

Term Paper: Answer Key

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August 17, 2024

1 The Dornbusch-Fischer-Samuelson model

No. Suppose that $\omega = 3$ and that the function A is defined by

$$A(z) = \begin{cases} -9z + 6 & \text{for } 0 \leq z \leq 1/3, \\ 3 & \text{for } 1/3 \leq z \leq 2/3, \\ -9z + 9 & \text{for } 2/3 \leq z \leq 1. \end{cases}$$

A is continuous and weakly decreasing. For any $z \in [1/3, 2/3]$, $\omega = A(z)$ holds.

2 Caliendo and Parro (2015)

1. [Eaton and Kortum \(2002\)](#) consider only one tradeable sector but [Caliendo and Parro \(2015\)](#) consider multiple tradeable sectors. Moreover, [Caliendo and Parro \(2015\)](#) have input-output linkages across such multiple sectors.
2. Let X_{ni}^j be the trade value of sector j goods from country i to country n . Let θ^j be the trade elasticity of sector j . $\tilde{\tau}_{ni}^j$ denotes the gross tariff¹ for sector j goods from i to n . Then the econometric specification that [Caliendo and Parro \(2015\)](#) adopt is

$$\log \left(\frac{X_{ni}^j X_{ih}^j X_{hn}^j}{X_{in}^j X_{hi}^j X_{nh}^j} \right) = -\theta^j \log \left(\frac{\tilde{\tau}_{ni}^j \tilde{\tau}_{ih}^j \tilde{\tau}_{hn}^j}{\tilde{\tau}_{in}^j \tilde{\tau}_{hi}^j \tilde{\tau}_{nh}^j} \right) + \tilde{\epsilon}^j,$$

where $\tilde{\epsilon}^j$ is the error term.

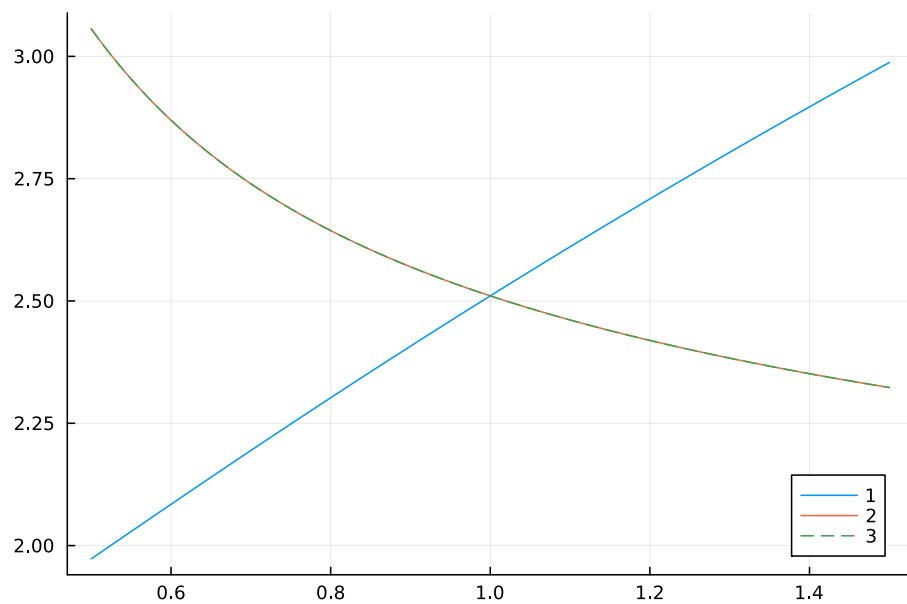
¹If the tariff rate is 30 percent, the gross tariff is 1.3.

3 Computing the Eaton-Kortum model

3.1 Comparative statics with respect to T_1

See Figure 1.

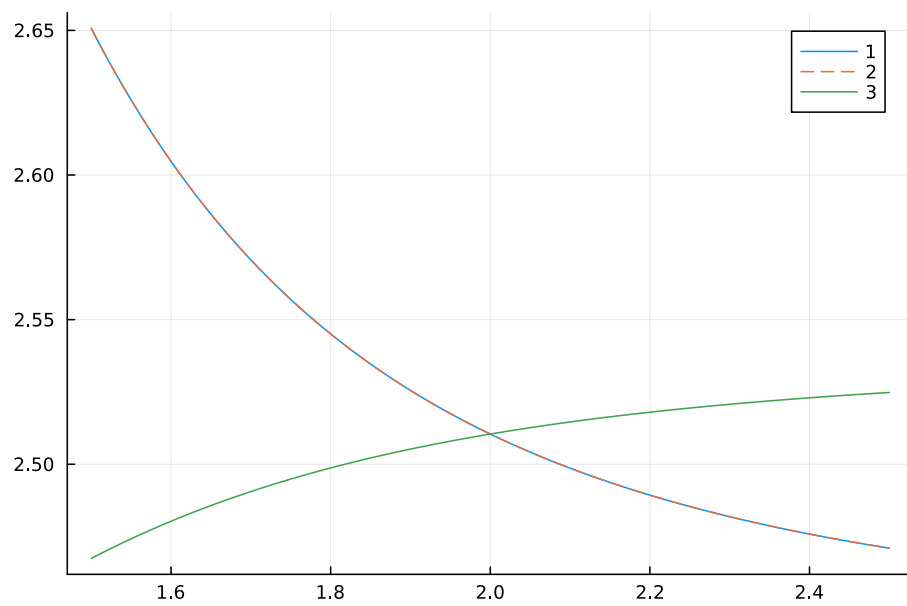
Figure 1: Comparative statics with respect to T_1



3.2 Comparative statics with respect to $d_{12} = d_{21}$

See Figure 2.

Figure 2: Comparative statics with respect to $d_{12} = d_{21}$



4 The Melitz-Chaney model

The cumulative distribution function of productivity in country i is

$$G_i(\varphi) = 1 - \left(\frac{\varphi}{\underline{\varphi}_i} \right)^{-\theta_i}$$

for $\varphi \geq \underline{\varphi}_i$. Then the probability density function is

$$g_i(\varphi) = \theta_i \underline{\varphi}_i^{\theta_i} \varphi^{-\theta_i-1}$$

for $\varphi \geq \underline{\varphi}_i$. The trade value from country i to j is

$$X_{ij} = \left(\frac{\sigma}{\sigma-1} \right)^{1-\sigma} \tau_{ij}^{1-\sigma} w_i^{1-\sigma} M_i \left(\int_{\varphi_{ij}^*}^{\infty} \varphi^{\sigma-1} dG_i(\varphi) \right) Y_j P_j^{\sigma-1}.$$

We need to compute $\int_{\varphi_{ij}^*}^{\infty} \varphi^{\sigma-1} dG_i(\varphi)$.

$$\begin{aligned} \int_{\varphi_{ij}^*}^{\infty} \varphi^{\sigma-1} dG_i(\varphi) &= \int_{\varphi_{ij}^*}^{\infty} \varphi^{\sigma-1} g_i(\varphi) d\varphi \\ &= \theta_i \underline{\varphi}_i^{\theta_i} \int_{\varphi_{ij}^*}^{\infty} \varphi^{\sigma-\theta_i-2} d\varphi \\ &= \frac{\theta_i \underline{\varphi}_i^{\theta_i}}{\theta_i + 1 - \sigma} (\varphi_{ij}^*)^{\sigma-\theta_i-1} \\ &= \frac{\theta_i \underline{\varphi}_i^{\theta_i}}{\theta_i + 1 - \sigma} \left(\frac{\sigma f_{ij}(\frac{\sigma}{\sigma-1} w_i \tau_{ij})^{\sigma-1}}{Y_j P_j^{\sigma-1}} \right)^{\frac{\sigma-\theta_i-1}{\sigma-1}}. \end{aligned}$$

Therefore, the trade value is

$$\begin{aligned} X_{ij} &= \left(\frac{\sigma}{\sigma-1} \right)^{1-\sigma} \tau_{ij}^{1-\sigma} w_i^{1-\sigma} M_i \left(\frac{\theta_i \underline{\varphi}_i^{\theta_i}}{\theta_i + 1 - \sigma} \left(\frac{\sigma f_{ij}(\frac{\sigma}{\sigma-1} w_i \tau_{ij})^{\sigma-1}}{Y_j P_j^{\sigma-1}} \right)^{\frac{\sigma-\theta_i-1}{\sigma-1}} \right) Y_j P_j^{\sigma-1} \\ &= C_{1,i} \underline{\varphi}_i^{\theta_i} (\tau_{ij} w_i)^{-\theta_i} f_{ij}^{\frac{\sigma-\theta_i-1}{\sigma-1}} M_i (Y_j P_j^{\sigma-1})^{\frac{\theta_i}{\sigma-1}}, \end{aligned}$$

where $C_{1,i} = \sigma^{\frac{\sigma-\theta_i-1}{\sigma-1}} \left(\frac{\sigma}{\sigma-1} \right)^{-\theta_i} \left(\frac{\theta_i}{\theta_i+1-\sigma} \right)$. Since $\theta_i > 0$, the trade value X_{ij} is increasing in $\underline{\varphi}_i$.

References

Caliendo, L. and Parro, F. (2015). Estimates of the trade and welfare effects of NAFTA. *The Review of Economic Studies*, 82(1):1–44.

Eaton, J. and Kortum, S. (2002). Technology, geography, and trade. *Econometrica*, 70(5):1741–1779.