Spatial Ver

$$C_r \cancel{U_k} = \cancel{\prod_{k \in K}} \prod_{k=1}^{K} (C_{r.k})^{\alpha_k} \qquad \sum_{k=1}^{K} \alpha_k = 1.$$

$$U_{i.r} = C_{r.i}$$

$$\text{s.t.} \sum_{k=1}^{K} \int_0^1 \underbrace{P_{r.k}(x)\, dx}_{C_{r.k}(x)} \leq I_{ia}$$

$$U_{i.r} = C_{r.i} \cdot A_r \cdot V_{i.r}$$

$$Q_{r.k}(x) = \varphi_{r.k}(x) \cdot \left(\frac{H_{r.k}(x)}{1-\gamma_{rk}} \right)^{1-\gamma_{rk}} \prod_{\cancel{k}\, s=1}^{K} \left(\frac{M_{r.s.k}(x)}{\gamma_{r.s.k}} \right)^{\gamma_{r.s.k}} \quad (1).$$

$$\left(\sum_{s=1}^{K} \gamma_{r.s.k} = \gamma_{r.k} \right)$$

$$F_{r.k}(\varphi) = \exp\left[-\left(\frac{\varphi}{\varphi_{r.k}} \right)^{-\theta_k} \right].$$

$$a = 0 \rightarrow B_k = \frac{1}{1-b_k}$$

$$\left(1 + \frac{1}{B_k}\right) w_{rk} = \left(1 + \frac{(1-b_k)}{\cancel{1-b_k}}\right) w_{rk} = (2-b_r) w_{rk}.$$

$$C_{rk} = (1-b_k) w_{rk} \leftarrow \text{hiring cost.}$$

given

$$E_{rk}(x) = p_{rk}(x)\, Q_{rk}(x),$$

$$r_{rk}(x) = \frac{\partial E_{rk}(x)}{\partial H_{rk}(x)}.$$

$$= p_{rk}(x) \frac{\partial Q_{rk}(x)}{\partial H_{rk}(x)}$$

$$= p_{rk}(x) \cdot \varphi_{rk}(x) \cdot H_{rk}(x)^{-\gamma_{rk}} \prod_{k=1}^{K} \left(\frac{M_{rsk}(x)}{\gamma_{rsk}} \right)^{\gamma_{rsk}}.$$

$$\pi_{r.k}(x) = p_{r.k}(x) Q_{r.k}(x) - [w_{r.k}(x) + C_{r.k}] H_{r.k}(x) - \sum_{s=1}^{K} p_{r.k} \cdot M_{r.s.k}(x).$$

The FOC: $\dfrac{\partial \pi_{r.k}(x)}{\partial H_{r.k}(x)} = 0.$

$$\therefore \frac{\partial p_{r.k}(x) Q_{r.k}(x)}{\partial H_{r.k}(x)} \equiv r_{r.k}(x) = w_{r.k}(x) + C_{r.k}.$$

Nash bargaining:

$$\max_{w} [r_{ck}(x) - w]^{1/2} [w - b_{rk}]^{1/2}.$$

$$\max_{w} \frac{1}{2} \log (r_{rk}(x) - w) + \frac{1}{2} \log (w - b_{rk}) \equiv F(w).$$

$$\text{FOC}: \frac{\partial F(w)}{\partial w} = \frac{1}{2} \frac{(-1)}{r_{r.k}(x) - w} + \frac{1}{2} \frac{1}{w - b_{r.k}} = 0.$$

$$w - b_{r.k} = \underbrace{r_{r.k}(x) - w}_{C_{r.k}}$$

$$\cancel{2w = r_{r.}}$$

$$w = b_{r.k} + C_{r.k}. \qquad w = b_r w_{r.k} + C_{r.k} \qquad C_{r.k} = (1 - b_r) w_{r.k}.$$

$$\text{Equilibrium}: \underset{1}{(\lambda,} \underset{N\times K}{w_{r.k},} \underset{N}{L_r,} \underset{N\times K}{p_{r.k},} \underset{N\times K}{\frac{L_{r.k}}{L_r}}, \underset{N\times K}{l_{r.k}}.$$

C^k_r ✓ $N\times K$

✓ $5NK + N + 1$ ✓ $2N$?

$$C^k_r = \left[\left(1 + \frac{1}{B_r}\right) w_{rk}\right]^{1-\gamma_{rk}} \prod_{k=1}^{K} (p_{rk})^{\gamma_{rsk}}. \qquad ① \quad N\times K -$$

$$\frac{w_{rk}}{p_r} = B_r \left[\frac{(l_{rk})^{\lambda}}{\mu_{rk}}\right]^{\frac{1}{1-\lambda}}. \qquad ② \quad N\times K -$$

$\parallel$ $\omega_{r.k}$. $\quad B_r = \dfrac{1}{1 - b_r}$ $\qquad \mu_{rk} \equiv \dfrac{\mu_{rk}}{(v_{rk})^{1-\lambda}}$

vacancy cost in terms of goods.

$$L_r = \sum_{k=1}^{K} L_{rk} \qquad (3) \; . \; N$$

$$\tilde{\omega}_r = \frac{\widetilde{\omega_{r.k}}}{\hat{P}_{r.k}} = \frac{\omega_{r.k}\,\beta_{r.k}}{\hat{P}_{r.k}}, \qquad (4) \qquad NK$$

where $\beta_{r.k} = l_{r.k} + (1 - l_{r.k})\, b_r$

$E_{ij.k} = \pi_{ij.k} E_{jk}$, &

$$\pi_{ij.k} = \left[\frac{\tau_{ijk} \frac{C_{ik}}{T_{ik}}}{\left[\sum_{i'=1}^{N} \left(\frac{\tau_{i'jk} C_{i'k}}{T_{i'k}} \right)^{-\theta_k} \right]^{-\frac{1}{\theta_k}}} \right]^{-\theta_k}.$$

$\cancel{(4)}$ (5) $N \times N \times K$

$$= \frac{T_{ik}^{\theta_k} (\tau_{ijk} C_{ik})^{-\theta_k}}{\sum_{n=1}^{N} T_{nk}^{\theta_k} (C_{nk} \tau_{njk})^{-\theta_k}}.$$

$$E_{rk} = \alpha_k I_r + \alpha_k \sum_{k=1}^{K} \mathbb{P}_r \cdot v_{r.k} \cdot V_{r.k} + \sum_{k=1}^{K} \gamma_{rks} \sum_{j=1}^{N} \pi_{rjs} E_{j.s}.$$

$$l_{r.k} = \frac{H_{r.k}}{L_{r.k}} = \frac{\tilde{\mu}_{r.k} V_{r.k}^{1-\lambda} L_{r.k}^{\lambda}}{L_{r.k}} = \tilde{\mu}_{r.k} \left(\frac{V_{r.k}}{L_{r.k}} \right)^{1-\lambda}.$$

$$l_{r.k}^{\frac{1}{1-\lambda}} = \tilde{\mu}_{r.k}^{\frac{1}{1-\lambda}} \cdot \frac{V_{r.k}}{L_{r.k}}$$

$$\therefore \; V_{r.k} = \left(\frac{l_{r.k}}{\tilde{\mu}_{r.k}} \right)^{\frac{1}{1-\lambda}} \cdot \frac{1}{L_{r.k}}.$$

$$\therefore \; E_{rk} = \alpha_k I_r + \alpha_k \sum_{k=1}^{K} \mathbb{P}_r \cdot v_{r.k} \cdot \underbrace{\left[\left(\frac{l_{r.k}}{\tilde{\mu}_{r.k}} \right)^{\frac{1}{1-\lambda}} \cdot \frac{1}{L_{r.k}} \right]}_{= V_{r.k}} \qquad NK \qquad \cancel{(5)}\ (6)$$

$$+ \sum_{k=1}^{K} \gamma_{r.k.s} \sum_{j=1}^{N} \pi_{rj}^{s} E_j^{s}.$$

$$I_r = \left(1 + \frac{1}{\beta_r} \right) \sum_{k=1}^{K} w_{r.k} H_{r.k}.$$

$$0 = \sum_{j=1}^{N} \sum_{k=1}^{K} \pi_{rj}^{k} E_j^{k} - \sum_{i=1}^{N} \sum_{k=1}^{K} \pi_{ir}^{k} E_r^{k} .$$ ~~(6)~~ (7) N ✓

$$0 = \nu \sum_{r=1}^{N} \sum_{k=1}^{K} H_{rk} w_{rk} - \sum_{r=1}^{N} \sum_{k=1}^{K} (1-\nu) b_r w_{rk} \left(L_{rk} - \overset{\text{"} L_{rk} \cdot l_{rk} \text{"}}{H_{rk}}\right)$$ ~~(7)~~ (8) 1 ✓

Recall:

$$\tilde{\omega}_r = \frac{\omega_{r.k} \beta_{r.k}}{f_{r.k}} = \frac{\frac{w_{r.k}}{\mathbb{P}_r} \cdot \left[l_{r.k} + (1-l_{r.k}) b_r \right]}{f_{r.k}} .$$

$$\frac{L_r}{L} = \frac{\frac{\beta_r}{\cancel{A_r}} \tilde{\omega}_r^{\varepsilon}}{\sum_{n=1}^{N} \frac{\cancel{A_n}}{\beta_n} \tilde{\omega}_n^{\varepsilon}} .$$ (9) . N ✓

$$\mathbb{P}_{rk} = \text{"}T_{j.k}\text{"} \cdot \left[\Gamma\left(1 - \frac{1-\sigma_k}{\theta_k}\right) \right]^{\frac{1}{1-\sigma_k}}$$

in Carrere, Grujovic & Robert-Nicoud.

$$= \left[\sum_{i=1}^{N} \left(\tau_{ijk} \frac{c_{ik}}{T_{ik}} \right)^{-\theta_k} \right]^{-\frac{1}{\theta_k}} \cdot \cancel{A_k} \cdot \overbrace{\left[\Gamma\left(1 - \frac{1-\sigma_k}{\theta_k}\right) \right]^{\frac{1}{1-\sigma_k}}}^{A^k} .$$ (10) NK –

Endogenous variables:

C_r^k (NK), w_r^k (NK), $\mathbb{P}_r^k$ (NK), l_r^k (NK), π_{ij}^k (N×N×K), E_j^k (NK), L_r^k (NK), I_r (N), L_r (N), ν (1).

$$\mathbb{P}_r^k \to \mathbb{P}_r \to \omega_{r.k}, \quad w_r^k \to \omega_{r.k}, \quad \omega_{r.k} \to \tilde{\omega}_r, \quad l_r^k \to \tilde{\omega}_r$$

- I've written a model that comprises
  - internal geography
  - unemployment.
- I haven't simulated an equilibrium.
- Regional variation in unemployment is largely higher than time variation.

- I've found a quantitative macro model of M&A.
  → trying to extend it to a many-country setting
  → but it's difficult, especially trade costs.

・モデルを書く.
・イートンにメール書く.
・コード書く.
(・employment share. x inequality.
(・labor union, minimum wage.

✓ M&A
・trade cost と productivity のかけ算を "真 productivity" として使う.
"destination-specific productivity."

- $\log(u_{it}) = \log(U_{it}) - \log(L_{it})$.

$$C_r^k = P_r v_r^k \frac{V_r^k}{H_r^k} \qquad H_r^k = \widetilde{\mu}_r^k (V_r^k)^{1-\lambda} (L_r^k)^{\lambda}.$$

$$= P_r v_r^k \frac{V_r^k}{\widetilde{\mu}_r^k (V_r^k)^{1-\lambda} (L_r^k)^{\lambda}} \qquad \ell_r^k = \frac{H_r^k}{L_r^k}.$$

$$\frac{H_r^k}{L_r^k} = \widetilde{\mu}_r^k (V_r^k)^{1-\lambda} (L_r^k)^{\lambda-1}$$

$$\therefore \frac{1}{\widetilde{\mu}_r^k} \cdot \ell_r^k \cdot (L_r^k)^{1-\lambda} = (V_r^k)^{1-\lambda}.$$

$$V_r^k = L_r^k \cdot (\ell_r^k)^{\frac{1}{1-\lambda}} \cdot \left(\frac{1}{\widetilde{\mu}_r^k}\right)^{\frac{1}{1-\lambda}}.$$

$$C_r^k = P_r V_r^k \, \frac{L_r^k \cdot (a_r^k)^{\frac{1}{1-\lambda}} \cdot \left(\frac{1}{\hat{a}_r^k}\right)^{\frac{1}{1-\lambda}}}{H_r^k}$$

$$= P_r V_r^k \cdot \left(\frac{H_r^k}{L_r^k}\right)^{-1} \cdot (a_r^k)^{\frac{1}{1-\lambda}} \cdot (\hat{a}_r^k)^{-\frac{1}{1-\lambda}}.$$

イートン大体 Munday.

$$= P_r V_r^k \cdot (a_r^k)^{-1} \cdot (a_r^k)^{\frac{1}{1-\lambda}} \cdot (\hat{a}_r^k)^{-\frac{1}{1-\lambda}}$$

$$= P_r V_r^k \cdot (a_r^k)^{\frac{1-(1-\lambda)}{1-\lambda}} \cdot (\hat{a}_r^k)^{-\frac{1}{1-\lambda}}$$

$$= P_r V_r^k \cdot (\hat{a}_r^k)^{-\frac{1}{1-\lambda}} \cdot (a_r^k)^{\frac{\lambda}{1-\lambda}}.$$

---

(2): $$w_{r,k} = P_r \cdot \left(\frac{1}{1-b_r}\right) \cdot \left[\frac{(a_{r,k})^\lambda}{\mu_{r,k}}\right]^{\frac{1}{1-\lambda}}.$$

$$= \left[\prod_{k=1}^{K} (p_r^k)^k\right] \cdot \left(\frac{1}{1-b_r}\right) \cdot \left[\frac{(a_r^k)^\lambda}{\mu_{r,k}}\right]^{\frac{1}{1-\lambda}}.$$

(10): $$p_{r,k} = A^k \cdot \left[\sum_{i=1}^{N} \left(\tau_{ij}^k \cdot \frac{c_i^k}{T_i^k}\right)^{-\theta_k}\right]^{-\frac{1}{\theta_k}}.$$

(1): $$c_r^k = \left[\left(1+\frac{1}{\beta_r}\right) w_r^k\right]^{1-\gamma_r^k} \prod_{k=1}^{K} (p_r^k)^{\gamma_r^{s,k}}.$$

Given (w_r^k), iterate (11) & (1) $\Rightarrow$ p_r^k & c_r^k
(as a function of w_r^k).

$w_{r,k}$ & $P_r \to \tilde{w}_r$: (4)

$c_r^k \to \begin{cases} \pi_{i,j,k} : (5) \\ E_{i,j}^k. \end{cases}$ Need E_{ij}^k.

(6): $$E_r^k = \alpha_k I_r + \alpha_k \sum_{k=1}^{K} P_r V_r^k \cdot \left[\left(\frac{l_r^k}{\hat{a}_r^k}\right)^{\frac{1}{1-\lambda}} \cdot \frac{1}{L_{r,k}}\right] + \sum_{k=1}^{K} \gamma_r^{k,s} \sum_{j=1}^{N} \pi_{r,j}^s E_j^s,$$

& $$I_r = \left(1+\frac{1}{\beta_r}\right) \sum_{k=1}^{K} w_r^k l_r^k L_r^k.$$

⑤ → ⑦.

$$⑥\ \nu \sum_{r=1}^{N} \sum_{k=1}^{K} \cancel{\ } l_r^k L_r^k w_{rk} - \sum_{r=1}^{N} \sum_{k=1}^{K} (1-\nu)\, b_r w_r^k \left(L_r^k - L_r^k l_r^k \right) = 0.$$

$$\therefore \nu \sum_{r=1}^{N} \sum_{k=1}^{K} l_r^k L_r^k w_r^k = (1-\nu) \sum_{r=1}^{N} \sum_{k=1}^{K} b_r w_r^k (1-l_r^k) L_r^k.$$

$$\therefore \nu \left[\sum_{r=1}^{N} \sum_{k=1}^{K} l_r^k L_r^k w_r^k + \sum_{r=1}^{N} \sum_{k=1}^{K} b_r w_r^k (1-l_r^k) L_r^k \right]$$

$$= \sum_{r=1}^{N} \sum_{k=1}^{K} b_r w_r^k (1-l_r^k) L_r^k$$

$$\nu = \frac{\sum_{r=1}^{N} \sum_{k=1}^{K} b_r w_r^k (1-l_r^k) L_r^k}{\sum_{r=1}^{N} \sum_{k=1}^{K} l_r^k L_r^k w_r^k + \sum_{r=1}^{N} \sum_{k=1}^{K} b_r w_r^k (1-l_r^k) L_r^k}.$$

$$\tilde{\omega}_r = \frac{\omega_{rk}^k \beta_r^k}{f_{r.k}} = \frac{\frac{w_r^k}{P_r} \left[l_{r.k} + (1-l_r^k) b_r \right]}{f_{r.k}}. \quad ⑨$$

$$\& \ \frac{\mu L_r}{L} = \frac{\beta_r \tilde{\omega}_r^{\varepsilon}}{\sum_{n=1}^{N} \beta_n \tilde{\omega}_n^{-\varepsilon}}.$$

$$\tilde{\omega}_r P_r f_r^k = w_r^k \left[l_{r.k} + (1-l_r^k) b_r \right].$$

– Guess w_r^k, $\tilde{\omega}_r$, P_r, E_r^k.

$$\frac{\tilde{\omega}_r P_r f_r^k}{w_r} = (1-b_r) l_r^k + b_r.$$

$$\therefore (1-b_r) l_r^k = \frac{\tilde{\omega}_r P_r f_r^k}{w_r} - b_r$$

$$\therefore l_r^k = \frac{1}{1-b_r} \left[\frac{\tilde{\omega}_r P_r f_r^k}{w_r} - b_r \right].$$

→ iterate $P_{r.k}$ ⑩ & C_r^k ① → P_r^k & C_r^k.

$P_r^k \rightarrow$ New P_r.

→ π_{ij}^k.

→ (6). New E_r^k.

$$E_r^k = \alpha_k \left(1 + \frac{1}{B_r}\right) \sum_{k=1}^{K} w_r^k l_r^k L_r^k$$
$$+ \alpha_k \sum_{s=1}^{K} P_r \cdot \nu_r^s \cdot \left[\left(\frac{l_r^s}{\pi_r^s} \right)^{\frac{1}{1-\lambda}} \cdot \frac{1}{L_r^s} \right]$$
$$+ \sum_{s=1}^{K} \gamma_r^{k,s} \sum_{j=1}^{N} \pi_{r,j}^s E_j^s .$$

(suppose that I can solve this for E_r^k s exactly.

(7) $$\text{Surplus} = 0 = \sum_{j=1}^{N} \sum_{k=1}^{K} \pi_{rj}^k E_j^k - \sum_{i=1}^{N} \sum_{k=1}^{K} \pi_{ir}^k E_r^k .$$

$$\sum_{t=0}^{\infty} \beta^t \log C_{i,t} + \nu_{i,t}^{(r,k),(r',k')}.$$

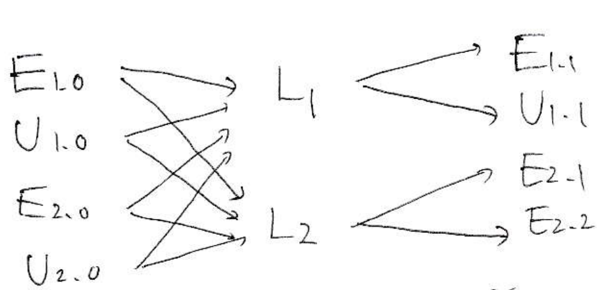
$$\cancel{V_{i,t}^{(r,k)} = \log C_{i,t}}$$

$$\cancel{V_{i,t}^{(e,r,k)} = \log (}$$

$$V_{i,t}^{(e,r)} = \log\left(\frac{w_r}{P_r}\right) + \beta \left\{ E V_{i,t+1} \right.$$

---

period 0:
$(E_{j,0})_{j=1}^{N}$, $(U_{j,0})_{j=1}^{N}$.



$$E_{1.0} \xrightarrow{1-\chi} E_{1.1}$$
$$E_{1.0} \xrightarrow{\chi} L_{1.1}$$
$$L_{1.1} \xrightarrow{e_{1.1}} E_{1.1}$$
$$L_{1.1} \xrightarrow{1-e_{1.1}} U_{1.1}.$$

$$v_0^{1,e} = U\left(\frac{w_0^1}{p_0^1}\right) + \max\left\{(1-\chi)\, v_1^{1,e} + \chi\, v_1^{1,u},\ \max_{j\in\{1,\dots,N\}\setminus\{1\}} E\, v_1^{j,L}\right\}.$$

$$\xcancel{E_{j,1.1} = (1-\chi)\, E_{1.0} + (L_{1.1})^{\mu} (V_{1.1})^{1-\mu}.}$$

$$v_t^{j,e} = U\left(\frac{w_t^j}{p_t^j}\right)$$
$$+\max\left\{(1-\chi)\, E\, v_{t+1}^{j,e} + \chi\, E\, v_{t+1}^{j,u} \underbrace{}_{-\tau^{jj}+\nu\varepsilon_t^{jj}},\ \max_{n\in\{1,\dots,N\}\setminus\{j\}} e_{t+1}^{n}\, E\, v_{t+1}^{n,e} + (1-e_{t+1}^{n})\, E\, v_{t+1}^{n,u} - \tau^{j,n} + \nu\varepsilon_t^{j,n}\right\}.$$

$-\tau^{j,n} + \nu\varepsilon_t^{j,n}$

$$v_t^{j,u} = U(b_t^j)$$
$$+\max_{n\in\{1,\dots,N\}}\left\{e_{t+1}^{n}\, v_{t+1}^{n,e} + (1-e_{t+1}^{n})\, v_{t+1}^{n,u} - \tau^{j,n}\right\}.$$

$$\mu_t^{j,n\neq j|e} = \frac{\exp\left(e_{t+1}^{n} V_{t+1}^{n,e} + (1-e_{t+1}^{n}) V_{t+1}^{n,u} - \tau^{j,n}\right)^{1/\nu}}{\exp\left(\chi V_{t+1}^{j,e} + (1-\chi) V_{t+1}^{j,u} - \tau^{jj}\right)^{1/\nu} + \sum_{n\neq j} \exp\left(e_{t+1}^{n} V_{t+1}^{n,e} + (1-e_{t+1}^{n}) V_{t+1}^{n,u} - \tau^{j,n}\right)^{1/\nu}}$$

$$\mu_t^{jj|e} = \frac{\exp\left(\chi v_{t+1}^{j,e} + (1-\chi) v_{t+1}^{j,u} - \tau^{jj}\right)^{1/\nu}}{\exp\left(\chi v_{t+1}^{j,e} + (1-\chi) v_{t+1}^{j,u} - \tau^{jj}\right)^{1/\nu} + \sum_{n\neq j} \exp\left(e_{t+1}^{n} v_{t+1}^{n,e} + (1-e_{t+1}^{n}) v_{t+1}^{n,u} - \tau^{j,n}\right)^{1/\nu}}.$$

$$E_{t+1}^{j} = \chi\, \mu_t^{jj|e} E_t^{j} + \underbrace{m^j (N_{t+1}^{j})^{\zeta} (V_{t+1}^{j})^{1-\zeta}}_{M_{t+1}^{j}}.$$

$$U_{t+1}^{j} = L_{t+1}^{j} - E_{t+1}^{j}$$
$$= (1-\chi)\, \mu_t^{jj|e} E_t^{j} + (N_{t+1}^{j} - M_{t+1}^{j}).$$

$$L_{t+1}^{j} = \cancel{\mu_t^{j,j|e} E_t^{j}} \sum_{n=1}^{N} \mu_t^{n,j|e} E_t^{n} + \sum_{n=1}^{N} \mu_t^{n,j|u} U_t^{n}.$$

$$\mu_t^{j,n|u} = \frac{\exp\left(e_{t+1}^{n} V_{t+1}^{n,e} + (1-e_{t+1}^{n}) V_{t+1}^{n,u} - \tau^{j,n}\right)^{1/\nu}}{\sum_{\ell} \exp\left(e_{t+1}^{\ell} V_{t+1}^{\ell,e} + (1-e_{t+1}^{\ell}) V_{t+1}^{\ell,u} - \tau^{j,\ell}\right)^{1/\nu}}.$$

for any n.

$$N_{t+1}^{j} = \sum_{n \neq j} \mu_t^{n,j|e} E_t^{n} + \sum_{n} \mu_t^{n,j|u} U_t^{n}$$

$$\boxed{\text{Asp}: \; \chi > e_t^{j} \quad \text{for any } t, j.}$$

$$\begin{cases} V_t^{j,e} = U\left(\frac{w_t^{j}}{P_t^{j}}\right) + \nu \log\left[\exp\left((1-\chi) V_{t+1}^{j,e} + \chi V_{t+1}^{j,u}\right)^{1/\nu} + \sum_{n \neq j} \exp\left(e_{t+1}^{n} V_{t+1}^{n,e} + (1-e_{t+1}^{n}) V_{t+1}^{n,u}\right)^{1/\nu}\right] \\ V_t^{j,u} = U(b_t^{j}) + \nu \log\left[\sum_{n} \exp\left(e_{t+1}^{n} V_{t+1}^{n,e} + (1-e_{t+1}^{n}) V_{t+1}^{n,u}\right)^{1/\nu}\right] \end{cases}$$

Static production.

$$q_t^{j} = T_t^{j} z^{j} \left(\frac{E_t^{j}}{1-\alpha}\right)^{\alpha^{j}} \left(\frac{M_t^{j}}{\alpha}\right)^{1-\alpha} + \Delta_t^{j}.$$

unit price

$$x_t^{j} = (w_t^{j})^{\alpha} (P_t^{j})^{1-\alpha} + \Delta_t^{j}.$$

unit cost: $$c_t^{j} = \frac{(w_t^{j})^{\alpha} (P_t^{j})^{1-\alpha}}{T_t^{j} z^{j}}.$$

$\exp\{-(z^n)^{-\theta}\} \to 1$

$z^n \to \infty$

$$p_t^{n}(z) = \min_{j} \left\{ \kappa^{jn} c_t^{jn} \right\}$$

$$Q_t^{n} = \left(\int_{z \in \mathbb{R}_+} \tilde{q}_t^{n}(z)^{1-1/\eta} \, d\phi(z)\right)^{\eta/(\eta-1)},$$

$$P_t^{n} = \Gamma^{n} \left(\sum_{j=1}^{N} (x_t^{j} \kappa_t^{jn})^{-\theta} (T_t^{j})^{\theta}\right)^{-1/\theta}.$$

joint distribution

$$\phi(z) = \exp\left\{-\sum_{n=1}^{N} (z^n)^{-\theta}\right\}.$$

$$\phi(z^n) = \exp\{-(z^n)^{-\theta}\}$$

marginal dist^{n}.

$$\pi_t^{nj} = \frac{(x_t^{n} \kappa_t^{nj})^{-\theta} (T_t^{n})^{\theta}}{\sum_{m=1}^{N} (x_t^{m} \kappa_t^{mj})^{-\theta} (T_t^{m})^{\theta}}.$$

$$X_t^{nj} = (1-\alpha) \sum_{m=1}^{N} \pi_t^{jm} X_t^{m} + \cancel{w^n E_t^n \left(1 + b_t^n P_t^n U_t^n\right)}.$$

Version with a constant replacement rate & a fixed federal labor income tax.

$$v_t^{j,e} = U\left((1-\tau)\frac{w_t^j}{P_t^j}\right)$$

$$+ \max\left\{(1-\chi) E v_{t+1}^{j,e} + \chi E v_{t+1}^{j,u} - \tau^{jj} + \nu \varepsilon_t^{jj}, \right.$$

$-\tau^{j,n} + \nu \varepsilon_t^{j,n}$

$$\left. \max_{n \in \{1,\dots,N\} \setminus \{j\}} \left\{ e_{t+1}^n E v_{t+1}^{n,e} + (1-e_{t+1}^n) E v_{t+1}^{n,u} \right\} \right\}$$

$$v_t^{j,u} = U\left(\frac{b^j w_t^j}{P_t^j}(1-\tau)\right)$$

$$+ \max_n \left\{ e_{t+1}^n E v_{t+1}^{n,e} + (1-e_{t+1}^n) E v_{t+1}^{n,u} - \tau^{j,n} + \nu \varepsilon_t^{j,n} \right\}.$$

$-\tau^{jj}$

$$\Rightarrow V_t^{j,e} = U\left((1-\tau)\frac{w_t^j}{P_t^j}\right) + \nu \log\left[\exp\left((1-\chi)V_{t+1}^{j,e} + \chi V_{t+1}^{j,u}\right)^{1/\nu}\right.$$

$$\left. + \sum_{n \neq j} \exp\left(e_{t+1}^n V_{t+1}^{n,e} + (1-e_{t+1}^n) V_{t+1}^{n,u}\right)^{1/\nu}\right] \quad -\tau^{j,n}$$

$-\tau^{j,n}$.

$$V_t^{j,u} = U\left((1-\tau)\frac{b^j w_t^j}{P_t^j}\right) + \nu \log\left[\sum_{n=1}^{N} \exp\left(e_{t+1}^n V_{t+1}^{n,e} + (1-e_{t+1}^n) V_{t+1}^{n,u}\right)^{1/\nu}\right].$$

$$\mu_t^{j,n|e} = \frac{\exp\left(e_{t+1}^n V_{t+1}^{n,e} + (1-e_{t+1}^n) V_{t+1}^{n,u} - \tau^{j,n}\right)^{1/\nu}}{\exp\left((1-\chi) V_{t+1}^{j,e} + \chi V_{t+1}^{j,u} - \tau^{jj}\right)^{1/\nu} + \sum_{n \neq j} \exp\left(e_{t+1}^n V_{t+1}^{n,e} + (1-e_{t+1}^n) V_{t+1}^{n,u} - \tau^{j,n}\right)^{1/\nu}}.$$

$$\mu_t^{jj|e} = \frac{\exp\left((1-\chi) V_{t+1}^{j,e} + \chi V_{t+1}^{j,u} - \tau^{jj}\right)^{1/\nu}}{\exp\left((1-\chi) V_{t+1}^{j,e} + \chi V_{t+1}^{j,u} - \tau^{jj}\right)^{1/\nu} + \sum_{n \neq j} \exp\left(e_{t+1}^n V_{t+1}^{n,e} + (1-e_{t+1}^n) V_{t+1}^{n,u} - \tau^{j,n}\right)^{1/\nu}}.$$

$$\mu_t^{j,n|u} = \frac{\exp\left(e_{t+1}^n V_{t+1}^{n,e} + (1-e_{t+1}^n) V_{t+1}^{n,u} - \tau^{j,n}\right)^{1/\nu}}{\sum_m \exp\left(e_{t+1}^m V_{t+1}^{m,e} + (1-e_{t+1}^m) V_{t+1}^{m,u} - \tau^{j,m}\right)^{1/\nu}}.$$

$$\cancel{\theta}_{t+1}^n = \frac{M_{t+1}^n}{N_{t+1}^n} = \cancel{m}\,\Xi^n \left(\frac{V_{t+1}^n}{N_{t+1}^n}\right)^{1-\xi}$$

$$\lambda_{t+1}^n = \frac{V_{t+1}^n}{N_{t+1}^n} \text{ : labor market tightness.}$$

$$= \Xi^n \left(\overset{\lambda}{\cancel{\theta}}{}_{t+1}^n\right)^{1-\xi}.$$

$$e_{t+1}^n = \frac{E_{t+1}^n}{L_{t+1}^n} = \frac{\chi \mu_t^{n,n|e} E_t^n + M_{t+1}^n}{\sum_m \mu_t^{m,n|e} E_t^m + \sum_m \mu_t^{m,n|u} U_t^m}.$$

@ Production & Labor market friction.

$$C_t^{*n} = (\underbrace{w_t^n}_{+\Delta_t^n})^\beta (P_t^n)^{1-\beta}.$$

$$\Delta_t^n \cdot \underbrace{E_t^n}_{\cancel{\text{predetermin}}} = \nu^n \cdot \underbrace{V_t^n}_{P_t^n}$$

predetermined.

$$= \nu^n \cdot P_{t*}^n \cdot \overset{\lambda}{\cancel{\theta}}{}_t^n \cdot N_t^n.$$

Think period t.

Predetermined variables in period $t-1$:

$\mu_t^{j,n|e}$, $\mu_t^{j,n|u}$, L_t^j, N_t^j.

Endogenous variables:

ν_t, X_t^{nj}, V_t^j, E_t^j, U_t^j, w_t^j, P_t^j, (Δ_t^j), $\overset{\lambda}{\cancel{\theta}}{}_t^j$, X_t^j, (m_t^j)

(π_t^{nj}) $\cancel{(I_t^j)}$

	X_t^{nj}	K	E	U	w	P	λ
1	$N\times N$	N	N	N	N	N.	N

$\cancel{5}\,6N + N^2 + 1.$

$$W_t^r = \overbrace{b^r \cdot w_t^r}^{b_t^r} + \Delta_t^r.$$

$$\Delta_t^r = (1-b^r) w_t^r.$$

Then the total labor cost is:

$$w_t^r + \Delta_t^r = (2-b^r) w_t^r.$$

$\cancel{C_t^r}$ $$q_t^r(\omega) = z_t^r(\omega) \cdot \left(\frac{E_t^r(\omega)}{1-\beta}\right)^{\beta} \left(\frac{M_t^r(\omega)}{\beta}\right)^{1-\beta}. \qquad z_t^r(\omega) \sim \text{Frechet}(T_t^r, \theta)$$

$$C_t^r = T_t^r \cdot (w_t^r + \Delta_t^r)^{\beta} (P_t^r)^{1-\beta}.$$

$$\pi_t^{nj} = \frac{(T_t^n)^{\theta} \left[(w_t^n + \Delta_t^n)^{\beta} (P_t^n)^{1-\beta} \cdot d^{nj}\right]^{-\theta}}{\sum_m (T_t^m)^{\theta} \left[(w_t^m + \Delta_t^m)^{\beta} (P_t^m)^{1-\beta} d^{mj}\right]^{-\theta}}$$

$$X_t^{j} = I_t^j + \frac{(1-\beta)}{\beta} \left[w_t^j \cdot E_t^j + \Delta_t^j E_t^j\right]$$

$$\left(= (w_t^j E_t^j + b^j w_t^j U_t^j) + \frac{1-\beta}{\beta} \left[w_t^j E_t^j + \nu^n P_t^n \theta_t^n \cdot N_t^n\right]. \right)$$

$$= (w_t^j E_t^j + b^j w_t^j U_t^j) + \frac{1-\beta}{\beta} (2-b^j) w_t^j E_t^j. \quad \leftarrow \text{balanced trade.} \qquad ① \; N$$

$$X_t^{nj} = \pi_t^{nj} X_t^j. \qquad \pi_t^{nj} = \frac{X_t^{nj}}{X_t^j}. \qquad ② \; N \times N$$

$$P_t^n = \Gamma\left(1 + \frac{1-\sigma}{\theta}\right)^{\frac{1}{1-\sigma}} \cdot \left(\sum_{j=1}^N (T_t^j)^{\theta} \left[((2-b^j) w_t^j)^{\beta} (P_t^j)^{1-\beta} d^{jn}\right]^{-\theta}\right)^{-1/\theta}. \qquad ③ \; N$$

$$\nu_t \sum_{n=1}^N \left[w_t^n E_t^n + b^r w_t^n U_t^n\right] = \sum_{n=1}^N b^r w_t^n U_t^n.$$

$$\nu_t = \frac{\sum_{n=1}^N b^r w_t^n U_t^n}{\sum_{n=1}^N \left[w_t^n E_t^n + b^r w_t^n U_t^n\right]} \qquad ④ \; 1$$

$$m_t^j = \Xi^j \cdot (\lambda_t^n)^{1-\xi}$$

$$\equiv \frac{M_t^j}{N_t^j}.$$

$$\boxed{\lambda_t^n = \frac{V_t^n}{N_t^n}}$$

$$E_t^j = \chi \mu_{t-1}^{j,j,le} E_{t-1}^j + M_t^j$$

$$= \chi \mu_{t-1}^{j,j,le} E_{t-1}^j + N_t^j m_t^j. \quad (5) \sim$$

$$U_t^j = L_t^j - E_t^j. \quad (7) \sim$$

$$\Delta_t^j \cdot E_t^j = \nu^j P_t^j V_t^j$$

$$\therefore (1-b^j) w_t^j \cdot E_t^j = \nu^j P_t^j V_t^j \quad (6) \sim$$

$$= \nu^j P_t^j N_t^j \lambda_t^j.$$

$$\underbrace{\cancel{w_t^j}}_{(2-b^j) w_t^j} E_t^j = \beta \cdot \sum_{n=1}^{N} \pi_{jn} X_n. \quad (8)$$

Given $\{\mu_{t-1}^{j,nle}, \mu_{t-1}^{j,nlu}, L_t^j, N_t^j, E_{t-1}^j, U_{t-1}^j\}$.

Def 1: a temporary equilibrium is ~~E~~ $\{\nu_t, X_t^j, X_t^{nj}, E_t^{nj}, U_t^j, w_t^j, P_t^j, \lambda_t^j\}_{j=1, n=1}^{N, N}$ given θ.

Not necessary

such that

$$X_t^j = (w_t^j E_t^j + b^j w_t^j U_t^j) + \frac{1-\beta}{\beta}(2-b^j) w_t^j E_t^j \quad (1).$$

$$X_t^{nj} = \pi_t^{nj} X_t^j, \quad (2)$$

where

$$\pi_t^{nj} = \frac{(T_t^n)^\theta \left[((2-b^n) w_t^n)^\beta (P_t^n)^{1-\beta} d^{nj} \right]^{-\theta}}{\sum_m (T_t^m)^\theta \left[((2-b^m) w_t^m)^\beta (P_t^m)^{1-\beta} d^{mj} \right]^{-\theta}}$$

$$P_t^j = G \cdot \left(\sum_{n=1}^{N} (T_t^n)^\theta \left[((2-b^n) w_t^n)^\beta (P_t^n)^{1-\beta} d^{nj} \right]^{-\theta} \right)^{-1/\theta}. \quad (3)$$

$$\nu_t = \frac{\sum_{n=1}^{N} b^n w_t^n U_t^n}{\sum_{n=1}^{N} \left[w_t^n E_t^n + b^n w_t^n U_t^n \right]}. \quad (4)$$

$$E_t^j = \chi \mu_{t-1}^{j,j|e} E_{t-1}^j + N_t^j \cdot \Xi^j (\lambda_t^j)^{1-\xi} \quad (5)$$

$$(1-b^{r,j}) w_t^j E_t^j = \nu^j P_t^j N_t^j \lambda_t^j \quad (6)$$

$$U_t^j = L_t^j - E_t^j \quad (7)$$ ← N_t necessary.

$$(2-b^j) w_t^j E_t^j = \beta \sum_{n=1}^{N} \pi_{jn} X_n \quad (8)$$

Def 2: Given $\{L_0^j, E_0^j, U_0^j\}$, a sequential equilibrium is $\{\mu_t^{j,n|e}, \mu_t^{j,n|u}\}_{t=0}^{\infty}$, $\{V_t^{j,e}, V_t^{j,u}\}_{t=1}^{\infty}$, $\{L_t^j, E_t^j, U_t^j, N_t^j\}_{t=1}^{\infty}$, $\{\tau_t, X_t^j, X_t^{nj}, E_t^{nj}, w_t^j, P_t^j, \lambda_t^j\}_{t=1}^{\infty}$ s.t.

$$V_t^{j,e} = U\left((1-\tau_t)\frac{w_t^j}{P_t^j}\right) + \nu \log\Big[\exp\left(\beta\left((1-\chi)V_{t+1}^{j,e} + \chi V_{t+1}^{j,u}\right) - \tau^{jj}\right)^{1/\nu} + \sum_{n\neq j} \exp\Big(\beta\big(\underbrace{\Xi^n(\lambda_t^n)^{1-\xi}}_{m_t^j} V_{t+1}^{n,e} + (1-\Xi^n(\lambda_t^n)^{1-\xi}) V_{t+1}^{n,u}\big) - \tau^{j,n}\Big)^{1/\nu}\Big] \quad (a)$$

$$V_t^{j,u} = U\left((1-\tau_t)\frac{b^j w_t^j}{P_t^j}\right) + \nu \log\Big[\sum_{n=1}^{N} \exp\Big(\beta\big(\underbrace{\Xi^n(\lambda_t^n)^{1-\xi}}_{m_t^n} V_{t+1}^{n,e} + (1-\Xi^n(\lambda_t^n)^{1-\xi}) V_{t+1}^{n,u}\big) - \tau^{j,n}\Big)^{1/\nu}\Big] \quad (b)$$

$$\mu_t^{j,n\neq j|e} = \frac{\exp\left(\beta\left(m_{t+1}^n V_{t+1}^{n,e} + (1-m_{t+1}^n) V_{t+1}^{n,u}\right) - \tau^{j,n}\right)^{1/\nu}}{\exp\left(\beta\left((1-\chi)V_{t+1}^{j,e} + \chi V_{t+1}^{j,u}\right) - \tau^{jj}\right)^{1/\nu} + \sum_{n\neq j}\exp\left(\beta\left(m_{t+1}^n V_{t+1}^{n,e} + (1-m_{t+1}^n) V_{t+1}^{n,u}\right) - \tau^{j,n}\right)^{1/\nu}} \quad (c)$$

$$\mu_t^{j,j|e} = \frac{\exp\left(\beta\left((1-\chi)V_{t+1}^{j,e} + \chi V_{t+1}^{j,u}\right) - \tau^{j,j}\right)^{1/\nu}}{\exp\left(\beta\left((1-\chi)V_{t+1}^{j,e} + \chi V_{t+1}^{j,u}\right) - \tau^{jj}\right)^{1/\nu} + \sum_{n\neq j}\exp\left(\beta\left(m_{t+1}^n V_{t+1}^{n,e} + (1-m_{t+1}^n) V_{t+1}^{n,u}\right) - \tau^{j,n}\right)^{1/\nu}} \quad (d)$$

$$\mu_t^{j,n|u} = \frac{\exp\left(\beta\left(m_{t+1}^n V_{t+1}^{n,e} + (1-m_{t+1}^n) V_{t+1}^{n,u}\right) - \tau^{j,n}\right)^{1/\nu}}{\sum_k \exp\left(\beta\left(m_{t+1}^k V_{t+1}^{k,e} + (1-m_{t+1}^k) V_{t+1}^{k,u}\right) - \tau^{j,k}\right)^{1/\nu}} \quad (e)$$

$$L_t^j = \sum_{n=1}^{N} \mu_{t-1}^{n,j|e} E_{t-1}^n + \sum_{n=1}^{N} \mu_{t-1}^{n,j|u} U_{t-1}^n \quad (f)$$

$$N_t^j = \sum_{n\neq j} \mu_{t-1}^{n,j|e} E_{t-1}^n + \sum_{n=1}^{N} \mu_{t-1}^{n,j|u} U_{t-1}^n = L_t^j - \mu_{t-1}^{j,j|e} E_{t-1}^j \quad (g)$$

Steady state: A stationary sequilibrium is a sequential equilibrium s.t. $\{\mu_t^e, \mu_t^u, V_t^e, V_t^u, L_t, E_t, U_t, N_t, \nu_t, X_t, \pi_t, w_t, P_t, \lambda_t\}$ are constant for all t.

$$X^j = \iota(1-\nu)(w^j E^j + b^j w^j U^j) + \frac{1-\beta}{\beta}(2-b^j) w^j E^j \quad ①$$

$$X^{nj} = \pi^{nj} X^j$$

where

$$\pi^{nj} = \frac{(T^n)^\theta [((2-b^n)w^n)^\beta (P^n)^{1-\beta} d^{nj}]^{-\theta}}{\sum_m (T^m)^\theta [((2-b^m)w^m)^\beta (P^m)^{1-\beta} d^{mj}]^{-\theta}} \quad ②$$

$$P^j = G \cdot \left(\sum_{n=1}^N (T^n)^\theta [((2-b^n)w^n)^\beta (P^n)^{1-\beta} d^{nj}]^{-\theta}\right)^{-1/\theta} \quad ③$$

$$\nu = \frac{\sum_{n=1}^N b^n w^n U^n}{\sum_{n=1}^N [w^n E^n + b^n w^n U^n]} \quad ④$$

$$(1-\chi\mu^{jj|e})E^j = N^j m^j$$

$$E^j = \chi\mu^{jj|e} E^j + N^j \cancel{\Xi^j(\lambda} \; m^j \quad ⑤$$

$$U^j = L^j - E^j \quad ⑥$$

$$m^j = \Xi^j (\lambda^m)^{1-\zeta} \qquad (\lambda^m) = \left(\frac{\Xi^j}{c}\right)^{-\frac{1}{1-\zeta}} (m^j)^{\frac{1}{1-\zeta}} \quad ⑦$$

$$(1-b^j) w^j E^j = \iota^j P^j N^j \lambda^j \quad ⑧$$

$$(2-b^j) w^j E^j = \beta \sum_{n=1}^N \pi^{jn} X_n \quad ⑨$$

$$V^{j,e} = U\left((1-\iota)\frac{w^j}{P^j}\right) + \nu \log\Big[\exp\left(\beta[(1-\chi)V^{j,e} + \chi V^{j,u}] - \tau^{jj}\right)^{1/\nu} + \sum_{n\neq j} \exp\left(\beta[m^j V^n + (1-m^j)V^n] - \tau^{j,n}\right)^{1/\nu}\Big] \quad ⓐ$$

$$V^{j,u} = U\left((1-\iota)\frac{b^j w^j}{P^j}\right) + \nu \log\left[\sum_{n=1}^N \exp\left(\beta(m^n V^{n,e} + (1-m^n)V^{n,u}) - \tau^{j,n}\right)^{1/\nu}\right] \quad ⓑ$$

$$\mu^{j,n\neq j|e} = \frac{\exp\left(\beta\left(m^n V^{n,e} + (1-m^n) V^{n,u}\right) - \tau^{j,n}\right)^{1/\kappa}}{\exp\left(\beta\left((1-\chi)V^{j,e} + \chi V^{j,u}\right) - \tau^{jj}\right)^{1/\kappa} + \sum_{m\neq j} \exp\left[\beta\left(m^m V^{m,e} + (1-m^m) V^{m,u}\right) - \tau^{j,m}\right]^{1/\kappa}} \quad (c)$$

$$\mu^{j,j|e} = \frac{\exp\left[\beta\left((1-\chi)V^{j,e} + \chi V^{j,u}\right) - \tau^{jj}\right]^{1/\kappa}}{\exp\left[\beta\left((1-\chi)V^{j,e} + \chi V^{j,u}\right) - \tau^{jj}\right]^{1/\kappa} + \sum_{n\neq j} \exp\left[\beta\left(m^n V^{n,e} + (1-m^n) V^{n,u}\right) - \tau^{j,n}\right]^{1/\kappa}} \quad (d)$$

$$\mu^{j,n|u} = \frac{\exp\left[\beta\left(m^n V^{n,e} + (1-m^n) V^{n,u}\right) - \tau^{j,n}\right]^{1/\kappa}}{\sum_{k=1}^{N} \exp\left[\beta\left(m^k V^{k,e} + (1-m^k) V^{k,u}\right) - \tau^{j,k}\right]^{1/\kappa}} \quad (e)$$

$$L^j = \sum_{n=1}^{N} \mu^{n,j|e} E^n + \sum_{n=1}^{N} \mu^{n,j|u} U^n \quad (f)$$

$$N^j = \sum_{n\neq j} \mu^{n,j|e} E^n + \sum_{n=1}^{N} \mu^{n,j|u} U^n \quad (g)$$

$$= L^j - \mu^{j,j|e} E^j$$

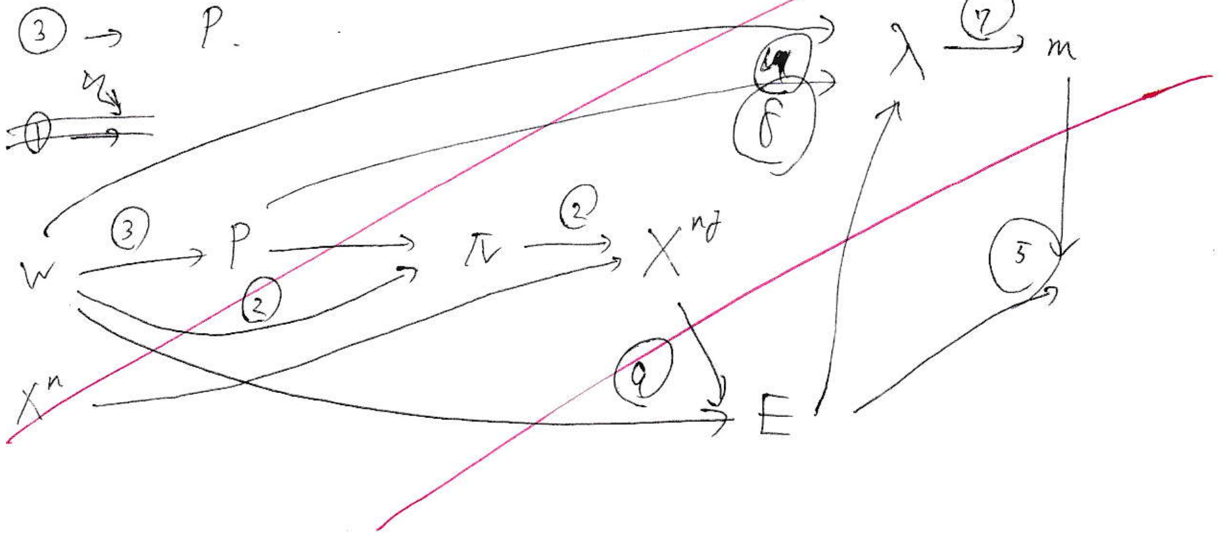
Outer loop $L, N, \mu., V$

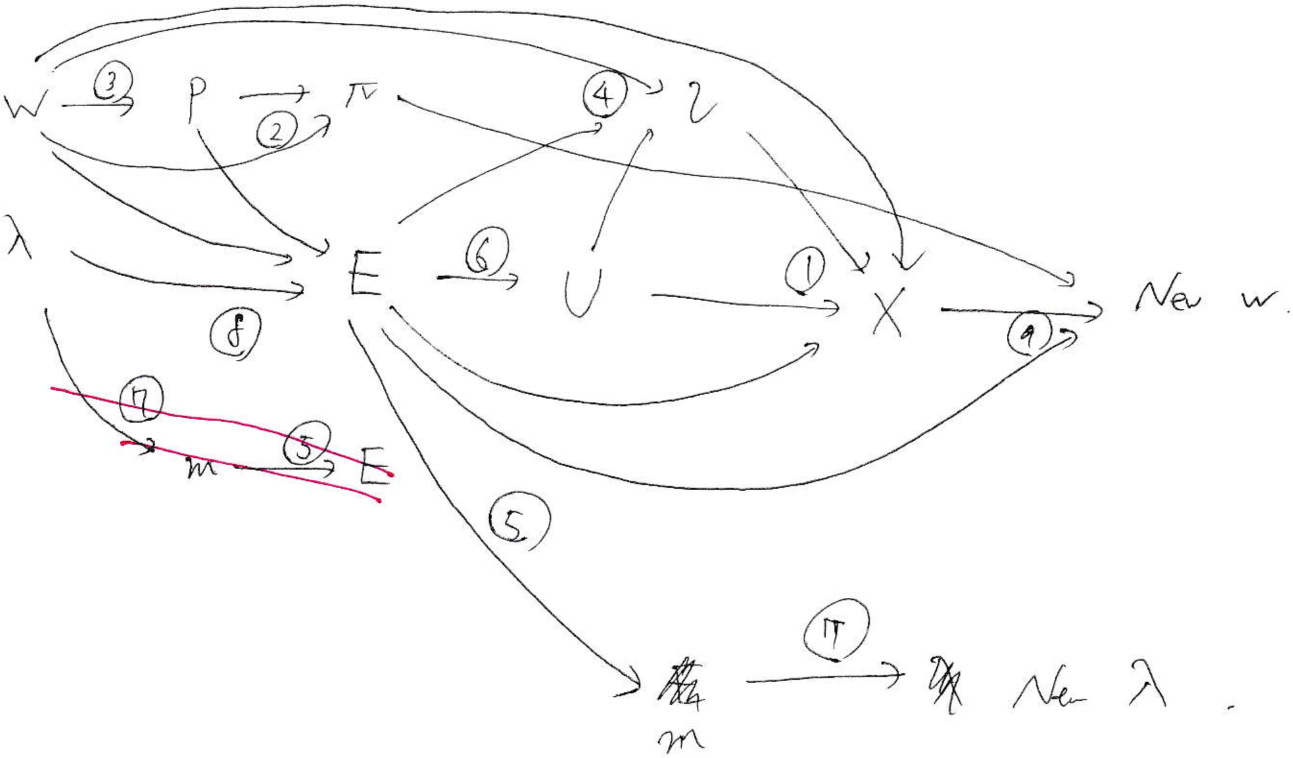
inner loop EK

Start w/ inner loop:

Guess w.

③ → P.





Outer loop:

~~$V^e \longrightarrow \mu^e$~~

~~$V^u \longrightarrow \mu^u$~~

Static variables that are used in the dynamic problems.

m, w, P, E, U.

V, L, N, ~~μ^e~~ → static problem → m, w, P, E, U

↓ (a), (b)

New V^e, V^u

↓ (c), (d), (e)

New μ^e, μ^u

↓ (f), (g)

New L, N.