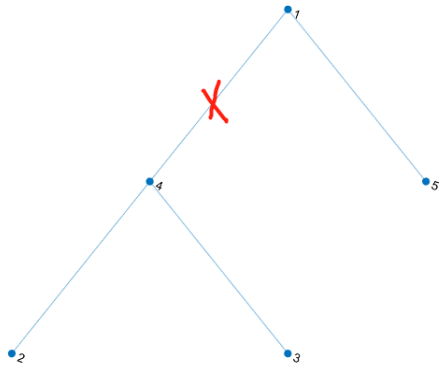
Colorem(A, L, samples) Test Report:

Case 1: Tree structure

Note: Training data are generated from Gibbs sampling, with w = [1 2 4 3];

Situation #1 : missing one variable (**color of edge(1,4) not observed**);



...

EM round # 14, w change: Δw = [3.158562e-03, 2.039755e-03, 4.906648e-04, 6.281418e-04,]

EM round # 15, w change: Δw = [2.248735e-03, 1.475814e-03, 3.425937e-04, 4.303272e-04,]

EM round # 16, w change: Δw = [1.602284e-03, 1.062974e-03, 2.409085e-04, 2.984015e-04,]

EM round # 17, w change: Δw = [1.142181e-03, 7.633074e-04, 1.701618e-04, 2.087118e-04,]

...

EM round # 36, w change: Δw = [1.859632e-06, 1.181991e-06, 3.107283e-07, 3.669128e-07,]

EM round # 37, w change: Δw = [1.354951e-06, 8.559857e-07, 2.293289e-07, 2.696360e-07,]

EM round # 38, w change: Δw = [9.362826e-07, 5.895306e-07, 1.592974e-07, 1.874546e-07,]

EM round # 39, w change: Δw = [6.724656e-07, 4.218494e-07, 1.152615e-07, 1.353547e-07,]

EM round # 40, w change: Δw = [4.374920e-07, 2.740940e-07, 7.507581e-08, 8.832220e-08,]

EM round # 41, w change: Δw = [3.204745e-07, 2.003114e-07, 5.526878e-08, 6.489430e-08,]

EM round # 42, w change: Δw = [2.108358e-07, 1.316635e-07, 3.640720e-08, 4.276508e-08,]

EM round # 43, w change: Δw = [2.062651e-07, 1.283984e-07, 3.590055e-08, 4.196617e-08,]

EM round # 44, w change: Δw = [1.028603e-07, 6.407084e-08, 1.785562e-08, 2.093382e-08,]

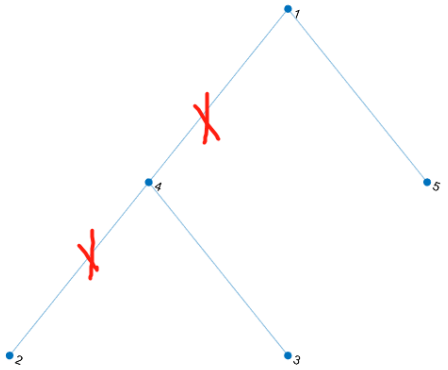
EM round # 45, w change: Δw = [1.017232e-07, 6.325876e-08, 1.772930e-08, 2.073513e-08,]

EM round # 46, w change: Δw = [1.005987e-07, 6.245711e-08, 1.760267e-08, 2.053892e-08,]

EM round # 47, w change: Δw = [0, 0, 0, 0,]

Test w is: [1.0, 2.0, 4.0, 3.0,]; Learned w is: [1.00, 1.97, 3.94, 2.95,]

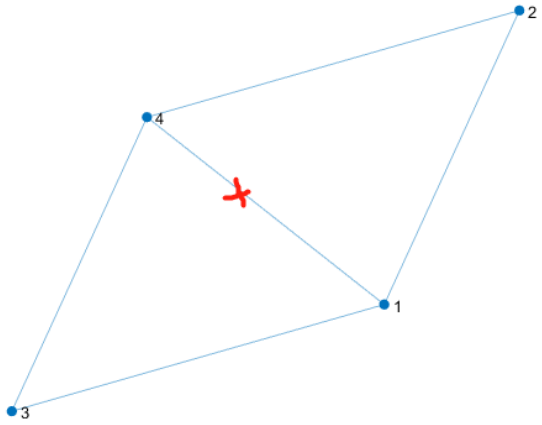
Situation #2 : missing one variable (color of edge(1,4) (2,4)not observed);



EM round # 1, w change: Δw = [7.851935e-01, 2.577424e-01, 8.673499e-01, 1.755859e-01,]
EM round # 2, w change: Δw = [4.579925e-01, 9.103765e-02, 3.655269e-01, 1.835033e-01,]
EM round # 3, w change: Δw = [1.948972e-01, 3.077350e-02, 1.167563e-01, 1.089145e-01,]
EM round # 4, w change: Δw = [6.781454e-02, 9.654502e-03, 3.357924e-02, 4.388980e-02,]
EM round # 5, w change: Δw = [2.170685e-02, 3.015194e-03, 9.805166e-03, 1.491688e-02,]
EM round # 6, w change: Δw = [6.753396e-03, 9.506691e-04, 2.939120e-03, 4.764945e-03,]
EM round # 7, w change: Δw = [2.082789e-03, 3.020209e-04, 8.950929e-04, 1.489717e-03,]
EM round # 8, w change: Δw = [6.407038e-04, 9.637813e-05, 2.747326e-04, 4.623492e-04,]
EM round # 9, w change: Δw = [1.969411e-04, 3.082361e-05, 8.462113e-05, 1.431436e-04,]
EM round # 10, w change: Δw = [6.051627e-05, 9.871647e-06, 2.610442e-05, 4.428350e-05,]
EM round # 11, w change: Δw = [1.858339e-05, 3.173536e-06, 8.059460e-06, 1.369746e-05,]
EM round # 12, w change: Δw = [5.698451e-06, 1.053417e-06, 2.503686e-06, 4.248182e-06,]
EM round # 13, w change: Δw = [1.768694e-06, 4.060162e-07, 8.175428e-07, 1.357167e-06,]
EM round # 14, w change: Δw = [5.570631e-07, 1.724331e-07, 2.807396e-07, 4.487565e-07,]
EM round # 15, w change: Δw = [1.732341e-07, 6.695189e-08, 9.416506e-08, 1.460210e-07,]
EM round # 16, w change: Δw = [5.384712e-08, 2.310779e-08, 3.044502e-08, 4.650989e-08,]
EM round # 17, w change: Δw = [2.037533e-08, 9.115177e-09, 1.171391e-08, 1.777659e-08,]
EM round # 18, w change: Δw = [0, 0, 0, 0,]
Test w is: [1.0, 2.0, 4.0, 3.0,]; Learned w is: [1.00, 2.00, 4.03, 3.03,]

Case 2: Non-tree structure

Situation #1 : missing one variable (**color of edge(1,4) not observed**);

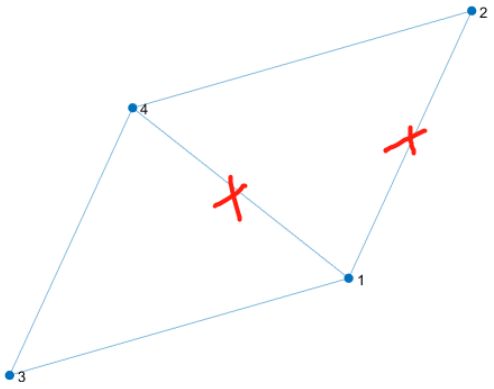


EM round # 1, w change: Δw = [5.393702e-01, 5.239073e-01, 2.573783e-02, 1.027485e-02,]
EM round # 2, w change: Δw = [7.562301e-02, 1.395593e-01, 3.743896e-02, 2.649733e-02,]
EM round # 3, w change: Δw = [1.304352e-02, 3.479126e-02, 1.265060e-02, 9.097151e-03,]
EM round # 4, w change: Δw = [2.726311e-03, 8.435732e-03, 3.266906e-03, 2.442514e-03,]
EM round # 5, w change: Δw = [6.243889e-04, 2.030200e-03, 7.951364e-04, 6.106749e-04,]
EM round # 6, w change: Δw = [1.476325e-04, 4.876557e-04, 1.910411e-04, 1.489821e-04,]
EM round # 7, w change: Δw = [3.526224e-05, 1.170721e-04, 4.580300e-05, 3.600687e-05,]
EM round # 8, w change: Δw = [8.449092e-06, 2.810024e-05, 1.098210e-05, 8.669048e-06,]
EM round # 9, w change: Δw = [2.025587e-06, 6.741804e-06, 2.632471e-06, 2.083747e-06,]
EM round # 10, w change: Δw = [4.885857e-07, 1.627817e-06, 6.331246e-07, 5.061070e-07,]
EM round # 11, w change: Δw = [1.159074e-07, 3.869088e-07, 1.488687e-07, 1.221326e-07,]
EM round # 12, w change: Δw = [2.709203e-08, 9.057735e-08, 3.407437e-08, 2.941097e-08,]
EM round # 13, w change: Δw = [6.468771e-09, 2.164283e-08, 8.038883e-09, 7.135171e-09,]
EM round # 14, w change: Δw = [3.019414e-09, 1.010516e-08, 3.732607e-09, 3.353137e-09,]
EM round # 15, w change: Δw = [0, 0, 0, 0,]

Test w is: [4.0, 1.0, 3.0, 2.0,]; Learned w is: [2.23, 1.00, 2.01, 1.60,]

(Although we didn't learned the exactly perfect w but the order of the weight is the same as true w.

Situation #2 : missing one variable (color of edge(1,4) (1,2)not observed);



EM round # 1, w change: Δw = [1.963766e-01, 3.413761e-01, 2.883757e-01, 1.433762e-01,]
EM round # 2, w change: Δw = [3.684133e-02, 9.494238e-02, 6.871672e-02, 1.061567e-02,]
EM round # 3, w change: Δw = [8.299718e-03, 3.038332e-02, 1.569555e-02, 6.388047e-03,]
EM round # 4, w change: Δw = [2.245198e-03, 1.062196e-02, 3.919753e-03, 4.457014e-03,]
EM round # 5, w change: Δw = [7.033657e-04, 3.894992e-03, 1.122526e-03, 2.069100e-03,]
EM round # 6, w change: Δw = [2.424732e-04, 1.461578e-03, 3.629158e-04, 8.561887e-04,]
EM round # 7, w change: Δw = [8.815644e-05, 5.543447e-04, 1.271852e-04, 3.390030e-04,]
EM round # 8, w change: Δw = [3.292544e-05, 2.112820e-04, 4.662049e-05, 1.317361e-04,]
EM round # 9, w change: Δw = [1.245578e-05, 8.069810e-05, 1.747879e-05, 5.076354e-05,]
EM round # 10, w change: Δw = [4.742399e-06, 3.085797e-05, 6.626077e-06, 1.948950e-05,]
EM round # 11, w change: Δw = [1.811093e-06, 1.180514e-05, 2.524994e-06, 7.469054e-06,]
EM round # 12, w change: Δw = [6.927921e-07, 4.515824e-06, 9.644240e-07, 2.858608e-06,]
EM round # 13, w change: Δw = [2.668591e-07, 1.733033e-06, 3.702169e-07, 1.095957e-06,]
EM round # 14, w change: Δw = [1.022231e-07, 6.595708e-07, 1.411332e-07, 4.162146e-07,]
EM round # 15, w change: Δw = [3.935861e-08, 2.501186e-07, 5.369337e-08, 1.570666e-07,]
EM round # 16, w change: Δw = [1.513588e-08, 9.454153e-08, 2.036878e-08, 5.903686e-08,]
EM round # 17, w change: Δw = [7.333435e-09, 4.505436e-08, 9.737419e-09, 2.798349e-08,]
EM round # 18, w change: Δw = [1.723671e-09, 1.056500e-08, 2.285992e-09, 6.555337e-09,]
EM round # 19, w change: Δw = [1.657698e-09, 1.010607e-08, 2.188612e-09, 6.259759e-09,]
EM round # 20, w change: Δw = [0, 0, 0, 0,]
Test w is: [4.0, 1.0, 3.0, 2.0,]; Learned w is: [2.12, 1.00, 1.85, 1.36,]

Q: what is the log likelyhood as a function of the weights?
A:

$$\nabla_\theta \log l(\theta) = \sum_C \sum_m \left(f_C(x_C^m, y^m) - \sum_{x_C} p_C(x_C | y^m, \theta) f_C(x_C, y^m) \right)$$

Here assume theta is the weight parameter