Trade and Unemployment

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Comparison

Helpman & Itskhoki (2010)	This model
Melitz+Search	Krugman+Search
Two countries	Many countries

Preferences

- N is the set of countries.
- Individual i in country j has a utility function

$$U_{i,j} = \left(\sum_{k \in N} \int_0^{M_k} C_{i,k,j}(\omega)^{\frac{\sigma-1}{\sigma}} d\omega \right)^{\frac{\sigma}{\sigma-1}}. \tag{1}$$

- Her income is

$$I_{i,j} = \begin{cases} w_j & \text{if } i \text{ is employed,} \\ 0 & \text{if } i \text{ is unemployed.} \end{cases} \tag{2}$$

- Individual i's demand for variety ω shipped from k to j is

$$C_{i,k,j}(\omega) = \left(\frac{p_k(\omega) t_{k,j}}{P_j} \right)^{-\sigma} \frac{I_{i,j}}{P_j}. \tag{3}$$

Revenue

- If firm ω hires h units of labor, the revenue is

$$R_j(\omega, h) = \Phi_j \left(z_j h\right)^{\frac{\sigma-1}{\sigma}} \tag{4}$$

where

$$\Phi_j = \left(\sum_{k \in N} t_{j,k}^{1-\sigma} P_k^{\sigma-1} X_k \right)^{\frac{1}{\sigma}}. \tag{5}$$

Stole-Zwiebel Bargaining

- Following Stole & Zwiebel (1996) and Helpman & Itskhoki (2010), workers and the firm engage in wage bargaining with equal weights.
- The firm and a worker equally divide the marginal surplus

$$\frac{\partial}{\partial h}[R_j(\omega, h) - w_j(\omega, h)h] = w_j(\omega, h). \tag{6}$$

- Solving the ODE, the wage as a function of labor input is

$$w_j(\omega, h) = \frac{\sigma - 1}{2\sigma - 1} \frac{R_j(\omega, h)}{h}. \tag{7}$$

- Workers get $\frac{\sigma-1}{2\sigma-1} R_j(\omega, h)$. The firm gets $\frac{\sigma}{2\sigma-1} R_j(\omega, h)$.

Profit Maximization

- Let d_j be the hiring cost per worker.
  - This is not a parameter, but determined in general equilibrium.
- Let $f_j P_j$ be the entry cost.
- Firm ω in j solves

$$\pi_j(\omega) = \max_{h \geq 0} \left\{ \frac{\sigma}{2\sigma - 1} \Phi_j z_j^{\frac{\sigma-1}{\sigma}} h^{\frac{\sigma-1}{\sigma}} - d_j h - f_j P_j \right\}. \tag{8}$$

  - The 1st term is $R_j(\omega, h) - w_j(\omega, h) h$.
- The FOC yields the labor input

$$h_j(\omega) = \left(\frac{\sigma - 1}{2\sigma - 1} \right)^{\sigma} \left(\frac{\Phi_j}{d_j} \right)^{\sigma} z_j^{\sigma-1}. \tag{9}$$

- The wage is

$$w_j(\omega) \equiv w_j(\omega, h_j(\omega)) = d_j. \tag{10}$$

Search and Matching

- The matching function is

$$H_j = L_j^{\chi} V_j^{1-\chi}. \tag{11}$$

$$\begin{aligned} H_j &: \text{ aggregate employment in } j, \\ L_j &: \text{ aggregate labor force in } j, \\ V_j &: \text{ aggregate vacancies in } j. \end{aligned}$$

- Let $\theta_j = \frac{V_j}{L_j}$.
- Let ζ_j be the cost of posting a vacancy.
  - This is a parameter.
- An identity about the aggregate hiring costs:

$$\begin{aligned} d_j H_j &= \zeta_j V_j \\ \Rightarrow (w_j =)\ d_j &= \zeta_j \theta_j^{\chi}. \end{aligned} \tag{12}$$

Optimal Price

- The f.o.b. price associated with $l_j(\omega)$ is

$$p_j(\omega) = \left(\frac{2\sigma - 1}{\sigma - 1}\right)\left(\frac{\zeta_j}{z_j}\right)\theta_j^{\chi}. \tag{13}$$

- The price index is

$$\begin{aligned} P_j &= \left[\sum_{k \in N} M_k (t_{k,j} p_k(\omega))^{1-\sigma}\right]^{\frac{1}{1-\sigma}} \\ &= \left[\sum_{k \in N} M_k t_{k,j}^{1-\sigma} \left(\frac{2\sigma - 1}{\sigma - 1}\right)^{1-\sigma} \left(\frac{\zeta_k}{z_k}\right)^{1-\sigma} \theta_k^{\chi(1-\sigma)}\right]^{\frac{1}{1-\sigma}}. \end{aligned} \tag{14}$$

Labor Input, Vacancies and the Mass of Firms

- I rewrite labor input as a function of θ_j

$$h_j(\omega) = \left(\frac{\sigma-1}{2\sigma-1}\right)^{-\sigma} \Phi_j^{\sigma} \zeta_j^{-\sigma} z_j^{\sigma-1} \theta_j^{-\chi\sigma}. \tag{15}$$

- An identity $d_j l_j(\omega) = \zeta_j v_j(\omega)$ leads to the firm's vacancies

$$v_j(\omega) = \left(\frac{\sigma-1}{2\sigma-1}\right)^{-\sigma} \Phi_j^{\sigma} \zeta_j^{-\sigma} z_j^{\sigma-1} \theta_j^{\chi(1-\sigma)}. \tag{16}$$

- Then $M_j v_j(\omega) = L_j \theta_j$ yields the mass of firms

$$M_j = \left(\frac{\sigma-1}{2\sigma-1}\right)^{-\sigma} \Phi^{-\sigma} \zeta_j^{\sigma} z_j^{1-\sigma} L_j \theta_j^{\chi(\sigma-1)+1}. \tag{17}$$

Aggregate Employment and Income

- The aggregate employment is

$$H_j = M_j h_j(\omega) = \theta_j^{1-\chi} L_j. \tag{18}$$

  - That is, the employment rate is $e_j = \theta_j^{1-\chi}$.
- The aggregate expenditure is

$$X_j = w_j e_j L_j + M_j f_j P_j = \zeta_j \theta_j L_j + M_j f_j P_j. \tag{19}$$

Zero Profit Condition

- Write the profits as a function of θ_j and Φ_j

$$\pi_j(\omega) = (\sigma-1)^{\sigma-1}(2\sigma-1)^{-\sigma}\Phi_j(\zeta_j\theta_j^{\chi})^{1-\sigma}z_j^{\sigma-1} - f_jP_j. \quad (20)$$

- The zero-profit condition $\pi_j(\omega) = 0$ yields

$$\theta_j = \left[(\sigma-1)(2\sigma-1)^{\frac{\sigma}{1-\sigma}}\Phi_j^{-\frac{1}{1-\sigma}}z_j\zeta_j^{-1}f_j^{\frac{1}{1-\sigma}}P_j^{\frac{1}{1-\sigma}}\right]^{\frac{1}{\chi}}. \quad (21)$$

Equilibrium

- Equilibrium conditions are condensed as follows.
- An equilibrium is $\{\theta_j, X_j, P_j, M_j, \Phi_j\}_{j\in N}$ that satisfies

$$\theta_j = \left[(\sigma-1)(2\sigma-1)^{\frac{\sigma}{1-\sigma}} \Phi_j^{\frac{1}{\sigma-1}} z_j \zeta_j^{-1} f_j^{\frac{1}{1-\sigma}} P_j^{\frac{1}{1-\sigma}}\right]^{\frac{1}{\chi}}, \tag{22}$$

$$X_j = \zeta_j \theta_j L_j + M_j f_j P_j, \tag{23}$$

$$P_j = \left[\sum_{k\in N} M_k t_{k,j}^{1-\sigma} \left(\frac{2\sigma-1}{\sigma-1}\right)^{1-\sigma} \left(\frac{\zeta_k}{z_k}\right)^{1-\sigma} \theta_k^{\chi(1-\sigma)}\right]^{\frac{1}{1-\sigma}}, \tag{24}$$

$$M_j = \left(\frac{2\sigma-1}{\sigma-1}\right)^{\sigma} \Phi^{-\sigma} \zeta_j^{\sigma} \theta_j^{\chi(\sigma-1)+1} z_j^{1-\sigma} L_j, \tag{25}$$

$$\Phi_j = \left(\sum_{k\in N} t_{j,k}^{1-\sigma} P_k^{\sigma-1} X_k\right)^{\frac{1}{\sigma}}. \tag{26}$$

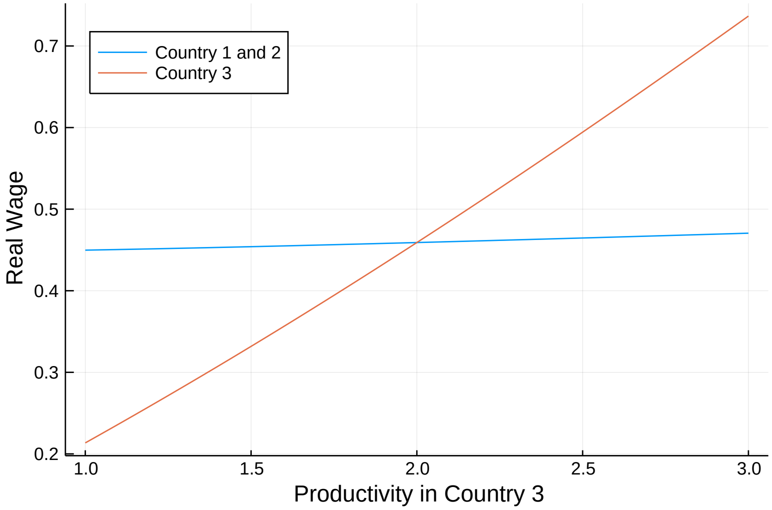
- Let $n = |N|$. This is a system of $5n$ equations with $5n$ unknowns.

Parameterization

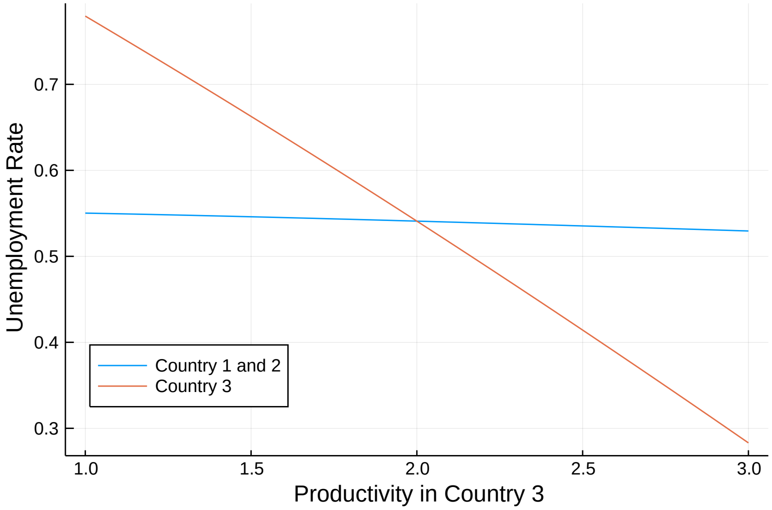
$n = 3$	number of countries
$\sigma = 4$	elasticity of substitution
$\chi = 0.5$	labor share in the matching function
$z_j = 2$	productivity
$\zeta_j = 2$	vacancy cost
$f_j = 1$	entry cost
$t_{j,j} = 1$	internal trade cost
$t_{j,k} = 1.1$	international trade cost
$L_j = 2$	labor

1. Let z_3 and L_3 vary from 1 to 3.
2. Let $t_{j,k}$ vary from 1 to 3.

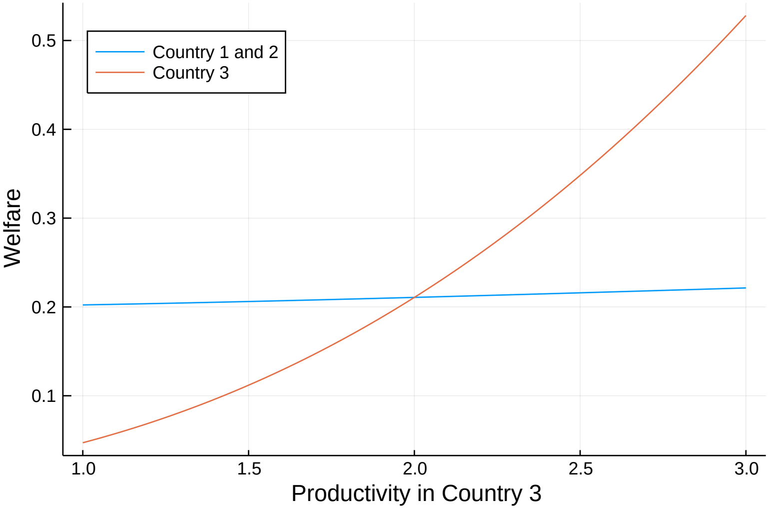
Real Wages against z_3



Unemployment against z_3

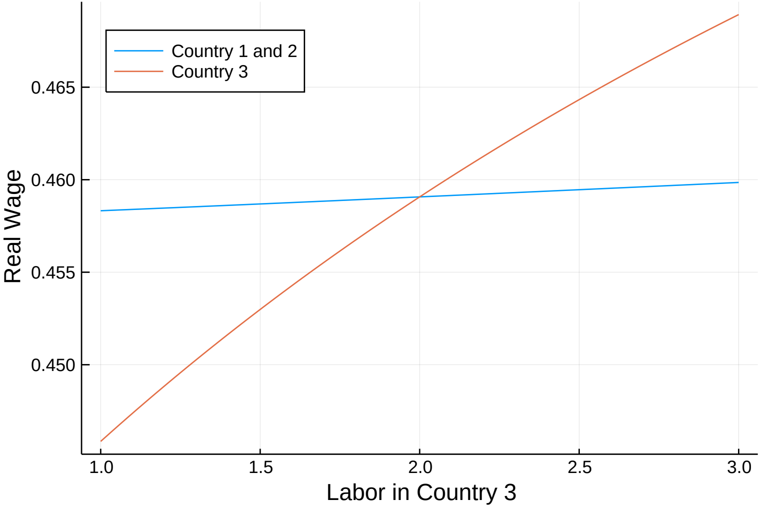


Welfare against z_3

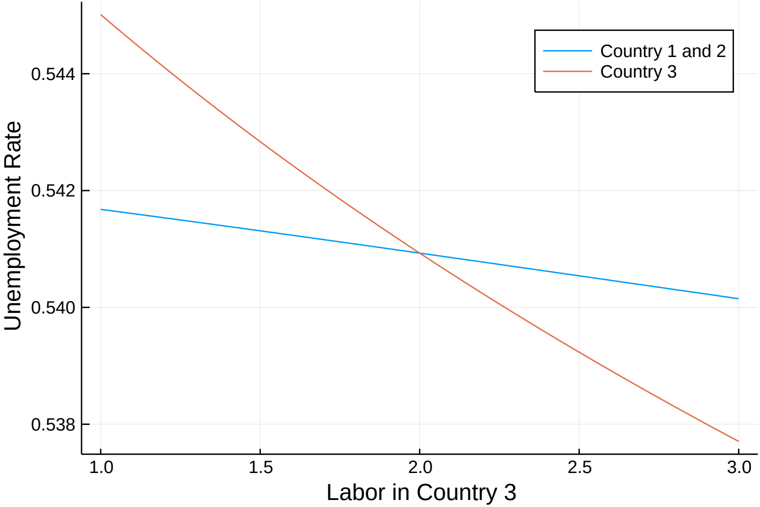


Welfare is $e_j \frac{w_j}{P_j}$.

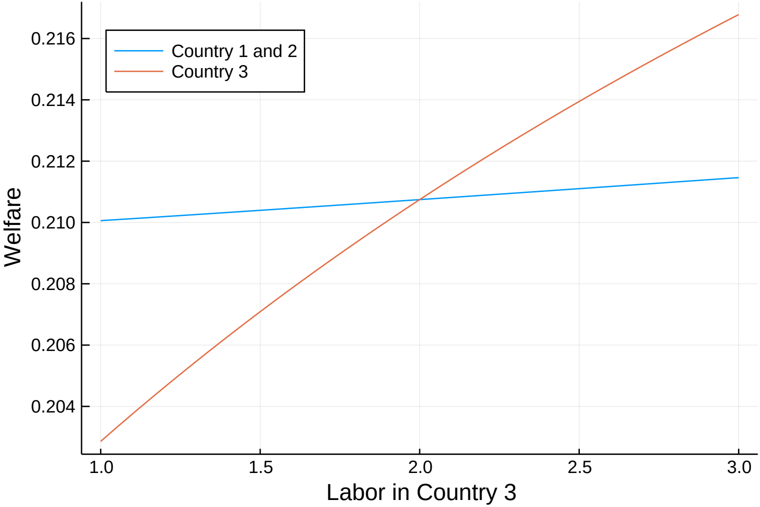
Real Wages against L_3



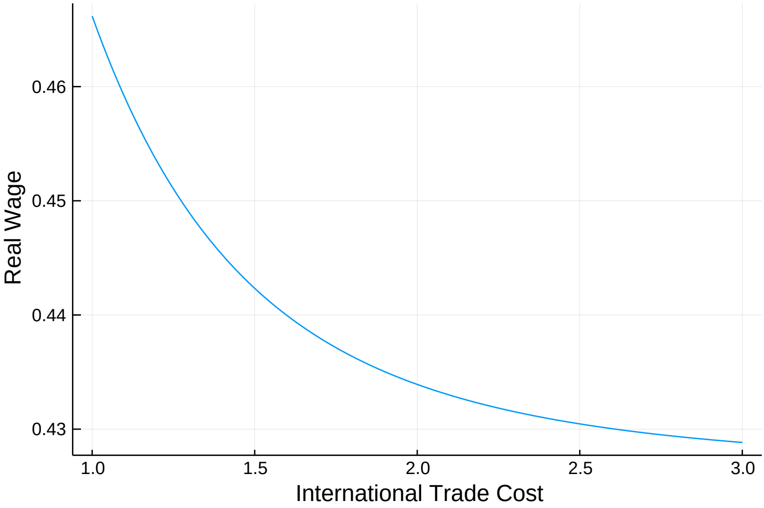
Unemployment against L_3



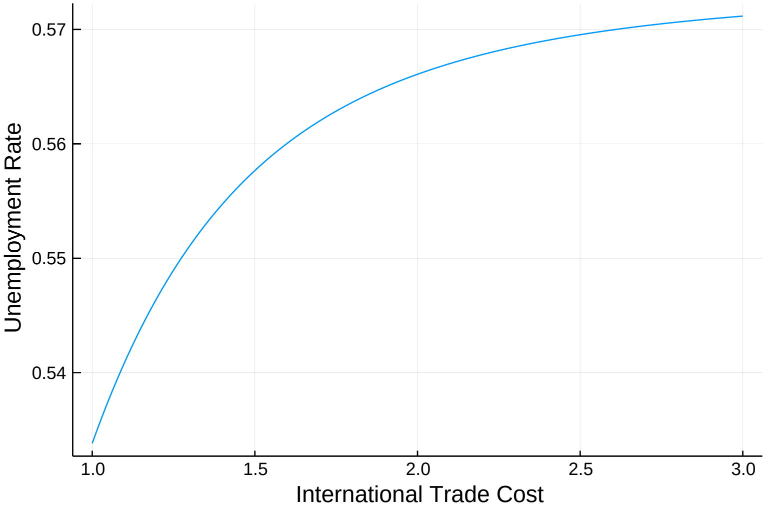
Welfare against L_3



Real Wages against $t_{j,k}$



Unemployment against $t_{j,k}$



Welfare against $t_{j,k}$