Comparison of day-by-day reproductivity of COVID-19 among countries by a newly-developed reproduction index R^{W9}

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Based on a new theory for virus infection, we have defined a reproduction index $R^{\mathrm{W9}}(i)$ as a function of date i. $R^{\mathrm{W9}}(i)$ means the day-by-day reproductivity, instantaneously responding to the social activity causing infection at data i. We can easily calculate $R^{\mathrm{W9}}(i)$ from the data of observed new cases. We show how $R^{\mathrm{W9}}(i)$ works, and what $R^{\mathrm{W9}}(i)$ implies in typical countries. $R^{\mathrm{W9}}(i)$ describes how the movement restriction in each country works against the spread of infections. We believe $R^{\mathrm{W9}}(i)$ will be very useful to monitor status of infection of COVID-19.

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I. INTRODUCTION

The global spread of COVID-19 is a serious problem for all human beings. Governments of countries carry out the different political implementations to inhibit the spread of infections. It is extremely important to access how the current implementations work effectively, for changing/retaining the future implementations. For the assessment, a comprehensive and useful analysis method is desirable.

In general studies of the transmission dynamics of virus infection, people often use the effective reproduction number R. If $R < 1$, we expect the epidemic will end. R is often evaluated via the SIR-like models. The SIR model is given as

$$\frac{dS(t)}{dt} = -\beta S(t)I(t)$$
$$\frac{dI(t)}{dt} = -\beta S(t)I(t) - \gamma I(t)$$
$$\frac{dR(t)}{dt} = \gamma I(t)$$

where $S(t)$, $I(t)$, and $R(t)$ are susceptible, infectious, and recovered (or removed) persons, respectively[1–4]. When we use the model for the analysis of actual data, we first extract β (infection rate) and γ (recovery rate) by the parametric fitting in manually divided time intervals[5,6]. Then we determine R from them. Therefore, determined R depends on the divisions to time intervals, thus somewhat arbitrary. Furthermore, we suspect the applicability of the SIR-like models based on the differential equation. Based on our theory, we should start from an integral equation as shown in Sec.III.

In this paper, we have newly developed a reproduction index $R^{\mathrm{W9}}(i)$ as a function of date i based on our theory given in the method section. This $R^{\mathrm{W9}}(i)$ is a replacement of R. We can easily calculate $R^{\mathrm{W9}}(i)$ directly from the observed number of new cases per day (= the number of newly-observed infected persons per day), without fitting procedures. $R^{\mathrm{W9}}(i)$ enables us to monitor the behavior of people for infection.

We organize this paper as follows. In Sec.II, we show $R^{\mathrm{W9}}(i)$ of COVID-19 for countries after the minimum explanation of $R^{\mathrm{W9}}(i)$. For convenience, we divide Sec.II into four cases; Sec.II A: South Korea; Sec.II B: Japan; Sec.II C: Germany, France, and Italy; Sec.II D: Brazil and Sweden. Data in other countries are available in the supplemental materials[7]. We see differences of the status of infection among countries. In Sec.III A-III C, we give our theory of infection followed by the derivation of $R^{\mathrm{W9}}(i)$. In addition, an alternative simple derivation of $R^{\mathrm{W9}}(i)$ is given in Sec.III D.

II. RESULT

In the top panels of Figs.1-6, we plot the numbers of new cases per day $N^{\mathrm{obs}}(i)$ for countries. Actual data is taken from European Centre for Disease Prevention and Control (ECDC)[8] except Japan. For Japan, we use the actual data compiled by Ogiwara[9] since data from ECDC for Japan is incomplete. We set $N^{\mathrm{obs}}(i) = 0$ when the actual data for date i are not available. In middle panels of Figs.1-6, we plot weekly average $N^{\mathrm{obs,W}}(i)$;

$$N^{\mathrm{obs,W}}(i) = \frac{1}{7}\sum_{k=i-3}^{i+3} N^{\mathrm{obs}}(k). \quad (1)$$

We see $N^{\mathrm{obs,W}}(i)$ are smooth without weekly oscillations.

In the bottom panels in Figs.1-6, we show our main results, the reproduction index $R^{\mathrm{W9}}(i)$ defined as

$$R^{\mathrm{W9}}(i) = \frac{N^{\mathrm{obs}}(i+9)}{N^{\mathrm{obs,W}}(i)}. \quad (2)$$

An intuitive rationale of $R^{\mathrm{W9}}(i)$ is given in Sec.III D, while formal derivation based on a theory of infection is given in Sec.III A-III C, which will be useful for further development along the line of the theory.

The meaning of $R^{\mathrm{W9}}(i)$ is quite clear. It just gives the ratio of newly-infected cases at date $i + 9$ relative to the newly-infected cases at date i. For example, if we have $R^{\mathrm{W9}}(i) = 0.5$, we expect the number of newly infected persons at date i will become halved at date

$i+9$. Thanks to a trick that Eq.(2) contains information nine-days ahead, we can identify $R^{\mathrm{W9}}(i)$ as the index for real-time monitoring at date i.

The assumption used for the definition of $R^{\mathrm{W9}}(i)$ is that some part of persons infected at data i are observed at date $i+9$ (See Sec.III). In addition, we assume the interval between generations of infection is nine days as well. However, note that the assumption can be modified. For example, C of $N^{\mathrm{obs}}(i+C)/N^{\mathrm{obs,W}}(i)$ in Eq.(2) is not necessarily to be $=9$. There are some other possibilities to improve the index. Thus we have to take $R^{\mathrm{W9}}(i)$ just as a day-by-day indicator with some ambiguities. Since the definition in Eq.(2) is simple but very useful as we show in the followings, we should use $R^{\mathrm{W9}}(i)$ to inform citizens of our current situation of COVID-19.

A. Case 1: South Korea

Let us see $R^{\mathrm{W9}}(i)$ for South Korea in Fig.1. $R^{\mathrm{W9}}(i)$ explosively increases around Feb.10th because of an outbreak occurred in Daegu city[10], and is suppressed by Feb.28th. This suppression may be given by movement control orders by the government. The small one-day oscillations seen at Mar.23th-24th and at Apr.2th-3rd may be due to the incompleteness of the actual data from ECDC. It seems data from ECDC contains such incompleteness in other countries as well. During March, $R^{\mathrm{W9}}(i)$ was gradually becoming larger to be 1.0 and went back to be 0.5 at the end of March. After Mar.25th, $R^{\mathrm{W9}}(i)$ was again going down to be 0.4. Overall, South Korea succeeded to reduce $R^{\mathrm{W9}}(i)$ very rapidly and to keep $R^{\mathrm{W9}}(i)$ stably less than ~ 1.0.

Latest minor enhancement after Apr.20th is not so meaningful since the absolute number of new cases are quite small (top panel), thus $R^{\mathrm{W9}}(i)$ can be easily fluctuated due to the number of imported persons and intrinsic fluctuations. Theoretically, $N^{\mathrm{obs}}(i)$ should not contain the number of imported persons. See Sec.III. However, the data from ECDC contains the total numbers including the imported persons. This type of fluctuation is observed in other countries in the case of small number of $N^{\mathrm{obs}}(i)$. We should keep this in mind to examine data in other countries in Supplemental materials[7].

B. Case 2: Japan

Let us see $R^{\mathrm{W9}}(i)$ for Japan in Fig.2. Japan suppressed initial explosion between Feb.8th and Feb.14th around Feb.15th. However, Japan could not make $R^{\mathrm{W9}}(i)$ less than 1.0, followed by small uncontrolled explosions during February. This is probably because no social control was introduced until the end of February. During the period between Mar.1st to Mar.15th, $R^{\mathrm{W9}}(i)$ tends to being reduced. This is because of the movement control order such as school closure[11] starting at the beginning of March may work effectively. Then the $R^{\mathrm{W9}}(i)$ were becoming higher after Mar.15th. This is reasonable because the last half of March in Japan is the season that people are moving actively because of the preparation of social new-year in Japan starting from April. Especially, the peak around Mar.20th corresponds to the national holiday Mar.20th-22th when people are highly communicated. On Apr.7th, Japanese government declared the state of emergency[12,13], containing business suspension orders. After the declaration, $R^{\mathrm{W9}}(i)$ is well suppressed and stabilized. Through the eyes of authors living in Japan, the behavior of $R^{\mathrm{W9}}(i)$ looks quite naturally corresponding to what happened in Japan.

We see regularly weekend drops of $R^{\mathrm{W9}}(i)$ in April. Since we see similar drops in other countries such as Germany, Italy, and Sweden, this drops may actually reflects the activity of societies. This observation may be supported a fact that Sweden with less social restrictions have larger size of weekly oscillations as shown in Fig.7.

C. Case 3: Germany, France, and Italy

Let us see $R^{\mathrm{W9}}(i)$ for countries of EU; Germany, France, and Italy. The governments of them started lockdown restrictions on March 22th, 17th, and 9th, respectively[14–16]. From the view of $R^{\mathrm{W9}}(i)$, it was a little too late. For example in Italy, we could not avoid for N^{obs} going over ~ 4000 at the data Mar.9th, if we take into account the latency of observation. The lockdown probably help suppressing the weekly peak of $R^{\mathrm{W9}}(i)$ expected at Mar.10th around (Compare it with previous week). However, problem was that it took two weeks to get $R^{\mathrm{W9}}(i)$ lower than 1.0; the overall speed of reduction of $R^{\mathrm{W9}}(i)$ is quite slow in such European countries, while it is relatively faster in Germany. If we were able to suppress $R^{\mathrm{W9}}(i)$ on date Mar.7th very sharply by the lockdown, the maximum N^{obs} had probably stopped at ~ 4000, and had resulted much smaller infections. At the end of April, we now see $R^{\mathrm{W9}}(i)$ is below 1.0 and stably reducing in these countries.

D. Case 4: Brazil and Sweden

Let us see Brazil and Sweden in Fig.6 and 7. These are not attempting any special movement control orders against COVID-19[17,18]. We see some similarities and differences of $R^{\mathrm{W9}}(i)$ between these countries. At first, both countries suppressed serious explosions well, and reduced $R^{\mathrm{W9}}(i)$ to ~ 1.0. However, it looks that both have similar difficulty to suppress enhancement of $R^{\mathrm{W9}}(i)$ on weekdays. Sweden is better than Brazil in the sense that $R^{\mathrm{W9}}(i)$ is kept around 1.0. However, it will be necessary to suppress $R^{\mathrm{W9}}(i)$ on weekdays more. Brazil is problematic in the sense $R^{\mathrm{W9}}(i)$ did not yet become below 1.0.

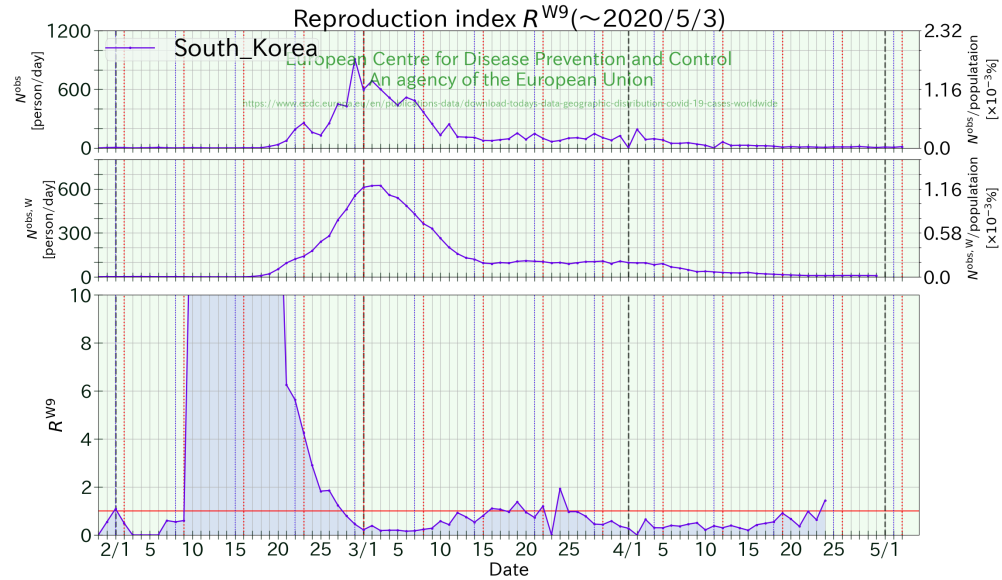

FIG. 1. Plots are for South Korea. In the top panel, we show the number of new cases $N^{\mathrm{obs}}(i)$. In the middle panel, we show the weekly average $N^{\mathrm{obs,W}}(i)$. In the bottom panel, we show the reproduction index $R^{\mathrm{W9}}(i)$, where hatch areas below data lines as a guide of eyes. The actual data $N^{\mathrm{obs}}(i)$ is taken from ECDC. $N^{\mathrm{obs,W}}(i)$ and $R^{\mathrm{W9}}(i)$ are calculated from $N^{\mathrm{obs}}(i)$.

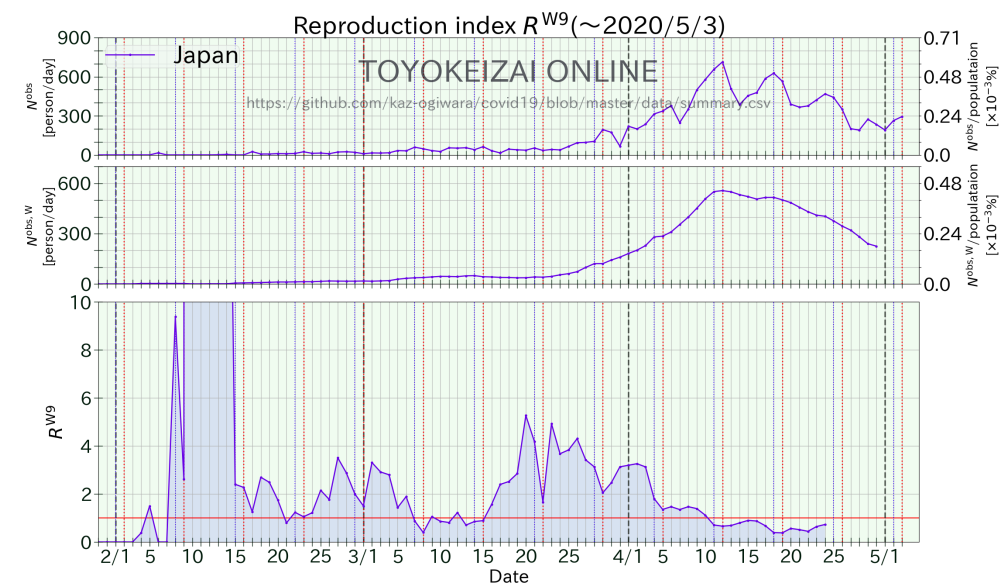

FIG. 2. Plots for Japan. See captions in Fig.1. Data of $N^{\mathrm{obs}}(i)$ is compiled by Ogiwara[9].

III. METHOD

A. Theory of statistical dynamics of virus infection

We consider a set Ω whose elements $p \in \Omega$ are all the infected persons by a virus in a country. For treating the statistical dynamics of virus infection theoretically, we consider following three quantities. Because we treat discretized dynamics of day by day, all the arguments of functions treated here are integers (denoted by i, j, n_p and so on) specifying date or period of days.

(1) The date of infection $\{n_p | p \in \Omega\}$.

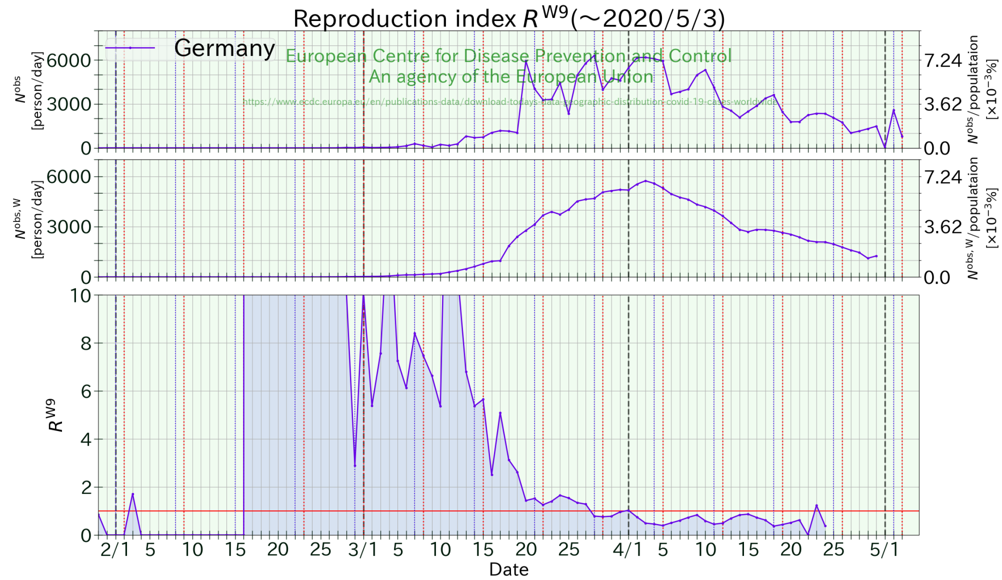

FIG. 3. Plots for Germany. See captions in Fig.1.

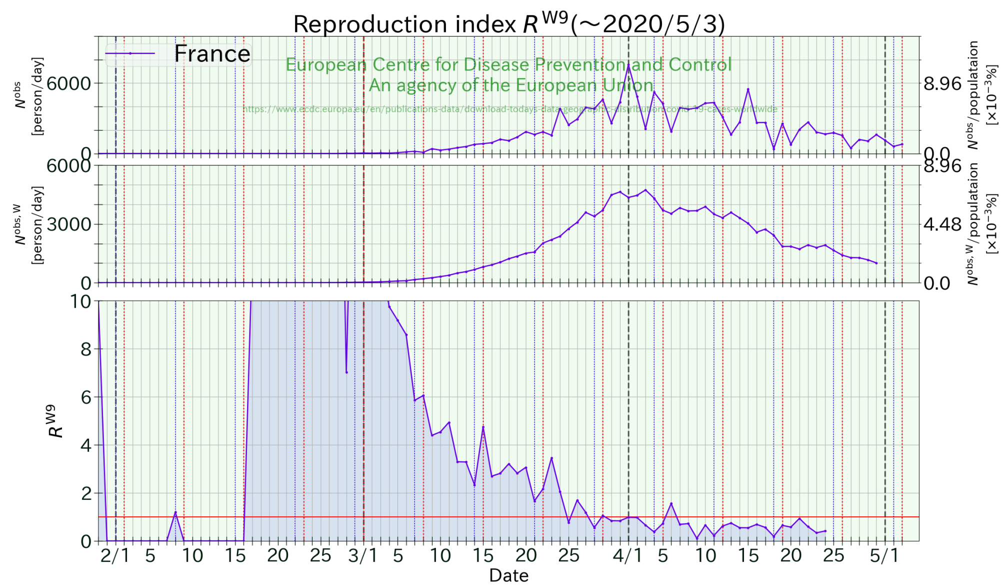

FIG. 4. Plots for France. See captions in Fig.1.

(2) The number of infected persons by virus exhaled by a person p, denoted as $\{N_p(i)|p \in \Omega\}$.

(3) The number of exhaled virus by a person p, $\{A^0_p(i)|p \in \Omega\}$. Here i is the day counted from the date of infection. Thus $A^0_p(i)$ must be zero for $i < 0$ even when we have no latency period.

In addition, we consider quantities $\{n^{\rm ext}_p, m^{\rm ext}_p, N^{\rm ext}_p(i), A^{0,\rm ext}_p(i)\}$ for $p \in \Omega^{\rm ext}$, where $\Omega^{\rm ext}$ is a set of persons imported from other countries. Here $m^{\rm ext}_p$ is the imported date of the person $p \in \Omega^{\rm ext}$. Persons $p \in \Omega^{\rm ext}$ are origins of all infected persons in the country. When we develop the theory as follows, we treat as if these quantities are already recorded and available. Let us concentrate on quantities of Ω at first. We will be back to a term related to $\Omega^{\rm ext}$ at Eq.(9).

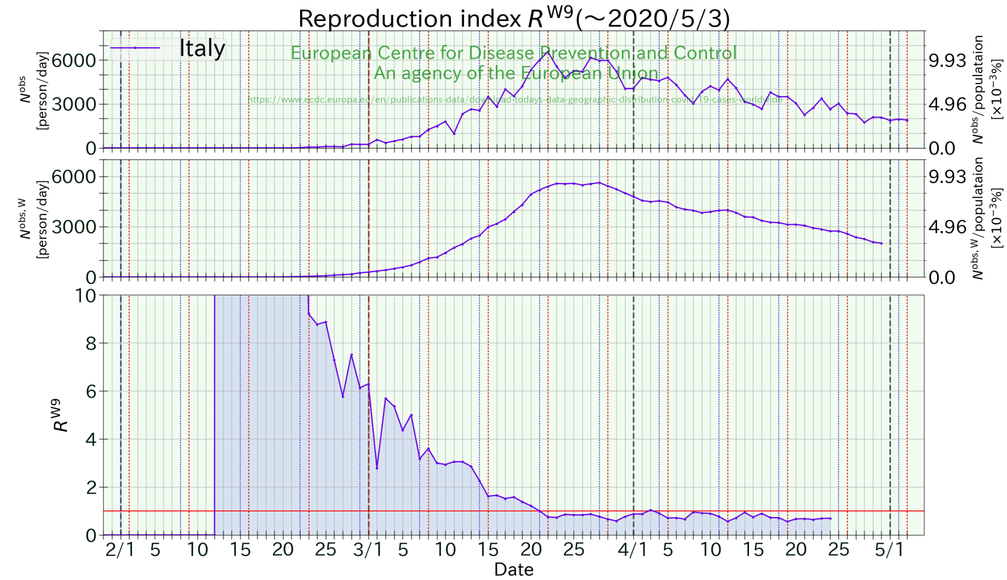

FIG. 5. Plots for Italy. See captions in Fig.1.

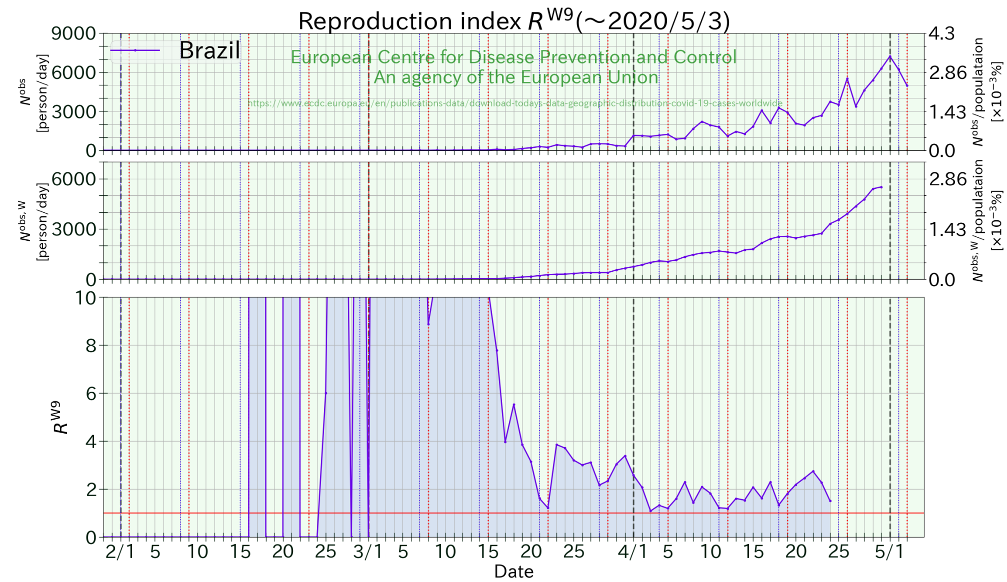

FIG. 6. Plots for Brazil. See captions in Fig.1.

Let us introduce two quantities for further discussion. First, we define the **normalized number of exhaled virus** (NNEV) $A_p(i)$. With the total number of exhaled virus by a person p, $A_p^{0,\mathrm{total}} = \sum_i A_p^0(i)$, we define $A_p(i) = (1/A_p^{0,\mathrm{total}})A_p^0(i)$. Thus we have the normalization of NNEV as

$$\sum_i A_p(i) = 1. \tag{3}$$

Second, we define $R_p(i)$ via

$$N_p(i) = R_p(i)A_p(i-n_p). \tag{4}$$

Thus $R_p(i)$ is defined for i of $A_p(i-n_p) \neq 0$; the value of $R_p(i)$ is irrelevant for $A_p(i-n_p) = 0$. $R_p(i)$ is the key quantity to relate $A_p(i-n_p)$ and $N_p(i)$. $R_p(i)$ is not necessarily a smooth function of i; after averaged on p, we have meaningful quantities for statistical dynamics as follows.

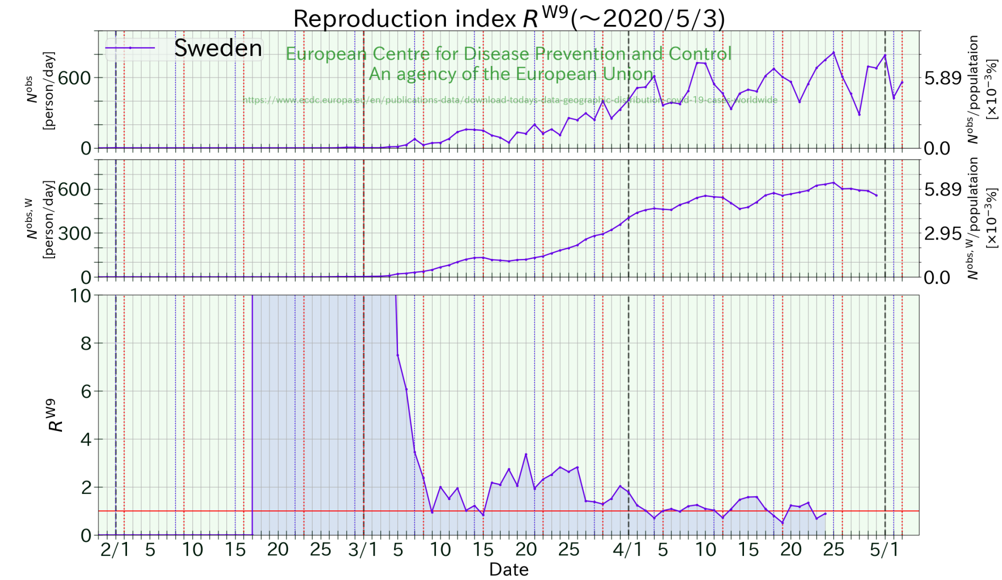

FIG. 7. Plots for Sweden. See captions in Fig.1.

By the sum of Eq.(4) for all p, we have the total number of new cases per day $N(i)$ as

$$N(i) = \sum_p R_p(i) A_p(i - n_p) = \sum_p R(i) A_p(i - n_p), \quad (5)$$

where we introduce the $A_p(i - n_p)$-weighted average of $R_p(i)$,

$$R(i) = \frac{\sum_p R_p(i) A_p(i - n_p)}{\sum_p A_p(i - n_p)}. \quad (6)$$

We name $R(i)$ the **day-by-day reproducibility**. With definitions of the **number of active members** $N^{\mathrm{A}}(i)$ and activation matrix $A(i, j)$ as

$$N^{\mathrm{A}}(i) = \sum_p A_p(i - n_p) = \sum_j A(i, j) N(j), \quad (7)$$

$$A(i, j) = \sum_{p \text{ for } n_p = j} A_p(i - j) \frac{1}{N(j)}, \quad (8)$$

we have $N(i) = \sum_p R(i) A_p(i - n_p) = R(i) \sum_j \sum_p A_p(i - j) \delta_{n_p j} = R(i) N^{\mathrm{A}}(i)$.

In addition, we have to add contribution from Ω^{ext}. Because we can treat the contribution from Ω^{ext} in the same manner, we ends up with the master equation of the infection written as

$$N(i) = R(i) N^{\mathrm{A}}(i) + R^{\mathrm{ext}}(i) N^{\mathrm{A,ext}}(i), \quad (9)$$

where

$$N^{\mathrm{A,ext}}(i) = \sum_p^{\mathrm{ext}} A_p^{\mathrm{ext}}(i - n_p) = \sum_j A^{\mathrm{ext}}(i, j) N^{\mathrm{ext}}(j), \quad (10)$$

$$A^{\mathrm{ext}}(i, j) = \sum_{p \text{ for } n_p = j}^{\mathrm{ext}} A_p^{\mathrm{ext}}(i - j) \frac{1}{N^{\mathrm{ext}}(j)}. \quad (11)$$

Here the term $N^{\mathrm{ext}}(i) = \sum_{p \text{ for } n_p = i}^{\mathrm{ext}} 1$ means the number of externally introduced persons as the 0th generation. The number of 1st generation, persons infected directly by the 0th generation, is given by the second term in Eq.(9). Note that we have the normalization condition $\sum_i A(i, j) = \sum_i A^{\mathrm{ext}}(i, j) = 1$. For the matrices $A, A^{\mathrm{ext}}, R, R^{\mathrm{ext}}$ whose elements are given by $A(i, j), A^{\mathrm{ext}}(i, j), R(i)\delta_{ij}, R^{\mathrm{ext}}(i)\delta_{ij}$, we can simply rewrite Eq.(9) as

$$N = RAN + R^{\mathrm{ext}} A^{\mathrm{ext}} N^{\mathrm{ext}}. \quad (12)$$

Until now, we have defined key quantities $A(i, j)$ and $R(i)$ as well as $A^{\mathrm{ext}}(i, j)$ and $R^{\mathrm{ext}}(i)$. Then we derive Eq.(12) without any approximation. Now we can interpret Eq.(12) as the dynamical equation of motion of N responding to the external source term N^{ext} if $R, A, R^{\mathrm{ext}} A^{\mathrm{ext}}$ are available in advance. Then Eq.(12) is easily solved as

$$N = \frac{1}{1 - RA} (R^{\mathrm{ext}} A^{\mathrm{ext}}) N^{\mathrm{ext}}. \quad (13)$$

Note that $N(i)$ is defined as the number of infected persons, not including the 0th generation $N^{\mathrm{ext}}(i)$.exhaling In the case that $R(i) = R$ and $R^{\mathrm{ext}}(i) = R^{\mathrm{ext}}$ are static, the total number of infected persons including the 0th generation is given as

$$\sum_i N(i) + \sum_i N^{\rm ext}(i) = \frac{R^{\rm ext}}{1-R}\sum_i N^{\rm ext}(i) + \sum_i N^{\rm ext}(i) \quad (14)$$

For the derivation of Eq.(14), we use $\sum_i A(i,j) = \sum_i A^{\rm ext}(i,j) = 1$. One interesting point is that Eq.(14) do not include the information of $A(i,j)$. This is because the knowledge of size of reproductivity is pushed into $R(i)$ and $R^{\rm ext}(i)$, while $A(i,j)$ contains information of time.

Let us consider how we supply $A, A^{\rm ext}, R$ and $R^{\rm ext}$ when we treat Eq.(13) as a dynamical equation of motion. Based on the definition, $A(i,j)$ is directly related to the nature of virus; the latent period (period to start exhaling virus after infected) and the period of communicability (exhaling period of virus). Thus $A(i,j)$ is given by medical research. Then we may simply assume $A(i,j) = A^{\rm ext}(i,j)$. $R(i)$ is the complex unknown quantities related to the nature of virus, to the social behavior of people, and to the climate changing day by day. $R^{\rm ext}(i)$ can be constructed based on $R(i)$ with taking into account the effect of some border measures. $R_p^{\rm ext}(i)$ should be zero for $i < m_p$. To set up $R(i)$ and $R^{\rm ext}(i)$ in Eq(13), we need a model which correctly capture the essence of the definition of Eq.(6).

Otherwise, we can determine $R(i)$ (assuming $R^{\rm ext}(i)$ is neglected here for simplicity) from $N(i)$ when $A(i,j)$ is given. This is what we did in this paper with the help of observation theory in Sec.III B.

$A, A^{\rm ext}, R$ and $R^{\rm ext}$ should contain statistical fluctuations. These can be strongly fluctuating when the size of members related to the sum in their definitions are very small. Such fluctuations should be evaluated along the line of formulation presented here. We will not go into this problem in this paper.

Our formulation contains key quantity $R(i)$ directly reflects the social behavior of people. Changes of the behavior gives the change of $R(i)$ instantaneously. If all the people stop to meet someone else at date i, $R(i)$ becomes zero. $R(i)$ has a clear meaning of day-by-day reproducibility. The average period of days to cause new infection can be calculated; see Eq.(20). If $R(i) = 1$ is kept, we have constant number of newly infected persons everyday. As we will see in Sec.III B, we will use Eq.(12) so as to calculate $R(i)$ from the newly-observed cases day by day, $N^{\rm obs}(i)$.

Our formulation can be extended to the case of age-resolved dynamics and space-resolved dynamics. In such cases, we need to consider multiple sets Ω^i in addition to the set for persons of transition as $\Omega^{i\to j}$.

B. observation of infection

We like to evaluate $R(i)$ based on Eq.(12). For simplicity, we assume that we have a good border measures satisfying $R^{\rm ext}(i) = 0$ for $i \geq i_{\rm BM}$ in the followings. Then we have $N = RAN$ after $i \geq i_{\rm BM}$. Based on $N = RAN$, we can evaluate $R(i)$ if we have $A(i,j)$ and $N(i)$. In order to obtain $N(i)$ from the observation data $N^{\rm obs}(i)$, we need the following theory.

To consider the theory of observation of infection, we need additional information for Ω. The information is the date of observation (date of infection detected) of a person p, $\{o_p | p \in \Omega'\}$. Because only limited part of $p \in \Omega$ is observed, we expect $\Omega' \subset \Omega$. We now define $O_p(i,j)$ as

$$O_p(i,j) = \delta_{o_p i}\delta_{n_p j}. \quad (15)$$

In the similar manner in Sec.III A, we reorganize the sum of $O_p(i,j)$ as

$$N^{\rm obs}(i) = \sum_j O(i,j)N(j), \quad (16)$$

$$O(i,j) = \sum_{p \text{ for } n_p=j}^{p\in\Omega'} O_p(i,j)\frac{1}{N(j)}. \quad (17)$$

Here, $O(i,j)$ is the observation matrix. Like $R(i)$ and $A(i,j)$ Sec.III A, $O(i,j)$ should have its own fluctuation identified as the 'observation fluctuation', which can be evaluated based on the definition of Eq.(17).

With Eq.(16), $N = RAN$ is written as

$$N^{\rm obs}(i) = \sum_{j,k,l} O(i,j)R(j)A(j,k)O^{-1}(k,l)N^{\rm obs}(l). \quad (18)$$

Note that $A(i,j)$ is a retarded response function while $O^{-1}(i,j)$ is an advanced response function. Eq.(18) means that $N^{\rm obs}(i)$ can be calculated from $N^{\rm obs}(l)(i \geq l)$ if $R(i)$ is given. Otherwise, we solve Eq.(18) for $R(i)$. Then we have

$$R(i) = \frac{\sum_j O^{-1}(i,j)N^{\rm obs}(j)}{\sum_{k,j} A(i,k)O^{-1}(k,j)N^{\rm obs}(j)}. \quad (19)$$

From Eq.(19), we can calculate $R(i)$ when $A(i,j)$ and $O(i,j)$ are given. Calculated $R(i)$ in Eq.(19) reflects the social behavior of people instantaneously. Changes of the behavior gives the change of $R(i)$ without latency.

We can calculate the average period of days $T(j)$ to cause a new infection after infected;

$$T(i) = \frac{\sum_{k,j}(i-k)A(i,k)O^{-1}(k,j)N^{\rm obs}(j)}{\sum_{k,j} A(i,k)O^{-1}(k,j)N^{\rm obs}(j)}. \quad (20)$$

This means that it takes $T(i)$ to reduce the newly infected persons by the factor $R(i)$. Thus we can evaluate the half-time period $T_2(j)$ as

$$T_2(j) = -\frac{\ln 2}{\ln R(j)}T(j). \quad (21)$$

C. Derivation of the reproduction index R^{W9}

To derive R^{W9}, we neglect fluctuations of $O(i,j)$ and $A(j,k)$ in Eq.(19). We use simple assumptions about $A(i,j)$ and $O(i,j)$ as

$$A(i,j) = \begin{cases} \frac{1}{7} & (i-j = 6, \ldots, 12) \\ 0 & (i-j = 1,2,3,4,5) \\ 0 & (i-j \geq 13) \end{cases}, \tag{22}$$

and

$$O(i,j) = \epsilon \delta_{i,j+9}, \tag{23}$$

where the ϵ is a constant. Eq.(22) means that infectivity of each person retains for seven days and the infectivity become active at the sixth day after the person is infected. This assumption is a simplification of the situation of COVID-19 shown in Fig.1 in Ref.2. Eq.(23) means that an infected person is observed at a rate of ϵ, at nine days after one is infected. By substituting Eq.(22) and Eq.(23) into Eq. (19), we have

$$R(i) = \frac{N^{\mathrm{obs}}(i+9)}{N^{\mathrm{obs,W}}(i)} \equiv R^{\mathrm{W9}}(i), \tag{24}$$

where $N^{\mathrm{obs,W}}(i)$ is the week average of $N^{\mathrm{obs}}(i)$ around the date i. By substituting Eq.(22) and Eq.(23) into Eq. (20), we have

$$T(i) = 9. \tag{25}$$

Thus R^{W9} is a reproduction index to show the ratio of reduction of the number of newly infected persons after nine days.

In principle, we cannot expect perfect border measures as we have assumed at the beginning of Sec.III B. Thus we need to do analysis taking into account the imported persons $p \in \Omega^{\mathrm{ext}}$. However, since we do not have data to analyze the effect of the imported persons, we simply use the number of actual data as N^{obs} in Eq.(24). This causes spiky behaviors due to the imported persons in the plot of $R^{\mathrm{W9}}(i)$. See the case of Taiwan in Supplemental materials[7].

D. Alternative derivation of R^{W9}

Here we give a simple derivation of R^{W9}, instead of the derivation via Sec.III A-III C. Let us describe R^{W9} using Fig.8. The bold lines in Fig.8 indicate the period of communicability for each infected person (specified by p) and the thin lines indicate incubation period, where we assume the latent period is the same with the incubation period. In our model, the infectivity emerges six days after the infection, and keeps the infectivity to others for seven days. In Fig.8, the possible infection routes at the date i are persons specified as $p = 2, 3, \ldots, 8$. We assume the infectivity is constant during the period of communicability. Then we use normalization so that R^{W9}=1 means constant number of new cases. Therefore, the effective number of infection route at the date i is equals to the average of $N^{\mathrm{orb}}(i-m)$ where $m = -3, -2, \ldots, 3$, as written in Eq.(1). As we assume the infection number at the date i are observed nine days after, a reproduction rate as Eq.(2).

The idea presented here is very simple. This can mean the robustness of the index R^{W9}. Some new improved index rather than R^{W9} will be invented, however, the essential as the reproductivity index may change not so much.

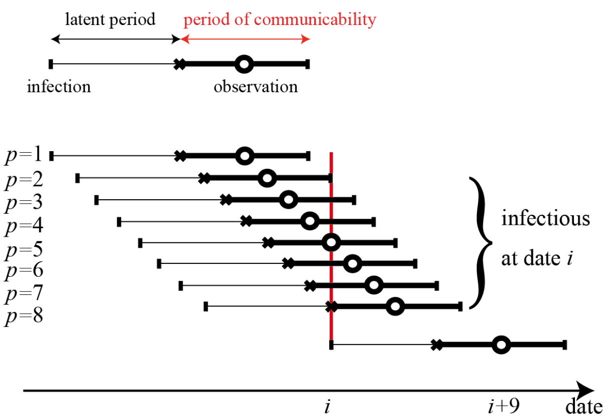

FIG. 8. Each horizontal line means a set of persons infected at the same date. The period between two short vertical lines at the ends of the horizontal lines means life time, from infected date to the date losing ability of infection to others. Crosses indicate the date that persons get ability of exhaling virus. Opened circles indicate the date of observation (confirmed by the medical test such as RT-PCR).

IV. SUMMARY

We have developed a new reproduction index $R^{\mathrm{W9}}(i)$. With $R^{\mathrm{W9}}(i)$, we compare status of infections of COVID-19 among varieties of countries. $R^{\mathrm{W9}}(i)$ can be simply calculated from the data of newly-observed number of infected persons everyday. With plot of $R^{\mathrm{W9}}(i)$, we can see how each country tries to suppress COVID-19. At a glance, $R^{\mathrm{W9}}(i)$ works well for countries, however, we need further examination to justify the index $R^{\mathrm{W9}}(i)$. We believe that it is important to improve $R^{\mathrm{W9}}(i)$ so as to capture the effect of political decisions correctly.

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