Department of molecular science and nanosystems
Prof. A. Alla

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Exercise Sheet 7

Exercise 1: Prepare a Matlab script that approximates $\int_a^b f(x)dx$

INPUT:

- a, b the boundary of the domain,
- m number of nodes of the partition of $[a, b]$,
- f the function you want to integrate.

OUTPUT:

- Absolute error when the integral is computable.
- the plot of the polynomial approximation and the exact solution with different colors,
- the plot of the error when increasing the degree n of the plynomial.

You might test your code on the following functions:

a) $\int_0^{2\pi} \sin(x) = 0,$

b) $\int_0^{2\pi} xe^{-x}\cos(2x) = \dfrac{3(e^{-2\pi}-1) - 10\pi e^{-2\pi}}{25},$

c) $\int_{-5}^{5} \dfrac{1}{1+x^2}\,dx,$

d) $\int_0^1 x^{5/2}\,dx,$

e) $\int_{-2}^{2} x\sin(x)\,dx,$

f) $\int_{-1}^{0} |x|\,dx = 0.5 \quad \int_{-1}^{1} |x|\,dx = 1,$ (what happens if you have an even or an odd number of nodes?)

g) $\int_0^{2\pi} |\sin(x)|\,dx,$

h) $\int_{-1}^{1} e^{-x^2}\,dx,$

i) $\int_{-3}^{5} sgn(x-\pi)\,dx,$

Exercise 2 Let us consider the integral $I(f) = \int_0^1 \sin(x)\cos(x)\,dx$ and estimate the minimum number m of subintervals needed to have $E_{1,m} \leq 5\cdot 10^{-4}$ with the composite trapezoidal formula.