One factor : labor
No intermediate goods.

- N locations.

$$U_{ij} = C_{ij} A_j V_{ij} \quad ①$$

$$C_{ij} = \left(\sum_{k \in N} \int_0^{M_k} C_{i.k.j}(\omega)^{\frac{\sigma-1}{\sigma}} d\omega \right)^{\frac{\sigma}{\sigma-1}} \quad ②$$

$$P_j = \left(\sum_{k \in N} \int_0^{M_k} (p_k(\omega) t_{kj})^{1-\sigma} d\omega \right)^{\frac{1}{1-\sigma}} \quad ③$$

- Think of Krugman. No profits.

$$I_{ij} = \begin{cases} w_j & \text{if } i \text{ is employed.} \\ b_j P_j & \text{if } i \text{ is unemployed.} \end{cases} \quad ④$$

$$\sum_{k \in N} \int_0^{M_k} p_k(\omega) t_{kj} C_{i.kj}(\omega) d\omega \leq I_{ij} \quad ⑤$$

$$C_{i.k.j}(\omega) = \left(\frac{p_k(\omega) t_{kj}}{P_j} \right)^{-\sigma} \left(\frac{I_{ij}}{P_j} \right) \quad ⑥$$

$$P_j C_{ij} = I_{ij} \quad ⑦$$

Pick up one (infinitesimal) firm ω in location j.

X_j : expenditure in location j.

$$C_{kj}(\omega) = \left(\frac{p_k(\omega) t_{kj}}{P_j} \right)^{-\sigma} \left(\frac{X_j}{P_j} \right) \quad ⑧$$

$$\overset{\text{HI}}{\beta} = \frac{\sigma - 1}{\sigma}^{\text{オレ.}}$$

$$y_k(\omega) = \sum_{j \in N} t_{kj} C_{kj}(\omega) \quad ⑨$$

$R_j(\omega)$: the revenue of firm ω in location j.

$$R_j(\omega) = \cancel{\sum_{k \in N}} p_j(\omega) \cdot y_j(\omega)$$

$$= p_j(\omega) \sum_{k \in N} t_{j.k} C_{j.k}(\omega) \quad ⑩$$

$$-\frac{\rho}{1-\rho} = -\frac{\frac{\sigma-1}{\sigma}}{1-\frac{\sigma-1}{\sigma}} = -\frac{\frac{\sigma-1}{\sigma}}{\frac{1}{\sigma}} = -(\sigma-1)$$

(8) $\Rightarrow$ $$p_k(\omega)^{\sigma} = \frac{1}{c_{kj}(\omega)} \left(\frac{t_{kj}}{P_j}\right)^{-\sigma} \left(\frac{X_j}{P_j}\right).$$

$$= c_{kj}(\omega)^{-1}\, t_{kj}^{-\sigma}\, P_j^{\sigma-1} X_j.$$

$$p_k(\omega) = c_{kj}(\omega)^{-\frac{1}{\sigma}}\, t_{kj}^{-1}\, P_j^{\frac{\sigma-1}{\sigma}} X_j^{\frac{1}{\sigma}} \qquad (11).$$

$R_{jk}(\omega)$: the revenue of firm ω in origin j from exporting to destination k.

$$R_{jk}(\omega) = p_j(\omega)\, t_{jk}\, c_{jk}(\omega).$$

$$= \left[c_{jk}(\omega)^{-\frac{1}{\sigma}}\, \cancel{t_{jk}^{-1}}\, P_k^{\frac{\sigma-1}{\sigma}} X_k^{\frac{1}{\sigma}}\right] \overbrace{\cancel{t_{jk}}\, c_{jk}(\omega)}^{q_{jk}(\omega)}.$$

$$q_j(\omega) = \sum_k q_{jk}(\omega) = \sum_k t_{jk}\, c_{jk}(\omega).$$

$$R_{jk}(\omega) = p_j(\omega)\, q_{jk}(\omega). \qquad \left(p_{kj}(\omega) = p_k(\omega)\, t_{kj}.\right)$$

(8) $\Rightarrow$ $$t_{kj}\, c_{kj}(\omega) = t_{kj} \left(\frac{\cancel{p_k(\omega)\, t_k}\; p_{kj}(\omega)}{P_j}\right)^{-\sigma} \left(\frac{X_j}{P_j}\right).$$

$$q_{kj}(\omega) = t_{kj}\, p_{kj}(\omega)^{-\sigma} P_j^{\sigma-1} X_j.$$

$$p_{kj}(\omega)^{\sigma} = q_{kj}(\omega)^{-1}\, t_{kj}\, P_j^{\sigma-1} X_j.$$

$$p_{kj}(\omega) = q_{kj}(\omega)^{-\frac{1}{\sigma}}\, t_{kj}^{\frac{1}{\sigma}}\, P_j^{\frac{\sigma-1}{\sigma}} X_j^{\frac{1}{\sigma}}.$$

$$R_{jk}(\omega) = p_{jk}(\omega)\, \frac{q_{jk}(\omega)}{t_{jk}}$$

$$= \left[q_{jk}(\omega)^{-\frac{1}{\sigma}}\, t_{jk}^{\frac{1}{\sigma}}\, P_k^{\frac{\sigma-1}{\sigma}} X_k^{\frac{1}{\sigma}}\right] \frac{q_{jk}(\omega)}{t_{jk}}.$$

$$= P_k^{\frac{\sigma-1}{\sigma}} X_k^{\frac{1}{\sigma}}\, t_{jk}^{\frac{1-\sigma}{\sigma}}\, q_{jk}(\omega)^{\frac{\sigma-1}{\sigma}}.$$

$$R_{jj}(\omega) = P_j^{\frac{\sigma-1}{\sigma}} X_j^{\frac{1}{\sigma}}\, q_{jj}(\omega)^{\frac{\sigma-1}{\sigma}} \qquad (\because t_{jj} = 1).$$

$$\frac{\partial R_{j\cdot k}(\omega)}{\partial q_{j\cdot k}(\omega)} = \frac{\sigma-1}{\sigma} P_k^{\frac{\sigma-1}{\sigma}} X_k^{\frac{1}{\sigma}} t_{j\cdot k}^{\frac{1-\sigma}{\sigma}} q_{j\cdot k}(\omega)^{-\frac{1}{\sigma}}.$$

$$\frac{\partial R_{jj}(\omega)}{\partial q_{jj}(\omega)} = \frac{\sigma-1}{\sigma} P_j^{\frac{\sigma-1}{\sigma}} X_j^{\frac{1}{\sigma}} q_{jj}(\omega)^{-\frac{1}{\sigma}}.$$

Marginal revenues are equalized (see HI footnote 14 p.1105).

$$\cancel{\frac{\sigma-1}{\sigma}} P_k^{\frac{\sigma-1}{\sigma}} X_k^{\frac{1}{\sigma}} t_{j\cdot k}^{\frac{1-\sigma}{\sigma}} q_{j\cdot k}(\omega)^{-\frac{1}{\sigma}} = \cancel{\frac{\sigma-1}{\sigma}} P_j^{\frac{\sigma-1}{\sigma}} X_j^{\frac{1}{\sigma}} q_{jj}(\omega)^{-\frac{1}{\sigma}}.$$

$$P_k^{\sigma-1} X_k t_{j\cdot k}^{1-\sigma} q_{j\cdot k}(\omega)^{-1} = P_j^{\sigma-1} X_j q_{jj}(\omega)^{-1}.$$

$$\cancel{q_{jj}} \qquad \frac{q_{j\cdot k}(\omega)}{q_{jj}(\omega)} = \frac{t_{j\cdot k}^{1-\sigma} P_k^{\sigma-1} X_k}{P_j^{\sigma-1} X_j}.$$

$$q_{j\cdot k}(\omega) = \frac{t_{j\cdot k}^{1-\sigma} P_k^{\sigma-1} X_k}{P_j^{\sigma-1} X_j} q_{jj}(\omega).$$

$$\begin{aligned} R_j(\omega) &= \sum_k R_{j\cdot k}(\omega) \\ &= \sum_k P_k^{\frac{\sigma-1}{\sigma}} X_k^{\frac{1}{\sigma}} t_{j\cdot k}^{\frac{1-\sigma}{\sigma}} q_{j\cdot k}(\omega)^{\frac{\sigma-1}{\sigma}} \end{aligned}$$

$$\begin{aligned} z_j l_j(\omega) = q_j(\omega) &= \sum_k q_{j\cdot k}(\omega) \\ &= \sum_k \frac{t_{j\cdot k}^{1-\sigma} P_k^{\sigma-1} X_k}{P_j^{\sigma-1} X_j} q_{jj}(\omega). \\ &= q_{jj}(\omega) \cdot \sum_k \frac{t_{j\cdot k}^{1-\sigma} P_k^{\sigma-1} X_k}{P_j^{\sigma-1} X_j}. \\ &= \frac{q_{jj}(\omega)}{P_j^{\sigma-1} X_j} \left(\sum_k t_{j\cdot k}^{1-\sigma} P_k^{\sigma-1} X_k \right). \end{aligned}$$

$$q_{jj}(\omega) = \frac{P_j^{\sigma-1} X_j}{\left(\sum_k t_{j\cdot k}^{1-\sigma} P_k^{\sigma-1} X_k \right)} z_j l_j(\omega).$$

$$q_{j\cdot k}(\omega) = \frac{t_{j\cdot k}^{1-\sigma} P_k^{\sigma-1} X_k}{\sum_n t_{j\cdot n}^{1-\sigma} P_n^{\sigma-1} X_n} z_j l_j(\omega). \quad (4.0)$$

$$R_j(\omega) = \sum_k P_k^{\frac{\sigma-1}{\sigma}} X_k^{\frac{1}{\sigma}} t_{j\cdot k}^{\frac{1-\sigma}{\sigma}} \cdot \left[\frac{t_{j\cdot k}^{1-\sigma} P_k^{\sigma-1} X_k}{\sum_n t_{j\cdot n}^{1-\sigma} P_n^{\sigma-1} X_n} z_j l_j(\omega) \right]^{\frac{\sigma-1}{\sigma}}$$

$$\cancel{= \left(\sum_n t_{j\cdot n}^{1-\sigma} P_n^{\sigma-1} X_n\right)^{\frac{\sigma}{\sigma-1}} \cdot \left(\sum_k t_{j\cdot k}\right.}$$

$$= \left(z_j l_j(\omega)\right)^{\frac{\sigma-1}{\sigma}} \cdot \left(\sum_n t_{j\cdot n}^{1-\sigma} P_n^{\sigma-1} X_n\right)^{\cancel{\frac{\sigma}{\sigma-1}} - \frac{\sigma-1}{\sigma}} \cdot \sum_k P_k^{\frac{\sigma-1}{\sigma}} X_k^{\frac{1}{\sigma}} t_{j\cdot k}^{\frac{1-\sigma}{\sigma}} \cdot t_{j\cdot k}^{-\frac{(1-\sigma)^2}{\sigma}} P_k^{\frac{(\sigma-1)^2}{\sigma}} X_k^{\frac{\sigma-1}{\sigma}}$$

$$= \left(z_j l_j(\omega)\right)^{\frac{\sigma-1}{\sigma}} \cdot \left(\sum_n t_{j\cdot n}^{1-\sigma} P_n^{\sigma-1} X_n\right)^{\cancel{\frac{\sigma}{\sigma-1}} - \frac{\sigma-1}{\sigma}}$$

$$\times \sum_k t_{j\cdot k}^{\frac{1-\sigma-(1-2\sigma+\sigma^2)}{\sigma}} P_k^{\frac{(\sigma-1)+(\sigma-1)^2}{\sigma}} X_k$$

$$= \left(z_j l_j(\omega)\right)^{\frac{\sigma-1}{\sigma}} \cdot \left(\sum_n t_{j\cdot n}^{1-\sigma} P_n^{\sigma-1} X_n\right)^{\cancel{\frac{\sigma}{\sigma-1}} - \frac{\sigma-1}{\sigma}} \sum_k t_{j\cdot k}^{\frac{\sigma-\sigma^2}{\sigma}} P_k^{\frac{\sigma \cancel{-1} + \sigma^2 - 2\sigma \cancel{+1}}{\sigma}} X_k.$$

$$= \left(z_j l_j(\omega)\right)^{\frac{\sigma-1}{\sigma}} \left(\sum_n t_{j\cdot n}^{1-\sigma} P_n^{\sigma-1} X_n\right)^{\cancel{\frac{\sigma}{\sigma-1}} - \frac{\sigma-1}{\sigma}} \left(\sum_k t_{j\cdot k}^{1-\sigma} P_k^{\sigma-1} X_k\right).$$

$$= \left(z_j l_j(\omega)\right)^{\frac{\sigma-1}{\sigma}} \underbrace{\left(\sum_k t_{j\cdot k}^{1-\sigma} P_k^{\sigma-1} X_k\right)^{\frac{1}{\sigma}}}_{= \Phi_j} = \Phi_j \left(z_j l_j(\omega)\right)^{\frac{\sigma-1}{\sigma}}.$$

According to footnote 16, HI p.1106,

$$\frac{\partial}{\partial l}\left[R_j(\omega, l) - w_j(\omega, l)\, l\right] = w_j(\omega, l). \leftarrow (4.1)$$

$$\boxed{\beta = \frac{\sigma-1}{\sigma}}$$

Drop ω b/c it's Krugman.

$$\frac{\partial}{\partial l}\left[R_j(l) - w_j(l)\, l\right] = w_j(l).$$

$$\Rightarrow \quad w_j(l) = \frac{\beta}{1+\beta} \frac{R_j(l)}{\cancel{L} l} = \frac{\frac{\sigma-1}{\sigma}}{1+\frac{\sigma-1}{\sigma}} \frac{R_j(l)}{\cancel{L} l} = \frac{\sigma-1}{2\sigma-1} \frac{R_j(l)}{\cancel{L} l}. \leftarrow (4.2)$$

solution to the differential equation

Firm's share: $1 - \frac{\sigma-1}{2\sigma-1} = \frac{2\sigma-1-\sigma+1}{2\sigma-1} = \frac{\sigma}{2\sigma-1}$.

$$(4.1) \Rightarrow \Phi_j \cdot z_j^{\frac{\sigma-1}{\sigma}} \cdot \left(\frac{\sigma-1}{\sigma}\right) l_j^{-\frac{1}{\sigma}} - w_j(l) - l \frac{\partial w_j(l)}{\partial l} = w_j(l).$$

$$\Rightarrow -l \frac{\partial w_j(l)}{\partial l} = 2 w_j(l) - \Phi_j z_j^{\frac{\sigma-1}{\sigma}} \cdot \left(\frac{\sigma-1}{\sigma}\right) \cdot l_j^{-\frac{1}{\sigma}}.$$

Want : $w_j(l)$ as a function.
$\uparrow$
$f(x)$ ← rewrite.

$$-x f'(x) = 2 f(x) - C \cdot x^{-\frac{1}{\sigma}}$$

$$\left(C \equiv \Phi_j z_j^{\frac{\sigma-1}{\sigma}} \left(\frac{\sigma-1}{\sigma} \right) \right)$$

Or
$$-x y' = 2y - C \cdot x^{-\frac{1}{\sigma}}$$
$$x y' = -2y + C \cdot x^{-\frac{1}{\sigma}}$$
$$x \frac{dy}{dx} = -2y + C \cdot x^{-\frac{1}{\sigma}}$$
$$\frac{dy}{dx} = -2\frac{y}{x} + C x^{-\frac{1}{\sigma}-1}$$
$$x\, dy = \left(-2y + C \cdot x^{-\frac{1}{\sigma}}\right) dx$$
$$\therefore \left(2y - C x^{-\frac{1}{\sigma}}\right) dx - x\, dy = 0$$
$$\therefore \left(2 x^{\frac{1}{\sigma}} y - C\right) dx - x^{\frac{1+\sigma}{\sigma}} dy = 0$$

Since I do not know how to solve this differential eq, verify (4.2) actually satisfies (4.1).

$$LHS = \frac{\partial R_j(l)}{\partial l} - w_j(l) - l \cdot \frac{\partial w_j(l)}{\partial l}$$
$$= \frac{\sigma-1}{\sigma} \cdot z_j^{\frac{\sigma-1}{\sigma}} \cdot l_j(\omega)^{-\frac{1}{\sigma}} - w_j(l) - l \cdot \left(\frac{\sigma-1}{2\sigma-1}\right) \cdot \frac{\frac{\partial R_j(l)}{\partial l} \cdot l - R_j(l)}{l^2}$$
$$= \frac{\sigma-1}{\sigma} \cdot z_j^{\frac{\sigma-1}{\sigma}} l^{-\frac{1}{\sigma}} \Phi_j - w_j(l) - \left(\frac{\sigma-1}{2\sigma-1}\right) \cdot \frac{\frac{\sigma-1}{\sigma} z_j^{\frac{\sigma-1}{\sigma}} l^{-\frac{1}{\sigma}} \Phi_j\, l - (z_j l)^{\frac{\sigma-1}{\sigma}} \Phi_j}{l}$$
$$= \frac{\sigma-1}{\sigma} z_j^{\frac{\sigma-1}{\sigma}} l^{-\frac{1}{\sigma}} \Phi_j - w_j(l) - \left(\frac{\sigma-1}{2\sigma-1}\right) \cdot \left[\frac{\sigma-1}{\sigma} z_j^{\frac{\sigma-1}{\sigma}} l^{-\frac{1}{\sigma}} \Phi_j - z_j^{\frac{\sigma-1}{\sigma}} l^{-\frac{1}{\sigma}} \Phi_j\right]$$
$$= \frac{\sigma-1}{\sigma} z_j^{\frac{\sigma-1}{\sigma}} l^{-\frac{1}{\sigma}} \Phi_j - w_j(l) - \left(\frac{\sigma-1}{2\sigma-1}\right) \cdot \left(-\frac{1}{\sigma} z_j^{\frac{\sigma-1}{\sigma}} l^{-\frac{1}{\sigma}} \Phi_j\right)$$

$$= \left[\frac{\sigma-1}{\sigma} + \frac{\sigma-1}{\sigma(2\sigma-1)}\right] \cdot z_j^{\frac{\sigma-1}{\sigma}} l^{-\frac{1}{\sigma}} \Phi_j - w_j(l)$$

$$= \frac{(\sigma-1)(2\sigma-1)+\sigma-1}{\sigma(2\sigma-1)} \cdot z_j^{\frac{\sigma-1}{\sigma}} l^{-\frac{1}{\sigma}} \Phi_j - w_j(l)$$

$$= \frac{(\sigma-1)(2\cancel{\sigma})}{\cancel{\sigma}(2\sigma-1)} z_j^{\frac{\sigma-1}{\sigma}} l^{-\frac{1}{\sigma}} \Phi_j - w_j(l).$$

$$= \frac{2(\sigma-1)}{2\sigma-1} z_j^{\frac{\sigma-1}{\sigma}} l^{-\frac{1}{\sigma}} \Phi_j - w_j(l)$$

LHS = RHS

$$\Rightarrow \frac{2(\sigma-1)}{2\sigma-1} z_j^{\frac{\sigma-1}{\sigma}} l^{-\frac{1}{\sigma}} \Phi_j - w_j(l) = w_j(l)$$

$$\therefore \quad w_j(l) = \frac{\sigma-1}{2\sigma-1} z_j^{\frac{\sigma-1}{\sigma}} l^{-\frac{1}{\sigma}} \Phi_j \qquad \cdots \quad (6.0)$$

Firm ω solves the following problem:

$$\pi_j(\omega) = \max_{l_j(\omega)\geq 0} \left\{ \frac{\sigma}{2\sigma-1} \Phi_j z_j^{\frac{\sigma-1}{\sigma}} l_j(\omega)^{\frac{\sigma-1}{\sigma}} - d_j l_j(\omega) - f_j P_j \right\} \quad \cdots \quad (6.1)$$

d_j: hiring cost per worker.

f_j: fixed entry cost (Krugman).

FOC: $$\frac{\partial \pi_j(\omega)}{\partial l_j(\omega)} = \frac{\sigma-1}{\cancel{\sigma}} \cdot \frac{\cancel{\sigma}}{2\sigma-1} \Phi_j z_j^{\frac{\sigma-1}{\sigma}} l_j(\omega)^{-\frac{1}{\sigma}} - d_j = 0.$$

$$\therefore \frac{\sigma-1}{2\sigma-1} \Phi_j z_j^{\frac{\sigma-1}{\sigma}} = d_j l_j(\omega)^{\frac{1}{\sigma}}.$$

$$\therefore l_j(\omega)^{\frac{1}{\sigma}} = \left(\frac{\sigma-1}{2\sigma-1}\right) \cdot \frac{\Phi_j}{d_j} z_j^{\frac{\sigma-1}{\sigma}}.$$

$$\therefore l_j(\omega) = \left(\frac{\sigma-1}{2\sigma-1}\right)^{\sigma} \cdot \left(\frac{\Phi_j}{d_j}\right)^{\sigma} \cdot z_j^{\sigma-1}. \qquad \cdots \quad (6.2)$$

Sub (6.2) into (6.1):

$$\pi_j(\omega) = \frac{\sigma}{2\sigma-1} \Phi_j z_j^{\frac{\sigma-1}{\sigma}} \left[\left(\frac{\sigma-1}{2\sigma-1}\right)^{\sigma} \cdot \left(\frac{\Phi_j}{d_j}\right)^{\sigma} z_j^{\sigma-1}\right]^{\frac{\sigma-1}{\sigma}} - d_j \cdot \left(\frac{\sigma-1}{2\sigma-1}\right)^{\sigma} \cdot \left(\frac{\Phi_j}{d_j}\right)^{\sigma} z_j^{\sigma-1}. \; - f_j P_j$$

$$= \frac{\sigma}{2\sigma-1} \Phi_j z_j^{\frac{\sigma-1}{\sigma}} \cdot \left(\frac{\sigma-1}{2\sigma-1}\right)^{\sigma-1} \cdot \left(\frac{\Phi_j}{d_j}\right)^{\sigma-1} z_j^{\frac{(\sigma-1)^2}{\sigma}} - \left(\frac{\sigma-1}{2\sigma-1}\right)^{\sigma} \cdot d_j^{1-\sigma} \Phi_j^{\sigma} z_j^{\sigma-1}. \; - f_j P_j$$

$$\frac{\cancel{\sigma-1}+\sigma^2-2\sigma+\cancel{1}}{\sigma}$$

$$= \sigma(\sigma-1)^{\sigma-1}(2\sigma-1)^{-\sigma}\cdot\Phi_j^{\sigma}d_j^{1-\sigma}z_j^{\frac{\cancel{\sigma-1}+\sigma^2-2\sigma+\cancel{1}}{\sigma}}$$

$$-(\sigma-1)^{\sigma}(2\sigma-1)^{-\sigma}\Phi_j^{\sigma}d_j^{1-\sigma}z_j^{\sigma-1} \quad - f_j\cdot P_j$$

$$= \sigma(\sigma-1)^{\sigma-1}(2\sigma-1)^{-\sigma}\Phi_j^{\sigma}d_j^{1-\sigma}z_j^{\sigma-1} - (\sigma-1)^{\sigma}(2\sigma-1)^{-\sigma}\Phi_j^{\sigma}d_j^{1-\sigma}z_j^{\sigma-1} - f_j P_j$$

$$= \left[\sigma(\sigma-1)^{\sigma-1} - (\sigma-1)^{\sigma}\right](2\sigma-1)^{-\sigma}\Phi_j^{\sigma}d_j^{1-\sigma}z_j^{\sigma-1} - f_j P_j$$

$$= (\sigma-\sigma+1)(\sigma-1)^{\sigma-1}(2\sigma-1)^{-\sigma}\Phi_j^{\sigma}d_j^{1-\sigma}z_j^{\sigma-1} - f_j P_j$$

$$= (\sigma-1)^{\sigma-1}(2\sigma-1)^{-\sigma}\Phi_j^{\sigma}d_j^{1-\sigma}z_j^{\sigma-1} - f_j P_j$$

Sub (6.2) into (6.0):

$$w_j(\omega) = \frac{\sigma-1}{2\sigma-1}z_j^{\frac{\sigma-1}{\sigma}}\Phi_j\cdot\left[\left(\frac{\sigma-1}{2\sigma-1}\right)^{\sigma}\cdot\left(\frac{\Phi_j}{d_j}\right)^{\sigma}z_j^{\sigma-1}\right]^{-\frac{1}{\sigma}}$$

$$= \frac{\sigma-1}{2\sigma-1}z_j^{\frac{\sigma-1}{\sigma}}\cdot\Phi_j\left(\frac{\sigma-1}{2\sigma-1}\right)^{-1}\cdot\Phi_j^{-1}d_j z_j^{-\frac{\sigma-1}{\sigma}}$$

$$= d_j.$$

$$H_j = L_j^{\chi}V_j^{1-\chi}. \qquad (\text{or } H_j = m_j L_j^{\chi}V_j^{1-\chi}). \qquad \boxed{\theta_j = \frac{V_j}{L_j}.}$$

- the probability that a worker finds a job:

$$x_j = \frac{H_j}{L_j} = L_j^{\chi-1}V_j^{1-\chi} = \left(\frac{V_j}{L_j}\right)^{1-\chi} = \theta_j^{1-\chi}.$$

- the probability that a firm finds a worker:

$$\phi_j = \frac{H_j}{V_j} = L_j^{\chi}V_j^{-\chi} = \left(\frac{V_j}{L_j}\right)^{-\chi} = \theta_j^{-\chi}.$$

- ξ_j $\cancel{v_j}$: the cost of posting a vacancy.

v_j: vacancies per firm.

~~By~~ $$d_j = \frac{\xi_j v_j}{l_j} \quad \cdots (7.1).$$

$$V_j = M_j v_j, \quad H_j = M_j l_j.$$

parameter

$$d_j = \frac{\xi_j V_j}{H_j} \;\leftarrow \text{endogenous}$$

endogenous

endogenous.

$$\frac{H_j}{V_j} = \frac{L_j^{\chi}}{V_j^{\chi}} = \theta_j^{-\chi}.$$

$$(w_j =)\ d_j = \zeta_j \theta_j^{\chi}. \quad \cdots (8.1)$$

$$\pi_j(\omega) = (\sigma-1)^{\sigma-1}(2\sigma-1)^{-\sigma}\cdot\left(\sum_k t_{j,k}^{1-\sigma} P_k^{\sigma-1} X_k\right)\cdot d_j^{1-\sigma} z_j^{\sigma-1} - f_j P_j. \quad \cdots (8.2).$$

Sub (8.1) into (8.2) &

$$\pi_j(\omega) = 0$$

endogenous.

$$\Rightarrow (\sigma-1)^{\sigma-1}(2\sigma-1)^{-\sigma}\underbrace{\left(\sum_k t_{j,k}^{1-\sigma} P_k^{\sigma-1} X_k\right)}_{\Phi_j}\cdot(\zeta_j \theta_j^{\chi})^{1-\sigma} z_j^{\sigma-1} = f_j P_j$$

$$\therefore (\zeta_j \theta_j^{\chi})^{1-\sigma} = (\sigma-1)^{-\sigma+1}(2\sigma-1)^{\sigma}\Phi_j^{-1} z_j^{-\sigma+1} f_j P_j$$

$$= (\sigma-1)^{1-\sigma}(2\sigma-1)^{\sigma}\Phi_j^{-1} z_j^{1-\sigma} f_j P_j$$

$$\zeta_j \theta_j^{\chi} = (\sigma-1)(2\sigma-1)^{\frac{\sigma}{1-\sigma}}\Phi_j^{-\frac{1}{1-\sigma}} z_j (f_j P_j)^{\frac{1}{1-\sigma}}.$$

$$\theta_j^{\chi} = (\sigma-1)(2\sigma-1)^{\frac{\sigma}{1-\sigma}}\Phi_j^{-\frac{1}{1-\sigma}} z_j \zeta_j^{-1} (f_j P_j)^{\frac{1}{1-\sigma}}.$$

$$\theta_j = \left[(\sigma-1)(2\sigma-1)^{\frac{\sigma}{1-\sigma}}\Phi_j^{-\frac{1}{1-\sigma}} z_j \zeta_j^{-1} f_j^{\frac{1}{1-\sigma}} P_j^{\frac{1}{1-\sigma}}\right]^{\frac{1}{\chi}}. \quad \cdots (8.3)$$

in GE, endogenous b/c it's a function of P_ks & X_ks.

Sub (8.1) into (6.2):

$$l_j(\omega) = (\sigma-1)^{\sigma}(2\sigma-1)^{-\sigma}\Phi_j^{\sigma} d_j^{-\sigma} z_j^{\sigma-1}$$

$$= (\sigma-1)^{\sigma}(2\sigma-1)^{-\sigma}\Phi_j^{\sigma}(\zeta_j \theta_j^{\chi})^{-\sigma} z_j^{\sigma-1}$$

$$= (\sigma-1)^{\sigma}(2\sigma-1)^{-\sigma}\Phi_j^{\sigma}\zeta_j^{-\sigma}\theta_j^{-\chi\sigma} z_j^{\sigma-1}. \quad \cdots (8.4).$$

Sub (8.1) & (8.4) into (7.1):

$$d_j l_j = \zeta_j v_j$$

$$\therefore (\cancel{\zeta_j}\theta_j^{\chi})\cdot\left[(\sigma-1)^{\sigma}(2\sigma-1)^{-\sigma}\Phi_j^{\sigma}\zeta_j^{-\sigma}\theta_j^{-\chi\sigma} z_j^{\sigma-1}\right] = \cancel{\zeta_j} v_j.$$

$$\therefore v_j = (\sigma-1)^{\sigma}(2\sigma-1)^{-\sigma}\Phi_j^{\sigma}\zeta_j^{-\sigma}\theta_j^{\chi(1-\sigma)} z_j^{\sigma-1}.$$

$$M_j v_j = L_j \theta_j$$

$$\therefore M_j = v_j^{-1} L_j \theta_j$$
$$= (\sigma-1)^{-\sigma}(2\sigma-1)^{\sigma}\Phi_j^{-\sigma}\zeta_j^{\sigma}\theta_j^{\chi(\sigma-1)} z_j^{1-\sigma} \cdot L_j \theta_j$$
$$= (\sigma-1)^{-\sigma}(2\sigma-1)^{\sigma}\Phi_j^{-\sigma}\zeta_j^{\sigma}\theta_j^{\chi(\sigma-1)+1} z_j^{1-\sigma} L_j \quad \cdots (9.1)$$

$$H_j = M_j \ell_j(\omega)$$
$$= (\sigma-1)^{-\sigma}(2\sigma-1)^{\sigma}\Phi_j^{-\sigma}\zeta_j^{\sigma}\theta_j^{\chi(\sigma-1)+1} z_j^{1-\sigma} L_j$$
$$\times (\sigma-1)^{\sigma}(2\sigma-1)^{-\sigma}\Phi_j^{\sigma}\zeta_j^{-\sigma}\theta_j^{-\chi\sigma} z_j^{\sigma-1}$$
$$= \theta_j^{1-\chi} L_j \qquad (9.2)$$
$$= \left(\frac{\cancel{H_j}\, V_j}{L_j}\right)^{1-\chi} L_j$$
$$= V_j^{1-\chi} L_j^{\chi}$$ ← the def of the matching function.

$$u_j = \frac{L_j - H_j}{L_j} \leftarrow \cancel{\ldots} = \frac{L_j - \theta_j^{1-\chi} L_j}{L_j} = (1-\theta_j^{1-\chi}). \qquad (9.3)$$

$$\overset{I}{\cancel{X}}_j = w_j \cancel{L_j} H_j \qquad \text{(w/o home production)}.$$
$$= \zeta_j \theta_j^{\chi} \cdot \theta_j^{1-\chi} L_j$$
$$= \zeta_j \theta_j L_j. \qquad (9.4)$$

$X_j = \zeta_j \theta_j L_j + M_j f_j P_j$

$$P_j = \left(\sum_{k\in N}\int_0^{M_k}(p_k(\omega)t_{k,j})^{1-\sigma}d\omega\right)^{\frac{1}{1-\sigma}} \qquad (9.5).$$

What's $p_k(\omega)$?

Sub (8.4) into (4.0):

件

$$q_{j,k}(\omega) = \frac{t_{j,k}^{1-\sigma}P_k^{\sigma-1}X_k}{\cancel{\Phi_j^{\sigma}}} z_j \cdot \left[(\sigma-1)^{\sigma}(2\sigma-1)^{-\sigma}\cancel{\Phi_j^{\sigma}}\cancel{\zeta_j}^{-\sigma}\theta_j^{-\chi\sigma}z_j^{\sigma-1}\right].$$

$$= t_{j,k}^{1-\sigma}P_k^{\sigma-1}X_k(\sigma-1)^{\sigma}(2\sigma-1)^{-\sigma}\zeta_j^{-\sigma}\theta_j^{-\chi\sigma}z_j^{\sigma}. \qquad (9.6)$$

Recall ⑧:

$$C_{k,j}(\omega) = \left(\frac{p_{k,j}(\omega)}{P_j}\right)^{-\sigma}\left(\frac{X_j}{P_j}\right).$$

$$t_{kj}\, C_{kj}(\omega) = q_{kj}(\omega)$$

$$= t_{kj} \cdot p_{kj}(\omega)^{-\sigma} P_j^{\sigma-1} X_j .$$

$$= t_{kj} \cdot \left(t_{kj}\, p_k(\omega)\right)^{-\sigma} P_j^{\sigma-1} X_j . \qquad (10.1)$$

$$\cancel{t_{j\cdot k}^{1-\sigma} P_k^{\sigma-1} X_k}$$

$$q_{j\cdot k}(\omega) = t_{j\cdot k}\, p_{j\cdot k}(\omega)^{-\sigma} P_k^{\sigma-1} X_k . \qquad (10.2)$$

Combine (9.6) & (10.2):

$$t_{j\cdot k}^{1-\sigma} \cancel{P_k^{\sigma-1}} \cancel{X_k} (\sigma-1)^{\sigma} (2\sigma-1)^{-\sigma} \xi_j^{-\sigma} \theta_j^{-\chi\sigma} z_j^{\sigma} = t_{j\cdot k}\, p_{j\cdot k}(\omega)^{-\sigma} \cancel{P_k^{\sigma-1}} \cancel{X_k} .$$

$$t_{j\cdot k}^{-\sigma} p_{j\cdot k}(\omega)^{\sigma} = (\sigma-1)^{-\sigma} (2\sigma-1)^{\sigma} \xi_j^{\sigma} \theta_j^{\chi\sigma} z_j^{-\sigma} .$$

$$\left(\frac{p_{j\cdot k}(\omega)}{t_{j\cdot k}}\right)^{\sigma} = (\sigma-1)^{-\sigma} (2\sigma-1)^{\sigma} \left(\frac{\xi_j}{z_j}\right)^{\sigma} \theta_j^{\chi\sigma}$$

$$\therefore \underbrace{\frac{p_{j\cdot k}(\omega)}{t_{j\cdot k}}}_{= p_j(\omega).} = (\sigma-1)^{-1} (2\sigma-1) \cdot \left(\frac{\xi_j}{z_j}\right) \theta_j^{\chi} . \qquad (10.3)$$

$$p_{j\cdot k}(\omega) = \left(\frac{2\sigma-1}{\sigma-1}\right) \cdot t_{j\cdot k} \cdot \left(\frac{\xi_j}{z_j}\right) \theta_j^{\chi} . \qquad (10.4)$$

$$P_j^{1-\sigma} = \sum_{k\in N} \int_0^{M_k} (\underbrace{p_k(\omega)\, t_{kj}}_{\leftarrow\ p_{k\cdot j}(\omega)})^{1-\sigma} d\omega$$

$$= \sum_{k\in N} \int_0^{M_k} \left(\frac{2\sigma-1}{\sigma-1}\right)^{1-\sigma} t_{kj}^{1-\sigma} \left(\frac{\xi_k}{z_k}\right)^{1-\sigma} \theta_k^{\chi(1-\sigma)} d\omega .$$

$$= \sum_{k\in N} M_k \cdot \left(\frac{2\sigma-1}{\sigma-1}\right)^{(1-\sigma)} t_{kj}^{1-\sigma} \left(\frac{\xi_k}{z_k}\right)^{(1-\sigma)} \theta_k^{\chi(1-\sigma)} . \qquad (10.5)$$

$$(9.1):\ M_k = (\sigma-1)^{-\sigma} (2\sigma-1)^{\sigma} \Phi_k^{-\sigma} \xi_k^{\sigma} \theta_k^{\chi(\sigma-1)+1} z_k^{1-\sigma} L_k . \qquad (10.6)$$

Sub (10.6) into (10.5):

$$P_j^{1-\sigma} = \sum_{k\in N} \left[\cancel{(\sigma-1)^{-\sigma}} \cancel{(2\sigma-1)^{\sigma}} \cancel{\Phi_k^{-\sigma}} \cancel{\xi_k^{\sigma}} \cancel{\theta_k}^{\chi(\sigma-1)+1} \cancel{z_k^{1-\sigma}} L_k\right] \cdot \cancel{(2\sigma-1)} \cancel{(\sigma-1)^{-1}} t_{kj} \cdot \cancel{\xi_k} \cancel{z_k^{-1}} \cancel{\theta_k^{\chi}} .$$

$$= \sum_{k\in N} (\sigma-1)^{-(\sigma+1)} (2\sigma-1)^{\sigma+1} \underline{\Phi_k^{-\sigma}}\, \xi_k^{\sigma+1} z_k^{-\sigma} \underline{\theta_k^{\chi\sigma+1}}\, t_{kj} L_k .$$

endogenous

$$X_j = \zeta_j \theta_j L_j .$$

$$P_j^{1-\sigma} = \sum_k (\sigma-1)^{-(\sigma+1)} (2\sigma-1)^{\sigma+1} \Phi_k^{-\sigma} \zeta_k^{\sigma+1} z_k^{-\sigma} \theta_k^{\chi\sigma+1} t_{k.j} L_k .$$

$$\Phi_j = \left(\sum_k t_{j.k} P_k^{\sigma-1} X_k \right)^{\frac{1}{\sigma}}$$

$$\begin{aligned}
\Phi_j^{\sigma} &= \sum_k t_{j.k} P_k^{\sigma-1} X_k \\
&= \sum_k t_{j.k} \zeta_k \theta_k L_k \cdot \left[\sum_{n\in N} (\sigma-1)^{-(\sigma+1)} (2\sigma-1)^{\sigma+1} \Phi_n^{-\sigma} \zeta_n^{\sigma+1} z_n^{-\sigma} \theta_n^{\chi\sigma+1} t_{n.k} L_n \right]^{-1} . \\
&= \frac{\sum_k t_{j.k} \zeta_k \theta_k L_k}{\sum_{n\in N} (\sigma-1)^{-(\sigma+1)} (2\sigma-1)^{\sigma+1} \Phi_n^{-\sigma} \zeta_n^{\sigma+1} z_n^{-\sigma} \theta_n^{\chi\sigma+1} t_{n.k} L_n} . \\
&= \left(\frac{\sigma-1}{2\sigma-1} \right)^{\sigma+1} \cdot \frac{\sum_k t_{j.k} \zeta_k \theta_k L_k}{\sum_n \Phi_n^{-\sigma} \zeta_n^{\sigma+1} z_n^{-\sigma} \theta_n^{\chi\sigma+1} t_{n.k} L_n} .
\end{aligned}$$

An equilibrium is $(\theta_j, X_j, P_j, \Phi_j)$ s.t.

$$\theta_j = \left[(\sigma-1)(2\sigma-1)^{\frac{\sigma}{1-\sigma}} \Phi_j^{-\frac{1}{1-\sigma}} z_j \zeta_j^{-1} t_j^{\frac{1}{1-\sigma}} P_j^{1-\sigma} \right]^{\frac{1}{\chi}} \quad \cdots (11.1).$$

$X_j = \zeta_j \theta_j L_j$ + $\to$ go to p. 13. (11.2).

$$P_j = \left[\sum_k (\sigma-1)^{-(\sigma+1)} (2\sigma-1)^{\sigma+1} \Phi_k^{-\sigma} \zeta_k^{\sigma+1} z_k^{-\sigma} \theta_k^{\chi\sigma+1} t_{k.j} L_k \right]^{\frac{1}{1-\sigma}} . \quad \cdots (11.3)$$

$$\Phi_j = \left(\sum_k \cancel{t_{k.j}}\, t_{j.k}^{1-\sigma} P_k^{\sigma-1} X_k \right)^{\frac{1}{\sigma}} . \quad \cdots (11.4)$$

Nominal wages & employment rates are

$$w_j = \zeta_j \theta_j^{\chi} \quad \cdots (11.5)$$

$$e_j = \theta_j^{1-\chi} . \quad \cdots (11.6).$$

Parameters : $\sigma, \chi, z_j, \zeta_j, t_j, t_{k.j}, L_k$.

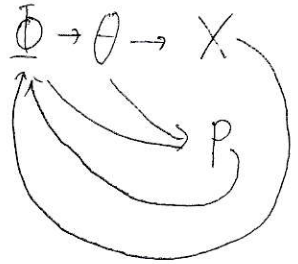
(11.1) ~ (11.4) is a system of $4N$ equations w/ $4N$ unknowns.

$$\theta_j = (\sigma-1)^{\frac{1}{\chi}} (2\sigma-1)^{\frac{\sigma}{(1-\sigma)\chi}} \Phi_j^{-\frac{1}{(1-\sigma)\chi}} z_j^{+\frac{1}{\chi}} \zeta_j^{-\frac{1}{\chi}} f_j^{\frac{1}{\chi(1-\sigma)}}$$

$$= (\sigma-1)^{\frac{1}{\chi}} (2\sigma-1)^{\frac{\sigma}{\chi(1-\sigma)}} \Phi_j^{\frac{1}{(\sigma-1)\chi}} z_j^{\frac{1}{\chi}} \zeta_j^{-\frac{1}{\chi}} f_j^{\frac{1}{\chi(1-\sigma)}}$$

$$P_j^{1-\sigma} = (\sigma-1)^{-(\sigma+1)} (2\sigma-1)^{\sigma+1} \sum_k t_{kj} \Phi_k^{-\sigma} \zeta_k^{\sigma+1} z_k^{-\sigma} \theta_k^{\chi\sigma+1} L_k$$

$$P_j = \left[(\sigma-1)^{-(\sigma+1)} (2\sigma-1)^{\sigma+1} \sum_k t_{kj} \Phi_k^{-\sigma} \zeta_k^{\sigma+1} z_k^{-\sigma} \theta_k^{\chi\sigma+1} L_k \right]^{\frac{1}{1-\sigma}}.$$



I need to set a numeraire.

$w_j = 1$

$\Rightarrow \zeta_j \theta_j^{\chi} = 1$

$\theta_j^{\chi} = \frac{1}{\zeta_j} \qquad \therefore \theta_j = \zeta_j^{-\frac{1}{\chi}}.$

$$w_k = \frac{w_k}{w_j} = \frac{\zeta_k \theta_k^{\chi}}{\zeta_j \theta_j^{\chi}}$$

$d_j h_j(\omega) = \zeta_j v_j(\omega).$

$d_j \int h_j(\omega) d\omega = \zeta_j \int v_j(\omega) d\omega$

$\therefore d_j H_j = \zeta_j V_j.$

Equilibrium: $(\theta_j, X_j, M_j, P_j, \Phi_j)$:

$$\theta_j = \left[(\sigma-1)(2\sigma-1)^{\frac{\sigma}{1-\sigma}} \Phi_j^{-\frac{1}{1-\sigma}} z_j \zeta_j^{-1} f_j^{\frac{1}{1-\sigma}} P_j^{\frac{1}{1-\sigma}}\right]^{\frac{1}{x}} \quad (13.1)$$

$$X_j = \zeta_j \theta_j L_j + M_j f_j P_j \quad (13.2)$$

$$\cancel{P_j = \left[\sum_{k\in N} M_k \left(\frac{2\sigma-1}{\sigma-1}\right) t_{kj} \left(\frac{\zeta_k}{z_k}\right) \theta_k^{x}\right]^{\frac{1}{1-\sigma}}}$$

$$M_j = \left(\frac{2\sigma-1}{\sigma-1}\right)^{\sigma} \Phi_j^{-\sigma} \zeta_j^{\sigma} \theta_j^{x(\sigma-1)+1} z_j^{1-\sigma} L_j \quad (13.3)$$

$$\Phi_j = \left(\sum_{k\in N} t_{jk}^{1-\sigma} P_k^{\sigma-1} X_k\right)^{\frac{1}{\sigma}}. \quad (13.4)$$

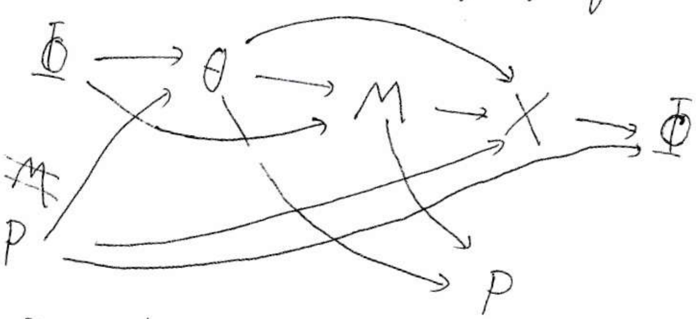
(13.3):

$$\cancel{P_j^{1-\sigma} = \sum_{k\in N} (2\sigma-1)\cdot(\sigma-1)^{-1}\cdot t_{kj}\cdot M_k\cdot \zeta_k \cdot z_k^{-1}\, \theta_k^{x}.}$$

$$\cancel{= (2\sigma-1)(\sigma-1)^{-1} \sum_{k\in N} t_{kj} M_k \zeta_k z_k^{-1} \theta_k^{x}.}$$

(13.4):

$$M_j = (2\sigma-1)^{\sigma}(\sigma-1)^{-\sigma} \Phi_j^{-\sigma} \zeta_j^{\sigma} \theta_j^{x(\sigma-1)+1} z_j^{1-\sigma} L_j.$$



$$(13.3):\ P_j^{1-\sigma} = \sum_{k\in N} M_k \cdot \left(\frac{2\sigma-1}{\sigma-1}\right)^{(1-\sigma)} t_{kj}^{1-\sigma} \zeta_k^{1-\sigma} z_k^{\sigma-1} \theta_k^{x(1-\sigma)}$$

$$= \sum_{k\in N} M_k (2\sigma-1)^{1-\sigma} (\sigma-1)^{\sigma-1} t_{kj}^{1-\sigma} \zeta_k^{1-\sigma} z_k^{\sigma-1} \theta_k^{x(1-\sigma)}$$

$$= (2\sigma-1)^{1-\sigma}(\sigma-1)^{\sigma-1} \sum_{k\in N} t_{kj}^{1-\sigma} M_k \zeta_k^{1-\sigma} z_k^{\sigma-1} \theta_k^{x(1-\sigma)}.$$

$$P_j = \left(\frac{\sigma-1}{2\sigma-1}\right)^{-1} \left[\sum_{k\in N} t_{kj}^{1-\sigma} M_k \zeta_k^{1-\sigma} z_k^{\sigma-1} \theta_k^{x(1-\sigma)}\right]^{\frac{1}{1-\sigma}}.$$