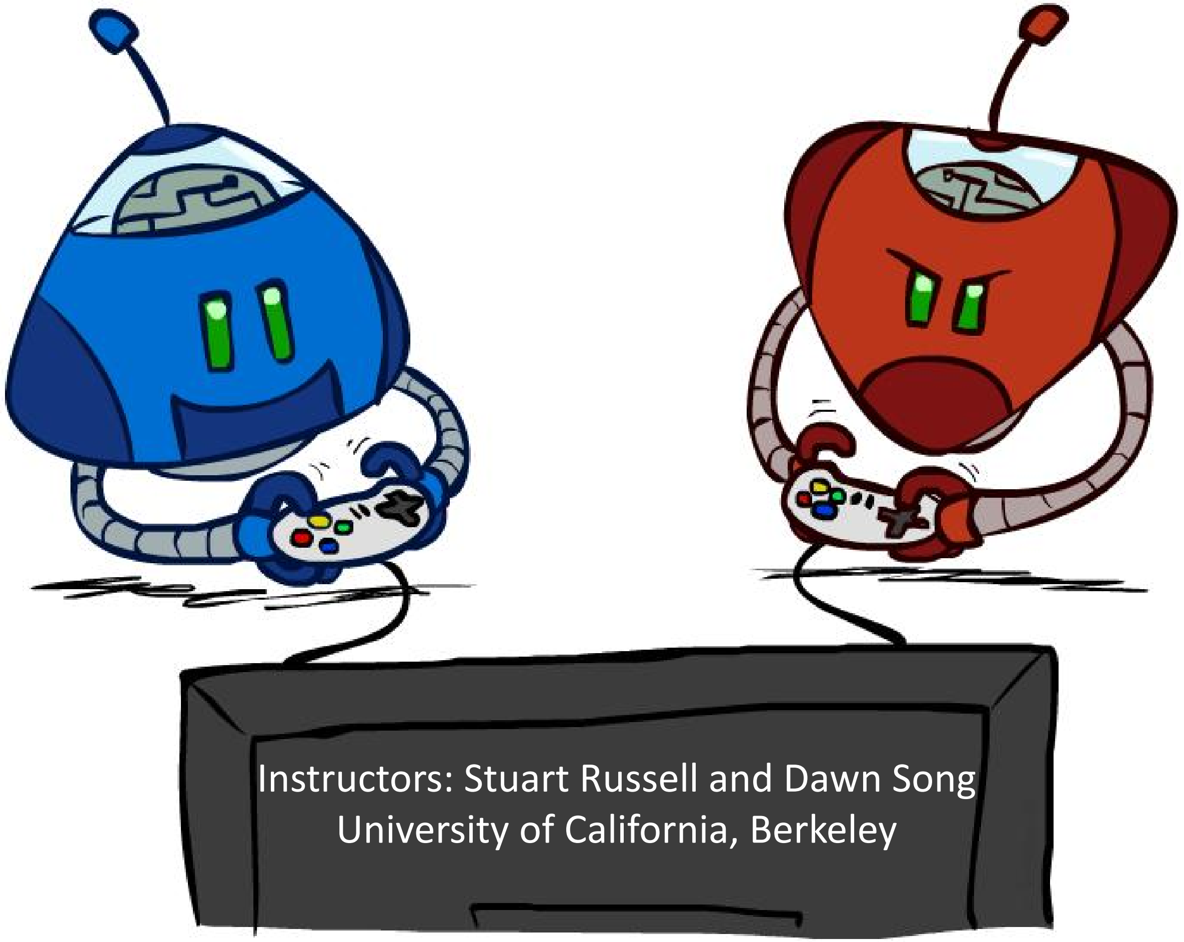
Artificial Intelligence

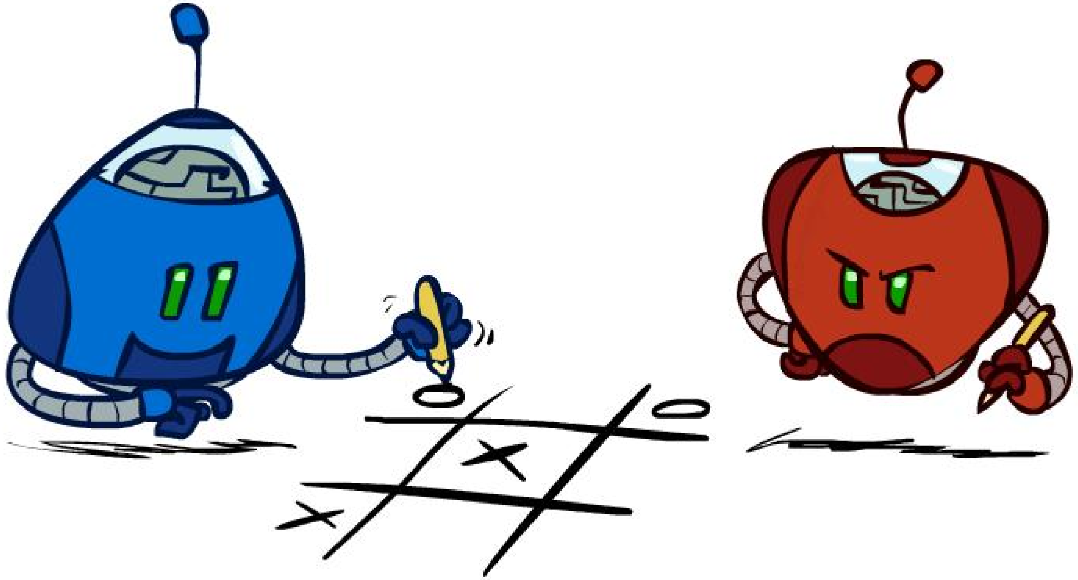
Adversarial Search



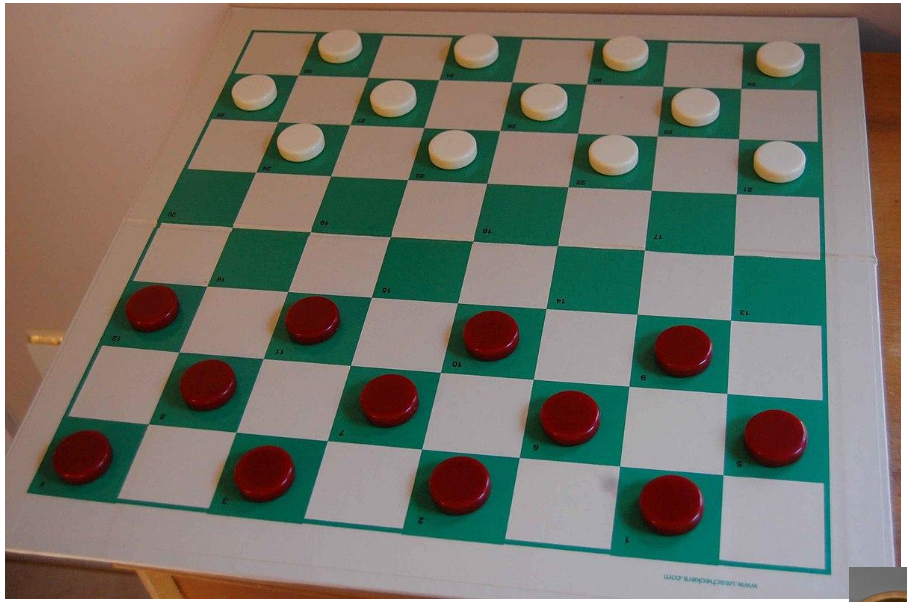
Instructors: Stuart Russell and Dawn Song
University of California, Berkeley

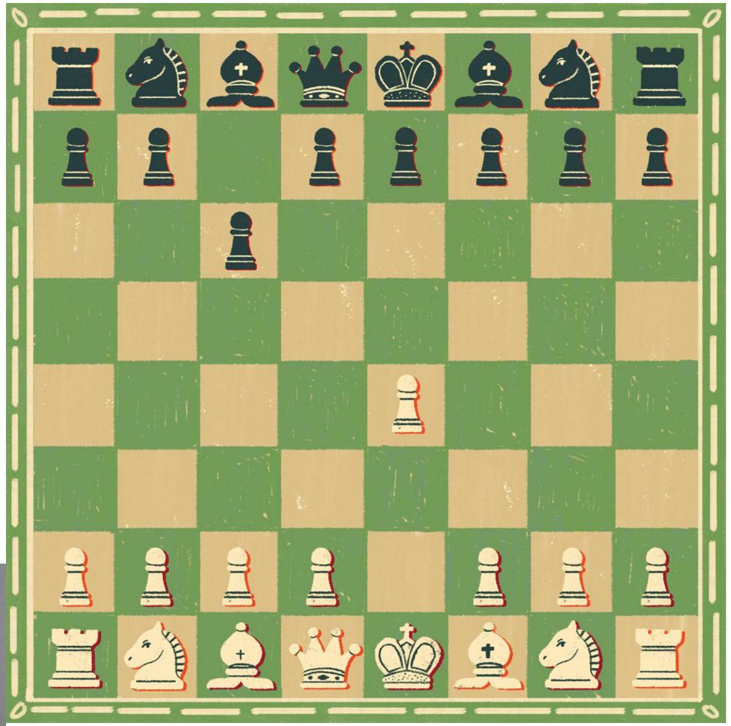
Outline

- History / Overview
- Minimax for Zero-Sum Games
- α-β Pruning
- Finite lookahead and evaluation



Checkers, Chess, Go





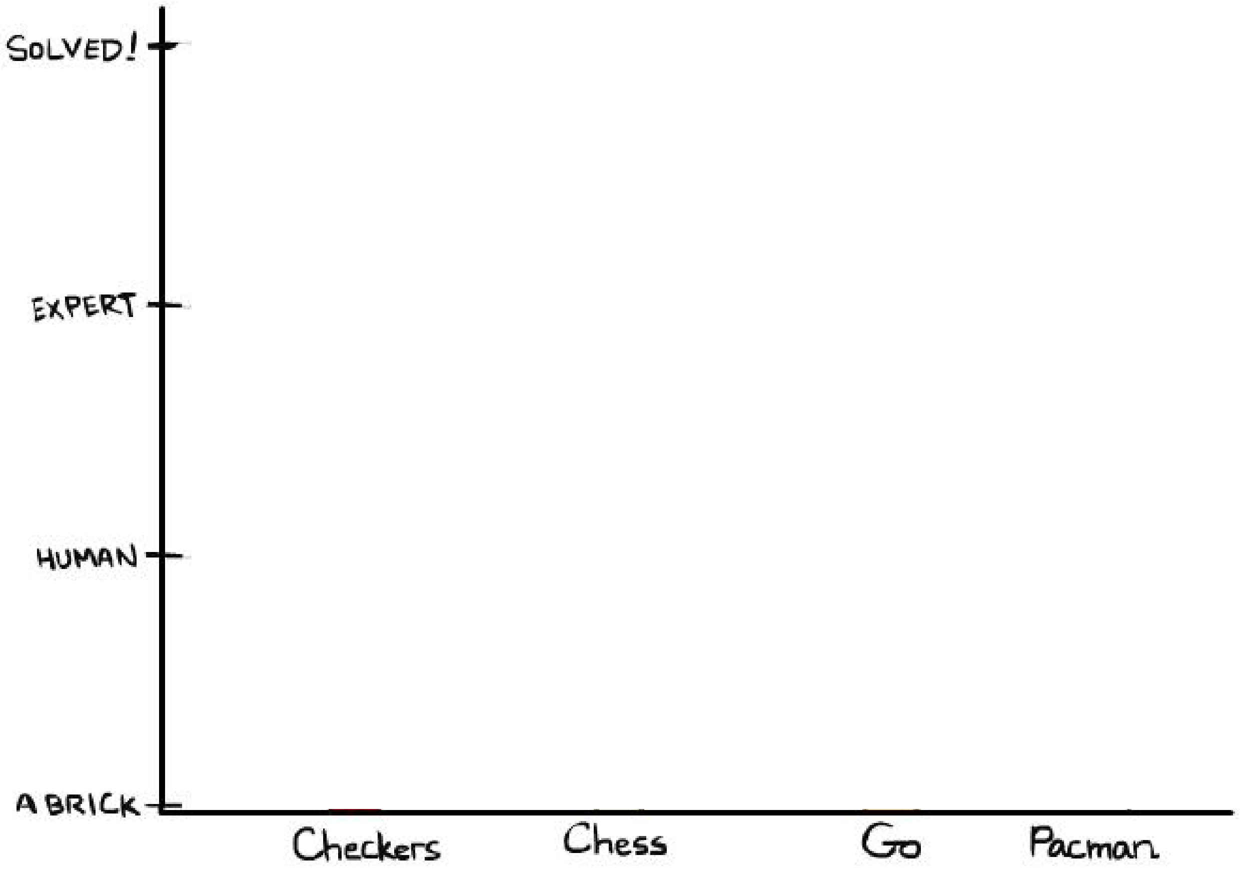
A brief history

- **Checkers:**
  - 1950: First computer player.
  - 1959: Samuel’s self-taught program.
  - 1994: First computer world champion: Chinook defeats Tinsley
  - 2007: Checkers solved! Endgame database of 39 trillion states

- **Chess:**
  - 1945-1960: Zuse, Wiener, Shannon, Turing, Newell & Simon, McCarthy.
  - 1960s onward: gradual improvement under “standard model”
  - 1997: Deep Blue defeats human champion Gary Kasparov
  - 2021: Stockfish rating 3551 (vs 2870 for Magnus Carlsen).

- **Go:**
  - 1968: Zobrist’s program plays legal Go, barely (b>300!)
  - 1968-2005: various ad hoc approaches tried, novice level
  - 2005-2014: Monte Carlo tree search -> strong amateur
  - 2016-2017: AlphaGo defeats human world champions

- **Pacman**



Types of Games

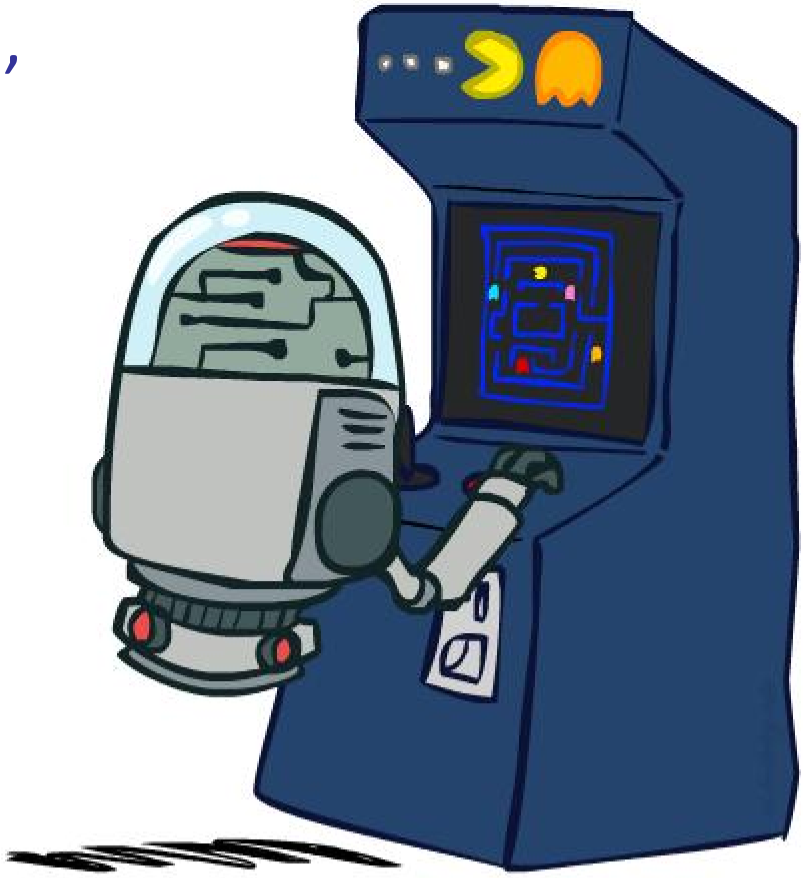
- Game = task environment with > 1 agent

- Axes:
  - Deterministic or stochastic?
  - Perfect information (fully observable)?
  - One, two, or more players?
  - Turn-taking or simultaneous?
  - Zero sum?

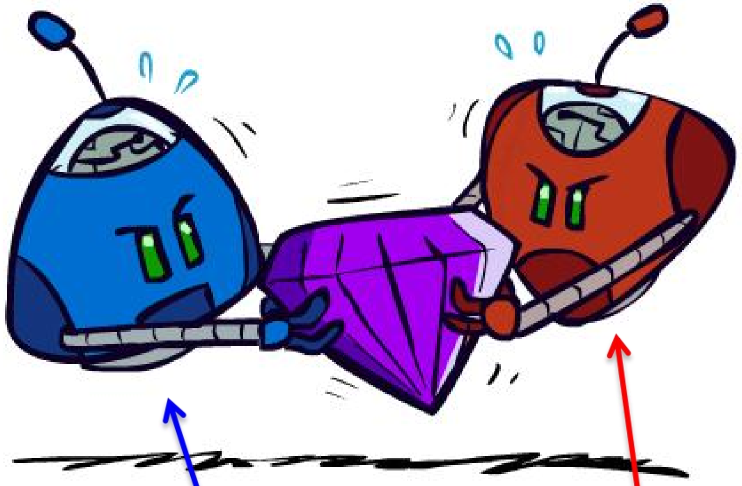
- Want algorithms for calculating a ***contingent plan*** (a.k.a. strategy or policy) which recommends a move for every possible eventuality

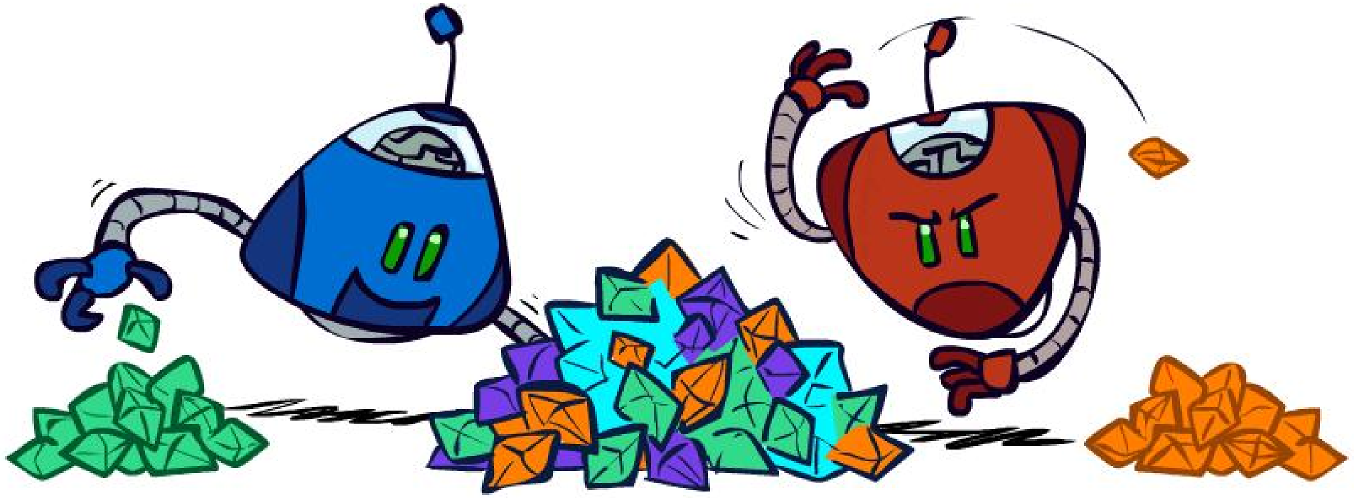
“Standard” Games

- Standard games are deterministic, observable, two-player, turn-taking, zero-sum
- Game formulation:
  - Initial state: s_0
  - Players: Player(s) indicates whose move it is
  - Actions: Actions(s) for player on move
  - Transition model: Result(s,a)
  - Terminal test: Terminal-Test(s)
  - Terminal values: Utility(s,p) for player p
    - Or just Utility(s) for player making the decision at root



Zero-Sum Games

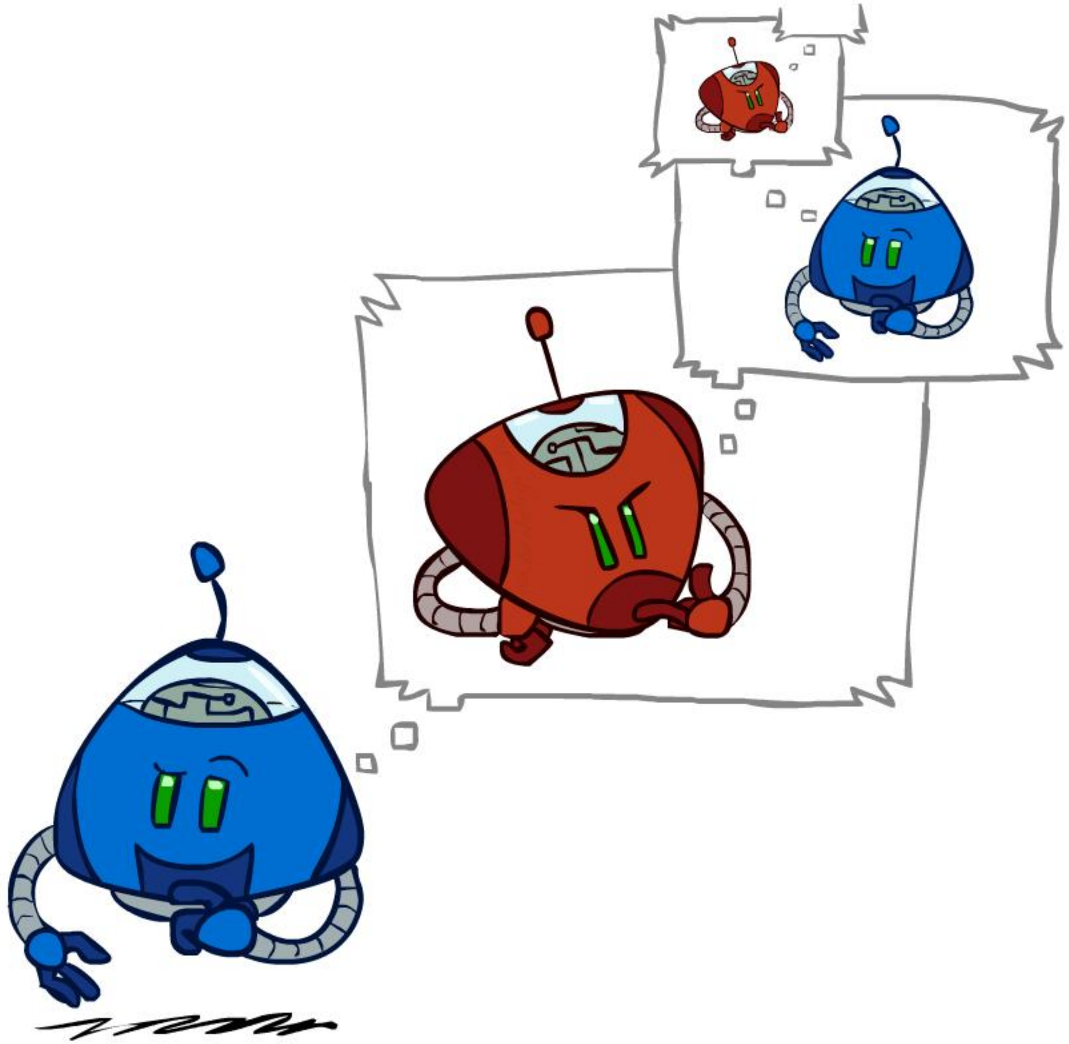




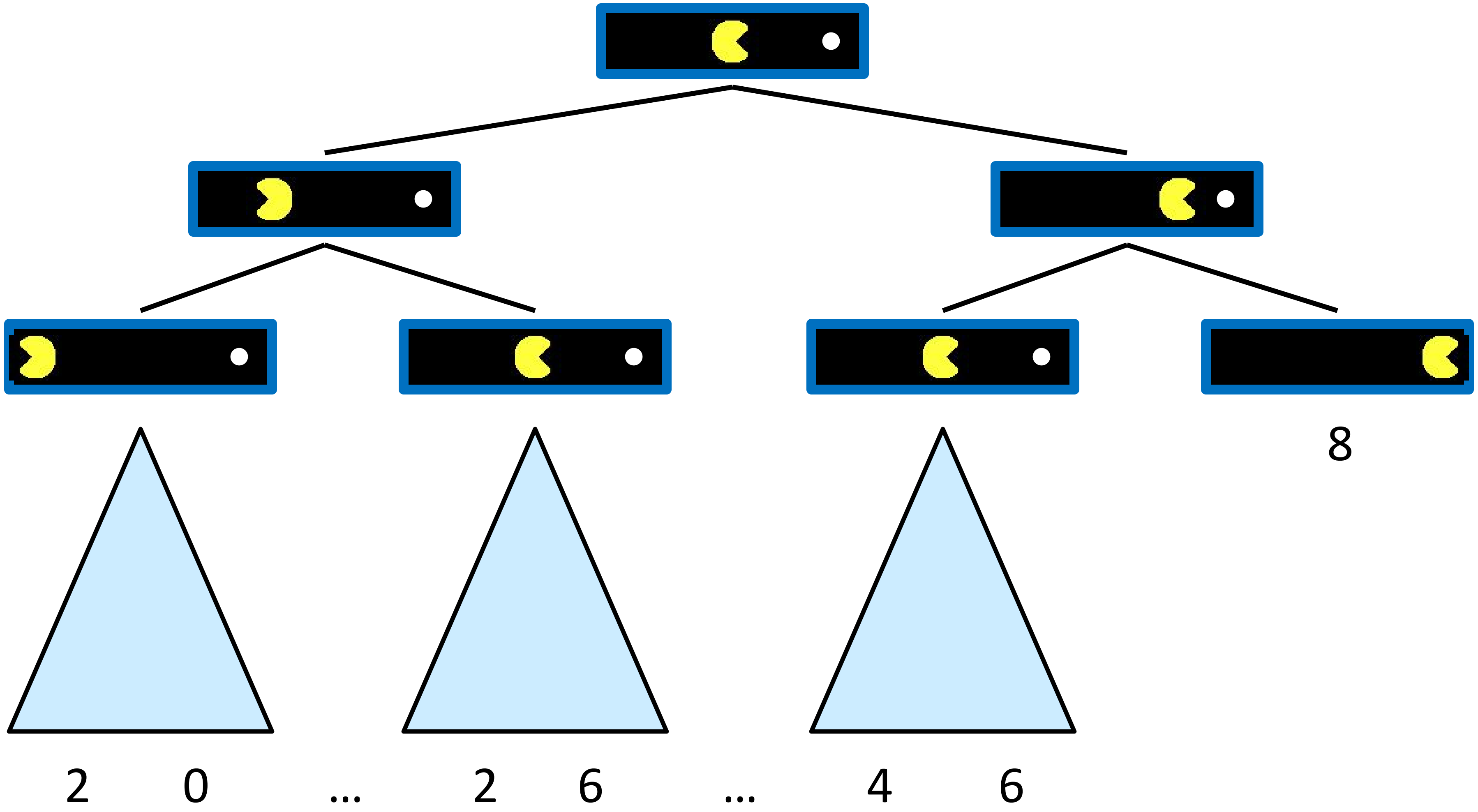
- Zero-Sum Games
  - Agents have ***opposite*** utilities
  - Pure competition:
    - One ***maximizes***, the other ***minimizes***

- General Games
  - Agents have ***independent*** utilities
  - Cooperation, indifference, competition, shifting alliances, and more are all possible

Adversarial Search

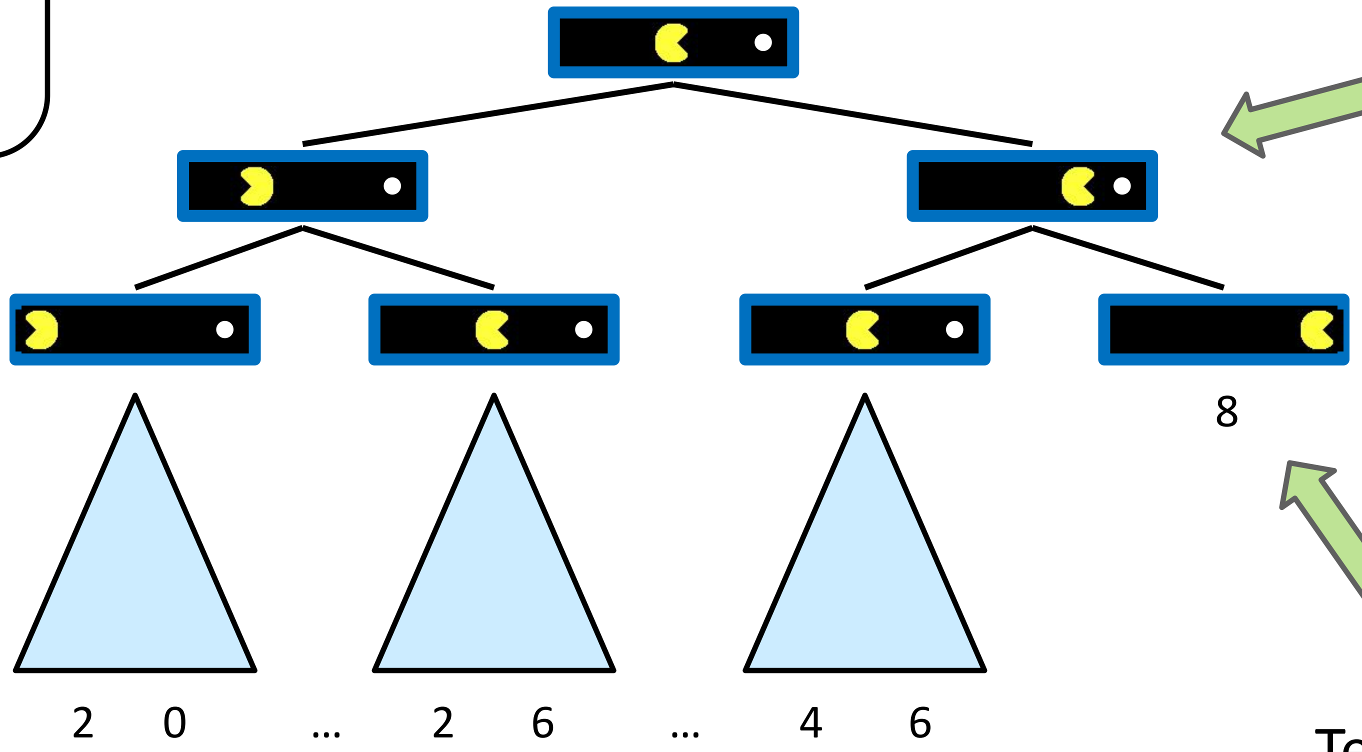


Single-Agent Trees



Value of a State

Value of a state:
The best achievable outcome (utility) from that state



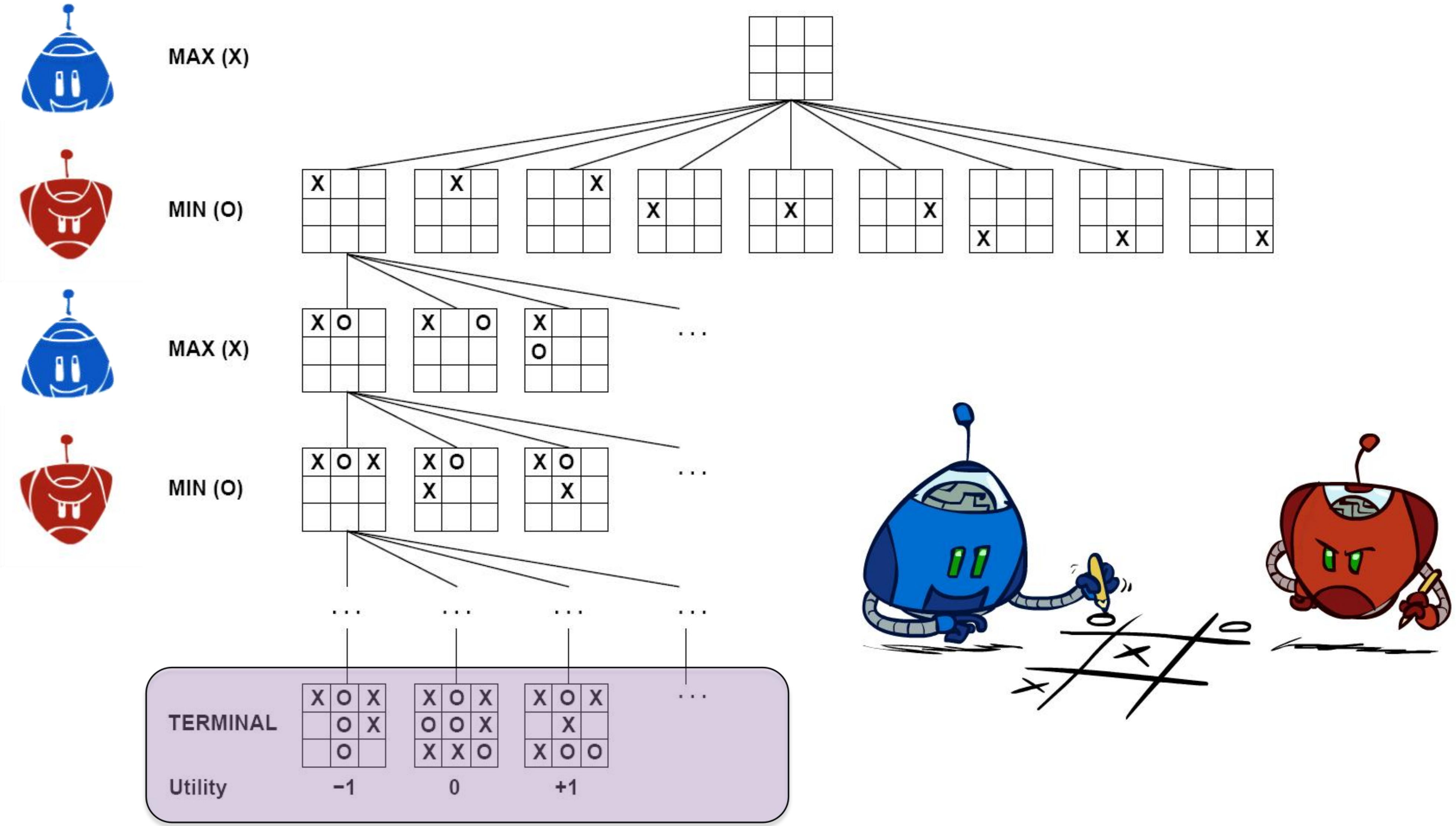
Non-Terminal States:

$$V(s) = \max_{s' \in \text{successors}(s)} V(s')$$

Terminal States:

$V(s)$ = known

Tic-Tac-Toe Game Tree



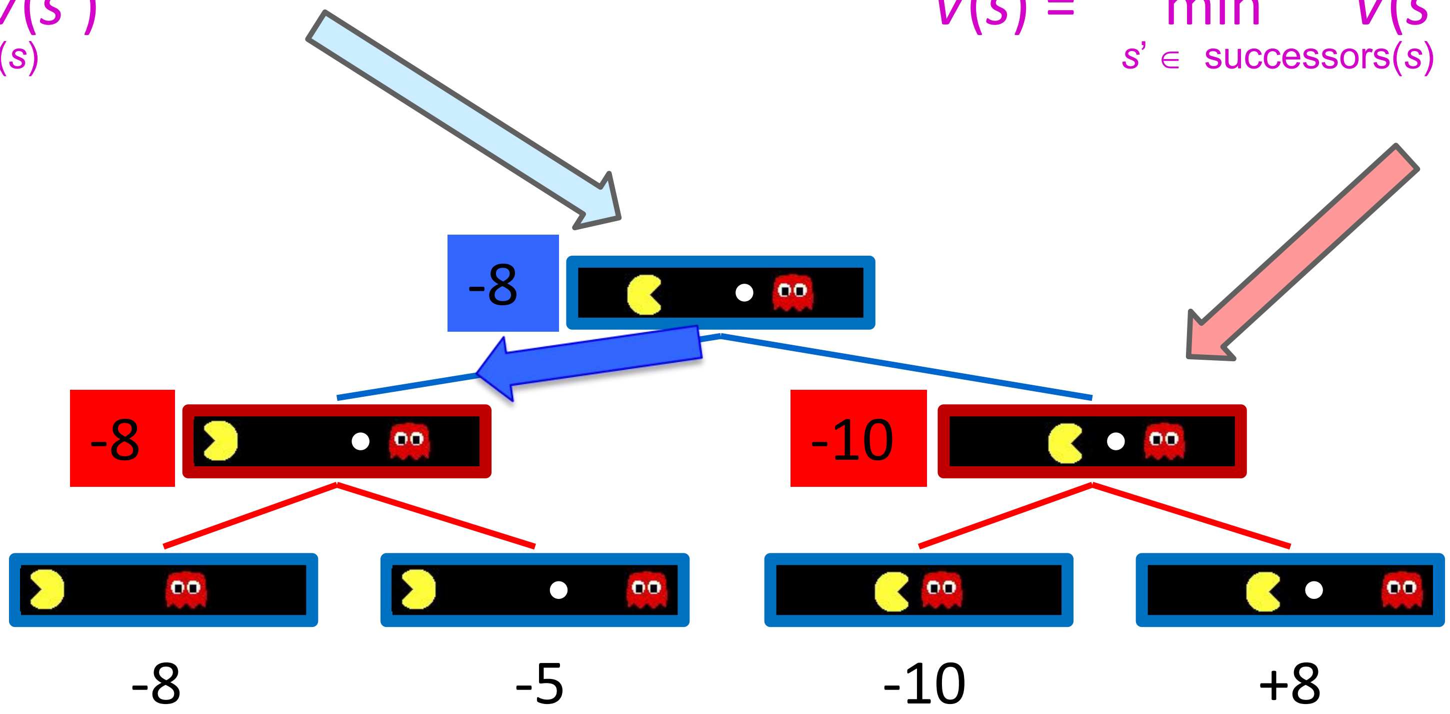
Minimax Values

MAX nodes: under Agent's control

$$V(s) = \max_{s' \in \text{successors}(s)} V(s')$$

MIN nodes: under Opponent's control

$$V(s) = \min_{s' \in \text{successors}(s)} V(s')$$



Terminal States:

$V(s)$ = known

Minimax algorithm

- Choose action leading to state with best ***minimax value***
- Assumes all future moves will be optimal
- => rational against a rational player

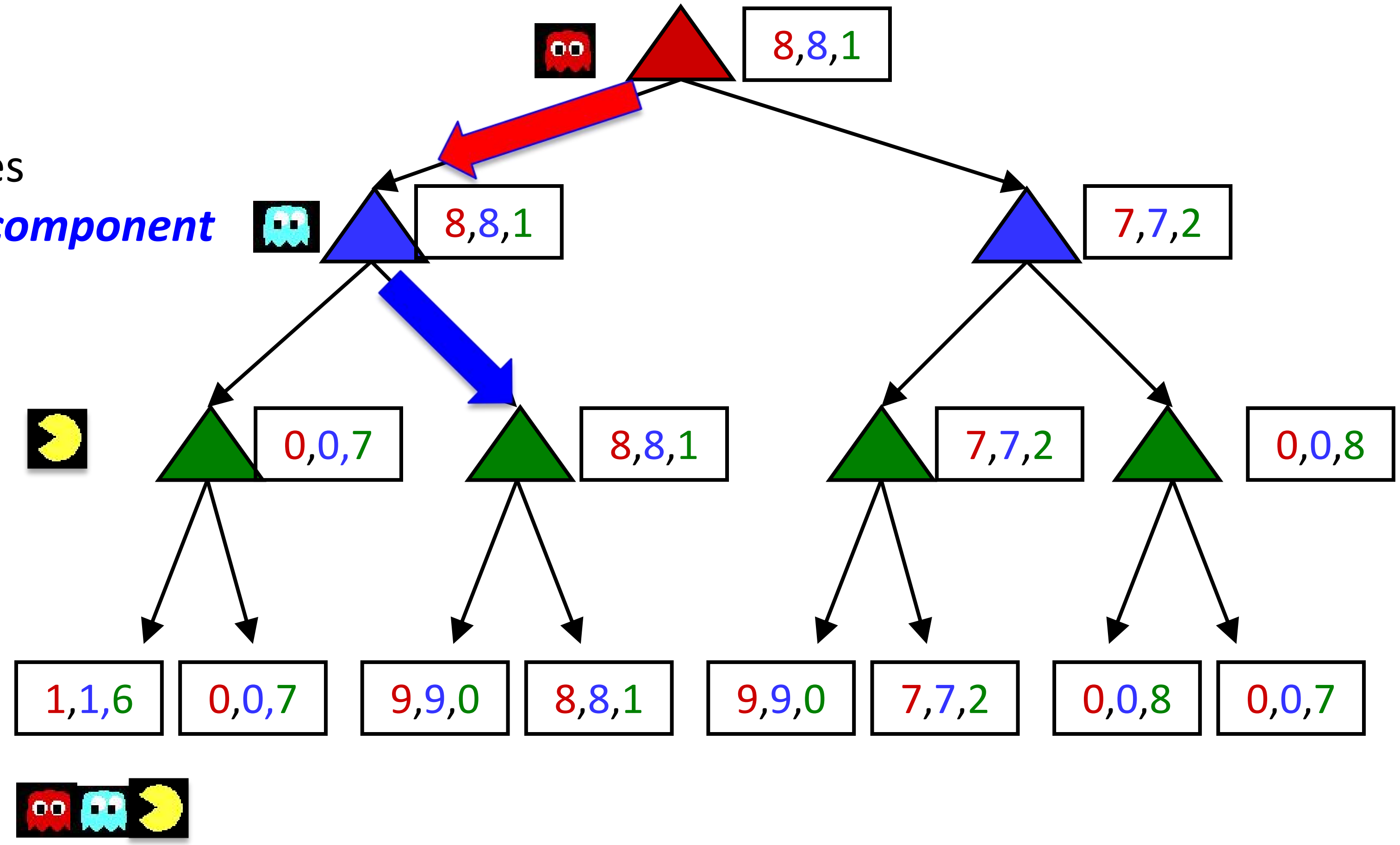
Implementation

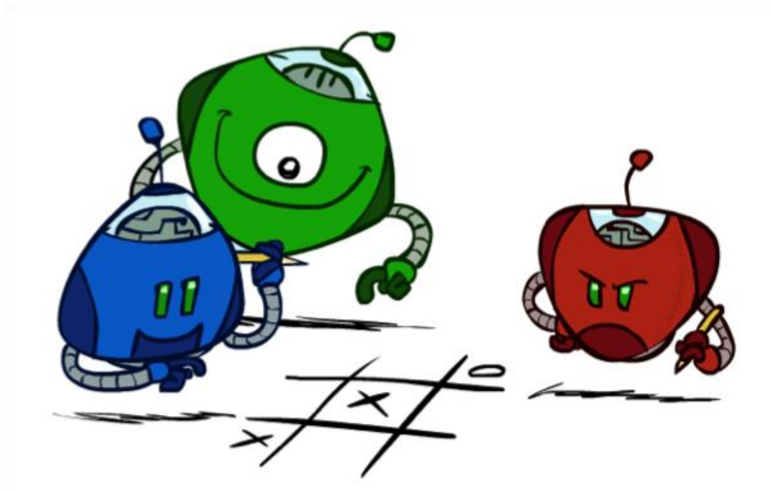
```
function minimax-decision(s) returns an action
    return the action a in Actions(s) with the highest
     minimax_value(Result(s,a))
```

```
function minimax_value(s) returns a value
    if Terminal-Test(s) then return Utility(s)
    if Player(s) = MAX then return max_{a in Actions(s)} minimax_value(Result(s,a))
    if Player(s) = MIN then return min_{a in Actions(s)} minimax_value(Result(s,a))
```

Generalized minimax

- What if the game is not zero-sum, or has multiple players?
- Generalization of minimax:
  - Terminals have ***utility tuples***
  - Node values are also utility tuples
  - ***Each player maximizes its own component***
  - Can give rise to cooperation and competition dynamically…

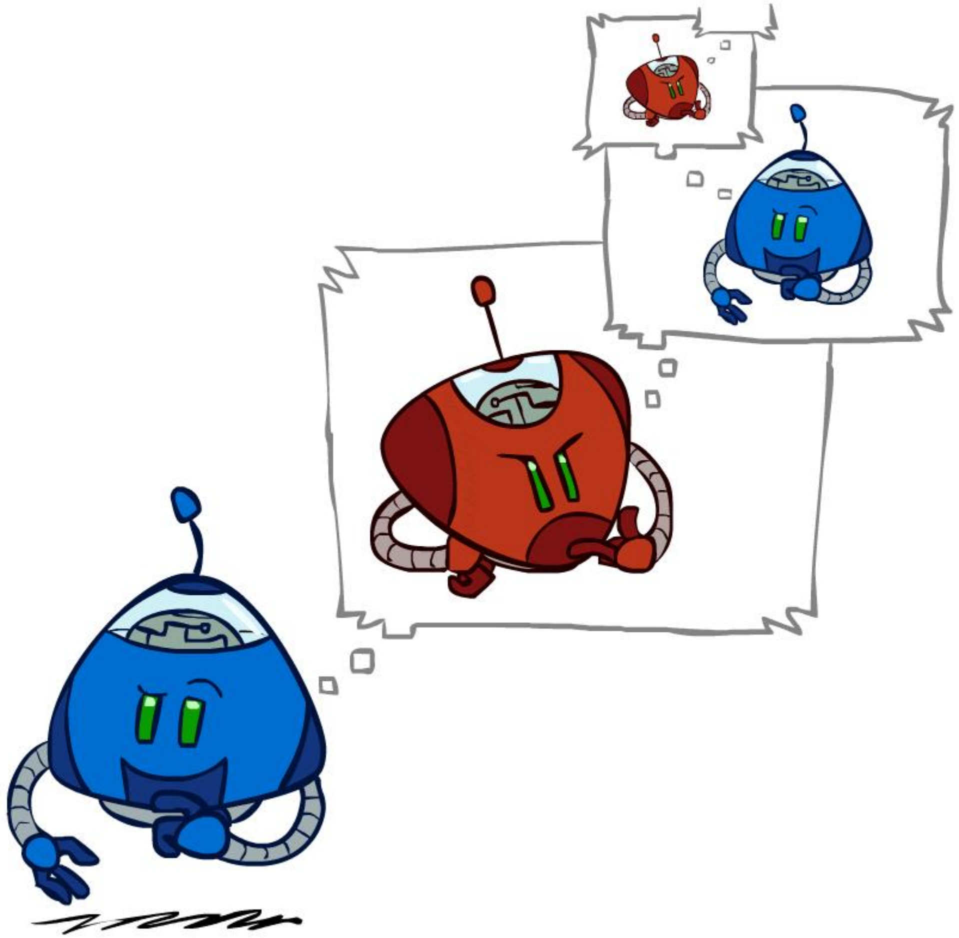




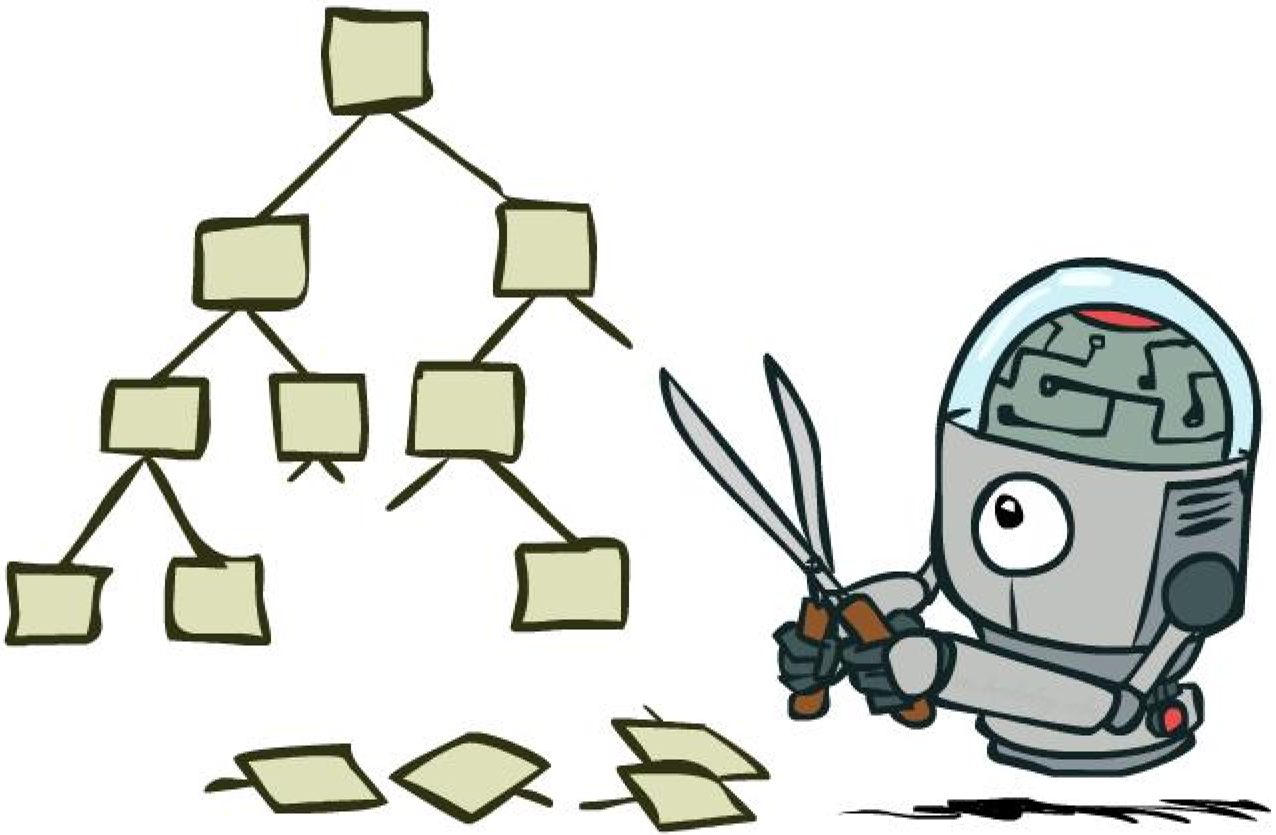
Minimax Efficiency

- How efficient is minimax?
  - Just like (exhaustive) DFS
  - Time: $O(b^m)$
  - Space: $O(bm)$

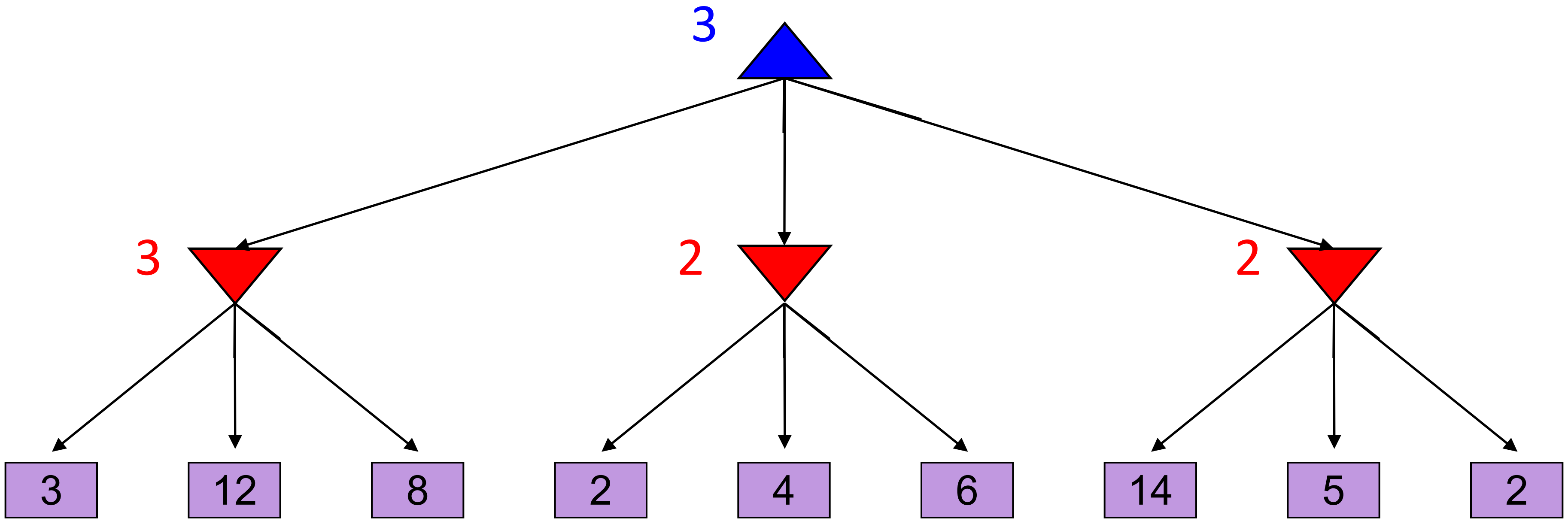
- Example: For chess, $b \approx 35$, $m \approx 100$
  - Exact solution is completely infeasible
  - Humans can't do this either, so how do we play chess?



Game Tree Pruning

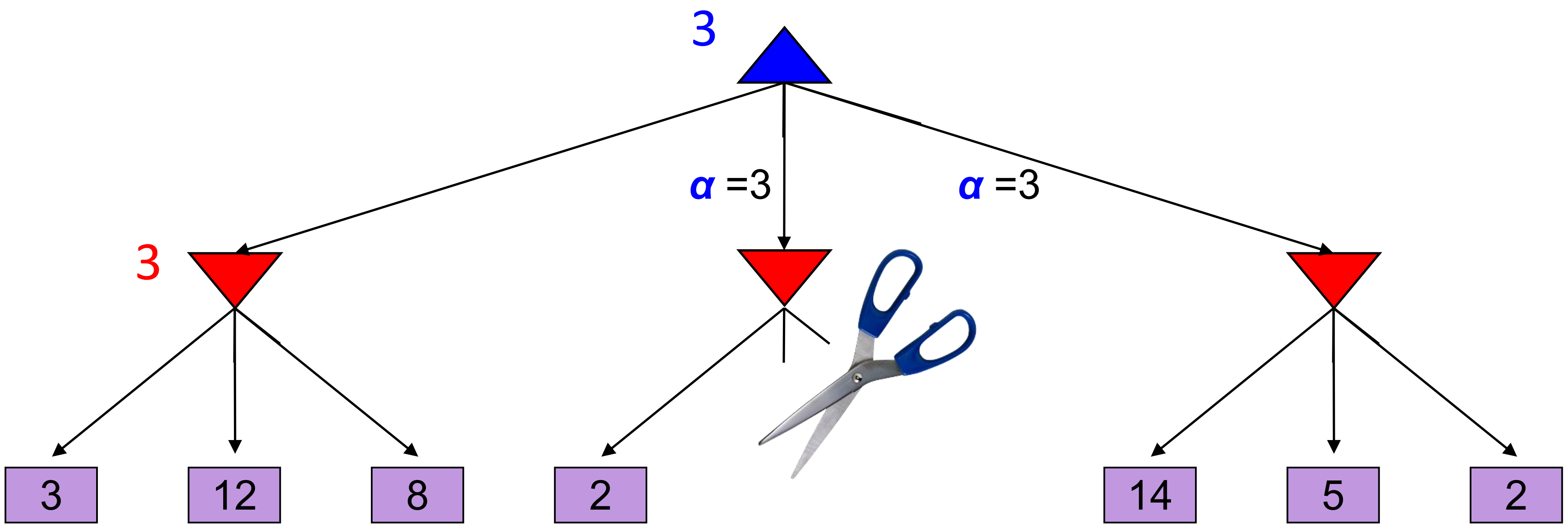


Minimax Example

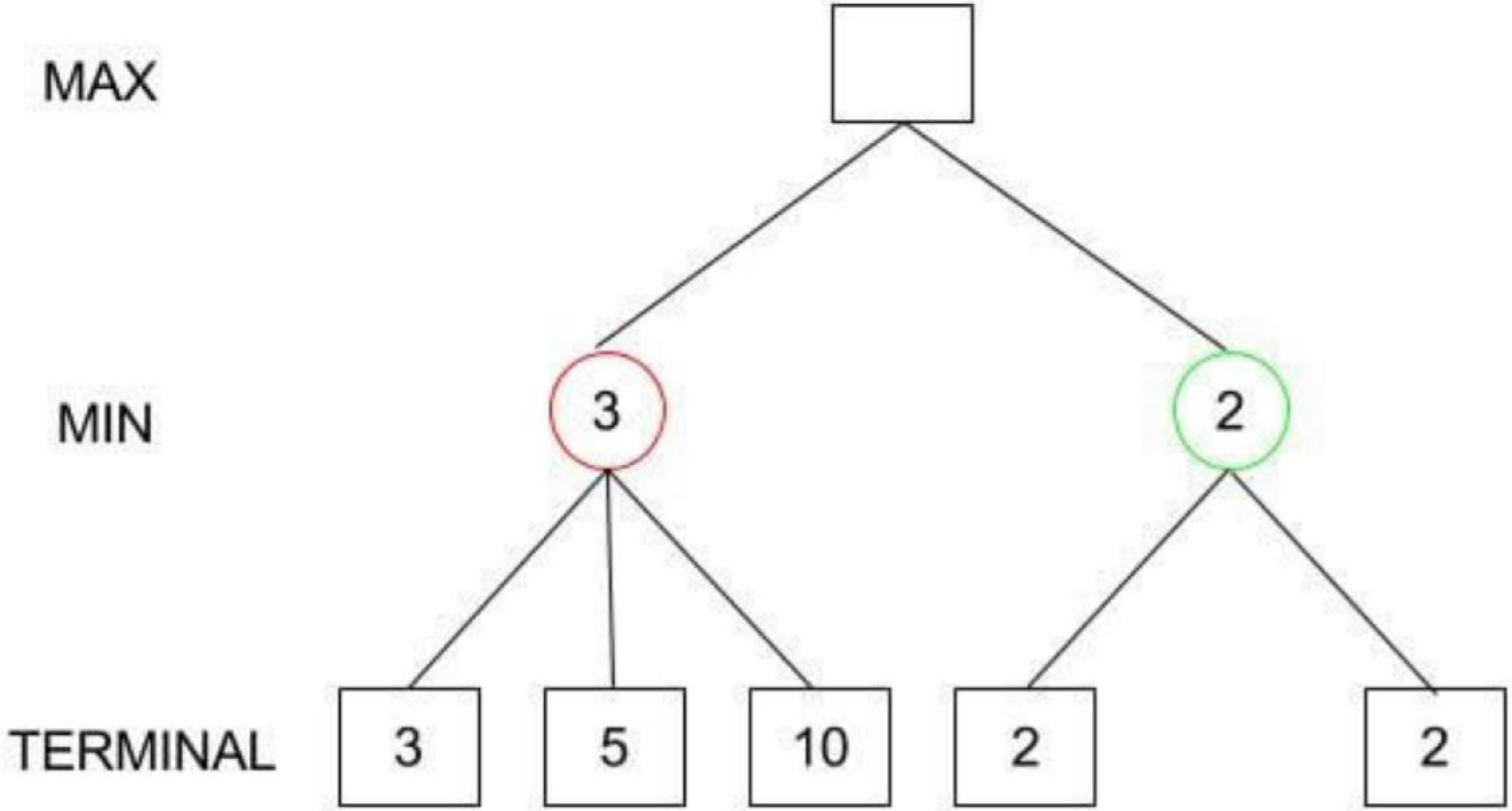


Alpha-Beta Example

α = best option so far from any MAX node on this path



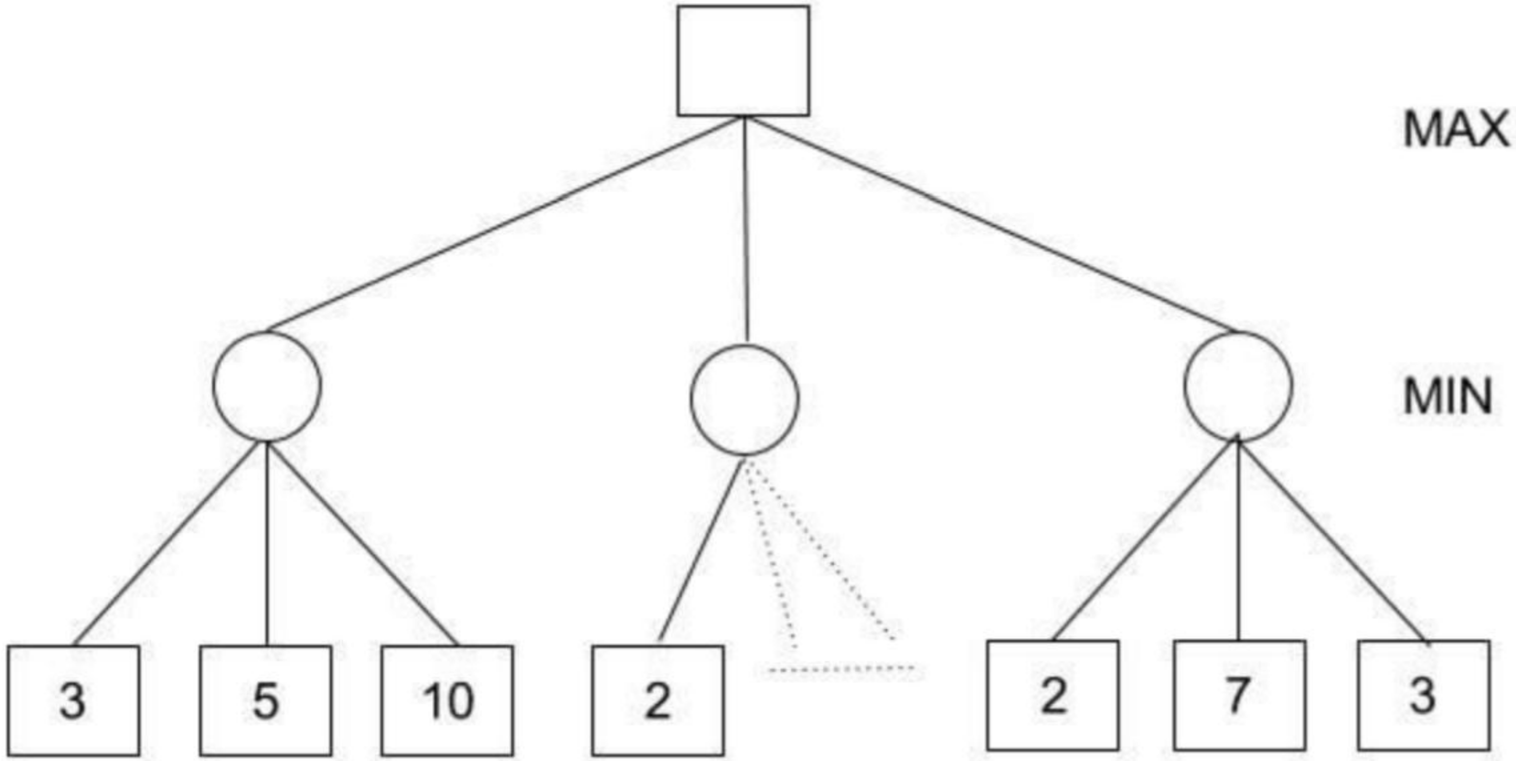
The order of generation matters: more pruning is possible if good moves come first



Minimax Decision = MAX{MIN{3,5,10},MIN{2,2}}

= MAX{3,2}

= 3

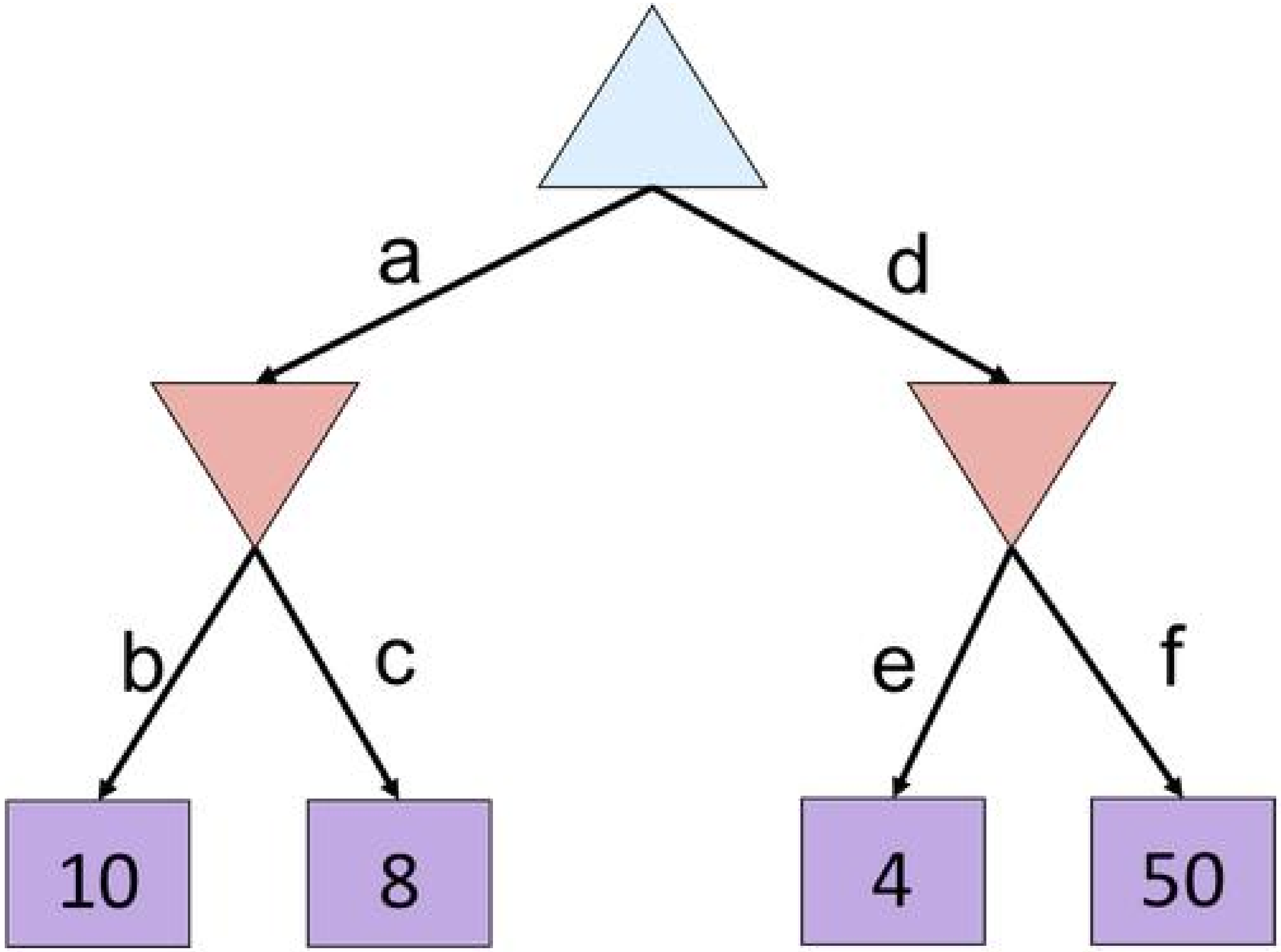


Minimax Decision = MAX{MIN{3,5,10}, MIN{2,a,b}, MIN{2,7,3}}

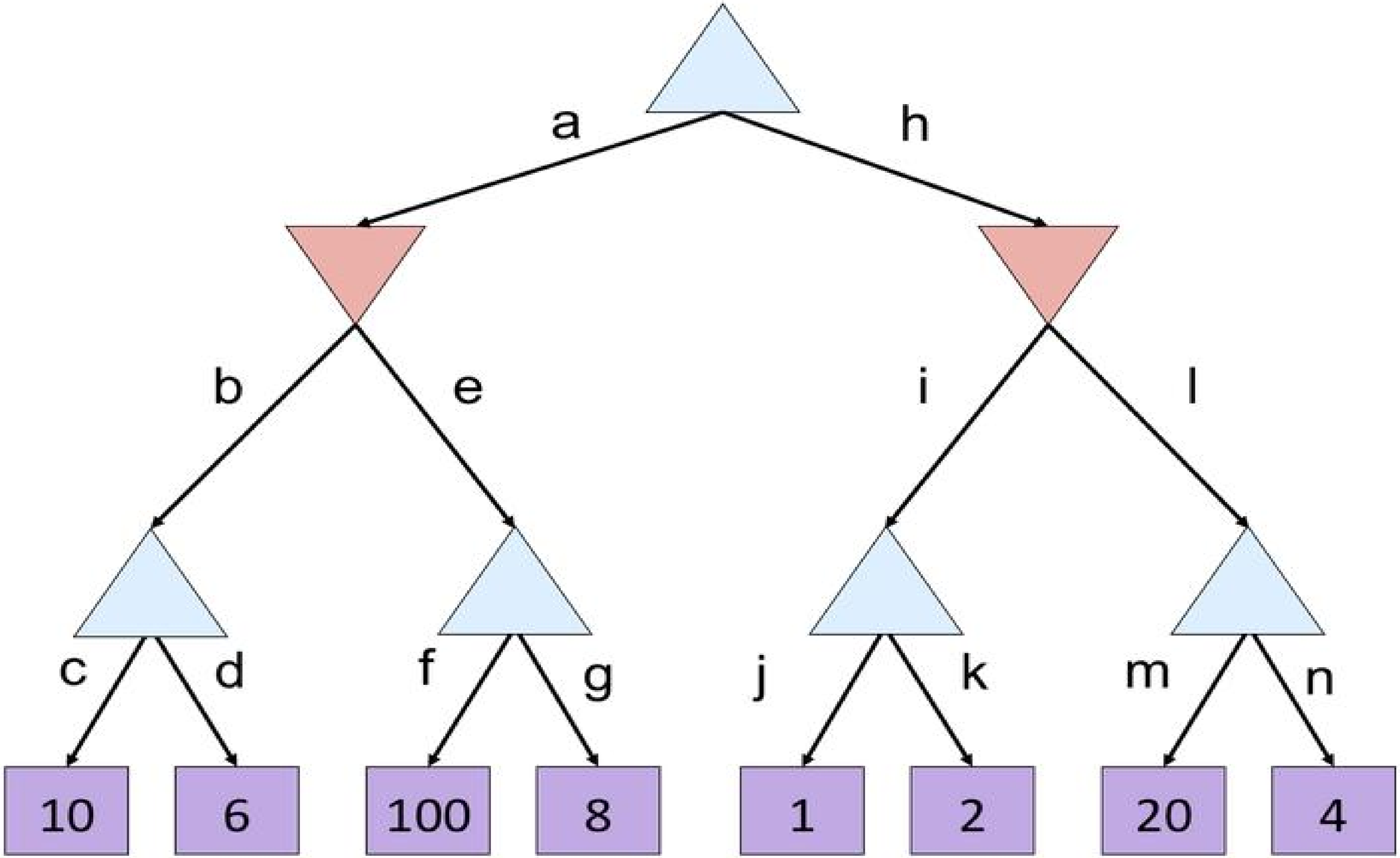
= MAX{3,c,2}

= 3

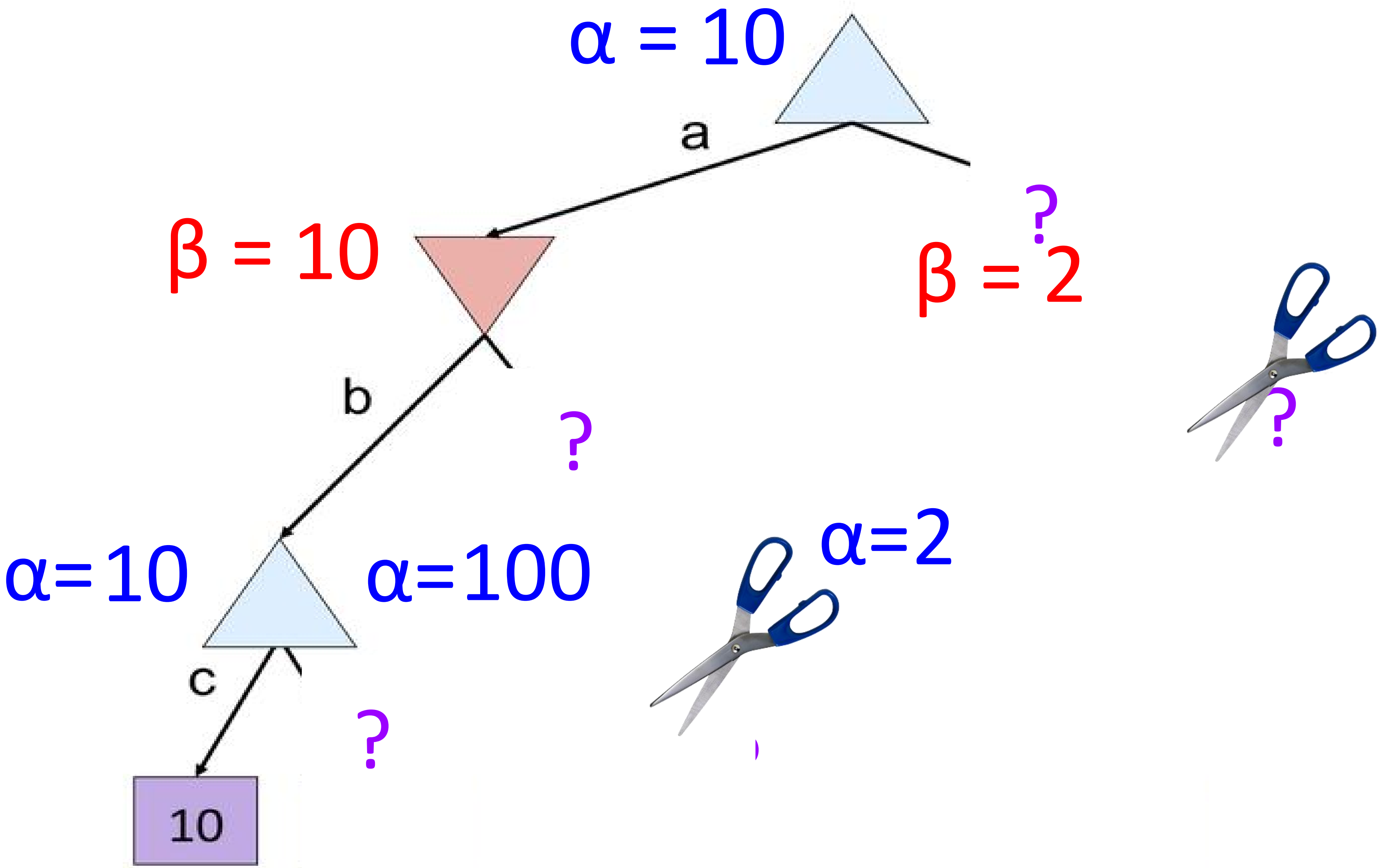
Alpha-Beta Quiz



Alpha-Beta Quiz 2

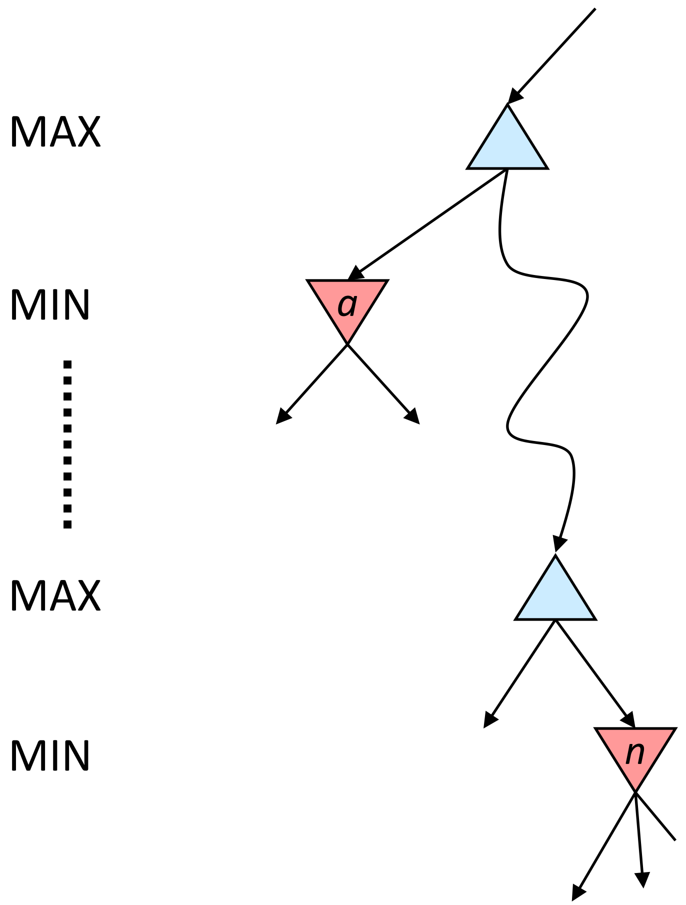


Alpha-Beta Quiz 2



Alpha-Beta Pruning

- General case (pruning children of MIN node)
  - We're computing the MIN-VALUE at some node *n*
  - We're looping over *n*'s children
  - *n*'s estimate of the childrens' min is dropping
  - Who cares about *n*'s value? MAX
  - Let ***α*** be the best value that MAX can get so far at any choice point along the current path from the root
  - If *n* becomes worse than ***α***, MAX will avoid it, so we can prune *n*'s other children (it's already bad enough that it won't be played)
- Pruning children of MAX node is symmetric
  - Let **β** be the best value that MIN can get so far at any choice point along the current path from the root



Alpha-Beta Implementation

α: MAX's best option on path to root
β: MIN's best option on path to root

```
def max-value(state, α, β):
    initialize v = -∞
    for each successor of state:
        v = max(v, value(successor, α, β))
        if v ≥ β
            return v
        α = max(α, v)
    return v
```

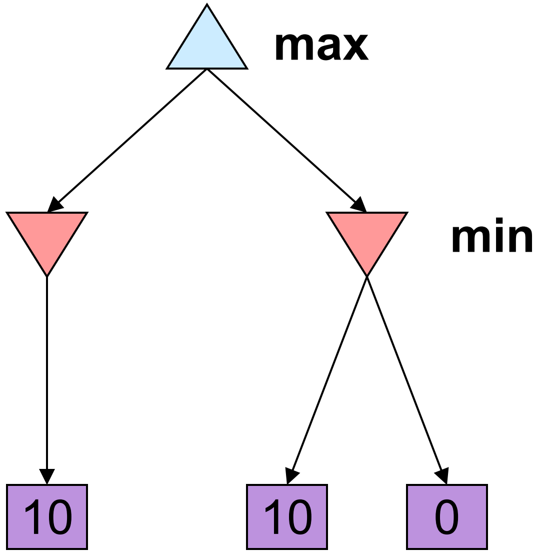
```
def min-value(state , α, β):
    initialize v = +∞
    for each successor of state:
        v = min(v, value(successor, α, β))
        if v ≤ α
            return v
        β = min(β, v)
    return v
```

Alpha-Beta Pruning Properties

- Theorem: This pruning has ***no effect*** on minimax value computed for the root!

- Good child ordering improves effectiveness of pruning
  - Iterative deepening helps with this

- With "perfect ordering":
  - Time complexity drops to $O(b^{m/2})$
  - Doubles solvable depth!



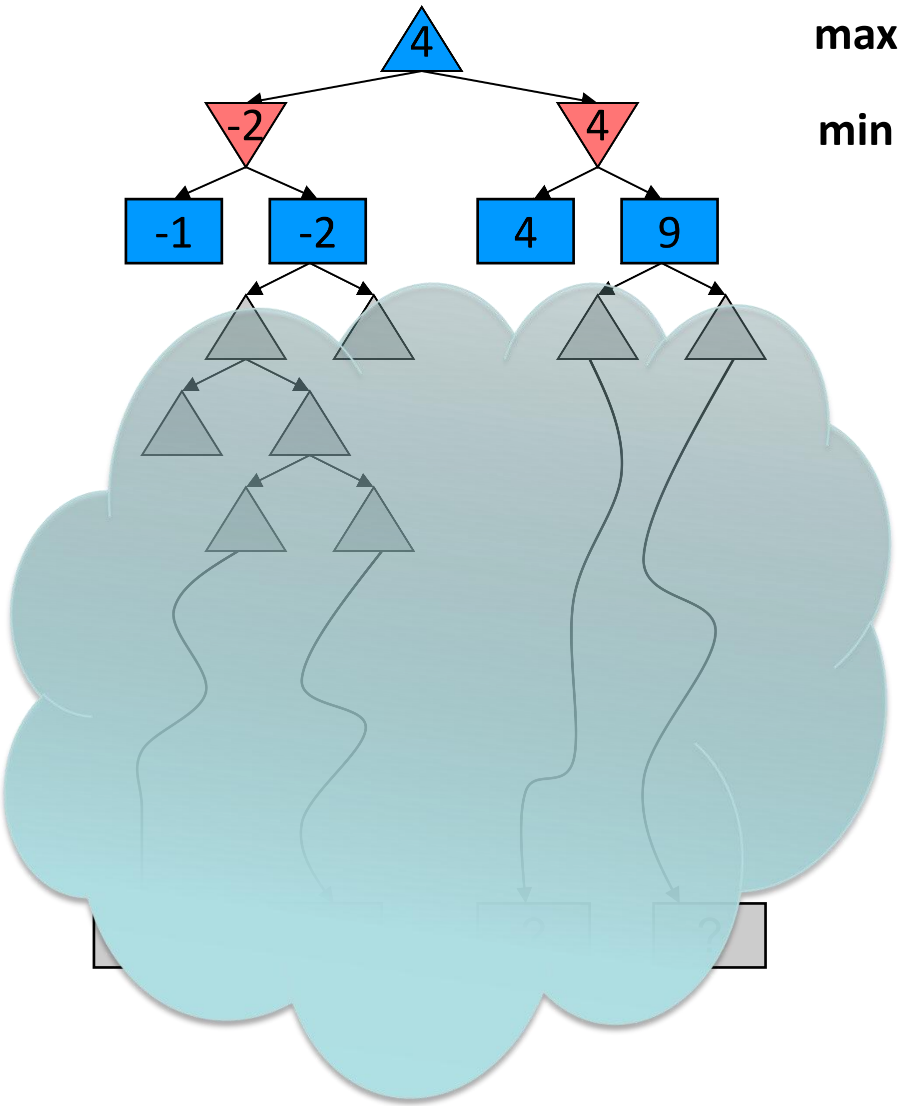
- This is a simple example of metareasoning (reasoning about reasoning)

- For chess: only 35^{50} instead of 35^{100}!! Yaaay!!!!!

Resource Limits

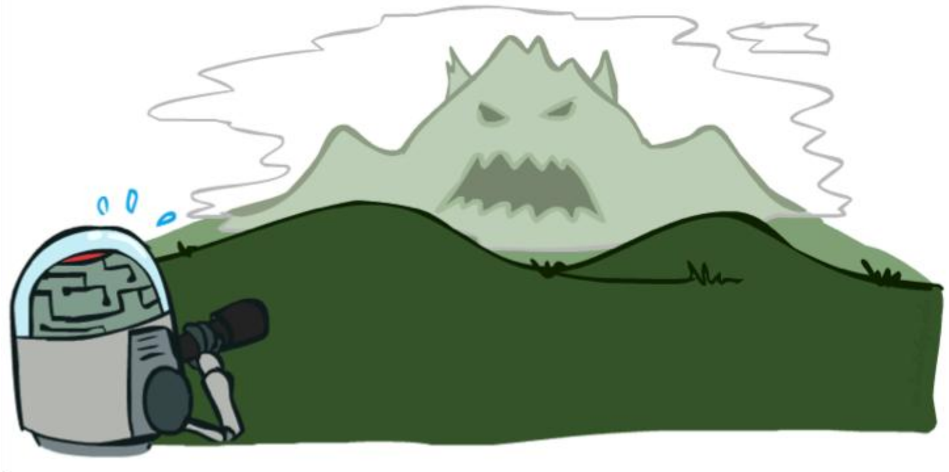
Resource Limits

- Problem: In realistic games, cannot search to leaves!
- Solution 1: Bounded lookahead
  - Search only to a preset ***depth limit*** or ***horizon***
  - Use an ***evaluation function*** for non-terminal positions
- Guarantee of optimal play is gone
- More plies make a BIG difference
- Example:
  - Suppose we have 100 seconds, can explore 10K nodes / sec
  - So can check 1M nodes per move
  - Chess with alpha-beta, $35^{(8/2)}$ =~ 1M; depth 8 is good



Depth Matters

- Evaluation functions are always imperfect
- Deeper search => better play (usually)
- Or, deeper search gives same quality of play with a less accurate evaluation function
- An important example of the tradeoff between complexity of features and complexity of computation

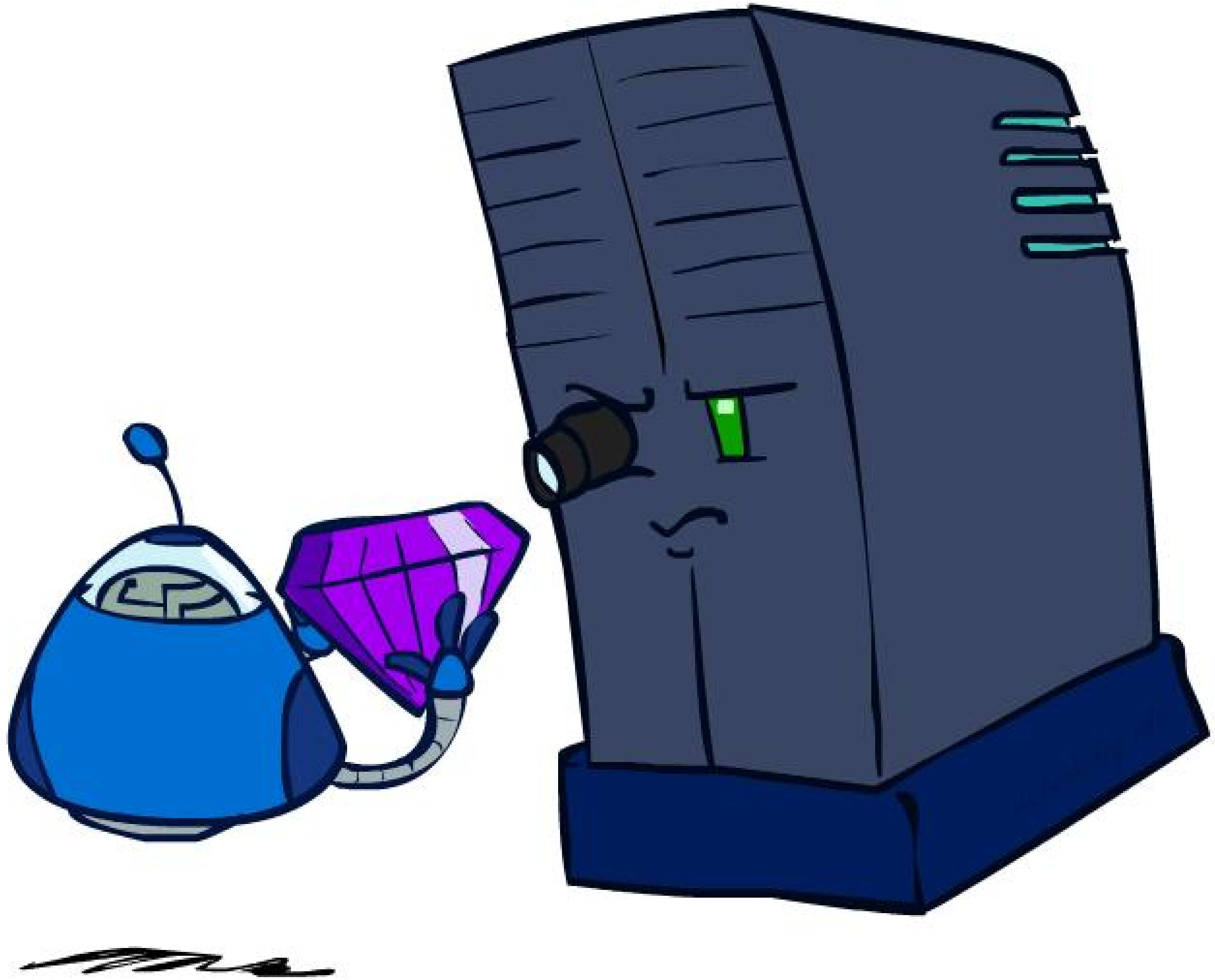


[Demo: depth limited (L6D4, L6D5)]

Pacman with Depth-2 Lookahead

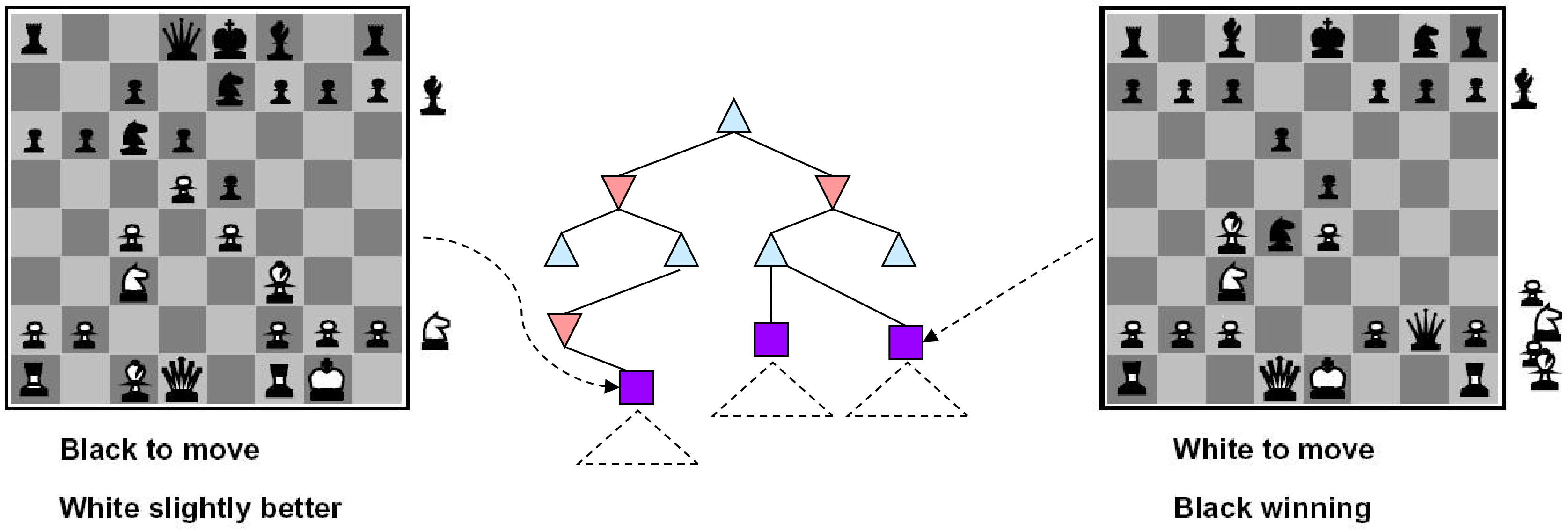
Pacman with Depth-10 Lookahead

Evaluation Functions



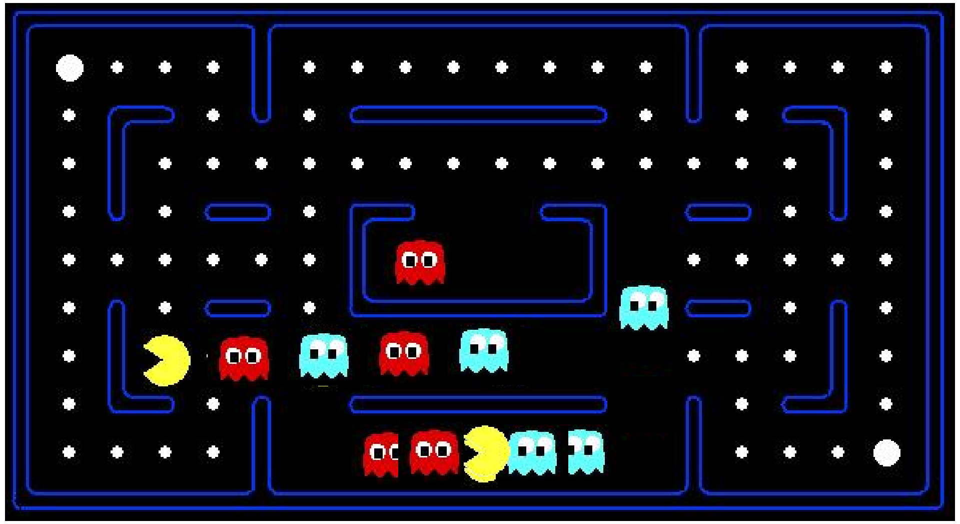
Evaluation Functions

- Evaluation functions score non-terminals in depth-limited search



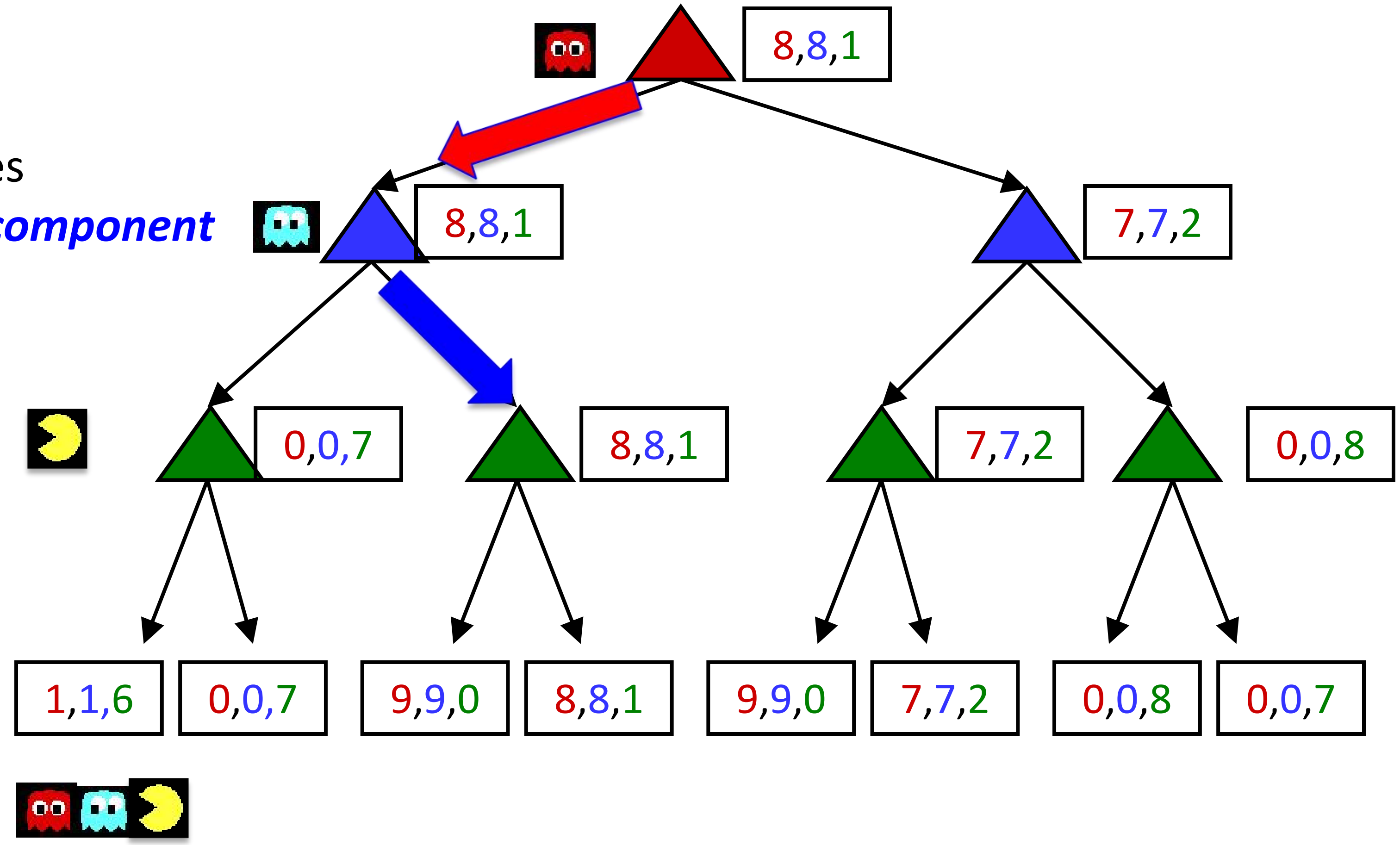
- Typically weighted linear sum of features:
  - EVAL(s) = $w_1 f_1(s) + w_2 f_2(s) + \ldots. + w_n f_n(s)$
  - E.g., $w_1 = 9$, $f_1(s)$ = (num white queens – num black queens), etc.
- Or a more complex nonlinear function (e.g., NN) trained by self-play RL
- Terminate search only in ***quiescent*** positions, i.e., no major changes expected in feature values

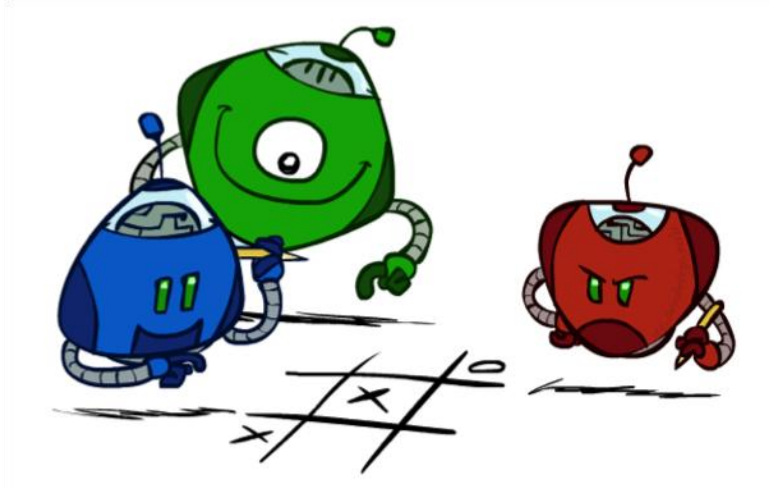
Evaluation for Pacman



Generalized minimax

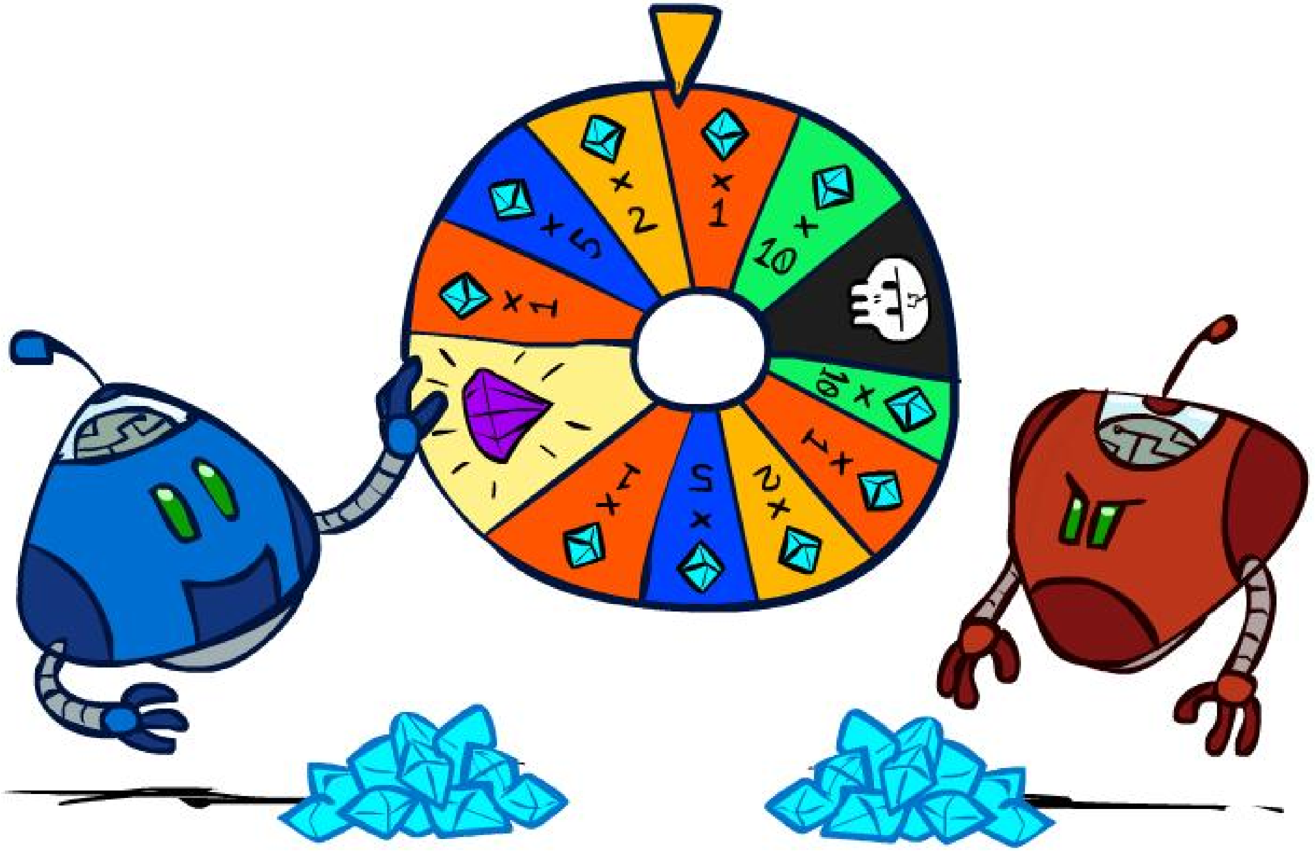
- What if the game is not zero-sum, or has multiple players?
- Generalization of minimax:
  - Terminals have ***utility tuples***
  - Node values are also utility tuples
  - ***Each player maximizes its own component***
  - Can give rise to cooperation and competition dynamically…



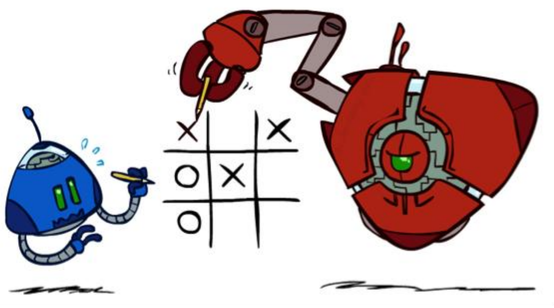


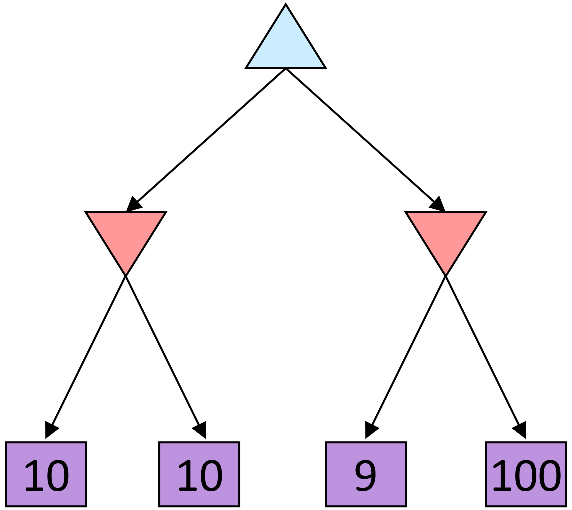
Emergent coordination in ghosts

Games with uncertain outcomes

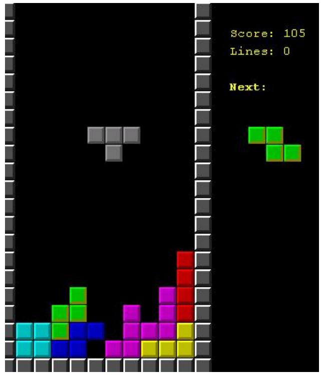


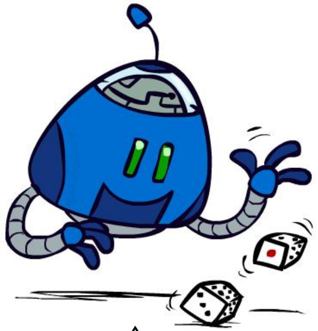
Chance outcomes in trees

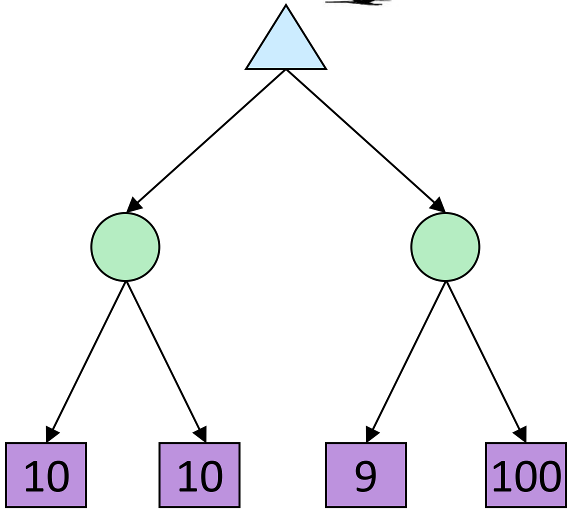




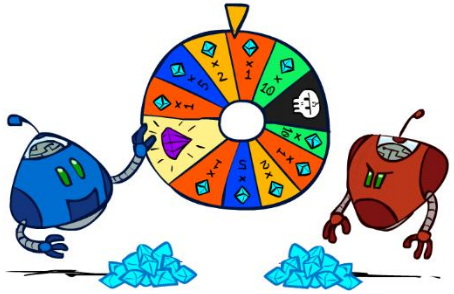
Tictactoe, chess
Minimax

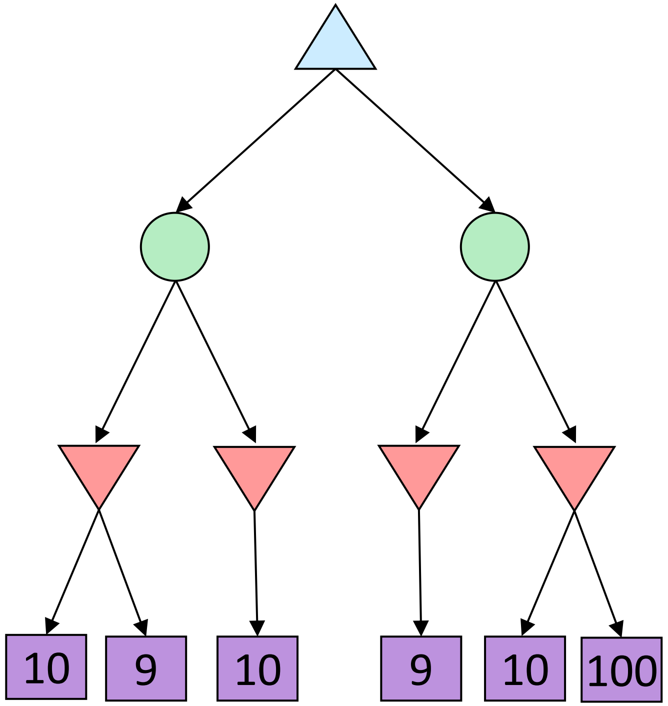






Tetris, investing
Expectimax





Backgammon, Monopoly
Expectiminimax

Minimax

function decision(s) returns an action

return the action a in Actions(s) with the highest minimax_value(Result(s,a))

function minimax_value(s) returns a value

if Terminal-Test(s) then return Utility(s)

if Player(s) = MAX then return $\max_{a \text{ in Actions}(s)}$ minimax_value(Result(s,a))

if Player(s) = MIN then return $\min_{a \text{ in Actions}(s)}$ minimax_value(Result(s,a))

Expectiminimax

```
function decision(s) returns an action
    return the action a in Actions(s) with the highest
     value(Result(s,a))
```

```
function value(s) returns a value
    if Terminal-Test(s) then return Utility(s)
    if Player(s) = MAX      then return max_{a in Actions(s)} value(Result(s,a))
    if Player(s) = MIN      then return min_{a in Actions(s)} value(Result(s,a))
    if Player(s) = CHANCE then return sum_{a in Actions(s)} Pr(a) * value(Result(s,a))
```

Summary

- Games require decisions when optimality is impossible
  - Bounded-depth search and approximate evaluation functions
- Games force efficient use of computation
  - Alpha-beta pruning
- Game playing has produced important research ideas
  - Reinforcement learning (checkers)
  - Iterative deepening (chess)
  - Rational metareasoning (Othello)
  - Monte Carlo tree search (Go)
  - Solution methods for partial-information games in economics (poker)
- Video games present much greater challenges – lots to do!
  - $b = 10^{500}$, $|S| = 10^{4000}$, $m = 10{,}000$