Spatial on-the-job search.

- $n = 1, \cdots, N$ locations.
- First consider a two-period model
  - the current period: $\leftarrow$ w.o. $'$
  - the next period $\leftarrow$ with $'$

$(L_n)_{n=1}^{N}$, $(U_n)_{n=1}^{N}$: given.
$(E_n)_{n=1}^{N}$.

- The separation rate in n : δ_n.
- The arrival rate of an offer from i to n : λ_{in}^{e}, λ_{in}^{u}. $\Big\}$ exogenous.
- Unemployed people make b_n of home production in n.
- The distribution of values offered from a firm in ~~in~~ n : $F_n(\cdot)$.
  with the connected support $[\underline{V_n}, \overline{V_n}]$.
  - Suppose that the continuous density $f_n(\cdot)$ exists.
- utility function:

$$\mathbb{E}_0 \left[\int_0^{\infty} e^{-\rho t} \left(\frac{c_t}{1-\omega} \right)^{1-\omega} \left(\frac{h_t}{\omega} \right)^{\omega} dt \right] \quad , \omega > 0.$$

- Constant housing supply $(H_n)_{n=1}^{N}$: parameters.

$$\rho U_n = b_n r_n^{-\omega} + \sum_{i=1}^{n} \lambda_{in} \int_{\underline{V_i}}^{\overline{V_i}} \max\{x - U_n, 0\}\, dF_{ni}(x) \qquad (1)$$

~~For $V_n \in [\underline{V_n}, \overline{V_n}]$,~~

$$\rho V_n = w_n r_n^{-\omega} + \sum_{i=1}^{n} \frac{\lambda_{i.n}}{\tau_{n.i}} \int_{\underline{V_i}}^{\overline{V_i}} \max\{x - V_n, 0\}\, dF_i(x) \qquad (2)$$

$$+ \delta_n (U_n - V_n).$$

$\tau_{i.n}$: migration cost from i to n.

In (2), w_n is the wage associated with $V_n \in [\underline{V_n}, \overline{V_n}]$.

$$w_n r_n^{-\omega} = \rho V_n - \sum_{i=1}^{n} \frac{\lambda_{in}}{\tau_{i.n}} \int_{\underline{V_i}}^{\overline{V_i}} \max\{x - V_n, 0\}\, dF_i(x)$$

$$- \delta_n (U_n - V_n).$$

$U_n = \min_i \frac{V_i}{\tau_{n.i}}$.

I assume that $(\tau_{n.i})_{n.i}$ are large enough such that

$U_n = \frac{V_n}{\tau_{n.n}}$.

That is, given $n \in \{1, \dots, N\}$.

$$\frac{V_i}{\tau_{n.i}} \leq \frac{V_n}{\tau_{n.n}} \quad \text{for any } i.$$

Unlike Hoffmann & Shi 2016, $U_n \neq \frac{V_n}{\tau_{n.n}}$ or $\frac{V_i}{\tau_{n.i}}$.

New (2).

for $V_n \in [\underline{V_n}, \overline{V_n}]$,

$$\rho V_n = w_n r_n^{-\alpha} + \sum_{i=1}^{n} \lambda_{i,n} \int_{\underline{V_i}}^{\overline{V_i}} \max\left\{\frac{x}{\tau_{n,i}} - V_n, 0\right\} dF_i(x) \qquad (2)$$
$$+ \delta_n (U_n - V_n).$$

New (1).

$$\rho U = b_n r_n^{-\alpha} + \sum_{i=1}^{n} \lambda_{i,n} \int_{\underline{V_i}}^{\overline{V_i}} \max\left\{\frac{x}{\tau_{n,i}} - U_n, 0\right\} dF_i(x) \qquad (1)$$

$$\begin{cases} \text{In (1), fix } i. & \text{If } \cancel{\tau_{n,i} U_n \in [\underline{V_i}, \overline{V_i}]} \ \ \tau_{n,i} U_n \le \overline{V_i}. \\ \underline{V_{n,i}^U} \equiv \tau_{n,i} U_n & : \text{the lowest offer such that the unemployed are willing to work in } i. \\ \text{If } \overline{V_i} < \tau_{n,i} U_n, & \underline{V_{n,i}^U} \equiv \overline{V_i}. \end{cases}$$

$$\int_{\underline{V_i}}^{\overline{V_i}} \max\left\{\frac{x}{\tau_{n,i}} - U_n, 0\right\} dF_i(x)$$

$$= \int_{\underline{V_{n,i}^U}}^{\overline{V_i}} \left(\frac{x}{\tau_{n,i}} - U_n\right) dF_i(x)$$

$$= \frac{1}{\tau_{n,i}} \int_{\underline{V_{n,i}^U}}^{\overline{V_i}} x f_i(x)\, dx - U_n \left(\overbrace{F_i(\overline{V_i})}^{=1} - F_i(\underline{V_{n,i}^U}) \right).$$

Suppose that $\cancel{\tau_{n,i} U_n \in [\underline{V_i}, \overline{V_i}]}$ for any n, i.
$\boxed{\tau_{n,i} U_n < \overline{V_i}}$.

That is, taking the best offer job in i is better than being unemployed in n, even if the unemployed themselves need to pay the migration cost.

$$\int_{\underline{V_i}}^{\overline{V_i}} \max\left\{\frac{x}{T_{n,i}} - U_n,\ 0\right\} dF_i(x)$$

$$= \int_{T_{n,i}U_n}^{\overline{V_i}} \left(\frac{x}{T_{n,i}} - U_n\right) f_i(x)\, dx.$$

$$= \int_{T_{n,i}U_n}^{\overline{V_i}} \left(\frac{x}{T_{n,i}} - U_n\right)\left(F_i(x)\right)' dx$$

$$= \left[\left(\frac{x}{T_{n,i}} - U_n\right) F_i(x)\right]_{T_{n,i}U_n}^{\overline{V_i}}$$

$$- \int_{T_{n,i}U_n}^{\overline{V_i}} \frac{1}{T_{n,i}} F_i(x)\, dx.$$

$$= \left(\frac{\overline{V_i}}{T_{n,i}} - U_n\right) \overbrace{F_i(\overline{V_i})}^{=1} - \int_{T_{n,i}U_n}^{\overline{V_i}} \frac{1}{T_{n,i}} F_i(x)\, dx.$$

The other expression:

$$\int_{\underline{V_i}}^{\overline{V_i}} \max\left\{\frac{x}{T_{n,i}} - U_n,\ 0\right\} dF_i(x)$$

$$= -\int_{T_{n,i}U_n}^{\overline{V_i}} \left(\frac{x}{T_{n,i}} - U_n\right)\left(1 - F_i(x)\right)' dx$$

$$= -\left[\left(\frac{x}{T_{n,i}} - U_n\right)\left(1 - F_i(x)\right)\right]_{T_{n,i}U_n}^{\overline{V_i}}$$

$$+ \int_{T_{n,i}U_n}^{\overline{V_i}} \frac{1}{T_{n,i}} \left(1 - F_i(x)\right) dx.$$

$$= -\left(\frac{\overline{V_i}}{T_{n,i}} - U_n\right)\left(1 - F_i(\overline{V_i})\right) + \frac{1}{T_{n,i}} \int_{T_{n,i}U_n}^{\overline{V_i}} \left(1 - F_i(x)\right) dx.$$

$$= \frac{\overline{V_i}}{T_{n,i}} - U_n - \frac{1}{T_{n,i}} \int_{T_{n,i}U_n}^{\overline{V_i}} F_i(x)\, dx. \qquad (3)$$

① & ③ :

$$\rho \cancel{U}^{V^u} = b_n r_n^{-\omega} + \sum_{i=1}^{n} \lambda_{i,n} \left[\frac{\overline{V_i}}{\tau_{n,i}} - U_{\cancel{n}} - \frac{1}{\tau_{n,i}} \int_{\tau_{n,i} U}^{\overline{V_i}} F_i(x)\, dx \right] \qquad ④$$

② :

$$\rho V_n = w_n r_n^{-\omega} + \sum_{i=1}^{n} \lambda_{i,n} \underbrace{\int_{\underline{V_i}}^{\overline{V_i}} \max\left\{ \frac{x}{\tau_{n,i}} - V_n,\ 0 \right\} dF_i(x)}_{A^{e}_{n,i}} \quad \text{for } V_n \in [\underline{V_n}, \overline{V_n}]$$

$$+ \delta_n (U - V_n).$$

$$\frac{\underline{V_{n,i}}}{\tau_{n,i}} = V_n \qquad \therefore \quad \underline{V^{e}_{n,i}} = \begin{cases} \underline{V_i} & \text{if } \tau_{n,i} V_n \le \underline{V_i} \\ \tau_{n,i} V_n & \text{if } \underline{V_i} < \tau_{n,i} V_n < \overline{V_i} \\ \overline{V_i} & \text{if } \tau_{n,i} V_n \ge \overline{V_i}. \end{cases}$$

Going back to ① :

$$\rho U = b_n r_n^{-\omega} + \sum_{i=1}^{n} \lambda_{i,n} \underbrace{\int_{\underline{V_i}}^{\overline{V_i}} \max\left\{ \frac{x}{\tau_{n,i}} - U_{\cancel{n}},\ 0 \right\} dF_i(x)}_{A^{u}_{n,i}}.$$

If $\frac{\overline{V_i}}{\tau_{n,i}} \le U$, $A^u_{n,i} = 0$.

If $\frac{\underline{V_i}}{\cancel{V_i}\,\tau_{n,i}} \ge U$, $A^u_{n,i} = \int_{\underline{V_i}}^{\overline{V_i}} \left(\frac{x}{\tau_{n,i}} - U \right) dF_i(x)$.

If $\frac{\underline{V_i}}{\tau_{n,i}} < U < \frac{\overline{V_i}}{\tau_{n,i}}$, $A^u_{n,i} = \int_{\tau_{n,i} U}^{\overline{V_i}} \left(\frac{x}{\tau_{n,i}} - U \right) dF_i(x)$.

$$\underline{V^{U}_{n,i}} = \begin{cases} \overline{V_i} & \text{if } \overline{V_i} \le \tau_{n,i} U \\ \tau_{n,i} U & \text{if } \underline{V_i} < \tau_{n,i} U < \overline{V_i} \\ \underline{V_i} & \text{if } \tau_{n,i} U \le \underline{V_i}. \end{cases}$$

$$A_{n,i}^{U} = \int_{\underline{V}_{n,i}^{U}}^{\overline{V_i}} \left(\frac{x}{T_{n,i}} - U\right) dF_i(x)$$

$$= \int_{\underline{V}_{n,i}^{U}}^{\overline{V_i}} \left(\frac{x}{T_{n,i}} - U\right) f_i(x)\, dx$$

$$= \int_{\underline{V}_{ni}^{U}}^{\overline{V_i}} \left(\frac{x}{T_{ni}} - U\right) F_i(x)'\, dx$$

$$= \left[\left(\frac{x}{T_{n,i}} - U\right) F_i(x)\right]_{\underline{V}_{n,i}^{U}}^{\overline{V_i}} - \frac{1}{T_{n,i}} \int_{\underline{V}_{n,i}^{U}}^{\overline{V_i}} F_i(x)\, dx.$$

$$= \left(\frac{\overline{V_i}}{T_{n,i}} - U\right) - \left(\frac{\underline{V}_{ni}^{U}}{T_{n,i}} - U\right) F_i\left(\underline{V}_{ni}^{U}\right)$$

$$- \frac{1}{T_{ni}} \int_{\underline{V}_{ni}^{U}}^{\overline{V_i}} F_i(x)\, dx.$$

Suppose that $\underline{V_i} = T_{ii} U_i$.

Asm $T_{ii} = 1$ for $\forall i$.

$\underline{V_i} = U$. ← In SS, the value of the unemployed is equalized across space.

Under the assumption that

$$\underline{V_i} = U \quad \& \quad T_{ni} U < \overline{V_i},$$

4 is right.

$$A_{ni}^{e} = \int_{\underline{V_i}}^{\overline{V_i}} \max\left\{\frac{x}{T_{ni}} - V_n,\ 0\right\} dF_i(x)$$

$$= \int_{\underline{V}_{ni}^{e}}^{\overline{V_i}} \left(\frac{x}{T_{ni}} - V_n\right) F_i(x)'\, dx.$$

$$= \left[\left(\frac{x}{I_{ni}} - V_n \right) F_i(x) \right]_{\underline{V_{ni}^e}}^{\overline{V_i}} - \frac{1}{I_{ni}} \int_{\underline{V_{ni}^e}}^{\overline{V_i}} F_i(x)\,dx.$$

$$\cancel{N(V_n) = \{ i \mid \overline{V_i} < I_{ni} V_n \}}$$

$$N(V_n) = \left\{ i = 1, \cdots, N \;\middle|\; V_n \leq \frac{\overline{V_i}}{I_{ni}} \right\}.$$

Sps $\dfrac{\underline{V_i}}{I_{ni}} \leq V_n$.

$$\frac{\underline{V_i}}{I_{ni}} = \frac{\underline{U}}{I_{ni}}$$

this automatically holds b/c $I_{ni} \geq 1$.

$$\rho V_n = w_n(V_n) r_n^{-\omega} + \sum_{i \in N(V_n)} \lambda_{i,n} \int_{I_{ni} V_n}^{\overline{V_i}} \left(\frac{x}{I_{ni}} - V_n \right) dF_i(x)$$

then,

$$A_{ni}^{e} = \left[\left(\frac{x}{I_{ni}} - V_n \right) F_i(x) \right]_{I_{ni} V_n}^{\overline{V_i}} - \frac{1}{I_{ni}} \int_{I_{ni} V_n}^{\overline{V_i}} F_i(x)\,dx$$

$$= \left(\frac{\overline{V_i}}{I_{ni}} - V_n \right) - \frac{1}{I_{ni}} \int_{I_{ni} V_n}^{\overline{V_i}} F_i(x)\,dx.$$

$$\therefore \rho V_n = w_n(V_n) r_n^{-\omega} + \sum_{i \in N(V_n)} \lambda_{i,n} \left[\left(\frac{\overline{V_i}}{I_{ni}} - V_n \right) - \frac{1}{I_{ni}} \int_{I_{ni} V_n}^{\overline{V_i}} F_i(x)\,dx \right] + \delta_n (U - V_n)$$

$$\therefore w_n(V_n) r_n^{-\omega} = \rho V_n - \sum_{i \in N(V_n)} \lambda_{i,n} \left[\left(\frac{\overline{V_i}}{I_{ni}} - V_n \right) - \frac{1}{I_{ni}} \int_{I_{ni} V_n}^{\overline{V_i}} F_i(x)\,dx \right] - \delta_n (U - V_n)$$

$$\therefore w_n(V_n) = r_n^{\omega} \left\{ \rho V_n + \boxed{\sum_{i \in N(V_n)}} \lambda_{i,n} \left[V_n - \frac{\overline{V_i}}{I_{ni}} + \frac{1}{I_{ni}} \int_{I_{ni} V_n}^{\overline{V_i}} F_i(x)\,dx \right] + \delta_n (U - V_n) \right\}.$$

source of discontinuity.

$$w_n'(V_n) = r_n^{\omega} \left\{ \rho + \sum_{\cancel{i \in N(V_n)}\; i=1}^{N} \lambda_{i,n} \left(1 - F_i(I_{ni} V_n) \right) - \delta_n \right\}$$

ok.

how did Hoffmann & Shi handle this?

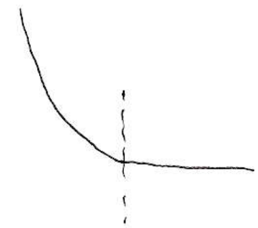
$\therefore$ ~~finitely~~ indifferentiable.

If $V_n < \frac{\bar{V}_i}{I_{ni}}$

$$A_{ni}^{e} = \int_{I_{ni}V_n}^{\bar{V}_i} \left(\frac{x}{I_{ni}} - v\right) dF_i(x)$$

$$= \left(\frac{\bar{V}_i}{I_{ni}} - V_n\right) - \frac{1}{I_{ni}}\int_{I_{ni}V_n}^{\bar{V}_i} F_i(x)\,dx.$$

If $V_n \geq \frac{\bar{V}_i}{I_{ni}}$



$A_{ni}^{e} = 0$ ← final conclusion.

$$= \int_{I_{ni}V_n}^{\bar{V}_i} \left(\frac{x}{I_{ni}} - v\right) dF_i(x).$$

$$= \left(\frac{\bar{V}_i}{I_{ni}} - V_n\right) - \frac{1}{I_{ni}}\int_{I_{ni}V_n}^{\bar{V}_i} \overbrace{F_i(x)}^{=1}\,dx$$

$$= \left(\frac{\bar{V}_i}{I_{ni}} - V_n\right) + \frac{1}{I_{ni}}\int_{\bar{V}_i}^{I_{ni}V_n} dx.$$

$$= \left(\frac{\bar{V}_i}{I_{ni}} - V_n\right) + \left(V_n - \frac{\bar{V}_i}{I_{ni}}\right) = 0.$$

integration of a continuous function is differentiable.

$+\rho_n(\bar{V} - V_n)$

$$\therefore W_n = w(V_n)\, r_n^{-\alpha} + \sum_{i=1}^{N} \lambda_{in} \left\{ \left(\frac{\bar{V}_i}{I_{ni}} - V_n\right) - \frac{1}{I_{ni}}\int_{I_{ni}V_n}^{\bar{V}_i} F_i(x)\,dx \right\}.$$

~~The RHS is con differentiable w.r.t. V_n except $V_n \in \{\frac{\bar{V}_i}{I_{ni}};\ i = 1, \cdots, N\}$.~~

according to Hoffmann & Shi, this is continuously differentiable.

$$W_n'(V_n) = r_n^{\alpha} \left\{ \rho + \sum_{i=1}^{N} \lambda_{in} \left(1 - F_i(I_{ni}V_n)\right) \right\}.$$

$+\rho_n$

$G_i(\cdot)$: the cdf of values at which workers are employed in i.

M_i : the measure of vacancies posted by firms in i.

A firm offering $V_n \in [\underline{V_n}, \bar{V_n}]$ fills a vacancy at the rate $h_{in}(V_n)/M_n$ where

$$h_n(V_n) \equiv \sum_{i=1}^{N} \lambda_{ni} U_i \mathbb{1}\left\{\frac{V_n}{\tau_{in}} \geq U\right\}$$

$$+ \sum_{i=1}^{N} \lambda_{ni} E_i G_i\left(\frac{V_n}{\tau_{in}}\right)$$

$$S_n(V_n) = \delta_n + \sum_{i=1}^{N} \lambda_{in}\left(1 - F_i(\tau_{ni} V_n)\right).$$

$\rho + S_n(V_n)$: the effective discount rate. for a firm of V_n in n.

the expected profit of a vacancy offering V_n :

$$\hat{\pi}_n(V_n) \equiv \left[\frac{y_n - w_n(V_n)}{\rho + S_n(V_n)}\right] \frac{h_n(V_n)}{M_n}.$$

$$L_i = U_i + E_i.$$

~~Discrete time ver.~~

$$\cancel{v_n^U = b_n r_n^{-\omega} + \beta \{ \max}$$

$$\cancel{\rho V_n^U = b_n r_n^{-\omega} + \zeta}$$ $\qquad E_\zeta v = V.$

$$\rho v_n^U = b_n r_n^{-\omega} + \zeta_n + \sum_{i=1}^{\hat{n}} \lambda_{in} \int_{\underline{V_i}}^{\overline{V_i}} \max\left\{ \frac{x}{\tau_{ni}} - U, 0 \right\} dF_i(x).$$

$$\cancel{\rho V^U =}$$

For any $V_n \in [\underline{V_n}, \overline{V_n}]$

$$\pi_n = \hat{\pi}_n(V_n) \equiv \left[\frac{y_n - w_n(V_n)}{\rho + \delta_n(V_n)} \right] \frac{h_n(V_n)}{M_n}.$$

For any $V_n \notin [\underline{V_n}, \overline{V_n}]$ that is $V_n < \underline{V_n}$ & $\overline{V_n} < V_n$,

$$\pi_n > \hat{\pi}_n(V_n) \equiv \left[\frac{y_n - w_n(V_n)}{\rho + \delta_n(V_n)} \right] \frac{h_n(V_n)}{M_n}.$$

(10.1)

$\because$ $w_n(U) = b_n$. (Since λ_{ni} is the same for the unemployed & the employed, the reservation wage is the same as home production).

$$\underline{V_n} = \cancel{U}.\ V^U$$

$$\therefore \pi_n = \hat{\pi}_r(\cancel{U}\ V^U) \equiv \left[\frac{y_n - b_n}{\rho + \delta_n(U)} \right] \frac{h_n(\cancel{U}\ V^U)}{M_n}$$ $\qquad (\tau_{nn} = 1.)$

$$= \left[\frac{y_n - b_n}{\rho + \delta_n(\cancel{U}\ V)} \right] \frac{\lambda_{nn}(U_n + E_n G_n(\cancel{U}\ V^U))}{M_n}$$

$$s_n(V^u) = \delta_n + \sum_{i=1}^{N} \lambda_{in}\left(1 - F_i(\tau_{ni} V^u)\right).$$

$$\pi_n = \hat{\pi}_n(V^u)$$

$$= \left[\frac{y_n - b_n}{\rho + \delta_n + \sum_{i} \lambda_{in}\left(1 - F_i(\tau_{ni} V^u)\right)}\right] \frac{\lambda_{nn}\left(U_n + E_n G_n(V^u)\right)}{M_n}$$

$$= \left[\frac{y_n - w_n(V_n)}{\rho + \delta_n + \sum_{i=1}^{N} \lambda_{in}\left(1 - F_i(\tau_{ni} V_n)\right)}\right] \frac{\sum_{i=1}^{N}\left(\lambda_{ni} U_i \mathbb{1}\left\{\frac{V_n}{\tau_{in}} \geq V^u\right\} + \lambda_{ni} E_i G_i\left(\frac{V_n}{\tau_{in}}\right)\right)}{M_n}$$

- A unit continuum of firms exists in ~~&~~ $n = 1, \cdots, N$.
- ~~Eea~~each firm casts ~~a~~ one vacancy at each moment in time.

Toward SS:

the inflow to $V_n \in [\underline{V_n}, \overline{V_n}]$:

$$\sum_{i=1}^{N} E_i \lambda_{ni} G_{\cancel{n}i}\left(\frac{V_n}{\tau_{in}}\right) + \sum_{i=1}^{N} U_i \lambda_{ni} \mathbb{1}\left\{\frac{V_n}{\tau_{in}} \geq V^u\right\}. \equiv h_n(V_n).$$

the outflow from V_n

$$g_n(V_n) \cdot E_n \underbrace{\left\{\delta_n + \sum_{i=1}^{N} \lambda_{in}\left(1 - F_i(\tau_{ni} V_n)\right)\right\}}_{= s_n(V_n)}.$$

or

$$G_n(V_n) - G_n(V_n^-)$$

$$G_n(V_n^-) \equiv \lim_{\varepsilon \downarrow 0} G_n(V_n - \varepsilon)$$

Asp 1:

F_n & G_n are differentiable in the $(\underline{V_i}, \overline{V_i})$ except possibly when

$\{ I_{ni}\underline{V_i},\ I_{ni}\overline{V_i} : i=1,\dots,N \}$.

SS conditions.

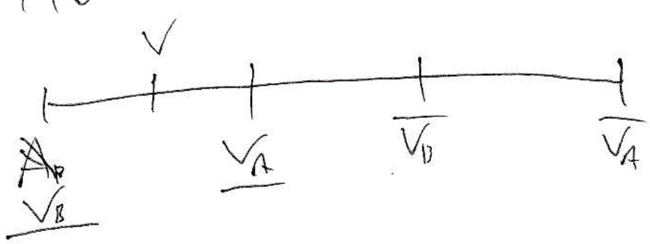
~~For~~

$$g(V_n-\varepsilon) = \lim_{\varepsilon\to 0} \frac{G(V_n)-G(V_n-\varepsilon)}{\varepsilon}$$

In a differentiable point of $G_n(\cdot)$ in $[\underline{V_n}, \overline{V_n}]$,

$h_n(V_n) = g_n(V_n)\, E_n \cdot s_n(V_n)$.

HS:



Order $I_{ni}\underline{V_i}$, $I_{ni}\overline{V_i}$ as

$$\begin{array}{l} \tilde V_n^1 \le \cdots \tilde V_n^2 \le \cdots \tilde V_n^{2N}. \\ \parallel \\ \underline{V_n} \\ \parallel \\ V^U \end{array}$$

Consider the workers in

$[\underline{v_n}, v]$. w/ $v \in [\underline{v_n}, \overline{v_n}]$.

$\underline{v_n} = v^U$

$$\int_{\underline{v_n}}^{v_n} h_n(\tilde{v}_n)\, d\tilde{v}_n = \int_{\underline{v_n}}^{v_n} E_n\, s_n(\tilde{v}_n)\, d\, G_n(\tilde{v}_n) \qquad (13.1)$$

$$= E_n \int_{\underline{v_n}}^{v_n} s_n(\tilde{v}_n)\, d\, G_n(\tilde{v}_n). \quad \text{for any } n, v_n.$$

~~$r_n H_n =$~~

$$c_t + r_t h_t = \begin{cases} b_t \\ w_t. \end{cases}$$

$$\omega\, w_t = r_t h_t.$$

$$\omega \left(E_n \int_{\underline{v_n}}^{\overline{v_n}} w_n(v_n)\, dG_n(v_n) + U_n b_n \right) = r_n H_n. \qquad (13.2)$$

inflow & outflow of U_i.

終わったらすぐやんべく。

~~$\hat{h}_n(V^U) = U_n$~~

the inflow to U_n:

$\delta_n E_n$.

the outflow from U_n:

$U_n\left\{\sum_{i=1}^{N} \lambda_{i,n}\left(1-F_i(\tau_{ni} V^U)\right)\right\}$.

$$\dot{U}_n = \cancel{\sum_n \dot{U}_n}$$
$$= \sum_n \left(\delta_n E_n - U_n \left\{\sum_{i=1}^{n} \lambda_{i,n}\left(1-F_i(\tau_{ni} V^U)\right)\right\}\right)$$
$$= 0. \quad (14.1)$$

$V_n \in [\underline{V_n}, \overline{V_n}]$, π_n, r_n, F_n, G_n, E_n, U_n.

$\underline{V_n} = V^U$

s.t.

(i) the optimal offers satisfy (10.1) π_n.

(ii) the value functions satisfy p. 5. V_n, V^U. $\underline{V_n} = V^U$.

(iii) the dist^n of workers & offers satisfy p. 13. (13.1)

(iv) the rent satisfies (13.2)

(v) the total unemployment is time-invariant.

(vi) $\sum_n (E_n + U_n) = L$.

significant

- in-mig & out-mig are positively correlated.
- heating

@ Patel - Vinay & Robin.

@ $x - V^U -$ Tai.

- An. amenity. ←
- V_n^U.
- gravity
  job-to-job transition }.

SY. seasonality.

- $\boxed{\text{age.}}$ ←).
- λ_{in} ← what is this?
- Comparative statistics.

detail ?

$$\rho V_n^U = b_n \overset{\text{exogenous}}{\cancel{(V_n^U)}}\, r_n^{-\omega} + \sum_{i=1}^{n} \lambda_{in} \left[\frac{\overline{V_i}}{\tau_{ni}} - V_n^U - \frac{1}{\tau_{ni}} \int_{\tau_{ni} V_n^U}^{\overline{V_i}} F_i(x)\,dx \right] \quad (16.1)$$

$N.$

$V_n \in [\underline{V_n}, \overline{V_n}]$.

$$\rho V_n = w_n(V_n) r_n^{-\omega} + \sum_{i=1}^{N} \lambda_{in} \left[\left(\frac{\overline{V_i}}{\tau_{ni}} - V_n \right) - \frac{1}{\tau_{ni}} \int_{\tau_{ni} V_n}^{\overline{V_i}} F_i(x)\,dx \right] \quad (16.2)$$

$$+ \delta_n (V_n^U - V_n).$$

$\cancel{2N}$.

$F \,\&\, V \to V.$

For any $V_n \in [\underline{V_n}, \overline{V_n}]$,

$$\pi_n = \hat{\pi}_n(V_n) \equiv \left[\frac{y_n - w_n(V_n)}{\rho + \delta_n(V_n)} \right] \frac{h_n(\cancel{V_n})}{}$$

For any $V_n \notin [\underline{V_n}, \overline{V_n}]$.

$\pi_n > \hat{\pi}_n(V_n) \not\leq$

$$w_n(V_n) = r_n^{\omega} \left\{ \rho V_n + \sum_{i=1}^{N} \lambda_{in} \left[V_n - \frac{\overline{V_i}}{\tau_{ni}} + \frac{1}{\tau_{ni}} \int_{\tau_{ni} V_n}^{\overline{V_i}} F_i(x)\,dx \right] - \delta_n (V_n^U - V_n) \right\}.$$

$$w_n'(V_n) = r_n^{\omega} \{ \underbrace{\rho + \sum_i \lambda_{in} (1 - F_i(\tau_{ni} V_n)) + \delta_n}_{S_n(V_n)} \}.$$

$$h_n(V_n) = \sum_{i=1}^{N} E_i \lambda_{ni} G_i\left(\frac{V_n}{\tau_{in}} \right) + \sum_i U_i \lambda_{ni} \mathbb{1}\left\{ \frac{V_n}{\tau_{in}} \geq V^U \right\}.$$

For any $V_n \in [\underline{V_n}, \overline{V_n}]$

$$\int_{\underline{V_n}}^{V_n} h_n(\tilde{V_n})\, d\tilde{V_n} = \int_{\underline{V_n}}^{V_n} E_n S_n(\tilde{V_n})\, d G_n(\tilde{V_n})$$

$$\text{LHS} = \int_{\underline{V_n}}^{V_n} \left[\sum_{i=1}^{n} E_i \lambda_{ni} G_i\left(\frac{\tilde{V_n}}{\tau_{in}} \right) + \sum_i U_i \lambda_{ni} \mathbb{1}\left\{ \frac{\tilde{V_n}}{\tau_{in}} \geq V^U \right\} \right] d\tilde{V_n}.$$

$$= \sum_{i=1}^{N} E_i \lambda_{ni} \int_{\underline{V_n}}^{V_n} G_i\left(\frac{\widetilde{V_n}}{T_{in}}\right) d\widetilde{V_n}$$

$$+ \sum_i U_i \lambda_{ni} \int_{\underline{V_n}}^{V_n} \mathbb{1}\{\widetilde{V_n} \geq T_{in} V_n^U\} \, d\widetilde{V_n}.$$

$$= \sum_i E_i \lambda_{ni} \int_{\underline{V_n}}^{V_n} G_i\left(\frac{\widetilde{V_n}}{T_{in}}\right) d\widetilde{V_n}$$

$$+ \sum_i U_i \lambda_{ni} \max\{0, V_n - T_{in} V_n^U\}.$$

$$\cancel{RHS = \int_{\underline{V_n}}^{V_n} E_n \left(\sum_i \lambda_{ni} (1-\right.}$$

$$RHS = \int_{\underline{V_n}}^{V_n} E_n \left(\sum_i \lambda_{in} \left(1 - F_i(T_{ni} \widetilde{V_n})\right) + \delta_n \right) d\, G_n(\widetilde{V_n}).$$

$$= E_n \int_{\underline{V_n}}^{V_n} \delta_n \, d\, G_n(\widetilde{V_n}) + E_n \sum_i \int_{\underline{V_n}}^{V_n} \lambda_{in} \, dG_n(\widetilde{V_n}).$$

$$- E_n \sum_{i=1}^{N} \lambda_{in} \int_{\underline{V_n}}^{V_n} F_i(T_{ni} \widetilde{V_n}) \, d\, G_n(\widetilde{V_n}).$$

$$= E_n \delta_n \int_{\underline{V_n}}^{V_n} d\, G_n(\widetilde{V_n}) + E_n \sum_i \lambda_{in} \int_{\underline{V_n}}^{V_n} d G_n(\widetilde{V_n})$$

$$- E_n \sum_i \lambda_{in} \int_{\underline{V_n}}^{V_n} F_i(T_{ni} \widetilde{V_n}) \, d\, G_n(\widetilde{V_n}).$$

$$\therefore \sum_i E_i \lambda_{ni} \int_{\underline{V_n}}^{V_n} G_i\left(\frac{\widetilde{V_n}}{T_{in}}\right) d\widetilde{V_n} + \sum_i U_i \lambda_{ni} \max\{0, V_n - T_{in} V_n^U\}$$

$$= E_n \delta_n \left(G_n(V_n) - G_n(\underline{V_n}) \right) + E_n \sum_i \lambda_{in} \boxed{\left(G_n(V_n) - G_n(\underline{V_n}) \right)}$$

$$- E_n \sum_i \lambda_{in} \int_{\underline{V_n}}^{V_n} F_i(T_{ni} \widetilde{V_n}) \, d\, G_n(\widetilde{V_n}).$$

$$\omega\left(\phi E_n \int_{\underline{V_n}}^{\overline{V_n}} w_n(v_n)\, d G_n(v_n) + U_n b_n\right) = r_n H_n.$$

$$\delta_n E_n = U_n \left\{ \sum_i \lambda_{in} \left(1 - F_i \left(I_{ni} V_n^U\right)\right)\right\}.$$

$$u_n = \frac{U_n}{E_n + U_n}$$

$$\delta_n \frac{E_n}{E_n + U_n} = \frac{U_n}{E_n + U_n} \left\{ \sum_i \lambda_{in} \left(1 - F_i \left(I_{ni} V_n^U\right)\right)\right\}.$$

$$\delta_n (1 - u_n) = u_n \left\{ \sum_i \lambda_{in} \left(1 - F_i \left(I_{ni} V_n^U\right)\right)\right\}.$$

$$\delta_n = u_n \left\{ \delta_n + \sum_i \lambda_{in} \left(1 - F_i \left(I_{ni} V_n^U\right)\right)\right\}.$$

$$u_n = \frac{\delta_n}{\delta_n + \sum_i \lambda_{in} \left(1 - F_i \left(I_{ni} V_n^U\right)\right)}.$$

$$E_{in} = \int_{\underline{V_{in}}}^{\overline{V_{in}}} E_i \lambda_{ni} G_i\left(\frac{\widetilde{V_n}}{I_{in}}\right) dF_n(\widetilde{V_n}).$$

$$= E_i \lambda_{ni} \int_{\underline{V_n}}^{\overline{V_n}} G_i\left(\frac{\widetilde{V_n}}{I_{in}}\right) dF_n(\widetilde{V_n}).$$

- job-to-job separation rates are different
  ← persistent at least for 25 states.

$$E_{in} = E_i \lambda_{ni} \int_{\underline{v_n}}^{\overline{v_n}} G_{ni}\left(\frac{\tilde{v}_n}{T_{in}}\right) dF_n(\tilde{v}_n)$$

$$= E_i \lambda_{ni} \underbrace{\int_{\underline{v_n}}^{\overline{v_n}} G_i\left(\frac{\tilde{v}_n}{T_{in}}\right) F_n(\tilde{v}_n)' \, d\tilde{v}_n}_{\star}$$

$$\star = \left[G_i\left(\frac{\tilde{v}_n}{T_{in}}\right) F_n(\tilde{v}_n)\right]_{\underline{v_n}}^{\overline{v_n}} - \frac{1}{T_{in}} \int_{\underline{v_n}}^{\overline{v_n}} g_i\left(\frac{\tilde{v}_n}{T_{in}}\right) F_n(\tilde{v}_n) \, d\tilde{v}_n .$$

$$= G_i\left(\frac{\overline{v_n}}{T_{in}}\right) - \frac{1}{T_{in}} \underbrace{\int_{\underline{v_n}}^{\overline{v_n}} \left(\cancel{g_i} G_i\left(\frac{\tilde{v}_n}{T_{in}}\right) T_{in}\right)' F_n(\tilde{v}_n) \, d\tilde{v}_n}_{\star\star}$$

$$\star\star = \left[T_{in} G_i\left(\frac{\tilde{v}_n}{T_{in}}\right) F_n(\tilde{v}_n)\right]_{\underline{v_n}}^{\overline{v_n}}$$

$$- T_{in} \int_{\underline{v_n}}^{\overline{v_n}} G_i\left(\frac{\tilde{v}_n}{T_{in}}\right) dF_n(\tilde{v}_n) .$$

$$= T_{in} G_i\left(\frac{\overline{v_n}}{T_{in}}\right) - T_{in} \star$$

macro evidence over business cycle
job to job

$\boxed{\text{Bilal}}$