

$$u' + (\lambda + \mu)u = \lambda$$

integrating factor :

$$I(t) = e^{(\lambda + \mu)t}$$

$$e^{(\lambda + \mu)t} u' + e^{(\lambda + \mu)t} (\lambda + \mu)u = e^{(\lambda + \mu)t} \lambda$$

$$\int (e^{(\lambda + \mu)t} u' + e^{(\lambda + \mu)t} (\lambda + \mu)u) dt = \int e^{(\lambda + \mu)t} \lambda dt$$

$$e^{(\lambda + \mu)t} u = \int e^{(\lambda + \mu)t} \lambda dt + C$$

$$u = e^{-(\lambda + \mu)t} \left[\int e^{(\lambda + \mu)t} \lambda dt + C \right]$$

$$u = e^{-(\lambda + \mu)t} \left[\lambda \int e^{(\lambda + \mu)t} dt + C \right]$$

$$u = e^{-(\lambda + \mu)t} \left[\lambda \frac{1}{\lambda + \mu} e^{(\lambda + \mu)t} + C \right]$$

$$= e^{-(\lambda + \mu)t} \left[\frac{\lambda}{\lambda + \mu} e^{(\lambda + \mu)t} + C \right]$$

$$= \frac{\lambda}{\lambda + \mu} + C e^{-(\lambda + \mu)t}$$

$$u(0) = \frac{\lambda}{\lambda + \mu} + C = u_0$$

$$C = u_0 - \underbrace{\frac{\lambda}{\lambda + \mu}}_{u^*}$$

$$u = u^* + e^{-(\lambda + \mu)t} (u_0 - u^*) //$$

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