The dynamics of asset-holding:

$$\dot{a}(t) = n(t) \cdot \pi(t) + n(t) \cdot \dot{v}(t) + w(t) - er(t),$$

- $\underbrace{}$ income gain from firms / varieties
- $\underbrace{\phantom{n(t) \cdot \dot{v}(t)}}$ capital gain from firms / varieties
- labor income
- expenditure for consumption

$$a(t) = n(t) v(t)$$

$$\dot{a}(t) = \dot{n}(t) v(t) + n(t) \dot{v}(t) \cdots \qquad \text{①}$$

$$\dot{n}(t) = \frac{\dot{a}(t) - n(t) \dot{v}(t)}{v(t)}$$

$$= \frac{n(t)\pi(t) + w(t) - er(t)}{v(t)} \cdots \qquad \text{②}$$

Substitute ② into ①:

$$\dot{a}(t) = n(t) \pi(t) + w(t) - er(t) + n(t) \dot{v}(t)$$

$$= (\pi(t) + \dot{v}(t)) \underbrace{n(t)}_{\substack{\| \\ \boxed{\dfrac{a(t)}{v(t)}}}} + w(t) - er(t)$$

$$= \frac{\pi(t) + \dot{v}(t)}{v(t)} a(t) + w(t) - er(t),$$

Let $r(t) = \dfrac{\pi(t) + \dot{v}(t)}{v(t)}$, then

$$\dot{a}(t) = r(t) a(t) + w(t) - er(t). \qquad //$$