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# Generalization of Gassmann equations for porous media saturated with a solid material

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## ABSTRACT

Gassmann equations predict effective elastic properties of an isotropic homogeneous bulk rock frame filled with a fluid. This theory has been generalized for an anisotropic porous frame by Brown and Korringa's equations. Here, we develop a new model for effective elastic properties of porous rocks — a generalization of Brown and Korringa's and Gassmann equations for a solid infill of the pore space. We derive the elastic tensor of a solid-saturated porous rock considering small deformations of the rock skeleton and the pore infill material upon loading them with the confining and pore-space stresses. In the case of isotropic material, the solution reduces to two generalized Gassmann equations for the bulk and shear moduli. The applicability of the new model is tested by independent numerical simulations performed on the microscale by finite-difference and finite-element methods. The results show very good agreement between the new theory and the numerical simulations. The generalized Gassmann model introduces a new heuristic parameter, characterizing the elastic properties of average deformation of the pore-filling solid material. In many cases, these elastic moduli can be substituted by the elastic parameters of the infill grain material. They can also represent a proper viscoelastic model of the pore-filling material. Knowledge of the effective elastic properties for such a situation is required, for example, when predicting seismic velocities in some heavy oil reservoirs, where a highly viscous material fills the pores. The classical Gassmann fluid substitution is inapplicable for a configuration in which the fluid behaves as a quasi-solid.

## INTRODUCTION

Physical properties of porous rocks, such as seismic velocity, depend on elastic properties of the porous frame and the pore-space filling material. Gassmann equation (Gassmann, 1951) commonly is applied to predict the bulk modulus of rocks saturated with different fluids. This equation assumes that all pores are interconnected and that the pore pressure is in equilibrium in the pore space. The porous frame is macroscopically and microscopically homogeneous and isotropic. Brown and Korringa (1975) generalize Gassmann equations for inhomogeneous anisotropic material of the porous frame.

In this paper, we further extend this theory to the case when the pore-filling material is an anisotropic elastic solid. In the isotropic case, i.e., in the case equivalent to Gassmann equation but with a possibility of solid pore-space infill, the effective bulk modulus is estimated from the mineral and dry bulk moduli of the porous frame, porosity, and a new parameter: bulk modulus of the pore space filled by a solid material. The saturated shear modulus differs significantly from the dry shear modulus. This is one of the main advantages of our approach in comparison with Gassmann, and Brown and Korringa's equations, which apply only when the pore-filling material is a fluid. The developed model has its applicability for computing effective elastic properties of heavy oil reservoirs when the highly viscous (and possibly non-Newtonian) liquid saturating the pores acts as a solid.

## STRESS/STRAIN THEORY

We consider a porous rock of porosity $\phi$. It is possible that an elastic solid fills up the pore space. Let $\Sigma$ be the external surface of the porous rock. It cuts and seals the pores (jacketed sample). The pore space is assumed to be interconnected, representing Biot's medium (Biot, 1962). We also define the surface of the pore space $\Psi$. The surface $\Sigma$ coincides with the surface $\Psi$ where it cuts the pores. Their normals are opposite in these points, i.e., $n_j' = -n_j$. The traction component $\tau_i$ at any given point $\mathbf{x}$ of a surface $\Sigma$ is given by

$$\tau_i = \sigma_{ij}^c n_j(\mathbf{x}), \tag{1}$$

where $n_j(\mathbf{x})$ are the components of the outward normal of $\Sigma$ (see Figure 1 in Shapiro and Kaselow, 2005) and $\sigma_{ij}^c$ is the uniform confining

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stress. We assume application of small uniform changes in both confining stress $\sigma^c_{ij}$ and pore stress $\sigma^f_{ij}$.

The pore stress term now represents the stress field in the solid filling the pore space. When the pore-space saturating material is a fluid, the pore stress reduces to the pore pressure. Because of the load, points of the external surface $\Sigma$ are displaced by $u_i(\mathbf{x})$ to their final position (see Figure 2 in Shapiro and Kaselow, 2005). This displacement is assumed to be very small in comparison to the size of the rock volume under consideration.

According to Brown and Korringa (1975) and Shapiro and Kaselow (2005), we can describe the deformation of a rock sample by symmetric tensors representing the deformation of the rock sample,

$$\eta_{ij} = \int_\Sigma \frac{1}{2}(u_i n_j + u_j n_i) d^2\mathbf{x}, \tag{2}$$

and the deformation of the pore space,

$$\zeta_{ij} = \int_\Psi \frac{1}{2}(u_i n'_j + u_j n'_i) d^2\mathbf{x}. \tag{3}$$

For a continuous elastic body replacing the porous matrix and the continuous pore-filling elastic material, Gauss's theorem applies:

$$\eta_{ij} = \int_V \frac{1}{2}(\partial_j u_i + \partial_i u_j) d^3\mathbf{x}, \tag{4}$$

$$\zeta_{ij} = \int_{V_\phi} \frac{1}{2}(\partial_j u_i + \partial_i u_j) d^3\mathbf{x}. \tag{5}$$

The integrands in equations 4 and 5 are the strain tensors. The quantity $\eta_{ij}/V$ represents a volume-averaged strain of the bulk volume, and the quantity $\zeta_{ij}/V_\phi$ denotes a volume-averaged strain of the pore volume. Here, $V$ is the volume of the porous body and $V_\phi$ is the volume of all its connected pores.

Three fundamental compliances of an anisotropic porous body can be introduced:

$$S^{\mathrm{dry}}_{ijkl} = \frac{1}{V}\left(\frac{\partial \eta_{ij}}{\partial \sigma^d_{kl}}\right)_{\sigma^f}, \tag{6}$$

$$S^{\mathrm{gr}}_{ijkl} = \frac{1}{V}\left(\frac{\partial \eta_{ij}}{\partial \sigma^f_{kl}}\right)_{\sigma^d}, \tag{7}$$

and

$$S^{\phi}_{ijkl} = -\frac{1}{V_\phi}\left(\frac{\partial \zeta_{ij}}{\partial \sigma^f_{kl}}\right)_{\sigma^d}, \tag{8}$$

where $\sigma^d_{kl} = \sigma^c_{kl} - \sigma^f_{kl}$ is the differential stress, and where indices dry, gr, and $\phi$ are related to the dry porous frame, the grain material of the frame, and the pore space of the dry porous frame, respectively. These quantities in expressions 6–8 represent the tensorial generalization of Brown and Korringa's (1975) expressions 4a–4c.

The fourth compliance tensor,

$$S'_{ijkl} = -\frac{1}{V}\left(\frac{\partial \zeta_{ij}}{\partial \sigma^d_{kl}}\right)_{\sigma^f}, \tag{9}$$

is not independent because of the reciprocity theorem (Shapiro and Kaselow, 2005):

$$S'_{ijkl} = S^{\mathrm{dry}}_{ijkl} - S^{\mathrm{gr}}_{ijkl}. \tag{10}$$

The fifth tensor is required to describe the compliance of the pore space filled by a solid material. We define it heuristically in the following way:

$$S^{\mathrm{if}}_{ijkl} = -\frac{1}{V_\phi}\left(\frac{\partial \zeta_{ij}}{\partial \sigma^f_{kl}}\right)_{\mathrm{con}}, \tag{11}$$

where the index if (infill) is related to the body of the pore-space infill and where con is constant infill mass. This generalized (in the sense that the infill can be solid) compliance tensor $S^{\mathrm{if}}_{ijkl}$ is related to the volume-averaged strain of the pore space and therefore differs from the compliance tensor of the grain material of the pore infill $S^{\mathrm{ifgr}}_{ijkl}$ (index ifgr denotes the pore-infill grain material). Later we explain how to estimate $S^{\mathrm{if}}_{ijkl}$. The effective compliance tensor of the composite porous rock with a solid infill $S^*_{ijkl}$, which is the subject of our consideration, is defined as follows:

$$S^*_{ijkl} = \frac{1}{V}\left(\frac{\partial \eta_{ij}}{\partial \sigma^c_{kl}}\right)_{\mathrm{con}}. \tag{12}$$

Further, we evaluate the changes of $\eta_{ij}$ and of $\zeta_{ij}$ by applying $\delta\sigma^d$ and by keeping the pore stress $\sigma^f$ constant and the effect of applying $\delta\sigma^f_{ij}$ from inside and outside while leaving $\sigma^d$ constant:

$$\delta\eta_{ij} = \left(\frac{\partial \eta_{ij}}{\partial \sigma^d_{kl}}\right)_{\sigma^f}\delta\sigma^d_{kl} + \left(\frac{\partial \eta_{ij}}{\partial \sigma^f_{kl}}\right)_{\sigma^d}\delta\sigma^f_{kl} \tag{13}$$

and

$$\delta\zeta_{ij} = \left(\frac{\partial \zeta_{ij}}{\partial \sigma^d_{kl}}\right)_{\sigma^f}\delta\sigma^d_{kl} + \left(\frac{\partial \zeta_{ij}}{\partial \sigma^f_{kl}}\right)_{\sigma^d}\delta\sigma^f_{kl}. \tag{14}$$

These two expressions serve for the derivation of the effective compliance tensor $S^*_{ijkl}$.

## GENERALIZED BROWN AND KORRINGA'S EQUATIONS FOR SOLID INFILL OF THE PORE SPACE

The definition of the effective compliance tensor in expression 12, the definitions in expressions 6–8, and the resulting change in the value of the frame deformation tensor $\eta_{ij}$ given by expression 13 yield

$$\frac{\delta\eta_{ij}}{V} \equiv S^*_{ijkl}\delta\sigma^c_{kl} = S^{\mathrm{dry}}_{ijkl}(\delta\sigma^c_{kl} - \delta\sigma^f_{kl}) + S^{\mathrm{gr}}_{ijkl}\delta\sigma^f_{kl}. \tag{15}$$

The requirement of the conservation of mass of a pore-filling material upon loading it with the pore stress $\delta\sigma^f_{kl}$, using expressions 3, 9, and 8, gives

$$-\frac{\delta\zeta_{ij}}{V_\phi} \equiv S^{\mathrm{if}}_{ijkl}\delta\sigma^f_{kl} = \frac{1}{\phi}S'_{ijkl}(\delta\sigma^c_{kl} - \delta\sigma^f_{kl}) + S^{\phi}_{ijkl}\delta\sigma^f_{kl}, \tag{16}$$

where $\phi = V_\phi/V$ denotes porosity. To obtain the effective compliance tensor $S^*_{ijkl}$, we eliminate $\delta\sigma^c_{kl}$ and $\delta\sigma^f_{kl}$ from equations 15 and

16 and use equation 10. The result yields the generalized anisotropic Brown and Korringa's (1975) equation for a solid filling the pore space:

$$S^*_{ijkl} = S^{dry}_{ijkl} - (S^{dry}_{ijkl} - S^{gr}_{ijkl})[\phi(S^{if} - S^{\phi}) + (S^{dry} - S^{gr})]^{-1}_{mnqp}(S^{dry}_{mnqp} - S^{gr}_{mnqp}). \quad (17)$$

Equation 17 is the main result of this paper. (Note the tensorial nature of this equation.)

## ISOTROPIC GENERALIZED BROWN AND KORRINGA'S EQUATIONS FOR A SOLID INFILL OF THE PORE SPACE

In the case of isotropic materials, the compliance tensor **S** can be expressed in terms of bulk and shear moduli $K$ and $\mu$ (Mavko et al., 1998). We substitute individual compliances $S^{dry}_{ijkl}$, $S^{gr}_{ijkl}$, $S^{\phi}_{ijkl}$, and $S^{if}_{ijkl}$ into expression 17. Solving the system of equations 17 for the isotropic case, we obtain the solid-saturated bulk and shear moduli:

$$K^{*-1}_{sat} = K^{-1}_{dry} - \frac{(K^{-1}_{dry} - K^{-1}_{gr})^2}{\phi(K^{-1}_{if} - K^{-1}_{\phi}) + (K^{-1}_{dry} - K^{-1}_{gr})} \quad (18)$$

and

$$\mu^{*-1}_{sat} = \mu^{-1}_{dry} - \frac{(\mu^{-1}_{dry} - \mu^{-1}_{gr})^2}{\phi(\mu^{-1}_{if} - \mu^{-1}_{\phi}) + (\mu^{-1}_{dry} - \mu^{-1}_{gr})}, \quad (19)$$

where $K^*_{sat}$ and $\mu^*_{sat}$ are solid saturated bulk and shear moduli, $K_{dry}$ and $\mu_{dry}$ denote drained bulk and shear moduli of the porous frame, $K_{gr}$ and $\mu_{gr}$ represent bulk and shear moduli of the grain material of the frame, $K_{\phi}$ and $\mu_{\phi}$ are bulk and shear moduli related to the pore space of the frame, and $K_{if}$ and $\mu_{if}$ are the newly defined bulk and shear moduli related to the solid body of the pore infill. Equations 18 and 19 represent the isotropic Gassmann equations for a solid-saturated porous rock. (Note that the equations for the solid-saturated bulk and shear moduli are of the same form.)

If the porous frame material is homogeneous, it follows (Brown and Korringa, 1975) $K_{\phi} = K_{gr}$ and $\mu_{\phi} = \mu_{gr}$. If the pore infill is a fluid ($\mu_{if} = 0$), equations 18 and 19 reduce to Brown and Korringa's equations (1975). In the case of a single mineral in the porous frame ($K_{\phi} = K_{gr}$) and a fluid in the pore space, equations 18 and 19 reduce to the standard equations of Gassmann (1951).

## COMPARISON WITH KNOWN THEORIES

When all constituents of an elastic composite have the same shear modulus $\mu$, Hill (1963) shows that the effective bulk modulus $K_{eff}$ is given by an exact formula:

$$\frac{1}{K_{eff} + \frac{4}{3}\mu} = \frac{1-\phi}{K_{gr} + \frac{4}{3}\mu} + \frac{\phi}{K_{ifgr} + \frac{4}{3}\mu}, \quad (20)$$

where $K_{ifgr}$ represents the grain bulk modulus of the pore-filling material.

In this case, $\mu_{gr} = \mu_{ifgr} = \mu_{if}$, and equation 19 reduces to $\mu^*_{sat} = \mu_{gr}$. For this case, $K_{\phi} = K_{gr}$ also. In equation 18, we still have an unknown heuristic parameter, $K_{if}$. This special case of identical shear moduli allows us to analyze this new parameter. We use equation 20 to calculate $K_{if}$.

Both equations 18 and 20 express the effective bulk moduli of the mixture of matrix and pore materials. Thus, $K^*_{sat} = K_{eff}$ and we obtain

$$K_{if} = \phi\left[\frac{\left(1 - \frac{K_{dry}}{K_{gr}}\right)^2}{K_{eff} - K_{dry}} - \frac{\left(1 - \frac{K_{dry}}{K_{gr}}\right) - \phi}{K_{gr}}\right]^{-1}. \quad (21)$$

To analyze equation 21, we keep constant elastic parameters of the porous frame and vary the grain bulk modulus of the saturating solid in the pore space. The porous frame has the following parameters: bulk and shear moduli of grains is $K_{gr} = 36.7$ GPa and $\mu_{gr} = 22$ GPa, porosity $\phi = 0.22$, and drained bulk and shear moduli of the frame is $K_{dry} = 10$ GPa and $\mu_{dry} = 7.6$ GPa. The grain shear moduli of the porous frame and the pore-filling material are equal: $\mu_{ifgr} = \mu_{gr} = 22$ GPa. (Note that $\mu_{ifgr}$ represents the shear modulus of grains of the pore infill, whereas $\mu_{if}$ denotes shear modulus related to the volume-averaged shear deformation of the pore-filling solid body.)

Analysis of equation 21 shows that for the low contrast in bulk moduli of the frame material and of the pore-filling material, the unknown modulus $K_{if}$ can be approximated very well by the bulk modulus $K_{ifgr}$ (i.e., bulk modulus of the infill material). This result greatly simplifies the use of equations 18 and 19. For the contrast in elastic moduli up to 20%, the unknown parameters $K_{if}$ and $\mu_{if}$ can be almost exactly substituted with the bulk and shear moduli of the pore-filling material, $K_{ifgr}$ and $\mu_{ifgr}$. This approximation is still applicable for the contrast in the elastic moduli in the range from 20% to 40%.

## COMPARISON WITH NUMERICAL SIMULATIONS

To check the correctness of expressions 18 and 19, in Figures 1 and 2 we compare the derived analytical model with the numerical simulations. For the comparison, we use two numerical approaches: finite-element and finite-difference (FD) modeling.

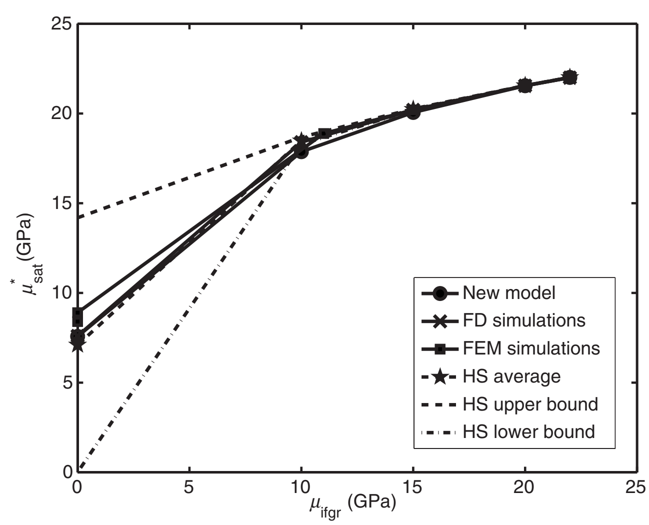


Figure 1. Effective shear modulus $\mu^*_{sat}$ in dependence of the shear modulus of the pore-filling solid $\mu_{ifgr}$. The results for the new model are obtained from equation 19, assuming $\mu_{if} = \mu_{ifgr}$, $\mu_{gr} = 22$ GPa, and $\mu_{dry} = 7.6$ GPa. FD = finite difference; FEM = finite-element modeling; HS = Hashin-Shtrikman.

## Finite-element modeling

The finite-element (FE) modeling algorithm was developed originally by Garboczi and Day (1995) and adopted for the study of porous rocks by Arns et al. (2002). The algorithm uses a formulation of the static linear elastic equations and finds the steady-state solution by minimizing the strain energy of the system. We implement this algorithm here to compute the effective elastic constants of the isotropic porous model saturated in the pore space with another elastic solid material. The numerical simulations provide the static effective bulk and shear moduli of such a model. The tests are performed on the Gaussian random field models (GRF5 and GRF1) created and analyzed by Saenger et al. (2005).

## Finite-difference modeling

The elastic version of the rotated staggered-grid FD algorithm, developed by Saenger et al. (2000), has been extended to viscoelasticity (Saenger et al., 2005). This algorithm is proven to be effective in simulations of wave propagation in porous media on the microscale. In this study, we perform several simulations on the artificial rock sample GRF5 to obtain the effective P- and S-wave velocities. This enables us to derive the dynamic effective bulk and shear moduli of the analyzed piece of porous rock saturated with elastic solid material. Of course, we expect a coincidence of the static and dynamic moduli for the case of low enough frequencies of propagation waves.

## Results

The comparisons of both numerical methods with the new model are shown in Figures 1 and 2. These figures illustrate the effective solid-saturated shear moduli dependent on the shear moduli of the pore-filling material $\mu_{\mathrm{ifgr}}$. For completeness, we also plot the Hashin-Shtrikman bounds and their average value (Hashin and Shtrikman, 1963). The porous matrix model is represented by the

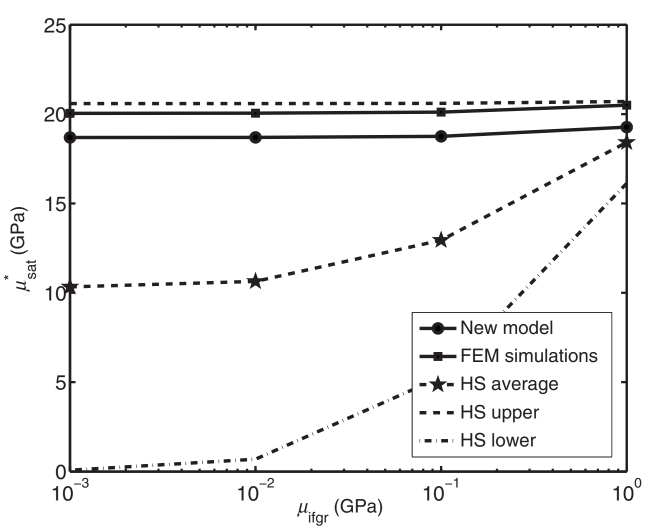


Figure 2. Effective shear modulus $\mu^*_{\mathrm{sat}}$ dependent of the shear modulus of the pore-filling solid $\mu_{\mathrm{ifgr}}$. The results for the new model are computed using $\mu_{\mathrm{if}} = \mu_{\mathrm{ifgr}}$. For the low-porosity medium and the high contrast in shear moduli of a porous matrix and a pore-space filling material, the new model performs better than the Hashin-Shtrikman (1963) average.

GRF5 (in Figure 1) and GRF1 (in Figure 2) models with the same frame parameters given after equation 21.

The results in Figure 1 show the six numerical simulations performed with the following bulk and shear moduli $[K_{\mathrm{ifgr}}, \mu_{\mathrm{ifgr}}]$: [36.7, 22], [25, 20], [20, 15], [13.34, 10], [2.25, 0], and [0,0] GPa, where the last one represents the drained porous sample. The numerical simulations are in very good agreement with the theoretical results. The theoretical effective shear moduli are obtained using equation 19, where the unknown shear parameter $\mu_{\mathrm{if}}$ is taken to be equal to its grain counterpart $\mu_{\mathrm{ifgr}}$. These results confirm the conclusions stated in the previous section. The low contrast in the elastic parameters of the elastic solid matrix and the pore-filling material enables us to carry out the substitutions $K_{\mathrm{if}} = K_{\mathrm{ifgr}}$ and $\mu_{\mathrm{if}} = \mu_{\mathrm{ifgr}}$. When the pore infill is a fluid, the new model reduces to the classical Gassmann equation, as shown earlier. This situation confirms the numerical simulation for the point [2.25,0] GPa.

Figure 2 shows an example where the Hashin-Shtrikman average deviates from the effective shear modulus obtained by numerical simulations. This is the case of rock (model GRF1) with low porosity and high contrast in the frame and the pore-filling materials. The parameters of the model are $\phi = 0.0342$, bulk and shear moduli of the frame grains are $K_{\mathrm{gr}} = 36.7$ GPa and $\mu_{\mathrm{gr}} = 22$ GPa, and drained moduli of the porous frame are $K_{\mathrm{dry}} = 29$ GPa and $\mu\mathrm{dry} = 18.7$ GPa. The results in Figure 2 show four numerical simulations performed with the following bulk and shear moduli $[K_{\mathrm{ifgr}}, \mu_{\mathrm{ifgr}}]$: [2.2, 0.001], [2.2, 0.01], [2.2, 0.1], and [2.2,1] GPa. This example represents the situation when the developed model performs better than the Hashin-Shtrikman average. Such a situation can be observed in the case of fractured (low-porosity) rocks filled with heavy oils.

## VISCOELASTIC EXTENSION

We heuristically extend the elastic equations 17–19 for the viscoelastic material filling the pore space. For this, complex bulk and shear moduli $K_{\mathrm{if}}$ and $\mu_{\mathrm{if}}$ are introduced into equations 18 and 19. The numerical code implements the viscoelastic infill as a generalized Maxwell body. The S-wave velocity in the rock is then given by

$$V_s = \sqrt{\frac{\mu^*_{\mathrm{sat}}}{\rho}}, \tag{22}$$

where $\mu^*_{\mathrm{sat}}$ is obtained from equation 19 using the viscoelastic extension of $\mu_{\mathrm{if}}$ (Maxwell fluid model):

$$\mu_{\mathrm{if}}(\omega) = \frac{\mu_\infty}{\dfrac{-i\mu_\infty}{\omega\eta} + 1}. \tag{23}$$

Here, $\mu_\infty$ is the real shear modulus of the infill medium at high frequencies, $\eta$ is the dynamic shear viscosity of the same medium, and $\rho = (1 - \phi)\rho_{\mathrm{gr}} + \phi\rho_f$ is the overall density.

In Figure 3, we show the results of numerical experiments for the S-wave transmission through porous rock filled with a viscous fluid for varying viscosity from 1 to $10^7$ kg/ms. We use the viscoelastic extension of the finite-difference code developed by Saenger et al. (2005). The porous rock is represented by the GRF5 model. The P- and S-wave velocities and the density of the porous frame grain material are $V_p = 5100$ m/s, $V_s = 2944$ m/s, and $\rho_{\mathrm{gr}} = 2540$ kg/m$^3$, respectively; $\phi = 0.22$; drained bulk and shear moduli are $K_{\mathrm{dry}}$

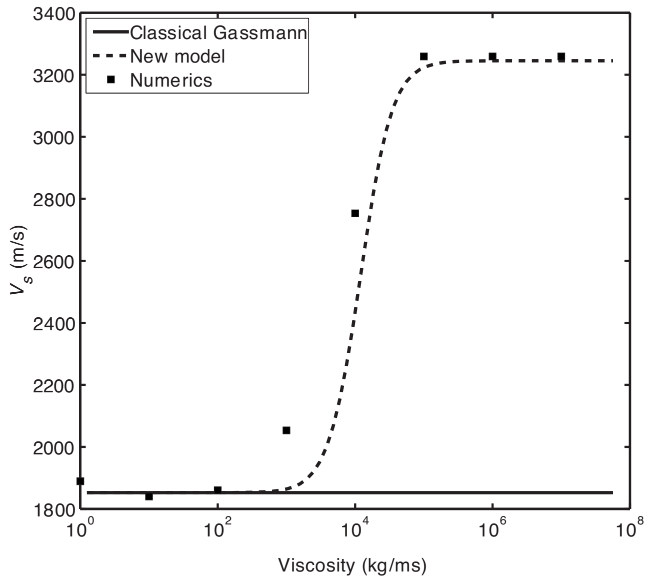


Figure 3. The S-wave velocity $V_s$ in dependence of the fluid viscosity. The classical Gassmann model computes S-wave velocity from drained bulk modulus, whereas the new model implements the new viscoelastic Gassmann equation for saturated shear modulus (equation 22).

$= 10$ GPa and $\mu_{\mathrm{dry}} = 7.6$ GPa; and fluid density is $\rho_f = 1000$ kg/m$^3$. A pressure seismic source radiates a Ricker wavelet with the dominant frequency $f_{\mathrm{dom}} = 80$ kHz.

Figure 3 compares the new viscoelastic Gassmann equation for the shear modulus and the results of our numerical simulations. The new model in equation 22 is in very good agreement with the numerical results. For comparison, we also plot the elastic solution given by the classical Gassmann equation, $V_s = \sqrt{\mu_{\mathrm{dry}}/\rho}$, which is frequency and viscosity independent. Thus, the new model developed here, along with its viscoelastic extension, is important for modeling seismic responses of rocks containing heavy oil in the pore space.

## CONCLUSIONS

The main result of this paper is a new analytical model of effective elastic properties of porous rock. This model extends Brown and Korringa's anisotropic version of Gassmann equations to the situation with an elastic solid filling the pore space. The pore space filled with elastic material is characterized by a new tensor of elastic compliances. This tensor is related to the pore-space volume-averaged strain. It can be approximated well by a tensor of infill compliances. For an isotropic material, our formalism reduces to two equations describing effective saturated bulk and shear moduli of the solid-saturated porous rock.

The numerical simulations support the validity of this model. The model is particularly suited for computing effective properties of rock saturated with heavy oil. For very heavy oils, the viscosity is high and the material behaves like a quasi-solid. The classical Gassmann equation is inapplicable if the pore-filling material is a solid or a liquid whose shear modulus has a finite component. Our model attempts to overcome these limitations and is extendable for the viscoelastic pore-space infill.

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