Chapter 6 - Section 7

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Fact. Given a set A of distinct elements in a random order, The positition of the maximum element of a subset $S \subset A$ is uniform in S.

Define indicator random variables L_i as

$$L_i = \begin{cases} 1 & a_i > a_{i-1}, a_{i-2}, \ldots, a_1 \\ 0 & a_i < a_j, \text{ for some } j = 1, 2, \ldots, i-1 \end{cases}$$

So $L_i = 1$ if and only if the ith item a_i is the maximum in subset $A[1:i]$.

It follows $Pr[L_i = 1] = 1/i$ and $Ex[Li] = 1/i$.

Let X be a random variable for the number of times the line `a[first] > a[max_loc]` returns `True`. Observe $X = L_2 + L_3 + \cdots + L_n$. So $Ex[X] = 1/2 + \cdots + 1/n = H(n) - 1 \approx \ln n - 1$.

$H(n)$ here is the nth harmonic sum.