Note: A square-root sign is to be understood over every coefficient, e.g., for -8/15 read $-\sqrt{8/15}$. Notation: $Y_1^0 = \sqrt{\frac{3}{4\pi}}\cos\theta$ 1/2×1/2 Coefficients +1/2 + 1/2+1/2 -1/2 1/2 1/2 -1/2 +1/2 1/2-1/2 $1 \times 1/2$ $Y_2^1 = -\sqrt{\frac{15}{8\pi}} \sin\theta \cos\theta \, e^{i\phi}$ +1/2 +1/2 3/2×1/2 2 +3/2 +1/2 1 -1 -1/2 -2 +1/2 4/5 3/2 1/2 $Y_2^2 = \frac{1}{4} \sqrt{\frac{15}{2\pi}} \sin^2 \theta \, e^{2i\phi}$ -1/2 - 1/22/3 1/3 1/3-2/3 +3/2 -1/2 1/4 3/4 +1/2 +1/2 3/4-1/4 $3/2 \times 1$ $\begin{bmatrix} 5/2 \\ +5/2 \end{bmatrix}$ 5/2 3/2-1 - 1/2+1/2-1/2 1/2 1/2 -1/2 +1/2 1/2 -1/2 +3/2 +3/2 +2 0 1/3 2/3 +1 +1 2/3 -1/3 2/5 3/5 5/2 3/2 1/2 3/5 -2/5 +1/2 +1/2 +1/2 -1/2 -1/2 3/4 1/4 -3/2 +1/2 1/4-3/4 1/3 +3/2-1 1/10 2/5 1/2 +1/2 0 3/5 1/15 -1/3 -1/2+1 3/10 -8/15 1/6 +2 -1 1/15 +1 0 8/15 1/6-3/10 5/2 0+1 2/5 -1/2 1/10 -1/2 -1/2 -1/2 +1 -1 1/5 1/2 3/10 0 0 3/5 0 -2/5 +1/2 -1 3/10 8/15 1/6 -1/2 0 3/5 -1/15 -1/3 5/2 3/2 -3/2 +1 1/10 -2/5 1/2 -3/2 -3/2 -1 +1 1/5 -1/2 3/10 -1 -1 -1 -1/2-1 3/5 2/5 -3/2 0 2/5 -3/5 +1 -1 1/6 1/2 1/3 0-1 2/5 1/2 1/10 0 0 2/3 0-1/3 2 **-1** 0 8/15 **-1/6-3/10** -1 +1 1/6 -1/2 1/3 -1 -1 -2 +1 1/15 -1/3 3/5 -1 -1 2/3 1/3 $\langle j_1 j_2 m_1 m_2 | j_1 j_2 JM \rangle$ $= (-1)^{J-j_1-j_2} \langle j_2 j_1 m_2 m_1 | j_2 j_1 JM \rangle$ $3/2 \times 3/2$ $\frac{3}{+3}$ $\frac{3}{+2}$ $d_{m'm}^{j} = (-1)^{m-m'} d_{mm'}^{j} = d_{-m-m'}^{j}$ $d_{0,0}^1 = \cos \theta$ $d_{1/2,1/2}^{1/2} = \cos \frac{\theta}{2}$ $d_{1,1}^1 = \frac{1 + \cos \theta}{2}$ $2 \times 3/2 \begin{array}{c|c} 7/2 \\ +7/2 \\ \hline +2+3/2 \end{array} \begin{array}{c|c} 7/2 & 5/2 \\ +5/2 +5/2 \end{array}$ $d_{1/2,-1/2}^{1/2} = -\sin\frac{\theta}{2}$ $d_{1,0}^{1} = -\frac{\sin\theta}{\sqrt{2}}$ +3/2+1/2 1/2 1/2 3 +1/2+3/2 1/2-1/2 +2+1/2 3/7 4/7 7/2 +1+3/2 4/7-3/7 +3/2 $d_{1,-1}^1 = \frac{1-\cos\theta}{2}$ +3/2 + 3/2+2-1/2 1/7 16/35 2/5 +1+1/2 4/7 1/35-2/5 0+3/2 2/7-18/35 1/5 +3/2 -3/2 1/20 1/4 9/20 1/4 7/2 +1/2 5/2 3/2 +1/2 +1/2 +1/2 -1/2 9/20 1/4 --1/2 +1/2 9/20 -1/4 --3/2 +3/2 1/20 -1/4 +1/2 +2 -3/2 1/35 +1 -1/2 12/35 1/5 1/2 3/10 +2 0 3/14 1/2 2/7 +1 +1 4/7 0 -3/7 0 +2 3/14 -1/2 2/7 0 3/10 2/5 –2/5 3/10 3/7 2/7 18/35 1/5 4/7 -1/35-2/5 0 -3/2 +1 3/7 -1/5 +2 1/14-3/10 1/70 1/10 2/7 -1 –3/2 -2 –1/2 2/5 1/14-1/10 0 -2/7 0 -1 +1 8/35 -2/5 1/14 1/10 -1/5 -2 +2 1/70-1/10 2/7 -2/5 1/5 $d_{3/2,3/2}^{3/2} = \frac{1 + \cos \theta}{2} \cos \frac{\theta}{2}$ $d_{2,2}^2 = \left(\frac{1+\cos\theta}{2}\right)^2$ $d_{\,3/2,1/2}^{\,3/2} = -\sqrt{3}\frac{1+\cos\theta}{2}\sin\frac{\theta}{2}$ $d_{3/2,-1/2}^{3/2} = \sqrt{3} \frac{1 - \cos \theta}{2} \cos \frac{\theta}{2}$ $d_{2,1}^2 = -\frac{1+\cos\theta}{2}\sin\theta$ 0 -2 3/14 1/2 2/7 -1 -1 4/7 0-3/7 $d_{3/2,-3/2}^{\,3/2} = -\frac{1-\cos\theta}{2}\sin\frac{\theta}{2} \qquad \quad d_{2,0}^{\,2} = \frac{\sqrt{6}}{4}\sin^2\theta \qquad \qquad d_{1,1}^{\,2} = \frac{1+\cos\theta}{2}\left(2\cos\theta-1\right)$ $d_{1/2,1/2}^{3/2} = \frac{3\cos\theta - 1}{2}\cos\frac{\theta}{2} \qquad \qquad d_{2,-1}^2 = -\frac{1-\cos\theta}{2}\sin\theta \qquad \quad d_{1,0}^2 = -\sqrt{\frac{3}{2}}\sin\theta\cos\theta$ $d_{1/2,-1/2}^{3/2} = -\frac{3\cos\theta + 1}{2}\sin\frac{\theta}{2} \qquad d_{2,-2}^2 = \left(\frac{1-\cos\theta}{2}\right)^2 \qquad \qquad d_{1,-1}^2 = \frac{1-\cos\theta}{2}\left(2\cos\theta + 1\right) \qquad d_{0,0}^2 = \left(\frac{3}{2}\cos^2\theta - \frac{1}{2}\right)^2$