

INDiC: Improved Non intrusive load monitoring using load Division and Calibration

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Abstract—Non-Intrusive appliance load monitoring (NIALM) is the process of disaggregating the overall electricity usage into constituent appliances. In this paper we extend the Combinatorial Optimization (CO) approach for disaggregation, which was originally proposed in the seminal work on NIALM, in following two ways: 1) Breaking the problem into subproblems and reducing the state space; 2) Applying additional constraints backed by sound domain expertise. We evaluate our approach using REDD dataset and show practical problems which need to be solved while dealing with the dataset. We also propose a metric for evaluating NILM, which we believe overcomes many shortcomings of commonly used metrics.

I. INTRODUCTION

- Motivate the importance of energy consumption in building
- Motivate that appliance level information is crucial - detailed feedback and optimized decision making [1]
- Challenges with getting appliance level information - introduce NIALM [2]
- introduce your proposed approach
- Enumerate the contributions

Primary contributions of our work are:

- Fill it up with 2-3 crisp points

Open source implementation of the proposed work is released for comparative analysis with other NIALM approaches as an IPython notebook¹. We believe this is the first extensive release of a generic NIALM

II. RELATED WORK

Non-intrusive appliance load monitoring was first studied by Hart [2] in the early 1990s by examining signatures in aggregated load to indicate activities of appliances. The problem has been well studied in recent years ([3], [4], [5]) and NIALM systems can broadly be divided into two categories based on whether supervised or unsupervised disaggregation methods are used. Supervised learning techniques include optimization-based methods such as integer programming [6] and genetic algorithms [7]. These approaches are compute intensive and appliances with similar or overlapping load signatures are

difficult to discern. Other machine learning techniques (such as Artificial Neural Network (ANNs) [8] and Hidden Markov Models (HMM) [9]), have also been shown to work well for the task. Variants of HMMs such as factorial hidden Markov models (FHMMs) [10], additive FHMMs [11], conditional factorial hidden semi-Markov models [12] and difference HMMs [13] have been studied. In factorial HMMs several models evolve independently and in parallel and the observed output is some joint function of all the hidden states; additive HMMs allow emission of a single real-valued (unobserved) output from each HMM and the output is the sum of these HMMs. In difference HMMs [13] each appliance's load is modeled using a graphical model and it is disaggregated from the aggregate power – this process is repeated iteratively until all appliances for which general models are available are disaggregated. It is therefore possible to infer the probability that a change in aggregated power was generated by two consecutive states of an appliance. The applicability of these sophisticated techniques are generally hindered by the difficulty of inference from models with a large number of HMMs. Rahayu et. al. [14] propose a discriminative model for energy disaggregation that predicts the most likely appliance state configuration from aggregated load using nonparametric classification algorithms. They posit that “subset sum” type techniques are not very effective since a large portion of the home energy is not monitored directly. In this paper, we put forward a simple idea – the aggregated load can be further split up by information on mains and knowing which appliance is assigned to which main. This would make the task of disaggregation inherently easier by solving a simpler optimization problem and most of the above sophisticated machine learning based modeling approaches can still be applied to the task².

Several datasets have been made publicly available for benchmarking energy disaggregation algorithms including REDD[15], Blued[16], Pecan[17] and Smart*[18]. Our empirical analysis is on the REDD data primarily because this is the most popular data set for evaluating state-of-the-art NIALM approaches.

III. NIALM

NIALM is the process of disaggregating the total electrical load into constituent appliances [2]. A typical NIALM setup

¹<http://www.ipython.org>

²The use of combinatorial optimization is only for the purpose of illustration and there is clearly no requirement to adhere to this modeling technique alone.

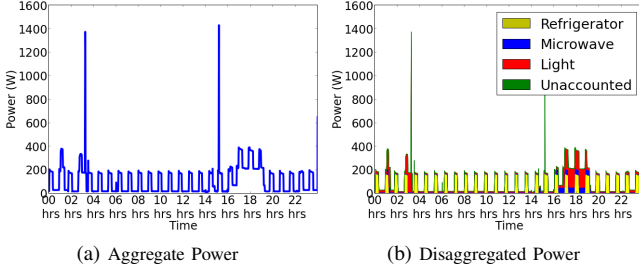


Fig. 1: The process of disaggregation

TABLE I: Terminologies and Functions

| Symbol | Meaning |
|--|--|
| $t \in 1, \dots, T$ | Time slice |
| $n \in 1, \dots, N$ | Appliance number |
| $\theta^n = \{\theta_1^n, \dots, \theta_T^n\}$ | Measured power sequence for each appliance |
| $\theta^{M_1} = \{\theta_1^{M_1}, \dots, \theta_T^{M_1}\}$ | Measured power sequence for Mains 1 |
| $\theta^{M_2} = \{\theta_1^{M_2}, \dots, \theta_T^{M_2}\}$ | Measured power sequence for Mains 2 |
| $x = \{x_1, \dots, x_T\} = \theta^{M_1} + \theta^{M_2}$ | Measured aggregate power sequence |
| $e = \{e_1, \dots, e_T\}$ | Aggregate noise power sequence |
| $y^n = \{y_1^n, \dots, y_T^n\}$ | Predicted power sequence for n^{th} appliance |
| $k \in 1, \dots, K$ | Appliance power state |
| $z^n = \{z_1^n, \dots, z_T^n\}$ | Appliance state sequence, $z_i^n \in [1, \dots, K]$ |
| $z_{t,k}^n \in 0, 1$ | Whether n^{th} appliance is in k^{th} state at time t |
| $\mu^n = \{\mu_1^n, \dots, \mu_K^n\}$ | Power draw by n^{th} appliance in k^{th} state |
| $Mapping[n] \in M^1, M^2$ | Mapping of n^{th} appliance to Mains |
| $Downsample(s, filter, interval)$ | Function to downsample a timeseries s to an <i>interval</i> according to specified <i>filter</i> |
| $Preprocess(\{s^1, \dots, s^n\}, method)$ | Function to ensure that n timeseries start and end at same times and handling missing data using specified <i>method</i> |
| $Sort(\{s^1, \dots, s^n\}, parameter, order)$ | Function to sort n timeseries according to <i>parameter</i> in specified <i>order</i> |
| $Event_Detection(s, threshold, interval)$ | Function returning magnitude and times of step events in time series s during an <i>interval</i> , whose absolute value is greater than <i>threshold</i> |

involves instrumenting the power mains with smart meters and individual appliances with appliance meters for ground truth. Figure 1 shows the house mains disaggregated into 3 appliances.

We use some terminologies from previous work [15], [13], [2] and extend them for our analysis in Table I. Based on these terminologies, the NIALM problem in supervised setting can be formulated as predicting the power sequence for n^{th} appliance, y^n , given the measured power sequence for each appliance θ^n and the total aggregate power sequence x .

Combinatorial optimization: At a given time an appliance can only be in a single state which is given by: $\sum_{k=1}^{K=K} z_{t,k}^n = 1$

The power consumption by n^{th} appliance in k^{th} state at time t is given by: $\hat{\theta}_{t,k}^n = \sum_{k=1}^K z_{t,k}^n \mu_k^n$

The overall power consumption of all appliances at a given time t is given by: $\hat{x}_t = \sum_{n=1}^N \sum_{k=1}^K z_{t,k}^n \mu_k^n$

The error in power signal after the load assignment explained above is given by:

$$e_t = |x_t - \sum_{n=1}^N \sum_{k=1}^K z_{t,k}^n \mu_k^n|$$

Combinatorial optimization tries to find the optimal combination of appliances in different states which will minimize this error term, by the following state assignment scheme:

$$z_t = \underset{z_t}{\operatorname{argmin}} |x_t - \sum_{n=1}^N \sum_{k=1}^K z_{t,k}^n \mu_k^n|$$

The corresponding predicted power draw by n^{th} appliance is given by: $y^n = \{\mu_{z_1^n}^n, \dots, \mu_{z_T^n}^n\}$

This optimization problem resembles subset sum problem [19] and is NP-complete. The state space of this optimization function is K^N , which means it is exponential in the number of appliances. Owing to the exponential nature of the state space and the fact that the algorithm requires all appliances be known, this approach has not been thoroughly studied in the past [13]. We chose to use this approach as a proof of concept of our contributions.

what to use consistently- load division or appliance

Load division: In the US, homes have 2 electrical mains corresponding to different phases. Many Asian countries have multiple meters per home. Different loads are electrically connected to different mains/meters. Since an NIALM deployment requires monitoring different electrical mains/meters, we leverage the load division to perform disaggregation on these mains separately to improve load disaggregation. We assign N_i different loads to p different mains given by where: $\sum_{i=1}^p N_i = N$ After load assignment to different mains, the state spaces for different mains are given by $K^{N_1} \dots K^p$. As a practical example, if a home has two mains and 20 appliances equally distributed across the two mains, state space before load division is given by 2^{20} and after load division is 2^{10} . Thus, there is an exponential reduction in state space, which also leads to an exponential decrease in running time. **Where to write that we can also benefit by parallelizing**

IV. INDiC NIALM

In this section we explain our algorithm- Improved Non intrusive load monitoring using load Division and Calibration (INDiC). While INDiC can be used with any modeling technique such as Hidden Markov Models[20], combinatorial optimization, etc., we present INDiC-CO (INDiC using Combinatorial optimization as modeling approach). The various steps of INDiC-CO NIALM shown in Algorithm 1 are described below.

Preprocessing: Since multiple data streams (appliances and mains power time series) from varied hardware are used to produce data for NIALM applications, it is highly likely that some hardware malfunctions during the data collection process. In the preprocessing step we ensure that all the time series start and end at the same time and handle missing data using techniques such as forward filling(padding).

Downsampling: While performing CO it is desired that transients and fluctuations in the power signal are filtered[2]. The transients occur due to the high starting current of the appliance, whereas the fluctuations are a consequence of minor voltage fluctuations and oscillatory nature of loads.

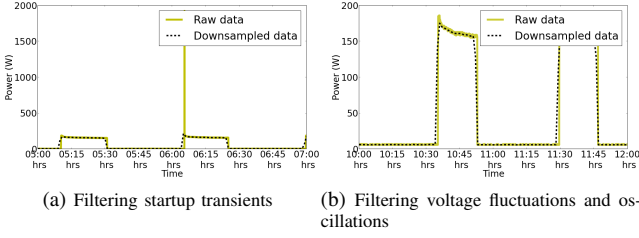
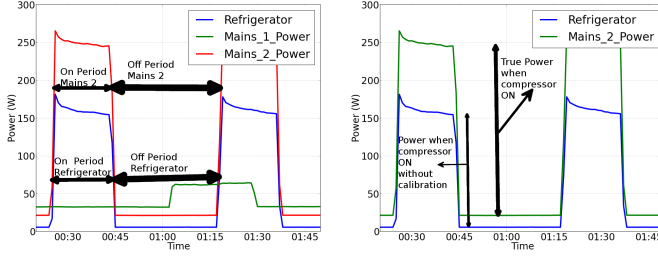


Fig. 2: Effect of downsampling appliance data



(a) Assigning refrigerator to Mains 2 since refrigerator power > Mains 1 power and on basis of duty cycle (b) Calibrating Refrigerator Power consumption

Fig. 3: Calibrating and assigning refrigerator to Mains 2

Figure 2a and Figure 2b show how starting current and voltage fluctuations can be filtered by down sampling. Filters such as mean/median can be used to down sample a time series, where the value assigned to a time window is the mean/median of values occurring during that time window.

Assigning Loads to Mains³: This step aims to identify the mapping between appliances and mains. Patterns corresponding to appliances having higher peak power are generally easier to extract from main signal, thus we sort the appliance in decreasing order of peak power. For all such background loads we see if the mean power of the appliance is greater than mean power of any mains for all time instances. If so, we can safely assign the appliance to the other mains. If this step is unable to provide conclusive evidence we look at the periodicity associated with such background loads during periods of low or no human activity (such as night time). Figure 3a shows how based on refrigerator duty cycle it is mapped to Mains 2. On similar lines assignment of foreground loads can be done.

Appliance Power Calibration: Power measured by appliance level meters may need calibration due to the following reasons:

- Difference in measurement of different measurement instrument (diagram showing CC, ZWave, etc) [21]
- Fluctuation in voltage cause power to fluctuate as well
- Missing meta data, whether real or apparent power is being measured at appliance level

³While we provide methods for 2 Mains, the approach can be easily extended

Since different hardware is used for measuring appliance and mains data there may be a need to calibrate the two. Since mains data is usually collected using better precision hardware, we keep mains data as a reference and calibrate appliance data against it. In practice we found appliance level monitors to usually provide only real power whereas the mains monitors can provide much more like reactive and active power. Like the previous step, time instances when an appliance in a particular mains is single used are identified. The ratio of mains and appliance power step changes occurring this window serve as the calibration factor for that appliance. Further each appliance power is corrected with the corresponding calibration factor.

Clustering to identify appliance states:

- 1) Step changes occurring in Mains vs Appliances
- 2) Isolating single appliance usage We use [22] to run our clustering

Appliance states identification using clustering (Possibly talk about unbalanced data, but leave it for future work) State space creation Applying CO for different mains Find energy distribution by appliance and assign weights (To be used in results)

V. EVALUATION

We use Reference Energy Disaggregation Data Set (REDD) [15] for validating our algorithms. This dataset contains power and voltage data for mains (2 phases) as well as appliances from 6 homes in Boston area collected in the summer of 2011. The data is made available as raw, high frequency (sampled at 15 KHz) and low frequency (Mains at 1 Hz, appliances at 0.3 Hz). Considering the practical implications of residential smart meter installation, we believe that low frequency data represents the most realistic scenario and thus we use this data for analysis.

A. Evaluation Metric

Commonly used metrics such as accuracy, sensitivity and specificity can be misleading when applied to NIALM. It can be seen from Figure 4 that since stove is mostly in state 0 (Off), accuracy will be largely decided by accuracy for this state. This does not tell how badly the prediction is misclassifying state 1 (On). Thus, we use the following metrics which have been used in the past work [13], [15]:

- **Mean Normalized Error (MNE %):** Normalized error in the energy assigned to an appliance n over time period T , given by:

$$MNE(n) = \frac{\sum_{t=1}^T |\theta_t^n - y_t^n|}{\sum_{t=1}^T \theta_t^n} \quad (1)$$

- **RMS Error (RE Watts):** RMS error in power assignment to an appliance n per time slice t given by:

$$RE(n) = \sqrt{\frac{1}{T} \sum_{t=1}^T (\theta_t^n - y_t^n)^2} \quad (2)$$

Since both these quantities represent error, the lesser they are the better is the prediction.

Algorithm 1: INDiC-CO

Input: $x, \theta^n, \theta^{M_1}, \theta^{M_2}, B, F, v$ **Output:** $y^n, \mu_k^n, \text{Calibrated } \theta^n$ **Preprocessing**

1 $\theta^1, \dots, \theta^n, \theta^{M_1}, \theta^{M_2} \leftarrow$
Preprocess($[\theta^1, \dots, \theta^n, \theta^{M_1}, \theta^{M_2}]$, forward fill)

Downsampling

2 **for** $n \in 1, N$ **do**
 $\theta^n \leftarrow \text{Downsample}(\theta^n, \text{mean}, 1 \text{ minute})$
3 $\theta^{M_1} \leftarrow \text{Downsample}(\theta^{M_1}, \text{mean}, 1 \text{ minute})$
4 $\theta^{M_2} \leftarrow \text{Downsample}(\theta^{M_2}, \text{mean}, 1 \text{ minute})$
5 $s \leftarrow \text{Sort}([\theta^1, \dots, \theta^n], \text{peak power})$

Appliance to Mains mapping

6 **for** Appliance $n \in s$ **do**
7 **if** $\theta_t^n > \theta_t^{M_1}$ for any $t \in 1, T$ **then**
 $\text{Mapping}[n] = 2$
8 **else if** $\theta_t^n > \theta_t^{M_2}$ for any $t \in 1, T$ **then**
 $\text{Mapping}[n] = 1$
9 **else**
10 **if** $\text{Event_Detection}(\theta^n, 15).Times \subset$
 $\text{Event_Detection}(\theta^{M_1}, 15, w).Times$ **then**
 $\text{Mapping}[n] = 1$
11 **else**
 $\text{Mapping}[n] = 2$
12 $\theta^{M_{\text{Mapping}[n]}} \leftarrow \theta^{M_{\text{Mapping}[n]}} - \theta^n$

Calibration

for $n \in 1, N$ **do**
13 $\theta^n \leftarrow \theta^n * \text{Step_Changes}(\theta^n, 15, w).Magnitude$
 $\div \text{Step_Changes}(\theta^{M_{\text{Mapping}[n]}}, 15, w).Magnitude$
14 $\theta^n \leftarrow \text{Normalize}(\theta^n, v)$

Clustering

for $n \in 1, N$ **do**
15 $\mu_k^n \leftarrow \text{Cluster}(\theta^n, K, kmeans++)$ for $k \in 1, K$
16 Solve combinatorial optimization using equations 1-6
17 **return** y^n

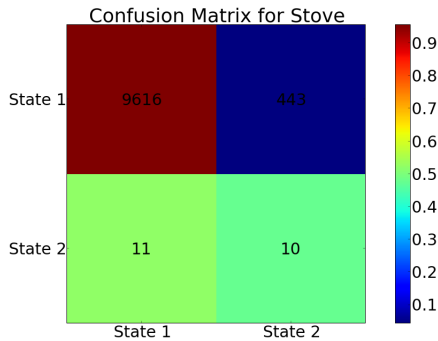


Fig. 4: Confusion Matrix showing predicted state accuracy for Stove

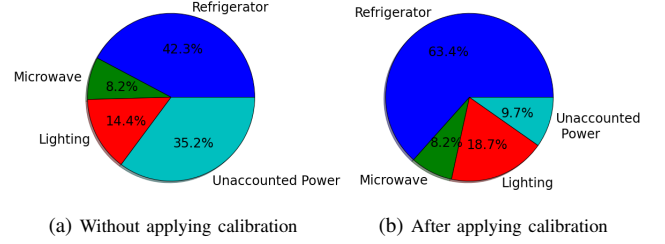


Fig. 5: Mains 2 Break down by load

B. Empirical Analysis

We performed empirical analysis on REDD dataset Home 2, which consists of 11 channels (including 2 mains and 9 appliances). We believe that the same analysis can be easily repeated across multiple homes. We aligned the data and downsampled it to one minute using mean filter. Since we had two weeks of clean data, we used the first week as the train set and the second week as the test set. We used INDiC algorithm described earlier to assign loads to different mains and calibrate appliance data. Table II shows the assignment of loads to different mains. Further this table also shows the learnt power states of these appliance via k-means++ clustering (which for several reasons is considered better than k-means) [22]. Refrigerator and lighting showed significant difference in power states post calibration. Based on prior experience and appliance circuitry, we believe that since only these two appliances needed calibration, it may be a case that the appliance level monitor measured real power instead of apparent power. These loads constitute a major chunk of Mains 2 power. Figure 5 shows the reduction in unassigned power due to calibrating these two appliances. Two loads - washer dryer and disposal did not have significant usage and we chose not to consider them in the analysis.

To show the significance of load division and calibration, we applied Combinatorial optimization on the test set, considering 4 possible cases: i) no calibration, no load division; ii) no calibration, load division; iii) calibration, no load division; iv) calibration, load division. These results are presented in Table III. For the overall dataset, it can be seen that MNE reduces from 187% to 39%, RE reduces from 478 W to 168 W, after applying INDiC. All appliances show reduction in MNE and RE after applying INDiC. However, there is significant improvement in correctly predicting refrigerator and lighting. Figure 6 shows the confusion matrix for refrigerator prediction pre and post applying INDiC. It can be seen that after applying INDiC there is a vast improvement in predicting refrigerator's state 0 and 1.

We had used Combinatorial Optimization which is the simplest NIALM technique to show the improvements which can be made by load division and appliance calibration. We believe that using state of the art NIALM algorithms will improve the results by leaps and bounds.

VI. CONCLUSION AND FUTURE WORK

The conclusion goes here. We also provide mains load assignment of all 6 homes from REDD to further the research

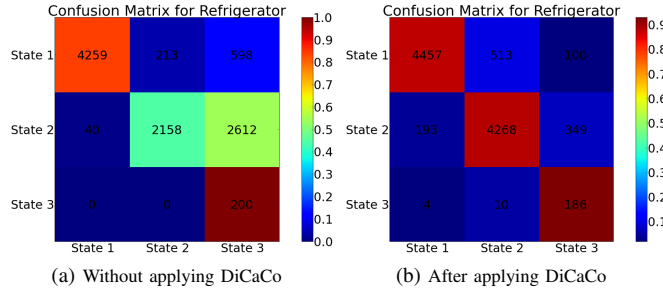


Fig. 6: Confusion Matrix for refrigerator disaggregation

TABLE II: Mains assignment and appliance states power

| Appliance | Mains | States Power (W) | | | | | |
|--------------|-------|------------------|-----|------|------------------|-----|------------------|
| | | Pre calibration | | | Post calibration | | |
| Refrigerator | 2 | 7 | 162 | 423 | 9 | 210 | 423 ⁴ |
| Microwave | 2 | 10 | 832 | 1730 | 10 | 832 | 1730 |
| Lighting | 2 | 9 | 96 | 156 | 10 | 110 | 178 |
| Dishwasher | 1 | 0 | 256 | 1195 | 0 | 256 | 1195 |
| Stove | 1 | 0 | 374 | - | 0 | 374 | - |
| Kitchen | 1 | 5 | 727 | - | 5 | 727 | - |
| Kitchen 2 | 1 | 1 | 204 | 1032 | 1 | 204 | 1032 |

in this direction.

- Applying model on noisy datasets
- 2 D CO (when Real and Reactive Power are known)
- Factoring in Time of Day etc.
- Factoring in Appliance Correlation
- Factor in switch continuity, essentially leads to Factorial HMM
- Distributed NILM
- Adaptive Learning

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TABLE III: MNE and RE with and without INDiC-CO

| | Without calibration | | | | With calibration | | | |
|--------------|-----------------------|----------|--------------------|----------|-----------------------|----------|--------------------|----------|
| | Without load division | | With load division | | Without load division | | With load division | |
| Appliance | R.E. Watts | M.N.E. % | R.E. Watts | M.N.E. % | R.E. Watts | M.N.E. % | R.E. Watts | M.N.E. % |
| Refrigerator | 136 | 109 | 71 | 32 | 130 | 95 | 59 | 21 |
| Microwave | 102 | 98 | 97 | 110 | 104 | 97 | 96 | 109 |
| Lighting | 51 | 164 | 48 | 195 | 44 | 83 | 38 | 60 |
| Dishwasher | 406 | 2947 | 63 | 100 | 377 | 2517 | 63 | 100 |
| Stove | 77 | 1191 | 36 | 281 | 75 | 1118 | 36 | 281 |
| Kitchen | 64 | 182 | 58 | 168 | 69 | 196 | 58 | 168 |
| Kitchen 2 | 95 | 267 | 91 | 117 | 92 | 230 | 91 | 117 |
| Overall | 478 | 187 | 161 | 58 | 450 | 157 | 168 | 39 |

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