Spatial Ver

$$\cancel{r U_k} = \cancel{\prod_{k\in A}} \prod_{k=1}^{K} (C_{r.k})^{\alpha_k} . \qquad \sum_{k=1}^{K} \alpha_k = 1 .$$

$$U_{i.r} = C_{r.i}$$

$$\text{s.t.} \sum_{k=1}^{K} \int_0^1 p_{r.k}(x)\, \underbrace{C_{r.k}(x)}\, dx \leq I_{i.a}.$$

$$U_{i.r} = C_{r.i} \cdot A_r \cdot V_{i.r}.$$

$$Q_{r.k}(x) = \varphi_{r.k}(x) \cdot \left(\frac{H_{r.k}(x)}{1-\gamma_{rk}}\right)^{1-\gamma_{rk}} \prod_{\cancel{k}\,s=1}^{K} \left(\frac{M_{r.s.k}(x)}{\gamma_{r.s.k}}\right)^{\gamma_{r.s.k}} \quad (1).$$

$$\left(\sum_{s=1}^{K} \gamma_{r.s.k} = \gamma_{r.k}.\right).$$

$$F_{r.k}(\varphi) = \exp\left[-\left(\frac{\varphi}{\varphi_{r.k}}\right)^{-\sigma_k}\right].$$

$$a = 0 \rightarrow B_k = \frac{1}{1-b_k}$$

$$\left(1+\frac{1}{B_k}\right) w_{rk} = \left(1+\frac{(1-b_k)}{\cancel{1-b_k}}\right) w_{rk} = (2-b_r)\, w_{rk}.$$

$$C_{rk} = (1-b_k)\, w_{rk} \leftarrow \text{hiring cost.}$$

given

$$E_{rk}(x) = p_{rk}(x)\, Q_{rk}(x),$$

$$r_{rk}(x) = \frac{\partial E_{rk}(x)}{\partial H_{rk}(x)}.$$

$$= p_{rk}(x) \frac{\partial Q_{rk}(x)}{\partial H_{rk}(x)}$$

$$= p_{rk}(x) \cdot \varphi_{rk}(x) \cdot H_{rk}(x)^{-\gamma_{rk}} \prod_{k=1}^{K} \left(\frac{M_{rsk}(x)}{\gamma_{rsk}}\right)^{\gamma_{rsk}}.$$

$$\pi_{r.k}(x) = \varphi_{r.k}(x) Q_{r.k}(x) - [w_{r.k}(x) + c_{r.k}] H_{r.k}(x)$$
$$- \sum_{s=1}^{K} P_{r.k} \cdot M_{r.s.k}(x).$$

The FoC: $\frac{\partial \pi_{r.k}(x)}{\partial H_{r.k}(x)} = 0.$

$$\therefore \frac{\partial \varphi_{r.k}(x) Q_{r.k}(x)}{\partial H_{r.k}(x)} \equiv r_{r.k}(x) = w_{r.k}(x) + c_{r.k}.$$

Nash bargaining:

$$\max_{w} [r_{ck}(x) - w]^{1/2} [w - b_{rk}]^{1/2}.$$
$$\max_{w} \frac{1}{2}\log(r_{rk}(x) - w) + \frac{1}{2}\log(w - b_{rk}) \equiv F(w).$$

$$\text{FoC}: \frac{\partial F(w)}{\partial w} = \frac{1}{2} \frac{(-1)}{r_{rk}(x) - w} + \frac{1}{2} \frac{1}{w - b_{r.k}} = 0.$$

$$w - b_{r.k} = \underbrace{r_{r.k}(x) - w}_{c_{r.k}}.$$
$$\cancel{2w = r_{r.}}$$

$$w = b_{r.k} + c_{r.k}.$$

Equilibrium: $(\lambda, w_{r.k}, L_r, P_{r.k}, \frac{L_{r.k}}{L_r}, l_{r.k})$ $\leftarrow C_r^k$

$1 \quad N\times K \quad N \quad N\times K \quad N\times K \quad N\times K.$

$$C_{ar}^{k} = \left[\left(1 + \frac{1}{\beta_{ar}}\right) w_{rk}\right]^{1-\gamma_{rk}} \prod_{k=1}^{K} (P_{rk})^{\gamma_{rsk}}. \quad (1)$$

$$\frac{w_{rk}}{P_r} = \beta_r \left[\frac{(l_{rk})^{\lambda}}{\mu_{rk}}\right]^{\frac{1}{1-\lambda}}. \quad (2)$$

$\parallel$

$a_{r.k}$. $\quad B_r = \frac{1}{1 - b_r}$ $\qquad \mu_{rk} \equiv \frac{\mu_{rk}}{(v_{rk})^{1-\lambda}}$

vacancy cost in terms of goods.

$$L_r = \sum_{k=1}^{K} L_{rk} \quad (3).$$

$$\tilde{\omega}_r = \frac{\widetilde{\omega_{r.k}}}{f_{r.k}} = \frac{\omega_{r.k}\,\beta_{r.k}}{f_{r.k}}, \qquad (4)$$

where $\beta_{r.k} = l_{r.k} + (1 - l_{r.k})\, b_r$.

$E_{ij.k} = \pi_{ij.k}\, E_{jk}$, &

$$\pi_{i.j.k} = \left[\frac{I_{ijk}\dfrac{C_{ik}}{T_{i.k}}}{\left[\sum_{i'=1}^{N} \left(\dfrac{I_{i'jk}\, C_{i'k}}{T_{i'k}} \right)^{-\theta_k} \right]^{-\frac{1}{\theta_k}}} \right]^{-\theta_k}. \qquad \cancel{(4)}\ (5)$$

$$= \frac{T_{i.k}^{\theta_k} \left(I_{ijk} C_{ik} \right)^{-\theta_k}}{\sum_{n=1}^{N} T_{n.k}^{\theta_k} \left(C_{n.k} I_{n.j.k} \right)^{-\theta_k}}.$$

$$E_{rk} = \alpha_k I_r + \alpha_k \sum_{k=1}^{K} P_r \cdot v_{r.k} \cdot V_{r.k} + \sum_{k=1}^{K} \gamma_{rks} \sum_{j=1}^{N} \pi_{rjs} E_{j.s}.$$

$$l_{r.k} = \frac{H_{r.k}}{L_{r.k}} = \frac{\tilde{\mu}_{r.k} V_{r.k}^{1-\lambda} L_{r.k}^{\lambda}}{L_{r.k}} = \tilde{\mu}_{r.k} \left(\frac{V_{r.k}}{L_{r.k}} \right)^{1-\lambda}.$$

$$l_{r.k}^{\frac{1}{1-\lambda}} = \tilde{\mu}_{r.k}^{\frac{1}{1-\lambda}} \cdot \frac{V_{r.k}}{L_{r.k}}$$

$$\therefore\ V_{r.k} = \left(\frac{l_{r.k}}{\tilde{\mu}_{r.k}} \right)^{\frac{1}{1-\lambda}} \cdot \frac{1}{L_{r.k}}.$$

$$\therefore\ E_{rk} = \alpha_k I_r + \alpha_k \sum_{k=1}^{K} P_r \cdot v_{r.k} \cdot \underbrace{\left[\left(\frac{l_{r.k}}{\tilde{\mu}_{r.k}} \right)^{\frac{1}{1-\lambda}} \cdot \frac{1}{L_{r.k}} \right]}_{= V_{r.k}} \qquad \cancel{(5)}\ (6)$$

$$+ \sum_{k=1}^{K} \gamma_{r.k.s} \sum_{j=1}^{N} \pi_{rj}^{s} E_j^{s}.$$

$$I_r = \left(1 + \frac{1}{\beta_r} \right) \sum_{k=1}^{K} w_{r.k} H_{r.k}.$$

$$0=\sum_{j=1}^{N}\sum_{k=1}^{K}\pi_{rj}^{k}E_j^{k}-\sum_{i=1}^{N}\sum_{k=1}^{K}\pi_{ir}^{k}E_r^{k} \qquad \cancel{(6)}\ (7)$$

$L_{rk}\cdot l_{rk}$

$$0=\nu\sum_{r=1}^{N}\sum_{k=1}^{K}H_{rk}w_{rk}-\sum_{r=1}^{N}\sum_{k=1}^{K}(1-\nu)b_r w_{rk}\left(L_{rk}-H_{rk}\right) \qquad \cancel{(7)}\ (8)$$

Recall:

$$\widetilde{\omega_r}=\frac{\omega_{r.k}\,\rho_{r.k}}{f_{r.k}}=\frac{\frac{w_{r.k}}{P_r}\cdot\left[l_{r.k}+(1-l_{r.k})b_r\right]}{f_{r.k}}.$$

$$\frac{L_r}{L}=\frac{\cancel{B}A_r\,\widetilde{\omega_r}^{\varepsilon}}{\sum_{n=1}^{N}A_n\,\widetilde{\omega_n}^{\varepsilon}} \qquad (9).$$

$$P_{rk}=\text{"}T_{j.k}\text{"}\cdot\left[\Gamma\left(1-\frac{1-\sigma_k}{\theta_k}\right)\right]^{\frac{1}{1-\sigma_k}}$$

in Carrere, Grujovic & Robert-Nicoud.

$$=\left[\sum_{i=1}^{N}\left(\tau_{ijk}\frac{c_{ik}}{T_{ik}}\right)^{-\theta_k}\right]^{-\frac{1}{\theta_k}}\cdot\cancel{A_k}\cdot\left[\Gamma\left(1-\frac{1-\sigma_k}{\theta_k}\right)\right]^{\frac{1}{1-\sigma_k}}. \qquad (10).$$

Endogenous variables:

$C_r^k,\ w_r^k,\ p_r^k,\ l_r^k,\ \pi_{ij}^k,\ E_j^k,\ L_{r.}^k,\ I_r,\ L_r,\ \nu$

$$\begin{tikzcd} & P_r \arrow[dl] & \\ \omega_{r.k} \arrow[rr] & & \widetilde{\omega_r} \end{tikzcd}$$

- I've written a model that comprises
    - internal geography
    - unemployment.
- I haven't simulated an equilibrium.
- Regional variation in unemployment is largely higher than time variation.

- I've found a quantitative macro model of M&A.
  - → trying to extend it to a many-country setting
  - → but it's difficult, especially trade costs.