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Exercise Class Numerical Methods

Eigenvalues and Eigenvectors

Exercise 1: Write a script that reads a matrix $n \times n$ and computes the largest eigenvalue and its correspondent eigenvector using the power method. Test your algorithm with the matrices below A, B, C, D and with the Hilbert matrix of dimensions: $n = \{2, 4, 10, 20, 50\}$.

Exercise 2: Write a script that reads a matrix $n \times n$ and computes the smallest eigenvalue and its correspondent eigenvector using the inverse iteration. Test your algorithm with the matrices below A, B, C, D.

Exercise 3: Write a script that reads a matrix $n \times n$ and computes all the eigenvalues of the matrix by means of the QR method. How could we approximate all the eigenvectors? Test your algorithm with the matrices below A, B, C, D.

$$A = \begin{pmatrix} 4 & -5 \\ 2 & 3 \end{pmatrix}, \; B = \begin{pmatrix} 0 & 11 & -5 \\ -2 & 17 & -7 \\ -4 & 26 & -10 \end{pmatrix},$$

$$C = \begin{pmatrix} 1 & 0.5 & -0.5 & 0.2 \\ 0.5 & 12 & 0.8 & 1 \\ -0.5 & 0.8 & -16 & 1 \\ 0.2 & 1 & 1 & -4 \end{pmatrix}, \; D = \begin{pmatrix} 2 & 0 & 0.5 & -1 \\ 0.5 & 7 & 6.5 & 21 \\ -2 & 1 & 12 & -0.5 \\ 0 & -.5 & 0 & 18 \end{pmatrix}.$$

Exercise 4. Localize the eigenvalues of the matrix

$$E = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 5 & 1 & 0 \\ 1 & 0 & 8 & 1 \\ 1 & 0 & 0 & 11 \end{pmatrix}.$$

Then use a proper algorithm to approximate all the pairs (λ, x) for the problem $Ex = \lambda x$.