

Homework 5: RRR elbow type Robot

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Github Link:

<https://github.com/mostafa-metwaly/DoNRS-HW5>

Forward Kinematics

$$H = R_z(q1) \cdot T_z(L1) \cdot R_y(q2) \cdot T_x(L2) \cdot R_y(q3) \cdot T_x(L3)$$

```
clear all
close all

% set angles as symbolical
syms q1 q2 q3 real
%Link lengths
L1=1
```

```
L1 = 1
```

```
L2=1
```

```
L2 = 1
```

```
L3=1
```

```
L3 = 1
```

```
L=[L1, L2 L3];
% FK symbolical
FK=simplify(Rz(q1)*Tz(L1)*Ry(q2)*Tx(L2)*Ry(q3)*Tx(L3))
```

```
FK =
```

$$\begin{pmatrix} \cos(q_2 + q_3) \cos(q_1) & -\sin(q_1) & \sin(q_2 + q_3) \cos(q_1) & \cos(q_1) \sigma_1 \\ \cos(q_2 + q_3) \sin(q_1) & \cos(q_1) & \sin(q_2 + q_3) \sin(q_1) & \sin(q_1) \sigma_1 \\ -\sin(q_2 + q_3) & 0 & \cos(q_2 + q_3) & 1 - \sin(q_2) - \sin(q_2 + q_3) \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

where

$$\sigma_1 = \cos(q_2 + q_3) + \cos(q_2)$$

```
q=[0.7854 -0.7854 1.3708 ]
```

```
q = 1x3
    0.7854    -0.7854     1.3708
```

```
FK= (Rz(q(1))*Tz(L1)*Ry(q(2))*Tx(L2)*Ry(q(3))*Tx(L3))
```

```
FK = 4x4
    0.5894    -0.7071     0.3907     1.0894
    0.5894     0.7071     0.3907     1.0894
   -0.5525         0     0.8335     1.1546
         0         0         0     1.0000
```

```
T=FK(1:3,4);
px=T(1)
```

```
px = 1.0894
```

```
py=T(2)
```

```
py = 1.0894
```

```
pz=T(3)
```

```
pz = 1.1546
```

Inverse Kinematics

```
q=InverseKinematics(FK)
```

```
q = 4x3
   -0.8760    -0.4014     1.9722
    0.8760    -0.4014     1.9722
    3.1416   -0.9854     2.5562
         0     0.5236     2.0944
```

Jacobian

Numerical derivatives

```
syms q1 q2 q3 real
% forward kinematics
H = Rz(q1)*Tz(L1)*Ry(q2)*Tx(L2)*Ry(q3)*Tx(L3);
H=simplify(H);
% extract rotation matrix
R = simplify(H(1:3,1:3));
% diff by q1
Td=Rzd(q1)*Tz(L1)*Ry(q2)*Tx(L2)*Ry(q3)*Tx(L3)*...
    [R^-1 zeros(3,1);0 0 0 1];
% extract 6 components from 4x4 Td matrix to Jacobian 1st column
J1 = [Td(1,4), Td(2,4), Td(3,4), Td(3,2), Td(1,3), Td(2,1)]'
```

J1 =

$$\begin{pmatrix} \sigma_1 - \cos(q_2) \sin(q_1) - \sigma_3 \\ \cos(q_1) \cos(q_2) + \sigma_4 - \sigma_2 \\ 0 \\ 0 \\ -\frac{\cos(q_2 + q_3) (\cos(q_2) \sin(q_1) \sin(q_3) + \cos(q_3) \sin(q_1) \sin(q_2))}{\sigma_7 + \sigma_6} - \frac{\sin(q_2 + q_3) (\sigma_1 - \sigma_3)}{\sigma_7 + \sigma_6} \\ \frac{\sin(q_1)^2}{\cos(q_1)^2 + \sin(q_1)^2} + \frac{\cos(q_2 + q_3) \cos(q_1) (\sigma_4 - \sigma_2)}{\sigma_5} + \frac{\sin(q_2 + q_3) \cos(q_1) (\cos(q_1) \cos(q_2) \sin(q_3) + \cos(q_2) \sin(q_1) \sin(q_3))}{\sigma_5} \end{pmatrix}$$

where

$$\sigma_1 = \sin(q_1) \sin(q_2) \sin(q_3)$$

$$\sigma_2 = \cos(q_1) \sin(q_2) \sin(q_3)$$

$$\sigma_3 = \cos(q_2) \cos(q_3) \sin(q_1)$$

$$\sigma_4 = \cos(q_1) \cos(q_2) \cos(q_3)$$

$$\sigma_5 = \sigma_7 \cos(q_1)^2 + \sigma_7 \sin(q_1)^2 + \sigma_6 \cos(q_1)^2 + \sigma_6 \sin(q_1)^2$$

$$\sigma_6 = \sin(q_2 + q_3)^2$$

$$\sigma_7 = \cos(q_2 + q_3)^2$$

```
% diff by q2
Td=Rz(q1)*Tz(L1)*Ryd(q2)*Tx(L2)*Ry(q3)*Tx(L3)*...
    [R^-1 zeros(3,1);0 0 0 1];
% extract 6 components from 4x4 Td matrix to Jacobian 2nd column
J2 = [Td(1,4), Td(2,4), Td(3,4), Td(3,2), Td(1,3), Td(2,1)]'
```

J2 =

$$\begin{pmatrix} -\cos(q_1) \sin(q_2) - \sigma_4 - \sigma_3 \\ -\sin(q_1) \sin(q_2) - \sigma_2 - \sigma_1 \\ \sigma_5 - \sigma_6 - \cos(q_2) \\ -\frac{\cos(q_2 + q_3) \sin(q_1) (\sigma_6 - \sigma_5)}{\sigma_7} - \frac{\sin(q_2 + q_3) \sin(q_1) (\cos(q_2) \sin(q_3) + \cos(q_3) \sin(q_2))}{\sigma_7} \\ \frac{\cos(q_2 + q_3) (\cos(q_1) \cos(q_2) \cos(q_3) - \cos(q_1) \sin(q_2) \sin(q_3))}{\sigma_9 + \sigma_8} + \frac{\sin(q_2 + q_3) (\sigma_4 + \sigma_3)}{\sigma_9 + \sigma_8} \\ -\frac{\cos(q_2 + q_3) \cos(q_1) (\sigma_2 + \sigma_1)}{\sigma_7} - \frac{\sin(q_2 + q_3) \cos(q_1) (\sin(q_1) \sin(q_2) \sin(q_3) - \cos(q_2) \cos(q_3) \sin(q_1))}{\sigma_7} \end{pmatrix}$$

where

$$\sigma_1 = \cos(q_3) \sin(q_1) \sin(q_2)$$

$$\sigma_2 = \cos(q_2) \sin(q_1) \sin(q_3)$$

$$\sigma_3 = \cos(q_1) \cos(q_3) \sin(q_2)$$

$$\sigma_4 = \cos(q_1) \cos(q_2) \sin(q_3)$$

$$\sigma_5 = \sin(q_2) \sin(q_3)$$

$$\sigma_6 = \cos(q_2) \cos(q_3)$$

$$\sigma_7 = \sigma_9 \cos(q_1)^2 + \sigma_9 \sin(q_1)^2 + \sigma_8 \cos(q_1)^2 + \sigma_8 \sin(q_1)^2$$

$$\sigma_8 = \sin(q_2 + q_3)^2$$

$$\sigma_9 = \cos(q_2 + q_3)^2$$

```
% diff by q3
Td=Rz(q1)*Tz(L1)*Ry(q2)*Tx(L2)*Ryd(q3)*Tx(L3)*...
[R^-1 zeros(3,1);0 0 0 1];
% extract 6 components from 4x4 Td matrix to Jacobian 2nd column
J3 = [Td(1,4), Td(2,4), Td(3,4), Td(3,2), Td(1,3), Td(2,1)]'
```

J3 =

$$\begin{pmatrix} -\sigma_4 - \sigma_3 \\ -\sigma_2 - \sigma_1 \\ \sigma_5 - \sigma_6 \\ -\frac{\cos(q_2 + q_3) \sin(q_1) (\sigma_6 - \sigma_5)}{\sigma_7} - \frac{\sin(q_2 + q_3) \sin(q_1) (\cos(q_2) \sin(q_3) + \cos(q_3) \sin(q_2))}{\sigma_7} \\ \frac{\cos(q_2 + q_3) (\cos(q_1) \cos(q_2) \cos(q_3) - \cos(q_1) \sin(q_2) \sin(q_3))}{\sigma_9 + \sigma_8} + \frac{\sin(q_2 + q_3) (\sigma_4 + \sigma_3)}{\sigma_9 + \sigma_8} \\ -\frac{\cos(q_2 + q_3) \cos(q_1) (\sigma_2 + \sigma_1)}{\sigma_7} - \frac{\sin(q_2 + q_3) \cos(q_1) (\sin(q_1) \sin(q_2) \sin(q_3) - \cos(q_2) \cos(q_3) \sin(q_1))}{\sigma_7} \end{pmatrix}$$

where

$$\sigma_1 = \cos(q_3) \sin(q_1) \sin(q_2)$$

$$\sigma_2 = \cos(q_2) \sin(q_1) \sin(q_3)$$

$$\sigma_3 = \cos(q_1) \cos(q_3) \sin(q_2)$$

$$\sigma_4 = \cos(q_1) \cos(q_2) \sin(q_3)$$

$$\sigma_5 = \sin(q_2) \sin(q_3)$$

$$\sigma_6 = \cos(q_2) \cos(q_3)$$

$$\sigma_7 = \sigma_9 \cos(q_1)^2 + \sigma_9 \sin(q_1)^2 + \sigma_8 \cos(q_1)^2 + \sigma_8 \sin(q_1)^2$$

$$\sigma_8 = \sin(q_2 + q_3)^2$$

$$\sigma_9 = \cos(q_2 + q_3)^2$$

```
% Full Jacobian 3x3
Jq = [simplify(J1), simplify(J2), simplify(J3)]
```

Jq =

$$\begin{pmatrix} -\sin(q_1) \sigma_2 & -\cos(q_1) \sigma_1 & -\sin(q_2 + q_3) \cos(q_1) \\ \cos(q_1) \sigma_2 & -\sin(q_1) \sigma_1 & -\sin(q_2 + q_3) \sin(q_1) \\ 0 & -\cos(q_2 + q_3) - \cos(q_2) & -\cos(q_2 + q_3) \\ 0 & -\sin(q_1) & -\sin(q_1) \\ 0 & \cos(q_1) & \cos(q_1) \\ 1 & 0 & 0 \end{pmatrix}$$

where

$$\sigma_1 = \sin(q_2 + q_3) + \sin(q_2)$$

$$\sigma_2 = \cos(q_2 + q_3) + \cos(q_2)$$

Trajectory

Task 1

```
% Q2
% define the initial and final conditions

t0 = 0; tf = 2;
q0 = [0 0 0]; qf = [2 3 4];
v0 = 0; vf = 0;
acc0 = 0; accf = 0;
tf = 2;
q_all=zeros(21,3);
v_all= zeros(21,3);
acc_all=zeros(21,3);

for i=1:3
%Given:
A = [1 t0 t0^2 t0^3 t0^4 t0^5
      0 1 2*t0 3*t0^2 4*t0^3 5*t0^4
      0 0 2 6*t0 12*t0^2 20*t0^3
      1 tf tf^2 tf^3 tf^4 tf^5
      0 1 2*tf 3*tf^2 4*tf^3 5*tf^4
      0 0 2 6*tf 12*tf^2 20*tf^3];
c = [q0(i);v0;acc0;qf(i);vf;accf];
b = A\c;
% assign the results to the coefficients
a0 = b(1); a1 = b(2); a2 = b(3); a3 = b(4); a4 = b(5); a5 = b(6);

t = 0:0.1:2;
q = a0+a1.*t+a2.*t.^2+a3.*t.^3+a4.*t.^4+a5.*t.^5;
v = a1+2*a2.*t+3*a3.*t.^2+4*a4.*t.^3+5*a5.*t.^4;
```

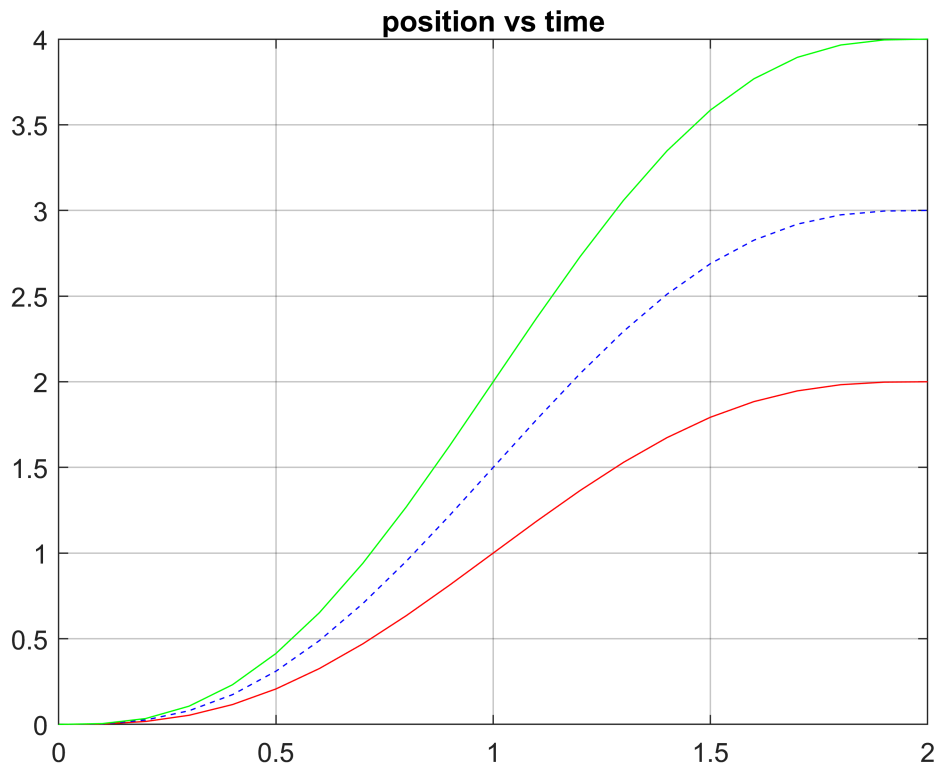
```

acc = 2*a2+6*a3.*t+12*a4.*t.^2+20*a5.*t.^3;

q_all(:,i) = q;
v_all(:,i) = v;
acc_all(:,i) = acc ;
end

figure
plot(t,q_all(:,1),'r-',t,q_all(:,2),'b--',t,q_all(:,3),'g-')
title('position vs time')
grid on

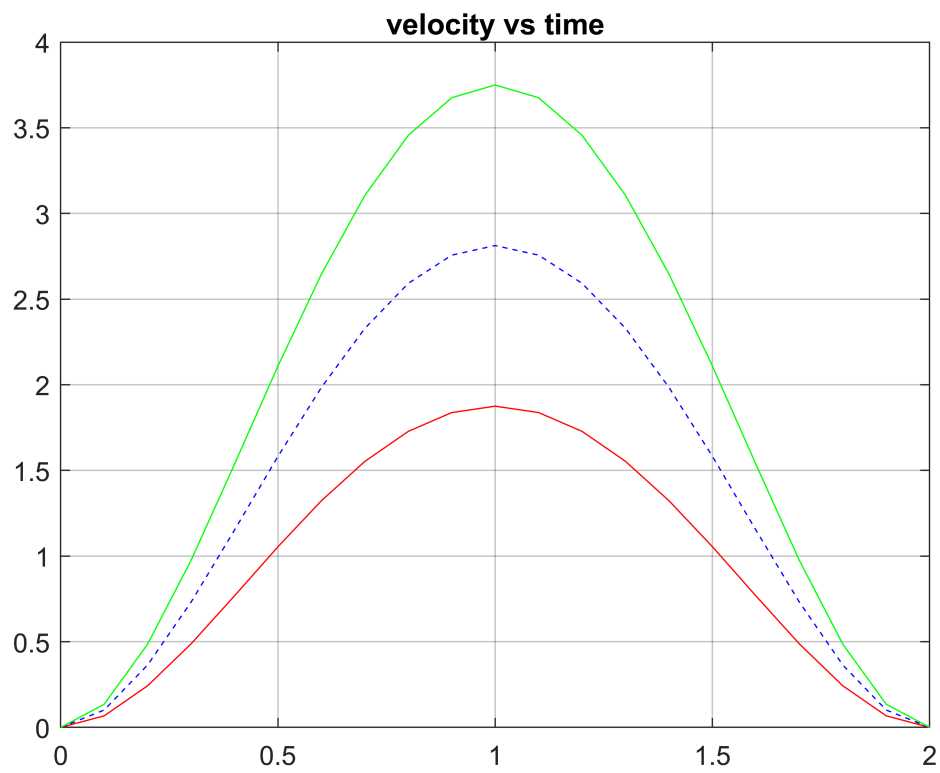
```



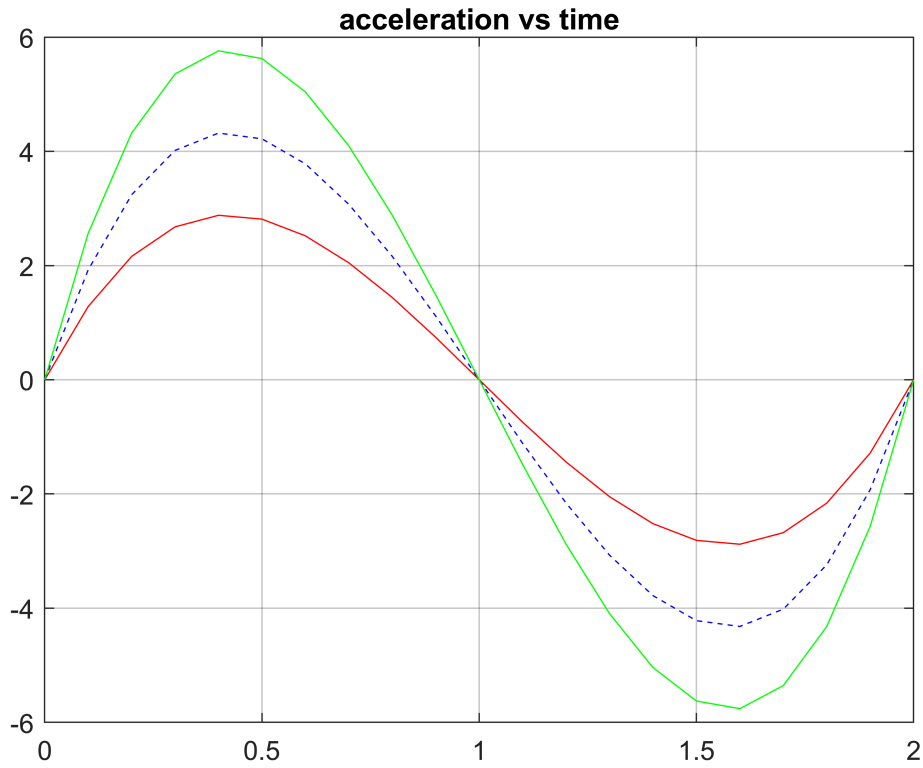
```

figure
plot(t,v_all(:,1),'r-',t,v_all(:,2),'b--',t,v_all(:,3),'g-')
title('velocity vs time')
grid on

```



```
figure
plot(t,acc_all(:,1),'r-',t,acc_all(:,2),'b--',t,acc_all(:,3),'g-')
title('acceleration vs time')
grid on
```

Task 2

Calculate Joint Trajectory (Trapezoidal) for PTP Given

$q(t)$ from $q(0) = (0, 0, 0)$ to $q(2) = (2, 3, 4)$

Controller command interpretation frequency $f = 100$ Hz

Maximum joint velocity = 1 rad/s

Maximum joint acceleration = 10 rad/s²

$$q(t) = \begin{cases} a_{10} + a_{11}t + a_{12}t^2 & 0 \leq t \leq t_a \\ a_{20} + a_{21}t & t_a < t < t_f - t_a \\ a_{30} + a_{31}t + a_{32}t^2 & t_f - t_a < t < t_f \end{cases}$$

where t_a and $t_f - t_a$ is the period when the joint moves in constant velocity and they computed as follows:

$$t_a = \frac{v_{\max}}{a_{\max}}$$

$$t_f = \frac{q_f - q_0}{v_{\max}} + t_a$$

```
clear all
close all;clc;
q0 = [0 0 0]; qf = [2 3 4]; v_max = [1 1 1]; a_max = [10 10 10];
v0= [0 0 0];
dt = 0.01;
n = 0;
```

```
while (floor(dt*10^n)~=dt*10^n)
    n=n+1;
```

```
end
E = 1*10^-n;
ta = v_max./a_max;
```

```
delta_q = (qf(:)-q0(:));
tf = (delta_q(:))./v_max + ta;
taw=delta_q ./v_max;
```

```
if rem(ta,dt)~=0
    ta_new = round(ta,n)+E;
else
    ta_new = round(ta,n);
end
tf = (qf(:)-q0(:))./v_max(:) + ta_new(:);
```

```
if rem(tf,dt)~=0
    tf_new = round(tf,n)+E;
else
    tf_new = round(tf,n);
end
```

```
ta_new=max(ta_new)
```

```
ta_new = 0.1000
```

```
tf_new=max(tf_new)
```

```
tf_new = 4.1000
```

```
v_old =((qf(:)-q0(:))./(tf-ta));
a_old = v_old/ta;
```

```
v = ((qf(:)-q0(:))./(tf_new-ta_new))
```

```
v = 3×1
    0.5000
    0.7500
    1.0000
```

```
a = v/ta_new
```

```
a = 3×1  
5.0000  
7.5000  
10.0000
```

```
% all joints - coefficients:  
for i =1:3  
  
% t0 --> ta:  
a10 = q0(i);  
a11 = v0(i);  
a12 = 0.5*a(i);  
  
% ta --> tf-ta:  
a20 = q0(i) + 0.5*a(i)*ta_new^2 - v(i)*ta_new;  
a21 = v(i);  
  
% tf-ta --> tf:  
a30 = qf(i) - 0.5*a(i)*tf_new^2;  
a31 = a(i)*tf_new;  
a32 = -0.5*a(i);  
  
b(i,:) = [a10; a11; a12; a20; a21; a30; a31; a32];  
end  
  
t = 0:dt:tf_new;  
v=zeros(3,length(t));  
for i=1:3  
  
q(i,:) = (b(i,1)+b(i,2).*t+b(i,3).*t.^2).*(t<=ta_new)...  
+(b(i,4)+b(i,5).*t).*(t>ta_new).*(t<=(tf_new-ta_new))...  
+(b(i,6)+b(i,7).*t+b(i,8).*t.^2).*(t>(tf_new-ta_new)).*(t<=tf_new);  
  
v(i,:) = (b(i,2)+2*b(i,3).*t).*(t<=ta_new)...  
+(b(i,5)).*(t>ta_new).*(t<=(tf_new-ta_new))...  
+(b(i,7)+2*b(i,8).*t).*(t>(tf_new-ta_new)).*(t<=tf_new);  
  
acc(i,:) = (2*b(i,3)).*(t<=ta_new)...  
+(0).*(t>ta_new).*(t<=(tf_new-ta_new))...  
+(2*b(i,8)).*(t>(tf_new-ta_new)).*(t<=tf_new);  
  
end
```

```
figure  
subplot(1,3,1)  
plot(t,q(1,:), 'b-', 'linewidth', 2)  
hold on  
plot(t,q(2,:), 'g.', 'linewidth', 2)
```

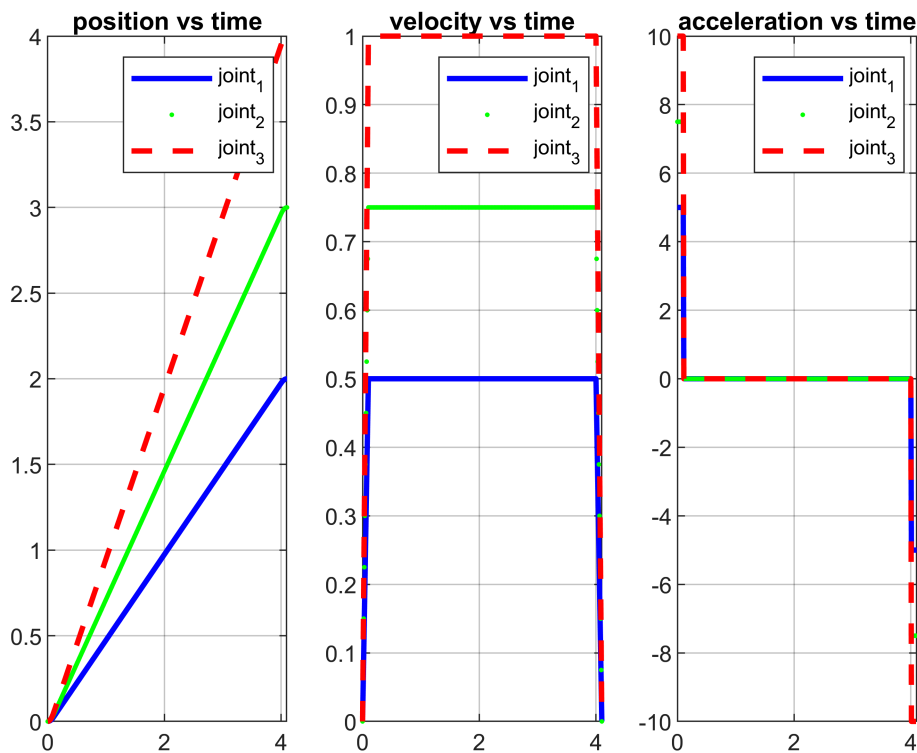
```

hold on
plot(t,q(3,:), 'r--', 'linewidth', 2)
grid on
title('position vs time')
legend('joint_1', 'joint_2', 'joint_3')
axis([0 tf_new -inf inf])

subplot(1,3,2)
plot(t,v(1,:), 'b-', 'linewidth', 2)
hold on
plot(t,v(2,:), 'g.', 'linewidth', 2)
hold on
plot(t,v(3,:), 'r--', 'linewidth', 2)
title('velocity vs time')
legend('joint_1', 'joint_2', 'joint_3')
grid on
axis([0 tf_new -inf inf])

subplot(1,3,3)
plot(t,acc(1,:), 'b-', 'linewidth', 2)
hold on
plot(t,acc(2,:), 'g.', 'linewidth', 2)
hold on
plot(t,acc(3,:), 'r--', 'linewidth', 2)
title('acceleration vs time')
legend('joint_1', 'joint_2', 'joint_3')
grid on
axis([0 tf_new -inf inf])

```



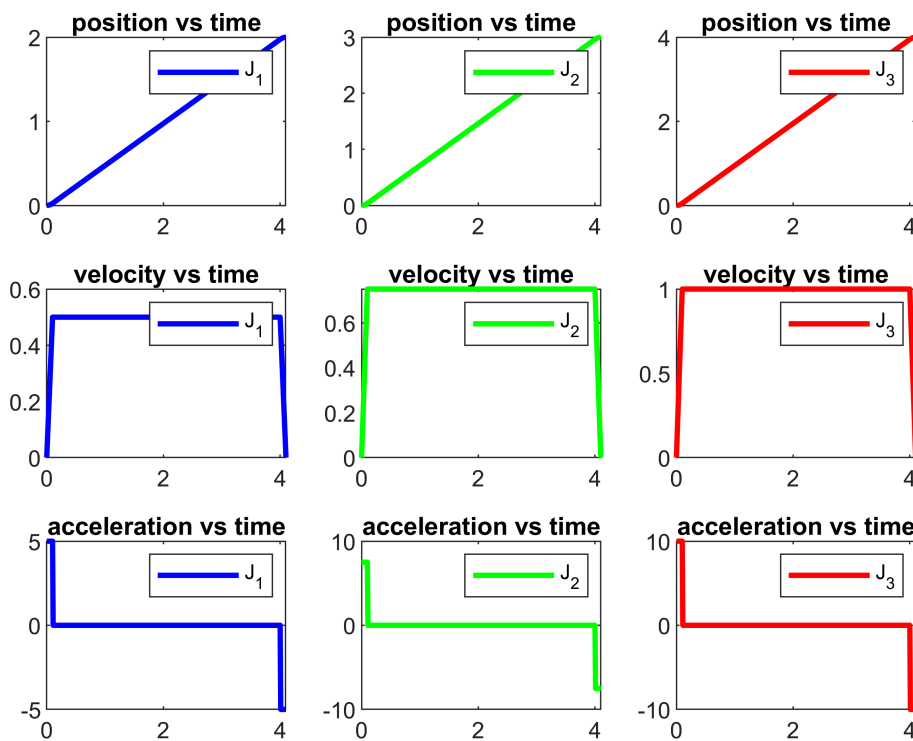
```

figure;
subplot(3,3,1);plot(t,q(1,:), 'b-', 'linewidth',2);title('position vs time');legend('J_1');
subplot(3,3,2);plot(t,q(2,:), 'g-', 'linewidth',2);title('position vs time');legend('J_2');
subplot(3,3,3);plot(t,q(3,:), 'r-', 'linewidth',2 );title('position vs time');legend('J_3');

subplot(3,3,4);plot(t,v(1,:), 'b-', 'linewidth',2);title('velocity vs time');legend('J_1');
subplot(3,3,5);plot(t,v(2,:), 'g-', 'linewidth',2);title('velocity vs time');legend('J_2');
subplot(3,3,6);plot(t,v(3,:), 'r-', 'linewidth',2 );title('velocity vs time');legend('J_3');

subplot(3,3,7);plot(t,acc(1,:), 'b-', 'linewidth',2);title('acceleration vs time');legend('J_1');
subplot(3,3,8);plot(t,acc(2,:), 'g-', 'linewidth',2);title('acceleration vs time');legend('J_2');
subplot(3,3,9);plot(t,acc(3,:), 'r-', 'linewidth',2 );title('acceleration vs time');legend('J_3')
hold off

```



```

figure
subplot(2,3,1:3)
plot(t,q(1,:), 'b-', 'linewidth',2)
hold on
plot(t,q(2,:), 'g.', 'linewidth',2)
hold on
plot(t,q(3,:), 'r--', 'linewidth',2)
grid on

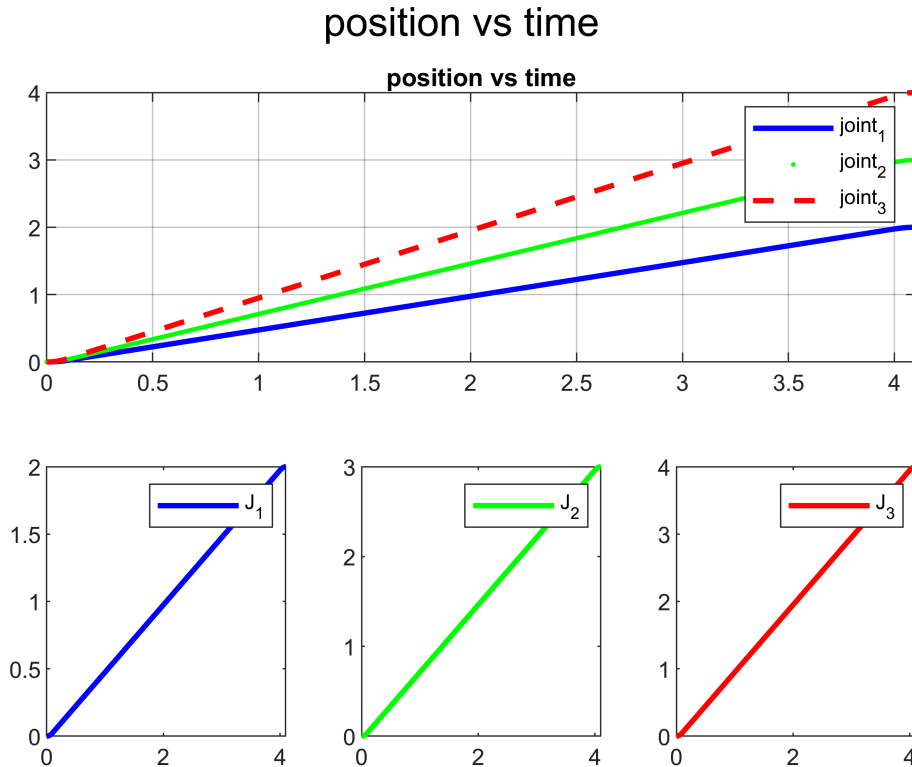
```

```

title('position vs time')
legend('joint_1','joint_2','joint_3')
axis([0 tf_new -inf inf])

subplot(2,3,4);plot(t,q(1,:), 'b-', 'linewidth',2);sgtitle('position vs time');legend('J_1');
subplot(2,3,5);plot(t,q(2,:), 'g-', 'linewidth',2);sgtitle('position vs time');legend('J_2');
subplot(2,3,6);plot(t,q(3,:), 'r-', 'linewidth',2 );sgtitle('position vs time');legend('J_3');

```

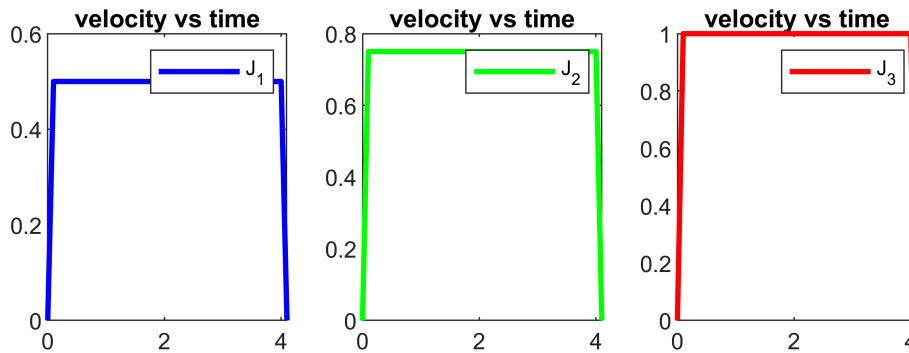
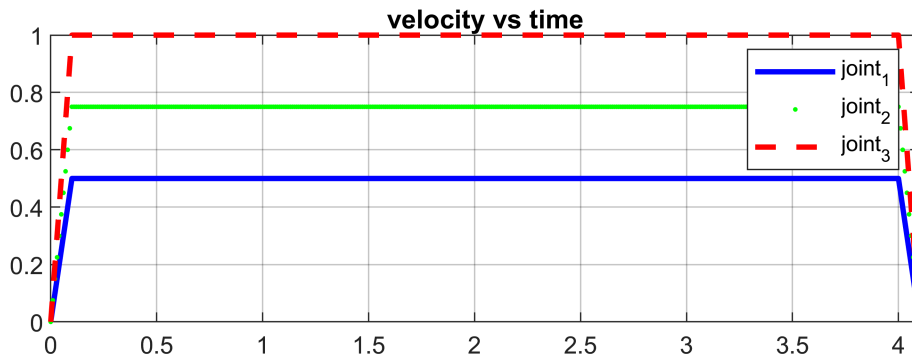


```

figure;
subplot(2,3,1:3)
plot(t,v(1,:), 'b-', 'linewidth',2)
hold on
plot(t,v(2,:), 'g.', 'linewidth',2)
hold on
plot(t,v(3,:), 'r--', 'linewidth',2)
title('velocity vs time')
legend('joint_1','joint_2','joint_3')
grid on
axis([0 tf_new -inf inf])

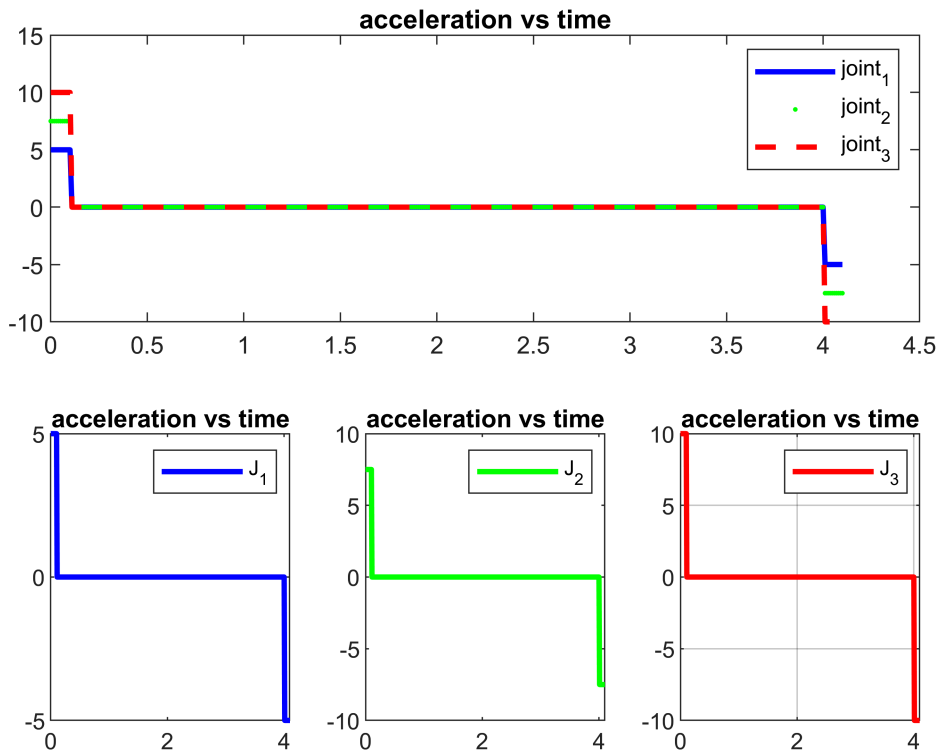
subplot(2,3,4);plot(t,v(1,:), 'b-', 'linewidth',2);title('velocity vs time');legend('J_1');
subplot(2,3,5);plot(t,v(2,:), 'g-', 'linewidth',2);title('velocity vs time');legend('J_2');
subplot(2,3,6);plot(t,v(3,:), 'r-', 'linewidth',2 );title('velocity vs time');legend('J_3');

```



```
figure;
subplot(2,3,1:3)
plot(t,acc(1,:), 'b-', 'linewidth', 2)
hold on
plot(t,acc(2,:), 'g.', 'linewidth', 2)
hold on
plot(t,acc(3,:), 'r--', 'linewidth', 2)
title('acceleration vs time')
legend('joint_1', 'joint_2', 'joint_3')

subplot(2,3,4);plot(t,acc(1,:), 'b-', 'linewidth', 2);title('acceleration vs time');legend('J_1');
subplot(2,3,5);plot(t,acc(2,:), 'g-', 'linewidth', 2);title('acceleration vs time');legend('J_2');
subplot(2,3,6);plot(t,acc(3,:), 'r-', 'linewidth', 2 );title('acceleration vs time');legend('J_3')
grid on
axis([0 tf_new -inf inf])
```



Task 3

Joint trajectory for the following commands: LIN – $p_1 = (1, 0, 1)$ to $p_2 = (\sqrt{2}/2, \sqrt{2}/2, 1.2)$ (trapezoidal)

- Controller command interpretation frequency – $f = 100$ Hz
- Maximum linear velocity – 1 m/s
- Maximum linear acceleration – 10 m/s²

```
clear all;
p1 = [ 1, 0, 1];
p2 = [sqrt(2)/2 sqrt(2)/2 1.2];
v_max = [1 1 1]; a_max = [10 10 10];
v0= [0 0 0];
dt = 0.01;
n = 0;
N = 100;

while (floor(dt*10^n)~=dt*10^n)
    n=n+1;
end
E = 1*10^-n;
```



```
ta = v_max./a_max
```

```
ta = 1×3  
    0.1000    0.1000    0.1000
```

```
delta_p = (p2(:)-p1(:));
```

we will calculate for the controller frequency and recalculate the time by using maximum time

```
if rem(ta,dt)~=0  
    ta_new = round(ta,n)+E;  
else  
    ta_new = round(ta,n);  
end  
  
tf = (delta_p(:))./v_max(:) + ta_new(:);
```

```
if rem(tf,dt)~=0  
    tf_new = round(tf,n)+E;  
else  
    tf_new = round(tf,n);  
end
```

```
ta_new=max(ta_new)
```

```
ta_new = 0.1000
```

```
tf_new=max(tf_new)
```

```
tf_new = 0.8200
```

```
% v_old =((p2(:)-p1(:))./(tf-ta));  
% a_old = v_old/ta;  
%  
% v = ((p2(:)-p1(:))./(tf_new-ta_new))  
% a = v/ta_new  
  
t_lin = linspace(0,tf_new,N);
```

we will then calculate all the waypoints positions and velocities that would make the straight line path between the given points in cartesian coordinates

```
% positions  
% x = ((p2(1)-p1(1))/tf_new).*t+p1(1);  
% y = ((p2(2)-p1(2))/tf_new).*t+p1(2);  
% z = ((p2(3)-p1(3))/tf_new).*t+p1(3);  
    waypoints = [((p2(:)-p1(:))./tf_new).*t_lin+p1(:)]';  
% velocities  
% vx = ((p2(1)-p1(1))/tf_new);  
% vy = ((p2(2)-p1(2))/tf_new);  
% vz = ((p2(3)-p1(3))/tf_new);  
    vel = ((p2(:)-p1(:))/tf_new);  
m = -1; % 1 elbow up, -1 elbow down
```

we then calculate these waypoints from the cartesian space to the joint space getting joint angles and velocities using robot inverse kinematics and jacobian matrix

```
for i=1:N
    waypnt = waypnts(i,:);
    jointangles(i,:) = InverseKinematics(waypnt);
    if i == 1 || i==N
        jointVelocity(i,:) = [0 0 0];
    else
        J = Jacobian(jointangles(i,:));
        jointVelocity(i,:) = (J\vel)';
    end
end
```

we calculate the position ,velocity and acceleration for each joint for each waypoint and store there values.

```
n = 15;
num=0;
Q = [];
Vel = [];
Acc = [];
acc =[];
vel =[];
for i=1:N-1
    t1 = t_lin(i); t2 = t_lin(i+1);
    A = [1 t1 t1^2 t1^3
         0 1 2*t1 3*t1^2
         1 t2 t2^2 t2^3
         0 1 2*t2 3*t2^2];
    for j = 1:3
        q1 = jointangles(i,j);
        q2 = jointangles(i+1,j);
        v1 = jointVelocity(i,j);
        v2 = jointVelocity(i+1,j);
        c = [q1;v1;q2;v2];
        b = A\c;
        t = linspace(t1,t2,n);
        q(j,:) = b(1)+b(2).*t+b(3).*t.^2+b(4).*t.^3;
        vel(j,:) = b(2)+2*b(3).*t+3*b(4).*t.^2;
        acc(j,:) = 2* b(3)+6* b(4).*t;
    end
    Q = [Q q];
    Vel = [Vel vel];
    Acc = [Acc acc];
end
```

we check our solution by getting the forward kinematics for the joints and calculate the path and its velocity and acceleration from the jacobian and visualize the robot motion

```
cartTrajectory = ForwardKinematics(Q');
```

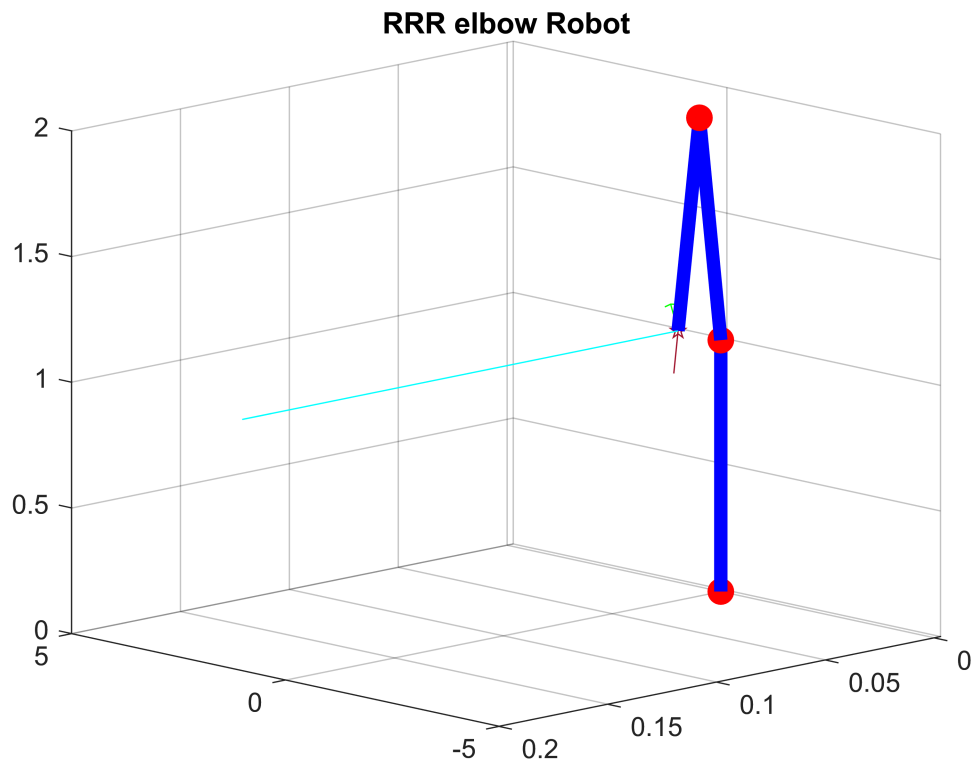
```

for i=1:length(Q)
    jac= Jacobian(Q(:,i)');
    CartVelocity(:,i)= jac * Vel(:,i);
    CartAcceleration(i,:)= jac * Acc(:,i);

end
CartAcceleration=CartAcceleration';
cartTrajectory=cartTrajectory';
figure;

hold on
for i=1:50:length(Q)
    draw_myrobot([1 1 1],Q(:,i)')

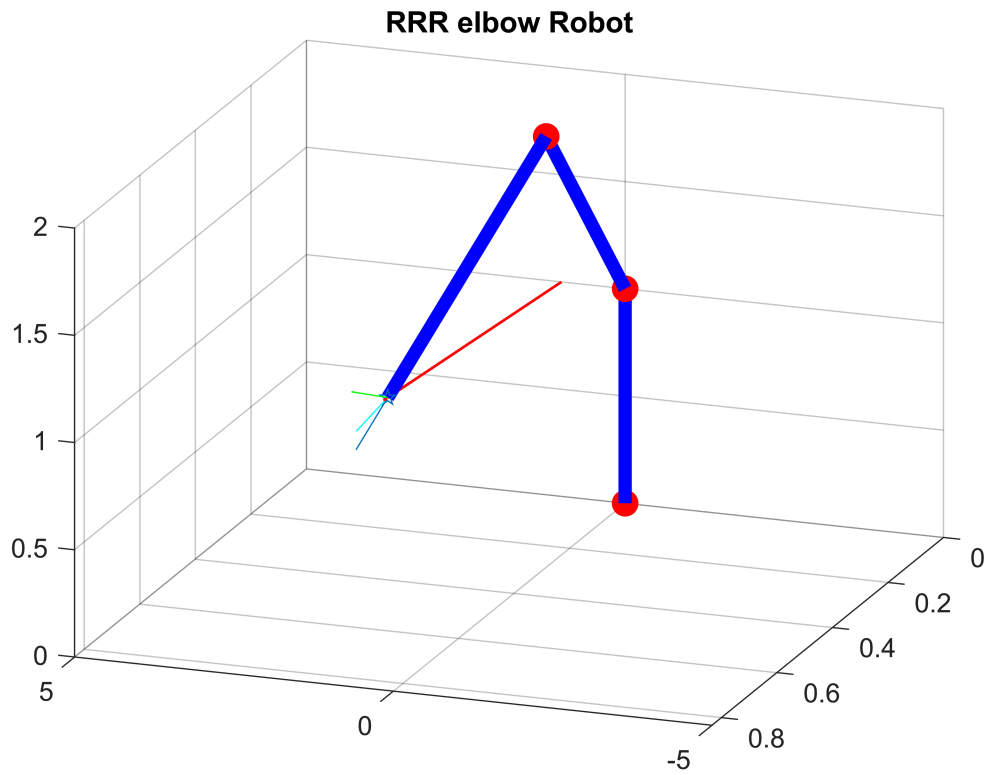
```



```

pause(0.01)
cla
plot3(waypnts(:,1),waypnts(:,2),waypnts(:,3),'r-','linewidth',1)
end
draw_myrobot([1 1 1],Q(:,i)')

```



```
num=n*N-n;
t = linspace(0,tf_new,num);
```

these graphs draw the relation between all the joints position velocity and acceleration and also the linear position , velocity and acceleration of the end-effector motion.

```
figure
subplot(1,3,1)
plot(t,Q(1,:), 'b-', 'linewidth', 2)
hold on
plot(t,Q(2,:), 'g-', 'linewidth', 2)
hold on
plot(t,Q(3,:), 'r-', 'linewidth', 2)
grid on
title('joints position vs time')
legend('joint_1', 'joint_2', 'joint_3')
axis([0 tf_new -inf inf])

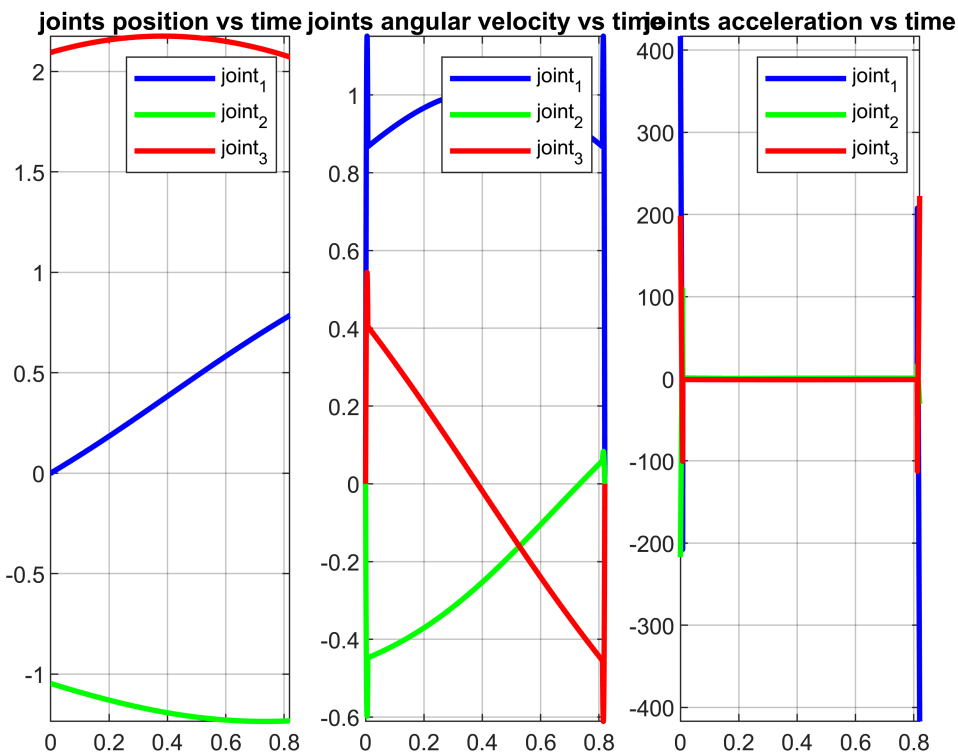
subplot(1,3,2)
plot(t,Vel(1,:), 'b-', 'linewidth', 2)
hold on
plot(t,Vel(2,:), 'g-', 'linewidth', 2)
hold on
plot(t,Vel(3,:), 'r-', 'linewidth', 2)
title('joints angular velocity vs time')
legend('joint_1', 'joint_2', 'joint_3')
```

```

grid on
axis([0 tf_new -inf inf])

subplot(1,3,3)
plot(t,Acc(1,:), 'b-', 'linewidth',2)
hold on
plot(t,Acc(2,:), 'g-', 'linewidth',2)
hold on
plot(t,Acc(3,:), 'r-', 'linewidth',2)
title('joints acceleration vs time')
legend('joint_1', 'joint_2', 'joint_3')
grid on
axis([0 tf_new -inf inf])

```



```

figure;
subplot(3,3,1);plot(t,cartTrajectory(1,:), 'b-', 'linewidth',2);title('Linear position vs time');
subplot(3,3,2);plot(t,cartTrajectory(2,:), 'g-', 'linewidth',2);sgtitle('Linear position vs time');
subplot(3,3,3);plot(t,cartTrajectory(3,:), 'r-', 'linewidth',2 );title('Linear position vs time');

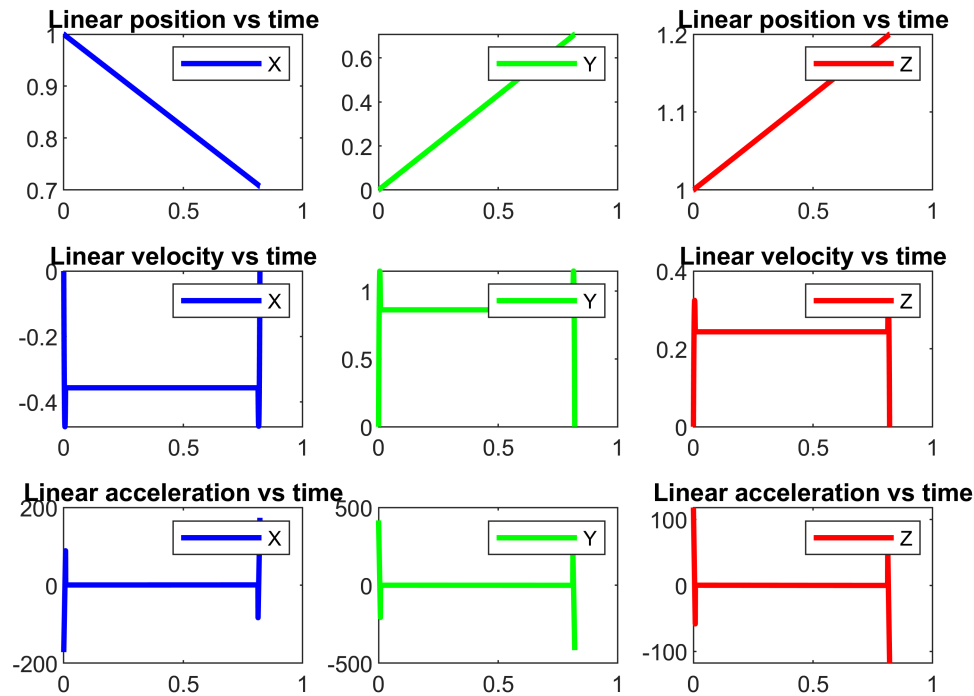
subplot(3,3,4);plot(t,CartVelocity(1,:), 'b-', 'linewidth',2);title('Linear velocity vs time');le
subplot(3,3,5);plot(t,CartVelocity(2,:), 'g-', 'linewidth',2);sgtitle('Linear velocity vs time');
subplot(3,3,6);plot(t,CartVelocity(3,:), 'r-', 'linewidth',2 );title('Linear velocity vs time');l

subplot(3,3,7);plot(t,CartAcceleration(1,:), 'b-', 'linewidth',2);title('Linear acceleration vs t
subplot(3,3,8);plot(t,CartAcceleration(2,:), 'g-', 'linewidth',2);sgtitle('Linear acceleration vs
subplot(3,3,9);plot(t,CartAcceleration(3,:), 'r-', 'linewidth',2 );title('Linear acceleration vs

```

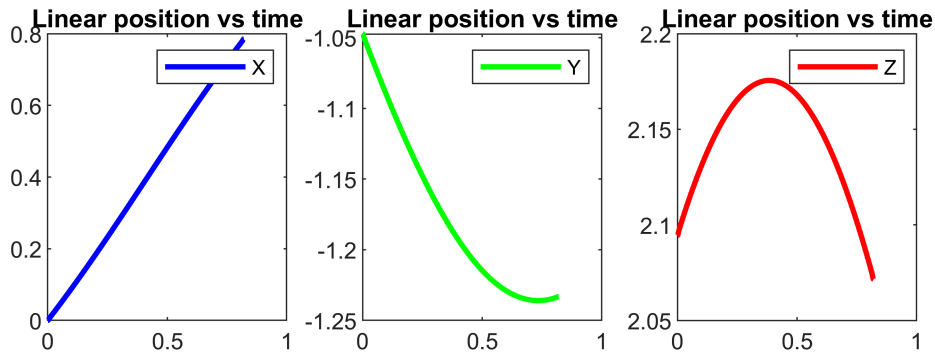
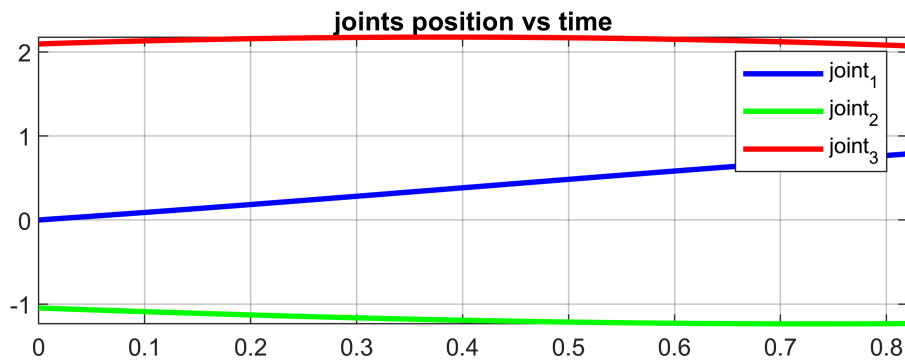
hold off

Linear acceleration vs time



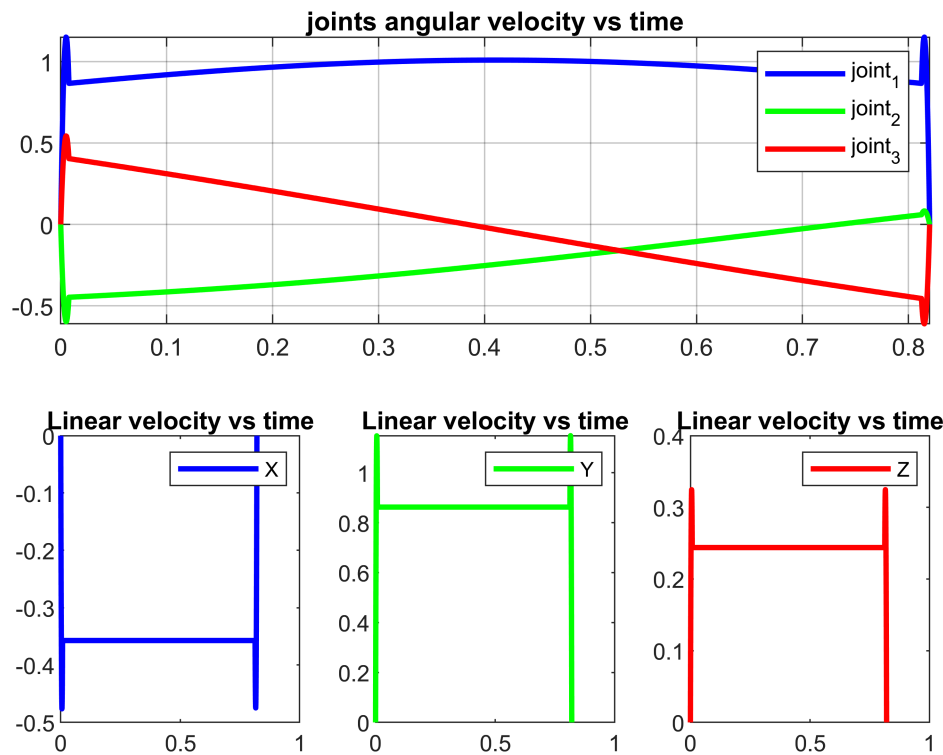
```
figure
subplot(2,3,1:3)
plot(t,Q(1,:), 'b-', 'linewidth', 2)
hold on
plot(t,Q(2,:), 'g-', 'linewidth', 2)
hold on
plot(t,Q(3,:), 'r-', 'linewidth', 2)
grid on
title('joints position vs time')
legend('joint_1', 'joint_2', 'joint_3')
axis([0 tf_new -inf inf])
```

```
subplot(2,3,4);plot(t,Q(1,:), 'b-', 'linewidth', 2);title('Linear position vs time');legend('X');
subplot(2,3,5);plot(t,Q(2,:), 'g-', 'linewidth', 2);title('Linear position vs time');legend('Y');
subplot(2,3,6);plot(t,Q(3,:), 'r-', 'linewidth', 2 );title('Linear position vs time');legend('Z');
```



```
figure;
subplot(2,3,1:3)
plot(t,Vel(1,:), 'b-', 'linewidth', 2)
hold on
plot(t,Vel(2,:), 'g-', 'linewidth', 2)
hold on
plot(t,Vel(3,:), 'r-', 'linewidth', 2)
title('joints angular velocity vs time')
legend('joint_1', 'joint_2', 'joint_3')
grid on
axis([0 tf_new -inf inf])
```

```
subplot(2,3,4);plot(t,CartVelocity(1,:), 'b-', 'linewidth', 2);title('Linear velocity vs time');le
subplot(2,3,5);plot(t,CartVelocity(2,:), 'g-', 'linewidth', 2);title('Linear velocity vs time');le
subplot(2,3,6);plot(t,CartVelocity(3,:), 'r-', 'linewidth', 2 );title('Linear velocity vs time');l
```



```
figure;
subplot(2,3,1:3)
plot(t,Acc(1,:), 'b-', 'linewidth', 2)
hold on
plot(t,Acc(2,:), 'g-', 'linewidth', 2)
hold on
plot(t,Acc(3,:), 'r-', 'linewidth', 2)
title('joints acceleration vs time')
legend('joint_1', 'joint_2', 'joint_3')

subplot(2,3,4);plot(t,CartAcceleration(1,:), 'b-', 'linewidth', 2);title('Linear acceleration vs t
subplot(2,3,5);plot(t,CartAcceleration(2,:), 'g-', 'linewidth', 2);title('Linear acceleration vs t
subplot(2,3,6);plot(t,CartAcceleration(3,:), 'r-', 'linewidth', 2 );title('Linear acceleration vs t
grid on
axis([0 tf_new -inf inf])
```