# Trade, Migration, and Unemployment

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# Questions

- Why is unemployment different across places?
- How do foreign shocks affect the spatial distribution of unemployment?

# Outline of the Model

- I integrate search and matching into Caliendo, Dvorkin & Parro (2019).
- Unemployment rates for newcomers and those who have been employed are different.

# Preferences (1)

The employed in $j$ in $t$ have privilege.

- They will have a higher chance to be hired in $j$ in $t+1$.

The value of the employed:

$$
\begin{aligned}
v_t^{j,e} = & U\left((1-\iota_t)\frac{w_t^j}{P_t^j}\right) \\
& + \max\Big\{\beta[e_{t+1}^{j,e}\mathbb{E}v_{t+1}^{j,e} + (1-e_{t+1}^{j,e})\mathbb{E}v_{t+1}^{j,u}] - \tau^{j,j} + \nu\varepsilon_t^{j,j}, \\
& \max_{n\neq j}\left\{\beta[e_{t+1}^{n,N}\mathbb{E}v_{t+1}^{n,e} + (1-e_{t+1}^{n,N})\mathbb{E}v_{t+1}^{n,u}] - \tau^{j,n} + \nu\varepsilon^{j,n}\right\}\Big\}
\end{aligned} \tag{1}
$$

# Preferences (2)

The value of the unemployed:

$$
\begin{aligned}
v_t^{j,u} =& U\left((1-\iota_t)\frac{b^j w_t^j}{P_t^j}\right) \\
&+ \max_n \left\{\beta[e_{t+1}^{n,N}\mathbb{E}v_{t+1}^{n,e} + (1-e_{t+1}^{n,N})\mathbb{E}v_{t+1}^{n,u}] - \tau^{j,n} + \nu\varepsilon_t^{j,n}\right\}
\end{aligned}
\tag{2}
$$

# Expected Values (1)

The expected value of the employed:

$$
\begin{aligned}
V_t^{j,e} =& U\left((1-\iota_t)\frac{w_t^j}{P_t^j}\right) \\
&+\nu \log\left[\exp\left(\beta[e_{t+1}^{j,e}V_{t+1}^{j,e}+(1-e_{t+1}^{j,e})V_{t+1}^{j,u}]-\tau^{j,j}\right)^{1/\nu}\right. \\
&\left.+\sum_{n\neq j}\exp\left(\beta[e_{t+1}^{n,N}V_{t+1}^{n,e}+(1-e_{t+1}^{n,N})V_{t+1}^{n,u}]-\tau^{j,n}\right)^{1/\nu}\right]
\end{aligned} \tag{3}
$$

# Expected Values (2)

The expected value of the unemployed:

$$
\begin{aligned}
V_t^{j,u} =& U\left((1-\iota_t)\frac{b^j w_t^j}{P_t^j}\right) \\
&+ \nu \log\left[\sum_n \exp\left(\beta[e_{t+1}^{n,N} V_{t+1}^{n,e} + (1-e_{t+1}^{n,N}) V_{t+1}^{n.u}] - \tau^{j,n}\right)^{1/\nu}\right]
\end{aligned}
\tag{4}
$$

# Migration of the Employed

The migration share from $j$ to $n \neq j$ of the employed:

$$\mu_t^{j,n\neq j|e}$$

$$= \frac{\exp\left(\beta[e_{t+1}^{n,N} V_{t+1}^{n,e} + (1 - e_{t+1}^{n,N}) V_{t+1}^{n,u}] - \tau^{j,n}\right)^{1/\nu}}{\exp\left(\beta[e_{t+1}^{j,e} V_{t+1}^{j,e} + (1 - e_{t+1}^{j,e}) V_{t+1}^{j,u}] - \tau^{j,j}\right)^{1/\nu} + \sum_{k\neq j} \exp\left(\beta[e_{t+1}^{k,N} V_{t+1}^{k,e} + (1 - e_{t+1}^{k,N}) V_{t+1}^{k,u}] - \tau^{j,k}\right)^{1/\nu}} \tag{5}$$

$j$ to $j$:

$$\mu_t^{j,j|e}$$

$$= \frac{\exp\left(\beta[e_{t+1}^{j,e} V_{t+1}^{j,e} + (1 - e_{t+1}^{j,e}) V_{t+1}^{j,u}] - \tau^{j,j}\right)^{1/\nu}}{\exp\left(\beta[e_{t+1}^{j,e} V_{t+1}^{j,e} + (1 - e_{t+1}^{j,e}) V_{t+1}^{j,u}] - \tau^{j,j}\right)^{1/\nu} + \sum_{k\neq j} \exp\left(\beta[e_{t+1}^{k,N} V_{t+1}^{k,e} + (1 - e_{t+1}^{k,N}) V_{t+1}^{k,u}] - \tau^{j,k}\right)^{1/\nu}} \tag{6}$$

# Migration of the Unemployed

The migration share from $j$ to $n$ of the unemployed:

$$\mu_t^{j,n|u} = \frac{\exp\left(\beta[e_{t+1}^{n,N} V_{t+1}^{n,e} + (1 - e_{t+1}^{n,N}) V_{t+1}^{n,u}] - \tau^{j,n}\right)^{1/\nu}}{\sum_k \exp\left(\beta[e_{t+1}^{k,N} V_{t+1}^{k} + (1 - e_{t+1}^{k,N}) V_{t+1}^{k}] - \tau^{j,k}\right)^{1/\nu}} \tag{7}$$

# Population Dynamics

Labor forces evolve as

$$L_t^j = \sum_n \mu_{t-1}^{n,j|e} E_{t-1}^n + \sum_n \mu_{t-1}^{n,j|u} U_{t-1}^n \tag{8}$$

"Newcomers" evolve as

$$\begin{aligned} N_t^j &= \sum_{n \neq j} \mu_{t-1}^{n,j|e} E_{t-1}^n + \sum_n \mu_{t-1}^{n,j|u} U_{t-1}^n \\ &= L_t^j - \mu_{t-1}^{j,j|e} E_{t-1}^j \end{aligned} \tag{9}$$

The remaining is EK+job search.

# Producers

- The unit cost of producing $x \in [0,1]$ in region $j$ is

$$\frac{(w_t^j + \Delta_t^j)^\beta (P_t^j)^{1-\beta}}{z_t^j(x)}. \tag{10}$$

  - $\Delta_t^j$: the cost of hiring one unit of labor
  - $z_t^j(x) \sim F_t^j(x) = \exp\left[-\left(\frac{x}{T_t^j}\right)^{-\theta}\right]$.

# Matching Function and Employment

- The matching function is

$$E_t^j = \Xi_t^j (a^e \mu_{t-1}^{j,j|e} E_{t-1}^j + a^N N_t^j)^\zeta (V_t^j)^{1-\zeta} \tag{11}$$

  - presumably $a^e > a^N$.
  - constant returns to scale.

- Unemployment is

$$U_t^j = L_t^j - E_t^j \tag{12}$$

# Expressions per Effective Job Seekers

- Define effective job seekers

$$\tilde{L}_t^j = a^e \mu_{t-1}^{j,j|e} E_{t-1}^j + a^N N_t^j. \tag{13}$$

- Let

$$m_t^j = \frac{E_t^j}{\tilde{L}_t^j}, \ \lambda_t^j = \frac{V_t^j}{\tilde{L}_t^j}. \tag{14}$$

- Then the matching function is

$$m_t^j = \Xi^j (\lambda_t^j)^{1-\zeta}. \tag{15}$$

- The employment rates for the two classes are

$$e_t^{j,e} = a^e m_t^j, \quad e_t^{j,N} = a^N m_t^j. \tag{16}$$

# Wage Bargaining

- Let $r_t^j(x)$ be the revenue of producer $x$ from the marginal employee.
- Suppose that producer $x$ in $j$ and its employees have the equal bargaining power.
- Then the Nash bargaining problem is

$$\max_{w_t^j(x)} \left[r_t^j(x) - w_t^j(x)\right]^{1/2} \left[w_t^j(x) - b_t^j\right]^{1/2}. \tag{17}$$

  - The non-cooperative foundation is from Brugemann, Gautier and Menzio (2020).
- The FOC is $w_t^j(x) - b_t^j = r_t^j(x) - w_t^j(x)$.
- Then $w_t^j(x) = b_t^j + \Delta_t^j \equiv w_t^j$. Proof
- Recall $b_t^j = b^j w_t^j$.
- The unit labor cost is $w_t^j + \Delta_t^j = (2 - b^j) w_t^j$.

# Vacancy Posting

- A producer casts a vacancy by paying $v_t^j$ in terms of final goods.
- An identity on the aggregate hiring costs:

$$\Delta_t^j E_t^j = v_t^j P_t^j V_t^j \tag{18}$$

Equivalently,

$$(1 - b_t^j) w_t^j E_t^j = v_t^j P_t^j \tilde{L}_t^j \lambda_t^j . \tag{19}$$

# EK formulas

- Trade shares are

$$\pi_t^{n,j} = \frac{T_t^n[((2-b^n)w_t^n)^\beta (P_t^n)^{1-\beta} d^{n,j}]^{-\theta}}{\sum_k T_t^k[((2-b^k)w_t^k)^\beta (P_t^k)^{1-\beta} d^{k,j}]^{-\theta}} \tag{20}$$

- Price indices are

$$P_t^j = G\left(\sum_k T_t^k[((2-b^k)w_t^k)^\beta (P_t^k)^{1-\beta} d^{k,j}]^{-\theta}\right)^{-1/\theta} \tag{21}$$

# Federal Tax

Unemployment benefits are financed by the labor income tax.

$$\iota_t = \frac{\sum_n b^n w_t^n U_t^n}{\sum_n \left(w_t^n E_t^n + b^n w_t^n U_t^n\right)} \tag{22}$$

# Balanced Trade

- Expenditures are

$$X_t^j = (w_t^j E_t^j + b^j w_t^j U_t^j) + \frac{1-\beta}{\beta}(2-b^j) w_t^j E_t^j. \tag{23}$$

- The labor cost is a constant share of revenue

$$(2-b^j) w_t^j E_t^j = \beta \sum_n \pi_t^{j,n} X_t^n. \tag{24}$$

# Temporary Equilibrium

- Let $(\pi_t)_{n,j} = \pi_t^{n,j}$, $(w_t)_j = w_t^j$ and so on.

Given $\{\mu_{t-1}^e, \mu_{t-1}^u, L_t, E_{t-1}, U_{t-1}\}$, a temporary equilibrium is a tuple of $\{\iota_t, X_t, \pi_t^{n,j}, E_t, U_t, w_t, P_t, \lambda_t\}$ such that all static conditions hold.

# Sequential Equilibrium

Given $\{L_0, E_0, U_0\}$, a sequential equilibrium is a tuple of $\{\mu_t^e, \mu_t^u\}_{t=0}^{\infty}$, $\{V_t^e, V_t^u\}_{t=1}^{\infty}$, $\{L_t, E_t, U_t\}_{t=1}^{\infty}$, $\{\iota_t, X_t, \pi_t, E_t, U_t, w_t, P_t, \lambda_t\}_{t=1}^{\infty}$ satisfying

1. dynamic location choices,
2. temporary equilibria

in any $t$.

# Stationary Equilibrium

A stationary equilibrium is a sequential equilibrium such that all objects are time-invariant.

# Wage Determination

Back

- Recall that the profits are

$$\Pi_t^j(x) = p_t^j(x)y_t^j(x) - [w_t^j(x) + \Delta_t^j]E_t^j(x) - P_t^j M_t^j(x). \tag{25}$$

- The FOC w.r.t. $E_t^j(x)$ is

$$r_t^j(x) \equiv \frac{\partial p_t^j(x)y_t^j(x)}{\partial E_t^j(x)} = w_t^j(x) + \Delta_t^j. \tag{26}$$

- Recall the FOC from the Nash bargaining

$$w_t^j(x) - b_t^j = r_t^j(x) - w_t^j(x). \tag{27}$$

- (26) and (27) imply

$$w_t^j(x) = b_t^j + \Delta_t^j. \tag{28}$$