Department of molecular science and nanosystems 2023/24
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Exercise Sheet 8

Exercise 1 Prepare a Matlab script that computes the numerical approximation of the Cauchy problem

$$\begin{aligned} \dot{y}(t) &= f(t, y(t)), \quad t \in I \\ y(t_0) &= y_0 \end{aligned}$$

utilizing the 4 methods studied during the classes: Forward and Backward Euler, Heun and Crank-Nicolson.

INPUT:

- f, the function which describes the vectorial field
- t_0, y_0 initial conditions;
- T final time
- Δt temporal step size;

OUTPUT:

- Absolute error when the solution is known.
- the plot of the numerical approximation and the exact solution (when known) with different colors,
- Order of convergence of the scheme.

You might test your code on the following ODEs

a) $\dot{y} = -y \log y$, $y(0) = 0.5$, exact solution $y(t) = e^{-e^{\log\log(2) - t}}$

b) $\dot{y} = -e^{-(t+y)}$, $y(0) = 1$, exact solution $y(t) = \log(e + e^{-t} - 1)$

c) $\dot{y} = y(1-y)$, $y(0) = 0.5$, exact solution $y(t) = e^t/(1 + e^t)$

d) $\dot{y} = 16y(1-y)\, y(0) = 1/1024$, exact solution $y(t) = e^{16t - \log 1023} / \left(1 + e^{16t - \log 1023}\right)$

e) Harmonic oscillator

$$y'' = -\omega^2 y, y(0) = x_0, y'(0) = y_0 \omega,$$

exact solution: $y(t) = x_0 \cos(\omega t) + y_0 \sin(\omega t)$

f) Lotka-Volterra

$$\begin{aligned} y_1' &= \alpha y_1 - \beta y_1 y_2 \\ y_2' &= -\gamma y_2 + \delta y_1 y_2 \end{aligned}$$

For example: $\alpha = 0.25, \beta = 0.01, \gamma = 1, \delta = 0.01, y_1(0) = 80, y_2(0) = 30, t_0 = 0, T = 30.$

g) Van der Pol

$$\begin{aligned} y_1' &= y_2 - f(y_1) \\ y_2' &= -y_1 \end{aligned}$$

with $f(x) = x^3 - x, y_1(0) = x_1, y_2(0) = x_2.$