

Assume that we have n nodes and that $P(n, k, h)$ is the probability that k nodes has the information after h handshakes. The probability that no node get the information in the next try if k nodes has it, is

$$p(n, k) = \frac{k(k-1) + (n-k)(n-k-1)}{n(n-1)}, \quad (1)$$

since $k(k-1)/2$ is the number of relations between nodes with the information, and $(n-k)(n-k-1)/2$ is the number of relations between nodes that don't. These relations/handshakes do not propagate the info.

So the probability that the information does spread is

$$1 - p(n, k) = 1 - \frac{k(k-1) + (n-k)(n-k-1)}{n(n-1)} \quad (2)$$

$$= 2 \frac{k}{n} \frac{n-k}{n-1}, \quad (3)$$

Then we have the recursion:

$$P(n, k, h) = (1 - 2 \frac{k}{n} \frac{n-k}{n-1}) P(n, k, h-1) + 2 \frac{(k-1)}{n} \frac{n-k+1}{n-1} P(n, k-1, h-1), \quad (4)$$

since we reach the state (n, k, h) only from $(n, k, h-1)$ (first term) or from $(n-1, k-1, h-1)$ (second term), with known probabilities. The iteration starts with

$$P(n, 2, 1) = 1, \quad (5)$$

assuming two nodes have the information after one handshake. As Pär said.

The n instances $h = Hn+1, \dots, (H+1)n$ here corresponds to H handshakes if we consider n handshakes during one step. The distribution in this latter case, denoted $P'(n, k, i)$, is approximated with the probabilities $P(n, k, iH) \approx P'(n, k, i)$, $i = 1, 2, 3, \dots$.

1 Simulation

If you have time to simulate, you can do as follows.

1. Pick one start node at random.
2. Pick one end node at random, not the same as the start node.
3. Check if information spreads (if exactly one has info) or not. Update the node state.
4. Pick one start node among the available, i.e. that is not picked before. If there is no such node, go to 6.
5. Do as in 2 and 3 for this node. Go to 4.

6. Now one step is completed in the multi-handshake scenario. Save the number of nodes that have info.
7. Make all nodes available again.
8. Go to 1.

You don't necessarily need to do it infinitely.