The Aggregate Effects of the Great Black Migration*

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Abstract

In the United States, four million Black Americans migrated from the South to the North between 1940 and 1970. How did this great Black migration impact aggregate US output and the welfare of Black and non-Black Americans? To answer this question, I quantify an overlapping generations model of the spatial economy in which cohorts of Black and non-Black Americans migrate across states. I compare the baseline equilibrium matched with US data from 1940 to 2010 with counterfactual equilibria in which Black or non-Black Americans cannot relocate across the North and the South between 1940 and 1970. The mobility of Black and non-Black Americans increased aggregate output by 0.7 and 0.3 percent, respectively. Although Black Americans accounted for about 10 percent of the US population, their relocation impacted the aggregate economy more than the relocation of the other 90 percent did. The mobility of Black Americans induced a large increase in the welfare of Black Americans in the South, a small decrease in the welfare of Black Americans in the North, and little change in the welfare of non-Black Americans.

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JEL Classification: R13, J61, N12, N32

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1 Introduction

Slavery was a place-based policy in the history of the United States. Before Emancipation, about 70 to 80 percent of the Black population in the US resided in the South, and the vast majority of them were enslaved. The largest change in the spatial distribution of the Black population occurred from 1940 to 1970. In this period, 4 million African Americans migrated from the South to the North, and the fraction of the Black population in the South dropped from 69 percent to 45 percent. This is called the great Black migration.

How did the great Black migration impact aggregate US output and the welfare of cohorts of African Americans and others? To answer this question, I develop and quantify a dynamic general equilibrium model in which cohorts of Black and non-Black Americans migrate across states. Specifically, I provide a unified general equilibrium framework that incorporates overlapping generations who live, work, and migrate over multiple periods, imperfect substitution between Black and non-Black workers, and imperfectly elastic housing supply in each location.

In the period of the great Black migration, Black Americans in the South were more likely to move to the North than non-Black Americans in the South were. 44 percent of Black Americans born in the South in the 1930s migrated to the North, whereas only 17 percent of non-Black Americans born in the South in the same decade did so. Accordingly, my model delivers different migration patterns across races.

In the great migration period, Black Americans who moved from the South to the North earned much higher wages than Black Americans who stayed put in the South. The degree of this wage gap between movers and stayers was higher for Black Americans from the South than for any other group of people. These facts suggest that there was pecuniary incentive for the great Black migration. I primarily attribute wages to productivity parameters flexibly varying at race, age, time, and location levels to capture heterogeneous pecuniary incentive for migration.

Higher housing rent in the North partly offset higher wages in the North. But, in the great migration period, only about one fourth of the wage gap between movers and stayers for Black Americans from the South was absorbed by higher rent in the North. In the model, I assume that individuals spend fixed expenditure shares on freely tradeable goods and locally supplied housing via the Cobb-Douglas period utility function. Therefore, in the model, real wages are nominal wages divided by the power function of housing rent. The supply of housing is imperfectly elastic, so it serves as a congestion force in the local economy.

The great Black migration did not make everyone better off. The migration of Black Americans to the North increased the Black labor force and put stronger downward pressure on Black Americans' wages than on non-Black Americans' wages in the North. Therefore the great Black migration made worse off Black Americans who had already lived in the North (Boustan, 2009). Allowing for imperfect substitutability across Black and non-Black Americans, my general equilibrium model generates such ramifications of the great Black migration in the North.

Since people of different cohorts and ages migrate differently in data, the model includes overlapping generations, so that the model has distinct notions of cohorts and ages. Black Americans have lower survival probabilities and life expectancies than non-Black Americans in US data. Accordingly, Black and non-Black Americans of the same cohort face different survival probabilities and life expectancies in the model.

I parameterize the model in two steps. In the first step, I estimate various elasticities. These elasticities include migration elasticity mapping percent changes in real wages and non-pecuniary amenities to percent changes in migration flows, elasticity of substitution across ages and races in the production function, and rent elasticity mapping a percent change in aggregate local income to a percent change in housing rent. In so doing, I follow standard methods in trade, labor, and urban economics literature (Artuc and McLaren, 2015; Borjas, 2003; Card, 2009; Glaeser et al., 2005; Saks, 2008). My estimate for the elasticity of substitution between Black and non-Black Americans is about 9.0, which falls in the range of the estimates by Boustan (2009) from 8.3 to 11.1.

In the second step, I back out the other parameters such as amenities, productivity, and migration costs for different age and racial groups across states over time. Rent shifters, which govern levels of housing rent given aggregate local income, are also recovered. The model delivers explicit formulae to pin down these parameters given elasticities and relevant data, as the models in Allen and Arkolakis (2014) and Ahlfeldt et al. (2015) do. The migration costs thus backed out are higher for Black Americans than for non-Black Americans, but the racial gap in the migration costs shrank over time.

Armed with the parameter values, I compare the baseline equilibrium that resembles the factual path of the US economy to two counterfactual equilibria. In the first counterfactual equilibrium, Black Americans could not migrate across the North and the South between 1940 and 1970 (the no Black migration scenario). In the second counterfactual equilibrium, non-Black Americans could not migrate across the North and the South for the same period (the no non-Black migration scenario). The first counterfactual helps me understand the role of the great Black migration in the US economy. Comparing the first and second counterfactuals contrast the role of Black migration to the role of non-Black migration.

In the no Black migration scenario, aggregate US output in 1970 would have been lower by 0.73 percent than in the baseline equilibrium. In the no non-Black migration scenario, aggregate output in 1970 would have been lower by 0.28 percent. Therefore, although Black Americans accounted for about 10 percent of the US population, their migration had a larger impact on the aggregate economy than the migration of the other 90 percent of the population did.

If the great Black migration did not occur, fewer Black Americans would have worked in the North. Because of the imperfect substitutability across races, this would have put upward pressure on the wage of Black workers in the North. Indeed, in the no Black migration scenario, the average wage of Black Americans in the North would have been higher by 5.2 percent than in the baseline equilibrium. This number is somewhat smaller than, but comparable to, the predictions made by Boustan (2009) from 7.2 to 9.6.[1]

I measure welfare changes from the baseline to counterfactual equilibriua by consumption

[1] See her table 6.

equivalent. In the no Black migration scenario, the welfare for Black Americans born in Mississippi in the 1930s would have been 2.9 percent lower than in the baseline equilibrium. This is because they lost opportunities of migrating to the productive, high-wage North. In the no Black migration scenario, the welfare for Black Americans born in Illinois in the 1930s would have been 0.2 percent higher. This is because the wage of Black Americans in the North would have been higher in the no great migration scenario than in the baseline equilibrium. The welfare of non-Black Americans in the no great Black migration scenario is not substantially different from the welfare of non-Black Americans in the baseline equilibrium.

In the no non-Black migration scenario, the welfare of non-Black Americans born in Mississippi in the 1930s would have been 1.1 percent lower than in the baseline equilibrium. This number is smaller than 2.9 percent, the welfare loss of Black Americans born in Mississippi from the baseline equilibrium to the no Black migration scenario. These two counterfactual experiments highlight Black Americans' strong incentive for the outmigration from the South. In the no non-Black migration scenario, the welfare of non-Black Americans born in Illinois in the 1930s would have been 0.3 percent lower than in the baseline equilibrium. Non-Black workers in Illinois were already in the productive location, but they forwent varieties in location choices in the no non-Black migration scenario, leading to the welfare loss.

My quantitative model speaks to racial inequality. In the no Black migration scenario, the nationwide average *nominal* wage ratio between Black and non-Black Americans in 1970 would have been 10.2 percent lower than in the baseline equilibrium. This is in line with the prediction by Smith and Welch (1989), who adopt a reduced-form decomposition technique to measure the impact of the great Black migration on nominal wage gaps between Black and white Americans. In the no Black migration scenario, the nationwide average *real* wage ratio between Black and non-Black Americans in 1970 would have been 8.8 percent lower than in the baseline equilibrium.[2] Therefore, my quantitative model suggests that the great Black migration substantially reduced the racial gap in nominal and real wages.

This paper relates to two strands of literature. First, this paper contributes to the literature on the great Black migration and economic geography of Black Americans in the US, including Smith and Welch (1989), Gregory (2006), Boustan (2009, 2010, 2017), Black et al. (2015), Chay and Munshi (2015), Derenoncourt (2022), Calderon et al. (2022), Althoff and Reichardt (2024). To the best of my knowledge, this paper is the first to quantify the aggregate, general equilibrium effects of the great Black migration.

Among the papers I have listed, the most related is Boustan (2009). She estimates a constant elasticity of substitution (CES) production function and finds imperfect substitutability between Black and white Americans. She extrapolates the estimated production function to predict what Black Americans' wages would have been in the North if the great Black migration did not occur. This paper integrates her idea of imperfect substitutability across races into a quantitative general equilibrium setting.

[2] Smith and Welch (1989) do not discuss the impact of the great Black migration on the racial gap in real wages.

Second, I take advantage of the recent development of quantitative general equilibrium models of the dynamic spatial economy, including Desmet and Rossi-Hansberg (2014), Caliendo et al. (2019), Kleinman et al. (2023), Allen and Donaldson (2022), Eckert and Peters (2022), Pellegrina and Sotelo (2022), and Takeda (2022). Following Allen and Donaldson (2022), Eckert and Peters (2022), Pellegrina and Sotelo (2022), and Takeda (2022), the model has an overlapping generations structure in the spatial economy. My model differs from theirs in that individuals work for more than one period because decennial US census data enable me to keep track of wages, residential places, and migration of cohorts for multiple decades. In comparison to the existing literature, my model allows for different productivity, amenities, migration costs, and survival probabilities across ages and races, delivering heterogeneous migration patterns across ages and races.[3]

After this paper came out, Yang (2024) also applied a dynamic spatial framework to the great Black migration. His focus is on capital-labor substitution and factor-biased technological change caused by the great Black migration. Regarding the models, there are three main differences. (i) Black and non-Black workers are imperfectly substitutable in my paper, whereas they are perfectly substitutable in his paper; (ii) overlapping generations exist in my paper, while he considers perpetually-lived individuals; (iii) there is no capital in my paper, while capital is accumulated in his paper.

The remainder of the paper is organized as follows. Section 2 describes motivating facts including the data mentioned in this introduction. Section 3 lays out the model. In Section 4, I estimate the elasticities and back out the other parameters. Section 5 discusses the fit of the model with the data. Section 6 compares the baseline equilibrium with counterfactual equilibria. Section 7 concludes.

2 Motivating Facts

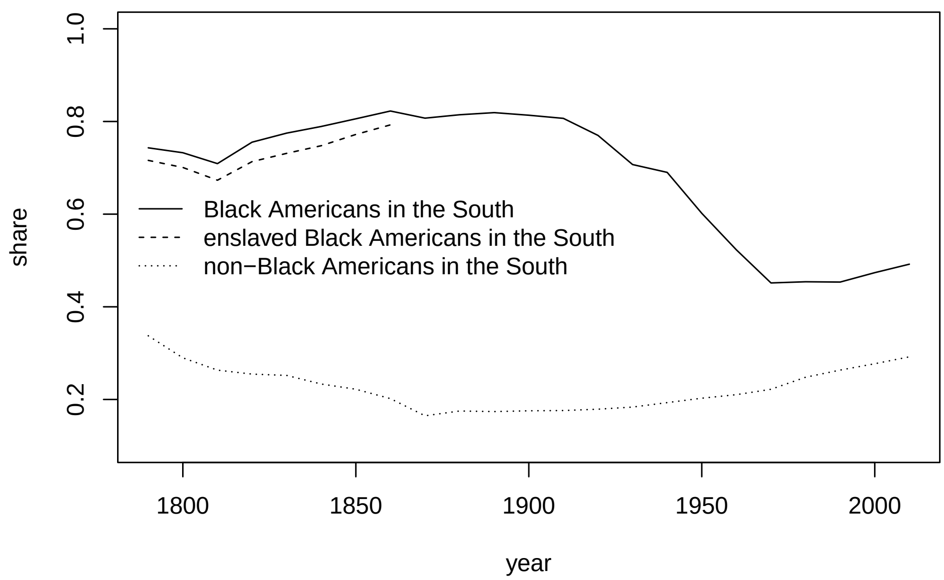
How have Black Americans been spatially distributed in the US? Before Emancipation, about 70 to 80 percent of Black Americans in the US lived in the South,[4] as the solid line in Figure 1 shows. The dashed line shows the fraction of enslaved Black Americans in the South relative to the total number of Black Americans in the US. The vast majority of Black Americans in the South were enslaved. In the meantime, only about 20 to 30 percent of the people other than Black Americans (henceforth, non-Black Americans) in the US resided in the South, as in the dotted line. The fraction of Black Americans in the South stayed high around 80 percent even after Emancipation.

Black Americans started leaving the South to the North circa 1910, and the fraction of Black Americans in the South dropped from 81 percent to 71 percent between 1910 and 1930. This amounts to the migration of 1.5 million Black Americans from the South to the North and is called the first great Black migration. Following the Great Depression, Black migration paused for a

[3] Suzuki (2021) integrates heterogeneous migration costs and survival probabilities across ages into the model of Caliendo et al. (2019).

[4] The US refers to the area of the current US states except Alaska and Hawaii (the contiguous US). The South refers to all confederate states. The North refers to the area of the contiguous US except the South.

Figure 1: Fractions of Black and non-Black Populations in the South



Notes: The solid line is the ratio of the number of Black Americans in the South to the total number of Black Americans in the US. The dashed line is the ratio of the number of enslaved Black Americans in the South to the total number of Black Americans in the US. The dotted line is the ratio of the number of non-Black Americans in the South to the total number of non-Black Americans in the US. Sources: US census 1940-2000, American Community Survey 2010.

decade. The largest migration occurred after the pause. The fraction of Black Americans in the South declined from 69 percent to 45 percent between 1940 and 1970. 4 million Black Americans left the South to the North in this period. This is called the second great Black migration on which this paper focuses.[5]

To see migration behavior by demographic group, I define movers and stayers for races (Black and non-Black Americans), birthplaces (the North or the South), and cohorts.[6] I consider 10-year windows as cohort bins and call those who were born in 1930-1939 as cohort 1930, and so on.[7] For each cohort c, I collect the individuals who lived in either the North or the South as of year $c+50$. Then for each race, birthplace, and cohort c,

- movers are the individuals who lived in the other place than the birthplace as of year $c+50$,
- stayers are the individuals who lived in the birthplace as of year $c+50$.

[5]The second great Black migration is often just referred to as the great Black migration. See footnote 1 of Derenoncourt (2022).

[6]See Figure 1 of Black et al. (2015) and Chapter 1 of Boustan (2017) for earlier tabulation of the great Black migration by cohort.

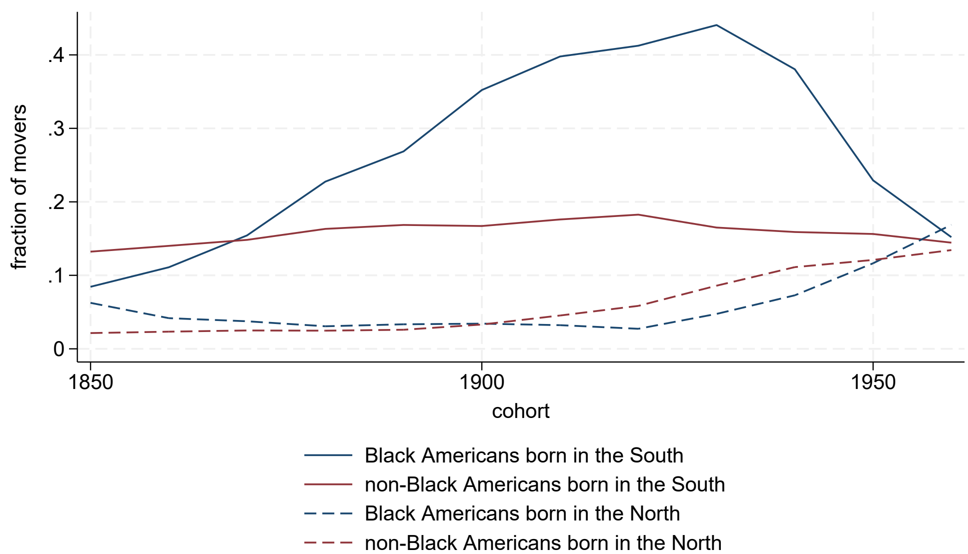
[7]Formally, I refer to the individuals who were born in the period from year c to year $c+9$ as cohort c for calendar year c whose 4th digit is zero.

For each race r, birthplace p, and cohort c, I compute

$$\frac{\text{movers}_{r,p,c}}{\text{movers}_{r,p,c} + \text{stayers}_{r,p,c}}, \tag{1}$$

where $\text{movers}_{r,p,c}$ and $\text{stayers}_{r,p,c}$ denote the numbers of movers and stayers for race r, birthplace p, and cohort c, respectively. I call the ratio (1) as the fraction of movers.

Figure 2: Fractions of Movers for Races, Cohorts, and Birthplaces

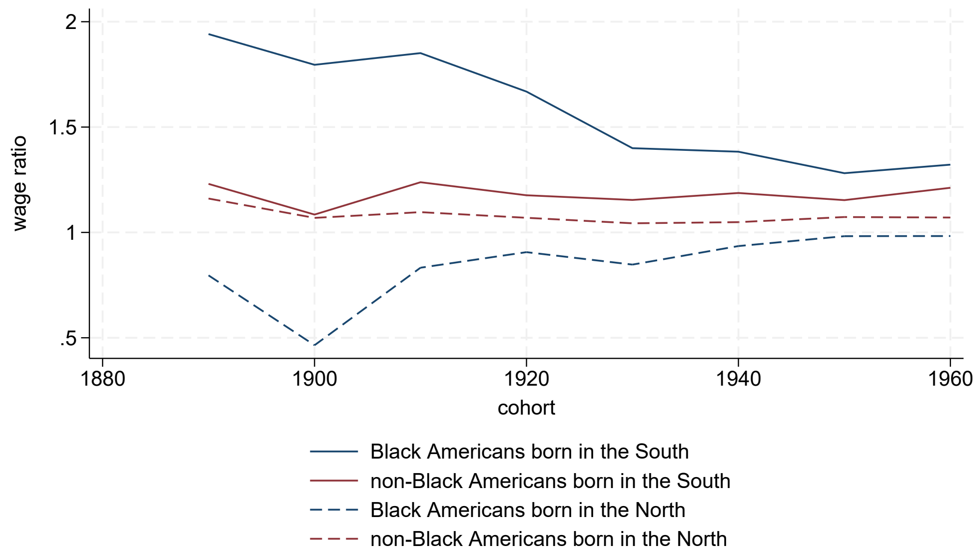


Notes: Cohort 1850 refers to those who were born in 1850-1859, and so on. For each cohort (say c), race, and birthplace (the North or the South) tuple, the fraction of movers is the ratio of the number of movers to the sum of the numbers of movers and stayers. Sources: US census 1900-2000, American Community Survey 2010.

The migration patterns of Black Americans from the South were different from the migration patterns of other groups of people. Figure 2 provides the fractions of movers for Black and non-Black Americans born in the North or the South. The fraction of movers for Black Americans born in the South exhibits remarkable changes over time. It steadily increased from cohort 1850 and peaked at 0.44 in cohort 1930. That is, over 40 percent of Black Americans born in the South in the 1930s moved to the North by 1980. After that, the fraction of movers for Black Americans born in the South sharply declined to 0.15. The trajectory of the fraction of movers for Black Americans born in the South highlights different migration behavior across cohorts within the race-birthplace bin. This motivates the model with a notion of cohorts in Section 3. The trajectory of the fraction of movers for Black Americans born in the South is clearly different from the trajectory of non-Black Americans born in the South, which was always around 0.15. This suggests that Black and non-Black Americans had different economic incentive for the migration from the South to the North. The fractions of movers for Black and non-Black Americans born in the North exhibit similar patterns. The fractions of movers for these two groups were stable and less than

0.05 from cohort 1860 to cohort 1910. After that, the fractions of movers increased and reached 0.15 and 0.13 for Black and non-Black Americans in cohort 1960, respectively. The recent net migration from the North to the South is called the reverse great migration. But its magnitude is smaller than the magnitude of the original great migration.

Figure 3: Mover-Stayer Wage Ratios for Cohorts, Races, and Birthplaces



Notes: For each cohort (say c), race, and birthplace (the North or the South), this graph provides the ratio of the average wage of movers to the average wage of stayers as of year $x + 50$. Sources: US census 1940-2000, American Community Survey 2010.

In the great migration period, Black Americans who moved from the South to the North earned much higher nominal wages than Black Americans who stayed put in the South. To see this, I compute a measure I call the mover-stayer wage ratio. For each cohort c, race, and birthplace, I compute the ratio of the average wage of movers to the average wage of stayers as of year $c + 50$.[8] Figure 3 provides mover-stayer wage ratios for race-birthplace tuples. As in the blue solid line, Black Americans who moved from the South to the North earned 79 to 94 percent higher wages than Black Americans who stayed in the South from cohort 1890 to cohort 1910. This wage differential is extremely high compared with the other race-cohort-birthplace tuples. Since cohort 1910, the mover-stayer wage ratio for Black Americans born in the South declined and reached 1.39 in cohort 1930. That is, on average, Black Americans who moved from the South to the North earned 39 percent higher wages than Black Americans who stayed in the South at the peak of the great migration. Since then, the mover-stayer wage ratio for Black Americans born in the South moderately declined. As the blue dashed line shows, Black Americans who moved from the North to the South earned 17 to 54 percent lower wages than Black Americans who stayed in the North

[8] The average wage means the average of the wages of all individuals who earn positive wages for each race, birthplace, and cohort tuple. In Appendix A, Figure 16 reports the mover-stayer ratios of per capita payrolls and yields a similar result.

from cohort 1890 to cohort 1910. The mover-stayer ratio for Black Americans born in the North increased after cohort 1910 and is 0.98 in cohort 1960. As in the red solid and dashed lines, the mover-stayer wage ratios for non-Black Americans were relatively stable over time: 1.08 to 1.24 for non-Black Americans born in the South and 1.04 to 1.16 for non-Black Americans born in the North.

Housing rent in the North was higher than housing rent in the South, but the rent gap absorbed only a small part of the mover-stayer wage gap for Black Americans from the South. For each cohort c, the first row of Table 1 shows

$$\text{movers' wage} - \text{stayers' wage}.$$

for Black Americans' born in the South. Similarly, the second row of Table 1 shows

$$\text{movers' rent} - \text{stayers' rent}.$$

Both of wages and rent are deflated by the consumer price index and measured in 2010 US dollars. For cohort 1890, the magnitude of the rent gap was comparable to the magnitude of the wage gap. But for cohorts 1920-1940, the rent gaps were only about one-fourth of the wage gaps. Therefore, at the peak of the great Black migration, the rent gap between the North and the South was unlikely to absorb the mover-stayer wage gaps for Black Americans born in the South.[9]

Table 1: Wage and Rent Gaps between Movers and Stayers for African Americans Born in the South

cohort	1890	1900	1910	1920	1930	1940	1950	1960
wage gaps	5,479	7,232	10,222	12,839	9,826	11,543	9,714	10,789
rent gaps	2,978	2,365	2,644	2,875	2,303	2,987	2,183	2,765

Notes: For cohort c, the first row refers to the average wage of movers minus the average wage of stayers as of year $c+50$ for Black Americans born in the South. Analogously, for cohort c. the second row refers to the average rent of movers minus the average rent of stayers as of year $c+50$ for Black Americans born in the South. Wages and rent are deflated by the consumer price index and measured in 2010 US dollars.

I summarize the empirical facts I have described so far to four points.

1. The migration rate of Black Americans from the South in the great migration period was higher than the migration rate of people of the other race-birthplace-cohort tuples.

2. Black Americans who moved from the South to the North in the great migration period earned much higher wages than Black Americans who stayed put in the South.

3. The mover-stayer wage gap for Black Americans from the South in the great migration period

[9]Wages are defined for individuals, but rent is defined for households. So I match housing rent with the household head's race, cohort, birthplace, and current place bins. If a household has multiple wage-earners, the mover-stayer rent gap relative to the mover-stayer wage gap can be even smaller at household levels.

was higher than the mover-stayer wage gap for people of the other race-birthplace-cohort tuples.

4. The mover-stayer rent gap accounted for only about one fourth of the mover-stayer wage gap for Black Americans from the South in the great migration period.

3 Model

I develop a dynamic general equilibrium model that delivers different migration patterns across races and cohorts over time. Individuals of different races and cohorts migrate, taking into account the future flows of real wages and non-pecuniary amenities in potential destinations, and migration costs across locations.

3.1 Environment

The economy consists of a finite set of locations $\mathcal{N}$. Let $N = |\mathcal{N}|$, that is, N is the number of locations. Time is discrete and denoted by $t = 0, 1, \cdots$. Goods are perishable in each period. Individuals cannot save their income.

Individuals are characterized by race r, age a, and location i in period t. The set of races is $\{b, o\}$, where b and n denote Black and non-Black Americans, respectively. The set of ages is $\{0, 1, \cdots, \bar{a}\}$, where $\bar{a} > 0$ denotes the age of the oldest group in each period. Individuals can live through at most age $\bar{a}$, but they may die before age $\bar{a}$ due to exogenous survival probabilities. Specifically, individuals of race r and age a in period t can survive to period $t+1$ with probability $s_{r,a,t}$. Note that the maximum periods of life is $\bar{a}+1$.

I can trace trajectories of individuals' behavior by cohort. Individuals of cohort c are born in period c. If all relevant survival probabilities are strictly greater than 0,[10] some of them survive up to period $c+\bar{a}$. Individuals of cohort c are age 0 in period c, age 1 in period $c+1$, $\cdots$, age $\bar{a}$ in period $c+\bar{a}$. Thus tracing behavior of individuals of these age-period pairs pins down the life course of cohort c.

Individuals' only source of income is their wages. They supply a fixed length of work hours in each period and earn the market wage (no intensive margin of the labor supply). Individuals of age 0 do not work. Individuals of ages $1, \cdots, \bar{a}$ work.

3.2 Period Utility

The period utility of individuals of race r and age a in period t and location i, $u^i_{r,a,t}$, is

$$u^i_{r,a,t} = \begin{cases} 0 & \text{for } a = 0, \\ \log C^i_{r,a,t} + \log B^i_{r,a,t} & \text{for } a = 1, \cdots, \bar{a}, \end{cases} \tag{2}$$

[10] Specifically, $s_{r,a,c+a} > 0$ for any $a = 0, 1, \cdots, \bar{a}-1$.

where $C^i_{r,a,t}$ is the consumption of individuals of race r, age a in period t and location i (henceforth individuals of (r,a,t,i)), and $B^i_{r,a,t}$ is the exogenous parameter of the amenites for individuals of (r,a,t,i).

For age $a = 1,\cdots,\bar{a}$, workers of (r,a,t,i) consume the Cobb-Douglas composite of homogeneous goods and housing

$$C^i_{r,a,t} = \left(\frac{G^i_{r,a,t}}{1-\gamma}\right)^{1-\gamma}\left(\frac{H^i_{r,a,t}}{\gamma}\right)^{\gamma}, \tag{3}$$

where $G^i_{r,a,t}$ and $H^i_{r,a,t}$ are the consumption of homogeneous goods and housing by the individuals of (r,a,t,i), and γ is the exogenous parameter for the expenditure share on housing. Homogeneous goods are freely tradeable across locations. Housing is not tradeable across locations. Homogeneous goods are the numeraire in each period. Let r^i_t be the unit rent in location i and period t. Then for age $a = 1,\cdots,\bar{a}$, individuals of (r,a,t,i) are subject to the following budget constraint

$$G^i_{r,a,t} + r^i_t H^i_{r,a,t} \leq w^i_{r,a,t}, \tag{4}$$

where $w^i_{r,a,t}$ is the nominal wage of the individuals of (r,a,t,i). Since the amenities $B^i_{r,a,t}$ are exogenous, the maximization of the period utility (2) (or, equivalently, (3)) subject to the budget constraint (4) yields the demand functions for homogeneous goods and housing service $G^i_{r,a,t} = (1-\gamma)w^i_{r,a,t}$ and $H^i_{r,a,t} = \gamma w^i_{r,a,t}/r^i_t$. Substituting these demands into the composite (3), the consumption level is equalized to the real wage

$$C^i_{r,a,t} = \frac{w^i_{r,a,t}}{(r^i_t)^{\gamma}}.$$

Substituting this into the period utility yields the indirect period utility

$$\bar{u}^i_{r,a,t} = \begin{cases} 0 & \text{for } a = 0, \\ \log\left(\frac{w^i_{r,a,t}}{(r^i_t)^{\gamma}}\right) + \log B^i_{r,a,t} & \text{for } a = 1,\cdots,\bar{a}. \end{cases}$$

3.3 Values

Individuals of age 0 to $\bar{a}-1$ make migration decisions, and arrive in destinations next period. Individuals of age $\bar{a}$ do not make migration decisions because they are not alive next period. The value of individuals of (r,a,t,i), $v^i_{r,a,t}$, is

$$v^i_{r,a,t} = \begin{cases} \bar{u}^i_{r,a,t} + \max_{j\in\mathcal{N}}\left\{s_{r,a,t}E[v^j_{r,a+1,t+1}] - \tau^{j,i}_{r,a,t} + \nu\epsilon^j_{r,a,t}\right\} & \text{for } a = 0,\cdots,\bar{a}-1, \\ \bar{u}^i_{r,a,t} & \text{for } a = \bar{a}. \end{cases}$$

where $\tau^{j,i}_{r,a,t}$ is the migration cost for individuals of race r and age a in period t from location i to location j, $\epsilon^j_{r,a,t}$ is the idiosyncratic preference shock, and ν adjusts the variance of the idiosyncratic preference shock. The expectation is taken over the next period's idiosyncratic preference shocks $\epsilon^k_{r,a+1,t+1}$ for $k \in \mathcal{N}$.

Assume that the idiosyncratic preference shock $\epsilon^j_{r,a,t}$ independently and identically follows the Type-I extreme value distribution $F(x) = \exp(-\exp(x))$ across all infinitesimal individuals. Then the expected value of workers of (r,a,t,i), $V^i_{r,a,t} = E[v^i_{r,a,t}]$, is

$$V^i_{r,a,t} = \begin{cases} \bar{u}^i_{r,a,t} + \nu \log\left(\sum_{j\in\mathcal{N}} \exp(s_{r,a,t} V^j_{r,a+1,t+1} - \tau^{j,i}_{r,a,t})^{1/\nu}\right) & \text{for } a = 0,\ldots,\bar{a}-1, \\ \bar{u}^i_{r,a,t} & \text{for } a = \bar{a}. \end{cases} \tag{5}$$

3.4 Migration

For age $a = 0,\cdots,\bar{a}-1$, the fraction of individuals of (r,a,t,i) who migrate to location j, $\mu^{j,i}_{r,a,t}$, is

$$\mu^{j,i}_{r,a,t} = \frac{\exp(s_{r,a,t} V^j_{r,a+1,t+1} - \tau^{j,i}_{r,a,t})^{1/\nu}}{\sum_{k\in\mathcal{N}} \exp\left(s_{r,a,t} V^k_{r,a+1,t+1} - \tau^{k,i}_{r,a,t}\right)^{1/\nu}}. \tag{6}$$

I call $\mu^{j,i}_{r,a,t}$ the migration share.

3.5 Populations

Then for $a = 1,\cdots,\bar{a}$, the population of (r,a,t,i) is

$$L^i_{r,a,t} = \sum_{j\in\mathcal{N}} \mu^{j,i}_{r,a-1,t-1} s_{r,a-1,t-1} L^j_{r,a-1,t-1} + I^i_{r,a,t}, \tag{7}$$

where $I^i_{r,a,t}$ denotes the number of immigrants of race r and age a who arrive from abroad in location i in period t.[11] Individuals of age 0 are born according to

$$L^i_{r,0,t} = \sum_{a=1}^{\bar{a}} \alpha_{r,a,t} L^i_{r,a,t}, \tag{8}$$

where, as before, the second subscript of $L^i_{r,0,t}$ denotes age (0), and $\alpha_{r,a,t}$ denotes the exogenous parameter of how many individuals of age 0 are born per person of race r and age a in period t.

3.6 Firms and Wages

A representative firm exists in each location. The firm sells homogeneous goods in the competitive good market and hires individuals of various races and ages from its location. The production function of the firm in location i is

$$Y^i_t = A^i_t L^i_t, \tag{9}$$

where A^i_t is the parameter of the productivity in location i and period t, and L^i_t is the labor input in location i and period t. L^i_t has a nested CES structure. At the outer nest, L^i_t aggregates labor of

[11] More precisely, this includes immigrants and US citizens (return migrants) who were not in the US in period $t-1$ but in period t. See Subsection 4.4 for its data counterpart.

different age groups within period t and location i

$$L_t^i = \left(\sum_{a=1}^{\bar{a}} (\kappa_{a,t}^i)^{\frac{1}{\sigma_0}} (L_{a,t}^i)^{\frac{\sigma_0 - 1}{\sigma_0}} \right)^{\frac{\sigma_0}{\sigma_0 - 1}}, \tag{10}$$

where $\kappa_{a,t}^i$ is the parameter of the productivity of individuals of age a in period t and location i, and σ_0 is the parameter of the elasticity of substitution across age groups within location-period bins. Then, for $a = 1, \cdots, \bar{a}$, $L_{a,t}^i$, in turn, aggregates labor of different racial groups within age a, period t, and location i

$$L_{a,t}^i = \left(\sum_{r \in \{b,n\}} (\kappa_{r,a,t}^i)^{\frac{1}{\sigma_1}} (L_{r,a,t}^i)^{\frac{\sigma_1 - 1}{\sigma_1}} \right)^{\frac{\sigma_1}{\sigma_1 - 1}}, \tag{11}$$

where $\kappa_{r,a,t}^i$ is the parameter of the productivity of individuals of race r and age a in period t and location i, and σ_1 is the parameter of the elasticity of substitution across races within age-period-location bins. This production function is similar to, but different from Boustan (2009). She controls for education, but I do not. She considers one representative producer in the entire North (which is her only geographic location), whereas I consider different producers in different geographic locations.

The firm in location i solves the following profit maximization problem

$$\max_{\{L_{r,a,t}^i\}_{r,a}} A_t^i L_t^i - \sum_a \sum_r w_{r,a,t}^i L_{r,a,t}^i.$$

The first-order conditions imply that wages are priced at the marginal product of labor

$$\begin{aligned} w_{r,a,t}^i &= A_t^i \frac{\partial L_t^i}{\partial L_{a,t}^i} \frac{\partial L_{a,t}^i}{\partial L_{r,a,t}^i} \\ &= A_t^i (L_t^i)^{\frac{1}{\sigma_0}} (\kappa_{a,t}^i)^{\frac{1}{\sigma_0}} (L_{a,t}^i)^{-\frac{1}{\sigma_0} + \frac{1}{\sigma_1}} (\kappa_{r,a,t}^i)^{\frac{1}{\sigma_1}} (L_{r,a,t}^i)^{-\frac{1}{\sigma_1}}. \end{aligned} \tag{12}$$

Note that migration decisions are made one period ahead, so L_t^i, $L_{a,t}^i$, $L_{r,a,t}^i$ are all predetermined from the viewpoint in period t.

3.7 Rent

Let H_t^i be the quantity of housing in location i and period t. Then the housing market clearing condition is

$$r_t^i H_t^i = \gamma \sum_{r \in \{b,n\}} \sum_{a=1}^{\bar{a}} L_{r,a,t}^i w_{r,a,t}^i, \tag{13}$$

where the right-hand side is the Cobb-Douglas expenditure share on housing service γ multiplied by the total income in location i and period t. Therefore, the right-hand side is the total housing expenditure in location i and period t. I assume that the quantity of housing service H_t^i is determined by the housing supply function

$$H_t^i = \frac{1}{\bar{r}^i}\left(\gamma \sum_{r\in\{b,n\}} \sum_{a=1}^{\bar{a}} L_{r,a,t}^i w_{r,a,t}^i\right)^{1-\eta},$$

where the inverse of $\bar{r}^i$ is the exogenous time-invariant and location-specific housing supply shifter, and η is the exogenous parameter governing the elasticity of housing service with respect to local housing expenditure. Substituting this into (13) yields

$$r_t^i = \bar{r}^i\left(\gamma \sum_{r\in\{b,n\}} \sum_{a=1}^{\bar{a}} L_{r,a,t}^i w_{r,a,t}^i\right)^{\eta}. \tag{14}$$

Rent r_t^i is decomposed into $\bar{r}^i$ and the power function of the local housing expenditure. I call $\bar{r}^i$ the location-specific rent shifter. Because $\eta = d\log r_t^i / d\log\left(\sum_{r\in\{b,o\}} \sum_{a=1}^{\bar{a}} L_{r,a,t}^i w_{r,a,t}^i\right)$ holds, I call η as the rent elasticity with respect to local income, or simply the rent elasticity.

3.8 Equilibrium and Steady State

Now I am equipped with all equilibrium conditions.

Equilibrium. Given populations in period 0 $\{L_{r,a,0}^i\}_{r,a}^i$, an equilibrium is a tuple of expected values $\{V_{r,a,t}^i\}_{r,a,t=0,1,\ldots}^i$, wages $\{w_{r,a,t}^i\}_{r,a,t=0,1,\ldots}^i$, populations $\{L_{r,a,t}^i\}_{r,a,t=1,2,\ldots}^i$, migration shares $\{\mu_{r,a,t}^{j,i}\}_{r,a,t=0,1,\ldots}^{j,i}$, housing rent $\{r_t^i\}_{t=0,1,\ldots}^i$ that satisfies (5), (6), (7), (8), (12), and (14).

I compute transition paths to steady states, given the initial populations of all demographic groups in all locations. For this purpose, I characterize steady states.

Steady state. A steady state is a tuple of time-invariant variables: expected values $\{V_{r,a}^i\}_{r,a}^i$, wages $\{w_{r,a}^i\}_{r,a}^i$, populations $\{L_{r,a}^i\}_{r,a}^i$, migration shares $\{\mu_{r,a}^{j,i}\}_{r,a}^{j,i}$, housing rent $\{r^i\}^i$ satisfying (5), (6), (7), (8), (12), and (14), dropping time subscripts in all equations.

4 Quantification

I load the parameter values from 1940 to 2010 into the model. The geographic units are 36 states including all confederate and border states, the District of Columbia, and the constructed rest of the North. In total, there are 38 locations in the sample. The rest of the North aggregates states with less than 5,000 Black population as of 1940.[12] The rest of the North accounts for 0.1 and 1 percent of the Black population in the US as of 1940 and 2010, respectively. One period is ten years.

[12] The rest of the North consists of Idaho, Maine, Montana, Nevada, New Hampshire, New Mexico, North Dakota, Oregon, South Dakota, Utah, Vermont, and Wyoming.

I estimate a set of parameters that I call elasticities: migration elasticity $1/\nu$, elasticities of substitution σ_0 and σ_1, and rent elasticity η. The model delivers explicit formulae mapping elasticities and relevant data to amenities, migration costs, productivity, and location-specific rent shifters. Specifically, with relevant data, migration elasticity $1/\nu$ pins down amenities $B^i_{r,a,t}$ and migration costs $\tau^{j,i}_{r,a,t}$. Elasticities of substitution σ_0 and σ_1 pin down productivity A^i_t, $\kappa^i_{a,t}$, $\kappa^i_{r,a,t}$. Rent elasticity η pins down location-specific rent shifters $\bar{r}^i$. Subsection 4.4 touches on survival probabilities, fertility, and immigrants from abroad. As in Ahlfeldt et al. (2015), I set the Cobb-Douglas share on housing $\gamma = 0.25$ following Davis and Ortalo-Magné (2011).

The main data source is the US censuses from 1940 to 2000 and the American Community Survey (ACS) from 2001 to 2019, both of which are tabulated in IPUMS (Ruggles et al., 2022; Manson et al., 2022). Migration shares, populations, wages, and fertility (babies per person) for race and age bins in states and time periods are from these data. Median rent across states from 1940 to 2010 is published by the US Census Bureau or IPUMS. All prices (wages and rent) are deflated by the consumer price index and measured in the 2010 US dollars. I use payrolls per capita as wages and head counts as populations for race, age, location, and time tuples. See Appendix B for further details on the data sources.

Table 2: Age Bins in the Model and in the Data

model	0	1	$\cdots$	$\bar{a} = 6$
data	1-10	11-20	$\cdots$	61-70

Notes: The first row lists age bins used in the model. The second row lists the corresponding age bins in the data.

Since US censuses are decennial, one period in the model corresponds to ten years in the data. Accordingly, age bins in the model correspond to 10-year windows as in Table 2. In the quantification of the model, ages run from 0 to 6, so the maximum length of life is 7 periods. Age 0 in the model corresponds to ages 1 to 10 in the data, $\cdots$, age $\bar{a} = 6$ in the model corresponds to ages 61 to 70 in the data.

4.1 Migration Elasticity, Migration Costs, and Amenities

I estimate migration elasticity $1/\nu$, following the two-step estimation developed by Artuc and McLaren (2015). Artuc and McLaren (2015) used this method to study sectoral and occupational choices by workers, and Caliendo et al. (2021) applied it to the context of migration.

Define the option value of race r, age a, period t (hereafter (r,a,t)), in location j, $\Omega^j_{r,a,t}$, by

$$\Omega^j_{r,a,t} = \nu \log\left(\sum_{k\in\mathcal{N}} \exp(s_{r,a,t} V^k_{r,a+1,t+1} - \tau^{k,j}_{r,a,t})^{1/\nu}\right).$$

Then the expected value of (r,a,t) in location j (5) is rewritten as

$$V^j_{r,a,t} = \bar{u}^j_{r,a,t} + \Omega^j_{r,a,t}. \tag{15}$$

The migration share of (r,a,t) from location i to j (6) is also rewritten as

$$\mu^{j,i}_{r,a,t} = \exp\left\{\frac{1}{\nu}(s_{r,a,t}V^j_{r,a+1,t+1} - \tau^{j,i}_{r,a,t}) - \frac{1}{\nu}\Omega^i_{r,a,t}\right\} \tag{16}$$

Multiplying both sides by the population of (r,a,t) in origin location i, $L^i_{r,a,t}$, I have the number of migrants

$$L^i_{r,a,t}\mu^{j,i}_{r,a,t} = \exp\left\{\frac{1}{\nu}(s_{r,a,t}V^j_{r,a+1,t+1} - \tau^{j,i}_{r,a,t}) - \frac{1}{\nu}\Omega^i_{r,a,t} + \log(L^i_{r,a,t})\right\}$$

I decompose the number of migrants into destination fixed effects $v^j_{r,a,t}$, origin fixed effects $\omega^i_{r,a,t}$, and the remaining variation $\tilde{\tau}^{j,i}_{r,a,t}$ by race r, age a, and period t

$$L^i_{r,a,t}\mu^{j,i}_{r,a,t} = \exp\{v^j_{r,a,t} + \omega^i_{r,a,t} + \tilde{\tau}^{j,i}_{r,a,t}\}. \tag{17}$$

Comparing equations (16) and (17) yields

$$v^j_{r,a,t} = \frac{1}{\nu}s_{r,a,t}V^j_{r,a+1,t+1}, \tag{18}$$

$$\omega^i_{r,a,t} = -\frac{1}{\nu}\Omega^i_{r,a,t} + \log(L^i_{r,a,t}), \tag{19}$$

$$\tilde{\tau}^{j,i}_{r,a,t} = -\frac{1}{\nu}\tau^{j,i}_{r,a,t}. \tag{20}$$

Note that $v^j_{r,a,t}$, $\omega^i_{r,a,t}$, and $\tilde{\tau}^{j,i}_{r,a,t}$ capture expected values, option value, and migration costs, respectively. Equations (15), (18), and (19) imply

$$\begin{aligned} \frac{v^j_{r,a,t}}{s_{r,a,t}} + \omega^j_{r,a+1,t+1} - \log(L^j_{r,a+1,t+1}) &= \frac{1}{\nu}\bar{u}^j_{r,a,t} \\ &= \frac{1}{\nu}\left\{\log\left(\frac{w^j_{r,a+1,t+1}}{(r^j_{t+1})^\gamma}\right) + \log(B^j_{r,a+1,t+1})\right\}. \end{aligned} \tag{21}$$

That is, regressing migration shares on origin and destination fixed effects recovers period utilities for each (r,a,t) in location j.

Guided by the derivations so far, I implement two step estimation. Following equation (17), in the first step, I run the following regression

$$L^i_{r,a,t}\mu^{j,i}_{r,a,t} = \exp\left\{v^j_{r,a,t} + \omega^i_{r,a,t} + \tilde{\tau}^{j\neq i}_t + \tilde{\tau}^{\{i,j\}}_{r,G(t)} + \tilde{\tau}^{\{i,j\}}_{a,G(t)}\right\} + \epsilon^{j,i}_{r,a,t}. \tag{22}$$

For race r, age a, and period t, $v^j_{r,a,t}$ is the destination fixed effect, and $\omega^i_{r,a,t}$ is the origin fixed effect. The terms $\tilde{\tau}$ with various subscripts and superscripts capture migration costs. $\tilde{\tau}_t^{j\neq i}$ denotes the fixed effect for year t and moving, that is, destination j is a different location from origin i. The sample years are decennial: $1930, 1940, \cdots, 2000, 2010$.[13] In the subscripts of $\tilde{\tau}_{r,G(t)}^{\{i,j\}}$ and $\tilde{\tau}_{a,G(t)}^{\{i,j\}}$, function $G(\cdot)$ groups years as in Table 3. I call partitions of years defined by G as year groups. In the superscripts of $\tilde{\tau}_{r,G(t)}^{\{i,j\}}$ and $\tilde{\tau}_{a,G(t)}^{\{i,j\}}$, $\{i,j\}$ represents the unordered pair of locations i and j.[14] Thus $\tilde{\tau}_{r,G(t)}^{\{i,j\}}$ is the race × year group × location pair fixed effect, and $\tilde{\tau}_{a,G(t)}^{\{i,j\}}$ is the age × year group × location pair fixed effect. Finally, $\epsilon_{r,a,t}^{j,i}$ is the error term. Notice that I assume symmetric migration costs.

Since one period is 10 years, migration shares in the regression (22) must be of 10-year windows. Migration is, however, reported in 1- or 5-year windows in the US censuses and ACS. Appendix D details how I map 1- or 5-year migration in the data to 10-year migration in the quantification of the model.

Table 3: Grouping Sample Years

year	1930	1940	1950	1960	1970	1980	1990	2000	2010
group	1		2		3		4		

Notes: This table defines function G used in equation (22).

Suppose that I obtained estimates of the destination fixed effects $\hat{v}_{r,a,t}^j$ and the origin fixed effects $\hat{\omega}_{r,a,t}^i$ by race r, age a, ant time t from the first step. In the second step, guided by equation (21), I run the regression (whose result is in column (3) in Table 4)

$$\frac{\hat{v}_{r,a,t}^j}{s_{r,a,t}} + \hat{\omega}_{r,a+1,t+1}^j - \log(L_{r,a+1,t+1}^j) = \frac{1}{\nu}\log(w_{r,a+1,t+1}^j) + \tilde{B}_{r,a+1}^j + \tilde{B}_{r,t+1}^j + \epsilon_{r,a,t}^j, \tag{23}$$

where $\log(w_{r,a+1,t+1}^j)$ is the log of the nominal wage of individuals of $(r, a+1, t+1)$ in location j, $\tilde{B}_{r,a+1}^j$ is the race × age × location fixed effect, $\tilde{B}_{r,t+1}^j$ is the race × year × location fixed effect, and $\epsilon_{r,a,t}^j$ is the error term. $\tilde{B}_{r,a+1}^j$ and $\tilde{B}_{r,t+1}^j$ are to control for the amenities $B_{r,a+1,t+1}^j$. Since rent r_{t+1}^j in equation (21) varies at location-time levels, it is absorbed by the race × year × location fixed effect $\tilde{B}_{r,t+1}^j$.

Nominal wages are directly from the data. Following Artuc and McLaren (2015), I instrument the log of the nominal wage for individuals of $(r, a+1, t+1)$ in location j $\log(w_{r,a+1,t+1}^j)$ by the log of the nominal wage for individuals of $(r, a+1, t)$ in location j $\log(w_{r,a+1,t}^j)$ (the lagged instrumental variable). Since one period is ten years, I instrument the nominal wage of each race-age-year-location quadruple by the nominal wage of the same race-age-location triple but ten years before.

Table 4 reports the results of the second step. Column (3) reports the result of the specification

[13] The migration shares in 1930 are necessary to compute amenities in 1940. See time subscripts in equation (21).

[14] More precisely, for locations $i \neq j$, I assume $\tilde{\tau}_{r,G(t)}^{\{i,i\}} = \tilde{\tau}_{r,G(t)}^{\{j,j\}}$ and $\tilde{\tau}_{a,G(t)}^{\{i,i\}} = \tilde{\tau}_{a,G(t)}^{\{j,j\}}$. Thus I have the fixed effects for staying and all unordered pairs of different locations.

Table 4: Migration Elasticity

Dependent variable:	period utility × migration elasticity		
	(1)	(2)	(3)
log(real wage)	0.4976***	0.6129***	0.7676***
	(0.1323)	(0.1665)	(0.1952)
fixed effects:			
race-location	✓	✓	✓
age-location	✓	✓	✓
year-location	✓	✓	✓
age-race	✓	✓	✓
year-race	✓	✓	✓
age-race-location		✓	✓
year-race-location			✓
Observations	2,660	2,660	2,660

Notes: The second step of the migration elasticity estimation. Dependent variables are constructed of the estimates from the first step and represent period utilities multiplied by the migration elasticity. Units of observations are year-age-race-location tuples. Robust standard errors clustered at locations are in parentheses. Significance codes: ***: 0.01, **: 0.05, *: 0.1.

(23), and the other columns report the results of the specifications with fewer fixed effects. The estimates for the migration elasticity range from 0.50 to 0.77, which are between 0.5 estimated by Caliendo et al. (2021) for EU countries and 2.0 estimated by Suzuki (2021) for Japanese prefectures. I use 0.77 as the value of the migration elasticity.

Given $\hat{\nu}$, I can back out migration costs. Recall that in the first step (22), I have estimated the fixed effects $\tilde{\tau}_t^{j\neq i}$, $\tilde{\tau}_{r,G(t)}^{\{i,j\}}$, and $\tilde{\tau}_{a,G(t)}^{\{i,j\}}$. Let $\hat{\tilde{\tau}}_t^{j\neq i}$, $\hat{\tilde{\tau}}_{r,G(t)}^{\{i,j\}}$, and $\hat{\tilde{\tau}}_{a,G(t)}^{\{i,j\}}$ be the estimates of these fixed effects. By equation (20), I obtain migration costs induced by these estimates of the fixed effects,

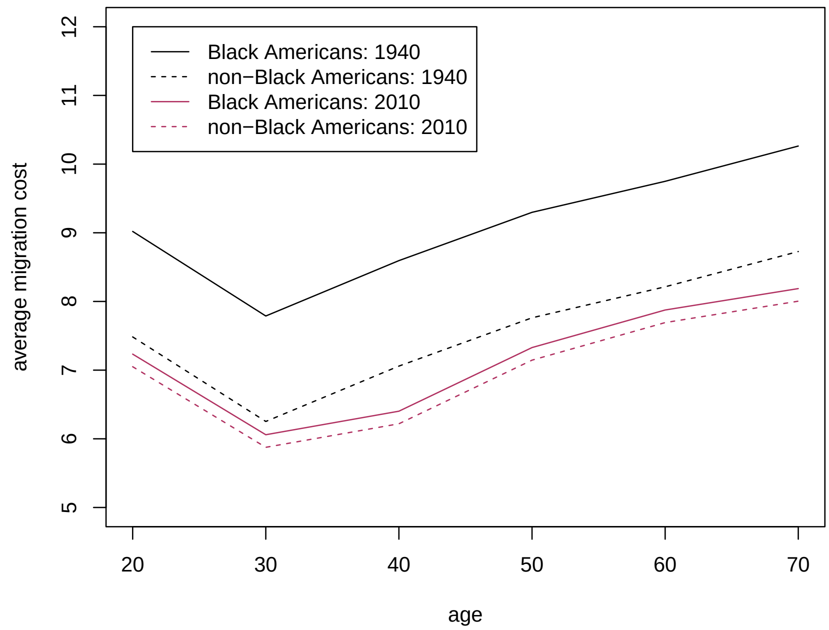
$$\hat{\tau}_{r,a,t}^{j,i} = -\hat{\nu}\left(\hat{\tilde{\tau}}_t^{j\neq i} + \hat{\tilde{\tau}}_{r,G(t)}^{\{i,j\}} + \hat{\tilde{\tau}}_{a,G(t)}^{\{i,j\}}\right).$$

Figure 4 shows the averages of the induced migration costs for races and ages. 20 in the horizontal axis refers to age bin 11-20, and so on.[15] The migration costs are the lowest for people of the ages of 21-30. Migration costs increase after the ages 21-30. The migration costs in 2010 increase with ages less steeply than the migration costs in 1940 do. This is perhaps because seniors are more physically mobile or infrastructure is better in 2010. Black Americans faced higher migration costs than others in 1940, but the racial gap in migration costs shrank by 2010.

Figure 5 illustrates the averages of the induced migration costs for races and years. The migration costs of Black Americans were always higher than those of non-Black Americans. The migration costs, however, have steadily declined over the sample periods, particularly for Black Americans. The racial gap in migration costs declined over time.

[15] Age $x \in \{20, \cdots, 70\}$ on the horizontal axis refers to ages from $x-9$ to x.

Figure 4: Average Migration Costs for Races and Ages: 1940 and 2010



Notes: For races, ages, and years, I compute the averages of the induced migration costs across location pairs. The migration costs are induced by the estimate of the migration elasticity and the fixed effects in equation (22).

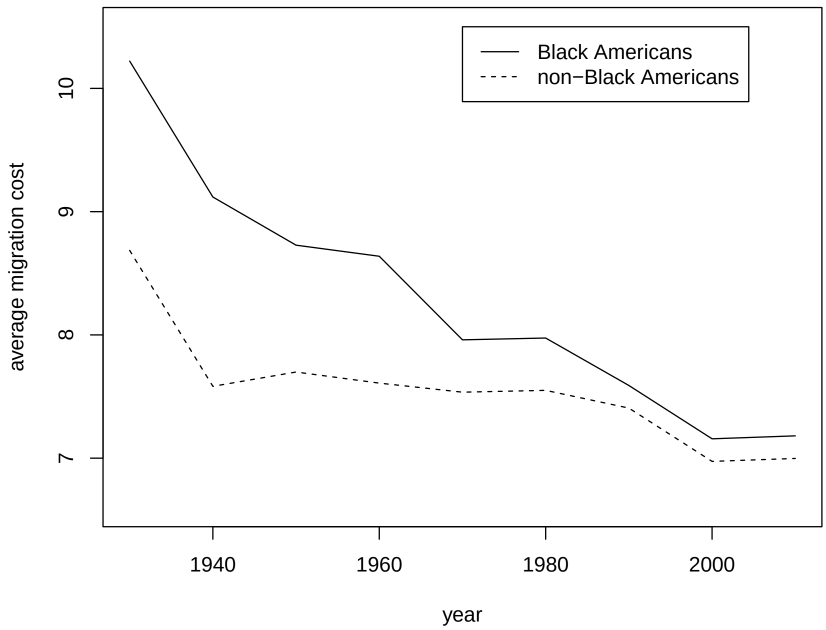
Using $\hat{\nu}$ and the fixed effects estimated in the second step (23), I back out amenities. Let $\hat{\tilde{B}}^j_{r,a+1}$ and $\hat{\tilde{B}}^j_{r,t+1}$ be the estimates of the fixed effects in the second step (23). Then by comparing equations (21) and (23), the induced amenities, $\hat{B}_{r,a,t}$, are

$$\hat{B}^j_{r,a,t} = \exp\left\{\hat{\nu}\left(\hat{\tilde{B}}^j_{r,a} + \hat{\tilde{B}}^j_{r,t}\right) + \gamma \log(r^j_t)\right\}, \tag{24}$$

where rent r^j_t is directly from the data, and I set $\gamma = 0.25$. I normalize amenities $\{\hat{B}^j_{r,a,t}\}^j$ so that the mean of $\{\hat{B}^j_{r,a,t}\}^j$ is 1 for each (r,a,t). In the model, migration decisions are made in period t foreseeing real wages and amenities in period $t+1$. I have the data on wages and rent from 1940 to 2019. So I can compute the induced amenities for the years 1950 to 2010 in this way. The reason that I cannot obtain the amenities in 1940 is that I do not have the data on wages as of 1930, so I do not have the lagged instrumental variable for wages. For 1940, I directly back out the amenities using equation (21)

$$\hat{B}^j_{r,a,1940} = \exp\left\{\nu\left(\frac{\hat{v}^j_{r,a-1,1930}}{s_{r,a-1,1930}} + \hat{\omega}^j_{r,a,1940} - \log(L_{r,a,1940})\right)\right\} \Big/ \left\{\frac{w^j_{r,a,1940}}{(r^j_{1940})^{\gamma}}\right\}.$$

Figure 5: Average Migration Costs for Races and Years

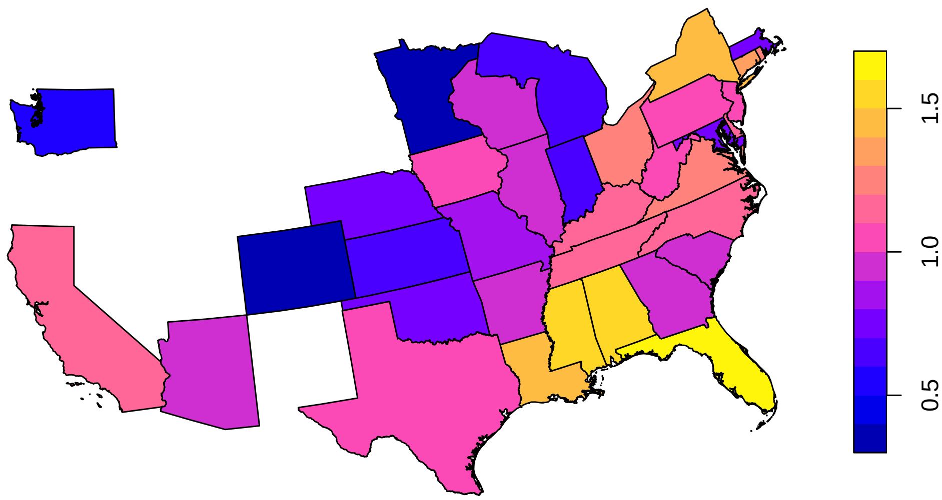


Notes: For races and years, I compute the averages of the induced migration costs across (ordered) location pairs and ages. The migration costs are induced by the estimate of the migration elasticity and the fixed effects in equation (22).

Recall that in Table 2, the age 6 in the model is the highest age and corresponds to the ages 61 to 70 in the data. I do not include the age 7 (71-80 year olds) in the sample because including the age 7 in the estimation makes estimates of the migration elasticity unstable. Since the origin fixed effect for the age 7 $\hat{\omega}^{j}_{r,7,t+1}$ is needed to induce the amenities for the age 6 (see (23)), I cannot obtain the amenities for the age 6 by using (23) and (24). I assume that within race r, location i, and period t, the amenities for the age 6 (61-70 year olds) are the same as the amenities for the age 5 (51-60 year olds). I use the estimates for the amenities for the age 5 for the amenities for the age 6.

Figures 6 and 7 show the induced amenities for Black and non-Black Americans for states in 1960 averaged across the age bins from 1 (11-20 year olds) to 5 (51-60 year olds). The peak of the great migration was in the 1950s, and in the model, individuals make migration decisions in 1950 foreseeing real wages and amenities from 1960 onward. This is why I pick up the year 1960. The amenities of the rest of the North are not in the figures, although they are assigned in the quantification of the model. As in Figure 6, the induced amenities for Black Americans were high in states in the South such as Florida, Alabama, Mississippi, and Louisiana in 1960. In contrast, states in the North such as Michigan, Minnesota, Kansas, Colorado, and Washington had low amenities for Black Americans then. Figure 7 shows a different geographic pattern of amenities

Figure 6: Amenities for Black Americans in 1960



Notes: The amenities for Black Americans in 1960 averaged across ages. The amenities are induced by equation (24). The rest of the North is excluded from the map.

for non-Black Americans. California provided high amenities for non-Black Americans, but there is not a clear North-South pattern in the amenities for non-Black Americans.

4.2 Elasticity of Substitution and Productivity

4.2.1 Races

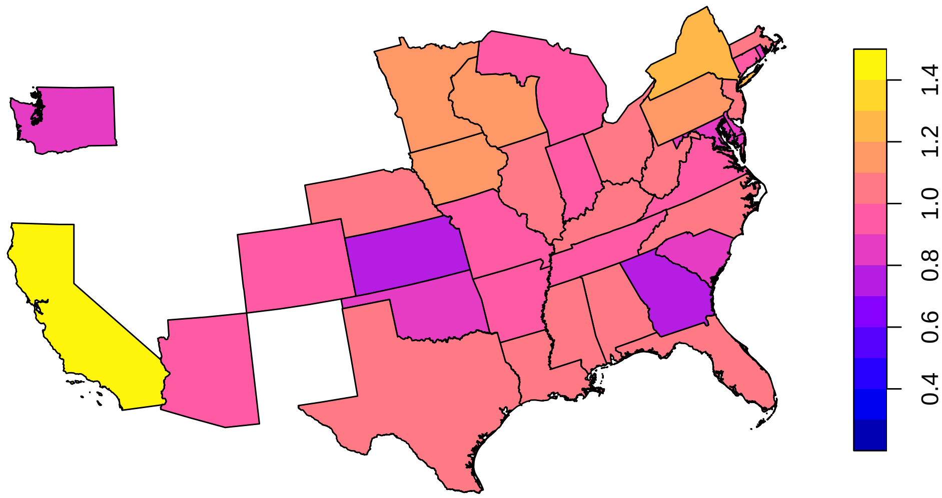
I turn to the estimation of the elasticities of substitution. I start with the elasticity of substitution across races within age, location, and time bins σ_1. Equation (12) implies

$$\frac{w^i_{b,a,t}}{w^i_{n,a,t}} = \frac{(\kappa^n_{b,a,t})^{\frac{1}{\sigma_1}} (L^i_{b,a,t})^{-\frac{1}{\sigma_1}}}{(\kappa^i_{n,a,t})^{\frac{1}{\sigma_1}} (L^i_{n,a,t})^{-\frac{1}{\sigma_1}}}, \tag{25}$$

where I recall that the first subscripts b and n denote Black and non-Black Americans, respectively. Taking logs of both sides,

$$\log\left(\frac{w^i_{b,a,t}}{w^i_{n,a,t}}\right) = -\frac{1}{\sigma_1}\log\left(\frac{L^i_{b,a,t}}{L^i_{n,a,t}}\right) + \frac{1}{\sigma_1}\log\left(\frac{\kappa^i_{b,a,t}}{\kappa^i_{n,a,t}}\right). \tag{26}$$

Figure 7: Amenities for non-Black Americans in 1960



Notes: The amenities for non-Black Americans in 1960 averaged across ages. The amenities are induced by equation (24). The rest of the North is excluded from the map.

Since productivity ratio between races $\kappa^i_{b,a,t}/\kappa^i_{n,a,t}$ is not observable in data, my main econometric specification is

$$\log\left(\frac{w^i_{b,a,t}}{w^i_{n,a,t}}\right) = -\frac{1}{\sigma_1}\log\left(\frac{L^i_{b,a,t}}{L^i_{n,a,t}}\right) + f_a + f_t + f_{a,t} + \epsilon^i_{a,t}, \tag{27}$$

where f_a denotes the age fixed effect, f_t denotes the time fixed effect, $f_{a,t}$ denotes the age $\times$ time fixed effect, and $\epsilon^n_{a,t}$ is the error term. Notice that age-time fixed effects $f_{a,t}$ capture cohorts. For earlier cohorts, the education gap between African Americans and others was larger. As the education gap is a reason for the productivity gap between races, controlling for cohorts is important in the regression (27).

In his seminal work, Borjas (2003) considers the nationwide labor market. But, here I consider different locations in the US as different labor markets. Suppose that African Americans migrate to a location where their productivity is high relative to others within age bins. Then productivity ratio $\kappa^n_{b,a,t}/\kappa^n_{o,a,t}$ is positively correlated with population ratio $L^n_{b,a,t}/L^n_{o,a,t}$, which causes an upward bias for the estimator of $-1/\sigma_1$ in ordinary least squares (OLS). Note that the concern here is that productivity ratio $\kappa^n_{b,a,t}/\kappa^n_{o,a,t}$ may work as a pull factor of migration.

To deal with this potential bias, I follow Card (2009). In Appendix E, I pursue a different approach following the first difference estimation of Monras (2020). Here I consider two instrumental variables. The first one is the ratio of shift-share predicted populations. The shift-share predicted population of race r and age a in period t and location n is

$$\hat{L}^i_{r,a,t} = \sum_{j \in \mathcal{N}} \mu^{i,j}_{r,a-1,t-1-X} \cdot s_{r,a-1,t-1} L^j_{r,a-1,t-1}. \tag{28}$$

If $X = 1$, this equation would be the same as equation (7), omitting immigrants. But I use the value of $X > 1$. That is, I interact the current (actually one period before) survival probability and populations with the old-time migration shares to make shift-share predicted populations. As in Goldsmith-Pinkham et al. (2020), the assumption for identification is that the error term $\epsilon^i_{a,t}$ is mean-independent from the old-time migration shares $\{\mu^{i,j}_{r,a-1,t-1-X}\}_{j \in \mathcal{N}}$

$$E[\epsilon^i_{a,t} \mid \{\mu^{i,j}_{r,a-1,t-1-X}\}_{j \in \mathcal{N}}] = 0.$$

This is satisfied if the old-time migration shares $\mu^{i,j}_{r,a-1,t-1-X}$ do not react to shocks to the current productivity ratio between races $\kappa^i_{b,a,t}/\kappa^i_{o,a,t}$. Here I am teasing out the push factor of migration because I am extracting variation in $L^i_{b,a,t}/L^i_{o,a,t}$ that is orthogonal to the pull factor of migration $\kappa^i_{b,a,t}/\kappa^i_{o,a,t}$. The relevance (correlation with the actual population ratio) of this IV hinges on the so-called network effect of migration; migrants tend to go to a destination to which their precursors went.

Table 5: Elasticity of Substitution across Races: Level Estimation

Dependent variable:		$\log(w^i_{b,a,t}/w^i_{n,a,t})$	
Model:	OLS	IV 1	IV 2
$\log(L^i_{b,a,t}/L^i_{n,a,t})$	-0.1154***	-0.1108***	-0.1120***
	(0.0120)	(0.0127)	(0.0139)
fixed effects:			
year-age	✓	✓	✓
Observations	1,368	1,368	1,328
First-stage F-statistic		91.24	62.09

Notes: The results of the level estimation of the elasticity of substitution across races. Block bootstrap standard errors are in parentheses. See Appendix F for the computation of standard errors. Significance codes: ***: 0.01.

I also consider a leave-one-out version of (28) as the second IV, removing $\mu^{i,i}_{r,a-1,t-1-X} \cdot s_{r,a-1,t-1} L^i_{r,a-1,t-1}$ from its right-hand side. Then I obtain

$$\hat{L}^{i,-i}_{r,a,t} = \sum_{j \neq i} \mu^{i,j}_{r,a-1,t-1-X} \cdot s_{r,a-1,t-1} L^j_{r,a-1,t-1}. \tag{29}$$

The economic interpretation for this is shift-share predicted gross inflows because the right-hand side collects inflows of people from all locations but n itself.

For either IV, I set $X = 2$. Since 1 period is 10 years, I use the migration shares 20 years before period $t-1$.

Table 5 provides the estimation results.[16] The first column shows the result of OLS.[17] The second and third columns show the results of two-step least squares using the first and second IVs (28) and (29), respectively. The OLS and the two IV estimations produce similar estimates around -0.11. From columns 1, 2, and 3, let the OLS estimate, the first IV estimate, and the second IV estimate for σ_1 be $\hat{\sigma}_r^{OLS}$, $\hat{\sigma}_r^{IV1}$, $\hat{\sigma}_r^{IV2}$, respectively. Then $\hat{\sigma}_1^{OLS} = 1/0.1154 = 8.67$, $\hat{\sigma}_1^{IV1} = 1/0.1108 = 9.02$, $\hat{\sigma}_1^{IV2} = 1/0.1120 = 8.93$. In the quantification of the model, I use $\hat{\sigma}_1^{IV1}$ as the value of the elasticity of substitution across races.

How do my estimates for the elasticity of substitution across races compare with estimates in the literature? The estimates of the elasticity of substitution across races range from 8.7 to 9.0. In Appendix E, the first difference estimation produces the estimate of 4.9. Boustan (2009) estimates the elasticity of substitution across races within education-experience bins in the entire US North. Her preferred values range from 8.3 to 11.1 and coincide with my estimates here. Her paper also includes an estimate of 5.4, which is somewhat similar to my estimate from the first difference estimation.

Given the estimate of the elasticity of substitution across races $\hat{\sigma}_r$, I can back out race-specific productivity $\kappa_{r,a,t}^i$. Rearranging equation (25), I obtain

$$\frac{\hat{\kappa}_{b,a,t}^i}{\hat{\kappa}_{n,a,t}^i} = \left(\frac{w_{b,a,t}^i}{w_{n,a,t}^i}\right)^{\hat{\sigma}_1} \cdot \left(\frac{L_{b,a,t}^i}{L_{n,a,t}^i}\right). \tag{30}$$

Since wages $w_{r,a,t}^i$ and populations $L_{r,a,t}^i$ are directly observable for $r = b, n$, I can back out productivity ratio between Black and non-Black Americans $\hat{\kappa}_{b,a,t}^i / \hat{\kappa}_{n,a,t}^i$. Comparing equations (10) and (11), multiplying all $\kappa_{r,a,t}^n$ $(r = b, n)$ by scalar $x > 0$ is equivalent to multiplying $\kappa_{a,t}^n$ by $x^{(\sigma_0-1)/(\sigma_1-1)}$ in the production function. Thus I normalize $\hat{\kappa}_{r,a,t}^i$ for $r = b, n$, so that $\sum_{r=b,n} \hat{\kappa}_{r,a,t}^i = 1$. With equation (30), this normalization pins down $\hat{\kappa}_{r,a,t}^i$ for $r = b, n$.

4.2.2 Ages

Dual to age-level labor (11) in the production function, age-level wages within location-time bins are

$$w_{a,t}^i = \left(\sum_r \kappa_{r,a,t}^i (w_{r,a,t}^i)^{1-\sigma_1}\right)^{\frac{1}{1-\sigma_1}}. \tag{31}$$

Then I obtain

$$\frac{w_{a,t}^i}{w_{a',t}^i} = \frac{(\kappa_{a,t}^i)^{\frac{1}{\sigma_0}} (L_{a,t}^i)^{-\frac{1}{\sigma_0}}}{(\kappa_{a,t}^i)^{\frac{1}{\sigma_0}} (L_{a',t}^i)^{-\frac{1}{\sigma_0}}}. \tag{32}$$

[16] Appendix F details the computation of standard errors.

[17] Since the IVs (28) and (29) use populations 20 years before ($X = 2$), wages and populations in the main specification (27) run from 1960 to 2010. For fair comparison, I use the data from 1960 to 2010 for the OLS, too.

Taking logs of both sides of (32), I have

$$\log\left(\frac{w^i_{a,t}}{w^i_{a',t}}\right) = -\frac{1}{\sigma_0}\log\left(\frac{L^i_{a,t}}{L^i_{a',t}}\right) + \frac{1}{\sigma_0}\log\left(\frac{\kappa^i_{a,t}}{\kappa^i_{a',t}}\right).$$

Fix an age bin a'. For any age $a \neq a'$, the econometric specification is

$$\log\left(\frac{w^i_{a,t}}{w^i_{a',t}}\right) = -\frac{1}{\sigma_0}\log\left(\frac{L^i_{a,t}}{L^i_{a',t}}\right) + f_a + f_t + f_{a,t} + \epsilon^i_{a,t}, \tag{33}$$

where f_a is the age fixed effect, f_t is the time fixed effect, and $f_{a,t}$ is the age × time fixed effect. Note that $w^i_{a,t}$ and $L^i_{a,t}$ are computed using $\hat{\sigma}_r$ and $\hat{\kappa}^i_{r,a,t}$ for $r = b, n$. A concern is that people of an age group migrate to a location where relative productivity of the age group is high, causing positive correlation between population ratios across ages $L^i_{a,t}/L^i_{a',t}$ and productivity ratios across ages $\kappa^i_{a,t}/\kappa^i_{a',t}$.

To deal with this endogeneity concern, I make use of the shift-share predicted populations and gross inflows at race-age-location-time levels in equations (28) and (29). The first IV is constructed of the aggregates of the shift-share predicted populations

$$\hat{L}^i_{a,t} = \left[\sum_{r\in\{b,n\}} (\hat{\kappa}^i_{r,a,t})^{\frac{1}{\hat{\sigma}_1}} (\hat{L}^i_{r,a,t})^{\frac{\hat{\sigma}_1-1}{\hat{\sigma}_1}}\right]^{\frac{\hat{\sigma}_1}{\hat{\sigma}_1-1}}, \tag{34}$$

where $\hat{L}^i_{r,a,t}$ is the shift-share predicted populations defined in equation (28). The second IV is constructed of the aggregates of the shift-share predicted gross inflows

$$\hat{L}^{i,-i}_{a,t} = \left[\sum_{r\in\{b,n\}} (\hat{\kappa}^i_{r,a,t})^{\frac{1}{\hat{\sigma}_1}} (\hat{L}^{i,-i}_{r,a,t})^{\frac{\hat{\sigma}_1-1}{\hat{\sigma}_1}}\right]^{\frac{\hat{\sigma}_1}{\hat{\sigma}_1-1}}, \tag{35}$$

where $\hat{L}^{i,-i}_{r,a,t}$ is the shift-share predicted gross inflows defined in equation (29). In either case, I instrument the population ratios across ages $L^i_{a,t}/L^i_{a',t}$ by the ratios of the aggregates of shift-share predicted populations $\hat{L}^i_{a,t}/\hat{L}^i_{a',t}$ or gross inflows $\hat{L}^{i,-i}_{a,t}/\hat{L}^{i,-i}_{a',t}$.

Age-level wages $w^i_{a,t}$, age-level labor $L^i_{a,t}$, and instruments (34) and (35) are all constructed with the estimate of σ_1 and the race-specific productivity. Therefore, to compute standard errors for estimates of σ_0, I need to take into account variability in the estimate of σ_1 and the race-specific productivity. I compute block bootstrap standard errors to address this issue. See Appendix F for details.

Table 6 reports three estimation results for the elasticity of substitution across ages. In the column 1, I use $\hat{\sigma}_1^{OLS}$ and its associated race-specific productivity to compute age-level wages $w^i_{a,t}$ and populations $L^i_{a,t}$, respectively. Here I use actual race-level populations $L^i_{r,a,t}$ to construct $L^i_{a,t}$ using equation (11). Likewise, in columns 2 and 3, I use $\hat{\sigma}_1^{IV1}$ and $\hat{\sigma}_1^{IV2}$ and the race-specific productivity induced by these two estimates to construct age-level wages $w^i_{a,t}$ and populations $L^i_{a,t}$. Columns 2 and 3 use the ratios of the aggregates of shift-share predicted populations (34) and the ratios of the aggregates of shift-share predicted gross inflows (35) as IVs, respectively. The two IVs seem to correct a positive bias in the OLS (column 1). In the quantification of the model, I use 1/0.3401 = 2.94 as the value of the elasticity of substitution across ages. The induced elasticities of substitution across ages range from 1.8 to 3.5. They are lower than the estimates in the prior literature in labor economics. Card and Lemieux (2001) and Ottaviano and Peri (2012) report estimates of 3.8-4.9 and 3.3-6.3 for the US, respectively; Manacorda et al. (2012) provide estimates of 5.1-5.2 for the UK. There are three differences between these papers and mine. First, they consider the nationwide labor market whereas I consider the state-level labor markets. Second, they control education levels, but I do not. Third, their age bins are five-year windows, whereas mine is of ten years because the wage and population data are from the decennial censuses. My age bins are twice as large as the bins in the literature, and it is possible that substitution across larger age bins exhibits a larger degree of imperfection.

Table 6: Elasticity of Substitution across Ages

Dependent variable:	$\log(w^i_{a,t}/w^i_{a',t})$		
Model:	OLS	IV 1	IV 2
$\log(L^i_{a,t}/L^i_{a',t})$	-0.2978***	-0.3401*	-0.5429***
	(0.0672)	(0.1922)	(0.1579)
fixed effects:			
year-age	✓	✓	✓
Observations	1,140	1,140	1,140
1st-stage F-statistic		426.8	219.8

Notes: The estimates of the elasticity of substitution across ages. Block bootstrap standard errors are in parentheses. See Appendix F for the computation of standard errors. Significance codes: ***: 0.01, **: 0.05, *: 0.1.

4.2.3 Locations

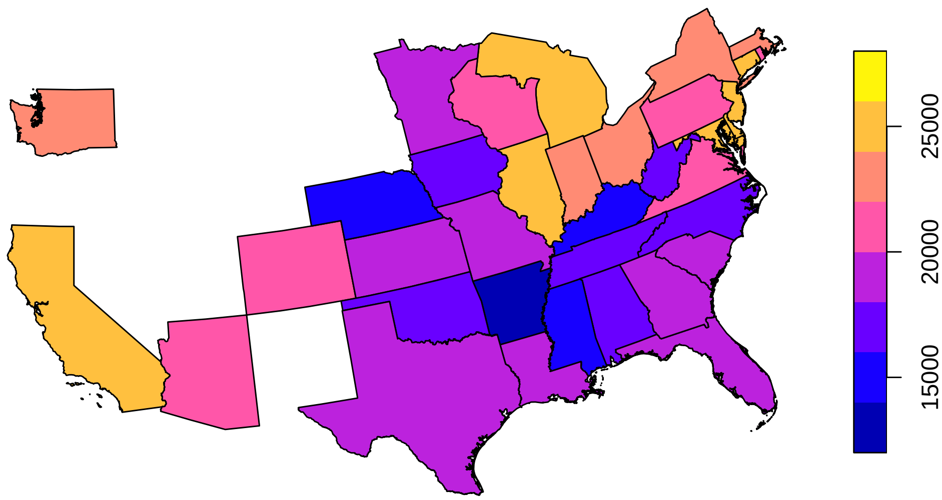
Dual to equation (10), location-time-level wages w^i_t are the aggregate of age-location-time level wages $w^i_{a,t}$

$$w^i_t = \left(\sum_{a=1}^{\bar{a}} \kappa^i_{a,t}(w^i_{a,t})^{1-\sigma_0}\right)^{\frac{1}{1-\sigma_0}}.$$

In equilibrium, the representative firm in location i makes zero profit. Thus the revenue equates the cost

$$A^i_t L^i_t = \sum_{a=1}^{\bar{a}} \sum_{r\in\{b,n\}} w^i_{r,a,t} L^i_{r,a,t}. \tag{36}$$

Figure 8: Productivity in 1960



Notes: The induced location-level productivity $\hat{A}_t^i$ in 1960. The rest of the North is excluded from the map.

By a property of the CES function, I have

$$\sum_{a=1}^{\bar{a}} \sum_{r \in \{b,n\}} w_{r,a,t}^i L_{r,a,t}^i = w_t^i L_t^i. \tag{37}$$

Equations (36) and (37) imply

$$A_t^i = w_t^i.$$

Thus I can back out location-level productivity A_t^i by computing location-level wages w_t^i.

I compute the induced location-level productivity $\hat{A}_t^i$ using $\hat{\sigma}_0^{IV1} = 1/0.3401 = 2.94$ from the column 2 of Table 6 and the age-level productivity $\hat{\kappa}_{a,t}^{i,IV1}$ induced by $\hat{\sigma}_0^{IV1}$. Figure 8 shows the induced location-level productivity across locations except for the rest of the North in 1960. Along with California, Northern manufacturing states such as Illinois, Michigan, and Ohio had higher productivity. These places were destinations of the great Black migration. In contrast, states in the South such as Arkansas and Mississippi had lower productivity. The two states were typical origins of the great Black migration.

I have backed out migration costs, amenities, and productivity using the formulae implied by the model. A possible story from the induced parameters is as follows. As in Figure 5, migration costs were high for Black Americans in the great migration period. Figure 6 showed states in the North provided lower amenities for Black Americans in the great migration period. Despite these impediments or disincentive, Black Americans made the journey from the South to the North for higher wages or workplaces of higher productivity as in Figure 8.

4.3 Rent Elasticity

I estimate the rent elasticity η, which governs how much local rent increases if aggregate local income increases by one percent. Taking logs of both sides in equation (14), I have

$$\log r_t^i = \log \bar{r}^i + \eta \log\left(\gamma \sum_{r\in\{b,n\}} \sum_{a=1}^{\bar{a}} L_{r,a,t}^i w_{r,a,t}^i\right). \tag{38}$$

Note that $\sum_r \sum_a L_{r,a,t}^i w_{r,a,t}^i$ is the total income in location i in my model. Taking time differences of equation (38), I obtain

$$\Delta \log r^i = \eta \Delta \log(\text{income}^i),$$

where income^i is the total income in location i. The location-specific rent shifter $\bar{r}^i$ and the expenditure share on housing γ are washed out by taking time differences.

Table 7: Rent Elasticity

Dependent variable:	$\Delta \log r^i$	
Model:	OLS	IV
$\Delta \log(\text{income}^i)$	0.3948***	0.4092***
	(0.0254)	(0.0264)
Observations	38	38
First-stage F-statistic		17.02

Notes: The estimation of the rent elasticity. The regressions are weighted by populations as of 1970. Robust standard errors are in parentheses. Significance code: ***: 0.01.

The econometric specification is

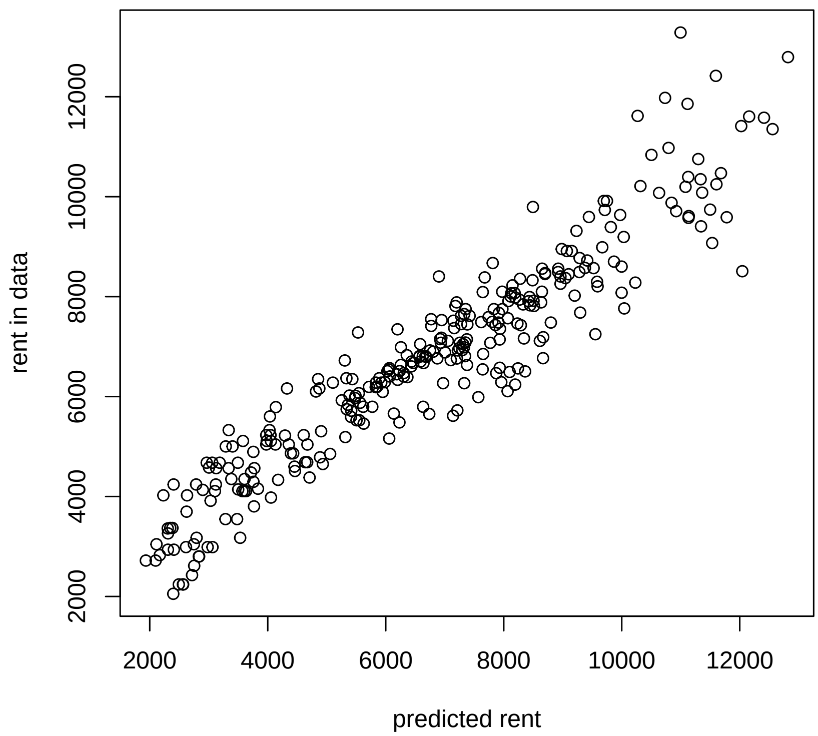
$$\Delta \log r^i = \eta \Delta \log(\text{income}^i) + \epsilon_i, \tag{39}$$

where ϵ_i is the error term. Time differences are taken between 1970 and 2010. I use median rent for rent in each location.[18] Threat to identification is that an increase in rent may increase local income, causing a positive correlation between the growth rate in local income $\Delta \log(\text{income}^i)$ and the error term ϵ_i. To deal with this threat, I instrument the growth rate in local income $\Delta \log(\text{income}^i)$ by the manufacturing share in employment and the share of college graduates in population as of 1950.[19] In this IV estimation, I pick up only the variation in the income growth predicted by sectoral and

[18]For the rest of the North, I take the mean of the median rents across the states within the rest of the North. The reason that I use median rent for the measure of local rent is that it is only the available rent measure for 1940.

[19]Glaeser et al. (2005) and Saks (2008) estimate the housing supply elasticity with an IV constructed of old-time sectoral shares.

Figure 9: Predicted and Actual Rent



Notes: Actual rent in the US state data from 1940 to 2010 on the vertical axis against the predicted rent on the horizontal axis. Units of observations are state-time.

educational composition in the old time. The regression is weighted by populations as of 1970. Table 7 reports the result. The OLS and IV estimations produce similar estimates around 0.4.

I back out location-specific rent shifters $\bar{r}^i$ given the estimate of rent elasticity $\hat{\eta} = 0.41$, the expenditure share on housing service $\gamma = 0.25$, and the income data. Rearranging equation (14) yields

$$\bar{r}^i = \frac{r_t^i}{\left(\gamma \sum_r \sum_a L_{r,a,t}^i w_{r,a,t}^i\right)^{\eta}}. \tag{40}$$

But if I replace the model objects with data counterparts, the left-hand side and the right-hand side of equation (40) cannot perfectly equate because the left-hand side depends on only location i, but the right-hand side depends on both location i and time t. By taking the averages of the numerator and the denominator on the right-hand side over time, I compute the sample counterpart to the location-specific rent shifter $\hat{\bar{r}}^i$

$$\hat{\bar{r}}^i = \frac{\frac{1}{8}\sum_{t=1940}^{2010} r_t^i}{\frac{1}{8}\sum_{t=1940}^{2010}\left(\gamma \cdot \text{income}_t^i\right)^{\hat{\eta}}},$$

where $t = 1940, \cdots, 2010$ runs the sample periods, and income_t^i denotes the total income in location i and period t.

Using the estimates $\hat{\eta}$ and $\{\hat{\bar{r}}^i\}^i$, I can predict rent by

$$\hat{r}_t^i = \hat{\bar{r}}^i\left(\gamma \cdot \text{income}_t^i\right)^{\hat{\eta}}. \tag{41}$$

Figure 9 plots actual rent in the data against rent predicted by equation (41). Predicted rent has a tight and linear relationship with actual rent. The correlation between actual and predicted rent is 0.94.

4.4 Fertility, Survival Probabilities, and Immigrants

Fertility. Recall that age 0 in the model corresponds to ages 1 to 10 in the data. I attribute each person of ages 1 to 10 to the parents (in the household including a married couple) or the single parent (in the single parent household) of race-age bins in each period. Averaging the number of children in each race-age bin in each period yields the data counterpart to the number of babies per person $\alpha_{r,a,t}$ for (r, a, t). This procedure is detailed in Appendix G.

Survival probabilities. The Centers for Disease Control and Prevention (CDC) publish life tables documented by several different government agencies. I use life tables for 1940, 1950, $\cdots$, 2010. These tables provide the annual survival probabilities for Black and white Americans at each age for these sample years. Since periods and age bins are of 10-year windows in the quantification of the model, I map annual survival probabilities for 1-year age bins in the life tables to 10-year survival probabilities for 10-year age bins. This procedure is detailed in Appendix H.

Immigrants from abroad. In the model, population dynamics (7) take into account immigrants from abroad. Recall that locations in my quantification cover all US states and DC except Alaska and Hawaii. Thus all migrants from outside of the contiguous US are regarded as immigrants from abroad. I tabulate the numbers of immigrants for each race r and age a in period t and location i using the census and ACS data. See Appendix I for details.

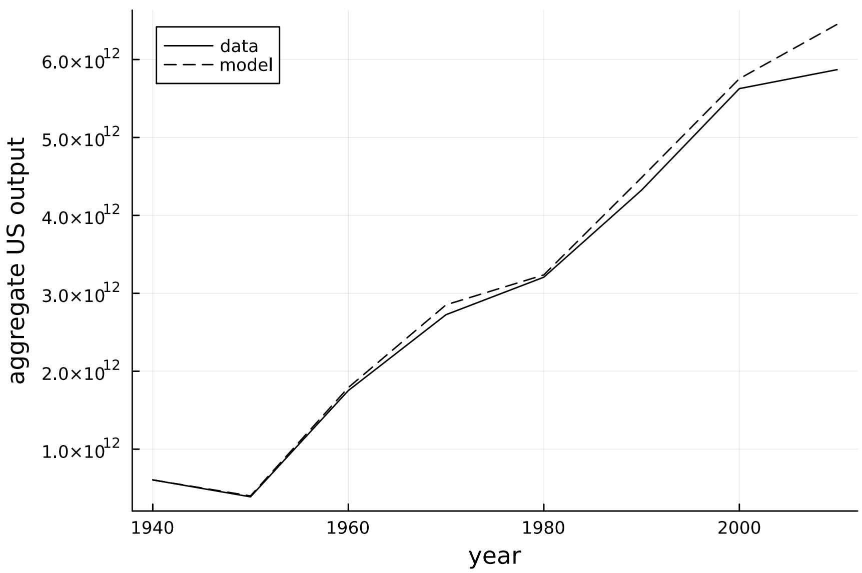
4.5 Computation of Steady States and Transition Paths

To compute transition paths, I first compute steady states toward which transition paths converge. Then I compute transition paths given an initial condition. Appendix J details the algorithm to compute steady states. Appendix K explains how to compute transition paths toward given steady states. I make technical assumptions on fertility parameters $\{\alpha_{r,a,t}\}_{r,a,t}$ so that populations smoothly converge to steady state levels. Appendix G delineates the assumptions on fertility parameters.

5 Model Fit

I compare variables in the baseline equilibrium of the model with their data counterparts.

Figure 10: Aggregate US Output: Model vs Data



Notes: Following (42), I plot aggregate US output generated by the baseline equilibrium of the model and its data counterpart.

First, I compare aggregate US output between the baseline equilibrium and the data. Aggregate output in period t, Y_t, is

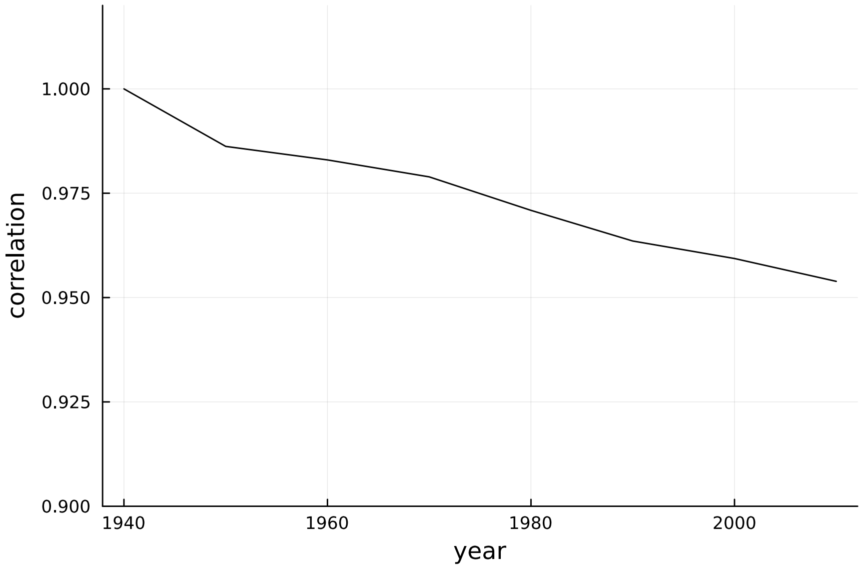
$$Y_t = \sum_{i \in \mathcal{N}} Y_t^i = \sum_{i \in \mathcal{N}} A_t^i L_t^i = \sum_{i \in \mathcal{N}} \sum_{r \in \{b,n\}} \sum_{a=1}^{\bar{a}} w_{r,a,t}^i L_{r,a,t}^i, \tag{42}$$

where the second and third equalities follow from equations (9) and (36), respectively.[20] The right-most object has the data counterpart because it depends on only wages and populations. Figure 10 plots aggregate US output (or labor income) in the baseline equilibrium of the model and in the data over time. From 1940 to 2000, the baseline equilibrium closely resembles the data in aggregate output. (the largest difference is 4.6 percent in 1970.) In 2010, the baseline equilibrium overstates aggregate output by 9.9 percent.

Second, I compare populations in the baseline equilibrium with those in the data. Pick up any sample year t from $\{1940, \cdots, 2010\}$. For t, let $(L_{r,a,t}^{i,\text{baseline}})_{r,a}^i$ and $(L_{r,a,t}^{i,\text{data}})_{r,a}^i$ be the vectors of the popu-

[20] Hsieh and Moretti (2019) discuss the impact of spatial misallocation on aggregate output defined similarly. See their equation (7).

Figure 11: Correlations between the Populations in the Baseline Equilibrium and in the Data



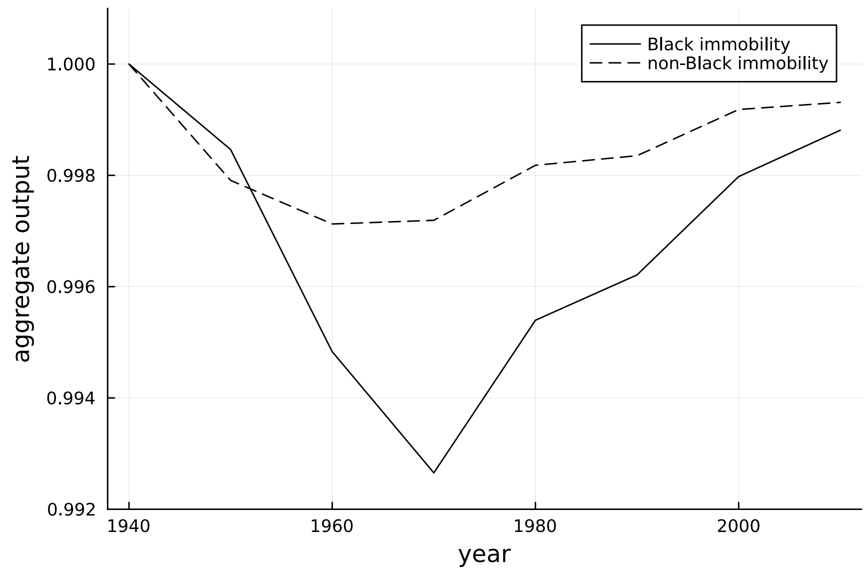
Notes: For each year, this graph shows the correlation between the population vector in the baseline equilibrium $(L_{r,a,t}^{i,\text{baseline}})_{r,a}^{i}$ and the population vector in the data $(L_{r,a,t}^{i,\text{data}})_{r,a}^{i}$. Since I load the actual population vector in 1940 to the model as the initial population vector, the correlation is one in 1940 by construction.

lations in the baseline equilibrium and in the data, respectively. Then for each sample year t, I compute the correlation coefficient between these two population vectors $Cor((L_{r,a,t}^{i,\text{baseline}})_{r,a}^{i}, (L_{r,a,t}^{i,\text{data}})_{r,a}^{i})$. Figure 11 plots such correlations over time. In 1940, the correlation is one because I load the actual populations in 1940 as the initial population. The correlation between the model and data population vectors declined over time. But, even in 2010, the correlation between the model and data population vectors is over 0.95. Throughout the sample years, the baseline equilibrium captures the spatial distribution of populations fairly well.

6 The Effects of the Great Black (and non-Black) Migration

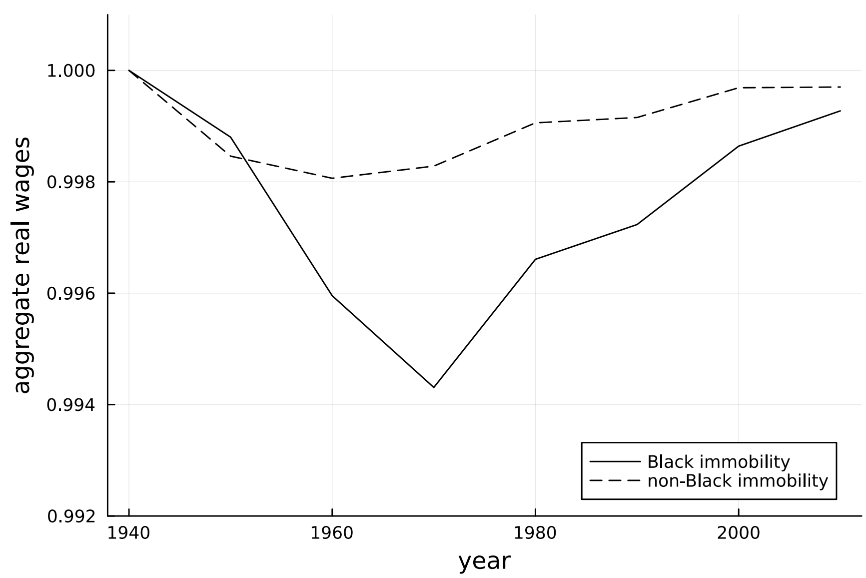
I compare the baseline equilibrium that resembles the US economy from 1940 to 2010 with two counterfactual equilibria. In the first counterfactual equilibrium, Black Americans cannot migrate across the North and the South from 1940 to 1960. That is, $\tau_{b,a,t}^{j,i} = \infty$ for any pair of locations j, i such that $(j,i) \in \mathcal{N}_N \times \mathcal{N}_S$ or $(j,i) \in \mathcal{N}_S \times \mathcal{N}_N$, any age a, and $t = 1940, \cdots, 1960$. Here I bilaterally shut down the migration of Black Americans from the North to the South and from the South to the North. Since migration decisions are made one period ahead of arrival, this shuts down Black Americans' relocation until 1970, the end of the great Black migration. I call this equilibrium the equilibrium of Black immobility. In the second counterfactual equilibrium, non-Black Americans cannot migrate across the North and the South from 1940 to 1960. That is, $\tau_{n,a,t}^{j,i} = \infty$ for any pair of locations j, i such that $(j,i) \in \mathcal{N}_N \times \mathcal{N}_S$ or $(j,i) \in \mathcal{N}_S \times \mathcal{N}_N$, any age a, and $t = 1940, \cdots, 1960$. I call this equilibrium the equilibrium of non-Black immobility.

Figure 12: Aggregate Output



Notes: Aggregate output in the equilibria of Black and non-Black immobility relative to the baseline equilibrium.

Figure 13: Aggregate Real Wages



Notes: Aggregate real wages in the equilibria of African Americans' or others' immobility relative to the baseline equilibrium.

Figure 12 plots aggregate output in the two counterfactual equilibria relative to the baseline equilibrium. In 1970, aggregate output in the equilibrium of Black immobility is 0.73 percent lower than aggregate output in the baseline equilibrium, as in the solid line. The dashed line shows that in the same year, aggregate output in the equilibrium of non-Black immobility is 0.28

percent lower than aggregate output in the baseline equilibrium. These two results imply that Black Americans' relocation across the North and the South increased aggregate output more than non-Black Americans' relocation did, although Black Americans accounted for only about 10 percent of the US population. The back-of-the-envelope calculation in Appendix L predicts that if Black Americans were spatially distributed as in 1940, aggregate labor income in 1970 would have been lower than actual aggregate labor income in the data by 0.86 percent. Therefore the quantitative model and the back-of-the-envelope calculation yield similar predictions for the aggregate impact of the great Black migration. And it is reasonable that the back-of-the-envelope calculation predicts a larger loss from the Black immobility than the GE model because the former does not take into account congestion force.

Figure 13 plots aggregate real wages

$$\sum_{i\in\mathcal{N}}\sum_{r\in\{b,n\}}\sum_{a=1}^{\bar{a}} L^i_{r,a,t}\frac{w^i_{r,a,t}}{(r^i_t)^\gamma}$$

in the two counterfactual equilibria relative to the baseline equilibrium. Shutting down the North-South migration of Black and non-Black Americans decreases aggregate real wages by 0.57 percent and 0.17 percent, respectively. Shutting down the North-South relocation decreases real wages less than output because higher nominal wages are partly offset by higher housing rent. But the difference between the decrease in aggregate output and the decrease in aggregate real wages is not large.

Table 8: Nominal Wage Changes due to the Great Black Migration

		this paper	Boustan (2009)	
			OLS	IV
North	Black Americans	0.052	0.096	0.072
	non-Black Americans	-0.002	-0.005	-0.004
South	Black Americans	-0.038	-	-
	non-Black Americans	0.005	-	-

Notes: Percent changes in nominal wages from the baseline equilibrium to the equilibrium of Black immobility. The result of Boustan (2009) is from her table 6.

What wages would Black and non-Black Americans have earned if the great Black migration did not occur? Let region g index the North N or the South S, $g \in \{N,S\}$. Then for $g \in \{N,S\}$, define the average nominal wage of race $r \in \{b,n\}$ in region g and period t by

$$\text{average nominal wage}^g_{r,t} = \frac{\sum_{i\in\mathcal{N}_g}\sum_{a=1}^{\bar{a}} L^i_{r,a,t} w^i_{r,a,t}}{\sum_{i\in\mathcal{N}_g}\sum_{a=1}^{\bar{a}} L^i_{r,a,t}}.$$

Let average nominal wage$^g_{r,t}$ and average nominal wage$^{g,\text{no mig}}_{r,t}$ be such average nominal wages of race r in region g and period t in the baseline equilibrium and in the equilibrium of Black immobility, respectively. Then the percent change in the average nominal wage of race r in region g and period t from the baseline equilibrium to the equilibrium of Black immobility is

$$\frac{\text{average nominal wage}^{g,\text{no mig}}_{r,t}}{\text{average nominal wage}^{g}_{r,t}} - 1.$$

Table 8 shows the percent changes in the average nominal wages in its first column. The year is 1970. According to my quantitative model, if the great Black migration did not occur, the average wage of Black Americans in the North would have been higher by 5.2 percent, and the average wage of non-Black Americans in the North would have been lower by 0.2 percent. As is common in Boustan (2009) and this paper, Black and non-Black Americans are imperfectly substitutable. In the baseline equilibrium, the inflow of Black Americans from the South to the North decreased the wages of Black Americans in the North. But in the equilibrium of Black immobility, Black Americans in the North would have had fewer competitors in their local labor markets than in the baseline equilibrium and would have received higher wages. The second and third columns show the predictions by Boustan (2009). My result for the change in Black Americans' wages in the North, 5.3 percent, is a little smaller than her predictions ranging from 7.2 percent to 9.6 percent.[21] My result for the change in others' wages in the North, -0.2 percent, is, again, a little smaller than her predictions, -0.4 or -0.5 percent, in the absolute values. I differ from Boustan (2009) in that I make wage predictions not only in the North but also in the South. If Black Americans could not migrate to the North, more Black Americans would have remained in the South. The average nominal wage of Black Americans in the South would have been lower by 3.8 percent in the no great Black migration scenario than in the baseline. Non-Black wages in the South would have been higher by 0.5 percent.

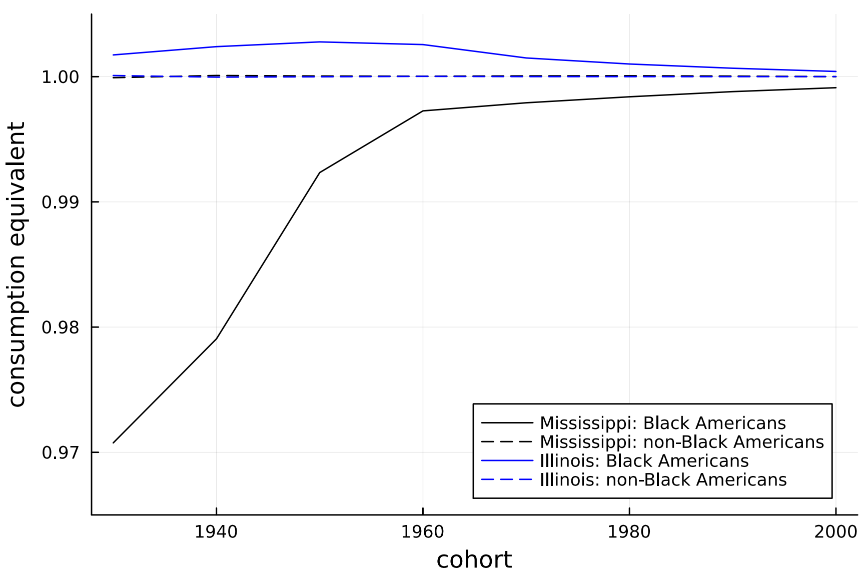
I turn to the welfare effects of the Black and non-Black migration across the North and the South. Figure 14 plots the welfare for each cohort of Black and non-Black Americans born in Mississippi and Illinois in the equilibrium of Black immobility relative to the baseline equilibrium. The welfare changes are measured by consumption equivalent, as detailed in Appendix M. As they lost opportunities of migrating to the high-wage North, in the equilibrium of Black immobility, the welfare of Black Americans born in Mississippi in the 1930s would have been 2.9 percent lower than in the baseline equilibrium. Although Black Americans born in Illinois lost opportunities of migrating to the South till 1970, in the equilibrium of Black immobility, their welfare for the cohort 1950 would have been 0.28 percent higher than in the baseline equilibrium. The welfare of non-Black Americans in the equilibrium of Black immobility are very similar to the welfare of non-Black Americans in the baseline equilibrium. As in the blue solid line, for Black Americans in Illinois, the welfare change from the baseline to the equilibrium of Black immobility for the cohorts 1950 and 1960 are higher than for the cohorts 1930 and 1940. This is because the earlier

[21] She considers wages of employed men between the ages of 18 and 64 whereas wages in my analysis are per-capita payrolls (the total payroll divided by the number of individuals in each race-age-location bin). My sample includes women and the nonemployed. This difference may drive the difference in wage predictions.

generations cannot move to the South in their youth (till 1970), but the later generations can move to the South in their youth and still benefit from fewer competitors in their local labor market. As in the blue solid line, if the great migration did not occur, Black Americans in the North would have been better off for generations. This is reminiscent of the result of Derenoncourt (2022), who argues intergenerational negative impacts of the great migration on locations in the North which received relatively large Black migrants from the South.

In Appendix O, Figure 21 shows the welfare of Black Americans born in the 1930s in the equilibrium of Black immobility relative to the one in the baseline equilibrium across states. In the equilibrium of Black immobility, the welfare of Black Americans born in the South would have been lower than in the baseline equilibrium by 1.3 to 2.9 percent, depending on the states. The welfare loss from the baseline equilibrium to the equilibrium of Black immobility is particularly large in South Carolina, Mississippi, and Arkansas. In the equilibrium of Black immobility, the welfare of Black Americans born in most states in the North would have been higher than in the baseline equilibrium by less than 1 percent.[22] As in Figure 22, moving from the baseline equilibrium to the equilibrium of Black immobility does not substantially change welfare of non-Black Americans born in the 1930s across states.

Figure 14: Welfare: Black Immobility

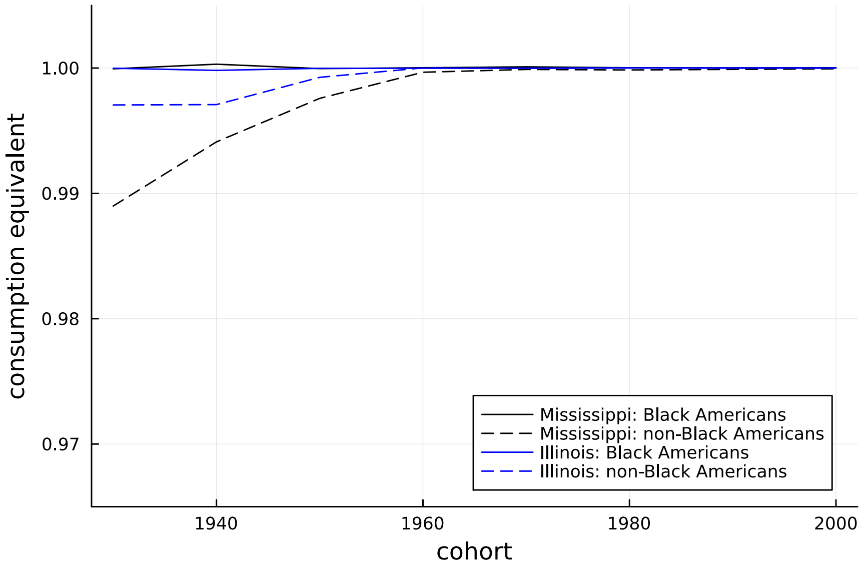


Notes: The welfare of Black and non-Black Americans born in Mississippi and Illinois in the equilibrium of Black immobility relative to those in the baseline equilibrium (consumption equivalent).

Figure 15 plots the welfare for each cohort of Black and non-Black Americans born in Mississippi and Illinois in the equilibrium of non-Black immobility relative to the baseline equilibrium. The

[22] In the equilibrium of Black immobility, the welfare of Black Americans born in 5 locations in the North would have been lower than in the baseline equilibrium. These 5 locations are DC, Colorado, the Rest of the North, Oklahoma, and Arizona in the order of the magnitude of the welfare loss from the baseline equilibrium to the equilibrium of Black immobility.

Figure 15: Welfare: Non-Black Immobility



Notes: The welfare of Black and non-Black Americans born in Mississippi and Illinois in the equilibrium of non-Black immobility relative to those in the baseline equilibrium (consumption equivalent).

welfare of non-Black Americans born in Mississippi in the 1930s in the equilibrium of non-Black immobility would have been 1.1 percent lower than that in the baseline equilibrium. Recall that in Figure 14, for this cohort, the welfare of Black Americans born in Mississippi would have been 2.9 percent lower in the equilibrium of Black Americans' immobility than in the baseline equilibrium. Thus these two figures jointly highlight Black Americans' strong incentive for outmigration from the South. Non-Black Americans in Illinois would also have been worse off by closing the North-South border for non-Black Americans because they lost varieties in location choices. The effects of non-Black immobility on Black Americans' welfare are small.

In Appendix O, Figure 24 shows the welfare changes of non-Black Americans born in the 1930s from the baseline equilibrium to the equilibrium of non-Black immobility across states. In the equilibrium of non-Black immobility, the welfare of non-Black Americans in the North and the South would have been lower than in the baseline equilibrium by 0.2 to 1.0 percent and 1.0 to 2.2 percent, respectively, depending on the states. Arkansas has the largest welfare loss of 2.2 percent for non-Black Americans born in the 1930s moving from the baseline to the equilibrium of non-Black immobility. As in Figure 23, moving from the baseline equilibrium to the equilibrium of non-Black immobility does not substantially change the welfare of Black Americans across states.

In the great migration period, the gaps in wages and living standards between Black and non-Black Americans shrank. How did the relocation of Black Americans across the North and the South contribute to reducing the racial gaps? For period t and race $r \in \{b, n\}$, I define the

Table 9: Real Wage Ratios between Black and non-Black Americans

	1940	1950	1960	1970	1980	1990	2000	2010
baseline	0.448	0.576	0.563	0.634	0.677	0.696	0.731	0.717
Black immobility	0.448	0.565	0.525	0.579	0.645	0.670	0.718	0.710

Notes: Nationwide average real wage ratios between Black and non-Black Americans in the baseline equilibrium and in the equilibrium of Black immobility.

(nationwide) average real wage by

$$\text{average real wage}_{r,t} = \frac{\sum_{i\in\mathcal{N}}\sum_{a=1}^{\bar{a}} L_{r,a,t}^{i}\left(\frac{w_{r,a,t}^{i}}{(r_t^i)^{\gamma}}\right)}{\sum_{i\in\mathcal{N}}\sum_{a=1}^{\bar{a}} L_{r,a,t}^{i}}.$$

Then I compute the ratio of average real wages between African American and others

$$\frac{\text{average real wage}_{b,t}}{\text{average real wage}_{n,t}}.$$

Table 9 reports the Black-non-Black average real wage ratios in the baseline equilibrium and in the equilibrium of Black immobility. If Black Americans could not migrate across the North and the South, the Black-non-Black average real wage ratio is smaller. In 1970, the Black-non-Black average real wage ratio was 0.634 in the baseline equilibrium. In the same year, the Black-non-Black average real wage ratio was 0.578 in the equilibrium of Black immobility. Therefore, the relocation of Black Americans across the North and the South decreased the racial gap in average real wages by 8.8 percent, because $(0.634-0.579)/0.634 = 0.088$.

Since the prior literature primarily addresses nominal wages, I similarly define the average nominal wage for race $r \in \{b, n\}$ and time t by

$$\text{average nominal wage}_{r,t} = \frac{\sum_{i\in\mathcal{N}}\sum_{a=1}^{\bar{a}} L_{r,a,t}^{i} w_{r,a,t}^{i}}{\sum_{i\in\mathcal{N}}\sum_{a=1}^{\bar{a}} L_{r,a,t}^{i}}.$$

I take the ratio of average nominal wages between African Americans and others

$$\frac{\text{average nominal wage}_{b,t}}{\text{average nominal wage}_{o,t}}.$$

Table 10 reports the Black-non-Black ratios of average nominal wages in the baseline equilibrium and in the equilibrium of Black immobility. In 1970, the Black-non-Black average nominal wage ratios were 0.629 and 0.566 in the baseline equilibrium and in the equilibrium of Black immobility, respectively. Therefore, the great Black migration reduced the racial gap in nominal wages by $(0.629-0.566)/0.629 = 0.102$, 10.2 percent. This is a similar number to Smith and Welch (1989) who use a reduced-form decomposition technique to compute the contribution of the great

Table 10: Nominal Wage Ratios between Black and non-Black Americans

	1940	1950	1960	1970	1980	1990	2000	2010
baseline	0.431	0.562	0.555	0.629	0.673	0.694	0.728	0.716
Black immobility	0.431	0.549	0.511	0.566	0.635	0.663	0.712	0.707

Notes: Nationwide average nominal wage ratios between Black and non-Black Americans in the baseline equilibrium and in the equilibrium of Black immobility

migration to reducing the racial gap in nominal wages.

7 Conclusion

4 million Black Americans migrated from the South to the North between 1940 and 1970, which is called the great Black migration. This paper has quantified the aggregate and distributional effects of the great Black migration. An overlapping generations model of the spatial economy has served this purpose. I have estimated the elasticities in the model and backed out the other parameters. My quantitative model and the existing reduced-form studies have produced comparable predictions about nominal wages and racial inequality in the counterfactual, no great migration scenario. The quantitative model revealed that between 1940 and 1970, the mobility of Black Americans across the North and the South increased aggregate US output more than the mobility of non-Black Americans did. I view this paper as the first step in understanding the connection between the geography of Black Americans and the aggregate performance of the US economy.

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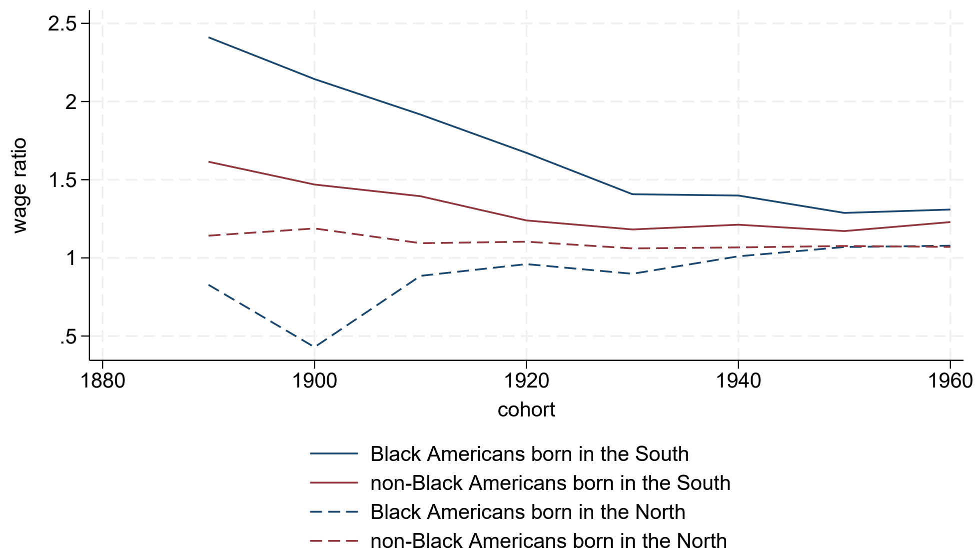
Appendices

A An Additional Figure on Motivating Facts

Figure 16: Mover-Stayer Ratios of Payrolls per Capita for Cohorts, Races, and Birthplaces



Notes: For each cohort (say c), race, and birthplace (the North or the South), this graph provides the ratio of the payroll per capita of movers to the payroll per capita of stayers as of year $c+50$. Source: US census 1940-2000, American Community Survey 2010.

B Details on Data Sources

Wages, populations, migration shares, and fertility. The data on wages, populations, migration shares, and fertility (babies per person) are from the US censuses from 1940 to 1990, and the ACS from 2000 to 2019. I use the full count data for the US census 1940, 5 percent samples for the US censuses 1960, 1980, and 1990, and 1 percent samples for the US censuses 1950 and 1970, and the ACS of all the sample years. All of them are tabulated in IPUMS USA (Ruggles et al., 2022). Figure 1 requires the data on the populations of African Americans, enslaved African Americans, and others in the North and the South since 1790. These data are from the US censuses tabulated in IPUMS NHGIS (Manson et al., 2022).

Rent. IPUMS does not provide the data on rent in 1950. The website of the US Census Bureau provides median rent in states from 1940 to 2000.[23] I obtain rent in 2010 and 2019 from IPUMS

[23] https://www.census.gov/data/tables/time-series/dec/coh-grossrents.html (accessed on 10/31/2022)

NHGIS (Manson et al., 2022).

Survival probabilities. Survival probabilities are from life tables published on the CDC's website.[24]

Aggregate income, college graduates, manufacturing shares. Aggregate incomes in states are used in the regression (39). In this regression, I use the manufacturing shares in employment and the shares of college graduates in the population in 1950 as IVs. All of these variables are from IPUMS NHGIS (Manson et al., 2022).

Consumer price index. Wages and rent are deflated by the consumer price index and measured in the 2010 US dollars. The Bureau of Labor Statistics publishes consumer price indices on its website.[25]

C Imputation of Wage Data

As I detailed in Appendix B, to compute wages at race-age-location-period levels, I use the full count data for 1940, 5 percent samples for 1960, 1980, and 1990, and 1 percent samples for 2000 and 2010. Here, by the wage of race r and age a in period t and location i, I mean the weighted average of per capita payrolls in the bin

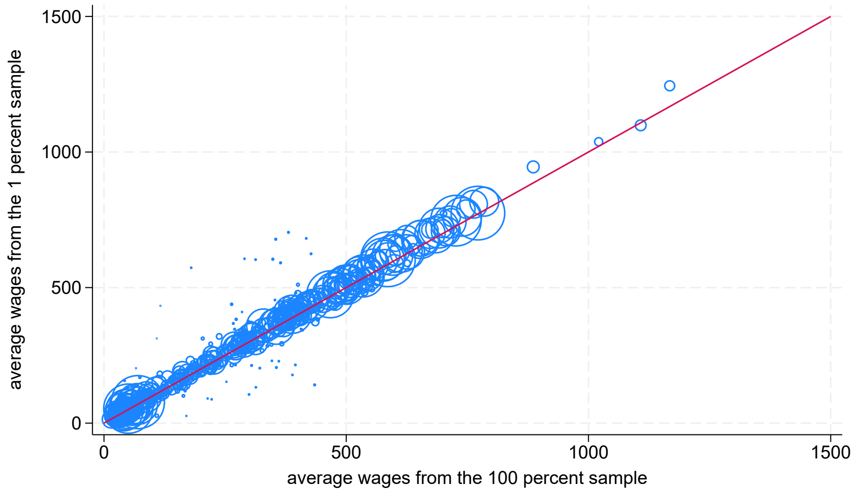
$$w^i_{r,a,t} = \frac{\sum_{d \in \mathcal{I}^i_{r,a,t}} \text{weight}_d \cdot \text{payroll}_d}{\sum_{d \in \mathcal{I}^i_{r,a,t}} \text{weight}_d},$$

where $\mathcal{I}^i_{r,a,t}$ denotes the set of individuals (observations) who fall in the race-age-state bin (r, a, i) in sample year t, d indexes observations in the bin, weight_d denotes the sampling weight of individual d, and payroll_d is the annual labor income of individual d. A problem is that if my sample is a 1 or 5 percent sample from the US population, such computed $w^i_{r,a,t}$ may be seriously affected by outliers in bins where the numbers of observations are small. This can occur for African Americans in some states in the North.

I can explore how average wages for race-age-state triples from the 1 percent sample defer from those from the 100 percent (full) sample using the data for the year 1940. Figure 17 plots average wages from the 1 percent sample against those from the 100 percent sample. Bubble sizes represent the numbers of observations in race-age-state bins. The average wages from the 1 percent sample largely align with those from the 100 percent, but several average wages from the 1 percent sample deviate from those from the 100 percent sample, and all of them are small bubbles. This is because average wages computed with small samples suffer from sampling errors. The correlation coefficient between averages wages from the 1 percent sample and those from the 100 percent sample is 0.9631.

[24] `https://www.cdc.gov/nchs/products/life_tables.htm` (accessed on 10/31/2022)
[25] `https://data.bls.gov/cgi-bin/surveymost?bls` (accessed on 11/01/2022)

Figure 17: Average Wages from the 1 and 100 Percent Samples in 1940



Notes: This figure plots average wages for race-age-state triples from the 1 percent sample against those from the 100 percent sample in 1940. Bubble sizes represent the numbers of observation in race-age-state bins.

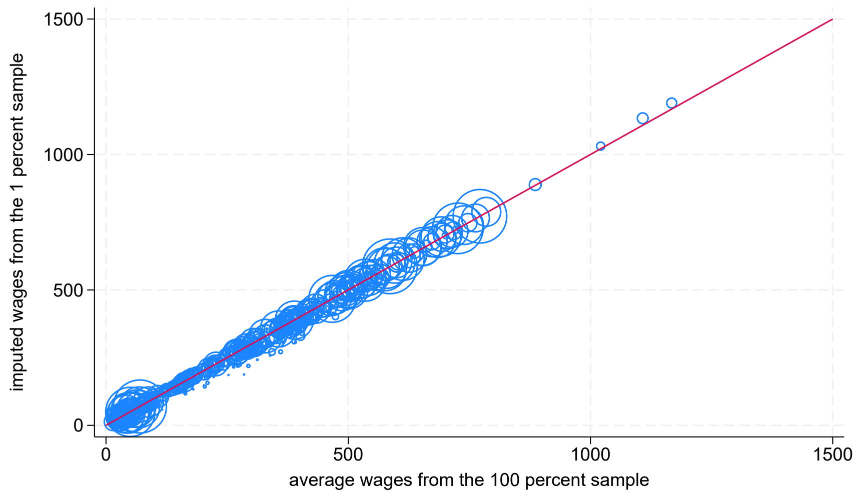
Now I take a different approach. For the 1 percent sample, after computing average wages for race-age-state bins, for each sample year, I regress log of average wages on three two-way fixed effects: state-race, state-cohort, and race-cohort. Note that since this regression is done for each year, controlling cohorts implies controlling ages. Then take the exponentials of the predicted values (predicted log-wages) to obtain the predicted wages. I call these predicted wages as the "imputed wages." Figure 18 plots imputed wages from the 1 percent sample against average wages from the 100 percent sample. Apparently, bubbles are more close to the 45 degree line (the red line) in this graph than in Figure 17. Indeed, the correlation coefficient between imputed wages from the 1 percent sample and average wages from the 100 percent sample is 0.9956, which is higher than the correlation coefficient between average wages from the 1 percent sample and those from the 100 percent sample, 0.9631.

I have full sample data for only the year 1940. Therefore, I cannot know how average wages computed with 1 or 5 percent samples in the other sample years suffer from sampling errors. Here I am conservative on imputing. I replace the average wage with the imputed wage only when the average wage is zero. The number of such race-age-state bins is only 16 out of 4,788 (0.3 percent). It might be the case that I should use only imputed wages for the entire estimation/calibration instead of using just 0.3 percent of them.

D Tabulation of Migration Shares

In the quantification of the model, time periods are 10 years. The US censuses and ACS report individuals' locations 1 or 5 years ago, depending on sample years. Table 11 reports which sample

Figure 18: Imputed Wages from the 1 Percent Sample and Average Wages from the 100 Percent Sample in 1940



Notes: This figure plots imputed wages for race-age-state triples from the 1 percent sample against average wages from the 100 percent sample in 1940. Bubble sizes represent the numbers of observation in race-age-state bins.

year includes 1- or 5-year migration shares. I need to map 1- or 5-year migration shares in the data to 10-year migration shares in the model. In the model, individuals make migration decisions in period t and arrive in destinations in period $t+1$. Thus, the census data of 1940 inform me of migration decisions as of 1930, the census data of 1950 inform me of migration decisions as of 1940, and so on.

Let $\mu_{r,a,t}^{i,j,10}$ be the model-consistent 10-year migration share for race r and age a in year t from location j to location i. $\mathbf{M}_{r,a,t}^{10}$ is the matrix whose (i,j) element is $\mu_{r,a,t}^{i,j,10}$.

The censuses in the years 1940, 1960, 1970, 1980, and 1990 yield 5-year migration shares. Let $\mu_{r,a,t}^{i,j,5,\text{data}}$ be such 5-year migration share of race r and age a in (the sample) year t from location j to location i. $\mu_{r,a,t}^{i,j,5,\text{data}}$ is directly computed from the census data of the aforementioned sample years. Let $\mathbf{M}_{r,a,t}^{5,\text{data}}$ be the matrix whose (i,j) element is $\mu_{r,a,t}^{i,j,5,\text{data}}$. I assume that for such census year $t+10$, 5-year migration shares are constant between years t and $t+10$. Then for $t = 1930, 1950, 1960, 1970, 1980$, the model-consistent migration matrix $\mathbf{M}_{r,a,t}^{10}$ is computed by

$$\mathbf{M}_{r,a,t}^{10} = \left(\mathbf{M}_{r,a,t+10}^{5,\text{data}}\right)^2.$$

The census in 1950 yields 1-year migration shares. Let $\mu_{r,a,t}^{i,j,1,\text{data}}$ be the 1-year migration share for race r and age a in the census or ACS year t from location j to location i. Let $\mathbf{M}_{r,a,t}^{1,\text{data}}$ be the matrix whose (i,j) element is $\mu_{r,a,t}^{i,j,1,\text{data}}$. I assume that from 1940 to 1950, 1-year migration shares are constant. Then the model-consistent migration matrix $\mathbf{M}^{10}_{r,a,1940}$ is computed by

$$\mathbf{M}^{10}_{r,a,1940} = \left(\mathbf{M}^{1,\text{data}}_{r,a,1950}\right)^{10}.$$

Since 2000, ACS reports current locations and locations 1 year ago for individuals every year. Then $\mathbf{M}^{10}_{r,a,2000}$ is computed by

$$\mathbf{M}^{10}_{r,a,2000} = \mathbf{M}^{1,\text{data}}_{r,a,2001}\mathbf{M}^{1,\text{data}}_{r,a,2002}\cdots\mathbf{M}^{1,\text{data}}_{r,a,2010}.$$

Since I would like to avoid picking up irregularity caused by the COVID-19 pandemic in 2020, $\mathbf{M}^{10}_{r,a,2010}$ is computed by

$$\mathbf{M}^{10}_{r,a,2010} = \mathbf{M}^{1,\text{data}}_{r,a,2011}\cdots\mathbf{M}^{1,\text{data}}_{r,a,2018}\mathbf{M}^{1,\text{data}}_{r,a,2019}\mathbf{M}^{1,\text{data}}_{r,a,2019},$$

where the 1-year migration shares in the 2019 data are double-counted and the 1-year migration shares in the 2020 data are excluded.

Table 11: 1- or 5-Year Migration Shares

year	source	location X years ago
1940	census	5
1950	census	1
1960	census	5
1970	census	5
1980	census	5
1990	census	5
2000-2019	ACS	1

Notes: The US censuses and American Community Survey report individuals' locations 1 or 5 years ago depending on sample years.

E First Difference Estimation of the Elasticity of Substitution across Races

I estimate the elasticity of substitution across races within age, time, and location bins σ_1 by the first difference estimation, following Monras (2020). Taking the time difference in equation (26), I have

$$\Delta\log\left(\frac{w^n_{b,a}}{w^n_{o,a}}\right) = -\frac{1}{\sigma_1}\Delta\log\left(\frac{L^n_{b,a}}{L_{o,a}}\right) + \frac{1}{\sigma_1}\Delta\log\left(\frac{\kappa^n_{b,a}}{\kappa^n_{o,a}}\right).$$

Since the growth rate of the productivity ratio $\Delta \log\left(\frac{\kappa^n_{b,a}}{\kappa^n_{o,a}}\right)$ is unobservable, the econometric specification is

$$\Delta \log\left(\frac{w^n_{b,a}}{w^n_{o,a}}\right) = -\frac{1}{\sigma_1}\Delta \log\left(\frac{L^n_{b,a}}{L_{o,a}}\right) + f_a + \epsilon^n_a, \tag{43}$$

where f_a is the age fixed effect, and ϵ^n_a is the error term. The time differences are taken between 1940 and 2010. If the population of African Americans relative to the population of others increase in a location with a high growth rate of productivity ratio between African Americans and others $\Delta \log\left(\frac{\kappa^n_{b,a}}{\kappa^n_{o,a}}\right)$, OLS estimation of equation (43) produces an upward bias for $-1/\sigma_1$. Following Monras (2020), I instrument $\Delta \log\left(\frac{L^n_{b,a}}{L_{o,a}}\right)$ by the level of population ratio between African Americans and others in the old time, 1930, $\log\left(\frac{L^n_{b,a,1930}}{L^n_{o,a,1930}}\right)$. I make two assumptions. One is that the population ratio between African Americans and others as of 1930 is not correlated with the growth rate of the productivity ratio between African Americans and others between 1940 and 2010. The other assumption is that the growth rate of the population ratio between African Americans and others between 1940 and 2010 is correlated with the level of the population ratio between the two racial groups as of 1930. An example of the latter assumption is that from 1940 to 2010, African Americans migrated to Illinois or New York where a moderate number of African Americans resided in 1930, but very few African Americans migrated to Wyoming or Montana where very few African Americans resided in 1930.

Table 12: Elasticity of Substitution across Races: First Difference Estimation

Dependent variable:	$\Delta \log(w^n_{b,a}/w^n_{o,a})$	
Model:	OLS	IV
$\Delta \log(L^n_{b,a}/L^n_{o,a})$	-0.1510***	-0.2048***
	(0.0235)	(0.0276)
fixed effects:		
age	✓	✓
Observations	228	228
First-stage F-statistic		621.3

Notes: The first difference estimation of the elasticity of substitution across races. Robust standard errors are in parentheses. Significance codes: ***: 0.01.

Table 12 reports the results. The first column reports the result of OLS, and the second column reports the result of the IV estimation. In line with my conjecture, the IV estimation seems to correct a positive bias in OLS.

F Standard Errors of the Elasticities of Substitution

The estimation of the elasticity of substitution across ages σ_0 involves an estimate of σ_1, as equations (11), (31), and (33) imply. Let $\hat{\sigma}_r$ and $\hat{\sigma}_a$ be estimates for σ_1 and σ_0, respectively. Then the standard error of $\hat{\sigma}_a$ need to take into account variability of $\hat{\sigma}_r$.

For this purpose, I compute block bootstrap standard errors. This is a similar approach to Glitz and Wissmann (2021). I have the data of wages and populations from 1940 to 2010 and the data of migration shares from 1930. To construct shift-share predicted populations (28) and gross inflows (29), I need migration shares 30 years before the year for wages and populations. The earliest migration data are of 1930, so my sample years are 1960 to 2010. I split the sample years to two groups. Group 1 consists of the years 1960, 1970, and 1980. Group 2 consists of the years 1990, 2000, and 2010. The reason that I resample block of years is that bootstrap samples would understate serial correlation over time if each year is resampled separately.[26]

The procedure of block bootstrap is the following. Set the number of bootstrap samples $B = 10,000$. Recall that the number of locations, N, is 38. Then for $b = 1, \cdots, B$,

1. Randomly choose one of N locations $2N$ times, allowing for replacement.[27] I get x_b, a $2N$-dimensional vector of locations . Treat them as $2N$ distinct locations.

2. Draw a Bernoulli random number $2N$ times with the success probability $1/2$. I get y_b, a $2N$-dimensional vector whose element is either 0 or 1.

3. Let $x_{b,i}$ and $y_{b,i}$ be the i-th elements of x_b and y_b, respectively. If $y_{b,i} = 0$, location $x_{b,i}$ has the sample years in group 1. If $y_{b,i} = 1$, location $x_{b,i}$ has the sample years in group 2. Then $y_{b,i} + 1$ is the group number of sample years.

4. Collect all observations in the pairs of $(x_{b,i}, y_{b,i} + 1)_{i=1}^{2N}$ from the original sample. Note that all age bins are collected within location-sample year group pairs $(x_{b,i}, y_{b,i} + 1)$. Call such bootstrap sample S_b.

5. For bootstrap sample S_b, compute the OLS, IV1, and IV2 estimates for σ_1, following Subsubsection 4.2.1. Denote such OLS, IV1, and IV2 estimates by $\hat{\sigma}_{r,b}^{OLS}$, $\hat{\sigma}_{r,b}^{IV1}$, and $\hat{\sigma}_{r,b}^{IV2}$, respectively.

6. Compute the race-specific productivity induced by each of $\hat{\sigma}_{r,b}^{OLS}$, $\hat{\sigma}_{r,b}^{IV1}$, and $\hat{\sigma}_{r,b}^{IV2}$.

7. Using $\hat{\sigma}_{r,b}^{OLS}$, $\hat{\sigma}_{r,b}^{IV1}$, and $\hat{\sigma}_{r,b}^{IV2}$ and the race-specific productivity induced by each of the three estimates, compute the OLS, IV1, and IV2 estimates for σ_0 following Subsubsection 4.2.2. Denote such OLS, IV1, and IV2 estimates by $\hat{\sigma}_{a,b}^{OLS}$, $\hat{\sigma}_{a,b}^{IV1}$, and $\hat{\sigma}_{a,b}^{IV2}$.

Now I have six vectors: $\vec{\hat{\sigma}}_r^{OLS} = (\hat{\sigma}_{r,b}^{OLS})_{b=1}^B$, $\vec{\hat{\sigma}}_r^{IV1} = (\hat{\sigma}_{r,b}^{IV1})_{b=1}^B$, $\vec{\hat{\sigma}}_r^{IV2} = (\hat{\sigma}_{r,b}^{IV2})_{b=1}^B$, $\vec{\hat{\sigma}}_a^{OLS} = (\hat{\sigma}_{a,b}^{OLS})_{b=1}^B$, $\vec{\hat{\sigma}}_a^{IV1} = (\hat{\sigma}_{a,b}^{IV1})_{b=1}^B$, and $\vec{\hat{\sigma}}_a^{IV2} = (\hat{\sigma}_{a,b}^{IV2})_{b=1}^B$. The standard deviations of $\vec{\hat{\sigma}}_r^{OLS}$, $\vec{\hat{\sigma}}_r^{IV1}$, and $\vec{\hat{\sigma}}_r^{IV2}$ are the standard errors in Table 5. The standard deviations of $\vec{\hat{\sigma}}_a^{OLS}$, $\vec{\hat{\sigma}}_a^{IV1}$, and $\vec{\hat{\sigma}}_a^{IV2}$ are the standard errors in Table 6.

[26] See Glitz and Wissmann (2021) for details.

[27] I collect $2N$ locations because the number of sample years in each group (1 or 2) is one half of the number of sample years in the original sample.

G Tabulation of Fertility

G.1 Fertility from 1940 to 2000

From year 1940 to 2010, I compute the data counterparts to babies per person of race r, age a and period t $\alpha_{r,a,t}$ in equation (8) in the following way. Note that the households are sampled in the census data, and information of all members in the sampled households are presumably recorded.

1. Fix census year t.
2. For each household i, count the number of 1-10 year-old children of the household head. Denote such number by b_i.
3. (a) If household i has both the household head and his or her spouse, apportion $x = 0.5b_i$ to the household head's race-age bin, and apportion $x = 0.5b_i$ to his or her spouse's race-age bin.

   (b) If household i has only the household head, and not his or her spouse, apportion $x = b_i$ to the household head's race-age bin.
4. Now I have a list of parents with various values of x. Sum x across all parents within each race-age bin. Let $b_{r,a,t}$ denote such summation of x for race-age bin (r, a).
5. Compute shares of babies of age bin a within race-time tuple (r, t), $\xi_{r,a,t}$,
$$\xi_{r,a,t} = \frac{b_{r,a,t}}{\sum_{a'} b_{r,a',t}}.$$
6. Let $L_{r,0,t}$ be the number of 1-10 year-old people of race r in period t. The babies per person of race-age tuple (r, a), $\alpha_{r,a,t}$ are
$$\alpha_{r,a,t} = L_{r,0,t} \cdot \xi_{r,a,t}.$$

Step 2 captures only 1-10 year-old children whose biological parent is the household head in their household. This may understate the number of children because they may live without biological parents or their parent may not be a household head. Step 5 and 6 correct this understatement of the number of children. First I compute the relative importance of age a in reproduction $\xi_{r,a,t}$ within race-time tuple (r, t). Then I attribute all children $L_{r,0,t}$ of race-time tuple (r, t) to various ages within (r, t) using $\xi_{r,a,t}$. This yields babies per person $\alpha_{r,a,t}$ for race-age-time bin (r, a, t).

G.2 Fertility from 2010 onward

For the baseline equilibrium and relevant counterfactual equilibria, I set fertility as below. First, I compute a steady state toward which the economy converges. In the steady state, I assume that Black and non-Black Americans have the survival probabilities as in 2010. For the steady state, I assume that Black and non-Black Americans of age 2 (ages from 21 to 30 in data) have children such that the population of each race is sustained

$$\alpha_{r,2,\infty} = \frac{1}{s_{r,0,7} \cdot s_{r,1,7}},$$

where period 7 is year 2010. By computing the steady state, I obtain the populations for race-age bins $L_{r,a,\infty} = \sum_{i \in \mathcal{N}} L^i_{r,a,\infty}$. Then for period 7, I set fertility as $\alpha_{r,2,7} = L_{r,0,\infty}/L_{r,2,7}$ and $\alpha_{r,a,7} = 0$ for any $a \neq 2$. So that in period 7, the number of babies for each race is the same as the one in the steady state. From period 8 onward, I assume there is no immigrant from abroad. Therefore, $L_{r,2,8} = s_{r,1,7}L_{r,1,7}$. Using this, to have as many babies as in the steady state, I set fertility in period 8 by $\alpha_{r,2,8} = L_{r,0,\infty}/L_{r,2,8} = L_{r,0,\infty}/s_{r,1,7}L_{r,1,7}$ and $\alpha_{r,a,8} = 0$ for any $a \neq 3$. For any $t \geq 9$, as in the steady state, I assume $\alpha_{r,2,t} = 1/(s_{r,0,7} \cdot s_{r,1,7})$ and $\alpha_{r,a,t} = 0$ for any $a \neq 2$. With these fertility parameters, from period 7 onward, each race always has the same nationwide number of babies as in the steady state.

G.3 The Forever 1940 Equilibrium

In Subsection N, I consider an equilibrium and its variant where almost all parameter values are as in 1940 forever. The procedure to set the fertility parameters for such equilibria is similar to the one for the baseline equilibrium. I assume that the survival probabilities are those in 1940 forever $s_{r,a,t} = s_{r,a,0}$ for any r, a, t, where period 0 is year 1940. For the steady state toward which the economy converges, I assume $\alpha_{r,2,\infty} = 1/(s_{r,0,0} \cdot s_{r,1,0})$ and $\alpha_{r,a,\infty} = 0$ for any $a \neq 2$. I compute the steady state and obtain populations at race-age levels $L_{r,a,\infty} = \sum_{i \in \mathcal{N}} L^i_{r,a,\infty}$. Then for period 0, I set fertility by $\alpha_{r,2,0} = L_{r,0,\infty}/L_{r,2,0}$ and $\alpha_{r,a,0} = 0$ for any $a \neq 2$. I assume that there is no immigrant from abroad throughout the forever 1940 equilibrium (and its variant). Therefore, the population of age 2 for each race in period 1 is $L_{r,2,1} = s_{r,1,0}L_{r,1,0}$. To have as many babies as in the steady state, I set fertility in period 1 by $\alpha_{r,2,1} = L_{r,0,\infty}/L_{r,2,1} = L_{r,0,\infty}/(s_{r,1,0}L_{r,1,0})$ and $\alpha_{r,a,1} = 0$ for any $a \neq 2$. For any period $t \geq 2$, as in the steady state, I assume $\alpha_{r,2,t} = 1/(s_{r,0,0} \cdot s_{r,1,0})$ and $\alpha_{r,a,t} = 0$ for any $a \neq 2$. With these fertility parameters, throughout the equilibrium, the nationwide number of babies $L_{r,0,t}$ is constant over time.

H Tabulation of Survival Probabilities

I assume that survival probabilities are common across locations within race-age-time bin (r, a, t). The source of survival probabilities is life tables in the website of CDC.[28] Age 0 in the model corresponds to age 1 to 10 in the data, so I do not consider infant mortality that is the probability of death before one becomes 1 year old. The life table provides the annual survival probability for each race-age bin (r, a), where ages are counted as $0, 1, \cdots$. But in my quantification of the model, one period is 10 years, and age bins are of 10-year windows.

[28] https://www.cdc.gov/nchs/products/life_tables.htm (accessd on 09/10/2022)

I map survival probabilities in life tables to those in the setting of my model in the following way. Take any census year t and race r. My model assumes that people of age $\bar{a} = 6$ cannot survive to the next period, so I need to compute survival probabilities from age 0 to age $\bar{a} - 1 = 5$. Pick up any age bin a from the 6 age bins that can survive to the next period. Notice that age bin a in the model includes people of the ages from $10a - 9$ to $10a$ in the data. For example, age bin 3 is the set of people who are 21 to 30 years old. According to the life table of year t, the oldest within 10-year-window age bin a survive to the next census year $t + 10$ with probability

$$s_{r,10a,t} \times s_{r,10a+1,t} \times \cdots \times s_{r,10a+9,t}, \tag{44}$$

where $s_{r,a',t}$ is the annual probability that people of race r, age a' (of 1-year windows) can survive to the next year in the life table of year t. The youngest within 10-year window age bin a survive to the next census year $t + 10$ with probability

$$s_{r,10a-9,t} \times s_{r,10a-8,t} \times \cdots \times s_{r,10a,t}. \tag{45}$$

I take the average of probabilities (44) and (45), and obtain the 10-year-window survival probability of people of race r, 10-year age bin a, and time t.

I Immigrants from Abroad

The US census 1950 and the ACS 2010 report residential places 1 year ago. For each race r, age a, period $t = 1950, 2010$, and location i, I count the number of individuals who came from abroad (including Alaska and Hawaii) to location i in the last one year. Assuming that the number of immigrants is constant every year within 10-year windows, I multiply the number of immigrants in the last one year by 10 to obtain the number of immigrants in 10-year windows. The US censuses from 1960 to 1990 and the ACS in 2000 report residential places 5 years ago. Similarly, for each race r, age a, period $t = 1960, \cdots, 2000$, and location i, I count the number of individuals who came from abroad to location i in the last five years. Assuming that the number of immigrants is constant every 5-year window within 10-year windows, I multiply the number of immigrants in the last 5 years by 2 to obtain the number of immigrants in 10-year windows.

J Computation of Steady States

Given parameter values, I compute steady states by iterating populations $\{L^i_{r,a}\}^i_{r,a}$. To achieve a steady state, fertility $\alpha_{r,a}$ and survival probabilities $s_{r,a}$ are such that populations will not explode or shrink.

1. Guess populations $\{L^i_{r,a}\}^i_{r,a}$.
2. Given populations $\{L^i_{r,a,t}\}^i_{r,a}$, compute wages $\{w^i_{r,a}\}^i_{r,a}$, rent $\{r^i\}^i$, and eventually period indirect utilities $\{\bar{u}^i_{r,a}\}^i_{r,a}$, using (12), (14), and (2) respectively.

3. In steady state, expected values $\{V^i_{r,a}\}^i_{r,a}$ are fully characterized by period indirect utilities $\{\bar{u}^i_{r,a}\}^i_{r,a}$ by (5). Thus I get expected values $\{V^i_{r,a}\}^i_{r,a}$.

4. Given expected values $\{V^i_{r,a}\}^i_{r,a}$, compute migration shares $\{\mu^{j,i}_{r,a}\}^{j,i}_{r,a}$ using (6).

5. Given populations $\{L^i_{r,a}\}^i_{r,a}$ and migration shares $\{\mu^{j,i}_{r,a}\}^{j,i}_{r,a}$, update populations $\{\tilde{L}^i_{r,a}\}^i_{r,a}$ using (7) and (8).

6. Let $\epsilon > 0$ be a prespecified small number.

   (a) Go back to Step 1 with updated guesses $\{\tilde{L}^i_{r,a}\}^i_{r,a}$ if

   $$\max_{r,a,i} \left| \frac{\tilde{L}^i_{r,a} - L^i_{r,a}}{L^i_{r,a}} \right| > \epsilon.$$

   (b) End the process with the converged populations $\{\tilde{L}^i_{r,a}\}^i_{r,a}$ otherwise.

K Computation of Transition Paths

I compute transition paths by value function iteration in the following way. Assume that the economy converges to a steady state in period T.

1. Compute the steady state expected values $\{V^i_{r,a,\infty}\}^i_{r,a}$ and populations $\{L_{r,a,\infty}\}^i_{r,a}$ as in Appendix J.

2. Load the steady state expected values and populations into those in period T. That is, for any race r, age a, location i,

   $$V^i_{r,a,T} = V^i_{r,a,\infty},$$
   $$L^i_{r,a,T} = L^i_{r,a,\infty}.$$

3. Load the populations from the 1940 data to those in the first period 0. That is, for any race r, age a, location i,

   $$L^i_{r,a,0} = L^i_{r,a,1940}.$$

4. Guess expected values $\{V^i_{r,a,t}\}^i_{r,a}$ for $t = 0, \cdots, T-1$.

5. Given expected values $\{V^i_{r,a,t}\}^i_{r,a}$ for $t = 0, \cdots, T$, compute populations $\{L^i_{r,a,t}\}^i_{r,a}$ for $t = 1, \cdots, T-1$ forward from period 1 to period $T-1$, using (6), (7), (8).

6. Given populations $\{L^i_{r,a,t}\}^i_{r,a}$ for $t = 0, \cdots, T-1$, compute wages $\{w^i_{r,a,t}\}^i_{r,a}$, rent $\{r^i_t\}^i$, and eventually period indirect utilities $\{\bar{u}^i_{r,a,t}\}^i_{r,a}$ for $t = 0, \cdots, T-1$, using (12), (14), and (2) respectively.

7. Given period indirect utilities $\{\bar{u}^i_{r,a,t}\}^i_{r,a}$ for $t = 0, \cdots, T-1$ and expected values in the last period $\{V^i_{r,a,118}\}^i_{r,a}$, compute new expected values $\{\tilde{V}^i_{r,a,t}\}^i_{r,a}$ for $t = 0, \cdots, T-1$.

8. Let $\epsilon > 0$ be a prespecified small number.

   (a) Go back to Step 4 with updated guesses $\{\tilde{V}^i_{r,a,t}\}^i_{r,a}$ for $t = 0, \cdots, T-1$ if

$$\max_{r,a,t,i} \left| \frac{\tilde{V}^i_{r,a,t} - V^i_{r,a,t}}{V^i_{r,a,t}} \right| > \epsilon.$$

   (b) End the process with the converged expected values $\{\tilde{V}^i_{r,a,t}\}^i_{r,a}$ for $t = 0, \cdots, T-1$ otherwise.

L Back-of-the-Envelope Calculation

I attempt to calculate the effects of the great Black migration on aggregate output (or labor income) (42) without a structural model. Individuals are classified to two races $r \in \{b, o\}$ and two regions $i \in \{N, S\}$, where b and n denote Black and non-Black Americans, and N and S denote the North and the South, respectively. Let $L^i_{r,t}$ and $w^i_{r,t}$ be the population and the wage of race r in region i and year t. As in Section 4, I use data of head counts and per capita payrolls as populations and wages, respectively. Then the actual aggregate labor income in 1970, labor income$_{1970}$, is

$$\text{labor income}_{1970} = L^N_{b,1970} \cdot w^N_{b,1970} + L^S_{b,1970} \cdot w^S_{b,1970} + L^N_{n,1970} \cdot w^N_{n,1970} + L^S_{n,1970} \cdot w^S_{n,1970}.$$

I seek the counterfactual aggregate labor income as of 1970 in the situation where the Black population is apportioned to the North and the South as in 1940. Let $L_{b,t} = L^N_{b,t} + L^S_{b,t}$ be the total Black population in year t. Then define $s^i_{b,1940}$ by

$$s^i_{b,1940} = \frac{L^i_{b,1940}}{L_{b,1940}}$$

for $i = N, S$. That is, $s^i_{b,1940}$ denotes the fraction of the Black population in region i in 1940. Then the counterfactual aggregate labor income in 1970, labor income$^{cf}_{1970}$, is

$$\begin{aligned} \text{labor income}^{cf}_{1970} = & L_{b,1970} \cdot s^N_{b,1940} \cdot w^N_{b,1970} + L_{b,1970} \cdot s^S_{b,1940} \cdot w^S_{b,1970} \\ & + L^N_{n,1970} \cdot w^N_{n,1970} + L^S_{n,1970} \cdot w^S_{n,1970}. \end{aligned}$$

From the census data, I obtain

$$\frac{\text{labor income}^{cf}_{1970}}{\text{labor income}_{1970}} = 0.9914.$$

Therefore, if the Black population was distributed across the North and the South as in 1940, aggregate labor income in 1970 would have been 0.86 percent lower. In Section 6, Figure 12 shows the quantitative model predicts that aggregate labor income would have been 0.74 percent lower without the North-South migration of Black Americans between 1940 and 1970. These two numbers are in the same ballpark.

M Consumption Equivalent

Welfare in the baseline equilibrium and a counterfactual equilibrium is compared by consumption equivalent. I derive the compensating variation in my model below. I largely follow Caliendo et al. (2019).

Let $V^j_{r,0,t}$ and $\tilde{V}^j_{r,0,t}$ be the expected values of race r, age 0 in period t and location j in the baseline equilibrium and a counterfactual equilibrium, respectively.

The consumption equivalent $\delta^j_{r,0,t}$ is the additional consumption flows to the baseline such that the expected values of the baseline and counterfactual equilibria equate

$$\tilde{V}_{r,0,t} = V^j_{r,0,t} + \sum_{a=0}^{\bar{a}} \left[\prod_{a'=-1}^{a-1} s_{r,a',t+a'} \log(\delta^j_{r,0,t}) \right],$$

where $s_{r,-1,t-1} = 1$ for any r and t for notational convenience. Note that the counterfactual equilibrium achieves higher welfare for $(r, 0, t, j)$ than the baseline equilibrium if $\delta^j_{r,0,t} > 1$.

Solving this, I obtain

$$\delta^j_{r,0,t} = \exp\left\{ \frac{\tilde{V}^j_{r,0,t} - V^j_{r,0,t}}{\sum_{a=0}^{\bar{a}} \prod_{a'=-1}^{a-1} s_{r,a',t+a'}} \right\}.$$

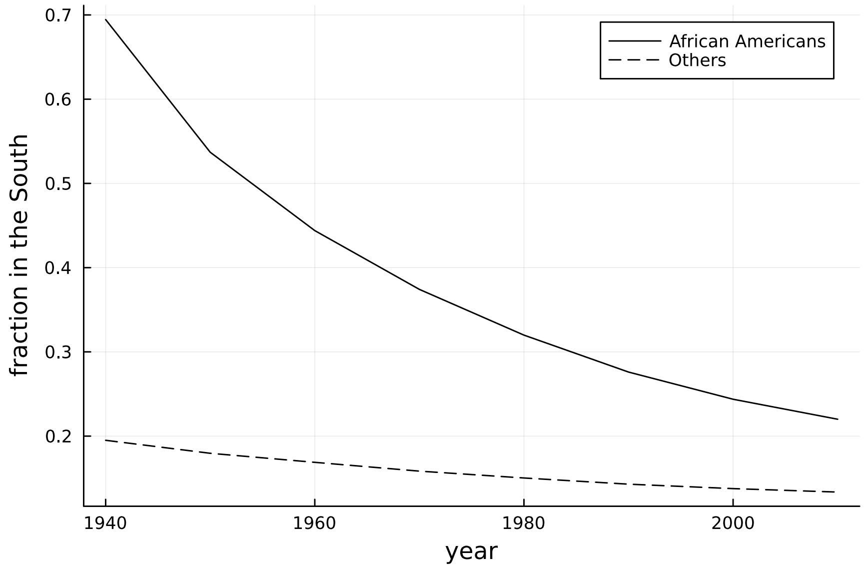
N Forever 1940 Equilibrium

I consider the counterfactual equilibrium in which all the parameter values except fertility are as of 1940 every period.[29] I call this equilibrium the forever 1940 equilibrium. Figure 19 plots the fractions of Black and non-Black Americans in the South in the forever 1940 equilibrium. As the solid line shows, 69 percent of Black Americans lived in the South in 1940, but the fraction of Black Americans in the South drops to 22 percent by 2010. The fraction of non-Black Americans in the South declines from 20 percent to 13 percent from 1940 to 2010 as in the dashed line. Although the parameter values such as productivity, amenities, and migration costs are constant over time, the spatial distribution of populations change drastically. This means that the US economy in 1940 was far from the steady state induced by the parameter values in 1940.

In addition to the forever 1940 equilibrium, I consider the equilibrium in which Black Americans cannot migrate across the North and the South for 5 periods since 1940. Let $\mathcal{N}_N$ be the set of locations in the North, and $\mathcal{N}_S$ be the set of locations in the South. Then I set $\tau^{j,i}_{b,a,t} = \infty$ for any pair of locations j, i such that $(j,i) \in \mathcal{N}_N \times \mathcal{N}_S$ or $(j,i) \in \mathcal{N}_S \times \mathcal{N}_N$ and $t = 1940, \cdots, 1980$. If individuals make migration decisions in 1980, they arrive in destinations in 1990. Therefore, this shuts down the relocation of individuals until 1990. All the other parameters are as in the forever 1940 equilibrium. For the forever 1940 equilibrium and the equilibrium without the Black North-South migration, Figure 20 shows (nationwide) per capita output Y_t/L_t, where $L_t = \sum_{i\in\mathcal{N}} \sum_{r\in\{b,n\}} \sum_{a=0}^{\bar{a}} L^i_{r,a,t}$. Per capita output is normalized by the initial level in 1940. The solid

[29] Fertility is such that the nationwide number of babies are constant over time for each race. See Appendix G.3 for details.

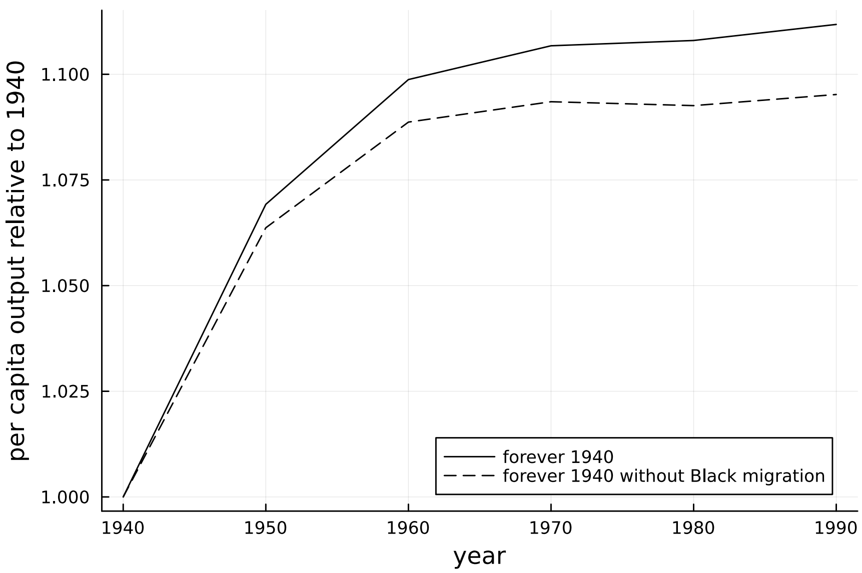
Figure 19: Fractions of Black and non-Black Americans in the South: Forever 1940



Notes: The fractions of the populations of Black and non-Black Americans in the South over time in the equilibrium in which the parameter values are as of 1940 forever.

line shows that even without any change in productivity, amenities, and migration costs, per capita output increases by 11.2 percent by 1990. The increase in per capita output is caused by migration because only dynamic change in the forever 1940 equilibrium is the relocation of individuals. This suggests that there was an opportunity to increase output by relocating the work force in 1940. If Black Americans cannot migrate across the North and the South, per capita output increases by 9.5 percent by 1990. Thus the remaining $11.2 - 9.5 = 1.7$ percentage point increase is explained by the migration of Black Americans across the North and the South. Putting differently, in the 11.3 percent increase in per capita output, the migration of Black Americans across the North and the South accounts for 14 percent of it because $(0.112 - 0.095)/0.112 = 0.15$. The remaining 85 percent is explained by the migration of non-Black Americans within or across the North and the South and the migration of Black Americans within the North and the South. The relocation of Black Americans accounts for a substantial part of room for improving aggregate output.

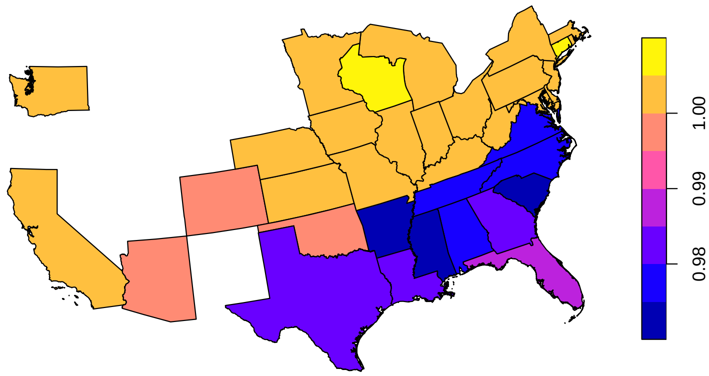
Figure 20: Per Capita Output in the Forever 1940 Equilibrium and the Equilibrium without Black Migration



Notes: Per capita output relative to the 1940 level for the forever 1940 equilibrium and the equilibrium in which Black Americans cannot migrate across the North and the South from 1940 to 1980.

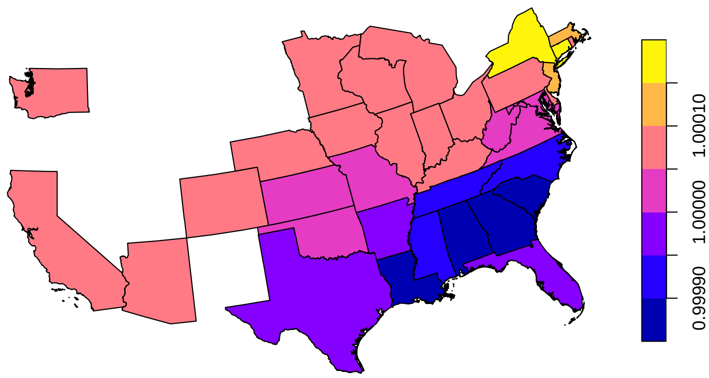
O Additional Figures on the Counterfactual Results

Figure 21: The Welfare of Black Americans in the Equilibrium of Black Immobility



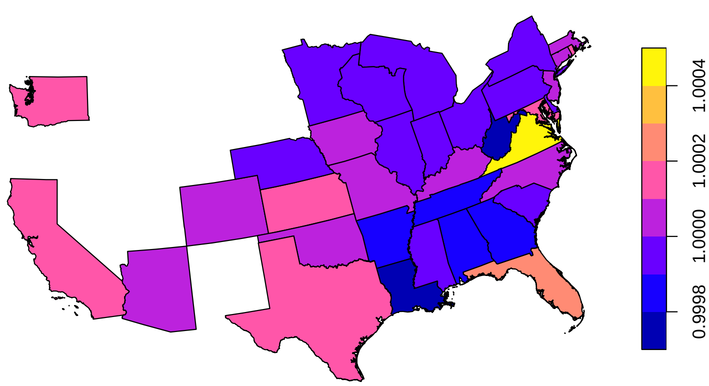
Notes: The welfare of Black Americans born in the 1930s in the equilibrium of Black immobility relative to that in the baseline equilibrium (consumption equivalent). The rest of the North is excluded from the map.

Figure 22: The Welfare of Non-Black Americans in the Equilibrium of Black Immobility



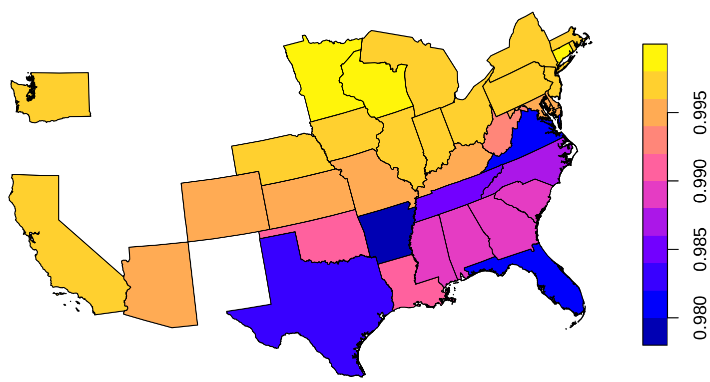
Notes: The welfare of non-Black Americans born in the 1930s in the equilibrium of Black immobility relative to that in the baseline equilibrium (compensating variation). The rest of the North is excluded from the map.

Figure 23: The Welfare of Black Americans in the Equilibrium of Non-Black Immobility



Notes: The welfare of Black Americans born in the 1930s in the equilibrium of non-Black immobility relative to that in the baseline equilibrium (consumption equivalent). The rest of the North is excluded from the map.

Figure 24: The Welfare of Non-Black Americans in the Equilibrium of Non-Black Immobility



Notes: The welfare of non-Black Americans born in the 1930s in the equilibrium of non-Black immobility relative to that in the baseline equilibrium (consumption equivalent). The rest of the North is excluded from the map.