GEOPHYSICS, VOL. 60, NO. 1 (JANUARY-FEBRUARY 1995); P. 97–107, 10 FIGS., 1 TABLE.

Squirt flow in fully saturated rocks

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ABSTRACT

We estimate velocity/frequency dispersion and attenuation in fully saturated rocks by employing the squirt-flow mechanism of solid/fluid interaction. In this model, pore fluid is squeezed from thin soft cracks into the surrounding large pores. Information about the compliance of these soft cracks at low confining pressures is extracted from high-pressure velocity data. The frequency dependence of squirt-induced pressure in the soft cracks is linked with the porosity and permeability of the soft pore space, and the characteristic squirt-flow length. These unknown parameters are combined into one expression that is assumed to be a fundamental rock property that does not depend on frequency. The appropriate value of this expression for a given rock can be found by matching our theoretical predictions with the experimental measurements of attenuation or velocity. The low-frequency velocity limits, as given by our model, are identical to those predicted by Gassmann's formula. The high-frequency limits may significantly exceed those given by the Biot theory: the high-frequency frame bulk modulus is close to that measured at high confining pressure. We have applied our model to D'Euville Limestone, Navajo Sandstone, and Westerly Granite. The model realistically predicts the observed velocity/frequency dispersion, and attenuation.

INTRODUCTION AND MODEL DESCRIPTION

This paper continues our previous work (Dvorkin et al., 1993) where we described the squirt-flow mechanisms of solid/fluid interaction through macroscopic rock and fluid characteristics. Considering the squirt-flow mechanism as an addition or alternative to the Biot mechanism is important because in many cases Biot's theory greatly underestimates velocity/frequency dispersion and attenuation. Mavko and Nur (1979), Palmer and Traviolia (1980), Murphy et al. (1986), Miksis (1988), Wang and Nur (1990), and Akbar et al. (1993) have shown that the squirt-flow mechanism can indeed be responsible for the measured high attenuation and velocity dispersion. By applying a combined Biot/squirt model to a set of experimental data, Dvorkin and Nur (1993) show that, in many rocks, it is the squirt part of the combined Biot/squirt flow that is responsible for the measured large attenuation and velocity dispersion.

The latter model has been developed for partially saturated rocks, and for apparently fully saturated rocks with only small amounts of high-compressibility gas in the pores—a situation quite typical in natural reservoirs. In such rocks, the squirting flow is realized as cross-flow between pore space with fluid and gas-occupied pockets. In this paper, we concentrate on the description of the squirt-flow mechanism in saturated rocks without residual gas where the squirt-flow pattern becomes more complex (Figure 1): pore fluid is squeezed from thin cracks into surrounding large pores or adjacent cracks of different orientation (Mavko and Nur, 1975).

Following this physical picture, we estimate the influence of the squirt-flow mechanism on velocity/frequency dispersion and attenuation by incorporating the effects of pressure variation in thin cracks into the calculations of the effective bulk and shear moduli and/or compressional- and shear-wave velocities. This effect of local pressure is incorporated into the global rock/wave interaction picture in the way suggested by Biot (1962), Stoll and Bryan (1970), and Keller (1989). The elastic, dry mineral frame in Gassmann's (1951) formulation or in Biot's (1956) model is replaced with a viscoelastic frame. Mavko and Jizba (1991) have used this approach to find the velocity difference between low-frequency and high-frequency measurements in saturated rocks. Their theoretical estimates agree well with the measured velocity dispersion in such rocks as Westerly Granite, tight gas Sandstone, Navajo Sandstone, and Fontainebleau Sandstone. Here, we derive formulas for compressional- and shear-wave velocities and attenuation versus frequency. By doing so, we fill the missing link between the low-frequency (Gassmann's) formulas and high-frequency (e.g., Kuster and

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Toksöz, 1974; O'Connel and Budiansky, 1977; and Mavko and Jizba, 1991) estimates.

LIST OF PRINCIPAL SYMBOLS

c_0	fluid acoustic velocity
G	shear modulus of dry rock
G_m	shear modulus of modified frame
K	bulk modulus of dry rock
K_S	bulk modulus of the mineral phase
K_r	bulk modulus of saturated rock
K_{ms}	bulk modulus of modified solid
K_{msd}	bulk modulus of dry modified solid
K_m	bulk modulus of modified frame
K_{hP}	bulk modulus of dry rock at high pressure
K_f	pore fluid bulk modulus
k	permeability of soft pore space
P	pore pressure
R	characteristic squirt-flow length
μ	pore fluid viscosity
σ	hydrostatic confining stress
ρ	rock density
ρ_f	pore fluid density
ϕ	total porosity
ϕ_{so}	soft porosity
ϕ_{st}	stiff porosity
ϕ_s	$\phi_{so}/(1-\phi_{st})$.

The approach used in our model is based on the experimental observation that at high confining pressure, velocity/frequency dispersion is small and can be adequately described by Biot's theory (Mavko and Jizba, 1991). Thus we conclude that the thin compliant pores and fractures that close at high pressure are responsible for the observed large velocity/frequency dispersion and attenuation at low pressures. This conclusion allows us to find the viscoelastic frame parameters by modeling the squirt flow of pore fluid between the compliant (soft) pores and the main (stiff) pore space.

In this model, information about the compliance of the soft cracks at low confining pressures is extracted from high-pressure velocity data. Describing the frequency dependence of squirt-induced pressure in the soft cracks requires additional knowledge. This additional information includes the porosity and permeability of the soft pore space, and the characteristic squirt-flow length (R). We combine these parameters into one expression R^2/κ where κ is the diffusivity of the soft pore space. This expression is assumed to be a fundamental rock property that does not depend on frequency. The appropriate value of this expression for a given rock can be found by matching our theoretical predictions with the experimental measurements of attenuation or velocity.

By applying our model to such rocks as Westerly Granite, D'Euville Limestone, and Navajo Sandstone, we show that it realistically predicts the observed velocity/frequency dispersion and attenuation.

MODIFIED FRAME AND MODIFIED SOLID

We consider a fully saturated isotropic, macroscopically homogeneous rock of porosity ϕ with a pore space that has compliant (soft) and stiff portions (Figure 2). Stiff portions of the pore space have relatively small variations of pore pressure when the rock is loaded by a passing wave. Soft portions of the pore space tend to transfer more of the stress to the fluid, which results in high variations of induced pore pressure. At low frequencies, one expects these pressures to equilibrate—an assumption leading to Gassmann's formula for the bulk modulus of saturated rock. At higher frequencies, intensive cross-flow between the soft and the stiff parts persists resulting in wave-energy dissipation and thus velocity/frequency dispersion. At very high frequencies, the fluid is unrelaxed and blocked in the compliant pores. Attenuation approaches zero and both P- and S-wave velocities approach their high-frequency limits.

Consider the modified solid phase of the rock—the phase that includes the actual solid phase plus the soft pore space (Figure 2). The modified solid has bulk modulus K_{ms}. The frame that corresponds to this modified solid is the modified dry frame of porosity ϕ_{st} and bulk modulus K_m (the modified solid may be dry, or saturated). It is clear that $\phi = \phi_{st} + \phi_{so}$, where ϕ_{so} is the porosity due to the soft-pore volume. The values of stiff porosity ϕ_{st} are typically very close to the total porosity ϕ (Mavko and Jizba, 1991).

In our further analysis, we will use the following relation among the bulk modulus of the mineral K_S, the bulk modulus of the dry frame K, and dry pore-volume V_P:

$$\frac{1}{K} - \frac{1}{K_S} = \frac{1}{V}\frac{dV_P}{d\sigma},$$

where σ is hydrostatic confining stress, and V is the total volume of the rock (Walsh, 1965). Applying this relation to

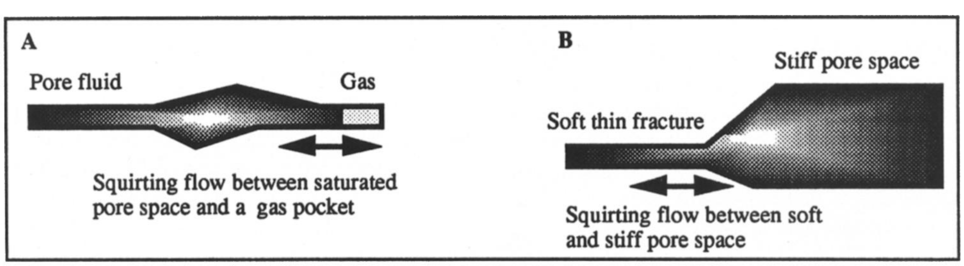

FIG. 1. (a) Squirting flow in undersaturated rocks–pore fluid moves between the saturated pore space and gas-filled pockets. (b) Squirt in fully saturated rocks–pore fluid exchange between the soft and stiff parts of pore space.

the dry modified frame (as we mentioned above, the modified solid may be saturated) we have:

$$\frac{1}{K_m} - \frac{1}{K_{ms}} = \frac{1}{V}\frac{dV_{P\,\mathrm{stiff}}}{d\sigma}, \tag{1}$$

where $V_{P\,\mathrm{stiff}}$ is the volume of the stiff pores.

Following Mavko and Jizba (1991), we assume that at very high confining pressure, the compliant, soft pores close and, therefore, the bulk modulus of the modified solid approaches that of the actual solid:

$$\frac{1}{K_{hP}} - \frac{1}{K_S} = \frac{1}{V}\frac{dV_{P\,\mathrm{stiff}}}{d\sigma}, \tag{2}$$

where K_{hP} is the bulk modulus of the dry frame at high pressure. In this formula we assume that the expression in the right-hand side is approximately pressure-independent.

We now use equations (1) and (2) to relate the bulk moduli of the modified solid and the modified frame:

$$\frac{1}{K_m} = \frac{1}{K_{ms}} + \frac{1}{K_{hP}} - \frac{1}{K_S}, \tag{3}$$

where both K_m and K_{ms} are as yet unknown.

If the modified solid is dry (its bulk modulus K_{msd}), the bulk modulus of the modified frame is that of the actual dry frame. Therefore, it follows from equation (3) that

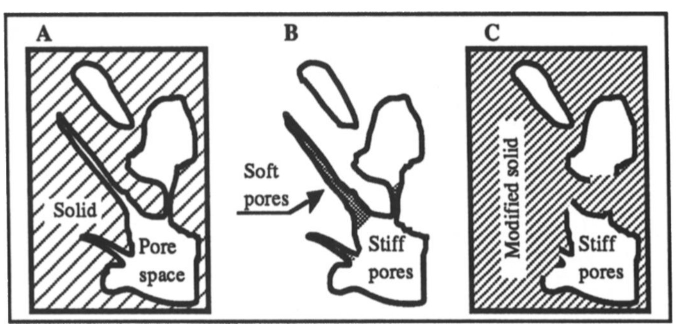

FIG. 2. (a) Porous rock. (b)Pore space—soft and stiff parts. (c) Modified solid includes the actual solid phase and the soft pore space; the modified frame has porosity which is equal to the stiff porosity of the rock.

$$\frac{1}{K_{msd}} \cong \frac{1}{K_S} - \frac{1}{K_{hP}} + \frac{1}{K}. \tag{4}$$

MODIFIED SOLID—VISCOELASTIC BEHAVIOR

To find the bulk modulus of the modified solid with pore fluid in the compliant cracks, we assume that it can be replaced by a microscopic representative volume of porosity $\phi_S = \phi_{so}/(1 - \phi_{st})$. This volume is chosen to be a cylinder of radius R—the characteristic squirt-flow length (Dvorkin and Nur, 1993). The choice of such representative volume geometry (Figure 3) is consistent with the physical picture of the radial flow of the pore fluid from compliant pores surrounding grain contacts into the stiff pore space (e.g., Murphy et al., 1986).

We model the radial 1-D squirting flow in the representative cylinder by using the equation of fluid mass conservation, the poroelastic (Biot's) relations among strains, stresses, and pore pressure, Darcy's law for filtration, and the constitutive law for the fluid's compressibility. As the boundary condition, we employ the relation between the confining-pressure increment $d\sigma$ and the pore-pressure increment dP at the cylinder's lateral surface. The resulting expression for the bulk modulus of the modified solid is (Appendix A):

$$K_{ms} = \frac{K_{msd} + \dfrac{\alpha^2 F}{\phi_S}[1 - f(\xi)]}{1 + \alpha f(\xi) dP/d\sigma}, \tag{5}$$

where

$$\frac{1}{F} = \frac{1}{K_f} + \frac{1}{\phi_S Q}, \quad \alpha = 1 - \frac{K_{msd}}{K_S}, \quad Q = \frac{K_S}{\alpha - \phi_S},$$

$$f(\xi) = \frac{2J_1(\xi)}{\xi J_0(\xi)}, \quad \xi = \sqrt{i\omega}\,Z, \quad Z = \sqrt{\frac{R^2\mu\phi_S}{kF}},$$

J_0 and J_1 are Bessel functions of zero and first order, μ is the fluid's viscosity, k is the permeability of the soft pore space, and K_f is the fluid's bulk modulus.

It is important that all these parameters are grouped into a single expression:

$$Z^2 = \frac{R^2\mu\phi_S}{kF} = \frac{R^2\mu\phi_S}{k}\left(\frac{1}{K_f} + \frac{1}{\phi_S Q}\right) = \frac{R^2}{\kappa},$$

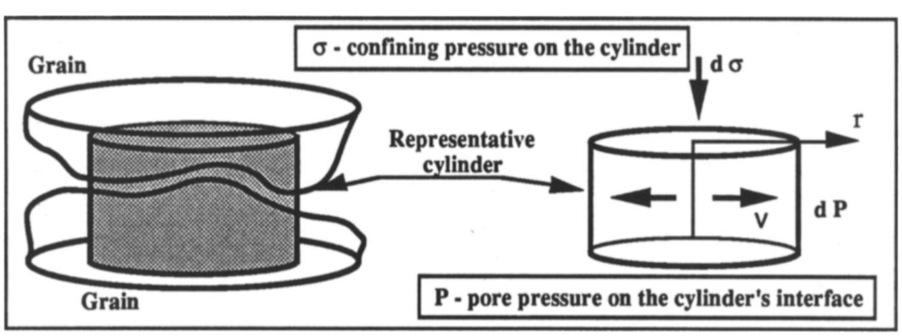

FIG. 3. The modified solid modeled as the representative cylinder.

where κ is the diffusivity of the soft pore space [the derivation of the diffusivity expression is given in Dvorkin et al. (1993)]. Therefore, there is no need for a separate determination of the characteristic squirt-flow length and the other parameters of the soft pore space. Indeed, if R is assumed to be a fundamental property of a rock, which does not depend on frequency or fluid viscosity, all we have to do is to find parameter Z from matching theoretical and experimental values of velocities and/or attenuation.

Strictly speaking, the ratio $dP/d\sigma$ has to be found by incorporating as yet unknown constants of the modified solid and modified frame into Gassmann's equation or Biot's formulas. For simplicity, we assume that the desired value of $dP/d\sigma$ is approximately equal to that given by Gassmann's formula in the actual rock (Appendix B):

$$\frac{dP}{d\sigma} = -\frac{1}{\alpha_0\left(1 + \dfrac{K\phi}{\alpha_0^2 F_0}\right)} \tag{6}$$

with

$$\frac{1}{F_0} = \frac{1}{K_f} + \frac{1}{\phi Q_0}, \quad \alpha_0 = 1 - \frac{K}{K_S}, \quad Q_0 = \frac{K_S}{\alpha_0 - \phi}.$$

This approximation is accurate since the variation of pore pressure predicted by Biot's theory often results in a relatively small increase in the high-frequency velocity above Gassmann's low-frequency limit.

Formula (5) can be simplified for $\phi_{so} \ll 1$ (Appendix C):

$$\alpha^2 F/\phi_S \approx \alpha K_S, \quad K_{ms} \approx \frac{K_{msd} + \alpha K_S[1 - f(\xi)]}{1 + \alpha f(\xi)\, dP/d\sigma},$$

$$Z \approx \sqrt{\frac{R^2 \mu \alpha}{k K_S}}.$$

Physically, this approximation means that at the high-frequency limit, K_{ms} is close to K_S, and K_m approaches K_{hP}. The latter result is consistent with the approximation given by Mavko and Jizba (1991).

MODIFIED FRAME—VISCOELASTIC BEHAVIOR

The bulk modulus of the modified frame K_m can now be found from equations (3), (4), (5), and (6). The bulk modulus of the saturated rock K_r can be found by using the moduli K_m and K_{ms} in Gassmann's equation (Appendix B):

$$K_r = \frac{K_m}{1 + \alpha_m\, dP/d\sigma}, \qquad \alpha_m = 1 - \frac{K_m}{K_{ms}}.$$

The use of Gassmann's formula here is again justified by the fact that Biot's theory gives small velocity/frequency dispersion.

We find the shear modulus of the modified frame G_m by using, similar to Mavko and Jizba (1991), the following assumptions: (1) the shear-induced pore pressure and pore volume compressibility of a soft thin pore are determined by the normal (to the pore's surface) component of the shear stress field, and (2) the effect of this normal component is identical to that for bulk compression. The resulting formula is (Appendix D):

$$\frac{1}{G} - \frac{1}{G_m} = \frac{4}{15}\left(\frac{1}{K} - \frac{1}{K_{md}}\right), \tag{7}$$

where

$$\frac{1}{K_{md}} = \frac{1}{\tilde{K}_{ms}} + \frac{1}{K_{hP}} - \frac{1}{K_S}, \tag{8}$$

$$\tilde{K}_{ms} = K_{msd} + \frac{\alpha^2 F}{\phi_S}[1 - f(\xi)],$$

and G is the shear modulus of the dry frame.

Again, similar to the previous section, we can use an approximation

$$\alpha^2 F/\phi_S \approx \alpha K_S.$$

When estimating the shear modulus of saturated rock, we assume (as in Gassmann's formalism) that it is equal to the shear modulus of the modified frame as given by equation (7).

VELOCITIES AND ATTENUATION

We calculate compressional- and shear-wave velocities V_P and V_S as (Toksöz and Johnston, 1981)

$$V_P = \sqrt{\frac{\mathrm{Re}\,(K_r + \frac{4}{3}G_m)}{\rho}}, \qquad V_S = \sqrt{\frac{\mathrm{Re}\,(G_m)}{\rho}},$$

where ρ is rock density. Inverse quality factors Q_P^{-1} and Q_S^{-1} are:

$$Q_P^{-1} = \left|\frac{\mathrm{Im}\,(K_r + \frac{4}{3}G_m)}{\mathrm{Re}\,(K_r + \frac{4}{3}G_m)}\right|, \qquad Q_S^{-1} = \frac{|\mathrm{Im}\,(G_m)|}{|\mathrm{Re}\,(G_m)|}.$$

A step-by-step procedural outline for implementing the theory developed in this paper is given in Appendix E.

ANALYSIS AND EXAMPLES

It follows from formulas (3), (4), (5), and (6) that at zero frequency, the bulk modulus of a saturated rock is that predicted by Gassmann's formula. The bulk modulus of the modified frame increases with increasing frequency and becomes close to the high-pressure dry modulus at the high-frequency limit. The decrease in shear compliance of the modified frame, as frequency changes from zero to infinity, is proportional to the decrease in its bulk compressibility. These results are consistent with the conclusions drawn by Mavko and Jizba (1991).

Example 1: D'Euville Limestone

In this example we use the data of Lucet (1989) obtained on a limestone sample of 18 percent porosity. Compressional- and shear-wave velocities and attenuation were measured at a frequency of 500 kHz using the pulse-transmission method. These were measured also at sonic frequencies of 2.5 kHz–6.1 kHz (the resonant-bar method). The measurements were conducted on a water-saturated sample at varying effective pressure (from 0 to 30 MPa).

We applied Gassmann's equation to the sonic data to obtain the dry elastic properties of the rock (Table 1). The assumption was that velocity/frequency dispersion is small at these relatively low frequencies. High-pressure (40 MPa) sonic velocities in the saturated rock were: V_P = 4643 m/s and V_S = 2228 m/s. The density of the solid phase was 2710 kg/m^3 with a bulk modulus of 62 GPa (Carmichael, 1989).

Then we determined the Z parameter in equation (5) by matching the theoretical to the observed V_P. The result was $Z = 0.001$ for 3 MPa differential pressure, and $Z = 0.0011$ for 5 MPa differential pressure. The theoretical results for velocities, attenuation, and ultrasonic measurements are given in Figure 4 for 3 MPa differential pressure. We observe a reasonable quantitative correlation between the predicted and the measured values of ultrasonic shear-wave velocity, and shear- and compressional-wave attenuation.

The results for a 5 MPa differential pressure are given in Figure 5. Again, by adjusting the compressional-wave velocity only, we were able to obtain a reasonable correlation between the predicted and the measured values of ultrasonic shear-wave velocity and shear- and compressional-wave attenuation.

Example 2: Navajo Sandstone

In this example we use the data of Coyner (1984) obtained on a Navajo Sandstone sample of 11.8 percent porosity. Compressional- and shear-wave velocities were measured versus differential pressure in the vacuum-dry and benzene-saturated rock at a frequency of 1 MHz (the pulse-transmission method). The density and bulk modulus of benzene as measured at 10 MPa pressure are 880 kg/cu m and 1.21 GPa, respectively. The measured density of the solid phase and its bulk modulus are 2630 kg/m^3 and 36 GPa, respectively. Dry-rock compressional- and shear-wave velocities at a 100 MPa differential pressure were 4863 m/s and 3210 m/s, respectively.

We use our theoretical model to predict velocity versus frequency at differential pressures of 2.5 MPa and 5.0 MPa (Figure 6). As in the previous example, we determined the Z parameter in equation (5) by matching the theoretical to the observed V_P. The result was Z = 0.00185 for 2.5 MPa differential pressure, and Z = 0.00275 for 5 MPa differential pressure. By adjusting the compressional-wave velocity only, we were able to obtain a good correlation between the predicted and the measured values of shear-wave velocity.

Example 3: Westerly Granite

In this example we use the data of Coyner (1984) obtained on a Westerly Granite sample of 0.8 percent porosity. Compressional- and shear-wave velocities were measured versus differential pressure in the vacuum-dry and water-saturated rock at a frequency of 1 MHz. The measured density of the solid phase and its bulk modulus are 2640 kg/m^3 and 56 GPa, respectively. Dry-rock compressional- and shear-wave velocities at a 100 MPa differential pressure were 5810 m/s and 3435 m/s, respectively.

We predict velocity versus frequency at differential pressures of 10 MPa and 25 MPa (Figure 7). Parameter Z for these two cases, as found by matching the V_P data, was 0.0016 and 0.0028, respectively. As in the previous examples, we observe a good correlation between the predicted and the measured values of shear-wave velocity.

DISCUSSION

Our theoretical model requires as input the following measurable properties of rock and fluid: porosity, dry-rock velocities at a given effective pressure, and at high differential pressure, solid-phase density and compressibility, and fluid density and compressibility. An additional input parameter Z, which describes the viscoelastic behavior of the rock is a combination of the characteristic squirt-flow length (R), "soft porosity," soft-pore-space permeability, fluid viscosity and compressibility, and elastic properties of the dry skeleton, and of the solid phase. All these constants are combined into a single expression:

$$Z = \sqrt{R^2/\kappa},$$

where κ is the diffusivity of the soft-pore space. Therefore, there is no need to measure all these (some of them are hard to determine) constants. All we have to do is fit our theoretical prediction of one of the velocities in a saturated rock to its measured value. The corresponding Z value is the desired one (under the assumption that it is a fundamental rock property that does not depend on frequency). We recommend that a P-wave velocity be used for this adjustment because it has a larger frequency dispersion than the S-wave velocity. The above examples show that it is practically possible to find the Z value by using V_P measurements, and then to reliably predict V_S and attenuation based on this value.

Parameter Z is proportional to the square root of fluid viscosity. Therefore, once determined for a given fluid, it can be adjusted for another fluid of known viscosity.

Notice that in all above examples, parameter Z consistently increases with increasing confining pressure. This behavior has a physical basis, because Z is inversely proportional to the square root of the permeability of the soft pore space. This permeability sharply decreases as the soft pores close with increasing differential pressure.

Table 1. Measured velocities and attenuation in a water-saturated limestone sample (after Lucet, 1989). Sonic V_P values were calculated from the extensional- and shear-wave velocities. Dry velocities were calculated from the sonic velocities using Gassmann's formula.

Dif. pres. MPa	Sonic V_P m/s	Sonic V_S m/s	Dry V_P m/s	Dry V_S m/s	Ultrasonic V_P m/s	Ultrasonic V_S m/s	Sonic Q_P	Sonic Q_S	Ultrasonic Q_P	Ultrasonic Q_S
3	3550	1981	3181	2059	4222	2209	14.7	58.0	5.0	7.0
5	3845	2059	3628	2140	4269	2229	16.2	80.0	6.0	7.0

Applications

The examples show that in water-saturated samples, the transition from low-frequency velocity values to high-frequency velocity values occurs at frequencies higher than 0.1 MHz. Such high frequencies are above the range used in seismic prospecting. In this case our theoretical model can be used to correct laboratory ultrasonic data for interpreting practical seismic measurements.

In heavy-oil-bearing reservoirs, the transition from low-frequency velocity values to high-frequency velocity values may occur within the seismic frequency range. Consider Navajo Sandstone (from our second example) saturated with heavy crude oil. For reservoir conditions at about 40° C, such oil may have a viscosity up to 200 centipoise. We assumed that its density is 920 kg/m^3 and its acoustic velocity is 1430 m/s (the data from Wang and Nur, 1988). P- and S-wave velocities are plotted versus frequency in Figure 8. In this case, velocity/frequency correction is needed for practical purposes of seismic prospecting: at the low frequencies used, for example, in 3-D seismic, V_P is about 4360 m/s. This velocity increases to about 4600 m/s at 1 kHz—a frequency commonly used in well-to-well tomography. The corresponding increase in V_S is about 86 m/s.

CONCLUSIONS

We have introduced theoretical formulas for relating compressional- and shear-wave velocities and attenuation in fully saturated rocks to frequency. The formulas are based on the squirt-flow mechanism with the squirting flow occurring between thin compliant pores and stiff pores. The information about the compliance of the soft-pore space is obtained by comparing dry-rock velocities at given (low)

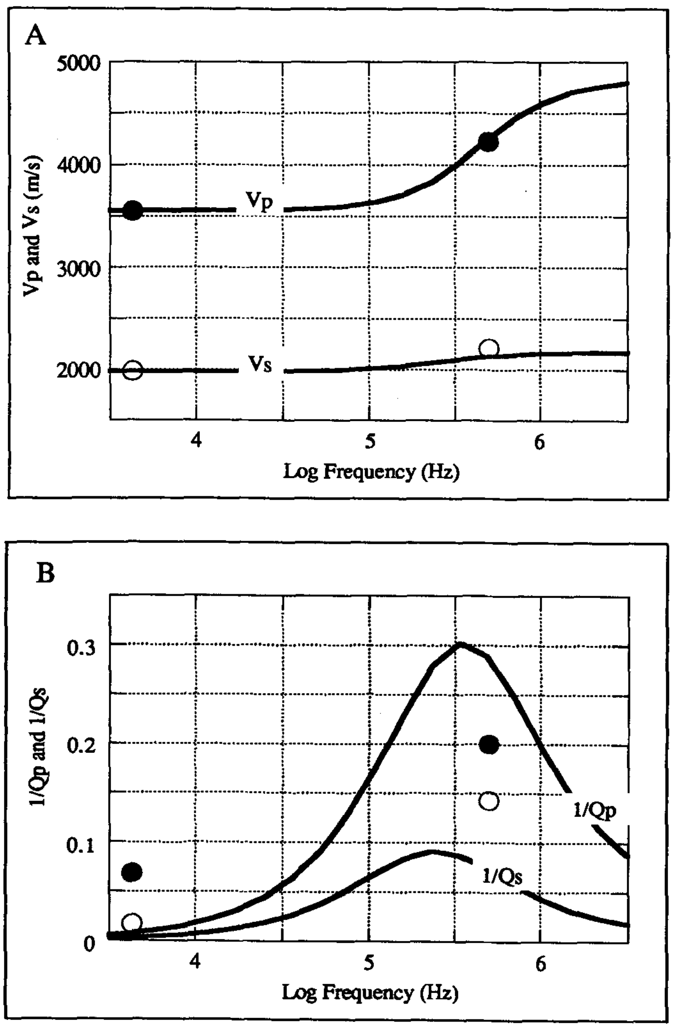

FIG. 4. Velocities and attenuation for the limestone sample from Lucet (1989). Differential pressure 3 MPa. (a) Theoretical and observed values for the velocities. (b) Theoretical and observed values for the attenuation. Filled circles represent the observed values for compressional waves; empty circles represent the observed values for shear waves.

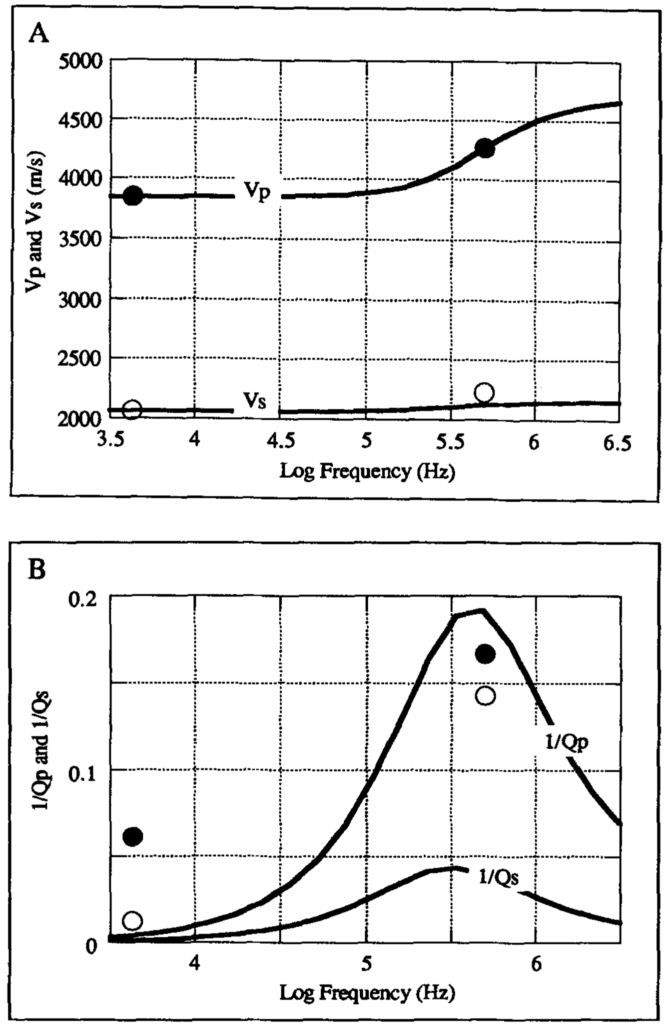

FIG. 5. Velocities and attenuation for the limestone sample from Lucet (1989). Differential pressure 5 MPa. (a) Theoretical and observed values for the velocities. (b) Theoretical and observed values for the attenuation. Filled circles represent the observed values for compressional waves, empty circles represent the observed values for shear waves.

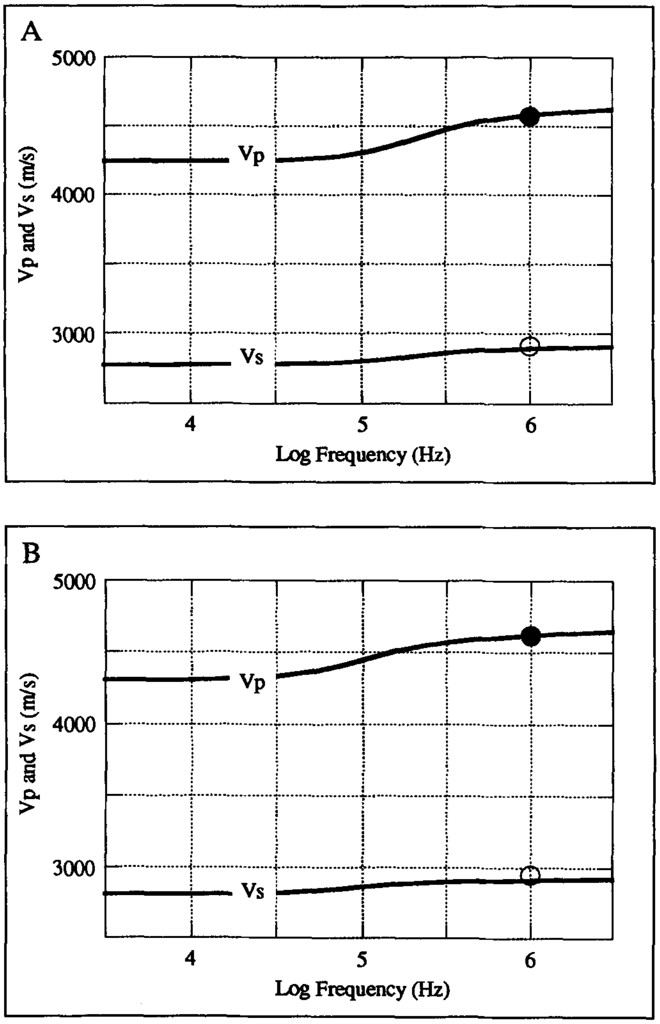

FIG. 6. Compressional- and shear-wave velocities for the Navajo Sandstone sample from Coyner (1984). (a) Theoretical and observed velocity values for a differential pressure of 2.5 MPa. (b) Differential pressure 5.0 MPa. Filled circles represent the observed values for compressional waves; empty circles represent the observed values for shear waves. The adjustment was done using the compressional-wave velocity only.

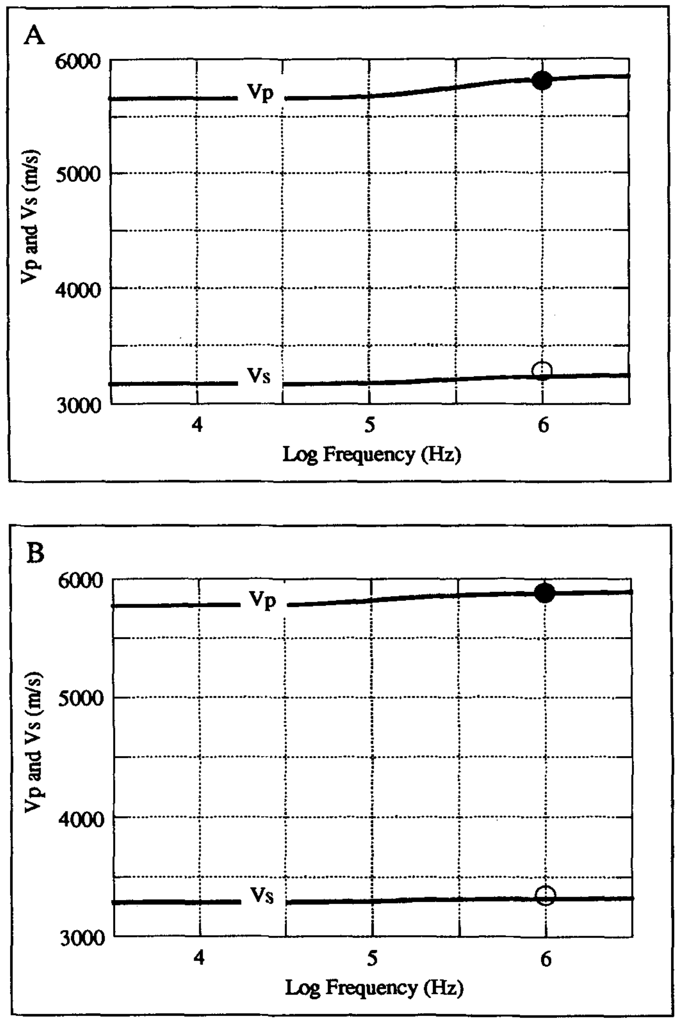

Fig. 7. Compressional- and shear-wave velocities for the Westerly Granite sample from Coyner (1984). (a) Theoretical and observed velocity values for a differential pressure of 10 MPa. (b) Differential pressure 25 MPa. Filled circles represent the observed values for compressional waves; empty circles represent the observed values for shear waves. The adjustment was done using the compressional-wave velocity only.

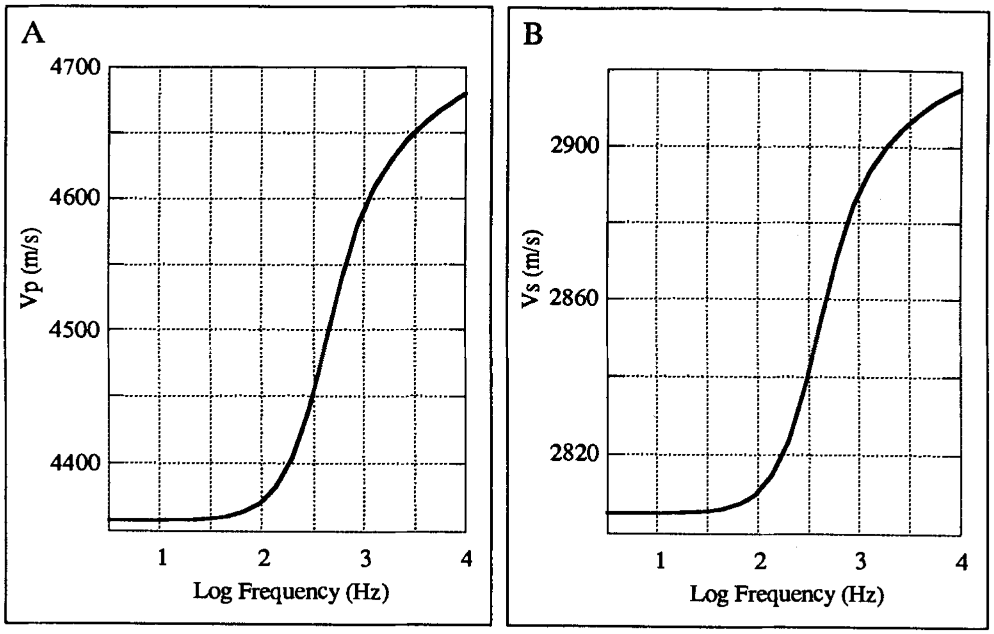

FIG. 8. Theoretical compressional- and shear-wave velocities in Navajo Sandstone saturated with heavy crude oil. (a) V_P and (b) V_S.

confining pressure with high-pressure velocity measurements. The low-frequency velocity limits, as given by our model, are identical to those predicted by Gassmann's formula. The high-frequency limits may significantly exceed those given by the Biot theory: the high-frequency frame bulk modulus is approximately equal to that measured at high confining pressure. The viscoelastic behavior of rocks in our formulation is determined by one fundamental parameter that depends on the characteristic squirt-flow length and diffusivity of the soft pore space. This parameter can be found by matching theoretical predictions with measured velocities and attenuation. The velocity/frequency correction offered by our model may be necessary for interpreting seismic signatures of reservoirs with heavy oil.

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APPENDIX A

VISCOELASTIC BEHAVIOR OF MODIFIED SOLID

We define the modified solid as the phase that includes the actual solid phase plus the soft-pore space (Figure 2). Its porosity, therefore, is

$$\phi_S = \frac{\phi_{so}}{1 - \phi_{st}}.$$

To explore the viscoelastic behavior of the modified solid, we model it as a representative cylinder of radius R (Figure 3). We assume that this cylinder undergoes uniform bulk deformation ε because of a uniform (total) external stress σ that acts on the rock. From Biot's constitutive equations (e.g., Dvorkin and Nur, 1993) we relate the changes in the deformation, porosity, and pore pressure P as:

$$d\phi_S = \alpha d\varepsilon + \frac{dP}{Q}, \tag{A-1}$$

where α and Q are defined following equation (5).

The change in the fluid density $d\rho_f$ is related to the pressure change dP as:

$$d\rho_f = \frac{dP}{c_0^2}, \tag{A-2}$$

where c_0 is the fluid's acoustic velocity.

We assume that pore fluid inside the cylinder can flow in the radial direction r only (Figure 3), similar to the fluid flowing in a radial gap around a grain-to-grain contact. For such axisymmetrical flow, the equation of the fluid's mass conservation is:

$$\frac{1}{\rho_f}\frac{\partial \rho_f}{\partial t} + \frac{1}{\phi_S}\frac{\partial \phi_S}{\partial t} + \frac{\partial}{\partial t}\left(\frac{\partial \nu}{\partial r} + \frac{\nu}{r}\right) = 0, \tag{A-3}$$

where t is time, and ν is the fluid's displacement.

Substituting equations (A-1) and (A-2) into (A-3) we have:

$$\frac{1}{\rho_f c_0^2}\frac{\partial P}{\partial t} + \frac{1}{\phi_S}\left(\alpha\frac{\partial \varepsilon}{\partial t} + \frac{1}{Q}\frac{\partial P}{\partial t}\right) + \frac{\partial}{\partial t}\left(\frac{\partial \nu}{\partial r} + \frac{\nu}{r}\right) = 0. \tag{A-4}$$

We relate the flow velocity $\partial\nu/\partial t$ to pore pressure gradient by Darcy's law:

$$\frac{\partial \nu}{\partial t} = -\frac{k}{\mu\phi_S}\frac{\partial P}{\partial r}, \tag{A-5}$$

where κ is the permeability of the soft pore space.

We assume in addition that all parameters are time-harmonic with time-factor

$$\exp(-i\omega t), \tag{A-6}$$

where ω is angular frequency. Using equation (A-6), we transform equations (A-4) and (A-5) to:

$$\left(\frac{1}{K_f} + \frac{1}{\phi_S Q}\right)P + \frac{\alpha\varepsilon}{\phi_S} + \frac{\partial \nu}{\partial r} + \frac{\nu}{r} = 0,$$

$$\nu = \frac{k}{i\omega\mu\phi_S}\frac{\partial P}{\partial r},$$

where $K_f = \rho_f c_0^2$ is the fluid's bulk modulus.

The last two equations give an ordinary differential equation for P:

$$\frac{d^2P}{dr^2} + \frac{1}{r}\frac{dP}{dr} + \frac{i\omega\mu\phi_S}{k}\left(\frac{1}{K_f} + \frac{1}{\phi_S Q}\right)P = -\varepsilon\frac{\alpha}{\phi_S}\frac{i\omega\mu\phi_S}{k}.$$

We solve this equation with the boundary condition $P = dP$ at $r = R$, where dP is the variation of pore pressure on the surface of the cylinder caused by the bulk deformation of the rock.

The solution is:

$$P = dP\frac{J_0(\lambda r)}{J_0(\lambda R)} - \varepsilon\frac{\alpha}{\phi_S}\frac{1}{\dfrac{1}{K_f} + \dfrac{1}{\phi_S Q}}\left[1 - \frac{J_0(\lambda r)}{J_0(\lambda R)}\right],$$

where

$$\lambda^2 = \frac{i\omega\mu\phi_S}{k}\left(\frac{1}{K_f} + \frac{1}{\phi_S Q}\right).$$

Average pressure (with respect to the r coordinate) is:

$$P_{av} = \frac{1}{\pi R^2}\int_0^R 2\pi r P(r)\,dr$$

$$= dP\frac{2J_1(\lambda R)}{\lambda R J_0(\lambda R)} - \varepsilon\frac{\alpha}{\phi}\frac{1}{\dfrac{1}{K_f} + \dfrac{1}{\phi_S Q}}\left[1 - \frac{2J_1(\lambda R)}{\lambda R J_0(\lambda R)}\right]. \tag{A-7}$$

In the modified solid, total external stress σ is related to deformation ε and average pore pressure P_{av} as (Biot, 1941; Rice and Cleary, 1976; and Dvorkin and Nur, 1993):

$$d\sigma = K_{msd}d\varepsilon - \alpha dP_{av}. \tag{A-8}$$

Now using the following definition of the bulk modulus of the modified solid:

$$K_{ms} = \frac{d\sigma}{d\varepsilon}, \tag{A-9}$$

we find from equations (A-7), (A-8), and (A-9):

$$K_{ms} = \frac{K_{msd} + \dfrac{\alpha^2 F}{\phi_S}\left[1 - \dfrac{2J_1(\lambda R)}{\lambda R J_0(\lambda R)}\right]}{1 + \alpha\dfrac{2J_1(\lambda R)}{\lambda R J_0(\lambda R)}\dfrac{dP}{d\sigma}},$$

which gives equation (5).

APPENDIX B

GASSMANN'S FORMULA

Consider rock of porosity ϕ, dry bulk modulus K, and solid phase bulk modulus K_S, saturated with fluid of density ρ_f, and acoustic velocity c_0. The rock is under uniform compression with deformation ε caused by total stress σ. Pore pressure is P.

Total stress, volumetric deformation and pore pressure are related as (Biot, 1941; Rice and Cleary, 1976; and Dvorkin and Nur, 1993):

$$d\sigma = Kd\varepsilon - \alpha dP, \qquad \alpha = 1 - K/K_S; \tag{B-1}$$

porosity, deformation, and pore pressure are related as:

$$d\phi = \alpha d\varepsilon + dP/Q, \qquad Q = K_S/(\alpha - \phi); \tag{B-2}$$

the fluid's compressibility law is:

$$d\rho_f = dP/c_0^2; \tag{B-3}$$

and the equation of the fluid's mass conservation (the zero-frequency case) is:

$$\phi d\rho_f + \rho_f d\phi = 0. \tag{B-4}$$

Equations (B-1), (B-2), (B-3), and (B-4) give the following two versions of Gassmann's formula (K_r is the bulk modulus of the saturated rock):

$$K_r = \frac{K}{1 + \alpha dP/d\sigma}, \quad \frac{dP}{d\sigma} = -\frac{1}{\alpha(1 + K\phi/\alpha^2 F)},$$

with $1/F = 1/K_f + 1/(\phi Q)$.

APPENDIX C

SIMPLIFIED EXPRESSION FOR MODIFIED SOLID BULK MODULUS

Typically, soft porosity in rocks is smaller than 1 percent (Mavko and Jizba, 1991). Therefore,

$$\phi_S = \phi_{so}/(1 - \phi_{st}) \ll 1. \tag{C-1}$$

Formula (C-1) leads to the following estimates:

$$\frac{\alpha^2 F}{\phi_S} = \alpha^2 \Big/ \left(\frac{\phi_S}{K_f} + \frac{\alpha - \phi_S}{K_S}\right) \approx \alpha^2 \Big/ \frac{\alpha}{K_S} = \alpha K_S.$$

Note that this estimate is correct if $K_S \phi_S / K_f \ll 1$.

APPENDIX D

SHEAR MODULUS OF THE MODIFIED FRAME

According to Gassmann's formalism, the shear modulus of saturated rock is not affected by the fluid present in the stiff pores. Therefore, this modulus can be calculated as the shear modulus of the partially saturated rock with fluid in the soft pores only. Below, we relate this shear modulus to the bulk modulus of the partially saturated rock with fluid in the soft pores only.

Consider first a partially saturated sample of rock, with fluid in the soft pores only, subject to uniform compression. We examine the two sets of tractions applied to the sample (Figure D-1): the system on the left is loaded by an externally applied uniform stress $d\sigma$ and by internally applied (in the soft pores) pressure dP; the system on the right is loaded by the external stress only (empty pores).

By applying the reciprocity theorem to the system (similar to Mavko and Jizba, 1991), we find:

$$d\sigma \frac{d\sigma}{K_{md}} V = d\sigma \frac{d\sigma}{K} V - \sum_{\substack{\text{soft}\\ \text{pores}}} dP\, dV_{\substack{\text{drysoft}\\ \text{pores}}}, \tag{D-1}$$

where V is the volume of the sample, and K_{md} is the bulk modulus of the partially saturated rock with the fluid in the soft pores only. This modulus can be found from formula (8) where the bulk modulus of the modified solid was found under the condition that stiff pores are empty.

It follows now from equation (D-1) that

$$\frac{1}{K} - \frac{1}{K_{md}} = \sum_{\substack{\text{soft}\\ \text{pores}}} \frac{dP}{d\sigma} \left(\frac{d\phi_{so}}{d\sigma}\right)_{\substack{\text{dry soft}\\ \text{pores}}}. \tag{D-2}$$

Consider now the same partially saturated sample of rock, with fluid in the soft pores only, subject to pure shear. We examine the two sets of tractions applied to the sample (Figure D-2): the system on the left is loaded by externally applied shear tractions and by internally applied (in the soft pores) pressure, the system on the right is loaded by the

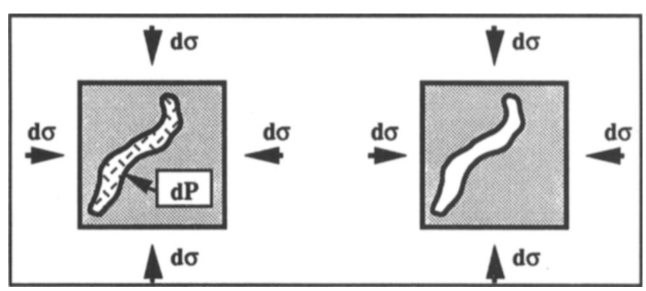

FIG. D-1. Two sets of traction applied to the same rock sample. The sample on the left is under uniform compression and pore pressure. The sample on the right has empty pores and is under uniform compression only.

external shear tractions only. The reciprocity theorem gives (analogous to the previous equation):

$$\frac{1}{G}-\frac{1}{G_m}=\sum_{\substack{\text{soft}\\\text{pores}}}\frac{dP}{d\tau}\left(\frac{d\phi_{so}}{d\tau}\right)_{\substack{\text{dry soft}\\\text{pores}}}, \qquad \text{(D-3)}$$

where G_m is the shear modulus of the modified frame.

We assume, following Mavko and Jizba (1991), that: (1) the shear-induced pore pressure and pore-volume compressibility of a soft thin pore are determined by the normal (to the pore's plane) component of the shear stress field, and (2) the effect of this normal component is identical to that for bulk compression. Now using the normal components of the stress field caused by pure shear (Walsh, 1965), we can find from formulas (D-2) and (D-3), similar to Mavko and Jizba (1991):

$$\frac{1}{G}-\frac{1}{G_m}=\frac{4}{15}\left(\frac{1}{K}-\frac{1}{K_{md}}\right).$$

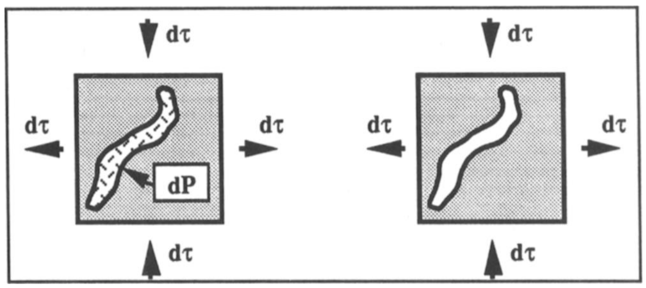

FIG. D-2. Two sets of traction applied to the same rock sample. The sample on the left is under pure shear and pore pressure. The sample on the right is under pure shear only.

APPENDIX E

STEP-BY-STEP PROCEDURE FOR IMPLEMENTING THE THEORY

1) Calculate K_{msd} from

$$1/K_{msd} = 1/K_S - 1/K_{hP} + 1/K.$$

2) Calculate $dP/d\sigma$ from

$$\frac{dP}{d\sigma} = -\left[\alpha_0\left(1+\frac{K\phi}{\alpha_0^2 F_0}\right)\right]^{-1},$$

where $1/F_0 = 1/K_f + 1/(\phi Q_0)$, $\alpha_0 = 1 - K/K_S$, $Q_0 = K_S/(\alpha_0 - \phi)$.

3) Assume a certain value for Z (start with $Z = 0.001$) and calculate K_{ms} from

$$K_{ms} = \frac{K_{msd} + \alpha K_S[1 - f(\xi)]}{1 + \alpha f(\xi) dP/d\sigma},$$

where $\alpha = 1 - K_{msd}/K_S$, $f(\xi) = 2J_1(\xi)/[\xi J_0(\xi)]$, $\xi = \sqrt{i\omega}Z$.

4) Calculate K_m from $1/K_m = 1/K_{ms} + 1/K_{hP} - 1/K_S$.

5) Calculate K_r from

$$K_r = K_m/(1 + \alpha_m\, dP/d\sigma),$$

where $\alpha_m = 1 - K_m/K_{ms}$.

6) Calculate G_m from

$$1/G - 1/G_m = (4/15)(1/K - 1/K_{md}),$$

where $1/K_{md} = 1/\bar{K}_{ms} + 1/K_{hP} - 1/K_S$ and $\bar{K}_{ms} = K_{msd} + \alpha K_S[1-f(\xi)]$.

7) Finally, calculate velocities V_P and V_S and inverse quality factors Q_P^{-1} and Q_S^{-1} from:

$$V_P = \sqrt{\mathrm{Re}\,(K_r + \tfrac{4}{3}G_m)/\rho}, \quad V_S = \sqrt{\mathrm{Re}\,(G_m)/\rho},$$

$$Q_P^{-1} = |\mathrm{Im}\,(K_r + \tfrac{4}{3}G_m)|/|\mathrm{Re}\,(K_r + \tfrac{4}{3}G_m)|,$$

$$Q_S^{-1} = |\mathrm{Im}\,(G_m)|/|\mathrm{Re}\,(G_m)|.$$

8) Adjust Z by matching one of the above four parameters (V_P, V_S, Q_P^{-1}, or Q_S^{-1}) with one experimental measurement (preferably V_P). Use thus obtained Z value for calculating velocities and attenuation at varying frequency, or for a different pore fluid. In the latter case, use the following value for Z: $Z_{\text{new}} = Z\sqrt{\mu_{\text{new}}/\mu}$, where subscript "new" indicates the new pore fluid.