

# Trade and Unemployment

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# Comparison

Helpman & Itskhoki (2010)	This model
Melitz+Search	Krugman+Search
Two countries	Many countries

# Preferences

- ▶  $N$  is the set of countries.
- ▶ Individual  $i$  in country  $j$  has a utility function

$$U_{i,j} = \left( \sum_{k \in N} \int_0^{M_k} C_{i,k,j}(\omega)^{\frac{\sigma-1}{\sigma}} d\omega \right)^{\frac{\sigma}{\sigma-1}}. \quad (1)$$

- ▶ Her income is

$$l_{i,j} = \begin{cases} w_j & \text{if } i \text{ is employed,} \\ 0 & \text{if } i \text{ is unemployed.} \end{cases} \quad (2)$$

- ▶ Individual  $i$ 's demand for variety  $\omega$  shipped from  $k$  to  $j$  is

$$C_{i,k,j}(\omega) = \left( \frac{p_k(\omega) t_{k,j}}{P_j} \right)^{-\sigma} \frac{l_{i,j}}{P_j}. \quad (3)$$

- ▶ If firm  $\omega$  hires  $h$  units of labor, the revenue is

$$R_j(\omega, h) = \Phi_j(z_j h)^{\frac{\sigma-1}{\sigma}} \quad (4)$$

where

$$\Phi_j = \left( \sum_{k \in N} t_{j,k}^{1-\sigma} P_k^{\sigma-1} X_k \right)^{\frac{1}{\sigma}}. \quad (5)$$

# Stole-Zwiebel Bargaining

- ▶ Following Stole & Zwiebel (1996) and Helpman & Itskhoki (2010), workers and the firm engage in wage bargaining with equal weights.
- ▶ The firm and a worker equally divide the marginal surplus

$$\frac{\partial}{\partial h} [R_j(\omega, h) - w_j(\omega, h)h] = w_j(\omega, h). \quad (6)$$

- ▶ Solving the ODE, the wage as a function of labor input is

$$w_j(\omega, h) = \frac{\sigma - 1}{2\sigma - 1} \frac{R_j(\omega, h)}{h}. \quad (7)$$

- ▶ Workers get  $\frac{\sigma-1}{2\sigma-1} R_j(\omega, h)$ . The firm gets  $\frac{\sigma}{2\sigma-1} R_j(\omega, h)$ .

# Profit Maximization

- ▶ Let  $d_j$  be the hiring cost per worker.
  - ▶ This is not a parameter, but determined in general equilibrium.
- ▶ Let  $f_j P_j$  be the entry cost.
- ▶ Firm  $\omega$  in  $j$  solves

$$\pi_j(\omega) = \max_{h \geq 0} \left\{ \frac{\sigma}{2\sigma - 1} \Phi_j z_j^{\frac{\sigma-1}{\sigma}} h^{\frac{\sigma-1}{\sigma}} - d_j h - f_j P_j \right\}. \quad (8)$$

- ▶ The 1st term is  $R_j(\omega, h) - w_j(\omega, h)h$ .
- ▶ The FOC yields the labor input

$$h_j(\omega) = \left( \frac{\sigma - 1}{2\sigma - 1} \right)^{\sigma} \left( \frac{\Phi_j}{d_j} \right)^{\sigma} z_j^{\sigma-1}. \quad (9)$$

- ▶ The wage is

$$w_j(\omega) \equiv w_j(\omega, h_j(\omega)) = d_j. \quad (10)$$

# Search and Matching

- ▶ The matching function is

$$H_j = L_j^\chi V_j^{1-\chi}. \quad (11)$$

$H_j$  : aggregate employment in  $j$ ,

$L_j$  : aggregate labor force in  $j$ ,

$V_j$  : aggregate vacancies in  $j$ .

- ▶ Let  $\theta_j = \frac{V_j}{L_j}$ .
- ▶ Let  $\zeta_j$  be the cost of posting a vacancy.
  - ▶ This is a parameter.
- ▶ An identity about the aggregate hiring costs:

$$\begin{aligned} d_j H_j &= \zeta_j P_j V_j \\ \Rightarrow (w_j =) d_j &= \zeta_j P_j \theta_j^\chi. \end{aligned} \quad (12)$$

## Optimal Price

- ▶ The f.o.b. price associated with  $l_j(\omega)$  is

$$p_j(\omega) = \left( \frac{2\sigma - 1}{\sigma - 1} \right) \left( \frac{\zeta_j P_j}{z_j} \right) \theta_j^\chi. \quad (13)$$

- ▶ The price index is

$$\begin{aligned} P_j &= \left[ \sum_{k \in N} M_k (t_{k,j} p_k(\omega))^{1-\sigma} \right]^{\frac{1}{1-\sigma}} \\ &= \left[ \sum_{k \in N} M_k t_{k,j}^{1-\sigma} \left( \frac{2\sigma - 1}{\sigma - 1} \right)^{1-\sigma} \left( \frac{\zeta_k P_k}{z_k} \right)^{1-\sigma} \theta_k^{\chi(1-\sigma)} \right]^{\frac{1}{1-\sigma}}. \end{aligned} \quad (14)$$



# Labor Input, Vacancies and the Mass of Firms

- I rewrite labor input as a function of  $\theta_j$

$$h_j(\omega) = \left( \frac{\sigma - 1}{2\sigma - 1} \right)^\sigma \left( \frac{\Phi_j}{\zeta_j P_j} \right)^\sigma z_j^{\sigma-1} \theta_j^{-\chi\sigma}. \quad (15)$$

- An identity  $d_j h_j(\omega) = \zeta_j v_j(\omega)$  leads to the firm's vacancies

$$v_j(\omega) = \left( \frac{\sigma - 1}{2\sigma - 1} \right)^\sigma \left( \frac{\Phi_j}{\zeta_j P_j} \right)^\sigma z_j^{\sigma-1} \theta_j^{\chi(1-\sigma)}. \quad (16)$$

- Then  $M_j v_j(\omega) = L_j \theta_j$  yields the mass of firms

$$M_j = \left( \frac{\sigma - 1}{2\sigma - 1} \right)^{-\sigma} \left( \frac{\Phi_j}{\zeta_j P_j} \right)^{-\sigma} z_j^{1-\sigma} L_j \theta_j^{\chi(\sigma-1)+1}. \quad (17)$$

# Aggregate Employment and Income

- ▶ The aggregate employment is

$$H_j = M_j h_j(\omega) = \theta_j^{1-\chi} L_j. \quad (18)$$

- ▶ That is, the employment rate is  $e_j = \theta_j^{1-\chi}$ .
- ▶ The aggregate expenditure is

$$\begin{aligned} X_j &= w_j e_j L_j + M_j f_j P_j + M_j d_j h_j(\omega) \\ &= \zeta_j P_j \theta_j L_j + M_j f_j P_j \\ &\quad + \left( \frac{\sigma - 1}{2\sigma - 1} \right)^\sigma \Phi_j^\sigma (\zeta_j P_j \theta_j^\chi)^{-\sigma+1} z_j^{\sigma-1} M_j. \end{aligned} \quad (19)$$

## Zero Profit Condition

- Write the profits as a function of  $\theta_j$  and  $\Phi_j$

$$\pi_j(\omega) = (\sigma - 1)^{\sigma-1} (2\sigma - 1)^{-\sigma} \Phi_j (\zeta_j P_j \theta_j^\chi)^{1-\sigma} z_j^{\sigma-1} - f_j P_j. \quad (20)$$

- The zero-profit condition  $\pi_j(\omega) = 0$  yields

$$\theta_j = \left[ (\sigma - 1)(2\sigma - 1)^{-\frac{\sigma}{\sigma-1}} \Phi_j^{\frac{\sigma}{\sigma-1}} z_j \zeta_j^{-1} f_j^{-\frac{1}{\sigma-1}} P_j^{-\frac{\sigma}{\sigma-1}} \right]^{\frac{1}{\chi}}. \quad (21)$$

# Equilibrium

- An equilibrium is  $\{\theta_j, X_j, P_j, M_j, \Phi_j\}_{j \in N}$  that satisfies

$$\theta_j = \left[ (\sigma - 1)(2\sigma - 1)^{-\frac{\sigma}{\sigma-1}} \Phi_j^{\frac{\sigma}{\sigma-1}} z_j \zeta_j^{-1} f_j^{-\frac{1}{\sigma-1}} P_j^{-\frac{\sigma}{\sigma-1}} \right]^{\frac{1}{\chi}}, \quad (22)$$

$$X_j = \zeta_j P_j \theta_j L_j + M_j f_j P_j + \left( \frac{\sigma - 1}{2\sigma - 1} \right)^{\sigma} \Phi_j^{\sigma} (\zeta_j P_j \theta_j^{\chi})^{-\sigma+1} z_j^{\sigma-1} M_j. \quad (23)$$

$$P_j = \left[ \sum_{k \in N} M_k t_{k,j}^{1-\sigma} \left( \frac{2\sigma - 1}{\sigma - 1} \right)^{1-\sigma} \left( \frac{\zeta_k P_k}{z_k} \right)^{1-\sigma} \theta_k^{\chi(1-\sigma)} \right]^{\frac{1}{1-\sigma}}, \quad (24)$$

$$M_j = \left( \frac{\sigma - 1}{2\sigma - 1} \right)^{-\sigma} \left( \frac{\Phi_j}{\zeta_j P_j} \right)^{-\sigma} z_j^{1-\sigma} L_j \theta_j^{\chi(\sigma-1)+1}. \quad (25)$$

$$\Phi_j = \left( \sum_{k \in N} t_{j,k}^{1-\sigma} P_k^{\sigma-1} X_k \right)^{\frac{1}{\sigma}}. \quad (26)$$

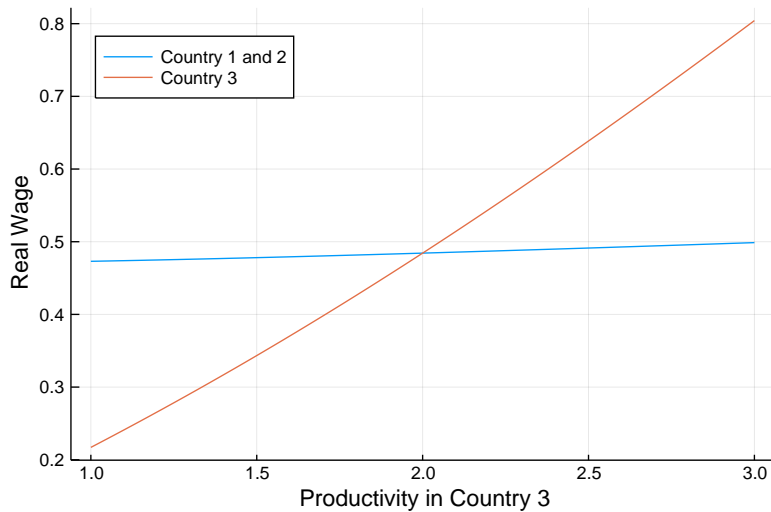
- This is a system of  $5n$  equations with  $5n$  unknowns ( $n = |N|$ ). 12 / 22

# Parameterization

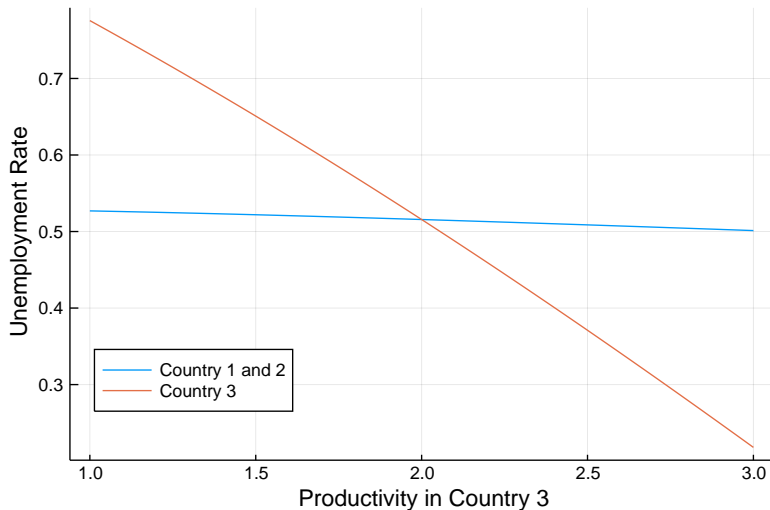
$n = 3$	number of countries
$\sigma = 4$	elasticity of substitution
$\chi = 0.5$	labor share in the matching function
$z_j = 2$	productivity
$\zeta_j = 2$	vacancy cost
$f_j = 1$	entry cost
$t_{j,j} = 1$	internal trade cost
$t_{j,k} = 1.1$	international trade cost
$L_j = 2$	labor

1. Let  $z_3$  and  $L_3$  vary from 1 to 3.
2. Let  $t_{j,k}$  vary from 1 to 3.

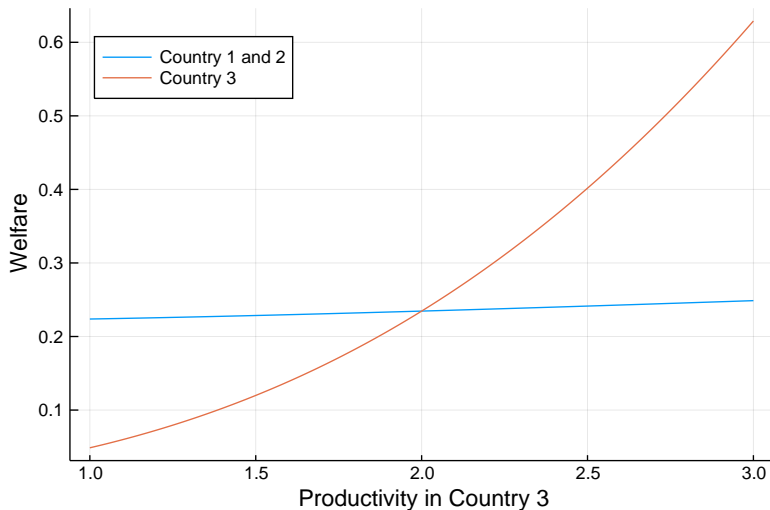
## Real Wages against $z_3$



## Unemployment against $z_3$



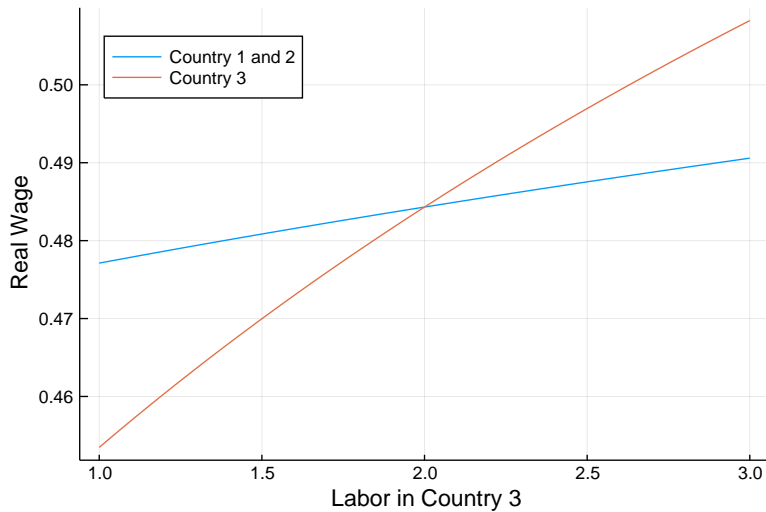
## Welfare against $z_3$



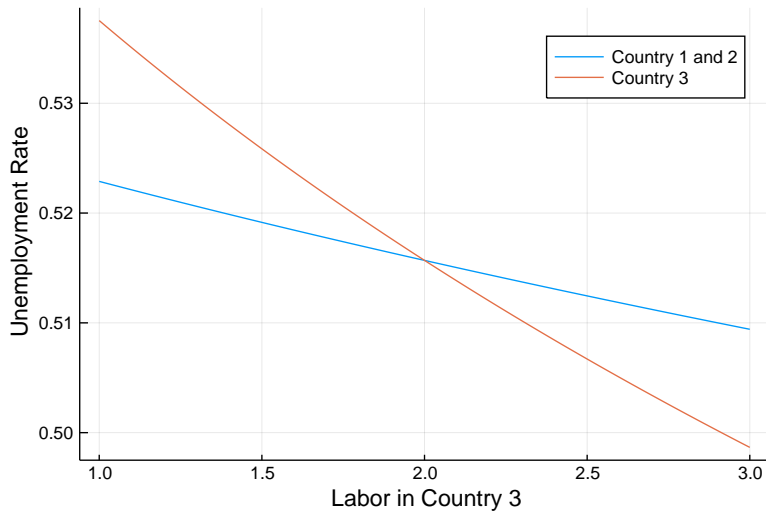
Welfare is  $e_j \frac{w_j}{P_j}$ .



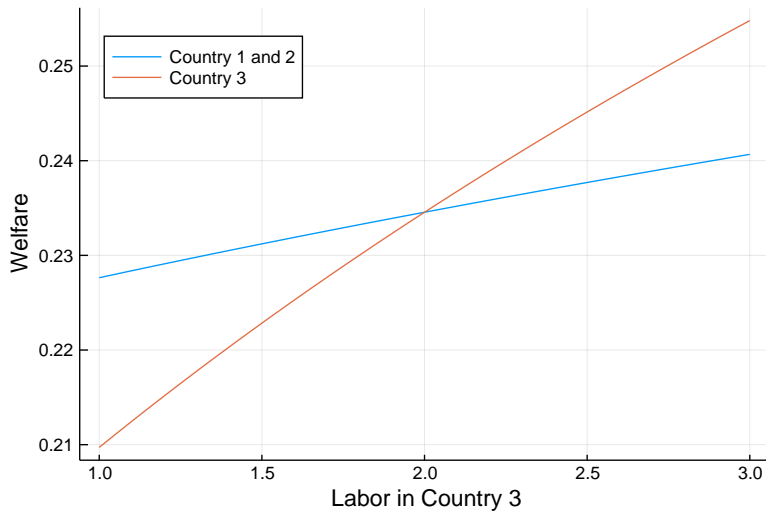
## Real Wages against $L_3$



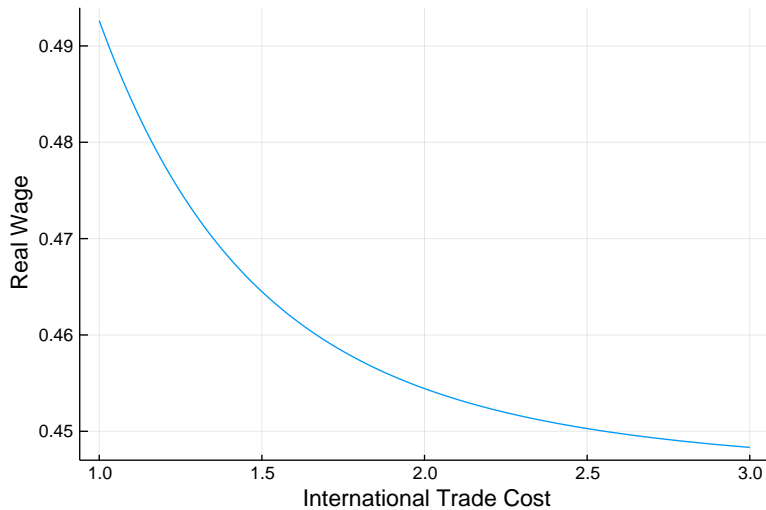
# Unemployment against $L_3$



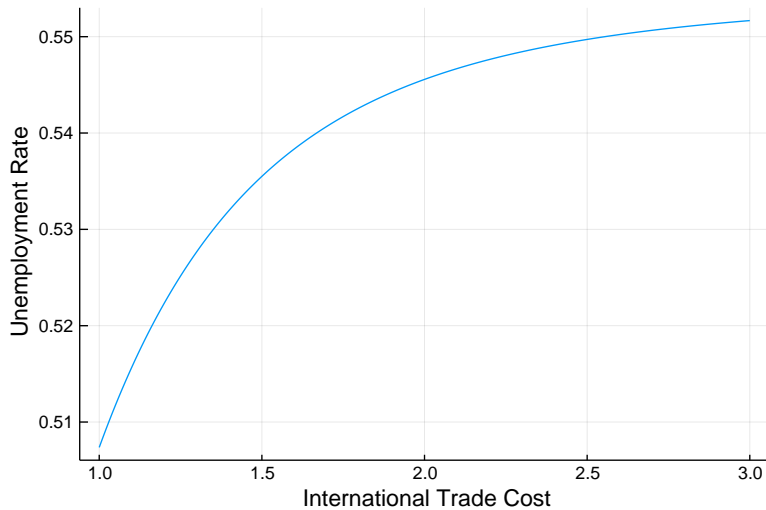
## Welfare against $L_3$



## Real Wages against $t_{j,k}$



## Unemployment against $t_{j,k}$



## Welfare against $t_{j,k}$

