MTH-632 PDEs
Assignment (2): Classification and Canonical Forms

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2.2.2

a.

$$\begin{aligned}\Delta &= B^2 - 4AC \\ &= 5^2 - 4(4 * 1) \\ &= 25 - 16 \\ &= 9 > 0 \\ \implies &\text{The problem is hyperbolic.}\end{aligned}$$

b.

$$\begin{aligned}\Delta &= B^2 - 4AC \\ &= 1^2 - 4(1 * 1) \\ &= 1 - 4 \\ &= -3 < 0 \\ \implies &\text{The problem is elliptic.}\end{aligned}$$

c.

$$\begin{aligned}\Delta &= B^2 - 4AC \\ &= 10^2 - 4(3 * 3) \\ &= 100 - 36 \\ &= 64 > 0 \\ \implies &\text{The problem is hyperbolic.}\end{aligned}$$

d.

$$\begin{aligned}\Delta &= B^2 - 4AC \\ &= 2^2 - 4(1 * 3) \\ &= 4 - 12 \\ &= -8 < 0 \\ \implies &\text{The problem is elliptic.}\end{aligned}$$

e.

$$
\begin{aligned}
\Delta &= B^2 - 4AC \\
&= (-4)^2 - 4(2 * 2) \\
&= 16 - 16 \\
&= 0 \\
\implies &\text{The problem is parabolic.}
\end{aligned}
$$

f.

$$
\begin{aligned}
\Delta &= B^2 - 4AC \\
&= 5^2 - 4(1 * 4) \\
&= 25 - 16 \\
&= 9 > 0 \\
\implies &\text{The problem is hyperbolic.}
\end{aligned}
$$

2.3.1

a.

$A = x,\ B = 0,\ C = 1.$

$$
\begin{aligned}
\Delta &= B^2 - 4AC \\
&= 0 - 4(x * 1) \\
&= -4x \\
\implies &x < 0 \text{ then hyperbolic} \\
&x = 0 \text{ then parabolic} \\
&x > 0 \text{ then elliptic.}
\end{aligned}
$$

1. $\underline{x < 0\ (\textit{hyperbolic})}$:

   Characteristic Equation:

$$
\begin{aligned}
\frac{dy}{dx} &= \frac{B^2 \pm \sqrt{B - 4AC}}{2A} \\
&= \frac{0^2 \pm \sqrt{0 - 4x}}{2x} \\
&= \pm\frac{2\sqrt{-x}}{2x} \\
&= \mp(x)^{-\frac{1}{2}}
\end{aligned}
$$

Characteristic Curves:

$$\xi = \phi_1(x,y) = y + 2x^{\frac{1}{2}} = C_1$$
$$\eta = \phi_2(x,y) = y - 2x^{\frac{1}{2}} = C_2$$

Canonical Form:

$$\xi_x = x^{-\frac{1}{2}} \qquad \xi_y = 1 \qquad \xi_{xy} = 0 \qquad \xi_{xx} = -\frac{1}{2}x^{-\frac{3}{2}} \qquad \xi_{yy} = 0$$
$$\eta_x = -x^{-\frac{1}{2}} \qquad \eta_y = 1 \qquad \xi_{xy} = 0 \qquad \xi_{xx} = \frac{1}{2}x^{-\frac{3}{2}} \qquad \xi_{yy} = 0$$

$$A^* = C^* = 0$$
$$B^* =$$