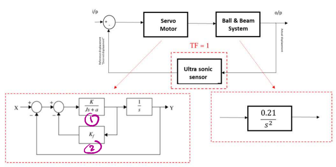
Steps:

For the unity feedback block diagram shown:



Block diagram of Servo Motor

Transfer function of Ball & Beam System

Loop 1

$$C.L.S = \frac{\frac{K}{JS+a}}{1+\frac{KK_f}{JS+a}} = \frac{K}{JS+a+KK_f}$$

$$\therefore \text{Closed loop system} = \frac{K}{JS+(a+KK_f)}$$

Loop 2 : $$C.L.S = \frac{\frac{K}{JS^2+(a+KK_f)S}}{1+\frac{K}{JS^2+(a+KK_f)S}} = \frac{K}{JS^2+(a+KK_f)S+K}$$

$$\frac{Y(S)}{X(S)} = \frac{K}{JS^2+(a+KK_f)S+K}$$

$F(S)$: open loop system $= \frac{K}{JS^2+(a+KK_f)S+K} \cdot \frac{0.21}{S^2}$

K : proportionality gain of motor $\equiv$ responsiveness / torque gain

K_f : friction / damping coefficient

J : Inertia

a : viscous damping coefficient

from description: $$\frac{\omega_n^2}{S^2+2\zeta\omega_n S+\omega_n^2} \cdot \frac{0.21}{S^2}$$

$$\frac{K}{JS^2+(a+KK_f)S+K} \cdot \frac{0.21}{S^2} \quad \begin{matrix}\div J \\ \div J\end{matrix}$$

$$\therefore \frac{K/J}{S^2+\frac{(a+KK_f)}{J}S+K/J} \cdot \frac{0.21}{S^2}$$

$\therefore \omega_n^2 = \frac{K}{J}$ $\qquad$ $2\zeta\omega_n = \frac{a+KK_f}{J}$

$\therefore \omega_n = \sqrt{\frac{K}{J}}$ $\qquad$ $\therefore \zeta = \frac{a+KK_f}{2J\omega_n}$

$$= \frac{a+KK_f}{2J\sqrt{\frac{K}{J}}}$$

$$= \frac{a+KK_f}{2\sqrt{KJ}}$$

ID = 1300577

$\boxed{\therefore \omega_n = 77}$

$77 = \sqrt{\frac{K}{J}}$ $\qquad$ & $\zeta = 0.5 = \frac{a+KK_f}{2\sqrt{KJ}}$

$5929 = \frac{K}{J}$ $\qquad$ & $1 = \frac{a+KK_f}{2\sqrt{KJ}}$

$J = 1 \; Kg/m^2$ & $a = 1 \; N \cdot m/rad \cdot sec$

$$\boxed{\therefore \; K = 5929 \; rad \cdot V/s}$$

$$1 = \frac{1 + 5929 \, K_f}{2\sqrt{5929}}$$

$$154 = 1 + 5929 \, K_f$$

$$\boxed{K_f = \frac{153}{5929} \; V \cdot s/rad}$$

$$\therefore G(S) = \frac{5929}{S^2 + 77S + 5929} \cdot \frac{0.21}{S^{2}}$$ ⇐ Feed Forward Transfer Function

$$= \boxed{\frac{1245.09}{S^2 \cdot (S^2 + 77S + 5929)}} = \frac{N(S)}{D(S)}$$

Closed loop system : $\frac{G(S)}{1 + G(S)} = \frac{N(S)/D(S)}{1 + N(S)/D(S)} = \frac{N(S)}{D(S) + N(S)}$

$$\boxed{H(S) : \text{Closed loop system} = \frac{1245.09}{S^2(S^2 + 77S + 5929) + 1245.09}}$$

Inputs on G(S) :

Analysis of G(S) ⟶ using Final Value Theorem

Steady state = $\lim_{S \to 0} S \, G(S)(1) = \frac{1245.09}{S(S^2 + 77S + 5929)} = \infty$ for impulse

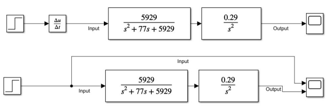
Steady State $\lim_{S \to 0} S \, G(S)(1/S) = \frac{1245.09}{S^2(S^2 + 77S + 5929)} = \infty$ for step input

* Expected as Function integrates the input making it increase Linearily with δ and exponentially with step input

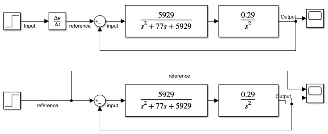
Analysis on H(S) : Total TF function

$$H(S) = \frac{1245.09}{S^2(S^2 + 77S + 5929) + 1245.09}$$

* Grows infinitly as the term of S^1 is missing (Hurwitz Criterion) for both impulse and step inputs, but will also be oscillating.



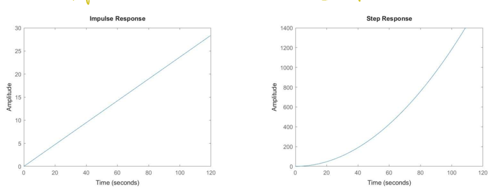
G(s)



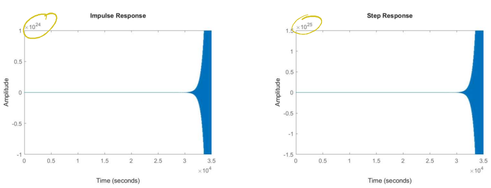
H(s)

The feedback introduced a pole in the right plane (+ve Real part) and so for any input (impulse/Step) the output grows exponentially in addition to also having an imaginary part and so introducing oscillations. So overall the response is worse as the system is now unstable.

Impulse Step

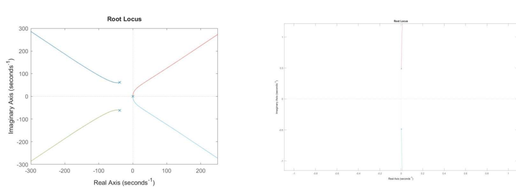


G(s)

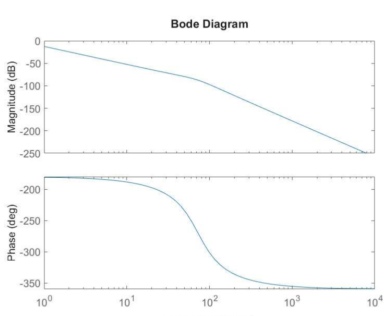


H(s)

* Gain adjustment will deteriorate the system's stability as 2 of the poles with increase of K, will move further into the right plane making it more unstable.



$$\frac{1245.09}{s^2(s^2+77s+5929)+1245.09}$$



System's phase is less than $-180°$ at Magnitude = 1 ∴ System is unstable

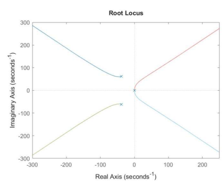
$$\boxed{\frac{1245.09}{S^2.(S^2+77S+5929)}} = G(S)$$

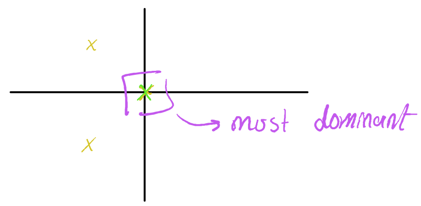
Poles of open loop : $S^2(S^2+77S+5929)$

$S=0 \quad n=2$

$S=-38.5+66.68j$

$S=-38.5-66.68j$





Using a PD Controller : A PD Controller is needed since the poles move to the right hand plane in the root locus.

$K_d = 100$

$K_p = 10$

New open loop: $$\frac{1245.09(100S+10)}{S^2(S^2+77S+5929)}$$

New closed loop: $$\frac{1245.09(100S+10)}{S^4+77S^2+5929S+1245098S+12450.9}$$

Poles: $S=-0.01 \quad S=-27.21 \quad S=-24.9\boxed{\pm}62.88j$ → Conjugate poles

∴ all in L.H.P ∴ Stable

- From the open loop : System is 2^{nd} type

∴ e_{ss} for Step & ramp $=0$

e_{ss} for Parabola ⟶ $e_{ss} = \frac{1}{1+\text{open loop}}$

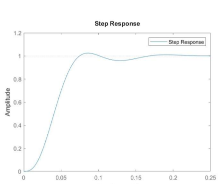
$$e_{ss} \overset{\lim_{S\to 0}}{=} \frac{S^2(S^2+77S+5929)}{S^2(S^2+77S+5929)+1245.09(100S+10)}$$

$$e_{ss} = \frac{0+5929}{0+(1245.09)(10)} = 0.4762$$

$\frac{1}{S^2}$ → parabola

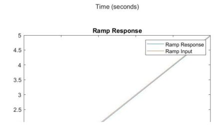
extra, not required

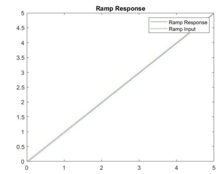
Using MATLAB:

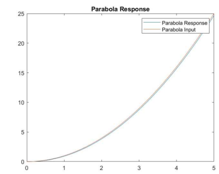


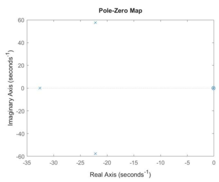
. As output is below 1.2 ∴ overshoot

. Steady State reached ≈ 0.25 seconds ∴ $t_s < 2.5$ s



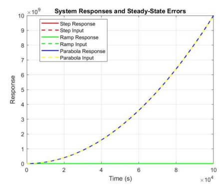






From the PZmap one can see that all poles are in the L.H.P

∴ System is stable




```
Numerical Steady-State Errors:
Step: 1.1102e-16
Ramp: 0
Parabolic: 0.49178
```

Step is practically zero

Ramp is zero

Parabolic e_{ss} is extremely close to value computed

Note: The steady state errors were computed numerically @ $t = 100000$

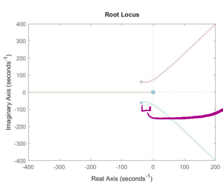
```
ans = struct with fields:
        RiseTime: 0.0432
   TransientTime: 0.1515
    SettlingTime: 0.1515
     SettlingMin: 0.9201
     SettlingMax: 1.0251
       Overshoot: 2.5124
      Undershoot: 0
            Peak: 1.0251
        PeakTime: 0.0852
```

Overshoot: 2.51 %

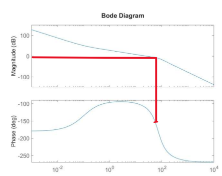
Settling time: 0.1515 s

Peak time: 0.0852 s

Rise time: 0.0432 s



The root locus has changed significantly as the poles at zero now are on the left side ∴ stable and for the corresponding values of K in $1+KL(s)=0$ the system is stable.



Note that for magnitude plot $=1$ the corresponding phase is larger than $-180°$

∴ System is stable

Lead Compensator design:

Compensator: $K\dfrac{(S+2)}{(S+P)}$ $\boxed{\dfrac{1245.09}{S^2.(S^2+77S+5929)}} = G(S)$

$M_p = 20\%$ $\quad t_s = 2.5\ s$

$$\zeta = \sqrt{\frac{\ln^2(0.2)}{\pi^2+\ln^2(0.2)}} = 0.456$$

$$t_s = \frac{4.6}{\zeta\omega_n} \qquad 2.5\,(0.456)\,\omega_n = 4.6 \qquad \omega_n = 4.035\ \text{rad/s}$$

$$1 + \frac{K(S+2)\,1245.09}{S^2(S^2+77S+5929)\,(S+P)} = 0$$

$$S^4 + 77S^3 + 5929S^2(S+P) + 1245.09\,K\,S + 1245.09\,2 = 0$$

$S^4 + 77S^3 + 5929S^2(S+P) + 1245.09KS + 1245.09Z = 0$

$S^5 + 77S^4 + 5929S^3 + PS^4 + 77PS^3 + 5929PS^2 + 1245.09KS + 1245.09Z = 0$

$S^5 + (77+P)S^4 + (5929+P)S^3 + 5929PS^2 + 1245.09KS + 1245.09Z = 0$

assume pole = 120

$+S^5 + 197S^4 + 6049S^3 + 711480S^2 + 1245.09KS + 1245.09Z = 0$

$+(S^3 + (a+b+c)S^2 + (ab+ac+bc)S + abc)(S^2 + 3.68S + 16.28) = 0$

$S^4 \quad 77+P = (a+b+c) + 3.68$ [1]

$S^3 \quad 5929+P = 16.28 + 3.68(a+b+c) + (ab+ac+bc)$

$S^2 \quad 5929P = 16.28(a+b+c) + 3.68(ab+ac+bc) + abc$ [3]

$S^1 \quad 1245.09K = 16.28(ab+ac+bc) + 3.68abc$ [4]

$S^0 \quad 1245.09Z = 16.28abc$ [5]

from [1] → [2]

$(a+b+c) = 77+P-3.68 = 73.32+P$

[6]

[1] → [3] & [6] → [3]

$5929+P = 16.28 + 3.68(73.32+P) + (ab+ac+bc)$

$5929 + \cancel{P} - 16.28 - 269.8176 \cancel{- 3.68P} = (ab+ac+bc)$

$-2.68P + 5642.9024 = (ab+ac+bc)$ [7]

[7] → [4] & [6] → [4]

$1245.09K = 16.28[-2.68P + 5642.9024] + 3.68abc$

$1245.09K = -43.6304P + 91866.45 + 3.68abc$

$1245.09K + 43.6304P - 91866.45 = 3.68abc \quad \div 3.68$

$338.34K + 11.856P - 24963.71 = abc$

$1245.09Z = 16.28[338.34K + 11.856P - 24963.71]$

$Z = 0.013075(\quad " \quad\quad " \quad)$

$$Z = 4.424K + 0.1550172P - 326.4$$

Using MATLAB plugging values into equation (trial & error)

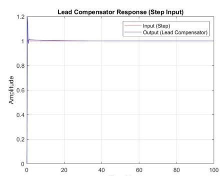
K: proportionality = 360

Z: Zero = 0.1

P: Pole = 10

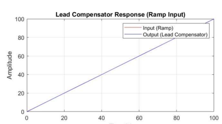
$$K \frac{(S+2)}{(S+P)}$$

Compensator Block

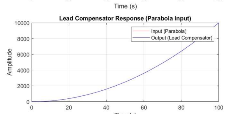


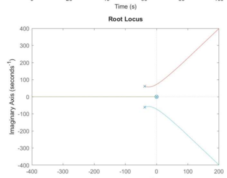

```
     RiseTime: 0.1722
TransientTime: 0.8434
 SettlingTime: 0.8434
  SettlingMin: 0.9098
  SettlingMax: 1.1955
    Overshoot: 19.5538
   Undershoot: 0
         Peak: 1.1955
     PeakTime: 0.4146
```

Note that the error is zero since the system type is 2

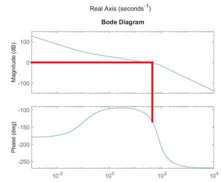


As system is of type 2 it can track ramp & Parabola too.





From the locus we can see that the system is stable since all poles are on the left hand plane.



Note that for Magnitude plot = 1 the Corresponding phase is larger than -180°

∴ System is stable