MTH-681 Analysis and Design of Algorithms
Assignment (4): Divide and Conquer
The Master Theorem

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Consider the following reucrrence:

$$T(n) = T(n/2) + 5^{\lfloor \log_5 n \rfloor}$$
$$T(1) = \Theta(1)$$

Can you solve it using the master method? If "yes", solve it; if "no", explain why.

Solution:

$$a = 1$$
$$b = 2$$
$$\log_b a = \log_2 1 = 0.$$

To simplify the analysis, assume n is an integer power of 5: $n = 5^k$.

$$\begin{aligned} \implies f(n) &= 5^{\lfloor \log_5 n \rfloor} \\ &= 5^{\log_5 n} \\ &= n. \end{aligned}$$

Comparing $f(n)$ with $n^{\log_b a}$:

$$\begin{aligned} f(n) &= n \\ &= \Omega(n^{0+\epsilon}) \quad \text{for } \epsilon = 0.5 \\ &= \Omega(n^{\log_b a+\epsilon}) \quad \text{for } \epsilon = 0.5. \end{aligned}$$

This matches the first condition for the third case of the master method.

Next, we check the regularity condition:

$$\begin{aligned} f(n/b) &= f(n/2) \\ &= n/2 \\ &\le cn \quad \text{for } c = 0.6 \\ &= cf(n) \quad \text{for } c = 0.6. \end{aligned}$$

Therefore the recurrence does match the third case of the master method.

Finally, we apply the master method to find a tight bound on $T(n)$:

$$T(n) = \Theta(f(n)) = \Theta(n).$$