Exercise Class Numerical methods

QR factorization and least squares problem

Exercise 1: Write a script that reads a matrix $A \in \mathbb{R}^{n \times m}$ and provides its QR factorization with the following methods:

- Gram-Schmidt's orthogonalization,
- Householder transformation matrices,
- Givens transformation matrices.

Exercise 2: Write a script that reads a matrix $A \in \mathbb{R}^{n \times m}$ (with $n \geq m$), a vector $b \in \mathbb{R}^n$ and solves the linear system Ax = b in a least square sense with the following methods:

- Normal equations,
- QR factorization,

At the end compare the results computing the residual $\|Ax - b\|_2$. Which method does it perform better? Why?