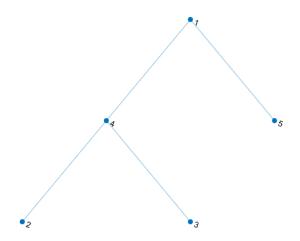
Case #1: Tree structure graph

Learning data M = 50,000; generate from Gibbs using $w = [3 \ 1 \ 5 \ 2 \ 4]$;

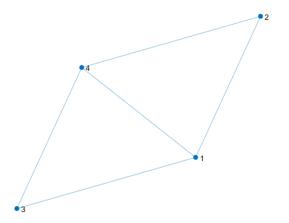


```
Gradient round #/06, parameter gradient: Δ(W) = [1.516916e-0/, -1.589386e-06, 5.599855e-0/, 3.34397/e-0/, 5.433116e-0/,
Gradient round #707, parameter gradient: Δ(w) = [1.492503e-07, -1.563638e-06, 5.508973e-07, 3.289995e-07, 5.344914e-07,
Gradient round #708, parameter gradient: Δ(w) = [1.468481e-07, -1.538307e-06, 5.419566e-07, 3.236882e-07, 5.258145e-07,
Gradient round #709, parameter gradient: Δ(w) = [1.444844e-07, -1.513387e-06, 5.331612e-07, 3.184625e-07, 5.172787e-07,
Gradient round #710, parameter gradient: \Delta(w) = [1.421586e-07, -1.488870e-06, 5.245086e-07, 3.133210e-07, 5.088815e-07,
Gradient round #711, parameter gradient: \Delta(w) = [1.398701e-07, -1.464750e-06, 5.159966e-07, 3.082623e-07, 5.006208e-07,
Gradient round #712, parameter gradient: Δ(w) = [1.376184e-07, -1.441021e-06, 5.076229e-07, 3.032851e-07, 4.924943e-07,
Gradient round #713, parameter gradient: \Delta(w) = [1.354027e-07, -1.417676e-06, 4.993851e-07, 2.983882e-07, 4.844999e-07,
Gradient round #714, parameter gradient: \Delta(w) = [1.332226e-07, -1.394709e-06, 4.912812e-07, 2.935701e-07, 4.766353e-07,
Gradient round #715, parameter gradient: \Delta(w) = [1.310775e-07, -1.372115e-06, 4.833088e-07, 2.888297e-07, 4.688986e-07,
Gradient round #716, parameter gradient: Δ(w) = [1.289668e-07, -1.349886e-06, 4.754660e-07, 2.841658e-07, 4.612875e-07,
Gradient round #717, parameter gradient: Δ(w) = [1.268900e-07, -1.328018e-06, 4.677505e-07, 2.795769e-07, 4.538001e-07,
Gradient round #718, parameter gradient: \Delta(w) = [1.248465e-07, -1.306503e-06, 4.601603e-07, 2.750621e-07, 4.464344e-07, -1.306503e-06, -1.306503e-07, -1.306506-07, -1.306506-07, -1.306506-07, -1.306506-07, -1.306506-07, -1.306506-07, -1.306506-07, -1.306506-07, -1.306506-07, -1.306506-07, -1.306506-07, -1.306506-07, -1.306506-07, -1.306506-07, -1.306506-07, -1.306506-07, -1.306506-07, -1.306506-07, -1.306506-07, -1.306506-07, -1.306506-07, -1.306506-07, -1.306506-07, -1.306506-07, -1.306506-07, -1.306506-07, -1.306506-07, -1.306506-07, -1.306506-07, -1.306506-07, -1.306506-07, -1.306506-07, -1.306506-07, -1.306506-07, -1.306506-07, -1.306506-07, -1.306506-07, -1.306506-07, -1.306506-07, -1.30606-07, -1.30606-07, -1.30606-07, -1.30606-07, -1.30606-07, -1.30606-07, -1.30606-07, -1.30606-07,
Gradient round #719, parameter gradient: \Delta(w) = [1.228358e-07, -1.285337e-06, 4.526934e-07, 2.706200e-07, 4.391883e-07,
Gradient round #720, parameter gradient: \Delta(w) = [1.208573e-07, -1.264515e-06, 4.453477e-07, 2.662495e-07, 4.320600e-07,
Gradient round #721, parameter gradient: Δ(w) = [1.189107e-07, -1.244029e-06, 4.381214e-07, 2.619495e-07, 4.250474e-07,
Gradient round #722, parameter gradient: Δ(w) = [1.169952e-07, -1.223875e-06, 4.310124e-07, 2.577188e-07, 4.181488e-07,
Gradient round #723, parameter gradient: Δ(w) = [1.151106e-07, -1.204048e-06, 4.240188e-07, 2.535563e-07, 4.113623e-07,
Gradient round #724, parameter gradient: Δ(w) = [1.132561e-07, -1.184542e-06, 4.171388e-07, 2.494609e-07, 4.046860e-07,
Gradient round #725, parameter gradient: Δ(w) = [1.114315e-07, -1.165352e-06, 4.103705e-07, 2.454316e-07, 3.981181e-07,
Gradient round #726, parameter gradient: Δ(w) = [1.096362e-07, -1.146472e-06, 4.037121e-07, 2.414672e-07, 3.916569e-07,
Gradient round #727, parameter gradient: Δ(w) = [1.078697e-07, -1.127899e-06, 3.971618e-07, 2.375667e-07, 3.853007e-07,
Gradient round #728, parameter gradient: Δ(w) = [1.061316e-07, -1.109626e-06, 3.907179e-07, 2.337292e-07, 3.790478e-07,
Gradient round #729, parameter gradient: Δ(w) = [1.044214e-07, -1.091650e-06, 3.843786e-07, 2.299535e-07, 3.728964e-07,
Gradient round #730, parameter gradient: Δ(w) = [1.027386e-07, -1.073965e-06, 3.781423e-07, 2.262387e-07, 3.668449e-07,
Gradient round #731, parameter gradient: Δ(w) = [1.010829e-07, -1.056566e-06, 3.720072e-07<u>, 2.225839e-07, 3.608917e-07</u>,
Gradient round #732, parameter gradient: Δ(w) = [9.945385e-08, -1.039449e-06, 3.659717e-07, 2.189880e-07, 3.550352e-07,
Gradient round #733, parameter gradient: Δ(w) = [9.785094e-08, -1.022609e-06, 3.600342e-07, 2.154500e-07, 3.492738e-07,
Gradient round #734, parameter gradient: Δ(w) = [9.627378e-08, -1.006042e-06, 3.541931e-07, 2.119692e-07, 3.436059e-07,
Gradient round #735, parameter gradient: Δ(w) = [9.472197e-08, -9.897434e-07, 3.484468e-07, 2.085445e-07, 3.380302e-07,
Test w is: [3.0, 1.0, 5.0, 2.0, 4.0, ]; Learned w is: [3.00, 1.00, 4.99, 2.00, 3.99, ]
```

As we can see, the algorithm learned out the perfect exact w.

Case #2 None-tree structure graph

Learning data M = 50,000; generate from Gibbs using $w = [3 \ 1 \ 5 \ 2 \ 4]$;



```
Gradient round #447, parameter gradient: \Delta(w) = [4.715987e-07, -2.076797e-06, 5.113054e-07, 6.130690e-07, 4.808242e-07, Gradient round #448, parameter gradient: \Delta(w) = [4.595951e-07, -2.023937e-06, 4.982913e-07, 5.974645e-07, 4.685859e-07, Gradient round #449, parameter gradient: \Delta(w) = [4.478971e-07, -1.972422e-06, 4.856084e-07, 5.822572e-07, 4.566591e-07,
 Gradient round #450, parameter gradient: \Delta(w) = [4.364969e-07, -1.922218e-06, 4.732483e-07, 5.674369e-07, 4.450358e-07, 5.674369e-07, 4.450358e-07, 5.674369e-07, 4.450358e-07, 5.674369e-07, 4.450358e-07, 5.674369e-07, 5.674369e-07, 4.450358e-07, 5.674369e-07, 5.674869e-07, 5.674869e-07, 5.674869e-07, 5.676869e-07, 5.676869e-07, 5.676869e-07, 5.676869e-07, 5.676869e-07, 5.676869e-07, 5.676869e-07, 5.676869e-07, 5.6768
 Gradient round #451, parameter gradient: \Delta(w) = [4.253868e-07, -1.873292e-06, 4.612028e-07, 5.529939e-07, 4.337084e-07, 6.529939e-07, 4.337084e-07, 6.529939e-07, 4.337084e-07, 6.529939e-07, 6.5299399-07, 6.529939e-07, 6.529999-07, 6.52999-07, 6.52999-07, 6.52999-07, 6.52999-07, 6.52999-07, 6.52999-07, 6.52999-07, 6.52999-07, 6.52999-07, 6.52999-07, 6.52999-07, 6.52999-07, 6.52999-07, 6.52999-07, 6.52999-07, 6.52999-07, 6.52999-07, 6.52999-07, 6.52999-07, 6.52999-07, 6.52999-07, 6.52999-07, 6.52999-07, 6.52999-07, 6.52999-07, 6.52999-07, 6.52999-07, 6.5299-07, 6.5299-07, 6.52999-07, 6.52999-07, 6.52999-07, 6.52999-07, 6.5299-07, 6.5299-07, 6.5299-07, 6.5299-07, 6.5299-07, 6.5299-07, 6.5299-07, 6.5299-07, 6.5299-07, 6.5299-07, 6.5299-07, 6.5299-07, 6.5299-07, 6.5299-07, 6.5299-07, 6.5299-07, 6.5299-07, 6.5299-07, 6.5299-07, 6.5299-07, 6.5299-07, 6.5299-07, 6.5299-07, 6.5299-07, 6.5299-07, 6.5299-07, 6.5299-07, 6.5299-07, 6.5299-07, 6.5299-07, 6.5299-07, 6.5299-07, 6.5299-07, 6.5299-07, 6.5299-07, 6.5299-07, 6.5299-07, 6.5299-07, 6.5299-07, 6.5299-07, 6.5299-07, 6.5299-07, 6.529
 Gradient round #452, parameter gradient: \Delta(w) = [4.145595e-07, -1.825611e-06, 4.494639e-07, 5.389185e-07, 4.226694e-07,
 Gradient round #453, parameter gradient: \Delta(w) = [4.040078e-07, -1.779144e-06, 4.380238e-07, 5.252014e-07, 4.119112e-07, 4.1191
 Gradient round #454, parameter gradient: \Delta(w) = [3.937246e-07, -1.733860e-06, 4.268749e-07, 5.118335e-07, 4.014270e-07, 5.118335e-07]
 Gradient round #455, parameter gradient: \Delta(w) = [3.837032e-07, -1.689728e-06, 4.160098e-07, 4.988057e-07, 3.912095e-07, 4.988057e-07, 4.988067e-07, 4.988
 Gradient round #457, parameter gradient: \Delta(w) = [3.644191e-07, -1.604806e-06, 3.951021e-07, 4.737367e-07, 3.715482e-07, -1.604806e-06, 3.95102e-07, -1.604806e-06, 3.95102e-07, -1.604806e-06, 3.95102e-07, 4.737367e-07, 3.715482e-07, -1.604806e-06, 3.95102e-07, -1.604806e-07, -1.604806e-06, 3.95102e-07, -1.604806e-07, -1.604806-07, -1.604806e-07, -1.604806e-07, -1.604806e-07, -1.604806e-07, 
 Gradient round #458, parameter gradient: \Delta(w) = [3.551436e-07, -1.563959e-06, 3.850456e-07, 4.616786e-07, 3.620913e-07, 0.00913e-07]
 Gradient round #459, parameter gradient: \Delta(w) = \begin{bmatrix} 3.461042e-07 & -1.524152e-06 & 3.752452e-07 & 4.499275e-07 & 3.528751e-07 \end{bmatrix}
   Gradient round #460, parameter gradient: \Delta(w) = [3.372949e-07, -1.485358e-06, 3.656941e-07, 4.384755e-07, 3.438934e-07, 3.43894e-07, 3.43896e-07, 3.43896e-07, 3.43896e-07, 3.43896e-07, 3.4
 Gradient round #461, parameter gradient: \Delta(w) = [3.287098e-07, -1.447551e-06, 3.563862e-07, 4.273150e-07, 3.351404e-07, 3.361404e-07, 3.36140e-07, 
 Gradient round #462, parameter gradient: \Delta(w) = [3.203432e-07, -1.410707e-06, 3.473152e-07, 4.164385e-07, 3.266102e-07, 4.164385e-07, 4.164386e-07, 4.16486e-07, 4.1
 Gradient round #463, parameter gradient: \Delta(w) = [3.121896e-07, -1.374801e-06, 3.384751e-07, 4.058389e-07, 3.182970e-07, 3.1829
 Gradient round #464, parameter gradient: \Delta(w) = [3.042435e-07, -1.339808e-06, 3.298600e-07, 3.955091e-07, 3.101955e-07, -1.339808e-06, 3.298600e-07, 3.998600e-07, 3.9986000e-07, 3.9986000e-07, 3.9986000e-07, 3.9986000e-07, 3.99860
 Gradient round #466, parameter gradient: \Delta(w) = [2.889529e-07, -1.272472e-06, 3.132820e-07, 3.756316e-07, 2.946058e-07,
 Gradient round #467, parameter gradient: \Delta(w) = [2.815982e-07, -1.240084e-06, 3.053081e-07, 3.660706e-07, 2.871073e-07, 3.660706e-07, 2.871073e-07, 3.660706e-07, 2.871073e-07, 3.660706e-07, 2.871073e-07, 3.660706e-07, 3.660
 Gradient round #468, parameter gradient: \Delta(w) = [2.744308e-07, -1.208521e-06, 2.975372e-07, 3.567530e-07, 2.797996e-07, 3.567530e-07, 2.797996e-07, 3.567530e-07, 2.797996e-07, 3.567530e-07, 3.567500e-07, 3.567
 Gradient round #469, parameter gradient: \Delta(w) = [2.674457e-07, -1.177760e-06, 2.899640e-07, 3.476726e-07, 2.726779e-07, Gradient round #470, parameter gradient: \Delta(w) = [2.606385e-07, -1.147783e-06, 2.825836e-07, 3.388233e-07, 2.657375e-07,
 Gradient round #471, parameter gradient: \Delta(w) = [2.540045e-07, -1.118569e-06, 2.753911e-07, 3.301993e-07, 2.589738e-07, 2.589786e-07, 2.5898
 Gradient round #472, parameter gradient: \Delta(w) = [2.475394e-07, -1.090098e-06, 2.683817e-07, 3.217947e-07, 2.523822e-07, 0.090098e-06]
 Gradient round #473, parameter gradient: \Delta(w) = [2.412388e-07, -1.062352e-06, 2.615506e-07, 3.136041e-07, 2.459584e-07, -1.062352e-06, 2.6166e-07, 3.136041e-07, 2.6166e-07, 3.136041e-07, 3.136041e-0
 Gradient round #474, parameter gradient: \Delta(w) = [2.350986e-07, -1.035312e-06, 2.548934e-07, 3.056220e-07, 2.396981e-07, 2.3969881e-07, 2.3969881e-07, 2.3969881e-07, 2.3
 Gradient round #475, parameter gradient: \Delta(w) = [2.291147e-07, -1.008960e-06, 2.484057e-07, 2.978430e-07, 2.335971e-07,
   Gradient round #476, parameter gradient: \Lambda(w) = [2.232831e-07, -9.832796e-07, 2.420831e-07, 2.902620e-07, 2.276514e-07, -9.832796e-07, 2.420831e-07, 2.902620e-07, 2.276514e-07, -9.832796e-07, 2.420831e-07, 2.902620e-07, 2.276514e-07, -9.832796e-07, 2.420831e-07, 2.902620e-07, 2.276514e-07, -9.832796e-07, 2.420831e-07, 2.902620e-07, 2.420831e-07, -9.832796e-07, -9.832796e-07, 2.420831e-07, 2.902620e-07, 2.420831e-07, -9.832796e-07, -9.832796e-07, 2.420831e-07, 2.902620e-07, 2.420831e-07, -9.832796e-07, -9.832796e-07, 2.420831e-07, 2.420831e-07, 2.420831e-07, -9.832796e-07, -9.832796e
             est w is: [3.0, 1.0, 5.0, 2.0, 4.0, ]; Learned w is: [2.63, 1.00, 3.52, 1.89, 3.19, ]
```