MAST30027 Modern Applied Statistics Assignment4

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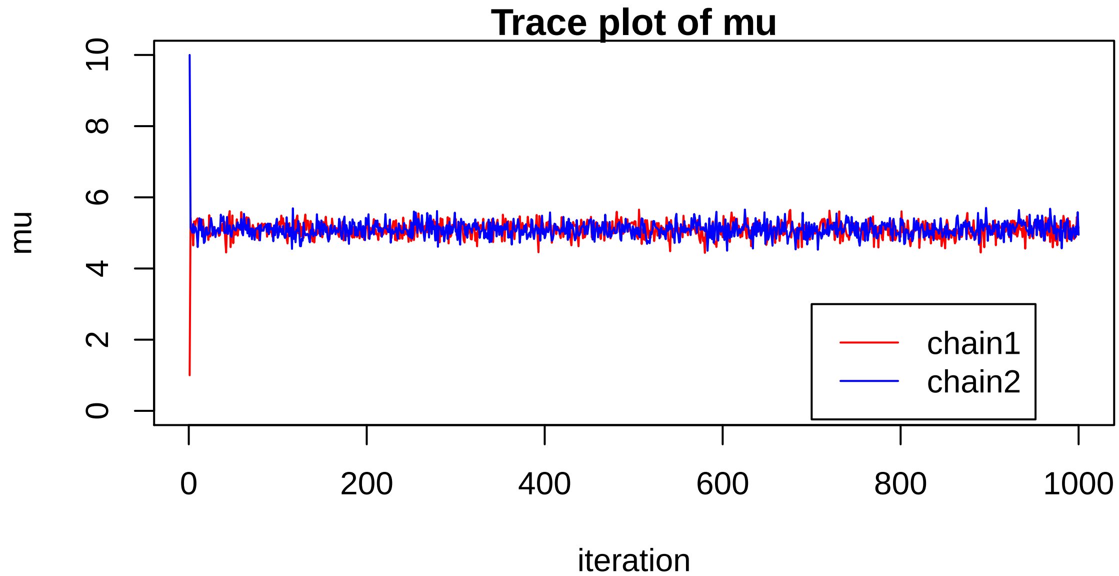
1(b)

```
#load the dataset
data <- scan("Assign4Data.txt")
n = 100
```

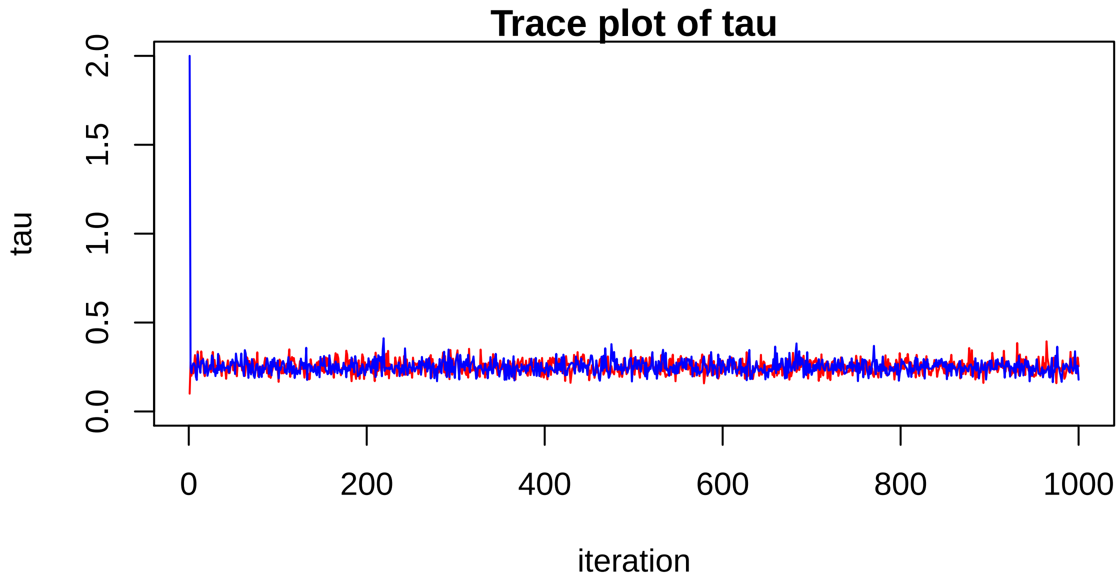
```
# gibbs sampler
GibbsS <- function(mu0, tau0, m){

  # create array
  mu.seq = rep(-1,m)
  tau.seq = rep(-1,m)

  # initial value
  mu.seq[1] = mu0
  tau.seq[1] = tau0

  # iterations
  for (i in 2:m) {
    mu.seq[i] = rnorm(1,mean(data),sd = sqrt(1/(tau.seq[i-1]*n)))
    tau.seq[i] = rgamma(1,n/2, scale = (2/sum((data-mu.seq[i])^2)))
  }

  # result as single list
  result  = list(mu = mu.seq, tau = tau.seq)
  return(result)
}
```

```
#set seed
set.seed(30027)

#number of iterations
m = 1000

#generate 2 samples from gibbs sampler with different initial value
gibbsam1 = GibbsS(1,0.1,m)
gibbsam2 = GibbsS(10,2,m)
#
# gibbsam1 = GibbsS(7,0.01,m)
# gibbsam2 = GibbsS(3,0.9,m)
```

```r
par(mfrow=c(1,1), mar = c(4,4,1,1))

#trace plot mu
plot(x=1:m, gibbsam1$mu, type="l", col="red", ylim = c(0, max(gibbsam1$mu,gibbsam2$mu)),
     xlab = "iteration", ylab ="mu",main = "Trace plot of mu")
points(1:m, gibbsam2$mu, type="l", col="blue")
legend(700,3,legend = c("chain1", "chain2"),col=c("red","blue"),lty = 1:1)
```



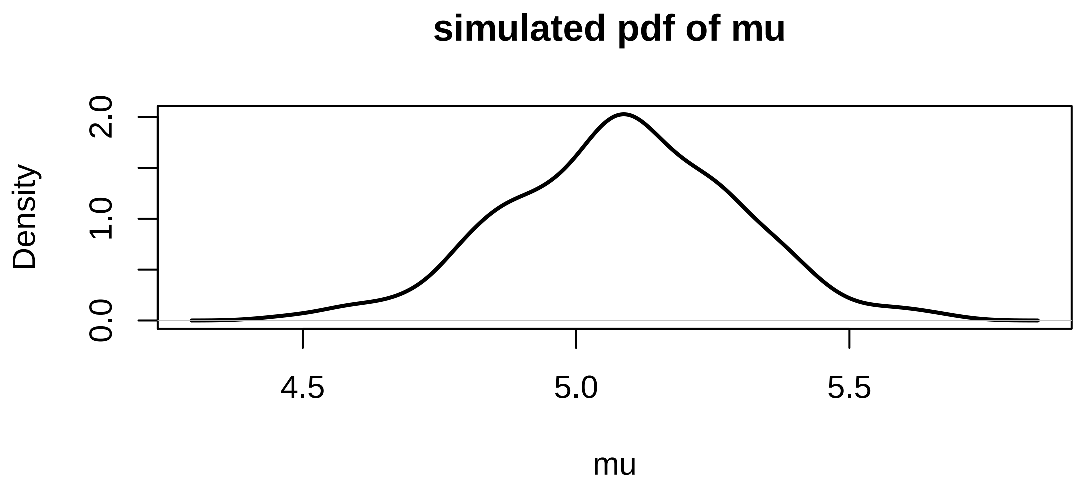

```r
#trace plot tau
plot(1:m, gibbsam1$tau, type="l", col="red", ylim = c(0, max(gibbsam1$tau,gibbsam2$tau)),
     xlab = "iteration", ylab ="tau",main = "Trace plot of tau")
points(1:m, gibbsam2$tau, type="l", col="blue")
```



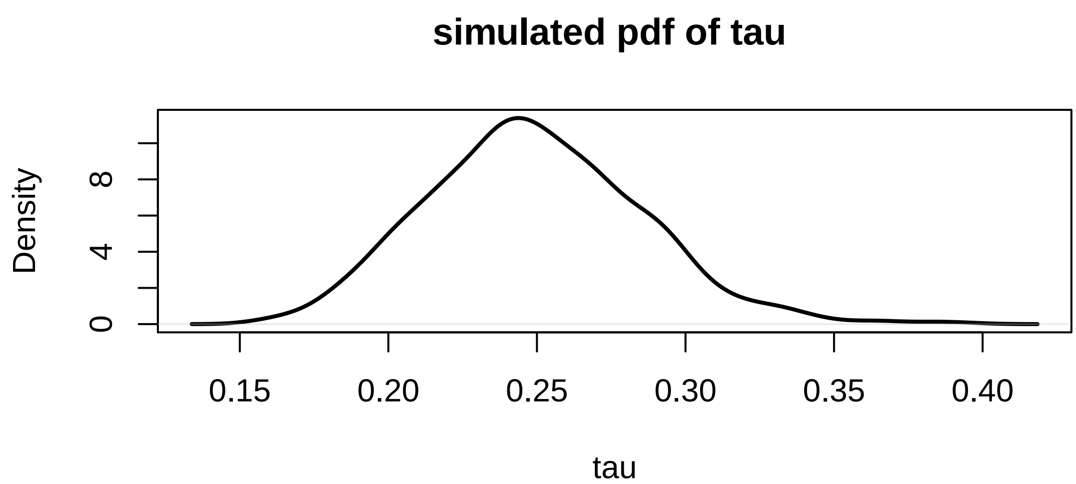
1(c)

1)

```
#combine 2 simulations
gibbsam = list(mu =c(tail(gibbsam1$mu,m/2),tail(gibbsam2$mu,m/2))
               ,tau =c(tail(gibbsam1$tau,m/2),tail(gibbsam2$tau,m/2)))

#marginal posterior distribution pf mu
plot(density(gibbsam$mu), main="simulated pdf of mu ", xlab="mu", lwd=2)
```




```
#marginal posterior distribution pf tau
plot(density(gibbsam$tau), main="simulated pdf of tau ", xlab="tau", lwd=2)
```



2)posterior mean

```
#posterior mean of mu
(mu_hat = mean(gibbsam$mu))
```

```
## [1] 5.083924
```

```r
#posterior mean of tau
(tau_hat = mean(gibbsam$tau))
```

```
## [1] 0.2490844
```

3)

```r
credible_interval <- function(simulation){

  #sort the simulation ascending order
  ascending = sort(simulation,decreasing = FALSE)

  x = 1:length(simulation)/length(simulation)

  #Conservative statistics
  interval = c(ascending[which(x>=0.05)[1]],ascending[which(x>=0.95)[1]])

  return(interval)
}

#credible interval of mu
(credible_interval(gibbsam$mu))
```

```
## [1] 4.744386 5.413589
```

```r
#credible interval of tau
(credible_interval(gibbsam$tau))
```

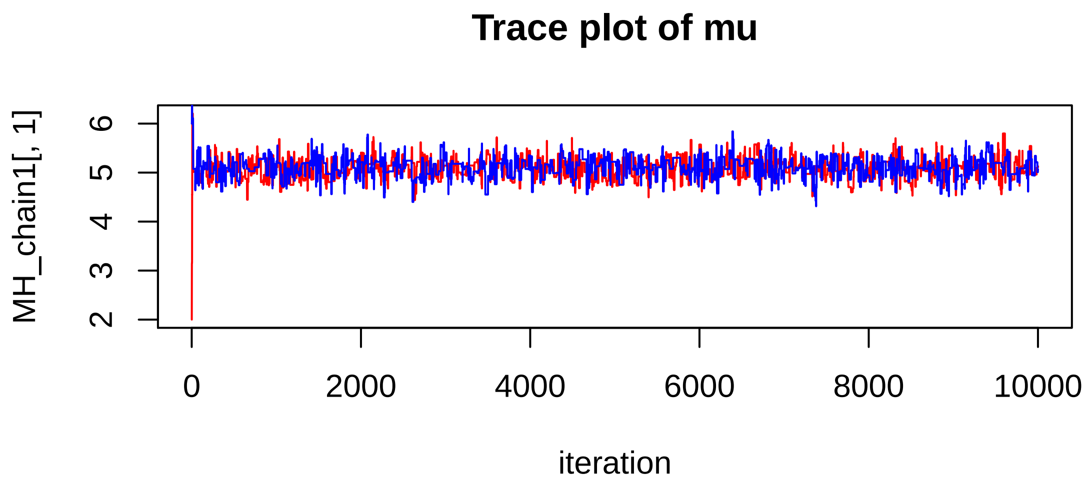
```
## [1] 0.1921695 0.3087699
```

2a)

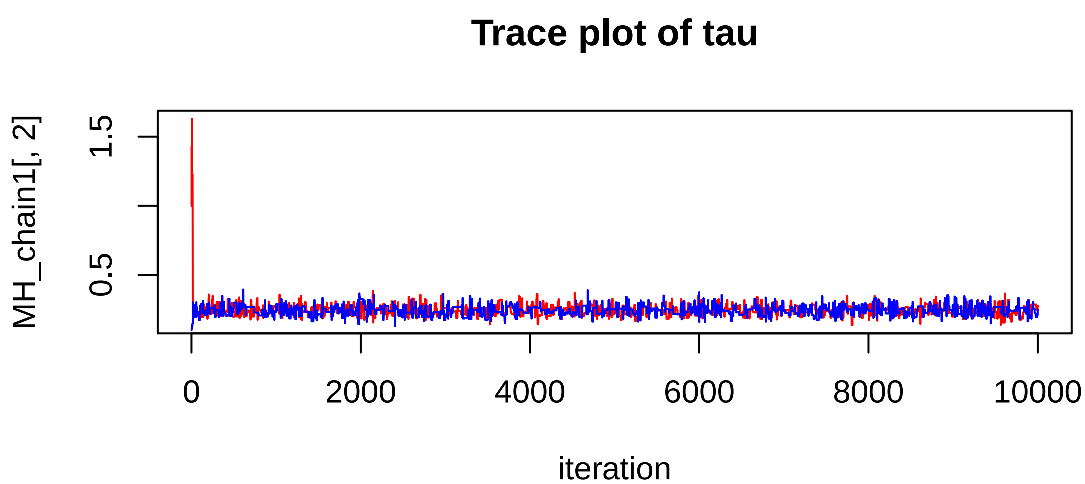
```r
#MH
# prior distribution
prior <- function(param){
  tau = param[2]
  return(log(1/tau))
}

# likelihood
likelihood <- function(param){
  mu = param[1]
  tau = param[2]

  #log likelihood
  logL = dnorm(data, mean = mu, sd = sqrt(1/tau),log = TRUE)
  return(sum(logL))
}

# posterior distribution
posterior <- function(param){
  return(likelihood(param) + prior(param))
}

# proposal function
proposalfunction <- function(param){
```

```r
  mu = param[1]
  tau = param[2]
  proposal_tau = rgamma(1,shape = 5*tau,rate = 5)
  proposal_mu = rnorm(1,mu,sd = sqrt(proposal_tau))
  return(c(proposal_mu, proposal_tau))
}

# metropolis hastings
metropolis_hastings <- function(startvalue, iterations){
  chain = array(dim = c(iterations+1, 2))
  chain[1,] = startvalue

  for(i in 1:iterations){
    proposal = proposalfunction(chain[i,])
    probab = exp(posterior(proposal) - posterior(chain[i,]))
    if(runif(1) < probab){
      chain[i+1,] = proposal
    }else{
      chain[i+1,] = chain[i,]
    }
  }
  return(chain)
}
```

```r
set.seed(30027)
#start value
startvalue1 = c(2,1)
startvalue2 = c(6,0.1)

iteration = 10000

#MH
MH_chain1 = metropolis_hastings(startvalue1,iteration)
MH_chain2 = metropolis_hastings(startvalue2,iteration)

# Trace plot of mu
plot(MH_chain1[,1], type = "l", xlab="iteration",
     main = "Trace plot of mu",col = "red" )
points(MH_chain2[,1],type="l", col="blue")
```



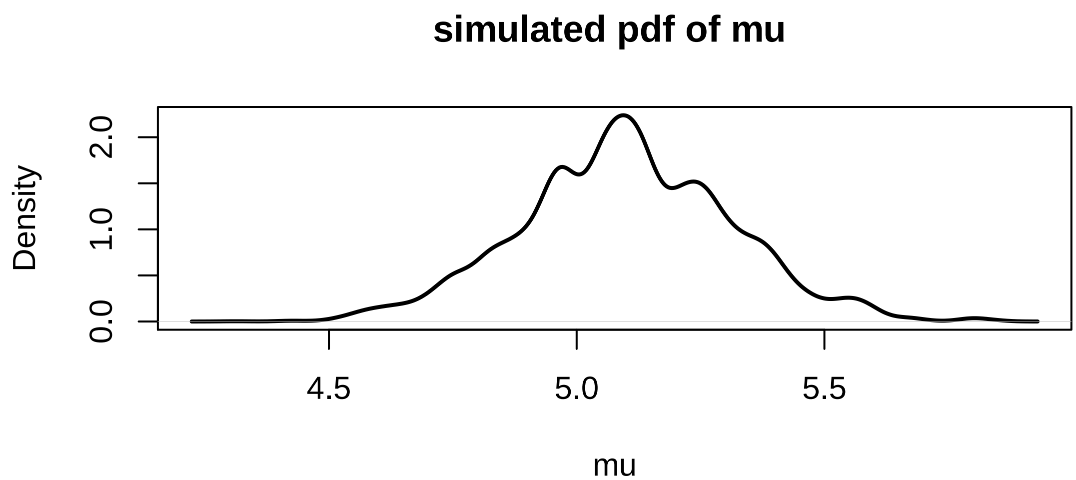

```r
# Trace plot of tau
plot(MH_chain1[,2], type = "l", xlab="iteration",
     main = "Trace plot of tau", col = "red")
points(MH_chain2[,2],type="l", col="blue")
```



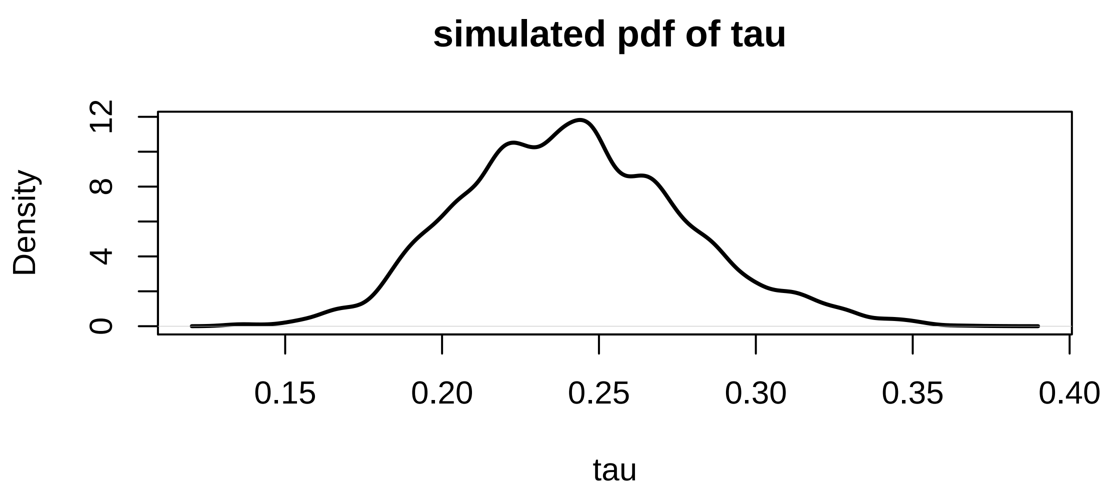
2(b)

1)Marginal posterior distribution

```
#combine 2 simulations
MH_chain = list(mu =c(tail(MH_chain1[,1],iteration/2),
                      tail(MH_chain2[,1],iteration/2))
                ,tau =c(tail(MH_chain1[,2],iteration/2),
                       tail(MH_chain2[,2],iteration/2)))

#marginal posterior distribution pf mu
plot(density(MH_chain$mu), main="simulated pdf of mu ", xlab="mu", lwd=2)
```




```
#marginal posterior distribution pf tau
plot(density(MH_chain$tau), main="simulated pdf of tau ", xlab="tau", lwd=2)
```



2)posterior mean

```r
#posterior mean of mu
(mu_hat = mean(MH_chain$mu))
```

```
## [1] 5.099941
```

```r
#posterior mean of tau
(tau_hat = mean(MH_chain$tau))
```

```
## [1] 0.2418788
```

3)

```r
#credible interval of mu
(credible_interval(MH_chain$mu))
```

```
## [1] 4.752370 5.441644
```

```r
#credible interval of tau
(credible_interval(MH_chain$tau))
```

```
## [1] 0.1874965 0.3064031
```