Integrate and Fire Neuron

Here we are going to simulate spike trains based on [Softky & Koch, 1993] and compare its statistics with real neural data. The simplest neuron model in this paper was a kind of ***integrate and fire*** neuron where it's inter-spike intervals (ISI) distribution explained by equation [9].

Temporarily, rather than direct implementation of equation [9] we are going to generate a spike train using renewal process. This procedure might help you understand why an Integrate and fire neuron, shows lower variability even with receiving irregular post synaptic inputs!

a. Generate a spike train using a Poisson process with r = 100 and $\Delta\tau$ = 1ms.

b. Plot spike count probability histogram calculated from many Poisson spike trains, each of 1 sec duration with r = 100, superimposed with the theoretical (Poisson) spike count density.

c. Plot Inter-spike interval (ISI) histogram calculated from the simulated Poisson spike trains, superimposed with the theoretical (exponential) inters-pike interval density.

A way to generate a renewal process spike train is to start with a Poisson spike train and delete all but every kth spike! This procedure is similar to integration over postsynaptic input with Poisson ISI distribution (Why?)

Repeat steps a-c but using the above deleting spike procedure. Now look at the new plot, what distribution it is look like?

d. Now use the generated data and find out what is the C_v (Coefficient of Variation) for this spike trains? Compare it with Poisson process?
e. Theoretically prove the C_v for the above distribution is equal to $\frac{1}{K^{0.5}}$
f. Compare variability of simulated spike trains with real data-set in [Softky & Koch, 1993].

Contradicting with generated spike trains, real neural data contain less spikes within small intervals. The biological support for this claim is that each neuron has a refractory period which limit neuron to generate spikes within this period, so there is an upper limit for firing frequency in small intervals.

g. Similar to equation [13] of paper, consider a refractory period for your spike trains and generate a plot similar to figure 6 of paper comparing C_v across different firing rates and different refractory periods.

Leaky Integrate and Fire Neuron

It is well known that depolarization do not persist forever, but that perturbations of membrane voltage tend to decay toward the resting potential. In this section we are going to implement a more realistic model of neuron, *Leaky Integrate and fire (LIF)* neuron, which consider the leakage of postsynaptic inputs.

The leaky integrate-and-fire (LIF) neuron is probably one of the simplest spiking neuron models, but it is still very popular due to the ease with which it can be analyzed and simulated. In its simplest form, a neuron is modeled as a "leaky integrator" of its input *I(t)*:

$$\tau_m \frac{dv}{dt} = -v(t) + RI(t)$$

where $v(t)$ represents the membrane potential at time t, τ_m is the membrane time constant and R is the membrane resistance.

a. With stimulation by a constant input current of 20 *mV,* simulate the time-course of the membrane potential for 100ms. Take resting potential v_r = 0 mV, and threshold voltage v_{th} = 15 mV. (the current is given in units of mV, because we assume R = 1 mΩ; thus, a more accurate way of expressing this would be RI = 20 mV, but for the sake of brevity we will simply omit R in what follows)

b. Write a mathematical equation explaining mean firing rate of neuron at a constant input current I, with considering a refractory period of Δt_r .

c. Repeat section a. with stimulating neuron by a time-varying input current $I(t)$. For a general time-varying input current $I(t)$, the solution of LIF equation, with the initial condition $v(t_0) = v_r$, is given by:

$$v(t) = v_r \exp(-\frac{t-t_0}{\tau_m}) + \frac{R}{\tau_m}\int_0^{t-t_0} \exp(-\frac{s}{\tau_m}) I(t-s)ds$$

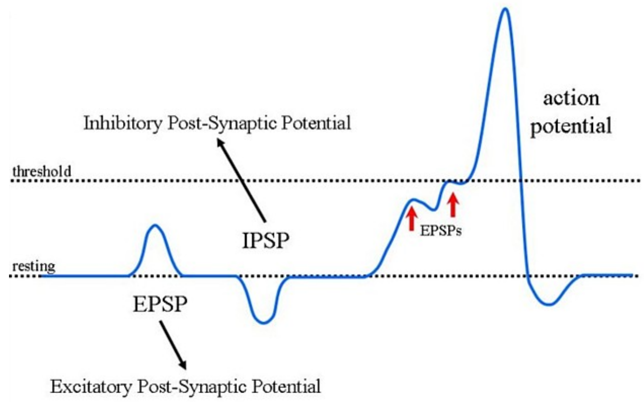
To generate a realistic $I(t)$, first define K spike train with Poisson distribution and then convolve them in an EPSC kernel, similar to what *Softky and Koch* did in their paper.
As long as the membrane potential is well below the synapse's reversal potential, we can approximate the synaptic current by:

$$I_s(\mathrm{t}) \propto \mathrm{t}\, e^{(-t/t_{peak})}$$

Accordingly

$$I(t) = \sum_i \delta(t-t_i) \,.\, I_s(\mathrm{t})$$

where t_i is the time of ith synaptic input spike.



Try to generate a contour plot similar to figure 8 of paper. Also make additional plots explaining the effect of width and magnitude of EPSCs on C_v

d. In this section we are going to include IPSPs effect in our simulations. To do so you just need to select a percentage of synaptic inputs to assign negative kernels. Repeat part c. for different percentages of inhibitory inputs and briefly explain (based on simulations!) how C_v depends on inhibition percentage of synaptic inputs and other simulation parameters.

e. Assume the neuron is doing coincidence detection of it excitatory inputs which have a Poisson distribution (assume neuron requires N out of M inputs to be active in a short D ms time window). What would be the C_v of the output neuron? How will it depend on the N/M and D? (you can use simulation to plot dependence, only consider excitatory inputs for this part).

f. What will happen if the neuron is coincidence detector but we also add inhibitory neurons. Here the neuron will fire if number of excitatory pulses received minus number of inhibitory pulses ($N_{net}=N_x-N_i$) is bigger than a threshold in D ms time window. What would be the C_v of the output neuron? How will it depend on the N_{net} and D? (you can use simulations to plot dependence).

Good luck!