KUKA-KR10-R1100-2 Robotic arm

Github Link : https://github.com/mostafa-metwaly/DoNRs-HW3

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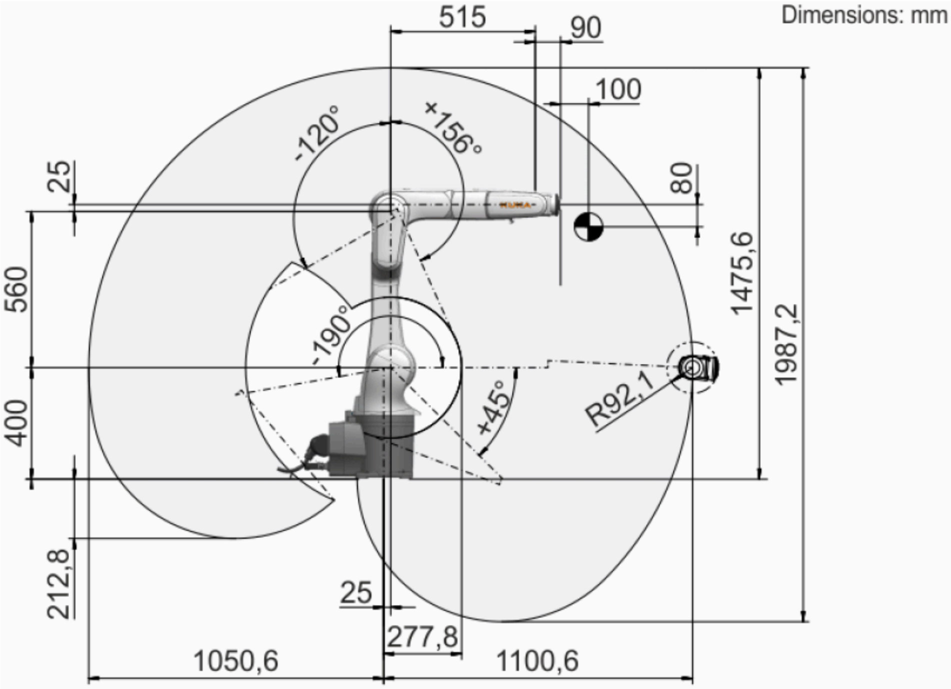
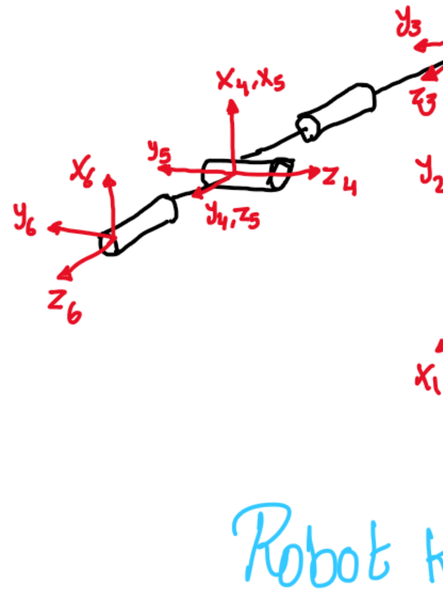
```
clear all
close all
clc
% set angles as symbolical
syms q1 q2 q3 q4 q5 q6 real
%Link lengths
d1=0.4;d2=0.025;d3=0.56;d4=0.025;d5=0.515;d6=0.09;
d=[d1, d2, d3, d4, d5, d6];
```

Workspace graphic





Forward Kinematics

$$H = R_z(\mathrm{q1}) \cdot T_z(\mathrm{d1}) \cdot R_x(-\mathrm{q2}) \cdot T_x(\mathrm{d2}) \cdot R_z(\mathrm{q3}) \cdot T_x(\mathrm{d3}) \cdot R_z(\mathrm{q4}) \cdot T_z(\mathrm{d4}) \cdot R_x(\mathrm{q5}) \cdot T_x(\mathrm{d5}) \cdot R_z(\mathrm{q6}) \cdot T_z(\mathrm{d6})$$

```
% FK symbolical
FK=simplify(Rz(q1)*Tz(d1)*Rx(-q2)*Tx(d2)*Rz(q3)*Tx(d3) *Rz(q4)*Tz(d4)*Rx(q5)*Tx(d5)*Rz(q6)*Tz(d
```

```
FK =
```

$$
\left(\begin{array}{cccc}
\cos(q_6)\,\sigma_2 - \sin(q_6)\,\sigma_4 & -\cos(q_6)\,\sigma_4 - \sin(q_6)\,\sigma_2 & \sin(q_5)\,\sigma_7 - \cos(q_5)\,\sin(q_1)\,\sin(q_2) & \frac{\cos(q_1)}{40} + \frac{14\cos}{} \\
\cos(q_6)\,\sigma_3 - \sin(q_6)\,\sigma_5 & -\cos(q_6)\,\sigma_5 - \sin(q_6)\,\sigma_3 & \sin(q_5)\,\sigma_8 + \cos(q_1)\,\cos(q_5)\,\sin(q_2) & \frac{\sin(q_1)}{40} + \frac{\cos(q_1)}{4} \\
\sin(q_6)\,\sigma_6 - \cos(q_6)\,\sigma_1 & \cos(q_6)\,\sigma_6 + \sin(q_6)\,\sigma_1 & \cos(q_2)\,\cos(q_5) - \sin(q_5)\,\sigma_9 & \frac{\cos(}{4} \\
0 & 0 & 0 &
\end{array}\right.
$$

where

$$\sigma_1 = \cos(q_3)\,\sin(q_2)\,\sin(q_4) + \cos(q_4)\,\sin(q_2)\,\sin(q_3)$$

$$\sigma_2 = \cos(q_4)\,\sigma_{10} - \sin(q_4)\,\sigma_{11}$$

$$\sigma_3 = \cos(q_4)\,\sigma_{13} - \sin(q_4)\,\sigma_{12}$$

$$\sigma_4 = \cos(q_5)\,\sigma_7 + \sin(q_1)\,\sin(q_2)\,\sin(q_5)$$

$$\sigma_5 = \cos(q_5)\,\sigma_8 - \cos(q_1)\,\sin(q_2)\,\sin(q_5)$$

$$\sigma_6 = \cos(q_2)\,\sin(q_5) + \cos(q_5)\,\sigma_9$$

$$\sigma_7 = \cos(q_4)\,\sigma_{11} + \sin(q_4)\,\sigma_{10}$$

$$\sigma_8 = \cos(q_4)\,\sigma_{12} + \sin(q_4)\,\sigma_{13}$$

$$\sigma_9 = \sin(q_2)\,\sin(q_3)\,\sin(q_4) - \cos(q_3)\,\cos(q_4)\,\sin(q_2)$$

$$\sigma_{10} = \cos(q_1)\,\cos(q_3) - \cos(q_2)\,\sin(q_1)\,\sin(q_3)$$

$$\sigma_{11} = \cos(q_1)\,\sin(q_3) + \cos(q_2)\,\cos(q_3)\,\sin(q_1)$$

$$\sigma_{12} = \sin(q_1)\,\sin(q_3) - \cos(q_1)\,\cos(q_2)\,\cos(q_3)$$

$$\sigma_{13} = \cos(q_3)\,\sin(q_1) + \cos(q_1)\,\cos(q_2)\,\sin(q_3)$$

```
disp('All 0 configuration')
```

```
All 0 configuration
```

```
q_test = [0 0 0 0 0 0] % all 0 configuration
```

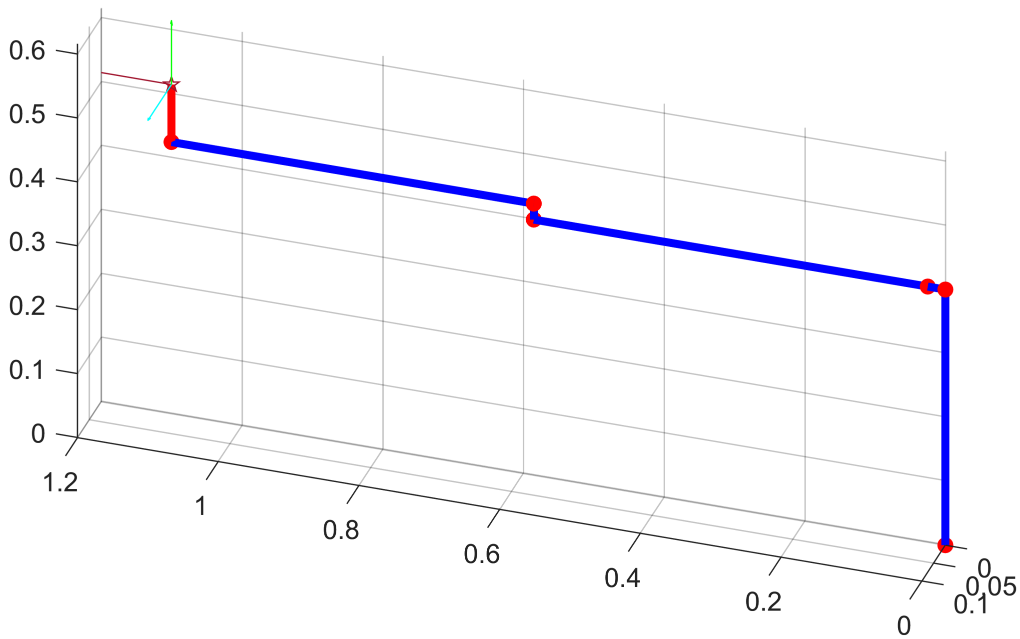
```
q_test = 1×6
```

```
     0     0     0     0     0     0
```

```
draw_robot(q_test,d)
```

```
q6 = 0
d6 = 0.0900
```

```
view([198.474 30.562])
```




```
% Substitute angles in symbolical form and convert to double
double(subs(FK, [q1 q2 q3 q4 q5 q6], q_test))
```

```
ans = 4×4
    1.0000         0         0    1.1000
         0    1.0000         0         0
         0         0    1.0000    0.5150
         0         0         0    1.0000
```

```
disp('Random configuration')
```

```
Random configuration
```

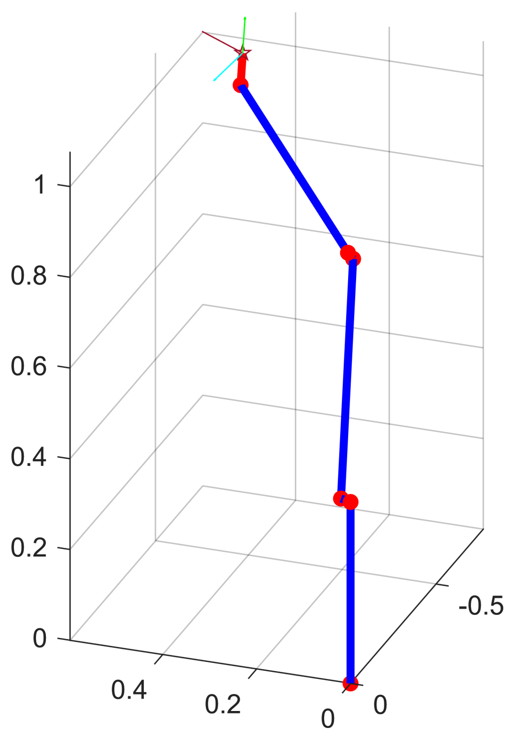
```
q1v=-20*pi/180;
q2v=40*pi/180;
q3v=-60*pi/180;
q4v=30*pi/180;
q5v=30*pi/180;
q6v=30*pi/180;
```

```
q_test=[q1v q2v q3v q4v q5v q6v]
```

```
q_test = 1×6
   -0.3491    0.6981   -1.0472    0.5236    0.5236    0.5236
```

```
draw_robot(q_test,d)
```

```
q6 = 0.5236
d6 = 0.0900
```




```
% Substitute angles in symbolical form and convert to double
double(subs(FK, [q1 q2 q3 q4 q5 q6], q_test))
```

```
ans = 4×4
    0.9480    0.2764   -0.1580    0.5025
   -0.2213    0.9289    0.2969   -0.7495
    0.2288   -0.2465    0.9417    0.9812
         0         0         0    1.0000
```

```

% disp('Test configuration')
% q_test = [pi/3 -pi/6 11 -1 -1 -1] % test configuration
% draw_robot(q_test,L)
% Substitute angles in symbolical form and convert to double
T=double(subs(FK, [q1 q2 q3 q4 q5 q6], q_test))
```

```
T = 4×4
    0.9480    0.2764   -0.1580    0.5025
   -0.2213    0.9289    0.2969   -0.7495
    0.2288   -0.2465    0.9417    0.9812
         0         0         0    1.0000
```

Jacobian

Numerical derivatives

Forward kinematics

```
syms q1 q2 q3 q4 q5 q6 real

H=simplify(Rz(q1)*Tz(d1)*Rx(-q2)*Tx(d2)*Rz(q3)*Tx(d3) *Rz(q4)*Tz(d4)*Rx(q5)*Tx(d5)*Rz(q6)*Tz(d6
```

```
H =
```

$$
\left(\begin{array}{cccc}
\cos(q_6)\,\sigma_2 - \sin(q_6)\,\sigma_4 & -\cos(q_6)\,\sigma_4 - \sin(q_6)\,\sigma_2 & \sin(q_5)\,\sigma_7 - \cos(q_5)\,\sin(q_1)\,\sin(q_2) & \frac{\cos(q_1)}{40} + \frac{14\cos}{} \\
\cos(q_6)\,\sigma_3 - \sin(q_6)\,\sigma_5 & -\cos(q_6)\,\sigma_5 - \sin(q_6)\,\sigma_3 & \sin(q_5)\,\sigma_8 + \cos(q_1)\,\cos(q_5)\,\sin(q_2) & \frac{\sin(q_1)}{40} + \frac{\cos(q_1)}{4} \\
\sin(q_6)\,\sigma_6 - \cos(q_6)\,\sigma_1 & \cos(q_6)\,\sigma_6 + \sin(q_6)\,\sigma_1 & \cos(q_2)\,\cos(q_5) - \sin(q_5)\,\sigma_9 & \frac{\cos(}{4} \\
0 & 0 & 0 &
\end{array}\right.
$$

where

$$\sigma_1 = \cos(q_3)\,\sin(q_2)\,\sin(q_4) + \cos(q_4)\,\sin(q_2)\,\sin(q_3)$$

$$\sigma_2 = \cos(q_4)\,\sigma_{10} - \sin(q_4)\,\sigma_{11}$$

$$\sigma_3 = \cos(q_4)\,\sigma_{13} - \sin(q_4)\,\sigma_{12}$$

$$\sigma_4 = \cos(q_5)\,\sigma_7 + \sin(q_1)\,\sin(q_2)\,\sin(q_5)$$

$$\sigma_5 = \cos(q_5)\,\sigma_8 - \cos(q_1)\,\sin(q_2)\,\sin(q_5)$$

$$\sigma_6 = \cos(q_2)\,\sin(q_5) + \cos(q_5)\,\sigma_9$$

$$\sigma_7 = \cos(q_4)\,\sigma_{11} + \sin(q_4)\,\sigma_{10}$$

$$\sigma_8 = \cos(q_4)\,\sigma_{12} + \sin(q_4)\,\sigma_{13}$$

$$\sigma_9 = \sin(q_2)\,\sin(q_3)\,\sin(q_4) - \cos(q_3)\,\cos(q_4)\,\sin(q_2)$$

$$\sigma_{10} = \cos(q_1)\,\cos(q_3) - \cos(q_2)\,\sin(q_1)\,\sin(q_3)$$

$$\sigma_{11} = \cos(q_1)\,\sin(q_3) + \cos(q_2)\,\cos(q_3)\,\sin(q_1)$$

$$\sigma_{12} = \sin(q_1)\,\sin(q_3) - \cos(q_1)\,\cos(q_2)\,\cos(q_3)$$

$$\sigma_{13} = \cos(q_3)\,\sin(q_1) + \cos(q_1)\,\cos(q_2)\,\sin(q_3)$$

where

$$H = \begin{bmatrix} R & T \\ 0 & 1 \end{bmatrix}$$

```
R = simplify(H(1:3,1:3))
```

```
R =
```

$$\begin{pmatrix} \cos(q_6)\,\sigma_4 - \sin(q_6)\,\sigma_2 & -\cos(q_6)\,\sigma_2 - \sin(q_6)\,\sigma_4 & \sin(q_5)\,\sigma_7 - \cos(q_5)\sin(q_1)\sin(q_2) \\ \cos(q_6)\,\sigma_5 - \sin(q_6)\,\sigma_3 & -\cos(q_6)\,\sigma_3 - \sin(q_6)\,\sigma_5 & \sin(q_5)\,\sigma_8 + \cos(q_1)\cos(q_5)\sin(q_2) \\ \sin(q_6)\,\sigma_6 - \cos(q_6)\,\sigma_1 & \cos(q_6)\,\sigma_6 + \sin(q_6)\,\sigma_1 & \cos(q_2)\cos(q_5) - \sin(q_5)\,\sigma_9 \end{pmatrix}$$

where

$$\sigma_1 = \cos(q_3)\sin(q_2)\sin(q_4) + \cos(q_4)\sin(q_2)\sin(q_3)$$

$$\sigma_2 = \cos(q_5)\,\sigma_7 + \sin(q_1)\sin(q_2)\sin(q_5)$$

$$\sigma_3 = \cos(q_5)\,\sigma_8 - \cos(q_1)\sin(q_2)\sin(q_5)$$

$$\sigma_4 = \cos(q_4)\,\sigma_{10} - \sin(q_4)\,\sigma_{11}$$

$$\sigma_5 = \cos(q_4)\,\sigma_{13} - \sin(q_4)\,\sigma_{12}$$

$$\sigma_6 = \cos(q_2)\sin(q_5) + \cos(q_5)\,\sigma_9$$

$$\sigma_7 = \cos(q_4)\,\sigma_{11} + \sin(q_4)\,\sigma_{10}$$

$$\sigma_8 = \cos(q_4)\,\sigma_{12} + \sin(q_4)\,\sigma_{13}$$

$$\sigma_9 = \sin(q_2)\sin(q_3)\sin(q_4) - \cos(q_3)\cos(q_4)\sin(q_2)$$

$$\sigma_{10} = \cos(q_1)\cos(q_3) - \cos(q_2)\sin(q_1)\sin(q_3)$$

$$\sigma_{11} = \cos(q_1)\sin(q_3) + \cos(q_2)\cos(q_3)\sin(q_1)$$

$$\sigma_{12} = \sin(q_1)\sin(q_3) - \cos(q_1)\cos(q_2)\cos(q_3)$$

$$\sigma_{13} = \cos(q_3)\sin(q_1) + \cos(q_1)\cos(q_2)\sin(q_3)$$

```
% forward kinematics
H =Rz(q1)*Tz(d1)*Rx(-q2)*Tx(d2)*Rz(q3)*Tx(d3) *Rz(q4)*Tz(d4)*Rx(q5)*Tx(d5)*Rz(q6)*Tz(d6);
H=simplify(H);
% extract rotation matrix
R = simplify(H(1:3,1:3));
% diff by q1
Td=Rzd(q1)*Tz(d1)*Rx(-q2)*Tx(d2)*Rz(q3)*Tx(d3) *Rz(q4)*Tz(d4)*Rx(q5)*Tx(d5)*Rz(q6)*Tz(d6)*...
    [R^-1 zeros(3,1);0 0 0 1];
% extract 6 components from 4x4 Td matrix to Jacobian 1st column
```

```
J1 = [Td(1,4), Td(2,4), Td(3,4), Td(3,2), Td(1,3), Td(2,1)]' ;
% diff by q2
Td=Rz(q1)*Tz(d1)*Rxd(-q2)*Tx(d2)*Rz(q3)*Tx(d3) *Rz(q4)*Tz(d4)*Rx(q5)*Tx(d5)*Rz(q6)*Tz(d6)*...
    [R^-1 zeros(3,1);0 0 0 1];
% extract 6 components from 4x4 Td matrix to Jacobian 2nd column
J2 = [Td(1,4), Td(2,4), Td(3,4), Td(3,2), Td(1,3), Td(2,1)]' ;
% diff by q3
Td=Rz(q1)*Tz(d1)*Rx(-q2)*Tx(d2)*Rzd(q3)*Tx(d3) *Rz(q4)*Tz(d4)*Rx(q5)*Tx(d5)*Rz(q6)*Tz(d6)*...
    [R^-1 zeros(3,1);0 0 0 1];
% extract 6 components from 4x4 Td matrix to Jacobian 3rd column
J3 = [Td(1,4), Td(2,4), Td(3,4), Td(3,2), Td(1,3), Td(2,1)]' ;
% diff by q4
Td=Rz(q1)*Tz(d1)*Rx(-q2)*Tx(d2)*Rz(q3)*Tx(d3) *Rzd(q4)*Tz(d4)*Rx(q5)*Tx(d5)*Rz(q6)*Tz(d6)*...
    [R^-1 zeros(3,1);0 0 0 1];
% extract 6 components from 4x4 Td matrix to Jacobian 4th column
J4 = [Td(1,4), Td(2,4), Td(3,4), Td(3,2), Td(1,3), Td(2,1)]' ;
Td=Rz(q1)*Tz(d1)*Rx(-q2)*Tx(d2)*Rz(q3)*Tx(d3) *Rz(q4)*Tz(d4)*Rxd(q5)*Tx(d5)*Rz(q6)*Tz(d6)*...
    [R^-1 zeros(3,1);0 0 0 1];
% extract 6 components from 4x4 Td matrix to Jacobian 5th column
J5 = [Td(1,4), Td(2,4), Td(3,4), Td(3,2), Td(1,3), Td(2,1)]' ;
Td=Rz(q1)*Tz(d1)*Rx(-q2)*Tx(d2)*Rz(q3)*Tx(d3) *Rz(q4)*Tz(d4)*Rx(q5)*Tx(d5)*Rzd(q6)*Tz(d6)*...
    [R^-1 zeros(3,1);0 0 0 1];
% extract 6 components from 4x4 Td matrix to Jacobian 6th column
J6 = [Td(1,4), Td(2,4), Td(3,4), Td(3,2), Td(1,3), Td(2,1)]' ;
% Full Jacobian 6x6
Jq1 = [simplify(J1), simplify(J2), simplify(J3), simplify(J4), simplify(J5), simplify(J6)]
```

```
Jq1 =
```

$$\left(\begin{array}{c}\frac{103\sin(q_4)\,\sigma_{13}}{200}-\frac{\sigma_2}{40}-\frac{14\cos(q_3)\sin(q_1)}{25}-\frac{103\cos(q_4)\,\sigma_{14}}{200}-\frac{\sin(q_1)}{40}-\frac{9\sin(q_5)\,\sigma_6}{100}-\frac{14\cos(q_1)\cos(q_2)\,\mathrm{s}}{25}\\ \frac{\cos(q_1)}{40}+\frac{14\cos(q_1)\cos(q_3)}{25}-\frac{\sigma_3}{40}+\frac{103\cos(q_4)\,\sigma_{15}}{200}-\frac{103\sin(q_4)\,\sigma_{12}}{200}+\frac{9\sin(q_5)\,\sigma_7}{100}-\frac{14\cos(q_2)\sin(q_1)}{25}\\ 0\\ 0\\ 0\\ 1\end{array}\right.$$

where

$$\sigma_1 = 112\sin(q_2)\sin(q_3) - 18\cos(q_2)\cos(q_5) - 5\cos(q_2) + 103\cos(q_3)\sin(q_2)\sin(q_4) + 103\cos(q_4)\sin($$

$$\sigma_2 = \cos(q_1)\sin(q_2)$$

$$\sigma_3 = \sin(q_1)\sin(q_2)$$

$$\sigma_4 = \frac{9\sin(q_5)\,\left(\cos(q_4)\,\sigma_{15} - \sin(q_4)\,\sigma_{12}\right)}{100}$$

$$\sigma_5 = \frac{9\sin(q_5)\,\left(\cos(q_4)\,\sigma_{14} - \sin(q_4)\,\sigma_{13}\right)}{100}$$

$$\sigma_6 = \cos(q_4)\,\sigma_{13} + \sin(q_4)\,\sigma_{14}$$

$$\sigma_7 = \cos(q_4)\,\sigma_{12} + \sin(q_4)\,\sigma_{15}$$

$$\sigma_8 = \frac{103\cos(q_4)\,\sigma_{12}}{200}$$

$$\sigma_9 = \frac{103\cos(q_4)\,\sigma_{13}}{200}$$

$$\sigma_{10} = \frac{103\sin(q_4)\,\sigma_{14}}{200}$$

$$\sigma_{11} = \frac{103\sin(q_4)\,\sigma_{15}}{200}$$

$$\sigma_{12} = \cos(q_1)\sin(q_3) + \cos(q_2)\cos(q_3)\sin(q_1)$$

$$\sigma_{13} = \sin(q_1)\sin(q_3) - \cos(q_1)\cos(q_2)\cos(q_3)$$

$$\sigma_{14} = \cos(q_3)\sin(q_1) + \cos(q_1)\cos(q_2)\sin(q_3)$$

$$\sigma_{15} = \cos(q_1)\cos(q_3) - \cos(q_2)\sin(q_1)\sin(q_3)$$

```
singularity=simplify(det(Jq1))
```

singularity =

$$\frac{49\cos(q_2)\sin(q_5)}{6250}+\frac{5047\cos(q_5)\sin(q_2)}{31250}+\frac{7\sin(q_2)\sin(q_3)\sin(q_5)}{20000}-\frac{721\sin(q_2)\sin(q_4)\sin(q_5)}{100000}-\frac{49\cos(}{}$$