Spatial Ver

$$C_r \cancel{U_r} = \cancel{\prod_{k\in\Omega}} \prod_{k=1}^{K} (C_{r.k})^{\alpha_k}. \qquad \sum_{k=1}^{K} \alpha_k = 1.$$

$$U_{i.r} = C_{r.i}$$

$$\text{s.t. } \sum_{k=1}^{K} \int_0^1 p_{r.k}(x)\, C_{r.k}(x)\, dx \leq I_{ia}$$

$$U_{i.r} = C_{r.i} \cdot A_r \cdot V_{i.r}.$$

$$Q_{r.k}(x) = \varphi_{r.k}(x) \cdot \left(\frac{H_{r.k}(x)}{1-\gamma_{rk}}\right)^{1-\gamma_{rk}} \prod_{\cancel{k}\, s=1}^{K} \left(\frac{M_{r.s.k}(x)}{\gamma_{r.s.k}}\right)^{\gamma_{r.s.k}} \quad (1).$$

$$\left(\sum_{s=1}^{K} \gamma_{r.s.k} = \gamma_{r.k}.\right)$$

$$F_{r.k}(\varphi) = \exp\left[-\left(\frac{\varphi}{\varphi_{r.k}}\right)^{-\theta_k}\right].$$

$$a = 0 \rightarrow B_k = \frac{1}{1-b_k}$$

$$\left(1+\frac{1}{B_k}\right) w_{rk} = \left(1+\frac{(1-b_k)}{\cancel{1-b_k}}\right) w_{rk} = (2-b_r)\, w_{rk}.$$

よ

$$C_{rk} = (1-b_k)\, w_{rk} \leftarrow \text{hiring cost.}$$

given.

$$E_{rk}(x) = p_{rk}(x)\, Q_{rk}(x),$$

$$r_{rk}(x) = \frac{\partial E_{rk}(x)}{\partial H_{rk}(x)}.$$

$$= p_{rk}(x)\, \frac{\partial Q_{rk}(x)}{\partial H_{rk}(x)}$$

$$= p_{rk}(x) \cdot \varphi_{rk}(x) \cdot H_{rk}(x)^{-\gamma_{rk}} \prod_{k=1}^{K} \left(\frac{M_{rsk}(x)}{\gamma_{rsk}}\right)^{\gamma_{rsk}}$$

$$\pi_{r.k}(x) = p_{r.k}(x) Q_{r.k}(x) - [w_{r.k}(x) + c_{r.k}] H_{r.k}(x)$$

$$- \sum_{s=1}^{K} \mathbb{P}_{r.k} \cdot M_{r.s.k}(x).$$

The FoC: $\frac{\partial \pi_{r.k}(x)}{\partial H_{r.k}(x)} = 0.$

$$\therefore \frac{\partial p_{r.k}(x) Q_{r.k}(x)}{\partial H_{r.k}(x)} \equiv r_{r.k}(x) = w_{r.k}(x) + c_{r.k}.$$

Nash bargaining:

$$\max_w [r_{ck}(x) - w]^{1/2} [w - b_{rk}]^{1/2}.$$

$$\max_w \frac{1}{2} \log (r_{rk}(x) - w) + \frac{1}{2} \log (w - b_{rk}) \equiv F(w).$$

$$\text{FoC: } \frac{\partial F(w)}{\partial w} = \frac{1}{2} \frac{(-1)}{r_{r.k}(x) - w} + \frac{1}{2} \frac{1}{w - b_{r.k}} = 0.$$

$$w - b_{r.k} = \underbrace{r_{r.k}(x) - w}_{c_{r.k}}.$$

$$\cancel{2w = r_{r.}}$$

$$w = b_{r.k} + c_{r.k}. \qquad w = b_r w_{r.k} + c_{r.k}$$

$$c_{r.k} = (1 - b_r) w_{r.k}.$$

Equilibrium: $\left(\begin{matrix} z, & w_{r.k}, & L_r, & p_{r.k}, & \frac{L_{r.k}}{L_r}, & l_{r.k} \\ 1 & N \times K & N & N \times K & N \times K & N \times K. \end{matrix} \right.$ C_r^k $N \times K$ $5NK + N + 1$ $2N$

$$C_{ar}^k = \left[\left(1 + \frac{1}{B_{ar}}\right) w_{rk} \right]^{1 - \gamma_{rk}} \cdot \prod_{k=1}^{K} (p_{rk})^{\gamma_{rsk}}. \qquad ① \quad N \times K$$

$$\frac{w_{rk}}{\mathbb{P}_r} = B_r \left[\frac{(l_{rk})^{\lambda}}{\mu_{rk}} \right]^{\frac{1}{1-\lambda}}. \qquad ② \quad N \times K$$

$$\omega_{r.k}. \quad B_r = \frac{1}{1 - b_r} \qquad \mu_{rk} \equiv \frac{\mu_{rk}}{(v_{rk})^{1-\lambda}}$$

vacancy cost in terms of goods.

$$L_r = \sum_{k=1}^{K} L_{rk} \qquad ③ \;.\; N$$

$$\tilde{\omega}_r = \frac{\widetilde{\omega_{r.k}}}{\mathfrak{f}_{r.k}} = \frac{\omega_{r.k}\beta_{r.k}}{\mathfrak{f}_{r.k}}, \qquad ④ \qquad NK$$

where $\beta_{r.k} = l_{r.k} + (1 - l_{r.k}) b_r$.

$E_{ij.k} = \pi_{i.j.k} E_{jk}$, &

$$\pi_{i.j.k} = \left[\frac{I_{ijk} \frac{C_{ik}}{T_{i.k}}}{\left[\sum_{i'=1}^{\tilde{N}} \left(\frac{I_{i'jk} C_{i'k}}{T_{i'k}} \right)^{-\theta_k} \right]^{-\frac{1}{\theta_k}}} \right]^{-\theta_k}.$$

$\cancel{④}$ ⑤ $N \times N \times K$

$$= \frac{T_{i.k}^{\theta_k} (I_{ijk} C_{ik})^{-\theta_k}}{\sum_{n=1}^{\tilde{N}} T_{n.k}^{\theta_k} (C_{n.k} I_{n.j.k})^{-\theta_k}}.$$

$$E_{rk} = \alpha_{k} I_r + \alpha_{k} \sum_{k=1}^{K} \mathbb{P}_r \cdot \cancel{\mathbb{P}} v_{r.k} \cdot V_{r.k} + \sum_{k=1}^{K} \gamma_{rks} \sum_{j=1}^{\tilde{N}} \pi_{rjs} E_{j.s}.$$

$$l_{r.k} = \frac{H_{r.k}}{L_{r.k}} = \frac{\tilde{\mu}_{r.k} V_{r.k}^{1-\lambda} L_{r.k}^{\lambda}}{L_{r.k}} = \tilde{\mu}_{r.k} \left(\frac{V_{r.k}}{L_{r.k}} \right)^{1-\lambda}.$$

$$l_{r.k}^{\frac{1}{1-\lambda}} = \tilde{\mu}_{r.k}^{\frac{1}{1-\lambda}} \cdot \frac{V_{r.k}}{L_{r.k}}$$

$$\therefore V_{r.k} = \left(\frac{l_{r.k}}{\tilde{\mu}_{r.k}} \right)^{\frac{1}{1-\lambda}} \cdot \frac{1}{L_{r.k}}.$$

$$\therefore E_{rk} = \alpha_k I_r + \alpha_k \sum_{k=1}^{K} \mathbb{P}_r \cdot v_{r.k} \cdot \underbrace{\left[\left(\frac{l_{r.k}}{\tilde{\mu}_{r.k}} \right)^{\frac{1}{1-\lambda}} \cdot \frac{1}{L_{r.k}} \right]}_{= V_{r.k}} \qquad NK \qquad \cancel{⑤}\, ⑥$$

$$+ \sum_{k=1}^{K} \gamma_{r.k.s} \sum_{j=1}^{\tilde{N}} \pi_{rj}^{s} E_j^{s}.$$

$$I_r = \left(1 + \frac{1}{\beta_r} \right) \sum_{k=1}^{K} w_{r.k} H_{r.k}.$$

$$0=\sum_{j=1}^{N}\sum_{k=1}^{K}\pi_{rj}^{k}E_j^{k}-\sum_{i=1}^{N}\sum_{k=1}^{K}\pi_{ir}^{k}E_r^{k}.$$ $\cancel{(6)}$ (7) N ✓

$L_{rk}\cdot l_{rk}$.

$$0=\nu\sum_{r=1}^{N}\sum_{k=1}^{K}H_{rk}w_{rk}-\sum_{r=1}^{N}\sum_{k=1}^{K}(1-\nu)b_r w_{rk}\underbrace{(L_{rk}-H_{rk})}_{}$$ $\cancel{(7)}$ (8) 1 ✓

Recall:

$$\tilde{\omega}_r=\frac{\omega_{r.k}\,\beta_{r.k}}{t_{r.k}}=\frac{\frac{w_{r.k}}{\mathbb{P}_r}\cdot\left[l_{r.k}+(1-l_{r.k})b_r\right]}{t_{r.k}}.$$

$$\frac{L_r}{L}=\frac{\beta_r\,\cancel{A_r}\,\tilde{\omega}_r^{\varepsilon}}{\sum_{n=1}^{N}\frac{\cancel{A_n}}{\beta_n}\tilde{\omega}_n^{\varepsilon}}$$ (9) . N ✓

$$p_{rk}=\text{“}T_{j.k}\text{”}\cdot\left[\Gamma\left(1-\frac{1-\sigma_k}{\theta_k}\right)\right]^{\frac{1}{1-\sigma_k}}$$

in Carrere, Grujovic & Robert-Nicoud.

$$=\left[\sum_{i=1}^{N}\left(\tau_{ijk}\frac{c_{ik}}{T_{ik}}\right)^{-\theta_k}\right]^{-\frac{1}{\theta_k}}\cdot\cancel{A_k}\cdot\overbrace{\left[\Gamma\left(1-\frac{1-\sigma_k}{\theta_k}\right)\right]^{\frac{1}{1-\sigma_k}}}^{A^k}.$$

(10)

NK –

Endogenous variables:

C_r^k,	w_r^k,	p_r^k,	l_r^k,	π_{ij}^k,	E_j^k,	L_r^k,	I_r,	L_r,	ν
NK	NK	NK	NK	N×N×K	NK	NK	N	N	1.

$$\begin{tikzcd}
w_r^k \arrow[dd] & p_r^k \arrow[d] & l_r^k \arrow[ddr] & \\
 & \mathbb{P}_r \arrow[dl] & & \\
\omega_{r.k} \arrow[rrr] & & & \tilde{\omega}_r
\end{tikzcd}$$

- I've written a model that comprises
  - internal geography
  - unemployment.
- I haven't simulated an equilibrium.
- Regional variation in unemployment is largely higher than time variation.

- I've found a quantitative macro model of M&A.
  → trying to extend it to a many-country setting
  → but it's difficult, especially trade costs.

・モデルを書く.
・イートンにメール書く.
・コード書く.
(・employment share. × inequality.
(・labor union, minimum wage).
✓ M&A
・trade cost と productivity のかけ算を "真 productivity" として使う.
"destination-specific productivity."
- $\log(u_{it}) = \log(U_{it}) - \log(L_{it})$.

$$C_r^k = P_r v_r^k \frac{V_r^k}{H_r^k} \qquad H_r^k = \tilde{\mu}_r^k (V_r^k)^{1-\lambda} (L_r^k)^{\lambda}.$$

$$= P_r v_r^k \frac{V_r^k}{\tilde{\mu}_r^k (V_r^k)^{1-\lambda} (L_r^k)^{\lambda}} \qquad l_r^k = \frac{H_r^k}{L_r^k}.$$

$$\frac{H_r^k}{L_r^k} = \tilde{\mu}_r^k (V_r^k)^{1-\lambda} (L_r^k)^{\lambda-1}$$

$$\therefore \frac{1}{\tilde{\mu}_r^k} \cdot l_r^k \cdot (L_r^k)^{1-\lambda} = (V_r^k)^{1-\lambda}.$$

$$V_r^k = L_r^k \cdot (l_r^k)^{\frac{1}{1-\lambda}} \cdot \left(\frac{1}{\tilde{\mu}_r^k}\right)^{\frac{1}{1-\lambda}}.$$

$$C_r^k = P_r V_r^k \frac{L_r^k \cdot (l_r^k)^{\frac{1}{1-\lambda}} \cdot \left(\frac{1}{\hat{\mu}_r^k}\right)^{\frac{1}{1-\lambda}}}{H_r^k}$$

$$= P_r V_r^k \cdot \left(\frac{H_r^k}{L_r^k}\right)^{-1} \cdot (l_r^k)^{\frac{1}{1-\lambda}} \cdot (\hat{\mu}_r^k)^{-\frac{1}{1-\lambda}}.$$

イートン大体 Mundlay.

$$= P_r V_r^k \cdot (l_r^k)^{-1} \cdot (l_r^k)^{\frac{1}{1-\lambda}} \cdot (\hat{\mu}_r^k)^{-\frac{1}{1-\lambda}}$$

$$= P_r V_r^k \cdot (l_r^k)^{\frac{1-(1-\lambda)}{1-\lambda}} \cdot (\hat{\mu}_r^k)^{-\frac{1}{1-\lambda}}$$

$$= P_r V_r^k \cdot (\hat{\mu}_r^k)^{-\frac{1}{1-\lambda}} \cdot (l_r^k)^{\frac{\lambda}{1-\lambda}}.$$

---

(2) : $$w_{r,k} = P_r \cdot \left(\frac{1}{1-b_r}\right) \cdot \left[\frac{(l_{r,k})^\lambda}{\mu_{r,k}}\right]^{\frac{1}{1-\lambda}}.$$

$$= \left[\prod_{k=1}^{K} (p_r^k)^k\right] \cdot \left(\frac{1}{1-b_r}\right) \cdot \left[\frac{(l_r^k)^\lambda}{\mu_{r,k}}\right]^{\frac{1}{1-\lambda}}.$$

(10) : $$p_{r,k} = A^k \cdot \left[\sum_{i=1}^{N} \left(\tau_{ij,k} \cdot \frac{C_i^k}{T_i^k}\right)^{-\theta_k}\right]^{-\frac{1}{\theta_k}}.$$

(1) : $$C_r^k = \left[\left(1+\frac{1}{B_r}\right) w_r^k\right]^{1-\gamma_r^k} \prod_{k=1}^{K} (p_r^k)^{\gamma_r^{s,k}}.$$

Given (w_r^k), iterate (11) & (1) $\Rightarrow$ p_r^k & C_r^k
(as a function of w_r^k).

$w_{r,k}$ & P_r $\rightarrow$ $\tilde{\omega}_r$: (4)

$$C_r^k \rightarrow \begin{cases} \pi_{r,j,k} : (5) \\ E_{r,j}^k. \end{cases}$$ Need E_{rj}^k.

(6) : $$E_r^k = \alpha_k I_r + \alpha_k \sum_{k=1}^{K} P_r V_r^k \cdot \left[\left(\frac{l_r^k}{\hat{\mu}_r^k}\right)^{\frac{1}{1-\lambda}} \cdot \frac{1}{L_{r,k}}\right] + \sum_{k=1}^{K} \gamma_r^{k,s} \sum_{j=1}^{N} \pi_{r,j}^s E_j^s,$$

& $$I_r = \left(1+\frac{1}{B_r}\right) \sum_{k=1}^{K} w_r^k l_r^k L_r^k.$$

(5) → (7).

$$(8)\quad \nu \sum_{r=1}^{N}\sum_{k=1}^{K} l_r^k L_r^k w_{rk} - \sum_{r=1}^{N}\sum_{k=1}^{K} (1-\nu)\, b_r w_r^k \left(L_r^k - L_r^k l_r^k\right) = 0.$$

$$\therefore \nu \sum_{r=1}^{N}\sum_{k=1}^{K} l_r^k L_r^k w_r^k = (1-\nu)\sum_{r=1}^{N}\sum_{k=1}^{K} b_r w_r^k (1-l_r^k) L_r^k.$$

$$\therefore \nu\left[\sum_{r=1}^{N}\sum_{k=1}^{K} l_r^k L_r^k w_r^k + \sum_{r=1}^{N}\sum_{k=1}^{K} b_r w_r^k (1-l_r^k) L_r^k\right]$$

$$= \sum_{r=1}^{N}\sum_{k=1}^{K} b_r w_r^k (1-l_r^k) L_r^k.$$

$$\nu = \frac{\sum_{r=1}^{N}\sum_{k=1}^{K} b_r w_r^k (1-l_r^k) L_r^k}{\sum_{r=1}^{N}\sum_{k=1}^{K} l_r^k L_r^k w_r^k + \sum_{r=1}^{N}\sum_{k=1}^{K} b_r w_r^k (1-l_r^k) L_r^k}.$$

$$\tilde{\omega}_r = \frac{a_{rk}^k\ \beta_r^k}{f_{r,k}} = \frac{\frac{w_r^k}{P_r}\left[l_{r,k} + (1-l_r^k) b_r\right]}{f_{r,k}}. \quad (9).$$

$$\& \ \underbrace{\frac{\mu L_r}{L}}_{3} = \frac{\beta_r\, \tilde{\omega}_r^{\ \varepsilon}}{\sum_{n=1}^{N} \beta_n\, \tilde{\omega}_n^{-\varepsilon}}.$$

$$\tilde{\omega}_r P_r f_r^k = w_r^k\left[l_{r,k} + (1-l_r^k) b_r\right].$$

- Guess w_r^k, $\tilde{\omega}_r$, P_r, E_r^k.

$$\frac{\tilde{\omega}_r P_r f_r^k}{w_r} = (1-b_r) l_r^k + b_r.$$

$$\therefore (1-b_r) l_r^k = \frac{\tilde{\omega}_r P_r f_r^k}{w_r} - b_r$$

$$\therefore l_r^k = \frac{1}{1-b_r}\cdot\left[\frac{\tilde{\omega}_r P_r f_r^k}{w_r} - b_r\right].$$

→ iterate $\overset{(10)}{P_{r,k}}$ & $\overset{(1)}{C_r^k}$ → P_r^k & C_r^k.

P_r^k → New P_r.

→ π_{ij}^k.

$\rightarrow$ (6). New E_r^k.

$$E_r^k = \alpha_k \left(1+\frac{1}{B_r}\right) \sum_{k=1}^{K} w_r^k l_r^k L_r^k$$
$$+ \alpha_k \sum_{s=1}^{K} P_r \cdot \nu_r^s \cdot \left[\left(\frac{l_r^s}{\pi_r^s}\right)^{\frac{1}{1-\lambda}} \cdot \frac{1}{L_r^s} \right]$$
$$+ \sum_{s=1}^{K} \gamma_r^{k,s} \sum_{j=1}^{N} \pi_{r,j}^s E_j^s .$$

(suppose that I can solve this for E_r^k s exactly.

(7) $$\text{Surplus} = 0 = \sum_{j=1}^{N} \sum_{k=1}^{K} \pi_{rj}^k E_j^k - \sum_{i=1}^{N} \sum_{k=1}^{K} \pi_{ir}^k E_r^k .$$

$$\sum_{t=0}^{\infty} \beta^t \log C_{i,t} + \nu_{i,t}^{(r,k),(r',k')}.$$

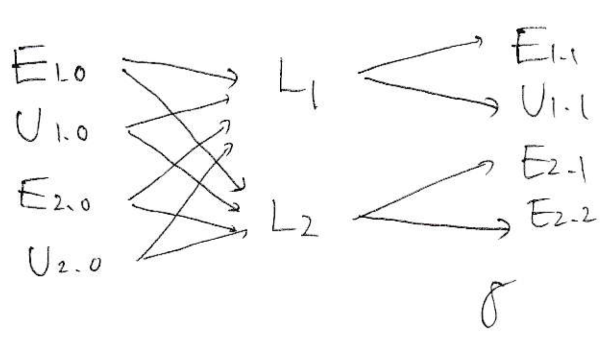
$$\cancel{\rightarrow V_{i,t}^{(r,k)} = \log C_{i,t}}$$

$$\cancel{V_{i,t}^{(e,r,k)} = \log (}$$

$$V_{i,t}^{(e,r)} = \log\left(\frac{w_r}{P_r}\right) + \beta \{ E V_{i,t+1}$$

---

period 0:
$(E_{j,0})_{j=1}^{N}$, $(U_{j,0})_{j=1}^{N}$.



$$E_{1.0} \xrightarrow{1-\chi} E_{1.1}$$

$$E_{1.0} \xrightarrow{\chi} L_{1.1}$$

$$L_{1.1} \xrightarrow{e_{1.1}} E_{1.1}$$

$$L_{1.1} \xrightarrow{1-e_{1.1}} U_{1.1}.$$

$$v_0^{1,e} = U\left(\frac{w_0^1}{p_0^1}\right) + \max\left\{(1-\chi)v_1^{1,e} + \chi v_1^{1,u},\ \max_{j\in\{1,\dots,N\}\setminus\{1\}} E v_1^{j,L}\right\}.$$

$$\cancel{E_{j,1} = (1-\chi)E_{1,0} + (L_{1,1})^{\mu}(V_{1,1})^{1-\mu}}.$$

$$v_t^{j,e} = U\left(\frac{w_t^j}{p_t^j}\right)$$

$$+\max\left\{(1-\chi)\overset{E}{v}{}_{t+1}^{j,e} + \chi \overset{E}{v}{}_{t+1}^{j,u} - \tau^{jj} + \nu\varepsilon_t^{jj},\ \max_{n\in\{1,\dots,N\}\setminus\{j\}} e_{t+1}^{n}\overset{E}{v}{}_{t+1}^{n,e} + (1-e_{t+1}^{n})\overset{E}{v}{}_{t+1}^{n,u} - \tau^{j,n} + \nu\varepsilon_t^{j,n}\right\}.$$

$$v_t^{j,u} = U(b_t^j)$$

$$+\max_{n\in\{1,\dots,N\}}\left\{e_{t+1}^{n} v_{t+1}^{n,e} + (1-e_{t+1}^{n}) v_{t+1}^{n,u} - \tau^{j,n}\right\}.$$

$$\mu_t^{j,n\neq j|e} = \frac{\exp\left(e_{t+1}^{n} V_{t+1}^{n,e} + (1-e_{t+1}^{n}) V_{t+1}^{n,u} - \tau^{j,n}\right)^{1/\nu}}{\exp\left(\chi V_{t+1}^{j,e} + (1-\chi)V_{t+1}^{j,u} - \tau^{jj}\right)^{1/\nu} + \sum_{n\neq j}\exp\left(e_{t+1}^{n} V_{t+1}^{n,e} + (1-e_{t+1}^{n}) V_{t+1}^{n,u} - \tau^{j,n}\right)^{1/\nu}}$$

$$\mu_t^{j,j|e} = \frac{\exp\left(\chi v_{t+1}^{j,e} + (1-\chi) v_{t+1}^{j,u} - \tau^{jj}\right)^{1/\nu}}{\exp\left(\chi v_{t+1}^{j,e} + (1-\chi) v_{t+1}^{j,u} - \tau^{jj}\right)^{1/\nu} + \sum_{n\neq j}\exp\left(e_{t+1}^{n} v_{t+1}^{n,e} + (1-e_{t+1}^{n}) v_{t+1}^{n,u} - \tau^{j,n}\right)^{1/\nu}}.$$

$$E_{t+1}^{j} = \chi\mu_t^{j,j|e} E_t^{j} + \underbrace{m^j (N_{t+1}^{j})^{\zeta}(V_{t+1}^{j})^{1-\zeta}}_{M_{t+1}^{j}}.$$

$$U_{t+1}^{j} = L_{t+1}^{j} - E_{t+1}^{j}$$

$$= (1-\chi)\mu_t^{j,j|e} E_t^{j} + (N_{t+1}^{j} - M_{t+1}^{j}).$$

$$L_{t+1}^{j} = \cancel{\mu_t^{j,j|e} E_t^{j}} \sum_{n=1}^{N} \mu_t^{n,j|e} E_t^{n} + \sum_{n=1}^{N} \mu_t^{n,j|u} U_t^{n}.$$

$$\mu_t^{j,n|u} = \frac{\exp\left(e_{t+1}^{n} V_{t+1}^{n,e} + (1-e_{t+1}^{n}) V_{t+1}^{n,u} - \tau^{j,n}\right)^{1/\nu}}{\sum_{l} \exp\left(e_{t+1}^{l} V_{t+1}^{l,e} + (1-e_{t+1}^{l}) V_{t+1}^{l,u} - \tau^{j,l}\right)^{1/\nu}}.$$

for any n.

$$N_{t+1}^{j} = \sum_{n\neq j} \mu_t^{n,j|e} E_t^{n} + \sum_{n} \mu_t^{n,j|u} U_t^{n}$$

$\boxed{\text{Asp}: \ \chi > e_t^{j} \ \text{for any } t, j.}$

$$\begin{cases} V_t^{j,e} = U\left(\frac{w_t^{j}}{P_t^{j}}\right) + \nu \log\left[\exp\left((1-\chi) V_{t+1}^{j,e} + \chi V_{t+1}^{j,u}\right)^{1/\nu} + \sum_{n\neq j} \exp\left(e_{t+1}^{n} V_{t+1}^{n,e} + (1-e_{t+1}^{n}) V_{t+1}^{n,u}\right)^{1/\nu}\right] \\ V_t^{j,u} = U(b_t^{j}) + \nu \log\left[\sum_{n} \exp\left(e_{t+1}^{n} V_{t+1}^{n,e} + (1-e_{t+1}^{n}) V_{t+1}^{n,u}\right)^{1/\nu}\right] \end{cases}$$

Static production.

$$q_t^{j} = T_t^{j} z^{j} \left(\frac{E_t^{j}}{1-\alpha}\right)^{\alpha^{j}} \left(\frac{M_t^{j}}{\alpha}\right)^{1-\alpha} + \Delta_t^{j}.$$

unit price

$$x_t^{j} = (w_t^{j})^{\alpha} (P_t^{j})^{1-\alpha} + \Delta_t^{j}.$$

unit cost: $c_t^{j} = \frac{(w_t^{j})^{\alpha} (P_t^{j})^{1-\alpha}}{T_t^{j} z^{j}}.$

$\exp(-(z^n)^{-\theta}) \to 1$, $z^n \to \infty$

$$p_t^{n}(z) = \min_{j} \left\{ \kappa^{jn} c_t^{jn} \right\}.$$

$$Q_t^{n} = \left(\int_{z\in\mathbb{R}_+} \tilde{q}_t^{n}(z)^{1-1/\eta} \, d\phi(z)\right)^{\eta/(\eta-1)},$$

$$P_t^{n} = \Gamma^{n} \left(\sum_{j=1}^{N} (x_t^{j} \kappa_t^{jn})^{-\theta} (T_t^{j})^{\theta}\right)^{-1/\theta}.$$

joint distribution

$$\phi(z) = \exp\left\{-\sum_{n=1}^{N} (z^n)^{-\theta}\right\}.$$

$$\phi(z^n) = \exp\left\{-(z^n)^{-\theta}\right\}$$

marginal dist^{n}.

$$\pi_t^{nj} = \frac{\left(x_t^{n} \kappa_t^{nj}\right)^{-\theta} \left(T_t^{n}\right)^{\theta}}{\sum_{m=1}^{N} \left(x_t^{m} \kappa_t^{mj}\right)^{-\theta} \left(T_t^{m}\right)^{\theta}}.$$

$$X_t^{nj} = (1-\alpha) \sum_{m=1}^{N} \pi_t^{jm} X_t^{m} + \cancel{w^n E_t^n \left(+ b_t^n P_t^n U_t^n\right)}.$$

Version with a constant replacement rate & a fixed federal labor income tax.

$$V_t^{j,e} = U\left((1-\tau)\frac{w_t^j}{P_t^j}\right)$$
$$+ \max\left\{(1-\chi) E V_{t+1}^{j,e} + \chi E V_{t+1}^{j,u} - \tau^{jj} + \nu \varepsilon_t^{jj},\ \max_{n\in\{1,\dots,N\}\setminus\{j\}} \left\{e_{t+1}^n E V_{t+1}^{n,e} + (1-e_{t+1}^n) E V_{t+1}^{n,u} - \tau^{jn} + \nu \varepsilon_t^{jn}\right\}\right\}$$

$$V_t^{j,u} = U\left(\frac{b^j w_t^j}{P_t^j}(1-\tau)\right)$$
$$+ \max_n \left\{ e_{t+1}^n E V_{t+1}^{n,e} + (1-e_{t+1}^n) E V_{t+1}^{n,u} - \tau^{jn} + \nu \varepsilon_t^{jn} \right\}.$$

$$\Rightarrow V_t^{j,e} = U\left((1-\tau)\frac{w_t^j}{P_t^j}\right) + \nu \log\left[\exp\left((1-\chi) V_{t+1}^{j,e} + \chi V_{t+1}^{j,u} - \tau^{jj}\right)^{1/\nu} + \sum_{n\neq j} \exp\left(e_{t+1}^n V_{t+1}^{n,e} + (1-e_{t+1}^n) V_{t+1}^{n,u} - \tau^{jn}\right)^{1/\nu}\right]$$

$$V_t^{j,u} = U\left((1-\tau)\frac{b^j w_t^j}{P_t^j}\right) + \nu \log\left[\sum_{n=1}^{N} \exp\left(e_{t+1}^n V_{t+1}^{n,e} + (1-e_{t+1}^n) V_{t+1}^{n,u} - \tau^{jn}\right)^{1/\nu}\right].$$

$$\mu_t^{j,n\neq j|e} = \frac{\exp\left(e_{t+1}^n V_{t+1}^{n,e} + (1-e_{t+1}^n) V_{t+1}^{n,u} - \tau^{jn}\right)^{1/\nu}}{\exp\left((1-\chi) V_{t+1}^{j,e} + \chi V_{t+1}^{j,u} - \tau^{jj}\right)^{1/\nu} + \sum_{n\neq j} \exp\left(e_{t+1}^n V_{t+1}^{n,e} + (1-e_{t+1}^n) V_{t+1}^{n,u} - \tau^{jn}\right)^{1/\nu}}.$$

$$\mu_t^{jj|e} = \frac{\exp\left((1-\chi) V_{t+1}^{j,e} + \chi V_{t+1}^{j,u} - \tau^{jj}\right)^{1/\nu}}{\exp\left((1-\chi) V_{t+1}^{j,e} + \chi V_{t+1}^{j,u} - \tau^{jj}\right)^{1/\nu} + \sum_{n\neq j} \exp\left(e_{t+1}^n V_{t+1}^{n,e} + (1-e_{t+1}^n) V_{t+1}^{n,u} - \tau^{jn}\right)^{1/\nu}}.$$

$$\mu_t^{j,n|u} = \frac{\exp\left(e_{t+1}^n V_{t+1}^{n,e} + (1-e_{t+1}^n) V_{t+1}^{n,u} - \tau^{j,n}\right)^{1/\nu}}{\sum_m \exp\left(e_{t+1}^m V_{t+1}^{m,e} + (1-e_{t+1}^n) V_{t+1}^{m,u} - \tau^{j,m}\right)^{1/\nu}}.$$

$$\underset{m}{\cancel{\theta}}{}_{t+1}^n = \frac{M_{t+1}^n}{N_{t+1}^n} = \cancel{m}\,\Xi^n \left(\frac{V_{t+1}^n}{N_{t+1}^n}\right)^{1-\beta}$$

$$= \Xi^n \left(\cancel{\theta}{}^{\lambda}{}_{t+1}^n\right)^{1-\beta}.$$

$\lambda_{t+1}^n = \frac{V_{t+1}^n}{N_{t+1}^n}$: labor market tightness.

$$e_{t+1}^n = \frac{E_{t+1}^n}{L_{t+1}^n} = \frac{\chi \mu_t^{n,n|e} E_t^n + M_{t+1}^n}{\sum_m \mu_t^{m,n|e} E_t^m + \sum_m \mu_t^{m,n|u} U_t^m}.$$

@ Production & Labor market friction.

$$C_t^{*n} = (w_t^n + \Delta_t^n)^\beta (P_t^n)^{1-\beta}.$$

$$\Delta_t^n \cdot E_t^n = \nu^n \cdot \underbrace{V_{t+1}^n}_{P_{t+1}^n}$$

~~predetermin~~

predetermined.

$$= \nu^n \cdot P_{t*}^n \cdot \cancel{\theta}{}^{\lambda}{}_t^n \cdot N_t^n.$$

Think period t.

Predetermined variables in period $t-1$:

$\mu_t^{j,n|e}$, $\mu_t^{j,n|u}$, L_t^j, N_t^j.

Endogenous variables:

r_t, X_t^{nj}, V_t^j, E_t^j, U_t^j, w_t^j, P_t^j, (Δ_t^j), $\cancel{\theta}{}^{\lambda}{}_t^j$, X_t^j, (m_t^j)

(π_t^{nj}) $\cancel{(I_t^j)}$

	X_t^{nj}	K	E	U	w	P	λ
1	N×N	N	N	N	N	N.	N

$\cancel{8}\,6N + N^2 + 1.$

$$w_t^r = \overbrace{b^r \cdot w_t^r}^{b_t} + \Delta_t^r .$$

$$\Delta_t^r = (1-b^r)\, w_t^r .$$

Then the total labor cost is:

$$w_t^r + \Delta_t^r = (2-b^r)\, w_t^r .$$

$\cancel{C_t^r}$ $\quad q_t^r(\omega) = z_t^r(\omega) \cdot \left(\frac{E_t^r(\omega)}{1-\beta}\right)^{\beta} \left(\frac{M_t^r(\omega)}{\beta}\right)^{1-\beta} . \qquad z_t^r(\omega) \sim \text{Frechet}(T_t^r, \theta)$

$$C_t^r = T_t^r \cdot (w_t^r + \Delta_t^r)^{\beta} (P_t^r)^{1-\beta} .$$

$$\pi_t^{nj} = \frac{(T_t^n)^{\theta} \left[(w_t^n + \Delta_t)^{\beta} (P_t^n)^{1-\beta} \cdot d^{nj}\right]^{-\theta}}{\sum_m (T_t^m)^{\theta} \left[(w_t^m + \Delta_t^m)^{\beta} (P_t^m)^{1-\beta} d^{mj}\right]^{-\theta}}$$

$$X_t^{mj} = I_t^j + \frac{(1-\beta)}{\beta}\left[w_t^j \cdot E_t^j + \Delta_t^j E_t^j\right]$$

$$\left(= \left(w_t^j E_t^j + b^j w_t^j U_t^j\right) + \frac{1-\beta}{\beta}\left[w_t^j E_t^j + \nu^n P_t^n \theta_t^n \cdot N_t^n\right] . \right) \qquad ① \; N$$

$$= \left(w_t^j E_t^j + b^j w_t^j U_t^j\right) + \frac{1-\beta}{\beta}(2-b^j)\, w_t^j E_t^j . \quad \Leftarrow \text{balanced trade.}$$

$$X_t^{nj} = \pi_t^{nj} X_t^j . \quad G. \qquad \pi_t^{nj} = \frac{X_t^{nj}}{X_t^j} . \qquad ② \; N \times N$$

$$P_t^n = \Gamma\left(1 + \frac{1-\sigma}{\theta}\right)^{\frac{1}{1-\sigma}} \cdot \left(\sum_{j=1}^{N} (T_t^j)^{\theta} \left[\left((2-b^r) w_t^j\right)^{\beta} (P_t^j)^{1-\beta} d^{jn}\right]^{-\theta}\right)^{-1/\theta} \qquad ③ \; N$$

$$\nu_t \sum_{n=1}^{N} \left[w_t^n E_t^n + b^r w_t^n U_t^n\right] = \sum_{n=1}^{N} b^r w_t^n U_t^n .$$

$$\nu_t = \frac{\sum_{n=1}^{N} b^r w_t^n U_t^n}{\sum_{n=1}^{N} \left[w_t^n E_t^n + b^r w_t^n U_t^n\right]} \qquad ④ \; 1$$

$$m_t^j = \Xi^j \cdot (\lambda_t^n)^{1-\xi}$$

$$\frac{M_t^j}{N_t^j}.$$

$$\boxed{\lambda_t^n = \frac{V_t^n}{N_t^n}}$$

$$E_t^j = \chi \mu_{t-1}^{j,j,le} \, E_{t-1}^j + M_t^j \qquad ⑤ \sim$$

$$= \chi \mu_{t-1}^{j,j,le} E_{t-1}^j + N_t^j m_t^j .$$

$$U_t^j = L_t^j - E_t^j . \qquad ⑦ \sim$$

$$\Delta_t^j \cdot E_t^j = \nu^j P_t^j V_t^j \qquad ⑥$$

$$\therefore (1-b^j) w_t^j \cdot E_t^j = \nu^j P_t^j V_t^j . \qquad \sim$$

$$= \nu^j P_t^j N_t^j \lambda_t^j .$$

$$\underbrace{\cancel{w_t^j}}_{(2-b^j)w_t^j} E_t^j = \beta \cdot \sum_{n=1}^{N} \pi_{jn} X_n . \qquad ⑧$$

Given $\{\mu_{t-1}^{j,nle}, \mu_{t-1}^{j,nlu}, L_t^j, N_t^j, E_{t-1}^j, U_{t-1}^j\}^N$.

Def 1: a temporary equilibrium is

~~EI~~ $\{\nu_t, X_t^j, X_t^{nj}, E_t^{nj}, U_t^j, w_t^j, P_t^j, \lambda_t^j\}_{j=1, n=1}^{N \quad N}$ ~~given~~

Not necessary

such that

$$X_t^j = (w_t^j E_t^j + b^j w_t^j U_t^j) + \frac{1-\beta}{\beta} (2-b^j) w_t^j E_t^j \qquad ①.$$

$$X_t^{nj} = \pi_t^{nj} X_t^j , \qquad ②$$

where

$$\pi_t^{nj} = \frac{(T_t^n)^\theta \left[((2-b^n) w_t^n)^\beta (P_t^n)^{1-\beta} \cdot d^{nj} \right]^{-\theta}}{\sum_m (T_t^m)^\theta \left[((2-b^m) w_t^m)^\beta (P_t^m)^{1-\beta} d^{mj} \right]^{-\theta}} \qquad ③.$$

$$P_t^j = G \cdot \left(\sum_{n=1}^{N} (T_t^n)^\theta \left[((2-b^n) w_t^n)^\beta (P_t^n)^{1-\beta} d^{nj} \right]^{-\theta} \right)^{-1/\theta}.$$

$$\nu_t = \frac{\sum_{n=1}^{N} b^n w_t^n U_t^n}{\sum_{n=1}^{N} \left[w_t^n E_t^n + b^n w_t^n U_t^n \right]} . \qquad ④$$

$$E_t^j = \chi \mu_{t-1}^{j,j|e} E_{t-1}^j + N_t^j \cdot \Xi^j (\lambda_t^j)^{1-\zeta} \quad (5)$$

$$(1-b^{wj}) w_t^j E_t^j = \nu^j P_t^j N_t^j \lambda_t^j \quad (6)$$

$$U_t^j = L_t^j - E_t^j \quad (7)$$

$\leftarrow N_t$ necessary.

$$(2-b^j) w_t^j E_t^j = \beta \sum_{n=1}^{N} \pi_{jn} X_n \quad (8)$$

Def 2: Given $\{L_0^j, E_0^j, U_0^j\}$, a sequential equilibrium is $\{\mu_t^{j,n|e}, \mu_t^{j,n|u}\}_{t=0}^{\infty}$, $\{V_t^{j,e}, V_t^{j,u}\}_{t=1}^{\infty}$, $\{L_t^j, E_t^j, U_t^j, N_t^j\}_{t=1}^{\infty}$, $\{\iota_t, X_t^j, X_t^{nj}, E_t^{nj}, w_t^j, P_t^j, \lambda_t^j\}_{t=1}^{\infty}$ s.t.

$$V_t^{j,e} = U\left((1-\iota_t)\frac{w_t^j}{P_t^j}\right) + \nu \log\Big[\exp\left((1-\chi)V_{t+1}^{j,e} + \chi V_{t+1}^{j,u} - \tau^{jj}\right)^{1/\nu} + \sum_{n\neq j} \exp\Big(\underbrace{\Xi^{n}(\lambda_t^{n})^{1-\zeta}}_{m_t^n} V_{t+1}^{n,e} + (1-\Xi^n(\lambda_t^n)^{1-\zeta}) V_{t+1}^{n,u} - \tau^{j,n}\Big)^{1/\nu}\Big] \quad (a)$$

$$V_t^{j,u} = U\left((1-\iota_t)\frac{b^j w_t^j}{P_t^j}\right) + \nu \log\Big[\sum_{n=1}^{N} \exp\Big(\underbrace{\Xi^n(\lambda_t^n)^{1-\zeta}}_{m_t^n} V_{t+1}^{n,e} + (1-\Xi^n(\lambda_t^n)^{1-\zeta}) V_{t+1}^{n,u} - \tau^{j,n}\Big)^{1/\nu}\Big] \quad (b)$$

$$\mu_t^{j,n\neq j|e} = \frac{\exp\left(m_{t+1}^n V_{t+1}^{n,e} + (1-m_{t+1}^n) V_{t+1}^{n,u} - \tau^{j,n}\right)^{1/\nu}}{\exp\left((1-\chi)V_{t+1}^{j,e} + \chi V_{t+1}^{j,u} - \tau^{jj}\right)^{1/\nu} + \sum_{n\neq j}\exp\left(m_{t+1}^n V_{t+1}^{n,e} + (1-m_{t+1}^n)V_{t+1}^{n,u} - \tau^{j,n}\right)^{1/\nu}} \quad (c)$$

$$\mu_t^{j,j|e} = \frac{\exp\left((1-\chi)V_{t+1}^{j,e} + \chi V_{t+1}^{j,u} - \tau^{j,j}\right)^{1/\nu}}{\exp\left((1-\chi)V_{t+1}^{j,e} + \chi V_{t+1}^{j,u} - \tau^{jj}\right)^{1/\nu} + \sum_{n\neq j}\exp\left(m_{t+1}^n V_{t+1}^{n,e} + (1-m_{t+1}^n)V_{t+1}^{n,u} - \tau^{j,n}\right)^{1/\nu}} \quad (d)$$

$$\mu_t^{j,n|u} = \frac{\exp\left(m_{t+1}^n V_{t+1}^{n,e} + (1-m_{t+1}^n)V_{t+1}^{n,u} - \tau^{j,n}\right)^{1/\nu}}{\sum_k \exp\left(m_{t+1}^k V_{t+1}^{k,e} + (1-m_{t+1}^k)V_{t+1}^{k,u} - \tau^{j,k}\right)^{1/\nu}} \quad (e)$$

$$L_t^j = \sum_{n=1}^{N} \mu_{t-1}^{n,j|e} E_{t-1}^n + \sum_{n=1}^{N} \mu_{t-1}^{n,j|u} U_{t-1}^n \quad (f)$$

$$N_t^j = \sum_{n\neq j} \mu_{t-1}^{n,j|e} E_{t-1}^n + \sum_{n=1}^{N} \mu_{t-1}^{n,j|u} U_{t-1}^n = L_t^j - \mu_{t-1}^{j,j|e} E_{t-1}^j \quad (g)$$