Homework 6

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Question 1

Given the initial stock of lumber k_0, let $\mathcal{K} = [0, k_0]$ be the set of possible values for a stock of lumber, and let $\mathcal{P} = \mathbb{R}$ be the set of possible prices. $\mathcal{K} \times \mathcal{P}$ is the state space. Let $(k, p) \in \mathcal{K} \times \mathcal{P}$. Then the Bellman equation is

$$V(k,p) = \max_{k'} p \cdot (k - k') - 0.2(k - k')^{1.5} + \delta \mathbb{E}_{p'|p} V(k', p') \tag{1}$$

subject to

$$p' = p_0 + \rho p + u,\ u \sim N(0, \sigma_u^2),$$

and

$$k' \in [0, k].$$

Question 2

The vector of grids is $(0.6536, 0.6882, 0.7229, 0.7575, 0.7922, 0.8268, 0.8614, \cdots, 1.3118, 1.3464)$.

Question 3

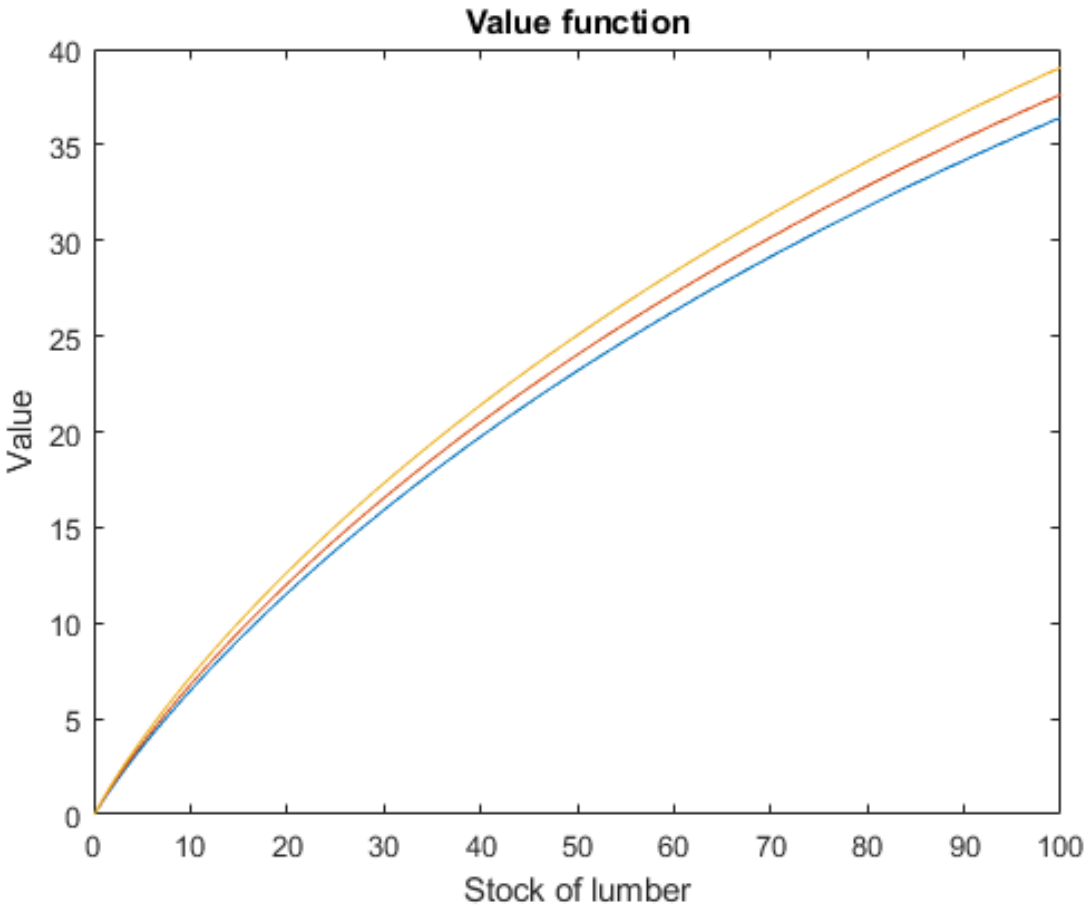

Figure 1: The values as a function of lumber stocks, for $p = 0.9, 1, 1.1$

Question 4

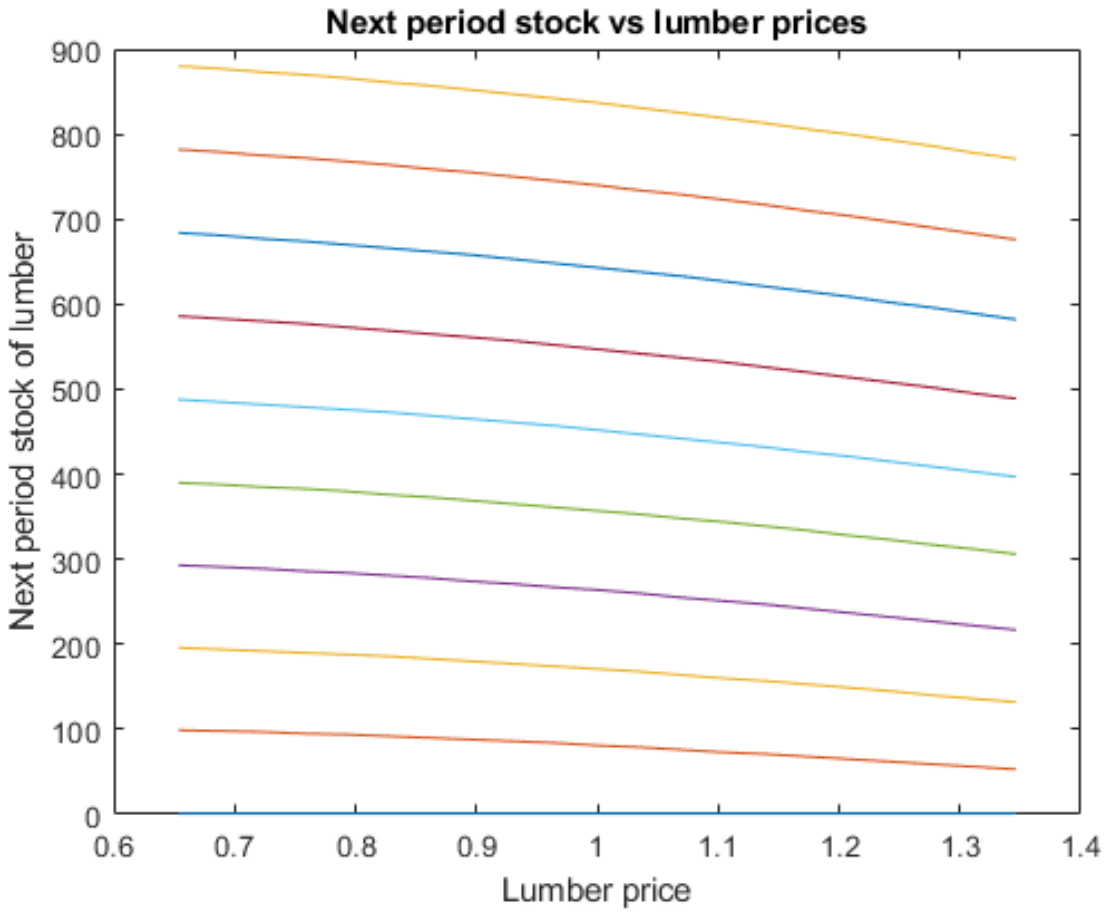

Figure 2: Next period optimal stocks as a function of lumber prices, for current period stock 0.1, 10.1, 20.1, ..., 90.1

Question 5

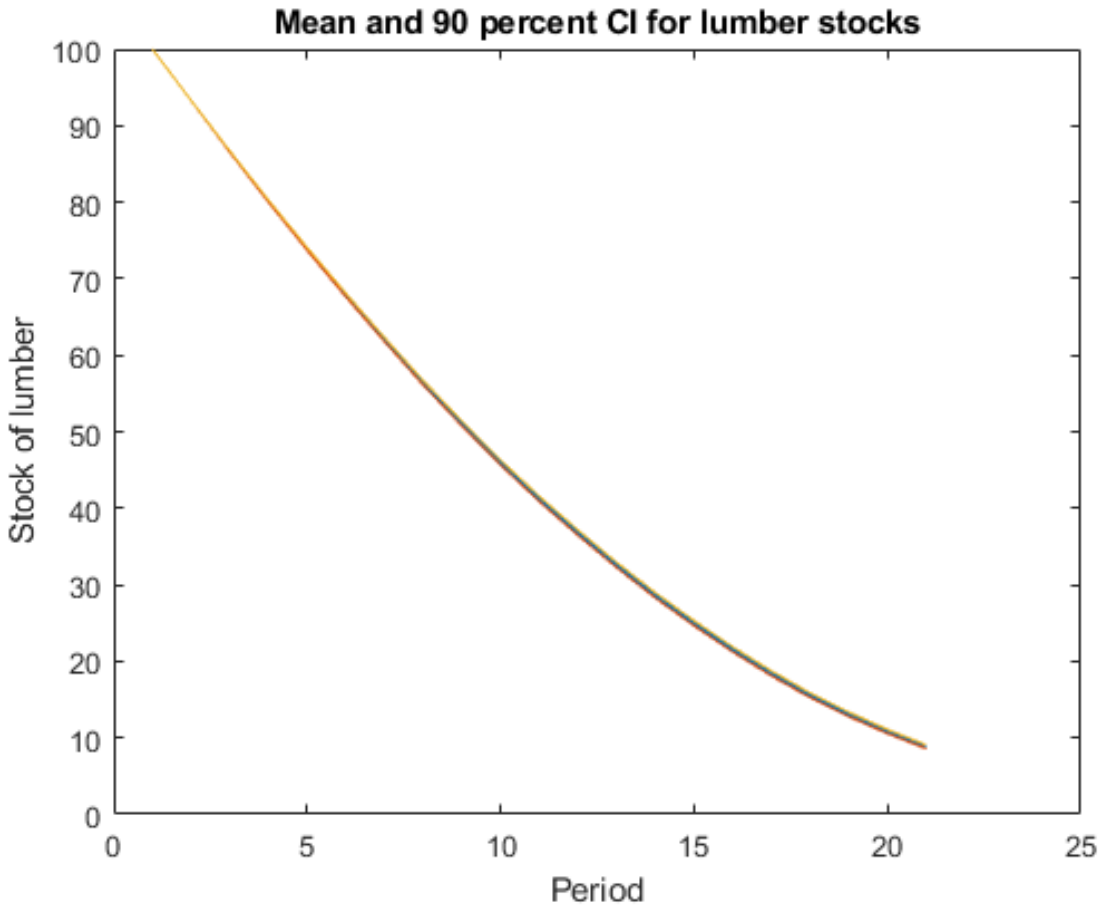

Figure 3: Expected stock and 90% confidence interval

Question 6

Since $p = 0.9, 1.1$ are not on the grid, I draw two curves associated with the closest prices to them in Fig. 4.

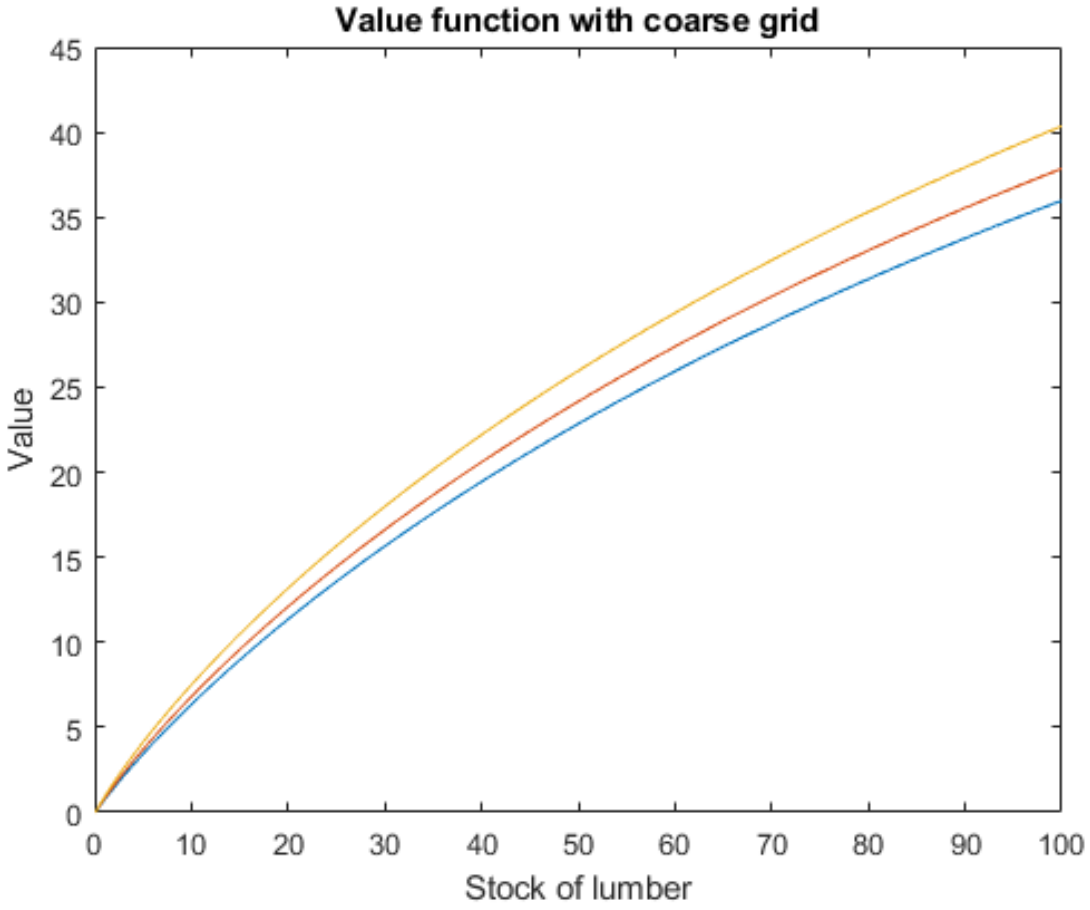

Figure 4: The values as a function of lumber stocks, for $p = 0.827, 1, 1.173$

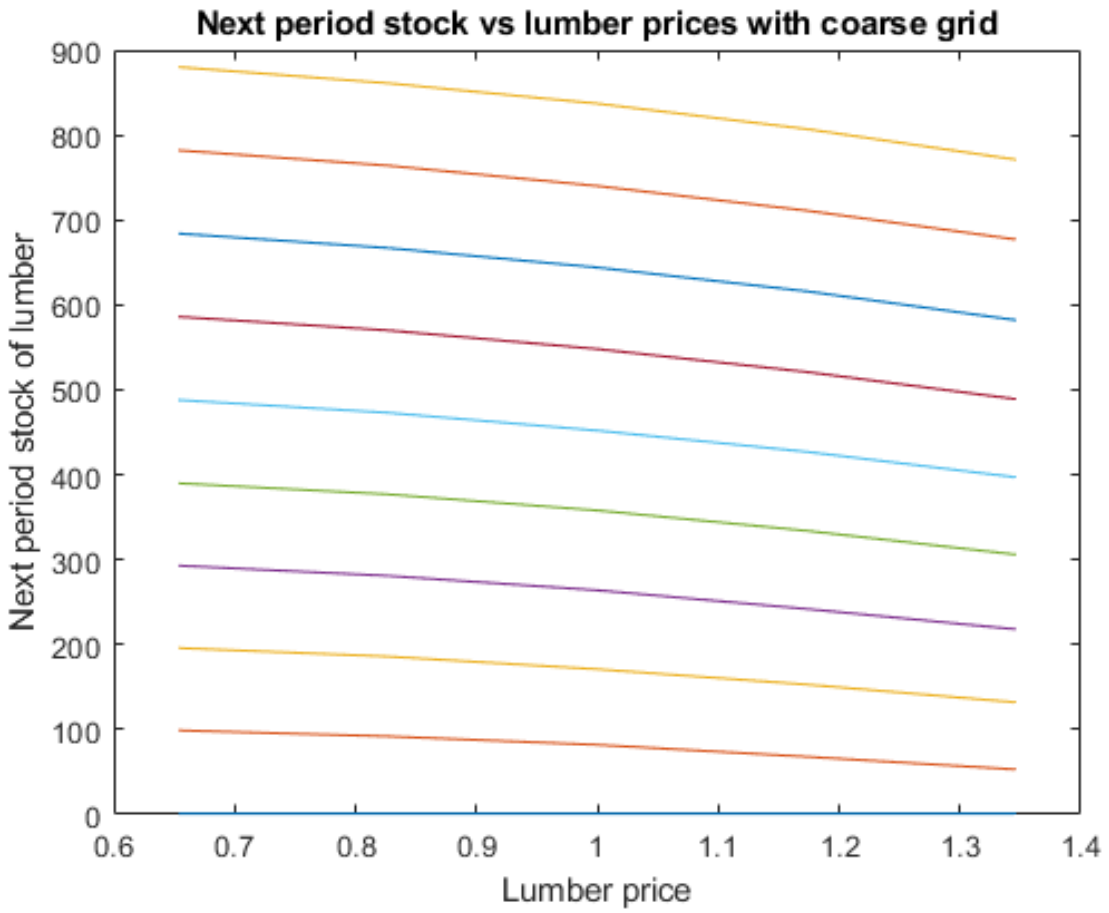

Figure 5: Next period optimal stocks as a function of lumber prices, for current period stock 0.1, 10.1, 20.1, ..., 90.1

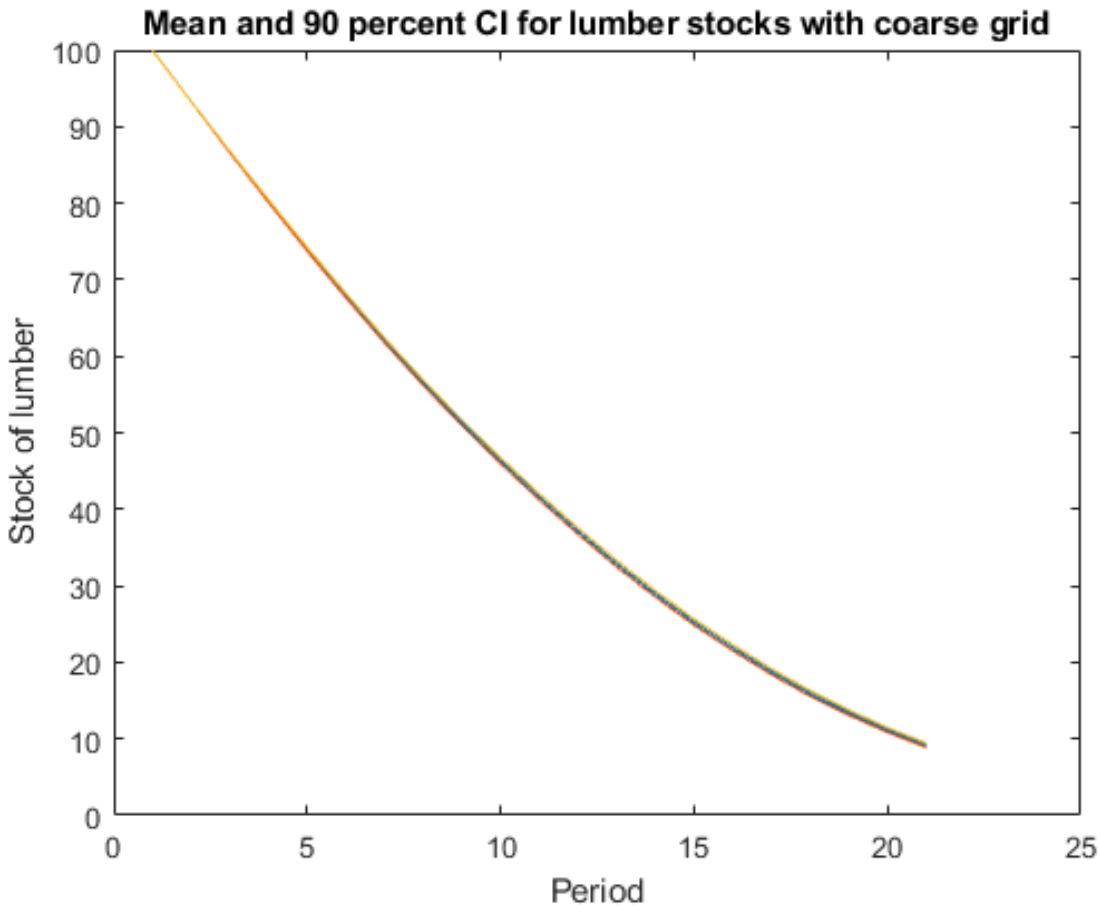

Figure 6: Expected stock and 90% confidence interval