

MTH-684 Logic
Assignment (4): Reasoning in Propositional Logic

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4-2

Prove that:

$$\{(A \implies B), ((B \vee C) \implies D), (E \wedge \neg D)\} \models (A \implies D)$$

Proof.

1. $(A \implies B)$	(Hypothesis)
2. $((B \vee C) \implies D)$	(Hypothesis)
3. $(E \wedge \neg D)$	(Hypothesis)
4. $\neg D$	(3, SIM)
5. $\neg(B \vee C)$	(2, 4, MT)
6. $(\neg A \vee B)$	(1, IMP1)
7. $((\neg A \vee B) \vee C)$	(6, AD)
8. $(\neg A \vee (B \vee C))$	(7, GR)
9. $(A \implies (B \vee C))$	(8, IMP2)
10. $\neg A$	(5, 9, MT)
11. $(\neg A \vee E)$	(10, AD)
12. $(A \implies E)$	(11, IMP2)

□