

Exercise Sheet 1

Exercise 1: Write a script which reads an upper triangular matrix $A \in \mathbb{R}^{n \times n}$ and a vector $b \in \mathbb{R}^n$ and solves the linear system $Ax = b$ with the backward substitution. Then compute the residual vector: $r = Ax - b$.

Exercise 2: Write a script which reads a lower triangular matrix $A \in \mathbb{R}^{n \times n}$ and a vector $b \in \mathbb{R}^n$ and solves the linear system $Ax = b$ with the forward substitution. Then compute the residual vector: $r = Ax - b$.

Exercise 3: Write a script that solves a linear system $Ax = b$ with the Gaussian elimination method. At the end print the solution \bar{x} , the residual $r = Ax - b$, the matrix L , U and $\|A - LU\|$. (Hint: start your code without pivoting, then partial pivoting and finally total pivoting)

Test your script with:

$$A = \begin{pmatrix} 1 & -1 & 4 \\ 3 & 1 & 5 \\ 1 & 3 & -1 \end{pmatrix}, \quad b = \begin{pmatrix} 10 \\ 15 \\ 6 \end{pmatrix}$$

$$A = \begin{pmatrix} 1 & -1 & 2 & -3 \\ 2 & 1 & 0 & -1 \\ 0 & 2 & 1 & 1 \\ 2 & 0 & 1 & 0 \end{pmatrix}, \quad b = \begin{pmatrix} 0 \\ 3 \\ -3 \\ 0 \end{pmatrix}$$

$$(A)_{ij} = \frac{1}{i+j-1}, i, j = 1, \dots, n \quad b = \sum_{j=1}^n a_{ij}, n > 4$$