KUKA-KR10-R1100-2 Robotic arm

Github Link : https://github.com/mostafa-metwaly/DoNRs-HW3

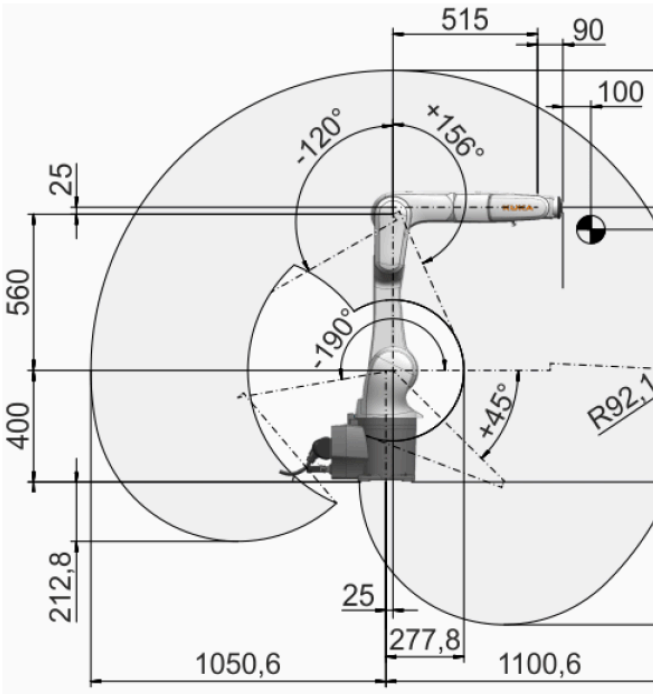
Mostafa Osama

Innopolis University

Table of Contents

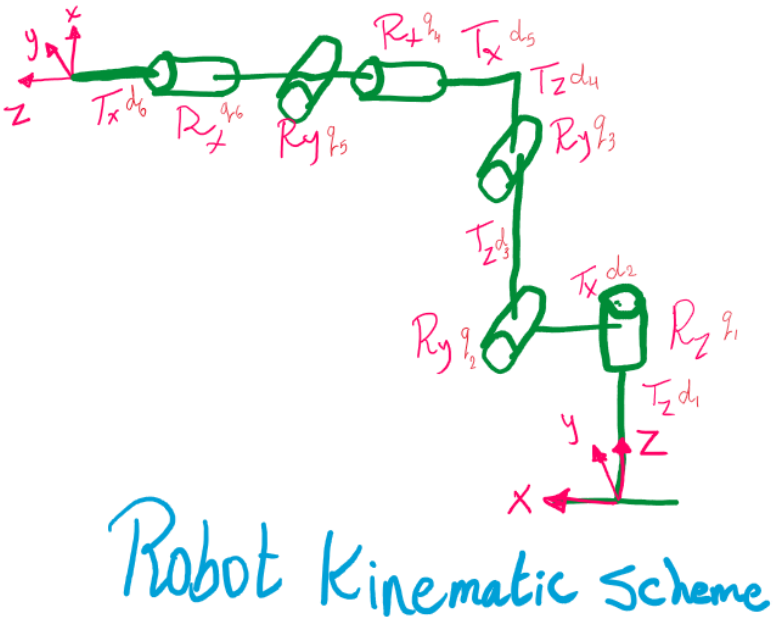
Forward Kinematics..... 2
Jacobian..... 6
Numerical derivatives..... 6

Workspace graphic




```
clear all
close all
clc
% set angles as symbolical
syms q q1 q2 q3 q4 q5 q6 real
%Link lengths
d1=400;d2=25;d3=560;d4=25;d5=515;d6=90;
d=[d1, d2, d3, d4, d5, d6];
```

```
q=[q1 q2 q3 q4 q5 q6];
```



Forward Kinematics

$$H = T_z(\mathrm{d1})\,.\,R_z(\mathrm{q1})\cdot T_x(\mathrm{d2})\cdot R_y(\mathrm{q2})\cdot T_z(\mathrm{d3})\cdot R_y(\mathrm{q3})\cdot T_z(\mathrm{d4})\ \cdot\ T_x(\mathrm{d5})\,.\,R_x(\mathrm{q4})\cdot R_y(\mathrm{q5})\cdot R_x(\mathrm{q6})\cdot T_x(\mathrm{d6})$$

```
% FK symbolical
FK=simplify(Tz(d1)*Rz(q(1))*Tx(d2)*Ry(q(2))*Tz(d3)*Ry(q(3))*Tz(d4)*Tx(d5)*Rx(q(4))*Ry(q(5))*Rx(
```

```
FK =
```

$$\begin{pmatrix} \cos(q_5)\,\sigma_9 - \sin(q_5)\,\sigma_8 & \sin(q_6)\,\sigma_3 - \cos(q_6)\,\sigma_5 \\ \sin(q_5)\,\sigma_6 - \cos(q_5)\,\sigma_7 & \cos(q_6)\,\sigma_4 - \sin(q_6)\,\sigma_2 \\ -\sin(q_2+q_3)\cos(q_5) - \cos(q_2+q_3)\cos(q_4)\sin(q_5) & \cos(q_2+q_3)\cos(q_6)\sin(q_4) - \sin(q_6)\,\sigma_1 \quad -\cos(q_6 \\ 0 & 0 \end{pmatrix}$$

where

$$\sigma_1 = \sin(q_2+q_3)\sin(q_5) - \cos(q_2+q_3)\cos(q_4)\cos(q_5)$$

$$\sigma_2 = \cos(q_5)\,\sigma_6 + \sin(q_5)\,\sigma_7$$

$$\sigma_3 = \cos(q_5)\,\sigma_8 + \sin(q_5)\,\sigma_9$$

$$\sigma_4 = \cos(q_1)\cos(q_4) + \sin(q_4)\,\sigma_{10}$$

$$\sigma_5 = \cos(q_4)\sin(q_1) - \sin(q_4)\,\sigma_{11}$$

$$\sigma_6 = \cos(q_1)\sin(q_4) - \cos(q_4)\,\sigma_{10}$$

$$\sigma_7 = \sin(q_1)\sin(q_2)\sin(q_3) - \cos(q_2)\cos(q_3)\sin(q_1)$$

$$\sigma_8 = \sin(q_1)\sin(q_4) + \cos(q_4)\,\sigma_{11}$$

$$\sigma_9 = \cos(q_1)\cos(q_2)\cos(q_3) - \cos(q_1)\sin(q_2)\sin(q_3)$$

$$\sigma_{10} = \cos(q_2)\sin(q_1)\sin(q_3) + \cos(q_3)\sin(q_1)\sin(q_2)$$

$$\sigma_{11} = \cos(q_1)\cos(q_2)\sin(q_3) + \cos(q_1)\cos(q_3)\sin(q_2)$$

```
disp('All 0 configuration')
```

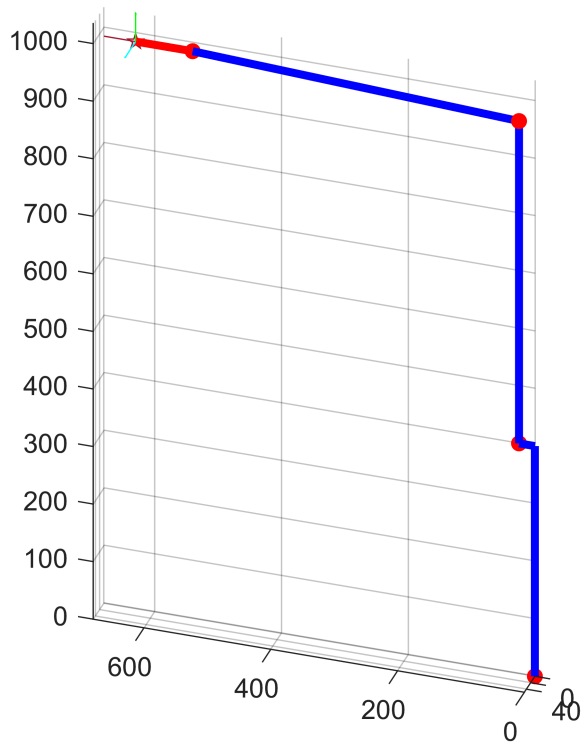
```
All 0 configuration
```

```
q_test = [0 0 0 0 0 0] % all 0 configuration
```

```
q_test = 1×6
     0     0     0     0     0     0
```

```
draw_robot(q_test,d)

view([198.474 30.562])
```




```
% Substitute angles in symbolical form and convert to double
double(subs(FK, [q1 q2 q3 q4 q5 q6], q_test))
```

```
ans = 4×4
     1     0     0   630
     0     1     0     0
     0     0     1   985
     0     0     0     1
```

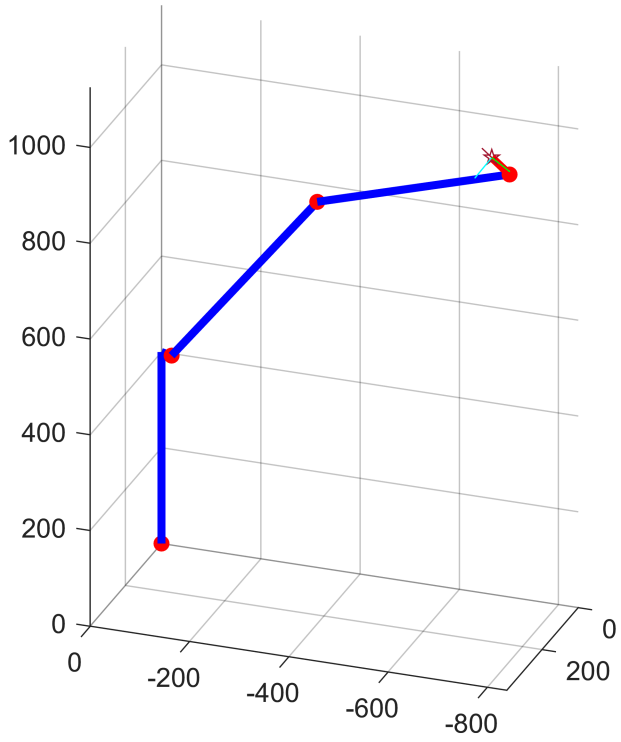
```
disp('Random configuration')
```

```
Random configuration
```

```
q1v=160*pi/180;
q2v=40*pi/180;
q3v=-60*pi/180;
q4v=-130*pi/180;
q5v=70*pi/180;
q6v=-30*pi/180;
q_test=[q1v q2v q3v q4v q5v q6v]
```

```
q_test = 1×6
    2.7925    0.6981   -1.0472   -2.2689    1.2217   -0.5236
```

```
draw_robot(q_test,d)
```

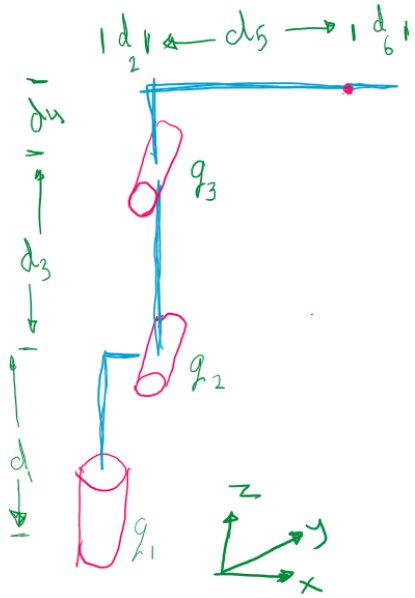



```
% Substitute angles in symbolical form an'''d convert to double
double(subs(FK, [q1 q2 q3 q4 q5 q6], q_test))
```

```
ans = 4×4
10^3 ×
    0.0001    0.0005   -0.0009   -0.7960
    0.0007    0.0006    0.0004    0.3587
    0.0007   -0.0007   -0.0003    1.0902
         0         0         0    0.0010
```

```
% disp('Test configuration')
% q_test = [pi/3 -pi/6 11 -1 -1 -1] % test configuration
% draw_robot(q_test,L)
% Substitute angles in symbolical form and convert to double
T=double(subs(FK, [q1 q2 q3 q4 q5 q6], q_test))
```

```
T = 4×4
10^3 ×
    0.0001    0.0005   -0.0009   -0.7960
    0.0007    0.0006    0.0004    0.3587
    0.0007   -0.0007   -0.0003    1.0902
         0         0         0    0.0010
```



Jacobian

Numerical derivatives

Forward kinematics

```
syms q1 q2 q3 q4 q5 q6 real

H=simplify(Tz(d1)*Rz(q(1))*Tx(d2)*Ry(q(2))*Tz(d3)*Ry(q(3))*Tz(d4)*Tx(d5)*Rx(q(4))*Ry(q(5))*Rx(q
```

```
H =
```

$$\begin{pmatrix} \cos(q_5)\,\sigma_9 - \sin(q_5)\,\sigma_8 & \sin(q_6)\,\sigma_3 - \cos(q_6)\,\sigma_5 \\ \sin(q_5)\,\sigma_6 - \cos(q_5)\,\sigma_7 & \cos(q_6)\,\sigma_4 - \sin(q_6)\,\sigma_2 \\ -\sin(q_2+q_3)\cos(q_5) - \cos(q_2+q_3)\cos(q_4)\sin(q_5) & \cos(q_2+q_3)\cos(q_6)\sin(q_4) - \sin(q_6)\,\sigma_1 \\ 0 & 0 \end{pmatrix} \quad -\cos(q_6$$

where

$$\sigma_1 = \sin(q_2+q_3)\sin(q_5) - \cos(q_2+q_3)\cos(q_4)\cos(q_5)$$

$$\sigma_2 = \cos(q_5)\,\sigma_6 + \sin(q_5)\,\sigma_7$$

$$\sigma_3 = \cos(q_5)\,\sigma_8 + \sin(q_5)\,\sigma_9$$

$$\sigma_4 = \cos(q_1)\cos(q_4) + \sin(q_4)\,\sigma_{10}$$

$$\sigma_5 = \cos(q_4)\sin(q_1) - \sin(q_4)\,\sigma_{11}$$

$$\sigma_6 = \cos(q_1)\sin(q_4) - \cos(q_4)\,\sigma_{10}$$

$$\sigma_7 = \sin(q_1)\sin(q_2)\sin(q_3) - \cos(q_2)\cos(q_3)\sin(q_1)$$

$$\sigma_8 = \sin(q_1)\sin(q_4) + \cos(q_4)\,\sigma_{11}$$

$$\sigma_9 = \cos(q_1)\cos(q_2)\cos(q_3) - \cos(q_1)\sin(q_2)\sin(q_3)$$

$$\sigma_{10} = \cos(q_2)\sin(q_1)\sin(q_3) + \cos(q_3)\sin(q_1)\sin(q_2)$$

$$\sigma_{11} = \cos(q_1)\cos(q_2)\sin(q_3) + \cos(q_1)\cos(q_3)\sin(q_2)$$

where

$$H = \begin{bmatrix} R & T \\ 0 & 1 \end{bmatrix}$$

```
R = simplify(H(1:3,1:3))
```

```
R =
```

$$\begin{pmatrix} \cos(q_5)\,\sigma_9 - \sin(q_5)\,\sigma_8 & \sin(q_6)\,\sigma_3 - \cos(q_6)\,\sigma_5 \\ \sin(q_5)\,\sigma_6 - \cos(q_5)\,\sigma_7 & \cos(q_6)\,\sigma_4 - \sin(q_6)\,\sigma_2 \\ -\sin(q_2+q_3)\cos(q_5) - \cos(q_2+q_3)\cos(q_4)\sin(q_5) & \cos(q_2+q_3)\cos(q_6)\sin(q_4) - \sin(q_6)\,\sigma_1 \quad -\cos(q_6 \end{pmatrix}$$

where

$$\sigma_1 = \sin(q_2+q_3)\sin(q_5) - \cos(q_2+q_3)\cos(q_4)\cos(q_5)$$

$$\sigma_2 = \cos(q_5)\,\sigma_6 + \sin(q_5)\,\sigma_7$$

$$\sigma_3 = \cos(q_5)\,\sigma_8 + \sin(q_5)\,\sigma_9$$

$$\sigma_4 = \cos(q_1)\cos(q_4) + \sin(q_4)\,\sigma_{10}$$

$$\sigma_5 = \cos(q_4)\sin(q_1) - \sin(q_4)\,\sigma_{11}$$

$$\sigma_6 = \cos(q_1)\sin(q_4) - \cos(q_4)\,\sigma_{10}$$

$$\sigma_7 = \sin(q_1)\sin(q_2)\sin(q_3) - \cos(q_2)\cos(q_3)\sin(q_1)$$

$$\sigma_8 = \sin(q_1)\sin(q_4) + \cos(q_4)\,\sigma_{11}$$

$$\sigma_9 = \cos(q_1)\cos(q_2)\cos(q_3) - \cos(q_1)\sin(q_2)\sin(q_3)$$

$$\sigma_{10} = \cos(q_2)\sin(q_1)\sin(q_3) + \cos(q_3)\sin(q_1)\sin(q_2)$$

$$\sigma_{11} = \cos(q_1)\cos(q_2)\sin(q_3) + \cos(q_1)\cos(q_3)\sin(q_2)$$

```
% forward kinematics
H =Tz(d1)*Rz(q(1))*Tx(d2)*Ry(q(2))*Tz(d3)*Ry(q(3))*Tz(d4)*Tx(d5)*Rx(q(4))*Ry(q(5))*Rx(q(6))*Tx(
H=simplify(H);
% extract rotation matrix
R = simplify(H(1:3,1:3));
% diff by q1
Td=Tz(d1)*Rzd(q(1))*Tx(d2)*Ry(q(2))*Tz(d3)*Ry(q(3))*Tz(d4)*Tx(d5)*Rx(q(4))*Ry(q(5))*Rx(q(6))*Tx
    [R^-1 zeros(3,1);0 0 0 1];
% extract 6 components from 4x4 Td matrix to Jacobian 1st column
J1 = [Td(1,4), Td(2,4), Td(3,4), Td(3,2), Td(1,3), Td(2,1)]' ;
% diff by q2
Td=Tz(d1)*Rz(q(1))*Tx(d2)*Ryd(q(2))*Tz(d3)*Ry(q(3))*Tz(d4)*Tx(d5)*Rx(q(4))*Ry(q(5))*Rx(q(6))*Tx
    [R^-1 zeros(3,1);0 0 0 1];
% extract 6 components from 4x4 Td matrix to Jacobian 2nd column
J2 = [Td(1,4), Td(2,4), Td(3,4), Td(3,2), Td(1,3), Td(2,1)]' ;
% diff by q3
Td=Tz(d1)*Rz(q(1))*Tx(d2)*Ry(q(2))*Tz(d3)*Ryd(q(3))*Tz(d4)*Tx(d5)*Rx(q(4))*Ry(q(5))*Rx(q(6))*Tx
    [R^-1 zeros(3,1);0 0 0 1];
```

```
% extract 6 components from 4x4 Td matrix to Jacobian 3rd column
J3 = [Td(1,4), Td(2,4), Td(3,4), Td(3,2), Td(1,3), Td(2,1)]' ;
% diff by q4
Td=Tz(d1)*Rz(q(1))*Tx(d2)*Ry(q(2))*Tz(d3)*Ry(q(3))*Tz(d4)*Tx(d5)*Rxd(q(4))*Ry(q(5))*Rx(q(6))*T>
    [R^-1 zeros(3,1);0 0 0 1];
% extract 6 components from 4x4 Td matrix to Jacobian 4th column
J4 = [Td(1,4), Td(2,4), Td(3,4), Td(3,2), Td(1,3), Td(2,1)]' ;
Td=Tz(d1)*Rz(q(1))*Tx(d2)*Ry(q(2))*Tz(d3)*Ry(q(3))*Tz(d4)*Tx(d5)*Rx(q(4))*Ryd(q(5))*Rx(q(6))*T>
    [R^-1 zeros(3,1);0 0 0 1];
% extract 6 components from 4x4 Td matrix to Jacobian 5th column
J5 = [Td(1,4), Td(2,4), Td(3,4), Td(3,2), Td(1,3), Td(2,1)]' ;
Td=Tz(d1)*Rz(q(1))*Tx(d2)*Ry(q(2))*Tz(d3)*Ry(q(3))*Tz(d4)*Tx(d5)*Rx(q(4))*Ry(q(5))*Rxd(q(6))*T>
    [R^-1 zeros(3,1);0 0 0 1];
% extract 6 components from 4x4 Td matrix to Jacobian 6th column
J6 = [Td(1,4), Td(2,4), Td(3,4), Td(3,2), Td(1,3), Td(2,1)]' ;
% Full Jacobian 6x6
Jq1 = [simplify(J1), simplify(J2), simplify(J3), simplify(J4), simplify(J5), simplify(J6)]
```

```
Jq1 =
```

$$\begin{pmatrix} 515\sin(q_1)\sin(q_2)\sin(q_3) - 560\sin(q_1)\sin(q_2) - 25\sin(q_1) - 90\cos(q_2+q_3)\cos(q_5)\sin(q_1) - 515\,cc \\ 25\cos(q_1) + 560\cos(q_1)\sin(q_2) - 90\sin(q_1)\sin(q_4)\sin(q_5) + 90\cos(q_2+q_3)\cos(q_1)\cos(q_5) + 515\cos \end{pmatrix}$$

where

$$\sigma_1 = \cos(q_2)\sin(q_1)\sin(q_3) + \cos(q_3)\sin(q_1)\sin(q_2)$$

$$\sigma_2 = \cos(q_1)\cos(q_2)\sin(q_3) + \cos(q_1)\cos(q_3)\sin(q_2)$$

$$\sigma_3 = 90\cos(q_2+q_3)\cos(q_5)$$

$$\sigma_4 = 25\sin(q_2+q_3)$$

$$\sigma_5 = 515\cos(q_2+q_3)$$

$$\sigma_6 = \cos(q_4)\sin(q_1)$$

$$\sigma_7 = \cos(q_1)\cos(q_4)$$

$$\sigma_8 = \sigma_{16} - \sigma_{17} - 112\cos(q_2) + \sigma_{15} + \sigma_{14} + \sigma_{13} + \sigma_{12} + \sigma_{11} - \sigma_{10}$$

$$\sigma_9 = \sigma_{16} - \sigma_{17} + \sigma_{15} + \sigma_{14} + \sigma_{13} + \sigma_{12} + \sigma_{11} - \sigma_{10}$$

$$\sigma_{10} = 18\cos(q_4)\sin(q_2)\sin(q_3)\sin(q_5)$$

$$\sigma_{11} = 18\cos(q_2)\cos(q_3)\cos(q_4)\sin(q_5)$$

$$\sigma_{12} = 18\cos(q_3)\cos(q_5)\sin(q_2)$$

$$\sigma_{13} = 18\cos(q_2)\cos(q_5)\sin(q_3)$$

$$\sigma_{14} = 5\sin(q_2)\sin(q_3)$$

$$\sigma_{15} = 103\cos(q_3)\sin(q_2)$$

$$\sigma_{16} = 103\cos(q_2)\sin(q_3)$$

$$\sigma_{17} = 5\cos(q_2)\cos(q_3)$$

```
singularity=simplify(det(Jq1))
```

singularity

$$= -14000\sin(q_5)\left(25\cos(q_2)+515\cos(q_3)-515\sin(q_2)+25\sin(q_3)+11536\cos(q_3)\sin(q_2)+560\sin(q_2\right.$$