38 | 2.31 | 2.23 | 2.19 | 2.15 | 2.10 | 2.06 | 2.01 | 1.96 |
| 18 | 2.41 | 2.34 | 2.27 | 2.19 | 2.15 | 2.11 | 2.06 | 2.02 | 1.97 | 1.92 |
| 19 | 2.38 | 2.31 | 2.23 | 2.16 | 2.11 | 2.07 | 2.03 | 1.98 | 1.93 | 1.88 |
| 20 | 2.35 | 2.28 | 2.20 | 2.12 | 2.08 | 2.04 | 1.99 | 1.95 | 1.90 | 1.84 |
| 21 | 2.32 | 2.25 | 2.18 | 2.10 | 2.05 | 2.01 | 1.96 | 1.92 | 1.87 | 1.81 |
| 22 | 2.30 | 2.23 | 2.15 | 2.07 | 2.03 | 1.98 | 1.94 | 1.89 | 1.84 | 1.78 |
| 23 | 2.27 | 2.20 | 2.13 | 2.05 | 2.01 | 1.96 | 1.91 | 1.86 | 1.81 | 1.76 |
| 24 | 2.25 | 2.18 | 2.11 | 2.03 | 1.98 | 1.94 | 1.89 | 1.84 | 1.79 | 1.73 |
| 25 | 2.24 | 2.16 | 2.09 | 2.01 | 1.96 | 1.92 | 1.87 | 1.82 | 1.77 | 1.71 |
| 26 | 2.22 | 2.15 | 2.07 | 1.99 | 1.95 | 1.90 | 1.85 | 1.80 | 1.75 | 1.69 |
| 27 | 2.20 | 2.13 | 2.06 | 1.97 | 1.93 | 1.88 | 1.84 | 1.79 | 1.73 | 1.67 |
| 28 | 2.19 | 2.12 | 2.04 | 1.96 | 1.91 | 1.87 | 1.82 | 1.77 | 1.71 | 1.65 |
| 29 | 2.18 | 2.10 | 2.03 | 1.94 | 1.90 | 1.85 | 1.81 | 1.75 | 1.70 | 1.64 |
| 30 | 2.16 | 2.09 | 2.01 | 1.93 | 1.89 | 1.84 | 1.79 | 1.74 | 1.68 | 1.62 |
| 40 | 2.08 | 2.00 | 1.92 | 1.84 | 1.79 | 1.74 | 1.69 | 1.64 | 1.58 | 1.51 |
| 60 | 1.99 | 1.92 | 1.84 | 1.75 | 1.70 | 1.65 | 1.59 | 1.53 | 1.47 | 1.39 |
| 120 | 1.91 | 1.83 | 1.75 | 1.66 | 1.61 | 1.55 | 1.50 | 1.43 | 1.35 | 1.25 |
| ∞ | 1.83 | 1.75 | 1.67 | 1.57 | 1.52 | 1.46 | 1.39 | 1.32 | 1.22 | 1.00 |

Table A.6 (continued) Critical Values of the *F*-Distribution

| v_2 | $f_{0.01}(v_1, v_2)$ | | | | | | | | |
|----------|----------------------|---------|---------|---------|---------|---------|---------|---------|---------|
| | v_1 | | | | | | | | |
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 1 | 4052.18 | 4999.50 | 5403.35 | 5624.58 | 5763.65 | 5858.99 | 5928.36 | 5981.07 | 6022.47 |
| 2 | 98.50 | 99.00 | 99.17 | 99.25 | 99.30 | 99.33 | 99.36 | 99.37 | 99.39 |
| 3 | 34.12 | 30.82 | 29.46 | 28.71 | 28.24 | 27.91 | 27.67 | 27.49 | 27.35 |
| 4 | 21.20 | 18.00 | 16.69 | 15.98 | 15.52 | 15.21 | 14.98 | 14.80 | 14.66 |
| 5 | 16.26 | 13.27 | 12.06 | 11.39 | 10.97 | 10.67 | 10.46 | 10.29 | 10.16 |
| 6 | 13.75 | 10.92 | 9.78 | 9.15 | 8.75 | 8.47 | 8.26 | 8.10 | 7.98 |
| 7 | 12.25 | 9.55 | 8.45 | 7.85 | 7.46 | 7.19 | 6.99 | 6.84 | 6.72 |
| 8 | 11.26 | 8.65 | 7.59 | 7.01 | 6.63 | 6.37 | 6.18 | 6.03 | 5.91 |
| 9 | 10.56 | 8.02 | 6.99 | 6.42 | 6.06 | 5.80 | 5.61 | 5.47 | 5.35 |
| 10 | 10.04 | 7.56 | 6.55 | 5.99 | 5.64 | 5.39 | 5.20 | 5.06 | 4.94 |
| 11 | 9.65 | 7.21 | 6.22 | 5.67 | 5.32 | 5.07 | 4.89 | 4.74 | 4.63 |
| 12 | 9.33 | 6.93 | 5.95 | 5.41 | 5.06 | 4.82 | 4.64 | 4.50 | 4.39 |
| 13 | 9.07 | 6.70 | 5.74 | 5.21 | 4.86 | 4.62 | 4.44 | 4.30 | 4.19 |
| 14 | 8.86 | 6.51 | 5.56 | 5.04 | 4.69 | 4.46 | 4.28 | 4.14 | 4.03 |
| 15 | 8.68 | 6.36 | 5.42 | 4.89 | 4.56 | 4.32 | 4.14 | 4.00 | 3.89 |
| 16 | 8.53 | 6.23 | 5.29 | 4.77 | 4.44 | 4.20 | 4.03 | 3.89 | 3.78 |
| 17 | 8.40 | 6.11 | 5.18 | 4.67 | 4.34 | 4.10 | 3.93 | 3.79 | 3.68 |
| 18 | 8.29 | 6.01 | 5.09 | 4.58 | 4.25 | 4.01 | 3.84 | 3.71 | 3.60 |
| 19 | 8.18 | 5.93 | 5.01 | 4.50 | 4.17 | 3.94 | 3.77 | 3.63 | 3.52 |
| 20 | 8.10 | 5.85 | 4.94 | 4.43 | 4.10 | 3.87 | 3.70 | 3.56 | 3.46 |
| 21 | 8.02 | 5.78 | 4.87 | 4.37 | 4.04 | 3.81 | 3.64 | 3.51 | 3.40 |
| 22 | 7.95 | 5.72 | 4.82 | 4.31 | 3.99 | 3.76 | 3.59 | 3.45 | 3.35 |
| 23 | 7.88 | 5.66 | 4.76 | 4.26 | 3.94 | 3.71 | 3.54 | 3.41 | 3.30 |
| 24 | 7.82 | 5.61 | 4.72 | 4.22 | 3.90 | 3.67 | 3.50 | 3.36 | 3.26 |
| 25 | 7.77 | 5.57 | 4.68 | 4.18 | 3.85 | 3.63 | 3.46 | 3.32 | 3.22 |
| 26 | 7.72 | 5.53 | 4.64 | 4.14 | 3.82 | 3.59 | 3.42 | 3.29 | 3.18 |
| 27 | 7.68 | 5.49 | 4.60 | 4.11 | 3.78 | 3.56 | 3.39 | 3.26 | 3.15 |
| 28 | 7.64 | 5.45 | 4.57 | 4.07 | 3.75 | 3.53 | 3.36 | 3.23 | 3.12 |
| 29 | 7.60 | 5.42 | 4.54 | 4.04 | 3.73 | 3.50 | 3.33 | 3.20 | 3.09 |
| 30 | 7.56 | 5.39 | 4.51 | 4.02 | 3.70 | 3.47 | 3.30 | 3.17 | 3.07 |
| 40 | 7.31 | 5.18 | 4.31 | 3.83 | 3.51 | 3.29 | 3.12 | 2.99 | 2.89 |
| 60 | 7.08 | 4.98 | 4.13 | 3.65 | 3.34 | 3.12 | 2.95 | 2.82 | 2.72 |
| 120 | 6.85 | 4.79 | 3.95 | 3.48 | 3.17 | 2.96 | 2.79 | 2.66 | 2.56 |
| ∞ | 6.63 | 4.61 | 3.78 | 3.32 | 3.02 | 2.80 | 2.64 | 2.51 | 2.41 |

Table A.6 (continued) Critical Values of the F -Distribution

| v_2 | $f_{0.01}(v_1, v_2)$ | | | | | | | | | |
|----------|----------------------|---------|---------|---------|---------|---------|---------|---------|---------|----------|
| | v_1 | | | | | | | | | |
| | 10 | 12 | 15 | 20 | 24 | 30 | 40 | 60 | 120 | ∞ |
| 1 | 6055.85 | 6106.32 | 6157.28 | 6208.73 | 6234.63 | 6260.65 | 6286.78 | 6313.03 | 6339.39 | 6365.86 |
| 2 | 99.40 | 99.42 | 99.43 | 99.45 | 99.46 | 99.47 | 99.47 | 99.48 | 99.49 | 99.50 |
| 3 | 27.23 | 27.05 | 26.87 | 26.69 | 26.60 | 26.50 | 26.41 | 26.32 | 26.22 | 26.13 |
| 4 | 14.55 | 14.37 | 14.20 | 14.02 | 13.93 | 13.84 | 13.75 | 13.65 | 13.56 | 13.46 |
| 5 | 10.05 | 9.89 | 9.72 | 9.55 | 9.47 | 9.38 | 9.29 | 9.20 | 9.11 | 9.02 |
| 6 | 7.87 | 7.72 | 7.56 | 7.40 | 7.31 | 7.23 | 7.14 | 7.06 | 6.97 | 6.88 |
| 7 | 6.62 | 6.47 | 6.31 | 6.16 | 6.07 | 5.99 | 5.91 | 5.82 | 5.74 | 5.65 |
| 8 | 5.81 | 5.67 | 5.52 | 5.36 | 5.28 | 5.20 | 5.12 | 5.03 | 4.95 | 4.86 |
| 9 | 5.26 | 5.11 | 4.96 | 4.81 | 4.73 | 4.65 | 4.57 | 4.48 | 4.40 | 4.31 |
| 10 | 4.85 | 4.71 | 4.56 | 4.41 | 4.33 | 4.25 | 4.17 | 4.08 | 4.00 | 3.91 |
| 11 | 4.54 | 4.40 | 4.25 | 4.10 | 4.02 | 3.94 | 3.86 | 3.78 | 3.69 | 3.60 |
| 12 | 4.30 | 4.16 | 4.01 | 3.86 | 3.78 | 3.70 | 3.62 | 3.54 | 3.45 | 3.36 |
| 13 | 4.10 | 3.96 | 3.82 | 3.66 | 3.59 | 3.51 | 3.43 | 3.34 | 3.25 | 3.17 |
| 14 | 3.94 | 3.80 | 3.66 | 3.51 | 3.43 | 3.35 | 3.27 | 3.18 | 3.09 | 3.00 |
| 15 | 3.80 | 3.67 | 3.52 | 3.37 | 3.29 | 3.21 | 3.13 | 3.05 | 2.96 | 2.87 |
| 16 | 3.69 | 3.55 | 3.41 | 3.26 | 3.18 | 3.10 | 3.02 | 2.93 | 2.84 | 2.75 |
| 17 | 3.59 | 3.46 | 3.31 | 3.16 | 3.08 | 3.00 | 2.92 | 2.83 | 2.75 | 2.65 |
| 18 | 3.51 | 3.37 | 3.23 | 3.08 | 3.00 | 2.92 | 2.84 | 2.75 | 2.66 | 2.57 |
| 19 | 3.43 | 3.30 | 3.15 | 3.00 | 2.92 | 2.84 | 2.76 | 2.67 | 2.58 | 2.49 |
| 20 | 3.37 | 3.23 | 3.09 | 2.94 | 2.86 | 2.78 | 2.69 | 2.61 | 2.52 | 2.42 |
| 21 | 3.31 | 3.17 | 3.03 | 2.88 | 2.80 | 2.72 | 2.64 | 2.55 | 2.46 | 2.36 |
| 22 | 3.26 | 3.12 | 2.98 | 2.83 | 2.75 | 2.67 | 2.58 | 2.50 | 2.40 | 2.31 |
| 23 | 3.21 | 3.07 | 2.93 | 2.78 | 2.70 | 2.62 | 2.54 | 2.45 | 2.35 | 2.26 |
| 24 | 3.17 | 3.03 | 2.89 | 2.74 | 2.66 | 2.58 | 2.49 | 2.40 | 2.31 | 2.21 |
| 25 | 3.13 | 2.99 | 2.85 | 2.70 | 2.62 | 2.54 | 2.45 | 2.36 | 2.27 | 2.17 |
| 26 | 3.09 | 2.96 | 2.81 | 2.66 | 2.58 | 2.50 | 2.42 | 2.33 | 2.23 | 2.13 |
| 27 | 3.06 | 2.93 | 2.78 | 2.63 | 2.55 | 2.47 | 2.38 | 2.29 | 2.20 | 2.10 |
| 28 | 3.03 | 2.90 | 2.75 | 2.60 | 2.52 | 2.44 | 2.35 | 2.26 | 2.17 | 2.06 |
| 29 | 3.00 | 2.87 | 2.73 | 2.57 | 2.49 | 2.41 | 2.33 | 2.23 | 2.14 | 2.03 |
| 30 | 2.98 | 2.84 | 2.70 | 2.55 | 2.47 | 2.39 | 2.30 | 2.21 | 2.11 | 2.01 |
| 40 | 2.80 | 2.66 | 2.52 | 2.37 | 2.29 | 2.20 | 2.11 | 2.02 | 1.92 | 1.80 |
| 60 | 2.63 | 2.50 | 2.35 | 2.20 | 2.12 | 2.03 | 1.94 | 1.84 | 1.73 | 1.60 |
| 120 | 2.47 | 2.34 | 2.19 | 2.03 | 1.95 | 1.86 | 1.76 | 1.66 | 1.53 | 1.38 |
| ∞ | 2.32 | 2.18 | 2.04 | 1.88 | 1.79 | 1.70 | 1.59 | 1.47 | 1.32 | 1.00 |

Table A.10 Critical Values for Bartlett's Test

| <i>n</i> | $b_k(0.01; n)$ | | | | | | | | |
|----------|---------------------------------|--------|--------|--------|--------|--------|--------|--------|--------|
| | Number of Populations, <i>k</i> | | | | | | | | |
| | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 3 | 0.1411 | 0.1672 | | | | | | | |
| 4 | 0.2843 | 0.3165 | 0.3475 | 0.3729 | 0.3937 | 0.4110 | | | |
| 5 | 0.3984 | 0.4304 | 0.4607 | 0.4850 | 0.5046 | 0.5207 | 0.5343 | 0.5458 | 0.5558 |
| 6 | 0.4850 | 0.5149 | 0.5430 | 0.5653 | 0.5832 | 0.5978 | 0.6100 | 0.6204 | 0.6293 |
| 7 | 0.5512 | 0.5787 | 0.6045 | 0.6248 | 0.6410 | 0.6542 | 0.6652 | 0.6744 | 0.6824 |
| 8 | 0.6031 | 0.6282 | 0.6518 | 0.6704 | 0.6851 | 0.6970 | 0.7069 | 0.7153 | 0.7225 |
| 9 | 0.6445 | 0.6676 | 0.6892 | 0.7062 | 0.7197 | 0.7305 | 0.7395 | 0.7471 | 0.7536 |
| 10 | 0.6783 | 0.6996 | 0.7195 | 0.7352 | 0.7475 | 0.7575 | 0.7657 | 0.7726 | 0.7786 |
| 11 | 0.7063 | 0.7260 | 0.7445 | 0.7590 | 0.7703 | 0.7795 | 0.7871 | 0.7935 | 0.7990 |
| 12 | 0.7299 | 0.7483 | 0.7654 | 0.7789 | 0.7894 | 0.7980 | 0.8050 | 0.8109 | 0.8160 |
| 13 | 0.7501 | 0.7672 | 0.7832 | 0.7958 | 0.8056 | 0.8135 | 0.8201 | 0.8256 | 0.8303 |
| 14 | 0.7674 | 0.7835 | 0.7985 | 0.8103 | 0.8195 | 0.8269 | 0.8330 | 0.8382 | 0.8426 |
| 15 | 0.7825 | 0.7977 | 0.8118 | 0.8229 | 0.8315 | 0.8385 | 0.8443 | 0.8491 | 0.8532 |
| 16 | 0.7958 | 0.8101 | 0.8235 | 0.8339 | 0.8421 | 0.8486 | 0.8541 | 0.8586 | 0.8625 |
| 17 | 0.8076 | 0.8211 | 0.8338 | 0.8436 | 0.8514 | 0.8576 | 0.8627 | 0.8670 | 0.8707 |
| 18 | 0.8181 | 0.8309 | 0.8429 | 0.8523 | 0.8596 | 0.8655 | 0.8704 | 0.8745 | 0.8780 |
| 19 | 0.8275 | 0.8397 | 0.8512 | 0.8601 | 0.8670 | 0.8727 | 0.8773 | 0.8811 | 0.8845 |
| 20 | 0.8360 | 0.8476 | 0.8586 | 0.8671 | 0.8737 | 0.8791 | 0.8835 | 0.8871 | 0.8903 |
| 21 | 0.8437 | 0.8548 | 0.8653 | 0.8734 | 0.8797 | 0.8848 | 0.8890 | 0.8926 | 0.8956 |
| 22 | 0.8507 | 0.8614 | 0.8714 | 0.8791 | 0.8852 | 0.8901 | 0.8941 | 0.8975 | 0.9004 |
| 23 | 0.8571 | 0.8673 | 0.8769 | 0.8844 | 0.8902 | 0.8949 | 0.8988 | 0.9020 | 0.9047 |
| 24 | 0.8630 | 0.8728 | 0.8820 | 0.8892 | 0.8948 | 0.8993 | 0.9030 | 0.9061 | 0.9087 |
| 25 | 0.8684 | 0.8779 | 0.8867 | 0.8936 | 0.8990 | 0.9034 | 0.9069 | 0.9099 | 0.9124 |
| 26 | 0.8734 | 0.8825 | 0.8911 | 0.8977 | 0.9029 | 0.9071 | 0.9105 | 0.9134 | 0.9158 |
| 27 | 0.8781 | 0.8869 | 0.8951 | 0.9015 | 0.9065 | 0.9105 | 0.9138 | 0.9166 | 0.9190 |
| 28 | 0.8824 | 0.8909 | 0.8988 | 0.9050 | 0.9099 | 0.9138 | 0.9169 | 0.9196 | 0.9219 |
| 29 | 0.8864 | 0.8946 | 0.9023 | 0.9083 | 0.9130 | 0.9167 | 0.9198 | 0.9224 | 0.9246 |
| 30 | 0.8902 | 0.8981 | 0.9056 | 0.9114 | 0.9159 | 0.9195 | 0.9225 | 0.9250 | 0.9271 |
| 40 | 0.9175 | 0.9235 | 0.9291 | 0.9335 | 0.9370 | 0.9397 | 0.9420 | 0.9439 | 0.9455 |
| 50 | 0.9339 | 0.9387 | 0.9433 | 0.9468 | 0.9496 | 0.9518 | 0.9536 | 0.9551 | 0.9564 |
| 60 | 0.9449 | 0.9489 | 0.9527 | 0.9557 | 0.9580 | 0.9599 | 0.9614 | 0.9626 | 0.9637 |
| 80 | 0.9586 | 0.9617 | 0.9646 | 0.9668 | 0.9685 | 0.9699 | 0.9711 | 0.9720 | 0.9728 |
| 100 | 0.9669 | 0.9693 | 0.9716 | 0.9734 | 0.9748 | 0.9759 | 0.9769 | 0.9776 | 0.9783 |

Reproduced from D. D. Dyer and J. P. Keating, "On the Determination of Critical Values for Bartlett's Test," *J. Am. Stat. Assoc.*, **75**, 1980, by permission of the Board of Directors.

Table A.10 (continued) Critical Values for Bartlett's Test

| <i>n</i> | $b_k(0.05; n)$ | | | | | | | | |
|----------|---------------------------------|--------|--------|--------|--------|--------|--------|--------|--------|
| | Number of Populations, <i>k</i> | | | | | | | | |
| | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 3 | 0.3123 | 0.3058 | 0.3173 | 0.3299 | | | | | |
| 4 | 0.4780 | 0.4699 | 0.4803 | 0.4921 | 0.5028 | 0.5122 | 0.5204 | 0.5277 | 0.5341 |
| 5 | 0.5845 | 0.5762 | 0.5850 | 0.5952 | 0.6045 | 0.6126 | 0.6197 | 0.6260 | 0.6315 |
| 6 | 0.6563 | 0.6483 | 0.6559 | 0.6646 | 0.6727 | 0.6798 | 0.6860 | 0.6914 | 0.6961 |
| 7 | 0.7075 | 0.7000 | 0.7065 | 0.7142 | 0.7213 | 0.7275 | 0.7329 | 0.7376 | 0.7418 |
| 8 | 0.7456 | 0.7387 | 0.7444 | 0.7512 | 0.7574 | 0.7629 | 0.7677 | 0.7719 | 0.7757 |
| 9 | 0.7751 | 0.7686 | 0.7737 | 0.7798 | 0.7854 | 0.7903 | 0.7946 | 0.7984 | 0.8017 |
| 10 | 0.7984 | 0.7924 | 0.7970 | 0.8025 | 0.8076 | 0.8121 | 0.8160 | 0.8194 | 0.8224 |
| 11 | 0.8175 | 0.8118 | 0.8160 | 0.8210 | 0.8257 | 0.8298 | 0.8333 | 0.8365 | 0.8392 |
| 12 | 0.8332 | 0.8280 | 0.8317 | 0.8364 | 0.8407 | 0.8444 | 0.8477 | 0.8506 | 0.8531 |
| 13 | 0.8465 | 0.8415 | 0.8450 | 0.8493 | 0.8533 | 0.8568 | 0.8598 | 0.8625 | 0.8648 |
| 14 | 0.8578 | 0.8532 | 0.8564 | 0.8604 | 0.8641 | 0.8673 | 0.8701 | 0.8726 | 0.8748 |
| 15 | 0.8676 | 0.8632 | 0.8662 | 0.8699 | 0.8734 | 0.8764 | 0.8790 | 0.8814 | 0.8834 |
| 16 | 0.8761 | 0.8719 | 0.8747 | 0.8782 | 0.8815 | 0.8843 | 0.8868 | 0.8890 | 0.8909 |
| 17 | 0.8836 | 0.8796 | 0.8823 | 0.8856 | 0.8886 | 0.8913 | 0.8936 | 0.8957 | 0.8975 |
| 18 | 0.8902 | 0.8865 | 0.8890 | 0.8921 | 0.8949 | 0.8975 | 0.8997 | 0.9016 | 0.9033 |
| 19 | 0.8961 | 0.8926 | 0.8949 | 0.8979 | 0.9006 | 0.9030 | 0.9051 | 0.9069 | 0.9086 |
| 20 | 0.9015 | 0.8980 | 0.9003 | 0.9031 | 0.9057 | 0.9080 | 0.9100 | 0.9117 | 0.9132 |
| 21 | 0.9063 | 0.9030 | 0.9051 | 0.9078 | 0.9103 | 0.9124 | 0.9143 | 0.9160 | 0.9175 |
| 22 | 0.9106 | 0.9075 | 0.9095 | 0.9120 | 0.9144 | 0.9165 | 0.9183 | 0.9199 | 0.9213 |
| 23 | 0.9146 | 0.9116 | 0.9135 | 0.9159 | 0.9182 | 0.9202 | 0.9219 | 0.9235 | 0.9248 |
| 24 | 0.9182 | 0.9153 | 0.9172 | 0.9195 | 0.9217 | 0.9236 | 0.9253 | 0.9267 | 0.9280 |
| 25 | 0.9216 | 0.9187 | 0.9205 | 0.9228 | 0.9249 | 0.9267 | 0.9283 | 0.9297 | 0.9309 |
| 26 | 0.9246 | 0.9219 | 0.9236 | 0.9258 | 0.9278 | 0.9296 | 0.9311 | 0.9325 | 0.9336 |
| 27 | 0.9275 | 0.9249 | 0.9265 | 0.9286 | 0.9305 | 0.9322 | 0.9337 | 0.9350 | 0.9361 |
| 28 | 0.9301 | 0.9276 | 0.9292 | 0.9312 | 0.9330 | 0.9347 | 0.9361 | 0.9374 | 0.9385 |
| 29 | 0.9326 | 0.9301 | 0.9316 | 0.9336 | 0.9354 | 0.9370 | 0.9383 | 0.9396 | 0.9406 |
| 30 | 0.9348 | 0.9325 | 0.9340 | 0.9358 | 0.9376 | 0.9391 | 0.9404 | 0.9416 | 0.9426 |
| 40 | 0.9513 | 0.9495 | 0.9506 | 0.9520 | 0.9533 | 0.9545 | 0.9555 | 0.9564 | 0.9572 |
| 50 | 0.9612 | 0.9597 | 0.9606 | 0.9617 | 0.9628 | 0.9637 | 0.9645 | 0.9652 | 0.9658 |
| 60 | 0.9677 | 0.9665 | 0.9672 | 0.9681 | 0.9690 | 0.9698 | 0.9705 | 0.9710 | 0.9716 |
| 80 | 0.9758 | 0.9749 | 0.9754 | 0.9761 | 0.9768 | 0.9774 | 0.9779 | 0.9783 | 0.9787 |
| 100 | 0.9807 | 0.9799 | 0.9804 | 0.9809 | 0.9815 | 0.9819 | 0.9823 | 0.9827 | 0.9830 |

Table A.11 Critical Values for Cochran's Test

| $\alpha = 0.01$ | | | | | | | | | | | | | | |
|-----------------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|----------|
| k | n | | | | | | | | | | | | | |
| | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 17 | 37 | 145 | ∞ |
| 2 | 0.9999 | 0.9950 | 0.9794 | 0.9586 | 0.9373 | 0.9172 | 0.8988 | 0.8823 | 0.8674 | 0.8539 | 0.7949 | 0.7067 | 0.6062 | 0.5000 |
| 3 | 0.9933 | 0.9423 | 0.8831 | 0.8335 | 0.7933 | 0.7606 | 0.7335 | 0.7107 | 0.6912 | 0.6743 | 0.6059 | 0.5153 | 0.4230 | 0.3333 |
| 4 | 0.9676 | 0.8643 | 0.7814 | 0.7212 | 0.6761 | 0.6410 | 0.6129 | 0.5897 | 0.5702 | 0.5536 | 0.4884 | 0.4057 | 0.3251 | 0.2500 |
| 5 | 0.9279 | 0.7885 | 0.6957 | 0.6329 | 0.5875 | 0.5531 | 0.5259 | 0.5037 | 0.4854 | 0.4697 | 0.4094 | 0.3351 | 0.2644 | 0.2000 |
| 6 | 0.8828 | 0.7218 | 0.6258 | 0.5635 | 0.5195 | 0.4866 | 0.4608 | 0.4401 | 0.4229 | 0.4084 | 0.3529 | 0.2858 | 0.2229 | 0.1667 |
| 7 | 0.8376 | 0.6644 | 0.5685 | 0.5080 | 0.4659 | 0.4347 | 0.4105 | 0.3911 | 0.3751 | 0.3616 | 0.3105 | 0.2494 | 0.1929 | 0.1429 |
| 8 | 0.7945 | 0.6152 | 0.5209 | 0.4627 | 0.4226 | 0.3932 | 0.3704 | 0.3522 | 0.3373 | 0.3248 | 0.2779 | 0.2214 | 0.1700 | 0.1250 |
| 9 | 0.7544 | 0.5727 | 0.4810 | 0.4251 | 0.3870 | 0.3592 | 0.3378 | 0.3207 | 0.3067 | 0.2950 | 0.2514 | 0.1992 | 0.1521 | 0.1111 |
| 10 | 0.7175 | 0.5358 | 0.4469 | 0.3934 | 0.3572 | 0.3308 | 0.3106 | 0.2945 | 0.2813 | 0.2704 | 0.2297 | 0.1811 | 0.1376 | 0.1000 |
| 12 | 0.6528 | 0.4751 | 0.3919 | 0.3428 | 0.3099 | 0.2861 | 0.2680 | 0.2535 | 0.2419 | 0.2320 | 0.1961 | 0.1535 | 0.1157 | 0.0833 |
| 15 | 0.5747 | 0.4069 | 0.3317 | 0.2882 | 0.2593 | 0.2386 | 0.2228 | 0.2104 | 0.2002 | 0.1918 | 0.1612 | 0.1251 | 0.0934 | 0.0667 |
| 20 | 0.4799 | 0.3297 | 0.2654 | 0.2288 | 0.2048 | 0.1877 | 0.1748 | 0.1646 | 0.1567 | 0.1501 | 0.1248 | 0.0960 | 0.0709 | 0.0500 |
| 24 | 0.4247 | 0.2871 | 0.2295 | 0.1970 | 0.1759 | 0.1608 | 0.1495 | 0.1406 | 0.1338 | 0.1283 | 0.1060 | 0.0810 | 0.0595 | 0.0417 |
| 30 | 0.3632 | 0.2412 | 0.1913 | 0.1635 | 0.1454 | 0.1327 | 0.1232 | 0.1157 | 0.1100 | 0.1054 | 0.0867 | 0.0658 | 0.0480 | 0.0333 |
| 40 | 0.2940 | 0.1915 | 0.1508 | 0.1281 | 0.1135 | 0.1033 | 0.0957 | 0.0898 | 0.0853 | 0.0816 | 0.0668 | 0.0503 | 0.0363 | 0.0250 |
| 60 | 0.2151 | 0.1371 | 0.1069 | 0.0902 | 0.0796 | 0.0722 | 0.0668 | 0.0625 | 0.0594 | 0.0567 | 0.0461 | 0.0344 | 0.0245 | 0.0167 |
| 120 | 0.1225 | 0.0759 | 0.0585 | 0.0489 | 0.0429 | 0.0387 | 0.0357 | 0.0334 | 0.0316 | 0.0302 | 0.0242 | 0.0178 | 0.0125 | 0.0083 |
| ∞ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

Reproduced from C. Eisenhart, M. W. Hastay, and W. A. Wallis, *Techniques of Statistical Analysis*, Chapter 15, McGraw-Hill Book Company, New, York, 1947. Used with permission of McGraw-Hill Book Company.

Table A.11 (continued) Critical Values for Cochran's Test

| $\alpha = 0.05$ | | | | | | | | | | | | | | |
|-----------------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|----------|
| k | n | | | | | | | | | | | | | |
| | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 17 | 37 | 145 | ∞ |
| 2 | 0.9985 | 0.9750 | 0.9392 | 0.9057 | 0.8772 | 0.8534 | 0.8332 | 0.8159 | 0.8010 | 0.7880 | 0.7341 | 0.6602 | 0.5813 | 0.5000 |
| 3 | 0.9669 | 0.8709 | 0.7977 | 0.7457 | 0.7071 | 0.6771 | 0.6530 | 0.6333 | 0.6167 | 0.6025 | 0.5466 | 0.4748 | 0.4031 | 0.3333 |
| 4 | 0.9065 | 0.7679 | 0.6841 | 0.6287 | 0.5895 | 0.5598 | 0.5365 | 0.5175 | 0.5017 | 0.4884 | 0.4366 | 0.3720 | 0.3093 | 0.2500 |
| 5 | 0.8412 | 0.6838 | 0.5981 | 0.5441 | 0.5065 | 0.4783 | 0.4564 | 0.4387 | 0.4241 | 0.4118 | 0.3645 | 0.3066 | 0.2513 | 0.2000 |
| 6 | 0.7808 | 0.6161 | 0.5321 | 0.4803 | 0.4447 | 0.4184 | 0.3980 | 0.3817 | 0.3682 | 0.3568 | 0.3135 | 0.2612 | 0.2119 | 0.1667 |
| 7 | 0.7271 | 0.5612 | 0.4800 | 0.4307 | 0.3974 | 0.3726 | 0.3535 | 0.3384 | 0.3259 | 0.3154 | 0.2756 | 0.2278 | 0.1833 | 0.1429 |
| 8 | 0.6798 | 0.5157 | 0.4377 | 0.3910 | 0.3595 | 0.3362 | 0.3185 | 0.3043 | 0.2926 | 0.2829 | 0.2462 | 0.2022 | 0.1616 | 0.1250 |
| 9 | 0.6385 | 0.4775 | 0.4027 | 0.3584 | 0.3286 | 0.3067 | 0.2901 | 0.2768 | 0.2659 | 0.2568 | 0.2226 | 0.1820 | 0.1446 | 0.1111 |
| 10 | 0.6020 | 0.4450 | 0.3733 | 0.3311 | 0.3029 | 0.2823 | 0.2666 | 0.2541 | 0.2439 | 0.2353 | 0.2032 | 0.1655 | 0.1308 | 0.1000 |
| 12 | 0.5410 | 0.3924 | 0.3264 | 0.2880 | 0.2624 | 0.2439 | 0.2299 | 0.2187 | 0.2098 | 0.2020 | 0.1737 | 0.1403 | 0.1100 | 0.0833 |
| 15 | 0.4709 | 0.3346 | 0.2758 | 0.2419 | 0.2195 | 0.2034 | 0.1911 | 0.1815 | 0.1736 | 0.1671 | 0.1429 | 0.1144 | 0.0889 | 0.0667 |
| 20 | 0.3894 | 0.2705 | 0.2205 | 0.1921 | 0.1735 | 0.1602 | 0.1501 | 0.1422 | 0.1357 | 0.1303 | 0.1108 | 0.0879 | 0.0675 | 0.0500 |
| 24 | 0.3434 | 0.2354 | 0.1907 | 0.1656 | 0.1493 | 0.1374 | 0.1286 | 0.1216 | 0.1160 | 0.1113 | 0.0942 | 0.0743 | 0.0567 | 0.0417 |
| 30 | 0.2929 | 0.1980 | 0.1593 | 0.1377 | 0.1237 | 0.1137 | 0.1061 | 0.1002 | 0.0958 | 0.0921 | 0.0771 | 0.0604 | 0.0457 | 0.0333 |
| 40 | 0.2370 | 0.1576 | 0.1259 | 0.1082 | 0.0968 | 0.0887 | 0.0827 | 0.0780 | 0.0745 | 0.0713 | 0.0595 | 0.0462 | 0.0347 | 0.0250 |
| 60 | 0.1737 | 0.1131 | 0.0895 | 0.0765 | 0.0682 | 0.0623 | 0.0583 | 0.0552 | 0.0520 | 0.0497 | 0.0411 | 0.0316 | 0.0234 | 0.0167 |
| 120 | 0.0998 | 0.0632 | 0.0495 | 0.0419 | 0.0371 | 0.0337 | 0.0312 | 0.0292 | 0.0279 | 0.0266 | 0.0218 | 0.0165 | 0.0120 | 0.0083 |
| ∞ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

Table A.12 Upper Percentage Points of the Studentized Range Distribution: Values of $q(0.05; k, v)$

| Degrees of Freedom, v | Number of Treatments k | | | | | | | | |
|----------------------------|--------------------------|------|------|-------|-------|-------|-------|-------|-------|
| | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 1 | 18.0 | 27.0 | 32.8 | 37.2 | 40.5 | 43.1 | 15.1 | 47.1 | 49.1 |
| 2 | 6.09 | 5.33 | 9.80 | 10.89 | 11.73 | 12.43 | 13.03 | 13.54 | 13.99 |
| 3 | 4.50 | 5.91 | 6.83 | 7.51 | 8.04 | 8.47 | 8.85 | 9.18 | 9.46 |
| 4 | 3.93 | 5.04 | 5.76 | 6.29 | 6.71 | 7.06 | 7.35 | 7.60 | 7.83 |
| 5 | 3.64 | 4.60 | 5.22 | 5.67 | 6.03 | 6.33 | 6.58 | 6.80 | 6.99 |
| 6 | 3.46 | 4.34 | 4.90 | 5.31 | 5.63 | 5.89 | 6.12 | 6.32 | 6.49 |
| 7 | 3.34 | 4.16 | 4.68 | 5.06 | 5.35 | 5.59 | 5.80 | 5.99 | 6.15 |
| 8 | 3.26 | 4.04 | 4.53 | 4.89 | 5.17 | 5.40 | 5.60 | 5.77 | 5.92 |
| 9 | 3.20 | 3.95 | 4.42 | 4.76 | 5.02 | 5.24 | 5.43 | 5.60 | 5.74 |
| 10 | 3.15 | 3.88 | 4.33 | 4.66 | 4.91 | 5.12 | 5.30 | 5.46 | 5.60 |
| 11 | 3.11 | 3.82 | 4.26 | 4.58 | 4.82 | 5.03 | 5.20 | 5.35 | 5.49 |
| 12 | 3.08 | 3.77 | 4.20 | 4.51 | 4.75 | 4.95 | 5.12 | 5.27 | 5.40 |
| 13 | 3.06 | 3.73 | 4.15 | 4.46 | 4.69 | 4.88 | 5.05 | 5.19 | 5.32 |
| 14 | 3.03 | 3.70 | 4.11 | 4.41 | 4.65 | 4.83 | 4.99 | 5.13 | 5.25 |
| 15 | 3.01 | 3.67 | 4.08 | 4.37 | 4.59 | 4.78 | 4.94 | 5.08 | 5.20 |
| 16 | 3.00 | 3.65 | 4.05 | 4.34 | 4.56 | 4.74 | 4.90 | 5.03 | 5.05 |
| 17 | 2.98 | 3.62 | 4.02 | 4.31 | 4.52 | 4.70 | 4.86 | 4.99 | 5.11 |
| 18 | 2.97 | 3.61 | 4.00 | 4.28 | 4.49 | 4.67 | 4.83 | 4.96 | 5.07 |
| 19 | 2.96 | 3.59 | 3.98 | 4.26 | 4.47 | 4.64 | 4.79 | 4.92 | 5.04 |
| 20 | 2.95 | 3.58 | 3.96 | 4.24 | 4.45 | 4.62 | 4.77 | 4.90 | 5.01 |
| 24 | 2.92 | 3.53 | 3.90 | 4.17 | 4.37 | 4.54 | 4.68 | 4.81 | 4.92 |
| 30 | 2.89 | 3.48 | 3.84 | 4.11 | 4.30 | 4.46 | 4.60 | 4.72 | 4.83 |
| 40 | 2.86 | 3.44 | 3.79 | 4.04 | 4.23 | 4.39 | 4.52 | 4.63 | 4.74 |
| 60 | 2.83 | 3.40 | 3.74 | 3.98 | 4.16 | 4.31 | 4.44 | 4.55 | 4.65 |
| 120 | 2.80 | 3.36 | 3.69 | 3.92 | 4.10 | 4.24 | 4.36 | 4.47 | 4.56 |
| ∞ | 2.77 | 3.32 | 3.63 | 3.86 | 4.03 | 4.17 | 4.29 | 4.39 | 4.47 |

Table A.13 Least Significant Studentized Ranges $r_p(0.05; p, v)$

| $\alpha = 0.05$ | | | | | | | | | |
|-----------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| v | p | | | | | | | | |
| | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 1 | 17.97 | 17.97 | 17.97 | 17.97 | 17.97 | 17.97 | 17.97 | 17.97 | 17.97 |
| 2 | 6.085 | 6.085 | 6.085 | 6.085 | 6.085 | 6.085 | 6.085 | 6.085 | 6.085 |
| 3 | 4.501 | 4.516 | 4.516 | 4.516 | 4.516 | 4.516 | 4.516 | 4.516 | 4.516 |
| 4 | 3.927 | 4.013 | 4.033 | 4.033 | 4.033 | 4.033 | 4.033 | 4.033 | 4.033 |
| 5 | 3.635 | 3.749 | 3.797 | 3.814 | 3.814 | 3.814 | 3.814 | 3.814 | 3.814 |
| 6 | 3.461 | 3.587 | 3.649 | 3.68 | 3.694 | 3.697 | 3.697 | 3.697 | 3.697 |
| 7 | 3.344 | 3.477 | 3.548 | 3.588 | 3.611 | 3.622 | 3.626 | 3.626 | 3.626 |
| 8 | 3.261 | 3.399 | 3.475 | 3.521 | 3.549 | 3.566 | 3.575 | 3.579 | 3.579 |
| 9 | 3.199 | 3.339 | 3.420 | 3.470 | 3.502 | 3.523 | 3.536 | 3.544 | 3.547 |
| 10 | 3.151 | 3.293 | 3.376 | 3.430 | 3.465 | 3.489 | 3.505 | 3.516 | 3.522 |
| 11 | 3.113 | 3.256 | 3.342 | 3.397 | 3.435 | 3.462 | 3.48 | 3.493 | 3.501 |
| 12 | 3.082 | 3.225 | 3.313 | 3.370 | 3.410 | 3.439 | 3.459 | 3.474 | 3.484 |
| 13 | 3.055 | 3.200 | 3.289 | 3.348 | 3.389 | 3.419 | 3.442 | 3.458 | 3.470 |
| 14 | 3.033 | 3.178 | 3.268 | 3.329 | 3.372 | 3.403 | 3.426 | 3.444 | 3.457 |
| 15 | 3.014 | 3.160 | 3.25 | 3.312 | 3.356 | 3.389 | 3.413 | 3.432 | 3.446 |
| 16 | 2.998 | 3.144 | 3.235 | 3.298 | 3.343 | 3.376 | 3.402 | 3.422 | 3.437 |
| 17 | 2.984 | 3.130 | 3.222 | 3.285 | 3.331 | 3.366 | 3.392 | 3.412 | 3.429 |
| 18 | 2.971 | 3.118 | 3.210 | 3.274 | 3.321 | 3.356 | 3.383 | 3.405 | 3.421 |
| 19 | 2.960 | 3.107 | 3.199 | 3.264 | 3.311 | 3.347 | 3.375 | 3.397 | 3.415 |
| 20 | 2.950 | 3.097 | 3.190 | 3.255 | 3.303 | 3.339 | 3.368 | 3.391 | 3.409 |
| 24 | 2.919 | 3.066 | 3.160 | 3.226 | 3.276 | 3.315 | 3.345 | 3.370 | 3.390 |
| 30 | 2.888 | 3.035 | 3.131 | 3.199 | 3.250 | 3.290 | 3.322 | 3.349 | 3.371 |
| 40 | 2.858 | 3.006 | 3.102 | 3.171 | 3.224 | 3.266 | 3.300 | 3.328 | 3.352 |
| 60 | 2.829 | 2.976 | 3.073 | 3.143 | 3.198 | 3.241 | 3.277 | 3.307 | 3.333 |
| 120 | 2.800 | 2.947 | 3.045 | 3.116 | 3.172 | 3.217 | 3.254 | 3.287 | 3.314 |
| ∞ | 2.772 | 2.918 | 3.017 | 3.089 | 3.146 | 3.193 | 3.232 | 3.265 | 3.294 |

Abridged from H. L. Harter, "Critical Values for Duncan's New Multiple Range Test," *Biometrics*, **16**, No. 4, 1960, by permission of the author and the editor.

Table A.13 (continued) Least Significant Studentized Ranges $r_p(0.01; p, v)$

| $\alpha = 0.01$ | | | | | | | | | |
|-----------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| v | p | | | | | | | | |
| | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 1 | 90.03 | 90.03 | 90.03 | 90.03 | 90.03 | 90.03 | 90.03 | 90.03 | 90.03 |
| 2 | 14.04 | 14.04 | 14.04 | 14.04 | 14.04 | 14.04 | 14.04 | 14.04 | 14.04 |
| 3 | 8.261 | 8.321 | 8.321 | 8.321 | 8.321 | 8.321 | 8.321 | 8.321 | 8.321 |
| 4 | 6.512 | 6.677 | 6.740 | 6.756 | 6.756 | 6.756 | 6.756 | 6.756 | 6.756 |
| 5 | 5.702 | 5.893 | 5.989 | 6.040 | 6.065 | 6.074 | 6.074 | 6.074 | 6.074 |
| 6 | 5.243 | 5.439 | 5.549 | 5.614 | 5.655 | 5.680 | 5.694 | 5.701 | 5.703 |
| 7 | 4.949 | 5.145 | 5.260 | 5.334 | 5.383 | 5.416 | 5.439 | 5.454 | 5.464 |
| 8 | 4.746 | 4.939 | 5.057 | 5.135 | 5.189 | 5.227 | 5.256 | 5.276 | 5.291 |
| 9 | 4.596 | 4.787 | 4.906 | 4.986 | 5.043 | 5.086 | 5.118 | 5.142 | 5.160 |
| 10 | 4.482 | 4.671 | 4.790 | 4.871 | 4.931 | 4.975 | 5.010 | 5.037 | 5.058 |
| 11 | 4.392 | 4.579 | 4.697 | 4.780 | 4.841 | 4.887 | 4.924 | 4.952 | 4.975 |
| 12 | 4.320 | 4.504 | 4.622 | 4.706 | 4.767 | 4.815 | 4.852 | 4.883 | 4.907 |
| 13 | 4.260 | 4.442 | 4.560 | 4.644 | 4.706 | 4.755 | 4.793 | 4.824 | 4.850 |
| 14 | 4.210 | 4.391 | 4.508 | 4.591 | 4.654 | 4.704 | 4.743 | 4.775 | 4.802 |
| 15 | 4.168 | 4.347 | 4.463 | 4.547 | 4.610 | 4.660 | 4.700 | 4.733 | 4.760 |
| 16 | 4.131 | 4.309 | 4.425 | 4.509 | 4.572 | 4.622 | 4.663 | 4.696 | 4.724 |
| 17 | 4.099 | 4.275 | 4.391 | 4.475 | 4.539 | 4.589 | 4.630 | 4.664 | 4.693 |
| 18 | 4.071 | 4.246 | 4.362 | 4.445 | 4.509 | 4.560 | 4.601 | 4.635 | 4.664 |
| 19 | 4.046 | 4.220 | 4.335 | 4.419 | 4.483 | 4.534 | 4.575 | 4.610 | 4.639 |
| 20 | 4.024 | 4.197 | 4.312 | 4.395 | 4.459 | 4.510 | 4.552 | 4.587 | 4.617 |
| 24 | 3.956 | 4.126 | 4.239 | 4.322 | 4.386 | 4.437 | 4.480 | 4.516 | 4.546 |
| 30 | 3.889 | 4.056 | 4.168 | 4.250 | 4.314 | 4.366 | 4.409 | 4.445 | 4.477 |
| 40 | 3.825 | 3.988 | 4.098 | 4.180 | 4.244 | 4.296 | 4.339 | 4.376 | 4.408 |
| 60 | 3.762 | 3.922 | 4.031 | 4.111 | 4.174 | 4.226 | 4.270 | 4.307 | 4.340 |
| 120 | 3.702 | 3.858 | 3.965 | 4.044 | 4.107 | 4.158 | 4.202 | 4.239 | 4.272 |
| ∞ | 3.643 | 3.796 | 3.900 | 3.978 | 4.040 | 4.091 | 4.135 | 4.172 | 4.205 |

Table A.14 Values of $d_{\alpha/2}(k, v)$ for Two-Sided Comparisons between k Treatments and a Control

| $\alpha = 0.05$ | | | | | | | | | |
|------------------------------------------------------------|------|------|------|------|------|------|------|------|------|
| $k = \text{Number of Treatment Means (excluding control)}$ | | | | | | | | | |
| v | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 5 | 2.57 | 3.03 | 3.29 | 3.48 | 3.62 | 3.73 | 3.82 | 3.90 | 3.97 |
| 6 | 2.45 | 2.86 | 3.10 | 3.26 | 3.39 | 3.49 | 3.57 | 3.64 | 3.71 |
| 7 | 2.36 | 2.75 | 2.97 | 3.12 | 3.24 | 3.33 | 3.41 | 3.47 | 3.53 |
| 8 | 2.31 | 2.67 | 2.88 | 3.02 | 3.13 | 3.22 | 3.29 | 3.35 | 3.41 |
| 9 | 2.26 | 2.61 | 2.81 | 2.95 | 3.05 | 3.14 | 3.20 | 3.26 | 3.32 |
| 10 | 2.23 | 2.57 | 2.76 | 2.89 | 2.99 | 3.07 | 3.14 | 3.19 | 3.24 |
| 11 | 2.20 | 2.53 | 2.72 | 2.84 | 2.94 | 3.02 | 3.08 | 3.14 | 3.19 |
| 12 | 2.18 | 2.50 | 2.68 | 2.81 | 2.90 | 2.98 | 3.04 | 3.09 | 3.14 |
| 13 | 2.16 | 2.48 | 2.65 | 2.78 | 2.87 | 2.94 | 3.00 | 3.06 | 3.10 |
| 14 | 2.14 | 2.46 | 2.63 | 2.75 | 2.84 | 2.91 | 2.97 | 3.02 | 3.07 |
| 15 | 2.13 | 2.44 | 2.61 | 2.73 | 2.82 | 2.89 | 2.95 | 3.00 | 3.04 |
| 16 | 2.12 | 2.42 | 2.59 | 2.71 | 2.80 | 2.87 | 2.92 | 2.97 | 3.02 |
| 17 | 2.11 | 2.41 | 2.58 | 2.69 | 2.78 | 2.85 | 2.90 | 2.95 | 3.00 |
| 18 | 2.10 | 2.40 | 2.56 | 2.68 | 2.76 | 2.83 | 2.89 | 2.94 | 2.98 |
| 19 | 2.09 | 2.39 | 2.55 | 2.66 | 2.75 | 2.81 | 2.87 | 2.92 | 2.96 |
| 20 | 2.09 | 2.38 | 2.54 | 2.65 | 2.73 | 2.80 | 2.86 | 2.90 | 2.95 |
| 24 | 2.06 | 2.35 | 2.51 | 2.61 | 2.70 | 2.76 | 2.81 | 2.86 | 2.90 |
| 30 | 2.04 | 2.32 | 2.47 | 2.58 | 2.66 | 2.72 | 2.77 | 2.82 | 2.86 |
| 40 | 2.02 | 2.29 | 2.44 | 2.54 | 2.62 | 2.68 | 2.73 | 2.77 | 2.81 |
| 60 | 2.00 | 2.27 | 2.41 | 2.51 | 2.58 | 2.64 | 2.69 | 2.73 | 2.77 |
| 120 | 1.98 | 2.24 | 2.38 | 2.47 | 2.55 | 2.60 | 2.65 | 2.69 | 2.73 |
| ∞ | 1.96 | 2.21 | 2.35 | 2.44 | 2.51 | 2.57 | 2.61 | 2.65 | 2.69 |

Reproduced from Charles W. Dunnett, "New Tables for Multiple Comparison with a Control," *Biometrics*, **20**, No. 3, 1964, by permission of the author and the editor.

Table A.14 (continued) Values of $d_{\alpha/2}(k, v)$ for Two-Sided Comparisons between k Treatments and a Control

| $\alpha = 0.01$ | | | | | | | | | |
|------------------------------------------------------------|------|------|------|------|------|------|------|------|------|
| $k = \text{Number of Treatment Means (excluding control)}$ | | | | | | | | | |
| v | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 5 | 4.03 | 4.63 | 4.98 | 5.22 | 5.41 | 5.56 | 5.69 | 5.80 | 5.89 |
| 6 | 3.71 | 4.21 | 4.51 | 4.71 | 4.87 | 5.00 | 5.10 | 5.20 | 5.28 |
| 7 | 3.50 | 3.95 | 4.21 | 4.39 | 4.53 | 4.64 | 4.74 | 4.82 | 4.89 |
| 8 | 3.36 | 3.77 | 4.00 | 4.17 | 4.29 | 4.40 | 4.48 | 4.56 | 4.62 |
| 9 | 3.25 | 3.63 | 3.85 | 4.01 | 4.12 | 4.22 | 4.30 | 4.37 | 4.43 |
| 10 | 3.17 | 3.53 | 3.74 | 3.88 | 3.99 | 4.08 | 4.16 | 4.22 | 4.28 |
| 11 | 3.11 | 3.45 | 3.65 | 3.79 | 3.89 | 3.98 | 4.05 | 4.11 | 4.16 |
| 12 | 3.05 | 3.39 | 3.58 | 3.71 | 3.81 | 3.89 | 3.96 | 4.02 | 4.07 |
| 13 | 3.01 | 3.33 | 3.52 | 3.65 | 3.74 | 3.82 | 3.89 | 3.94 | 3.99 |
| 14 | 2.98 | 3.29 | 3.47 | 3.59 | 3.69 | 3.76 | 3.83 | 3.88 | 3.93 |
| 15 | 2.95 | 3.25 | 3.43 | 3.55 | 3.64 | 3.71 | 3.78 | 3.83 | 3.88 |
| 16 | 2.92 | 3.22 | 3.39 | 3.51 | 3.60 | 3.67 | 3.73 | 3.78 | 3.83 |
| 17 | 2.90 | 3.19 | 3.36 | 3.47 | 3.56 | 3.63 | 3.69 | 3.74 | 3.79 |
| 18 | 2.88 | 3.17 | 3.33 | 3.44 | 3.53 | 3.60 | 3.66 | 3.71 | 3.75 |
| 19 | 2.86 | 3.15 | 3.31 | 3.42 | 3.50 | 3.57 | 3.63 | 3.68 | 3.72 |
| 20 | 2.85 | 3.13 | 3.29 | 3.40 | 3.48 | 3.55 | 3.60 | 3.65 | 3.69 |
| 24 | 2.80 | 3.07 | 3.22 | 3.32 | 3.40 | 3.47 | 3.52 | 3.57 | 3.61 |
| 30 | 2.75 | 3.01 | 3.15 | 3.25 | 3.33 | 3.39 | 3.44 | 3.49 | 3.52 |
| 40 | 2.70 | 2.95 | 3.09 | 3.19 | 3.26 | 3.32 | 3.37 | 3.41 | 3.44 |
| 60 | 2.66 | 2.90 | 3.03 | 3.12 | 3.19 | 3.25 | 3.29 | 3.33 | 3.37 |
| 120 | 2.62 | 2.85 | 2.97 | 3.06 | 3.12 | 3.18 | 3.22 | 3.26 | 3.29 |
| ∞ | 2.58 | 2.79 | 2.92 | 3.00 | 3.06 | 3.11 | 3.15 | 3.19 | 3.22 |