Department of Molecular Sciences and Nanosystems 2023/24
Prof. A. Alla

Exercise Sheet 6

Exercise 1: Prepare a script that computes

- Lagrange polynomial Π_n in standard form,
- Piecewise Lagrange approximation.

INPUT:

- a, b the boundary of the domain,
- N number of subintervals of the partition of $[a, b]$,
- n degree of the polynomial (for piecewise approximation),
- f the function you want to interpolate.

OUTPUT:

- The error in $\|\cdot\|_\infty$ between the function f and the polynomail approximation.,
- the plot of the polynomial approximation and the exact solution with different colors,
- the plot of the error when increasing the degree n of the plynomial.

You might test your code on the following functions

a) $f(x) = \sin(3x) \quad 0 \le x \le 2\pi,$

b) $f(x) = \log(x) \quad 1 \le x \le 1.5,$

c) $f(x) = e^x, \quad 0 \le x \le 1,$

d) $f(x) = x^5 + 3x^4 + 2x^3 - x^2 - 5 + 1, \quad -5 \le x \le 5,$

e) $f(x) = \dfrac{1}{1+x^2}, \quad -5 \le x \le 5,$

f) $f(x) = |x|, \quad 0 \le x \le 5,$ and $-3 \le x \le 2,$

g) $f(x) = \sin(x^2), \quad -5 \le x \le 5,$

h) $f(x) = |\sin(x^2)|, \quad -5 \le x \le 5,$

i) $f(x) = sgn(x), \quad -5 \le x \le 5,$

Exercise 2: Provide an estimate of the error (independent from the distribution of the nodes) approximating the function $f(x) = \sin^2(x)$ in the interval $[-5, 5]$ with a polynomial interpolation of degree 7 . (This is a theoretical exercise)

Exercise 3:[optional] Write a script that computes the Hermite polynomial. Test your script with the functions provided in Exercise 1.