# MTH-684 Logic
# Assignment (6): Inference in First-Order Predicate Logic (FOPL)

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## 6-1

Using the FOPL system of natural deduction introduced in class, prove the following.

$$\vdash \exists x\,[P(x) \lor Q(x)] \iff \exists x P(x) \lor \exists x Q(x)$$

*Proof.*

$\underline{\implies:}$

| | |
|---|---|
| $*\,1.\exists x\,[P(x) \lor Q(x)]$ | (Assumption) |
| $*\,2.P(c) \lor Q(c)$ | (1, $\exists$-elim) |
| $*\,3.\exists x P(x) \lor Q(c)$ | (2, $\exists$-intro) |
| $*\,4.\exists x P(x) \lor \exists x Q(x)$ | (3, $\exists$-intro) |
| $5.\exists x\,[P(x) \lor Q(x)] \implies \exists x P(x) \lor \exists x Q(x)$ | (1, 4, $\implies$-intro). |

$\underline{\impliedby:}$

| | |
|---|---|
| $*\,i.\exists x P(x) \lor \exists x Q(x)$ | (Assumption) |
| $**ii.\neg\,[\exists x\,[P(x) \lor Q(x)]]$ | (Assumption) |
| $**iii.\neg\,[P(c) \lor Q(c)]$ | ($ii$, $\exists$-elim) |
| $**iv.\neg\,[\exists x P(x) \lor Q(e)]$ | ($iii$, $\exists$-intro) |
| $**v.\neg\,[\exists x P(x) \lor \exists x Q(x)]$ | ($iv$, $\exists$-intro) |
| $*\,vi.\neg\neg\,[\exists x\,[P(x) \lor Q(x)]]$ | ($i$, $ii$, $v$, $\neg$-intro) |
| $*\,vii.\exists x\,[P(x) \lor Q(x)]$ | (vi, $\neg$-elim) |
| $viii.\exists x P(x) \lor \exists x Q(x) \implies \exists x\,[P(x) \lor Q(x)]$ | ($i$, $vii$, $\implies$-intro). |

$\underline{\iff:}$

$\exists x\,[P(x) \lor Q(x)] \iff \exists x P(x) \lor \exists x Q(x)$ (5, $viii$, $\iff$-intro).

$\square$