Homework 5: RRR elbow type Robot

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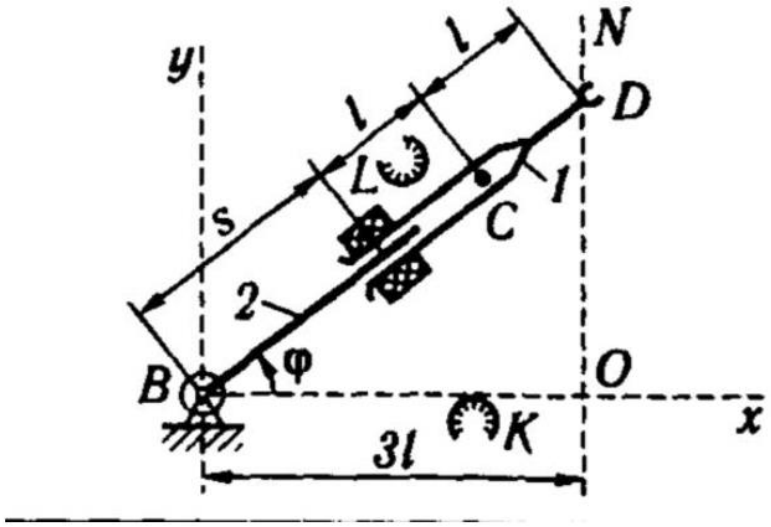
Github Link:

https://github.com/mostafa-metwaly/DoNRs-HW6

Robot:



m1 = 2 kg (C – center mass)
m2 = 2 kg (B – center mass)
I1 = 1 kg*m^2
I2 = 2 kg*m^2
L = 0,2 m

Solving direct dynamic problem using Lagrange-Euler method

Tasks:

1. Solve direct dynamic problem using Lagrange-Euler method. Suppose that robot located in a vertical plane, it means that we have gravity force with $g = 9.81\frac{m}{c^2}$.
2. Get the dynamic equation in matrix form:

$$M(q) * \ddot{q} + C(q, \dot{q}) * \dot{q} + G(q) = \tau(t)$$

   Where q is a vector of yours generalize coordinates.
3. Apply any force and torque functions $f(t)$ and $\tau(t)$ for the dynamic model. As result you will get new $q(t), \dot{q}(t), \ddot{q}(t)$ function.

```
clear all;close all;clc;


syms m1 m2 q1 q2 dq1 dq2 ddq1 ddq2 I1 I2 L1 L2 d1 d2 g real
```

```
% m1=2;
% m2=2;
% I1=1;
% I2=2;
% L1=0.2;
% g=9.81;
```

Skew theory

First find origin of each joint

FK:

```
T00= eye(4)
```

```
T00 = 4×4
     1     0     0     0
     0     1     0     0
     0     0     1     0
     0     0     0     1
```

```
T01= Rz(q1)
```

T01 =

$$\begin{pmatrix} \cos(q_1) & -\sin(q_1) & 0 & 0 \\ \sin(q_1) & \cos(q_1) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
T0c1= Rz(q1)
```

T0c1 =

$$\begin{pmatrix} \cos(q_1) & -\sin(q_1) & 0 & 0 \\ \sin(q_1) & \cos(q_1) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
T02= Rz(q1)*Tx(q2)*Tx(L2)
```

T02 =

$$\begin{pmatrix} \cos(q_1) & -\sin(q_1) & 0 & L_2\cos(q_1)+q_2\cos(q_1) \\ \sin(q_1) & \cos(q_1) & 0 & L_2\sin(q_1)+q_2\sin(q_1) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
T0c2= Rz(q1)*Tx(q2)*Tx(d2)
```

T0c2 =

$$\begin{pmatrix} \cos(q_1) & -\sin(q_1) & 0 & d_2\cos(q_1)+q_2\cos(q_1) \\ \sin(q_1) & \cos(q_1) & 0 & d_2\sin(q_1)+q_2\sin(q_1) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Origins as position component of FK

```
O0 = T00(1:3,4)
```

```
O0 = 3×1
     0
     0
     0
```

```
O1 = T01(1:3,4)
```

O1 =

$$\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

```
Oc1 = T0c1(1:3,4)
```

Oc1 =

$$\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

```
O2 = simplify(T02(1:3,4))
```

O2 =

$$\begin{pmatrix} \cos(q_1)\ (L_2+q_2) \\ \sin(q_1)\ (L_2+q_2) \\ 0 \end{pmatrix}$$

```
Oc2 = simplify(T0c2(1:3,4))
```

Oc2 =

$$\begin{pmatrix} \cos(q_1)\ (d_2+q_2) \\ \sin(q_1)\ (d_2+q_2) \\ 0 \end{pmatrix}$$

Find rotation (translation in case of prismatic joint) axis Z from transformation, note the coloumn! Its should correspond to the joint axis

```
Z0 = T00(1:3,3) % 3rd coloumn corresponds to Rz
```

```
Z0 = 3×1
     0
```

```
     0
     1
```

```
Zc1 = T0c1(1:3,3) % 3rd coloumn corresponds to Rz
```

```
Zc1 =
```

$$\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

```
Zc2 = T0c2(1:3,1) % 1st coloumn corresponds to Tx
```

```
Zc2 =
```

$$\begin{pmatrix} \cos(q_1) \\ \sin(q_1) \\ 0 \end{pmatrix}$$

```
R00=T00(1:3,1:3)
```

```
R00 = 3×3
     1     0     0
     0     1     0
     0     0     1
```

```
R0c1=T0c1(1:3,1:3)
```

```
R0c1 =
```

$$\begin{pmatrix} \cos(q_1) & -\sin(q_1) & 0 \\ \sin(q_1) & \cos(q_1) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

```
R0c2=T0c2(1:3,1:3)
```

```
R0c2 =
```

$$\begin{pmatrix} \cos(q_1) & -\sin(q_1) & 0 \\ \sin(q_1) & \cos(q_1) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Full Jacobian

```
Zer = [0 0 0]';

Jv1 = [cross(Zc1,(Oc1-O0)), Zer]
```

```
Jv1 =
```

$$\begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$$

```
Jv2 = [ cross(Zc1,(Oc2-O0)),Zc2]
```

Jv2 =

$$\begin{pmatrix} -\sin(q_1)\ (d_2+q_2) & \cos(q_1) \\ \cos(q_1)\ (d_2+q_2) & \sin(q_1) \\ 0 & 0 \end{pmatrix}$$

```
Jw1 = [Zc1, Zer]
```

Jw1 =

$$\begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \end{pmatrix}$$

```
Jw2 = [Zc1, Zer]
```

Jw2 =

$$\begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \end{pmatrix}$$

Euler-Lagrange Equation

```
q = [q1;q2];
```

Kinetic Energy Equation

```
D1 = m1*Jv1'*Jv1 + Jw1'*R0c1*I1*R0c1'*Jw1
```

D1 =

$$\begin{pmatrix} I_1 & 0 \\ 0 & 0 \end{pmatrix}$$

```
D2 = m2*Jv2'*Jv2 + Jw2'*R0c2*I2*R0c2'*Jw2
```

D2 =

$$\begin{pmatrix} I_2 + m_2\cos(q_1)^2\ (d_2+q_2)^2 + m_2\sin(q_1)^2\ (d_2+q_2)^2 & 0 \\ 0 & m_2\cos(q_1)^2 + m_2\sin(q_1)^2 \end{pmatrix}$$

```
D=D1+D2;

D=simplify(D)
```

D =

$$\begin{pmatrix} m_2 d_2^2 + 2 m_2 d_2 q_2 + m_2 q_2^2 + I_1 + I_2 & 0 \\ 0 & m_2 \end{pmatrix}$$

Potentioal Energy Equation

```
P1 = 0;
P2 = m2*g*((d2 +q2)*sin(q1));
P = P1+P2
```

P = $g\, m_2 \sin(q_1)\ (d_2 + q_2)$

```
G1 = diff(P, q1)
```

G1 = $g\, m_2 \cos(q_1)\ (d_2 + q_2)$

```
G2 = diff(P, q2)
```

G2 = $g\, m_2 \sin(q_1)$

```
G = [G1; G2]
```

G =

$$\begin{pmatrix} g\, m_2 \cos(q_1)\ (d_2 + q_2) \\ g\, m_2 \sin(q_1) \end{pmatrix}$$

```
dq = [dq1; dq2];
ddq = [ddq1; ddq2];
```

calculating coriolios force

```
C = Coriolis(D,q,dq,2);
C=simplify(C)
```

C =

$$\begin{pmatrix} \mathrm{dq}_2\, m_2\ (d_2 + q_2) & \mathrm{dq}_1\, m_2\ (d_2 + q_2) \\ -\mathrm{dq}_1\, m_2\ (d_2 + q_2) & 0 \end{pmatrix}$$

Direct Dynamic equation

Equation for Torque:

$$D(q_1, q_2) \begin{bmatrix} \ddot{q_1} \\ \ddot{q_2} \end{bmatrix} + C\left(q_1, q_2, \dot{q_1}, \dot{q_2}\right) \begin{bmatrix} \dot{q_1} \\ \dot{q_2} \end{bmatrix} + G(q_1, q_2) = \begin{bmatrix} \tau \\ F \end{bmatrix}$$

```
tor =simplify( D*ddq+C*dq+G)
```

tor =

$$\begin{pmatrix} \mathrm{ddq}_1\,\left(m_2\,d_2^2+2\,m_2\,d_2\,q_2+m_2\,q_2^2+I_1+I_2\right)+g\,m_2\cos(q_1)\,\left(d_2+q_2\right)+2\,\mathrm{dq}_1\,\mathrm{dq}_2\,m_2\,\left(d_2+q_2\right) \\ -m_2\,\left(d_2+q_2\right)\,{\mathrm{dq}_1}^2+\mathrm{ddq}_2\,m_2+g\,m_2\sin(q_1) \end{pmatrix}$$

```
D(q1,q2) = subs(D,{m1, m2, I1, I2, L2, d2},{2 2 1 2 0.2 0.1})
```

D(q1, q2) =

$$\begin{pmatrix} 2\,q_2^2+0.4000\,q_2+3.0200 & 0 \\ 0 & 2 \end{pmatrix}$$

```
C(q1,q2,dq1,dq2) = subs(C*dq ,{m1, m2, I1, I2, L2, d2},{2 2 1 2 0.2 0.1})
```

C(q1, q2, dq1, dq2) =

$$\begin{pmatrix} 4\,\mathrm{dq}_1\,\mathrm{dq}_2\,\left(q_2+0.1000\right) \\ -2\,{\mathrm{dq}_1}^2\,\left(q_2+0.1000\right) \end{pmatrix}$$

```
G(q1,q2) = subs(G ,{m1, m2, I1, I2, L2, d2, g},{2 2 1 2 0.2 0.1 9.81})
```

G(q1, q2) =

$$\begin{pmatrix} 19.6200\cos(q_1)\,\left(q_2+0.1000\right) \\ 19.6200\sin(q_1) \end{pmatrix}$$

```
n=200;
t = 0:0.1:(0.1*(n-1));
q1_0 = deg2rad(90);
q2_0 = 0;
dq1_0 = 0;
dq2_0 = 0;
dt=0.01;
```

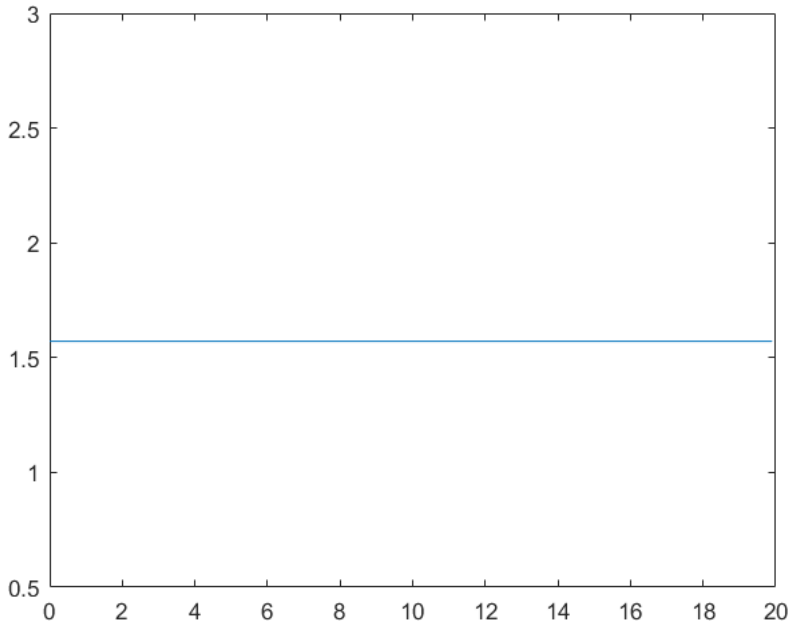
Adding forces and torques values

```
U = [ 0;  0];



for i = 1:n
    q1p(i)=q1_0;
    q2p(i)=q2_0;
    dq1p(i)=dq1_0;
    dq2p(i)=dq2_0;
```

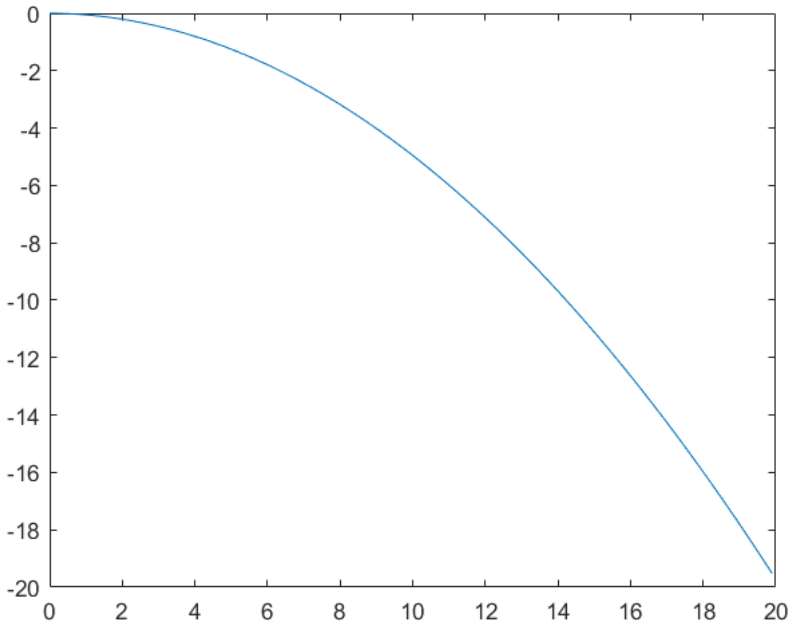
```
    ddq(:,i) = inv(D(q1_0, q2_0))*(U(:,1)-C(q1_0, q2_0,dq1_0,dq2_0)-
G(q1_0,q2_0));
    dq1_0=dq1p(i) + double(ddq(1)*dt);
    dq2_0=dq2p(i) + double(ddq(2)*dt);
    q1_0 = q1p(i) + dq1_0*dt;
    q2_0 = q2p(i) + dq2_0*dt;
end
```

Plotting the Position , velocity and acceleration of joints.

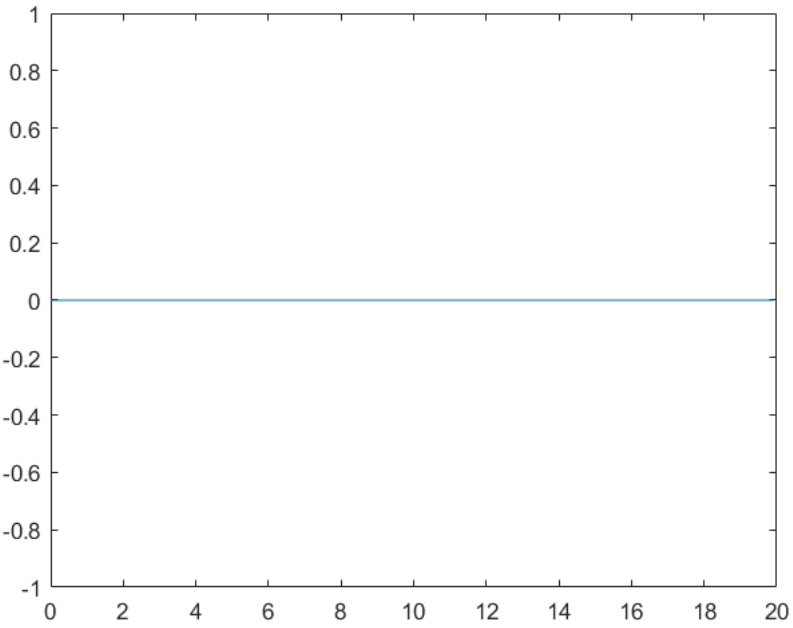
```
figure
plot(t,q1p)
```



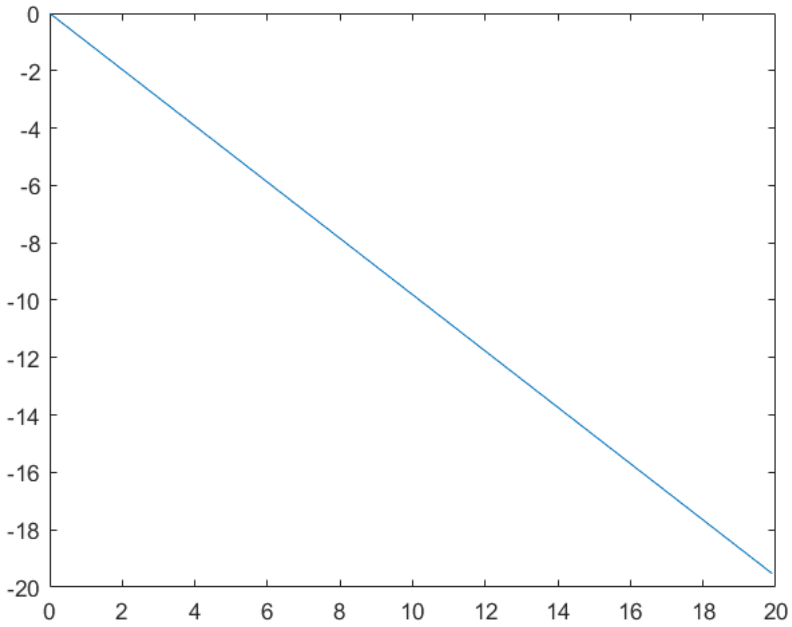

```
figure
plot(t,q2p)
```



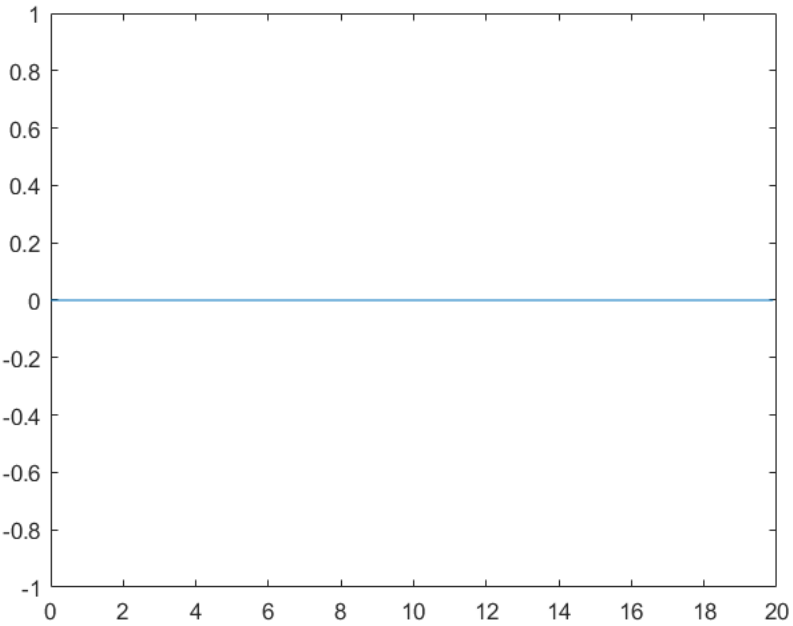

```
figure
plot(t,dq1p)
```



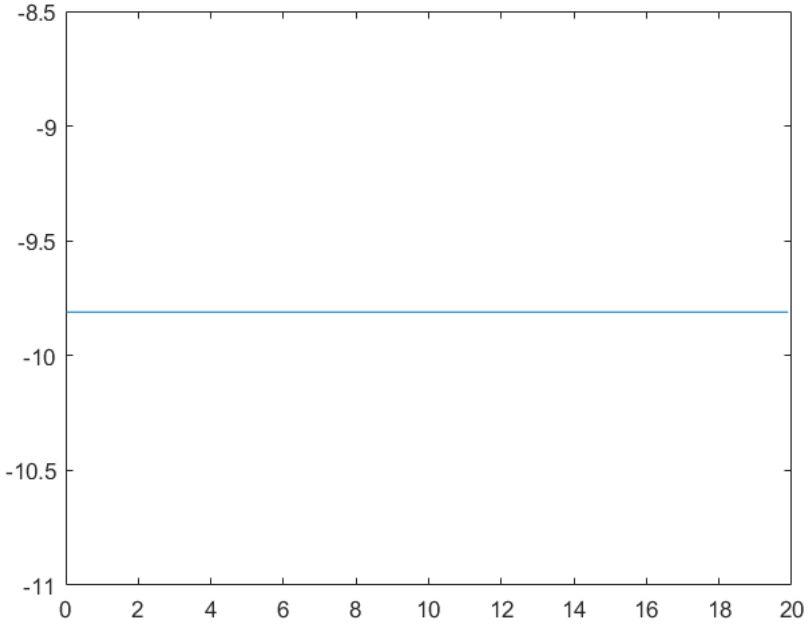

```
figure
plot(t,dq2p)
```



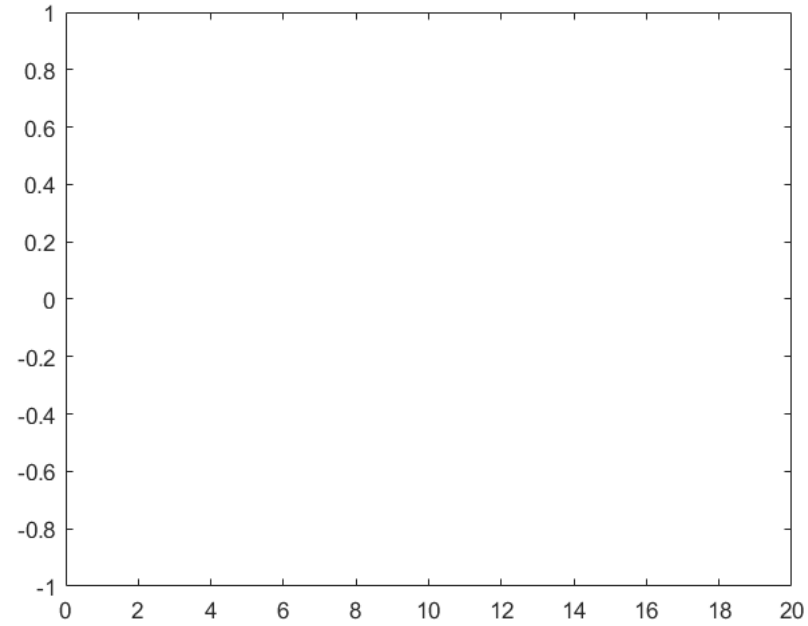

```
figure
plot(t,ddq(1,:))
```



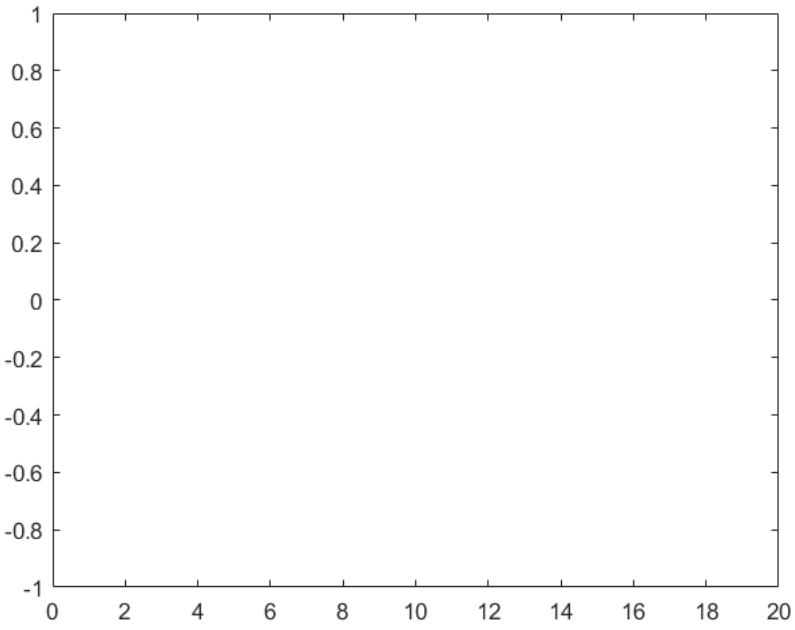

```
figure
plot(t,ddq(2,:))
```




```
figure
plot(t,U(1,:))
```

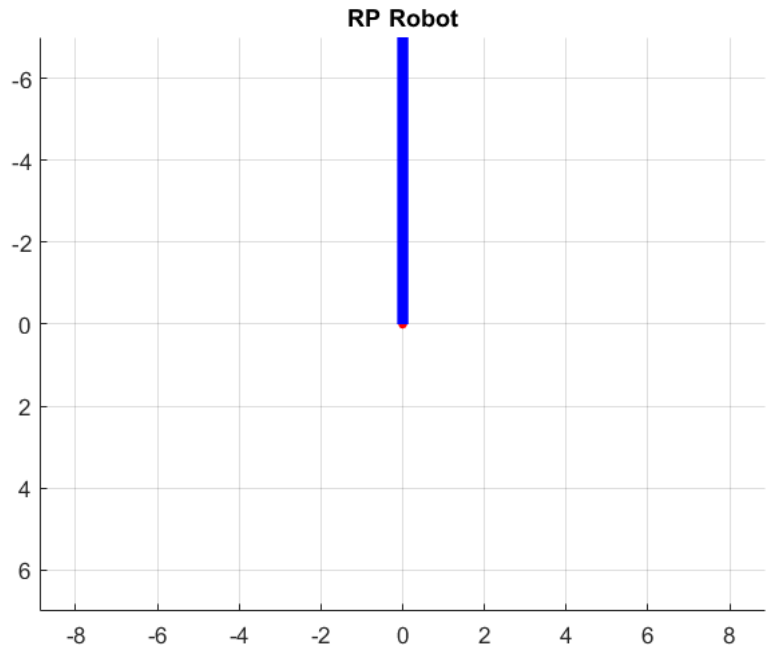



```
figure
plot(t,U(2,:))
```



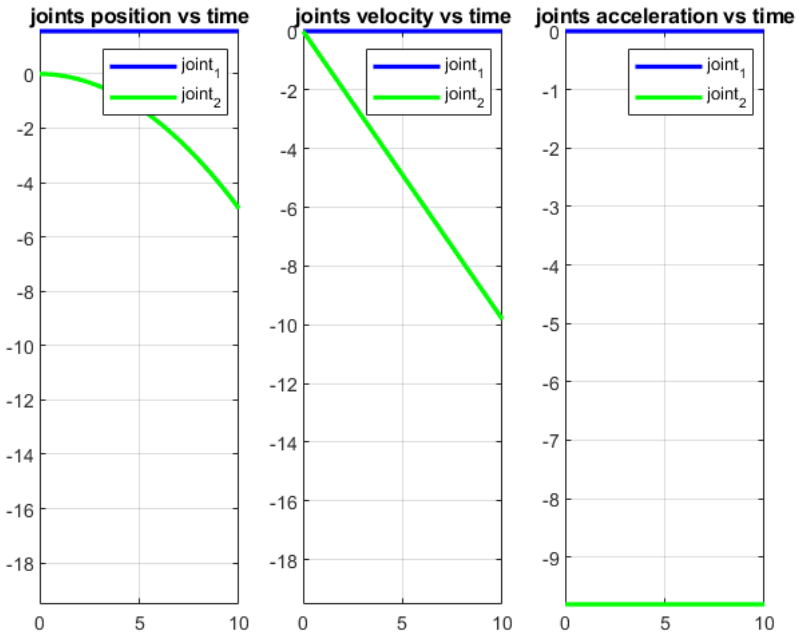
viewing the robot motion

```
Q=[q1p ;q2p];
Vel=[dq1p ; dq2p ];
Acc=[ddq(1,:); ddq(2,:) ];
figure
for i=1:1:length(Q)
   draw_myrobot( Q(:,i)')

   pause(0.01)
   cla
   end
 draw_myrobot( Q(:,i)')
```

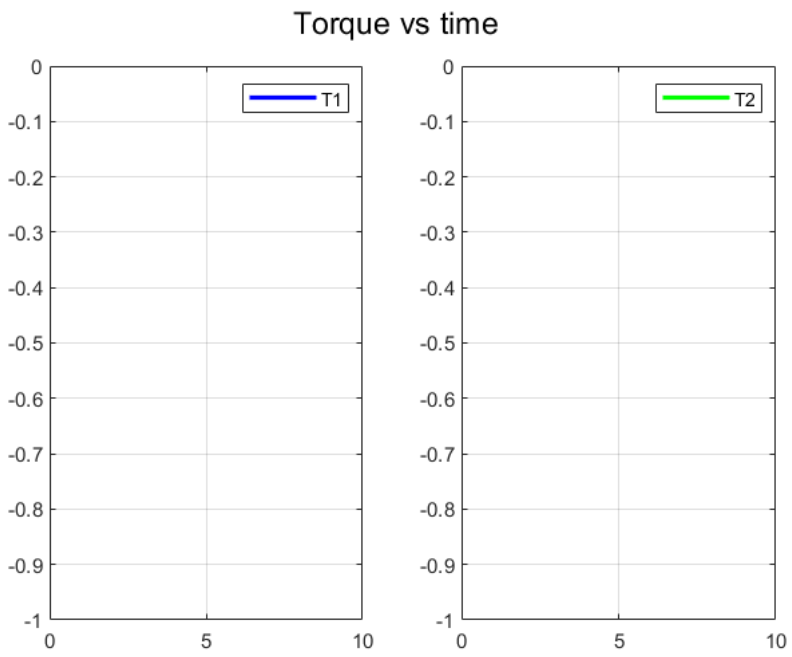



```
figure
subplot(1,3,1)
plot(t,Q(1,:),'b-','linewidth',2)
hold on
plot(t,Q(2,:),'g-','linewidth',2)
  grid on
title('joints position vs time')
legend('joint_1','joint_2' )
axis([0 10 -inf inf])

subplot(1,3,2)
plot(t,Vel(1,:),'b-','linewidth',2)
hold on
plot(t,Vel(2,:),'g-','linewidth',2)
 title('joints velocity vs time')
legend('joint_1','joint_2' )
grid on
axis([0 10 -inf inf])

subplot(1,3,3)
plot(t,Acc(1,:),'b-','linewidth',2)
hold on
plot(t,Acc(2,:),'g-','linewidth',2)
hold on
```

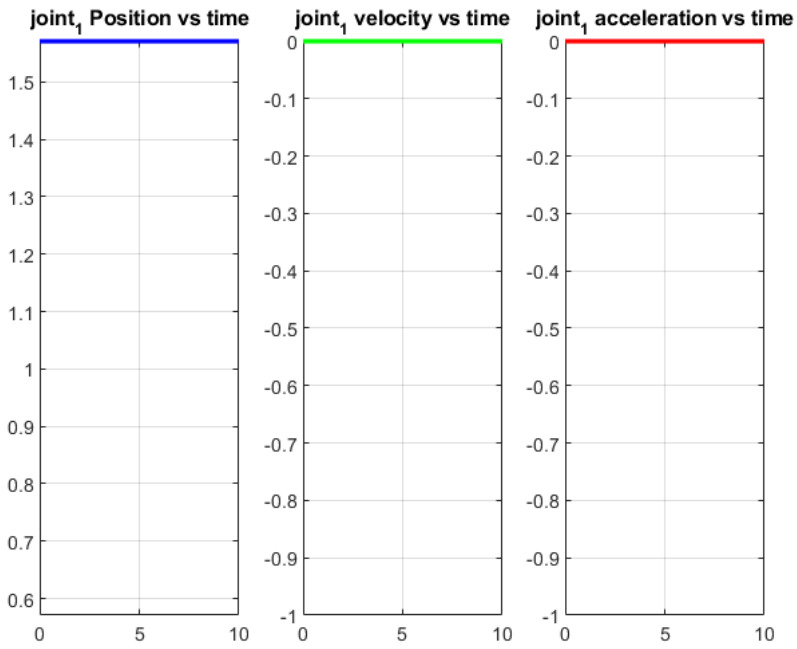
```
 title('joints acceleration vs time')
legend('joint_1','joint_2')
grid on
axis([0 10 -inf inf])
```



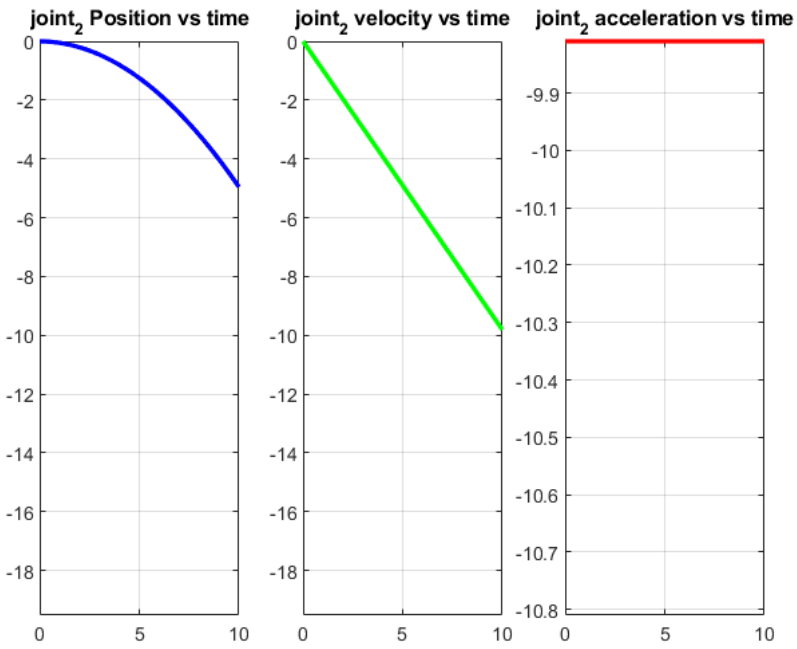

```
figure
subplot(1,2,1)
plot(t,U(1,:),'b-','linewidth',2)
hold on
legend('T1')
grid on
axis([0 10 -inf inf])
subplot(1,2,2)
plot(t,U(2,:),'g-','linewidth',2)
hold on
sgtitle('Torque vs time')
legend('T2')
grid on
axis([0 10 -inf inf])
```



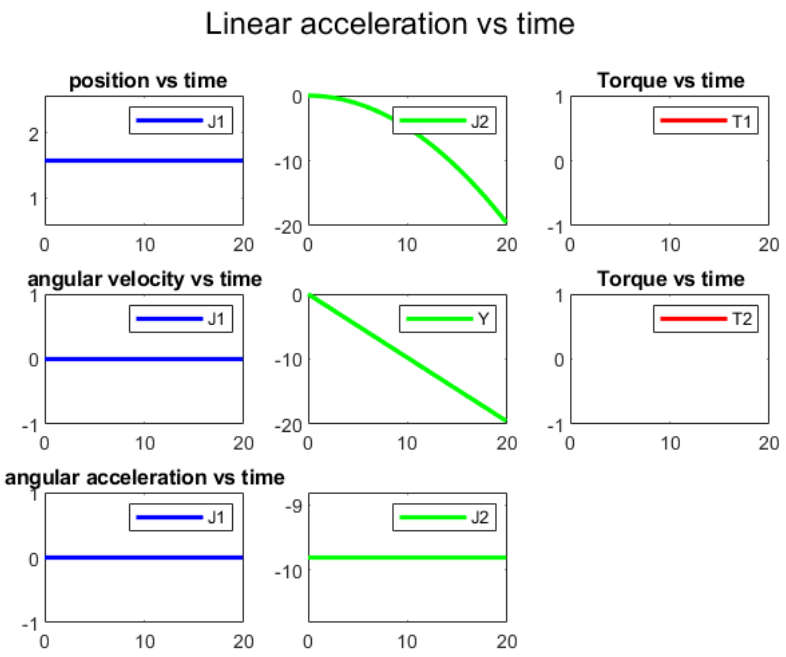

```
figure
hold on
subplot(1,3,1)
plot(t,Q(1,:),'b-','linewidth',2)
hold on
title('joint_1 Position vs time')
grid on
axis([0 10 -inf inf])
hold on
subplot(1,3,2)
plot(t,Vel(1,:),'g-','linewidth',2)
hold on
title('joint_1 velocity vs time')
grid on
axis([0 10 -inf inf])
subplot(1,3,3)
plot(t,Acc(1,:),'r-','linewidth',2)
title('joint_1 acceleration vs time')
grid on
axis([0 10 -inf inf])
```




```
figure

hold on
subplot(1,3,1)
plot(t,Q(2,:),'b-','linewidth',2)
hold on
title('joint_2 Position vs time')
grid on
axis([0 10 -inf inf])
hold on
subplot(1,3,2)
plot(t,Vel(2,:),'g-','linewidth',2)
hold on
title('joint_2 velocity vs time')
grid on
axis([0 10 -inf inf])
subplot(1,3,3)
plot(t,Acc(2,:),'r-','linewidth',2)
title('joint_2 acceleration vs time')
grid on
axis([0 10 -inf inf])
```




```
 figure;
 subplot(3,3,1);plot(t,Q(1,:),'b-','linewidth',2);title(' position vs
time');legend('J1');
 subplot(3,3,2);plot(t,Q(2,:),'g-','linewidth',2);sgtitle(' position vs
time');legend('J2');
 subplot(3,3,3);plot(t,U(1,:),'r-','linewidth',2 );title('Torque vs
time');legend('T1');

 subplot(3,3,4);plot(t,Vel(1,:),'b-','linewidth',2);title('angular velocity vs
time');legend('J1');
 subplot(3,3,5);plot(t,Vel(2,:),'g-','linewidth',2);sgtitle('Linear velocity vs
time');legend('Y');
 subplot(3,3,6);plot(t,U(2,:),'r-','linewidth',2 );title('Torque vs
time');legend('T2');

 subplot(3,3,7);plot(t,Acc(1,:),'b-','linewidth',2);title('angular acceleration
vs time');legend('J1');
 subplot(3,3,8);plot(t,Acc(2,:),'g-','linewidth',2);sgtitle('Linear acceleration
vs time');legend('J2');
  hold off
```



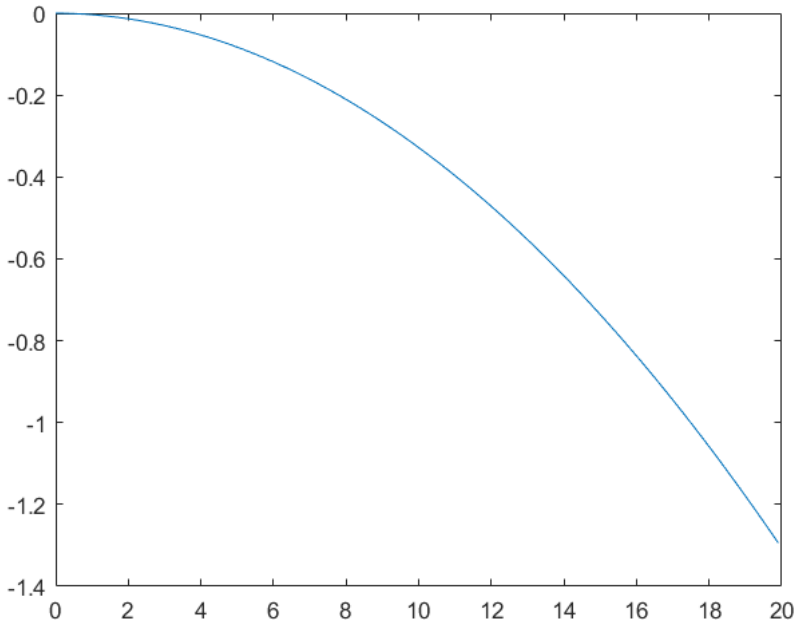
Adding forces and torques functions

```
n=200;
t = 0:0.1:(0.1*(n-1));
q1_0 = deg2rad( 0);
q2_0 = 0;
dq1_0 = 0;
dq2_0 = 0;
dt=0.01;

 U2 = [ 30*sin(t); 2*cos(t)];



for i = 1:n
    q1p(i)=q1_0;
    q2p(i)=q2_0;
    dq1p(i)=dq1_0;
    dq2p(i)=dq2_0;

    ddq(:,i) = inv(D(q1_0, q2_0))*(U2(:,i)-C(q1_0, q2_0,dq1_0,dq2_0)-
G(q1_0,q2_0));
    dq1_0=dq1p(i) + double(ddq(1)*dt);
    dq2_0=dq2p(i) + double(ddq(2)*dt);
    q1_0 = q1p(i) + dq1_0*dt;
```

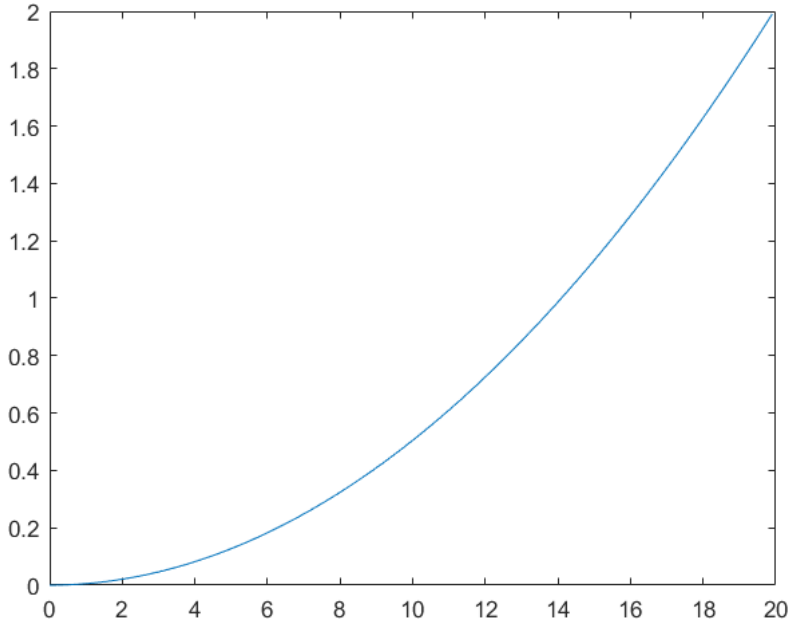
```
    q2_0 = q2p(i) + dq2_0*dt;
end
```

Plotting the Position , velocity and acceleration of joints.

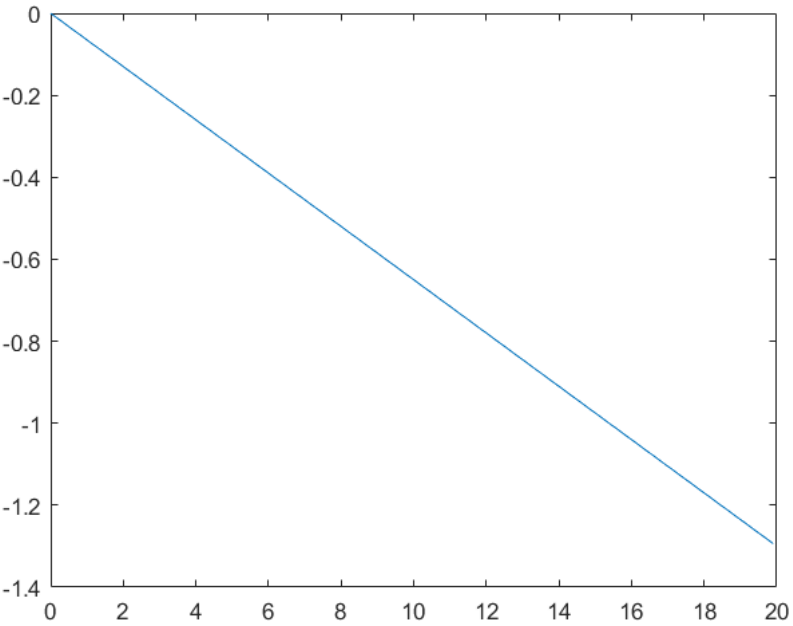
```
figure
plot(t,q1p)
```



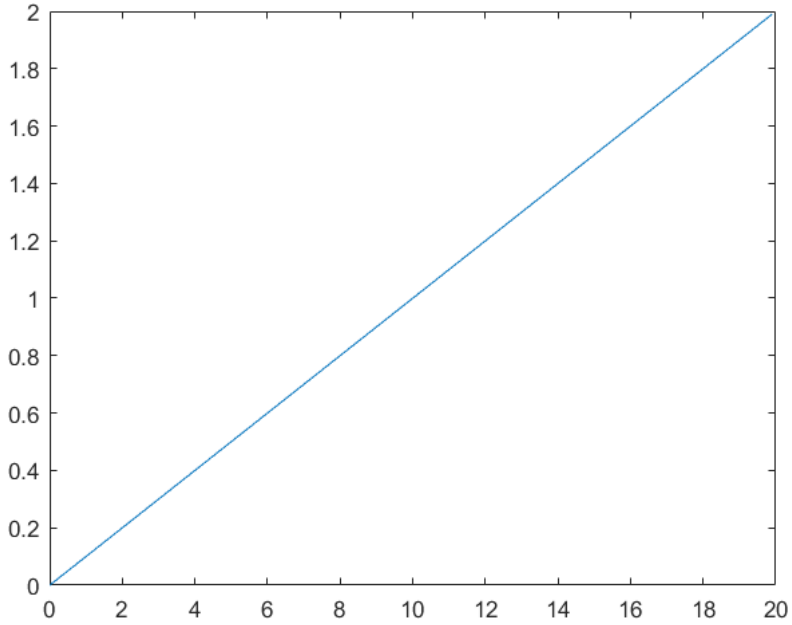

```
figure
plot(t,q2p)
```



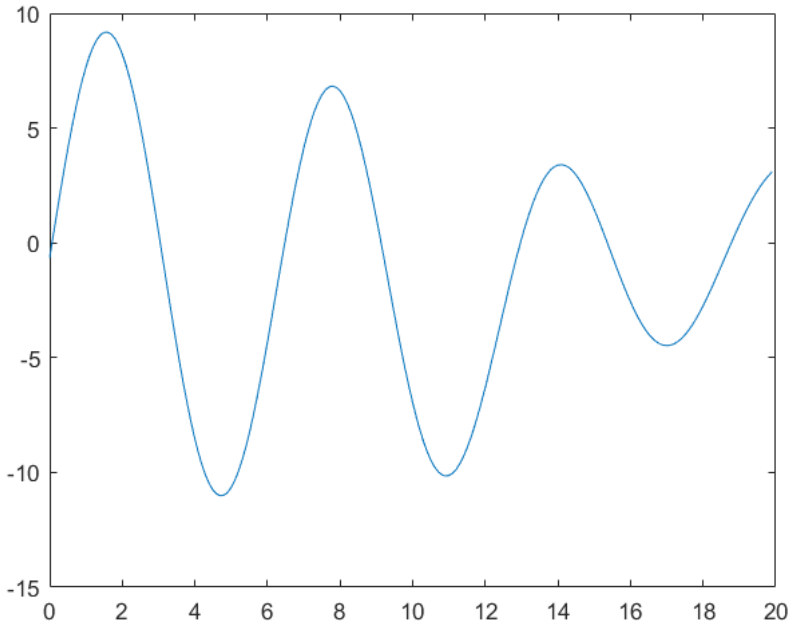

```
figure
plot(t,dq1p)
```



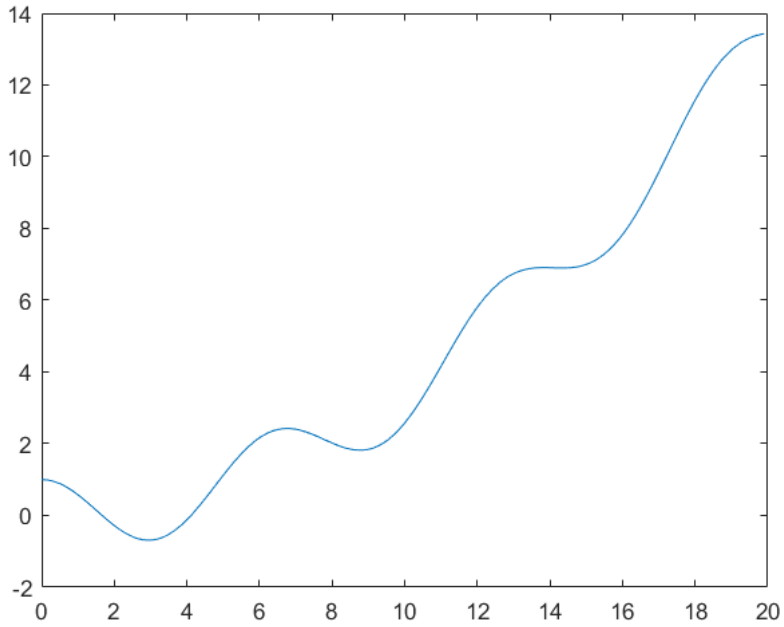

```
figure
plot(t,dq2p)
```



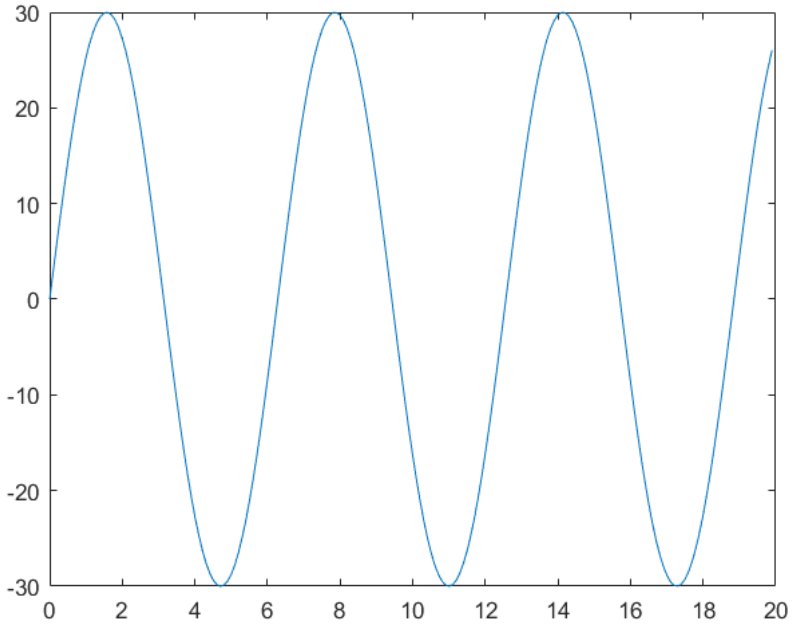

```
figure
plot(t,ddq(1,:))
```



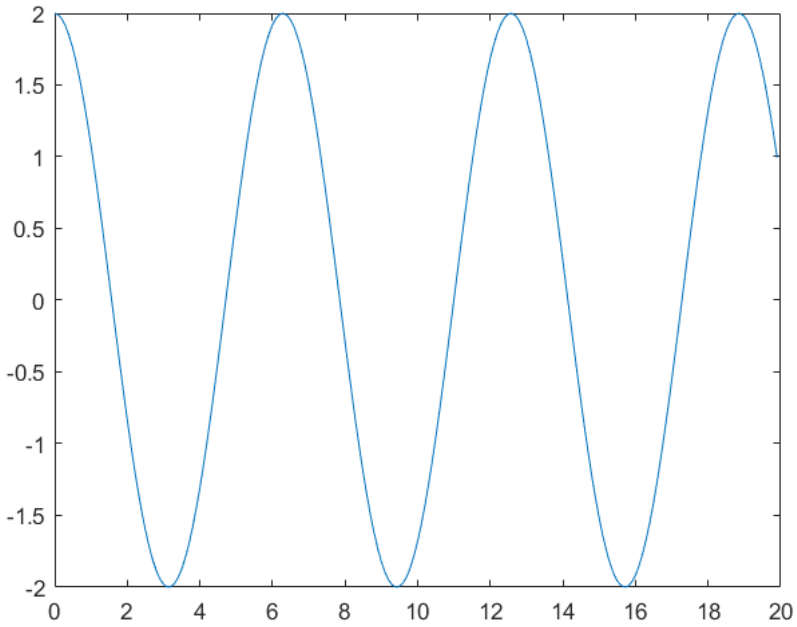

```
figure
plot(t,ddq(2,:))
```




```
figure
plot(t,U2(1,:))
```

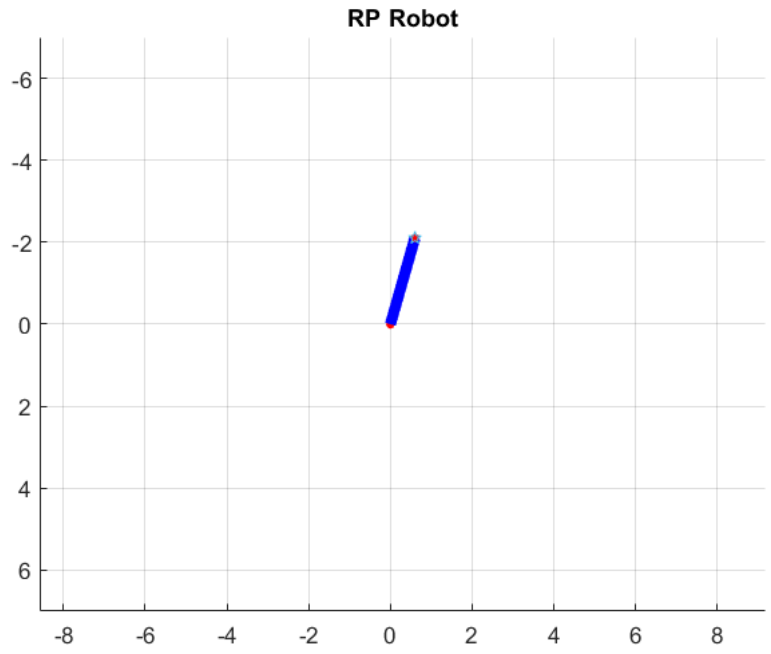



```
figure
plot(t,U2(2,:))
```



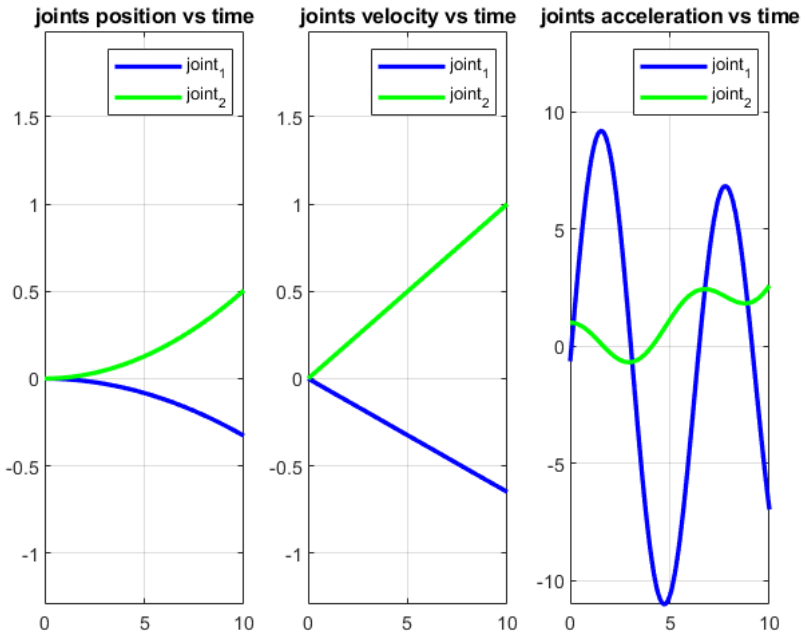
viewing the robot motion

```
Q=[q1p ;q2p];
Vel=[dq1p ; dq2p ];
Acc=[ddq(1,:); ddq(2,:) ];
figure
for i=1:1:length(Q)
   draw_myrobot( Q(:,i)')

   pause(0.01)
   cla
   end
 draw_myrobot( Q(:,i)')
```

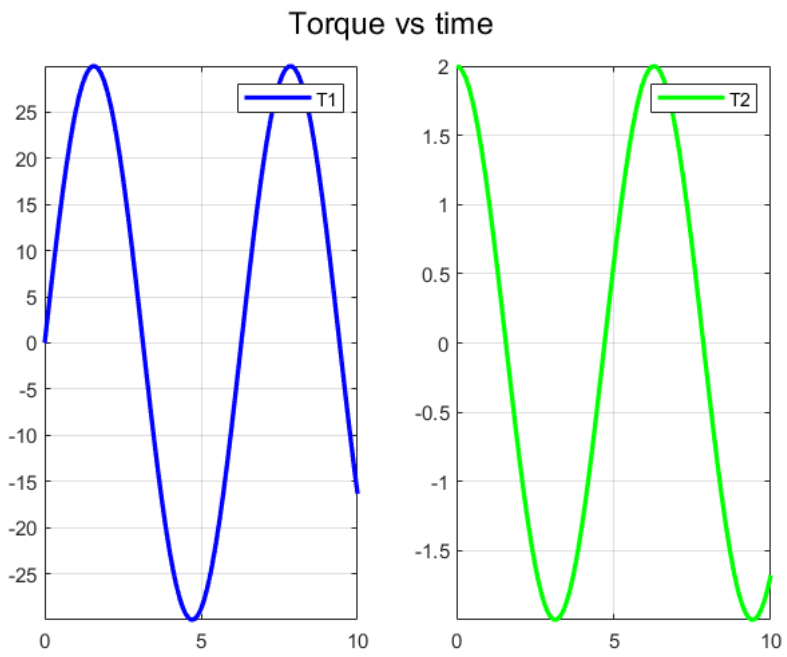



```
figure
subplot(1,3,1)
plot(t,Q(1,:),'b-','linewidth',2)
hold on
plot(t,Q(2,:),'g-','linewidth',2)
  grid on
title('joints position vs time')
legend('joint_1','joint_2' )
axis([0 10 -inf inf])

subplot(1,3,2)
plot(t,Vel(1,:),'b-','linewidth',2)
hold on
plot(t,Vel(2,:),'g-','linewidth',2)
 title('joints velocity vs time')
legend('joint_1','joint_2' )
grid on
axis([0 10 -inf inf])

subplot(1,3,3)
plot(t,Acc(1,:),'b-','linewidth',2)
hold on
plot(t,Acc(2,:),'g-','linewidth',2)
hold on
```

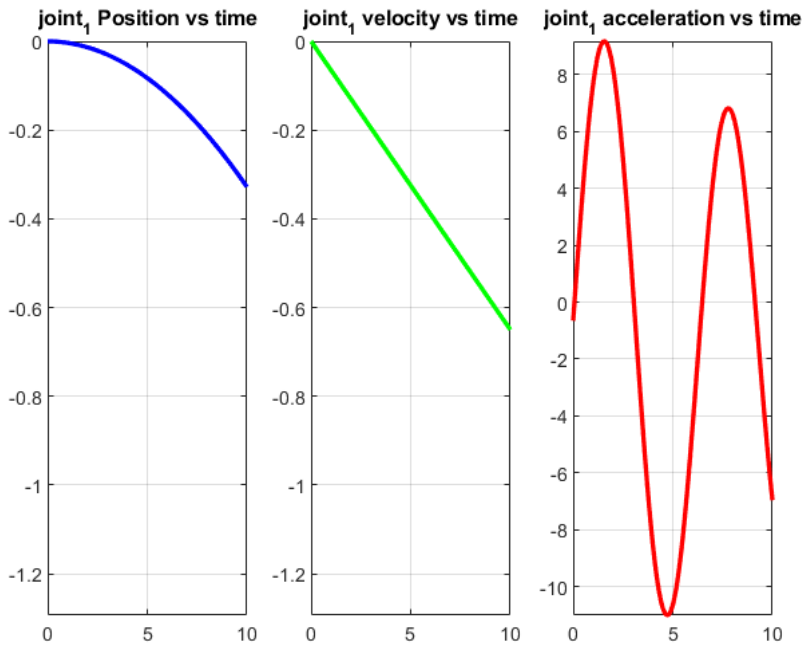
```
 title('joints acceleration vs time')
legend('joint_1','joint_2')
grid on
axis([0 10 -inf inf])
```



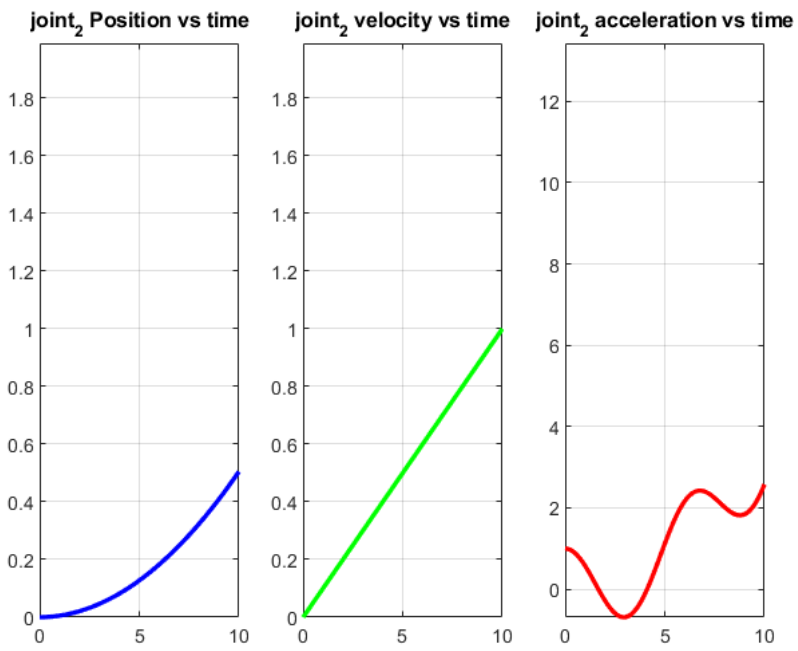

```
figure
subplot(1,2,1)
plot(t,U2(1,:),'b-','linewidth',2)
hold on
legend('T1')
grid on
axis([0 10 -inf inf])
subplot(1,2,2)
plot(t,U2(2,:),'g-','linewidth',2)
hold on
sgtitle('Torque vs time')
legend('T2')
grid on
axis([0 10 -inf inf])
```



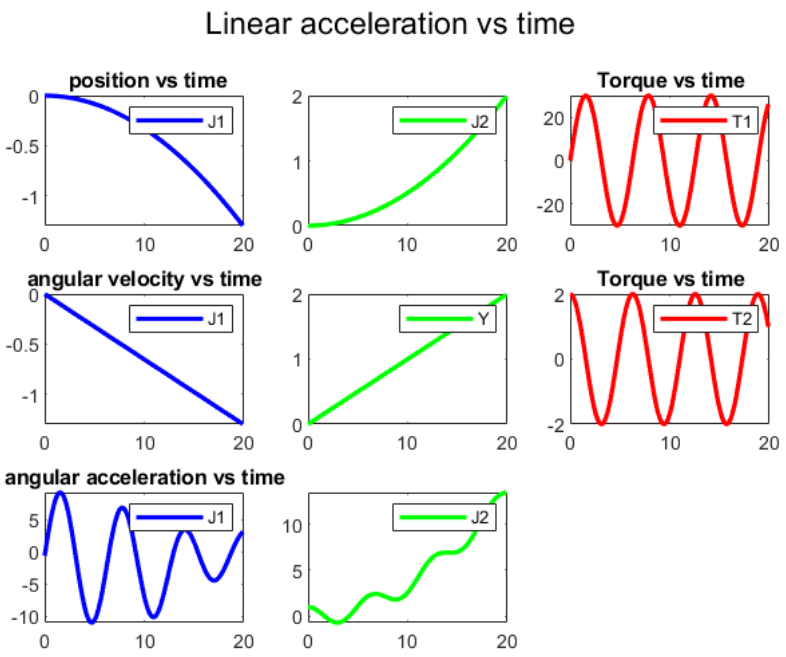

```
figure
hold on
subplot(1,3,1)
plot(t,Q(1,:),'b-','linewidth',2)
hold on
title('joint_1 Position vs time')
grid on
axis([0 10 -inf inf])
hold on
subplot(1,3,2)
plot(t,Vel(1,:),'g-','linewidth',2)
hold on
title('joint_1 velocity vs time')
grid on
axis([0 10 -inf inf])
subplot(1,3,3)
plot(t,Acc(1,:),'r-','linewidth',2)
title('joint_1 acceleration vs time')
grid on
axis([0 10 -inf inf])
```




```
figure

hold on
subplot(1,3,1)
plot(t,Q(2,:),'b-','linewidth',2)
hold on
title('joint_2 Position vs time')
grid on
axis([0 10 -inf inf])
hold on
subplot(1,3,2)
plot(t,Vel(2,:),'g-','linewidth',2)
hold on
title('joint_2 velocity vs time')
grid on
axis([0 10 -inf inf])
subplot(1,3,3)
plot(t,Acc(2,:),'r-','linewidth',2)
title('joint_2 acceleration vs time')
grid on
axis([0 10 -inf inf])
```




```
 figure;
 subplot(3,3,1);plot(t,Q(1,:),'b-','linewidth',2);title(' position vs
time');legend('J1');
 subplot(3,3,2);plot(t,Q(2,:),'g-','linewidth',2);sgtitle(' position vs
time');legend('J2');
 subplot(3,3,3);plot(t,U2(1,:),'r-','linewidth',2 );title('Torque vs
time');legend('T1');

 subplot(3,3,4);plot(t,Vel(1,:),'b-','linewidth',2);title('angular velocity vs
time');legend('J1');
 subplot(3,3,5);plot(t,Vel(2,:),'g-','linewidth',2);sgtitle('Linear velocity vs
time');legend('Y');
 subplot(3,3,6);plot(t,U2(2,:),'r-','linewidth',2 );title('Torque vs
time');legend('T2');

 subplot(3,3,7);plot(t,Acc(1,:),'b-','linewidth',2);title('angular acceleration
vs time');legend('J1');
 subplot(3,3,8);plot(t,Acc(2,:),'g-','linewidth',2);sgtitle('Linear acceleration
vs time');legend('J2');
  hold off
```


Linear acceleration vs time
position vs time
J1
J2
Torque vs time
T1
angular velocity vs time
J1
Y
Torque vs time
T2
angular acceleration vs time
J1
J2