The elastic properties of carbonates

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Although carbonates contain about 60% of the world's oil reserves, the characterization of carbonate reservoirs remains challenging because of their heterogeneity and complex microstructure. Central to the seismic characterization of carbonates is an understanding of the effect of porosity on the elastic moduli and seismic-wave velocities. Figure 1 shows the compressional and shear-wave velocities measured by Rafavich et al. (1984) for limestones and dolomites from the Williston Basin, North Dakota, measured on air-saturated samples. It is seen that although the velocities decrease with increasing porosity, there is considerable scatter of the data. In this article, the possible contribution of variations in pore shape to the elastic moduli of carbonates is discussed, the role of microcracks and fractures is assessed, and differences between the static and dynamic moduli are highlighted.

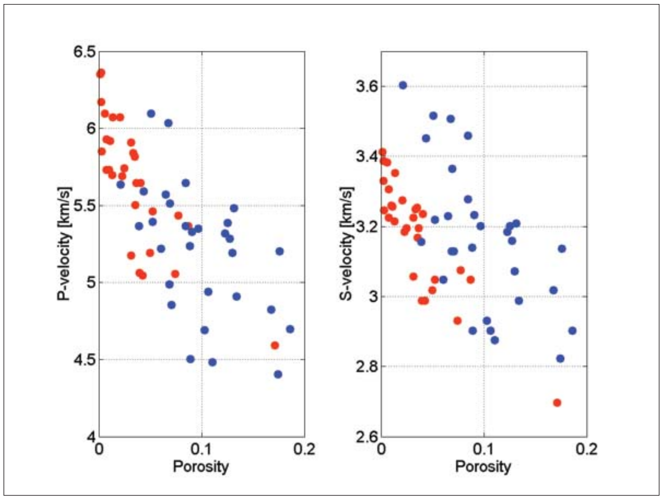

Figure 1. Compressional velocities (left) and shear velocities (right) as a function of porosity measured by Rafavich et al. on air-saturated limestone (red) and dolomite (blue) samples from the Williston Basin, North Dakota.

Mineral	Bulk modulus	Shear modulus	Density	Investigators
Calcite	76.8 GPa	32.0 GPa	2.71 g/cc	Simmons (1965)
Dolomite	76.4 GPa	49.7 GPa	2.87 g/cc	Nur and Simmons (1969)

Table 1. Bulk modulus K, shear modulus μ, and density ρ used in the calculations (references in Mavko et al., 1998).

Effect of pore shape on the elastic moduli of carbonates

In the following, effective field theory is used to examine the effect of pore shape on the elastic properties of carbonates. To calculate the effective elastic stiffness of a porous rock, effective field theories place a representative pore in an effective stress field. This contrasts with effective medium theories such as the self-consistent and differential medium schemes. In the self-consistent scheme proposed by O'Connell and Budiansky (1974) and Budiansky and O'Connell (1976), each pore is assumed to be embedded in a medium with the effective stiffness of the porous rock. Bruner (1976) and Henyey and Pomphrey (1982) pointed out that the self-consistent scheme may overestimate the interactions between pores, and proposed an alternative differential scheme in which the porosity is increased in small steps, and the elastic properties are recalculated incrementally. However, this scheme has the disadvantage of depending on the order in which differently oriented pores are input into the model (Sayers and Kachanov, 1991). In the following, the effective field theory of Mori and Tanaka (1973) is used. In this method the effective field is taken as the average stress in the matrix (see also Benveniste, 1987, and Zhao et al., 1989).

To calculate the elastic moduli requires knowledge of the elastic moduli of calcite and dolomite. The values of the bulk modulus K and shear modulus μ used in the computations are listed in Table 1. The pores are modeled as ellipsoidal voids with major and minor axes $a = b \geq c$. While pores in carbonates are unlikely to be exactly ellipsoidal, this geometry encompasses a wide range of possible shapes, from spherical to crack-like, in the limit $a = b >> c$. A random distribution of orientations is assumed. The results are shown in Figure 2 for limestones and dolomites. The bulk and shear moduli corresponding to the measurements were calculated from the compressional- and shear-wave velocities shown in Figure 1 using the density measurements reported by Rafavich et al. It is seen that the spread in the bulk and shear moduli can be explained by a distribution in aspect ratio of the pores as has been discussed by others (see, for example, Eberli et al., 2003; Kumar and Han, 2005; Baechle et al., 2007; Rossebø et al., 2007, and references therein).

Figure 3 compares a plot of the shear-wave velocity versus the compressional velocity for the air-filled samples with that predicted by the model. It is seen that despite the large influence of pore shape on the variation of velocity with porosity shown in Figure 2, the effect of pore shape on the relation between V_P and V_S shown in Figure 3 is relatively minor, the dominant effect being the variation in mineral composition of the samples.

Rafavich et al. also measured the elastic wave velocities for brine-saturated specimens. Figure 4 compares the saturated bulk and shear moduli calculated using Gassmann's theory with those calculated from the measured velocities using the saturated densities reported by Rafavich et al.

Figure 5 compares a plot of the shear wave velocity versus the compressional velocity for the brine-filled samples with that predicted by the model. In contrast to the case of dry carbonates, it is seen in Figure 5 that the effect of pore shape on the relation between V_P and V_S for saturated carbonates is significant.

Fracture characterization

It is clear from Figures 2 and 4 that, for a given porosity, low-aspect-ratio pores have the most significant effect in reducing the elastic moduli. Thus, seismic waves are expected to be

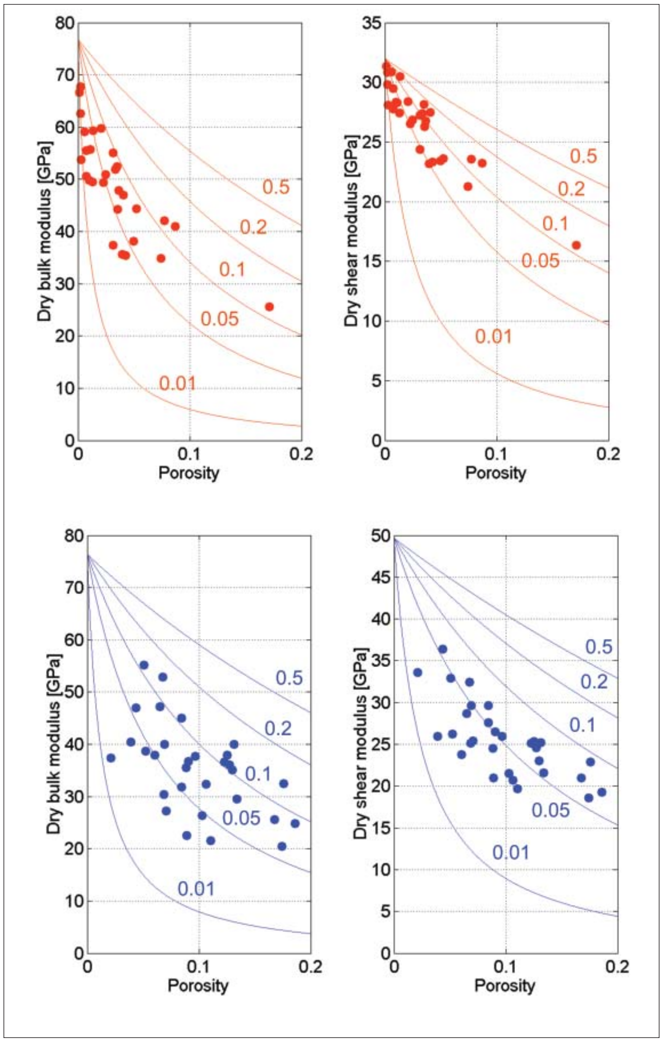

Figure 2. *Comparison of the values of the bulk modulus K (left) and shear modulus μ (right) computed using the effective field theory of Mori and Tanaka with the values obtained from the measurements of Rafavich et al. on air-saturated limestone (red) and dolomite (blue) samples from the Williston Basin, North Dakota. The aspect ratio of the voids is indicated on the curves.*

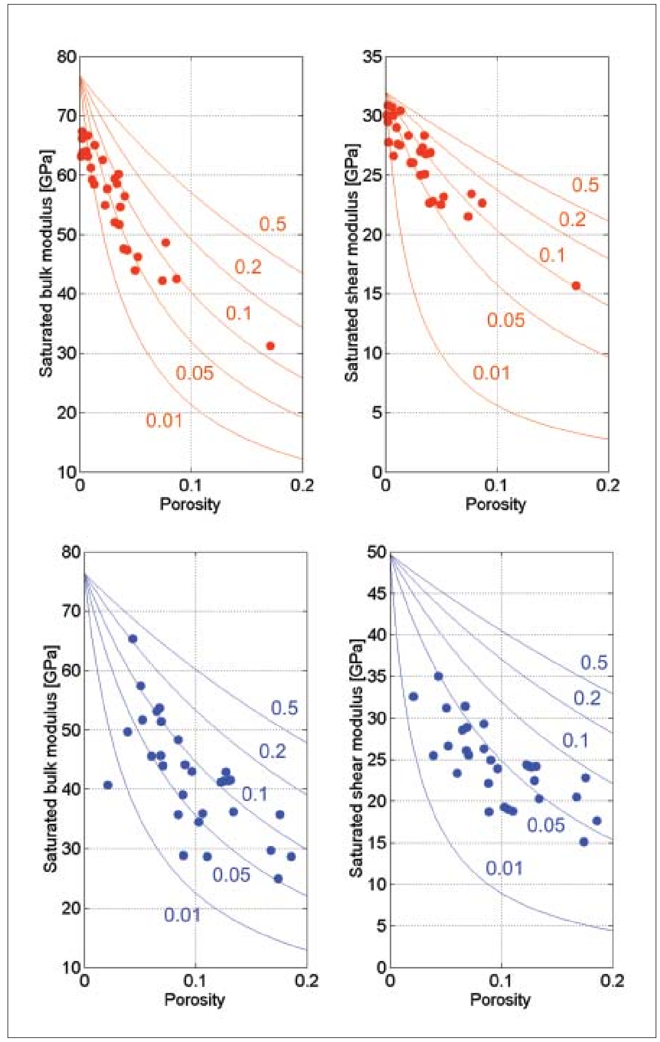

Figure 4. *Comparison of the values of the bulk modulus K (left) and shear modulus μ (right) computed using Gassmann's theory together with frame moduli calculated using the effective field theory of Mori and Tanaka with the values obtained from the measurements of Rafavich et al. on brine-saturated limestone (red) and dolomite (blue) samples from the Williston Basin, North Dakota. The aspect ratio of the voids is indicated on the curves.*

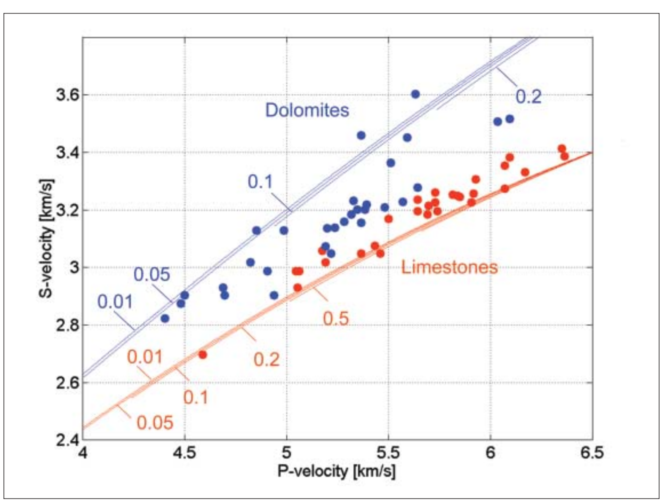

Figure 3. *Comparison of a plot of the shear-wave velocity versus the compressional velocity for the air-filled samples with that predicted by the model. The aspect ratio of the pores is indicated on the curves.*

particularly sensitive to the presence of low-aspect-ratio microcracks and fractures in the reservoir. Naturally occurring fractures in carbonate reservoirs may play a significant role in determining fluid flow during production, and areas of high fracture density may represent "sweet spots" of high permeability. Reflection amplitudes offer advantages over the use of seismic velocities for characterizing fractured reservoirs because they have higher vertical resolution and are more sensitive to the properties of the reservoir. Because carbonates have a significantly higher Poisson's ratio than sandstones, vertical fractures in carbonates have a much larger effect on the normal incidence acoustic impedance than in sandstones and may therefore be studied using a narrower range of incidence angles than is required to study vertical fractures in sandstones. This can be seen by considering vertically propagating P-waves in an isotropic background rock containing vertical fractures (Schoenberg and Sayers, 1995). Defining a

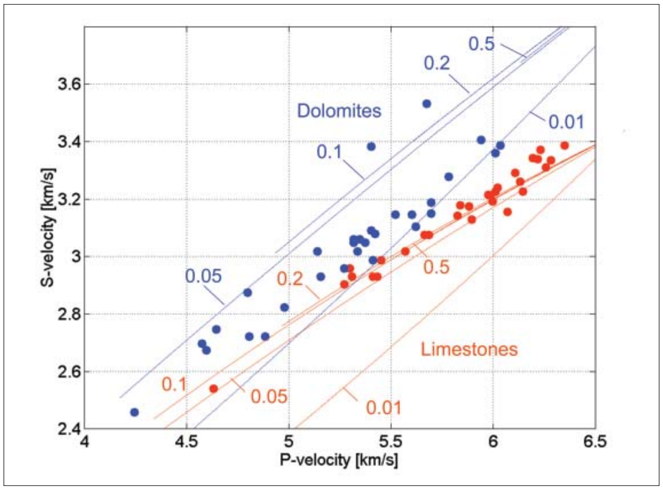

Figure 5. *Comparison of a plot of the shear-wave velocity versus the compressional velocity for the brine-saturated samples with that predicted by the model. The aspect ratio of the pores is indicated on the curves.*

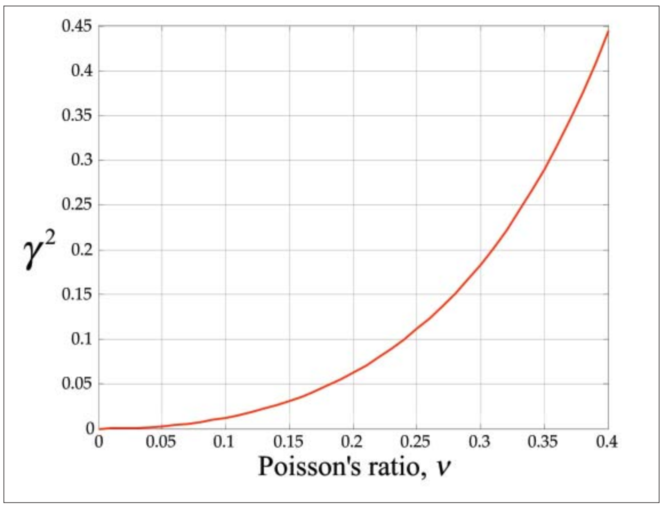

Figure 6. *Variation of γ^2 where $\gamma = v/(1-v)$ versus Poisson's ratio v.*

reference set of axes $x_1x_2x_3$ with x_3 vertical, the velocity v_{33} of vertically propagating P-waves is given by

$$v_{33} = \sqrt{c_{33}/\rho}$$

where ρ is the density, and the elastic stiffness coefficient c_{33} is given by

$$c_{33} = (\lambda + 2\mu)(1 - \gamma^2 \delta_N)$$

where λ and μ are the second order Lamé elastic stiffnesses of the background rock (assumed isotropic), $v = \lambda/(2(\lambda + \mu))$ is Poisson's ratio, $\gamma = v/(1-v)$, and δ_N is given by

$$\delta_N = \frac{(\lambda + 2\mu)Z_N}{1 + (\lambda + 2\mu)Z_N}$$

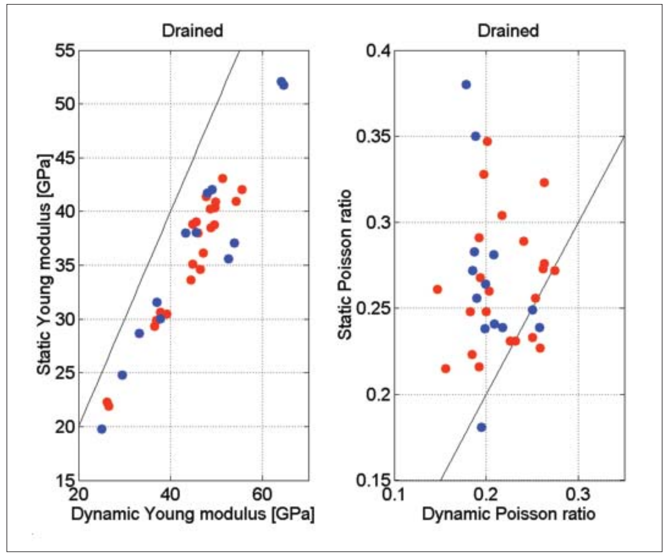

Figure 7. *Comparison of the dynamic and static Young's modulus (left) and Poisson's ratio (right) for limestones (red) and dolostones (blue) measured by Yale and Jamieson (1994) for air-filled samples from the Hugoton and Panoma fields, Kansas.*

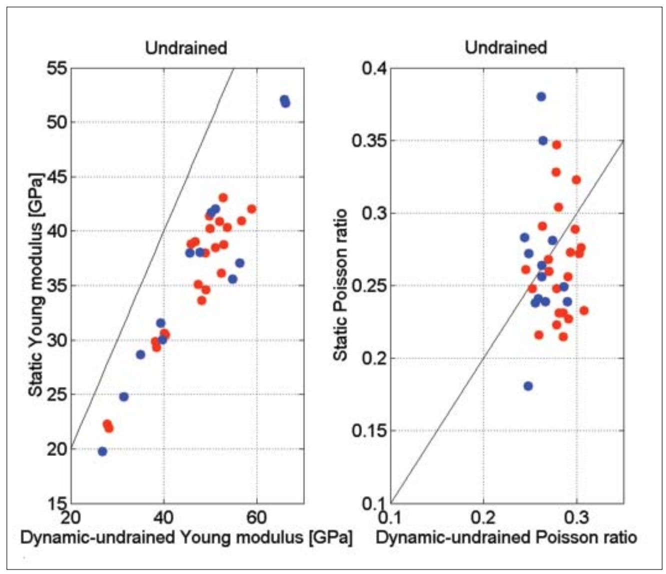

Figure 8. *Comparison of the dynamic undrained Young's modulus (left) and Poisson's ratio (right) calculated for water-filled samples using Gassmann's theory for limestones (red) and dolostones (blue) with the drained values measured by Yale and Jamieson on air-filled samples from the Hugoton and Panoma fields, Kansas.*

Here Z_N is the normal compliance of the fractures (Schoenberg and Sayers). Figure 6 plots γ^2 as a function of Poisson's ratio v. It is seen that a limestone with Poisson's ratio of 0.3 has a value of γ^2 that is about 15 times larger than a clean sandstone with Poisson's ratio 0.1. It follows that vertical fractures in carbonates have a much larger effect on the normal incidence acoustic impedance than in sandstones, and may therefore be studied using a narrower range of incidence angles than is required to study vertical fractures in sandstones.

Static elastic moduli

Of more relevance than dynamic moduli for geomechanics are the static moduli. The dynamic and static moduli for air-filled carbonate samples from the Hugoton and Panoma fields, Kansas, have been measured by Yale and Jamieson (1994), the results being shown in Figure 7. It is seen that the static Young's moduli are lower than the dynamic moduli, while for Poisson's ratio the reverse is true.

Yale and Jamieson suggest that the bulk of the difference between the static and dynamic elastic moduli is due to viscoelastic effects, since the frequency difference between the dynamic and static measurements is large enough to allow significant viscoelastic deformation in static tests that would not occur on the deformation timescale of the elastic wave used to measure the dynamic moduli. An example of such a viscoelastic deformation would be the inelastic deformation of microcracks and grain boundaries within the rock.

Figure 8 shows estimates of the dynamic undrained brine-saturated elastic moduli made using Gassmann's theory with the static drained measurements. While the effect of saturation by water, with assumed bulk modulus 2.25 GPa, on the Young's modulus as estimated using Gassmann theory is seen to be small, Poisson's ratio is significantly increased as expected. Coincidentally, the saturated Poisson's ratio estimates are much closer to the static drained Poisson's ratio than are the dynamic drained values (Yale and Jamieson).

Conclusion

The effect of ellipsoidal pores of varying aspect ratio on the elastic properties of carbonates predicted by the effective field theory of Mori and Tanaka have been compared with the measurements of Rafavich et al. While pores in carbonates are unlikely to be exactly ellipsoidal, this geometry encompasses a wide range of possible shapes, from spherical to crack-like, in the limit of one of the dimensions being much smaller than the other two. Although pore shape can account for the scatter in the velocity versus porosity trends for carbonates, it only has a small effect on the variation of P-wave versus S-wave velocity for dry rocks, the effect of mineral composition being greater. However, for fluid-saturated rocks the effect of pore shape on the relation between V_P and V_S is much greater. In agreement with the results of other workers, low-aspect-ratio pores cause the largest decrease in elastic moduli for a given porosity. Elastic wave velocities in carbonate reservoirs are particularly sensitive to the presence of low-aspect-ratio microcracks and fractures; therefore, because of the higher Poisson's ratio of carbonates in comparison to sandstones, vertical fractures in carbonates have a much larger effect on the normal incidence acoustic impedance than in sandstones and may therefore be studied using a narrower range of incidence angles than is required to study vertical fractures in sandstones. Because fractures are sensitive to changes in reservoir pressure, they may hold the key to time-lapse seismic monitoring of carbonates if the change in reservoir pressure is sufficient to cause opening or closure of the fractures. Finally, the presence of low-aspect-ratio pores may contribute to the observed difference in the dynamic and static elastic properties of carbonates.

Suggested reading. "Modeling velocity in carbonates using a dual-porosity DEM model" by Baechle et al. (SEG 2007 *Expanded Abstracts*). "A new approach to the application of Mori-Tanaka's theory in composite materials" by Benveniste (*Mechanics of Materials*, 1987). "Comment on 'Seismic velocities in dry and saturated cracked solids' by Richard J. O'Connell and Bernard Budiansky" by Bruner (*Journal of Geophysical Research*, 1976). "Elastic moduli of a cracked solid" by Budiansky and O'Connell (*International Journal of Solids and Structures*, 1976). "Factors controlling elastic properties in carbonate sediments and rocks" by Eberli et al. (*TLE,* 2003). "Self-consistent elastic moduli of a cracked solid" by Henyey and Pomphrey (*Geophysical Research Letters*, 1982). "Pore shape effect on elastic properties of carbonate rocks" by Kumar and Han (SEG 2005 *Expanded Abstracts*). *The Rock Physics Handbook* by Mavko et al. (Cambridge, 1998). "Average stress in matrix and average elastic energy of materials with misfitting inclusions" by Mori and Tanaka (*Acta Metallurgica*, 1973). "Seismic velocities in dry and saturated cracked solids" by O'Connell and Budiansky (*Journal of Geophysical Research*, 1974). "The relationship between acoustic properties and the petrographic character of carbonate rocks" by Rafavich et al. (GEOPHYSICS, 1984). "Modeling of acoustic properties in carbonate rocks" by Rossebø et al. (SEG 2005 *Expanded Abstracts*). "A simple technique for finding effective elastic constants of cracked solids for arbitrary crack orientation statistics" by Sayers and Kachanov (*International Journal of Solids Structures*, 1991). "Seismic anisotropy of fractured rock" by Schoenberg and Sayers (GEOPHYSICS, 1995). "Static and dynamic mechanical properties of carbonates" by Yale and Jamieson (*Rock Mechanics*, Balkema, 1994). "Elastic moduli for a class of porous materials" by Zhao et al. (*Acta Mechanica*, 1989). TLE

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