Fundamentals of theory of computation I
Sample for Test 1

There will be only 5 exercises in the midterm test.

Exercise 1: Let $L_1 = \{a^n b^n \mid n \in \mathbb{N}\}$, $L_2 = \{a^n b^k \mid n \geq 0, k \equiv 1 \pmod 3\}$, Determine the following languages.

$$L_1 L_2 = ? \qquad L_1 \setminus L_2 = ? \qquad \mathrm{Pre}(L_2) = ?$$

($\mathrm{Pre}(L_2)$ is the prefix language of L_2, the set of prefixes of the words of L_2.)

Exercise 2: Let $N_t(u)$ denote the number of t's occuring in the word u. Make a type 3 grammar generating the language $L = \{u \in \{a,b,c\}^* \mid bb \not\subseteq u, N_a(u) \leq 1\}$. (I.e., there are at most 1 a and no consecutive b's in the words of L.)

Exercise 3: Determine $L(G)$ for the grammar $G = \langle \{S, X, Y\}, \{a, b\}, P, S \rangle$, where P is as follows.

$$S \to aXa \mid bSb \mid a \mid b \mid \varepsilon$$
$$X \to aYa \mid bSb \mid b \mid \varepsilon$$
$$Y \to bSb \mid b$$

List all the Chomsky grammar classes having G as a member.

Exercise 4: Give a regular expression describing the following languages.

$L_1 = \{u \in \{a,b\}^* \mid u$ contains an even number of a's, and if u contains a b, then the number of a's after the last b is odd$\}$.

$L_2 = \{u \in \{a,b,c\}^* \mid u$ does not contain the subword $ab\}$.

Exercise 5: Construct a type 3 grammar in normal form, according to the algorithm we learnt, generating the same language as the following type 3 grammar G. $G = \langle \{S, A, B, C\}, \{a, b\}, P, S \rangle$, where the set of production rules P consists of the following rules.

$$S \to aabA \mid bB$$
$$A \to C \mid aS$$
$$B \to bC \mid \varepsilon$$
$$C \to B \mid a$$