- Symmetric matrix:
  - eigenvector inner product (or dot) with each other is 0.
  - Think about transpose equal itself, or more specifically, each (i,j) entry and (j,i) entry
  - orthornormal eigen basis
  - $Q^T DQ$, $Q^{-1}DQ$
- Determinant is the **product** of all eigenvalues, and trace is the **sum** of all eigenvalues. For 2 x 2 matrix A, $x^2 - tr(A)x + det(A)$ is the characteristic polynomial
- Orthogonal Projection P
  - Let A be an m x n matrix with linearly independent columns, and let $W = Col(A)$. Then the projection matrix is
    $$x_W = A(A^T A)^{-1} A^T x$$
  - P is linear and symmetric
  - $P^2 = P, P^T = P$
  - $P = QQ^T = A(A^T A)^{-1} A^T$, where columns of $A(Q)$ form an basis (ONB) of V
  - If $(u_1, \ldots, u_r)$ is an ONB of $V \subseteq \mathbb{R}^n$, then for all $\vec{x} \in \mathbb{R}^n$,
    $$\text{proj}_V(\vec{x}) = \sum_{i=1}^{r} \langle \vec{x}, \vec{u}_i \rangle \vec{u}_i$$
  - eigenvalue 0 or 1
- matrix multiplication
  - A(b1 b2) = (Ab1 Ab2)
  - first row in C is the product of first column of A with the first row in B
- Pythagorean theorem and Cauchy inequality
  - Pythagorean theorem:
    $||\vec{x} + \vec{y}||^2 = ||\vec{x}||^2 + ||\vec{y}||^2$
    holds iff x and y are orthogonal
  - Cauchy-Schwardz inequality
    $|\langle \vec{x}, \vec{y} \rangle| \leq ||\vec{x}|| ||\vec{y}||$
    equality iff x and y are parallel
- Orthogonal Matrix A
  - A has absolute eigenvalue of 1

  - if eigenvalue of orthogonal matrix is 1, meaning it preserves geometry, and -1 if it reverses the geometry
  - orthogonoal transformation preserves geometry and length

- l.s.s and some theorems
  - $\ker A^T = \text{im} A^{\perp}$
  - $\ker A^T A = \ker A$
  - $A\vec{x} = \text{proj}_{im(A)}\vec{x}$
  - $A^T A\vec{x} = A^T\vec{x}$

- Some common approaches:
  - if we see words like "if every vector xxx", then we are going to find some special matrices that make our proof easy, such as all-ones' or all-zeros matrix.
  - Usually 8(b) follows directly from 8(a). Try come up with the association.
  - when stuck, always think about contrapositives and contradictions.
  - do not panic, do problems step by step.