The Dornbusch-Fischer-Samuelson Model

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Outline

- The world consists of two countries: home (H) and foreign (F).
- Both countries can produce many goods (a unit continuum of goods).
  - But, whether they actually produce all goods in a continuum is a different problem.
  - In equilibrium, they do not.
- Each country has different productivity in producing a good.
- The world economy is perfectly competitive.
  - No externality, no tax, no friction, no distortion.
- How is the pattern of international trade and payment determined in such an economy?

Production

- Goods are distributed over $[0, 1]$.
- The only production factor is labor.
- For $z \in [0, 1]$, the unit labor requirements in H and F are $a(z)$ and $a^*(z)$, respectively.
- Unit labor requirement: how much labor is required to produce one unit of a good.
  - Three people are needed to produce a desk,
  - 123.45 work hours are needed to produced a software.
- For any good z, define relative unit labor requirement $A(z)$ by

$$A(z) = \frac{a^*(z)}{a(z)}.$$

- By reordering indices for goods, without loss of generality, we can assume that $A(\cdot)$ is weakly decreasing.
- We further assume that $A(\cdot)$ is strictly decreasing and continuous.

Free Trade and the Cutoff Good (1)

- Assume free trade; no cost is incurred to ship a good from one country to the other.
- Let the wage rates in H and F be w and w^*, respectively.
  - These wage rates can be measured in *any* common unit.
  - For example, good 0, good $\sqrt{2}/2$, labor in H or F.
- For any $z \in [0, 1]$, consumers in both countries purchase good z from the country serving it at a lower price.
- Producers in both countries sell goods at their marginal costs.

Free Trade and the Cutoff Good (2)

- Define relative wage ω by $\omega = \frac{w}{w^*}$.
- H produces good z if

$$a(z)w \leq a^*(z)w^* \;\Leftrightarrow\; \omega \leq A(z).$$

- F produces good z if

$$a^*(z)w^* \leq a(z)w \;\Leftrightarrow\; A(z) \leq \omega.$$

- Suppose that $A(0) \geq \omega \geq A(1)$.
  - Later we will verify this. That is, in equilibrium, $A(0) \geq \omega \geq A(1)$ does hold.
- Then, since $A(\cdot)$ is continuous and strictly decreasing, given ω, there exists one and only one element in $[0, 1]$, $\tilde{z}$, such that

$$\omega = A(\tilde{z}). \tag{1}$$

- Since $A(\cdot)$ is strictly decreasing and therefore invertible, we can define cutoff good $\tilde{z}$ as a function of ω

$$\tilde{z}(\omega) = A^{-1}(\omega).$$

Free Trade and the Cutoff Good (3)

- Note that H produces goods $z \in [0, \tilde{z}(\omega)]$ and that F produces goods $z \in [\tilde{z}(\omega), 1]$.

Consumers' Demands (1)

- We move on to the consumers' side.
- Consumers in both countries have the identical Cobb-Douglas utility function[1]

$$U = \int_0^1 b(z) \cdot \log(C(z)) dz, \tag{2}$$

where

  - $b(z) > 0$ is the parameter of the expenditure share on good z and satisfies

$$\int_0^1 b(z) dz = 1,$$

  - $C(z)$ is the consumption of good z.

[1] Here we are omitting the subscripts (or superscripts) for countries.

Consumers' Demands (2)

- Consumers maximize (2) subject to the budget constraint

$$\int_0^1 P(z)C(z)dz \leq Y,$$

  where
  - $P(z)$ is the price of good z,
  - Y is the income or total expenditure.
- Solving this utility maximization problem, the demand function satisfies

$$b(z) = \frac{P(z)C(z)}{Y}.$$

- The expenditure share on the goods produced in H, $\vartheta(\tilde{z})$, is

$$\vartheta(\tilde{z}) = \int_0^{\tilde{z}} b(z)dz \geq 0. \tag{3}$$

- By the fundamental theorem of calculus,

$$\vartheta'(\tilde{z}) = b(\tilde{z}) > 0.$$

Labor Market Clearing

or Trade Balance

- Both H and F spend the share $\vartheta(\tilde{z})$ on the goods produced in H.
- The incomes of H and F are wL and w^*L^*, respectively.
- Then how much H earns must be equal to how much the whole world pays to H

$$wL = \vartheta(\tilde{z})(wL + w^*L^*).$$

- Solving this for ω, we obtain

$$\omega = \frac{\vartheta(\tilde{z})}{1 - \vartheta(\tilde{z})} \cdot \frac{L^*}{L} = B(\tilde{z}; L^*/L). \tag{4}$$

  - $B(\cdot;\cdot)$ is strictly increasing in both arguments.
  - If $\tilde{z} = 0$, $B(\tilde{z}; L^*/L) = 0$. (See (3).)
  - As $\tilde{z} \uparrow 1$, $B(\tilde{z}; L^*/L) \to \infty$.[2]

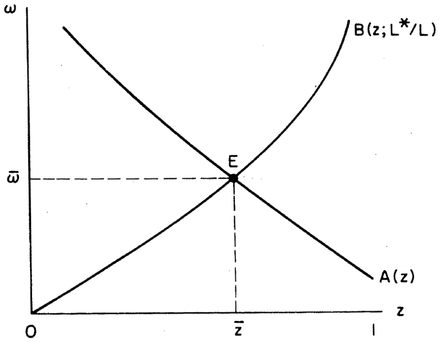
[2] $\tilde{z} \uparrow 1$ means that $\tilde{z}$ tends to one from below.

General Equilibrium

- Two equations (1) and (4) jointly determine the equilibrium cutoff good $\bar{z}$ and relative wage $\bar{\omega}$:

$$\bar{\omega} = A(\bar{z}) = B(\bar{z}; L^*/L).$$

- This closes the model.
- The equilibrium $(\bar{z}, \bar{\omega})$ is illustrated as point E below.



Showing $A(0) > \bar{\omega} > A(1)$: (1)

- In p.5, we conjectured that $A(0) \geq \bar{\omega} \geq A(1)$ in equilibrium.
- This holds. More strongly, $A(0) > \bar{\omega} > A(1)$ holds.
- For any $z \in [0, 1]$, define function $F(z; L^*/L)$ by

$$F(z; L^*/L) = A(z) - B(z; L^*/L).$$

- Since $A(\cdot)$ and $B(\cdot; L^*/L)$ are continuous, $F(\cdot; L^*/L)$ is continuous.
- Since $A(\cdot)$ is strictly decreasing and $B(\cdot; L^*/L)$ is strictly increasing, $F(\cdot; L^*/L)$ is strictly decreasing.

Showing $A(0) > \bar{\omega} > A(1)$: (2)

- Note that

$$F(0; L^*/L) = A(0) - B(0; L^*/L) = A(0) - 0 = A(0) > 0$$

$$\lim_{z \uparrow 1} F(z; L^*/L) = \lim_{z \uparrow 1} [A(z) - B(z; L^*/L)] = A(1) - \lim_{z \uparrow 1} B(z; L^*/L) = -\infty.$$

- Since $F(\cdot; L^*/L)$ is continuous and strictly decreasing, there exists one and only one $\bar{z} \in (0, 1)$ such that $F(\bar{z}; L^*/L) = 0$.
- For such $\bar{z} \in (0, 1)$, we define $\bar{\omega}$ by

$$\bar{\omega} = A(\tilde{z}) = B(\tilde{z}; L^*/L).$$

- Since $A(\cdot)$ is strictly decreasing,

$$A(0) > \bar{\omega} > A(1).$$