The Aggregate Effects of the Great Black Migration*

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Abstract

In the United States, four million African Americans migrated from the South to the North between 1940 and 1970. How did it impact aggregate US output and the welfare of cohorts of African Americans and others? To answer this question, I quantify a dynamic general equilibrium model in which cohorts of African Americans and others migrate across states. I compare the baseline equilibrium matched with the US data from 1940 to 2010 with counterfactual equilibria in which African Americans or others cannot relocate across the North and the South between 1940 and 1970. The mobility of African Americans and the mobility of others increased aggregate US output in 1970 by 0.7 and 0.3 percent, respectively. Therefore, although African Americans accounted for about 10 percent of the US population, their migration had a larger impact than the migration of the other 90 percent of the population did. The mobility of African Americans increased the welfare of African Americans born in Mississippi in the 1930s by 3.5 percent, decreased the welfare of African Americans born in Illinois in the 1930s by 0.2 percent, and did not change the welfare of others substantially.

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1 Introduction

Slavery was a place-based policy in the history of the United States. Before Emancipation, about 70 to 80 percent of the Black population in the US resided in the South, and the vast majority of them were enslaved. The largest change in the spatial distribution of the Black population occurred from 1940 to 1970. In this period, 4 million African Americans migrated from the South to the North, and the fraction of the Black population in the South dropped from 69 percent to 45 percent. This is called the great Black migration, the largest internal migration in US history.

This paper quantifies the effects of the great Black migration on aggregate US output and the welfare of African Americans and others. For this purpose, I develop and quantify a dynamic general equilibrium model of the spatial economy in which cohorts of African Americans and others migrate across states. In the period of the great Black migration, the migration pattern of African Americans was quite different from the migration pattern of others; 44 percent of African Americans born in the South in the 1930s migrated to the North, whereas only 17 percent of others born in the South in the same decade did so. Accordingly, my model generates different migration patterns across races. Since people of different cohorts and ages migrate differently, the model includes overlapping generations, so that the model has distinct notions of cohorts and ages. In the model, African Americans and others of the same cohort face different survival probabilities and life expectancies, because African Americans have lower survival probabilities and life expectancies than others in data.

In the great migration period, African Americans who moved from the South to the North earned much higher wages than African Americans who stayed put in the South. The degree of this wage gap between movers and stayers was higher for African Americans from the South than for any other group of people in this period. These facts suggest that there was pecuniary incentive for the great Black migration. I primarily attribute wages to productivity parameters varying flexibly at race, age, time, and location levels to capture heterogeneous pecuniary incentive for migration.

The great Black migration, however, did not make everyone better off. The migration of African Americans to the North shifts up the labor supply of the Black labor force and puts stronger downward pressure on African Americans' wages than on others' wages in the North. Therefore the great Black migration made worse off African Americans who had already lived in the North (Boustan, 2009). Allowing for imperfect substitution across African Americans and others, my general equilibrium model generates such ramifications of the great Black migration in the North.

I parameterize the model in two steps. First, I estimate elasticities: parameters mapping a percent change in one endogenous variable to another. Elasticities include migration elasticity governing how migration flows react to real wages and non-pecuniary amenities, and elasticity of substitution across ages and races in the production function. In so doing, I follow standard methods in trade and labor economics literature (Artuc and McLaren, 2015; Borjas, 2003; Card, 2009). Specifically, I estimate the elasticity of substitution between African Americans and others using shift-share instruments. My estimate is about 9.0, which falls in the range of estimates from 8.3 to 11.1 by Boustan (2009). Second, I back out the other parameters such as amenities, productivity, and migration costs for different age and racial groups in locations over time. The model delivers explicit formulae to pin down these parameters given elasticities and relevant data, as the models in Allen and Arkolakis (2014) and Ahlfeldt et al. (2015) do. The migration costs thus backed out are higher for African Americans than for others, but the racial gap in the migration costs shrank over time.

Armed with the parameter values, I compare the baseline equilibrium that resembles the factual path of the US economy to counterfactual equilibria. I mainly consider two counterfactual equilibria. First, I shut down the North-South migration of African Americans between 1940 and 1970. In such counterfactual equilibrium, aggregate US output as of 1970 would have been 0.74 percent lower than that of the baseline equilibrium. Although African Americans accounted for about 10 percent of the US population, their migration from the South to the North increased aggregate US output substantially. Welfare is measured as the expected value at the beginning of life. The welfare for African Americans born in Mississippi and Illinois in the 1930s would have been 3.5 percent lower and 0.2 percent higher, respectively. The lower welfare of African Americans in the South in the no great migration scenario is because they lose opportunities of migrating to the productive, high-wage North. The higher welfare of African Americans in the North is mainly explained by higher wages in the no great migration scenario than in the baseline equilibrium. If the great Black migration did not occur, the average wage of (incumbent) African Americans in the North would have been higher by 8.0 percent. This number falls in the range of predictions made by Boustan (2009) from 7.2 to 9.6.[1] The wage and welfare of others would not have been substantially different from the baseline equilibrium. Boustan (2009) also predicts that the inflow of African Americans to the North did not change the wage of white workers in the North substantially. The nationwide nominal and real wage ratio between African Americans and others would have been lower by 10 and 8.9 percent, respectively. The result about the nominal wage ratio is in line with the prediction by Smith and Welch (1989).[2] The North-South migration of African Americans

[1] See her table 6.

[2] See their table 20.

contributed to reducing racial gaps in nominal and real wages.

Second, I shut down the North-South migration of others for between 1940 and 1970. In such counterfactual, aggregate US output in 1970 would have been 0.28 percent lower. Although others constitute 90 percent of the US population, their relocation across the North and the South had a smaller impact than African Americans' relocation did. The welfare of others born in Mississippi in the 1930s would have been 1.4 percent lower. In this counterfactual equilibrium, others in Mississippi lose opportunities of migrating to the productive North, thus the welfare declines by 1.4 percent. This is, however, smaller than the welfare loss for African Americans in Mississippi in the no great Black migration scenario explained above. These two counterfactual experiments highlight African Americans' strong incentive for the outmigration from the South. The welfare of others born in Illinois in the 1930s would have been 0.37 percent lower. Others in Illinois are already in the productive location, but they still lose extensive margins in location choices, leading to the welfare loss.

This paper relates to two strands of literature. First, this paper contributes to the literature on the great Black migration and economic geography of African Americans in the US, including Smith and Welch (1989), Gregory (2006), Boustan (2009, 2010, 2017), Black et al. (2015), Chay and Munshi (2015), Derenoncourt (2022), Calderon et al. (2022), Althoff and Reichardt (2022). To the best of my knowledge, this paper is the first to quantify the aggregate, general equilibrium effects of the great Black migration. Among the papers I have listed, the most related is Boustan (2009). She estimates a constant elasticity of substitution (CES) production function and finds imperfect substitutability between African Americans and white people. She extrapolates the estimated production function to predict what African Americans' wages would have been in the North if the great Black migration did not occur. This paper integrates her idea of imperfect substitutability across races into a quantitative general equilibrium setting, making similar predictions for the counterfactual wage of African Americans in the North in the no great Black migration scenario. I quantify the contribution of the great Black migration to the reduction in racial wage gaps, and the result is in the same ballpark of Smith and Welch (1989). Moreover, because of the overlapping generations structure, the model speaks to the effects of the great migration on not only migrants and their contemporaries, but also the later generations. In this sense, this paper relates to Derenoncourt (2022) who studies the intergenerational effects of the great Black migration.

Second, I take advantage of the recent development of quantitative general equilibrium models of the dynamic spatial economy, including Desmet and Rossi-Hansberg (2014), Caliendo et al. (2019), Monras (2020), Kleinman et al. (2022), Allen and Donaldson (2022), Eckert and Peters (2022), Nagy (2022). Following Allen and Donaldson (2022) and Eckert and Peters (2022), the model has the overlapping generations structure in the spatial economy. I differ from them in that individuals work for more than one period in my model. In comparison to the existing literature, my model allows different productivity, amenities, migration costs and survival probabilities across ages and races, hence generates heterogeneous migration patterns across these groups.[3]

The remainder of the paper is organized as follows. Section 2 describes motivating facts including the data mentioned in this introduction. Section 3 lays out the model. In Section 4, I estimate the elasticities and back out the other parameters. Section 5 compares some of the variables in the baseline equilibrium of the model with the data counterparts. Section 6 compares the baseline equilibrium with counterfactual equilibria. Section 7 concludes.

2 Motivating Facts

How have African Americans been spatially distributed in the US? Before Emancipation, about 70 to 80 percent of African Americans in the US lived in the South,[4] as the solid line in Figure 1 shows. The dashed line shows the fraction of enslaved African Americans in the South out of African Americans in the US. The vast majority of African Americans in the South were enslaved. In the meantime, only about 20 to 30 percent of the people other than African Americans (henceforth, others) in the US resided in the South, as in the dotted line. The fraction of African Americans in the South stayed high around 80 percent even after Emancipation.

African Americans started leaving the South to the North circa 1910, and the fraction of African Americans in the South dropped from 81 percent to 71 percent by 1930. This amounts to the migration of 1.5 million African Americans and is called the first great Black migration. Following the Great Depression, Black migration paused for a decade. The largest migration occurred after the pause. The fraction of African Americans in the South declined from 69 percent to 45 percent between 1940 and 1970. 4 million African Americans left the South to the North in this period. This is called the second great Black migration on which this paper focuses.[5]

To see migration behavior by demographic groups, I define movers and stayers for races (African Americans or others), birthplaces (the North or the South), and cohorts.[6] I

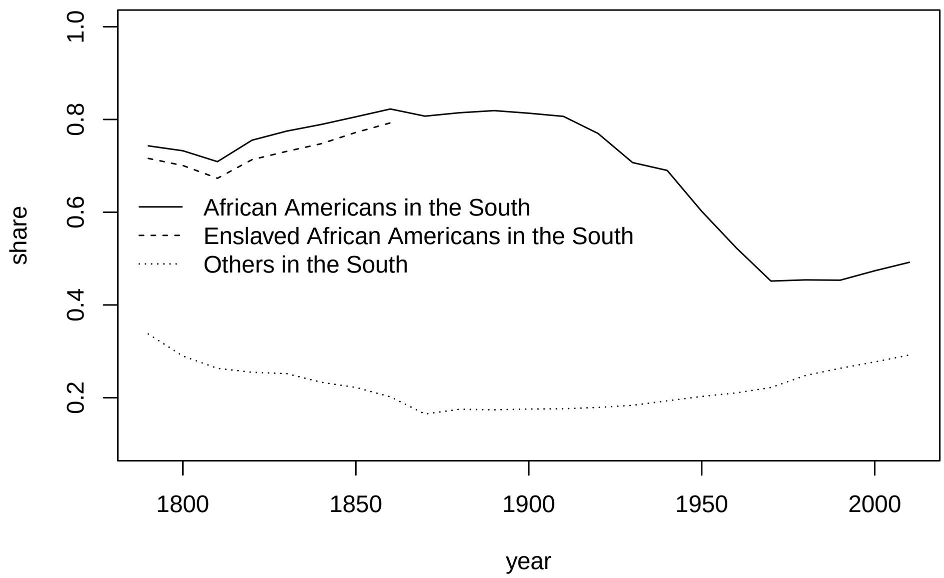
[3] Suzuki (2021) integrates heterogeneous migration costs and survival probabilties across ages into a dynamic spatial model.

[4] The South refers to all confederate states. The US refers to the area of the current US states except Alaska and Hawaii. The North refers to the area of the US except the South.

[5] The second great Black migration is often just referred to as the great Black migration. See footnote 1 of Derenoncourt (2022).

[6] See Figure 1 of Black et al. (2015) and Chapter 1 of Boustan (2017) for earlier tabulation of the great

Figure 1: Fractions of Populations in the South for African Americans and Others



Notes: The solid line is the ratio of the number of African Americans in the South to the number of African Americans in the US. The dashed line is the ratio of the number of enslaved African Americans in the South to the number of African Americans in the US. The dotted line is the ratio of the number of the people other than African Americans (others) in the South to the number of others in the US. Source: US census 1940-2000, American Community Survey 2010.

consider 10-year windows as cohort bins and call those who were born in 1930-1939 as cohort 1930, and so on.[7] For each cohort c, I collect the individuals who lived in either the North or the South as of year $c+50$. Then for each race, birthplace, and cohort c,

- movers are the individuals who lived in the other place than the birthplace as of year $c+50$,
- stayers are the individuals who lived in the birthplace as of year $c+50$.

For each race r, birthplace p, and cohort c, I compute

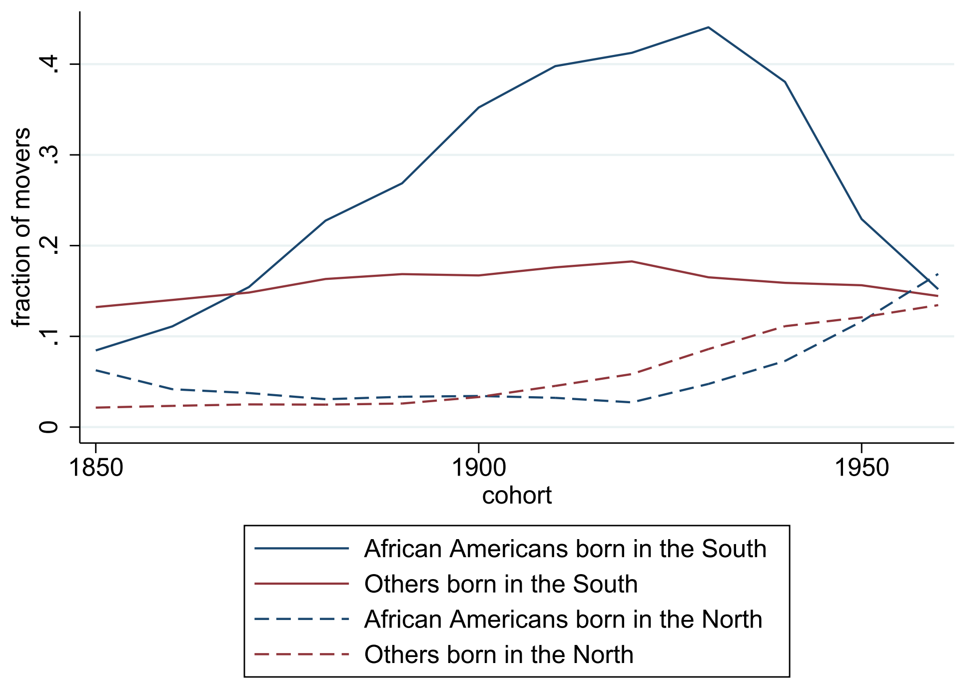
$$\frac{\text{movers}_{r,p,c}}{\text{movers}_{r,p,c}+\text{stayers}_{r,p,c}}, \tag{1}$$

Black migration by cohort.

[7]Formally, I refer to the individuals who were born in the period from year c to year $c+9$ as cohort c for calendar year c whose 4th digit is zero.

where $\text{movers}_{r,p,c}$ and $\text{stayers}_{r,p,c}$ denote the numbers of movers and stayers for race r, birthplace p, and cohort c. I call the ratio (1) as the fraction of movers.

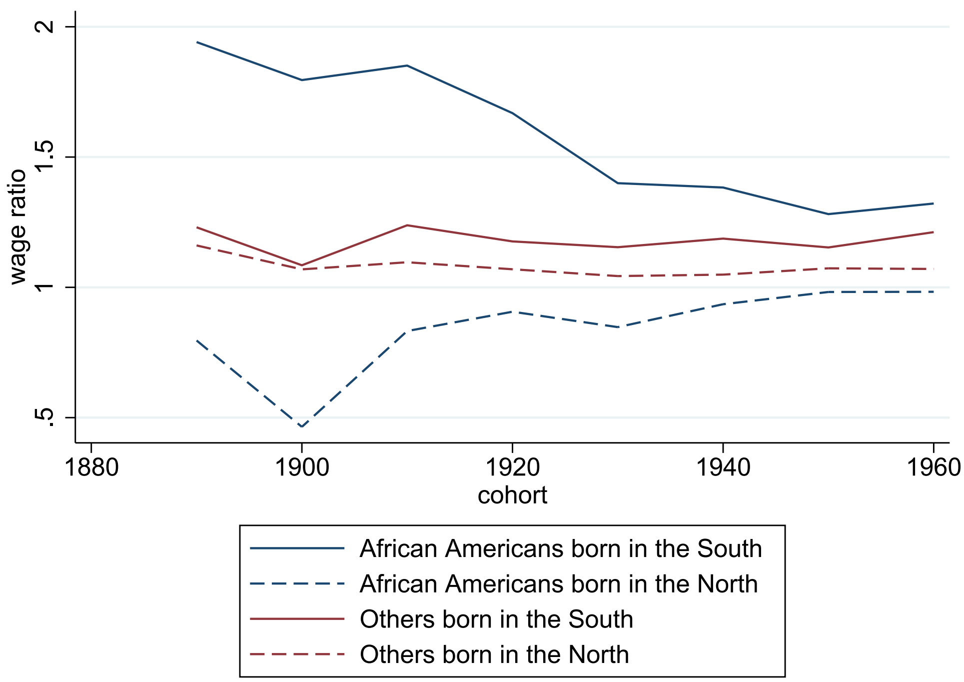
Figure 2: Fractions of Movers for Races, Cohorts, and Birthplaces



Notes: Cohort 1850 refers to those who were born in 1850-1859, and so on. For each cohort (say c), race, and birthplace (the North or the South) tuple, the fraction of movers is the ratio of the number of movers to the sum of the numbers of movers and stayers. Source: US census 1900-2000, American Community Survey 2010.

The migration patterns of African Americans from the South were different from the migration patterns of other groups of people. Figure 2 provides the fractions of movers for African Americans and others born in the North or the South. The fraction of movers for African Americans born in the South exhibits remarkable changes over time. It steadily increased from cohort 1850 and peaked at 0.44 in cohort 1930. That is, over 40 percent of African Americans born in the South in the 1930s moved to the North by 1980. After that, the fraction of movers for African Americans born in the South sharply declined to 0.15. The trajectory of the fraction of movers for African Americans born in the South highlights different migration behavior across cohorts within the race-birthplace bin. This motivates the model with a notion of cohorts in Section 3. The trajectory of the fraction of movers for African Americans born in the South is clearly different from the trajectory of others born in the South, which was always around 0.15. This suggests that African Americans and others had different economic incentive for South-to-North migration. The fraction of movers for African Americans and others born in the North exhibit similar patterns. The fraction of movers for them was stable and less than 0.05 from cohort 1860 to cohort 1910. After that, the fraction of movers increased and reached 0.15 and 0.13 for African Americans and others in cohort 1960, respectively. The recent net migration from the North to the South is called the reverse great migration. But its magnitude is smaller than the one of the original great migration.

Figure 3: Mover-Stayer Wage Ratios for Cohorts, Races, and Birthplaces



Notes: For each cohort (say c), race, and birthplace (the North or the South), this graph provides the ratio of the average wage of the individuals who lived in the place other than the birthplace as of year $c+50$ to the average wage of the individuals who lived in the birthplace as of year $x+50$. Source: US census 1940-2000, American Community Survey 2010.

In the great migration period, African Americans who moved from the South to the North earned much higher nominal wages than African Americans who stayed put in the South. To see this, I compute a measure I call the mover-stayer wage ratio. For each cohort c, race, and birthplace, I compute the ratio of the average wage of movers to the average wage of stayers as of year $c+50$.[8] Figure 3 provides mover-stayer wage ratios for race-birthplace tuples. As in the blue solid line, African Americans who moved from the

[8] The average wage means the average of the wages of all individuals who earn positive wages for each race, birthplace, and cohort tuple. In Appendix A, Figure 20 reports the mover-stayer ratios of per capita payrolls and yields similar results.

South to the North earned 79 to 94 percent higher wages than African Americans who stayed in the South from cohort 1890 to cohort 1910. This wage differential is extremely high compared with the other race-cohort-birthplace tuples. Since cohort 1910, the mover-stayer wage ratio for African Americans born in the South declined and reached 1.39 in cohort 1930. That is, on average, African Americans who moved from the South to the North earned 39 percent higher wages than African Americans who stayed in the South at the peak of the great migration. Since then, the mover-stayer wage ratio for African Americans born in the South moderately declined. As the blue dashed line shows, African Americans who moved from the North to the South earned 54 to 17 percent lower wages than African Americans who stayed in the North from cohort 1890 to cohort 1910. The mover-stayer ratio for African Americans born in the North increased after cohort 1910 and is 0.98 in cohort 1960. As in the red solid and dashed lines, the mover-stayer wage ratios for others were relatively stable over time: 1.08 to 1.24 for others born in the South and 1.04 to 1.16 for others born in the North.

Housing rent in the North was higher than housing rent in the South, but the rent gap did not fully absorb the mover-stayer wage gap for African Americans from the South. For each cohort c, the second row of Table 1 shows

$$\text{movers' wage} - \text{stayers' wage}.$$

for African Americans' born in the South. Similarly, the third row of Table 1 shows

$$\text{movers' rent} - \text{stayers' rent}.$$

Both of wages and rent are deflated by the consumer price index and measured in 2010 US dollars. For cohort 1890, the magnitude of the rent gap was comparable to the magnitude of the wage gap. But for cohorts 1920-1940, the rent gaps were only about one-fourth of the wage gaps. Therefore, at the peak of the great Black migration, the rent gap between the North and the South was unlikely to absorb the mover-stayer wage gaps for African Americans born in the South.[9]

I summarize the empirical facts I have described so far to four points.

1. The migration rate of African Americans from the South in the great migration period was higher than the migration rate of people of the other race-birthplace-cohort tuples.

2. African Americans who moved from the South to the North in the great migration

[9] Wages are defined for individuals, but rent is defined for households. So I match housing rent with the household head's race, cohort, birthplace, and current place bins. If a household has multiple wage-earners, the mover-stayer rent gap relative to the mover-stayer wage gap can be even smaller at household levels.

Table 1: Wage and Rent Gaps between Movers and Stayers for African Americans Born in the South

cohort	1890	1900	1910	1920	1930	1940	1950	1960
wage gaps	4,693	6,986	9,957	11,985	8,748	9,782	8,406	7,008
rent gaps	3,080	2,616	2,512	2,711	2,052	2,506	1,841	2,038

Notes: For cohort c, the second row refers to the average wage of movers minus the average wage of stayers for African Americans born in the South. Analogously, for cohort c. the third row refers to the average rent of African Americans who were born in the South and lived in the North as of year $c+50$ minus the average rent of African Americans who lived in the birthplace, the South, as of year $c+50$. Wages and rent are deflated by the consumer price index and measured in 2010 US dollars.

period earned much higher wages than African Americans who stayed put in the South.

3. The mover-stayer wage gap for African Americans from the South in the great migration period was higher than the mover-stayer wage gap for people of the other race-birthplace-cohort tuples.

4. The mover-stayer rent gap accounted for only about one fourth of the mover-stayer wage gap for African Americans from the South in the great migration period.

3 Model

I develop a dynamic general equilibrium model that generates different migration patterns across races and cohorts over time. Individuals of different races and cohorts migrate, taking into account the future flows of real wages and non-pecuniary amenities in potential destinations, and migration costs across locations.

3.1 Environment

The economy consists of a finite set of locations $\mathcal{N}$. Let $N = |\mathcal{N}|$, that is, N is the number of locations. Time is discrete and denoted by $t = 0, 1, \cdots$. Goods are perishable in each period. Individuals cannot save their income.

Individuals are characterized by race r, age a, and location i in period t. The set of races is $\{b, o\}$, where b denotes African Americans, and o denotes others. The set of ages is $\{0, 1, \cdots, \bar{a}\}$, where $\bar{a} > 0$ denotes the age of the oldest group in each period. Individuals can live through at most age $\bar{a}$, but they may die before age $\bar{a}$ due to exogenous survival probabilities. Specifically, individuals of race r and age a in period t can survive to period $t+1$ with probability $s_{r,a,t}$. Note that the maximum periods of life is $\bar{a}+1$.

I can trace trajectories of individuals' behavior by cohort. Individuals of cohort c are born in period c. If all relevant survival probabilities are strictly greater than 0,[10] some of them survive up to period $c+\bar{a}$. Individuals of cohort c are age 0 in period c, age 1 in period $c+1$, $\cdots$, age $\bar{a}$ in period $c+\bar{a}$. Thus tracing behavior of individuals of these age-period pairs pins down the life course of cohort c.

Individuals' only source of income is their wages. They supply fixed length of work hours in each period and earn the market wage (no intensive margin of the labor supply). Individuals of age 0 do not work. Individuals of ages $1,\cdots,\bar{a}$ work.

3.2 Period Utility

The period utility of individuals of race r, age a, period t, and location i, $u^i_{r,a,t}$, is

$$u^i_{r,a,t} = \begin{cases} 0 & \text{for } a=0, \\ \log C^i_{r,a,t} + \log B^i_{r,a,t} & \text{for } a=1,\cdots,\bar{a}, \end{cases} \tag{2}$$

where $C^i_{r,a,t}$ is the consumption of individuals of race r, age a in period t and location i (henceforth individuals of (r,a,t,i)), and $B^i_{r,a,t}$ is the exogenous parameter of the amenites for individuals of (r,a,t,i).

For age $a=1,\cdots,\bar{a}$, workers of (r,a,t,i) consume the Cobb-Douglas composite of homogeneous goods and housing service

$$C^i_{r,a,t} = \left(\frac{G^i_{r,a,t}}{1-\gamma}\right)^{1-\gamma}\left(\frac{H^i_{r,a,t}}{\gamma}\right)^{\gamma}, \tag{3}$$

where $G^i_{r,a,t}$ and $H^i_{r,a,t}$ are the consumption of homogeneous goods and housing service by the individuals of (r,a,t,i), and γ is an exogenous parameter for the expenditure share on housing service. Homogeneous goods are freely tradeable across locations. Housing service is not tradeable across locations. Homogeneous goods are the numeraire in each period. Let r^i_t be the unit rent in location i and period t. Then for age $a=1,\cdots,\bar{a}$, individuals of (r,a,t,i) are subject to the following budget constraint

$$G^i_{r,a,t} + r^i_t H^i_{r,a,t} \leq w^i_{r,c,t}, \tag{4}$$

where $w^i_{r,c,t}$ is the nominal wage of the individuals of (r,a,t,i). Since the amenities $B^i_{r,a,t}$

[10]Specifically, $s_{r,a,c+a} > 0$ for any $a=0,1,\cdots,\bar{a}-1$.

are exogenous, the maximization of the period utility (2) (or, equivalently, (3)) subject to the budget constraint (4) yields the demand functions for homogeneous goods and housing service $G^i_{r,a,t} = (1-\gamma)w^i_{r,a,t}$ and $H^i_{r,a,t} = \gamma w^i_{r,a,t}/r^i_t$. Substituting these demands into the composite (3), the consumption level is equalized to the real wage

$$C^i_{r,a,t} = \frac{w_{r,a,t}}{(r^i_t)^\gamma}.$$

Substituting this into the period utility yields the indirect period utility

$$\bar{u}^i_{r,a,t} = \begin{cases} 0 & \text{for } a = 0, \\ \log\left(\frac{w_{r,a,t}}{(r^i_t)^\gamma}\right) + \log B^i_{r,a,t} & \text{for } a = 1, \cdots, \bar{a}. \end{cases}$$

3.3 Values

Individuals of age 0 to $\bar{a}-1$ make migration decisions, and arrive in destinations next period. Individuals of age $\bar{a}$ do not make migration decisions because they are not alive next period. The value of individuals of (r,a,t,i), $v^i_{r,a,t}$, is

$$v^i_{r,a,t} = \begin{cases} \bar{u}^i_{r,a,t} + \max_{j\in\mathcal{N}} \left\{ s_{r,a,t} E[v^j_{r,a+1,t+1}] - \tau^{j,i}_{r,a,t} + \nu\epsilon^j_{r,a,t} \right\} & \text{for } a = 0, \cdots, \bar{a}-1, \\ \bar{u}^i_{r,a,t} & \text{for } a = \bar{a}. \end{cases}$$

where $\tau^{j,i}_{r,a,t}$ is the migration cost for individuals of race r and age a in period t from location i to location j, $\epsilon^j_{r,a,t}$ is the idiosyncratic preference shock, ν adjusts the variance of the idiosyncratic preference shock. The expectation is taken over the next period's idiosyncratic preference shocks $\epsilon^k_{r,a+1,t+1}$ for $k \in \mathcal{N}$.

Assume that the idiosyncratic preference shock $\epsilon^j_{r,a,t}$ is independently and identically distributed Type-I extreme value distribution $F(x) = \exp(-\exp(x))$ across all infinitesimal individuals. Then the expected value of workers of (r,a,t,i), $V^i_{r,a,t} = E[v^i_{r,a,t}]$, is

$$V^i_{r,a,t} = \begin{cases} \bar{u}^i_{r,a,t} + \nu \log\left(\sum_{j\in\mathcal{N}} \exp(s_{r,s,t} V^j_{r,a+1,t+1} - \tau^{j,i}_{r,a,t})^{1/\nu}\right) & \text{for } a = 0, \ldots, \bar{a}-1, \\ \bar{u}^i_{r,a,t} & \text{for } a = \bar{a}. \end{cases} \tag{5}$$

3.4 Migration

For age $a = 0, \cdots, \bar{a}-1$, the fraction of individuals of (r, a, t, i) who migrate to location j, $\mu^{j,i}_{r,a,t}$, is

$$\mu^{j,i}_{r,a,t} = \frac{\exp(s_{r,a,t} V^{j}_{r,a+1,t+1} - \tau^{j,i}_{r,a,t})^{1/\nu}}{\sum_{k \in \mathcal{N}} \exp\left(s_{r,a,t} V^{k}_{r,a+1,t+1} - \tau^{k,i}_{r,a,t}\right)^{1/\nu}}. \tag{6}$$

I call $\mu^{j,i}_{r,a,t}$ the migration share.

3.5 Populations

Then for $a = 1, \cdots, \bar{a}$, the population of (r, a, t, i) is

$$L^{i}_{r,a,t} = \sum_{j \in \mathcal{N}} \mu^{j,i}_{r,a-1,t-1} s_{r,a-1,t-1} L^{j}_{r,a-1,t} + I^{i}_{r,a,t}, \tag{7}$$

where $I^{i}_{r,a,t}$ denotes the number of immigrants of race r and age a who arrive from abroad in location i in period t. Individuals of age 0 are born according to

$$L^{i}_{r,0,t} = \sum_{a=1,\cdots,\bar{a}} \alpha^{i}_{r,a,t} L^{i}_{r,a,t}, \tag{8}$$

where, as before, the second subscript of $L^{i}_{r,0,t}$ on the left-hand side denotes age (0), and $\alpha^{i}_{r,a,t}$ denotes the exogenous parameter of how many individuals of age 0 are born per person of (r, a, t, i).

3.6 Firms and Wages

A representative firm exists in each location. The firm sells homogeneous goods in the competitive goods market and hires individuals of various races and ages from its location. The production function of the firm in location i is

$$Y^{i}_{t} = A^{i}_{t} L^{i}_{t}, \tag{9}$$

where A^{i}_{t} is the parameter of the productivity in location i and period t, and L^{i}_{t} is the labor input in location i and period i. L^{i}_{t} has the nested CES structure. At the outer nest, L^{i}_{t} aggregates labor of different age groups within period t and location i

$$L^{i}_{t} = \left(\sum_{a=1}^{\bar{a}} (\kappa^{i}_{a,t})^{\frac{1}{\sigma_a}} (L^{i}_{a,t})^{\frac{\sigma_a - 1}{\sigma_a}} \right)^{\frac{\sigma_a}{\sigma_a - 1}}, \tag{10}$$

where $\kappa^i_{a,t}$ is the parameter of the productivity of individuals of age a, period t, and location i, and σ_a is parameter of the elasticity of substitution across age groups within location-period bins. Then $L^i_{a,t}$, in turn, aggregates labor of different racial groups within age a, period t and location i

$$L^i_{a,t} = \left(\sum_{r \in \{b,o\}} (\kappa^i_{r,a,t})^{\frac{1}{\sigma_r}} (L^i_{r,a,t})^{\frac{\sigma_r - 1}{\sigma_r}} \right)^{\frac{\sigma_r}{\sigma_r - 1}}, \tag{11}$$

where $\kappa^i_{r,a,t}$ is the parameter of the productivity of individuals of race r and age a in period t, and location i, and σ_r is the parameter of the elasticity of substitution across races within age-period-location bins. This production function is similar to, but different from Boustan (2009). She controls for education, but I do not. She considers the single producer in the entire North, whereas I consider different producers in different geographic locaitons.

The firm in location i solves the following profit maximization problem

$$\max_{\{L^i_{r,a,t}\}_{r,a}} A^i_t L^i_t - \sum_a \sum_r w^i_{r,a,t} L^i_{r,a,t}.$$

The first-order conditions imply that wages are priced at the marginal product of labor

$$\begin{aligned} w^i_{r,a,t} &= A^i_t \frac{\partial L^i_t}{\partial L^i_{a,t}} \frac{\partial L^i_{a,t}}{\partial L^i_{r,a,t}} \\ &= A^i_t (L^i_t)^{\frac{1}{\sigma_a}} (\kappa^i_{a,t})^{\frac{1}{\sigma_a}} (L^i_{a,t})^{-\frac{1}{\sigma_a} + \frac{1}{\sigma_r}} (\kappa^i_{r,a,t})^{\frac{1}{\sigma_r}} (L^i_{r,a,t})^{-\frac{1}{\sigma_r}}. \end{aligned} \tag{12}$$

Note that migration decisions are made one period ahead, so L^i_t, $L^i_{a,t}$, $L^i_{r,a,t}$ are all predetermined from the viewpoint in period t.

3.7 Rent

Let H^i_t be the quantity of housing service in location i and period t. Then the housing market clearing condition is

$$r^i_t H^i_t = \gamma \sum_{r \in \{b,o\}} \sum_{a=1}^{\bar{a}} L^i_{r,a,t} w^i_{r,a,t}, \tag{13}$$

where the right-hand side is the Cobb-Douglas expenditure share on housing service γ multiplied by the total income in location i and period t, that is, the total housing expenditure in location i and period t. I assume that the quantity of housing service H^i_t is determined by the housing supply function

$$H_t^i = \frac{1}{\bar{r}^i}\left(\gamma \sum_{r\in\{b,o\}} \sum_{a=1}^{\bar{a}} L_{r,a,t}^i w_{r,a,t}^i\right)^{1-\eta},$$

where the inverse of $\bar{r}^i$ is the exogenous time-invariant and location-specific housing supply shifter, and η is the exogenous parameter governing the elasticity of housing service with respect to local housing expenditure. Substituting this into (13) yields

$$r_t^i = \bar{r}^i\left(\gamma \sum_{r\in\{b,o\}} \sum_{a=1}^{\bar{a}} L_{r,a,t}^i w_{r,a,t}^i\right)^{\eta}. \tag{14}$$

Rent r_t^i is decomposed into $\bar{r}^i$ and the power function of the local housing expenditure. I call $\bar{r}^i$ the location-specific rent shifter. Because $\eta = d\log r_t^i / d\log\left(\sum_{r\in\{b,o\}}\sum_{a=1}^{\bar{a}} L_{r,a,t}^i w_{r,a,t}^i\right)$ holds, I call η as the rent elasticity with respect to local income, or simply the rent elasticity.

3.8 Equilibrium and Steady State

Now I am equipped with all equilibrium conditions.

Equilibrium. Given populations in period 0 $\{L_{r,a,0}^i\}_{r,a}^i$, an equilibrium is a tuple of expected values $\{V_{r,a,t}^i\}_{r,a,t=0,1,\ldots}^i$, wages $\{w_{r,a,t}^i\}_{r,c,t=0,1,\ldots}^i$, populations $\{L_{r,a,t}^i\}_{r,a,t=1,2,\ldots}^i$, migration shares $\{\mu_{r,a,t}^{j,i}\}_{r,a,t=0,1,\ldots}^{j,i}$, housing rent $\{r_t^i\}_{t=0,1,\ldots}^i$ that satisfies (5), (6), (7), (8), (12), and (14).

I compute transition paths to steady states, given the initial populations of all demographic groups in all locations. For this purpose, I characterize steady states. Recall that given race r, individuals of age a in period t do not necessarily behave in the same way as individuals of age a in period $t' \neq t$. However, in steady state, within race r, individuals of age a behave in exactly the same way period by period.

Steady state. A steady state is a tuple of time-invariant variables: expected values $\{V_{r,a}^i\}_{r,a}^i$, wages $\{w_{r,a}^i\}_{r,a}^i$, populations $\{L_{r,a}^i\}_{r,a}^i$, migration shares $\{\mu_{r,a}^{j,i}\}_{r,a}^{j,i}$, housing rent $\{r^i\}^i$ satisfying (5), (6), (7), (8), (12), and (14), dropping time subscripts in all equations.

4 Quantification

I take the model to US data from 1940 to 2010. The geographic units are 36 states including all confederate and border states, the District of Columbia, and the constructed rest of the North. In total, there are 38 locations in the sample. The rest of the North aggregates states with less than 5,000 Black population as of 1940.[11] The rest of the North accounts for 0.1 and 1 percent of the Black population in the US as of 1940 and 2010, respectively. One period is ten years.

I estimate a set of parameters which I call elasticities: migration elasticity $1/\nu$, elasticities of substitution σ_a and σ_r, and rent elasticity η. The model delivers explicit formulas mapping elasticities and relevant data to amenities, migration costs, productivity, and location-specific rent shifters. Specifically, with relevant data, migration elasticity $1/\nu$ pins down amenities $B^i_{r,a,t}$ and migration costs $\tau^{j,i}_{r,a,t}$. Elasticities of substitution σ_a and σ_r pin down productivity at various levels of aggregation A^i_t, $\kappa^i_{a,t}$, $\kappa^i_{r,a,t}$. Rent elasticity η pins down location-specific rent shifters $\bar{r}^i_t$. Subsection 4.4 touches on survival probabilities, fertility, and immigrants from abroad. As in Ahlfeldt et al. (2015), I set the Cobb-Douglas share on housing service $\gamma = 0.25$ following Davis and Ortalo-Magné (2011). Recall that the mover-stayer rent gap was about one-fourth of the mover-stayer wage gap in Section 2.

The main data source is the US census from 1940 to 2000 and the American Community Survey (ACS) from 2001 to 2019, both of which are tabulated in IPUMS (Ruggles et al., 2022; Manson et al., 2022). Migration shares, populations, wages, and fertility (babies per person) for race and age bins in states and time periods are from these data. Median rent across states from 1940 to 2010 is published by the US Census Bureau or IPUMS. All prices (wages and rent) are deflated by the consumer price index and measured in the 2010 US dollars. I use payrolls per capita as wages and head counts as populations for race, age, location, and time tuples. See Appendix B for further details on the data sources.

Table 2: Age Bins in the Model and in the Data

model	0	1	$\cdots$	$\bar{a} = 6$
data	1-10	11-20	$\cdots$	61-70

Notes: The first row lists age bins used in the model. The second row lists the corresponding age bins in the data.

Since US censuses are decennial, one period in the model corresponds to ten years in the data. Accordingly, age bins in the model correspond to 10-year windows as in Table 2.

[11] The rest of the North consists of Idaho, Maine, Montana, Nevada, New Hampshire, New Mexico, North Dakota, Oregon, South Dakota, Utah, Vermont, and Wyoming.

In the quantification of the model, ages run from 0 to 6, so the maximum length of life is 7 periods. Age 0 in the model corresponds to age 1 to 10 in the data, $\cdots$, age $\bar{a} = 6$ in the model corresponds to age 61 to 70 in the data.

4.1 Migration Elasticity, Migration Costs, and Amenities

I estimate migration elasticity $1/\nu$, following the two-step estimation developed by Artuc and McLaren (2015). Artuc and McLaren (2015) used this method to study sectoral and occupational choices by workers, and Caliendo et al. (2021) applied it to the context of migration.

Define the option value of race r, age a, period t (hereafter (r,a,t)), in location j, $\Omega^j_{r,a,t}$, by

$$\Omega^j_{r,a,t} = \nu \log\left(\sum_{j\in\mathcal{N}} \exp(s_{r,a,t} V^j_{r,a+1,t+1} - \tau^{j,i}_{r,a,t})^{1/\nu}\right).$$

Then the expected value of (r,a,t) in location j (5) is rewritten as

$$V^j_{r,a,t} = \bar{u}^j_{r,a,t} + \Omega^j_{r,a,t}. \tag{15}$$

The migration rate of (r,a,t) from location i to j (6) is also rewritten as

$$\mu^{j,i}_{r,a,t} = \exp\left\{\frac{1}{\nu}(s_{r,a,t} V^j_{r,a+1,t+1} - \tau^{j,i}_{r,a,t}) - \frac{1}{\nu}\Omega^i_{r,a,t}\right\} \tag{16}$$

Multiplying both sides by the population of (r,a,t) in origin location i, $L^i_{r,a,t}$, I have the number of migrants

$$L^i_{r,a,t}\mu^{j,i}_{r,a,t} = \exp\left\{\frac{1}{\nu}(s_{r,a,t} V^j_{r,a+1,t+1} - \tau^{j,i}_{r,a,t}) - \frac{1}{\nu}\Omega^i_{r,a,t} + \log(L^i_{r,a,t})\right\}$$

I decompose the number of migrants into destination fixed effects $v^j_{r,a,t}$, origin fixed effects $\omega^i_{r,a,t}$, and the remaining variation $\tilde{\tau}^{j,i}_{r,a,t}$ by race r, age a and period t

$$L^i_{r,a,t}\mu^{j,i}_{r,a,t} = \exp\{v^j_{r,a,t} + \omega^i_{r,a,t} + \tilde{\tau}^{j,i}_{r,a,t}\}. \tag{17}$$

Comparing equations (16) and (17), I have

$$v_{r,a,t}^{j} = \frac{1}{\nu} s_{r,a,t} V_{r,a+1,t+1}^{j}, \tag{18}$$

$$\omega_{r,a,t}^{i} = -\frac{1}{\nu} \Omega_{r,a,t}^{i} + \log(L_{r,a,t}^{i}), \tag{19}$$

$$\tilde{\tau}_{r,a,t}^{j,i} = -\frac{1}{\nu} \tau_{r,a,t}^{j,i}. \tag{20}$$

Note that $v_{r,a,t}^{j}$, $\omega_{r,a,t}^{i}$, and $\tilde{\tau}_{r,a,t}^{j,i}$ capture expected values, option value, and migration costs, respectively. Equations (15), (18), and (19) imply

$$\begin{aligned} \frac{v_{r,a,t}^{j}}{s_{r,a,t}} + \omega_{r,a+1,t+1}^{j} - \log(L_{r,a+1,t+1}^{j}) &= \frac{1}{\nu} \bar{u}_{r,a,t}^{j} \\ &= \frac{1}{\nu} \left\{ \log\left(\frac{w_{r,a+1,t+1}^{j}}{(r_{t+1}^{j})^{\gamma}} \right) + \log(B_{r,a+1,t+1}^{j}) \right\}. \end{aligned} \tag{21}$$

That is, regression migration shares on origin and destination fixed effects recovers period utilities for each (r,a,t) in location j.

Guided by the derivations so far, I do the two step estimation. Following equation (17), in the first step, I run the following regression

$$L_{r,a,t}^{i} \mu_{r,a,t}^{j,i} = \exp\left\{ v_{r,a,t}^{j} + \omega_{r,a,t}^{i} + \tilde{\tau}_{t}^{j\neq i} + \tilde{\tau}_{r,G(t)}^{\{i,j\}} + \tilde{\tau}_{a,G(t)}^{\{i,j\}} \right\} + \epsilon_{r,a,t}^{j,i}. \tag{22}$$

For race r, age a, and period t, $v_{r,a,t}^{j}$ is the destination fixed effect, and $\omega_{r,a,t}^{i}$ is the origin fixed effect. The terms $\tilde{\tau}$ with various subscripts and superscripts aim to capture migration costs. $\tilde{\tau}_{t}^{j\neq i}$ denotes the fixed effect for year t and moving, that is, destination j is a different location from origin i. The sample years are decennial: $1930, 1940, \cdots, 2000, 2010$.[12] In the subscripts of $\tilde{\tau}_{r,G(t)}^{\{i,j\}}$ and $\tilde{\tau}_{a,G(t)}^{\{i,j\}}$, function $G(\cdot)$ groups years as in Table 3. I call partitions of years defined by G as year groups. In the superscripts of $\tilde{\tau}_{r,G(t)}^{\{i,j\}}$ and $\tilde{\tau}_{a,G(t)}^{\{i,j\}}$, $\{i,j\}$ represents the unordered pair of locations i and j.[13] Thus $\tilde{\tau}_{r,G(t)}^{\{i,j\}}$ is the race-year group-location pair fixed effect, and $\tilde{\tau}_{a,G(t)}^{\{i,j\}}$ is the age-year group-location pair fixed effect. Finally, $\epsilon_{r,a,t}^{j,i}$ is the error term. Notice that I assume symmetric migration costs.

Since one period is 10 years, migration shares in the regression (22) must be of 10-year windows. The data from the US censuses and ACS, however, migration in 1- or 5-year

[12]The migration shares in 1930 are necessary to compute amenities in 1940. See time subscripts in equation (21).

[13]More precisely, for locations $i \neq j$, I assume $\tilde{\tau}_{r,G(t)}^{\{i,i\}} = \tilde{\tau}_{r,G(t)}^{\{j,j\}}$ and $\tilde{\tau}_{a,G(t)}^{\{i,i\}} = \tilde{\tau}_{a,G(t)}^{\{j,j\}}$. Thus I have the fixed effects for staying and all unordered pairs of different locations.

windows. Appendix C details how I map 1- or 5-year migration in the data to 10-year migration in the quantification of the model.

Table 3: Grouping Sample Years

year	1930	1940	1950	1960	1970	1980	1990	2000	2010
group	1		2		3		4		

Notes: This table defines function G used in equation (22).

Suppose that I obtain estimates of destination fixed effects $\hat{v}^j_{r,a,t}$ and fixed effects $\hat{\omega}^i_{r,a,t}$ by race r, age a, ant time t from the first step. In the second step, guided by equation (21), I run the regression (which corresponds to column (3) in Table 4)

$$\frac{\hat{v}^j_{r,a,t}}{s_{r,a,t}} + \hat{\omega}^j_{r,a+1,t+1} - \log(L^j_{r,a+1,t+1}) = \frac{1}{\nu}\log(w^j_{r,a+1,t+1}) + \tilde{B}^j_{r,a+1} + \tilde{B}^j_{r,t+1} + \epsilon^j_{r,a,t}, \tag{23}$$

where $\log(w^j_{r,a+1,t+1})$ is the log of the nominal wage of individuals of $(r, a+1, t+1)$ in location j, $\tilde{B}^j_{r,a+1}$ is the race-age-location fixed effect, $\tilde{B}^j_{r,t+1}$ is the race-year-location fixed effect, and $\epsilon^j_{r,a,t}$ is the error term. $\tilde{B}^j_{r,a+1}$ and $\tilde{B}^j_{r,t+1}$ are to control for the amenities $B^j_{r,a+1,t+1}$. Since rent r^j_{t+1} in equation (21) varies at location-time levels, it is absorbed by the fixed effect $\tilde{B}^j_{r,t+1}$.

Nominal wages is directly from the data. Following Artuc and McLaren (2015), I instrument the log of the nominal wage for individuals of (r, a+1, t+1) in location j, $\log(w^j_{r,a+1,t+1})$, by the log of the nominal wage for individuals of $(r, a+1, t)$ in location j, $\log(w^j_{r,a+1,t})$. Since one period is ten years, I instrument the nominal wage of each race-age-year-location quadruple by the nominal wage of the same race-age-location triple but ten years before.

Table 4 reports the results of the second step. Column (3) reports the result of specification (23), and the other columns report the results of the specifications with fewer fixed effects. The estimates for the migration elasticity ranges from 0.50 to 0.77, which are between 0.5 estimated by Caliendo et al. (2021) for EU countries and 2.0 estimated by Suzuki (2021) for Japanese prefectures. I use 0.77 as the value of the migration elasticity.

Given $\hat{\nu}$, I can back out migration costs. Recall that in the first step (22), I have estimated the fixed effects $\tilde{\tau}^{j\neq i}_t$, $\tilde{\tau}^{\{i,j\}}_{r,G(t)}$, and $\tilde{\tau}^{\{i,j\}}_{a,G(t)}$. Let $\hat{\tilde{\tau}}^{j\neq i}_t$, $\hat{\tilde{\tau}}^{\{i,j\}}_{r,G(t)}$, and $\hat{\tilde{\tau}}^{\{i,j\}}_{a,G(t)}$ be the estimates of these fixed effects. Then by equation (20), I obtain migration costs induced by these estimates of the fixed effects,

$$\hat{\tau}^{j,i}_{r,a,t} = -\hat{\nu}\left(\hat{\tilde{\tau}}^{j\neq i}_t + \hat{\tilde{\tau}}^{\{i,j\}}_{r,G(t)} + \hat{\tilde{\tau}}^{\{i,j\}}_{a,G(t)}\right).$$

Table 4: Migration Elasticity

Dependent variable:	period utility × migration elasticity		
	(1)	(2)	(3)
log(real wage)	0.4976***	0.6129***	0.7676***
	(0.1323)	(0.1665)	(0.1952)
fixed effects:			
race-location	✓	✓	✓
age-location	✓	✓	✓
year-location	✓	✓	✓
age-race	✓	✓	✓
year-race	✓	✓	✓
age-race-location		✓	✓
year-race-location			✓
Observations	2,660	2,660	2,660

Notes: The second step of the migration elasticity estimation. Dependent variables are constructed with the estimates from the first step and represent period utilities multiplied by the migration elasticity. Units of observations are year-age-race-location tuples. Robust standard errors clustered at locations are in parentheses. Significance codes: ***: 0.01, **: 0.05, *: 0.1.

Figure 4 shows the averages of the induced migration costs for races and ages. 20 in the horizontal axis refers to age bin 21-30, and so on.[14] The migration costs are the lowest for people of the ages 20s. Migration costs increase as people age after the 20s. The migration costs in 2010 increase with ages less steeply than the migration costs in 1940 do. This is perhaps because seniors are more physically mobile or infrastructure became better in 2010. African Americans faced higher migration costs than others in 1940, but the racial gap in migration costs shrunk by 2010.

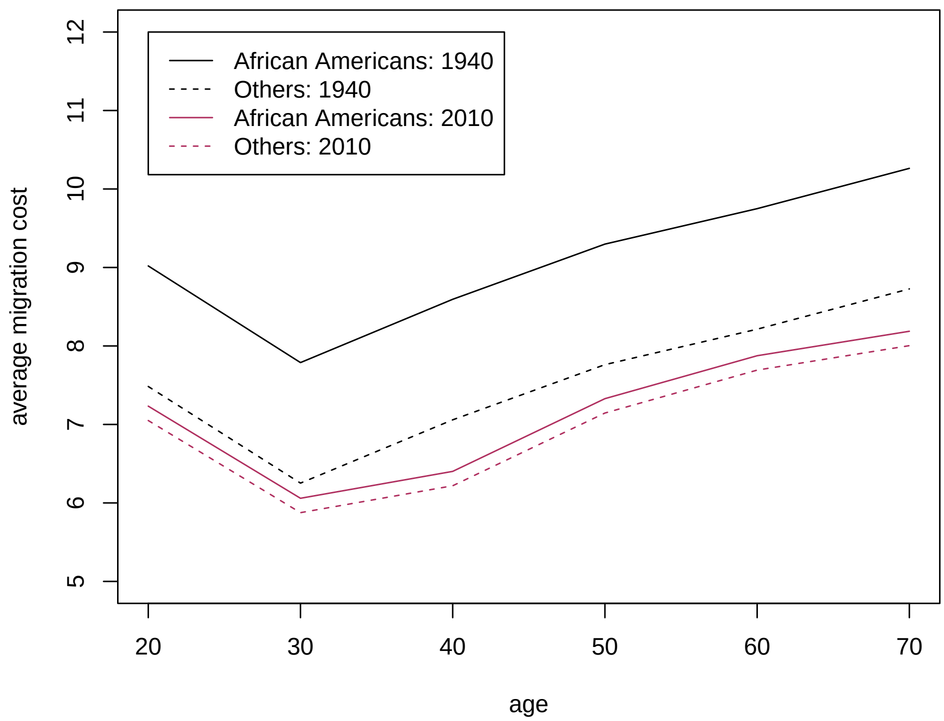
Figure 5 illustrates the averages of the induced migration costs for races and years. The migration costs of African Americans were always higher than those of others. The migration costs, however, have steadily declined over the sample periods particularly for African Americans. The racial gap in the migration costs declined over time.

I back out amenities using $\hat{\nu}$ and the fixed effects estimated in the second step (23). Let $\hat{\hat{B}}^j_{r,a+1}$ and $\hat{\hat{B}}^j_{r,t+1}$ be the estimates of the fixed effects in the second step (23). Then by comparing equations (21) and (23), the induced amenities, $\hat{B}_{r,a,t}$, are

$$\hat{B}^j_{r,a,t} = \exp\left\{\hat{\nu}\left(\hat{\hat{B}}^j_{r,a} + \hat{\hat{B}}^j_{r,t}\right) + \gamma \log(r^j_t)\right\}, \tag{24}$$

[14] Age $x \in \{20, \cdots, 70\}$ refers to ages from $x-9$ to x on the horizontal axis.

Figure 4: Average Migration Costs for Races and Ages: 1940 and 2010



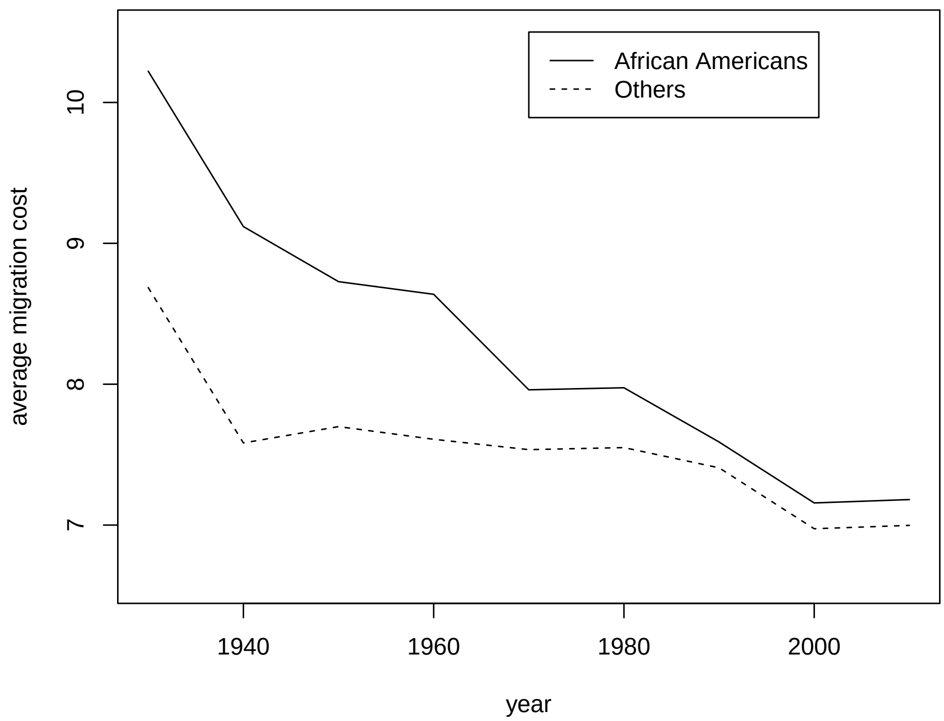
Notes: For races, ages, and years, I compute the averages of the induced migration costs across location pairs. The migration costs are induced by the estimate of the migration elasticity and the fixed effects in equation (22).

where rent r_t^j is directly from the data, and I set $\gamma = 0.25$. I normalize amenities $\{\hat{B}_{r,a,t}^j\}^j$ so that the mean of $\{\hat{B}_{r,a,t}^j\}^j$ is 1 for each (r, a, t). In the model, migration decisions are made in period t foreseeing real wages and amenities in period $t+1$. I have wages and rent data from 1940 to 2019. So I can compute the induced amenities for years 1950 to 2010 in this way. The reason that I cannot obtain the amenities in 1940 is that I do not have data of wages as of 1930, so I do not have the lagged instrumental variable for wages. For 1940, I directly back out the amenities using equation (21)

$$\hat{B}_{r,a,1940}^j = \exp\left\{\nu\left(\frac{\hat{\nu}_{r,a-1,1930}^j}{s_{r,a-1,1930}} + \hat{\omega}_{r,a,1940}^j - \log(L_{r,a,1940})\right)\right\} \Big/ \left\{\frac{w_{r,a,1940}^j}{(r_{1940}^j)^\gamma}\right\}.$$

Recall that in Table 2, the age 6 is the highest age and corresponds to the ages 61 to 70 in the data. The regression (23) does not yield information about amenities for the age

Figure 5: Average Migration Costs for Races and Years

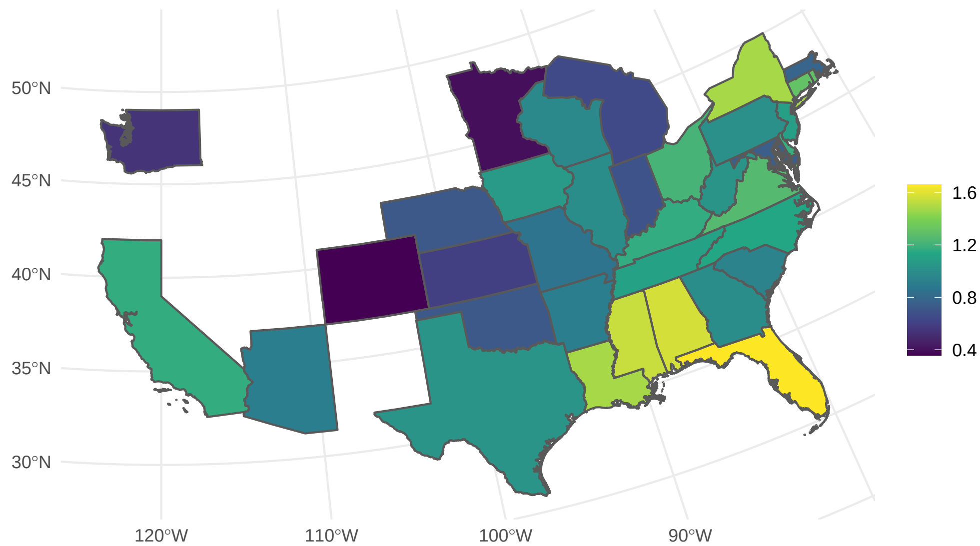


Notes: For races and years, I compute the averages of the induced migration costs across (ordered) location pairs and ages. The migration costs are induced by the estimate of the migration elasticity and the fixed effects in equation (22).

6, because the origin fixed effect for the age 7 $\hat{\omega}^{j}_{r,7,t+1}$ is needed to run this regression. I do not include the age 7 (71-80 year olds) in the sample because including them in the estimation makes estimates of the migration elasticity unstable. I assume that within race r, location i, and period t, amenities for the age 6 (61-70 year olds) are the same as amenities for the age 5 (51-60 year olds). I use the estimates for amenities for the age 5 for amenities for the age 6.

Figures 6 and 7 show the induced amenities for African Americans and others for states in 1960 averaged across age bins. The peak of the great migration was in the 1950s, and in the model, individuals make migration decision in 1950 foreseeing real wages and amenities from 1960 onward. The amenities of the rest of the North are not in the figures, although they are assigned in the quantification of the model. As in Figure 6, the induced amenities for African Americans were high in states in the South such as Florida, Alabama, Mississippi, and Louisiana in 1960. In contrast, states in the North such as Michigan,

Figure 6: Amenities for African Americans in 1960



Notes: The amenities for African Americans in 1960 averaged across ages. The amenities are induced by equation (24). The rest of the North is excluded from the map.

Minnesota, Kansas, Colorado, and Washington had low amenities for African Americans then. Figure 7 shows a different geographic pattern of amenities for others. California provided high amenities for them, but there is not a clear North-South pattern in the amenities for others.

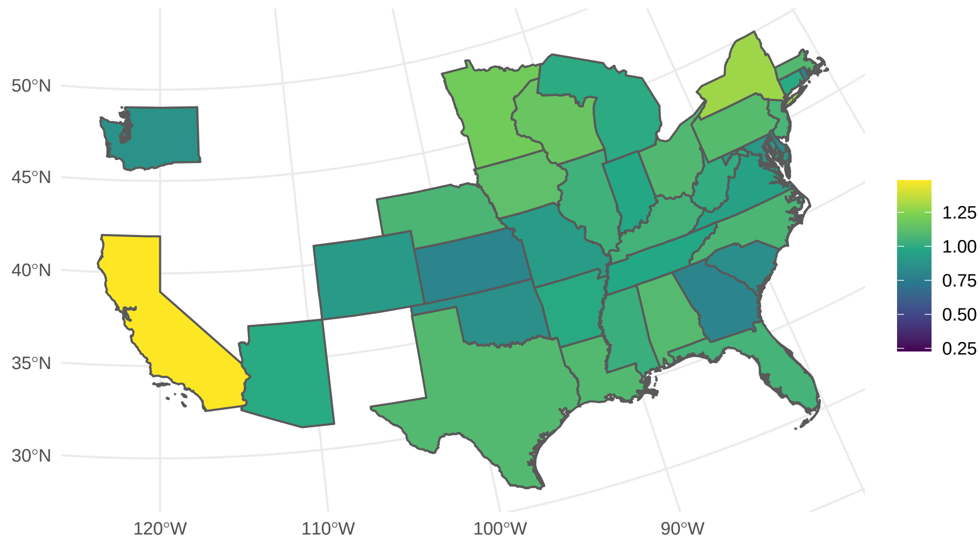
4.2 Elasticity of Substitution and Productivity

4.2.1 Races

I turn to the estimation of the elasticities of substitution. I start with the elasticity of substitution across races within age, locaiton, and time bins σ_r. Equation (12) implies

$$\frac{w^n_{b,a,t}}{w^n_{o,a,t}} = \frac{(\kappa^n_{b,a,t})^{\frac{1}{\sigma_r}}(L^n_{b,a,t})^{-\frac{1}{\sigma_r}}}{(\kappa^n_{o,a,t})^{\frac{1}{\sigma_r}}(L^n_{o,a,t})^{-\frac{1}{\sigma_r}}}, \tag{25}$$

Figure 7: Amenities for Others in 1960



Notes: The amenities for others in 1960 averaged across ages. The amenities are induced by equation (24). The rest of the North is excluded from the map.

where I recall that the first subscripts b and o denote African Americans and others respectively. Taking logs of both sides,

$$\log\left(\frac{w^n_{b,a,t}}{w^n_{o,a,t}}\right) = -\frac{1}{\sigma_r}\log\left(\frac{L^n_{b,a,t}}{L^n_{o,a,t}}\right) + \frac{1}{\sigma_r}\log\left(\frac{\kappa^n_{b,a,t}}{\kappa^n_{o,a,t}}\right). \tag{26}$$

Since productivity ratio between races $\kappa^n_{b,a,t}/\kappa^n_{o,a,t}$ is not observable in data, my main econometric specification is

$$\log\left(\frac{w^n_{b,a,t}}{w^n_{o,a,t}}\right) = -\frac{1}{\sigma_r}\log\left(\frac{L^n_{b,a,t}}{L^n_{o,a,t}}\right) + f_a + f_t + f_{a,t} + \epsilon^n_{a,t}, \tag{27}$$

where f_a denotes the age fixed effect, f_t denotes the time fixed effect, $f_{a,t}$ denotes the age-time fixed effect, and $\epsilon^n_{a,t}$ is the error term. Notice that age-time fixed effects $f_{a,t}$ capture cohorts. For earlier cohorts, the education gap between African Americans and others were higher. As the education gap is a reason of the productivity gap between races, controlling for cohorts is important in the regression (27).

In his seminal work, Borjas (2003) considers the nation-wide labor market. But, here

I consider different locations in the US. Suppose that African Americans migrate to a location where their productivity is high relative to the people of the other races within age bins. Then productivity ratio $\kappa^n_{b,a,t}/\kappa^n_{o,a,t}$ is positively correlated with population ratio $L^n_{b,a,t}/L^n_{o,a,t}$, which causes an upward bias for the estimator of $-1/\sigma_r$ in ordinary least squares (OLS). Note that the concern here is that productivity ratio $\kappa^n_{b,a,t}/\kappa^n_{o,a,t}$ may work as a pull factor of migration.

To deal with this potential bias, I follow Card (2009). In Appendix D, I pursue a different approach following the first difference estimation of Monras (2020). Here I consider two instrumental variables. The first one is the ratio of shift-share predicted populations. The shift-share predicted population of race r, age a, period t, and location n is

$$\hat{L}^n_{r,a,t} = \sum_{j\in\mathcal{N}} \mu^{n,j}_{r,a-1,t-1-X} \cdot s_{r,a-1,t-1} L^j_{r,a-1,t-1}. \tag{28}$$

If $X = 1$, this equation would be exactly the same as equation (7). But I use the value of $X > 1$. That is, I interact the current (actually one period before) survival probability and populations with the old-time migration shares to make shift-share predicted populations. The shift-share predicted populations aim to tease out the push factors of migration. I assume that neither the old-time migration shares $\mu^{n,j}_{r,a-1,t-1-X}$ nor the last period population $L^j_{r,a-1,t-1}$ would react to shocks to the current productivity ratio between races $\kappa^n_{b,a,t}/\kappa^n_{o,a,t}$. If so, the shift-share predicted populations are orthogonal to the current productivity ratios between races which may work as a pull factor of migration to destination n. The relevance (correlation with the actual population ratio) of this IV hinges on the so-called network effect of migration; migrants from an origin tend to go to a destination to which their precursors went.

However, on the right-hand side of equation (28), $L^n_{r,a-1,t-1}$, population in destination n itself in the previous period, is included, and it may be correlated with the current productivity ratio $\kappa^n_{b,a,t}/\kappa^n_{o,a,t}$. Thus I remove $L^n_{r,a-1,t-1}$ from the right-hand side. Then I obtain

$$\hat{L}^{n,-n}_{r,a,t} = \sum_{j\neq n} \mu^{n,j}_{r,a-1,t-1-X} \cdot s_{r,a-1,t-1} L^j_{r,a-1,t-1}. \tag{29}$$

The economic interpretation for this is shift-share predicted gross inflows, because the right-hand side collects inflows of people from all locations but n itself.

For either IV, I set $X = 2$. Since 1 period is 10 years, I use the migration shares 20 years before period $t-1$.

Table 5 provides the estimation results.[15] The first column shows the result of OLS. The second and third columns show the results of two-step least squares using the first

[15] Appendix E details the computation of standard errors.

Table 5: Elasticity of Substitution across Races: Level Estimation

Dependent variable:	$\log(w^n_{b,a,t}/w^n_{o,a,t})$		
Model:	OLS	IV 1	IV 2
$\log(L^n_{b,a,t}/L^n_{o,a,t})$	-0.1154***	-0.1108***	-0.1120***
	(0.0120)	(0.0127)	(0.0139)
fixed effects:			
year	✓	✓	✓
age	✓	✓	✓
year-age	✓	✓	✓
Observations	1,368	1,368	1,328
First-stage F-statistic		3,112.5	2,107.5

Notes: The results of the level estimation of the elasticity of substitution across races. Block bootstrap standard errors are in parentheses. See Appendix E for the computation of standard errors. Significance codes: ***: 0.01.

and second IVs (28) and (29), respectively. The OLS and the two IV estimation produce similar estimates around -0.11. From the columns 1, 2, and 3, let the OLS estimate, the first IV estimate, and the second IV estimate for σ_r be $\hat{\sigma}_r^{OLS} = 1/0.1154 = 8.67$, $\hat{\sigma}_r^{IV1} = 1/0.1108 = 9.02$, and $\hat{\sigma}_r^{IV2} = 1/0.1120 = 8.93$, respectively. In the quantification of the model, I use $\hat{\sigma}_r^{IV1}$ as the value of the elasticity of substitution across races.

How do my estimates for the elasticity of substitution across races compare with estimates in literature? From Table 5, the estimates of the elasticity of substitution across races range from 8.7 to 9.0. In Appendix D, the first difference estimation produces the estimate of 4.9. Boustan (2009) estimates the elasticity of substitution across races within education-experience bins in the entire US North. Her preferred value ranges from 8.3 to 11.1, which coincide with my estimates here. Her paper also includes an estimate of 5.4, which is somewhat similar to my estimate from the first difference estimation.

Given the estimate of elasticity of substitution across races $\hat{\sigma}_r$, I can back out race-specific productivity $\kappa^n_{r,a,t}$. Rearranging equation (25), I obtain

$$\frac{\hat{\kappa}^n_{b,a,t}}{\hat{\kappa}^n_{o,a,t}} = \left(\frac{w^n_{b,a,t}}{w^n_{o,a,t}}\right)^{\hat{\sigma}_r} \cdot \left(\frac{L^n_{b,a,t}}{L^n_{o,a,t}}\right). \tag{30}$$

Since wages $w^n_{r,a,t}$ and populations $L^n_{r,a,t}$ are directly observable for $r = b, o$, I can back out productivity ratio between African Americans and others $\hat{\kappa}^n_{b,a,t}/\hat{\kappa}^n_{o,a,t}$. Comparing equations (10) and (11), multiplying all $\kappa^n_{r,a,t}$ ($r = b, o$) by scalar $x > 0$ is equivalent to multiplying $\kappa^n_{a,t}$ by $x^{(\sigma_a-1)/(\sigma_r-1)}$ in the production function. Thus I normalize $\hat{\kappa}^n_{r,a,t}$ for $r = b, o$, so that

$\sum_{r=b,o} \hat{\kappa}^n_{r,a,t} = 1$. With equation (30), this normalization pins down $\hat{\kappa}^n_{r,a,t}$ for $r = b, o$.

4.2.2 Ages

Dual to age-lavel labor (11) in the production function, age-lavel wages within location-time bins are

$$w^i_{a,t} = \left(\sum_r \kappa^i_{r,a,t} (w^i_{r,a,t})^{1-\sigma_r} \right)^{\frac{1}{1-\sigma_r}}. \tag{31}$$

Then I obtain

$$\frac{w^n_{a,t}}{w^n_{a',t}} = \frac{(\kappa^n_{a,t})^{\frac{1}{\sigma_a}} (L^n_{a,t})^{-\frac{1}{\sigma_a}}}{(\kappa^n_{a,t})^{\frac{1}{\sigma_a}} (L^n_{a',t})^{-\frac{1}{\sigma_a}}}. \tag{32}$$

Taking logs of both sides of (32), I have

$$\log\left(\frac{w^n_{a,t}}{w^n_{a',t}}\right) = -\frac{1}{\sigma_a} \log\left(\frac{L^n_{a,t}}{L^n_{a',t}}\right) + \frac{1}{\sigma_a} \log\left(\frac{\kappa^n_{a,t}}{\kappa^n_{a',t}}\right).$$

Fix an age bin a'. For any age $a \neq a'$, the econometric specification is

$$\log\left(\frac{w^n_{a,t}}{w^n_{a',t}}\right) = -\frac{1}{\sigma_a} \log\left(\frac{L^n_{a,t}}{L^n_{a',t}}\right) + f_a + f_t + f_{a,t} + \epsilon^n_{a,t}, \tag{33}$$

where f_a is the age fixed effect, f_t is the time fixed effect, and $f_{a,t}$ is the age time fixed effect. Note that $w^n_{a,t}$ and $L^n_{a,t}$ are computed using $\hat{\sigma}_r$ and $\hat{\kappa}^n_{r,a,t}$ for $r = b, o$. A concern is that people of an age group migrate to a location where relative productivity of the age group is high, causing positive correlation between population ratios across ages $L^n_{a,t}/L^n_{a',t}$ and productivity ratios across ages $\kappa^n_{a,t}/\kappa^n_{a',t}$.

To deal with this endogeneity concern, I make use of shift-share predicted populations and gross inflows at race-age-location-time levels in equations (28) and (29). The first IV is the aggregate of the shift-share predicted populations

$$\hat{L}^n_{a,t} = \left[\sum_{r \in \{b,o\}} (\hat{\kappa}^n_{r,a,t})^{\frac{1}{\hat{\sigma}_r}} (\hat{L}^n_{r,a,t})^{\frac{\hat{\sigma}_r - 1}{\hat{\sigma}_r}} \right]^{\frac{\hat{\sigma}_r}{\hat{\sigma}_r - 1}}, \tag{34}$$

where $\hat{L}^n_{r,a,t}$ is the shift-share predicted populations defined in equation (28). The second IV is the aggregate of the shift-share predicted gross outflows

$$\hat{L}^{n,-n}_{a,t} = \left[\sum_{r \in \{b,o\}} (\hat{\kappa}^n_{r,a,t})^{\frac{1}{\hat{\sigma}_r}} (\hat{L}^{n,-n}_{r,a,t})^{\frac{\hat{\sigma}_r - 1}{\hat{\sigma}_r}} \right]^{\frac{\hat{\sigma}_r}{\hat{\sigma}_r - 1}}, \tag{35}$$

where $\hat{L}^{n,-n}_{r,a,t}$ is the shift-share predicted gross inflows defined in equation (29). In either case, I instrument population ratios across ages $L^n_{a,t}/L^n_{a',t}$ by the aggregates of shift-share predicted populations $\hat{L}^n_{a,t}/\hat{L}^n_{a',t}$ or gross inflows $\hat{L}^{n,-n}_{a,t}/\hat{L}^{n,-n}_{a',t}$.

Age-level wages $w^i_{a,t}$, age-level labor $L^i_{a,t}$, and instruments (34) and (35) are all constructed with the estimate of σ_r and the race-specific productivity. Therefore, to compute standard errors for estimates of σ_a, I need to take into account variability in the estimate of σ_r and the race-specific productivity. I compute block bootstrap standard errors to address this issue. See Appendix E for details.

Table 6 reports three estimation results for the elasticity of substitution across ages.[16] In the column 1, I use $\hat{\sigma}^{OLS}_r$ and its associated race-specific productivity to compute age-level wages $w^n_{a,t}$ and populations $L^n_{a,t}$. Here I use actual race-level populations $L^n_{r,a,t}$ to construct $L^n_{a,t}$ using equation (11). Likewise, in the columns 2 and 3, I use $\hat{\sigma}^{IV1}_r$ and $\hat{\sigma}^{IV2}_r$ and the race-specific productivity induced by these two estimates to construct age-level wages $w^n_{a,t}$ and populations $L^n_{a,t}$. The columns 2 and 3 use the aggregate of shift-share predicted populations (34) and the aggregate of shift-share predicted gross inflows (35), respectively. The two IVs seem to correct a positive bias in the OLS (column 1). In the quantification of the model, I use $1/0.3401 = 2.94$ as the value of the elasticity of substitution across ages. The induced elasticity of substitution across ages ranges from 1.8 to 3.5. They are lower than the estimates in prior literature in labor economics. Card and Lemieux (2001) and Ottaviano and Peri (2012) report estimates of 3.8-4.9 and 3.3-6.3 respectively for the US; Manacorda et al. (2012) provide estimates of 5.1-5.2 for the UK. There are three differences between these papers and mine. First, they consider the nation-wide labor market whereas I consider the state-level labor markets. Second, they control education levels, but I do not. Third, their age bins are five-year windows, whereas mine is of ten years, because wage and population data is from the decennial censuses. My age bins are twice as large as the bins in the literature, and it is possible that substitution across larger age bins exhibits a larger degree of imperfection.

4.2.3 Locations

Dual to equation (10), location-time-level wage w^i_t is the aggregate of age-location-time=level wage $w^i_{a,t}$

$$w^i_t = \left(\sum_{a=1}^{\bar{a}} \kappa^i_{a,t} (w^i_{a,t})^{1-\sigma_a} \right)^{\frac{1}{1-\sigma_a}}.$$

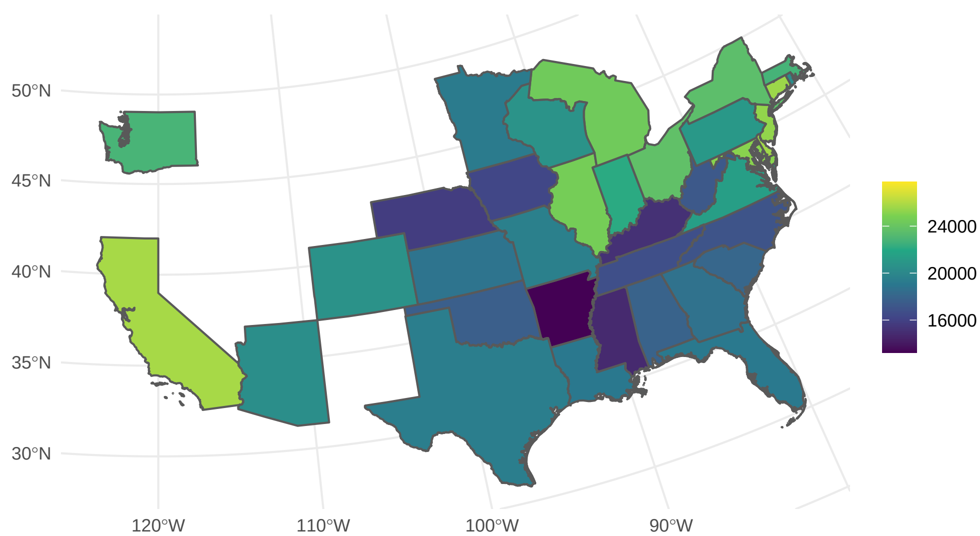
[16]See Appendix E for the computation of standard errors.

Table 6: Elasticity of Substitution across Ages

Dependent variable:	$\log(w^n_{a,t}/w^n_{a',t})$		
Model:	OLS	IV 1	IV 2
$\log(L^n_{a,t}/L^n_{a',t})$	-0.2849***	-0.3401*	-0.5429***
	(0.0672)	(0.1922)	(0.1579)
fixed effects:			
year	✓	✓	✓
age	✓	✓	✓
year-age	✓	✓	✓
Observations	1,520	1,140	1,140
1st-stage F-statistic		1,247.3	186.4

Notes: The estimates of the elasticity of substitution across ages. Block bootstrap standard errors are in parentheses. See Appendix E for the computation of standard errors. Significance codes: ***: 0.01, **: 0.05, *: 0.1.

Figure 8: Productivity in 1960



Notes: The induced location-level productivity $\hat{A}^i_t$ across states in 1960.

In equilibrium, the representative firm in location i makes zero profit. Thus the revenue equates the cost

$$A^i_t L^i_t = \sum_{a=1}^{\bar{a}} \sum_{r\in\{b,o\}} w^i_{r,a,t} L^i_{r,a,t}. \tag{36}$$

By a property of the CES function, I have

$$\sum_{a=1}^{\bar{a}} \sum_{r \in \{b,o\}} w_{r,a,t}^{i} L_{r,a,t}^{i} = w_t^i L_t^i. \tag{37}$$

Equations (36) and (37) imply

$$A_t^i = w_t^i.$$

Thus I can back out location-level productivity A_t^i by computing location-level wages w_t^i.

I compute the induced location-level productivity $\hat{A}_t^i$ using $\hat{\sigma}_a^{IV1} = 1/0.3401 = 2.94$ from the column 2 of Table 6 and the age-level productivity $\kappa_{a,t}^{i,IV1}$ induced by such $\hat{\sigma}_a^{IV1}$. Figure 8 shows the induced location-level productivity across locations except the rest of the North in 1960. Along with California, Northern manufacturing states such as Illinois, Michigan, and Ohio had higher productivity. These places were destinations of the great Black migration. In contrast, state in the South such as Arkansas and Mississippi had lower productivity. The two states were typical origins of the great Black migration.

I have backed out migration costs, amenities, and productivity using the formulae implied by the model. A possible story from the induced parameters is as follows. As in Figure 5, migration costs were high for African Americans in the great migration period. Figure 6 showed states in the North provided lower amenities for African Americans in the great migration period. Despite these impediments or disincentive, African Americans made the journey from the South to the North for higher wages or workplaces of higher productivity as in Figure 8.

4.3 Rent Elasticity

I estimate the rent elasticity η, which governs how much local rent increases if local income increases by one percent. Taking logs of both sides in equation (14), I have

$$\log r_t^i = \log \bar{r}^i + \eta \log\left(\gamma \sum_r \sum_c L_{r,c,t}^i w_{r,c,t}^i\right). \tag{38}$$

Note that $\sum_r \sum_c L_{r,c,t}^i w_{r,c,t}^i$ is the total income in location i in my model. Taking time differences of equation (38), I obtain

$$\Delta \log r^i = \eta \Delta \log(\text{income}^i),$$

where income^i is the total income in location i. The location-specific rent shifter $\bar{r}^i$ and the expenditure share on housing service γ are washed out by taking time differences.

Table 7: Rent Elasticity

Dependent variable:	$\Delta \log r^i$	
Model:	OLS	IV
$\Delta \log(\text{income}^i)$	0.3948***	0.4092***
	(0.0254)	(0.0264)
Observations	38	38
First-stage F-statistic		162.4

Notes: The result of the estimation of the rent elasticity. The regressions are weighted by populations as of 1970. Robust standard errors are in parentheses. Significance code: ***: 0.01.

The econometric specification is

$$\Delta \log r^i = \eta \Delta \log(\text{income}^i) + \epsilon_i, \tag{39}$$

where ϵ_i is the error term. Time differences are taken between 1970 and 2010. I use median rent for rent in each location.[17] Threat to identification is that an increase in rent may increase local income, causing positive correlation between the growth rate in local income $\Delta \log(\text{income}^i)$ and the error term ϵ_i. To deal with this threat, I instrument the growth rate in local income $\Delta \log(\text{income}^i)$ by the manufacturing share in employment and the share of college graduates in population as of 1950. In this IV estimation, I pick up only the variation in the income growth predicted by sectoral and educational composition in the old time. The regression is weighted by populations as of 1970. Table 7 reports the result. The OLS and IV estimations produce similar estimates around 0.4.

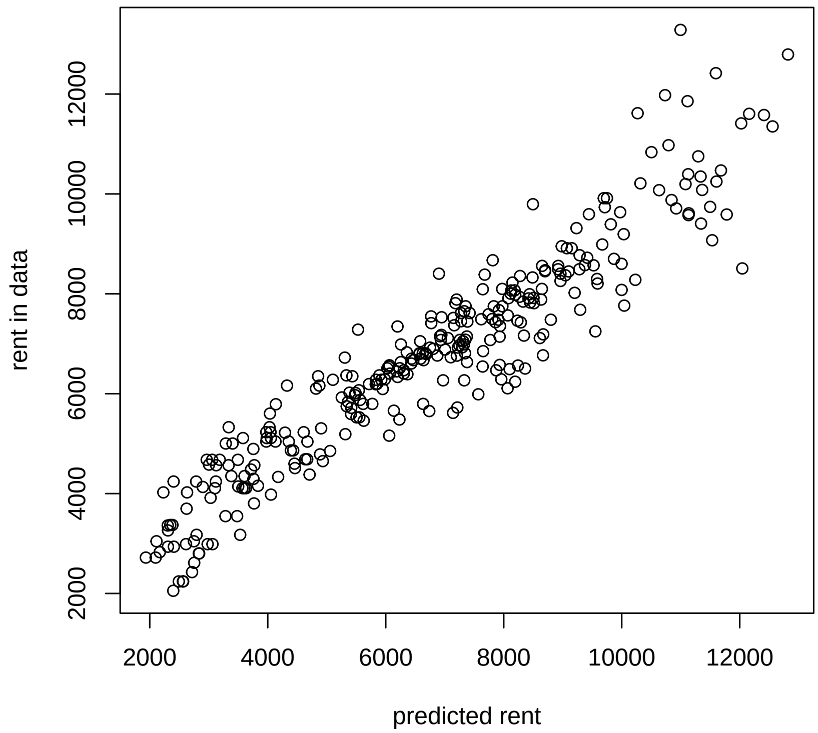
I back out location-specific rent shifters $\bar{r}^i$ given the estimate of rent elasticity $\hat{\eta} = 0.41$, the expenditure share on housing service $\gamma = 0.25$, and the income data. Rearranging equation (14) yields

$$\bar{r}^i = \frac{r_t^i}{\left(\gamma \sum_r \sum_c L_{r,c,t}^i w_{r,c,t}^i\right)^{\eta}}. \tag{40}$$

But if I replace the model objects with data counterparts, the left-hand side and the right-hand side of equation (40) cannot perfectly equate because the left-hand side depends on only location i, but the right-hand side depends on both location i and time t. By taking averages over time, I compute the sample counterpart to the location-specific rent shifter

[17] For the rest of the North, I take the mean of the median rents across the states. The reason that I use median rent for the measure of local rent is that it is only the available rent measure for 1940.

Figure 9: Predicted and Actual Rent



Notes: Actual rent in the US state data from 1940 to 2010 on the vertical axis against the predicted rent on the horizontal axis. Units of observations are state-time.

$\hat{\bar{r}}^i$

$$\hat{\bar{r}}^i = \frac{\frac{1}{8}\sum_{t=1940}^{2010} r_t^i}{\frac{1}{8}\sum_{t=1940}^{2010}\left(\gamma \cdot \text{income}^i\right)^{\hat{\eta}}},$$

where $t = 1940, \cdots, 2010$ runs the sample periods.

Using the estimates $\hat{\eta}$ and $\{\hat{\bar{r}}^i\}^i$, I can predict rent by

$$\hat{r}_t^i = \hat{\bar{r}}^i \left(\gamma \cdot \text{income}^i\right)^{\hat{\eta}}. \tag{41}$$

Figure 9 plots actual rent in data against rent predicted by equation (41). Predicted rent has a tight and linear relationship with actual rent. The correlation between actual and predicted rent is 0.94.

4.4 Fertility, Survival Probabilities, and Immigrants

Fertility. Recall that age 0 in the model corresponds to age 1 to 10 in the data. I attribute each person of age 1 to 10 to the parents (in the household including a married couple) or the single parent (in the single parent household) of race-age bins in each period. Aggregating the number of children for each race-age bin in each period yields the data counterpart to the number of babies per person $\alpha_{r,a,t}$ for (r,a,t). This procedure is detailed in Appendix F.

Survival probabilities. The Centers for Disease Control and Prevention (CDC) publish life tables documented by several different government agencies. I use life tables for 1940, 1950, $\cdots$, 2010. These tables provide the annual survival probabilities for white Americans and African Americans at each age for these sample years. Since periods and age bins are of 10-year windows in the quantification of the model, I map annual survival probabilities for 1-year age bins in the life tables to 10-year survival probabilities for 10-year age bins. This procedure is detailed in Appendix G.

Immigrants from abroad. In the model, population dynamics (7) take into account immigrants from abroad. Recall that locations in my quantification cover all US states and DC except Alaska and Hawaii. Thus all migrants from outside of these locations are regarded as immigrants from abroad. I tabulate the numbers of immigrants for each race r and age a in period t and location i using the census and ACS data. See Appendix H for details.

4.5 Computation of Steady States and Transition Paths

I consider two types of equilibria in Section 6. First, I keep all the parameters except fertility as in 1940 to see whether the US economy in 1940 was close to the steady state. I compute another equilibrium replacing parameter values for the first 5 periods in such forever 1940 scenario.[18] I set fertility such that individuals of the age 2 (21 to 30 year olds) have children such that the same size of the population of African Americans and others will be sustained, that is, $\alpha_{r,2,t} = 1/(s_{r,0,1940} \cdot s_{r,1,1940})$. Loading the populations in 1940 as the initial populations, $L^i_{r,a,0} = L^i_{r,a,1940}$, I compute the equilibrium forward. I assume that the economy converges to the steady state in period 105.[19]

Second, I compute the baseline equilibrium that resembles the US economy from 1940 to 2010. I compute counterfactual equilibria, too, replacing a subset of parameters in the

[18] See Section 6 for the purposes of such exercises.

[19] The initial period is period 0, so I have 106 periods.

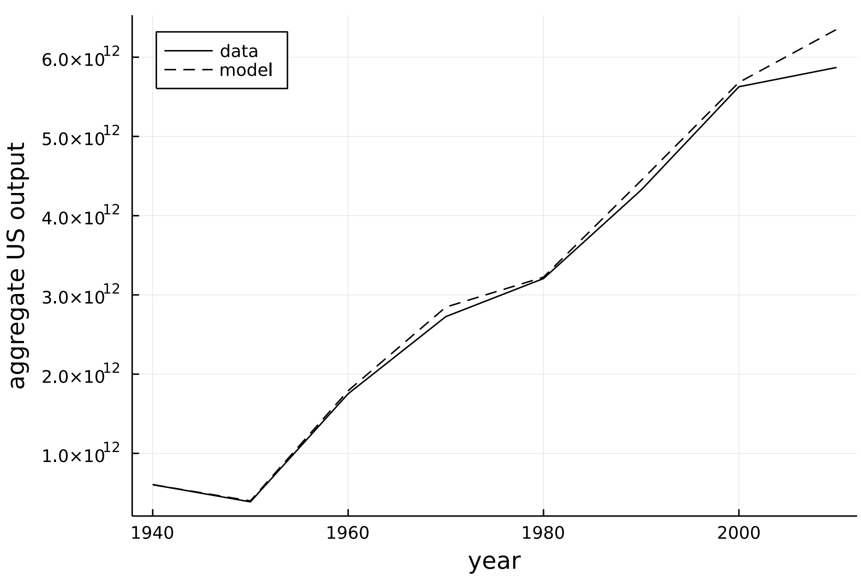
baseline equilibrium with hypothetical ones. For the baseline equilibrium, I load all the parameter values from 1940 to 2000. From 2010 onward, I assume that all parameters are as of 2010 with one exception. The exception is, again, fertility. If I use the actual fertility in the 2010 data for fertility from 2010 onward, the US population would shrink forever. To avoid this, I assume that individuals of the age 2 (21 to 30 year olds) have children such that the populations of African Americans and others will be sustained, that is, $\alpha_{r,2,t} = 1/(s_{r,0,2010} \cdot s_{r,1,2010})$. I assume that the economy converges to the steady state in period 107. Period 0 is the year 1940, ..., period 7 is the year 2010. So in the remaining 100 periods, the economy converges to the steady state with time-invariant parameters. Replacing a subset of parameters, I similarly compute counterfactual equilibria.

For either type of equilibria, I first compute steady states toward which the economy converges. Then I compute transition paths. Appendix I details the algorithm to compute steady states. Appendix J explains how to compute transition paths toward given steady states.

5 Model Fit

I compare variables generated by the baseline equilibrium of the model with the data counterparts.

Figure 10: Aggregate US Output: Model vs Data



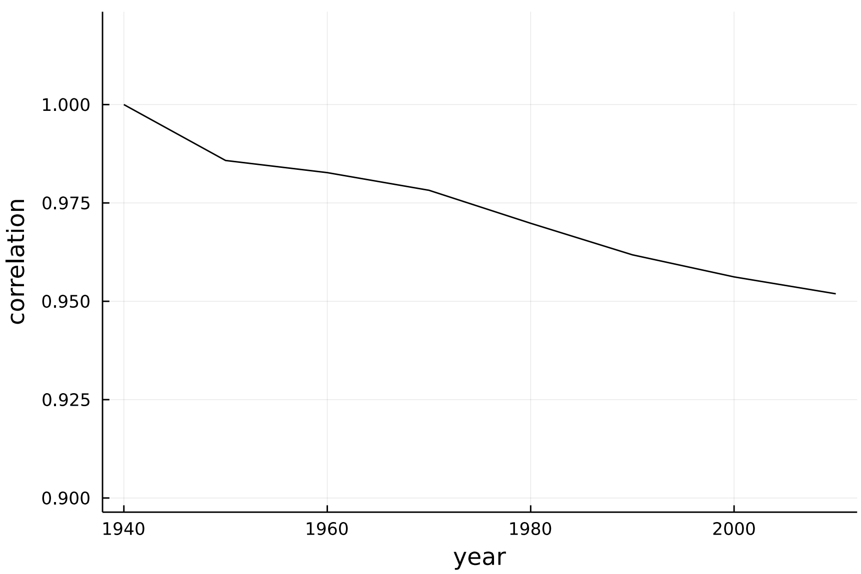
Notes: Following (42), I plot aggregate US output generated by the baseline equilibrium of the model and the one from the data.

First I compare aggregate US output between the baseline equilibrium and data. Aggregate output in period t, Y_t, is

$$Y_t = \sum_{i \in \mathcal{N}} Y_t^i = \sum_{i \in \mathcal{N}} A_t^i L_t^i = \sum_{i \in \mathcal{N}} \sum_{r \in \{b,o\}} \sum_{a=1}^{\bar{a}} w_{r,a,t}^i L_{r,a,t}^i, \tag{42}$$

where the second and third equalities follow from equations (9) and (36), respectively.[20] The right-most object has data counterparts because it depends on only wages and populations. Figure 10 plots aggregate US output (or labor income) in the baseline equilibrium of the model and in the data over time. From 1940 to 2000, the baseline equilibrium closely resembles the data in aggregate output. In 2010, the baseline equilibrium overstates aggregate output by 8 percent.

Figure 11: Correlations between the Populations in the Baseline Equilibrium and in



Notes: For each year, this graph shows the correlation between the population vector in the baseline equilibrium $(L_{r,a,t}^{i,\text{baseline}})_{r,a}^i$ and the population vector in the data $(L_{r,a,t}^{i,\text{data}})_{r,a}^i$. Since I load the actual population vector in 1940 as the initial population vector to the model, the correlation is one in 1940 by construction.

Second, I compare populations in the baseline equilibrium with those in the data. Pick up any sample year t from $\{1940, \cdots, 2010\}$. For t, let $(L_{r,a,t}^{i,\text{baseline}})_{r,a}^i$ and $(L_{r,a,t}^{i,\text{data}})_{r,a}^i$ be the vectors of the populations in the baseline equilibrium and in the data, respectively. Then for each sample year t, I compute the correlation coefficient between these two population vectors $Cor((L_{r,a,t}^{i,\text{baseline}})_{r,a}^i, (L_{r,a,t}^{i,\text{data}})_{r,a}^i)$. Figure 11 plots such correlations over time. In 1940, the correlation is one because I load the actual populations in 1940 as the initial population.

[20] Hsieh and Moretti (2019) also focus on aggregate output defined similarly. See their equation (7).

The correlation between the model and the data population vectors. But even in 2010, the correlation between the model and the data population vectors is over 0.95. Throughout the sample years, the baseline equilibrium captures the spatial distribution of populations fairly well.

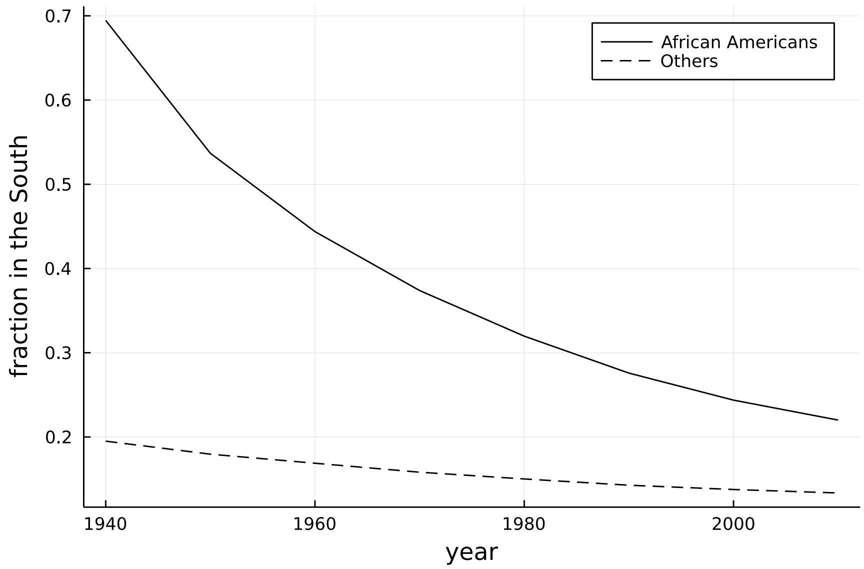
6 Counterfactual Results

6.1 Forever 1940

First, I consider the counterfactual equilibrium in which all the parameter values except fertility are as of 1940 every period. Fertility is such that individuals of the age 2 (21 to 30 year olds) have children such that the population size will not change for each race r, $\alpha_{r,2,t} = 1/(s_{r,0,t} \cdot s_{r,1,t})$. I call this equilibrium the forever 1940 equilibrium. Figure 12 plots the fractions of African Americans and others in the South in the forever 1940 equilibrium. As the solid line shows, 69 percent of African Americans lived in the South in 1940, but the fraction of African Americans in the South drops to 22 percent in 2010. The fraction of others in the South declines from 20 percent to 13 percent from 1940 to 2010 as in the dashed line. Although the parameter values of productivity, amenities, and migration costs are constant over time, the spatial distribution of populations change drastically. This suggests that the US economy in 1940 was far from the steady state induced by the parameter values in 1940.

Along with the forever 1940 equilibrium, I consider the equilibrium in which African Americans cannot migrate across the North and the South for 5 periods since 1940. Let $\mathcal{N}_N$ be the set of locations in the North, and $\mathcal{N}_S$ be the set of locations in the South. Then I set $\tau^{j,i}_{b,a,t} = \infty$ for any pair of locations j,i such that $(j,i) \in \mathcal{N}_N \times \mathcal{N}_S$ or $(i,j) \in \mathcal{N}_S \times \mathcal{N}_N$ and $t = 1940, \cdots, 1980$. Since individuals make migration decisions in 1980 and arrive in destinations in 1990, this shuts down the relocation of individuals until 1990. All the other parameters are as in the forever 1940 equilibrium. For the forever 1940 equilibrium and the equilibrium without the North-South migration of African Americans, Figure 13 shows (nation-level) per capita output Y_t/L_t, where $L_t = \sum_{i\in\mathcal{N}} \sum_{r\in\{b,o\}} \sum_{a=0}^{\bar{a}} L^i_{r,a,t}$. Per capita output is normalized by the initial level in 1940. The solid line shows that even without any change in productivity, amenities, and migration costs, per capita output increases by 11.3 percent by 1990. The increase in per capita output is caused by migration because only dynamic change in the forever 1940 equilibrium is relocation of individuals. This suggest that there was an opportunity to increase output by relocating the work force in 1940. If African Americans cannot migrate across the North and the South, per capita output increases by 9.7 percent by 1990. Thus the remaining $11.3 - 9.7 = 1.6$ percent

Figure 12: Fractions in the South for African Americans and Others: Forever 1940



Notes: The fractions of the populations of African Americans and others in the South over time in the equilibrium in which the parameter values are as of 1940 forever.

increase is explained by the migration of African Americans across the North and the South. Putting differently, in the 11.3 percent increase in per capita output, the migration of African Americans across the North and the South accounts for 14 percent of it because

$$\frac{0.113 - 0.097}{0.113} = 0.14.$$

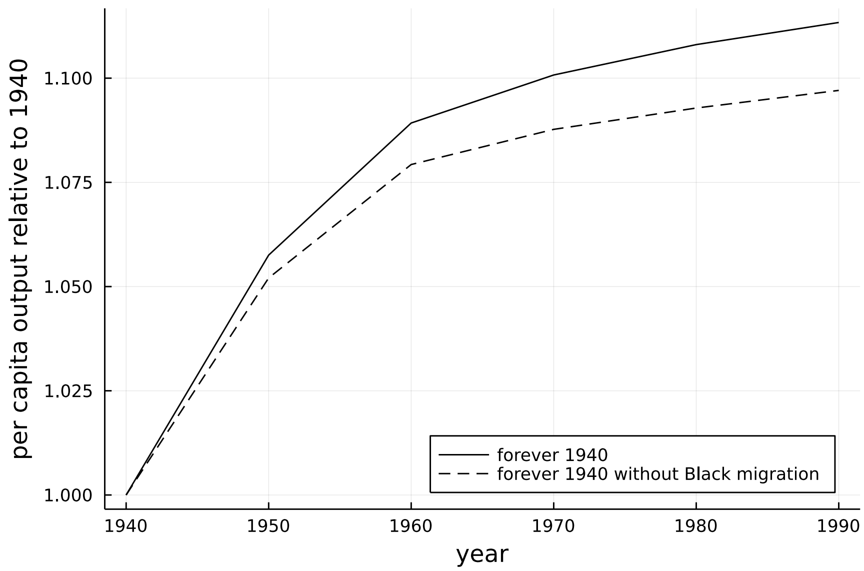
The remaining 86 percent is explained by the migration of others within or across the North and the South and the migration of African Americans within the North and the South. The relocation of African Americans accounts for a substantial part of room for improving aggregate output.

So far I have considered the role of migration, especially of Afrifan Americans, in equilibria in which parameters are time-invariant. In the actual US economy, however, all of productivity, amenities, and migration costs changed over time across locations. I turn to equilibria with time-varying parameters.

6.2 The Effects of the Great Black (and Others') Migration

I compare the baseline equilibrium that loads all the parameter values from Section 4 with two counterfactual equilibria. In the first counterfactual equilibrium, African Americans cannot migrate across the North and the South from 1940 to 1960. That is, $\tau_{b,a,t}^{j,i} = \infty$

Figure 13: Per Capita Output in the Forever 1940 Equilibrium and the Equilibrium without Black Migration

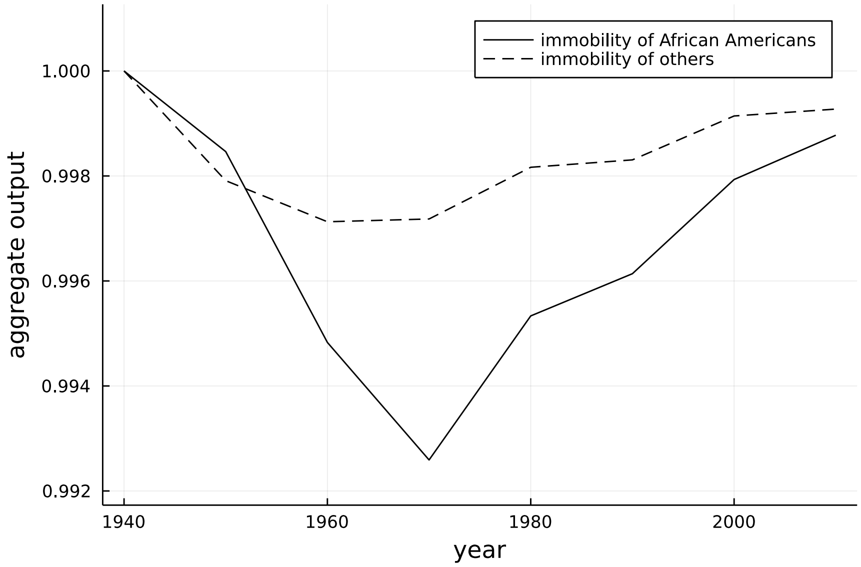


Notes: Per capita output relative to the 1940 level for the forever 1940 equilibrium and the equilibrium in which African Americans cannot migrate across the North and the South from 1940 to 1980.

for any pair of locations j, i such that $(j,i) \in \mathcal{N}_N \times \mathcal{N}_S$ or $(j,i) \in \mathcal{N}_S \times \mathcal{N}_N$, any age a, and $t = 1940, \cdots, 1960$. Here I bilaterally shut down the migration of African Americans from the North to the South and from the South to the North. Since migration decisions are made one period ahead of arrival, this shuts down African Americans' relocation until 1970, the end of the great Black migration. I call this equilibrium as the equilibrium of African Americans' immobility. In the second counterfactual equilibrium, others cannot migrate across the North and the South from 1940 to 1960. That is, $\tau_{o,a,t}^{j,i} = \infty$ for any pair of locations j, i such that $(j,i) \in \mathcal{N}_N \times \mathcal{N}_S$ or $(j,i) \in \mathcal{N}_S \times \mathcal{N}_N$, any age a, and $t = 1940, \cdots, 1960$. I call this equilibrium as the equilibrium of others' immobility.

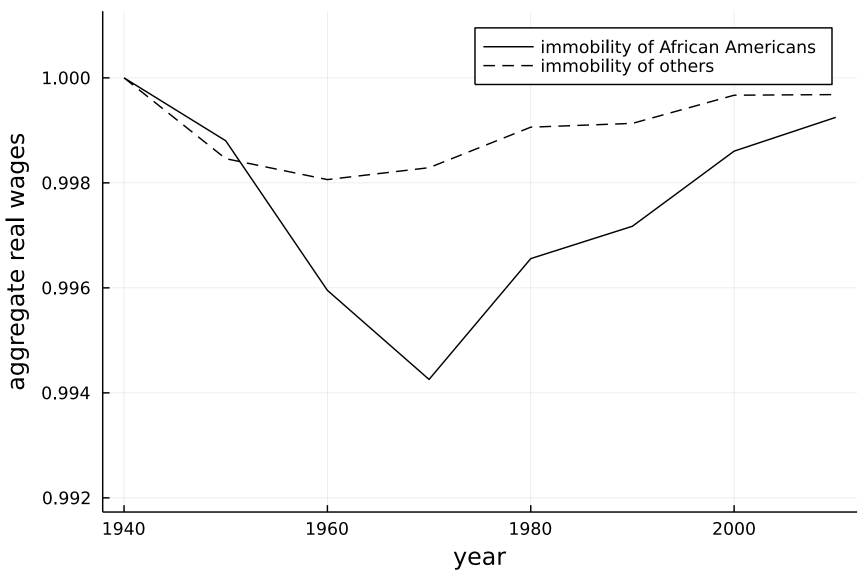
Figure 14 plots aggregate output in the two counterfactual equilibria relative to the baseline equilibrium. In 1970, aggregate output in the equilibrium of African Americans' immobility is 0.74 percent lower than aggregate output in the baseline equilibrium, as in the solid line. The dashed line shows that in the same year, aggregate output in the equilibrium of others' immobility is 0.28 percent lower than aggregate output in the baseline equilibrium. These two results imply that African Americans' relocation across the North and the South increased aggregate output more than others' relocation did, although African Americans accounted for only about 10 percent of the US population. Back-of-the-envelope calculation in Appendix K predicts that if African Americans were

Figure 14: Aggregate Output



Notes: Aggregate output in the equiribria of African Americans' or others' immobility relative to the baseline equilibrium.

Figure 15: Aggregate Real Wages



Notes: Aggregate real wages in the equiribria of African Americans' or others' immobility relative to the baseline equilibrium.

spatially distributed as in 1940, aggregate labor income in 1970 would have been lower by 0.84 percent. Therefore the quantitative model and the back-of-the-envelope calculation yield similar predictions for the aggregate impact of the great Black migration. Figure 15 plots aggregate real wages

$$\sum_{i\in\mathcal{N}}\sum_{r\in\{b,o\}}\sum_{a=1}^{\bar{a}} L^i_{r,a,t}\frac{w^i_{r,a,t}}{(r^i_t)^\gamma}$$

in the two counterfactual equilibria relative to the baseline equilibrium. Shutting down the North-South migration of African Americans and others decreases aggregate real wages by 0.63 percent and 0.17 percent, respectively. Shutting down the North-South relocation decreases real wages less than output because higher nominal wages are partly absorbed by higher housing rent. But the difference in the magnitude is not large.

Table 8: Nominal Wage Changes due to the Great Black Migration

		this paper	Boustan (2009)	
			OLS	IV
North	African Americans	0.079	0.096	0.072
	Others	-0.002	-0.005	-0.004
South	African Americans	-0.053	-	-
	Others	0.005	-	-

Notes: Percent changes in nominal wages from the baseline equilibrium to the equilibrium of African Americans' immobility (or the no great Black migration scenario). The result of Boustan (2009) is from her table 6.

How much wages would African Americans and others have earned if the great Black migration did not occur? Let region g index the North N or the South S, $g \in \{N,S\}$. Then for $g \in \{N,S\}$, difine the nominal wage of race $r \in \{b,o\}$ in region g and period t by

$$\text{nominal wage}^g_{r,t} = \frac{\sum_{i\in\mathcal{N}_g} L^i_{r,a,t} w^i_{r,a,t}}{\sum_{i\in\mathcal{N}_g} L^i_{r,a,t}}.$$

Let nominal wage$^g_{r,t}$ and nominal wage$^{g,\text{no mig}}_{r,t}$ be such nominal wages of race r in region g and period t in the baseline equilibrium and in the equilibrium of African Americans' immobility, respectively. Then the percent change in the nominal wage of race r in region g and period t from the baseline equilibrium to the equilibrium of African Americans' immobility is

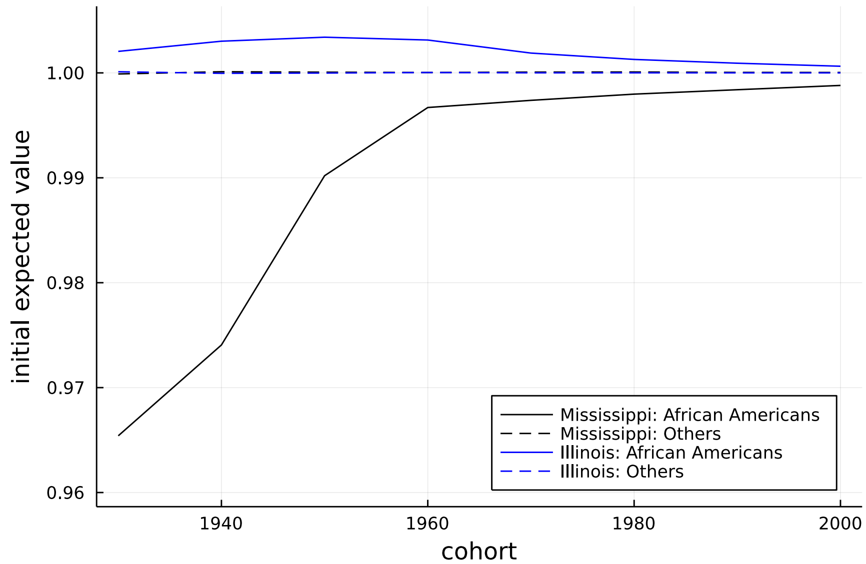
$$\frac{\text{nominal wage}^{g,\text{no mig}}_{r,t}}{\text{nominal wage}^g_{r,t}} - 1.$$

Table 8 shows such percent changes in the nominal wages in its first column. The year is 1970. According to my quantitative model, if the great Black migration did not occur, the wage of African Americans would have been higher by 7.9 percent, and the wage of others would have been lower by 0.2 percent. As is common in Boustan (2009) and this paper, African Americans and others are imperfectly substitutable. In the baseline equilibrium, the inflow of African Americans from the South to the North decreases the wages of African Americans in the North. But in the equilibrium of African Americans' immobility, African Americans in the North have fewer competitors in their local labor market than in the baseline equilibrium and receive higher wages. The second and third columns show the predictions by Boustan (2009). My result for the change in African Americans' wages in the North, 7.9 percent, is in the range of her predictions from 7.2 percent to 9.6 percent. My result for the change in others' wages in the North, -0.2 percent, is smaller than her predictions, -0.4 or -0.5 percent, in the absolute values. I differ from Boustan (2009) in that I make wage predictions not only in the North but also in the South. If African Americans could not migrate to the North, more African Americans would have remained in the South. The nominal wage of African Americans in the South would have been lower by 5.3 percent in the no great Black migration scenario than in the baseline. Others' wages in the South would have been higher by 0.5 percent.

I turn to the welfare effects of the migration of African Americans and others across the North and the South. Figure 16 plots the initial expected values for each cohort of African Americans and others born in Mississippi and Illinois in the equilibrium of African Americans' immobility relative to the baseline equilibrium. As they lose opportunities to migrating to the high-wage North, the expected value of African Americans born in Mississippi in the 1930s declines by 3.5 percent. Although African Americans born in Illinois lose opportunities of migrating to the South till 1970, their welfare increases by 0.34 percent for the cohort 1950. The expected values of others in the equilibrium of African Americans' immobility are very similar to the expected values of others in the baseline equilibrium. As in the blue solid line, for African Americans in Illinois, the expected values for the cohorts 1950 and 1960 are higher than the expected values for the cohorts 1930 and 1940 in the equilibrium of African Americans' immobility relative to the baseline equilibrium. This is because the earlier generations cannot move to the South in their youth (till 1970), but the later generations can move to the South in their youth and still benefit from fewer competitors in the labor market.

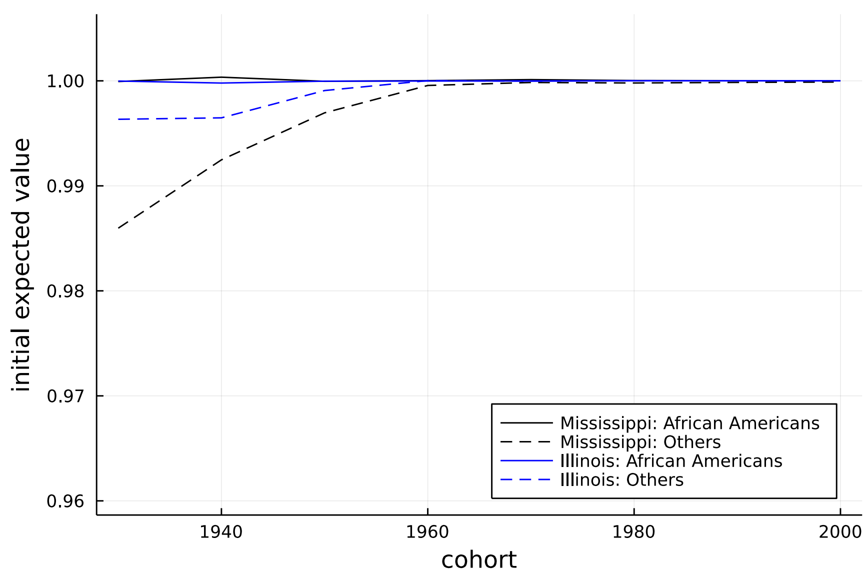
Figure 17 plots the expected values in the equilibrium of others' immobility relative to the baseline equilibrium. The expected value of others born in Mississippi in the 1930s in the equilibrium of others' immobility is 1.4 percent lower than that in the baseline equilibrium. Recall that in Figure 16, for this cohort, the welfare of African Americans born in Mississippi is 3.5 percent lower in the no great Black migration scenario than in the baseline equilibrium. Thus these two figures jointly highlight African Americans' strong incentive for outmigration from the South. Others in Illinois are also worse off by

Figure 16: Initial Expected Values: African Americans' Immobility



Notes: Initial expected values of African Americans and others in Mississippi and Illinois in the equilibrium of African Americans' immobility relative to the baseline equilibrium.

Figure 17: Initial Expected Values: Others' Immobility

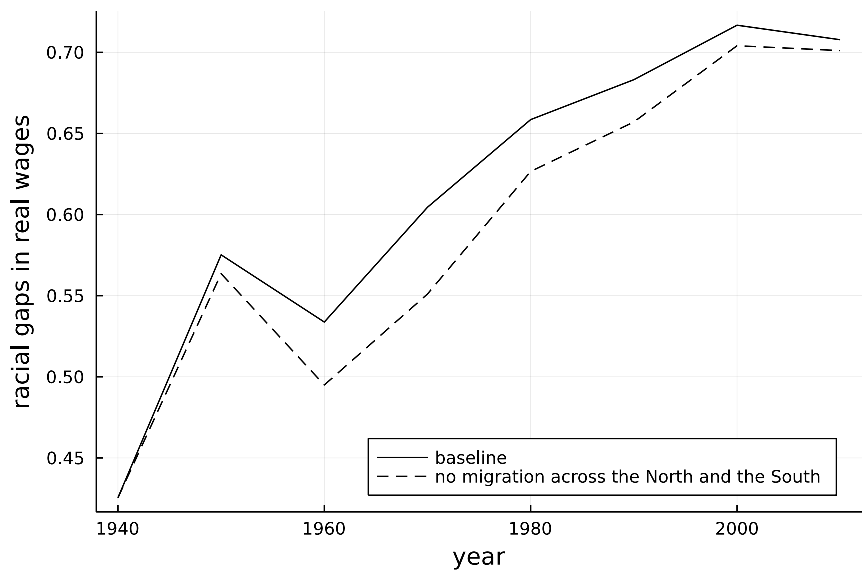


Notes: Initial expected values of African Americans and others in Mississippi and Illinois in the equilibrium of others' immobility relative to the baseline equilibrium.

closing the North-South border for others because they lose varieties in location choices. The effects of others' immobility on African Americans' welfare are small.

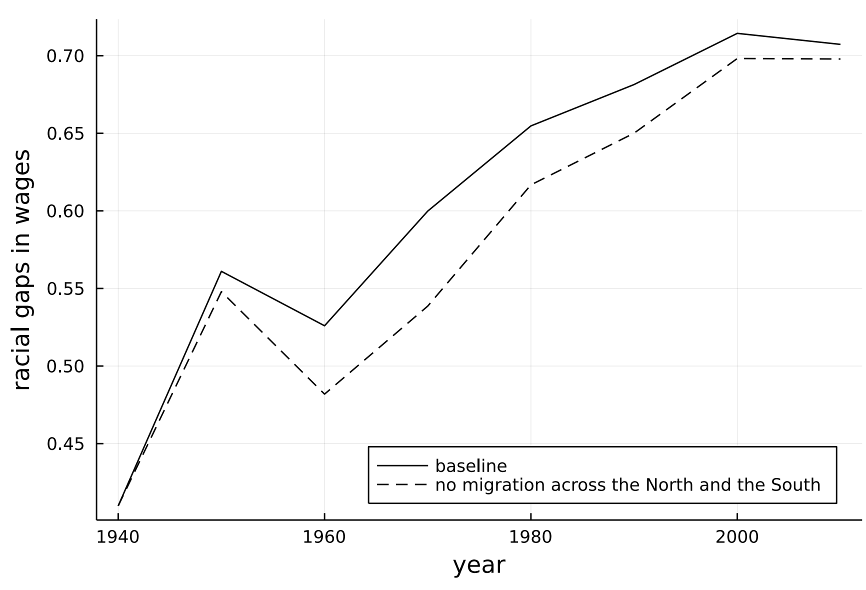
In the great migration period, the gaps in wages and living standards between Black

Figure 18: Racial Gaps in Real Wages



Notes: Per capita real wages of African Americans relative to those of others in the baseline equilibrium and the equilibrium of African Americans' immobility

Figure 19: Racial Gaps in Nominal Wages



Notes: Notes: Per capita nominal wages of African Americans relative to those of others in the baseline equilibrium and the equilibrium of African Americans' immobility

and white Americans shrank. How did the relocation of African Americans across the North and the South contribute to reducing the racial gaps? For period t and race $r \in \{b, o\}$, I define the average real wage by

$$\text{average real wage}_{r,t} = \frac{\sum_{i\in\mathcal{N}} \sum_{a=1}^{\bar{a}} L_{r,a,t}^{i} \left(\frac{w_{r,a,t}^{i}}{(r_t^i)^{\gamma}}\right)}{\sum_{i\in\mathcal{N}} \sum_{a=0}^{\bar{a}} L_{r,a,t}^{i}}.$$

Then I compute the ratio of average real wages between African American and others

$$\frac{\text{average real wage}_{b,t}}{\text{average real wage}_{o,t}}.$$

Figure 18 plots the Black-other average real wage ratios in the baseline equilibrium (the solid line) and the equilibrium of African Americans' immobility (the dashed line). If African Americans could not migrate across the North and the South, the racial gap in average real wages would have been larger. In 1970, the Black-other average real wage ratio was 0.605 in the baseline equilibrium. In the same year, the Black-other average real wage ratio was 0.551. Therefore, the relocation of African Americans across the North and the South decreased the racial gap in average real wages by 9 percent, because

$$\frac{0.605 - 0.551}{0.605} = 0.089.$$

To compare my result with the prior literature concerning nominal wages, I similarly define the average nominal wage for race $r \in \{b, o\}$ and time t by

$$\text{average nominal wage}_{r,t} = \frac{\sum_{i\in\mathcal{N}} \sum_{a=1}^{\bar{a}} L_{r,a,t}^{i} w_{r,a,t}^{i}}{\sum_{i\in\mathcal{N}} \sum_{a=0}^{\bar{a}} L_{r,a,t}^{i}}.$$

I take the ratio of average nominal wages between African Americans and others

$$\frac{\text{average nominal wage}_{b,t}}{\text{average nominal wage}_{o,t}}.$$

Figure 19 plots the Black-other ratios of average nominal wages in the baseline equilibrium and in the equilibrium of African Americans' immobility. In 1970, the Black-other average nominal wage ratios were 0.600 and 0.539 in the baseline equilibrium and in the equilibrium of African Americans' immobility, respectively. Therefore, the great Black migration reduced the racial gap in nominal wages by $(0.600 - 0.539)/0.600 = 0.101$, 10.1 percent. This is a similar number to Smith and Welch (1989) who use a reduced-form decomposition technique to compute the contribution of the great migration to reducing the racial gap in nominal wages.

7 Conclusion

4 million African Americans migrated from the South to the North between 1940 and 1970, which is called the great Black migration. This paper has quantified the aggregate and distributional effects of the great Black migration. A dynamic general equilibrium model of the spatial economy has served for this purpose. I have estimated the elasticities in the model and backed out other parameters. My quantitative model and the existing reduced-form studies have produced similar predictions about nominal wages and racial inequality in the no great migration scenario. The quantitative model revealed that between 1940 and 1970, the mobility of African Americans across the North and the South increased aggregate US output more than the mobility of others did. I view this paper as the first step in understanding the connection between the geography of African Americans and the aggregate performance of the US economy.

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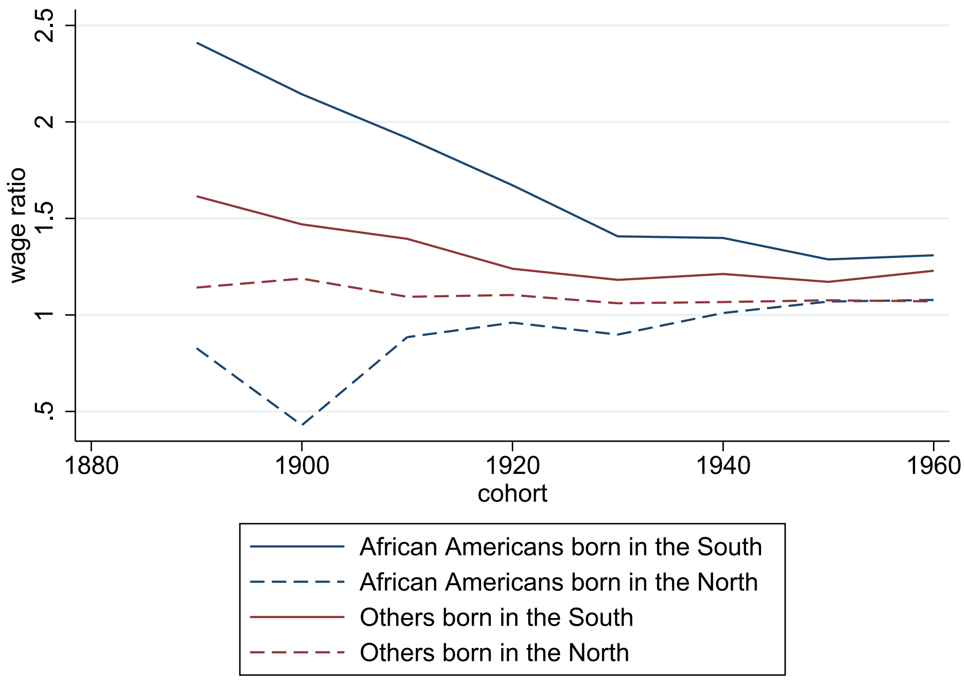
Appendices

A An Additional Figure on Motivating Facts

Figure 20: Mover-Stayer Ratios of Payrolls per Capita for Cohorts, Races, and Birthplaces



Notes: For each cohort (say c), race, and birthplace (the North or the South), this graph provides the ratio of the payroll per capita of the individuals who lived in the place other than the birthplace as of year $c+50$ to the payroll per capita of the individuals who lived in the birthplace as of year $x+50$. Source: US census 1940-2000, American Community Survey 2010.

B Details on Data Sources

Wages, populations, migration shares, and fertility. The data of wages, populations, migration shares, and fertility (babies per person) are from the US censuses from 1940 to 1990, and the ACS from 2000 to 2019. I use the full count data for the US census 1940, 5 percent samples for the US censuses 1960, 1980, and 1990, and 1 percent samples for the US censuses 1950 and 1970 and the ACS of all the sample years. All of them are tabulated in IPUMS USA (Ruggles et al., 2022). Figure 1 requires the data of the populations of African Americans, enslaved African Americans, and others in the North and the South since 1790. These data are from the US censuses tabulated in IPUMS NHGIS (Manson et al., 2022).

Rent. IPUMS does not provide the data of rent in 1950. The website of the US Census Bureau provides median rent in states from 1940 to 2000.[21] I obtain rent in 2010 and 2019 from IPUMS NHGIS (Manson et al., 2022).

Survival probabilities. Survival probabilities are from life tables published on the CDC's website.[22]

Aggregate income, college graduates, manufacturing shares. Aggregate incomes in states are used in the regression (39). In this regression, I use the manufacturing shares in employment and the shares of college graduates in population in 1950 as IVs. All of these variables are from IPUMS NHGIS (Manson et al., 2022).

Consumer price index. Wages and rent are deflated by the consumer price index and measured in the 2010 US dollars. The Bureau of Labor Statistics publishes consumer price indices at its website.[23]

C Tabulation of Migration Shares

In the quantification of the model, time periods are 10 years. The US censuses and ACS report individuals' locations 1 or 5 years ago, depending on sample years. Table 9 reports which sample year includes 1- or 5-year migration shares. I need to map 1- or 5-year migration shares in the data to 10-year migration shares in the model. In the model, individuals make migration decisions in period t and arrive in destinations in period $t+1$. Thus, the census data of 1940 inform me of migration decisions as of 1930, the census data of 1950 inform me of migration decisions as of 1940, and so on.

Let $\mu_{r,a,t}^{i,j,10}$ be the model-consistent 10-year migration share for race r and age a in year t from location j to location i. $\mathbf{M}_{r,a,t}^{10}$ is the matrix whose (i,j) element is $\mu_{r,a,t}^{i,j,10}$.

The censuses in the years 1940, 1960, 1970, 1980, and 1990 yield 5-year migration shares. Let $\mu_{r,a,t}^{i,j,5,\text{data}}$ be such 5-year migration share of race r and age a in (the sample) year t from location j to location i. $\mu_{r,a,t}^{i,j,5,\text{data}}$ is directly computed from the census data of the

[21] https://www.census.gov/data/tables/time-series/dec/coh-grossrents.html (accessed on 10/31/2022)
[22] https://www.cdc.gov/nchs/products/life_tables.htm (accessed on 10/31/2022)
[23] https://data.bls.gov/cgi-bin/surveymost?bls (accessed on 11/01/2022)

aforementioned sample years. Let $\mathbf{M}^{5,\text{data}}_{r,a,t}$ be the matrix whose (i,j) element is $\mu^{i,j,5,\text{data}}_{r,a,t}$. I assume that for such census year $t+10$, 5-year migration shares are constant between years t and $t+10$. Then for $t = 1930, 1950, 1960, 1970, 1980$, the model-consistent migration matrix $\mathbf{M}^{10}_{r,a,t}$ is computed by

$$\mathbf{M}^{10}_{r,a,t} = \left(\mathbf{M}^{5,\text{data}}_{r,a,t+10}\right)^2.$$

The census in 1950 yields 1-year migration shares. Let $\mu^{i,j,1,\text{data}}_{r,a,t}$ be the 1-year migration share for race r and age a in the census or ACS year t from location j to location i. Let $\mathbf{M}^{1,\text{data}}_{r,a,t}$ be the matrix whose (i,j) element is $\mu^{i,j,1,\text{data}}_{r,a,t}$. I assume that from 1940 to 1950, 1-year migration shares are constant. Then the model-consistent migration matrix $\mathbf{M}^{10}_{r,a,1940}$ is computed by

$$\mathbf{M}^{10}_{r,a,1940} = \left(\mathbf{M}^{1,\text{data}}_{r,a,1950}\right)^{10}.$$

Since 2000, ACS reports current locations and locations 1 year ago for individuals every year. Then $\mathbf{M}^{10}_{r,a,2000}$ is computed by

$$\mathbf{M}^{10}_{r,a,2000} = \mathbf{M}^{1,\text{data}}_{r,a,2001}\mathbf{M}^{1,\text{data}}_{r,a,2002}\cdots\mathbf{M}^{1,\text{data}}_{r,a,2010}.$$

Since I would like to avoid picking up irregularity caused by the COVID-19 pandemic in 2020, $\mathbf{M}^{10}_{r,a,2010}$ is computed by

$$\mathbf{M}^{10}_{r,a,2010} = \mathbf{M}^{1,\text{data}}_{r,a,2011}\cdots\mathbf{M}^{1,\text{data}}_{r,a,2018}\mathbf{M}^{1,\text{data}}_{r,a,2019}\mathbf{M}^{1,\text{data}}_{r,a,2019},$$

where the 1-year migration shares in the 2019 data are double-counted and the 1-year migration shares in the 2020 data are excluded.

Table 9: 1- or 5-Year Migration Shares

year	source	location X years ago
1940	census	5
1950	census	1
1960	census	5
1970	census	5
1980	census	5
1990	census	5
2000-2019	ACS	1

Notes: The US censuses and American Community Survey report individuals' locations 1 or 5 years ago depending on sample years.

D First Difference Estimation of the Elasticity of Substitution across Races

I estimate the elasticity of substitution across races within age, time, and location bins σ_r by the first difference estimation, following Monras (2020). Taking the time difference in equation (26), I have

$$\Delta \log\left(\frac{w_{b,a}^n}{w_{o,a}^n}\right) = -\frac{1}{\sigma_1}\Delta \log\left(\frac{L_{b,a}^n}{L_{o,a}}\right) + \frac{1}{\sigma_1}\Delta \log\left(\frac{\kappa_{b,a}^n}{\kappa_{o,a}^n}\right).$$

Since the growth rate of the productivity ratio $\Delta \log\left(\frac{\kappa_{b,a}^n}{\kappa_{o,a}^n}\right)$ is unobservable, the econometric specification is

$$\Delta \log\left(\frac{w_{b,a}^n}{w_{o,a}^n}\right) = -\frac{1}{\sigma_1}\Delta \log\left(\frac{L_{b,a}^n}{L_{o,a}}\right) + f_a + \epsilon_a^n, \tag{43}$$

where f_a is the age fixed effect, and ϵ_a^n is the error term. The time differences are taken between 1940 and 2010. If population of African Americans relative to population of the others increase in a location with a high growth rate of productivity ratio between African Americans and the others $\Delta \log\left(\frac{\kappa_{b,a}^n}{\kappa_{o,a}^n}\right)$, OLS estimation of equation (43) produces an upward bias for $-1/\sigma_1$. Following Monras (2020), I instrument $\Delta \log\left(\frac{L_{b,a}^n}{L_{o,a}}\right)$ by the level of population ratio between African Americans and the others in the old time, 1930, $\log\left(\frac{L_{b,a,1930}^n}{L_{o,a,1930}^n}\right)$. I make two assumptions. One is that the population ratio between African Americans and the others as of 1930 is not correlated with the growth rate of the productivity ratio between African Americans and the others between 1940 and 2010. The other assumption is that the growth rate of the population ratio between African Americans and the others between 1940 and 2010 is correlated with the level of the population ratio between the two racial groups as of 1930. An example for the latter assumption is that from 1940 to 2010, African Americans migrated to Illinois or New York where a moderate number of African Americans resided in 1930, but very few African Americans migrated to Wyoming or Montana where very few African Americans resided in 1930.

Table 10 reports the results. The first column reports the result of OLS, and the second column reports the result of the IV estimation. In line with my conjecture, the IV estimation seems to correct a positive bias in OLS.

Table 10: Elasticity of Substitution across Races: First Difference Estimation

Dependent variable:	$\Delta \log(w^n_{b,a}/w^n_{o,a})$	
Model:	OLS	IV
$\Delta \log(L^n_{b,a}/L^n_{o,a})$	-0.1510***	-0.2048***
	(0.0235)	(0.0276)
fixed effects:		
age	✓	✓
Observations	228	228
First-stage F-statistic		621.3

Notes: The results of the first difference estimation of the elasticity of substitution across races. Robust standard errors are in parentheses. Significance codes: ***: 0.01.

E Standard Errors of the Elasticities of Substitution

The estimation of the elasticity of substitution across ages σ_a involves an estimate of σ_r, as equations (11), (31), and (33) imply. Let $\hat{\sigma}_r$ and $\hat{\sigma}_a$ be estimates for σ_r and σ_a, respectively. Then the standard error of $\hat{\sigma}_a$ need to take into account variability of $\hat{\sigma}_r$.

For this purpose, I compute block bootstrap standard errors. This is a similar approach to Glitz and Wissmann (2021). I have the data of wages and populations from 1940 to 2010 and the data of migration shares from 1930. To construct shift-share predicted populations (28) and gross inflows (29), I need migration shares 30 years before the year for wages and populations. The earliest migration data are of 1930, so my sample years are 1960 to 2010. I split the sample years to two groups. Group 1 consists of the years 1960, 1970, and 1980. Group 2 consists of the years 1990, 2000, and 2010. The reason that I resample block of years is that bootstrap samples would understate serial correlation over time if each year is resampled separately.[24]

The procedure of block bootstrap is the following. Set the number of bootstrap samples $B = 10,000$. Recall that the number of locations, N, is 38. Then for $b = 1, \cdots, B$,

1. Randomly choose one of N locations $2N$ times, allowing for replacement.[25] I get x_b, a $2N$-dimensional vector of locations . Treat them as $2N$ distinct locations.

2. Draw a Bernoulli random number $2N$ times with the success probability $1/2$. I get y_b, a $2N$-dimensional vector whose element is either 0 or 1.

[24] See Glitz and Wissmann (2021) for details.

[25] I collect $2N$ locations because the number of sample years in each group (1 or 2) is one half of the number of sample years in the original sample.

3. Let $x_{b,i}$ and $y_{b,i}$ be the i-th elements of x_b and y_b, respectively. If $y_{b,i} = 0$, location $x_{b,i}$ has the sample years in group 1. If $y_{b,i} = 1$, location $x_{b,i}$ has the sample years in group 2. Then $y_{b,i} + 1$ is the group number of sample years.

4. Collect all observations in the pairs of $(x_{b,i}, y_{b,i} + 1)_{i=1}^{2N}$ from the original sample. Note that all age bins are collected within location-sample year group pairs $(x_{b,i}, y_{b,i} + 1)$. Call such bootstrap sample S_b.

5. For bootstrap sample S_b, compute the OLS, IV1, and IV2 estimates for σ_r, following Subsubsection 4.2.1. Denote such OLS, IV1, and IV2 estimates by $\hat{\sigma}_{r,b}^{OLS}$, $\hat{\sigma}_{r,b}^{IV1}$, and $\hat{\sigma}_{r,b}^{IV2}$, respectively.

6. Compute the race-specific productivity induced by each of $\hat{\sigma}_{r,b}^{OLS}$, $\hat{\sigma}_{r,b}^{IV1}$, and $\hat{\sigma}_{r,b}^{IV2}$.

7. Using $\hat{\sigma}_{r,b}^{OLS}$, $\hat{\sigma}_{r,b}^{IV1}$, and $\hat{\sigma}_{r,b}^{IV2}$ and the race-specific productivity induced by each of the three estimates, compute the OLS, IV1, and IV2 estimates for σ_a following Subsubsection 4.2.2. Denote such OLS, IV1, and IV2 estimates by $\hat{\sigma}_{a,b}^{OLS}$, $\hat{\sigma}_{a,b}^{IV1}$, and $\hat{\sigma}_{a,b}^{IV2}$.

Now I have six vectors: $\vec{\hat{\sigma}}_r^{OLS} = (\hat{\sigma}_{r,b}^{OLS})_{b=1}^{B}$, $\vec{\hat{\sigma}}_r^{IV1} = (\hat{\sigma}_{r,b}^{IV1})_{b=1}^{B}$, $\vec{\hat{\sigma}}_r^{IV2} = (\hat{\sigma}_{r,b}^{IV2})_{b=1}^{B}$, $\vec{\hat{\sigma}}_a^{OLS} = (\hat{\sigma}_{a,b}^{OLS})_{b=1}^{B}$, $\vec{\hat{\sigma}}_a^{IV1} = (\hat{\sigma}_{a,b}^{IV1})_{b=1}^{B}$, and $\vec{\hat{\sigma}}_a^{IV2} = (\hat{\sigma}_{a,b}^{IV2})_{b=1}^{B}$. The standard deviations of $\vec{\hat{\sigma}}_r^{OLS}$, $\vec{\hat{\sigma}}_r^{IV1}$, and $\vec{\hat{\sigma}}_r^{IV2}$ are the standard errors in Table 5. The standard deviations of $\vec{\hat{\sigma}}_a^{OLS}$, $\vec{\hat{\sigma}}_a^{IV1}$, and $\vec{\hat{\sigma}}_a^{IV2}$ are the standard errors in Table 6.

F Tabulation of Fertility

I compute the data counterparts to babies per person of race r, age a and period t $\alpha_{r,a,t}$ in equation (8) in the following way. Note that the households are sampled in the census data, and information of all members in the sampled households are presumably recorded.

1. Fix census year t.

2. For each household i, count the number of 1-10 year-old children of the household head. Denote such number by b_i.

3. (a) If household i has both the household head and his or her spouse, apportion $x = 0.5b_i$ to the household head's race-age bin, and apportion $x = 0.5b_i$ to his or her spouse's race-age bin.[26]

[26]Since the household head and his or her spouse are in the same household, their locations must be the same.

(b) If household i has only the household head, and not his or her spouse, apportion $x = b_i$ to the household head's race-age bin.

4. Now I have a list of parents with various values of x. Sum x across all parents within each race-age-location bin. Let $b^i_{r,a,t}$ denote such summation of x for race-age-location bin (r, a, i).

5. Compute shares of babies of age bin a within race-time tuple (r, t), $\xi_{r,a,t}$,

$$\xi_{r,a,t} = \frac{b_{r,a,t}}{\sum_{a'} b_{r,a',t}}.$$

6. Let $L_{r,0,t}$ be the number of 1-10 year-old people of race r in period t. The babies per person of race-age tuple (r, a), $\alpha_{r,a,t}$ are

$$\alpha_{r,a,t} = L_{r,0,t} \cdot \xi_{r,a,t}.$$

Step 2 captures only 1-10 year-old children whose biological parent is the household head in their household. This may understate the number of children because they may live without biological parents or their parent may not be a household head. Step 5 and 6 aim to correct this understatement of the number of children. First I compute the relative importance of age a in reproduction $\xi_{r,a,t}$ within race-time tuple (r, t). Then I attribute all children $L_{r,0,t}$ of race-time tuple (r, t) to various ages within (r, t) using $\xi_{r,a,t}$. This yields babies per person $\alpha_{r,a,t}$ for race-age-time bin (r, a, t).

G Tabulation of Survival Probabilities

I assume that survival probabilities are common across locations within race-age-time bin (r, a, t). The source of survival probabilities is life tables in the website of CDC.[27] Age 0 in the model corresponds to age 1 to 10 in the data, so I do not consider infant mortality that is the probability of death before one becomes 1 year old. The life table provides the annual survival probability for each race-age bin (r, a), where ages are counted as $0, 1, \cdots$. But in my quantification of the model, one period is 10 years, and age bins are of 10-year windows.

I map survival probabilities in life tables to those in the setting of my model in the following way. Take any census year t and race r. My model assumes that people of age $\bar{a} = 6$ cannot survive to the next period, so I need to compute survival probabilities from

[27] https://www.cdc.gov/nchs/products/life_tables.htm (accessd on 09/10/2022)

age 0 to age $\bar{a}-1=5$. Pick up any age bin a from the 6 age bins that can survive to the next period. Notice that age bin a in the model includes people of the ages from $10a-9$ to $10a$ in the data. For example, age bin 3 is the set of people who are 21 to 30 years old. According to the life table of year t, the oldest within 10-year-window age bin a survive to the next census year $t+10$ with probability

$$s_{r,10a,t} \times s_{r,10a+1,t} \times \cdots \times s_{r,10a+9,t}, \tag{44}$$

where $s_{r,a',t}$ is the annual probability that people of race r, age a' (of 1-year windows) can survive to the next year in the life table of year t. The youngest within 10-year window age bin a survive to the next census year $t+10$ with probability

$$s_{r,10a-9,t} \times s_{r,10a-8,t} \times \cdots \times s_{r,10a,t}. \tag{45}$$

I take the average of probabilities (44) and (45), and obtain the 10-year-window survival probability of people of race r, 10-year age bin a, and time t.

H Immigrants from Abroad

The US census 1950 and the ACS 2010 report residential places 1 year ago. For each race r, age a, period $t=1950,2010$, and location i, I count the number of individuals who came from abroad (including Alaska and Hawaii) to location i in the last one year. Assuming that the number of immigrants is constant every year within 10-year windows, I multiply the number of immigrants in the last one year by 10 to obtain the number of immigrants in 10-year windows. The US censuses from 1960 to 1990 and the ACS in 2000 report residential places 5 years ago. Similarly, for each race r, age a, period $t=1960,\cdots,2000$, and location i, I count the number of individuals who came from abroad to location i in the last five years. Assuming that the number of immigrants is constant every 5-year window within 10-year windows, I multiply the number of immigrants in the last 5 years by 2 to obtain the number of immigrants in 10-year windows.

I Computation of Steady States

Given parameter values, I compute steady states by iterating populations $\{L^i_{r,a}\}^i_{r,a}$. To achieve a steady state, fertility $\alpha_{r,a}$ and survival probabilities $s_{r,a}$ are such that populations will not explode or shrink.

1. Guess populations $\{L^i_{r,a}\}^i_{r,a}$.

2. Given populations $\{L^i_{r,a,t}\}^i_{r,a}$, compute wages $\{w^i_{r,a}\}^i_{r,a}$, rent $\{r^i\}^i$, and eventually period indirect utilities $\{\bar{u}^i_{r,a}\}^i_{r,a}$, using (12), (14), and (2) respectively.

3. In steady state, expected values $\{V^i_{r,a}\}^i_{r,a}$ are fully characterized by period indirect utilities $\{\bar{u}^i_{r,a}\}^i_{r,a}$ by (5). Thus I get expected values $\{V^i_{r,a}\}^i_{r,a}$.

4. Given expected values $\{V^i_{r,a}\}^i_{r,a}$, compute migration shares $\{\mu^{j,i}_{r,a}\}^{j,i}_{r,a}$ using (6).

5. Given populations $\{L^i_{r,a}\}^i_{r,a}$ and migration shares $\{\mu^{j,i}_{r,a}\}^{j,i}_{r,a}$, update populations $\{\tilde{L}^i_{r,a}\}^i_{r,a}$ using (7) and (8).

6. Let $\epsilon > 0$ be a prespecified small number.

   (a) Go back to Step 1 with updated guesses $\{\tilde{L}^i_{r,a}\}^i_{r,a}$ if

$$\max_{r,a,i} \left| \frac{\tilde{L}^i_{r,a} - L^i_{r,a}}{L^i_{r,a}} \right| > \epsilon.$$

   (b) End the process with the converged populations $\{\tilde{L}^i_{r,a}\}^i_{r,a}$ otherwise.

J Computation of Transition Paths

I compute transition paths by value function iteration in the following way. Assume that the economy converges to a steady state in period T.

1. Compute the steady state expected values $\{V^i_{r,a,\infty}\}^i_{r,a}$ and populations $\{L_{r,a,\infty}\}^i_{r,a}$ as in Appendix I.

2. Load the steady state expected values and populations into those in period T. That is, for any race r, age a, location i,

$$V^i_{r,a,T} = V^i_{r,a,\infty},$$

$$L^i_{r,a,T} = L^i_{r,a,\infty}.$$

3. Load the populations from the 1940 data to those in the first period 0. That is, for any race r, age a, location i,

$$L^i_{r,a,0} = L^i_{r,a,1940}.$$

4. Guess expected values $\{V^i_{r,a,t}\}^i_{r,a}$ for $t = 0, \cdots, T-1$.

5. Given expected values $\{V^i_{r,a,t}\}^i_{r,a}$ for $t = 0, \cdots, T$, compute populations $\{L^i_{r,a,t}\}^i_{r,a}$ for $t = 1, \cdots, T-1$ forward from period 1 to period $T-1$, using (6), (7), (8).

6. Given populations $\{L^i_{r,a,t}\}^i_{r,a}$ for $t = 0, \cdots, T-1$, compute wages $\{w^i_{r,a,t}\}^i_{r,a}$, rent $\{r^i_t\}^i$, and eventually period indirect utilities $\{\bar{u}^i_{r,a,t}\}^i_{r,a}$ for $t = 0, \cdots, T-1$, using (12), (14), and (2) respectively.

7. Given period indirect utilities $\{\bar{u}^i_{r,a,t}\}^i_{r,a}$ for $t = 0, \cdots, T-1$ and expected values in the last period $\{V^i_{r,a,118}\}^i_{r,a}$, compute new expected values $\{\tilde{V}^i_{r,a,t}\}^i_{r,a}$ for $t = 0, \cdots, T-1$.

8. Let $\epsilon > 0$ be a prespecified small number.

   (a) Go back to Step 4 with updated guesses $\{\tilde{V}^i_{r,a,t}\}^i_{r,a}$ for $t = 0, \cdots, T-1$ if

   $$\max_{r,a,t,i} \left| \frac{\tilde{V}^i_{r,a,t} - V^i_{r,a,t}}{V^i_{r,a,t}} \right| > \epsilon.$$

   (b) End the process with the converged expected values $\{\tilde{V}^i_{r,a,t}\}^i_{r,a}$ for $t = 0, \cdots, T-1$ otherwise.

K Back-of-the-Envelope Calculation

I attempt to calculate the effects of the great Black migration on aggregate output (or labor income) (42) without a structural model. Individuals are classified to two races $r \in \{b, o\}$ and two regions $i \in \{N, S\}$, where b and o denote African Americans and others, and N and S denote the North and the South, respectively. Let $L^i_{r,t}$ and $w^i_{r,t}$ be the population and the wage of race r in region i and year t. As in Section 4, I use data of head counts and per capita payrolls as populations and wages, respectively. Then the actual aggregate labor income in 1970, labor income$_{1970}$, is

$$\text{labor income}_{1970} = L^N_{b,1970} \cdot w^N_{b,1970} + L^S_{b,1970} \cdot w^S_{b,1970} + L^N_{o,1970} \cdot w^N_{o,1970} + L^S_{o,1970} \cdot w^S_{o,1970}.$$

I seek the counterfactual aggregate labor income as of 1970 in the situation where the Black population is apportioned to the North and the South as in 1940. Let $L_{b,t} = L^N_{b,t} + L^S_{b,t}$ be the total Black population in year t. Then define $s^i_{b,1940}$ by

$$s^i_{b,1940} = \frac{L^i_{b,1940}}{L_{b,1940}}$$

for $i = N, S$. That is, $s^i_{b,1940}$ denotes the fraction of the Black population in region i in 1940. Then the counterfactual aggregate labor income in 1970, labor income$^{cf}_{1970}$, is

$$\text{labor income}^{cf}_{1970} = L_{b,1970} \cdot s^N_{b,1940} \cdot w^N_{b,1970} + L_{b,1970} \cdot s^S_{b,1940} \cdot w^S_{b,1970} + L^N_{o,1970} \cdot w^N_{o,1970} + L^S_{o,1970} \cdot w^S_{o,1970}.$$

From the census data, I obtain

$$\frac{\text{labor income}^{cf}_{1970}}{\text{labor income}_{1970}} = 0.9914.$$

Therefore, if the Black population was distributed across the North and the South as in 1940, aggregate labor income in 1970 would have been 0.84 percent lower. In Section 6, Figure 14 shows the quantitative model predicts that aggregate labor income would have been 0.74 percent lower without the North-South migration of African Americans between 1940 and 1970. These two numbers are in the same ballpark.