Non-uniform rational B-spline | concepts and practice

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Concepts

Spline

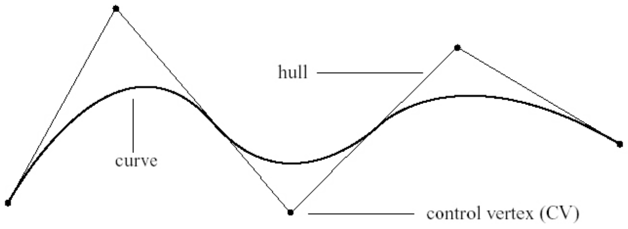
A special function defined piecewise by polynomials.

Linear Splines (1st degree)

- The most straight-forward way of drawing a curve is by connecting a sequence of points.
- The resulting curve is a linear spline, and is equivalent to a polygon.
- There are 2 major drawbacks to this method of producing a curve.
    - In order to produce anything that actually appears curved, you would need a large number of points. Storing and computing all those points is not an efficient use of the computer's resources.
    - Manipulating a curve created in this fashion is very cumbersome because, once a point is moved, you lose the smoothness of the shape.

Higher degree splines (2nd, 3rd… degree)

- The way around the jaggedness produced by linear connectivity is through a series of blending functions.
- The blending functions generate smooth connection between the control vertices (CV) of the curve.
- A spline curve generates a smooth transition between its CV through a blending function that operates on these points.
- The set of CVs controlling the curve is referred to as the "hull".



(source https://www.derivative.ca/wiki088/index.php?title=Spline)

Concepts

Nurbs

Stands for:

- **N**on-**U**niform: uniformity controlled by knots values (can be non-uniformly spaced).
- The "**R**" in NURBS stands for rational and indicates that a NURBS curve has the possibility of being rational (later explained).
- **B-S**pline: or basis spline (function that has minimal support with respect to a given degree, smoothness, and domain partition).

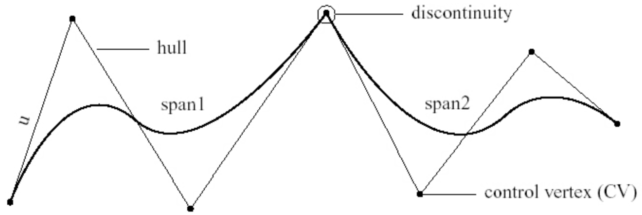
The NURBS evaluation is a formula that uses basis spline functions which feed on input parameters: <u>degree</u>, <u>control points</u>, and <u>knots</u>.

(sources:
https://www.derivative.ca/wiki088/index.php?title=Spline,
http://developer.rhino3d.com/guides/opennurbs/nurbs-geometry-overview/)

Concepts

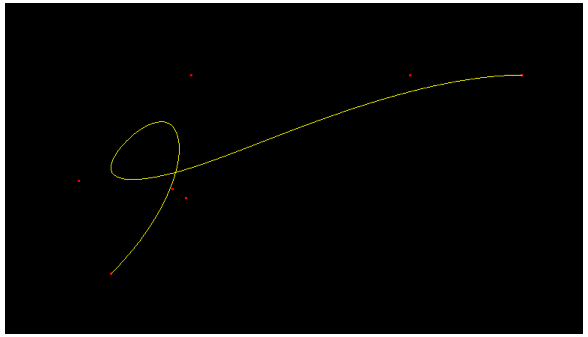
Nurbs and Bezier curves

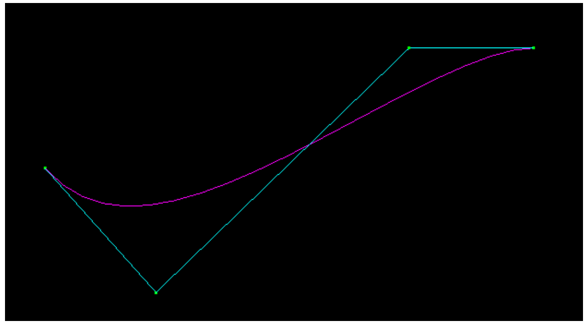
- Both are piecewise curves made of a number of connected curve segments.
- Differ in the level of continuity at the points where the curve segments touch.
- A NURBS curve will typically be very smooth at these joints (the higher the degree of the blending function, the smoother the connection).
- Bézier curves have a discontinuity every **degree plus one points**.



Source: https://www.derivative.ca/wiki088/index.php?title=Spline

Sources: download and execute nurbs and Bezier examples at https://nccastaff.bournemouth.ac.uk/jmacey/RobTheBloke/www/opengl_programming.html#3





Concepts

Degree

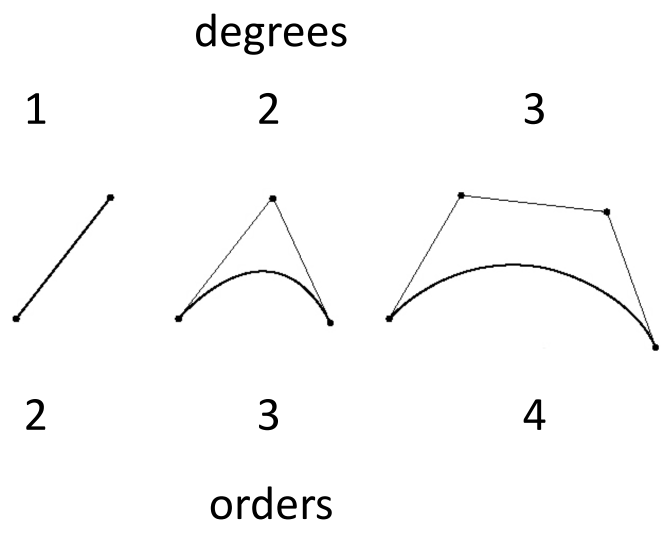
(http://developer.rhino3d.com/guides/opennurbs/nurbs-geometry-overview/)

- The degree of the spline is given by the degree of the underlying blending functions. It is a positive whole number.
- This number is usually 1, 2, 3 or 5, but can be any positive whole number.
- NURBS lines are usually degree 1,
- NURBS circles are degree 2, and most free-form curves are degree 3 or 5.
- Sometimes the terms linear, quadratic, cubic, and quintic are used.
- Linear means degree 1, quadratic means degree 2, cubic means degree 3, and quintic means degree 5.
- Cubic splines are usually sufficiently smooth and well behaved for most applications.

Order

(https://www.derivative.ca/wiki088/index.php?title=Spline)

- The "degree plus one" formulation is often referred to as the order of the curve.
- A cubic curve, for example, has a degree of three and, therefore, an order of four.



(adapted from source
https://www.derivative.ca/wiki088/index.php?title=Spline)

Concepts

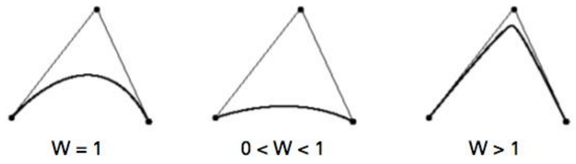
Control points (CP)

(http://developer.rhino3d.com/guides/opennurbs/nurbs-geometry-overview/, https://www.derivative.ca/wiki088/index.php?title=Spline)

- Each control point of the curve has X, Y, and Z coordinates that determine its position in world space.
- The control points are a list of at least degree + 1 points.
- The control points have an associated number called a weight (next section).

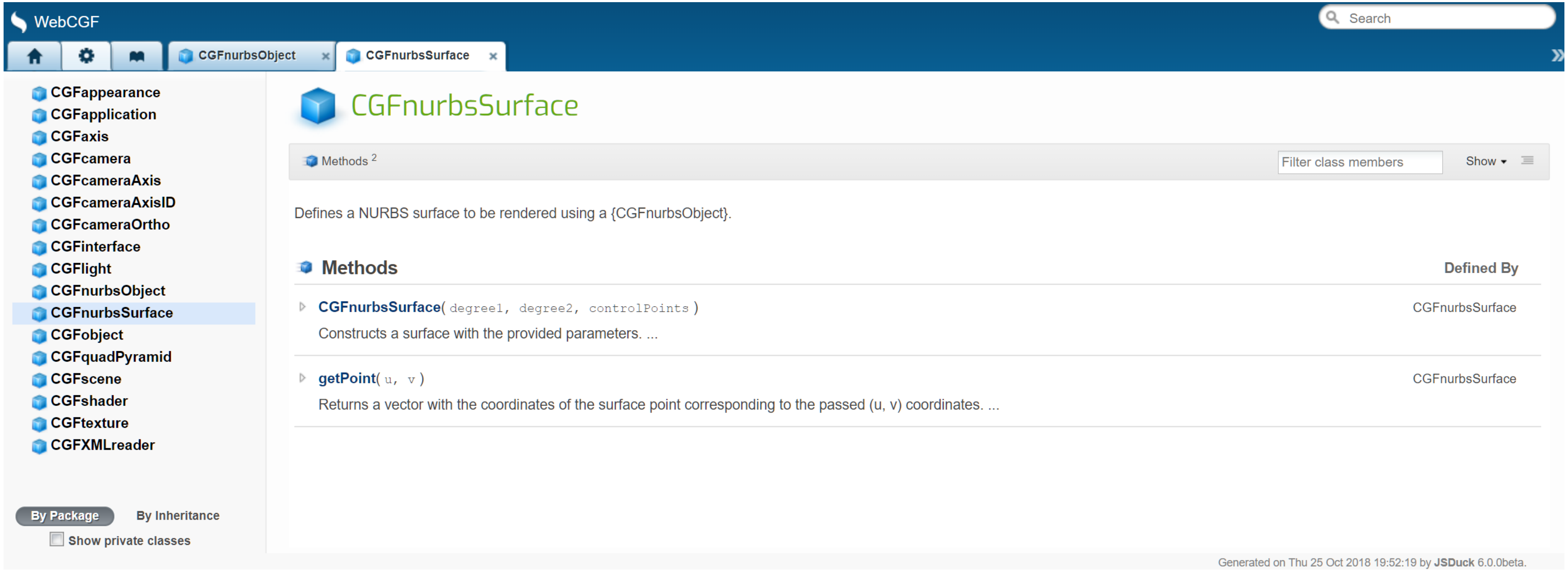
Rational / non-rational Spline

- Besides X,Y,Z coordinates, each control point has an additional fourth component, W.
- The W component determines a CP's weight. The weight determines the "pull" (like a magnet) of a CP on the spline curve.
- The value of the W component makes a spline rational or non-rational. A non-rational spline has only equal weights (typically, W=1), while a rational spline contains at least one different weight.
- With a few exceptions, weights are positive numbers. When a curve’s control points all have the same weight (usually 1), the curve is called non-rational, otherwise the curve is called rational.



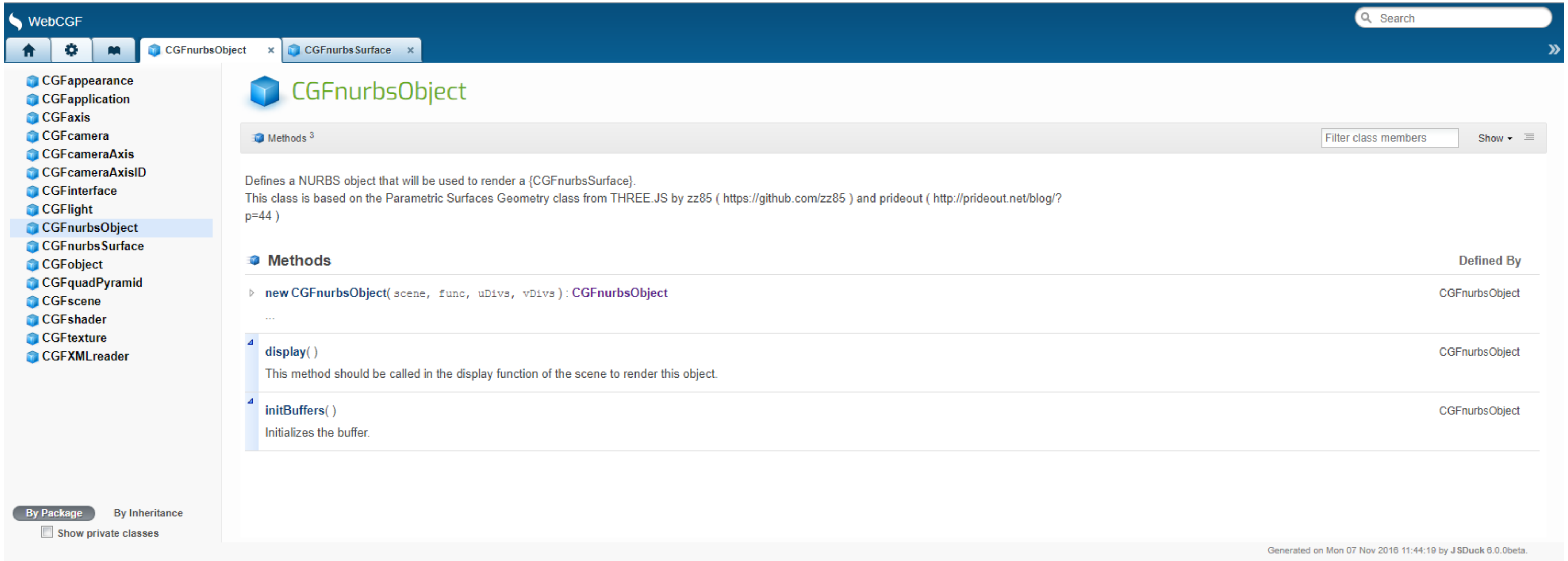
Practice (requires WebCGF 0.21+)

WebCGF: Parametric surface support



Practice (requires WebCGF 0.21+)

WebCGF: Curve rendering support



Practice

Putting all together

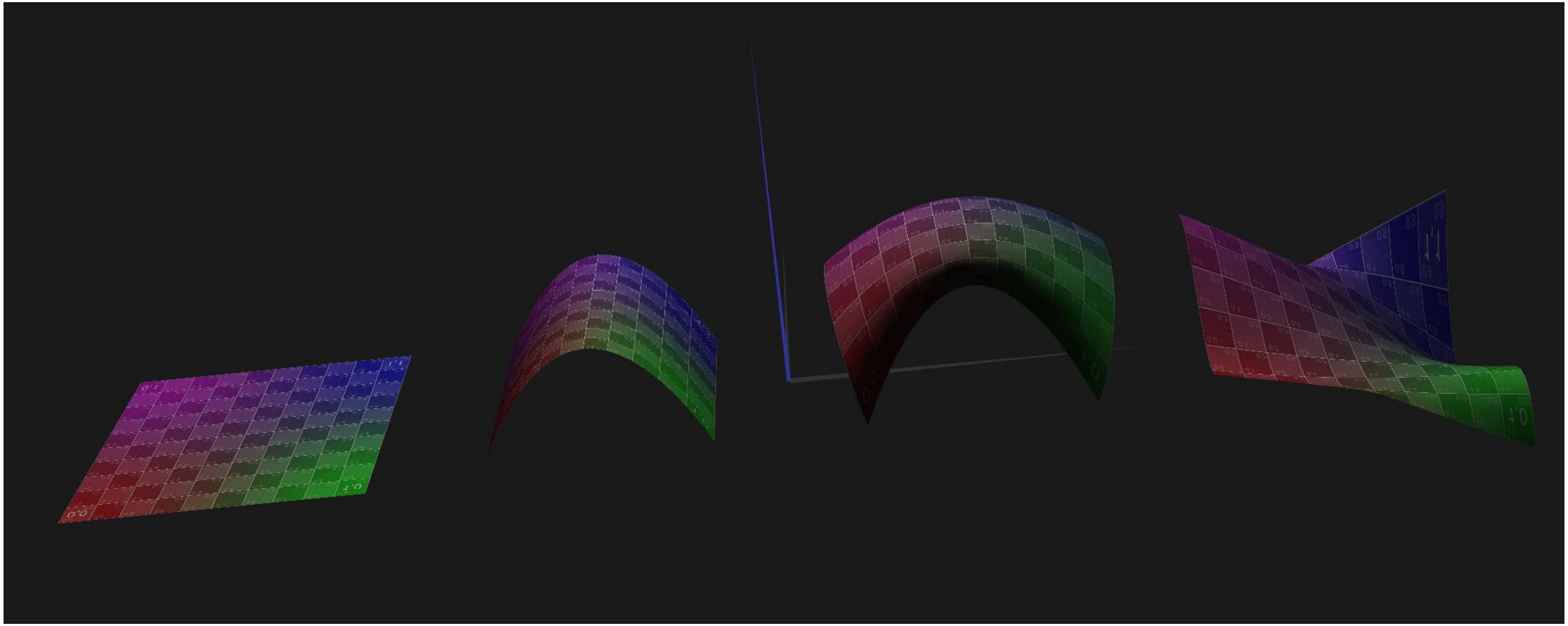
```
this.makeSurface("0", 1, // degree on U: 2 control vertexes U
                 1, // degree on V: 2 control vertexes on V
                [   // U = 0
                    [ // V = 0..1;
                        [-2.0, -2.0, 0.0, 1 ],
                        [-2.0,  2.0, 0.0, 1 ]

                    ],
                    // U = 1
                    [ // V = 0..1
                        [ 2.0, -2.0, 0.0, 1 ],
                        [ 2.0,  2.0, 0.0, 1 ]
                    ]
                ]);

this.makeSurface("1", 2, // degree on U: 3 control vertexes U
                 1, // degree on V: 2 control vertexes on V
                [   // U = 0
                    [ // V = 0..1;
                        [ -1.5, -1.5, 0.0, 1 ],
                        [ -1.5,  1.5, 0.0, 1 ]

                    ],
                    // U = 1
                    [ // V = 0..1
                        [ 0, -1.5, 3.0, 1 ],
                        [ 0,  1.5, 3.0, 1 ]
                    ],
                    // U = 2
                    [ // V = 0..1
                        [ 1.5, -1.5, 0.0, 1 ],
                        [ 1.5,  1.5, 0.0, 1 ]
                    ]
                ]);
```

Source code available in SurfaceDemo, from LAIG moodle website.

```
makeSurface(id, degree1, degree2, controlvertexes, translation) {

    var nurbsSurface = new CGFnurbsSurface(degree1, degree2, controlvertexes);

    var obj = new CGFnurbsObject(this, 20, 20, nurbsSurface ); // must provide an object with the function getPoint(u, v) (CGFnurbsSurface has it)

    this.surfaces.push(obj);
    this.translations.push(translation);

}
```

Number of parts in U and in V to generate geometry.

Demonstration (live)



Annex

Concepts [deprecated in webCGF 0.21]

Knots

- A list of numbers. The list size is:

  degree + control points -1 elements

- Usually called the knot vector (vector, as in unidimensional array).
- The knot vector is important to characterize the parametric space of a curve.
- A common misconception is that each knot is paired with a control point. Only true for degree 1 nurbs where 1 + 2 -1 = 2 = number of control points.
- For simplification, in this class we adopt:
  - Degree 1 [0, 1]
  - Degree 2 [0, 0, 1, 1]
  - Degree 3 [0, 0, 0, 1, 1, 1]
  - Degree 4 [0, 0, 0, 0, 1, 1, 1, 1]
  - Degree 5 [0, 0, 0, 0, 0, 1, 1, 1, 1, 1]