Homework 4: RRP Robot

Table of Contents

Github Link:

https://github.com/mostafa-metwaly/DoNRs-HW4

q1, q2, d3 are joint space variables, px, py, pz are operational space variables and Parameters

d1, a2 are known

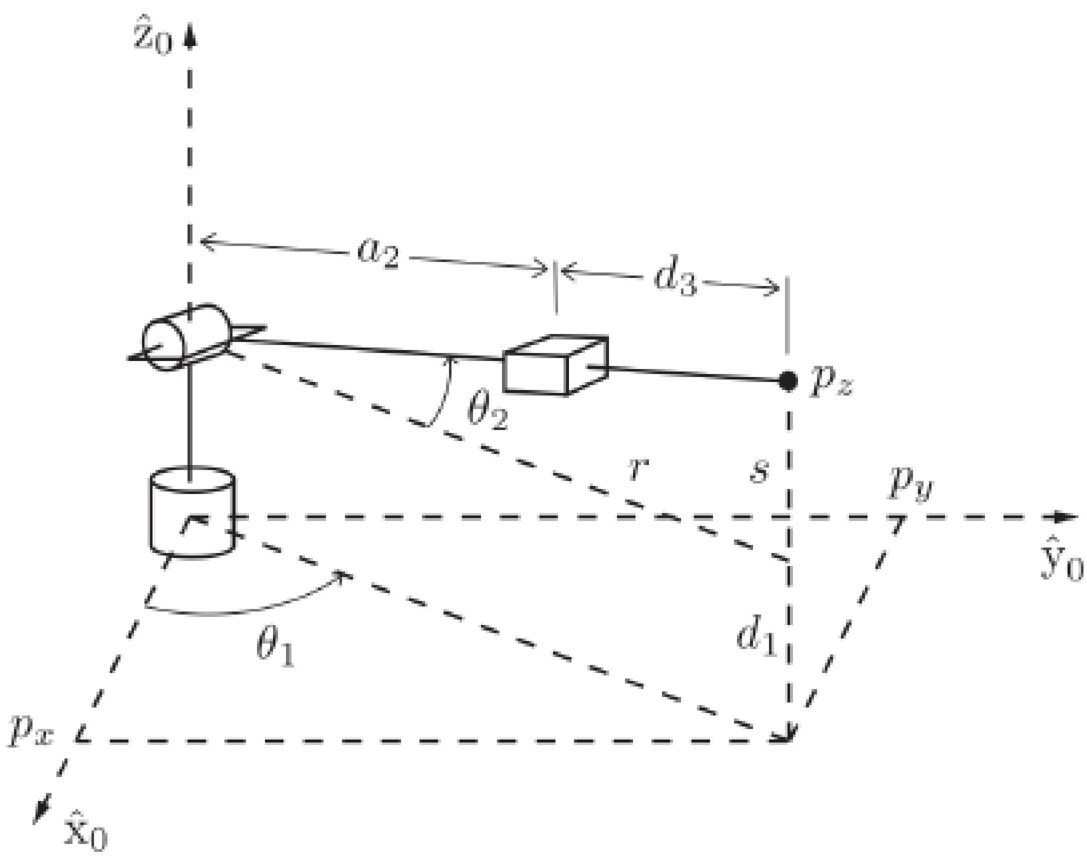
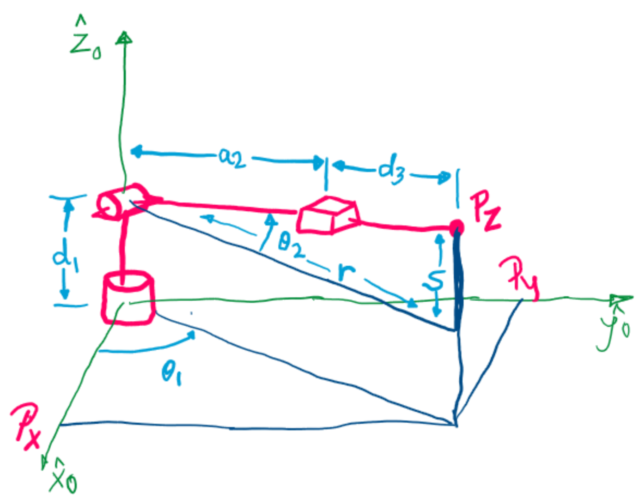


Figure 1: RRP robot.

1.Forward Kinematics

$$H = R_z(\mathrm{q1}) \cdot T_z(\mathrm{d1}) \cdot R_y(\mathrm{q2}) \cdot T_x(\mathrm{a2}) \cdot T_x(\mathrm{d3})$$



H =

$$\begin{pmatrix} \cos(q_1)\cos(q_2) & -\sin(q_1) & -\cos(q_1)\sin(q_2) & \cos(q_1)\cos(q_2)\ (a_2+d_3) \\ \cos(q_2)\sin(q_1) & \cos(q_1) & -\sin(q_1)\sin(q_2) & \cos(q_2)\sin(q_1)\ (a_2+d_3) \\ \sin(q_2) & 0 & \cos(q_2) & d_1 + a_2\sin(q_2) + d_3\sin(q_2) \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

R =

$$\begin{pmatrix} \cos(q_1)\cos(q_2) & -\sin(q_1) & -\cos(q_1)\sin(q_2) \\ \cos(q_2)\sin(q_1) & \cos(q_1) & -\sin(q_1)\sin(q_2) \\ \sin(q_2) & 0 & \cos(q_2) \end{pmatrix}$$

Px and Py depends on the value of r :

Px=r*cos(q1) , Py=r*sin(q1)

and r=(a2+d3)*cos(q2)

where for Pz value depends on s :

Pz=d1+s   , s=(a2+d3)*sin(q2)

Position of the End effector for RRP robot in operational space variables.can be described by the vector T

$$\begin{bmatrix} \text{Px} \\ \text{Py} \\ \text{Pz} \end{bmatrix} = \begin{bmatrix} r\cos(\text{q1}) \\ r\sin(\text{q1}) \\ \text{d1} + s \end{bmatrix} = \begin{bmatrix} \cos(q_1)\cos(q_2)\ (a_2 + d_3) \\ \cos(q_2)\sin(q_1)\ (a_2 + d_3) \\ d_1 + a_2\sin(q_2) + d_3\sin(q_2) \end{bmatrix}$$

T =

$$\begin{pmatrix} \cos(q_1)\cos(q_2)\ (a_2 + d_3) \\ \cos(q_2)\sin(q_1)\ (a_2 + d_3) \\ d_1 + a_2\sin(q_2) + d_3\sin(q_2) \end{pmatrix}$$

2. Inverse Kinematics

The robot has 3 joints 2 revolute and one prismatic

Px Py Pz - sets the position of EE we have already calculated it from vector T

q1 q2 d3 - sets the orientation of EE

there are only 2 solutions for solving this q1 and another 2 solutions for q2 and d3 can be found with one way the positive value where we neglected the negative value.

since we know vector T we can callculate the angles:

q1:

q11=atan2(Px,Py) or q12= π + atan2(Px,Py)

q1, 1st solution

q11 = $\text{angle}(\cos(q_2)\ (a_2 + d_3)\ (\cos(q_1) + \sin(q_1)\ \text{i}))$

q1, 2nd solution

q12 = $\pi + \text{angle}(\cos(q_2)\ (a_2 + d_3)\ (\cos(q_1) + \sin(q_1)\ \text{i}))$

we then could get the values of s, r :

$r = \sqrt{\text{Px}^2 + \text{Py}^2}$ and $s = \text{Pz} - \text{d1}$

q2:

q21=$\frac{\pi}{2}$+atan2(s,r) or q22= $\frac{3\pi}{2}$- atan2(s,r)

r = $\sqrt{-(a_2 + d_3)^2\ (\sin(q_2)^2 - 1)}$

s = $\sin(q_2)\ (a_2 + d_3)$

q2, for the 1st solution

q21 =

$$\frac{\pi}{2} + \text{atan2}\left(\sin(q_2)\ (a_2 + d_3), \sqrt{-(a_2 + d_3)^2\ (\sin(q_2)^2 - 1)}\right)$$

```
q2, for the 2nd solution
```

q22 =

$$\frac{3\pi}{2} - \mathrm{atan2}\left(\sin(q_2)\ (a_2 + d_3), \sqrt{-(a_2 + d_3)^2\ \left(\sin(q_2)^2 - 1\right)}\right)$$

d3 can be found by:

$$\mathrm{d3} = \sqrt{r^2 + s^2} - \mathrm{a2}$$

```
q3, same for both solutions
```

d3 = $\sqrt{(a_2 + d_3)^2} - a_2$

Orientation

$$q = \begin{bmatrix} \mathrm{q1} \\ \mathrm{q2} \\ \mathrm{d3} \end{bmatrix} = \begin{bmatrix} \mathrm{atan2}(\mathrm{Py}, \mathrm{Px}) \\ \frac{\pi}{2} + \mathrm{atan2}\left(s, r\right) \\ \sqrt{r^2 + s^2} - \mathrm{a2} \end{bmatrix} \quad \text{or} \quad q = \begin{bmatrix} \mathrm{q1} \\ \mathrm{q2} \\ \mathrm{d3} \end{bmatrix} = \begin{bmatrix} \pi + \mathrm{atan2}(\mathrm{Py}, \mathrm{Px}) \\ \frac{3\pi}{2} - \mathrm{atan2}\left(s, r\right) \\ \sqrt{r^2 + s^2} - \mathrm{a2} \end{bmatrix}$$

q_1 = $\left(\mathrm{angle}(\cos(q_2)\ (a_2 + d_3)\ (\cos(q_1) + \sin(q_1)\ \mathrm{i}))\quad \pi + \mathrm{angle}(\cos(q_2)\ (a_2 + d_3)\ (\cos(q_1) + \sin(q_1)\ \mathrm{i}))\quad \sqrt{(a_2}\right.$

q_2 =

$$\left(\frac{\pi}{2} + \mathrm{atan2}\left(\sin(q_2)\ (a_2 + d_3), \sqrt{-(a_2 + d_3)^2\ \left(\sin(q_2)^2 - 1\right)}\right)\quad \frac{3\pi}{2} - \mathrm{atan2}\left(\sin(q_2)\ (a_2 + d_3), \sqrt{-(a_2 + d_3)^2}\right.\right.$$

q =

$$\begin{pmatrix} q_1 \\ q_2 \end{pmatrix}$$

checking1 = $\begin{pmatrix} 0 & \pi & 2 \end{pmatrix}$

checking2 =

$$\begin{pmatrix} \frac{\pi}{2} & \frac{3\pi}{2} & 2 \end{pmatrix}$$

3. Jacobian

Classical approach

the 6-vector consisting of the linear and angular velocities of the end-effector

$$\begin{bmatrix} \dot{P}_x \\ \dot{P}_y \\ \dot{P}_z \\ \dot{\omega}_X \\ \dot{\omega}_X \\ \dot{\omega}_X \end{bmatrix}$$

Jacobian equal the partial derivative of vector T wrt each of the q1,q2,d3

$$J = \begin{bmatrix} J_\upsilon \\ J_\omega \end{bmatrix}$$

we have calculated the linearJacobian part J_υ

$$J_\upsilon = \begin{bmatrix} \delta\mathrm{Px}/\delta\mathrm{q1} & \delta\mathrm{Px}/\delta\mathrm{q2} & \delta\mathrm{Px}/\delta\mathrm{d3} \\ \delta\mathrm{Py}/\delta\mathrm{q1} & \delta\mathrm{Py}/\delta\mathrm{q2} & \delta\mathrm{Py}/\delta\mathrm{d3} \\ \delta\mathrm{Pz}/\delta\mathrm{q1} & \delta\mathrm{Pz}/\delta\mathrm{q2} & \delta\mathrm{Pz}/\delta\mathrm{d3} \end{bmatrix}$$

J1 =

$$\begin{pmatrix} -\cos(q_2)\sin(q_1)\ (a_2+d_3) \\ \cos(q_1)\cos(q_2)\ (a_2+d_3) \\ 0 \end{pmatrix}$$

J2 =

$$\begin{pmatrix} -\cos(q_1)\sin(q_2)\ (a_2+d_3) \\ -\sin(q_1)\sin(q_2)\ (a_2+d_3) \\ a_2\cos(q_2)+d_3\cos(q_2) \end{pmatrix}$$

J3 =

$$\begin{pmatrix} \cos(q_1)\cos(q_2) \\ \cos(q_2)\sin(q_1) \\ \sin(q_2) \end{pmatrix}$$

Jq1 =

$$\begin{pmatrix} -\cos(q_2)\sin(q_1)\ (a_2+d_3) & \cos(q_1)\sin(q_2)\ (a_2+d_3) & \cos(q_1)\cos(q_2) \\ \cos(q_1)\cos(q_2)\ (a_2+d_3) & \sin(q_1)\sin(q_2)\ (a_2+d_3) & \cos(q_2)\sin(q_1) \\ 0 & -\cos(q_2)\ (a_2+d_3) & \sin(q_2) \\ 0 & -\sin(q_1) & 0 \\ 0 & \cos(q_1) & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

Skew theory

or geometric approach

First find origin of each joint

FK:

```
T00 = 4×4
     1     0     0     0
     0     1     0     0
     0     0     1     0
     0     0     0     1
```

T01 =

$$\begin{pmatrix} \cos(q_1) & -\sin(q_1) & 0 & 0 \\ \sin(q_1) & \cos(q_1) & 0 & 0 \\ 0 & 0 & 1 & d_1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

T02 =

$$\begin{pmatrix} \cos(q_1)\cos(q_2) & -\sin(q_1) & -\cos(q_1)\sin(q_2) & 0 \\ \cos(q_2)\sin(q_1) & \cos(q_1) & -\sin(q_1)\sin(q_2) & 0 \\ \sin(q_2) & 0 & \cos(q_2) & d_1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

T03 =

$$\begin{pmatrix} \cos(q_1)\cos(q_2) & -\sin(q_1) & -\cos(q_1)\sin(q_2) & a_2\cos(q_1)\cos(q_2) + d_3\cos(q_1)\cos(q_2) \\ \cos(q_2)\sin(q_1) & \cos(q_1) & -\sin(q_1)\sin(q_2) & a_2\cos(q_2)\sin(q_1) + d_3\cos(q_2)\sin(q_1) \\ \sin(q_2) & 0 & \cos(q_2) & d_1 + a_2\sin(q_2) + d_3\sin(q_2) \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Origins as position component of FK

```
O0 = 3×1
     0
     0
     0
```

O1 =

$$\begin{pmatrix} 0 \\ 0 \\ d_1 \end{pmatrix}$$

O2 =

$$\begin{pmatrix} 0 \\ 0 \\ d_1 \end{pmatrix}$$

O3 =

$$\begin{pmatrix} a_2\cos(q_1)\cos(q_2)+d_3\cos(q_1)\cos(q_2) \\ a_2\cos(q_2)\sin(q_1)+d_3\cos(q_2)\sin(q_1) \\ d_1+a_2\sin(q_2)+d_3\sin(q_2) \end{pmatrix}$$

Find rotation (translation in case of prismatic joint) axis Z from transformation, note the coloumn! Its should correspond to the joint axis

```
Z0 = 3×1
     0
     0
     1
```

```
Z1 =
```

$$\begin{pmatrix} -\sin(q_1) \\ \cos(q_1) \\ 0 \end{pmatrix}$$

```
Z2 =
```

$$\begin{pmatrix} \cos(q_1)\cos(q_2) \\ \cos(q_2)\sin(q_1) \\ \sin(q_2) \end{pmatrix}$$

Full Jacobian

for revolute joint:

$$J_i = \begin{bmatrix} Z_{i-1} \times (O_n - O_{i-1}) \\ Z_{i-1} \end{bmatrix}$$

for prismatic joint:

$$J_i = \begin{bmatrix} Z_{i-1} \\ 0 \end{bmatrix}$$

```
Jq2 =
```

$$\begin{pmatrix} -\cos(q_2)\sin(q_1)\ (a_2+d_3) & \cos(q_1)\sin(q_2)\ (a_2+d_3) & \cos(q_1)\cos(q_2) \\ \cos(q_1)\cos(q_2)\ (a_2+d_3) & \sin(q_1)\sin(q_2)\ (a_2+d_3) & \cos(q_2)\sin(q_1) \\ 0 & -\cos(q_2)\ (a_2+d_3) & \sin(q_2) \\ 0 & -\sin(q_1) & 0 \\ 0 & \cos(q_1) & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

Numerical derivatives

J1 =

$$\begin{pmatrix} -a_2\cos(q_2)\sin(q_1) - d_3\cos(q_2)\sin(q_1) \\ a_2\cos(q_1)\cos(q_2) + d_3\cos(q_1)\cos(q_2) \\ 0 \\ 0 \\ 0 \\ \frac{\sin(q_1)^2}{\cos(q_1)^2+\sin(q_1)^2} + \frac{\sigma_3}{\sigma_1} + \frac{\sigma_2}{\sigma_1} \end{pmatrix}$$

where

$$\sigma_1 = \sigma_3 + \sigma_2 + \cos(q_2)^2\sin(q_1)^2 + \sin(q_1)^2\sin(q_2)^2$$

$$\sigma_2 = \cos(q_1)^2\sin(q_2)^2$$

$$\sigma_3 = \cos(q_1)^2\cos(q_2)^2$$

J2 =

$$\begin{pmatrix} a_2\cos(q_1)\sin(q_2) + d_3\cos(q_1)\sin(q_2) \\ a_2\sin(q_1)\sin(q_2) + d_3\sin(q_1)\sin(q_2) \\ -a_2\cos(q_2) - d_3\cos(q_2) \\ -\frac{\cos(q_2)^2\sin(q_1)}{\sigma_1} - \frac{\sin(q_1)\sin(q_2)^2}{\sigma_1} \\ \frac{\cos(q_1)\cos(q_2)^2}{\sigma_2} + \frac{\cos(q_1)\sin(q_2)^2}{\sigma_2} \\ 0 \end{pmatrix}$$

where

$$\sigma_1 = \cos(q_1)^2\cos(q_2)^2 + \cos(q_1)^2\sin(q_2)^2 + \cos(q_2)^2\sin(q_1)^2 + \sin(q_1)^2\sin(q_2)^2$$

$$\sigma_2 = \cos(q_2)^2 + \sin(q_2)^2$$

J3 =

$$\begin{pmatrix} \cos(q_1)\cos(q_2) \\ \cos(q_2)\sin(q_1) \\ \sin(q_2) \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Jq3 =

$$\begin{pmatrix} -\cos(q_2)\sin(q_1)\ (a_2+d_3) & \cos(q_1)\sin(q_2)\ (a_2+d_3) & \cos(q_1)\cos(q_2) \\ \cos(q_1)\cos(q_2)\ (a_2+d_3) & \sin(q_1)\sin(q_2)\ (a_2+d_3) & \cos(q_2)\sin(q_1) \\ 0 & -\cos(q_2)\ (a_2+d_3) & \sin(q_2) \\ 0 & -\sin(q_1) & 0 \\ 0 & \cos(q_1) & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

checking the different Jacobian methods:

check1 =

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

check2 =

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

check3 =

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

check4 =

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

check5 =

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

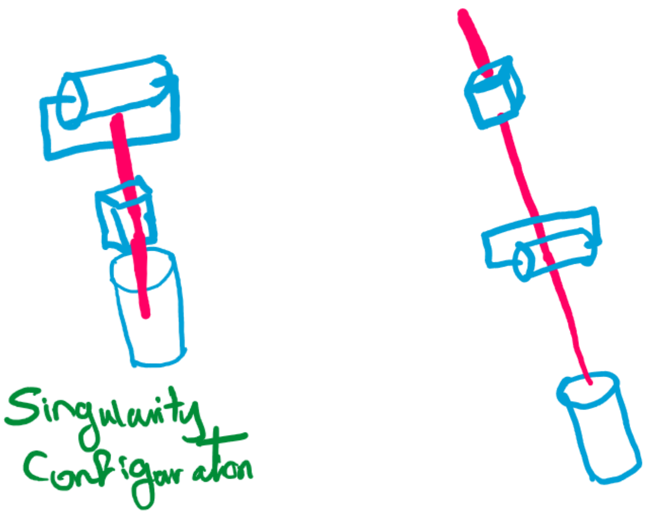
check6 =

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

then all the Jacobian Methods are the same.

4. Singularities

for Jacobian singularity analysis we need to get the determinant of the jacobian matrix in order to estimate the case which will make the matrix equal zero and we will calculate it for the upper part which is the linear part



analyze1 = $-\cos(q_2)\ (a_2 + d_3)^2$

$$\det\ \mathrm{Jq3}_v = \cos(\mathrm{q2})\ (\mathrm{a2} + \mathrm{d3})^2$$

this equation from the det of the matrix will be equal to zero in two cases only when

when case1: d3 = -a2 and case2: cos(q2) = 0

case1:

d3 can be found by:

$d3 = \sqrt{r^2 + s^2} - a2$ so this equation will be equal to zero if the square root part is equal to zero that means r and s are equal to zero where

$r = \sqrt{Px^2 + Py^2}$ and $s = Pz - d1$

this could happen if Px and Py points are equal to zero which is the origin point of the robot.

case2:

when cos(q2)=0 which happens when q2 is equal to pi/2 or -pi/2 and this will reduce the number of DOF by one and the endeffector position is intersecting the Z axis and any rotation around the robot base will not affect the position of end effector.

5. velocity of the tool frame

joint variables are changing with time as follows:

q1(t) = sin(t); q2(t) = cos(2t); d3(t) = sin(3t)

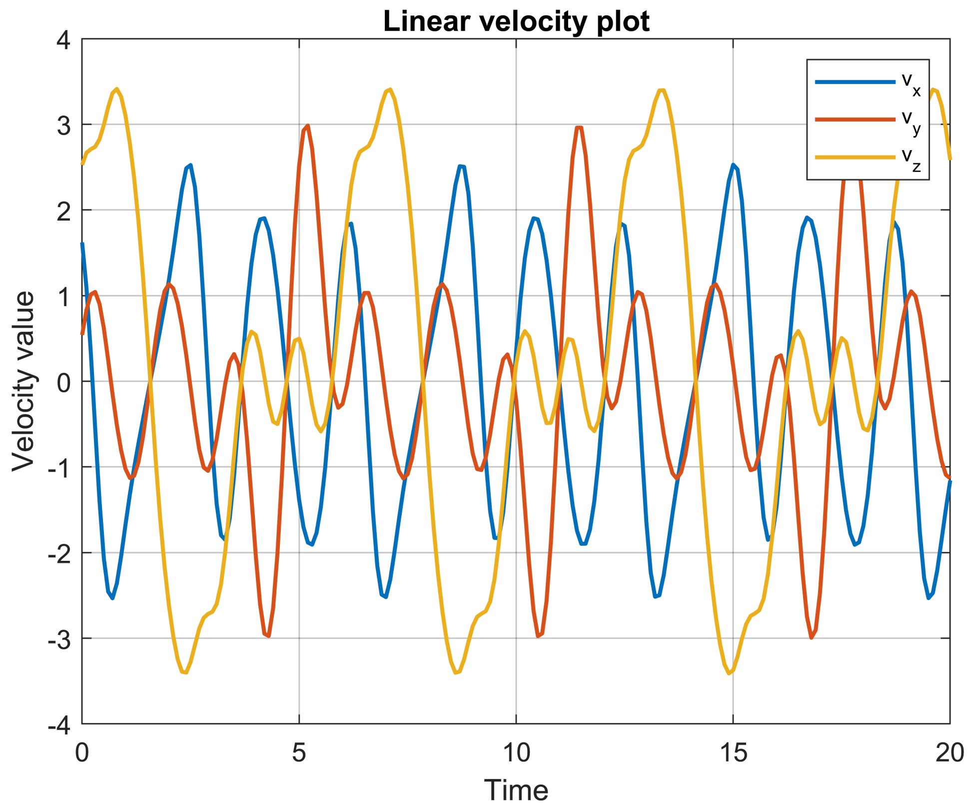
we will subistitute in the jacobian and multiply them by the derivatives of these equations

```
q_dot =
```

$$\begin{pmatrix} \cos(t) \\ -2\sin(2t) \\ 3\cos(3t) \end{pmatrix}$$

getting there derivative:

q_1(t) = cos (t) ; q_2(t) = -2 sin (2t) ; d_3(t) = 3 cos (3t)



Linear velocity graph is shown